University of Oulu
Astronomy and Space Physics
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Exercise Problems — Part 6, second batch

(Due Date: Tuesday, 24.11.15, *before* the lecture. For the computer exercise send me the IDL code per mail.)

2. Dynamics in the field of an axisymmetric planet — cnt'd

(a) Show that from the potential of an axisymmetric mass, including the effect of the J_2 term

$$\phi(\vec{r}) = -\frac{\mu}{r}\left[1 - \frac{1}{2}J_2\left(\frac{R_p}{r}\right)^2(3\cos^2\theta - 1)\right],$$

we get the acceleration

$$\ddot{\vec{r}} = -\vec{\nabla}\phi(\vec{r}) = -\frac{\mu}{r^2}\frac{\vec{r}}{r} + \frac{3}{2}J_2\frac{\mu}{r^2}\left(\frac{R_p}{r}\right)^2\left[\left(5\frac{z^2}{r^2} - 1\right)\frac{\vec{r}}{r} - 2\frac{z}{r}\vec{e_z}\right].$$

Here, R_p is the (equatorial) radius of the planet and $\cos\theta = z/r$.

(d) *Advanced:* Derive form the force in part (a) the conserved energy.