University of Oulu
Astronomy and Space Physics
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Exercise Problems — Part 4

(Due Date: Tuesday, 20.10.15, *before* the lecture. Computer problem 3b: Wednesday 21.10.15 before the exercise. You can also put the solutions in the red box outside office FY277.)

1. Laplace and Poisson Equation

(a) Show by direct calculation that the potential for N mass points

$$\phi(\vec{r}) = -G \sum_{i=1}^{N} \frac{m_i}{|\vec{r} - \vec{r}_i|}$$

fulfills the Laplace equation.
(b) In the lecture we defined the Poisson-Integral

$$\phi(\vec{r}) = -G \int \mathrm{d}^3 r' \, \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} \, .$$

Use the relation

$$\Delta \frac{1}{|\vec{r}|} = -4\pi\delta(\vec{r})$$

to show that Poisson-Integral is indeed a solution of the Poisson-equation.

2. Advanced: Force-free Interior of a Spherical Mass Shell

Show by direct calculation that the gravity force inside a massive shell (inner radius R_1, outer radius R_2) vanishes.

3. Orbital Decay of Semi-Major Axis and Eccentricity due to Atmospheric Drag

Consider the equations for the *orbit averaged* decay rates for semi-major axis and eccentricity due to atmospheric drag, as we derived them in the lecture

$$\begin{aligned} \left\langle \frac{\mathrm{d}a}{\mathrm{d}t} \right\rangle &= -\frac{C_d A}{m} \sqrt{\mu a} \rho \left[1 + \frac{3}{4} e^2 \right] \\ \left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle &= -\frac{C_d A}{m} \sqrt{\frac{\mu}{a}} \rho \frac{e}{2} \end{aligned} \tag{1}$$

where $e \ll 1$ and $\rho = const.$
(a) Introduce the same scaling for time and semi-major axis as we had it in the solution of problem 1c from Exercise 2

$$t = \theta\tau, \ \frac{1}{\theta} = \frac{CdA\rho}{m} \sqrt{\frac{\mu}{a_0}}, \quad a = \alpha a_0$$

to bring the equations in dimensionless form.
(b) Solve the coupled dimensionless equations form part (a) numerically. To this end make a copy of the routine `HarmOscillator.pro` and modify it to produce the numerical solution. Try different timesteps to make sure that your solution has converged.