Appendix B: Motor calculations

The motor calculations will be done in the following steps:

1. Choose the number of cells that will be used
2. Determine the stall current of the motor
3. Select a motor

Choose the number of cells that will be used

The motor specification document state that the maximum voltage **15V** and the cell specification document state that the nominal voltage is **3.2V**.

Maximum voltage of that the motor can take: $V_{max} := 15\ V$

The nominal cell voltage: $V_{nominal} := 3.2\ V$

Therefore, using the above information the maximum number of cells that should be connected in series can be calculated.

Maximum number of cells in series: $N_{max} := \frac{V_{max}}{V_{nominal}}$

$$N_{max} = 4.688$$

Thus, the maximum number of cells in series should be **4.**

Number of cells chosen for the design: $N_{cells} := 4$

Supply voltage to the motor: $V_{supply} := V_{nominal} \cdot N_{cells}$

$$V_{supply} = 12.8\ V$$

Determine the stall current of the motor

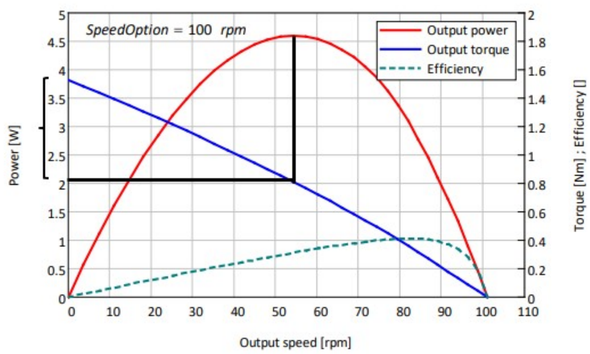

Figure 1: Curves generated from the motor specification document

The above figure is. The black lines depict that the maximum motor current occurs at half the stall current. Stall current occurs when the output speed is **0rpm**.

Thus, in order to determine the maximum motor operating current, stall current and torque must be calculated first.

Stall current occurs at the output speed of **0rpm**.

Output speed $[rad/s]$	$\omega_m = \dfrac{V_b - I_m R_a}{K_T r_g}$	V_b = supply voltage

Figure 2: Output speed from the motor specification document

$$I = \frac{V_b}{R_a}$$

The equation from figure 2 reduces to the one on the left due to the output speed equating to 0rpm.

Armature resistance [Ω]	$R_a = \begin{cases} 1.61 \;\; if \;\; I < I_{sat} \\ 1.61 + 0.15(I - I_{sat}) \;\; otherwise \end{cases}$ $I_{sat} = 1.2A$	I_{sat} is the magnetic saturation current

Figure 3: armature resistance equation from the motor specification document

Using the reduced equation from figure 2 and the armature resistance equation from figure 3, the below calculations are done.

$$R_a = 1.61 + 0.15(I - 1.2)$$
$$R_a = 1.61 + 0.15I - 0.18$$
$$R_a = 0.15I + 1.43$$

$$I = \frac{V_b}{0.15I + 1.43}$$

$$0.15I^2 + 1.43I = V_b$$
$$0.15I^2 + 1.43I - V_b = 0$$

Quadratic equation in standard form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic equation will be used to solve for the stall current.

Stall current:

$$I_{stall} := \left(\frac{-1.43 + \sqrt{1.43^2 - 4 \cdot (0.15) \cdot (-12.8)}}{2 \cdot 0.15} \right) A$$

$$I_{stall} = 5.628 \; A$$

Select a motor

In order to select the correct motor, the maximum conditions of all the motors will be calculated.

Now that stall current is determined. Stall torque can be calculated.

Output torque $[Nm]$	$T_m = I_m K_T r_g \eta_g - T_0$	I_m = motor current

Figure 4: Output torque equation from the motor specification document

The constants in the above equation are given in the motor specification document, they are listed below.

Torque constant: $K_T := 4.1 \cdot N \cdot \frac{mm}{A}$

For the 100 rpm motor

Gearbox reduction ratio: $r_{g\_100} := 156.8$

Gearbox efficiency: $\eta_{g\_100} := 0.7$

Static drag: $T_{0\_100} := 110 \cdot N \cdot mm$

Therefore, using the properties above stall torque can be calculated.

Stall current for 100rpm motor: $T_{stall\_100} := I_{stall} \cdot K_T \cdot r_{g\_100} \cdot \eta_{g\_100} - T_{0\_100}$

$$T_{stall\_100} = 2.423\ N \cdot m$$

As explained in figure 1 that the maximum motor operating current is when the motor current is at half the stall current. The output torque equation from figure 4 will be used to solve for the maximum motor current.

The maximum motor current for 100rpm motor:

$$I_{max\_100} := \frac{\left(\frac{T_{stall\_100}}{2}\right) + T_{0\_100}}{K_T \cdot r_{g\_100} \cdot \eta_{g\_100}}$$

$$I_{max\_100} = 2.936\ A$$

Armature resistance for 100rpm motor:

$$R_{a\_100} := \left(1.61 + 0.15 \cdot \left(\frac{I_{max\_100}}{1\ A} - 1.2\right)\right)\ \Omega$$

$$R_{a\_100} = 1.87\ \Omega$$

The maximum output torque for 100rpm motor:

$$T_{max\_100} := I_{max\_100} \cdot K_T \cdot r_{g\_100} \cdot \eta_{g\_100} - T_{0\_100}$$

$$T_{max\_100} = 1.211\ N \cdot m$$

Overload time to trip for 100rpm motor:

$$t_{trip\_100} := 140 \cdot \left(\frac{I_{max\_100}}{1 \cdot A}\right)^{-3.54} \cdot s$$

$$t_{trip\_100} = 3.091\ s$$

Output speed for 100rpm motor:

$$w_{m\_100} := \frac{V_{supply} - I_{max\_100} \cdot R_{a\_100}}{K_T \cdot r_{g\_100}}$$

$$w_{m\_100} = 11.367\ \frac{rad}{s}$$

For the 160 rpm motor

Gearbox reduction ratio: $r_{g\_160} := 98$

Gearbox efficiency: $\eta_{g\_160} := 0.72$

Static drag: $T_{0\_160} := 85 \cdot N \cdot mm$

As explained in figure 1 that the maximum motor operating current is when the motor current is at half the stall current. The output torque equation from figure 4 will be used to solve for the maximum motor current.

Stall current for 160rpm motor:

$$T_{stall\_160} := I_{stall} \cdot K_T \cdot r_{g\_160} \cdot \eta_{g\_160} - T_{0\_160}$$

$$T_{stall\_160} = 1.543\ N \cdot m$$

The maximum motor current for 160rpm motor:

$$I_{max\_160} := \frac{\left(\frac{T_{stall\_160}}{2}\right) + T_{0\_160}}{K_T \cdot r_{g\_160} \cdot \eta_{g\_160}}$$

$$I_{max\_160} = 2.961\ A$$

Armature resistance for 160rpm motor:

$$R_{a\_160} := \left(1.61 + 0.15 \cdot \left(\frac{I_{max\_160}}{1\ A} - 1.2\right)\right)\ \Omega$$

$$R_{a\_160} = 1.874\ \Omega$$

The maximum output torque for 160rpm motor:

$$T_{max\_160} := I_{max\_160} \cdot K_T \cdot r_{g\_160} \cdot \eta_{g\_160} - T_{0\_160}$$

$$T_{max\_160} = 0.772\ N \cdot m$$

Overload time to trip for 160rpm motor:

$$t_{trip\_160} := 140 \cdot \left(\frac{I_{max\_160}}{1 \cdot A}\right)^{-3.54} \cdot s$$

$$t_{trip\_160} = 3.001\ s$$

Output speed for 160rpm motor:

$$w_{m\_160} := \frac{V_{supply} - I_{max\_160} \cdot R_{a\_160}}{K_T \cdot r_{g\_160}}$$

$$w_{m\_160} = 18.045\ \frac{rad}{s}$$

For the 240 rpm motor

Gearbox reduction ratio: $r_{g\_240} := 65.33$

Gearbox efficiency: $\eta_{g\_240} := 0.75$

Static drag: $T_{0\_240} := 65 \cdot N \cdot mm$

As explained in figure 1 that the maximum motor operating current is when the motor current is at half the stall current. The output torque equation from figure 4 will be used to solve for the maximum motor current.

Stall current for 240rpm motor:

$$T_{stall\_240} := I_{stall} \cdot K_T \cdot r_{g\_240} \cdot \eta_{g\_240} - T_{0\_240}$$

$$T_{stall\_240} = 1.066\ N \cdot m$$

The maximum motor current for 240rpm motor:

$$I_{max\_240} := \frac{\left(\frac{T_{stall\_240}}{2}\right) + T_{0\_240}}{K_T \cdot r_{g\_240} \cdot \eta_{g\_240}}$$

$$I_{max\_240} = 2.976\ A$$

Armature resistance for 240rpm motor:

$$R_{a\_240} := \left(1.61 + 0.15 \cdot \left(\frac{I_{max\_240}}{1\ A} - 1.2\right)\right)\ \Omega$$

$$R_{a\_240} = 1.876\ \Omega$$

The maximum output torque for 240rpm motor:

$$T_{max\_240} := I_{max\_240} \cdot K_T \cdot r_{g\_240} \cdot \eta_{g\_240} - T_{0\_240}$$

$$T_{max\_240} = 0.533\ N \cdot m$$

Overload time to trip for 240rpm motor:

$$t_{trip\_240} := 140 \cdot \left(\frac{I_{max\_240}}{1 \cdot A}\right)^{-3.54} \cdot s$$

$$t_{trip\_240} = 2.948\ s$$

Output speed for 240rpm motor:

$$w_{m\_240} := \frac{V_{supply} - I_{max\_240} \cdot R_{a\_240}}{K_T \cdot r_{g\_240}}$$

$$w_{m\_240} = 26.94\ \frac{rad}{s}$$

$$rpm\_motor := 1$$

The results for the different motors can be summarised in the below table.

$Compare$	$Maximum\_current$ (A)	$Trip\_time$ (s)	$Output\_torque$ $(N \cdot m)$	$Output\_speed$ $\left(\frac{rad}{s}\right)$
$100\ rpm\_motor$	2.936	3.091	1.211	11.367
$160\ rpm\_motor$	2.961	3.001	0.772	18.045
$240\ rpm\_motor$	2.976	2.948	0.533	26.94

Thus, for this design the **100rpm motor** will be used because it has the largest output torque and longest trip time.

Largest output torque allows one to design the gearbox with a low gear ratio.

Longest trip time gives the design more time to achieve the goal before it stops working.