## Computing Project - Lasso Algorithm

Adam Lee, Viviana Rosales, Carlos Rodriquez, Katrina Walker March 22, 2017

## Implementation for a Time-Series Analysis

Prior to implementing the GLASSO algorithm on a a panel of financial returns, we created the following function in R:

```
GLASSO \leftarrow function(Y, lambda = 0.05, t = 0.001, max_iter = 5000, beta_tol = 0.001, beta_to
                       0.001) {
  2
                   # Functions ------
  3
  5
                  # Soft thresholding
                  S_{\text{-}}func \leftarrow function(x, t)
  6
                          sign(x) * max((abs(x) - t), 0)
  7
  8
  9
10
                   # update beta
                  update_beta ← function(u, V, beta, lambda){
11
12
                          for (i in seq(1, (p-1)))
                                 arg \leftarrow u[j] - V[-i,i] \% \% beta[-i]
13
                                beta[i] ← S_func(arg, lambda)
14
15
                         return (beta)
16
17
18
                   # GLASSO -----
19
20
                   # determine dimension
21
                  p \leftarrow ncol(Y)
22
23
                   # compute sample covariance matrix
24
                  S \leftarrow \mathbf{cov}(Y)
25
26
                   # compute tolerance
27
                  S_{temp} \leftarrow S
28
                   \operatorname{diag}(S_{-\operatorname{temp}}) \leftarrow \operatorname{rep}(0, p)
29
                   tol \leftarrow t * mean(abs(S_temp))
30
31
                   # initialise W & W_new for loop
32
                 W \leftarrow S + lambda * diag(p)
33
                 W_new \leftarrow W
34
35
                  # intialise B, Theta
36
                  B \leftarrow \mathbf{matrix}(\mathbf{data} = NA, \mathbf{nrow} = (p - 1), \mathbf{ncol} = p)
```

```
Theta \leftarrow matrix (data = NA, nrow = p, ncol = p)
38
39
          # set intial error and iteration counter
40
41
          it ←
42
         beta_err \leftarrow 1
43
          # loop
44
          while (err > tol & it < max_iter){
45
46
             for (j \text{ in } seq(1, p)){
47
48
                V \leftarrow W[-j \ , \ -j \ ]
49
                 u \leftarrow S[-j, j]
50
51
                 # initalise beta
52
                 \mathbf{beta} \leftarrow \mathbf{rep}(1, (p-1))
53
54
                 # cyclical co-ordinate-descent
55
                 while (beta_err > beta_tol){
56
                     \mathbf{beta\_new} \leftarrow \mathbf{update\_beta}(u\,,\ V,\ \mathbf{beta}\,,\ \mathrm{lambda})
57
                     beta_err = norm(as.matrix(beta_new - beta), type = 'f')
58
59
                     beta \leftarrow beta\_new
                 }
60
61
                 # update w_{-}12
62
                W_{new}[-j, j] \leftarrow V \% *\% beta_new
63
64
                 B[,j] \leftarrow \mathbf{beta\_new}
65
66
             # update error and iteration counter
67
68
             err \leftarrow mean(abs(W-W_new))
             it \leftarrow it + 1
69
70
             # update W
71
            W \leftarrow W_{-}\mathbf{new}
72
73
74
75
          # recover Theta
          for (j \text{ in } seq(1, p)){
76
             \label{eq:theta_bound} \operatorname{Theta}\left[\begin{smallmatrix} j \end{smallmatrix}, \begin{smallmatrix} j \end{smallmatrix}\right] \leftarrow 1 \; / \; \left(W[\begin{smallmatrix} j \end{smallmatrix}, \begin{smallmatrix} j \end{smallmatrix}\right] - W[-\begin{smallmatrix} j \end{smallmatrix}, \begin{smallmatrix} j \end{smallmatrix}\right] \; \% * \% \; B[\begin{smallmatrix} , \end{smallmatrix} \; j \end{smallmatrix})
77
             Theta[-j, j] \leftarrow -B[, j] * Theta[j, j]
78
79
80
         return (Theta)
81
     }
82
```

## Demo

Problem: Given... Inputs: ...

Outputs: ...

The Algorithm:

## How it Works

The graphical lasso (GLASSO) algorithm is a variant of the least absolute shrinkage and selection operator (LASSO) which estimates a sparse inverse covariance matrix using a lasso (L1) penalty. The L1 penalty constrains the size of the coefficients by adding a penalty equivalent to the absolute value of the magnitude of the coefficients.....BLABLABLA

Specifically, ..... BLABLABLA