Assignment 2: Detail Design

Pipe bending machine.

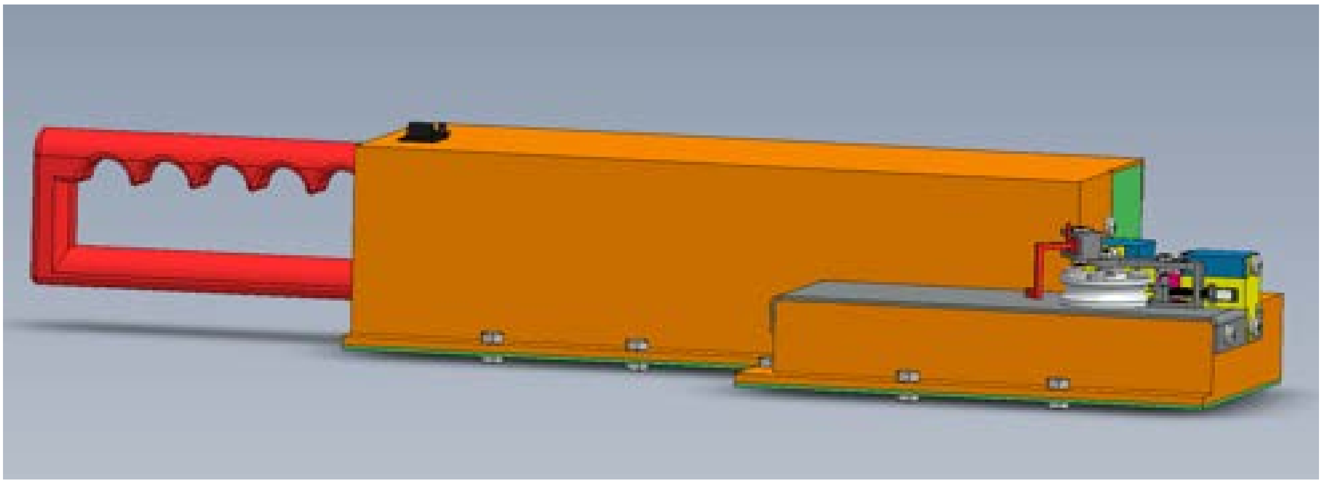

Figure 1 CAD Model of a pipe bending tool

The model shows a red handle at the back, with two switches (push and control switch) close to it. The lower casing is shown in green and the upper casing is shown in orange. At the front there's a front casing made with sheet metal shown in grey. On top of the front casing there's a big roller, clamp, angle control mechanism and limit switches safely secured in their holders. On the outside there are fasteners to hold the part together.

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1. I know that plagiarism is wrong. Plagiarism is to use another's work and pretends that it is one's own.
2. Each significant contribution to, and quotation in this assignment from the work(s) of other people has been attributed and has been cited and referenced.
3. This assignment is my/our own work.
4. I have not allowed and will not allow anyone to copy this work with the intention of passing it off as his or her own work.
5. By filling in my full names, I am formally signing this declaration and declares I have abided by the rules.

DOCUMENT AUTHOR

Student Number	Full Names
MFKKAT007	Katleho Mofokeng

CONTENTS

1. Introduction

The final concept of assignment is shown the figure below.

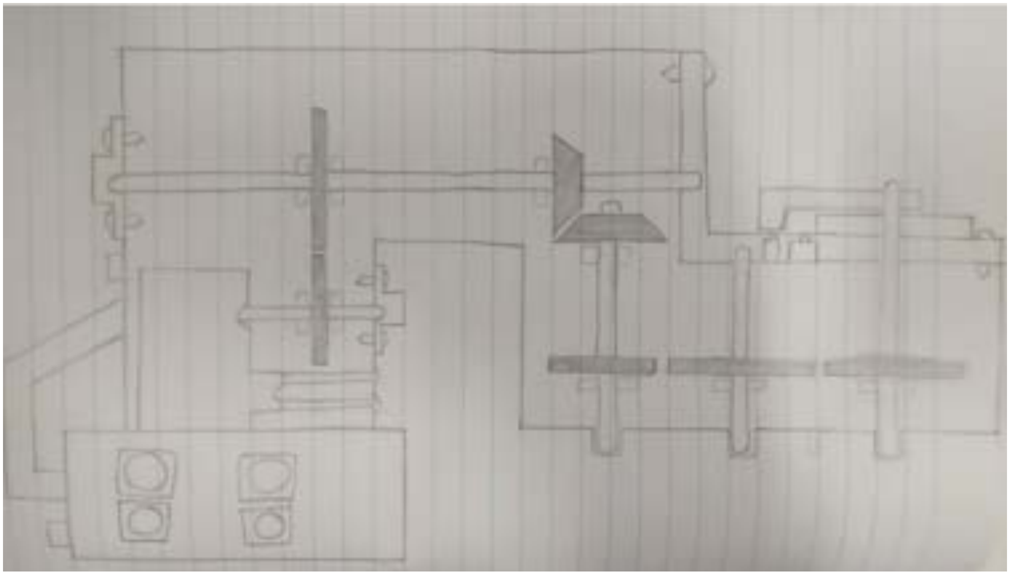


Figure 2: The final concept of assignment 1

The final concept od assignment 1 had the following issues:

- It could not bend the pipe fixed 40 mm the surface
- It could not bend the pipe in every direction
- The image of the concept was not clear enough
- The shaft support is not shown
- The angle control mechanism is not clear shown
- The concept did not the shape of the design into consideration

The new design is made using a drawing software so that the image are clear and every part can be seen. The following parts were redesigned:

- The housing
- The gearbox
- The battery holder
- The shaft supports
- The custom shaft

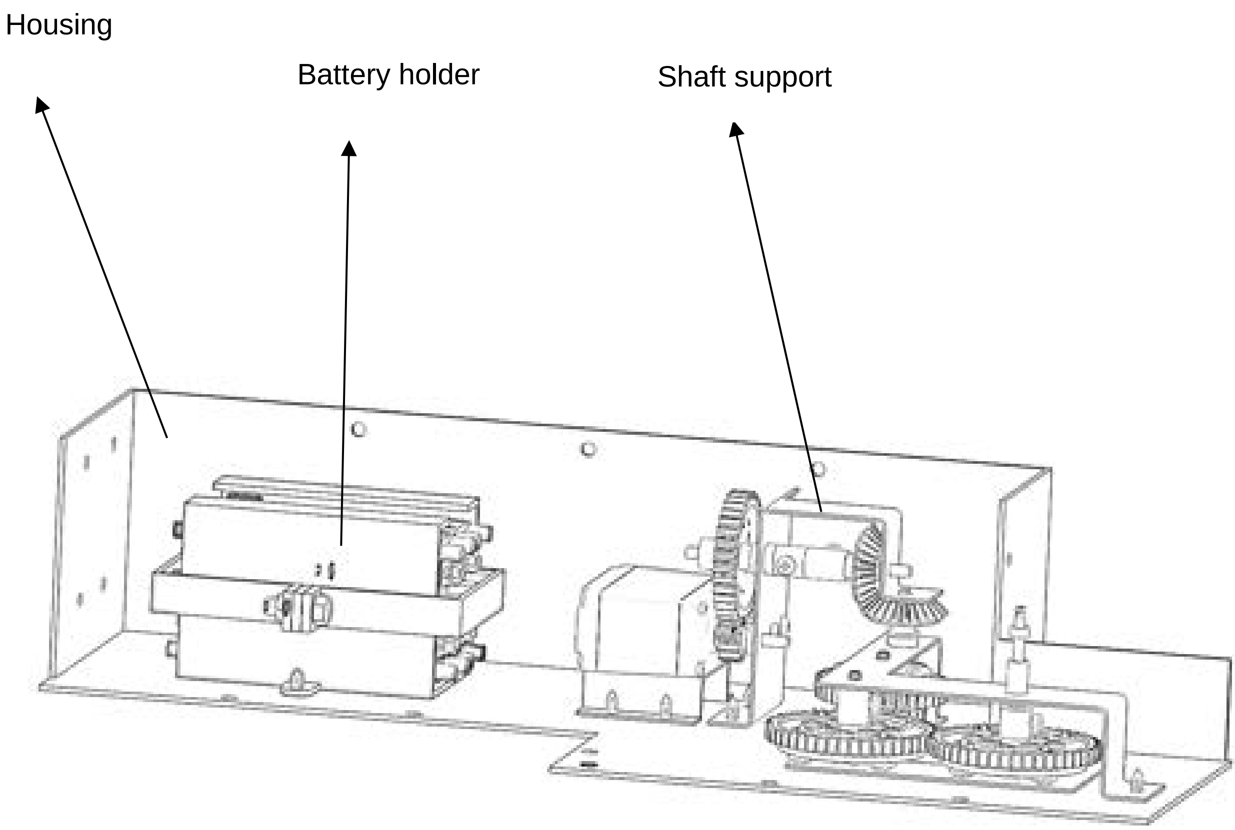


Figure 3: New concept gearbox

The top view of the new concept is attached below to show the position of all the internal parts

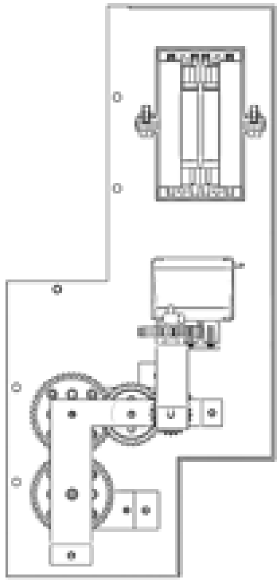


Figure 4: Top view of the new design

The shape of the housing because it has to allow the device to meet the following conditions:

- It shall be possible to bend a pipe that is fixed 40mm from a surface in any direction except towards the surface.
- It shall be possible to have a free end after the bend of minimum 60 mm.

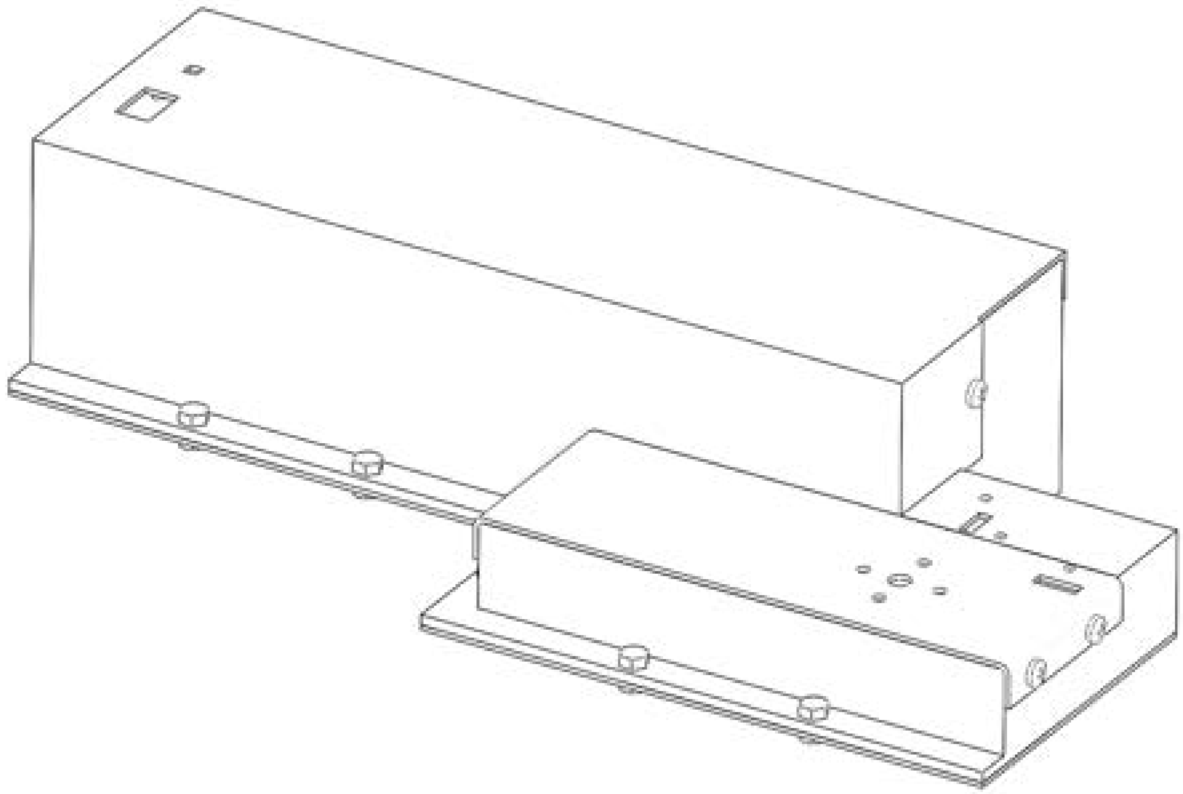


Figure 5: How the device looks from the outside

The angle control mechanism is explained in section 2.5

2. Detail design description

2.1 Motor mounting, shaft connection and gears alignment

The motor rest upon a 3D printed part named "motor lifter", the motor lifter is secured to be casing using self-tapping screws. The motor lifter is used to provide adequate vertical space for the last 4 gears to fit in properly.

To properly located the motor, the mounting motor inserts are measured to be 4.6mm in diameter. The output socket for the shaft is measured to be 4.2mm in diameter.

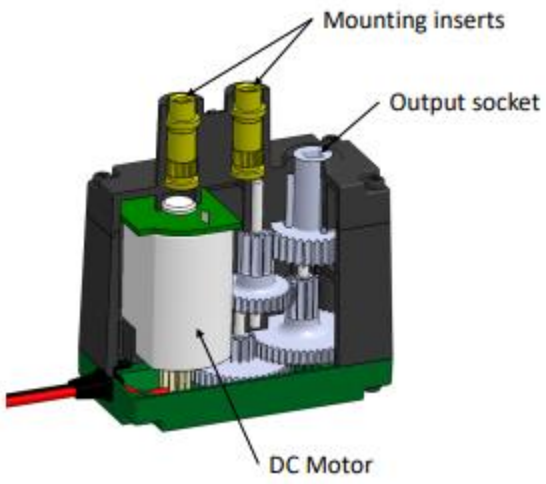


Figure 6: The inside of the motor

Thus, two 4.6mm diameter holes are cut into the motor support sheet metal to mount the motor. And a 4.2mm diameter hole is cut into the motor support sheet metal to support the shaft connecting to the motor.

The motor support is secured to the housing using self-tapping screws. The motor is mounted to the sheet metal using button head screws. The sheet metal is made with hot rolled mild steel. Thus, the button head screws together with the sheet metal part prevent the motor from moving.

The motor specification shows that it operates at low speeds. Therefore, the shafts can be simply supported through a sheet metal hole of 4.2mm in diameter.

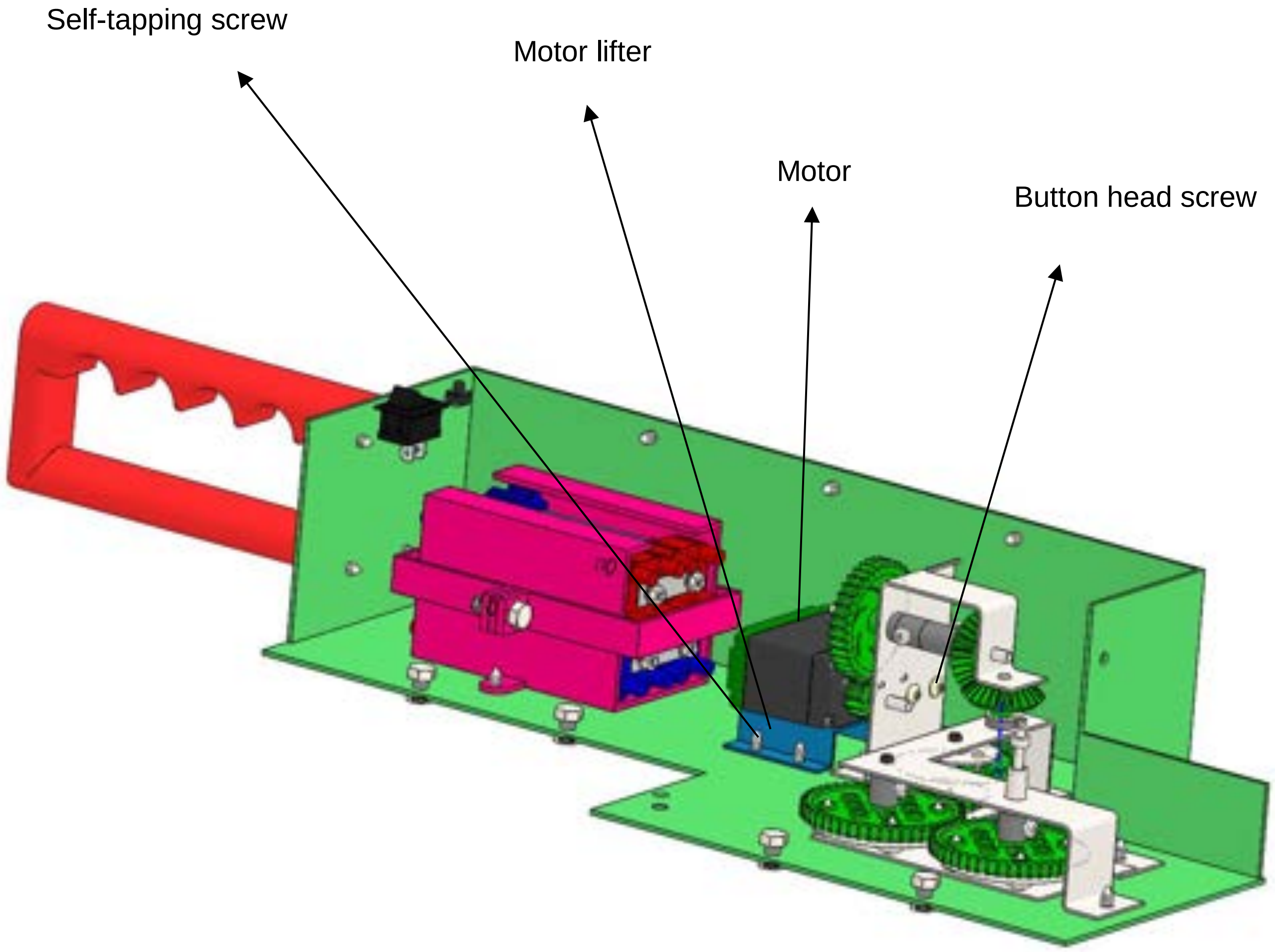


Figure 7: Motor mounting

The gearbox has 3 low carbon steel shafts, 2 high strength shaft and 1 custom shaft. So, in total there are 6 shafts. To properly align the shafts, the gears must be taken into considerations. The table below gives information about which gears are driven and which gears driving. It also shows which gears are on the same shaft. Each gear has its own colour, to make it easy to follow.

Gears	Driven by	Drives	On the same shaft as
12T spur gear	The motor	36T spur gear	-
36T spur gear	12T spur gear	-	24T bevel gear
24T bevel gear	-	24T bevel gear	36T spur gear
24T bevel gear	24T bevel gear	-	12T spur gear
12T spur gear	-	36T spur gear	24T bevel gear
36T spur gear	12T spur gear	-	24T spur gear
24T spur gear	-	48T spur gear	36T spur gear
48T spur gear	24T spur gear	-	-

The table below will be used to determine how the gears should be aligned. It shows the diameter of each gear.

Gear	Diameter [mm]
12 teeth spur gear	12.7
24 teeth spur gear	25.4
36 teeth spur gear	38.1
48 teeth spur gear	50.8
24 teeth bevel gear	25.4

The first spur gear of 12 teeth connects to the first shaft. The 12 teeth spur gear meshes with the 36 teeth spur gear. The distance between the centres will be used to position the second shaft so that the two gears can mesh properly.

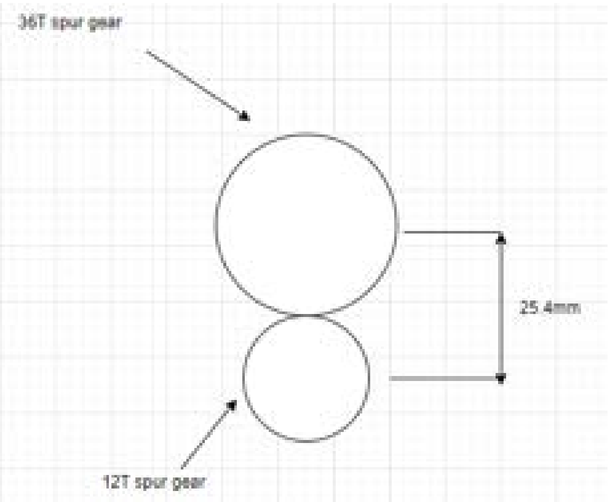


Figure 8: First meshing gears

The second shaft will be **25.4mm** above the first shaft.

The third gear is a 24 teeth bevel gear which is on the same shaft as the 36T spur gear. The third gear is meshing with another 24 teeth bevel gear. Therefore, a shaft needs to be correctly positioned to allow the gears to mesh properly.

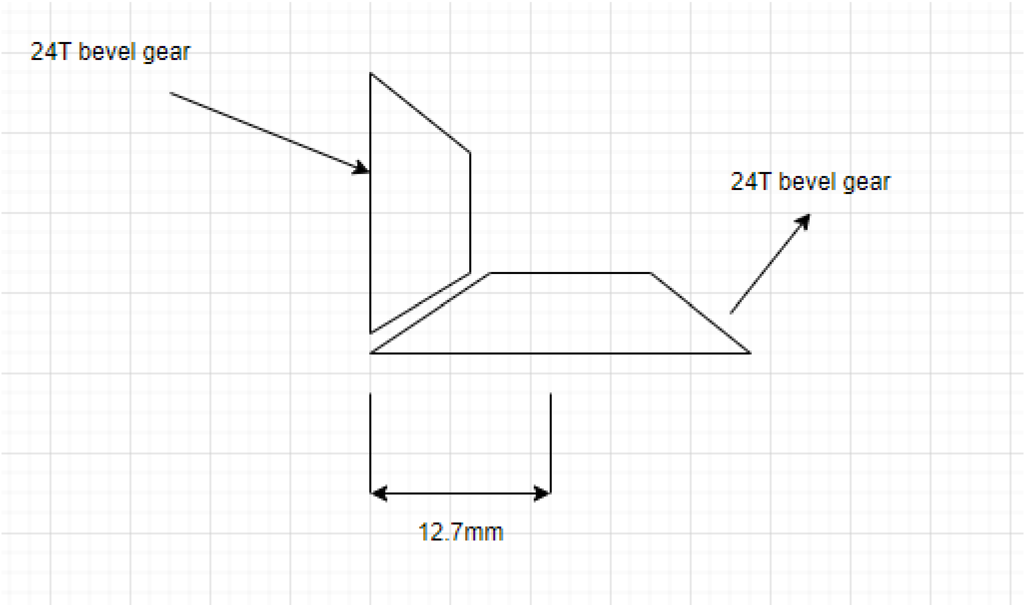


Figure 9: Second meshing gears

Therefore, the centre of the third shaft should be positioned **12.7mm** away from the edge of gear 3.

Gear 5 is a 12 teeth spur gear, and it is on the same shaft as gear 4. Gear 5 will mesh with gear 6 which is a 36 teeth spur gear. Therefore, a shaft needs to be correctly positioned to allow the gears to mesh properly.

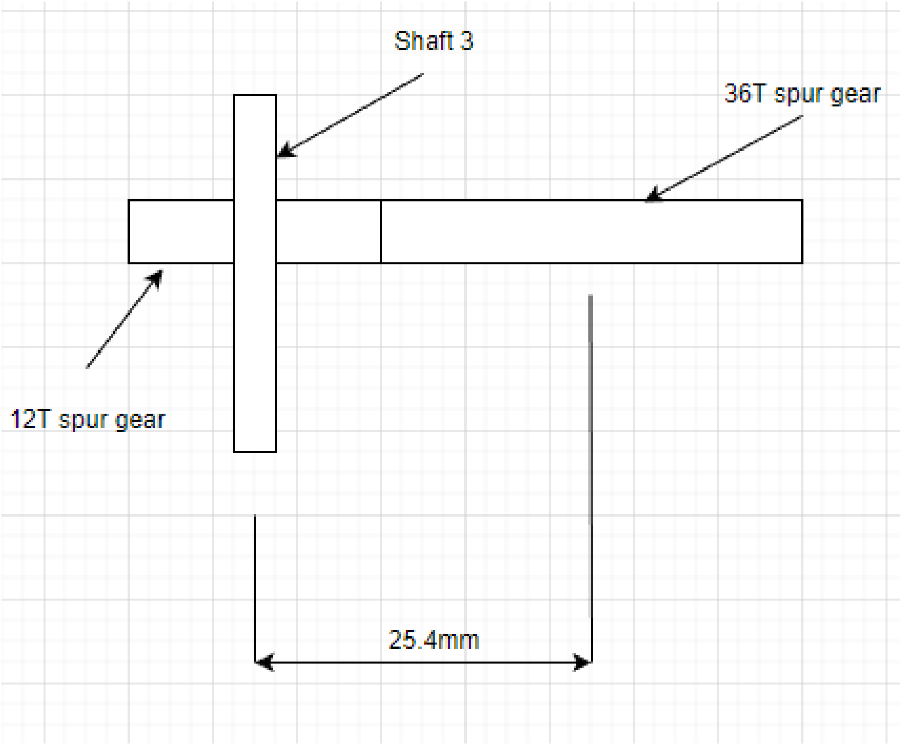


Figure 10: Third meshing gears

Therefore, the centre from shaft 3 to shaft 4 should be **25.4mm**.

Gear 7 is a 24 teeth spur gear, and it is on the same shaft as gear 6. Gear 7 will mesh with gear 8 which is a 48 teeth spur gear. Therefore, a shaft needs to be correctly positioned to allow the gears to mesh properly.

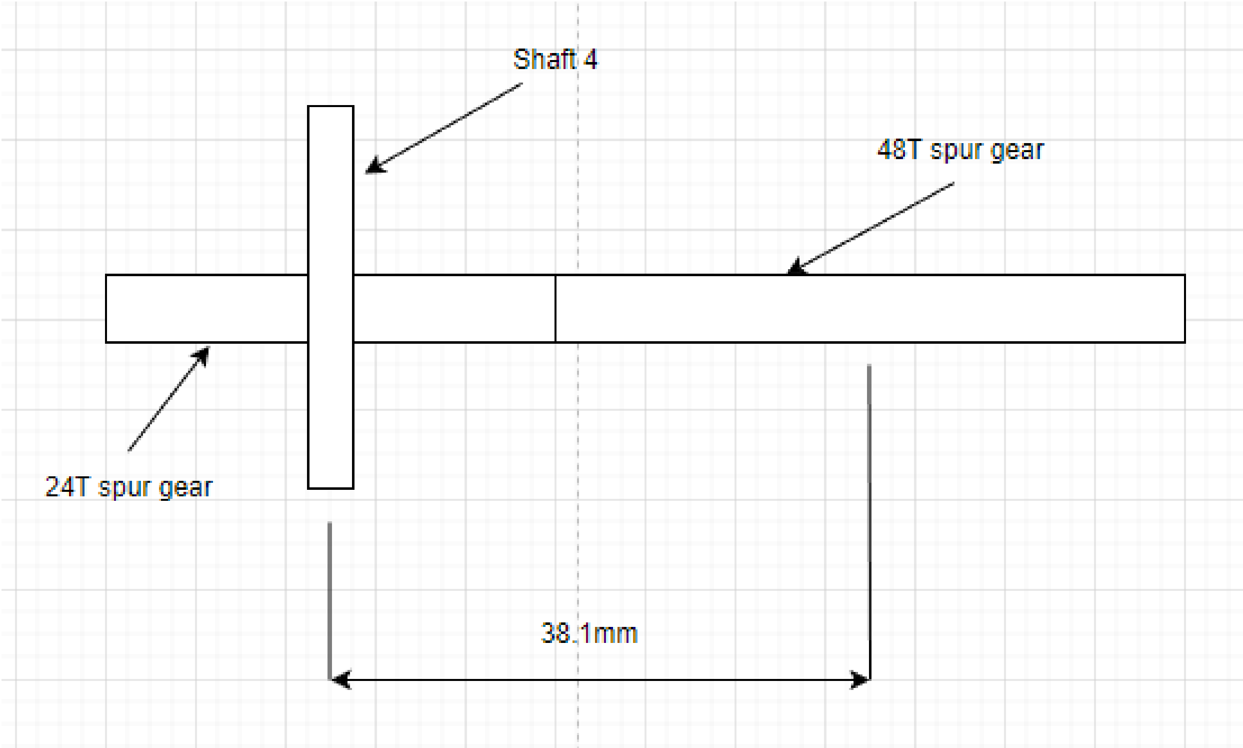


Figure 11: Fourth meshing gears

Therefore, the centre from shaft 4 to shaft 5 should be **38.1mm**.

Gear 8 is a 48 teeth spur gear and it will mesh with gear 9 which is a 48 teeth spur gear. Therefore, a shaft needs to be correctly positioned to allow the gears to mesh properly.

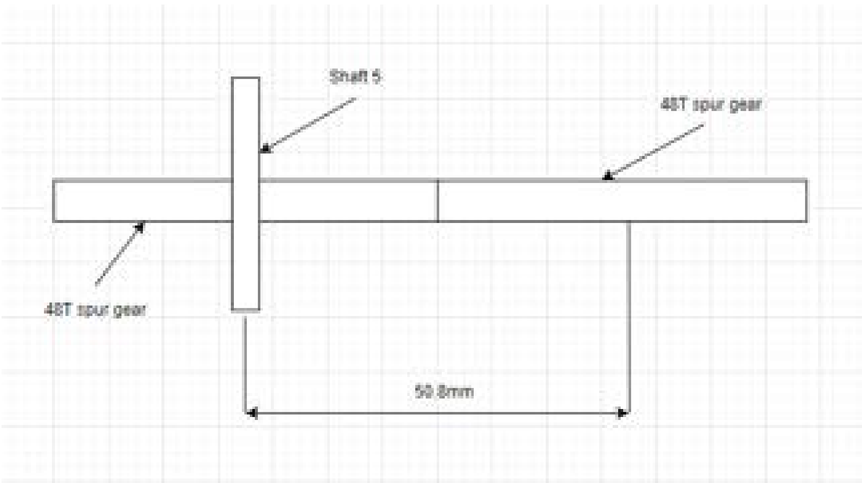


Figure 12: Fifth meshing gears

Therefore, the centre from shaft 4 to shaft 5 should be **38.1mm**.

To ensure that the shafts will not move, sheet metal support component are included in the design to hold them in place. The spacers and washers will be used to ensure that the gears are always aligned. This can be in the figure below.

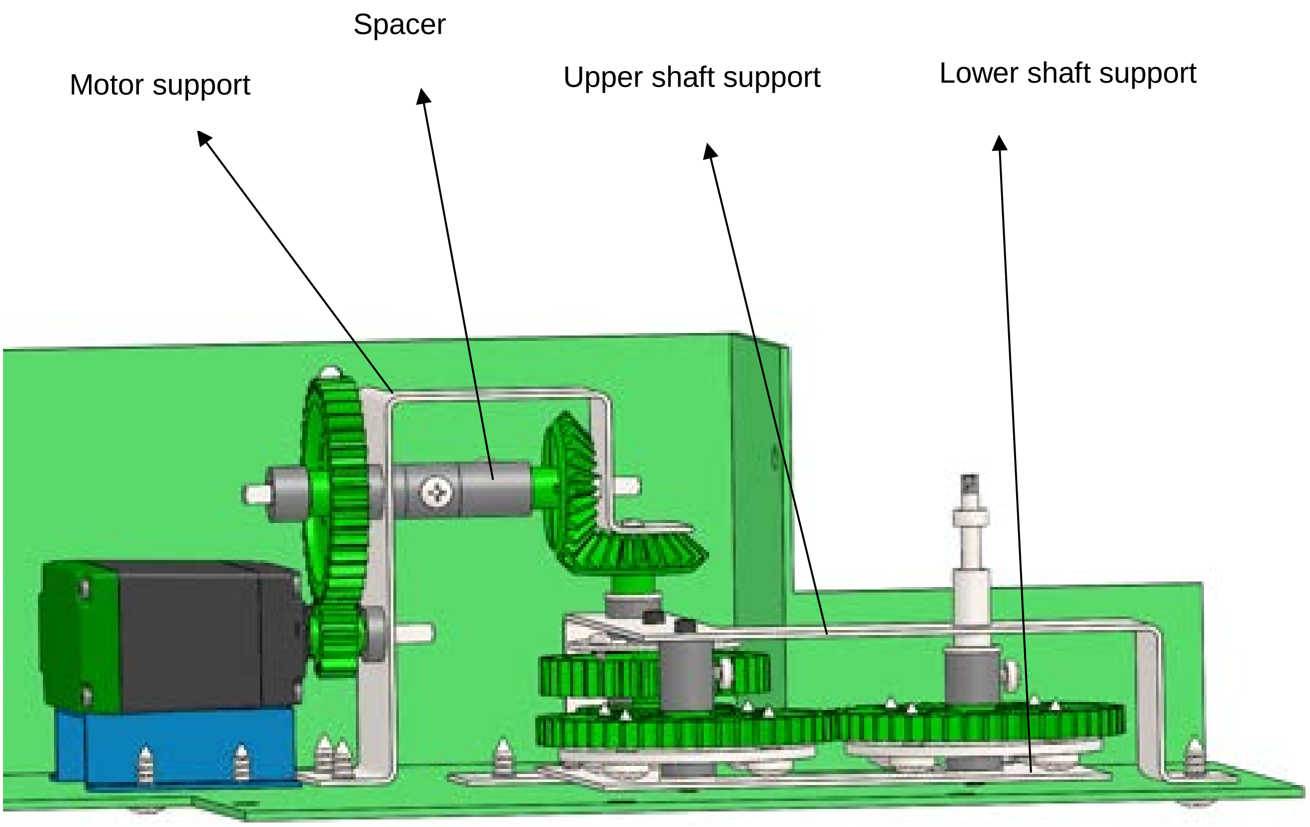


Figure 13: Spacers, washers and supports

2.2 Force/torque transfer and shaft/s support with related strength and deflections

Strengthening

The motor transfers torque to the first gear through a shaft. The torque experienced by each gear is determined (**See the detail calculation in Appendix B**). The torque that each gear will experience is then compared to its maximum allowable torque. Here's a table below showing the calculation results.

	Maximum Torque [Nm]	Experienced Torque [Nm]	Rotational speed [rad/s]
Gear 1 [12T]	3	0.941	14.537
Gear 2 [36T]	3.4	2.627	4.508
Gear 3 [24T]	3	2.627	4.508
Gear 4 [24T]	3	2.489	4.272
Gear 5 [12T]	3	2.489	4.272
Gear 6 [36T]	3.4	6.946	1.324
Gear 7 [24T]	1	6.946	1.324
Gear 8 [48T]	1.2	13.269	0.638
Gear 9 [48T]	1.2	12.922	0.621

The above table shows that Gear 6, 7, 8 and 9 have to reinforced because the torque that they will experience is greater than its maximum allowable torque. The figure below shows how the gears should be reinforced.

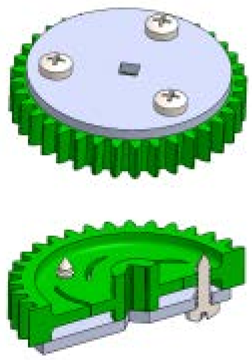


Figure 14: Reinforced gears

Thus, a doubler-plate cut from sheet metal to one side of the gear, and fastened with self-tapping screws will be used to reinforce the gear. The double plate thickness is determined using the method below.

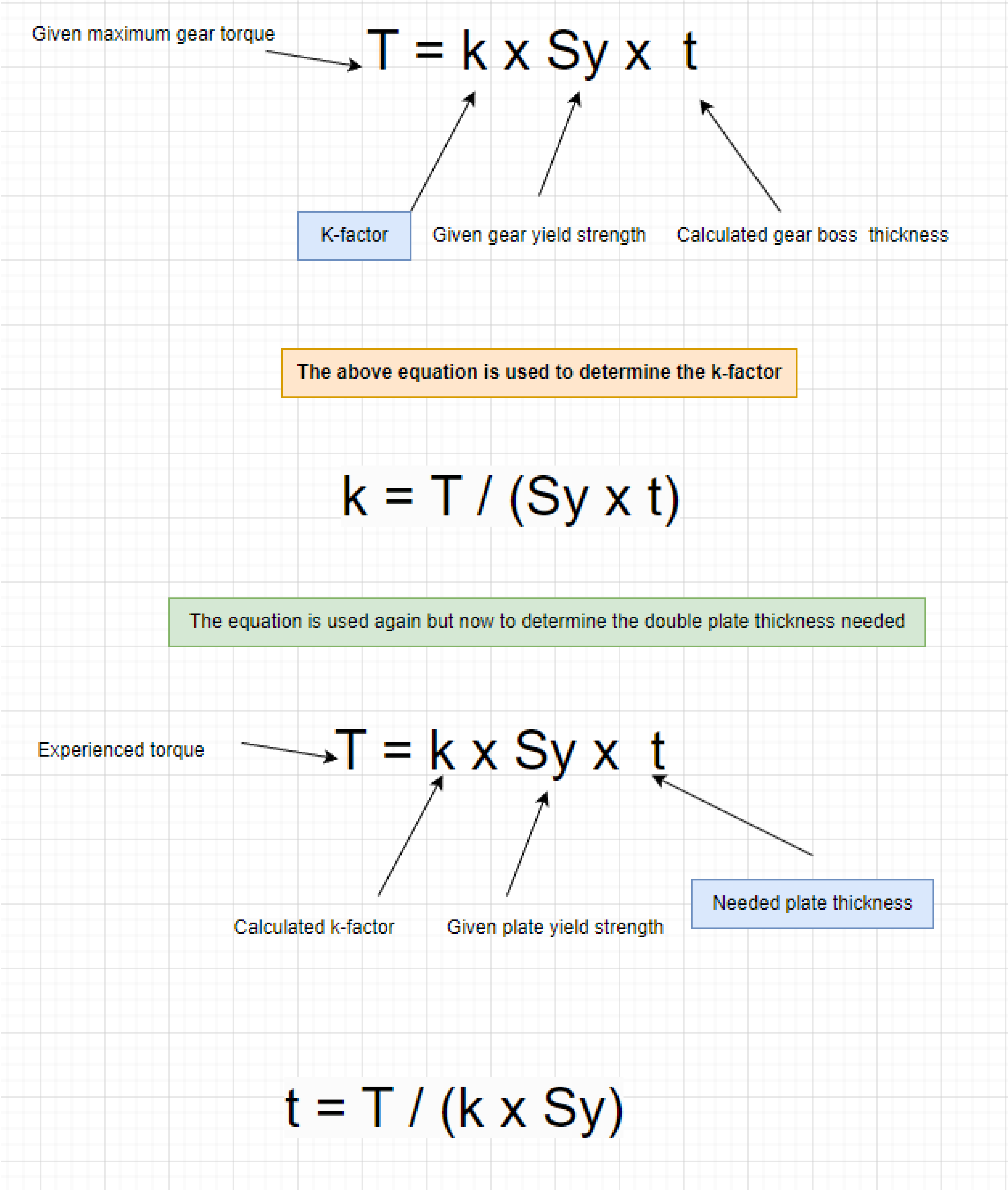


Figure 15: Method for double plate thickness approximation

The following results from **Appendix F**, show how much thickness is needed for each double plate.

	Maximum torque [Nm]	**Experienced torque [Nm]**	**Plate thickness needed [mm]**
Gear 6 [36T]	3.4	6.946	0.712
Gear 7 [24T]	1	6.946	0.958
Gear 8 [48T]	1.2	13.269	1.659
Gear 9 [48T]	1.2	12.922	1.615

The figure below show how the double plates are used to support the gears, taking into account the minimum thickness they need.

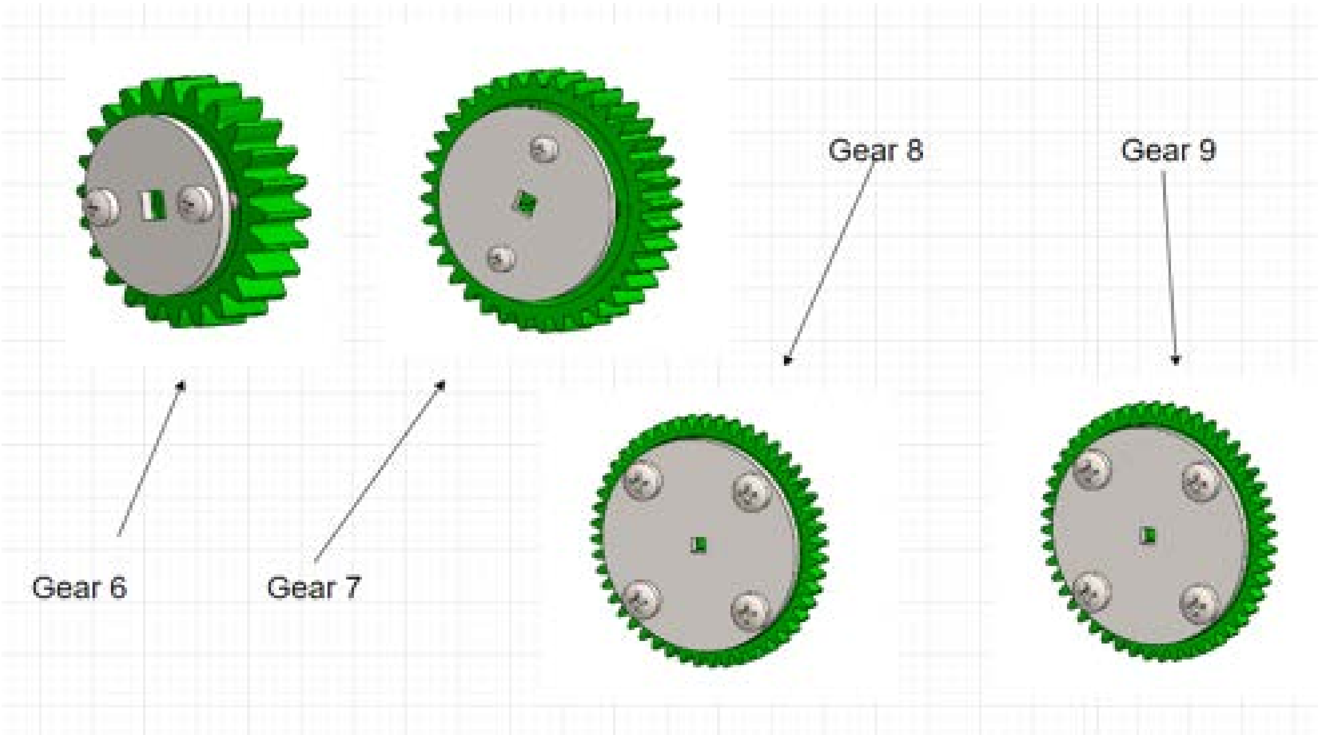


Figure 16: Reinforced gears

Each gear will have a tangential and force due meshing. The following equations are used to determine the different forces.

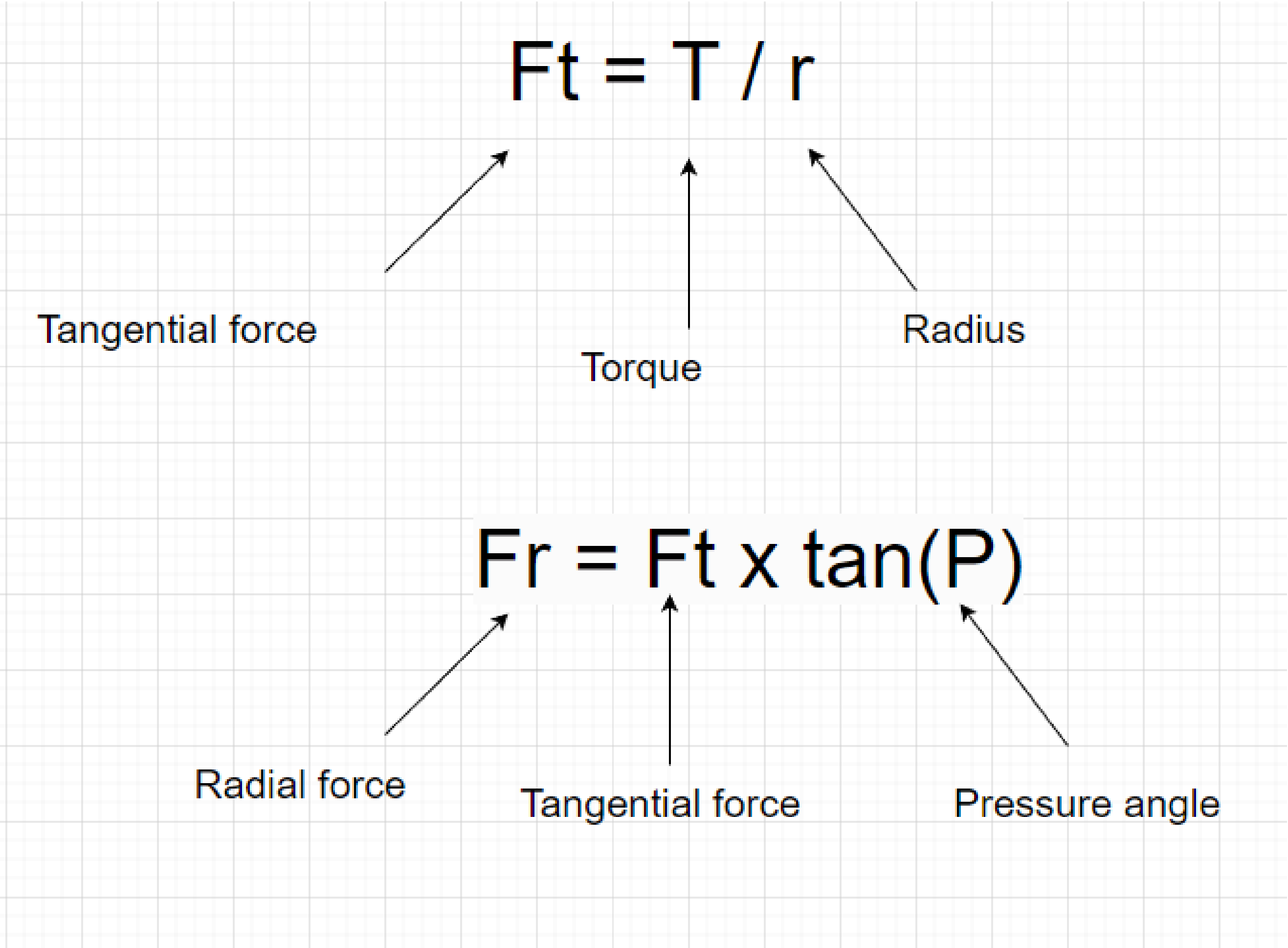


Figure 17: Forces equation

The following figure shows an example of gear 1 calculations from **Appendix C.**

Gear 1

Gear 1 is a spur gear driven by the motor through a shaft.

Number of teeth of gear 1: $N_{gear\_1} := 12$

Diameter of gear 1: $D_{gear\_1} := 12.7\ mm$

Pressure angle of gear 1: $\theta_{gear\_1} := 20\ °$

Speed of gear 1: $w_{gear\_1} := w_{actual}$

$w_{gear\_1} = 14.573\ \frac{rad}{s}$

Torque of gear 1: $T_{gear\_1} := T_{actual}$

$T_{gear\_1} = 0.936\ N \cdot m$

Tangential force of gear 1: $F_{tan\_gear\_1} := \frac{T_{gear\_1}}{D_{gear\_1} \cdot 0.5}$

$F_{tan\_gear\_1} = 147.374\ N$

Radial force of gear 1: $F_{rad\_gear\_1} := F_{tan\_gear\_1} \cdot \tan\left(\theta_{gear\_1}\right)$

$F_{rad\_gear\_1} = 53.64\ N$

Figure 18: Gear 1 calculations

The rest of the calculations are in **appendix C.** The table below shows a summary of the calculations.

	Maximum tangential force [N]	Experienced tangential force [N]	Experienced radial force [N]
Gear 1 [12T]	450	148.228	53.95
Gear 2 [36T]	700	137.879	50.184
Gear 3 [24T]	300	206.819	75.276
Gear 4 [24T]	300	195.99	71.335
Gear 5 [12T]	450	391.98	142.669
Gear 6 [36T]	700	364	132.709
Gear 7 [24T]	580	546.923	199.064
Gear 8 [48T]	580	522.417	190.144
Gear 9 [48T]	580	508.74	185.166

The table above shows the forces that will be exerted by each gear. Therefore, the forces are going to act on the gears. So, the deflection each shaft has to be determined to ensure that it won't deflect to such a point that it affects gear alignment and meshing.

The following method will be used to determine the deflection of gears.

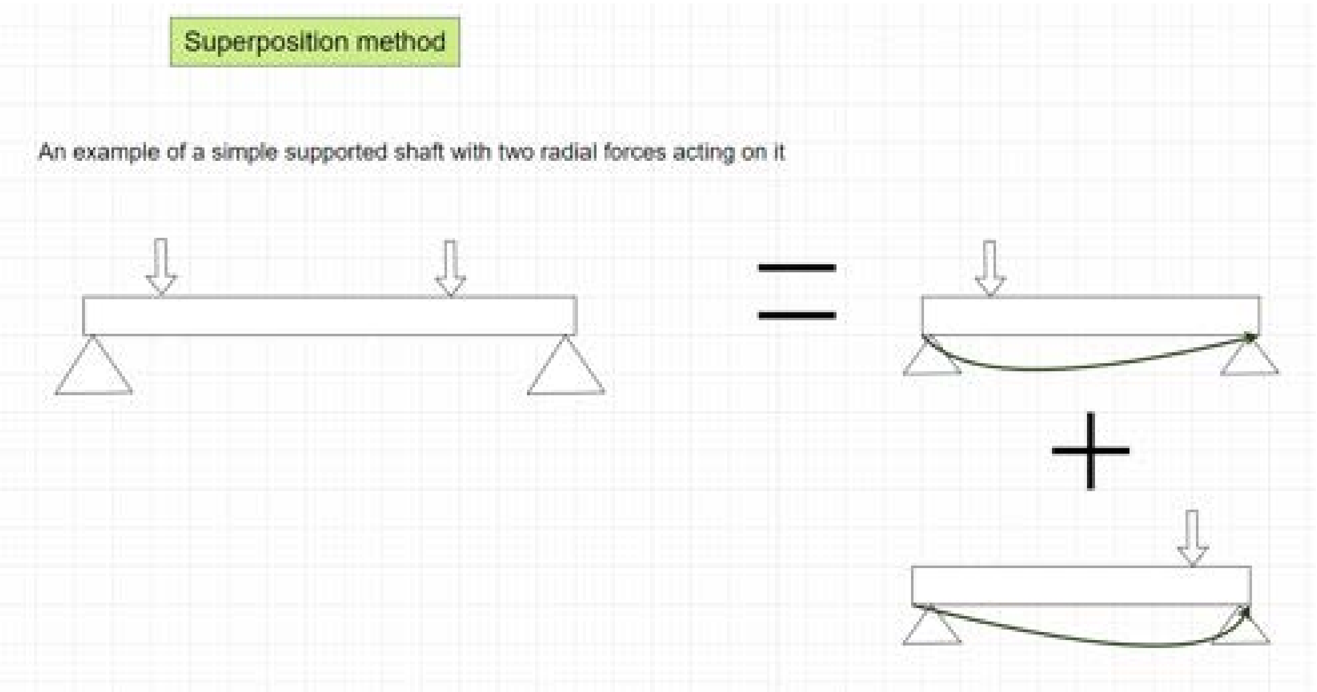


Figure 19: Superposition method

The above figure shows that the superposition method used will evaluate the deflection caused by each gear force at point of interest. Then the resulted deflection will be calculated by just adding the deflection caused by each force at a point of interest. The point of interest will be where the gears are meshing / the forces are acting.

Simply supported Beams:

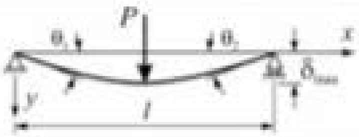
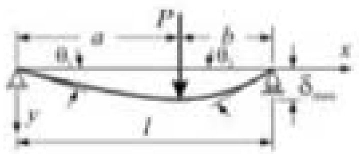
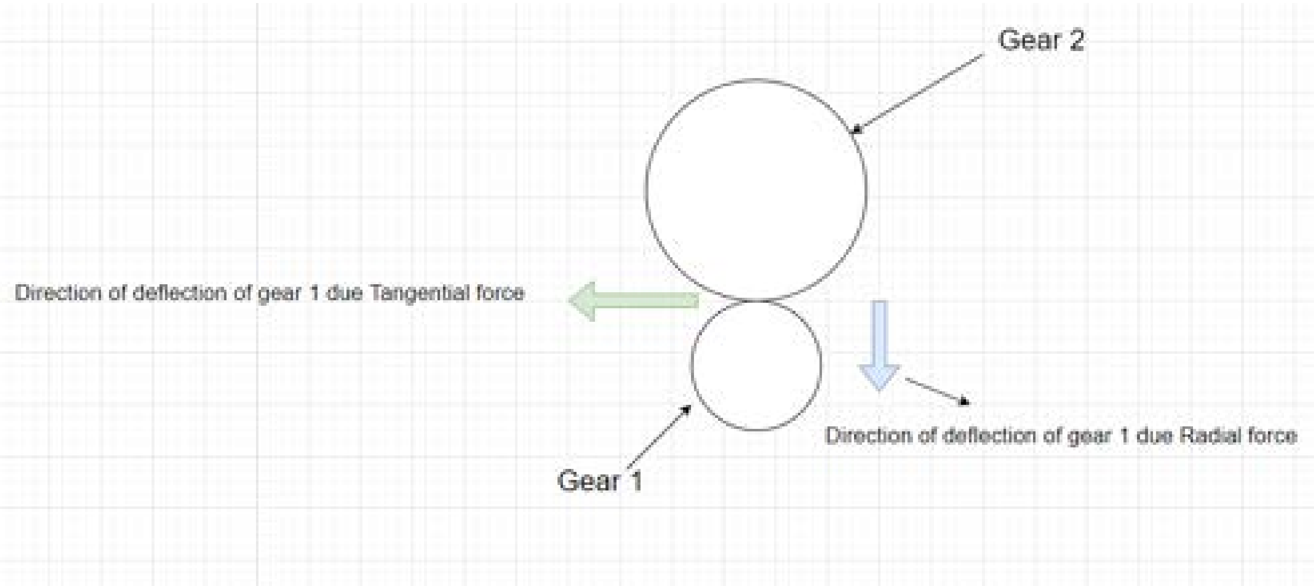
BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x
6. Beam Simply Supported at Ends – Concentrated load P at the center		
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$
7. Beam Simply Supported at Ends – Concentrated load P at any point		
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}\left(l^2 - x^2 - b^2\right)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x-a)^3 + (l^2 - b^2)x - x^3\right]$ for $a < x < l$

Figure 20: Deflection of beams equations

The above equation is from Mechanics of material textbook [1]

The figure below shows the deflection direction due to each for force



The results from the deflection calculations in **Appendix D** is the following.

Shafts	First point of interest		Second point of interest	
	Deflection due to radial forces [mm]	Deflection due to tangential forces [mm]	Deflection due to radial forces [mm]	Deflection due to tangential forces [mm]
Shaft 1	0.047	0.017	-	-
Shaft 2	0.196	0.537	0.162	0.446
Shaft 3	0.103	0.282	0.189	0.519
Shaft 4	0.089	0.0246	0.089	0.246
Shaft 5	0.056	0.153	-	-
Shaft 6	0.151	0.415	-	-

The above table shows the deflection caused by the radial forces and tangential forces at each point of interest.

The deflection values in the above table are too small to affect the gear alignment and meshing. Therefore, the shafts are strong enough to not get negatively affected by deflection.

Torsion

The effect of torsion on each should be determined. The torque that each shaft experiences is calculated in **Appendix B**. The results are in the table below.

Shafts	Torque experienced [Nm]
Shaft 1	0.941
Shaft 2	2.627
Shaft 3	2.489
Shaft 4	6.946
Shaft 5	13.269
Shaft 6	12.922

Therefore, using the above table, the shear stress applied to each shaft can be determined. It can then be compared to the shaft's shear strength and yield strength. The detail calculations are **Appendix D**. The results of the calculation are in the table below.

Shaft	Shear stress experience [GPa]	Shear strength [GPa]	Yield strength [MPa]
Shaft 1	0.702	3.732	450
Shaft 2	1.961	3.732	450
Shaft 3	1.858	3.732	450
Shaft 4	5.184	3.732	450
Shaft 5	9.903	3.732	450
Shaft 6	9.644	3.732	450

The above table show that each shaft will not fail due shear stress because the shear stress it is experiencing is less than the shaft shear strength and yield strength.

Component packaging

The components should in the following way:

- The electronics such as switches and wires should have its own package.
- The fasteners such as self-tapping screws and bolts should have its own package.
- The 3D printed parts and gears should have its own package.
- The sheet metal parts should have its own package.
- The machined parts should have its own package.

2.3 Gearbox / mechanical advantage device design with strength and efficiency considerations

The gearbox is designed in such a way that it will be able to output enough torque to bend the pipe. Therefore, the required bending moment need to bend the pipe has to be determined. In Appendix A, it was determined to be **12.922Nm**.

Now the correct motor must be selected before the gearbox is designed. The motor will be selected based on the following criteria:

- Peak current
- Peak output torque
- Peak output speed
- Peak trip time

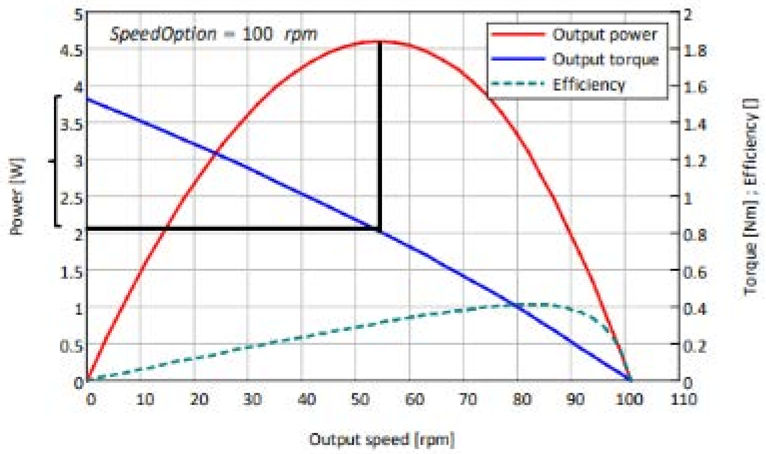


Figure 21: Graph from motor specification document

In order to determine the peak motor current, stall torque has to be used. The above figure shows that the motor will approximately have peak current at half the stall torque.

The stall torque is calculated using the equation below.

Output torque $[Nm]$	$T_m = I_m K_T r_g \eta_g - T_0$	I_m = motor current

Figure 22: Output torque formula

Thus, now the peak current, output torque, output speed and trip time can be determined. (See detail calculations in **Appendix B**) The results of the calculations are in the table below.

$Compare$	$Maximum\_current$ (A)	$Trip\_time$ (s)	$Output\_torque$ $(N \cdot m)$	$Output\_speed$ $\left(\frac{rad}{s}\right)$
$100\ rpm\_motor$	2.936	3.091	1.211	11.367
$160\ rpm\_motor$	2.961	3.001	0.772	18.045
$240\ rpm\_motor$	2.976	2.948	0.533	26.94

Figure 23: Summary of results

Thus, for this design the **100rpm motor** will be used because it has the largest output torque and longest trip time.

Largest output torque allows one to design the gearbox with a low gear ratio. Longest trip time gives the design more time to achieve the goal before it stops working.

Now the required gear train ratio for the gearbox can be calculated based on the required bending moment and the peak output motor torque.

The required gear train ratio was determined to be **10.671 (in Appendix C)**.

The gearbox has to be designed that will meet the required gear ratio of 10.671. Through trial and error, a gearbox was designed that will provided a gear train ratio that is larger than the required gear ratio. Here's a table below showing how the gearbox is designed.

Gears	**Driven by**	**Drives**	**On the same shaft as**
12T spur gear	The motor	36T spur gear	-
36T spur gear	12T spur gear	-	24T bevel gear
24T bevel gear	-	24T bevel gear	36T spur gear
24T bevel gear	24T bevel gear	-	12T spur gear
12T spur gear	-	36T spur gear	24T bevel gear
36T spur gear	12T spur gear	-	24T spur gear
24T spur gear	-	48T spur gear	36T spur gear
48T spur gear	24T spur gear	-	-

The gear train efficiency has to be determined based on the number of meshing gears. The equation below will be used to determine the efficiency.

$$\eta_{spur} = 1 - \mu\pi\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

Figure 24: Efficiency equation

Here's a table below showing which gears are meshing.

	Meshing gears
Mesh 1	12T spur gear with 36T spur gear
Mesh 2	24T bevel gear with 24T bevel gear
Mesh 3	12T spur gear with 36T spur gear
Mesh 4	24T spur gear with 48T spur gear
Mesh 5	48T spur gear with 48T spur gear

The efficiency for each meshing gear is determined. The gear train efficiency is determined by multiplying all the 5 meshing gears efficiencies, this resulted in a gear train efficiency of **0.767**.

Therefore, the gear ratio including the gear train efficiency was determined to be **13.808**. This is greater than the required gear train efficiency of 10.671.

Now the following has to be determined using the actual gear train ratio:

- Motor operating current
- Motor output torque
- Motor output speed
- Motor trip time

The operating output torque is determined by dividing the required bending moment with the actual gear train ratio. Here's a table showing the results of operating conditions in comparison to the peak condition.

	Peak condition	Actual condition
Current [A]	2.936	1.847
Output speed [rad/s]	11.367	14.537
Output torque [Nm]	1.211	0.941
Trip time [s]	3.091	15.948

The gearbox mechanical advantage is the gear train ratio **13.808.**

The strength and the forces of gears is determined in **section 2.2** above this.

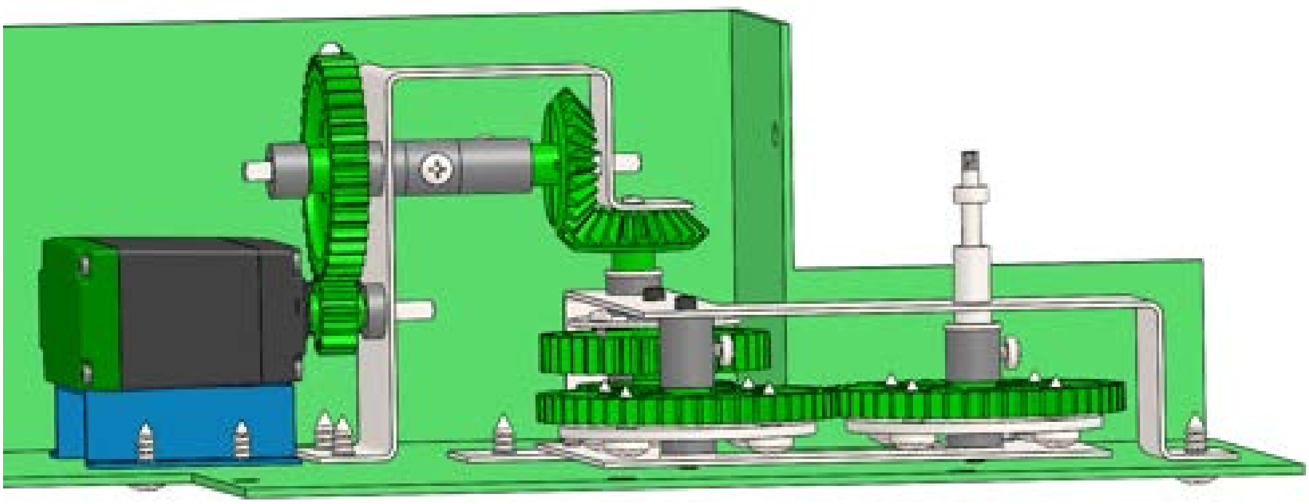


Figure 25: Gearbox design with all the parts in place

2.4 Final actuator design, clearly showing pipe support and operation.

The URS state that the device shall bend the pipe to an inner radius of 15mm without pinching. Thus, a big roller of inside 15mm will be used to provided surface that the will around.

A custom shaft is made for the last gear that will drive the actuator. It is made using the following components:

- 30mm high strength shaft
- Shaft coupler
- Custom circular shaft

The 30mm high strength shaft will merge with the custom circular shaft using a shaft coupler in between, they will be soldered onto the shaft coupler. The figure below shows how the custom shaft should look.

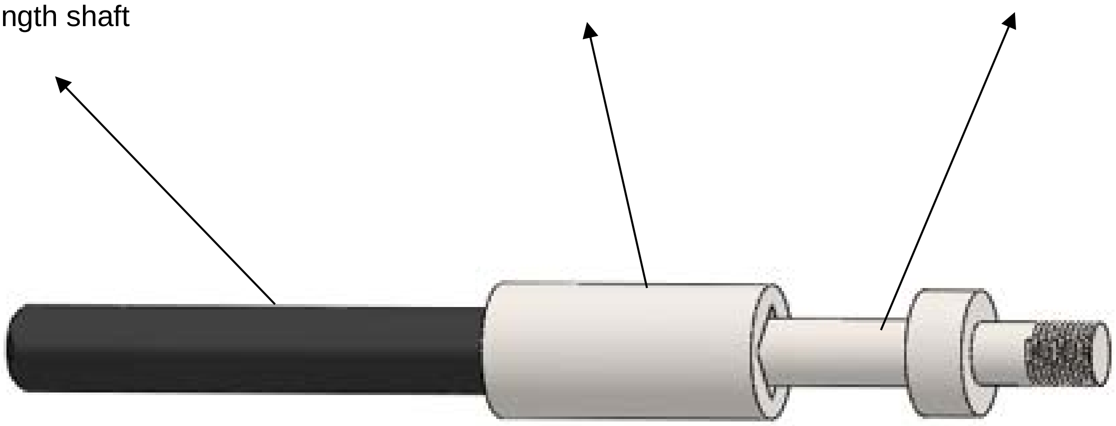


Figure 26: Custom shaft

The bending die that will get in contact with die will be machined. It will be designed like the figure below.

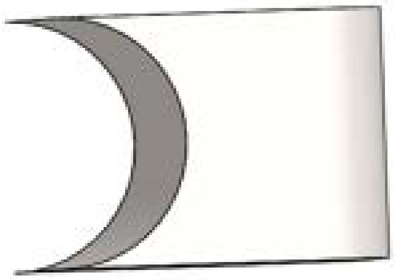


Figure 27: Bending die

The platform for the bending die to sit on has to be designed in such a way that it will also provide the platform for the angle control mechanism.

Considering that it will be machined using a CNC machine, it shows also allow a 3mm ball nose mill to be able to craft. The figure below shows how the bending platform is made.

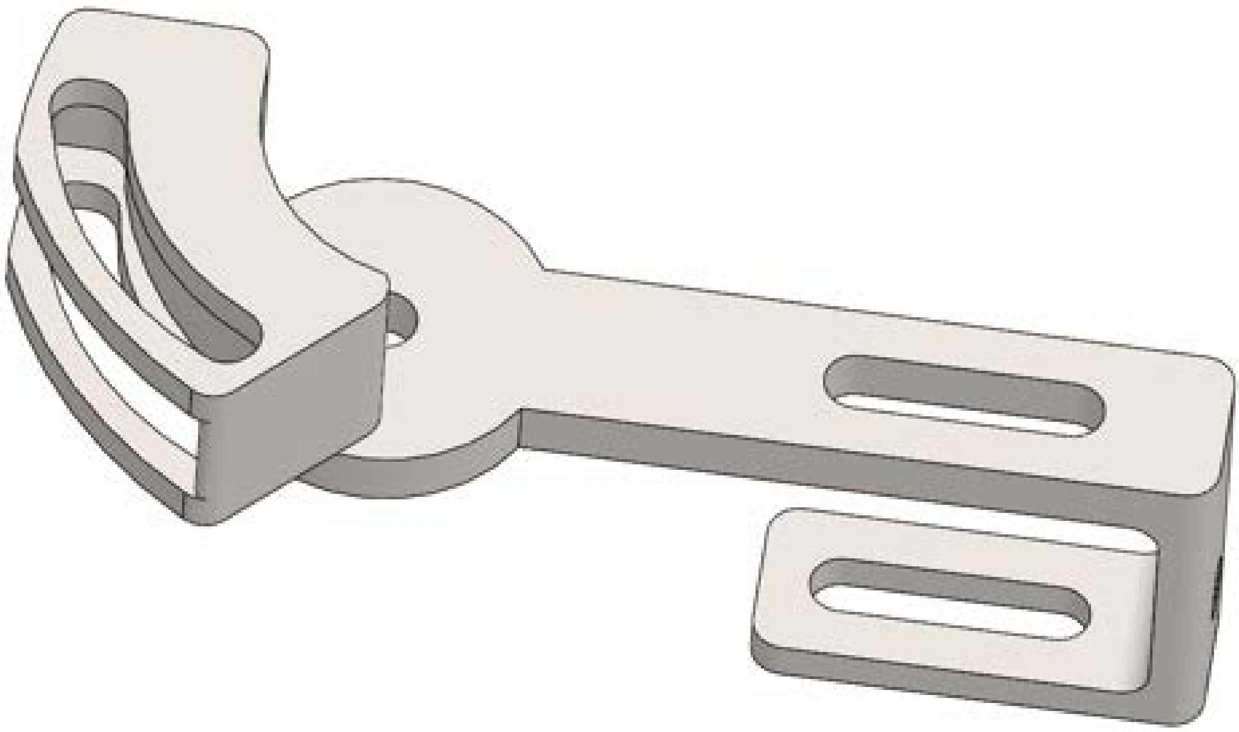


Figure 28: Bending platform

Now the following parts needed to be designed:

- Bending die position (To allow the bending die to move to a specific position
- Clamp (To clamp the bending die to a pipe)
- The angle lock (Will be used for angle control mechanism)
- Angle positioner (To set the angle)

The figure below shows how the final actuator design looks.

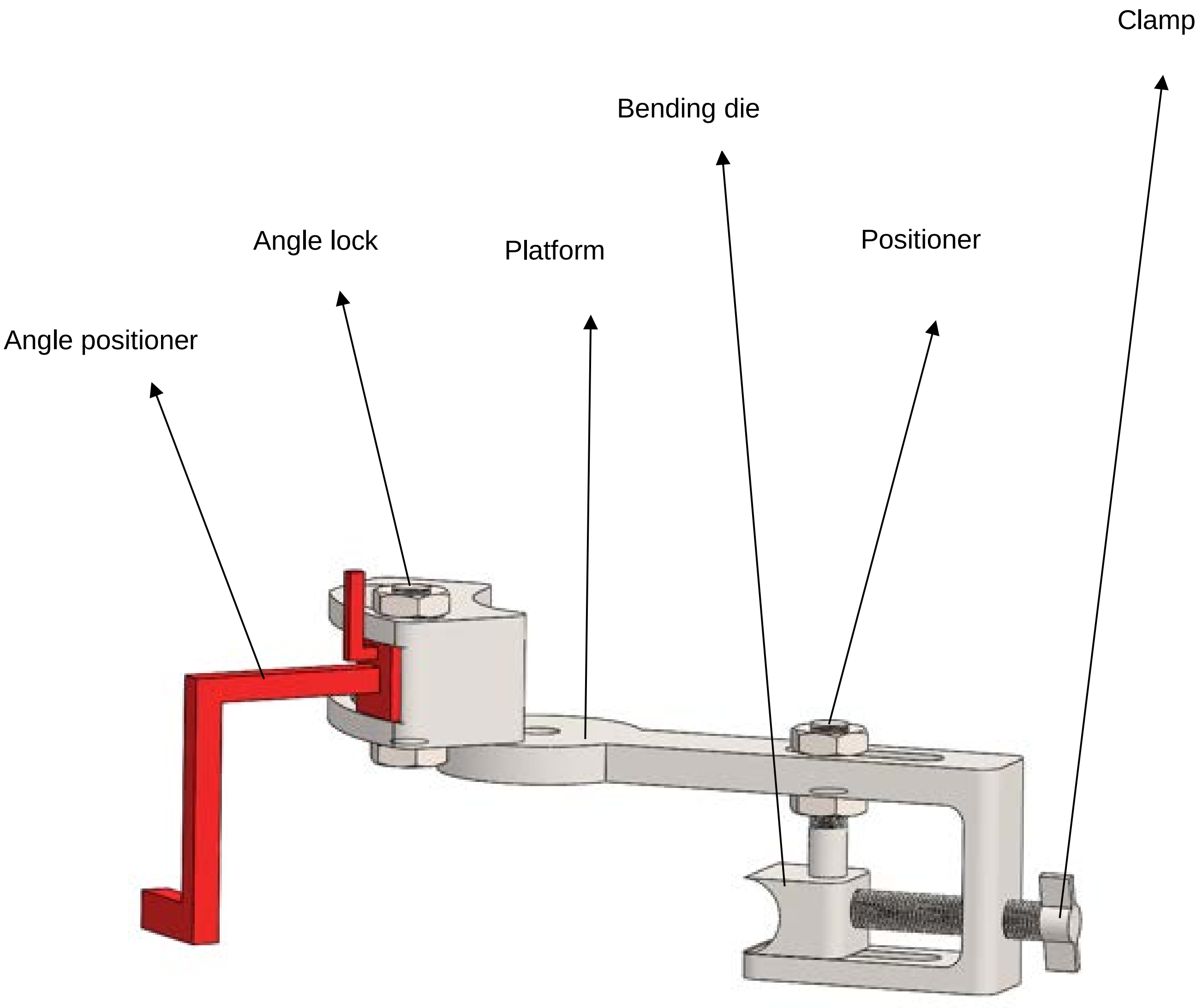


Figure 29: Actuator design

The custom shaft is designed in such a way that the actuator will not be in contact with the big roller and the base to avoid the frictional forces. This can be seen in the figure below.

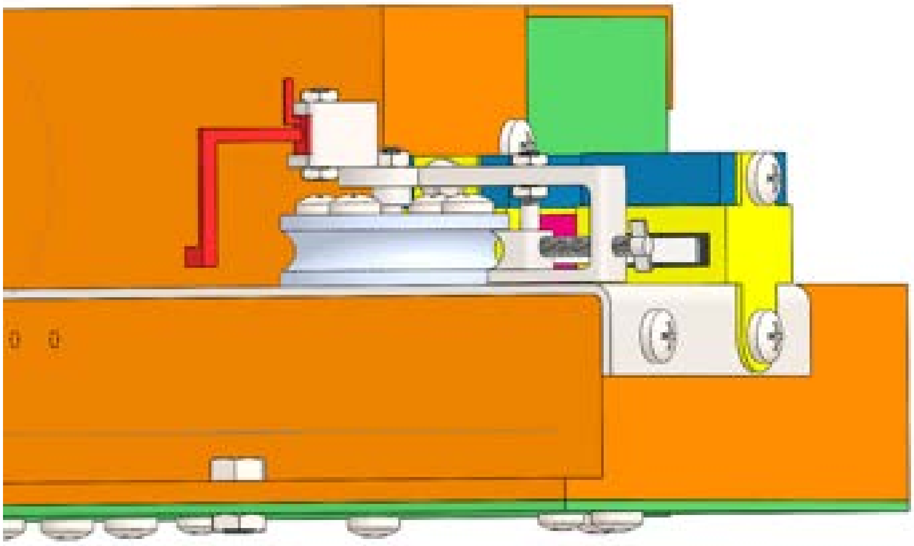


Figure 30: Model showing the actuator, big roller and base

The component that will support the pipe as it is getting bend has to designed. It must be considered that a pipe of 6mm diameter will be bent (state the URS). The figure below shows the pipe support design.

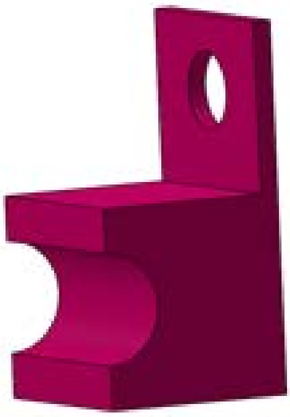


Figure 31: pipe support

Operation

The following figures show the model will work it is bent by doing the following:

- Clamping the pipe tight between the big roller and the bending
- Support the pipe.

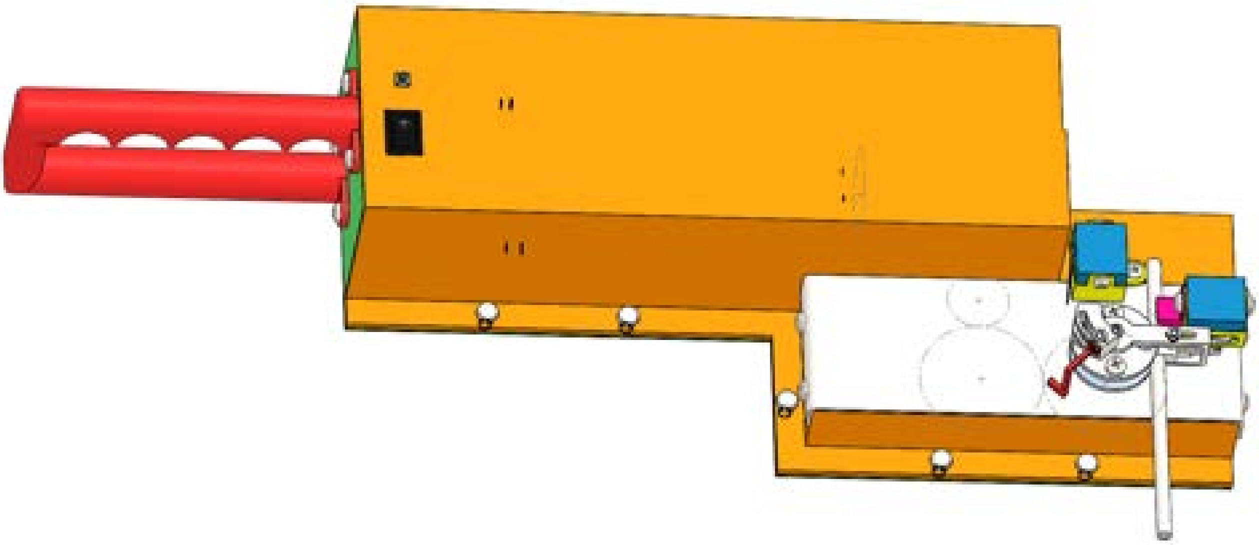


Figure 32: Pipe in operation

2.5 Angle control mechanism design and functionality

The angle control mechanism needs the following components:

- Microswitches for stopping the actuator at a certain point
- Push switch to start the motor rotation
- Control switch to change rotational direction of the motor.

The microswitch that will be used is in the figure below:

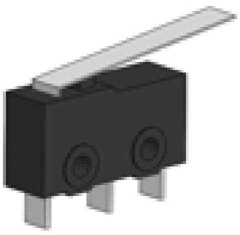


Figure 33: Microswitch

The push button that will be used is in the figure below:

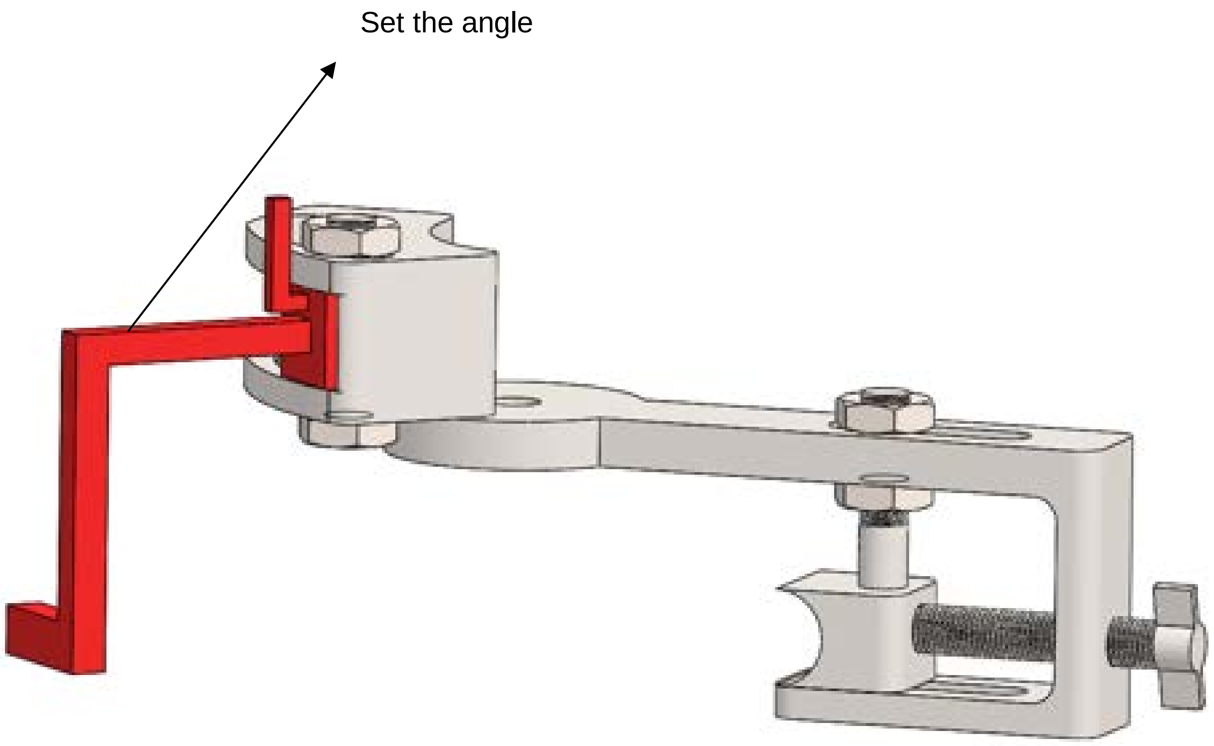
Figure 34: Push button from RS components (RS Components, 2023)

The control switch that will be used is in the figure below:

Figure 35: Control switch from RS components (RS components, 2023)

This is how the angle control should work:

- The first limit microswitch will stop the actuator at a desired angle
- The second microswitch will stop the actuator at the starting position
- The push switch will start the motor
- The control switch will the rotational direction of the motor



The highlighted part in the above figure will be used to set the desired angle and it will also trigger the limit switch.

In the event that the user will want to bend the pipe to an angle of 90 degrees, here are the steps that should be followed:

- Move the angle positioner to the desired angle, this is shown in the figure below

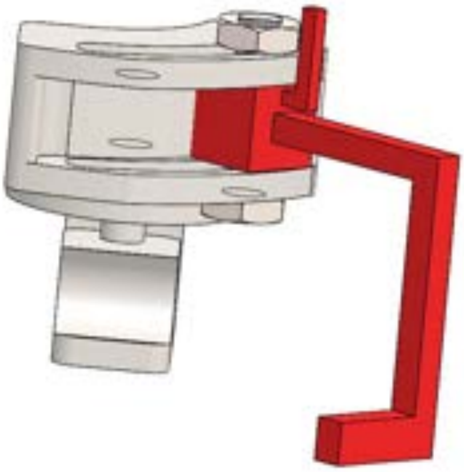


Figure 36: Moving the angle positioner to the very end will give an angle of 90 degrees

- The user will then hit the push button
- The actuator will rotate until it hits the limit switch, this can be seen in the figure below

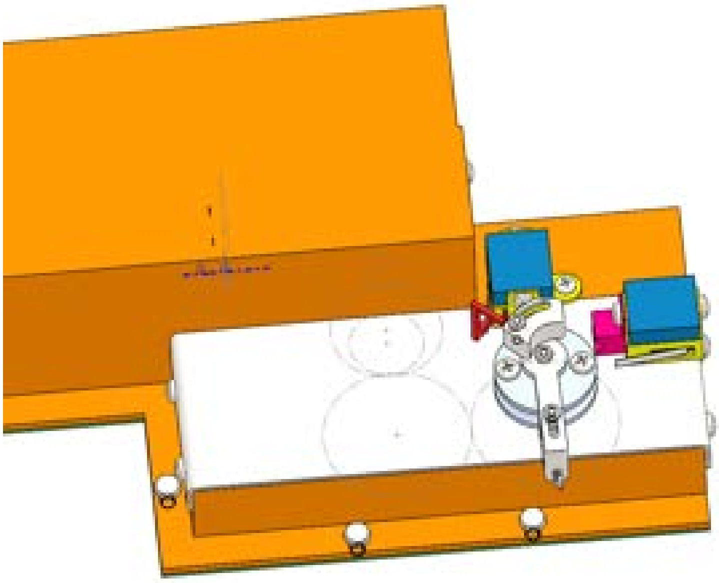


Figure 37: Actuator stopped at 90 degrees

- The user will then hit the control switch to change the rotational direction of the motor
- After, the user will press the push button to allow the actuator to rotate in a reverse direction
- The actuator will reverse until it hits the limit switch at the starting position, this ca be seen in the figure below

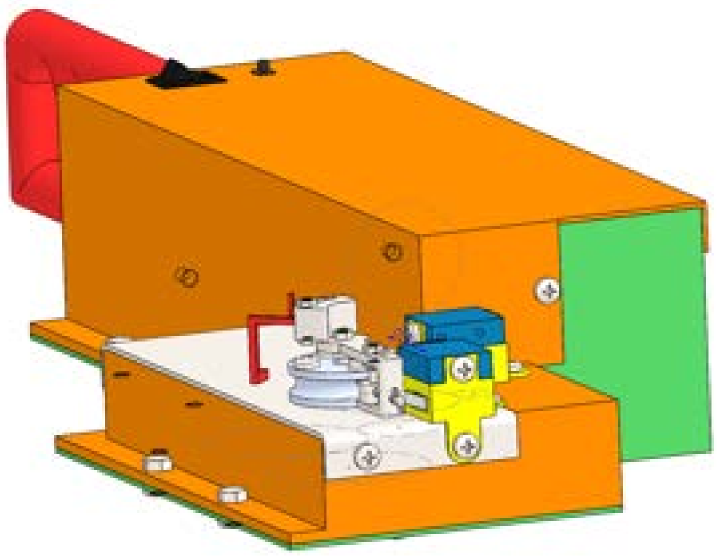
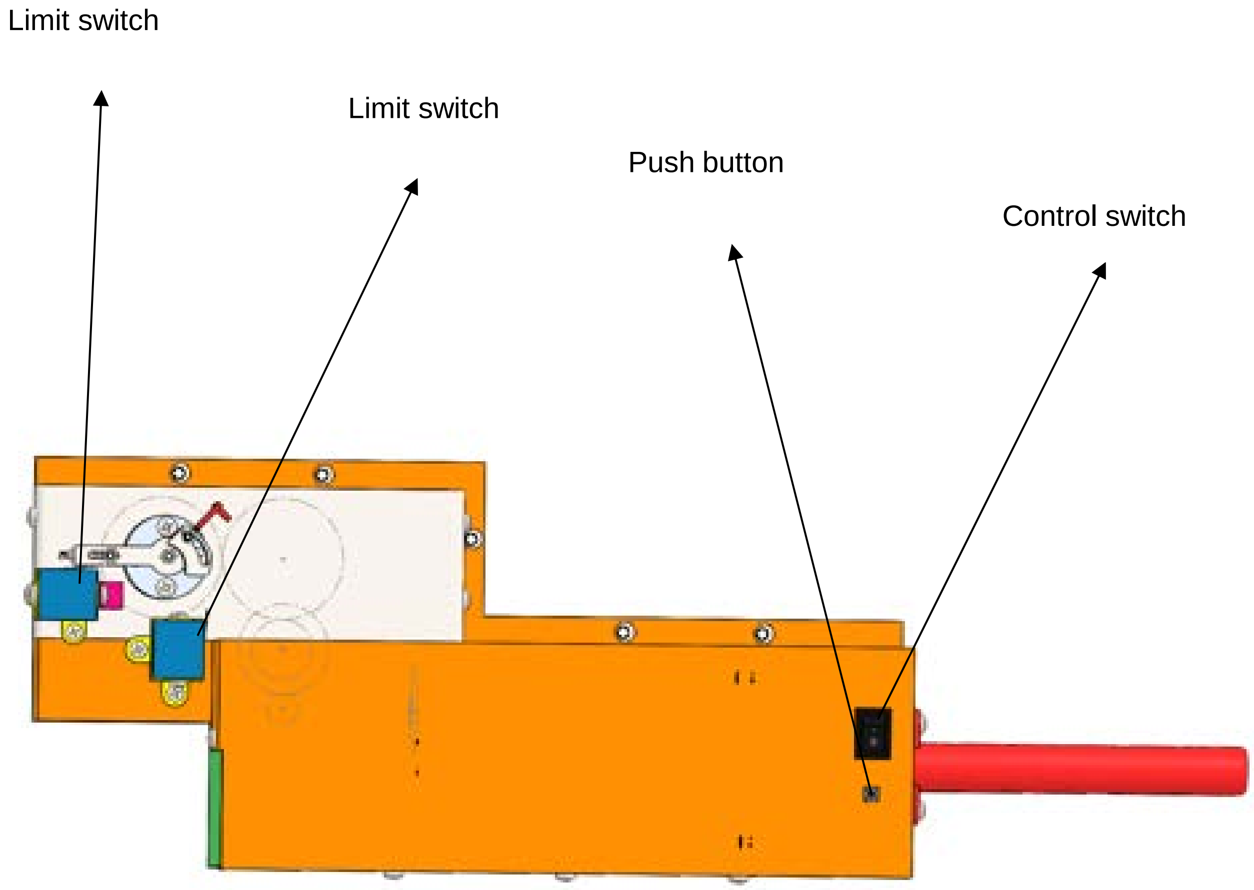


Figure 38: Actuator triggers the limit switch at the starting position

The figure below shows the position of all the switches.



2.6 Component packaging, manufacturing/assembly considerations and ergonomics considerations

Material selection

The parts that are not providing any support do not need to be strong will be 3D printed using PLA plastic.

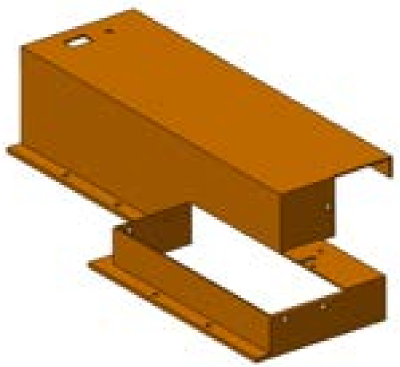


Figure 39: Example of a 3D printed part

The parts that will be providing support will be manufactured using hot rolled sheet metal.

Figure 40: Example of a sheet metal part

The important that are involved in bending the pipe need to be machine using low carbon.

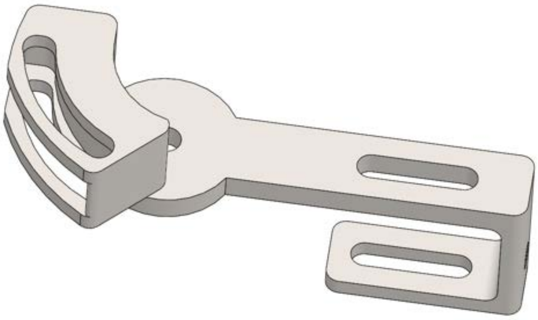


Figure 41: Example of a machined part

Assembly considerations

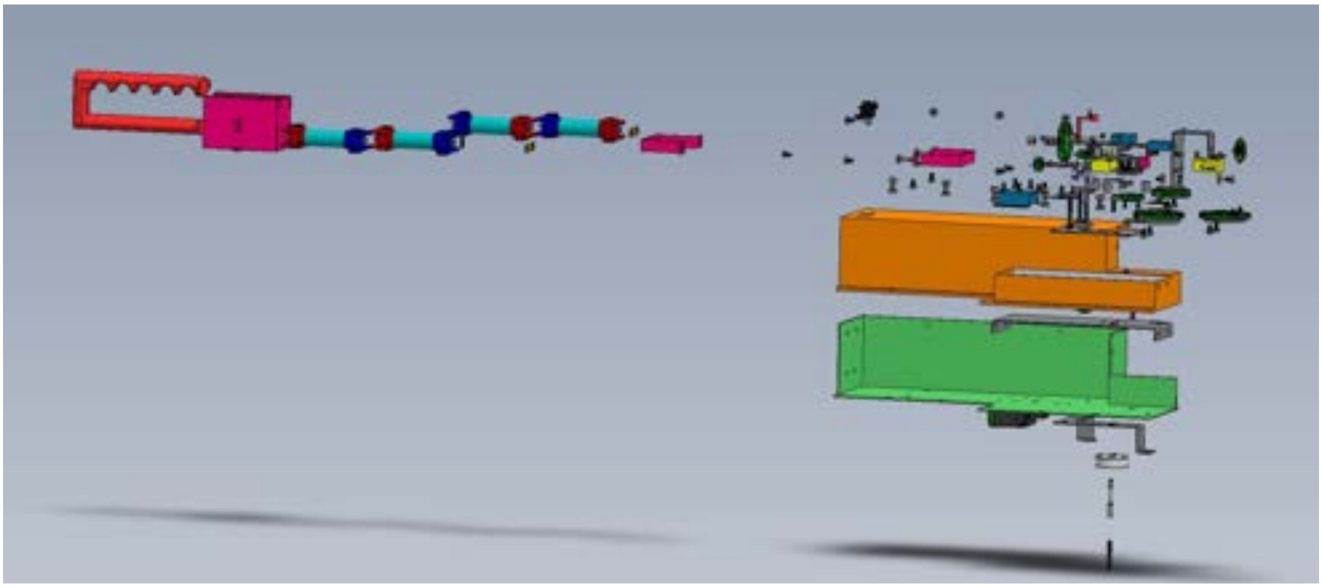


Figure 42: Exploded view showing all the parts.

The above figure shows all the parts that need to be assembled to form a complete model. The lower housing is designed particularly to allow all the internal the assemble without a problem.

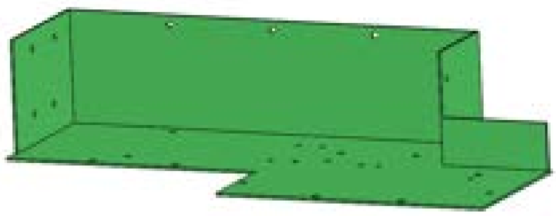


Figure 43: Lower housing

Before the upper housing is mounted on the lower housing. The wires must be passed through its holes on the upper housing to ensure that the switches and the motor can be connected.

Then the upper housing can be mounted to the lower housing using nuts, bolts, and self-tapping screws.

The upper housing is designed in such a way that it leaves spaces for where the bending base will sit. It also allows all the internal parts to assembled first before the external parts are assembled.

The bending base which is a sheet metal will be mounted on the upper housing using self-tapping screws.

Then the external parts can be assembled and mounted onto the bending base. After that the wiring can be done, which is exposed to the outside.

Ergonomics consideration

The handle is designed in such a way that it can be held with one hand, and it does not have any sharp edges. Thus, it will not damage the user when it being used.

Figure 44: Handle

The machined bending platform part is designed in such a way that it will allow a 3mm ball nose mill to fit through all the holes. It does have any sharp edges that the ball nose mill won’t be able to make.

Figure 45: Bending platform

The handle and two switches (control switch and push switch) are very close to each other to allow for one hand operation.

The two switches are also close to the battery and motor to allow for easy wiring.

There's enough space inside the model to allow the wires to go to the microswitches in front without interfering with the internal parts.

The housing shape is designed to allow the pipe to fully bend without interfering with any external parts.

3. BOM costing with prototype and production cost

Mass calculations

The mass of the parts that will be bought (e.g. Motor, switches) is given by the supplier and does not need to be calculated.

The mass of custom parts that need to be manufactured is determined in the following way:

Mass = Density x Volume

Density of the following materials is used for calculations:

- 3D printed components – PLA plastic
- Sheet metal components – Hot rolled mild steel
- Machined components – Low carbon steel

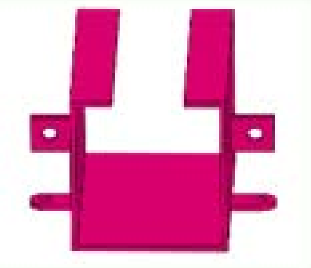
The volume used is calculated using solidworks, which is accurate for complex components.

Here's an example of the mass calculation.

The material that will be used for 3D printed parts is PLA plastic.

$$\rho_{PLA} := 1240 \cdot \frac{kg}{m^3}$$

<u>Battery holder</u>



$$V_{bh} := 30227.85 \cdot mm^3$$

$$m_{bh} := \rho_{PLA} \cdot V_{bh}$$

$$m_{bh} = 0.037\ kg$$

Figure 46: Example of mass calculation

The rest of the mass calculations are in **Appendix G**.

<u>Cost calculations</u>

Prototyping

- Machined component : raw material cost + R300 / h, in increments of 15 min
- Raw material :
  - Low carbon steel : R25 / kg
  - Brass : R155 / kg
  - Aluminium : R95 / kg
- 3D printing using PLA plastic : R700 / kg + R65 setup cost per part
- For hot rolled carbon steel 1.0 to 3.0 mm sheetmetal parts:

$$\text{Cost } [R] = 2.2M + 114L \cdot t^{0.2} + 9.6B$$

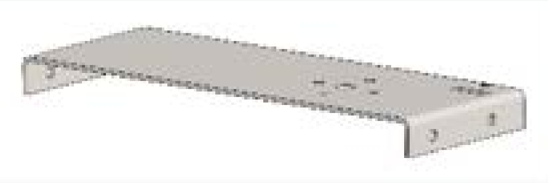
with M = mass $[kg]$ of plate without internal cut – outs
L = total length $[m]$ of cutting, incl outer perimeter
t = plate thickness $[m]$
B = number of simple bends

Figure 47: Typical costing inputs

The above typical costing inputs are used to determine the costs of all the components.

Sheet metal parts

Base



$$m_{base} := 0.163\ kg$$

$$L_{base} := 593.79\ mm$$

$$t_{base} := 1.6\ mm$$

$$B_{base} := 2$$

$$C_{base} := 2.2 \cdot \frac{m_{base}}{kg} + 114 \cdot \frac{L_{base}}{m} \cdot \left(\frac{t_{base}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{base}$$

$$C_{base} = 38.238$$

Figure 48: Example of cost calculations

The above figure shows an example of how cost will be determined using the inputs in Figure 5. The detail cost calculations are in **Appendix G**.

Here's a bill of material below, showing the cost and mass of each component

NO.	PART NUMBER	DESRIPTION	SPECIFICATION	MASS [kg]	COST [rands]	QTY
1	MEC4124-004-30	SQ shaft 30 long	Square Shaft 0.125" x 30mm (VEX 276-1149)	0.002	4.8	1
2	MEC4124-010	12T Gear	12T spur gear, 24/"P (VEX 276-2169-001)	0.002	9	2
3	MEC4124-004-70	SQ shaft 70 long	Square Shaft 0.125" x 70mm (VEX 276-1149)	0.005	11.2	1
4	MEC4124-016	24T Bevel Gear	24T bevel gear, 24/"P (VEX 276-2184-001)	0.002	12	2
5	MEC4124-004-45	SQ shaft 45 long	Square Shaft 0.125" x 70mm (VEX 276-	0.003	7.2	1

			1149)			
6	MEC4124-001	100rpm Motor	2-Wire Motor 393 (VEX 276-2177)	0.09	315	1
7	MFKKAT007-001	Motor lifter	See DWG for details, PLA 3D printed	0.003	67.1	1
8	MFKKAT007-002	Battery holder	See DWG for details, PLA 3D printed	0.037	90.9	1
9	MEC4124-030	Cell w caps	Li-ion LFP 18650 cell with Vruzend connector caps	0.216	216	4
10	MEC4124-031	Cell link	0.5mm Plated Brass link (Vruzend)	0.002	2	2
11	MEC4124-032	Fused link	PCB with 5A resettable fuse	0.0015	25	1
12	MEC4124-008-30	HS SQ shaft 30 long	Square Shaft 0.125" x 30mm, heat treated (VEX 276-1149)	0.007	14.4	2
13	MFKKAT007-003	Lower Housing	See DWG for details, PLA 3D printed	0.16	177	1
14	MFKKAT007-004	Upper Housing	See DWG for details, PLA 3D printed	0.149	169.3	1
15	MFKKAT007-005	Base	See DWG for details, 1.6mm Hot Rolled Mild Steel	0.163	38.24	1
16	MEC4124-052	Big Roller	30mm roller, 6.5dia groove	0.065	65	1
17	MFKKAT007-006	Microswitch lower holder two	See DWG for details, PLA 3D printed	0.004	67.8	1
18	MFKKAT007-007	Microswitch Lower Holder	See DWG for details, PLA 3D printed	0.004	67.8	1
19	MEC4124-036	Microswitch	Sub-Miniature Micro Switch 1C-SPDT(CO), 5A, 250VAC (Comm SS5GL111)	0.004	34	2
20	MFKKAT007-008	Motor Support	See DWG for details, 1mm Hot Rolled Mild Steel	0.037	57.85	1
21	MFKKAT007-009	Upper Shaft Support	See DWG for details, 1mm Hot Rolled Mild Steel	0.041	50.72	1
22	MFKKAT007-	Lower Shaft	See DWG for details, 1mm Hot Rolled Mild	0.034	13.79	1

	010	Support	Steel			
23	MFKKAT007-011	Handle	See DWG for details, PLA 3D printed	0.097	132.9	1
24	MFKKAT007-012	Double plate for 48T gear	See DWG for details, 2mm Hot Rolled Mild Steel	0.049	6.91	2
25	MEC4124-013	48T Gear	48T spur gear, 24/"P (VEX 228-3450-228)	0.0002	18	2
26	UCT-19516	Self-tapping screw	DIN 7049 ST4.2 X 9.5, Carbon Steel Gr 8.8	0.0004	4	4
27	MFKKAT007-013	Modified 36T Gear	Modified 36T gear MEC4124-012 see DWG for details	0.0052	36	1
28	MFKKAT007-014	Double plate for 36T gear	See DWG for details, 1mm Hot Rolled Mild Steel	0.006	3.47	1
29	UCT-19502	Self-tapping screw	DIN 7049 ST2.2 X 9.5, Carbon Steel Gr 8.8	0.002	2	2
30	MFKKAT007-015	Modified 24T Gear	Modified 24T gear MEC4124-011 see DWG for details	0.002	12	1
31	MFKKAT007-016	Double plate for 24T gear	See DWG for details, 1mm Hot Rolled Mild Steel	0.0025	3.46	1
32	UCT-19501	Self-tapping screw	DIN 7049 ST2.2 X 6.5, Carbon Steel Gr 8.8	0.0001	2	2
33	MEC4124-008-30	HS SQ shaft 30 long	Square Shaft 0.125" x 30mm, heat treated (VEX 276-1149)	0.007	14.4	1
34	MEC4124-007	Shaft Coupler	Shaft Coupler (VEX 276-1843-001)	0.003	25.5	1
35	MFKKAT007-017	Custom circular shaft	See DWG for details, Low Carbon Steel	0.002	150.05	1
36	MFKKAT007-018	Clamp platform	See DWG for details, Low Carbon Steel	0.024	450.6	1
37	MFKKAT007-019	Bending Die	See DWG for details, Low Carbon Steel	0.002	75.05	1
38	MFKKAT007-	Die positioner	See DWG for details,	0.00089	75.02	1

	020		Low Carbon Steel			
39	MFKKAT007-021	Die lock	See DWG for details, Low Carbon Steel	0.00096	150.024	
40	MFKKAT007-022	Angle lock	See DWG for details, PLA 3D printed	0.00051	65.36	1
41	MFKKAT007-023	Angle positioner	See DWG for details, Low Carbon Steel	0.00074	75.02	1
42	UCT-11203	Hex thin nut M3	ISO 4032 - M3, Carbon Steel Gr 8.8	0.001	1.2	1
43	MFKKAT007-024	Microswitch upper holder	See DWG for details, PLA 3D printed	0.004	67.8	2
44	MFKKAT007-025	Control Switch	Marquardt DPDT, (On)-Off-(On) Rocker Switch Panel Mount	0.03	65	1
45	MFKKAT007-026	Push button	RS PRO Momentary Miniature Push Button Switch, Panel Mount, SPST, 13.6mm Cutout, 32/50/125V ac, IP67	0.027	75	1
46	MFKKAT007-027	Pipe support	See DWG for details, PLA 3D printed	0.00126		1
47	MFKKAT007-028	Sheet support	See DWG for details, 1mm Hot Rolled Mild Steel	0.009	57.85	1
48	MEC4124-040	Spacer 3.2 long	3/8" OD x 0.125" Nylon Spacer (VEX 276-6340-001)	0.003	30	10
49	MEC4124-042	Spacer 9.5 long	3/8" OD x 0.375" Nylon Spacer (VEX 276-6340-003)	0.002	9	3
50	MEC4124-043	Spacer 12.7 long	3/8" OD x 0.5" Nylon Spacer (VEX 276-6340-004)	0.002	6	2
51	MEC4124-041	Spacer 6.4 long	3/8" OD x 0.25" Nylon Spacer (VEX 276-6340-002)	0.0005	3	1

52	MEC4124-012	36T Gear	36T spur gear, 24/"P (VEX 276-2169-002)	0.01	36	1
53	MEC4124-006	6-32x1/2" screw	#6-32x1/2" button-head screw (VEX 275-1169)	0.003	7	2
54	MFKKAT007-028	Battery Holder Lock	See DWG for details, PLA 3D printed	0.011	72.7	2
55	UCT-15006	Washer M5	ISO 7089 - 5.3 x 10 x 1THK, Carbon Steel Gr 5.8	0.003	6	6
56	UCT-19505	Self-tapping screw	DIN 7049 ST2.9 X 6.5, Carbon Steel Gr 8.8	0.008	6	6
57	UCT-19510	Self-tapping screw	DIN 7049 ST3.5 X 9.5, Carbon Steel Gr 8.8	0.018	12	12
58	UCT-19518	Self-tapping screw	DIN 7049 ST4.2 X 16, Carbon Steel Gr 8.8	0.019	4	4
59	UCT-01027	Hex Bolt M5	ISO 4015 - M5 X 8 x 8, Carbon Steel Gr 8.8	0.004	5	5
60	UCT-11205	Hex thin nut M5	ISO 4035 - M5, Carbon Steel Gr 8.8	0.019	7	7
61	UCT-19516	Self-tapping screw	DIN 7049 ST4.2 X 9.5, Carbon Steel Gr 8.8	0.034	21	21
62	UCT-11103	Hex nut M3	ISO 4032 - M3, Carbon Steel Gr 8.8	0.004	1	1
63	UCT-01031	Hex Bolt M5	ISO 4015 - M5 X 16 x 10, Carbon Steel Gr 8.8	0.0025	2	2

Here's the summary below of the mass and cost calculation.

	Bought parts	3D printed parts	Sheet metal parts	Machined parts
Total mass [kg]	0.607	0.469	0.341	0.03
Total cost [Rands]	694.1	978.656	178.241	975.765

The total mass of all the components is **1.447kg**

The total cost of all the components is **R2 827**

When determining the prototype cost, labour has to be included. In detail calculations (Appendix E) labour cost is determined to be **R162.**

Therefore, the prototype cost is **R 2 989.**

4. Development plan

Development costs

Development costs are commonly referred to as research and development costs. These costs can include a host of expenses, such as marketing analysis, developmental engineering and customer surveying.

Activities that are typically considered as research and development include:

Research to bring about new knowledge
Creation of product and process designs
Testing processes and products
Modifying processes and products
Designing prototypes
Testing prototypes
Designing new tools

In the costing calculations (**Appendix H**), the following was determined:

- 2 Engineers will work on research and development
- They will cost 400/hr
- They will work for 2 months
- Four demonstrations need to be made @prototype cost

The development cost came up to **R140 000.**

Ramp-up costs

Figure 49: Tooling cost

The above figure shows the formulae that will be used to determine the costs.

The tooling cost needed to be determined for the following:

- Single cavity simple injection mould
- Steel stamping tool

Thus, the following was determined in the costing calculations (Appendix H):

- The total tooling cost came to **R424 000**.
- Three alpha prototypes will be made @R2 989
- Six beta prototypes will be made @R2 242

The total ramp-up cost came up to **R446 400**

Volume production cost per product

The volume production costs are determined using the formulae in Figure 50.

The results of the calculations are (detail calculations in Appendix H):

- Total cost of injection moulded ABS plastic is R19.38
- Total cost of CNC machined components is R2.09
- Total cost of steel stamped sheet metal components is R14.96
- Total cost of off the shelf components is R694.10
- Labour cost is R162

The volume production cost came up to **R892.53**

The table below shows a summary of values used to determine the development plan.

Development costs	R140 000	2 man-months @R400/h plus, 4 demonstrations prototypes @R2989 each, spread over 2 months
Ramp-up costs	R446 400	Tooling @R424000, plus 3 alpha prototypes @R2989 each and 6 beta prototypes @R2242 each
Volume production cost per product	R892.53	See production costing in Appendix H
Sales price to distributor per product	R1 400	As per URS
Steady state production / sales per month	260	Given that 5 complete prototypes are made each day

Month	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Expenses [kR]	70,00	70,00	494,00	8,97	13,45	44,63	133,88	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06
Development	70,00	70,00	70,00																	
Ramp-up [kR]			424,00	8,97	13,45															
Production [kR]						44,63	133,88	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06	232,06
Quatity						50	150	260	260	260	260	260	260	260	260	260	260	260	260	260
Income [kR]								R280,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00	R364,00
Sales [kR]						0	0	280	364	364	364	364	364	364	364	364	364	364	364	364
Quatity								200	260	260	260	260	260	260	260	260	260	260	260	260
Nett Cost [kR]	-70,00	-140,00	-634,00	-642,97	-656,42	-701,05	-834,93	-786,98	-655,04	-523,10	-391,16	-259,21	-127,27	4,67	136,61	268,55	400,50	532,44	664,38	796,32
Stock inventory						50	200	260	260	260	260	260	260	260	260	260	260	260	260	260

Figure 51: Cash flow analysis

The above cash flow analysis shows that the break-even point after **14 months**.

5. Risks and drawbacks

The main drawback of the design is that it uses too many gears and it is because of how it is shaped. But the required moment to bend the pipe can be achieved with a smaller number of gears. Using less number benefit this design because it would make the following improvements:

- Reduce prototype cost
- Lower the prototype mass
- Reduce the number of gears that need to be supported
- Decrease the number of parts used

The main risk of the design is that if the wiring is not done correctly, it will affect the angle control mechanism. That will have the following effects:

- Fail to stop the pipe at a desired angle
- Fail to start the motor
- Fail to stop the actuator at the starting position

6. Reflection

The product is designed successfully because it meets all the URS requirements. Here's a table below showing how it meets each user requirement.

User requirements	How it meets the requirement
B1. The manufacturing of the first prototype has a target cost of R3 000	The prototype cost was determined to be R2989
B2. For the production product, the sales target price is R1 400.	Sales target of R1400 was used to determine the break-even point
B3. Break-even point for the product is required within 2 years from start of volume production	The break-even point is reach after 14 months
C1. The device shall be battery operated.	The device is battery operated
C2. The battery shall be integrated into the device	The battery is integrated into the device
C3. The bending force shall be from an electrical motor via mechanical means only.	The bending force is due to the actuator
C4. Bending control shall be by means of limit switches only.	Only limit switches are used for angle control mechanism
F1. The device shall perform the bending and retracting action using an electric switch.	The device is able to retract using a control switch
F2. It shall be possible to bend a pipe that is fixed 40mm from a surface in any direction except towards the surface.	The device is able to bend the pipe fixed 40mm from a surface in any direction except toward the wall
F3. It shall be possible to have a free end after the bend of minimum 60 mm.	The way actuator bends the pipe and the shape of the housing allows for a minimum 60mm free end after bend
F4. The device shall bend the pipe to an inner radius of 15mm without pinching	A big roller with an inner radius of 15mm is used
. Bend angle range 45° to 90°. Fully adjustable with at least 5° increments	The device can bend the pipe from 45° to 90° and it is fully adjustable in any increments.
P3. Bend angle precision ± 2.5°	The device meets this precision
P4. Total mass < 1.5 kg	The device has a total mass of 1.447kg
P5. Permanent deformation of any component when fully loaded	The strength calculations are done to show that the device will not deform plasticly
E1. All gears and electronics shall be covered such that a rod of minimum 2mm diameter cannot contact it.	All gears are covered, and there's no space for a rod of minimum 2mm diameter.
E2. The device shall be operatable by a 95% percentile adult male or female using only one	The device meets this requirement because it has small mass and handle to allow for one

hand	operation
E3. The device ergonomics shall enable maximum maneuverability and control to the user.	The switches are position close to the handle to allow for easy access and the handle is designed to allow the user to grip it.

7. References

[1] R. Hibbeler, Mechanics of Materials, SI Edition (Paperback, 10th edition), United Kingdom: Pearson Education, 2018.

[2] RS Components, "RS PRO Momentary Miniature Push Button Switch, Panel Mount, SPST, 13.6mm Cutout, 32/50/125V ac, IP67," 6 April 2023. [Online]. Available: https://za.rs-online.com/web/p/push-button-switches/7346716.

[3] RS components, "Marquardt DPDT, (On)-Off-(On) Rocker Switch Panel Mount," RS Components, 6 April 2023. [Online]. Available: https://za.rs-online.com/web/p/rocker-switches/7410820. [Accessed 6 April 2023].

Appendix A: Bending requirements

The user requirements document gives the properties of the pipe that must be bent by the battery operated bending tool. Those properties are:

Pipe yield strength: $T_{yield} := 650 \cdot MPa$

Pipe thickness: $t := 1.5\ mm$

Pipe outer diameter: $OD := 6\ mm$

Using the pipe outer diameter and thickness, the inner diameter can be calculated.

Pipe inner diameter: $ID := OD - 2 \cdot t$

$ID = 3\ mm$

Using the above pipe properties, the required bending moment to bend the pipe can be calculated.

To determine the maximum stress due to bending the flexure formula is used:

$$\sigma_{max} = \frac{M \times c}{I_x} = \frac{M}{Z_x}$$

where:

- σ_{max} is the maximum stress at the farthest surface from the neutral axis (it can be top or bottom)
- M is the bending moment along the length of the beam where the stress is calculated
  - if the maximum bending stress is required then M is the maximum bending moment acting on the beam
- I_x is the moment of inertia about x (horizontal) centroidal axis
- c is the maximum distance from the centroidal axis to the extreme fiber (again, this can be to the top or bottom of the shape)
- Z_x is called section modulus and is a term that combines the moment of inertia and the distance to the extreme fiber ($Z_x = I_x / c$)

Figure 1: Bending stress formula

The above formula is taken from a book called "Mechanic of Materials" 11th edition, written by Russell C. Hibbeler

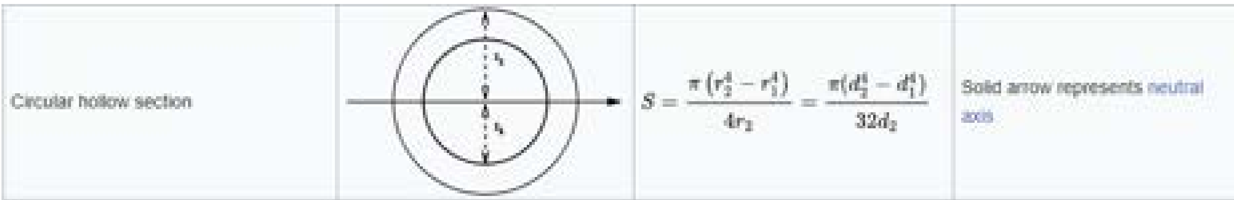


Figure 1: Section modulus formula

The above formula is taken from a book called "Mechanic of Materials" 11th edition, written by Russell C. Hibbeler.

Section Modulus:

$$SM := \frac{\pi \cdot \left(OD^4 - ID^4\right)}{32 \cdot OD}$$

$$SM = 19.88\ mm^3$$

Bending moment required:

$$M_{required} := SM \cdot T_{yield}$$

$$M_{required} = 12.922\ N \cdot m$$

Now that the required bending moment is calculated, the gearbox should be designed such that the torque of the last gears is greater or equal to the required bending moment.

Appendix B: Motor calculations

The motor calculations will be done in the following steps:

1. Choose the number of cells that will be used
2. Determine the stall current of the motor
3. Select a motor

<u>Choose the number of cells that will be used</u>

The motor specification document state that the maximum voltage **15V** and the cell specification document state that the nominal voltage is **3.2V**.

Maximum voltage of that the motor can take: $V_{max} := 15\ V$

The nominal cell voltage: $V_{nominal} := 3.2\ V$

Therefore, using the above information the maximum number of cells that should be connected in series can be calculated.

Maximum number of cells in series: $N_{max} := \frac{V_{max}}{V_{nominal}}$

$N_{max} = 4.688$

Thus, the maximum number of cells in series should be **4.**

Number of cells chosen for the design: $N_{cells} := 4$

Supply voltage to the motor: $V_{supply} := V_{nominal} \cdot N_{cells}$

$V_{supply} = 12.8\ V$

Determine the stall current of the motor

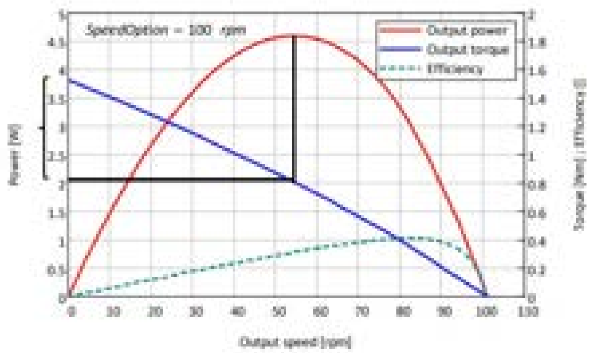


Figure 1: Curves generated from the motor specification document

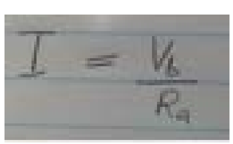
The above figure is. The black lines depict that the maximum motor current occurs at half the stall current. Stall current occurs when the output speed is **0rpm**.

Thus, in order to determine the maximum motor operating current, stall current and torque must be calculated first.

Stall current occurs at the output speed of **0rpm**.

Output speed $[rad/s]$	$\omega_m = \dfrac{V_b - I_m R_a}{K_T r_g}$	V_b = supply voltage

Figure 2: Output speed from the motor specification document



The equation from figure 2 reduces to the one on the left due to the output speed equating to 0rpm.

Armature resistance [Ω]	$R_a = \begin{cases} 1.61 \ \text{if} \ I < I_{sat} \\ 1.61 + 0.15(I - I_{sat}) \ \text{otherwise} \end{cases}$ $I_{sat} = 1.2A$	I_{sat} is the magnetic saturation current

Figure 3: armature resistance equation from the motor specification document

Using the reduced equation from figure 2 and the armature resistance equation from figure 3, the below calculations are done.

$$R_a = 1.61 + 0.15(I - 1.2)$$
$$R_a = 1.61 + 0.15I - 0.18$$
$$R_a = 0.15I + 1.43$$

$$I = \frac{V_{b}}{0.15I + 1.43}$$

$$0.15I^2 + 1.43I = V_{b}$$
$$0.15I^2 + 1.43I - V_{b} = 0$$

Quadratic equation in standard form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic equation will be used to solve for the stall current.

Stall current:

$$I_{stall} := \left(\frac{-1.43 + \sqrt{1.43^2 - 4 \cdot (0.15) \cdot (-12.8)}}{2 \cdot 0.15} \right) A$$

$$I_{stall} = 5.628 \ A$$

Select a motor

In order to select the correct motor, the maximum conditions of all the motors will be calculated.

Now that stall current is determined. Stall torque can be calculated.

Output torque $[Nm]$	$T_m = I_m K_T r_g \eta_g - T_0$	I_m = motor current

Figure 4: Output torque equation from the motor specification document

The constants in the above equation are given in the motor specification document, they are listed below.

Torque constant: $K_T := 4.1 \cdot N \cdot \frac{mm}{A}$

For the 100 rpm motor

Gearbox reduction ratio: $r_{g\_100} := 156.8$

Gearbox efficiency: $\eta_{g\_100} := 0.7$

Static drag: $T_{0\_100} := 110 \cdot N \cdot mm$

Therefore, using the properties above stall torque can be calculated.

Stall current for 100rpm motor: $T_{stall\_100} := I_{stall} \cdot K_T \cdot r_{g\_100} \cdot \eta_{g\_100} - T_{0\_100}$

$$T_{stall\_100} = 2.423\ N \cdot m$$

As explained in figure 1 that the maximum motor operating current is when the motor current is at half the stall current. The output torque equation from figure 4 will be used to solve for the maximum motor current.

The maximum motor current for 100rpm motor:

$$I_{max\_100} := \frac{\left(\frac{T_{stall\_100}}{2}\right) + T_{0\_100}}{K_T \cdot r_{g\_100} \cdot \eta_{g\_100}}$$

$$I_{max\_100} = 2.936\ A$$

Armature resistance for 100rpm motor:

$$R_{a\_100} := \left(1.61 + 0.15 \cdot \left(\frac{I_{max\_100}}{1\ A} - 1.2\right)\right)\ \Omega$$

$$R_{a\_100} = 1.87\ \Omega$$

The maximum output torque for 100rpm motor:

$$T_{max\_100} := I_{max\_100} \cdot K_T \cdot r_{g\_100} \cdot \eta_{g\_100} - T_{0\_100}$$

$$T_{max\_100} = 1.211\ N \cdot m$$

Overload time to trip for 100rpm motor:

$$t_{trip\_100} := 140 \cdot \left(\frac{I_{max\_100}}{1 \cdot A}\right)^{-3.54} \cdot s$$

$$t_{trip\_100} = 3.091\ s$$

Output speed for 100rpm motor:

$$w_{m\_100} := \frac{V_{supply} - I_{max\_100} \cdot R_{a\_100}}{K_T \cdot r_{g\_100}}$$

$$w_{m\_100} = 11.367\ \frac{rad}{s}$$

For the 160 rpm motor

Gearbox reduction ratio: $r_{g\_160} := 98$

Gearbox efficiency: $\eta_{g\_160} := 0.72$

Static drag: $T_{0\_160} := 85 \cdot N \cdot mm$

As explained in figure 1 that the maximum motor operating current is when the motor current is at half the stall current. The output torque equation from figure 4 will be used to solve for the maximum motor current.

Stall current for 160rpm motor:

$$T_{stall\_160} := I_{stall} \cdot K_T \cdot r_{g\_160} \cdot \eta_{g\_160} - T_{0\_160}$$

$$T_{stall\_160} = 1.543 \; N \cdot m$$

The maximum motor current for 160rpm motor:

$$I_{max\_160} := \frac{\left(\dfrac{T_{stall\_160}}{2}\right) + T_{0\_160}}{K_T \cdot r_{g\_160} \cdot \eta_{g\_160}}$$

$$I_{max\_160} = 2.961 \; A$$

Armature resistance for 160rpm motor:

$$R_{a\_160} := \left(1.61 + 0.15 \cdot \left(\frac{I_{max\_160}}{1 \; A} - 1.2\right)\right) \; \Omega$$

$$R_{a\_160} = 1.874 \; \Omega$$

The maximum output torque for 160rpm motor:

$$T_{max\_160} := I_{max\_160} \cdot K_T \cdot r_{g\_160} \cdot \eta_{g\_160} - T_{0\_160}$$

$$T_{max\_160} = 0.772 \; N \cdot m$$

Overload time to trip for 160rpm motor:

$$t_{trip\_160} := 140 \cdot \left(\frac{I_{max\_160}}{1 \cdot A}\right)^{-3.54} \cdot s$$

$$t_{trip\_160} = 3.001\ s$$

Output speed for 160rpm motor:

$$w_{m\_160} := \frac{V_{supply} - I_{max\_160} \cdot R_{a\_160}}{K_T \cdot r_{g\_160}}$$

$$w_{m\_160} = 18.045\ \frac{rad}{s}$$

For the 240 rpm motor

Gearbox reduction ratio: $r_{g\_240} := 65.33$

Gearbox efficiency: $\eta_{g\_240} := 0.75$

Static drag: $T_{0\_240} := 65 \cdot N \cdot mm$

As explained in figure 1 that the maximum motor operating current is when the motor current is at half the stall current. The output torque equation from figure 4 will be used to solve for the maximum motor current.

Stall current for 240rpm motor:

$$T_{stall\_240} := I_{stall} \cdot K_T \cdot r_{g\_240} \cdot \eta_{g\_240} - T_{0\_240}$$

$$T_{stall\_240} = 1.066\ N \cdot m$$

The maximum motor current for 240rpm motor:

$$I_{max\_240} := \frac{\left(\frac{T_{stall\_240}}{2}\right) + T_{0\_240}}{K_T \cdot r_{g\_240} \cdot \eta_{g\_240}}$$

$$I_{max\_240} = 2.976\ A$$

Armature resistance for 240rpm motor:

$$R_{a\_240} := \left(1.61 + 0.15 \cdot \left(\frac{I_{max\_240}}{1\ A} - 1.2\right)\right)\ \Omega$$

$$R_{a\_240} = 1.876\ \Omega$$

The maximum output torque for 240rpm motor:

$$T_{max\_240} := I_{max\_240} \cdot K_T \cdot r_{g\_240} \cdot \eta_{g\_240} - T_{0\_240}$$

$$T_{max\_240} = 0.533\ N \cdot m$$

Overload time to trip for 240rpm motor:

$$t_{trip\_240} := 140 \cdot \left(\frac{I_{max\_240}}{1 \cdot A}\right)^{-3.54} \cdot s$$

$$t_{trip\_240} = 2.948\ s$$

Output speed for 240rpm motor:

$$w_{m\_240} := \frac{V_{supply} - I_{max\_240} \cdot R_{a\_240}}{K_T \cdot r_{g\_240}}$$

$$w_{m\_240} = 26.94\ \frac{rad}{s}$$

$$rpm\_motor := 1$$

The results for the different motors can be summarised in the below table.

$Compare$	$Maximum\_current$ (A)	$Trip\_time$ (s)	$Output\_torque$ $(N \cdot m)$	$Output\_speed$ $\left(\frac{rad}{s}\right)$
100 rpm_motor	2.936	3.091	1.211	11.367
160 rpm_motor	2.961	3.001	0.772	18.045
240 rpm_motor	2.976	2.948	0.533	26.94

Thus, for this design the **100rpm motor** will be used because it has the largest output torque and longest trip time.

Largest output torque allows one to design the gearbox with a low gear ratio.

Longest trip time gives the design more time to achieve the goal before it stops working.

Appendix C: Gear calculations

In **Appendix B** the 100 rpm motor was selected. Its calculated properties are listed below:

The maximum motor current:

$$I_{max} := 2.936\ A$$

The maximum output torque:

$$T_{max} := 1.211\ N \cdot m$$

The maximum output speed:

$$w_{max} := 11.367\ \frac{rad}{s}$$

The maximum overload time to trip:

$$t_{max\_trip} := 3.091\ s$$

In **Appendix A** the required bending moment to bend the pipe was determined to be the following:

Bending moment required to bend the pipe:

$$M_{required} := 12.922\ N \cdot m$$

Before selecting the gears, the required gear ratio needs to be determined. The required gear ratio will be determined using the required bending moment and the peak output torque of the motor.

Required gear ratio:

$$GR_{required} := \frac{M_{required}}{T_{max}}$$

$$GR_{required} = 10.671$$

Therefore, the gear train ratio has to be equal to the required gear ratio taking into account all the efficiencies.

Through trial and error, and taking into account the shape of the model and the user requirements. Below is how the gear train is designed.

Gears	Driven by	Drives	On the same shaft as
12T spur gear	The motor	36T spur gear	-
36T spur gear	12T spur gear	-	24T bevel gear
24T bevel gear	-	24T bevel gear	36T spur gear
24T bevel gear	24T bevel gear	-	12T spur gear
12T spur gear	-	36T spur gear	24T bevel gear
36T spur gear	12T spur gear	-	24T spur gear
24T spur gear	-	48T spur gear	36T spur gear
48T spur gear	24T spur gear	-	-

Figure 2: Table summarising the gearbox design.

In the table above each gear is given its own colour to make it easy for the reader to follow. The table also shows which gears are driven, which gears drives and which gears are on the same shaft.

Using figure 1 and 2, the meshing gears can be shown in the table below.

	Meshing gears
Mesh 1	12T spur gear with 36T spur gear
Mesh 2	24T bevel gear with 24T bevel gear
Mesh 3	12T spur gear with 36T spur gear
Mesh 4	24T spur gear with 48T spur gear
Mesh 5	48T spur gear with 48T spur gear

Figure 3: Table showing meshing gears

Using the above table, the gear train efficiency can be calculated

$$\eta_{spur} = 1 - \mu\pi\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

The above equation from the gearbox efficiency document will be used to calculate the efficiency between two meshing spur gears.

The same equation will be used to calculate the efficiency for bevel gears because the friction on bevel gears is very similar to spur gears.

Assuming that the gear that will be used is made up of Dry Acetal on Acetal plastic, the coefficient that will be used is 0.2.

Mesh 1 (12T spur gear with 36T spur gear):

$$\eta_{mesh\_1} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{12} + \frac{1}{36}\right)$$

$$\eta_{mesh\_1} = 0.93$$

Mesh 2 (24T bevel gear with 24T bevel gear):

$$\eta_{mesh\_2} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{24} + \frac{1}{24}\right)$$

$$\eta_{mesh\_2} = 0.948$$

Mesh 3 (12T spur gear with 36T spur gear):

$$\eta_{mesh\_3} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{12} + \frac{1}{36}\right)$$

$$\eta_{mesh\_3} = 0.93$$

Mesh 4 (24T spur gear with 48T spur gear):

$$\eta_{mesh\_4} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{24} + \frac{1}{48}\right)$$

$$\eta_{mesh\_4} = 0.961$$

Mesh 5 (48T spur gear with 48T spur gear):

$$\eta_{mesh\_5} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{48} + \frac{1}{48}\right)$$

$$\eta_{mesh\_5} = 0.974$$

Now that the efficiency for all the meshing gears is determined, the gear train efficiency can be determined.

Gear train efficiency:

$$\eta_{gear\_train} := \eta_{mesh\_1} \cdot \eta_{mesh\_2} \cdot \eta_{mesh\_3} \cdot \eta_{mesh\_4} \cdot \eta_{mesh\_5}$$

$$\eta_{gear\_train} = 0.767$$

The gear ratio of the gearbox can be determined using figure 3, taking into only meshing gears.

Gear 1 (12T spur gear with 36T spur gear):

$$GR_1 := \frac{36}{12}$$

$$GR_1 = 3$$

Gear 2 (24T bevel gear with 24T bevel gear):

$$GR_2 := \frac{24}{24}$$

$$GR_2 = 1$$

Gear 3 (12T spur gear with 36T spur gear):

$$GR_3 := \frac{36}{12}$$

$$GR_3 = 3$$

Gear 4 (24T spur gear with 48T spur gear):

$$GR_4 := \frac{48}{24}$$

$$GR_4 = 2$$

Gear 5 (48T spur gear with 48T spur gear):

$$GR_5 := \frac{48}{48}$$

$$GR_5 = 1$$

Now that the gear ratio for all the meshing gears is determined, the gear train ratio can be determined, taking into account the efficiency.

Gear train ratio:

$$GR_{gear\_train} := GR_1 \cdot GR_2 \cdot GR_3 \cdot GR_4 \cdot GR_5 \cdot \eta_{gear\_train}$$

$$GR_{gear\_train} = 13.808$$

The gear train ratio works out perfectly because it greater than the required gear ratio, which is $GR_{required} = 10.671$.

Now the actual current, output torque, output speed and trip time for the motor can be calculated.

Actual output motor torque:

$$T_{actual} := \frac{M_{required}}{GR_{gear\_train}}$$

$$T_{actual} = 0.936\ N \cdot m$$

Output torque $[Nm]$	$T_m = I_m K_T r_g \eta_g - T_0$	I_m = motor current

Figure 4: Motor torque equation

The above equation is from the motor specification document, it will be used to calculate the operating motor current. The motor specification document also gives the values for the constant in the equation.

Gearbox reduction ratio: $r_g := 156.8$

Gearbox efficiency: $\eta_g := 0.7$

Static drag: $T_0 := 110 \cdot N \cdot mm$

Torque constant: $K_T := 4.1\ N \cdot \frac{mm}{A}$

Armature resistance: $R_a := 1.87\ \Omega$

Operating motor current:

$$I_{actual} := \frac{T_{actual} - T_0}{K_T \cdot r_g \cdot \eta_g}$$

$$I_{actual} = 1.835\ A$$

Output speed $[rad/s]$	$\omega_m = \frac{V_b - I_m R_a}{K_T r_g}$	V_b = supply voltage

Figure 5: Motor speed equation

The above equation is from the motor specification document, it will be used to calculate the motor output speed.

Supply voltage to the motor calculated in Appendix B: $V_{supply} := 12.8\ V$

Motor output speed:

$$w_{actual} := \frac{V_{supply} - I_{actual} \cdot R_a}{K_T \cdot r_g}$$

$$w_{actual} = 14.573\ \frac{rad}{s}$$

Overload time to trip: $t_{actual\_trip} := 140 \cdot \left(\frac{I_{actual}}{1 \cdot A}\right) \cdot s$

$t_{actual\_trip} = 16.322\ s$

	Peak condition	Actual condition
Current [A]	2.936	1.847
Output speed [rad/s]	11.367	14.537
Output torque [Nm]	1.211	0.941
Trip time [s]	3.091	15.948

Figure 6: Comparing peak conditions and actual conditions

The table above the shows the difference when the motor is operating at its peak and at its actual point.

Now the torque and forces experienced by each gear can calculated. The efficiencies for all the meshing gears are calculated.

Gear 1

Gear 1 is a spur gear driven by the motor through a shaft.

Number of teeth of gear 1: $N_{gear\_1} := 12$

Diameter of gear 1: $D_{gear\_1} := 12.7\ mm$

Pressure angle of gear 1: $\theta_{gear\_1} := 20\ °$

Speed of gear 1: $w_{gear\_1} := w_{actual}$

$w_{gear\_1} = 14.573\ \frac{rad}{s}$

Torque of gear 1:

$$T_{gear\_1} := T_{actual}$$

$$T_{gear\_1} = 0.936\ N \cdot m$$

Tangential force of gear 1:

$$F_{tan\_gear\_1} := \frac{T_{gear\_1}}{D_{gear\_1} \cdot 0.5}$$

$$F_{tan\_gear\_1} = 147.374\ N$$

Radial force of gear 1:

$$F_{rad\_gear\_1} := F_{tan\_gear\_1} \cdot \tan\left(\theta_{gear\_1}\right)$$

$$F_{rad\_gear\_1} = 53.64\ N$$

Gear 2

Gear 2 is a spur gear that meshes with gear 1. The efficiency is already calculated earlier in this section.

Number of teeth of gear 2:

$$N_{gear\_2} := 36$$

Diameter of gear 2:

$$D_{gear\_2} := 38.1\ mm$$

Pressure angle of gear 2:

$$\theta_{gear\_2} := 20\ °$$

Speed of gear 2:

$$w_{gear\_2} := \frac{N_{gear\_1}}{N_{gear\_2}} \cdot w_{gear\_1} \cdot \eta_{mesh\_1}$$

$$w_{gear\_2} = 4.518\ \frac{rad}{s}$$

Torque of gear 2:

$$T_{gear\_2} := \frac{N_{gear\_2}}{N_{gear\_1}} \cdot T_{gear\_1} \cdot \eta_{mesh\_1}$$

$$T_{gear\_2} = 2.611 \ N \cdot m$$

Tangential force of gear 2:

$$F_{tan\_gear\_2} := \frac{T_{gear\_2}}{D_{gear\_2} \cdot 0.5}$$

$$F_{tan\_gear\_2} = 137.085 \ N$$

Radial force of gear 2:

$$F_{rad\_gear\_2} := F_{tan\_gear\_2} \cdot \tan\left(\theta_{gear\_2}\right)$$

$$F_{rad\_gear\_2} = 49.895 \ N$$

Gear 3

Gear 3 is a bevel gear and is on the shaft as gear 2.

Number of teeth of gear 3: $N_{gear\_3} := 24$

Diameter of gear 3: $D_{gear\_3} := 25.4 \ mm$

Pressure angle of gear 3: $\theta_{gear\_3} := 20 \ °$

Speed of gear 3:

$$w_{gear\_3} := w_{gear\_2}$$

$$w_{gear\_3} = 4.518 \ \frac{rad}{s}$$

Torque of gear 3: $T_{gear\_3} := T_{gear\_2}$

$$T_{gear\_3} = 2.611\ N \cdot m$$

Tangential force of gear 3:

$$F_{tan\_gear\_3} := \frac{T_{gear\_3}}{D_{gear\_3} \cdot 0.5}$$

$$F_{tan\_gear\_3} = 205.627\ N$$

Radial force of gear 3:

$$F_{rad\_gear\_3} := F_{tan\_gear\_3} \cdot \tan\left(\theta_{gear\_1}\right)$$

$$F_{rad\_gear\_3} = 74.842\ N$$

Gear 4

Gear 4 is a bevel gear that meshes with gear 3. The efficiency is already calculated earlier in this section.

Number of teeth of gear 4: $N_{gear\_4} := 24$

Diameter of gear 4: $D_{gear\_4} := 25.4\ mm$

Pressure angle of gear 4: $\theta_{gear\_4} := 20\ °$

Speed of gear 4:

$$w_{gear\_4} := \frac{N_{gear\_3}}{N_{gear\_4}} \cdot w_{gear\_3} \cdot \eta_{mesh\_2}$$

$$w_{gear\_4} = 4.282\ \frac{rad}{s}$$

Torque of gear 4:

$$T_{gear\_4} := \frac{N_{gear\_4}}{N_{gear\_3}} \cdot T_{gear\_3} \cdot \eta_{mesh\_2}$$

$$T_{gear\_4} = 2.475\ N \cdot m$$

Tangential force of gear 4:

$$F_{tan\_gear\_4} := \frac{T_{gear\_4}}{D_{gear\_4} \cdot 0.5}$$

$$F_{tan\_gear\_4} = 194.861\ N$$

Radial force of gear 4:

$$F_{rad\_gear\_4} := F_{tan\_gear\_4} \cdot \tan\left(\theta_{gear\_4}\right)$$

$$F_{rad\_gear\_4} = 70.924\ N$$

Gear 5

Gear 5 is a spur gear and it's on the shaft as gear 4.

Number of teeth of gear 5: $N_{gear\_5} := 12$

Diameter of gear 5: $D_{gear\_5} := 12.7\ mm$

Pressure angle of gear 5: $\theta_{gear\_5} := 20\ °$

Speed of gear 5:

$$w_{gear\_5} := w_{gear\_4}$$

$$w_{gear\_5} = 4.282\ \frac{rad}{s}$$

Torque of gear 5:

$$T_{gear\_5} := T_{gear\_4}$$

$$T_{gear\_5} = 2.475\ N \cdot m$$

Tangential force of gear 5:

$$F_{tan\_gear\_5} := \frac{T_{gear\_5}}{D_{gear\_5} \cdot 0.5}$$

$$F_{tan\_gear\_5} = 389.722\ N$$

Radial force of gear 5:

$$F_{rad\_gear\_5} := F_{tan\_gear\_5} \cdot \tan\left(\theta_{gear\_5}\right)$$

$$F_{rad\_gear\_5} = 141.847\ N$$

Gear 6

Gear 6 is a spur gear that meshes with gear 5. The efficiency is already calculated earlier in this section.

Number of teeth of gear 6:

$$N_{gear\_6} := 36$$

Diameter of gear 6:

$$D_{gear\_6} := 38.1\ mm$$

Pressure angle of gear 6:

$$\theta_{gear\_6} := 20\ °$$

Speed of gear 6:

$$w_{gear\_6} := \frac{N_{gear\_5}}{N_{gear\_6}} \cdot w_{gear\_5} \cdot \eta_{mesh\_3}$$

$$w_{gear\_6} = 1.328\ \frac{rad}{s}$$

Torque of gear 6:

$$T_{gear\_6} := \frac{N_{gear\_6}}{N_{gear\_5}} \cdot T_{gear\_5} \cdot \eta_{mesh\_3}$$

$$T_{gear\_6} = 6.906 \; N \cdot m$$

Tangential force of gear 6:

$$F_{tan\_gear\_6} := \frac{T_{gear\_6}}{D_{gear\_6} \cdot 0.5}$$

$$F_{tan\_gear\_6} = 362.514 \; N$$

Radial force of gear 6:

$$F_{rad\_gear\_6} := F_{tan\_gear\_6} \cdot \tan\left(\theta_{gear\_6}\right)$$

$$F_{rad\_gear\_6} = 131.944 \; N$$

Gear 7

Gear 7 is a spur gear and it's on the shaft as gear 6.

Number of teeth of gear 7: $N_{gear\_7} := 24$

Diameter of gear 7: $D_{gear\_7} := 25.4 \; mm$

Pressure angle of gear 7: $\theta_{gear\_7} := 20 \; °$

Speed of gear 7:

$$w_{gear\_7} := w_{gear\_6}$$

$$w_{gear\_7} = 1.328 \; \frac{rad}{s}$$

Torque of gear 7:

$$T_{gear\_7} := T_{gear\_6}$$

$T_{gear\_7} = 6.906\ N \cdot m$

Tangential force of gear 7:

$$F_{tan\_gear\_7} := \frac{T_{gear\_7}}{D_{gear\_7} \cdot 0.5}$$

$F_{tan\_gear\_7} = 543.771\ N$

Radial force of gear 7:

$$F_{rad\_gear\_7} := F_{tan\_gear\_7} \cdot \tan\left(\theta_{gear\_7}\right)$$

$F_{rad\_gear\_7} = 197.916\ N$

Gear 8

Gear 8 is a spur gear that meshes with gear 7. The efficiency is already calculated earlier in this section.

Number of teeth of gear 8: $N_{gear\_8} := 48$

Diameter of gear 8: $D_{gear\_8} := 50.8\ mm$

Pressure angle of gear 8: $\theta_{gear\_8} := 20\ °$

Speed of gear 8:

$$w_{gear\_8} := \frac{N_{gear\_7}}{N_{gear\_8}} \cdot w_{gear\_7} \cdot \eta_{mesh\_4}$$

$$w_{gear\_8} = 0.638\ \frac{rad}{s}$$

Torque of gear 8:

$$T_{gear\_8} := \frac{N_{gear\_8}}{N_{gear\_7}} \cdot T_{gear\_7} \cdot \eta_{mesh\_4}$$

$$T_{gear\_8} = 13.269\ N \cdot m$$

Tangential force of gear 8:

$$F_{tan\_gear\_8} := \frac{T_{gear\_8}}{D_{gear\_8} \cdot 0.5}$$

$$F_{tan\_gear\_8} = 522.417\ N$$

Radial force of gear 8:

$$F_{rad\_gear\_8} := F_{tan\_gear\_8} \cdot \tan\left(\theta_{gear\_8}\right)$$

$$F_{rad\_gear\_8} = 190.144\ N$$

Gear 9

Gear 9 is a spur gear that meshes with gear 8. The efficiency is already calculated earlier in this section.

Number of teeth of gear 8:

$$N_{gear\_9} := 48$$

Diameter of gear 8:

$$D_{gear\_9} := 50.8\ mm$$

Pressure angle of gear 8:

$$\theta_{gear\_9} := 20\ °$$

Speed of gear 8:

$$w_{gear\_9} := \frac{N_{gear\_8}}{N_{gear\_9}} \cdot w_{gear\_8} \cdot \eta_{mesh\_5}$$

$$w_{gear\_9} = 0.621\ \frac{rad}{s}$$

Torque of gear 8:

$$T_{gear\_9} := \frac{N_{gear\_9}}{N_{gear\_8}} \cdot T_{gear\_8} \cdot \eta_{mesh\_5}$$

$$T_{gear\_9} = 12.922\ N \cdot m$$

Tangential force of gear 8:

$$F_{tan\_gear\_9} := \frac{T_{gear\_9}}{D_{gear\_9} \cdot 0.5}$$

$$F_{tan\_gear\_9} = 508.74\ N$$

Radial force of gear 8:

$$F_{rad\_gear\_9} := F_{tan\_gear\_9} \cdot \tan\left(\theta_{gear\_9}\right)$$

$$F_{rad\_gear\_9} = 185.166\ N$$

	Maximum Torque [Nm]	Experienced Torque [Nm]	Maximum tangential force [N]	Experienced tangential force [N]	Experienced radial force [N]	Rotational speed [rad/s]
Gear 1 [12T]	3	0.941	450	148.228	53.95	14.537
Gear 2 [36T]	3.4	2.627	700	137.879	50.184	4.508
Gear 3 [24T]	3	2.627	300	206.819	75.276	4.508
Gear 4 [24T]	3	2.489	300	195.99	71.335	4.272
Gear 5 [12T]	3	2.489	450	391.98	142.669	4.272
Gear 6 [36T]	3.4	6.946	700	364	132.709	1.324
Gear 7 [24T]	1	6.946	580	546.923	199.064	1.324
Gear 8 [48T]	1.2	13.269	580	522.417	190.144	0.638
Gear 9 [48T]	1.2	12.922	580	508.74	185.166	0.621

Figure 6: Summary of gear calculations

The above highlights which gears must be support in red, this is because the torque that their experiencing is more than the torque it can withstand.

From the above table the following must be supported:
Gear 6 (36T spur gear)
Gear 7 (24T spur gear)
Gear 8 (48T spur gear)
Gear 9 (48T spur gear)

Appendix D: Deflection calculations

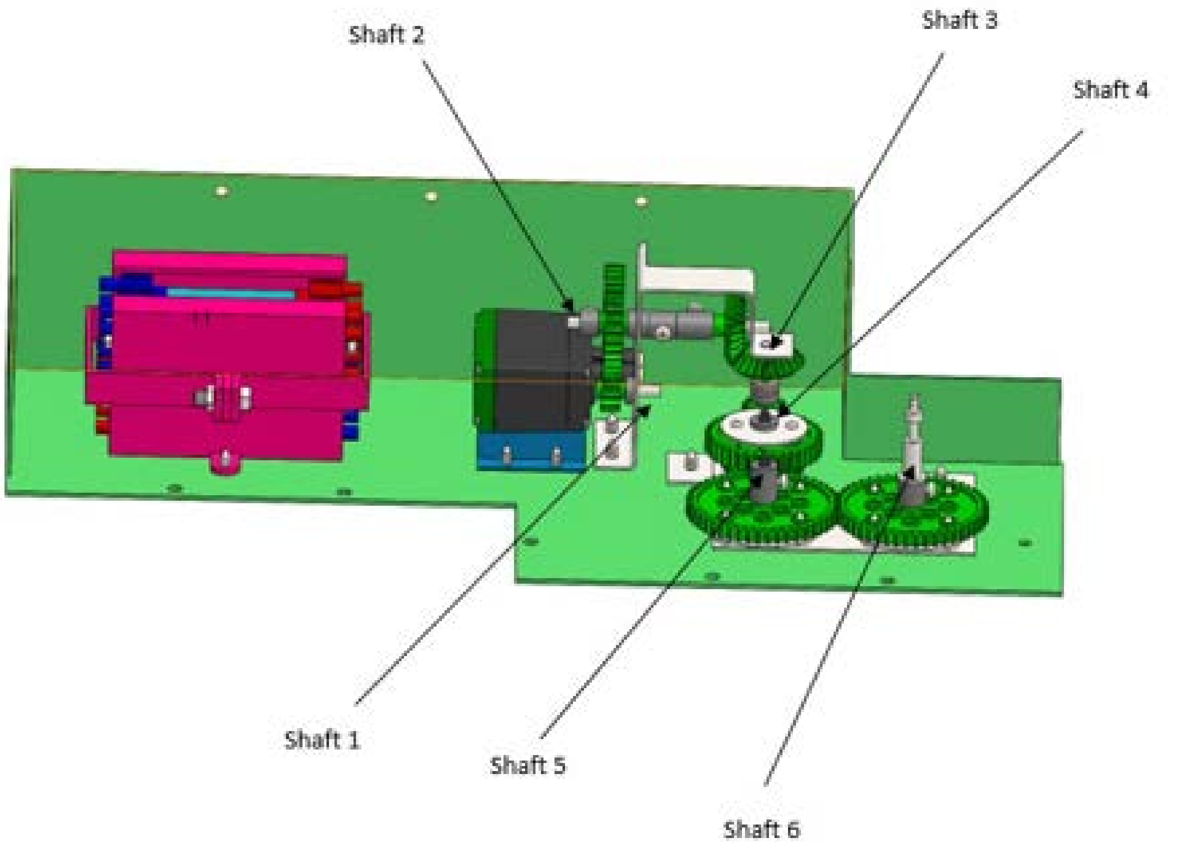


Figure 1: Shafts

Figure 1 above shows all the shafts that will be used in the prototype. It also shows all the the gears on the each shaft.

	Maximum Torque [Nm]	Experienced Torque [Nm]	Maximum tangential force [N]	Experienced tangential force [N]	Experienced radial force [N]	Rotational speed [rad/s]
Gear 1 [12T]	3	0.941	450	148.228	53.95	14.537
Gear 2 [36T]	3.4	2.627	700	137.879	50.184	4.508
Gear 3 [24T]	3	2.627	300	206.819	75.276	4.508
Gear 4 [24T]	3	2.489	300	195.99	71.335	4.272
Gear 5 [12T]	3	2.489	450	391.98	142.669	4.272
Gear 6 [36T]	3.4	6.946	700	364	132.709	1.324
Gear 7 [24T]	1	6.946	580	546.923	199.064	1.324
Gear 8 [48T]	1.2	13.269	580	522.417	190.144	0.638
Gear 9 [48T]	1.2	12.922	580	508.74	185.166	0.621

Figure 1: Summary of the forces

The above table the shows all the forces calculated in **Appendix C** which will be used in this section to determine the deflection of each gear.

$\theta_1 = -\frac{Pb(L^2-b^2)}{6LEI}$ $\theta_2 = +\frac{Pa(L^2-a^2)}{6LEI}$	$v = -\frac{Pa^2b^2}{3LEI}$ at $x = a$	$v = -\frac{Pbx}{6LEI}(L^2-b^2-x^2)$ for $0 \le x \le a$

The above deflection equation from "Mechanic of Materials" 11th edition, written by Russell C. Hibbeler, will be used to determine the deflection of the shaft.

All the shafts are made with low carbon steel and will have the same elastic modulus. All the square shafts have the same cross section and will the same moment of inertia.

Low carbon steel elastic modulus: $E_{shaft} := 200 \cdot GPa$

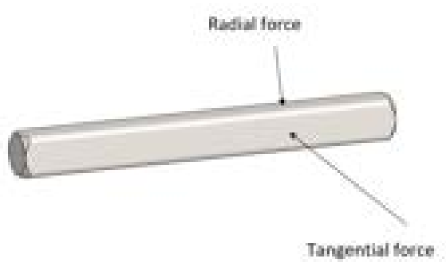
Cross sectional distance: $D_{shaft} := 3.18\ mm$

Moment of inertia:

$$I_{shaft} := \frac{D_{shaft}^{\ 4}}{12}$$

$$I_{shaft} = 8.522\ mm^4$$

Shaft 1



Shaft 1 has 12T spur gear on it. Thus, it will experience both radial and tangential forces.
Shaft 1 has low carbon steel material.

Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally.

Shaft 1 length: $L_{shaft\_1} := 30\ mm$

<u>Deflection due to radial force</u>

Radial force due to gear 1: $F_{R\_1} := 53.95\ N$

The distance from the boundary the radial force: $a_{r1} := 17.27\ mm$

The distance from the other boundary the radial force: $b_{r1} := L_{shaft\_1} - a_{r1}$

$$b_{r1} = 12.73\ mm$$

Maximum deflection caused by the radial force:

$$\delta_{r1} := \frac{F_{R\_1} \cdot b_{r1} \cdot a_{r1}}{6 \cdot L_{shaft\_1} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_1}{}^{2} - b_{r1}{}^{2} - a_{r1}{}^{2}\right)$$

$$\delta_{r1} = 0.017\ mm$$

<u>Deflection due to tangential force</u>

Radial force due to gear 1: $F_{T\_1} := 148.228\ N$

The distance from the boundary the radial force: $a_{t1} := 17.27\ mm$

The distance from the other boundary the radial force: $b_{t1} := L_{shaft\_1} - a_{r1}$

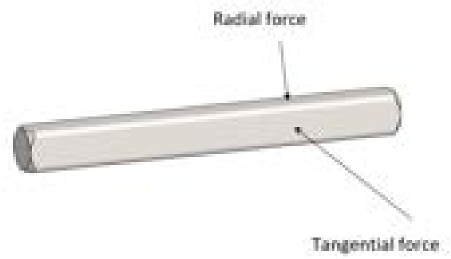
$$b_{t1} = 12.73\ mm$$

Maximum deflection caused by the radial force:

$$\delta_{t1} := \frac{F_{T\_1} \cdot b_{t1} \cdot a_{t1}}{6 \cdot L_{shaft\_1} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_1}{}^{2} - b_{t1}{}^{2} - a_{t1}{}^{2}\right)$$

$$\delta_{t1} = 0.047\ mm$$

Shaft 2



Shaft 2 has 36T spur gear and 24T bevel gear on it. Thus, it will experience both radial and tangential forces.

Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally.

Shaft 2 length: $L_{shaft\_2} := 70\ mm$

Shaft 2 will experience radial forces caused by the 36T spur gear and 24T bevel gear. The deflection of interest will be at the two points at where the gears are meshing.

First meshing distance from the left: $b_{2\_1l1} := 16\ mm$

First meshing distance from the right: $a_{2\_1r1} := L_{shaft\_2} - b_{2\_1l1}$

$a_{2\_1r1} = 54\ mm$

Second meshing distance from the left: $a_{2\_2l1} := 58\ mm$

Second meshing distance from the right: $b_{2\_2r2} := L_{shaft\_2} - a_{2\_2l1}$

$b_{2\_2r2} = 12\ mm$

Deflection due to radial force

Radial force due to 36T spur gear: $F_{R\_2\_36T} := 50.184\ N$

Radial force due to 24T bevel gear: $F_{R\_2\_24T} := 75.276\ N$

For 36T gear

Deflection caused by the radial force of gear 2 at the mesh 2:

$$\delta_{r2\_36T} := \frac{F_{R\_2\_36T} \cdot b_{2\_1l1} \cdot a_{2\_1r1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_1l1}{}^2 - a_{2\_1r1}{}^2\right)$$

$$\delta_{r2\_36T} = 0.105\ mm$$

Deflection caused by the radial force of gear 2 at the mesh 3:

$$\delta_{r3\_36T} := \frac{F_{R\_2\_36T} \cdot b_{2\_2r2} \cdot b_{2\_1l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_2r2}{}^2 - b_{2\_1l1}{}^2\right)$$

$$\delta_{r3\_36T} = 0.061\ mm$$

For 24T gear

Deflection caused by the radial force of gear 3 at the mesh 2:

$$\delta_{r2\_24T} := \frac{F_{R\_2\_24T} \cdot b_{2\_2r2} \cdot b_{2\_1l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_2r2}{}^2 - b_{2\_1l1}{}^2\right)$$

$$\delta_{r2\_24T} = 0.091\ mm$$

Deflection caused by the radial force of gear 3 at the mesh 3:

$$\delta_{r3\_24T} := \frac{F_{R\_2\_24T} \cdot b_{2\_2r2} \cdot a_{2\_2l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_2r2}{}^2 - a_{2\_2l1}{}^2\right)$$

$$\delta_{r3\_24T} = 0.102\ mm$$

Maximum deflection at mesh 2 due to radial forces:

$$\delta_{r2\_max} := \delta_{r2\_36T} + \delta_{r2\_24T}$$

$$\delta_{r2\_max} = 0.196\ mm$$

Maximum deflection at mesh 3 due to radial forces:

$$\delta_{r3\_max} := \delta_{r3\_36T} + \delta_{r3\_24T}$$

$$\delta_{r3\_max} = 0.162\ mm$$

Deflection due to tangential force

Tangential force due to 36T spur gear:

$$F_{T\_2\_36T} := 137.879\ N$$

Tangential force due to 24T bevel gear:

$$F_{T\_2\_24T} := 206.819\ N$$

For 36T gear

Deflection caused by the tangential force of gear 2 at the mesh 2:

$$\delta_{t2\_36T} := \frac{F_{T\_2\_36T} \cdot b_{2\_1l1} \cdot a_{2\_1r1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_1l1}{}^2 - a_{2\_1r1}{}^2\right)$$

$$\delta_{t2\_36T} = 0.288\ mm$$

Deflection caused by the tangential force of gear 2 at the mesh 3:

$$\delta_{t3\_36T} := \frac{F_{T\_2\_36T} \cdot b_{2\_2r2} \cdot b_{2\_1l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_2r2}{}^2 - b_{2\_1l1}{}^2\right)$$

$$\delta_{t3\_36T} = 0.166\ mm$$

For 24T gear

Deflection caused by the tangential force of gear 3 at the mesh 2:

$$\delta_{t2\_24T} := \frac{F_{T\_2\_24T} \cdot b_{2\_2r2} \cdot b_{2\_1l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_2r2}{}^2 - b_{2\_1l1}{}^2\right)$$

$$\delta_{t2\_24T} = 0.25\ mm$$

Deflection caused by the tangential force of gear 3 at the mesh 3:

$$\delta_{t3\_24T} := \frac{F_{T\_2\_24T} \cdot b_{2\_2r2} \cdot a_{2\_2l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}{}^2 - b_{2\_2r2}{}^2 - a_{2\_2l1}{}^2\right)$$

$$\delta_{t3\_24T} = 0.28\ mm$$

Maximum deflection at mesh 2 due to tangential forces:

$$\delta_{t2\_max} := \delta_{t2\_36T} + \delta_{t2\_24T}$$

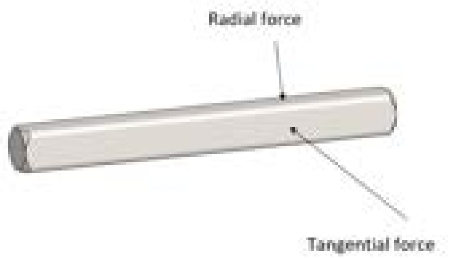
$$\delta_{t2\_max} = 0.537\ mm$$

Maximum deflection at mesh 3 due to tangential forces:

$$\delta_{t3\_max} := \delta_{t3\_36T} + \delta_{t3\_24T}$$

$$\delta_{t3\_max} = 0.446\ mm$$

Shaft 3



Shaft 3 has 24T spur gear and 12T bevel gear on it. Thus, it will experience both radial and tangential forces.

Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally.

Shaft 3 length: $L_{shaft\_3} := 45\ mm$

Shaft 3 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing.

First meshing distance from the left: $b_{3\_1l1} := 8\ mm$

First meshing distance from the right: $a_{3\_1r1} := L_{shaft\_3} - b_{3\_1l1}$

$a_{3\_1r1} = 37\ mm$

Second meshing distance from the left: $a_{3\_2l1} := 26\ mm$

Second meshing distance from the right: $b_{3\_2r2} := L_{shaft\_3} - a_{3\_2l1}$

$b_{3\_2r2} = 19\ mm$

Deflection due to radial force

Radial force due to 24T bevel gear: $F_{R\_3\_24T} := 71.335\ N$

Radial force due to 12T spur gear: $F_{R\_3\_12T} := 142.669\ N$

For 24T bevel gear

Deflection caused by the radial force of gear 2 at the mesh 3:

$$\delta_{m3\_r3\_24T} := \frac{F_{R\_3\_24T} \cdot b_{3\_1l1} \cdot a_{3\_1r1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_3}^{\ 2} - b_{3\_1l1}^{\ 2} - a_{3\_1r1}^{\ 2}\right)$$

$$\delta_{m3\_r3\_24T} = 0.027\ mm$$

Deflection caused by the radial force of gear 2 at the mesh 4:

$$\delta_{m4\_r3\_24T} := \frac{F_{R\_3\_24T} \cdot b_{3\_2r2} \cdot b_{3\_1l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_3}^{\ 2} - b_{3\_2r2}^{\ 2} - b_{3\_1l1}^{\ 2}\right)$$

$$\delta_{m4\_r3\_24T} = 0.038\ mm$$

For 12T spur gear

Deflection caused by the radial force of gear 3 at the mesh 3:

$$\delta_{m3\_r3\_12T} := \frac{F_{R\_3\_12T} \cdot b_{3\_2r2} \cdot b_{3\_1l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_3}^{\ 2} - b_{3\_2r2}^{\ 2} - b_{3\_1l1}^{\ 2}\right)$$

$$\delta_{m3\_r3\_12T} = 0.075\ mm$$

Deflection caused by the radial force of gear 4 at the mesh 4:

$$\delta_{m4\_r3\_12T} := \frac{F_{R\_3\_12T} \cdot b_{3\_2r2} \cdot a_{3\_2l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_3}{}^2 - b_{3\_2r2}{}^2 - a_{3\_2l1}{}^2\right)$$

$$\delta_{m4\_r3\_12T} = 0.151\ mm$$

Maximum deflection at mesh 3 due to radial forces:

$$\delta_{m3\_r3\_max} := \delta_{m3\_r3\_24T} + \delta_{m3\_r3\_12T}$$

$$\delta_{m3\_r3\_max} = 0.103\ mm$$

Maximum deflection at mesh 4 due to radial forces:

$$\delta_{m4\_r3\_max} := \delta_{m4\_r3\_24T} + \delta_{m4\_r3\_12T}$$

$$\delta_{m4\_r3\_max} = 0.189\ mm$$

Deflection due to tangential force

Tangential force due to 36T spur gear:

$$F_{T\_3\_24T} := 195.99\ N$$

Tangential force due to 24T bevel gear:

$$F_{T\_3\_12T} := 391.98\ N$$

For 24T bevel gear

Deflection caused by the tangential force of gear 2 at the mesh 3:

$$\delta_{m3\_t3\_24T} := \frac{F_{T\_3\_24T} \cdot b_{3\_1l1} \cdot a_{3\_1r1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_3}{}^2 - b_{3\_1l1}{}^2 - a_{3\_1r1}{}^2\right)$$

$$\delta_{m3\_t3\_24T} = 0.075\ mm$$

Deflection caused by the tangential force of gear 2 at the mesh 4:

$$\delta_{m4\_t3\_24T} := \frac{F_{T\_3\_24T} \cdot b_{3\_2r2} \cdot b_{3\_1l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_3}}^2 - {b_{3\_2r2}}^2 - {b_{3\_1l1}}^2 \right)$$

$$\delta_{m4\_t3\_24T} = 0.104 \; mm$$

For 12T spur gear

Deflection caused by the tangential force of gear 3 at the mesh 3:

$$\delta_{m3\_t3\_12T} := \frac{F_{T\_3\_12T} \cdot b_{3\_2r2} \cdot b_{3\_1l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_3}}^2 - {b_{3\_2r2}}^2 - {b_{3\_1l1}}^2 \right)$$

$$\delta_{m3\_t3\_12T} = 0.207 \; mm$$

Deflection caused by the tangential force of gear 3 at the mesh 4:

$$\delta_{m4\_t3\_12T} := \frac{F_{T\_3\_12T} \cdot b_{3\_2r2} \cdot a_{3\_2l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_3}}^2 - {b_{3\_2r2}}^2 - {a_{3\_2l1}}^2 \right)$$

$$\delta_{m4\_t3\_12T} = 0.416 \; mm$$

Maximum deflection at mesh 3 due to tangential forces:

$$\delta_{m3\_t3\_max} := \delta_{m3\_t3\_24T} + \delta_{m3\_t3\_12T}$$

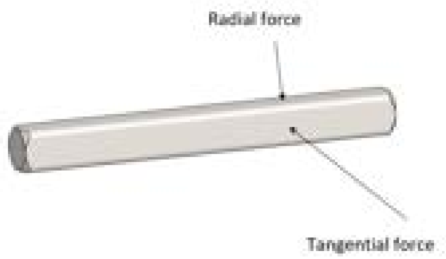
$$\delta_{m3\_t3\_max} = 0.282 \; mm$$

Maximum deflection at mesh 4 due to tangential forces:

$$\delta_{m4\_t3\_max} := \delta_{m4\_t3\_24T} + \delta_{m4\_t3\_12T}$$

$$\delta_{m4\_t3\_max} = 0.519 \; mm$$

Shaft 4



Shaft 4 has 36T spur gear and 24T bevel gear on it. Thus, it will experience both radial and tangential forces.

Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally.

Shaft 4 length: $L_{shaft\_4} := 30\ mm$

Shaft 3 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing.

First meshing distance from the left: $b_{4\_1l1} := 11\ mm$

First meshing distance from the right: $a_{4\_1r1} := L_{shaft\_4} - b_{4\_1l1}$

$a_{4\_1r1} = 19\ mm$

Second meshing distance from the left: $a_{4\_2l1} := 19\ mm$

Second meshing distance from the right: $b_{4\_2r2} := L_{shaft\_4} - a_{4\_2l1}$

$b_{4\_2r2} = 11\ mm$

Deflection due to radial force

Radial force due to 36T spur gear:

$F_{R\_4\_36T} := 132.709\ N$

Radial force due to 24T spur gear:

$F_{R\_4\_24T} := 199.064\ N$

For 36T spur gear

Deflection caused by the radial force of gear 2 at the mesh 4:

$$\delta_{m4\_r4\_36T} := \frac{F_{R\_4\_36T} \cdot b_{4\_1l1} \cdot a_{4\_1r1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}{}^2 - b_{4\_1l1}{}^2 - a_{4\_1r1}{}^2\right)$$

$\delta_{m4\_r4\_36T} = 0.038\ mm$

Deflection caused by the radial force of gear 2 at the mesh 5:

$$\delta_{m5\_r4\_36T} := \frac{F_{R\_4\_36T} \cdot b_{4\_2r2} \cdot b_{4\_1l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}{}^2 - b_{4\_2r2}{}^2 - b_{4\_1l1}{}^2\right)$$

$\delta_{m5\_r4\_36T} = 0.034\ mm$

For 24T spur gear

Deflection caused by the radial force of gear 3 at the mesh 4:

$$\delta_{m4\_r4\_24T} := \frac{F_{R\_4\_24T} \cdot b_{4\_2r2} \cdot b_{4\_1l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}{}^2 - b_{4\_2r2}{}^2 - b_{4\_1l1}{}^2\right)$$

$\delta_{m4\_r4\_24T} = 0.052\ mm$

Deflection caused by the radial force of gear 4 at the mesh 5:

$$\delta_{m5\_r4\_24T} := \frac{F_{R\_4\_24T} \cdot b_{4\_2r2} \cdot a_{4\_2l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}{}^{2} - b_{4\_2r2}{}^{2} - a_{4\_2l1}{}^{2}\right)$$

$$\delta_{m5\_r4\_24T} = 0.057\ mm$$

Maximum deflection at mesh 4 due to radial forces:

$$\delta_{m4\_r4\_max} := \delta_{m4\_r4\_36T} + \delta_{m4\_r4\_24T}$$

$$\delta_{m4\_r4\_max} = 0.089\ mm$$

Maximum deflection at mesh 5 due to radial forces:

$$\delta_{m5\_r4\_max} := \delta_{m5\_r4\_36T} + \delta_{m5\_r4\_24T}$$

$$\delta_{m4\_r4\_max} = 0.089\ mm$$

Deflection due to tangential force

Tangential force due to 36T spur gear:

$$F_{T\_4\_36T} := 364\ N$$

Tangential force due to 24T bevel gear:

$$F_{T\_4\_24T} := 546.923\ N$$

For 36T spur gear

Deflection caused by the tangential force of gear 2 at the mesh 4:

$$\delta_{m4\_t4\_36T} := \frac{F_{T\_4\_36T} \cdot b_{4\_1l1} \cdot a_{4\_1r1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}{}^{2} - b_{4\_1l1}{}^{2} - a_{4\_1r1}{}^{2}\right)$$

$$\delta_{m4\_t4\_36T} = 0.104\ mm$$

Deflection caused by the tangential force of gear 2 at the mesh 5:

$$\delta_{m5\_t4\_36T} := \frac{F_{T\_4\_36T} \cdot b_{4\_2r2} \cdot b_{4\_1l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_4}}^2 - {b_{4\_2r2}}^2 - {b_{4\_1l1}}^2 \right)$$

$$\delta_{m5\_t4\_36T} = 0.094\ mm$$

For 24T spur gear

Deflection caused by the tangential force of gear 3 at the mesh 4:

$$\delta_{m4\_t4\_24T} := \frac{F_{T\_4\_24T} \cdot b_{4\_2r2} \cdot b_{4\_1l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_4}}^2 - {b_{4\_2r2}}^2 - {b_{4\_1l1}}^2 \right)$$

$$\delta_{m4\_t4\_24T} = 0.142\ mm$$

Deflection caused by the tangential force of gear 3 at the mesh 5:

$$\delta_{m5\_t4\_24T} := \frac{F_{T\_4\_24T} \cdot b_{4\_2r2} \cdot a_{4\_2l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_4}}^2 - {b_{4\_2r2}}^2 - {a_{4\_2l1}}^2 \right)$$

$$\delta_{m5\_t4\_24T} = 0.156\ mm$$

Maximum deflection at mesh 4 due to tangential forces:

$$\delta_{m4\_t4\_max} := \delta_{m4\_t4\_36T} + \delta_{m4\_t4\_24T}$$

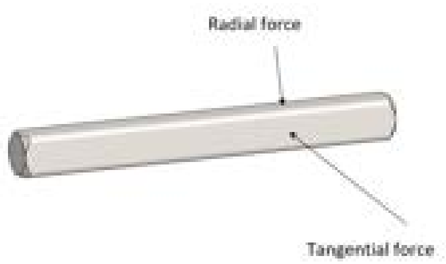
$$\delta_{m4\_t4\_max} = 0.246\ mm$$

Maximum deflection at mesh 5 due to tangential forces:

$$\delta_{m5\_t4\_max} := \delta_{m5\_t4\_36T} + \delta_{m5\_t4\_24T}$$

$$\delta_{m4\_t4\_max} = 0.246\ mm$$

Shaft 5



Shaft 5 has 48T spur gear on it. Thus, it will experience both radial and tangential forces.

Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally.

Shaft 5 length: $L_{shaft\_5} := 30\ mm$

Shaft 5 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing.

First meshing distance from the left: $b_{5\_1l1} := 18.57\ mm$

First meshing distance from the right: $a_{5\_1r1} := L_{shaft\_5} - b_{5\_1l1}$

$a_{5\_1r1} = 11.43\ mm$

Deflection due to radial force

Radial force due to 48T spur gear:

$$F_{R\_5\_48T} := 190.144\ N$$

Deflection caused by the radial force of gear 2 at the mesh :

$$\delta_{m5\_r5\_48T} := \frac{F_{R\_5\_48T} \cdot b_{5\_1l1} \cdot a_{5\_1r1}}{6 \cdot L_{shaft\_5} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_5}^{\ 2} - b_{5\_1l1}^{\ 2} - a_{5\_1r1}^{\ 2}\right)$$

$$\delta_{m5\_r5\_48T} = 0.056\ mm$$

Deflection due to tangential force

Tangential force due to 48T spur gear:

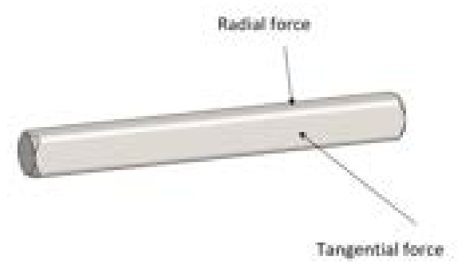
$$F_{T\_5\_48T} := 522.417\ N$$

Deflection caused by the tangential force of gear 2 at the mesh 5:

$$\delta_{m5\_t5\_48T} := \frac{F_{T\_5\_48T} \cdot b_{5\_1l1} \cdot a_{5\_1r1}}{6 \cdot L_{shaft\_5} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_5}^{\ 2} - b_{5\_1l1}^{\ 2} - a_{5\_1r1}^{\ 2}\right)$$

$$\delta_{m5\_t5\_48T} = 0.153\ mm$$

Shaft 6



Shaft 6 has 48T spur gear on it. Thus, it will experience both radial and tangential forces.

Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally.

Shaft 5 length: $L_{shaft\_6} := 54.2\ mm$

Shaft 5 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing.

First meshing distance from the left: $b_{6\_1l1} := 11\ mm$

First meshing distance from the right: $a_{6\_1r1} := L_{shaft\_6} - b_{6\_1l1}$

$a_{6\_1r1} = 43.2\ mm$

Deflection due to radial force

Radial force due to 48T spur gear:

$$F_{R\_6\_48T} := 185.166\ N$$

Deflection caused by the radial force of gear 2 at the mesh 6:

$$\delta_{m6\_r6\_48T} := \frac{F_{R\_6\_48T} \cdot b_{6\_1l1} \cdot a_{6\_1r1}}{6 \cdot L_{shaft\_6} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_6}}^2 - {b_{6\_1l1}}^2 - {a_{6\_1r1}}^2 \right)$$

$$\delta_{m6\_r6\_48T} = 0.151\ mm$$

Deflection due to tangential force

Tangential force due to 48T spur gear:

$$F_{T\_6\_48T} := 508.74\ N$$

Deflection caused by the tangential force of gear 2 at the mesh 5:

$$\delta_{m6\_t6\_48T} := \frac{F_{T\_6\_48T} \cdot b_{6\_1l1} \cdot a_{6\_1r1}}{6 \cdot L_{shaft\_6} \cdot E_{shaft} \cdot I_{shaft}} \left({L_{shaft\_6}}^2 - {b_{6\_1l1}}^2 - {a_{6\_1r1}}^2 \right)$$

$$\delta_{m6\_t6\_48T} = 0.415\ mm$$

Appendix E: Torsion calculations

	Maximum Torque [Nm]	Experienced Torque [Nm]	Maximum tangential force [N]	Experienced tangential force [N]	Experienced radial force [N]	Rotational speed [rad/s]
Gear 1 [12T]	3	0.941	450	148.228	53.95	14.537
Gear 2 [36T]	3.4	2.627	700	137.879	50.184	4.508
Gear 3 [24T]	3	2.627	300	206.819	75.276	4.508
Gear 4 [24T]	3	2.489	300	195.99	71.335	4.272
Gear 5 [12T]	3	2.489	450	391.98	142.669	4.272
Gear 6 [36T]	3.4	6.946	700	364	132.709	1.324
Gear 7 [24T]	1	6.946	580	546.923	199.064	1.324
Gear 8 [48T]	1.2	13.269	580	522.417	190.144	0.638
Gear 9 [48T]	1.2	12.922	580	508.74	185.166	0.621

Figure 1: Table from appendix C

The table above shows the summary torque calculated

Torsional shear stress

- We can state the torsional shear stress formula as:

$$\tau_{max} = \frac{TC}{J}$$

$$J = \frac{\pi D^4}{32} \text{ (for solid bar)}$$

τ = applied Torque
C = radius of the cross section
J = Polar moment of ineratia of the cross section

The above equation from "Mechanic of Materials" 11th edition, written by Russell C. Hibbeler will be used to determine shear strength

Shaft 1

Given yield strength of the shaft:

$$\sigma_{shaft\_1} := 450\ MPa$$

Given maximum torque the shaft can withstand:

$$T_{max\_shaft1} := 5\ N \cdot m$$

Radius of the cross section:

$$c_{shaft\_1} := 1.59\ mm$$

Polar moment of inertia of a square shaft:

$$J_{shaft\_1} := \frac{2 \cdot c_{shaft\_1}{}^{4}}{6}$$

$$J_{shaft\_1} = 2.13\ mm^{4}$$

Maximum shear stress the shaft can withstand:

$$\tau_{max\_shaft1} := \frac{T_{max\_shaft1} \cdot c_{shaft\_1}}{J_{shaft\_1}}$$

$$\tau_{max\_shaft1} = 3.732\ GPa$$

The shear strength that the shaft will experience will be calculated using the torque it is experiencing from table 1

Shaft 1 experienced torque:

$$T_{shaft1\_ex} := 0.941\ N \cdot m$$

Shaft 1 experienced shear stress:

$$\tau_{shaft1\_ex} := \frac{T_{shaft1\_ex} \cdot c_{shaft\_1}}{J_{shaft\_1}}$$

$$\tau_{shaft1\_ex} = 702.295\ MPa$$

Shaft 2

Given yield strength of the shaft:

$$\sigma_{shaft\_2} := 450\ MPa$$

Given maximum torque the shaft can withstand:

$$T_{max\_shaft2} := 5\ N \cdot m$$

Radius of the cross section:

$$c_{shaft\_2} := 1.59\ mm$$

Polar moment of inertia of a square shaft:

$$J_{shaft\_2} := \frac{2 \cdot c_{shaft\_2}{}^{4}}{6}$$

$$J_{shaft\_2} = 2.13\ mm^{4}$$

Maximum shear stress the shaft can withstand:

$$\tau_{max\_shaft2} := \frac{T_{max\_shaft2} \cdot c_{shaft\_2}}{J_{shaft\_2}}$$

$$\tau_{max\_shaft2} = 3.732\ GPa$$

The shear strength that the shaft will experience will be calculated using the torque it is experiencing from table 1

Shaft 2 experienced torque:

$$T_{shaft2\_ex} := 2.627\ N \cdot m$$

Shaft 2 experienced shear stress:

$$\tau_{shaft2\_ex} := \frac{T_{shaft2\_ex} \cdot c_{shaft\_2}}{J_{shaft\_2}}$$

$$\tau_{shaft2\_ex} = 1.961\ GPa$$

Shaft 3

Given yield strength of the shaft:

$$\sigma_{shaft\_3} := 450\ MPa$$

Given maximum torque the shaft can withstand:

$$T_{max\_shaft3} := 5\ N \cdot m$$

Radius of the cross section:

$$c_{shaft\_3} := 1.59\ mm$$

Polar moment of inertia of a square shaft:

$$J_{shaft\_3} := \frac{2 \cdot c_{shaft\_2}{}^{4}}{6}$$

$$J_{shaft\_3} = 2.13\ mm^{4}$$

Maximum shear stress the shaft can withstand:

$$\tau_{max\_shaft3} := \frac{T_{max\_shaft3} \cdot c_{shaft\_3}}{J_{shaft\_3}}$$

$$\tau_{max\_shaft3} = 3.732\ GPa$$

The shear strength that the shaft will experience will be calculated using the torque it is experiencing from table 1

Shaft 3 experienced torque:

$$T_{shaft3\_ex} := 2.489\ N \cdot m$$

Shaft 3 experienced shear stress:

$$\tau_{shaft3\_ex} := \frac{T_{shaft3\_ex} \cdot c_{shaft\_3}}{J_{shaft\_3}}$$

$$\tau_{shaft3\_ex} = 1.858\ GPa$$

Shaft 4

Given yield strength of the shaft:

$$\sigma_{shaft\_4} := 450\ MPa$$

Given maximum torque the shaft can withstand:

$$T_{max\_shaft4} := 13\ N \cdot m$$

Radius of the cross section:

$$c_{shaft\_4} := 1.59\ mm$$

Polar moment of inertia of a square shaft:

$$J_{shaft\_4} := \frac{2 \cdot {c_{shaft\_2}}^4}{6}$$

$$J_{shaft\_4} = 2.13\ mm^4$$

Maximum shear stress the shaft can withstand:

$$\tau_{max\_shaft4} := \frac{T_{max\_shaft4} \cdot c_{shaft\_4}}{J_{shaft\_4}}$$

$$\tau_{max\_shaft4} = 9.702\ GPa$$

The shear strength that the shaft will experience will be calculated using the torque it is experiencing from table 1

Shaft 3 experienced torque:

$$T_{shaft4\_ex} := 6.946\ N \cdot m$$

Shaft 3 experienced shear stress:

$$\tau_{shaft4\_ex} := \frac{T_{shaft4\_ex} \cdot c_{shaft\_4}}{J_{shaft\_4}}$$

$$\tau_{shaft4\_ex} = 5.184\ GPa$$

Shaft 5

Given yield strength of the shaft: $\sigma_{shaft\_5} := 450\ MPa$

Given maximum torque the shaft can withstand: $T_{max\_shaft5} := 15\ N \cdot m$

Radius of the cross section: $c_{shaft\_5} := 1.59\ mm$

Polar moment of inertia of a square shaft:

$$J_{shaft\_5} := \frac{2 \cdot c_{shaft\_2}{}^{4}}{6}$$

$$J_{shaft\_5} = 2.13\ mm^{4}$$

Maximum shear stress the shaft can withstand:

$$\tau_{max\_shaft5} := \frac{T_{max\_shaft5} \cdot c_{shaft\_5}}{J_{shaft\_5}}$$

$$\tau_{max\_shaft5} = 11.195\ GPa$$

The shear strength that the shaft will experience will be calculated using the torque it is experiencing from table 1

Shaft 3 experienced torque: $T_{shaft5\_ex} := 13.269\ N \cdot m$

Shaft 3 experienced shear stress:

$$\tau_{shaft5\_ex} := \frac{T_{shaft5\_ex} \cdot c_{shaft\_5}}{J_{shaft\_5}}$$

$$\tau_{shaft5\_ex} = 9.903\ GPa$$

Shaft 6

Given yield strength of the shaft:

$$\sigma_{shaft\_6} := 450\ MPa$$

Given maximum torque the shaft can withstand:

$$T_{max\_shaft6} := 15\ N \cdot m$$

Radius of the cross section:

$$c_{shaft\_6} := 1.59\ mm$$

Polar moment of inertia of a square shaft:

$$J_{shaft\_6} := \frac{2 \cdot c_{shaft\_6}{}^{4}}{6}$$

$$J_{shaft\_6} = 2.13\ mm^{4}$$

Maximum shear stress the shaft can withstand:

$$\tau_{max\_shaft6} := \frac{T_{max\_shaft6} \cdot c_{shaft\_6}}{J_{shaft\_6}}$$

$$\tau_{max\_shaft6} = 11.195\ GPa$$

The shear strength that the shaft will experience will be calculated using the torque it is experiencing from table 1

Shaft 3 experienced torque:

$$T_{shaft6\_ex} := 12.922\ N \cdot m$$

Shaft 3 experienced shear stress:

$$\tau_{shaft6\_ex} := \frac{T_{shaft6\_ex} \cdot c_{shaft\_6}}{J_{shaft\_6}}$$

$$\tau_{shaft6\_ex} = 9.644\ GPa$$

Appendix F: Strength calculations

	Maximum Torque [Nm]	Experienced Torque [Nm]	Maximum tangential force [N]	Experienced tangential force [N]	Experienced radial force [N]	Rotational speed [rad/s]
Gear 1 [12T]	3	0.941	450	148.228	53.95	14.537
Gear 2 [36T]	3.4	2.627	700	137.879	50.184	4.508
Gear 3 [24T]	3	2.627	300	206.819	75.276	4.508
Gear 4 [24T]	3	2.489	300	195.99	71.335	4.272
Gear 5 [12T]	3	2.489	450	391.98	142.669	4.272
Gear 6 [36T]	3.4	6.946	700	364	132.709	1.324
Gear 7 [24T]	1	6.946	580	546.923	199.064	1.324
Gear 8 [48T]	1.2	13.269	580	522.417	190.144	0.638
Gear 9 [48T]	1.2	12.922	580	508.74	185.166	0.621

Figure 1: Summary of calculations from Appendix C

In the summary calculations from figure 1, it can be seen that Gear 6, 7, 8 and 9 need to be reinforced because the torque they experience is greater that the torque it can withstand.

Gear 6

Gear 6 is 36 teeth spur gear.

Gear 6 yield strength: $S_{y\_gear6} := 66\ MPa$

Gear 6 maximum torque: $T_{max\_gear6} := 3.4\ N \cdot m$

Gear 6 boss thickness: $t_{boss\_gear6} := 9.5\ mm$

The k-factor for gear 6 can be calculated using the above properties.

Gear 6 k-factor:

$$k_{gear6} := \frac{T_{max\_gear6}}{t_{boss\_gear6} \cdot S_{y\_gear6}}$$

$$k_{gear6} = 5.423\ mm^2$$

Now the thickness of the sheet metal plate can be determined. Sheet metal is made with hot rolled mild plate

The sheet metal plate yield strength: $S_{y\_plate1} := 250\ MPa$

Torque experienced by the plate: $T_{plate1} := 6.946\ N \cdot m$

Plate thickness needed by the plate:

$$t_{plate1} := \frac{T_{plate1}}{k_{gear6} \cdot S_{y\_plate1}}$$

$$t_{plate1} = 0.712\ mm$$

Gear 7

Gear 7 is 24 teeth spur gear.

Gear 7 yield strength: $S_{y\_gear7} := 66\ MPa$

Gear 7 maximum torque: $T_{max\_gear7} := 1\ N \cdot m$

Gear 7 boss thickness: $t_{boss\_gear7} := 9.5\ mm$

The k-factor for gear 7 can be calculated using the above properties.

Gear 7 k-factor:

$$k_{gear7} := \frac{T_{max\_gear7}}{t_{boss\_gear7} \cdot S_{y\_gear7}}$$

$$k_{gear7} = 1.595\ mm^2$$

Now the thickness of the sheet metal plate can be determined. Sheet metal is made with hot rolled mild plate

The sheet metal plate yield strength:

$$S_{y\_plate7} := 250\ MPa$$

Torque experienced by the plate:

$$T_{plate7} := 6.946\ N \cdot m$$

Plate thickness needed by the plate:

$$t_{plate7} := \frac{T_{plate7}}{k_{gear7} \cdot S_{y\_plate7}}$$

$$t_{plate7} = 0.958\ mm$$

Gear 8

Gear 8 is 48 teeth spur gear.

Gear 8 yield strength:

$$S_{y\_gear8} := 66\ MPa$$

Gear 8 maximum torque:

$$T_{max\_gear8} := 1\ N \cdot m$$

Gear 8 boss thickness:

$$t_{boss\_gear8} := 9.5\ mm$$

The k-factor for gear 8 can be calculated using the above properties.

Gear 8 k-factor:

$$k_{gear8} := \frac{T_{max\_gear8}}{t_{boss\_gear8} \cdot S_{y\_gear8}}$$

$$k_{gear8} = 1.595\ mm^2$$

Now the thickness of the sheet metal plate can be determined. Sheet metal is made with hot rolled mild plate

The sheet metal plate yield strength:

$$S_{y\_plate8} := 250\ MPa$$

Torque experienced by the plate:

$$T_{plate8} := 13.269\ N \cdot m$$

Plate thickness needed by the plate:

$$t_{plate8} := \frac{T_{plate8}}{k_{gear8} \cdot S_{y\_plate8}}$$

$$t_{plate8} = 1.659\ mm$$

Gear 9

Gear 9 is 48 teeth spur gear.

Gear 9 yield strength:

$$S_{y\_gear9} := 66\ MPa$$

Gear 9 maximum torque:

$$T_{max\_gear9} := 1\ N \cdot m$$

Gear 8 boss thickness: $t_{boss\_gear9} := 9.5\ mm$

The k-factor for gear 8 can be calculated using the above properties.

Gear 8 k-factor:

$$k_{gear9} := \frac{T_{max\_gear9}}{t_{boss\_gear9} \cdot S_{y\_gear9}}$$

$$k_{gear9} = 1.595\ mm^2$$

Now the thickness of the sheet metal plate can be determined. Sheet metal is made with hot rolled mild plate

The sheet metal plate yield strength: $S_{y\_plate9} := 250\ MPa$

Torque experienced by the plate: $T_{plate9} := 12.922\ N \cdot m$

Plate thickness needed by the plate:

$$t_{plate9} := \frac{T_{plate9}}{k_{gear9} \cdot S_{y\_plate9}}$$

$$t_{plate9} = 1.615\ mm$$

	Maximum torque [Nm]	Experienced torque [Nm]	Plate thickness needed [mm]
Gear 6 [36T]	3.4	6.946	
Gear 7 [24T]	1	6.946	
Gear 8 [48T]	1.2	13.269	
Gear 9 [48T]	1.2	12.922	

Figure 2: Summary table of plate thickness need

The above table shows how much thickness is needed for the plates to reinforce the gears that need to be supported.

Base

The base of the pipe bending platform has a lot of parts on it. Thus, it will experience forces due to the weight of the parts. The forces can be converted to stresses.

Big roller

Big roller mass: $M_{big\_roller} := \frac{22.72}{1000} \cdot kg$

Big roller force: $F_{big\_roller} := M_{big\_roller} \cdot \left(9.81 \frac{m}{s^2}\right)$

$F_{big\_roller} = 0.223 \ N$

Big roller outer radius: $OD_{big\_roller} := \frac{35}{2} \ mm$

Big roller inner radius: $ID_{big\_roller} := \frac{8}{2} \ mm$

Big roller area: $A_{big\_roller} := \pi \cdot \left(OD_{big\_roller}{}^{2} - ID_{big\_roller}{}^{2}\right)$

Big roller stress: $\sigma_{big\_roller} := \dfrac{F_{big\_roller}}{A_{big\_roller}}$

$\sigma_{big\_roller} = 244.43\ Pa$

Pipe support

Pipe support mass: $M_{pipe\_s} := \dfrac{1.26}{1000} \cdot kg$

Pipe support force: $F_{pipe\_s} := M_{pipe\_s} \cdot \left(9.81\ \dfrac{m}{s^2}\right)$

$F_{pipe\_s} = 0.012\ N$

Pipe support area: $A_{pipe\_s} := 26 \cdot 21\ mm^2$

Pipe support stress: $\sigma_{pipe\_s} := \dfrac{F_{pipe\_s}}{A_{pipe\_s}}$

$\sigma_{pipe\_s} = 22.638\ Pa$

Lower & upper microswitch holder and microswitch

Lower & upper microswitch holder and microswitch mass [lum]: $M_{lum} := \dfrac{19.1}{1000} \cdot kg$

Lower & upper microswitch holder and microswitch force [lum]: $F_{lum} := M_{lum} \cdot \left(9.81\ \dfrac{m}{s^2}\right)$

$F_{lum} = 0.187\ N$

Lower & upper microswitch holder and microswitch area [lum]:

$$A_{lum} := 26 \cdot 7\ mm^2$$

Lower & upper microswitch holder and microswitch stress [lum]:

$$\sigma_{lum} := \frac{F_{lum}}{A_{lum}}$$

$$\sigma_{lum} = \left(1.03 \cdot 10^3\right)\ Pa$$

Total stress on the base

Total stress on the base:

$$\sigma_{base\_ex} := \sigma_{lum} + \sigma_{pipe\_s} + \sigma_{big\_roller}$$

$$\sigma_{base\_ex} = 0.001\ MPa$$

The base tensile strength at yield:

$$\sigma_{base\_yield} := 220\ MPa$$

Factor of safety:

$$FOC_{base} := \frac{\sigma_{base\_yield}}{\sigma_{base\_ex}}$$

$$FOC_{base} = 1.697 \cdot 10^5$$

Lower casing

The base of the lower housing has a lot components, it will be 3D printed using PLA plastic. Thus, it needs to be ensured that it will not fail due bending.

Total mass of the parts acting on the lower housing:

$$M_{lower\_s} := \frac{560}{1000} \cdot kg$$

Total mass of the parts acting on the lower housing:

$$F_{lower\_s} := M_{lower\_s} \cdot \left(9.81 \ \frac{m}{s^2}\right)$$

$$F_{lower\_s} = 5.494 \ N$$

Total mass of the parts acting on the lower housing:

$$A_{lower\_s} := 44899.54 \ mm^2$$

Total mass of the parts acting on the lower housing:

$$\sigma_{lower\_s} := \frac{F_{lower\_s}}{A_{lower\_s}}$$

$$\sigma_{lower\_s} = 122.353 \ Pa$$

The stress experienced by the lower housing is very small compared to the lower housing flexural strength. Thus, the lower housing will be able to withstand bending forces applied perpendicular to its longitudinal axis.

Flexural strength of the lower housing:

$$\sigma_{flexural\_lower\_s} := 80 \ MPa$$

Factor of safety:

$$FOC_{lower\_s} := \frac{\sigma_{flexural\_lower\_s}}{\sigma_{lower\_s}}$$

$$FOC_{lower\_s} = 6.538 \cdot 10^5$$

Bending die

Bending die is responsible for bending the pipe and it will be in contact with the pipe. Thus, the strength of the bending die needs to be determined.

The bending die will be machined using low carbon steel

Bending die tensile strength: $\sigma_{tensile\_die} := 550\ MPa$

Bending die yield strength: $\sigma_{yield\_die} := 250\ MPa$

The tensile stress experienced by the die can be determined. By first determining the force the will be experienced by the die.

The calculate bending moment of the die: $M_{die} := 12.922\ N \cdot m$

The distance that the force is acting: $D_{die} := 10\ mm$

The area that the force is acting: $A_{die} := 75.4\ mm^2$

The force acting on the die:

$$F_{die} := \frac{M_{die}}{D_{die}}$$

$$F_{die} = \left(1.292 \cdot 10^3\right)\ N$$

The tensile stress acting on the die:

$$\sigma_{die\_ex} := \frac{F_{die}}{A_{die}}$$

$$\sigma_{die\_ex} = 17.138 \; MPa$$

Factor of safety:

$$FOC_{die} := \frac{\sigma_{tensile\_die}}{\sigma_{die\_ex}}$$

$$FOC_{die} = 32.093$$

<u>Handle</u>

The handle of the design will be responsible for allowing the user to lift the design, it will be 3D printed using PLA plastic. Thus, it needs to be ensured that it will not .

Total mass of the parts acting on the handle:

$$M_{handle} := 1.11 \cdot kg$$

Total mass of the parts acting on the handle:

$$F_{handle} := M_{handle} \cdot \left(9.81 \; \frac{m}{s^2}\right)$$

$$F_{handle} = 10.889 \; N$$

Total mass of the parts acting on the handle:

$$A_{handle} := 44899.54 \; mm^2$$

Total mass of the parts acting on the handle:

$$\sigma_{handle\_ex} := \frac{F_{handle}}{A_{handle}}$$

$$\sigma_{handle\_ex} = 242.521 \; Pa$$

The stress experienced by the lower housing is very small compared to the lower housing flexural strength. Thus, the lower housing will be able to withstand bending forces applied perpendicular to its longitudinal axis.

Flexural strength of the lower housing:

$$\sigma_{flexural\_handle} := 80\ MPa$$

Factor of safety:

$$FOC_{handle} := \frac{\sigma_{flexural\_handle}}{\sigma_{handle\_ex}}$$

$$FOC_{handle} = 3.299 \cdot 10^{5}$$

1. **Appendix G: Mass calculations**

Notation:

p :	density
m :	mass
V :	volume

The mass of each part will be calculated using the below equation:

Mass = density x volume

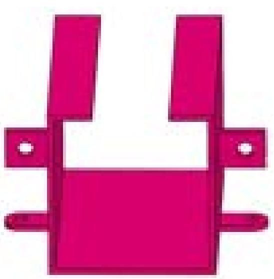
Density will be a value specific to the material used. The volume will be a value from solidworks, because solidworks is able to calculate the volume of complex parts accurately.

3D printed parts

The material that will be used for 3D printed parts is PLA plastic.

$$\rho_{PLA} := 1240 \cdot \frac{kg}{m^3}$$

Battery holder

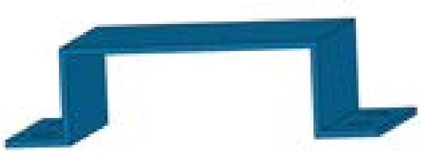


$$V_{bh} := 30227.85 \cdot mm^3$$

$$m_{bh} := \rho_{PLA} \cdot V_{bh}$$

$$m_{bh} = 0.037\ kg$$

Motor lifter

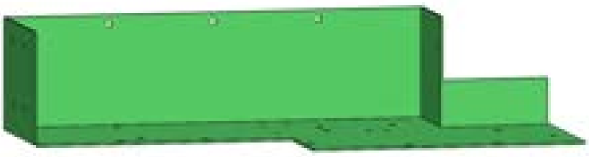


$$V_{ml} := 2810.52 \cdot mm^3$$

$$m_{ml} := \rho_{PLA} \cdot V_{ml}$$

$$m_{ml} = 0.003\ kg$$

Lower housing

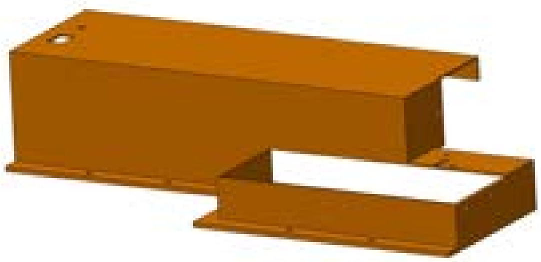


$$V_{lh} := 128903.53 \cdot mm^3$$

$$m_{lh} := \rho_{PLA} \cdot V_{lh}$$

$$m_{lh} = 0.16 \ kg$$

Upper housing

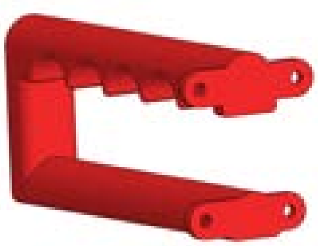


$$V_{uh} := 120305.23 \cdot mm^3$$

$$m_{uh} := \rho_{PLA} \cdot V_{uh}$$

$$m_{uh} = 0.149 \ kg$$

Handle

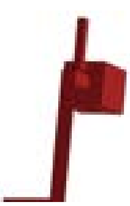


$$V_h := 77838.31 \cdot mm^3$$

$$m_h := \rho_{PLA} \cdot V_h$$

$$m_h = 0.097 \ kg$$

Angle lock

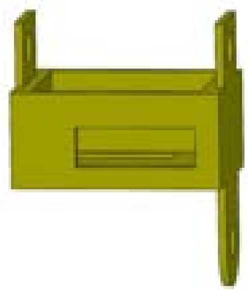


$$V_{al} := 409.63 \cdot mm^3$$

$$m_{al} := \rho_{PLA} \cdot V_{al}$$

$$m_{al} = \left(5.079 \cdot 10^{-4}\right) \ kg$$

Microswitch holder

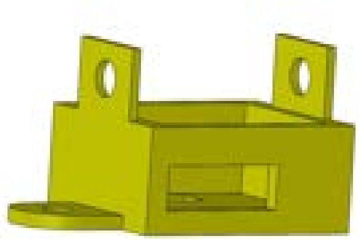


$$V_{mch} := 3068.22 \cdot mm^3$$

$$m_{mch} := \rho_{PLA} \cdot V_{mch}$$

$$m_{mch} = 0.004 \; kg$$

Microswitch holder two

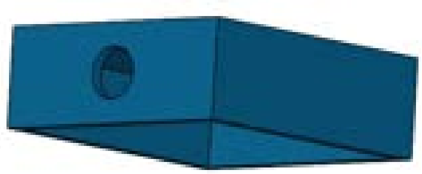


$$V_{mcht} := 3223.97 \cdot mm^3$$

$$m_{mcht} := \rho_{PLA} \cdot V_{mcht}$$

$$m_{mcht} = 0.004 \; kg$$

Two microswitch upper holders



$$V_{muh} := 1494.40 \cdot mm^3$$

$$m_{muh} := 2 \cdot \rho_{PLA} \cdot V_{muh}$$

$$m_{muh} = 0.004 \; kg$$

Two battery clamps

$$V_{b\_clamp} := 4286.19 \cdot mm^3$$

$$m_{b\_c} := 2 \cdot \rho_{PLA} \cdot V_{b\_clamp}$$

$$m_{b\_c} = 0.011 \; kg$$

$M_{3D\_p} := m_{bh} + m_{ml} + m_{lh} + m_{uh} + m_h + m_{al} + m_{mch} + m_{mcht} + m_{muh} + m_{b\_c}$

$M_{3D\_p} = 0.469\ kg$

Sheet metal parts

Sheet metal will be made using hot rolled mild steel. The density varies based on the alloying constituents but usually ranges between **7750** and **8050 kg/m3.**

To design for maximum conditions, I'll assume that the density of hot rolled mild steel is 8050 kg/m3.

$\rho_{mild\_steel} := 8050 \cdot \frac{kg}{m^3}$

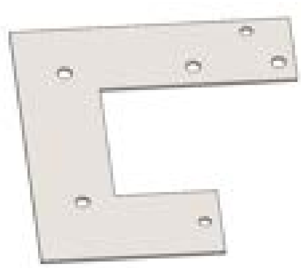
Base

$V_{base} := 20288.17 \cdot mm^3$

$m_{base} := \rho_{mild\_steel} \cdot V_{base}$

$m_{base} = 0.163\ kg$

Lower shaft support



$V_{lss} := 4197.34 \cdot mm^3$

$m_{lss} := \rho_{mild\_steel} \cdot V_{lss}$

$m_{lss} = 0.034\ kg$

Upper shaft support

$$V_{uss} := 5116.73 \cdot mm^3$$

$$m_{uss} := \rho_{mild\_steel} \cdot V_{uss}$$

$$m_{uss} = 0.041\ kg$$

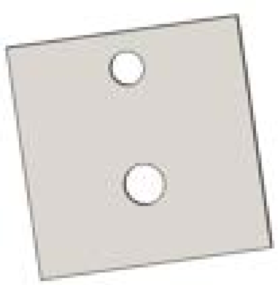
Motor support

$$V_{motor\_s} := 4596.91 \cdot mm^3$$

$$m_{motor\_s} := \rho_{mild\_steel} \cdot V_{motor\_s}$$

$$m_{motor\_s} = 0.037\ kg$$

Two sheet support



$$V_{sheet\_s} := 552.52 \cdot mm^3$$

$$m_{sheet\_s} := 2 \cdot \rho_{mild\_steel} \cdot V_{sheet\_s}$$

$$m_{sheet\_s} = 0.009\ kg$$

Two double plate for 48T gear

$$V_{D\_48T} := 3049.80 \cdot mm^3$$

$$m_{D\_48T} := 2 \cdot \rho_{mild\_steel} \cdot V_{D\_48T}$$

$$m_{D\_48T} = 0.049\ kg$$

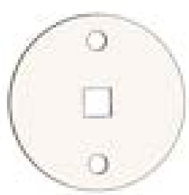
Double plate for 36T gear

$$V_{D\_36T} := 689.14 \cdot mm^3$$

$$m_{D\_36T} := \rho_{mild\_steel} \cdot V_{D\_36T}$$

$$m_{D\_36T} = 0.006\ kg$$

Double plate for 24T gear



$$V_{D\_24T} := 296.44 \cdot mm^3$$

$$m_{D\_24T} := \rho_{mild\_steel} \cdot V_{D\_24T}$$

$$m_{D\_24T} = 0.002\ kg$$

Total mass of the machined parts:

$$M_s := m_{base} + m_{lss} + m_{uss} + m_{motor\_s} + m_{sheet\_s} + m_{D\_48T} + m_{D\_36T} + m_{D\_24T}$$

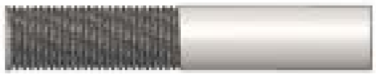
$$M_s = 0.341\ kg$$

Machined parts

Machined components will be made up of low carbon steel. Low carbon steel has a density of
7850 kg/m3.

$$\rho_{carbon\_steel} := 7850 \cdot \frac{kg}{m^3}$$

Die positioner



$$V_{Die\_p} := 113.22 \cdot mm^3$$

$$m_{Die\_p} := \rho_{carbon\_steel} \cdot V_{Die\_p}$$

$$m_{Die\_p} = \left(8.888 \cdot 10^{-4}\right)\ kg$$

Die lock

$$V_{Die\_l} := 122.48 \cdot mm^3$$

$$m_{Die\_l} := \rho_{carbon\_steel} \cdot V_{Die\_l}$$

$$m_{Die\_l} = \left(9.615 \cdot 10^{-4}\right)\ kg$$

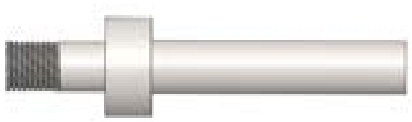
Clamp platform

$$V_{Clamp\_p} := 3055.02 \cdot mm^3$$

$$m_{Clamp\_p} := \rho_{carbon\_steel} \cdot V_{Clamp\_p}$$

$$m_{Clamp\_p} = 0.024\ kg$$

Circular custom shaft



$$V_{Custom\_s} := 242.83 \cdot mm^3$$

$$m_{Custom\_s} := \rho_{carbon\_steel} \cdot V_{Custom\_s}$$

$$m_{Custom\_s} = 0.002\ kg$$

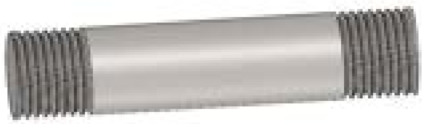
Angle positioner

$$V_{Angle\_p} := 94.76 \cdot mm^3$$

$$m_{Angle\_p} := \rho_{carbon\_steel} \cdot V_{Angle\_p}$$

$$m_{Angle\_p} = \left(7.439 \cdot 10^{-4}\right)\ kg$$

Bending die



$$V_{Die\_b} := 229.77 \cdot mm^3$$

$$m_{Die\_b} := \rho_{carbon\_steel} \cdot V_{Die\_b}$$

$$m_{Die\_b} = 0.002\ kg$$

Total mass of the machined parts:

$$M_{machine} := m_{Die\_p} + m_{Die\_l} + m_{Clamp\_p} + m_{Custom\_s} + m_{Angle\_p} + m_{Die\_b}$$

$$M_{machine} = 0.03\ kg$$

Bought parts

Some parts will be bought. Their mass is given.

100rpm Motor: $m_{motor} := 0.09 \cdot kg$

Two 12T spur gears: $m_{12T\_s} := 2 \cdot \frac{1.2}{1000} \cdot kg = 0.002\ kg$

Two 24T bevel gears: $m_{24T\_b} := 2 \cdot \frac{1.2}{1000} \cdot kg = 0.002\ kg$

Two 36T spur gears: $m_{36T\_s} := 2 \cdot \frac{5.2}{1000} \cdot kg = 0.01\ kg$

Two 48T spur gears: $m_{48T\_s} := 2 \cdot \frac{7.5}{1000} \cdot kg = 0.015\ kg$

Square shaft 30 long: $m_{30\_sq} := \frac{77}{1000} \cdot \frac{30}{1000} \cdot kg = 0.002\ kg$

Square shaft 70 long: $m_{70\_sq} := \frac{77}{1000} \cdot \frac{70}{1000} \cdot kg = 0.005\ kg$

Square shaft 45 long: $m_{45\_sq} := \frac{77}{1000} \cdot \frac{45}{1000} \cdot kg = 0.003\ kg$

Three high strength square shaft 30 long: $m_{30\_hsq} := 3 \cdot \frac{77}{1000} \cdot \frac{30}{1000} \cdot kg = 0.007\ kg$

Shaft coupler: $m_{shaft\_c} := \frac{2.3}{1000} \cdot kg = 0.002\ kg$

Four cells with caps: $m_{cell\_w\_caps} := 4 \cdot \frac{54}{1000} \cdot kg = 0.216\ kg$

Two cell links: $m_{cell\_link} := 2 \cdot \frac{1}{1000} \cdot kg = 0.002\ kg$

Fused link: $m_{fused\_link} := \frac{1.5}{1000} \cdot kg = 0.002\ kg$

Two microswitches: $m_{microswitch} := 2 \cdot \frac{2.2}{1000} \cdot kg = 0.004\ kg$

Ten spacers 3.2 long: $m_{3.2\_spacer} := 10 \cdot \frac{0.25}{1000} \cdot kg = 0.003\ kg$

Spacer 6.4 long: $m_{6.4\_spacer} := \frac{0.5}{1000} \cdot kg = \left(5 \cdot 10^{-4}\right)\ kg$

Three spacers 9.5 long: $m_{9.5\_spacer} := 3 \cdot \frac{0.75}{1000} \cdot kg = 0.002\ kg$

Two spacers 12.7 long: $m_{12.7\_spacer} := 2 \cdot \frac{1}{1000} \cdot kg = 0.002\ kg$

Two button head screws: $m_{b\_screws} := 2 \cdot \frac{1.3}{1000} \cdot kg = 0.003\ kg$

Six (2.9x6.5) self $m_{2.9x6.5\_screws} := 6 \cdot \frac{1.3}{1000} \cdot kg = 0.008\ kg$

tapping screws: $\quad 1000$

Twelve (3.5x9.5) self tapping screws:

$$m_{3.5x9.5\_screws} := 12 \cdot \frac{1.5}{1000} \cdot kg = 0.018\ kg$$

Four (4.2x16) self tapping screws:

$$m_{4.2x16\_screws} := 12 \cdot \frac{1.6}{1000} \cdot kg = 0.019\ kg$$

Twenty one (4.2x9.5) self tapping screws:

$$m_{4.2x9.5\_screws} := 21 \cdot \frac{1.6}{1000} \cdot kg = 0.034\ kg$$

Seven hex M5 (16x10) bolts:

$$m_{16x10\_bolt} := 7 \cdot \frac{2.72}{1000} \cdot kg = 0.019\ kg$$

Seven hex M5 nut:

$$m_{M5\_nut} := 7 \cdot \frac{0.85}{1000} \cdot kg = 0.006\ kg$$

Five hex M3 nut:

$$m_{M3\_nut} := 5 \cdot \frac{0.85}{1000} \cdot kg = 0.004\ kg$$

Six M5 washers:

$$m_{M5\_washer} := 6 \cdot \frac{0.44}{1000} \cdot kg = 0.003\ kg$$

Big roller:

$$m_{Big\_roller} := \frac{65}{1000} \cdot kg = 0.065\ kg$$

Push button:

$$m_{push\_b} := \frac{27}{1000} \cdot kg = 0.027\ kg$$

Control switch:

$$m_{control\_s} := \frac{30}{1000} \cdot kg = 0.03\ kg$$

$$M_1 := m_{control\_s} + m_{push\_b} + m_{Big\_roller} + m_{M5\_washer} + m_{M5\_nut} + m_{M3\_nut} + m_{16x10\_bolt} + m_{4.2x9.5\_screws}$$

$$M_2 := m_{4.2x16\_screws} + m_{3.5x9.5\_screws} + m_{2.9x6.5\_screws} + m_{b\_screws} + m_{12.7\_spacer} + m_{9.5\_spacer} + m_{6.4\_spacer}$$

$$M_3 := m_{3.2\_spacer} + m_{microswitch} + m_{fused\_link} + m_{cell\_link} + m_{cell\_w\_caps} + m_{shaft\_c} + m_{30\_hsq} + m_{45\_sq}$$

$$M_4 := m_{70\_sq} + m_{30\_sq} + m_{48T\_s} + m_{36T\_s} + m_{24T\_b} + m_{12T\_s} + m_{motor}$$

Total mass of the bought parts:

$$M_{bought} := M_1 + M_2 + M_3 + M_4$$

$$M_{bought} = 0.607\ kg$$

Total CAD mass:

$$M_{tot} := M_{bought} + M_{machine} + M_s + M_{3D\_p}$$

$$M_{tot} = 1.447\ kg$$

Appendix H: Cost calculations

Prototyping

- Machined component : raw material cost + R300 / h, in increments of 15 min
- Raw material :
  - Low carbon steel : R25 / kg
  - Brass : R155 / kg
  - Aluminium : R95 / kg
- 3D printing using PLA plastic : R700 / kg + R65 setup cost per part
- For hot rolled carbon steel 1.0 to 3.0 mm sheetmetal parts:

$$Cost\ [R] = 2.2M + 114L \cdot t^{0.2} + 9.6B$$

with M = mass [kg] of plate without internal cut – outs

L = total length [m] of cutting, incl outer perimeter

t = plate thickness [m]

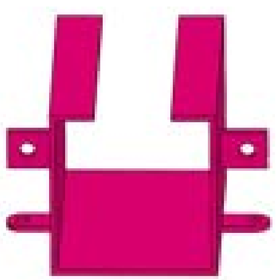
B = number of simple bends

Figure 1: Typical costing inputs

The above costing formulae will be used to calculate cost, the of each part is already determined in **Appendix E.**

3D printed parts

<u>Battery holder</u>



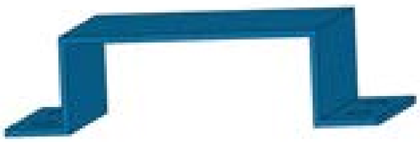
$$m_{bh} := 0.037\ kg$$

$$C_{bh} := \frac{700}{kg} \cdot m_{bh} + 65$$

$$C_{bh} = 90.9$$

Motor lifter

$m_{ml} := 0.003\ kg$

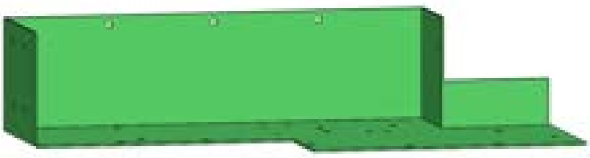


$$C_{ml} := \frac{700}{kg} \cdot m_{ml} + 65$$

$$C_{ml} = 67.1$$

Lower housing

$m_{lh} := 0.16\ kg$

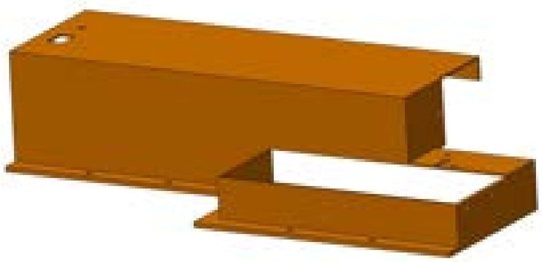


$$C_{lh} := \frac{700}{kg} \cdot m_{lh} + 65$$

$$C_{lh} = 177$$

Upper housing

$m_{uh} := 0.149\ kg$

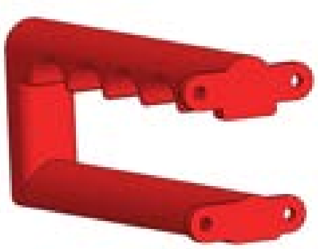


$$C_{uh} := \frac{700}{kg} \cdot m_{uh} + 65$$

$$C_{uh} = 169.3$$

Handle

$m_h := 0.097\ kg$



$$C_h := \frac{700}{kg} \cdot m_h + 65$$

$$C_h = 132.9$$

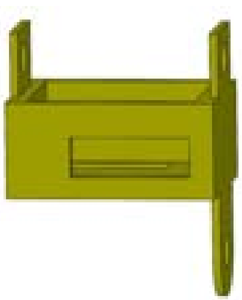
Angle lock

$$m_{al} := \left(5.079 \cdot 10^{-4}\right)\ kg$$

$$C_{al} := \frac{700}{kg} \cdot m_{al} + 65$$

$$C_{al} = 65.356$$

Microswitch holder

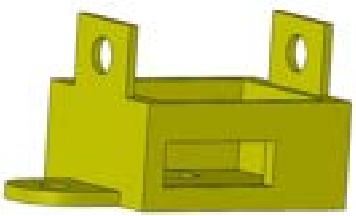


$$m_{mch} := 0.004\ kg$$

$$C_{mch} := \frac{700}{kg} \cdot m_{mch} + 65$$

$$C_{mch} = 67.8$$

Microswitch holder two

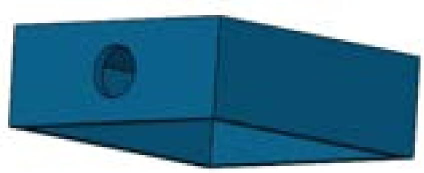


$$m_{mcht} := 0.004\ kg$$

$$C_{mcht} := \frac{700}{kg} \cdot m_{mcht} + 65$$

$$C_{mcht} = 67.8$$

Two microswitch upper holders



$m_{muh} := 0.004\ kg$

$C_{muh} := \frac{700}{kg} \cdot m_{muh} + 65$

$C_{muh} = 67.8$

Two battery clamps

$m_{b\_c} := 0.011\ kg$

$C_{b\_c} := \frac{700}{kg} \cdot m_{b\_c} + 65$

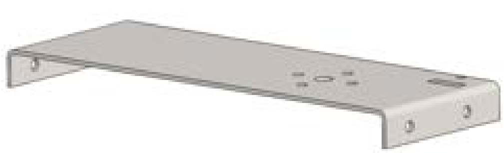
$C_{b\_c} = 72.7$

$C_{3D\_p} := C_{bh} + C_{ml} + C_{lh} + C_{uh} + C_h + C_{al} + C_{mch} + C_{mcht} + C_{muh} + C_{b\_c}$

$C_{3D\_p} = 978.656$

Sheet metal parts

Base



$m_{base} := 0.163\ kg$

$L_{base} := 593.79\ mm$

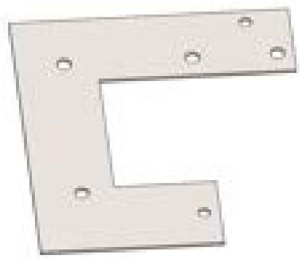
$t_{base} := 1.6\ mm$

$B_{base} := 2$

$$C_{base} := 2.2 \cdot \frac{m_{base}}{kg} + 114 \cdot \frac{L_{base}}{m} \cdot \left(\frac{t_{base}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{base}$$

$C_{base} = 38.238$

Lower shaft support



$$m_{lss} := 0.034\ kg$$

$$L_{lss} := 478.77\ mm$$

$$t_{lss} := 1\ mm$$

$$B_{lss} := 0$$

$$C_{lss} := 2.2 \cdot \frac{m_{lss}}{kg} + 114 \cdot \frac{L_{lss}}{m} \cdot \left(\frac{t_{lss}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{lss}$$

$$C_{lss} = 13.785$$

Upper shaft support

$$m_{uss} := 0.041\ kg$$

$$L_{uss} := 427.18\ mm$$

$$t_{uss} := 1\ mm$$

$$B_{uss} := 4$$

$$C_{uss} := 2.2 \cdot \frac{m_{uss}}{kg} + 114 \cdot \frac{L_{uss}}{m} \cdot \left(\frac{t_{uss}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{uss}$$

$$C_{uss} = 50.723$$

Motor support

$$m_{m\_s} := 0.037\ kg$$

$$L_{m\_s} := 615.6\ mm$$

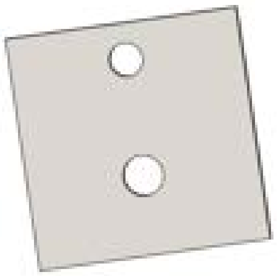
$$t_{m\_s} := 1.6\ mm$$

$$B_{m\_s} := 4$$

$$C_{m\_s} := 2.2 \cdot \frac{m_{m\_s}}{kg} + 114 \cdot \frac{L_{m\_s}}{m} \cdot \left(\frac{t_{m\_s}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{m\_s}$$

$$C_{m\_s} = 57.847$$

Two sheet support



$$m_{sheet\_s} := 0.009\ kg$$

$$L_{sheet\_s} := 120.19\ mm$$

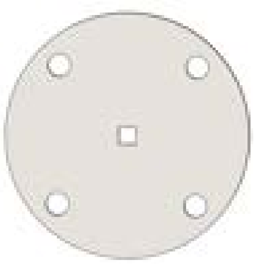
$$t_{sheet\_s} := 1.6\ mm$$

$$B_{sheet\_s} := 0$$

$$C_{sheet\_s} := 2.2 \cdot \frac{m_{sheet\_s}}{kg} + 114 \cdot \frac{L_{sheet\_s}}{m} \cdot \left(\frac{t_{sheet\_s}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{sheet\_s}$$

$$C_{m\_s} = 57.847$$

Two double plate for 48T gear



$$m_{D\_48T} := 0.049\ kg$$

$$L_{D\_48T} := 206.87\ mm$$

$$t_{D\_48T} := 2\ mm$$

$$B_{D\_48T} := 0$$

$$C_{D\_48T} := 2.2 \cdot \frac{m_{D\_48T}}{kg} + 114 \cdot \frac{L_{D\_48T}}{m} \cdot \left(\frac{t_{D\_48T}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{D\_48T}$$

$$C_{D\_48T} = 6.912$$

Double plate for 36T gear

$m_{D\_36T} := 0.006\ kg$

$L_{D\_36T} := 120.79\ mm$

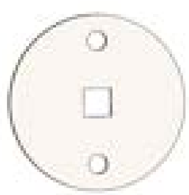
$t_{D\_36T} := 1\ mm$

$B_{D\_36T} := 0$

$$C_{D\_36T} := 2.2 \cdot \frac{m_{D\_36T}}{kg} + 114 \cdot \frac{L_{D\_36T}}{m} \cdot \left(\frac{t_{D\_36T}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{D\_36T}$$

$C_{D\_36T} = 3.472$

Double plate for 24T gear



$m_{D\_24T} := 0.002\ kg$

$L_{D\_24T} := 120.79\ mm$

$t_{D\_24T} := 1\ mm$

$B_{D\_24T} := 0$

$$C_{D\_24T} := 2.2 \cdot \frac{m_{D\_24T}}{kg} + 114 \cdot \frac{L_{D\_24T}}{m} \cdot \left(\frac{t_{D\_24T}}{1\ m}\right)^{0.2} + 9.6 \cdot B_{D\_24T}$$

$C_{D\_24T} = 3.463$

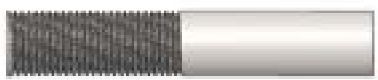
Total cost of the machined parts:

$$C_s := C_{base} + C_{lss} + C_{uss} + C_{m\_s} + C_{sheet\_s} + C_{D\_48T} + C_{D\_36T} + C_{D\_24T}$$

$C_s = 178.241$

Machined parts

Die positioner



$$m_{Die\_p} := \left(8.888 \cdot 10^{-4}\right)\ kg$$

$$C_{Die\_p} := \frac{25}{kg} \cdot m_{Die\_p} + 300 \cdot \frac{15}{60}$$

$$C_{Die\_p} = 75.022$$

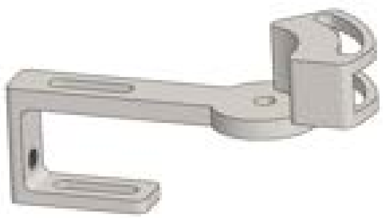
Die lock

$$m_{Die\_l} := \left(9.615 \cdot 10^{-4}\right)\ kg$$

$$C_{Die\_l} := \frac{25}{kg} \cdot m_{Die\_l} + 300 \cdot \frac{30}{60}$$

$$C_{Die\_l} = 150.024$$

Clamp platform

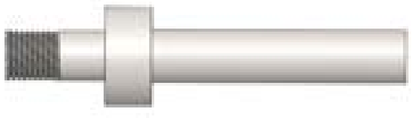


$$m_{Clamp\_p} := 0.024\ kg$$

$$C_{C\_p} := \frac{25}{kg} \cdot m_{Clamp\_p} + 300 \cdot \frac{90}{60}$$

$$C_{C\_p} = 450.6$$

Circular custom shaft

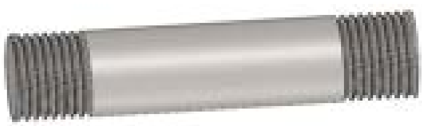


$$m_{Custom\_s} := 0.002\ kg$$

$$C_{C\_s} := \frac{25}{kg} \cdot m_{Custom\_s} + 300 \cdot \frac{30}{60}$$

$$C_{C\_s} = 150.05$$

Angle positioner

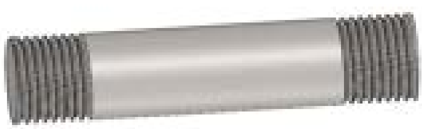


$$m_{Angle\_p} := \left(7.439 \cdot 10^{-4}\right)\ kg$$

$$C_{A\_p} := \frac{25}{kg} \cdot m_{Angle\_p} + 300 \cdot \frac{15}{60}$$

$$C_{A\_p} = 75.019$$

Bending die



$$m_{Die\_b} := 0.002\ kg$$

$$C_{Die\_b} := \frac{25}{kg} \cdot m_{Die\_b} + 300 \cdot \frac{15}{60}$$

$$C_{Die\_b} = 75.05$$

Total cost of the machined parts:

$$C_{machine} := C_{Die\_p} + C_{Die\_l} + C_{C\_p} + C_{C\_s} + C_{A\_p} + C_{Die\_b}$$

$$C_{machine} = 975.765$$

Bought parts

Some parts will be bought. Their mass is given.

100rpm Motor: $C_{motor} := 315$

Two 12T spur gears: $C_{12T\_s} := 2 \cdot 4.5 = 9$

Two 24T bevel gears: $C_{24T\_b} := 2 \cdot 6 = 12$

Two 36T spur gears: $C_{36T\_s} := 2 \cdot 18 = 36$

Two 48T spur gears: $C_{48T\_s} := 2 \cdot 9 = 18$

Square shaft 30 long: $C_{30\_sq} := 160 \cdot \frac{30}{1000} = 4.8$

Square shaft 70 long: $C_{70\_sq} := 160 \cdot \frac{70}{1000} = 11.2$

Square shaft 45 long: $C_{45\_sq} := 160 \cdot \frac{45}{1000} = 7.2$

Three high strength square shaft 30 long: $C_{30\_hsq} := 3 \cdot 160 \cdot \frac{30}{1000} = 14.4$

Shaft coupler: $C_{shaft\_c} := 25.5$

Four cells with caps: $C_{cell\_w\_caps} := 4 \cdot 54 = 216$

Two cell links: $C_{cell\_link} := 2 \cdot 1 = 2$

Fused link: $C_{fused\_link} := 25$

Two microswitches: $C_{microswitch} := 2 \cdot 17 = 34$

Ten spacers 3.2 long: $C_{3.2\_spacer} := 10 \cdot 3 = 30$

Spacer 6.4 long: $C_{6.4\_spacer} := 3$

Three spacers 9.5 long: $C_{9.5\_spacer} := 3 \cdot 3 = 9$

Two spacers 12.7 long: $C_{12.7\_spacer} := 2 \cdot 3 = 6$

Two button head screws: $C_{b\_screws} := 2 \cdot 3.5 = 7$

Six (2.9x6.5) self tapping screws: $C_{2.9x6.5\_screws} := 6 \cdot 1 = 6$

Twelve (3.5x9.5) self tapping screws: $C_{3.5x9.5\_screws} := 12 \cdot 1 = 12$

Four (4.2x16) self tapping screws: $C_{4.2x16\_screws} := 12 \cdot 1 = 12$

Twenty one (4.2x9.5) self tapping screws: $C_{4.2x9.5\_screws} := 21 \cdot 1 = 21$

Seven hex M5 (16x10) bolts: $C_{16x10\_bolt} := 7 \cdot 2 = 14$

Seven hex M5 nut: $C_{M5\_nut} := 7 \cdot 2 = 14$

Five hex M3 nut: $C_{M3\_nut} := 5 \cdot 2 = 10$

Six M5 washers: $C_{M5\_washer} := 6 \cdot 1 = 6$

Big roller: $C_{Big\_roller} := 65 = 65$

Push button: $C_{push\_b} := 75$

Control switch: $C_{control\_s} := 65$

$$C_1 := C_{control\_s} + C_{push\_b} + C_{Big\_roller} + C_{M5\_washer} + C_{M5\_nut} + C_{M3\_nut} + C_{16x10\_bolt} + C_{4.2x9.5\_screws}$$

$$C_2 := C_{4.2x16\_screws} + C_{3.5x9.5\_screws} + C_{2.9x6.5\_screws} + C_{b\_screws} + C_{12.7\_spacer} + C_{9.5\_spacer} + C_{6.4\_spacer}$$

$$C_3 := C_{3.2\_spacer} + C_{microswitch} + C_{fused\_link} + C_{cell\_link} + C_{cell\_w\_caps} + C_{shaft\_c} + C_{30\_hsq} + C_{45\_sq}$$

$$C_4 := C_{70\_sq} + C_{30\_sq} + C_{48T\_s} + C_{36T\_s} + C_{24T\_b} + C_{12T\_s} + C_{motor}$$

Total cost of the bought parts in rands:

$$C_{bought} := C_1 + C_2 + C_3 + C_4$$

$$C_{bought} = 694.1$$

Labour cost

The labour cost needs to be taken into account. The design needs to be assembled properly. An artisan will be perfect for this because they are skilled in manual labour fields such as:

water technology (plumbing)
engineering (electrical/mechanical/automotive etc)
fashion
building technology

Figure 1: average artisan salary

The above figure is from talent.com

Number of artisan needed: $N_{artisan} := 1$

Artisan cost: $C_{artisan} := \frac{108}{hr}$

Number of hours that an artisan will work: $N_{hours} := 1.5\ hr$

Labour cost: $C_{labour} := N_{artisan} \cdot C_{artisan} \cdot N_{hours}$

$C_{labour} = 162$

The prototype cost in rands: $C_{tot} := C_{bought} + C_{machine} + C_s + C_{3D\_p} + C_{labour}$

$$C_{tot} = 2.989 \cdot 10^3$$

Development costs

Development costs are commonly referred to as research and development costs. These costs can include a host of expenses, such as marketing analysis, developmental engineering and customer surveying.

Activities that are typically considered as research and development include:

Research to bring about new knowledge
Creation of product and process designs
Testing processes and products
Modifying processes and products
Designing prototypes
Testing prototypes
Designing new tools

Certain number of engineers need to be assigned to perform the above mentioned activities.

The cost per engineer needs to be stated and also how long it will take to complete research and development

Number of engineers that will be assigned for R&D: $N_{engineer} := 2$

The cost of one engineer: $C_{engineer} := \frac{200}{hr}$

Number of days dedicated for R&D per month:

$N_{days} := 20$

Number of hours dedicated for R&D day:

$\boxed{N_{hours}} := 8\ hr$

Number of months it will take for research and development:

$N_{dev\_months} := 2$

Number of demonstrations needed:

$N_{demo} := 4$

Prototype cost:

$C_{prototype} := C_{tot}$

$C_{prototype} = 2.989 \cdot 10^{3}$

Using the above information, the development cost can be determined.

Development cost:

$$C_{development} := N_{demo} \cdot C_{prototype} + N_{engineer} \cdot C_{engineer} \cdot N_{days} \cdot N_{hours} \cdot N_{dev\_months}$$

$$C_{development} = 1.4 \cdot 10^{5}$$

Ramp-up costs

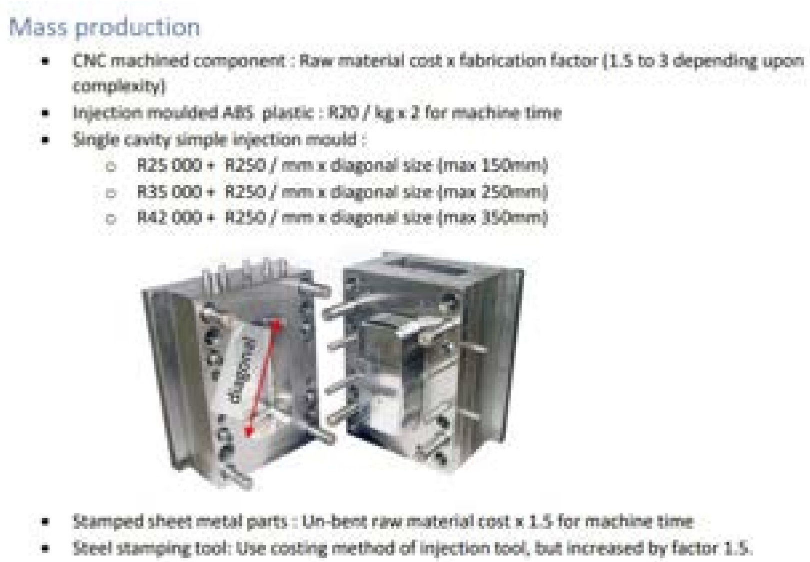
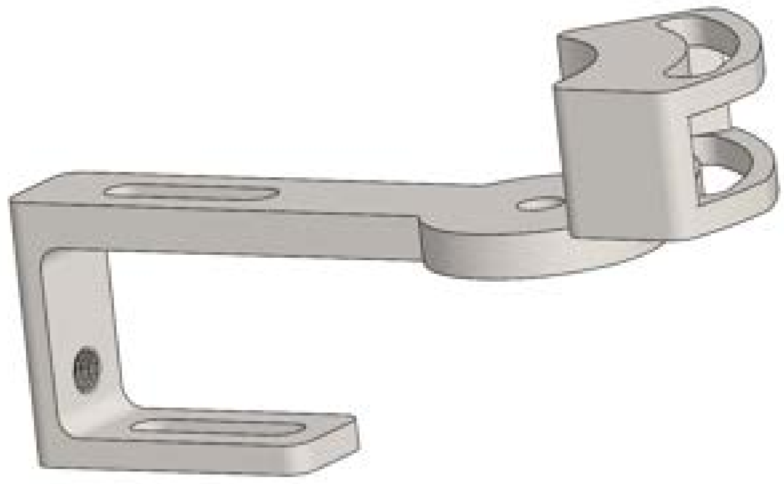


Figure 1: Mass production

The above figure will be used to determine the cost of tooling.

Complex parts will be made using single cavity simple injection mould

Clamp platform



Calculated mass: $m_{Clamp\_p} = 0.024\ kg$

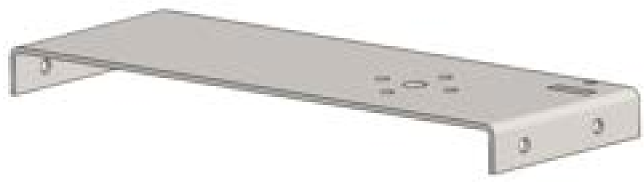
Diagonal size: $D_{Clamp\_p} := 96.52\ mm$

Tooling cost in rands: $T_{Clamp\_p} := 25000 + \frac{250}{mm} \cdot D_{Clamp\_p}$

$$T_{Clamp\_p} = 4.913 \cdot 10^4$$

Stamped sheet metal parts needs steel stamping tool to create the desired shape

Base



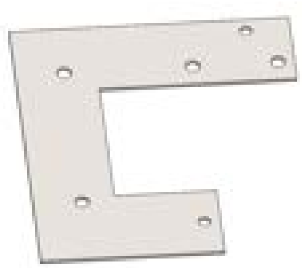
Calculated mass: $m_{base} = 0.163\ kg$

Diagonal size: $D_{base} := 189.2\ mm$

Tooling cost in rands: $T_{base} := 35000 + \frac{250}{mm} \cdot D_{base}$

$T_{base} = 8.23 \cdot 10^4$

Lower shaft support



Calculated mass: $m_{lss} = 0.034\ kg$

Diagonal size: $D_{lss} := 121\ mm$

Tooling cost in rands: $T_{lss} := 25000 + \frac{250}{mm} \cdot D_{lss}$

$T_{lss} = 5.525 \cdot 10^4$

Upper shaft support

Calculated mass: $m_{uss} = 0.041\ kg$

Diagonal size: $D_{uss} := 142\ mm$

Tooling cost in rands: $T_{uss} := 25000 + \frac{250}{mm} \cdot D_{uss}$

$T_{uss} = 6.05 \cdot 10^{4}$

Motor support

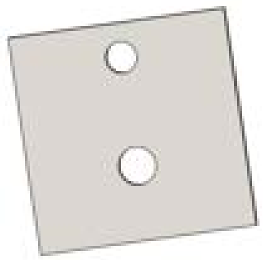
Calculated mass: $m_{m\_s} = 0.037\ kg$

Diagonal size: $D_{m\_s} := 75.47\ mm$

Tooling cost in rands: $T_{m\_s} := 25000 + \frac{250}{mm} \cdot D_{m\_s}$

$T_{m\_s} = 4.387 \cdot 10^{4}$

Two sheet support



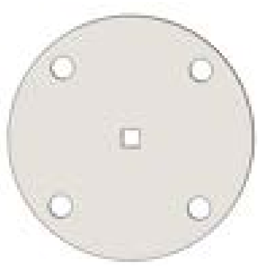
Calculated mass: $m_{sheet\_s} = 0.009\ kg$

Diagonal size: $D_{sheet\_s} := 33.94\ mm$

Tooling cost in rands: $T_{sheet\_s} := 25000 + \frac{250}{mm} \cdot D_{sheet\_s}$

$T_{sheet\_s} = 3.349 \cdot 10^4$

Two double plate for 48T gear



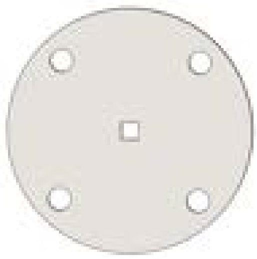
Calculated mass: $m_{D\_48T} = 0.049\ kg$

Diagonal size: $D_{D\_48T} := 48\ mm$

Tooling cost in rands: $T_{D\_48T} := 25000 + \frac{250}{mm} \cdot D_{D\_48T}$

$T_{D\_48T} = 3.7 \cdot 10^4$

Double plate for 36T gear



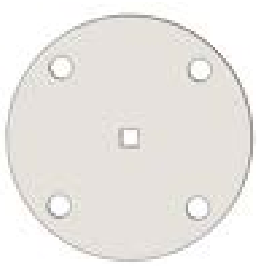
Calculated mass: $m_{D\_36T} = 0.006\ kg$

Diagonal size: $D_{D\_36T} := 30\ mm$

Tooling cost in rands: $T_{D\_36T} := 25000 + \frac{250}{mm} \cdot D_{D\_36T}$

$$T_{D\_36T} = 3.25 \cdot 10^{4}$$

Double plate for 24T gear



Calculated mass: $m_{D\_24T} = 0.002\ kg$

Diagonal size: $D_{D\_24T} := 20\ mm$

Tooling cost in rands: $T_{D\_24T} := 25000 + \frac{250}{mm} \cdot D_{D\_24T}$

$$T_{D\_24T} = 3 \cdot 10^{4}$$

Total tooling cost in rands:

$$T_{tot} := T_{D\_24T} + T_{D\_36T} + T_{D\_48T} + T_{sheet\_s} + T_{m\_s} + T_{uss} + T_{lss} + T_{base} + T_{Clamp\_p}$$

$$T_{tot} = 4.24 \cdot 10^5$$

The following prototypes will be made:

The number of the alpha prototypes: $N_{alpha} := 3$

The cost of each alpha prototype: $C_{alpha} := C_{tot}$

$$C_{alpha} = 2.989 \cdot 10^3$$

The number of the beta prototypes: $N_{beta} := 6$

The cost of each alpha prototype: $C_{beta} := C_{tot} \cdot 0.75$

$$C_{beta} = 2.242 \cdot 10^3$$

Total ramp-up cost: $C_{ramp\_up} := T_{tot} + N_{alpha} \cdot C_{alpha} + N_{beta} \cdot C_{beta}$

$$C_{ramp\_up} = 4.464 \cdot 10^5$$

Volume production cost

Figure 1: Mass production

The above figure will be used to determine the cost of the volume production cost

The following parts will be made with injection moulded ABS plastic

<u>Battery holder</u>

Calculated mass: $m_{bh} = 0.037\ kg$

Production cost in rands: $P_{bh} := \frac{20}{kg} \cdot m_{bh} \cdot 2$

$P_{bh} = 1.48$

<u>Motor lifter</u>

Calculated mass: $m_{ml} = 0.003\ kg$

Production cost in rands: $P_{ml} := \frac{20}{kg} \cdot m_{ml} \cdot 2$

$P_{ml} = 0.12$

Lower housing

Calculated mass: $m_{lh} = 0.16\ kg$

Production cost in rands: $P_{lh} := \frac{20}{kg} \cdot m_{lh} \cdot 2$

$P_{lh} = 6.4$

Upper housing

Calculated mass: $m_{uh} = 0.149\ kg$

Production cost in rands: $P_{uh} := \frac{20}{kg} \cdot m_{uh} \cdot 2$

$P_{uh} = 5.96$

Handle

Calculated mass: $m_h = 0.097\ kg$

Production cost in rands: $P_h := \frac{20}{kg} \cdot m_h \cdot 2$

$P_h = 3.88$

Angle lock

Calculated mass: $m_{al} = \left(5.079 \cdot 10^{-4}\right)\ kg$

Production cost in rands: $P_{al} := \frac{20}{kg} \cdot m_{al} \cdot 2$

$P_{al} = 0.02$

Microswitch holder

Calculated mass: $m_{msh} = 0.004\ kg$

Production cost in rands:

$$P_{mch} := \frac{20}{kg} \cdot m_{mch} \cdot 2$$

$$P_{mch} = 0.16$$

<u>Microswitch holder two</u>

Calculated mass: $m_{mcht} = 0.004\ kg$

Production cost in rands:

$$P_{mcht} := \frac{20}{kg} \cdot m_{mcht} \cdot 2$$

$$P_{mcht} = 0.16$$

<u>Two microswitch upper holders</u>

Calculated mass: $m_{muh} = 0.004\ kg$

Production cost in rands:

$$P_{muh} := 2 \cdot \frac{20}{kg} \cdot m_{mcht} \cdot 2$$

$$P_{muh} = 0.32$$

<u>Two battery clamps</u>

Calculated mass: $m_{b\_c} = 0.011\ kg$

Production cost in rands:

$$P_{b\_c} := 2 \cdot \frac{20}{kg} \cdot m_{b\_c} \cdot 2$$

$$P_{b\_c} = 0.88$$

Total cost of injection
moulded parts in rands:

.

$$P_{injection} := P_{bh} + P_{ml} + P_{lh} + P_{uh} + P_h + P_{al} + P_{mch} + P_{mcht} + P_{muh} + P_{b\_c}$$

$$P_{injection} = 19.38$$

The following parts will be made by stamped sheet metal

<u>Base</u>

Calculated mass: $m_{base} = 0.163\ kg$

Production cost in rands: $P_{base} := \frac{25}{kg} \cdot m_{base} \cdot 1.5$

$$P_{base} = 6.113$$

<u>Lower shaft support</u>

Calculated mass: $m_{lss} = 0.034\ kg$

Production cost in rands: $P_{lss} := \frac{25}{kg} \cdot m_{lss} \cdot 1.5$

$$P_{lss} = 1.275$$

<u>Upper shaft support</u>

Calculated mass: $m_{uss} = 0.041\ kg$

Production cost in rands: $P_{uss} := \frac{25}{kg} \cdot m_{uss} \cdot 1.5$

$$P_{uss} = 1.538$$

Motor support

Calculated mass: $m_{m\_s} = 0.037\ kg$

Production cost in rands: $P_{m\_s} := \frac{25}{kg} \cdot m_{m\_s} \cdot 1.5$

$P_{m\_s} = 1.388$

Two sheet support

Calculated mass: $m_{sheet\_s} = 0.009\ kg$

Production cost in rands: $P_{sheet\_s} := 2 \cdot \frac{25}{kg} \cdot m_{sheet\_s} \cdot 1.5$

$P_{sheet\_s} = 0.675$

Two double plate for 48T gear

Calculated mass: $m_{D\_48T} = 0.049\ kg$

Production cost in rands: $P_{D\_48T} := 2 \cdot \frac{25}{kg} \cdot m_{D\_48T} \cdot 1.5$

$P_{D\_48T} = 3.675$

Double plate for 36T gear

Calculated mass: $m_{D\_36T} = 0.006\ kg$

Production cost in rands: $P_{D\_36T} := \frac{25}{kg} \cdot m_{D\_36T} \cdot 1.5$

$P_{D\_36T} = 0.225$

Double plate for 24T gear

Calculated mass: $m_{D\_24T} = 0.002\ \boldsymbol{kg}$

Production cost in rands:

$$P_{D\_24T} := \frac{25}{\boldsymbol{kg}} \cdot m_{D\_24T} \cdot 1.5$$

$$P_{D\_24T} = 0.075$$

Total production cost of the stamped sheet metal parts in rands:

$$P_{sheet} := P_{base} + P_{lss} + P_{uss} + P_{m\_s} + P_{sheet\_s} + P_{D\_48T} + P_{D\_36T} + P_{D\_24T}$$

$$P_{sheet} = 14.963$$

The following parts will be made with a CNC machine.

Die positioner

Calculated mass: $m_{Die\_p} = \left(8.888 \cdot 10^{-4}\right)\ \boldsymbol{kg}$

Production cost in rands:

$$P_{Die\_p} := \frac{25}{\boldsymbol{kg}} \cdot m_{Die\_p} \cdot 1.5$$

$$P_{Die\_p} = 0.033$$

Die lock

Calculated mass: $m_{Die\_l} = \left(9.615 \cdot 10^{-4}\right)\ \boldsymbol{kg}$

Production cost in rands:

$$P_{Die\_l} := \frac{25}{} \cdot m_{Die\_l} \cdot 1.9$$

$P_{Die\_l} = 0.046$

Clamp platform

Calculated mass: $m_{Clamp\_p} = 0.024\ kg$

Production cost in rands: $P_{Clamp\_p} := \frac{25}{kg} \cdot m_{Clamp\_p} \cdot 3$

$P_{Clamp\_p} = 1.8$

Circular custom shaft

Calculated mass: $m_{Custom\_s} = 0.002\ kg$

Production cost in rands: $P_{Custom\_s} := \frac{25}{kg} \cdot m_{Custom\_s} \cdot 2.1$

$P_{Custom\_s} = 0.105$

Angle positioner

Calculated mass: $m_{Angle\_p} = \left(7.439 \cdot 10^{-4}\right)\ kg$

Production cost in rands: $P_{Angle\_p} := \frac{25}{kg} \cdot m_{Angle\_p} \cdot 1.5$

$P_{Angle\_p} = 0.028$

Bending die

Calculated mass: $m_{Die\_b} = 0.002\ kg$

Production cost in rands: $P_{Die\_b} := \frac{25}{kg} \cdot m_{Die\_b} \cdot 1.5$

$P_{Die\_b} = 0.075$

Total production cost of the CNC machined parts:

$$P_{CNC} := P_{Die\_p} + P_{Die\_l} + P_{Clamp\_p} + P_{Custom\_s} + P_{Angle\_p} + P_{Die\_b}$$

$$P_{CNC} = 2.087$$

The cost of parts that will be bought "off the shelf" will remain the same.

Total production cost of the "off the shelf" parts:

$$P_{bought} := C_{bought}$$

$$P_{bought} = 694.1$$

The labour cost will remain the same.

Total production cost due to labour in rands:

$$P_{labour} := C_{labour}$$

$$P_{labour} = 162$$

Now the volume production cost per product can be determined by adding all the production costs.

Volume production per product in rands:

$$P_{volume} := P_{bought} + P_{CNC} + P_{sheet} + P_{injection} + P_{labour}$$

$$P_{volume} = 892.53$$