Introduction to AI and ML

Matrix Project

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If the lines 2x+3y+1=0 and 3x-y-4=0 lie along diameter of a circle of circumference 10π, then the equation of the circle is

Given line equation is $2x + 3y + 1 = 0$

Line equation in matrix form $\begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{X} = \text{-1}$ **(1)**

Given line equation is $3x - y - 4 = 0$

Line equation in matrix form $\begin{bmatrix} 3 & -1 \end{bmatrix} \mathbf{X} = 4$ **(2)**

As given two lines lie along the diameters of the circle, so thier intersection point is the centre of the circle

Solving equation **(1)** and**(2)**

$$\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} \mathbf{X} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

To find the inverse we use Guass-Jordan method

Consider the matrix

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - \frac{3}{2} R_1$$

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 0 & -\frac{11}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

$$R_1 \leftarrow R_1 + \frac{6}{11} R_2$$

$$\begin{bmatrix} 2 & 0 & \frac{2}{11} & \frac{6}{11} \\ 0 & -\frac{11}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

$$R_1 \leftarrow \frac{R_1}{2}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{11} & \frac{3}{11} \\ 0 & -\frac{11}{2} & -\frac{3}{2} & 1 \end{bmatrix}$$

$$R_2 \leftarrow \frac{R_2}{2}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{11} & \frac{3}{11} \\ 0 & 1 & -\frac{3}{11} & -\frac{2}{11} \end{bmatrix}$$

Therefore

$$\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{3}{11} \\ -\frac{3}{11} & -\frac{2}{11} \end{bmatrix}$$

$$\mathbf{X} = \frac{1}{11}\begin{bmatrix}1 & 3\\ 3 & -2\end{bmatrix}\begin{bmatrix}-1\\ 4\end{bmatrix}$$

$$\mathbf{X} = \frac{1}{11}\begin{bmatrix}11\\ -11\end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix}1\\ -1\end{bmatrix}$$

Therefore centre of circle is $\mathbf{C}=\begin{bmatrix}1\\ -1\end{bmatrix}$

Given circumference of the circle is 10π

$\therefore 2\pi r = 10\pi$
We get r = 5
The general equation of the circle is

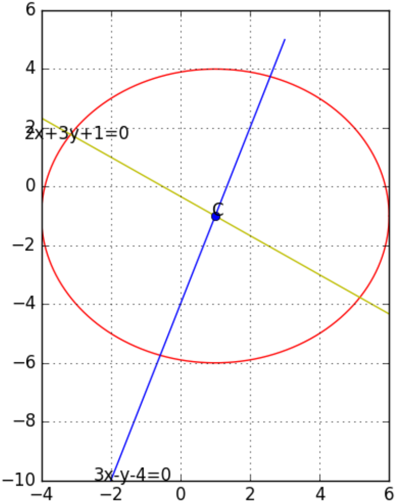
$$\mathbf{X}^T\mathbf{X} - 2\mathbf{C}^T\mathbf{X} = r^2 - \mathbf{C}^T\mathbf{C}$$

$$\mathbf{X}^T\mathbf{X} - 2\begin{bmatrix}1 & -1\end{bmatrix}\mathbf{X} = 25 - 2$$

$$\mathbf{X}^T\mathbf{X} - 2\begin{bmatrix}1 & -1\end{bmatrix}\mathbf{X} = 23$$

Above circle equation can also be written as

$$x^2 + y^2 - 2x + 2y - 23 = 0$$



2x+3y+1=0
C
3x-y-4=0