Let V denote the matrix to define the normal mode, i.e.,

$$U = \mathrm{V}u, \tag{68}$$

where $U = (X, P_x, Y, P_y, Z, P_z)$ and $u = (x, p_x, y, p_y, z, \delta \equiv p-1)$ are the normal and physical coordinates, respectively. The matrix V can be expressed as

$$V = \mathrm{PBR_6 H}, \tag{69}$$

where

$$\mathrm{H} = \begin{pmatrix} \left(1 - \dfrac{\det \mathrm{H}_x}{1+a}\right)\mathrm{I} & \dfrac{\mathrm{H}_x \mathrm{J}_2 \mathrm{H}_y^T \mathrm{J}_2}{1+a} & -\mathrm{H}_x \\ \dfrac{\mathrm{H}_y \mathrm{J}_2 \mathrm{H}_x^T \mathrm{J}_2}{1+a} & \left(1 - \dfrac{\det \mathrm{H}_y}{1+a}\right)\mathrm{I} & -\mathrm{H}_y \\ -\mathrm{J}_2 \mathrm{H}_x^T \mathrm{J}_2 & -\mathrm{J}_2 \mathrm{H}_y^T \mathrm{J}_2 & a\mathrm{I} \end{pmatrix}, \tag{70}$$

$$\mathrm{R}_6 = \begin{pmatrix} \mathrm{R} & 0 & 0 \\ 0 & 0 & \mathrm{I} \end{pmatrix} = \begin{pmatrix} b\mathrm{I} & \mathrm{J}_2 r^T \mathrm{J}_2 & 0 \\ r & b\mathrm{I} & 0 \\ 0 & 0 & \mathrm{I} \end{pmatrix}, \tag{71}$$

$$\mathrm{PB} = \begin{pmatrix} \mathrm{P}_x \mathrm{B}_x & 0 & 0 \\ 0 & \mathrm{P}_y \mathrm{B}_y & 0 \\ 0 & 0 & \mathrm{P}_z \mathrm{B}_z \end{pmatrix}, \tag{72}$$

with

$$a^2 + \det \mathrm{H}_x + \det \mathrm{H}_y = 1, \tag{73}$$

$$b^2 + \det \mathrm{R} = 1. \tag{74}$$

Symbols $\mathrm{I}, \mathrm{J}_2, \mathrm{H}_{x,y}, r, \mathrm{B}_{x,y,z}, \mathrm{P}_{x,y,z}$ above are 2 by 2 matrices:

$$\mathrm{I} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{75}$$

$$\mathrm{J}_2 \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{76}$$

$$r \equiv \begin{pmatrix} \mathrm{R}1 & \mathrm{R}2 \\ \mathrm{R}3 & \mathrm{R}4 \end{pmatrix}, \tag{77}$$

$$\mathrm{B}_{x,y} \equiv \begin{pmatrix} \dfrac{1}{\sqrt{\beta_{x,y}}} & 0 \\ \dfrac{\alpha_{x,y}}{\sqrt{\beta_{x,y}}} & \sqrt{\beta_{x,y}} \end{pmatrix}, \tag{78}$$

$$\mathrm{P}_{x,y,z} \equiv \begin{pmatrix} \cos \psi_{x,y,z} & \sin \psi_{x,y,z} \\ -\sin \psi_{x,y,z} & \cos \psi_{x,y,z} \end{pmatrix}. \tag{79}$$

Matrices $\mathrm{H}_{x,y}$ define dispersions as

$$\begin{pmatrix} \mathrm{ZX} & \mathrm{EX} \\ \mathrm{ZPX} & \mathrm{EPX} \\ \mathrm{ZY} & \mathrm{EY} \\ \mathrm{ZPY} & \mathrm{EPY} \end{pmatrix} \equiv \mathrm{R}\begin{pmatrix} \mathrm{H}_x \\ \mathrm{H}_y \end{pmatrix}. \tag{80}$$