University of Oulu
Astronomy and Space Physics
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Celestial Mechanics II, Fall 2015

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Exercise Problems — Part 2

(Due Date: Tuesday, 06.10.15, *before* the lecture. You can also put the solutions in the red box outside office FY277.)

1. Atmospheric Drag

(a) Check from a source of your choice what is the structure of the *upper* part of Earth's atmosphere. Describe briefly the different parts and altitude ranges. What densities are expected there, roughly?

(b) Derive the equation given in the lecture

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{C_\mathrm{d}\, A\, \rho}{m}\, v\, (e + \cos f) \ .$$

for the slow evolution of the eccentricity under the influence of atmospheric drag.

(c) Solve the equation for the slow evolution of the semi-major axis

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{C_\mathrm{d}\, A\, \rho}{m}\frac{a^2}{\mu}\, v^3$$

for the case of constant density ρ and $e = 0$.

2. *Advanced*: Slow evolution of the longitude of the ascending node

Show that the general evolution equation for the longitude of the ascending node reads

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \sqrt{\frac{a(1-e^2)}{\mu}}\, N\, \frac{\sin\varphi}{1 + e\cos f} \, .$$

Hint: Use the relations given in the lecture and express the longitude of the ascending node in terms of components of the angular momentum vector as

$$\tan\Omega = -\frac{h_x}{h_y} \, .$$

Also, you must express the unit vectors normal to the orbital plane and in azimuthal direction, $\vec{e}_N$ and $\vec{e}_\varphi$, respectively, in terms of Ω, i, and $\varphi = \varpi + f$ as

$$\vec{e}_N = \begin{pmatrix} \sin i\, \sin\Omega \\ -\sin i\, \cos\Omega \\ \cos i \end{pmatrix} , \quad \textit{and} \quad \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi\, \cos\Omega - \cos i\, \sin\Omega\, \cos\varphi \\ -\sin\varphi\, \sin\Omega + \cos i\, \cos\Omega\, \cos\varphi \\ \sin i\, \cos\varphi \end{pmatrix} .$$

Keep in mind that the angular momentum vector points in the direction of $\vec{e}_N$.