Appendix C: Gear calculations

In **Appendix B** the 100 rpm motor was selected. Its calculated properties are listed below:

The maximum motor current: $I_{max} := 2.936\ A$

The maximum output torque: $T_{max} := 1.211\ N \cdot m$

The maximum output speed: $w_{max} := 11.367\ \frac{rad}{s}$

The maximum overload time to trip: $t_{max\_trip} := 3.091\ s$

In **Appendix A** the required bending moment to bend the pipe was determined to be the following:

Bending moment required to bend the pipe: $M_{required} := 12.922\ N \cdot m$

Before selecting the gears, the required gear ratio needs to be determined. The required gear ratio will be determined using the required bending moment and the peak output torque of the motor.

Required gear ratio:

$$GR_{required} := \frac{M_{required}}{T_{max}}$$

$$GR_{required} = 10.671$$

Therefore, the gear train ratio has to be equal to the required gear ratio taking into account all the efficiencies.

Through trial and error, and taking into account the shape of the model and the user requirements. Below is how the gear train is designed.

Gears	Driven by	Drives	On the same shaft as
12T spur gear	The motor	36T spur gear	-
36T spur gear	12T spur gear	-	24T bevel gear
24T bevel gear	-	24T bevel gear	36T spur gear
24T bevel gear	24T bevel gear	-	12T spur gear
12T spur gear	-	36T spur gear	24T bevel gear
36T spur gear	12T spur gear	-	24T spur gear
24T spur gear	-	48T spur gear	36T spur gear
48T spur gear	24T spur gear	-	-

Figure 2: Table summarising the gearbox design.

In the table above each gear is given its own colour to make it easy for the reader to follow. The table also shows which gears are driven, which gears drives and which gears are on the same shaft.

Using figure 1 and 2, the meshing gears can be shown in the table below.

	Meshing gears
Mesh 1	12T spur gear with 36T spur gear
Mesh 2	24T bevel gear with 24T bevel gear
Mesh 3	12T spur gear with 36T spur gear
Mesh 4	24T spur gear with 48T spur gear
Mesh 5	48T spur gear with 48T spur gear

Figure 3: Table showing meshing gears

Using the above table, the gear train efficiency can be calculated

$$\eta_{spur} = 1 - \mu\pi\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

The above equation from the gearbox efficiency document will be used to calculate the efficiency between two meshing spur gears.

The same equation will be used to calculate the efficiency for bevel gears because the friction on bevel gears is very similar to spur gears.

Assuming that the gear that will be used is made up of Dry Acetal on Acetal plastic, the coefficient that will be used is 0.2.

Mesh 1 (12T spur gear with 36T spur gear):

$$\eta_{mesh\_1} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{12} + \frac{1}{36}\right)$$

$$\eta_{mesh\_1} = 0.93$$

Mesh 2 (24T bevel gear with 24T bevel gear):

$$\eta_{mesh\_2} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{24} + \frac{1}{24}\right)$$

$$\eta_{mesh\_2} = 0.948$$

Mesh 3 (12T spur gear with 36T spur gear):

$$\eta_{mesh\_3} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{12} + \frac{1}{36}\right)$$

$$\eta_{mesh\_3} = 0.93$$

Mesh 4 (24T spur gear with 48T spur gear):

$$\eta_{mesh\_4} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{24} + \frac{1}{48}\right)$$

$$\eta_{mesh\_4} = 0.961$$

Mesh 5 (48T spur gear with 48T spur gear):

$$\eta_{mesh\_5} := 1 - 0.2 \cdot \pi \cdot \left(\frac{1}{48} + \frac{1}{48}\right)$$

$$\eta_{mesh\_5} = 0.974$$

Now that the efficiency for all the meshing gears is determined, the gear train efficiency can be determined.

Gear train efficiency:

$$\eta_{gear\_train} := \eta_{mesh\_1} \cdot \eta_{mesh\_2} \cdot \eta_{mesh\_3} \cdot \eta_{mesh\_4} \cdot \eta_{mesh\_5}$$

$$\eta_{gear\_train} = 0.767$$

The gear ratio of the gearbox can be determined using figure 3, taking into only meshing gears.

Gear 1 (12T spur gear with 36T spur gear):

$$GR_1 := \frac{36}{12}$$

$$GR_1 = 3$$

Gear 2 (24T bevel gear with 24T bevel gear):

$$GR_2 := \frac{24}{24}$$

$$GR_2 = 1$$

Gear 3 (12T spur gear with 36T spur gear):

$$GR_3 := \frac{36}{12}$$

$$GR_3 = 3$$

Gear 4 (24T spur gear with 48T spur gear):

$$GR_4 := \frac{48}{24}$$

$$GR_4 = 2$$

Gear 5 (48T spur gear with 48T spur gear):

$$GR_5 := \frac{48}{48}$$

$$GR_5 = 1$$

Now that the gear ratio for all the meshing gears is determined, the gear train ratio can be determined, taking into account the efficiency.

Gear train ratio:

$$GR_{gear\_train} := GR_1 \cdot GR_2 \cdot GR_3 \cdot GR_4 \cdot GR_5 \cdot \eta_{gear\_train}$$

$$GR_{gear\_train} = 13.808$$

The gear train ratio works out perfectly because it greater than the required gear ratio, which is $GR_{required} = 10.671$.

Now the actual current, output torque, output speed and trip time for the motor can be calculated.

Actual output motor torque:

$$T_{actual} := \frac{M_{required}}{GR_{gear\_train}}$$

$$T_{actual} = 0.936\ N \cdot m$$

Output torque $[Nm]$	$T_m = I_m K_T r_g \eta_g - T_0$	I_m = motor current

Figure 4: Motor torque equation

The above equation is from the motor specification document, it will be used to calculate the operating motor current. The motor specification document also gives the values for the constant in the equation.

Gearbox reduction ratio: $r_g := 156.8$

Gearbox efficiency: $\eta_g := 0.7$

Static drag: $T_0 := 110 \cdot N \cdot mm$

Torque constant: $K_T := 4.1\ N \cdot \frac{mm}{A}$

Armature resistance: $R_a := 1.87\ \Omega$

Operating motor current:

$$I_{actual} := \frac{T_{actual} - T_0}{K_T \cdot r_g \cdot \eta_g}$$

$$I_{actual} = 1.835\ A$$

Output speed $[rad/s]$	$\omega_m = \frac{V_b - I_m R_a}{K_T r_g}$	V_b = supply voltage

Figure 5: Motor speed equation

The above equation is from the motor specification document, it will be used to calculate the motor output speed.

Supply voltage to the motor calculated in Appendix B: $V_{supply} := 12.8\ V$

Motor output speed:

$$w_{actual} := \frac{V_{supply} - I_{actual} \cdot R_a}{K_T \cdot r_g}$$

$$w_{actual} = 14.573\ \frac{rad}{s}$$

Overload time to trip: $t_{actual\_trip} := 140 \cdot \left(\frac{I_{actual}}{1 \cdot A} \right) \cdot s$

$t_{actual\_trip} = 16.322\ s$

	Peak condition	Actual condition
Current [A]	2.936	1.847
Output speed [rad/s]	11.367	14.537
Output torque [Nm]	1.211	0.941
Trip time [s]	3.091	15.948

Figure 6: Comparing peak conditions and actual conditions

The table above the shows the difference when the motor is operating at its peak and at its actual point.

Now the torque and forces experienced by each gear can calculated. The efficiencies for all the meshing gears are calculated.

Gear 1

Gear 1 is a spur gear driven by the motor through a shaft.

Number of teeth of gear 1: $N_{gear\_1} := 12$

Diameter of gear 1: $D_{gear\_1} := 12.7\ mm$

Pressure angle of gear 1: $\theta_{gear\_1} := 20\ °$

Speed of gear 1: $w_{gear\_1} := w_{actual}$

$w_{gear\_1} = 14.573\ \frac{rad}{s}$

Torque of gear 1: $T_{gear\_1} := T_{actual}$

$T_{gear\_1} = 0.936\ N \cdot m$

Tangential force of gear 1: $F_{tan\_gear\_1} := \dfrac{T_{gear\_1}}{D_{gear\_1} \cdot 0.5}$

$F_{tan\_gear\_1} = 147.374\ N$

Radial force of gear 1: $F_{rad\_gear\_1} := F_{tan\_gear\_1} \cdot \tan\left(\theta_{gear\_1}\right)$

$F_{rad\_gear\_1} = 53.64\ N$

Gear 2

Gear 2 is a spur gear that meshes with gear 1. The efficiency is already calculated earlier in this section.

Number of teeth of gear 2: $N_{gear\_2} := 36$

Diameter of gear 2: $D_{gear\_2} := 38.1\ mm$

Pressure angle of gear 2: $\theta_{gear\_2} := 20\ °$

Speed of gear 2: $w_{gear\_2} := \dfrac{N_{gear\_1}}{N_{gear\_2}} \cdot w_{gear\_1} \cdot \eta_{mesh\_1}$

$w_{gear\_2} = 4.518\ \dfrac{rad}{s}$

Torque of gear 2: $T_{gear\_2} := \frac{N_{gear\_2}}{N_{gear\_1}} \cdot T_{gear\_1} \cdot \eta_{mesh\_1}$

$T_{gear\_2} = 2.611\ N \cdot m$

Tangential force of gear 2: $F_{tan\_gear\_2} := \frac{T_{gear\_2}}{D_{gear\_2} \cdot 0.5}$

$F_{tan\_gear\_2} = 137.085\ N$

Radial force of gear 2: $F_{rad\_gear\_2} := F_{tan\_gear\_2} \cdot \tan\left(\theta_{gear\_2}\right)$

$F_{rad\_gear\_2} = 49.895\ N$

Gear 3

Gear 3 is a bevel gear and is on the shaft as gear 2.

Number of teeth of gear 3: $N_{gear\_3} := 24$

Diameter of gear 3: $D_{gear\_3} := 25.4\ mm$

Pressure angle of gear 3: $\theta_{gear\_3} := 20\ °$

Speed of gear 3: $w_{gear\_3} := w_{gear\_2}$

$w_{gear\_3} = 4.518\ \frac{rad}{s}$

Torque of gear 3: $T_{gear\_3} := T_{gear\_2}$

$$T_{gear\_3} = 2.611\ N \cdot m$$

Tangential force of gear 3:

$$F_{tan\_gear\_3} := \frac{T_{gear\_3}}{D_{gear\_3} \cdot 0.5}$$

$$F_{tan\_gear\_3} = 205.627\ N$$

Radial force of gear 3:

$$F_{rad\_gear\_3} := F_{tan\_gear\_3} \cdot \tan\left(\theta_{gear\_1}\right)$$

$$F_{rad\_gear\_3} = 74.842\ N$$

Gear 4

Gear 4 is a bevel gear that meshes with gear 3. The efficiency is already calculated earlier in this section.

Number of teeth of gear 4: $N_{gear\_4} := 24$

Diameter of gear 4: $D_{gear\_4} := 25.4\ mm$

Pressure angle of gear 4: $\theta_{gear\_4} := 20\ °$

Speed of gear 4:

$$w_{gear\_4} := \frac{N_{gear\_3}}{N_{gear\_4}} \cdot w_{gear\_3} \cdot \eta_{mesh\_2}$$

$$w_{gear\_4} = 4.282\ \frac{rad}{s}$$

Torque of gear 4:

$$T_{gear\_4} := \frac{N_{gear\_4}}{N_{gear\_3}} \cdot T_{gear\_3} \cdot \eta_{mesh\_2}$$

$$T_{gear\_4} = 2.475\ N \cdot m$$

Tangential force of gear 4:

$$F_{tan\_gear\_4} := \frac{T_{gear\_4}}{D_{gear\_4} \cdot 0.5}$$

$$F_{tan\_gear\_4} = 194.861\ N$$

Radial force of gear 4:

$$F_{rad\_gear\_4} := F_{tan\_gear\_4} \cdot \tan\left(\theta_{gear\_4}\right)$$

$$F_{rad\_gear\_4} = 70.924\ N$$

Gear 5

Gear 5 is a spur gear and it's on the shaft as gear 4.

Number of teeth of gear 5: $N_{gear\_5} := 12$

Diameter of gear 5: $D_{gear\_5} := 12.7\ mm$

Pressure angle of gear 5: $\theta_{gear\_5} := 20\ °$

Speed of gear 5:

$$w_{gear\_5} := w_{gear\_4}$$

$$w_{gear\_5} = 4.282\ \frac{rad}{s}$$

Torque of gear 5:

$$T_{gear\_5} := T_{gear\_4}$$

$$T_{gear\_5} = 2.475\ N \cdot m$$

Tangential force of gear 5:

$$F_{tan\_gear\_5} := \frac{T_{gear\_5}}{D_{gear\_5} \cdot 0.5}$$

$$F_{tan\_gear\_5} = 389.722\ N$$

Radial force of gear 5:

$$F_{rad\_gear\_5} := F_{tan\_gear\_5} \cdot \tan\left(\theta_{gear\_5}\right)$$

$$F_{rad\_gear\_5} = 141.847\ N$$

Gear 6

Gear 6 is a spur gear that meshes with gear 5. The efficiency is already calculated earlier in this section.

Number of teeth of gear 6: $N_{gear\_6} := 36$

Diameter of gear 6: $D_{gear\_6} := 38.1\ mm$

Pressure angle of gear 6: $\theta_{gear\_6} := 20\ °$

Speed of gear 6:

$$w_{gear\_6} := \frac{N_{gear\_5}}{N_{gear\_6}} \cdot w_{gear\_5} \cdot \eta_{mesh\_3}$$

$$w_{gear\_6} = 1.328\ \frac{rad}{s}$$

Torque of gear 6:

$$T_{gear\_6} := \frac{N_{gear\_6}}{N_{gear\_5}} \cdot T_{gear\_5} \cdot \eta_{mesh\_3}$$

$$T_{gear\_6} = 6.906 \; N \cdot m$$

Tangential force of gear 6:

$$F_{tan\_gear\_6} := \frac{T_{gear\_6}}{D_{gear\_6} \cdot 0.5}$$

$$F_{tan\_gear\_6} = 362.514 \; N$$

Radial force of gear 6:

$$F_{rad\_gear\_6} := F_{tan\_gear\_6} \cdot \tan\left(\theta_{gear\_6}\right)$$

$$F_{rad\_gear\_6} = 131.944 \; N$$

Gear 7

Gear 7 is a spur gear and it's on the shaft as gear 6.

Number of teeth of gear 7: $N_{gear\_7} := 24$

Diameter of gear 7: $D_{gear\_7} := 25.4 \; mm$

Pressure angle of gear 7: $\theta_{gear\_7} := 20 \; °$

Speed of gear 7:

$$w_{gear\_7} := w_{gear\_6}$$

$$w_{gear\_7} = 1.328 \; \frac{rad}{s}$$

Torque of gear 7: $T_{gear\_7} := T_{gear\_6}$

$$T_{gear\_7} = 6.906\ N \cdot m$$

Tangential force of gear 7:

$$F_{tan\_gear\_7} := \frac{T_{gear\_7}}{D_{gear\_7} \cdot 0.5}$$

$$F_{tan\_gear\_7} = 543.771\ N$$

Radial force of gear 7:

$$F_{rad\_gear\_7} := F_{tan\_gear\_7} \cdot \tan\left(\theta_{gear\_7}\right)$$

$$F_{rad\_gear\_7} = 197.916\ N$$

Gear 8

Gear 8 is a spur gear that meshes with gear 7. The efficiency is already calculated earlier in this section.

Number of teeth of gear 8:

$$N_{gear\_8} := 48$$

Diameter of gear 8:

$$D_{gear\_8} := 50.8\ mm$$

Pressure angle of gear 8:

$$\theta_{gear\_8} := 20\ °$$

Speed of gear 8:

$$w_{gear\_8} := \frac{N_{gear\_7}}{N_{gear\_8}} \cdot w_{gear\_7} \cdot \eta_{mesh\_4}$$

$$w_{gear\_8} = 0.638\ \frac{rad}{s}$$

Torque of gear 8:

$$T_{gear\_8} := \frac{N_{gear\_8}}{N_{gear\_7}} \cdot T_{gear\_7} \cdot \eta_{mesh\_4}$$

$$T_{gear\_8} = 13.269 \ N \cdot m$$

Tangential force of gear 8:

$$F_{tan\_gear\_8} := \frac{T_{gear\_8}}{D_{gear\_8} \cdot 0.5}$$

$$F_{tan\_gear\_8} = 522.417 \ N$$

Radial force of gear 8:

$$F_{rad\_gear\_8} := F_{tan\_gear\_8} \cdot \tan\left(\theta_{gear\_8}\right)$$

$$F_{rad\_gear\_8} = 190.144 \ N$$

Gear 9

Gear 9 is a spur gear that meshes with gear 8. The efficiency is already calculated earlier in this section.

Number of teeth of gear 8:

$$N_{gear\_9} := 48$$

Diameter of gear 8:

$$D_{gear\_9} := 50.8 \ mm$$

Pressure angle of gear 8:

$$\theta_{gear\_9} := 20 \ °$$

Speed of gear 8:

$$w_{gear\_9} := \frac{N_{gear\_8}}{N_{gear\_9}} \cdot w_{gear\_8} \cdot \eta_{mesh\_5}$$

$$w_{gear\_9} = 0.621 \ \frac{rad}{s}$$

Torque of gear 8:

$$T_{gear\_9} := \frac{N_{gear\_9}}{N_{gear\_8}} \cdot T_{gear\_8} \cdot \eta_{mesh\_5}$$

$$T_{gear\_9} = 12.922 \ N \cdot m$$

Tangential force of gear 8:

$$F_{tan\_gear\_9} := \frac{T_{gear\_9}}{D_{gear\_9} \cdot 0.5}$$

$$F_{tan\_gear\_9} = 508.74 \ N$$

Radial force of gear 8:

$$F_{rad\_gear\_9} := F_{tan\_gear\_9} \cdot \tan\left(\theta_{gear\_9}\right)$$

$$F_{rad\_gear\_9} = 185.166 \ N$$

	Maximum Torque [Nm]	Experienced Torque [Nm]	Maximum tangential force [N]	Experienced tangential force [N]	Experienced radial force [N]	Rotational speed [rad/s]
Gear 1 [12T]	3	0.941	450	148.228	53.95	14.537
Gear 2 [36T]	3.4	2.627	700	137.879	50.184	4.508
Gear 3 [24T]	3	2.627	300	206.819	75.276	4.508
Gear 4 [24T]	3	2.489	300	195.99	71.335	4.272
Gear 5 [12T]	3	2.489	450	391.98	142.669	4.272
Gear 6 [36T]	3.4	6.946	700	364	132.709	1.324
Gear 7 [24T]	1	6.946	580	546.923	199.064	1.324
Gear 8 [48T]	1.2	13.269	580	522.417	190.144	0.638
Gear 9 [48T]	1.2	12.922	580	508.74	185.166	0.621

Figure 6: Summary of gear calculations

The above highlights which gears must be support in red, this is because the torque that their experiencing is more than the torque it can withstand.

From the above table the following must be supported:
Gear 6 (36T spur gear)
Gear 7 (24T spur gear)
Gear 8 (48T spur gear)
Gear 9 (48T spur gear)