

The problem is to solve the 'steady state' heat equation on a rectangular region, representing a plate heated along three edges to 100°C, and cooled to 0°C along the fourth edge. The physical region, and the boundary conditions, are suggested by the diagram in figure 1, in which W represents temperature:

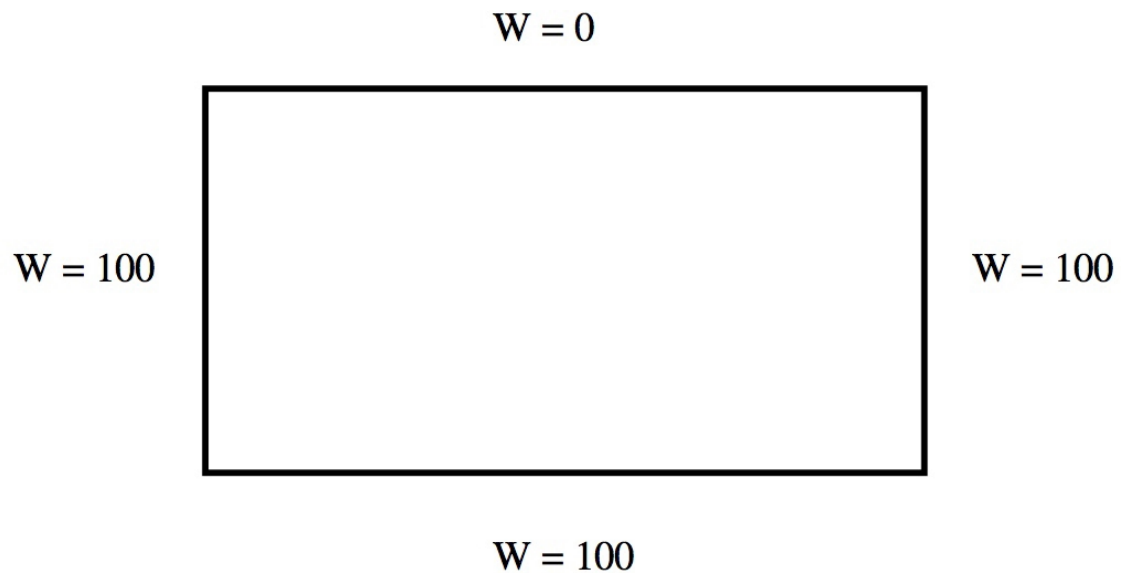


Figure 1: Physical region indicating temperature W maintained along each boundary. The region is covered with a grid of M by N nodes, and an M by N Array W is used to record the temperature. The correspondence between array indices and locations in the region is suggested by giving the indices of the four corners, as is shown in figure 2.

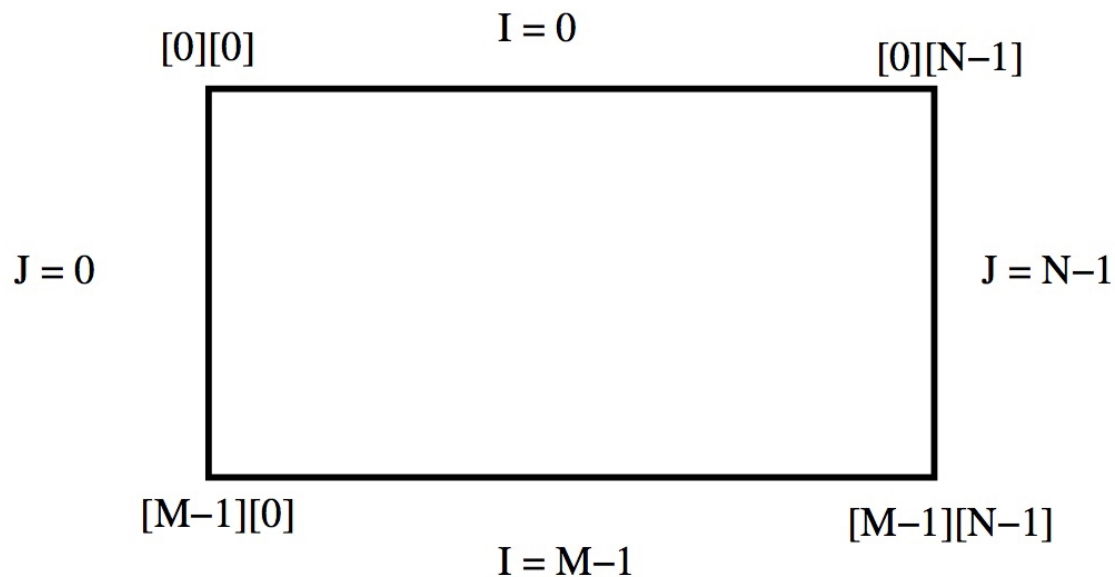


Figure 2: Correspondence between the array indices and locations in the region. The steady state solution to the discrete heat equation satisfies the following condition at an interior grid point:
 $W[\text{Central}] = (1/4) * (W[\text{North}] + W[\text{South}] + W[\text{East}] + W[\text{West}])$ where 'Central' is the index of the grid point, 'North' is the index of its immediate neighbor to the 'north', and so

on. Given an approximate solution of the steady state heat equation, a 'better' solution is given by replacing each interior point by the average of its 4 neighbors - in other words, by using the condition as an assignment statement. If this process is repeated often enough, the difference between successive estimates of the solution will go towards zero. Your program should carry out such an iteration, using a tolerance specified by the user, and write the final estimate of the solution to a file that can be used to generate graphic representation of the solution.

1. State the two possible methods that you could use to distribute this processing over multiple processors.
2. Write a program to implement your chosen method to solve the given heat diffusion problem in a distributed manner.
 - a. Document a detailed description of the implementation of the method you have chosen, including how the system was tested.
 - b. For an implementation using a fixed problem size (size of plate) and fixed tolerance (specified by the user), calculate the temperature distribution calculated by (a) a single processor, and (b) four processors. For both solutions show the calculated temperature distribution as an image. Comparing the two solutions, calculate the standard deviation of differences in calculated temperature at each point in the two solutions.