Neural networks STAT 4710

Where we are

Unit 1: R for data mining



Unit 2: Prediction fundamentals



Unit 3: Regression-based methods



Unit 4: Tree-based methods

Unit 5: Deep learning

Lecture 1: Deep learning preliminaries

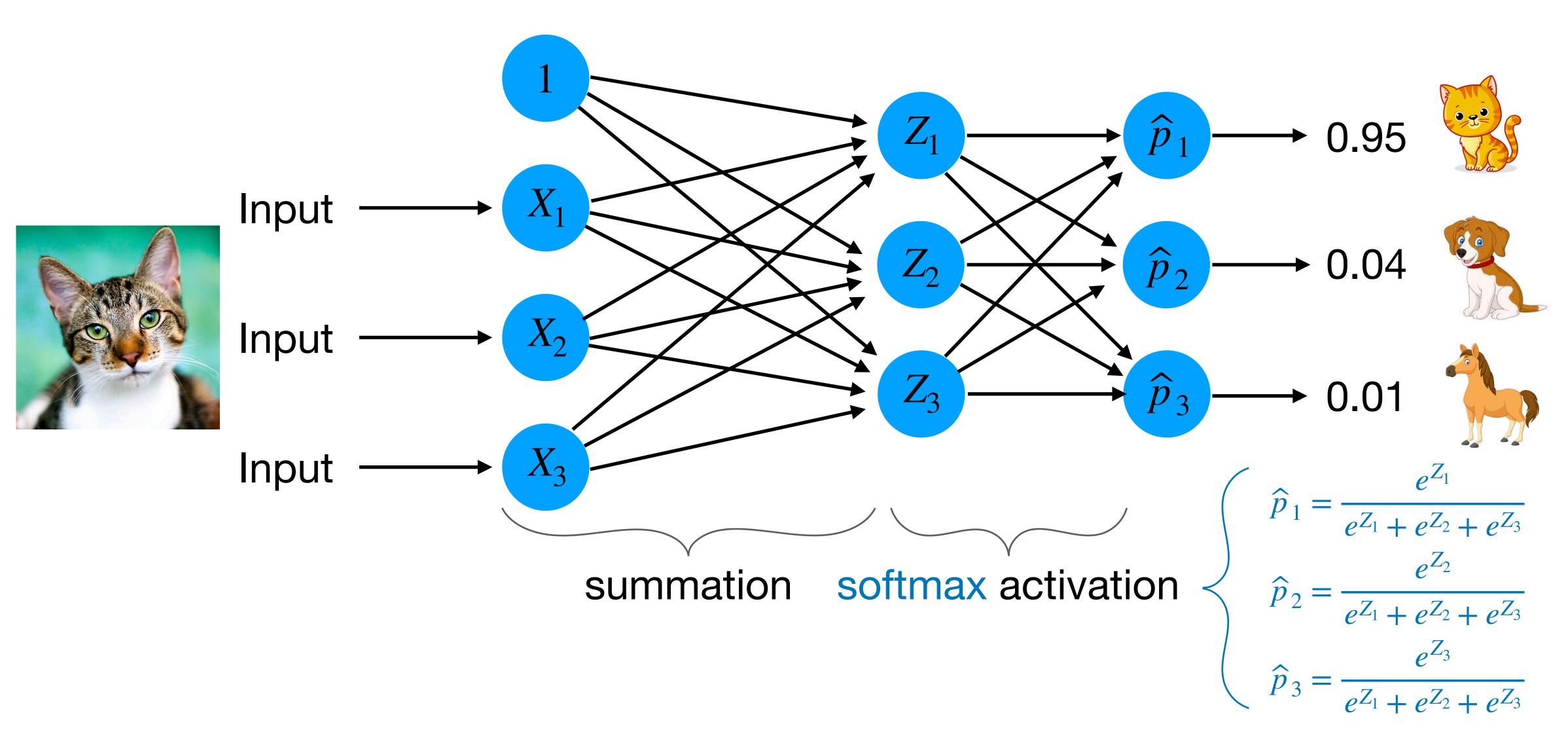
Lecture 2: Neural networks

Lecture 3: Deep learning for images

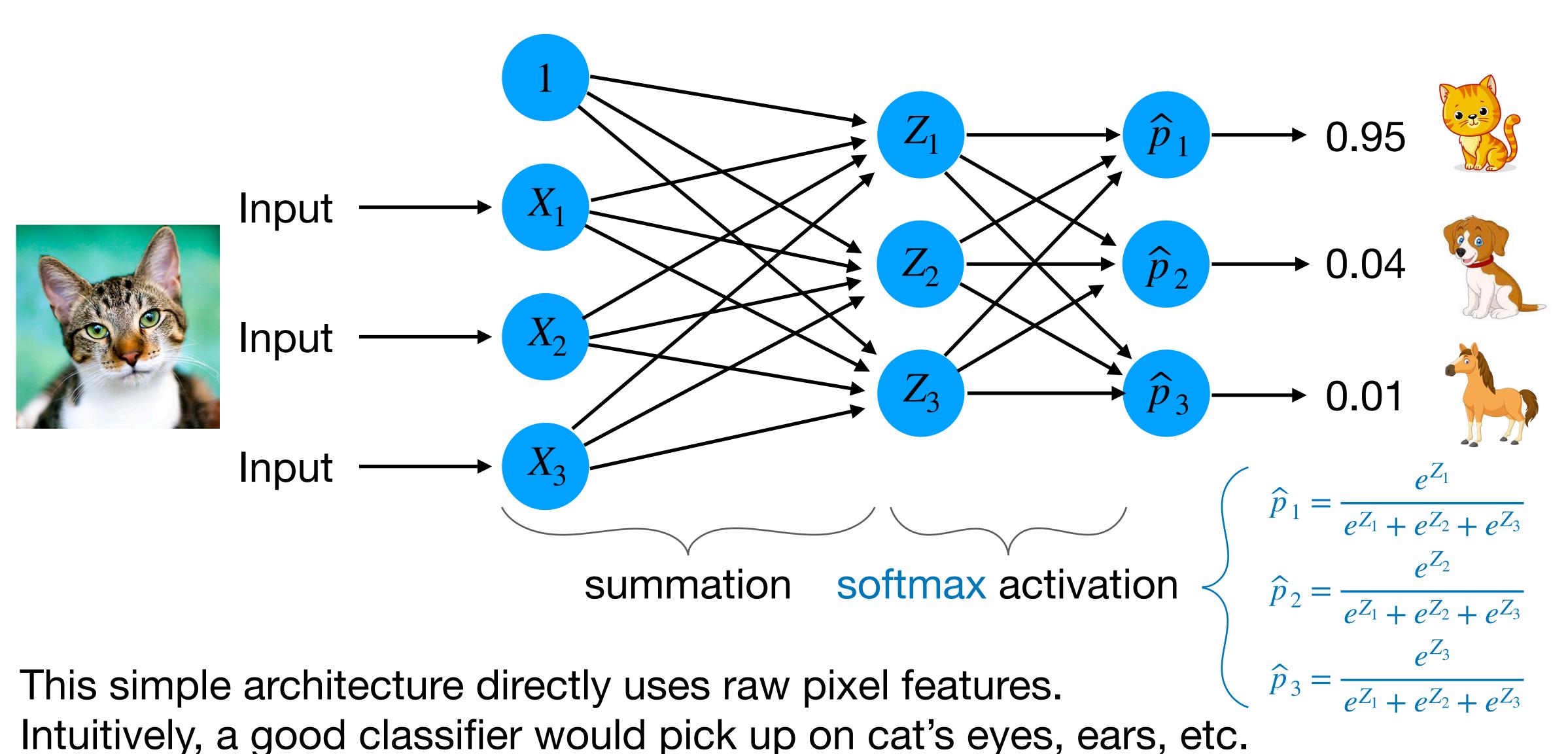
Lecture 4: Deep learning for text

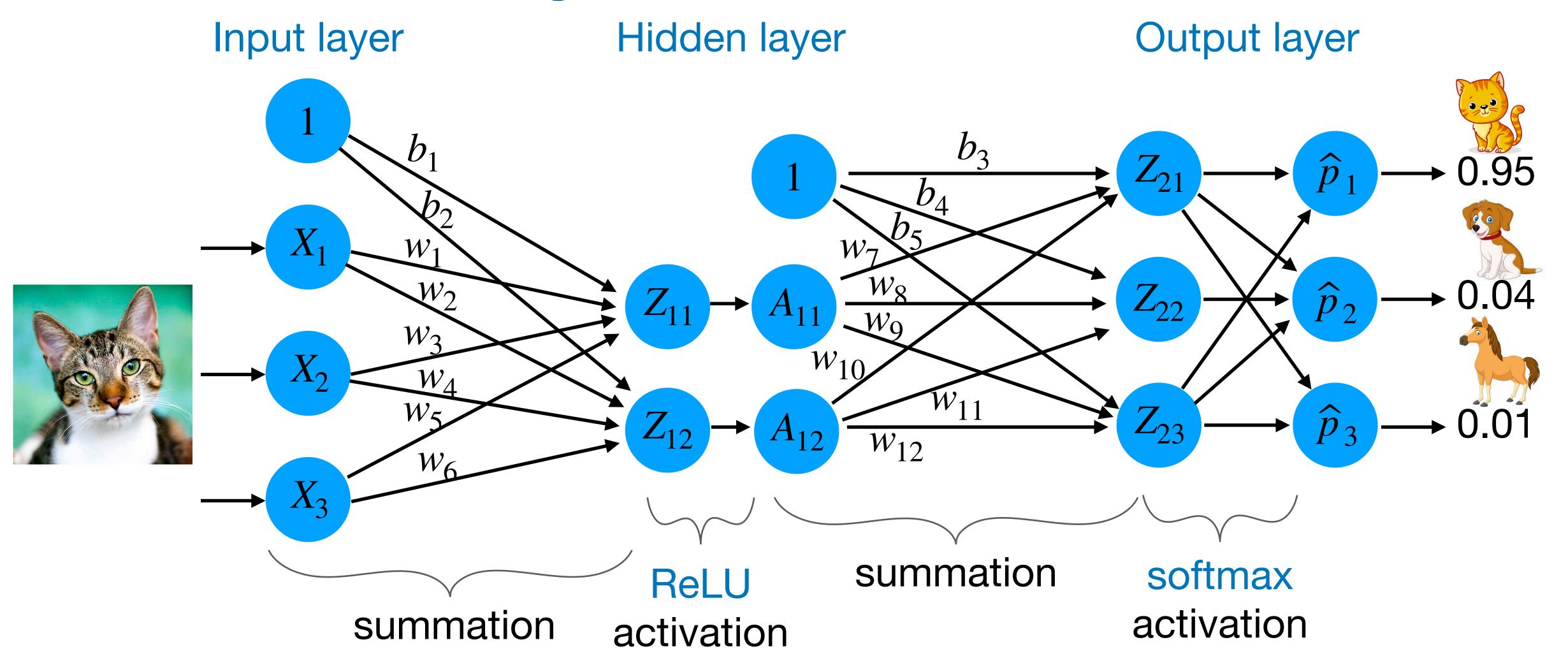
Lecture 5: Unit review and quiz in class

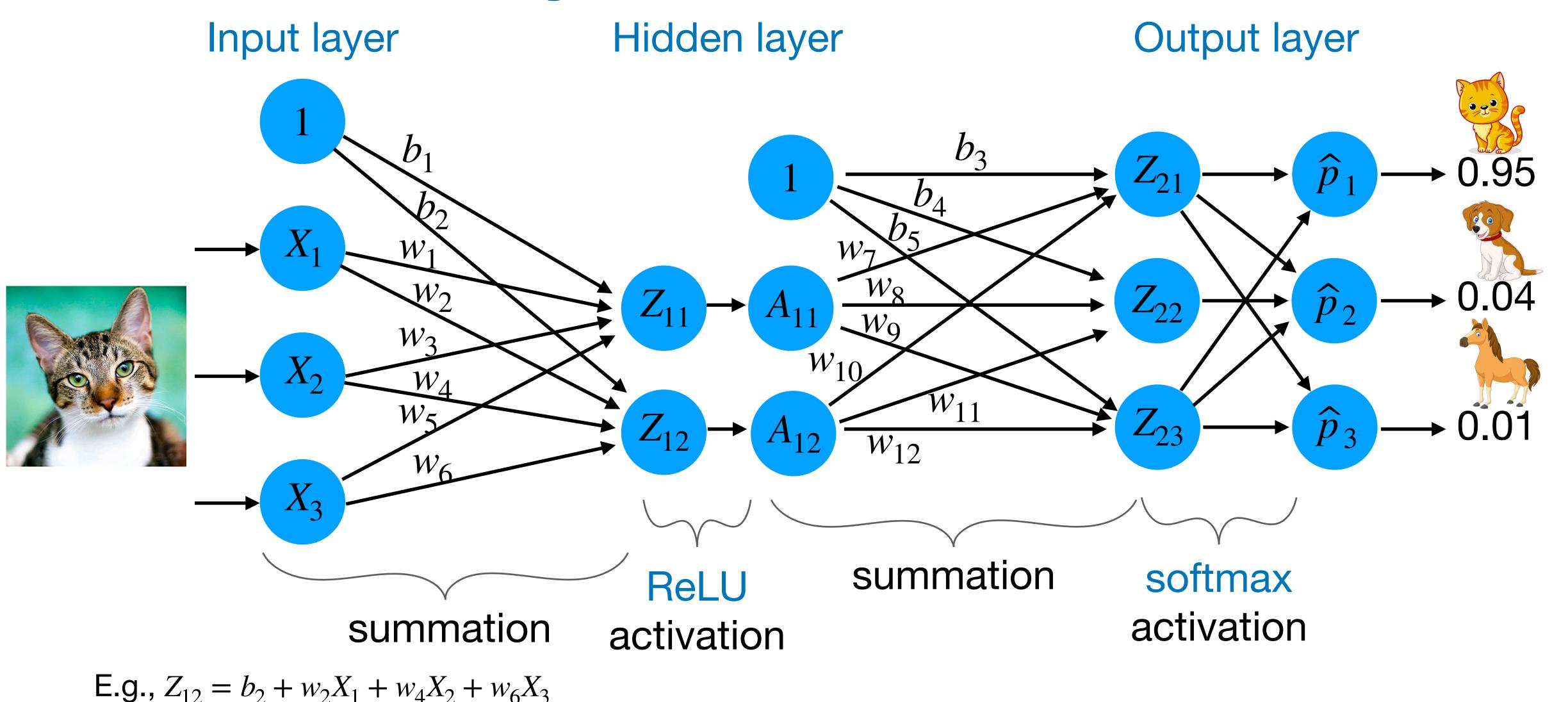
Recall: Multi-class logistic model

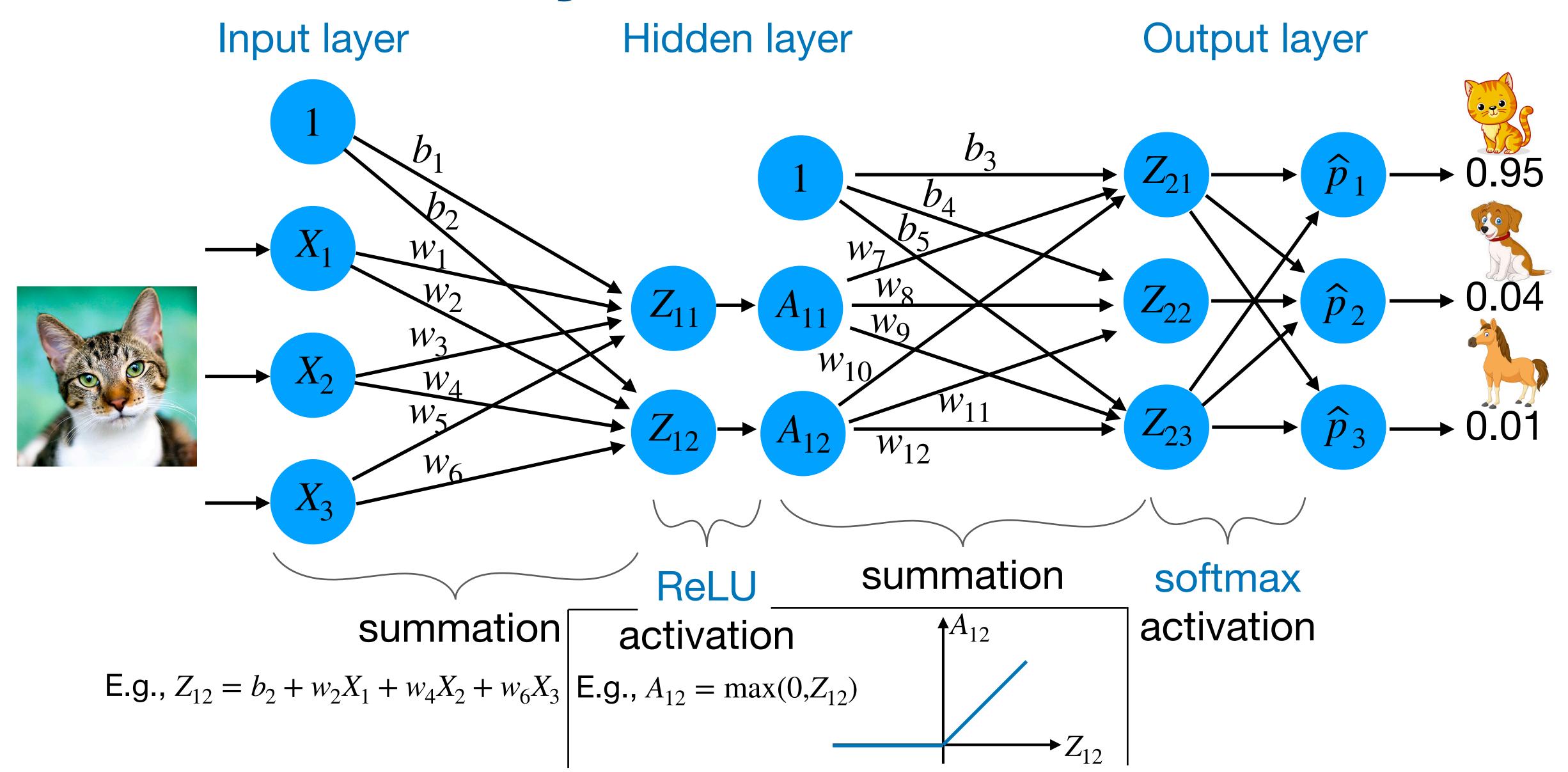


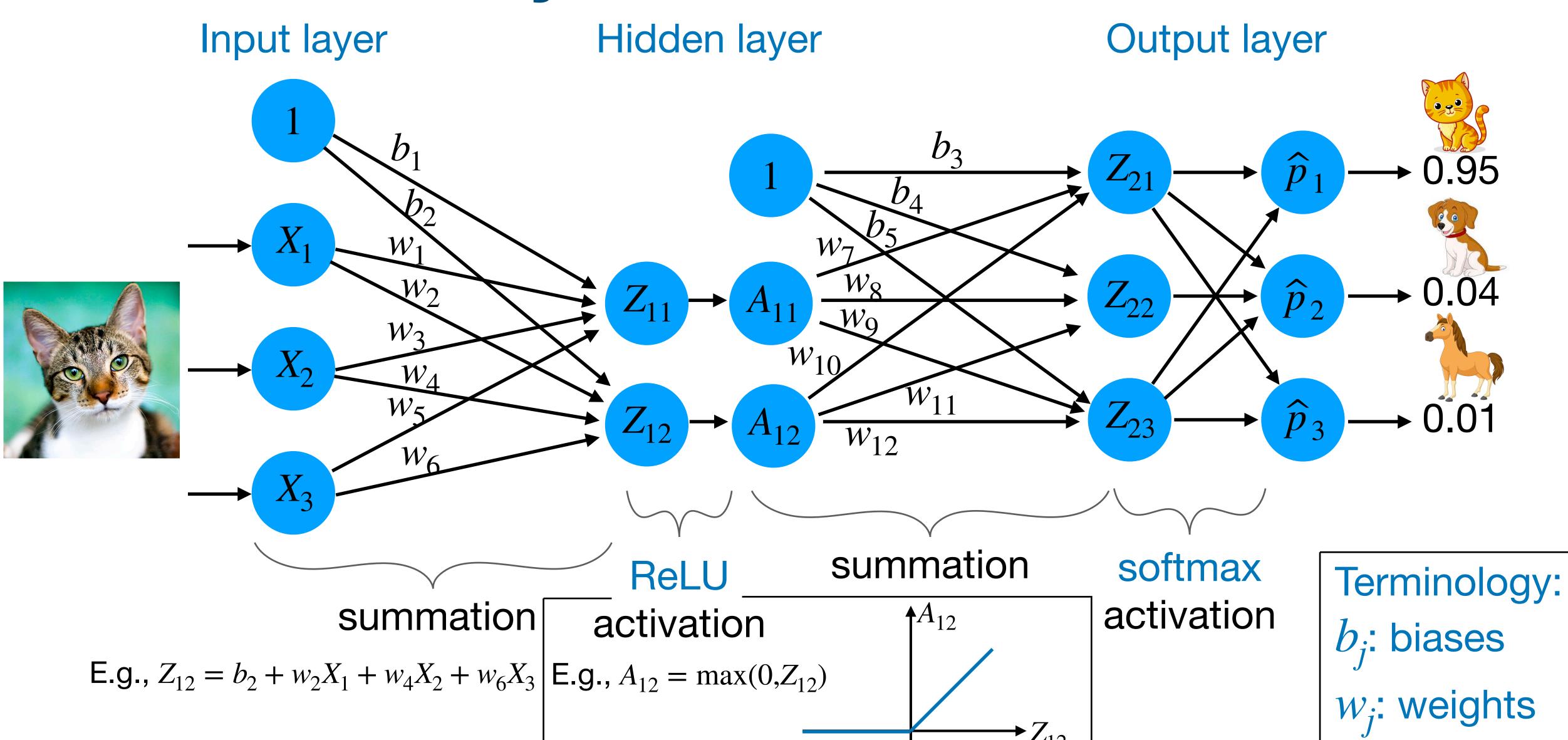
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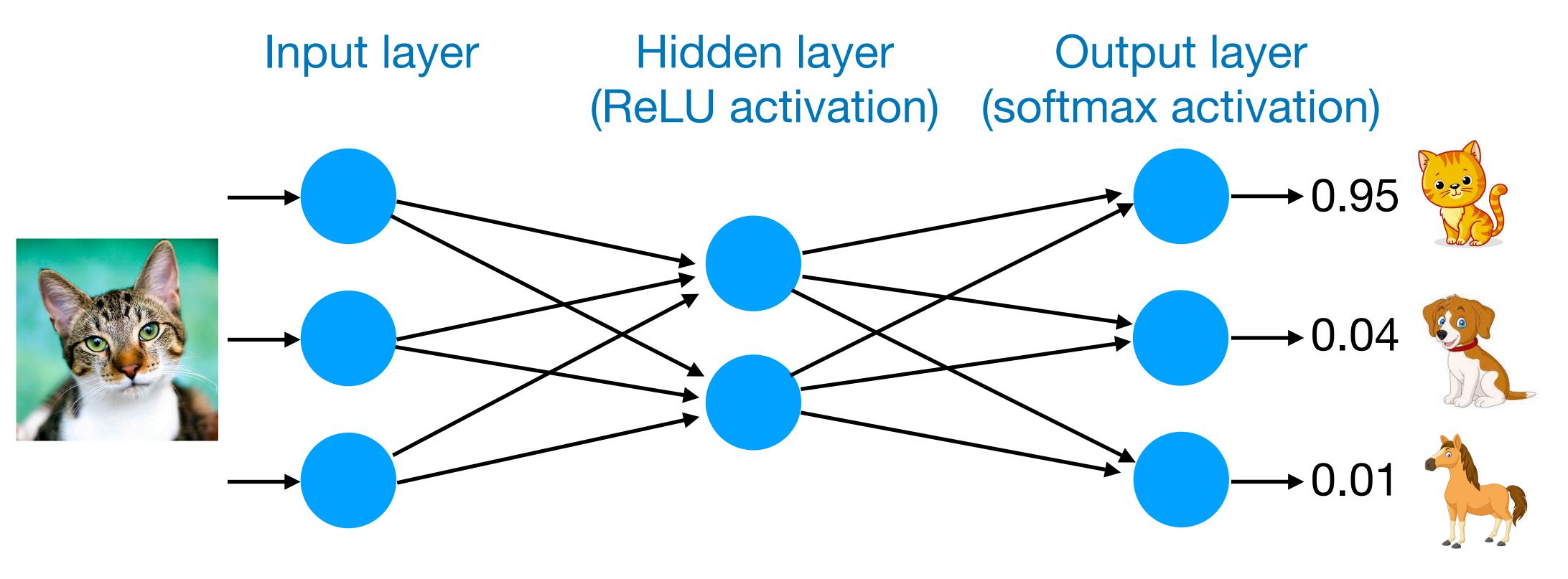




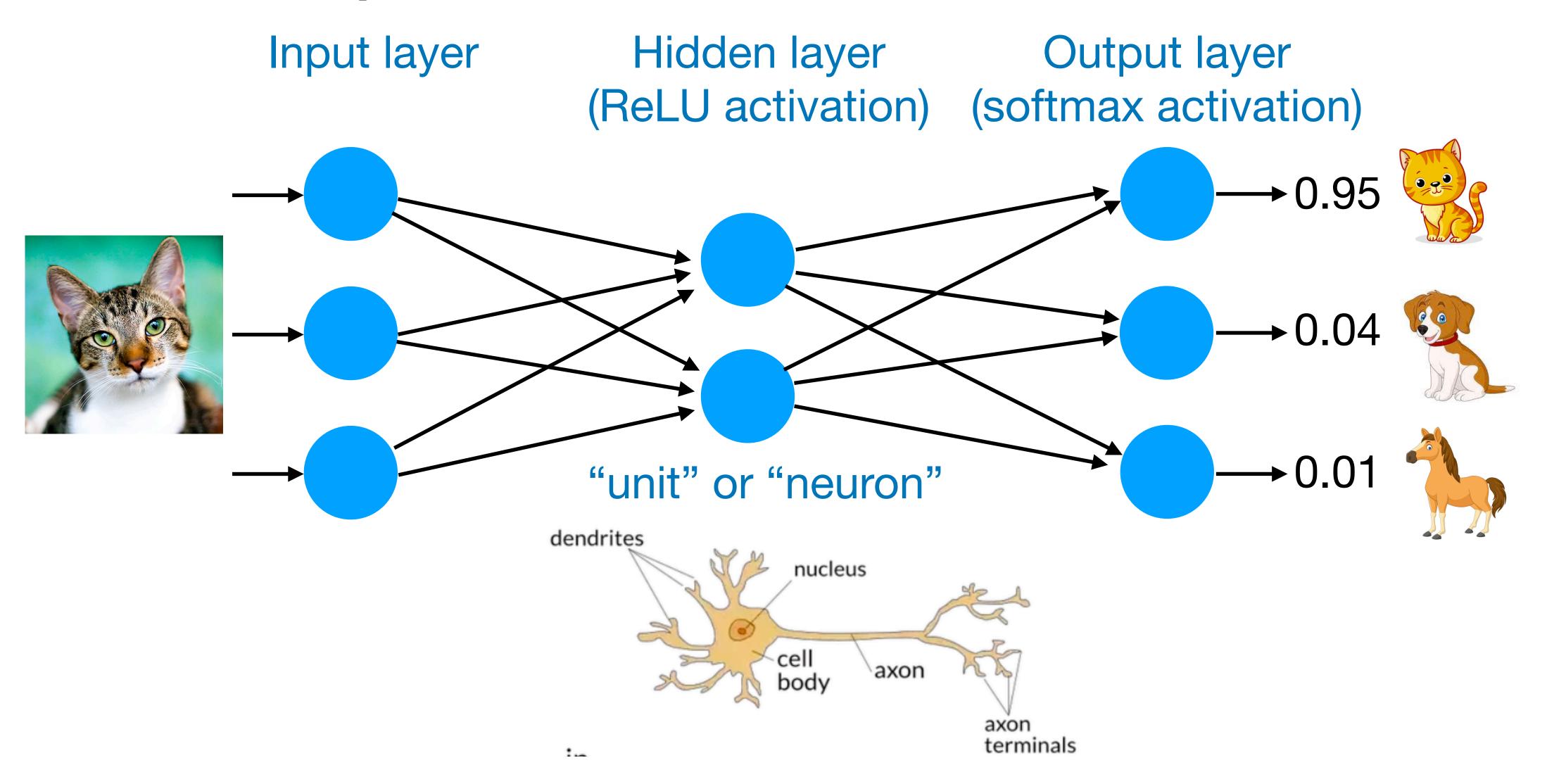




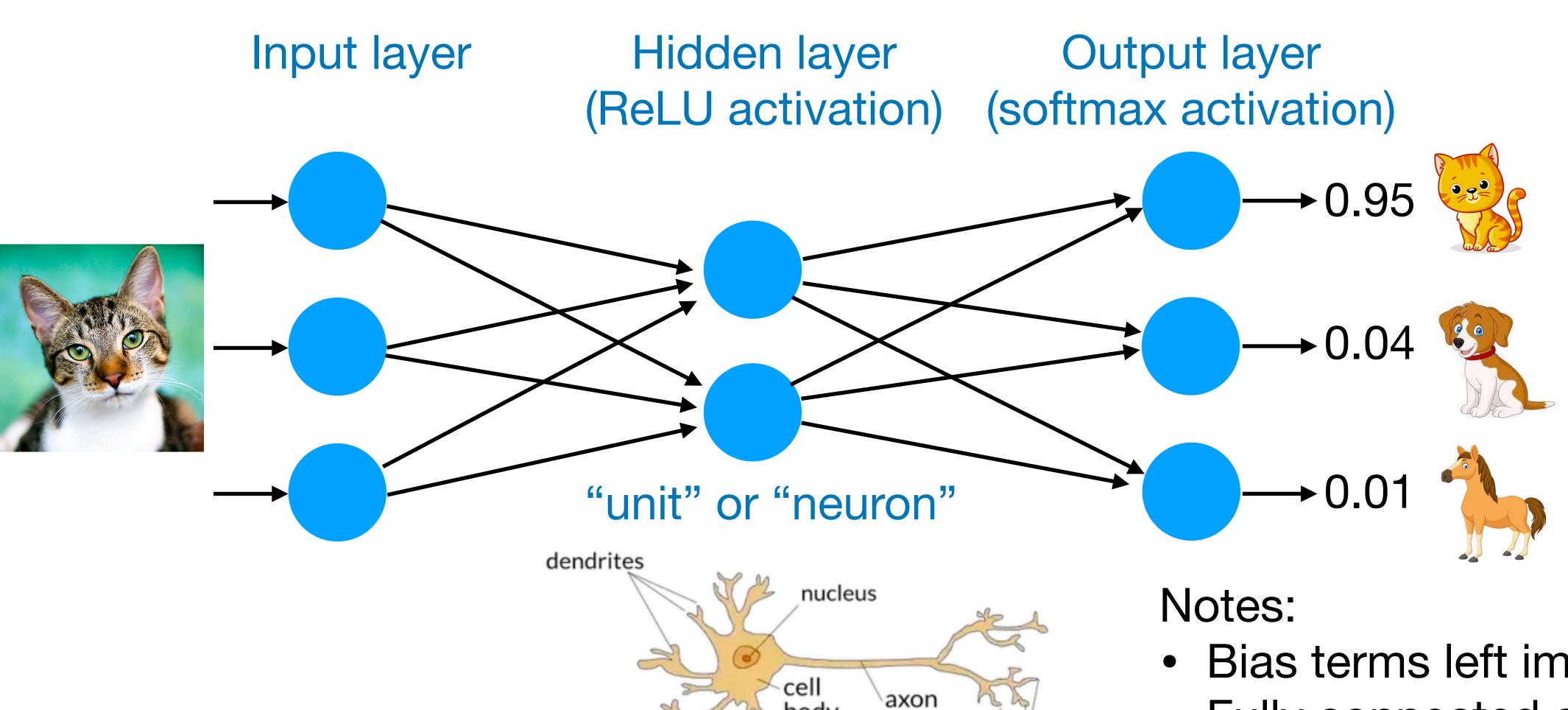
Concise representation



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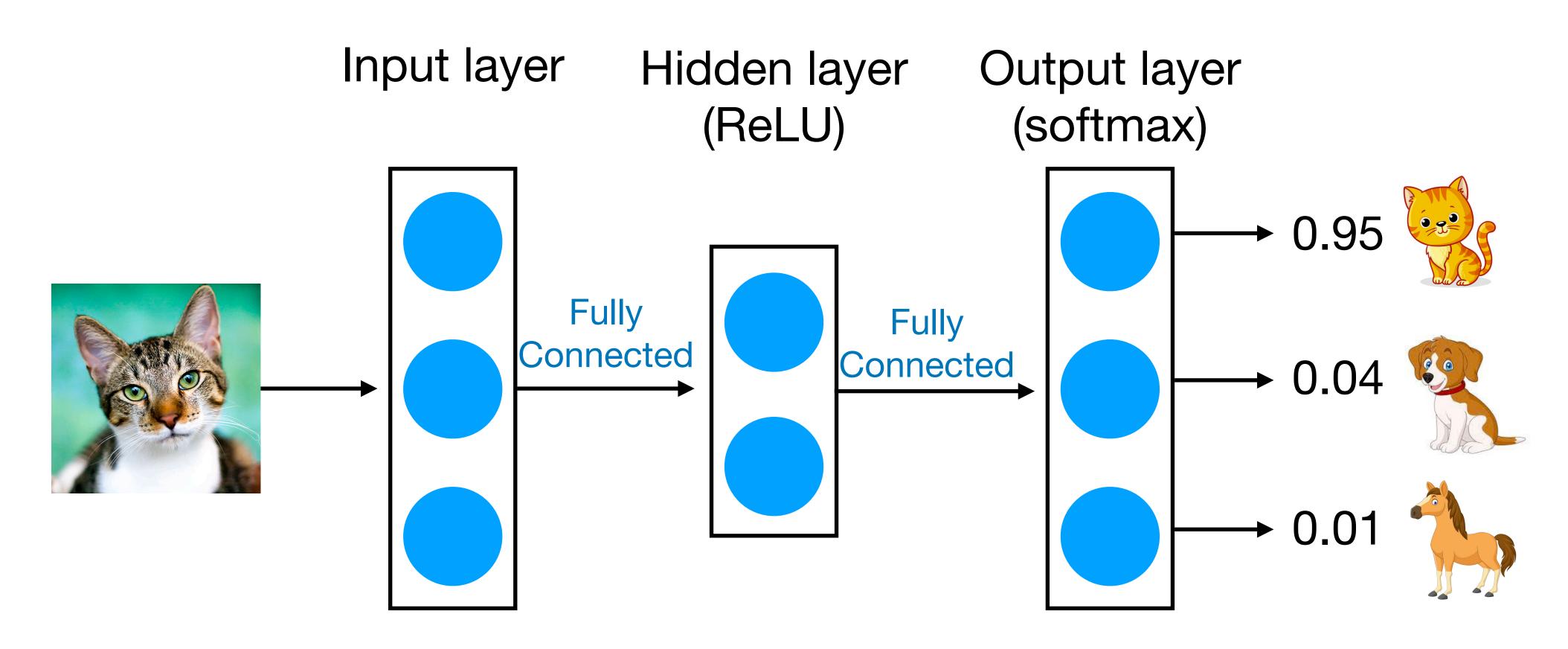
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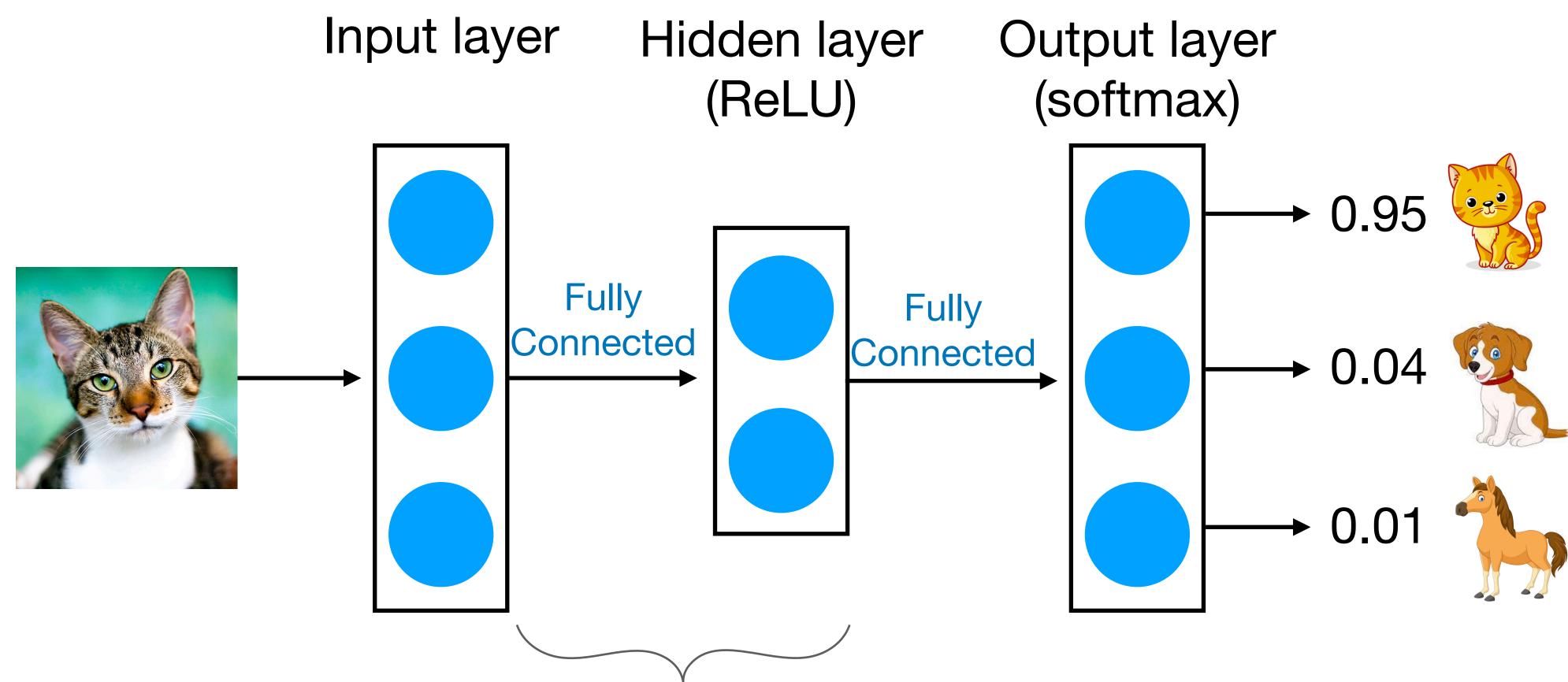
terminals

- Bias terms left implicit.
- Fully connected architecture.
- Hidden layer: learned features.

Even more concise representation

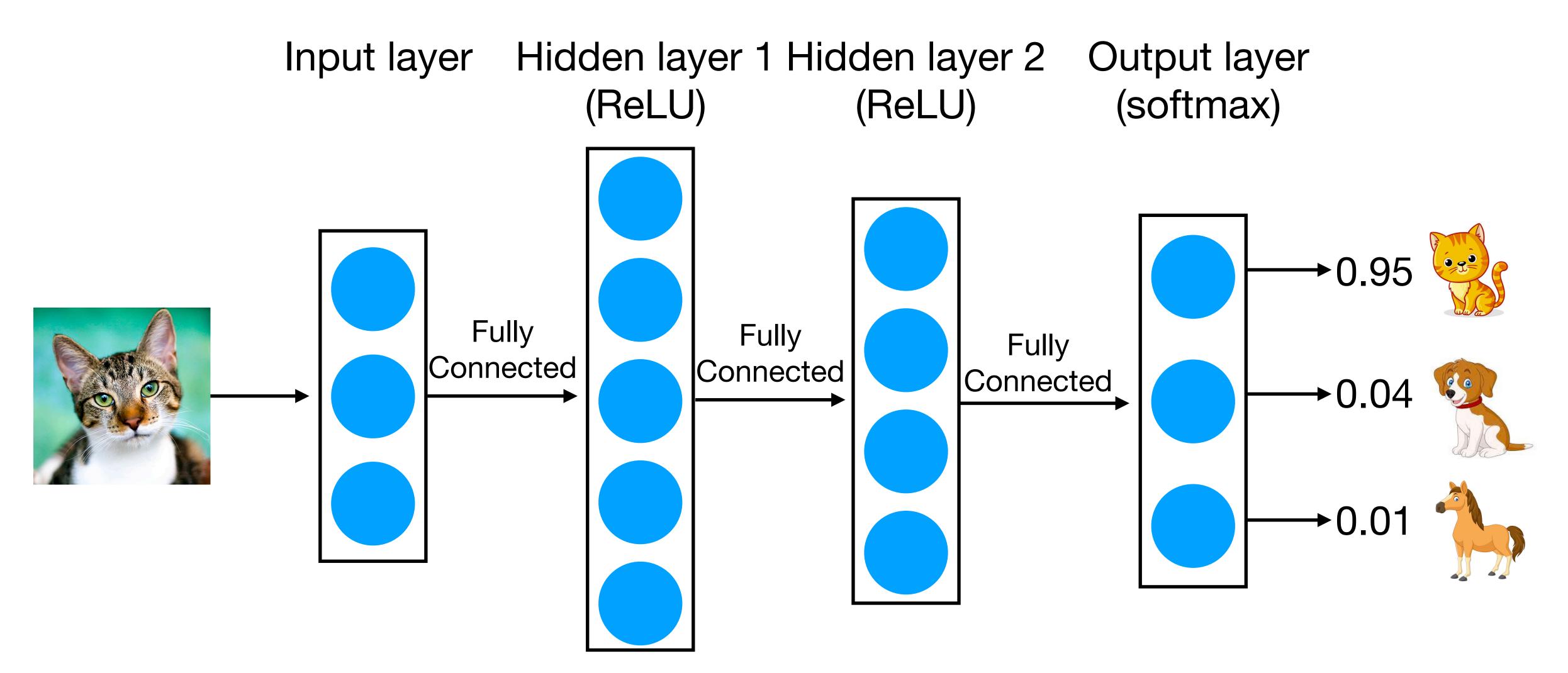


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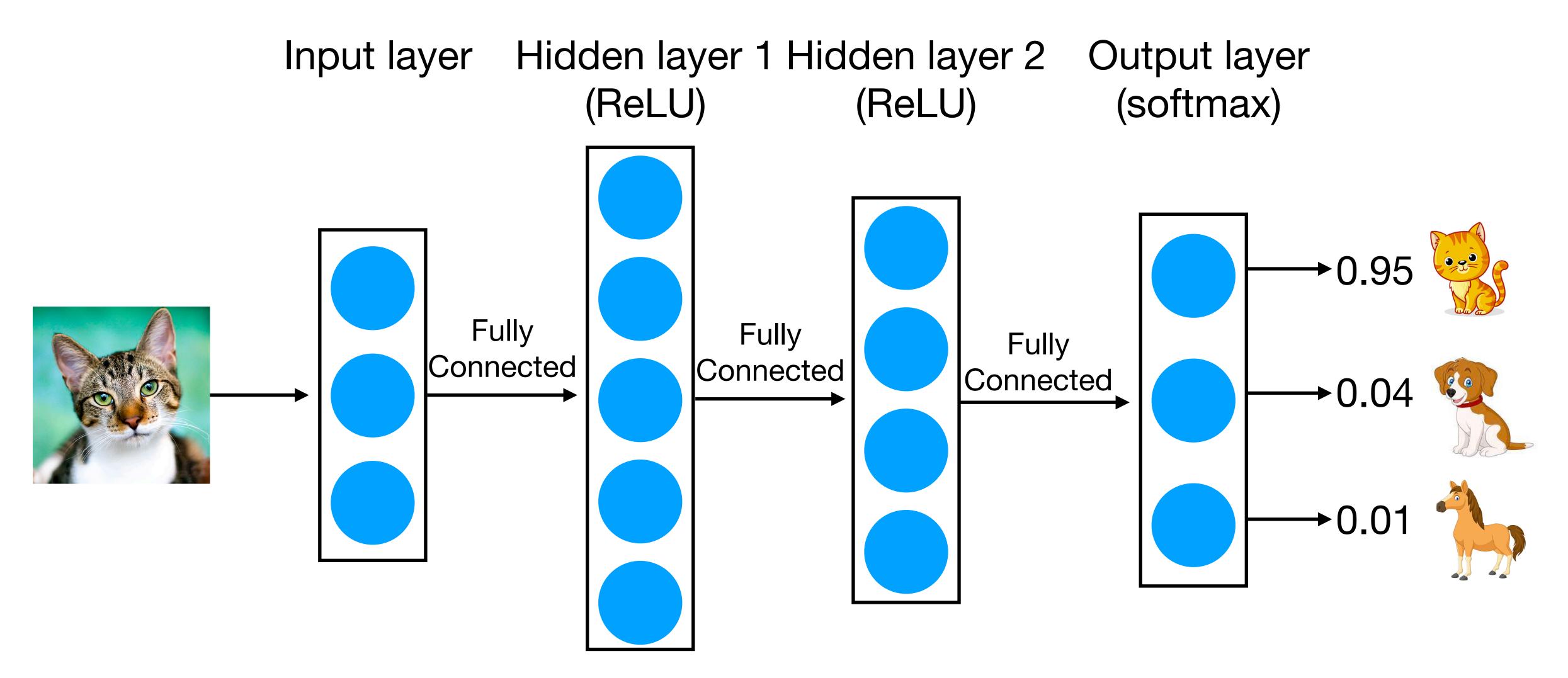


Number of parameters in FC layer = (# units in previous layer + 1) × (# units in this layer)

Multi-layer fully connected neural networks



Multi-layer fully connected neural networks



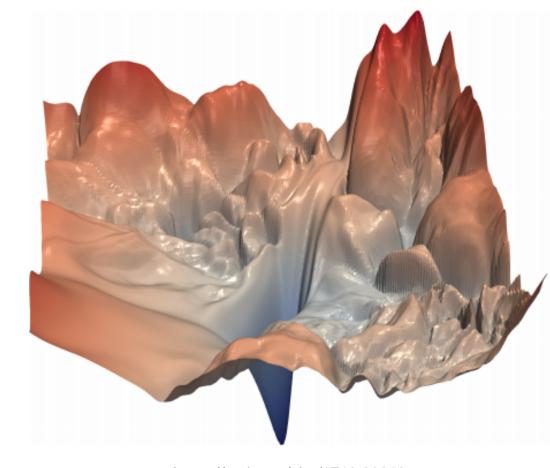
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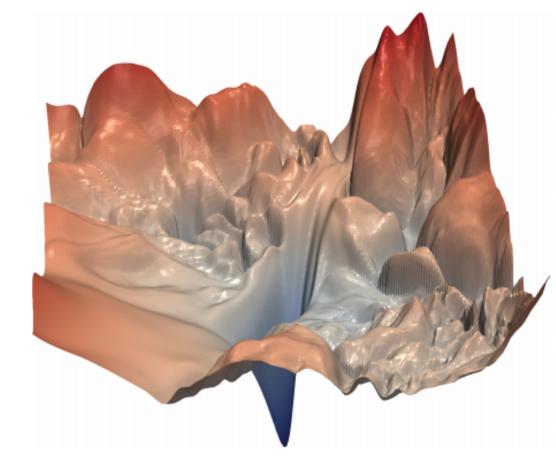


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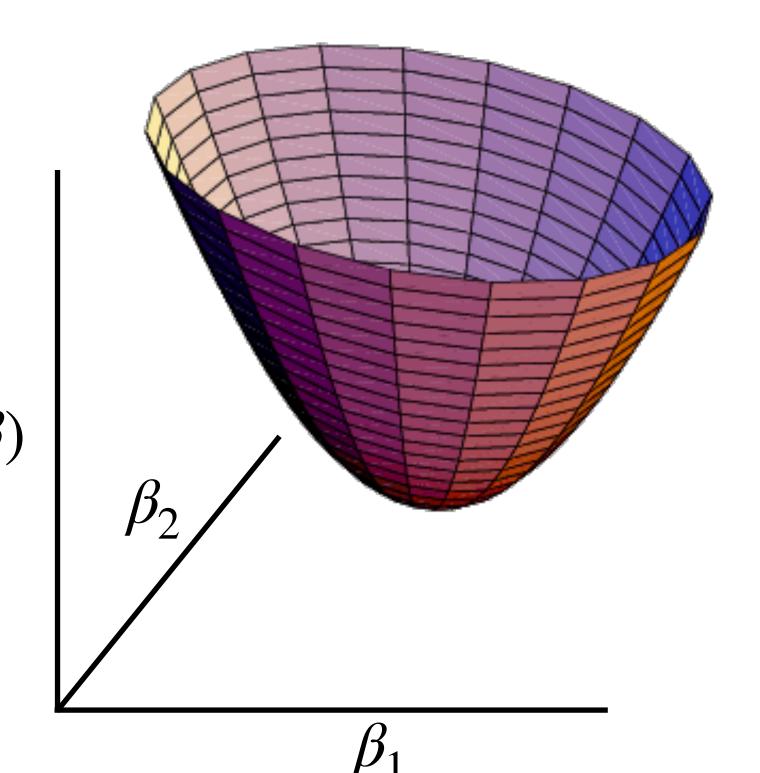
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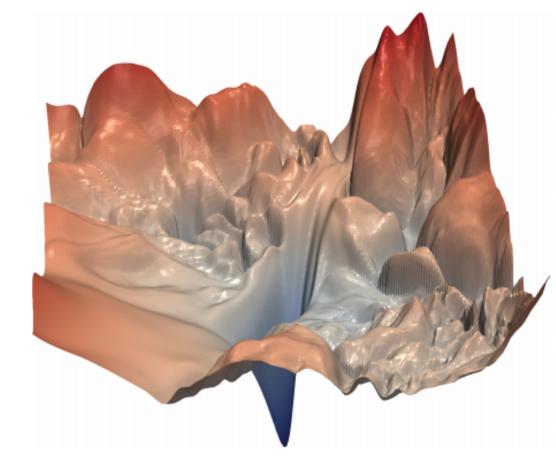
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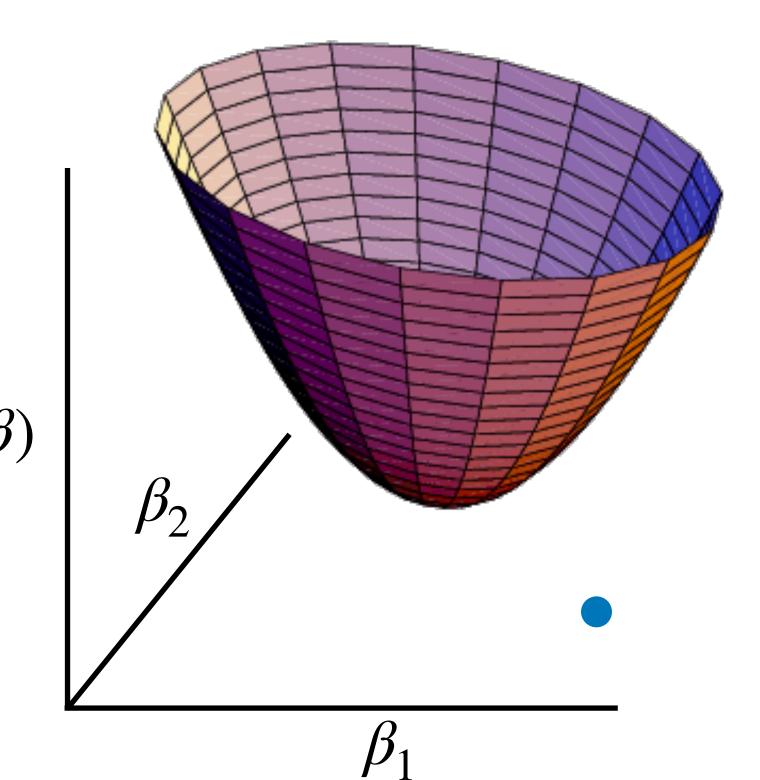
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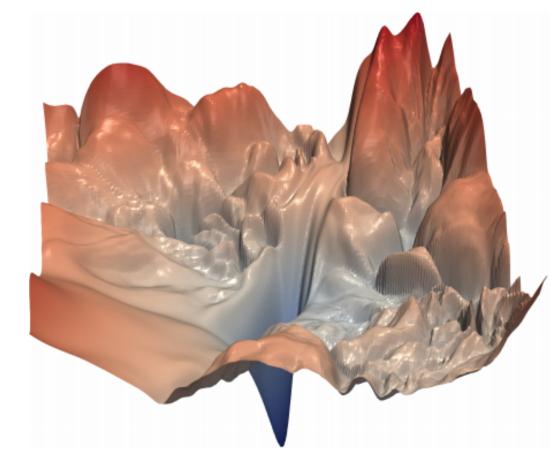
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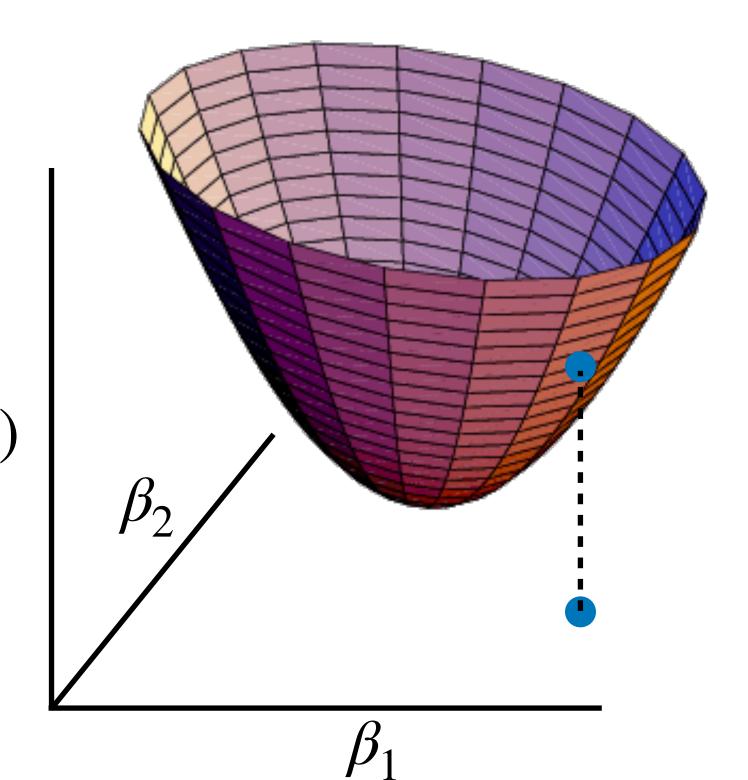
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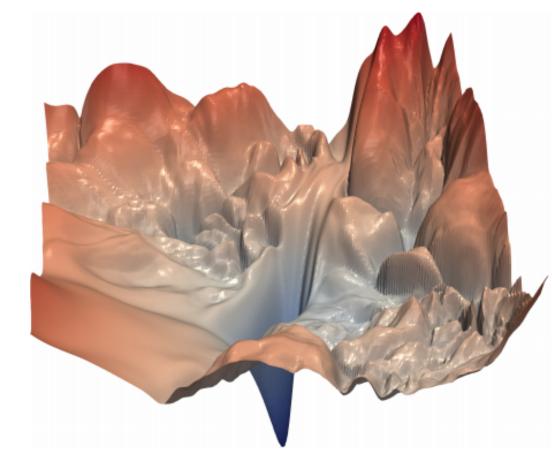
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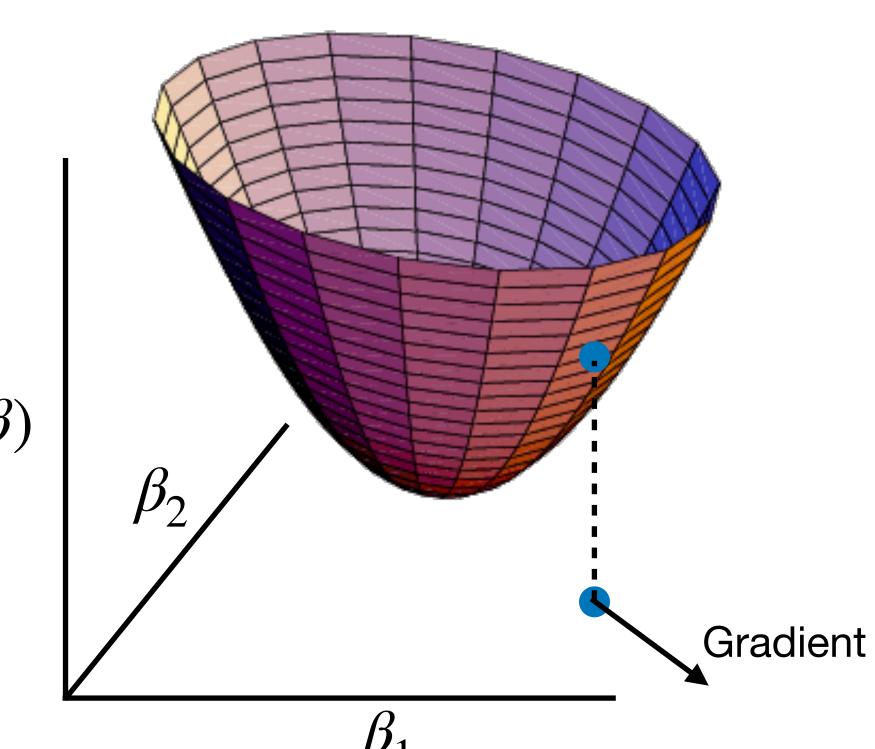
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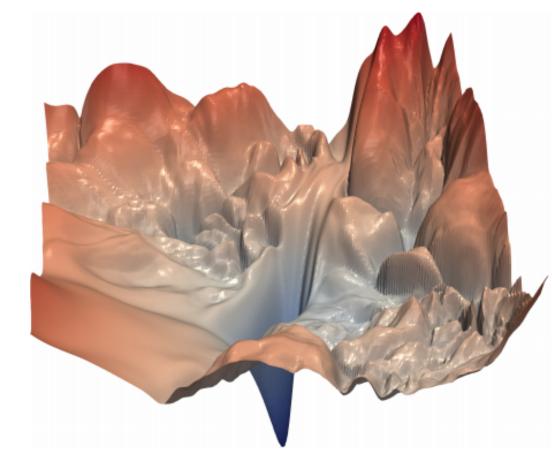
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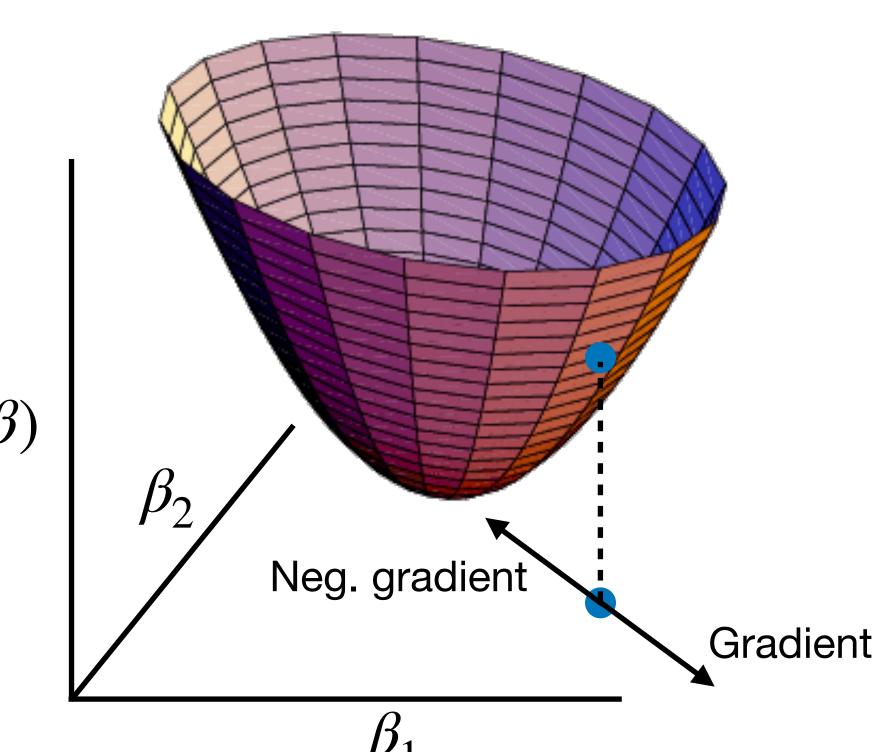
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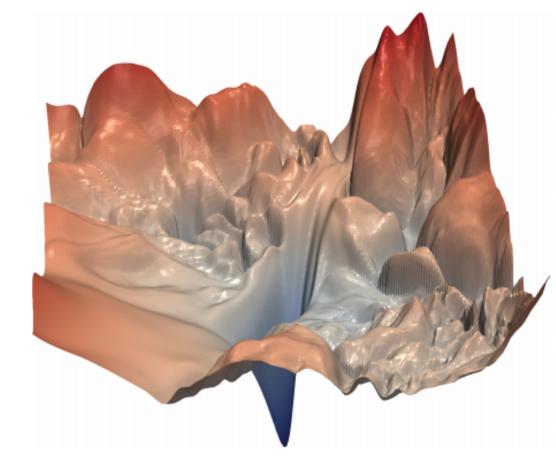
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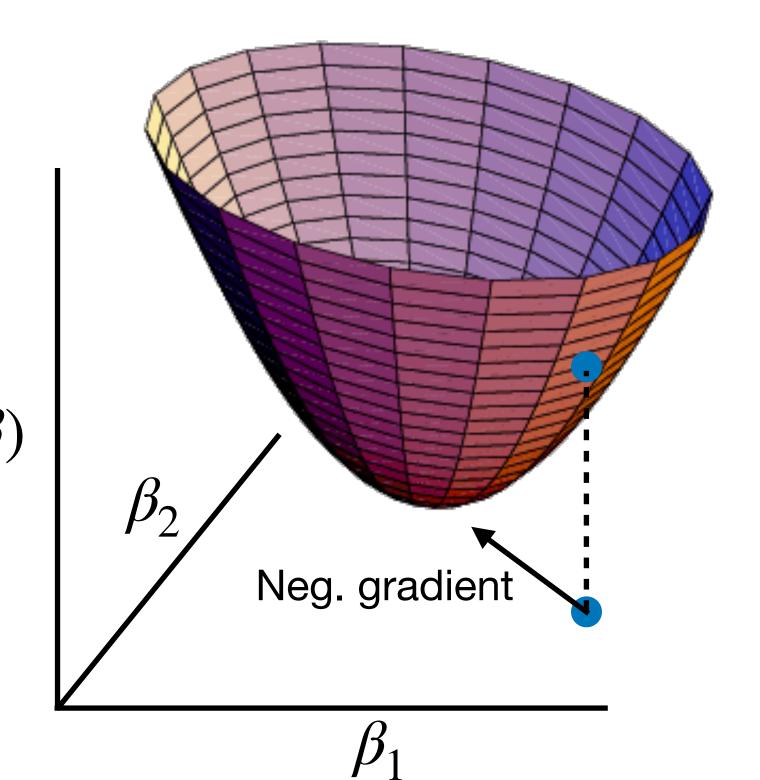
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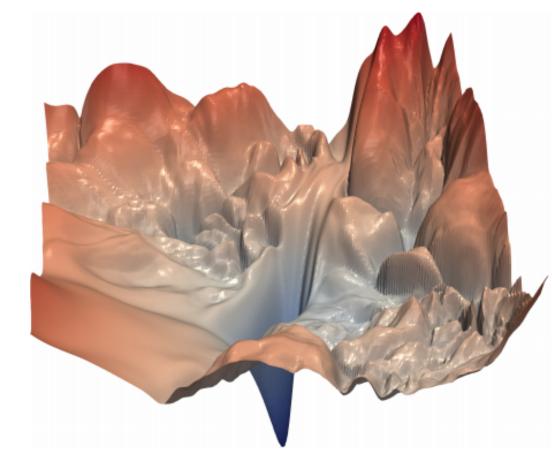
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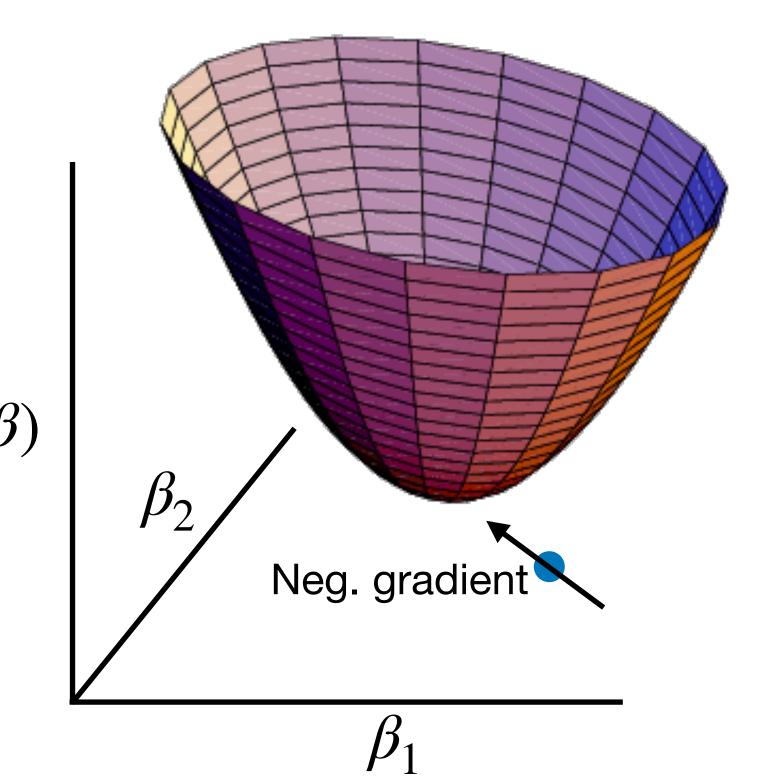
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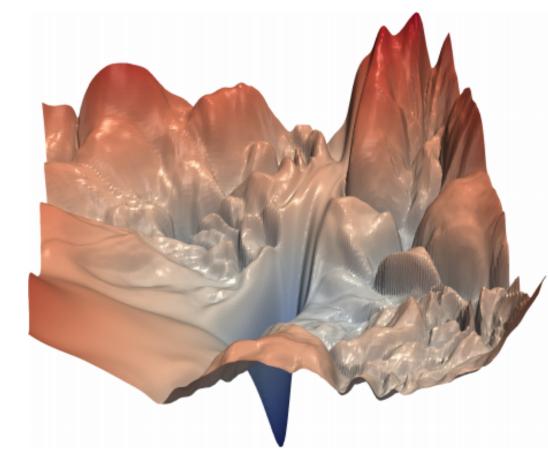


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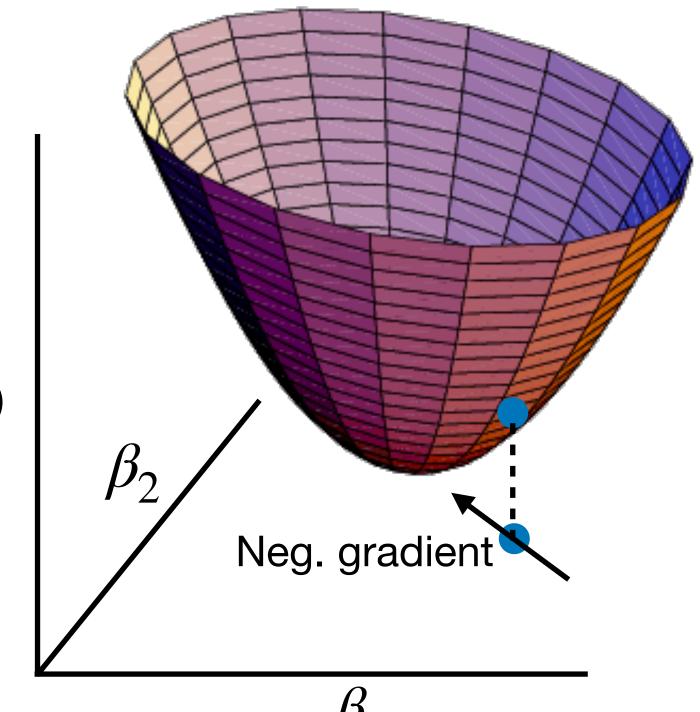
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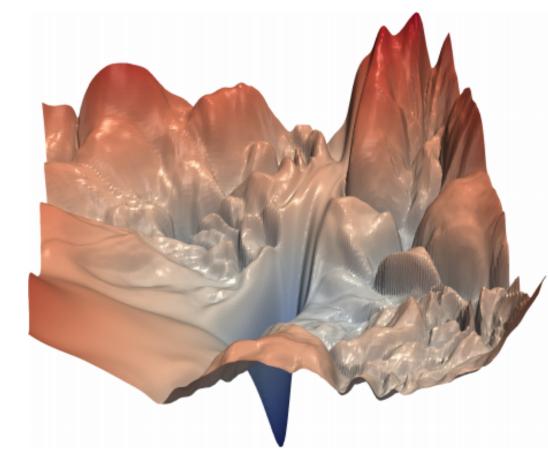


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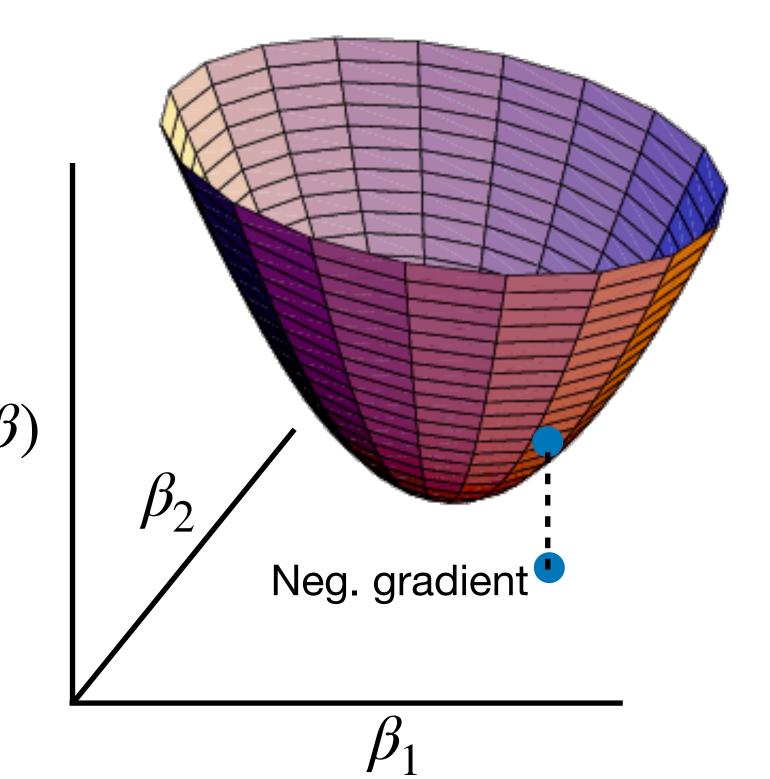
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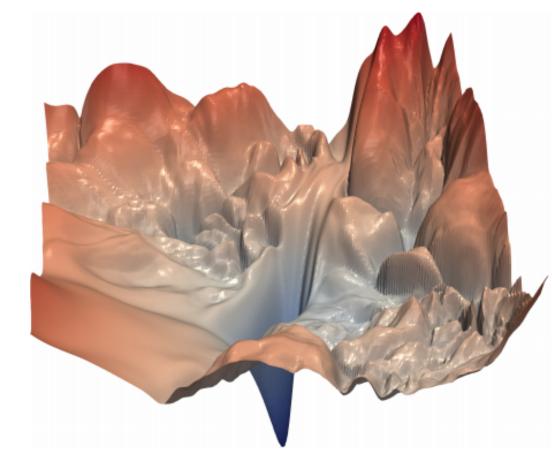
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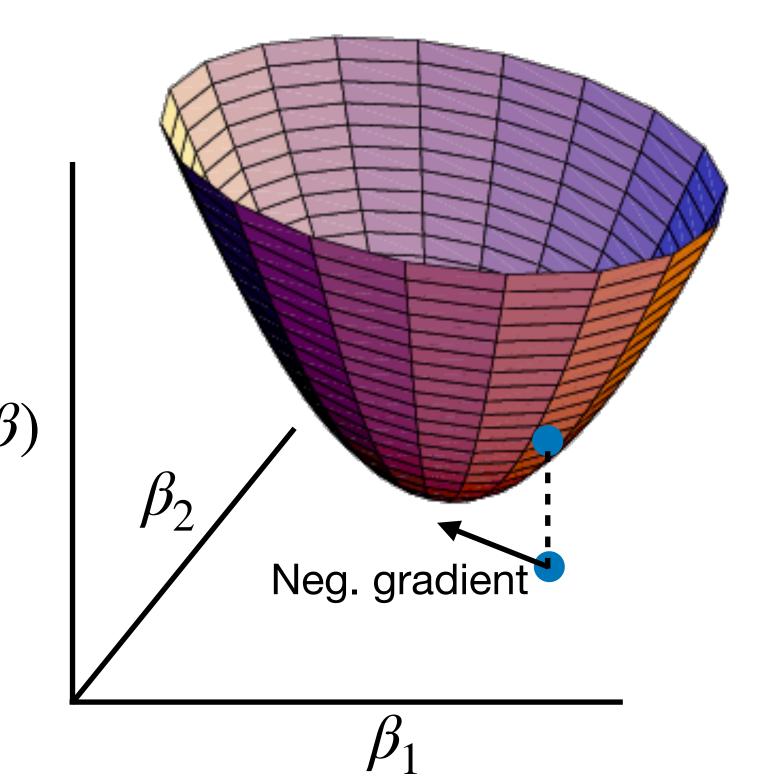
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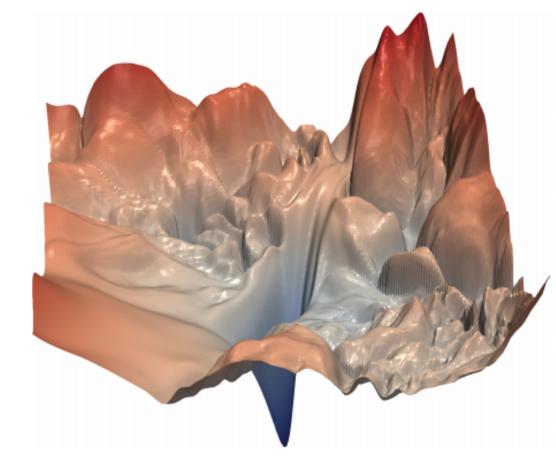


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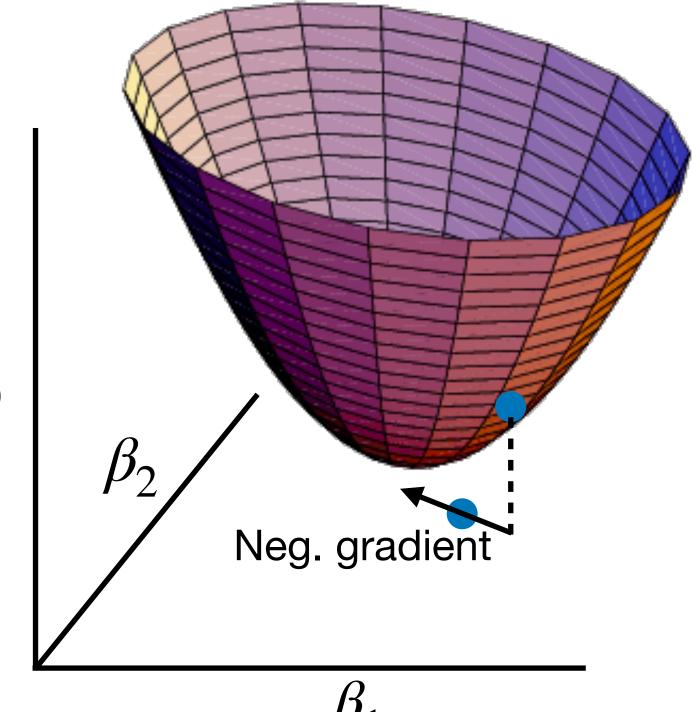
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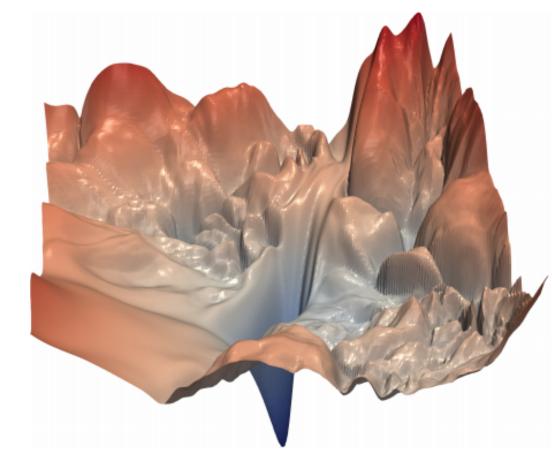


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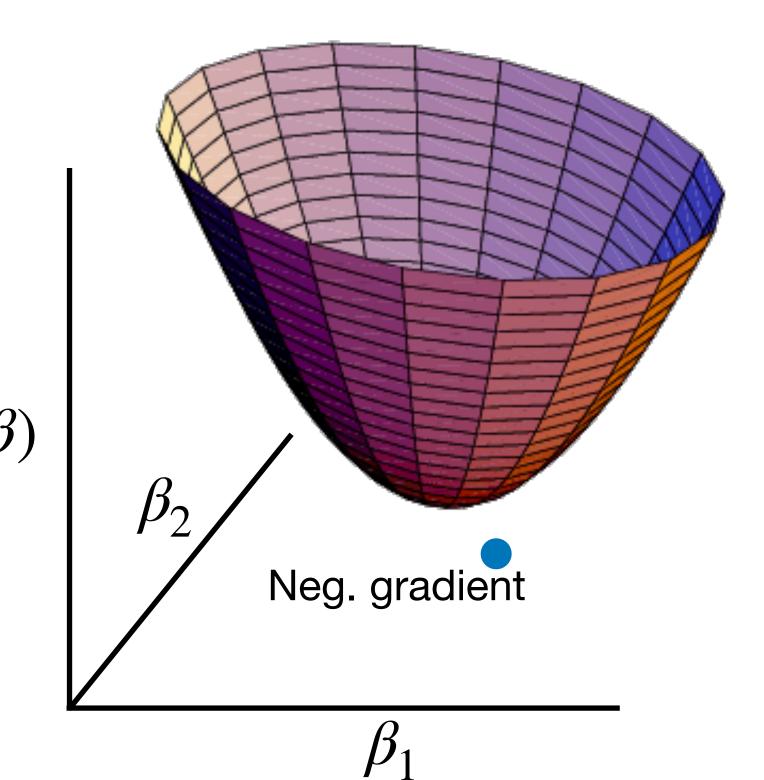
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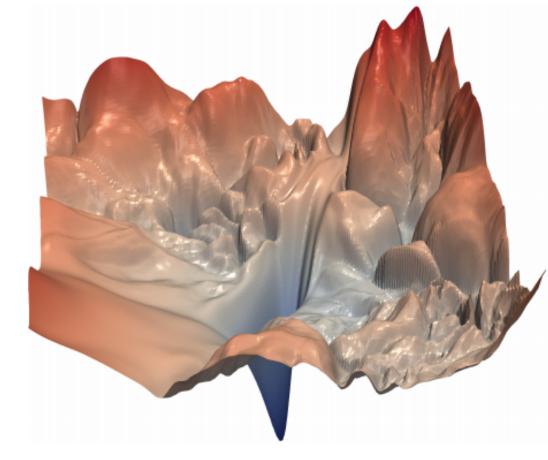
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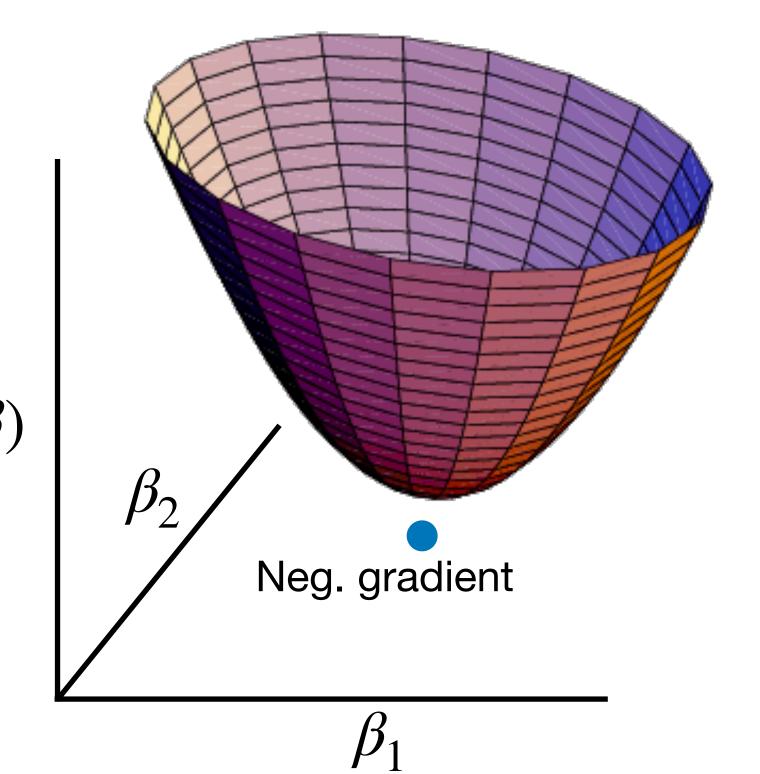
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$$F(\beta) = \frac{1}{n} \sum_{i=1}^{n} L(Y_i, f_{\beta}(X_i))$$
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This can make gradient descent prohibitively expensive.

$$\nabla F(\beta) = \frac{1}{n} \sum_{i=1}^{n} \nabla L(Y_i, f_{\beta}(X_i)) \approx \frac{1}{|S|} \sum_{i \in S} \nabla L(Y_i, f_{\beta}(X_i)).$$

Use subset $S \subseteq \{1, ..., n\}$ (mini-batch) of observations to approximate gradient:

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 - For $m=1,\ldots,M$, update $\beta \leftarrow \beta \gamma \cdot \frac{1}{|S_m|} \sum_{i \in S_m} \nabla L(Y_i,f_\beta(X_i)).$

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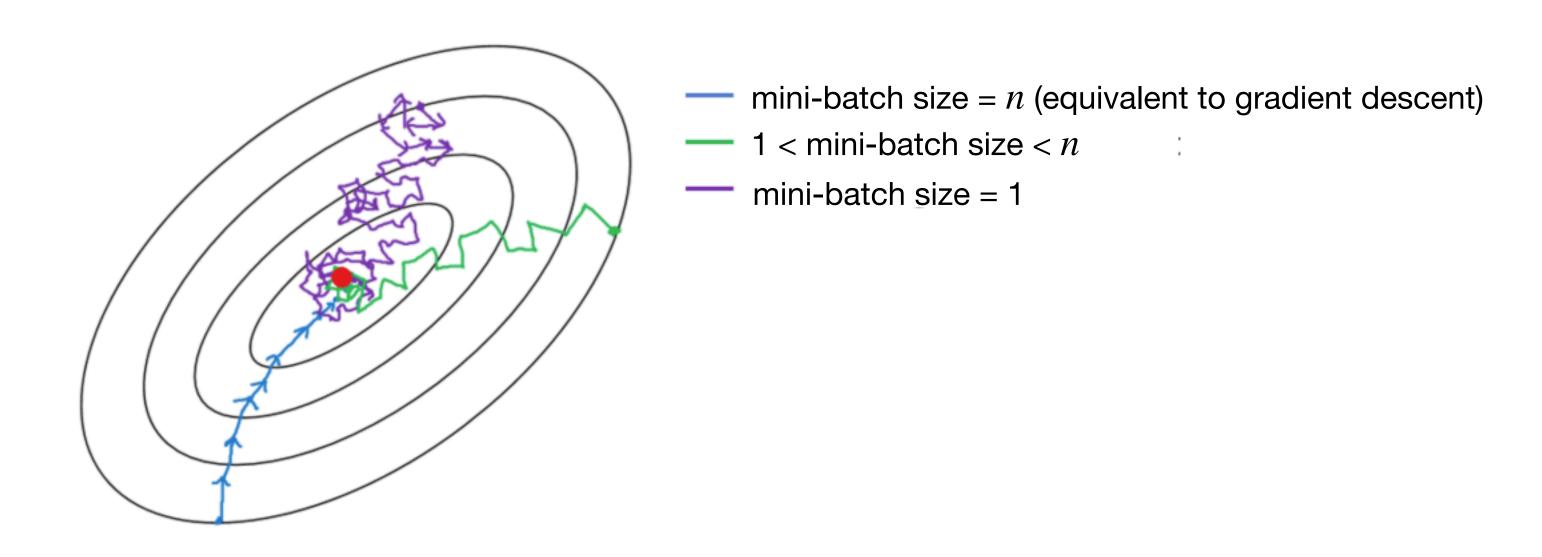
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One "epoch"

Backpropagation: An efficient algorithm to compute $\nabla L(Y_i, f_{\beta}(X_i))$

Behavior of stochastic gradient descent

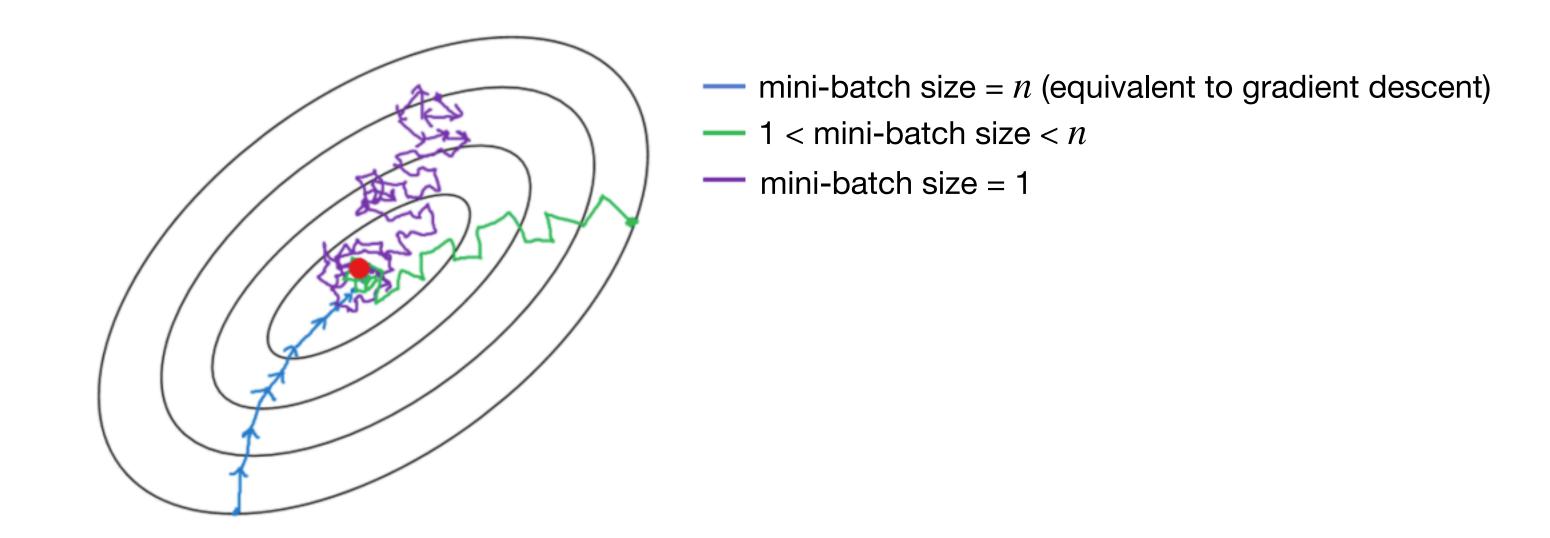
Stochastic gradient descent wobbles toward decreasing values of the objective:



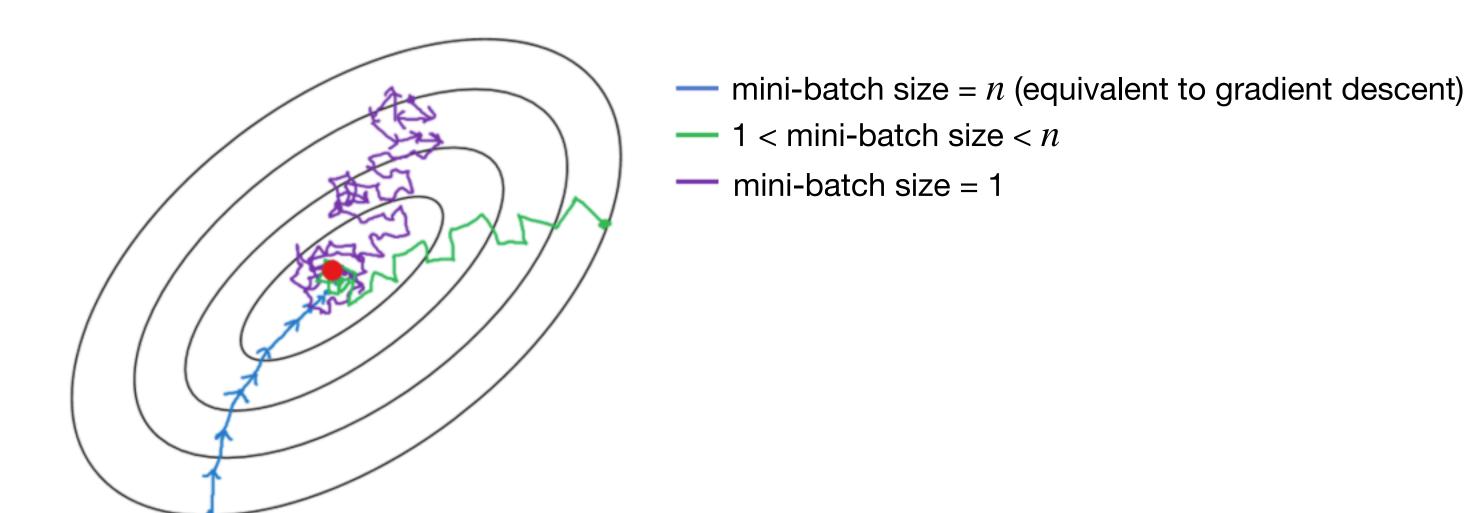
Source: https://medium.com/analytics-vidhya/gradient-descent-vs-stochastic-gd-vs-mini-batch-sgd-fbd3a2cb4ba4

The smaller the mini-batch, the cheaper and more wobbly each step is; Intermediate mini-batch sizes tend to work well, e.g. mini-batch size = 32.

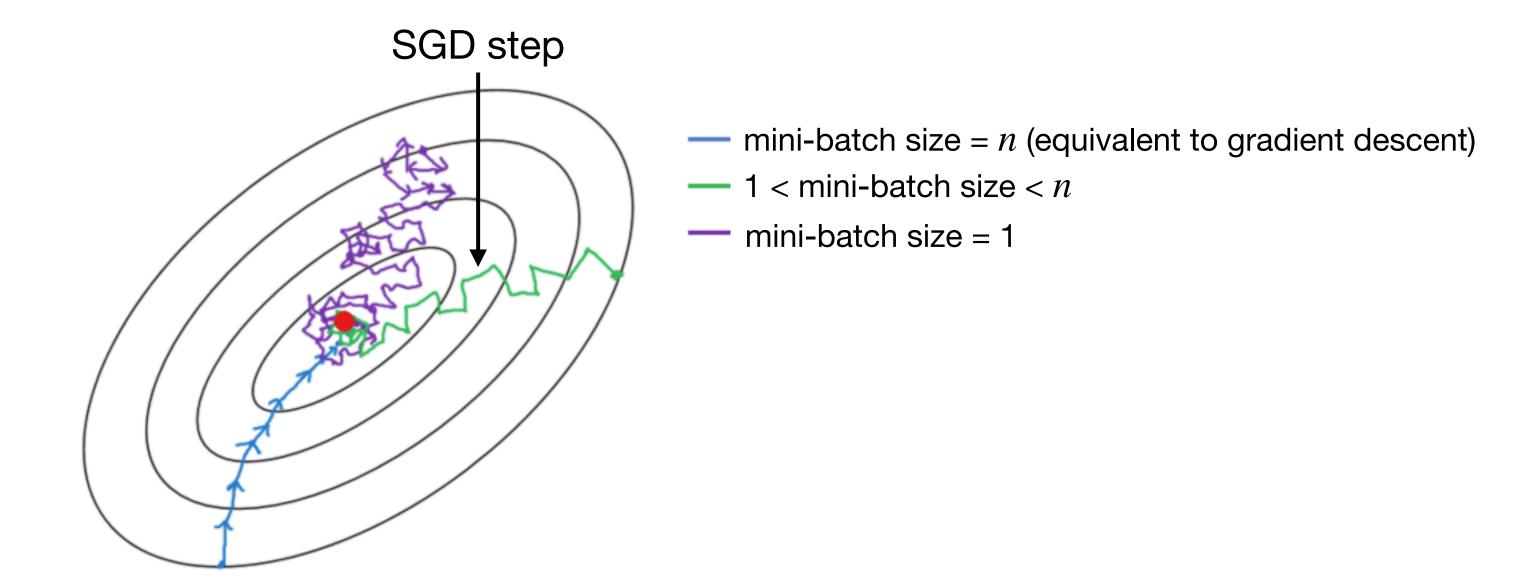
Bonus: The extra randomness sometimes allows SGD to wobble past local minima.



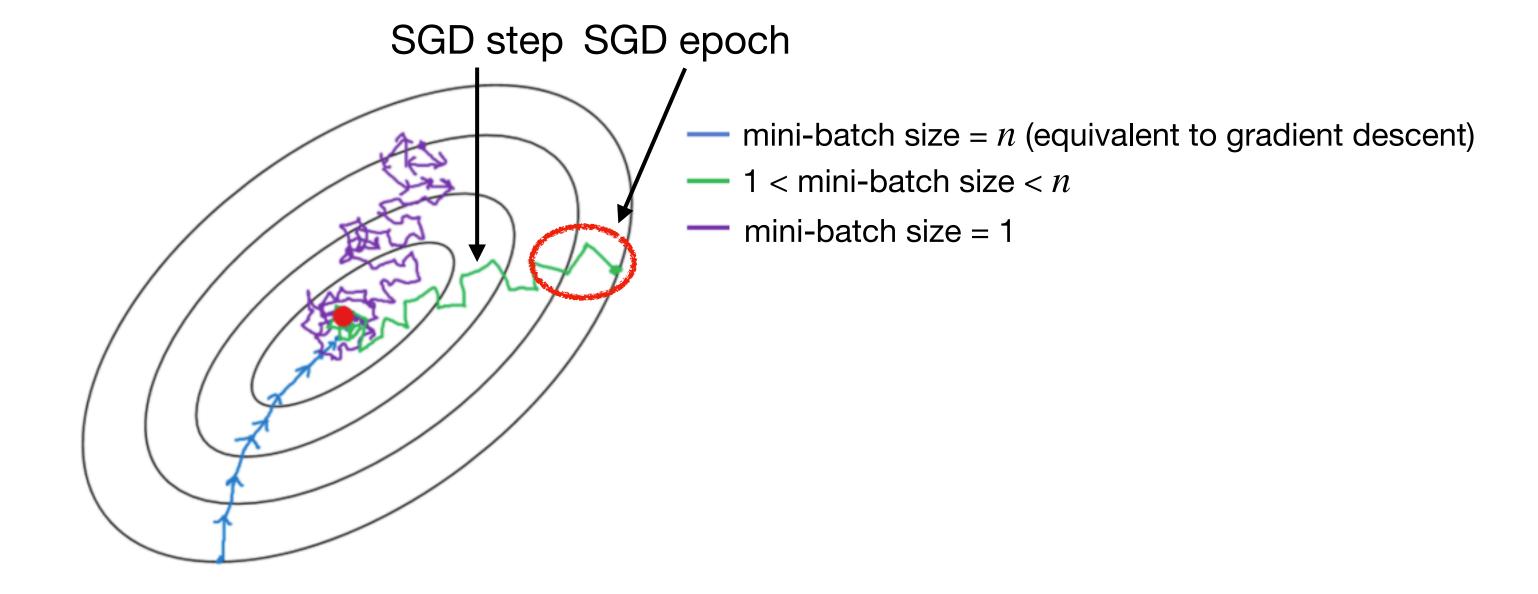
An epoch consists of SGD steps that cycle through all of the observations once.



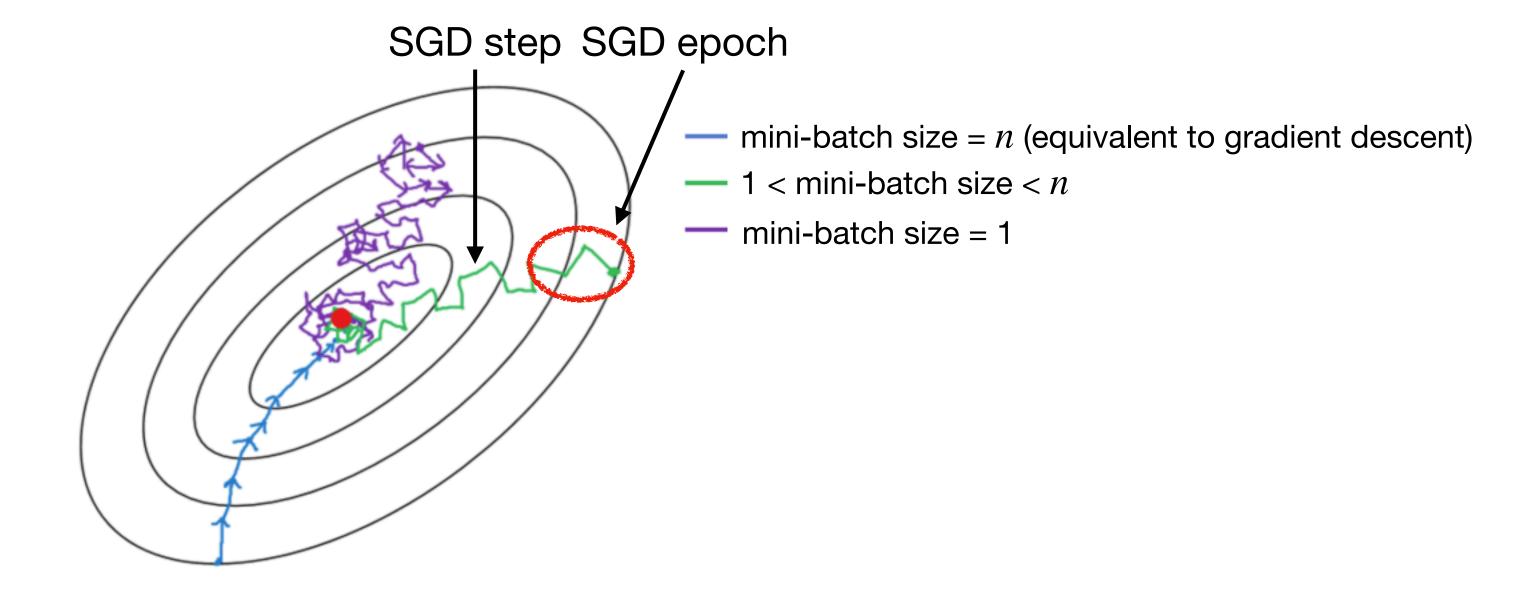
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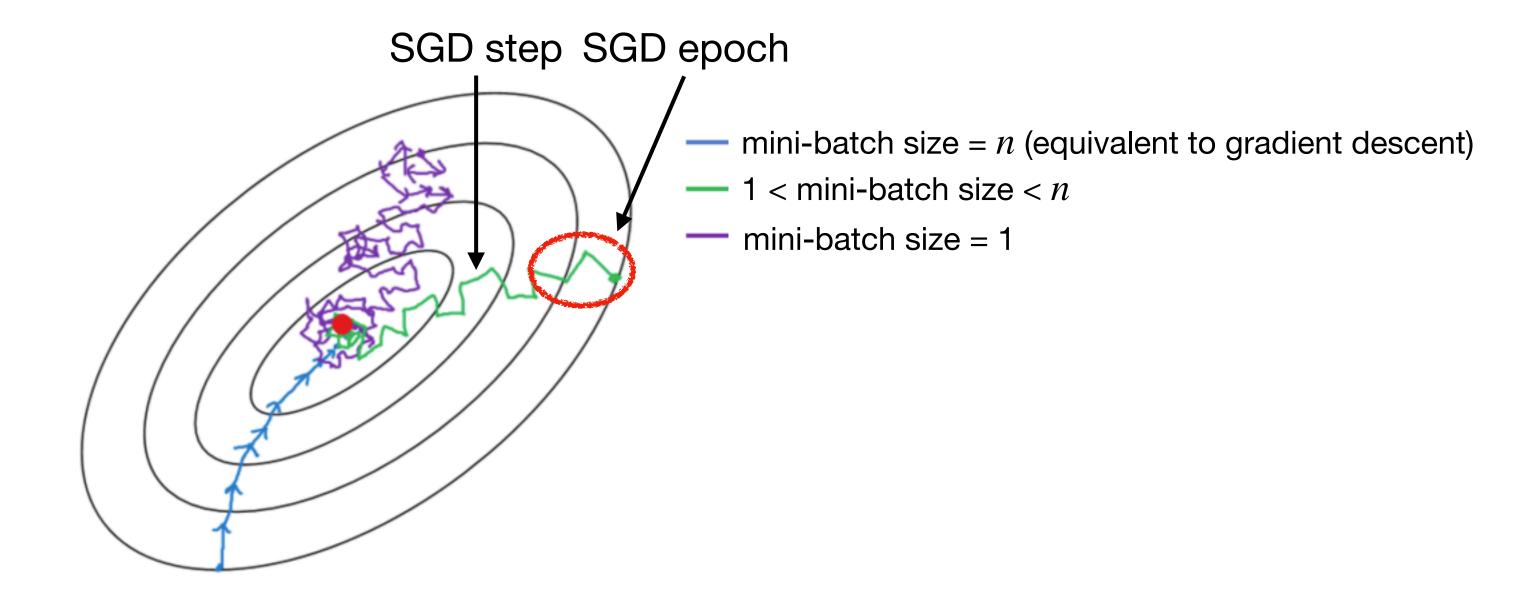


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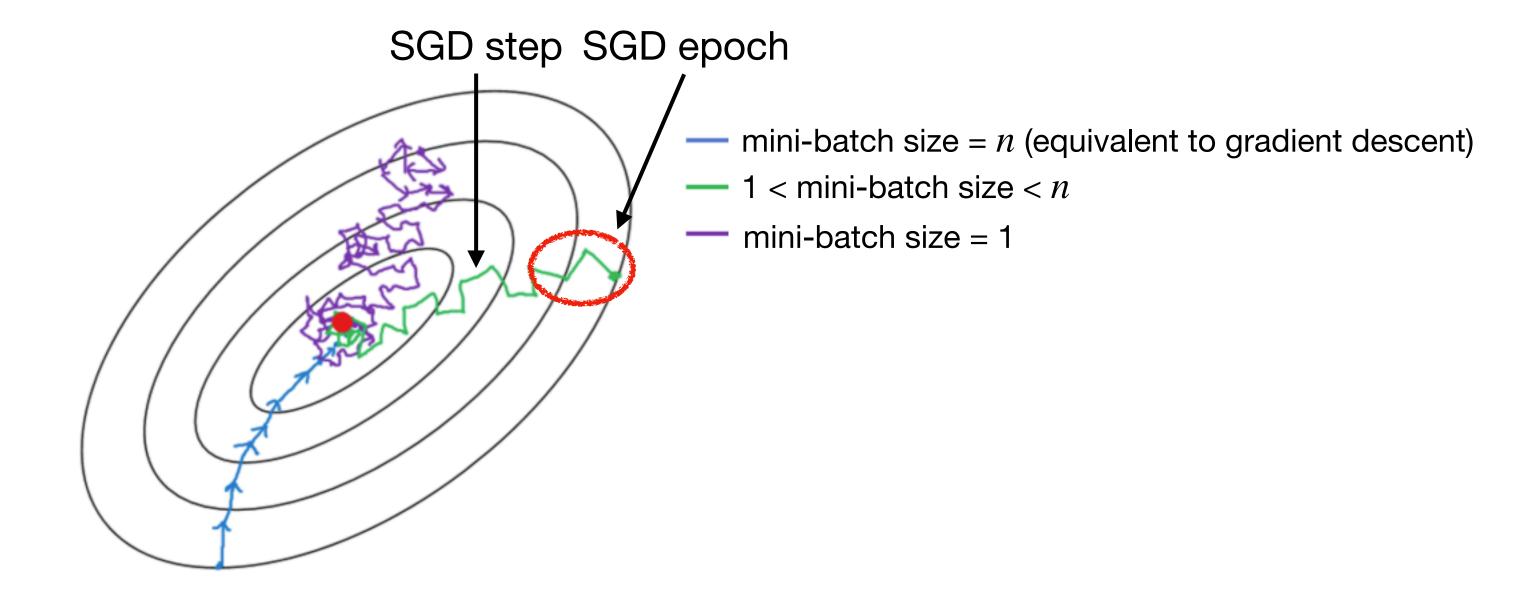
Observation	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
Epoch				Еp	ocł	า 1							Ер	ocł	า 2			

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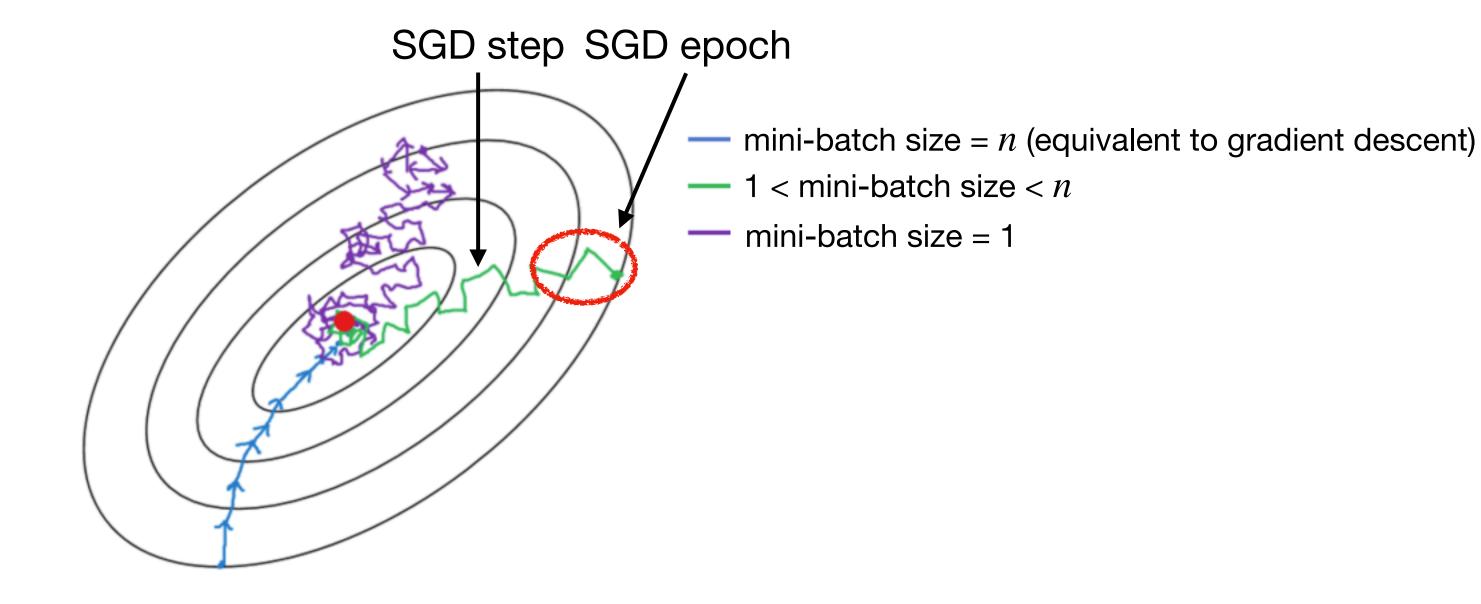
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Epoch				Ep	och	า 1			Epoch 2										
SGD step (mini-batch size 1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	

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SGD step (mini-batch size 1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
SGD step (mini-batch size 3)	Step 1			Step 2			Step 3			S	tep	4	S	tep	5	S	tep	6	

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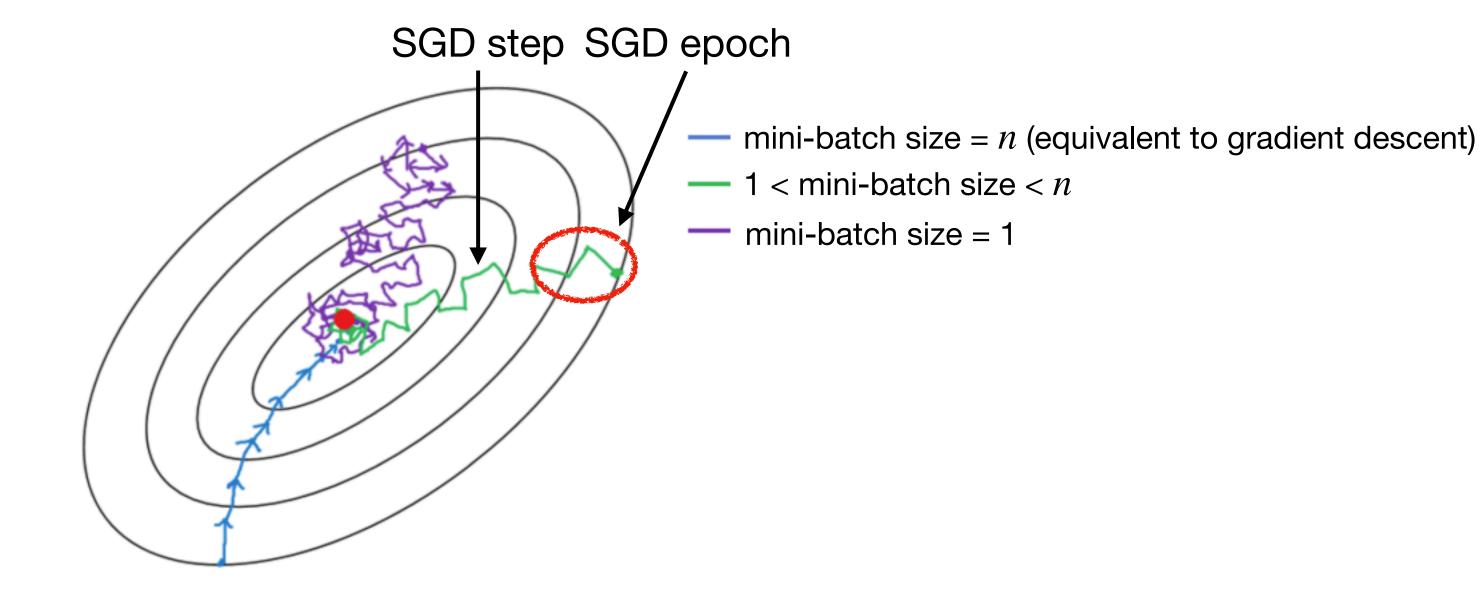


Source: https://medium.com/analytics-vidhya/gradient-descent-vs-stochastic-gd-vs-mini-batch-sgd-fbd3a2cb4ba4

E.g. n = 9, mini-batch size 3. Then each epoch consists of three SGD steps.

Observation	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9			
Epoch	Epoch 1										Epoch 2										
SGD step (mini-batch size 1)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18			
SGD step (mini-batch size 3)	Step 1			Step 2				Step 3			Step 4			Step 5			tep	6			

An epoch consists of SGD steps that cycle through all of the observations once.

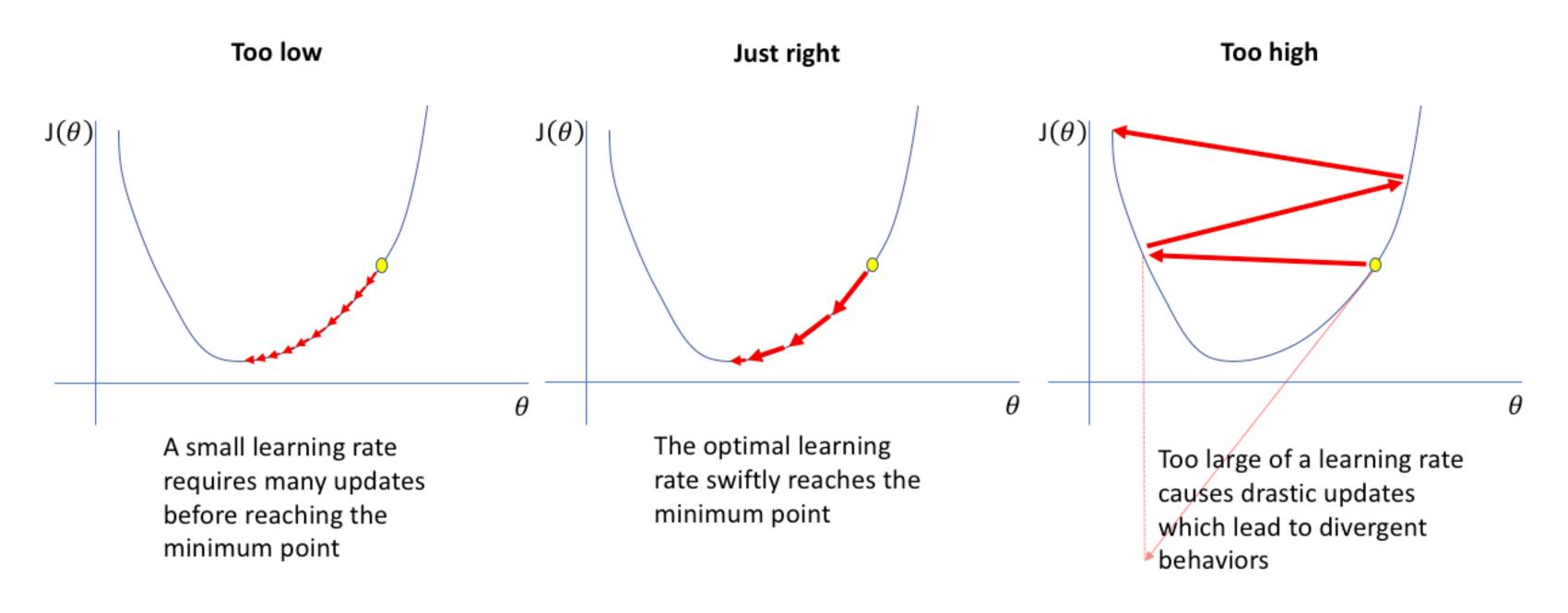


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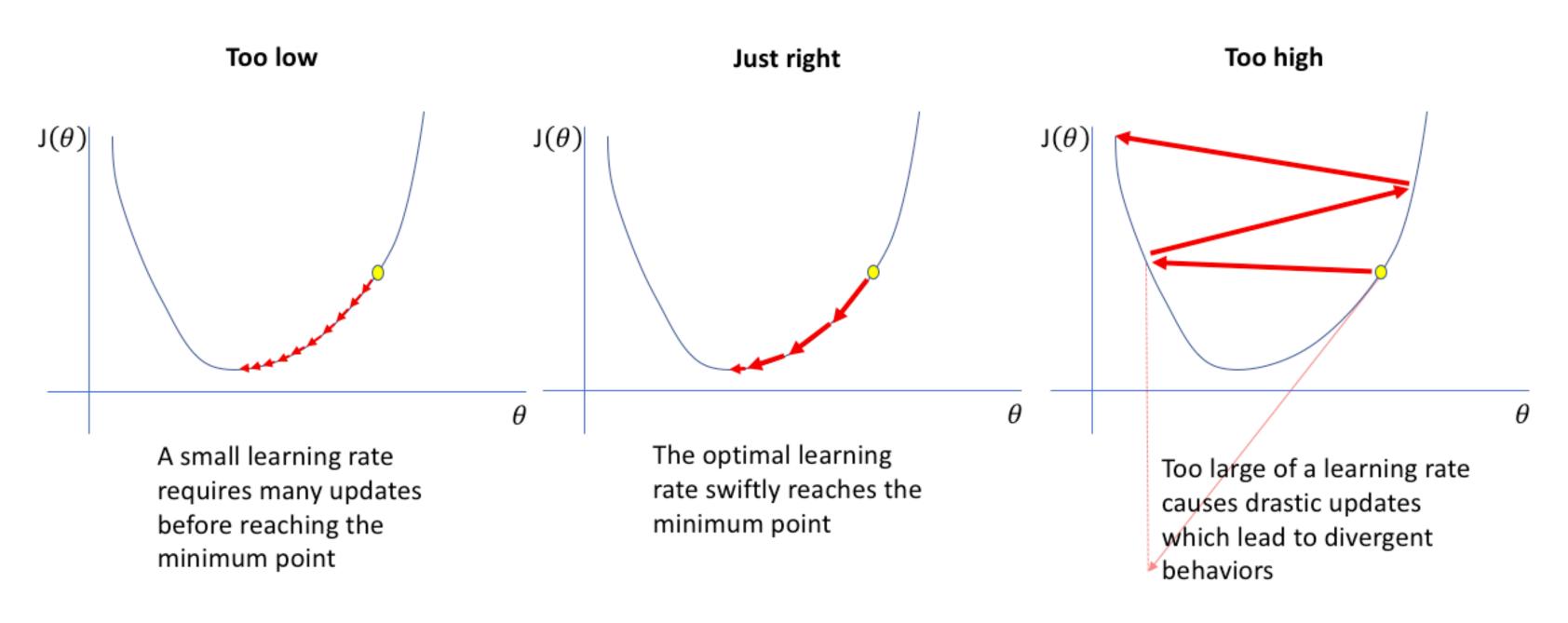
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Epoch				Ep	och	า 1			Epoch 2										
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SGD step (mini-batch size 3)	S	tep	1	Step 2 St					Step 3			4	S	tep	5	St	tep	6	
Gradient step	Step 1												S	tep	2				

The learning rate for (stochastic) gradient descent



Source: https://www.jeremyjordan.me/nn-learning-rate/

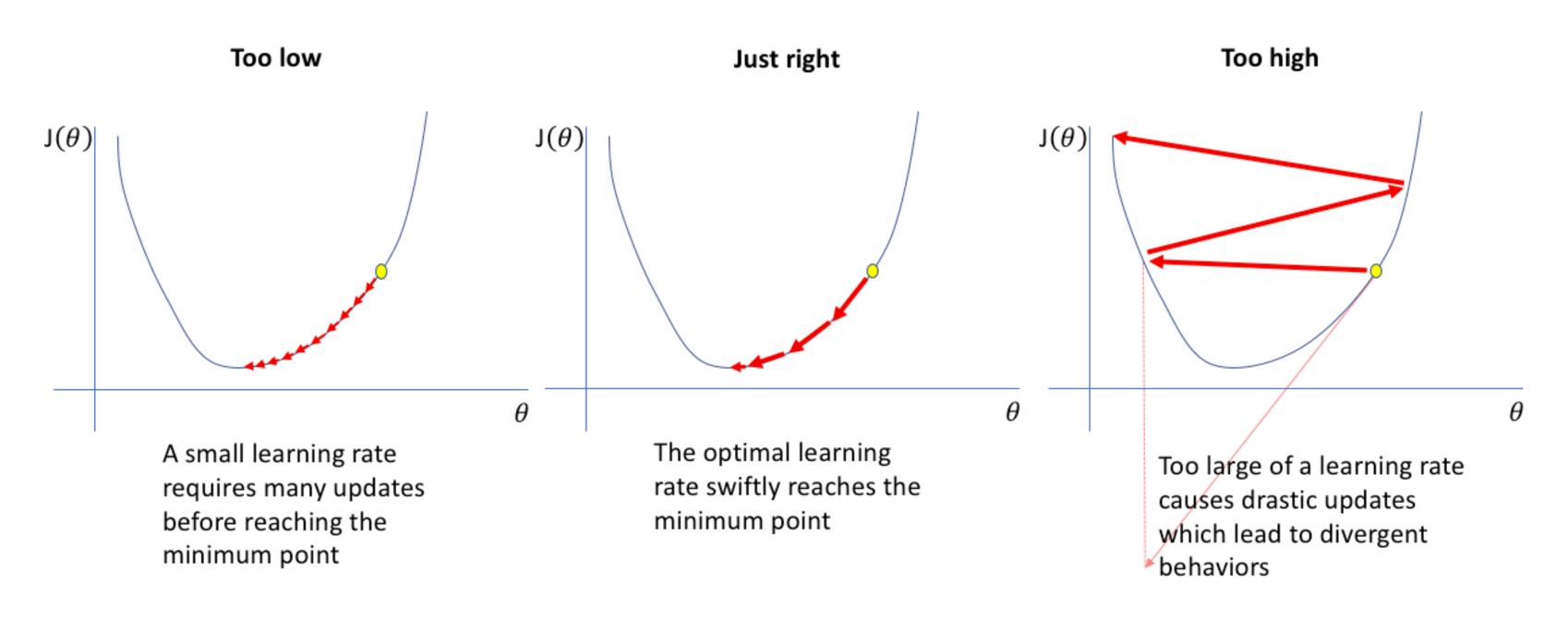
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- Setting the learning rate is more of an art than a science; might need to try a few values to get a good one.
- Especially for non-convex optimization, people come up with clever strategies like shrinking learning rates, cycling learning rates, adaptive learning rates, etc. (RMSprop, Adam, AdaGrad, AdaDelta, ...)

Explicit regularization via penalization. Ridge regression penalty is the most common; this kind of penalization is known as weight decay.

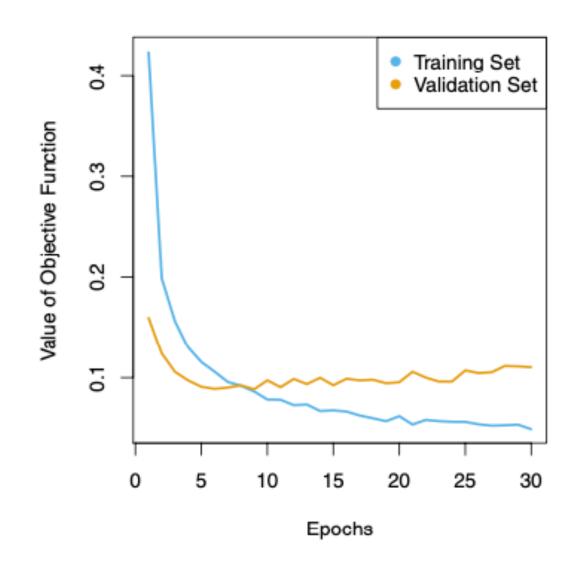
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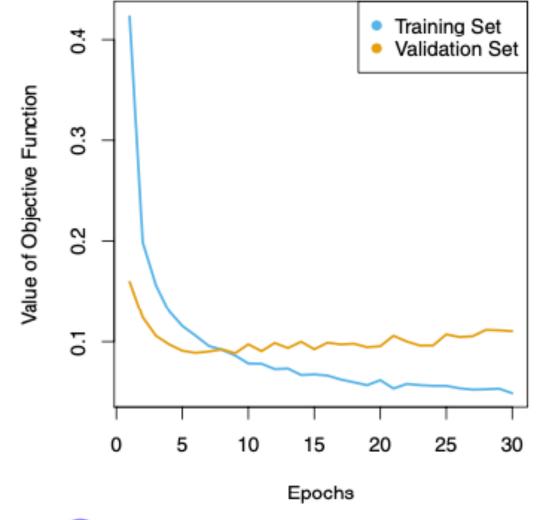


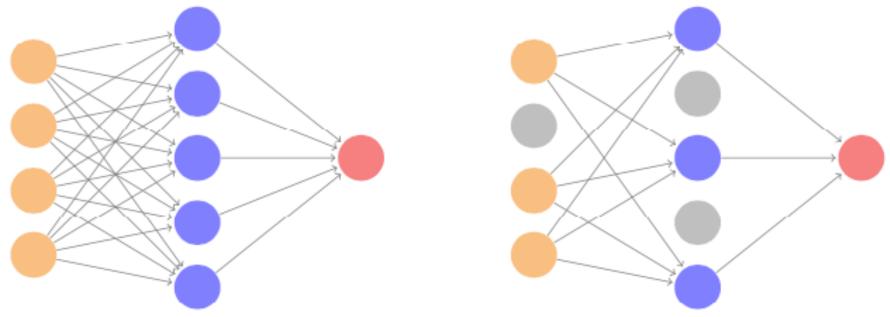
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Implicit regularization: Other techniques to control complexity of the model.

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 Dropout: At each SGD iteration, remove a randomly selected set of nodes from the network (analogous to sub-sampling features for random forests).





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For computational savings, validation sets are typically employed instead of cross-validation to tune neural networks.

Summary

- Adding hidden layers to logistic regression gives multilayer neural networks.
- Multilayer neural networks build increasingly complex representations of the input data (feature learning).
- Stochastic gradient descent is used to train neural networks; it uses noisy approximations to the gradient but is much faster than gradient descent.
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Quiz Practice