Homework 4 - Continuous Optimization

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holà les gus
ce devoir, Sam'enchante pas, vous?
(un peu cass-é comme blague je l'avoue, mais gardez ez-poir en moi please)

Question 1

The feasible set is $S = \{x, y \in \mathbb{R}^n | h(x, y) = 0\} = \{x, y \in \mathbb{R}^n | 1 - x^\top x = 0, 1 - y^\top y = 0, x^\top y = 0\}$.
It is not convex. Indeed, we will give two points z_1 and $z_2 \in S$, but such that $z = \lambda z_1 + (1 - \lambda) z_2 \notin S$ for a given λ. We will work with these $z_i \in \mathbb{R}^2 \times \mathbb{R}^2$ i.e. $n = 2$.

We will take $z_1 = (x_1, y_1) = ((1, 0), (0, 1))$. First we check that $z_1 \in S$.

- $1 - x_1^\top x_1 = 1 - \langle (1, 0), (1, 0) \rangle = 1 - 1 = 0$
- $1 - y_1^\top y_1 = 1 - \langle (0, 1), (0, 1) \rangle = 1 - 1 = 0$
- $x_1 \top y_1 = \langle (1, 0), (0, 1) \rangle = 0$

And we will take $z_2 = (x_2, y_2) = ((0, 1), (1, 0))$. we also check that $z_2 \in S$.

- $1 - x_2^\top x_2 = 1 - \langle (0, 1), (0, 1) \rangle = 1 - 1 = 0$
- $1 - y_2^\top y_2 = 1 - \langle (1, 0), (1, 0) \rangle = 1 - 1 = 0$
- $x_2 \top y_2 = \langle (0, 1), (1, 0) \rangle = 0$

Question 2

Question 3

Question 4

Question 5

Question 6

Question 7

Question 8