The transformation matrix from the physical coordinate $(x, p_x, y, p_y)$ to the $x$-$y$ decoupled coordinate $(X, P_X, Y, P_Y)$ is written as

$$\mathrm{R} = \begin{pmatrix} \mu\mathrm{I} & \mathrm{J}\mathrm{r}^T\mathrm{J} \\ \mathrm{r} & \mu\mathrm{I} \end{pmatrix} = \begin{pmatrix} \mu & . & -\mathtt{R4} & \mathtt{R2} \\ . & \mu & \mathtt{R3} & -\mathtt{R1} \\ \mathtt{R1} & \mathtt{R2} & \mu & . \\ \mathtt{R3} & \mathtt{R4} & . & \mu \end{pmatrix} \tag{60}$$

with a submatrix

$$\mathrm{r} = \begin{pmatrix} \mathtt{R1} & \mathtt{R2} \\ \mathtt{R3} & \mathtt{R4} \end{pmatrix}, \tag{61}$$

where

$$\mu^2 + \det(\mathrm{r}) = 1, \tag{62}$$

$$\mathrm{I} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{63}$$

$$\mathrm{J} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{64}$$

The inverse of R is obtained by reversing the sign of r:

$$\mathrm{R}^{-1} = \begin{pmatrix} \mu\mathrm{I} & -\mathrm{J}\mathrm{r}^T\mathrm{J} \\ -\mathrm{r} & \mu\mathrm{I} \end{pmatrix} = \begin{pmatrix} \mu & . & \mathtt{R4} & -\mathtt{R2} \\ . & \mu & -\mathtt{R3} & \mathtt{R1} \\ -\mathtt{R1} & -\mathtt{R2} & \mu & . \\ -\mathtt{R3} & -\mathtt{R4} & . & \mu \end{pmatrix} \tag{65}$$

.

The value of the function `DETR` is equal to det(r) in this case.

Let T stand for the physical transfer matrix from location 1 to location 2, then the transformation in the decoupled coordinate is diagonalized as

$$\mathrm{R}_2\mathrm{T}\mathrm{R}_1^{-1} = \begin{pmatrix} \mathrm{T}_X & 0 \\ 0 & \mathrm{T}_Y \end{pmatrix}. \tag{66}$$

The Twiss parameters are defined for the 2 by 2 matrices $\mathrm{T}_X$ and $\mathrm{T}_Y$.

If $\det(\mathrm{r}) \geq 1$, the above condition for $\mu$ is violated. In such a case, an alternative form of R is used:

$$\mathrm{R} = \begin{pmatrix} \mathrm{J}\mathrm{r}^T\mathrm{J} & \mu\mathrm{I} \\ \mu\mathrm{I} & \mathrm{r} \end{pmatrix}, \tag{67}$$

where $\mu^2 + \det(\mathrm{r}) = 1$. The function `DETR` shows a number $a - \det(\mathrm{r})$, where $a = 1.375$. thus the alternative form is used when det(r) >= 0.625.