University of Oulu
Astronomy and Space Physics
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Exercise Problems — Part 1

(Due Date: Tuesday, 22.09.15, *before* the exercise. You can hand me the solutions in the exercise or put them in the red box outside office FY277.)

1. Two body problem: Conic Sections.

In the lecture we derived the equation

$$r = \frac{a(1-e^2)}{1+e\cos f} \tag{1}$$

describing the dependence of radial distance in the two body problem on the mean anomaly f. It is well known that $r(f)$ describes so called *conic sections.* This means that these curves are the lines of intersection of a cone mantle (*half* opening angle θ) and a plane (see figure (1)). In general, the plane has an angle Φ with the central axis of the cone. Look up sources in the internet (or in the literature) to explain how the types of orbits

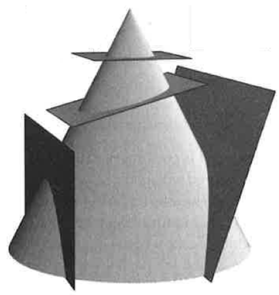

Figure 1: Conic Sections (from Murray and Dermott, Figure 2.4).

described by equation (1), namely ellipses, circles, parabolas, and hyperbolas, arise for different choices of the angle Φ for given cone half opening angle θ.

2. Two body problem: Energy and the *vis viva* equation.

(a) Use equation (1) and the angular momentum conservation to verify the *vis viva* equation

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) ,$$

for the square of the velocity $v^2 = \dot{r}^2 + r^2\dot{f}^2$.
(b) Use the result from (a) and relations from the lecture to show that

$$E = -\frac{\mu}{2a}$$

describes the energy of the orbit.

3. *Advanced*: Two body problem: Throw distance.

Consider an atmosphereless, spherical moon of radius R and mass M. A meteoroid is striking the surface and material gets ejected from the impact location. Let one particle be ejected with speed v at an angle θ from the surface normal (see figure). Typically, the

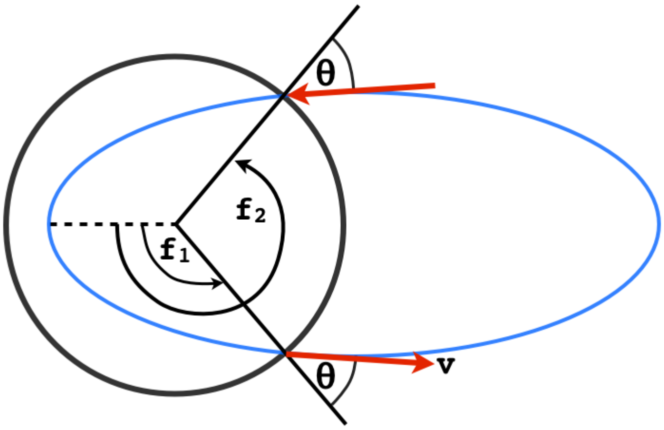

Figure 2: Particle ejected from the surface of an atmosphereless, spherical body.

particle will fall back to the moon surface. Calculate a formula for the throw distance of the particle measured on the surface. *Hint: Between ejection and re-impact the particle travels on a Kepler ellipse. So, you can use equation (1) to determine the angles* f_1 *and* f_2*. Express the semi-major axis* a *(or equivalently the angular momentum* h*) and the eccentricity in terms of* v *and* θ *alone. The particle mass can be neglected relative to the mass of the moon.*