Introduction to AI and ML

Matrix Project

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If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

Given equation of parabola is

$$y^2 - 4x = 0$$

Given equation in matrix form:

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 2 \begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + 0 = 0$$

General form of parabola is

$$\mathbf{x}^T \mathbf{v} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + F = 0$$

Comparing given equation with general form of parabola

$$\mathbf{v}=\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

$$\mathbf{u}^T = \begin{bmatrix}-2 & 0\end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix}-2\\0\end{bmatrix}$$

$$F = 0$$

Let P and Q are the point of contacts of the tangents drawn on to the parabola

Then P and Q are of form $P(a^2, 2a)$ and $Q(b^2, 2b)$
Equation of tangent to parabola at point P is

$$(\mathbf{P}^T\mathbf{v} + \mathbf{u}^T)\mathbf{x} + \mathbf{P}^T\mathbf{u} + F = 0$$

$$\left(\begin{bmatrix} a^2 & 2a \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \end{bmatrix}\right)\mathbf{x} + \begin{bmatrix} a^2 & 2a \end{bmatrix}\begin{bmatrix} -2 \\ 0 \end{bmatrix} = 0$$

$\begin{bmatrix}-2 & 2a\end{bmatrix}\mathbf{x}$ - 2a$^2 = 0$

$$\begin{bmatrix}-1 & a\end{bmatrix}\mathbf{x} = a^2 \qquad \textbf{(1)}$$

Similarly, tangent at point Q is

$$\begin{bmatrix}-1 & b\end{bmatrix}\mathbf{x} = b^2 \qquad \textbf{(2)}$$

Given two tangents are perpendicular, we have $\begin{bmatrix}-1 & -a\end{bmatrix}\begin{bmatrix}-1\\ b\end{bmatrix} = 0$

$1 + ab = 0$

$ab = -1$

From above two tangent equations we have

$$\implies \begin{bmatrix} -1 & a \\ -1 & b \end{bmatrix} \mathbf{x} = \begin{bmatrix} a^2 \\ b^2 \end{bmatrix}$$

$$\implies \mathbf{x} = \begin{bmatrix} -1 & a \\ -1 & b \end{bmatrix}^{-1} \begin{bmatrix} a^2 \\ b^2 \end{bmatrix}$$

$$\implies \mathbf{x} = \frac{1}{a-b} \begin{bmatrix} b & -a \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a^2 \\ b^2 \end{bmatrix}$$

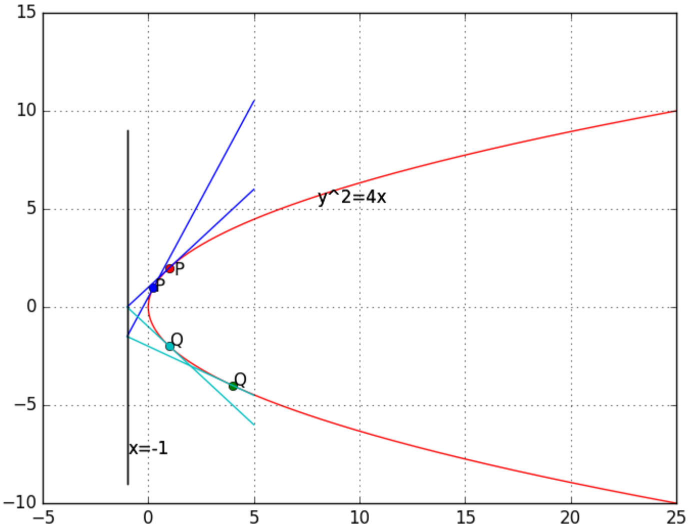
$$\implies \mathbf{x} = \frac{1}{a-b}\begin{bmatrix} ab(a-b) \\ a^2-b^2 \end{bmatrix}$$

$$\implies \mathbf{x} = \begin{bmatrix} ab \\ a+b \end{bmatrix}$$

$$\implies \mathbf{x} = \begin{bmatrix} -1 \\ a+b \end{bmatrix}$$

$$\implies x = -1$$

x = -1 is the required locus



15
10
5
0
−5
−10
−5
0
5
10
15
20
25
y^2=4x
P
P
Q
Q
x=-1