

# Stabilization of Linear Switched Systems with Long Constant Input Delay via Average or Averaging Predictor Feedbacks

## 1 Purpose of this Document

This document describes the MATLAB code that was used to produce the results in Section 4 of the paper “*Stabilization of Linear Switched Systems with Long Constant Input Delay via Average or Averaging Predictor Feedbacks*”. It explains:

- the system setup and the main equations implemented in the scripts,
- the structure of the repository and the role of each folder,
- how to run all simulations in order to reproduce the figures and numerical values,
- how the theoretical constants  $\epsilon$ ,  $\bar{\epsilon}$ ,  $\epsilon^*$ ,  $\bar{\epsilon}^*$  and dwell-time bounds  $\tau_d^*$ ,  $\bar{\tau}_d^*$  are computed.

## 2 System Setup

We consider a continuous-time switched linear system with input delay  $D > 0$  of the form

$$\dot{X}(t) = A_{\sigma(t)} X(t) + B_{\sigma(t)} U(t - D), \quad (1)$$

where

- $X(t) \in \mathbb{R}^2$  is the state,
- $U(t) \in \mathbb{R}$  is the control input,
- $\sigma(t) \in \{1, 2, 3\}$  is the switching signal indicating the active mode,
- for each  $i \in \{1, 2, 3\}$ ,  $(A_i, B_i)$  is a stabilizable pair.

In the numerical examples, we use the specific matrices

$$\begin{aligned} A_1 &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, & A_2 &= \begin{bmatrix} 0.97 & 1.15 \\ 1.06 & 2.09 \end{bmatrix}, & A_3 &= \begin{bmatrix} 1.08 & 1.2 \\ 1.14 & 2.13 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ 1.05 \end{bmatrix}, & B_3 &= \begin{bmatrix} 0 \\ 1.1 \end{bmatrix}. \end{aligned}$$

For each subsystem we choose a state-feedback gain  $K_i \in \mathbb{R}^{1 \times 2}$  such that  $A_i + B_i K_i$  is Hurwitz. Throughout the code this is done by pole placement:

$$K_i = -\text{place}(A_i, B_i, \{\lambda_1, \lambda_2\}), \quad \lambda_1 = -3, \lambda_2 = -2, \quad i = 1, 2, 3,$$

via the auxiliary function `computeK.m`.

In the simulations, we use:

- input/nominal delay  $D = 1$  s,
- minimum dwell time  $\tau_d = 0.9$  s, maximum dwell time  $\bar{\tau}_d = 3$  s
- total simulation time  $T = 10$  s,
- sampling period  $\Delta t = 10^{-3}$  s.

The switching signal  $\sigma(\cdot)$  is a random signal in  $\{1, 2, 3\}$  respecting the minimum and maximum dwell time conditions. The specific realization used in the paper is stored in the file `swsig_values.mat` as a vector `swsig_values`. It was generated once using `generateSwSignal.m` and is reused in all simulations for exact reproducibility.

### 3 Feedback Laws

The paper considers three control laws:

- Controller  $U_1$ , as defined in equation (4) in [KBL25]
- Controller  $U_2$ , as defined in equation (10) in [KBL25]
- The exact predictor  $U_{\text{exact}}$ , as defined in equation (140) in [KBL25]

#### 3.1 Average Predictor-Based Controller $U_1$

The average predictor feedback builds a single “average” pair  $(\bar{A}, \bar{B})$  and an associated gain  $\bar{K}$ . They are computed by solving convex problems of the form

$$\min_{\bar{Y}} R \quad \text{subject to} \quad |\bar{Y} - Y_i|_2 \leq R, \quad i = 1, 2, 3, \quad (2)$$

for  $Y = A, B, K$ , using the auxiliary function `optimizeL2.m`. That is,

$$\bar{A} = \text{optimizeL2}(A_1, A_2, A_3), \quad \bar{B} = \text{optimizeL2}(B_1, B_2, B_3), \quad \bar{K} = \text{optimizeL2}(K_1, K_2, K_3).$$

The average predictor controller  $U_1$  is then constructed as follows. Let  $\tau_0(t)$  denote the last switching instant and let  $\tau(t)$  be the “remaining dwell-time” variable satisfying

$$\tau(t) = \max\{0, \tau_0(t) + \tau_d - t\} \in [0, \tau_d].$$

Define

$$X(t + \tau(t)) = e^{A_{\sigma(\tau_0(t))}\tau(t)} X(t) + \int_{t-D}^{t+\tau(t)-D} e^{A_{\sigma(\tau_0(t))}(t+\tau(t)-D-\theta)} B_{\sigma(\tau_0(t))} U(\theta) d\theta. \quad (3)$$

Then  $U_1$  control is given by

$$U_1(t) = \bar{K} \left( e^{\bar{A}(D-\tau(t))} X(t + \tau(t)) + \int_{t+\tau(t)-D}^t e^{\bar{A}(t-s)} \bar{B} U(s) ds \right). \quad (4)$$

In the code, equations (3)–(4) are implemented using functions that compute numerically (via discretization) the matrix exponentials and the integrals are solved with the left end point Rule:

$$e^{A_i \Delta t} \approx \text{expm}(A_i \Delta t), \quad e^{\bar{A} \Delta t} \approx \text{expm}(\bar{A} \Delta t).$$

- `U1_dwell.m` implements (1) under (4) under dwell-time knowledge.
- `U1_nodwell.m` implements the same law but with  $\tau(t) \equiv 0 \forall t \geq 0$ , i.e. without using dwell time information.

### 3.2 Averaging Predictors Controller $U_2$

The averaging of predictors method constructs three separate exact predictors, one per subsystem  $(A_i, B_i)$ . Their dynamics are based on the fixed matrices  $A_i, B_i$  and the same structure as in (3). The corresponding controls are

$$U_i(t) = K_i \left( e^{A_i(D-\tau(t))} X(t + \tau(t)) + \int_{t+\tau(t)-D}^t e^{A_i(t-s)} B_i U(s) ds \right), \quad i = 1, 2, 3,$$

and the implemented control is the arithmetic average

$$U(t) = \frac{1}{3} (U_1(t) + U_2(t) + U_3(t)). \quad (5)$$

In the repository:

- `U2_dwell.m` implements (1) with (5) under dwell-time knowledge.
- `U2_nodwell.m` implements the same law but with  $\tau(t) \equiv 0 \forall t \geq 0$ , i.e. without using dwell time information.

### 3.3 Exact Predictor $U_{\text{exact}}$

The exact predictor uses for each mode  $i = \sigma(t)$  the nominal delay  $D = 1$  and reconstructs the future state  $P(t) = X(t + D)$  under the exact mode-dependent dynamics. The corresponding predictor state (for a fixed interval) satisfies equation (18) in [KBL25]. The exact predictor feedback is then of the form

$$U_{\text{exact}}(t) = K_{\sigma(t+D)} X(t + D), \quad (6)$$

It is not implementable in practice because it requires full knowledge of future modes.

In the repository, the routine `ExactPredictor.m` implements (6) along the stored switching signal `swsig_values`. It stores the resulting trajectories in `U_ex.mat`.

### 3.4 Robustness to Delay Perturbations

To test robustness to delay mismatch, we consider the case where the actual delay  $D_{\text{actual}} = D$  differs from the delay  $D_{\text{controller}}$  used inside the two proposed control laws.

The script `U1_robust_0_95.m` implements the proposed  $U_1$  controller, but with  $D_{\text{actual}} = D = 0.95$  in the delayed input and using  $D_{\text{controller}} = 1$  in the prediction-based controller.

The script `U1_robust_1_05.m` implements the proposed  $U_1$  controller, but with  $D_{\text{actual}} = D = 1.05$  in the delayed input and using  $D_{\text{controller}} = 1$  in the prediction-based controller.

The script `U2_robust_0_95.m` implements the proposed  $U_2$  controller, but with  $D_{\text{actual}} = D = 0.95$  in the delayed input and using  $D_{\text{controller}} = 1$  in the prediction-based controller.

The script `U1_robust_1_05.m` implements the proposed  $U_2$  controller, but with  $D_{\text{actual}} = D = 0.95$  in the delayed input and using  $D_{\text{controller}} = 1$  in the prediction-based controller.

## 4 Computation of Parameters

Theorems 1 and 2 in the paper ([KBL25]) define two main parameters:

- $\epsilon$  (equation (14) in paper), which is the maximal deviation between the triples  $(A_i, B_i, K_i)$  and their averaged counterparts  $(\bar{A}, \bar{B}, \bar{K})$ ,
- $\bar{\epsilon}$  (equation (16) in paper), which is the maximal deviation between subsystems  $(A_i, B_i, K_i)$  amongst themselves.

They determine admissible ranges  $\epsilon^*$ ,  $\bar{\epsilon}^*$  and corresponding dwell time bounds  $\tau_d^*$ ,  $\bar{\tau}_d^*$ .

The script `epsilon_star.m` computes  $\epsilon$  and then evaluates the rest of the constants defined in Theorem 1 (following the notation in the paper). Eventually, it prints the numerical values of  $\epsilon$ ,  $\epsilon^*$ ,  $\tau_d^*$ . The dwell time used in the simulations ( $\tau_d = 0.9$  s) can then be compared with  $\tau_d^*$  to determine how conservative the bound is.

In the same manner, script `barepsilon_star.m` computes  $\bar{\epsilon}$  and then evaluates the rest of the constants defined in Theorem 2 (following the notation in the paper). Eventually, it prints the numerical values of  $\bar{\epsilon}$ ,  $\bar{\epsilon}^*$ ,  $\bar{\tau}_d^*$  so that the user can compare them with the simulation dwell time  $\tau_d$ .

## 5 Performance Index

To compare the performance of different controllers  $U_1$ ,  $U_2$  and the exact predictor, we use the quadratic performance index

$$J = \int_0^T (|X(t)|^2 + |U(t)|^2) dt, \quad (7)$$

approximated via the trapezoidal rule:

$$J \approx \int_0^T |X(t)|^2 dt + \int_0^T |U(t)|^2 dt \approx \text{trapz}(T, |X(t)|^2) + \text{trapz}(T, |U(t)|^2).$$

The script `Performance_Index.m`:

- loads the files `U1_dwell.mat`, `U1_nodwell.mat`, `U2_dwell.mat`, `U2_nodwell.mat`, and `U_ex.mat`,
- extracts the trajectories  $(X, U)$  in each case,
- computes  $J$  according to (7),
- prints the resulting values in a table.

## 6 Repository Structure

The repository is organized as follows:

- **Auxiliary Functions/**
  - `computeK.m` (feedback gain design for each mode)
  - `optimizeL2.m` (CVX-based computation of  $\bar{A}$ ,  $\bar{B}$ ,  $\bar{K}$ )
  - `generateSwSignal.m` (generation of random dwell-time switching signal)
  - `swsig_values.mat` (fixed switching signal used in all simulations)
- **Section\_4.1/** (System (1) under  $U_1$  and  $U_2$  with dwell time knowledge)
  - `U1_dwell_time.m` (controller  $U_1$ )
  - `U2_dwell_time.m` (controller  $U_2$ )
  - `epsilon_star.m` (computes  $\epsilon$ ,  $\epsilon^*$ ,  $\tau_d^*$ )
  - `barepsilon_star.m` (computes  $\bar{\epsilon}$ ,  $\bar{\epsilon}^*$ ,  $\bar{\tau}_d^*$ )
- **Section\_4.2/** (Comparison with exact predictor and no dwell time knowledge cases)

- `U1_nodwell_time.m` ( $U_1$  without dwell time)
- `U2_nodwell_time.m` ( $U_2$  without dwell time)
- `ExactPredictor_Uex.m` (exact predictor simulation)
- `Performance_Index.m` (computes  $J$  performance index for all controllers)
- `Section_4.3/` (Delay perturbation / robustness)
  - `U1_robust_0_95.m` ( $U_1$  with  $D = 0.95$  and  $D_{\text{controller}} = 1$ )
  - `U1_robust_1_05.m` ( $U_1$  with  $D = 1.05$  and  $D_{\text{controller}} = 1$ )
  - `U2_robust_0_95.m` ( $U_2$  with  $D = 0.95$  and  $D_{\text{controller}} = 1$ )
  - `U2_robust_1_05.m` ( $U_2$  with  $D = 1.05$  and  $D_{\text{controller}} = 1$ )
- `README.pdf`
- `LICENSE`

## 7 How to Run the Simulations

### Requirements

- MATLAB
- CVX toolbox (for `optimizeL2.m`), with a compatible SDP solver (e.g. SeDuMi, SDPT3, MOSEK).

### General Steps

1. Open MATLAB and set the repository root as the current folder.
2. Ensure that `swsig_values.mat` is present in `Auxiliary Functions/` and that CVX is correctly installed and set up.

### Section 4.1: Implement $U_1(t)$ and $U_2(t)$

1. Navigate to `Section_4.1` in MATLAB.
2. Run

```
U1_dwell_time.m
U2_dwell_time.m
epsilon_star.m
barepsilon_star.m
```

3. Scripts `U1_dwell_time.m` and `U2_dwell_time.m` will produce plots of  $X(t)$ ,  $U(t)$  and  $\sigma(t)$ , and save the trajectories as `U1_dwell.mat` and `U2_dwell.mat` if needed.
4. Scripts `epsilon_star.m` and `barepsilon_star.m` will print the theoretical constants derived from Theorem 1 and 2, respectively.

## Section 4.2: Comparison with Exact Predictor and Performance Index

1. Navigate to `Section_4.2`.
2. Run the following scripts in any order:

```
U1_nodwell_time.m
U2_nodwell_time.m
ExactPredictor.m
```

They will create files `U1_nodwell.mat`, `U2_nodwell.mat`, and `U_ex.mat`.

3. Then run

```
Performance_Index.m
```

to compute and display the values of the performance index  $J$  in (7) for all controllers.

## Section 4.3: Delay Perturbations

1. Navigate to `Section_4.3`.
2. Run, in any order,

```
U1_robust_0_95.m
U1_robust_1_05.m
U2_robust_0_95.m
U2_robust_1_05.m
```

The scripts generate plots of  $X(t)$ ,  $U(t)$ , and  $\sigma(t)$  for the perturbed-delay cases.

## References

- [KBL25] Andreas Katsanikakis and Nikolaos Bekiaris-Liberis. Stabilization of linear switched systems with long constant input delay via average or averaging predictor feedbacks. *Revised version submitted in Systems and Control Letters*, 2025.