Introduction to AI and ML

Matrix Project

A.AVINASH, EE17BTECH11005
K.DEVENDER, EE17BTECH11015

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Q.no.55 in JEE Mains(2005)

An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

Let 'a' be the length of semi major axis.
Let 'b' be the length of semi minor axis.
$O(0,0)$ is centre of ellipse
F and F' are focii
we know,

$$\|OF\| = \sqrt{a^2 - b^2}$$

Let $\mathbf{X}$ be the direction vector of OF

$$OF = \sqrt{a^2 - b^2}\mathbf{X}$$

$$\text{where } \|X\| = 1$$

$$So, \quad \mathbf{X}^T\mathbf{X} = 1 \qquad \textbf{(1)}$$

Similarly for OF'

$$OF' = -\sqrt{a^2 - b^2}\mathbf{X}$$

As OB is perpendicular to OF and as $\|OB\| = b$

So,

$$OB = b \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{X}$$

And also

$$(OB)^T (OF) = 0$$

$$\mathbf{X}^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{X} = 0 \qquad \textbf{(2)}$$

We know

$$BF = OF - OB$$

$$\implies BF = \sqrt{a^2 - b^2}\mathbf{X} - b\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{X}$$

$$BF' = OF' - OB$$

$$\implies BF' = -\sqrt{a^2 - b^2}\mathbf{X} - b\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{X}$$

As BF is perpendicular to BF', we have

$$(BF)^T(BF') = 0$$

$$\implies (\sqrt{a^2 - b^2}\mathbf{X}^T - b\mathbf{X}^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix})$$
$$(\text{-}\sqrt{a^2 - b^2}\mathbf{X} - b \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{X}) = 0$$

$$\implies -(a^2-b^2)\mathbf{X}^T\mathbf{X} - b\sqrt{a^2-b^2}\mathbf{X}^T\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\mathbf{X} +$$

$$b\sqrt{a^2-b^2}\mathbf{X}^T\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\mathbf{X} + b^2\mathbf{X}^T\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\mathbf{X} = 0$$

$$\implies -(a^2-b^2) + b^2\mathbf{X}^T(\mathbf{I})\mathbf{X} = 0$$

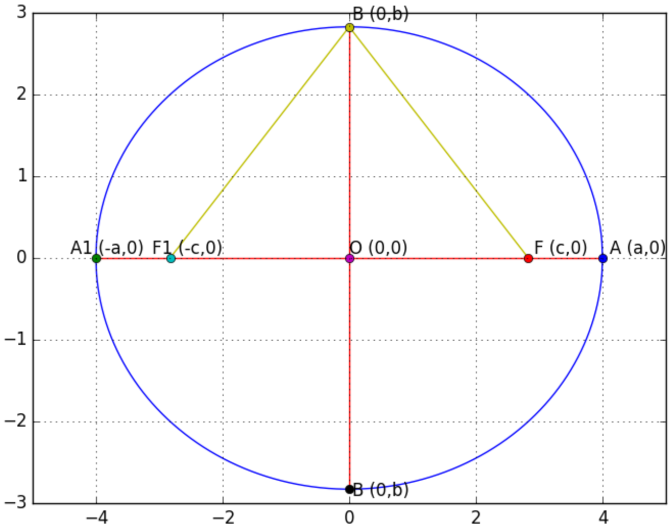
$$2b^2 = a^2$$

$$\frac{b^2}{a^2} = \frac{1}{2}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \frac{1}{\sqrt{2}}$$

Therefore eccentricity of ellipse $= \dfrac{1}{\sqrt{2}}$


B (0,b)
A1 (-a,0)
F1 (-c,0)
O (0,0)
F (c,0)
A (a,0)
B (0,b)
3
2
1
0
−1
−2
−3
−4
−2
0
2
4