

$$\min_x f(x) = -\min\left\{\frac{x}{2}, 2-(x-3)^2, 2-\frac{x}{2}\right\} \text{ on } [0, 8].$$

解: 令 $a_1 = 0, d_1 = 8$

$\therefore f(x)$ 在 $[0, 8]$ 上是单峰函数,

\therefore 在 $[0, 8]$ 上存在一个局部最小 s^* , $f(x)$ 在 $[a_1, s^*]$ 上单调递减, 在 $[s^*, d]$ 上单调递增

令 $\lambda = \frac{1}{2}(\sqrt{5}-1) \approx 0.6180$, 构造 $[a_1, d_1]$ 的三个子区间:

$$\begin{aligned} b_1 &= \lambda a_1 + (1-\lambda)d_1 \\ c_1 &= (1-\lambda)a_1 + \lambda d_1 \end{aligned} \Rightarrow \begin{cases} b_1 = 3.056 \\ c_1 = 4.944 \end{cases}$$

$$\text{得 } f(b_1) = -\min\{1.528, 1.997, 0.472\} = -0.472$$

$$f(c_1) = -\min\{2.492, -1.779, -0.472\} = 1.779$$

$$\therefore f(b_1) < f(c_1),$$

\therefore 最小值处于 $[a_1, c_1]$ 上

第1次迭代: 定义 $a_2 = a_1 = 0, d_2 = c_1 = 4.944$

$$\therefore b_2 = \lambda a_2 + (1-\lambda)d_2 = 1.889$$

$$c_2 = (1-\lambda)a_2 + \lambda d_2 = 3.055$$

$$\text{得 } f(b_2) = -\min\{0.945, 0.766, 1.056\} = -0.766$$

$$f(c_2) = -\min\{1.528, 1.997, 0.473\} = -0.473$$

$$\therefore f(b_2) < f(c_2)$$

\therefore 最小值在 $[a_2, c_2]$ 上

第2次迭代: 定义 $a_3 = a_2 = 0, d_3 = c_2 = 3.055$

$$\therefore b_3 = \lambda a_3 + (1-\lambda)d_3 = 1.167$$

$$c_3 = (1-\lambda)a_3 + \lambda d_3 = 1.888$$

$$\text{得 } f(b_3) = -\min\{0.584, -1.360, 1.417\} = 1.360$$

$$f(c_3) = -\min\{0.944, 0.763, 1.056\} = -0.763$$

$$\therefore f(b_3) > f(c_3)$$

\therefore 最小值处于 $[b_3, d_3]$

第3次迭代: 定义 $a_4 = b_3 = 1.167$, $d_4 = d_3 = 3.055$

$$\therefore b_4 = \lambda a_4 + (1-\lambda)d_4 = 1.888$$

$$c_4 = (1-\lambda)a_4 + \lambda d_4 = 2.334$$

$$\therefore f(b_4) = -\min\{0.944, 0.763, 1.056\} = -0.763$$

$$f(c_4) = -\min\{1.167, 1.556, 0.833\} = -0.833$$

$$\therefore f(b_4) > f(c_4)$$

\therefore 最小值处于 $[b_4, d_4]$

第4次迭代: 定义 $a_5 = b_4 = 1.888$, $d_5 = d_4 = 3.055$

$$\therefore b_5 = \lambda a_5 + (1-\lambda)d_5 = 2.334$$

$$c_5 = (1-\lambda)a_5 + \lambda d_5 = 2.609$$

$$\text{得 } f(b_5) = -\min\{1.167, 1.556, 0.833\} = -0.833$$

$$f(c_5) = -\min\{1.305, 1.747, 0.696\} = -0.696$$

$$\therefore f(b_5) < f(c_5)$$

\therefore 最小值处于 $[a_5, c_5]$

第5次迭代: 定义 $a_6 = a_5 = 1.888$, $d_6 = c_5 = 2.609$

$$\therefore b_6 = \lambda a_6 + (1-\lambda)d_6 = 2.163$$

$$c_6 = (1-\lambda)a_6 + \lambda d_6 = 2.334$$

$$\therefore f(b_6) = -\min\{1.082, 1.299, 0.919\} = -0.918$$

$$f(c_6) = -\min\{1.167, 1.556, 0.833\} = -0.833$$

\therefore 经过5次迭代, $f(x)$ 的最小值为 -0.918