Appendix D: Deflection calculations

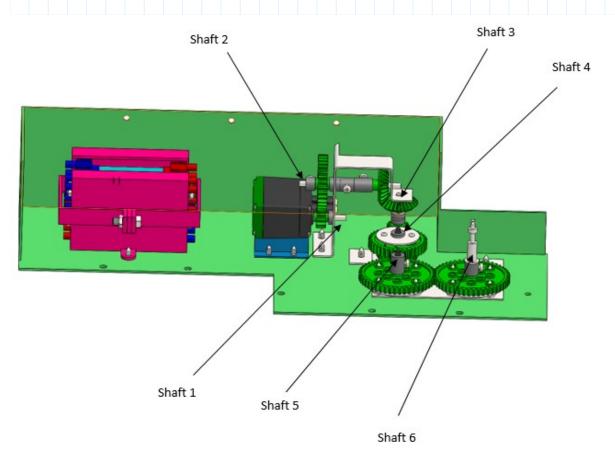


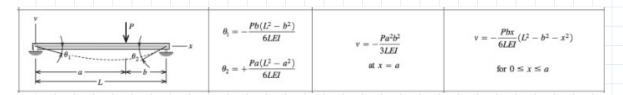
Figure 1: Shafts

Figure 1 above shows all the shafts that will be used in the prototype. It also shows all the the gears on the each shaft.

	Maximum	Experience	Maximum	Experience	Experience	Rotational
	Torque	d	tangential	d tangential	d radial	speed
	[Nm]	Torquo	force [N]	force [N]	force [N]	[rad/s]
		Torque [Nm]				
		[INIII]				
Gear 1 [12T]	3	0.941	450	148.228	53.95	14.537
Gear 2 [36T]	3.4	2.627	700	137.879	50.184	4.508
Gear 3 [24T]	3	2.627	300	206.819	75.276	4.508
Gear 4 [24T]	3	2.489	300	195.99	71.335	4.272
Gear 5 [12T]	3	2.489	450	391.98	142.669	4.272
Gear 6 [36T]	3.4	6.946	700	364	132.709	1.324
Gear 7 [24T]	1	6.946	580	546.923	199.064	1.324
Gear 8 [48T]	1.2	13.269	580	522.417	190.144	0.638
Gear 9 [48T]	1.2	12.922	580	508.74	185.166	0.621

Figure 1: Summary of the forces

The above table the shows all the forces calculated in **Appendix C** which will be used in this section to determine the deflection of each gear.



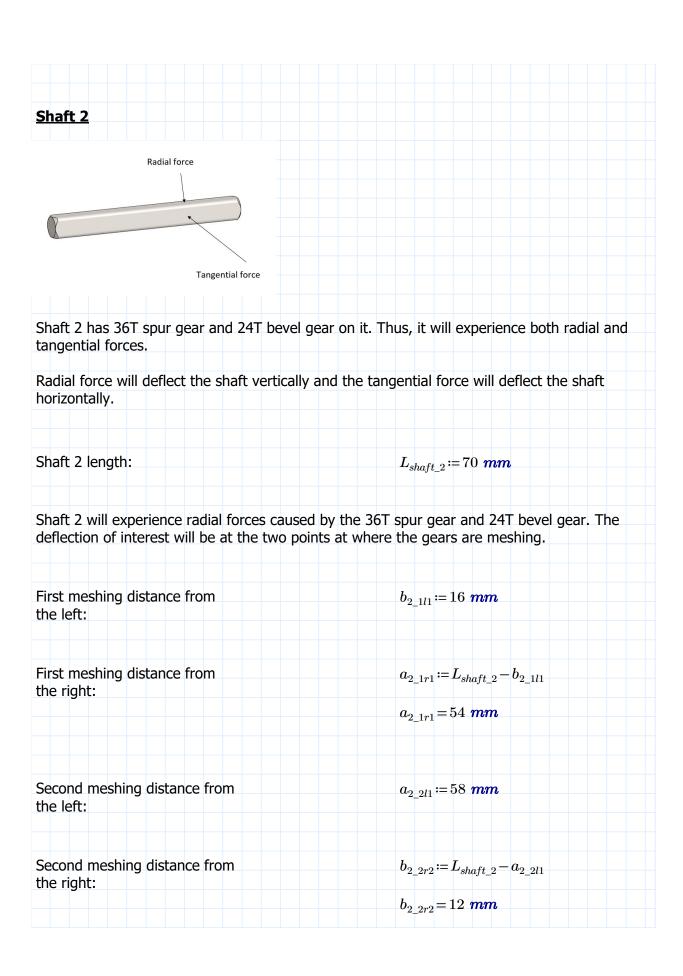
The above deflection equation from "Mechanic of Materials" 11th edition, written by Russell C. Hibbeler, will be used to determine the deflection of the shaft.

All the shafts are made with low carbon steel and will have the same elastic modulus. All the square shafts have the same cross section and will the same moment of inertia.

Low carbon steel elastic modulus:  $E_{shaft} \coloneqq 200 \cdot \textbf{\textit{GPa}}$  Cross sectional distance:  $D_{shaft} \coloneqq 3.18 \ \textbf{\textit{mm}}$ 

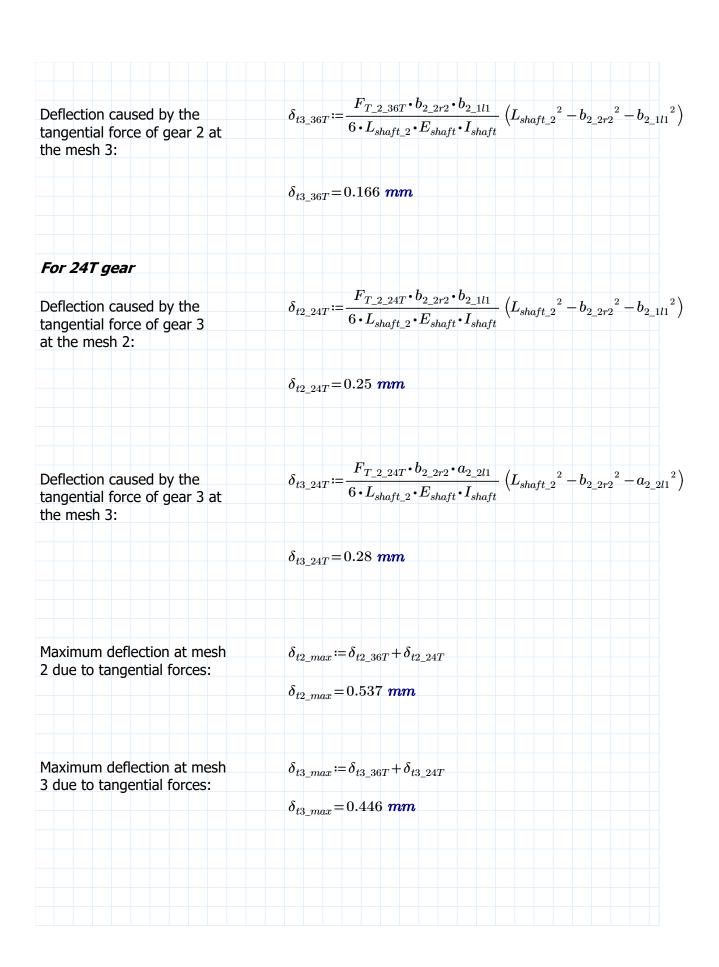
Moment of inertia:	$I_{shaft}\!\coloneqq\!rac{D_{shaft}^4}{12}$
	$I_{shaft}\!=\!8.522$ $m{mm}^4$
Shaft 1	
Radial force	
Tangential force	
Shaft 1 has 12T spur gear on it. Thus Shaft 1 has low carbon steel material	s, it will experience both radial and tangential forces.
	tically and the tangential force will deflect the shaft
horizontally.	
	$L_{shaft\_1}\!\coloneqq\!30\;m{mm}$
Shaft 1 length:	
Shaft 1 length:  Deflection due to radial force	
Shaft 1 length:  Deflection due to radial force  Radial force due to gear 1:  The distance from the	$L_{shaft\_1}\!\coloneqq\!30\;mm$
Shaft 1 length:  Deflection due to radial force  Radial force due to gear 1:  The distance from the boundary the radial force:	$L_{shaft\_1}$ := $30~mm$ $F_{R\_1}$ := $53.95~N$ $a_{r1}$ := $17.27~mm$
horizontally.  Shaft 1 length:  Deflection due to radial force  Radial force due to gear 1:  The distance from the boundary the radial force:  The distance from the other boundary the radial force:	$L_{shaft\_1}\!\coloneqq\!30~mm$ $F_{R\_1}\!\coloneqq\!53.95~N$

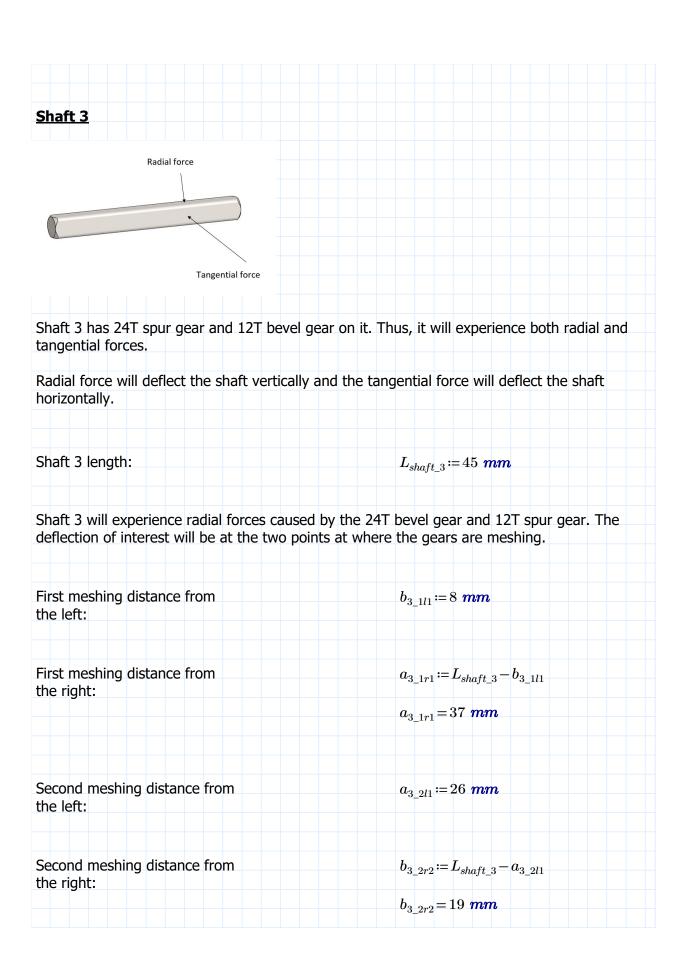
Maximum deflection caused by the radial force:	$\delta_{r1}$ :	$=rac{F_{R\_1}\!\cdot\! b_{r1}\!\cdot\! a_{r1}}{6\!\cdot\! L_{shaft\_1}\!\cdot\! E_{shaft}\!\cdot\! I_{shaft}}\left(L_{shaft\_1}^{\phantom$
	$\delta_{r1}$ =	=0.017 <i>mm</i>
Deflection due to tangential force		
Radial force due to gear 1:		$F_{T\_1} \coloneqq 148.228 \; \textbf{\textit{N}}$
The distance from the boundary the radial force:		$a_{t1}\!\coloneqq\!17.27\;m{mm}$
The distance from the other boundary the radial force:		$b_{t1}\!\coloneqq\!L_{shaft\_1}\!-\!a_{r1}$
		$b_{t1}$ = 12.73 $mm$
Maximum deflection caused by the radial force:	$\delta_{t1}$ :	$=rac{F_{T\_1}ullet b_{t1}ullet a_{t1}}{6ullet L_{shaft\_1}ullet E_{shaft}ullet I_{shaft}}\left(L_{shaft\_1}^2-b_{t1}^2-a_{t1}^2 ight)$
	$\delta_{t1}$ =	= 0.047 <i>mm</i>



Deflection due to radial force	
Radial force due to 36T spur gear:	$F_{R\_2\_36T}\coloneqq 50.184~ extbf{ extf{ extbf{ extbf{ ex} etf{ extbf{ ex}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$
Radial force due to 24T bevel gear:	$F_{R\_2\_24T}\!\coloneqq\!75.276~ extbf{ extit{N}}$
For 36T gear	
Deflection caused by the radial force of gear 2 at the mesh 2:	$\delta_{r2\_36T} \coloneqq \frac{F_{R\_2\_36T} \cdot b_{2\_1l1} \cdot a_{2\_1r1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}^{2} - b_{2\_1l1}^{2} - a_{2\_1r1}^{2}\right)$
	$\delta_{r2\_36T}\!=\!0.105~m{mm}$
Deflection caused by the radial force of gear 2 at the mesh 3:	$\delta_{r3\_36T} \coloneqq \frac{F_{R\_2\_36T} \cdot b_{2\_2r2} \cdot b_{2\_1l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}^{2} - b_{2\_2r2}^{2} - b_{2\_1l1}^{2}\right)$
	$\delta_{r3\_36T} \! = \! 0.061 \; m{mm}$
For 24T gear	
Deflection caused by the radial force of gear 3 at the mesh 2:	$\delta_{r2\_24T} \coloneqq \frac{F_{R\_2\_24T} \cdot b_{2\_2r2} \cdot b_{2\_1l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left( L_{shaft\_2}^{2} - b_{2\_2r2}^{2} - b_{2\_1l1}^{2} \right)$
	$\delta_{r2\_24T} \! = \! 0.091 \; m{mm}$

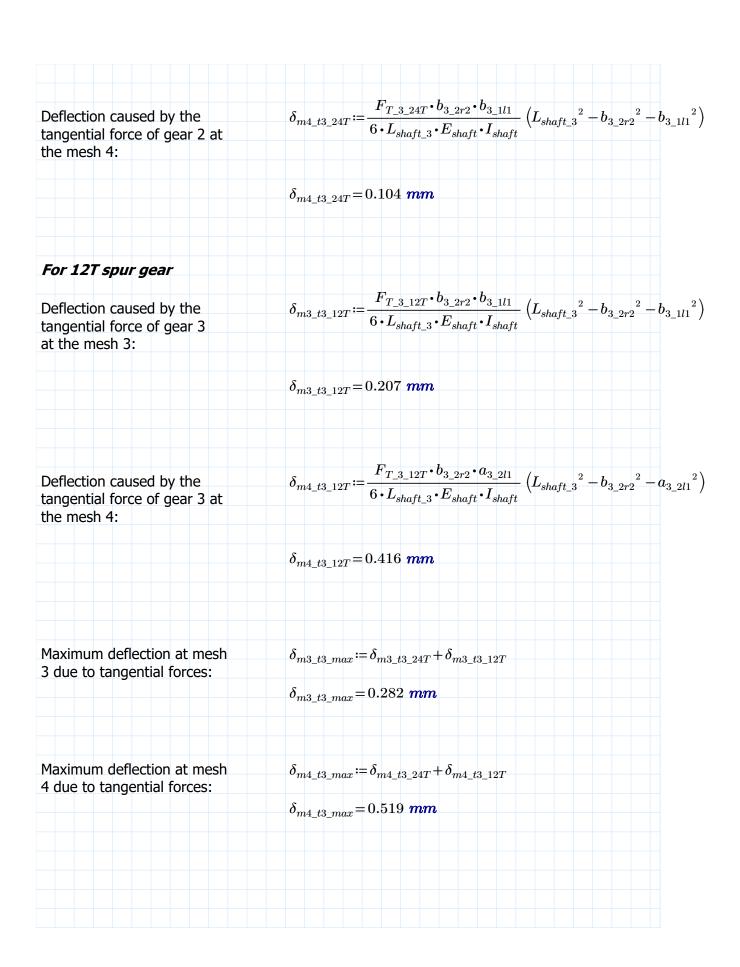
Deflection caused by the radial force of gear 3 at the mesh 3:	$\delta_{r3\_24T}$ :	$= \frac{F_{R\_2\_24T} \cdot b_{2\_2r2} \cdot a_{2\_2l1}}{6 \cdot L_{shaft\_2} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_2}^{2} - b_{2\_2r2}^{2} - a_{2\_2l1}^{2}\right)$
	$\delta_{r3\_24T}$ =	= 0.102 <i>mm</i>
Maximum deflection at mesh 2 due to radial forces:	_	$=\delta_{r2\_36T} + \delta_{r2\_24T}$
	$\delta_{r2\_max}$ =	=0.196 <i>mm</i>
Maximum deflection at mesh 3 due to radial forces:	$\delta_{r3\_max}$ :	$=\delta_{r3\_36T} + \delta_{r3\_24T}$
	$\delta_{r3\_max}$ :	=0.162 <i>mm</i>
Deflection due to tangential force		
Tangential force due to 36T spur gear:		$F_{T\_2\_36T} \coloneqq 137.879 \; \textit{N}$
Tangential force due to 24T bevel gear:		$F_{T\_2\_24T}$ := 206.819 $N$
For 36T gear		
Deflection caused by the tangential force of gear 2 at the mesh 2:	$\delta_{t2\_36T}$ :=	$=rac{F_{T\_2\_36T}ullet b_{2\_1l1}ullet a_{2\_1r1}}{6ullet L_{shaft\_2}ullet E_{shaft}ullet I_{shaft}}\left(L_{shaft\_2}^{2}-b_{2\_1l1}^{2}-a_{2\_1r1}^{2} ight)$
	$\delta_{t2\_36T}$ =	= 0.288 <i>mm</i>





Deflection due to radial force	
Radial force due to 24T bevel gear:	$F_{R\_3\_24T}$ := $71.335~ extbf{ extf{ extbf{ extf{ extbf{ ex}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$
Radial force due to 12T spur gear:	$F_{R\_3\_12T}$ $\coloneqq$ $142.669~N$
For 24T bevel gear	
Deflection caused by the radial force of gear 2 at the mesh 3:	$\delta_{m3\_r3\_24T} \coloneqq \frac{F_{R\_3\_24T} \cdot b_{3\_1l1} \cdot a_{3\_1r1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left( L_{shaft\_3}^{2} - b_{3\_1l1}^{2} - a_{3\_1r1}^{2} \right)$
	$\delta_{m3\_r3\_24T}\!=\!0.027\;m{mm}$
Deflection caused by the radial force of gear 2 at the mesh 4:	$\delta_{m4\_r3\_24T} \coloneqq \frac{F_{R\_3\_24T} \cdot b_{3\_2r2} \cdot b_{3\_1l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left( L_{shaft\_3}^2 - b_{3\_2r2}^2 - b_{3\_1l1}^2 \right)$
	$\delta_{m4\_r3\_24T} = 0.038 \; mm$
For 12T spur gear	
Deflection caused by the radial force of gear 3 at the mesh 3:	$\delta_{m3\_r3\_12T} \coloneqq \frac{F_{R\_3\_12T} \cdot b_{3\_2r2} \cdot b_{3\_1l1}}{6 \cdot L_{shaft\_3} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_3}^{2} - b_{3\_2r2}^{2} - b_{3\_1l1}^{2}\right)$
	$\delta_{m3\_r3\_12T}\!=\!0.075\;m{mm}$

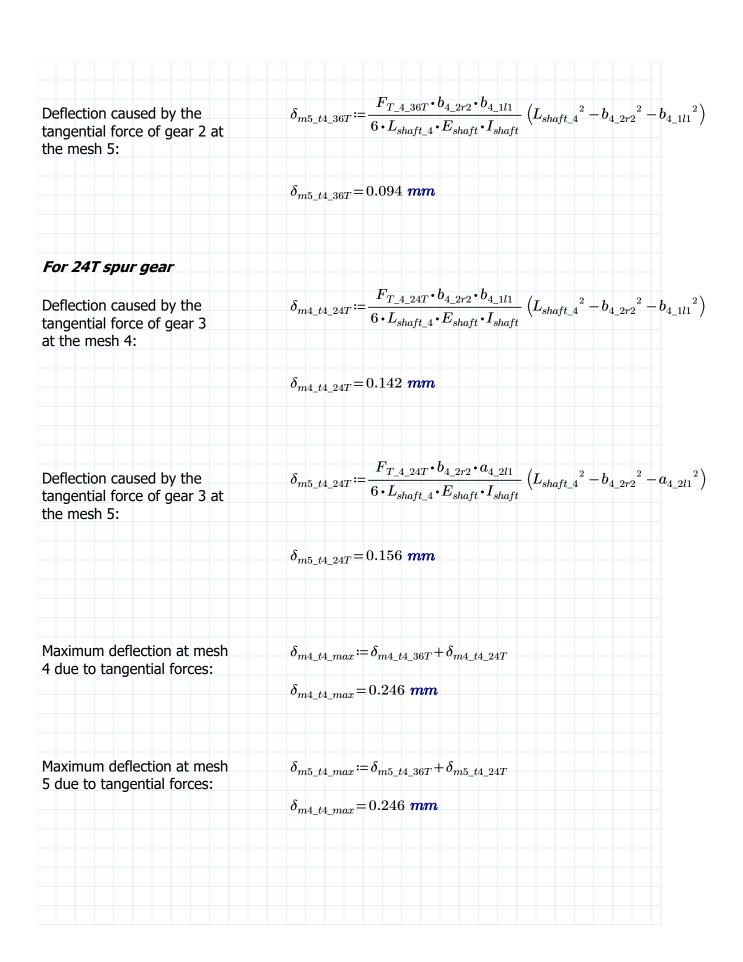
Deflection caused by the radial force of gear 4 at the mesh 4:	$\delta_{m4\_r3\_12T}$ := $rac{F_{R\_3\_12T} ullet b_{3\_2r2} ullet a_{3\_2l1}}{6 ullet L_{shaft\_3} ullet E_{shaft} ullet I_{shaft}} \left(L_{shaft\_3} ullet$	$a_{1}^{2} - b_{3\_2r2}^{2} - a_{3\_2l1}^{2}$
	$\delta_{m4\_r3\_12T} = 0.151 \; mm$	
Maximum deflection at mesh 3 due to radial forces:	$\delta_{m3\_r3\_max} := \delta_{m3\_r3\_24T} + \delta_{m3\_r3\_12T}$	
	$\delta_{m3\_r3\_max}$ $=$ $0.103~mm$	
Maximum deflection at mesh 4 due to radial forces:	$\delta_{m4\_r3\_max}\!:=\!\delta_{m4\_r3\_24T}\!+\!\delta_{m4\_r3\_12T}$	
	$\delta_{m4\_r3\_max}$ $=$ $0.189$ $m{mm}$	
Deflection due to tangential force		
Tangential force due to 36T spur gear:	$F_{T\_3\_24T}\!\coloneqq\!195.99~N$	
Tangential force due to 24T bevel gear:	$F_{T\_3\_12T}$ := $391.98~N$	
For 24T bevel gear		
Deflection caused by the tangential force of gear 2 at the mesh 3:	$\delta_{m3\_t3\_24T} \coloneqq rac{F_{T\_3\_24T} \! \cdot b_{3\_1l1} \! \cdot \! a_{3\_1r1}}{6 \! \cdot \! L_{shaft\_3} \! \cdot \! E_{shaft} \! \cdot \! I_{shaft}} \left( L_{shaft\_3} \! \cdot \! E_{shaft} \! \cdot \! I_{shaft}  ight)$	$_{lft\_3}{}^2-b_{3\_1l1}{}^2-a_{3\_1r1}{}^2$
	$\delta_{m3\_t3\_24T} \! = \! 0.075 \; mm$	



## Shaft 4 Radial force Tangential force Shaft 4 has 36T spur gear and 24T bevel gear on it. Thus, it will experience both radial and tangential forces. Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally. Shaft 4 length: $L_{shaft} = 30 \ \boldsymbol{mm}$ Shaft 3 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing. First meshing distance from $b_4 _{1l1} = 11 \ mm$ the left: First meshing distance from $a_4 _{1r1} := L_{shaft} _4 - b_4 _{1l1}$ the right: $a_{4-1r1} = 19 \ mm$ Second meshing distance from $a_4 _{2l1} = 19 \ mm$ the left: Second meshing distance from $b_{4\_2r2}\!\coloneqq\!L_{shaft\_4}\!-\!a_{4\_2l1}$ the right: $b_{4}|_{2r2} = 11 \ mm$

Deflection due to radial force	
Radial force due to 36T spur gear:	$F_{R\_4\_36T}$ := 132.709 $N$
Radial force due to 24T spur gear:	$F_{R\_4\_24T}$ := 199.064 $N$
For 36T spur gear	
Deflection caused by the radial force of gear 2 at the mesh 4:	$\delta_{m4\_r4\_36T} \coloneqq \frac{F_{R\_4\_36T} \cdot b_{4\_1l1} \cdot a_{4\_1r1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}^2 - b_{4\_1l1}^2 - a_{4\_1r1}^2 $
	$\delta_{m4\_r4\_36T}\!=\!0.038\;m{mm}$
Deflection caused by the radial force of gear 2 at the mesh 5:	$\delta_{m5\_r4\_36T} \coloneqq \frac{F_{R\_4\_36T} \cdot b_{4\_2r2} \cdot b_{4\_1l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left( L_{shaft\_4}^2 - b_{4\_2r2}^2 - b_{4\_1l1}^2 \right)$
	$\delta_{m5\_r4\_36T} \!=\! 0.034 \; m{mm}$
For 24T spur gear	
Deflection caused by the radial force of gear 3 at the mesh 4:	$\delta_{m4\_r4\_24T} \coloneqq \frac{F_{R\_4\_24T} \cdot b_{4\_2r2} \cdot b_{4\_1l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}^{2} - b_{4\_2r2}^{2} - b_{4\_1l1}^{2}\right)$
	$\delta_{m4\_r4\_24T}\!=\!0.052\;m{mm}$

Deflection caused by the radial force of gear 4 at the mesh 5:	$\delta_{m5\_r4\_24T}$ :=	$=\frac{F_{R\_4\_24T} \cdot b_{4\_2r2} \cdot a_{4\_2l1}}{6 \cdot L_{shaft\_4} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_4}^{2} - b_{4\_2r2}^{2} - a_{4\_2l1}^{2}\right)$
	$\delta_{m5\_r4\_24T}$ =	= 0.057 <i>mm</i>
Maximum deflection at mesh 4 due to radial forces:		$=\delta_{m4\_r4\_36T} + \delta_{m4\_r4\_24T}$
	$\delta_{m4\_r4\_max}$ =	= 0.089 <i>mm</i>
Maximum deflection at mesh 5 due to radial forces:	$\delta_{m5\_r4\_max}$ :	$=\delta_{m5\_r4\_36T} + \delta_{m5\_r4\_24T}$
	$\delta_{m4\_r4\_max}$ =	= 0.089 <i>mm</i>
Deflection due to tangential force		
Tangential force due to 36T spur gear:		$F_{T\_4\_36T}$ := $364~m{N}$
Tangential force due to 24T bevel gear:		$F_{T\_4\_24T} \!\coloneqq\! 546.923~ extbf{ extbf{N}}$
For 36T spur gear		
Deflection caused by the tangential force of gear 2 at the mesh 4:	$\delta_{m4\_t4\_36T}$ :=	$=rac{F_{T\_4\_36T}\! \cdot b_{4\_1l1}\! \cdot a_{4\_1r1}}{6\cdot L_{shaft\_4}\! \cdot \! E_{shaft}\! \cdot \! I_{shaft}}\left(L_{shaft\_4}\! ^2 - b_{4\_1l1}\! ^2 - a_{4\_1r1}\! ^2 ight)$
	$\delta_{m4\_t4\_36T} =$	=0.104 <i>mm</i>



## Shaft 5 Radial force Tangential force Shaft 5 has 48T spur gear on it. Thus, it will experience both radial and tangential forces. Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally. Shaft 5 length: $L_{shaft\_5} \coloneqq 30 \ \boldsymbol{mm}$ Shaft 5 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing. First meshing distance from $b_{5\ 1l1} = 18.57 \ mm$ the left: First meshing distance from $a_{5\_1r1}\!\coloneqq\!L_{shaft\_5}\!-\!b_{5\_1l1}$ the right: $a_{5\ 1r1} = 11.43 \ mm$

Deflection due to radial force	
Radial force due to 48T spur gear:	$F_{R\_5\_48T}\!\coloneqq\!190.144~N$
Deflection caused by the radial force of gear 2 at the mesh :	$\delta_{m5\_r5\_48T} \coloneqq \frac{F_{R\_5\_48T} \cdot b_{5\_1l1} \cdot a_{5\_1r1}}{6 \cdot L_{shaft\_5} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_5}^{2} - b_{5\_1l1}^{2} - a_{5\_1r1}^{2} \right)$ $\delta_{m5\_r5\_48T} = 0.056 \; \boldsymbol{mm}$
Deflection due to tangential force	
Tangential force due to 48T spur gear:	$F_{T\_5\_48T}\!\coloneqq\!522.417~N$
Deflection caused by the tangential force of gear 2 at the mesh 5:	$\delta_{m5\_t5\_48T} \coloneqq \frac{F_{T\_5\_48T} \cdot b_{5\_1l1} \cdot a_{5\_1r1}}{6 \cdot L_{shaft\_5} \cdot E_{shaft} \cdot I_{shaft}} \left( L_{shaft\_5}^{2} - b_{5\_1l1}^{2} - a_{5\_1r1}^{2} \right)$
	$\delta_{m5\_t5\_48T}\!=\!0.153\;m{mm}$

## Shaft 6 Radial force Tangential force Shaft 6 has 48T spur gear on it. Thus, it will experience both radial and tangential forces. Radial force will deflect the shaft vertically and the tangential force will deflect the shaft horizontally. Shaft 5 length: $L_{shaft 6} := 54.2 \ \boldsymbol{mm}$ Shaft 5 will experience radial forces caused by the 24T bevel gear and 12T spur gear. The deflection of interest will be at the two points at where the gears are meshing. First meshing distance from $b_{6\ 1l1} \coloneqq 11\ \boldsymbol{mm}$ the left: First meshing distance from $a_{6\_1r1}\!\coloneqq\!L_{shaft\_6}\!-\!b_{6\_1l1}$ the right: $a_{6\ 1r1} = 43.2 \ mm$

Deflection due to radial force	
Radial force due to 48T spur gear:	$F_{R\_6\_48T}$ $\coloneqq$ 185.166 $ extbf{ extit{N}}$
Deflection caused by the radial force of gear 2 at the mesh 6:	$\delta_{m6\_r6\_48T} \coloneqq rac{F_{R\_6\_48T} \! \cdot \! b_{6\_1l1} \! \cdot \! a_{6\_1r1}}{6 \cdot L_{shaft\_6} \cdot E_{shaft} \cdot I_{shaft}} \left(L_{shaft\_6}^{\ \ 2} - b_{6\_1l1}^{\ \ 2} - a_{6\_1r1}^{\ \ 2} - a_{6\_1r1}^{\ \ 2} \right) \ \delta_{m6\_r6\_48T} = 0.151 \ m{mm}$
Deflection due to tangential force	
Tangential force due to 48T spur gear:	$F_{T\_6\_48T}\!\coloneqq\!508.74~ extbf{ extit{N}}$
Deflection caused by the tangential force of gear 2 at the mesh 5:	$\delta_{m6\_t6\_48T} \coloneqq \frac{F_{T\_6\_48T} \! \cdot \! b_{6\_1l1} \! \cdot \! a_{6\_1r1}}{6 \! \cdot \! L_{shaft\_6} \! \cdot \! E_{shaft} \! \cdot \! I_{shaft}} \left( L_{shaft\_6}^{}^{2} - b_{6\_1l1}^{}^{2} - a_{6\_1r1}^{}^{2} \right)$
	$\delta_{m6\_t6\_48T}\!=\!0.415\;m{mm}$