University of Oulu
Astronomy and Space Physics
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# Exercise Problems — Part 6, first batch, computer exercise

(Due Date: Tuesday, 25.11.15, *before* the exercise. For the computer exercise send me the IDL code per mail. )

## 1. Dynamics in the field of an axisymmetric planet

Download from the noppa page of the course the `IDL` routine `PlanetAndSatellites.pro`. This routine provides a template for the integration of the dynamical equations of the two body problem. The routine contains already the main loop for the numerical integration and it is setting up initial conditions.

(a) In a first step you need to code up in the function `rhs` the set of *first order* ordinary differential equations that result from Newton's dynamic law for the motion of a satellite in the field of a spherically symmetric planet

$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\frac{\vec{r}}{r}\,.$$

These are second order and we must transform them into a set of first order equations.

(b) Now we consider the motion in the field of an *axisymmetric*, non-spherical planet. Specifically, we implement the effect of the $J_2$ term that arises from the expansion in terms of Legendre polynomials we did in the lecture. In this case the potential reads

$$\phi(\vec{r}) = -\frac{\mu}{r}\left[1 - \frac{1}{2}J_2\left(\frac{R_p}{r}\right)^2\left(3\cos^2\theta - 1\right)\right]\,,$$

where $R_p$ is the (equatorial) radius of the planet and $\cos\theta = z/r$. The equations of motion are obtained from the potential as

$$\ddot{\vec{r}} = -\vec{\nabla}\phi(\vec{r}) = -\frac{\mu}{r^2}\frac{\vec{r}}{r} + \frac{3}{2}J_2\frac{\mu}{r^2}\left(\frac{R_p}{r}\right)^2\left[\left(5\frac{z^2}{r^2} - 1\right)\frac{\vec{r}}{r} - 2\frac{z}{r}\vec{e_z}\right]\,.$$

Here, the second term on the right hand side represents the deviation from the spherically symmetric case. Code up this second term (the first term you did in part a) in the function `rhs` of the routine by adding the respective terms to the right hand sides of the ordinary differential equations.

(c) Debug and run the routine. The initial conditions implemented here are for $z = 0$, i.e. for motion in the equatorial plane. Briefly describe the main effect of the $J_2$ term on the motion of the satellite.

(d) *Advanced:* Make a copy of the routine. Remove from that copy all code that deals with the argument of pericenter $\varpi$. Instead, modify the initial conditions so that $\dot{z}(0) \neq 0$ and an inclined trajectory is obtained. Now add code that determines along the trajectory the longitude of ascending node $\Omega$. Print out the evolution of that angle on the screen and plot the directions of $\Omega$, as it was done in the original routine for $\varpi$.