Classification

STAT 4710

September 27, 2022

Where we are

Unit 1: R for data mining

Unit 2: Prediction fundamentals

Unit 3: Regression-based methods

Unit 4: Tree-based methods

Unit 5: Deep learning

Lecture 1: Model complexity

Lecture 2: Bias-variance trade-off

Lecture 3: Cross-validation

$\boxed{\textbf{Lecture 4: }\text{Classification}}$

Lecture 5: Unit review and quiz in class

Recall: Clinical decision support

A patient comes into the emergency room with stroke symptoms. Based on her CT scan, is the stroke ischemic or hemorrhagic?

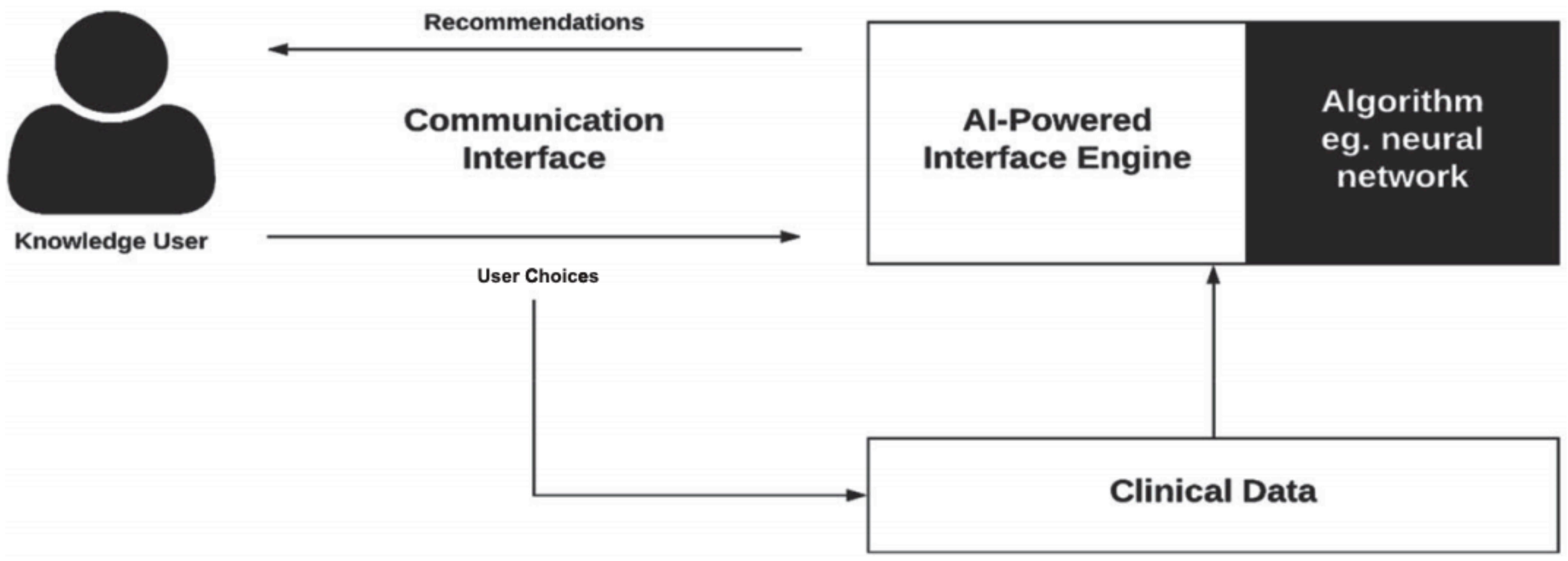


Image source: Sutton et al. 2020 (npj Digit. Med.)

Recall: Clinical decision support

A patient comes into the emergency room with stroke symptoms. Based on her CT scan, is the stroke ischemic or hemorrhagic?

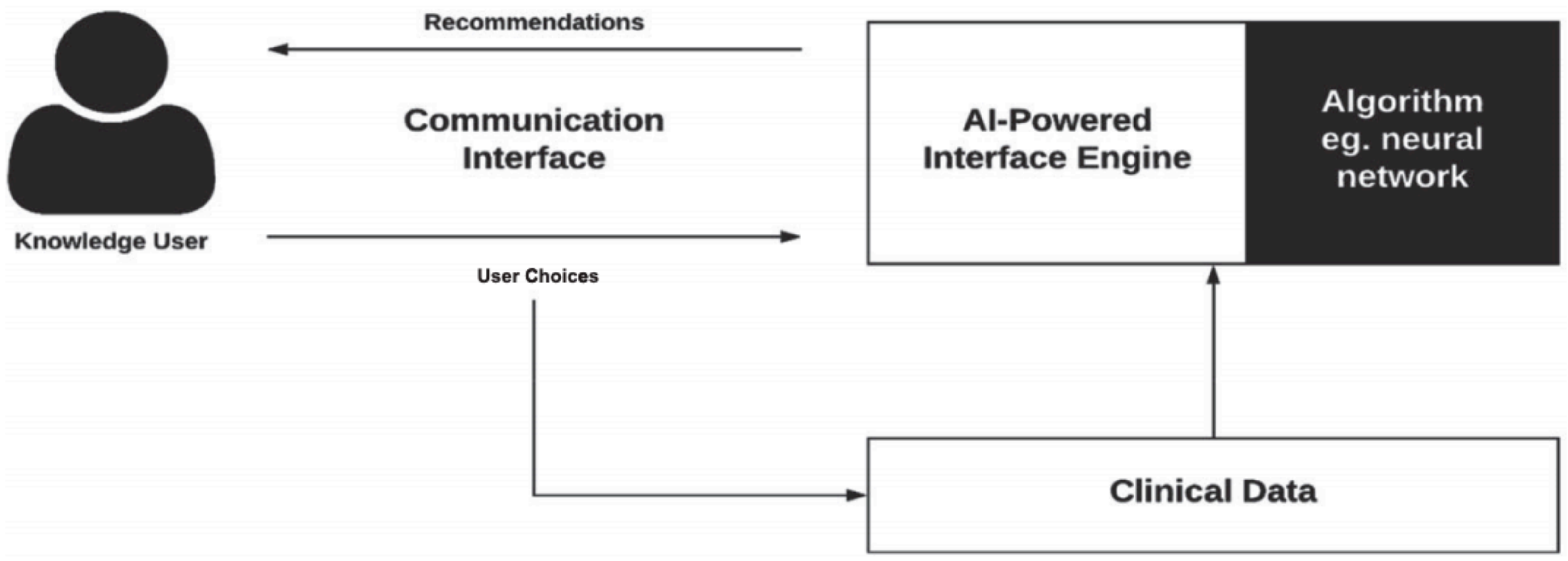


Image source: Sutton et al. 2020 (npj Digit. Med.)

This is a binary classification problem: $Y \in \{0,1\}$.

Given features $X = (X_1, \ldots, X_p)$, the goal is to guess a response $\widehat{Y} = \hat{f}(X)$ that is close to the true response, i.e. $\widehat{Y} \approx Y$. Measure of success is usually the

$$\text{test misclassification error} = \frac{1}{N}\sum_{i=1}^{N} I(Y_i^{\text{test}} \neq \hat{f}(X_i^{\text{test}})).$$

Classification via probability estimation

Suppose that the true relationship between Y and X is

$$\mathbb{P}[Y = 1 \,|\, X] = p(X), \quad \text{for some function } p.$$

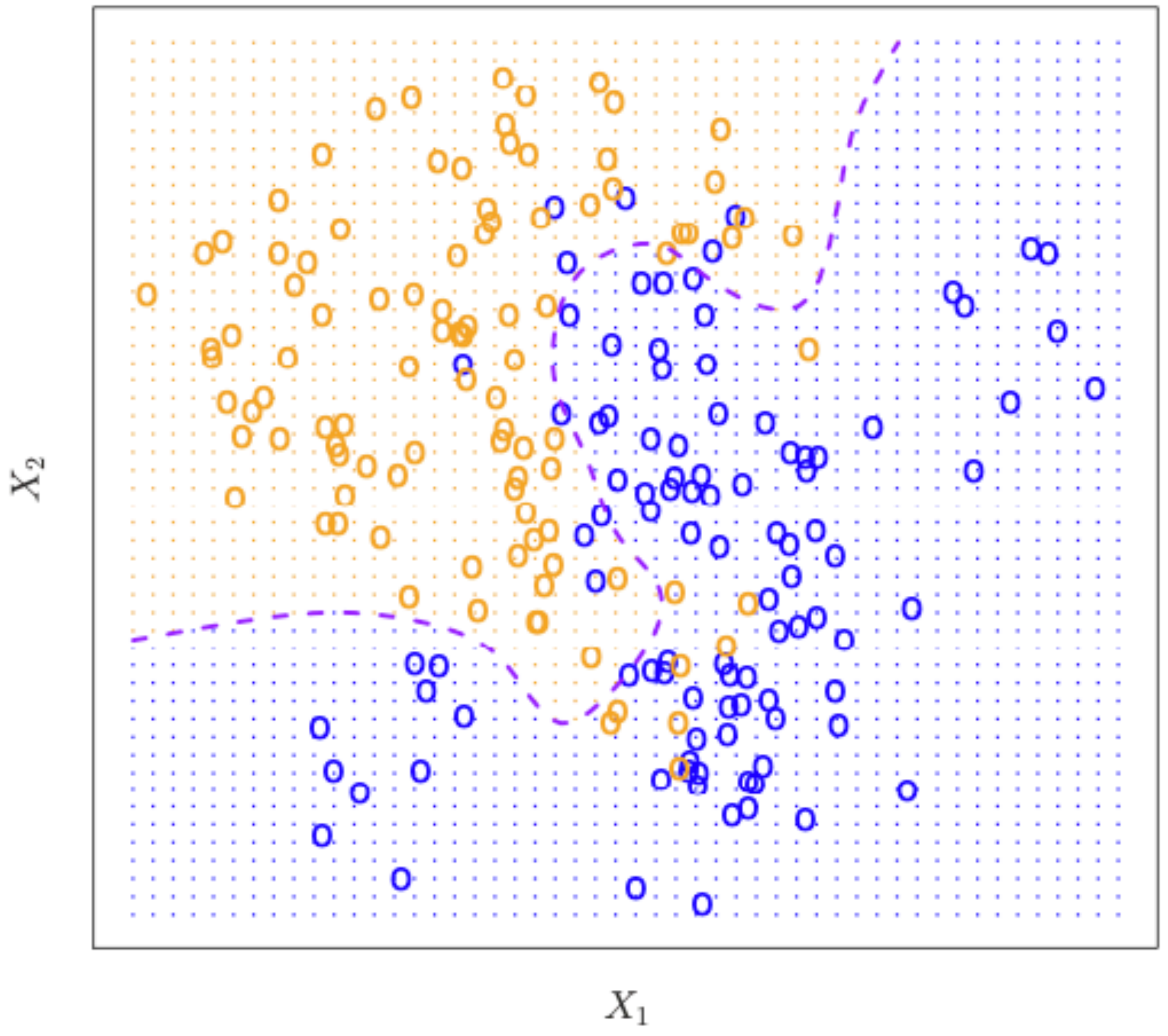
Then, the optimal classifier (called the Bayes classifier) is

$$\hat{f}^{\text{Bayes}}(X) = \begin{cases} 1, & \text{if } p(X) \geq 0.5; \\ 0 & \text{if } p(X) < 0.5. \end{cases}$$

Classifiers usually build an approximation $\widehat{p}(X) \approx \mathbb{P}[Y = 1 \,|\, X]$, and define

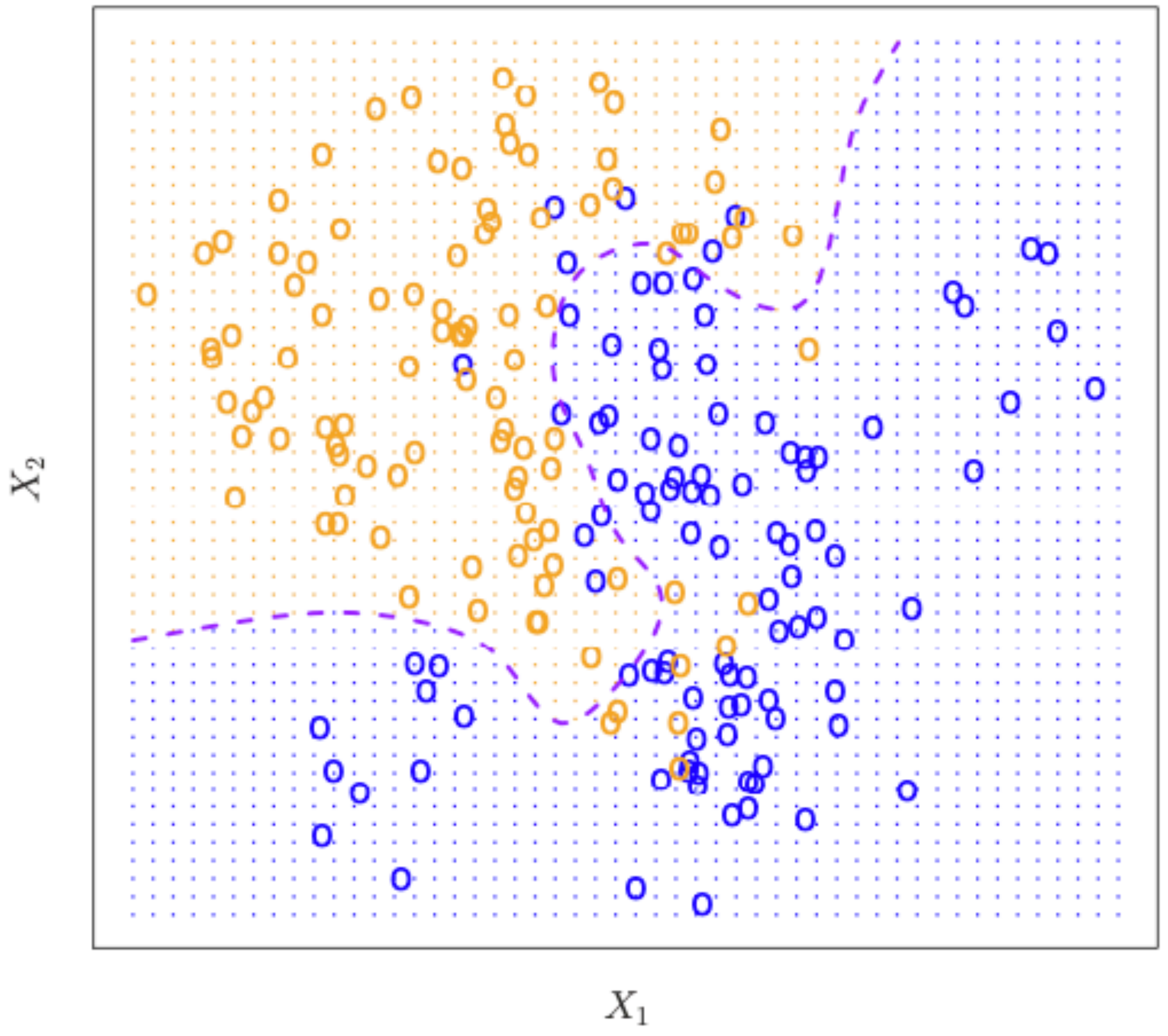
$$\hat{f}(X) = \begin{cases} 1, & \text{if } \widehat{p}(X) \geq 0.5; \\ 0 & \text{if } \widehat{p}(X) < 0.5. \end{cases}$$

Example: K-nearest neighbors

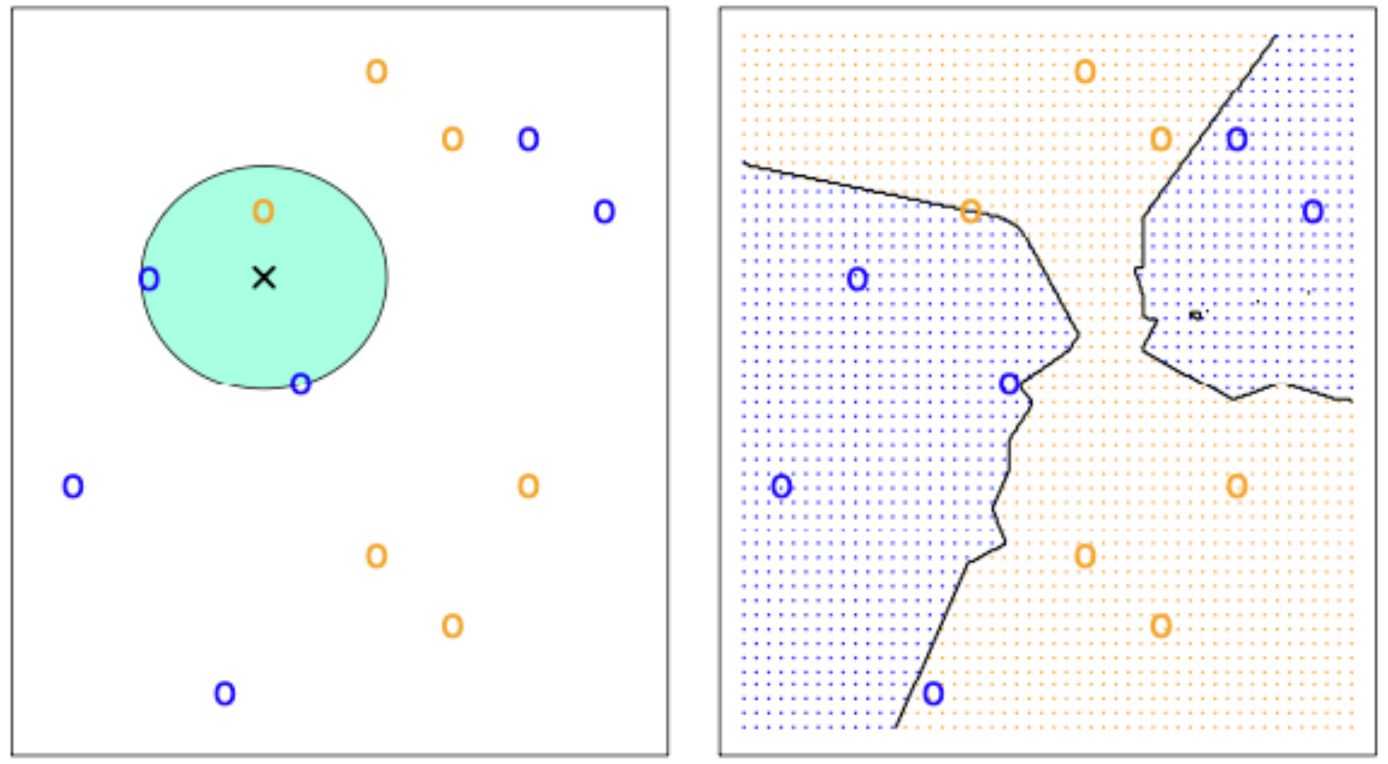


Simulated binary classification data.
Bayes classifier in purple.
E.g., color = stroke type, (X_1, X_2) = CT image.

Example: K-nearest neighbors

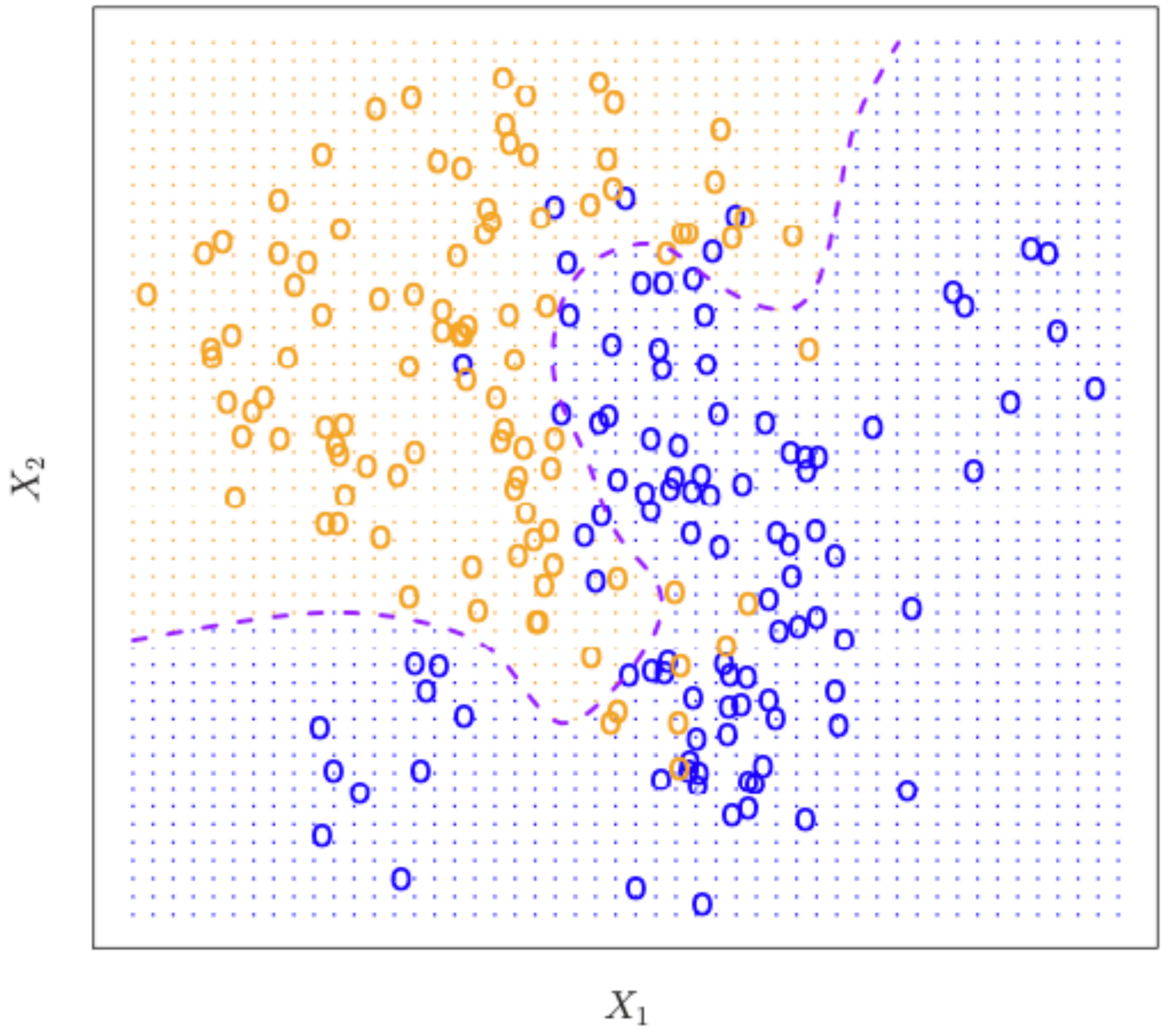


Simulated binary classification data.
Bayes classifier in purple.
E.g., color = stroke type, (X_1, X_2) = CT image.

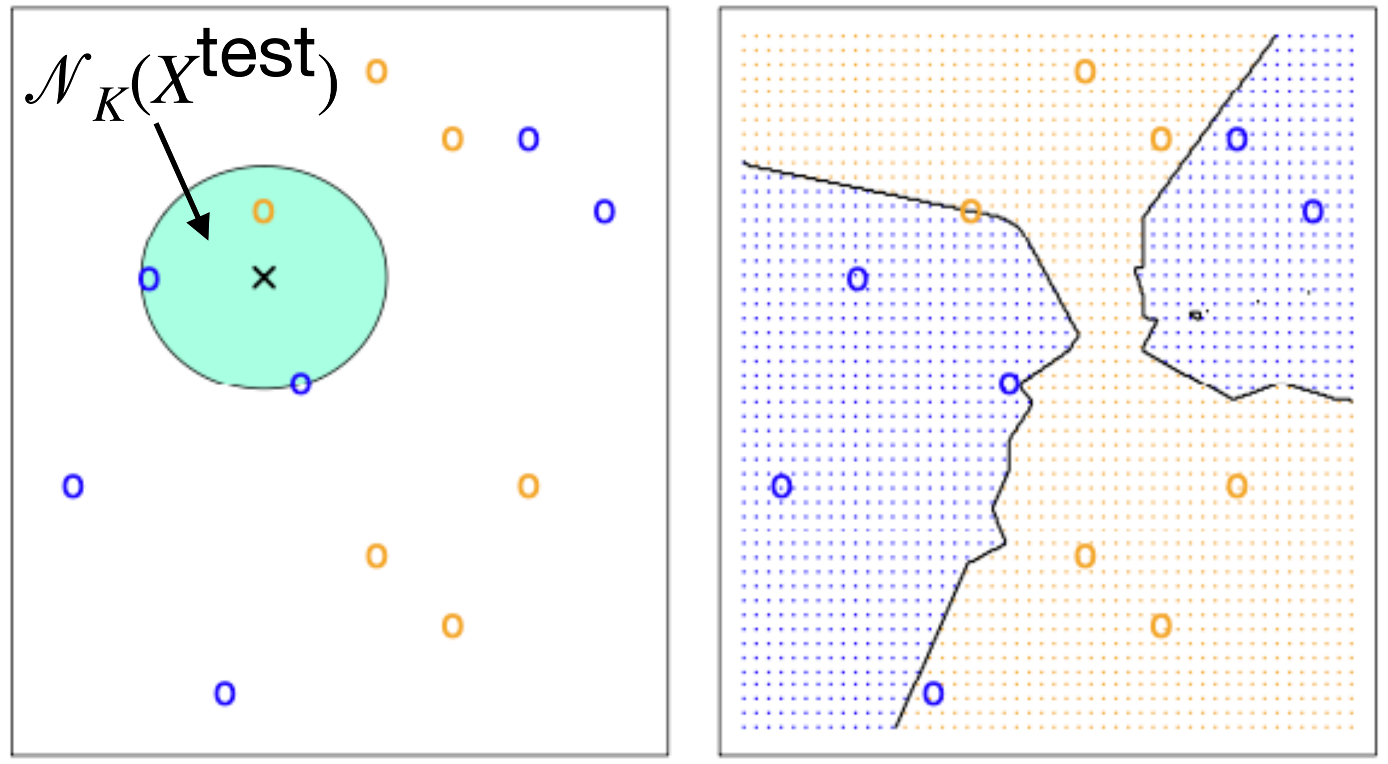


KNN illustration: Classify a test point based on majority vote among 3 nearest neighbors.

Example: K-nearest neighbors



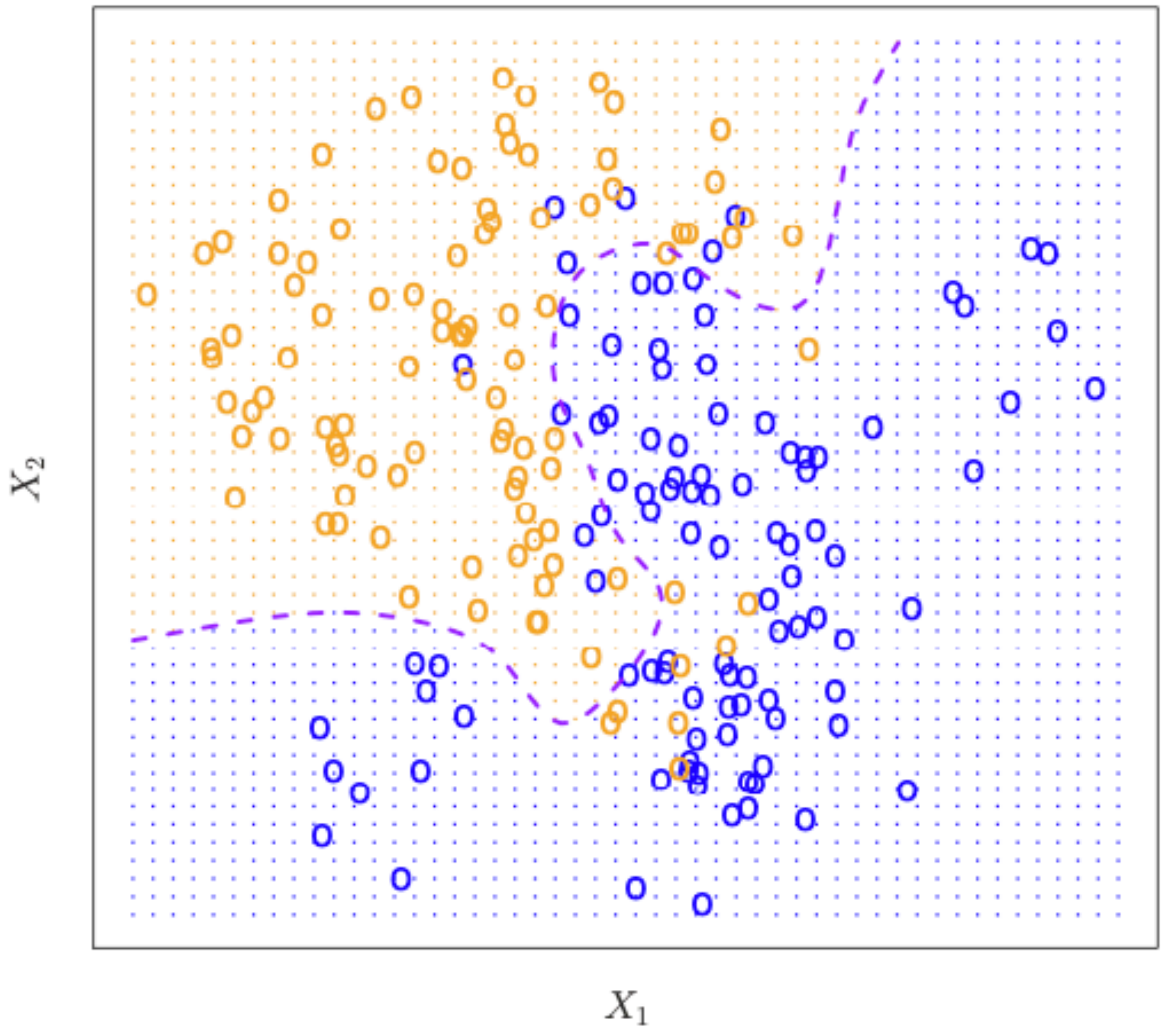
Simulated binary classification data.
Bayes classifier in purple.
E.g., color = stroke type, (X_1, X_2) = CT image.



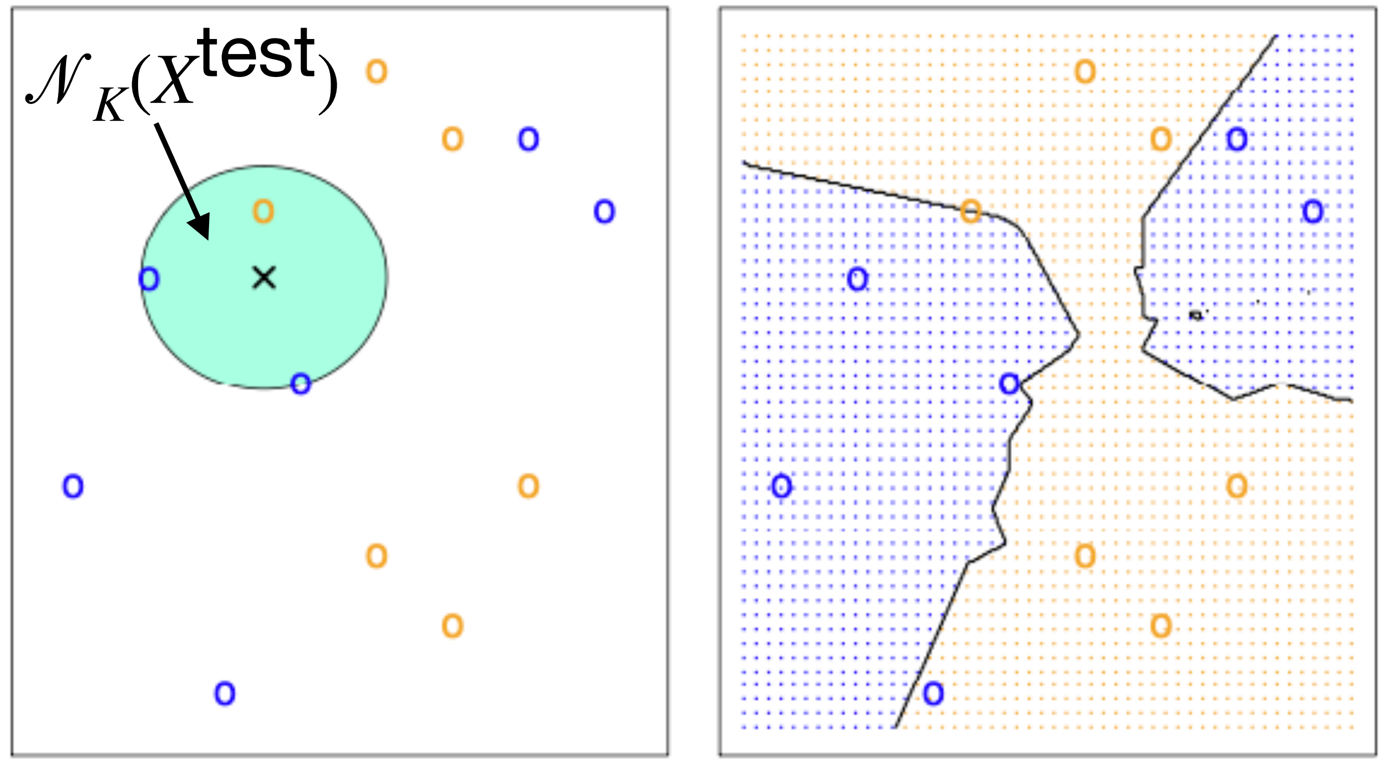
KNN illustration: Classify a test point based on majority vote among 3 nearest neighbors.

$$\hat{p}(X^{\text{test}}) = \frac{1}{K} \sum_{i \in \mathcal{N}_K} I(Y_i^{\text{train}} = 1)\,.$$

Example: K-nearest neighbors

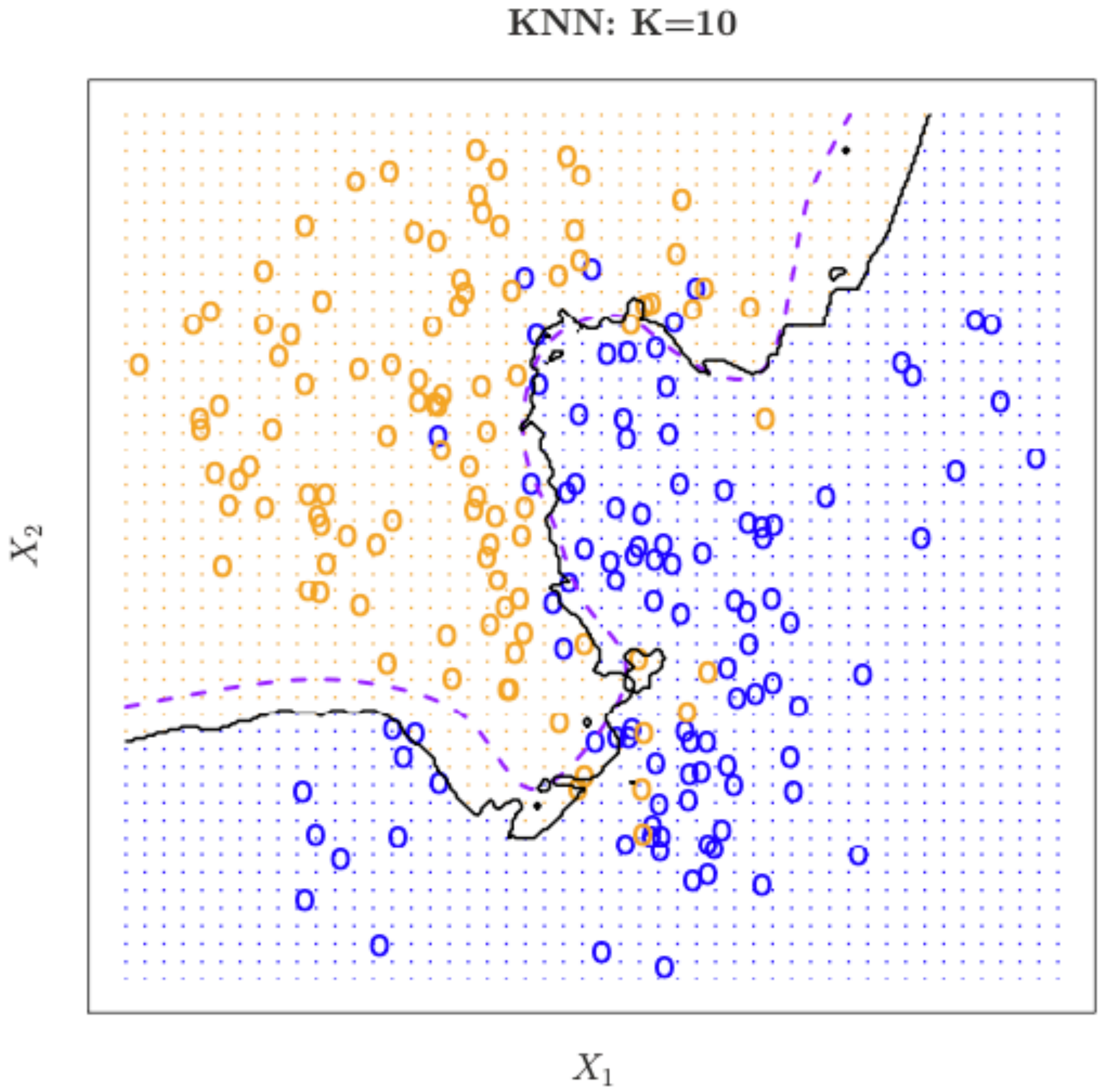


Simulated binary classification data.
Bayes classifier in purple.
E.g., color = stroke type, (X_1, X_2) = CT image.



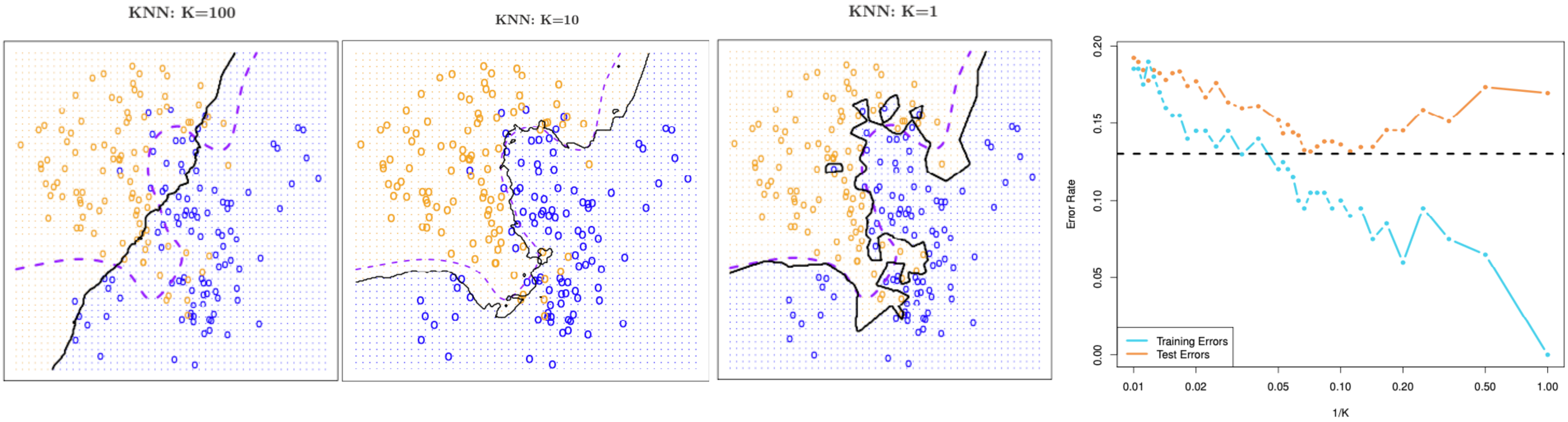
KNN illustration: Classify a test point based on majority vote among 3 nearest neighbors.

$$\hat{p}(X^{\text{test}}) = \frac{1}{K} \sum_{i \in \mathcal{N}_K} I(Y_i^{\text{train}} = 1)\,.$$



Applying KNN with K = 10 to simulated data.

Model complexity and misclassification error



Same Goldilocks principle as in regression case:

- Too little complexity: Can’t capture the true trend in the data.
- Too much complexity: Too sensitive to noise in the training data (overfitting).

Bias-variance tradeoff

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

For the estimate $\widehat{p}(X)$

For classifying $\widehat{Y} = I(p(X) \geq 0.5)$

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

For the estimate $\widehat{p}(X)$

- Bias: $\mathbb{E}[\widehat{p}(X)] - p(X)$

For classifying $\widehat{Y} = I(p(X) \geq 0.5)$

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

For the estimate $\widehat{p}(X)$

- Bias: $\mathbb{E}[\widehat{p}(X)] - p(X)$
- Variance: Var$[\widehat{p}(X)]$

For classifying $\widehat{Y} = I(p(X) \geq 0.5)$

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

For the estimate $\widehat{p}(X)$

- Bias: $\mathbb{E}[\widehat{p}(X)] - p(X)$ $\longrightarrow$

- Variance: Var$[\widehat{p}(X)]$

For classifying $\widehat{Y} = I(p(X) \geq 0.5)$

- Bias: Predict wrong class on average, to the extent $\widehat{p}$ on wrong side of 0.5

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

For the estimate $\widehat{p}(X)$

- Bias: $\mathbb{E}[\widehat{p}(X)] - p(X)$ $\longrightarrow$
- Variance: Var$[\widehat{p}(X)]$ $\longrightarrow$

For classifying $\widehat{Y} = I(p(X) \geq 0.5)$

- Bias: Predict wrong class on average, to the extent $\widehat{p}$ on wrong side of 0.5
- Variance: Prediction varies with training set, to the extent $\widehat{p}$ fluctuates above or below 0.5

Bias-variance tradeoff

Mathematically: Applies only to continuous response variables and MSE.

Intuitively: Applies to any prediction problem, including classification.

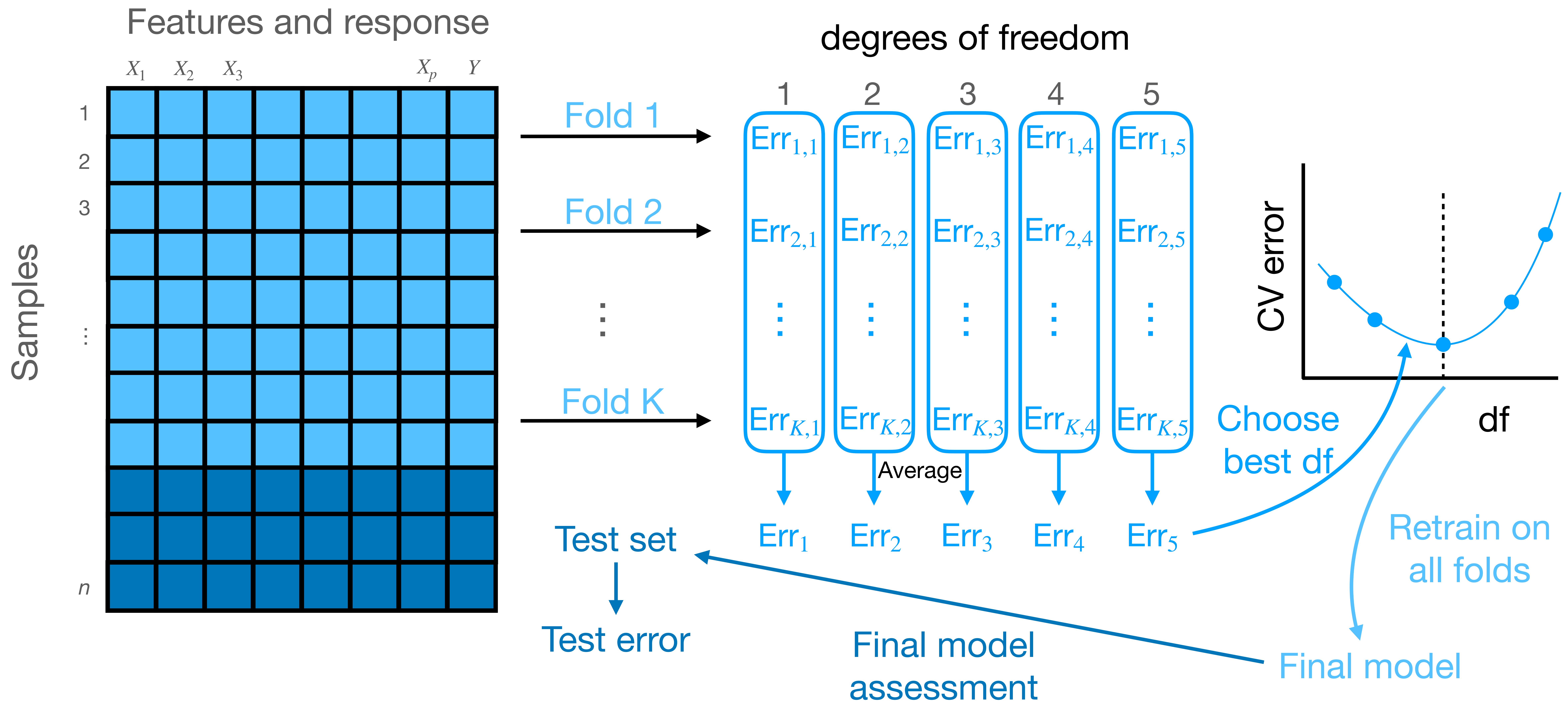
For the estimate $\widehat{p}(X)$

- Bias: $\mathbb{E}[\widehat{p}(X)] - p(X)$ $\longrightarrow$
- Variance: $\text{Var}[\widehat{p}(X)]$ $\longrightarrow$

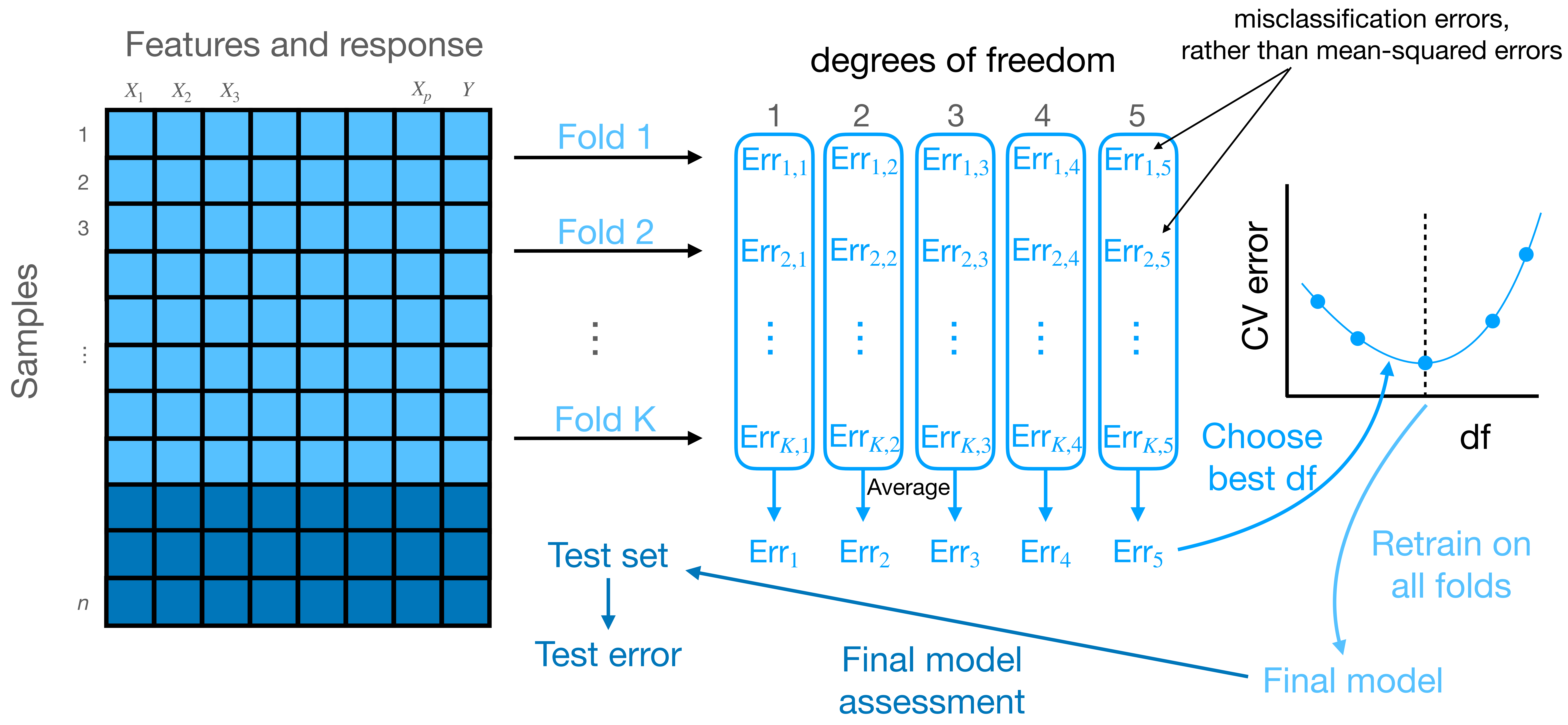
For classifying $\widehat{Y} = I(p(X) \geq 0.5)$

- Bias: Predict wrong class on average, to the extent $\widehat{p}$ on wrong side of 0.5
- Variance: Prediction varies with training set, to the extent $\widehat{p}$ fluctuates above or below 0.5
- Irreducible error (AKA Bayes error): Error incurred by Bayes classifier because $0 < \mathbb{P}[Y = 1 \mid X] < 1$.

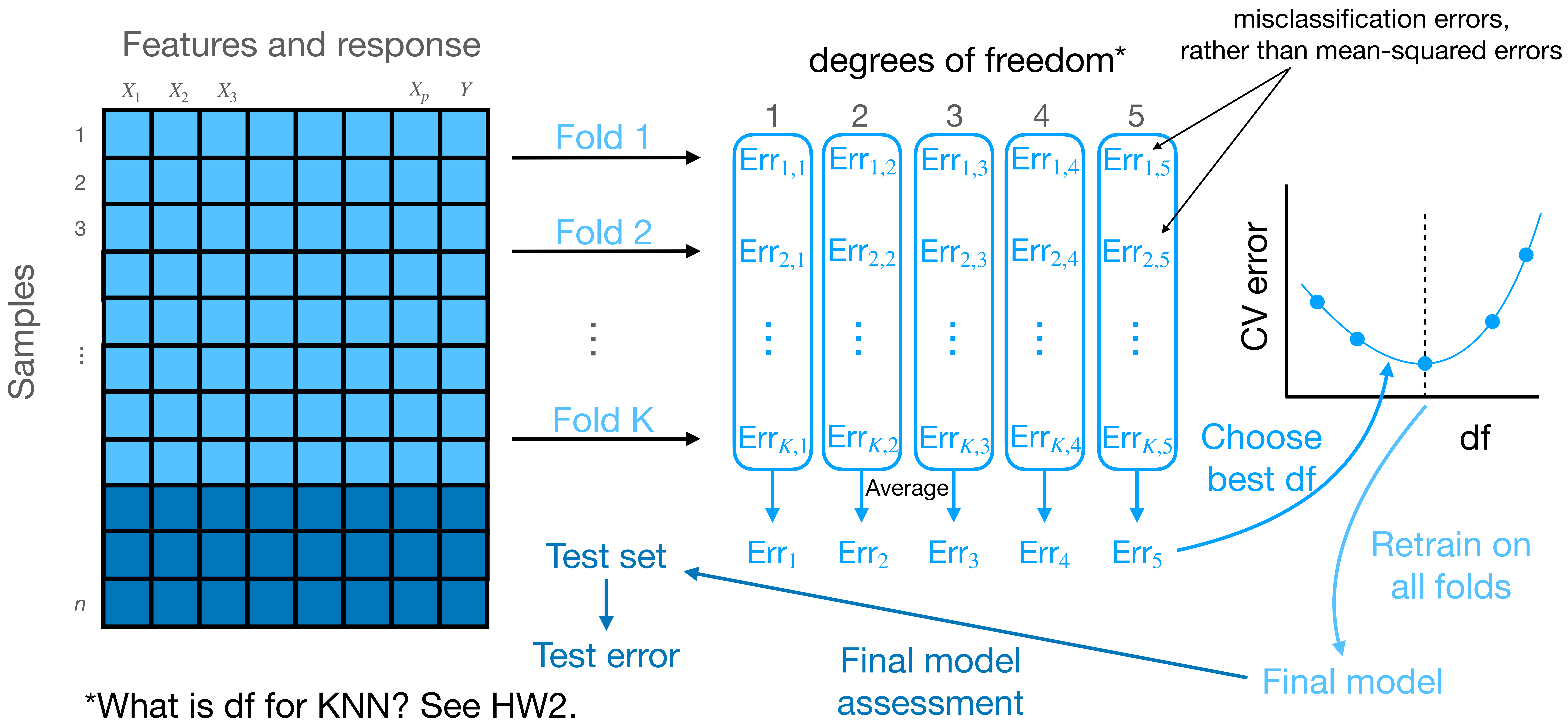
Cross-validation based on misclassification error
(otherwise same as before)



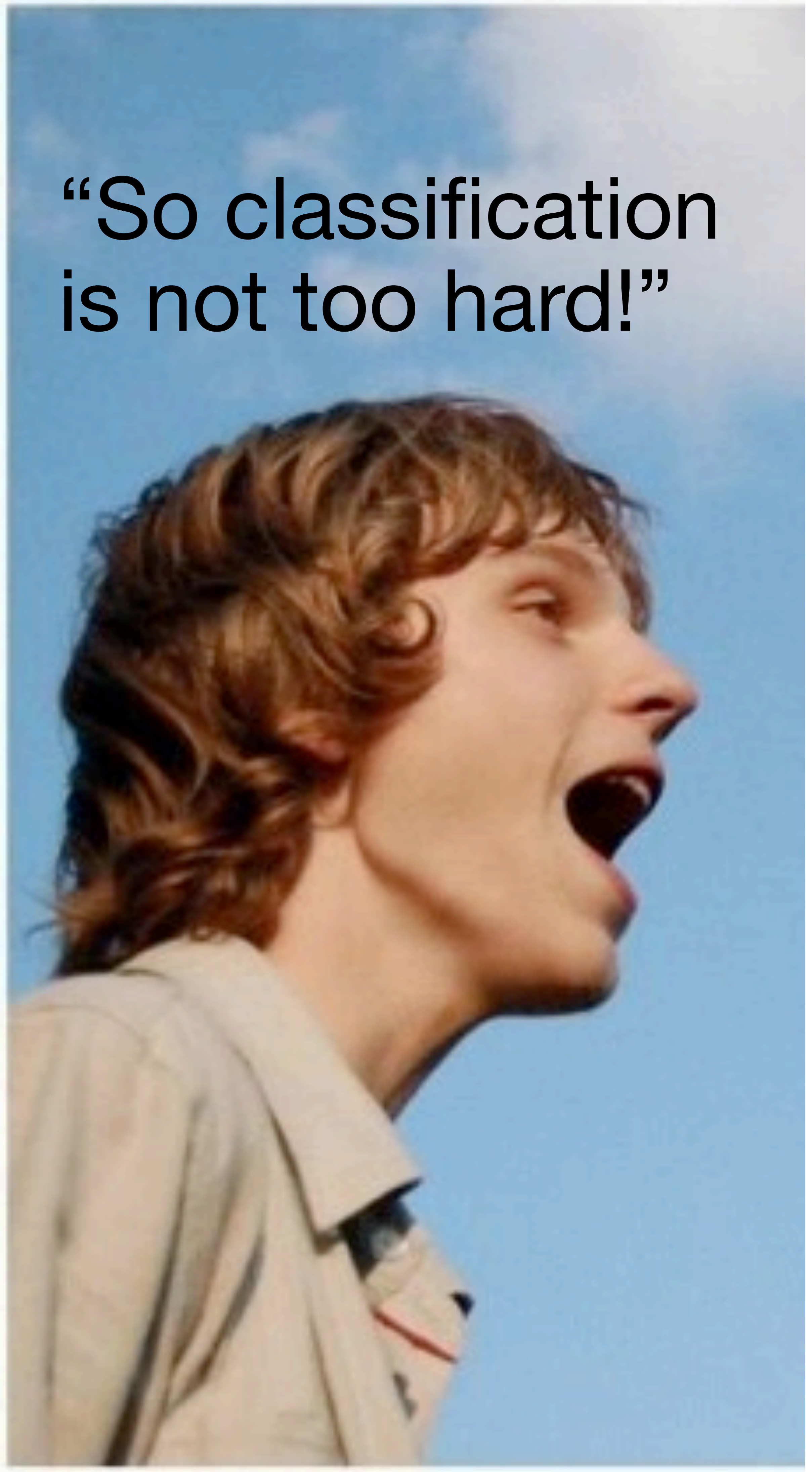
Cross-validation based on misclassification error
(otherwise same as before)

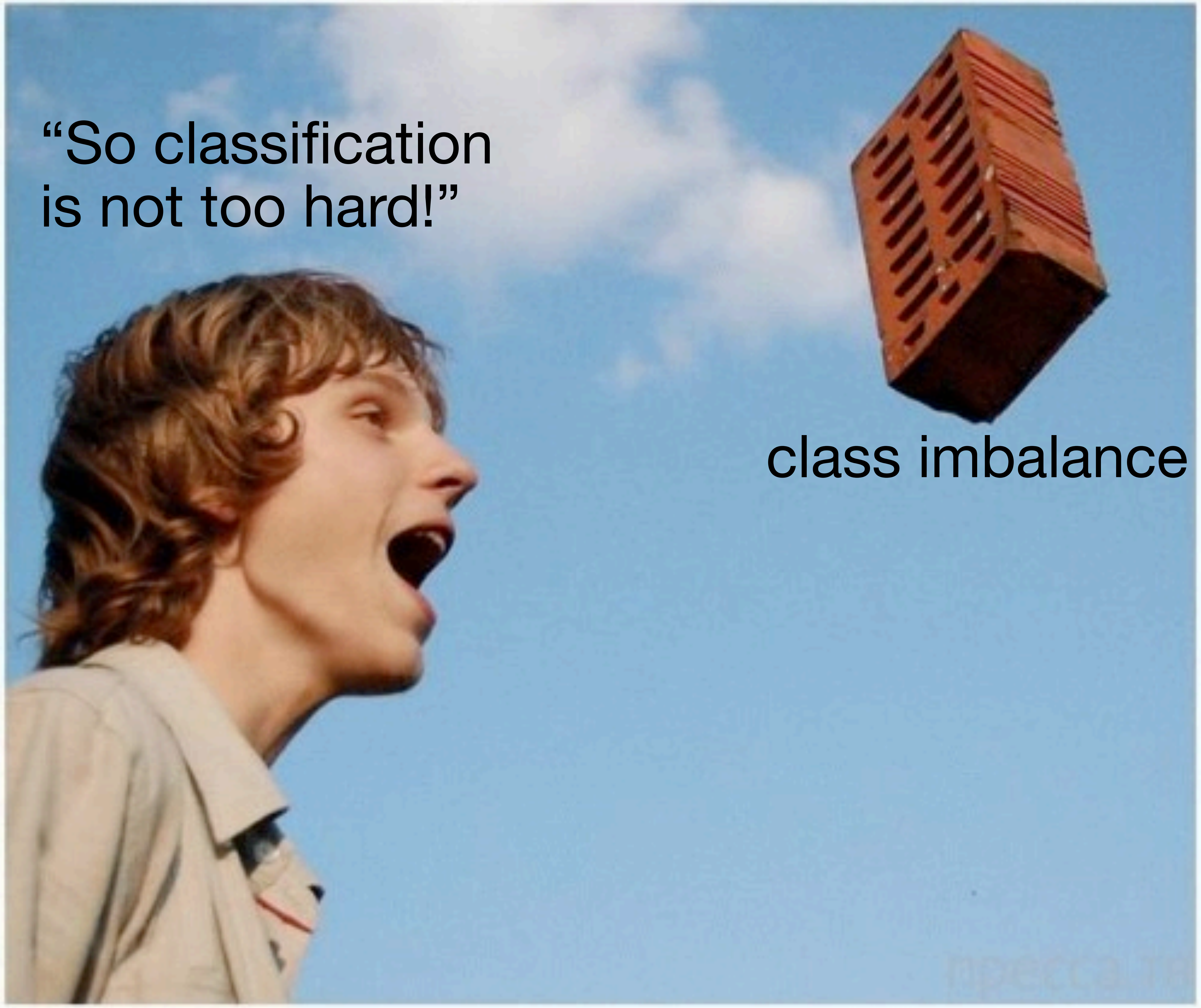


Cross-validation based on misclassification error
(otherwise same as before)



*What is df for KNN? See HW2.


"So classification is not too hard!"


“So classification is not too hard!”
class imbalance

Class imbalance

Class imbalance

In many real-world classification problems, one class (say $Y = 1$) is significantly less frequent than the other. For example:

- Credit card transaction classification: normal versus fraudulent
- COVID testing: negative versus positive

Class imbalance

In many real-world classification problems, one class (say $Y = 1$) is significantly less frequent than the other. For example:

- Credit card transaction classification: normal versus fraudulent

- COVID testing: negative versus positive

Often in these cases, the costs of misclassification are also asymmetric, i.e. the misclassification error is not the right metric.

Class imbalance

In many real-world classification problems, one class (say $Y = 1$) is significantly less frequent than the other. For example:

- Credit card transaction classification: normal versus fraudulent
- COVID testing: negative versus positive

Often in these cases, the costs of misclassification are also asymmetric, i.e. the misclassification error is not the right metric.

Let's say 1% of credit card transactions are fraudulent. Then, the classifier that always predicts "not fraudulent" will have a misclassification error of only 1%.

Class imbalance

In many real-world classification problems, one class (say $Y = 1$) is significantly less frequent than the other. For example:

- Credit card transaction classification: normal versus fraudulent
- COVID testing: negative versus positive

Often in these cases, the costs of misclassification are also asymmetric, i.e. the misclassification error is not the right metric.

Let's say 1% of credit card transactions are fraudulent. Then, the classifier that always predicts "not fraudulent" will have a misclassification error of only 1%.

Cross-validation based on misclassification error leads to overly simple models that ignore the minority class.

Binary classification terminology

Positive: $Y = 1$
(e.g. COVID positive)

Negative: $Y = 0$
(e.g. COVID negative)

Binary classification terminology

Positive: $Y = 1$
(e.g. COVID positive)

Negative: $Y = 0$
(e.g. COVID negative)

	Actually Positive	**Actually Negative**
Predicted Positive	True Positive (TP) (E.g. Sick person testing positive)	False Positive (FP) (E.g. Healthy person testing positive)
Predicted Negative	False negative (FN) (E.g. Sick person testing negative)	True Negative (TN) (E.g. Healthy person testing negative)

Thinking about misclassification costs

Thinking about misclassification costs

The cost of a false negative might be much greater than a false positive:

Thinking about misclassification costs

The cost of a false negative might be much greater than a false positive:

- Undetected fraudulent credit card transaction (false negative)
  $\rightarrow$ drained bank account. Cost: $C_{\mathrm{FN}} = \$10{,}000$.

Thinking about misclassification costs

The cost of a false negative might be much greater than a false positive:

- Undetected fraudulent credit card transaction (false negative)
  → drained bank account. Cost: C_{FN} = \$10,000.
- False alarm of fraud (false positive)
  → annoying text message and/or replaced credit card. Cost: C_{FP} = \$10.

Thinking about misclassification costs

The cost of a false negative might be much greater than a false positive:

- Undetected fraudulent credit card transaction (false negative)
  $\rightarrow$ drained bank account. Cost: C_{FN} = \$10,000.

- False alarm of fraud (false positive)
  $\rightarrow$ annoying text message and/or replaced credit card. Cost: C_{FP} = \$10.

Weighted misclassification error:

$$\frac{1}{N}\sum_{i=1}^{N} w_i \cdot I(\hat{Y}_i^{\text{test}} \neq Y_i^{\text{test}}), \quad \text{where} \quad w_i = \begin{cases} C_{\text{FP}} & \text{if } Y_i = 0 \\ C_{\text{FN}} & \text{if } Y_i = 1 \end{cases}.$$

Thinking about misclassification costs

The cost of a false negative might be much greater than a false positive:

- Undetected fraudulent credit card transaction (false negative)
  → drained bank account. Cost: C_{FN} = \$10,000.

- False alarm of fraud (false positive)
  → annoying text message and/or replaced credit card. Cost: C_{FP} = \$10.

Weighted misclassification error:

$$\frac{1}{N}\sum_{i=1}^{N} w_i \cdot I(\hat{Y}_i^{\text{test}} \neq Y_i^{\text{test}}), \quad \text{where} \quad w_i = \begin{cases} C_{\text{FP}} & \text{if } Y_i = 0 \\ C_{\text{FN}} & \text{if } Y_i = 1 \end{cases}.$$

w_i are called observation weights; integer weights like replicating observations.

Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:

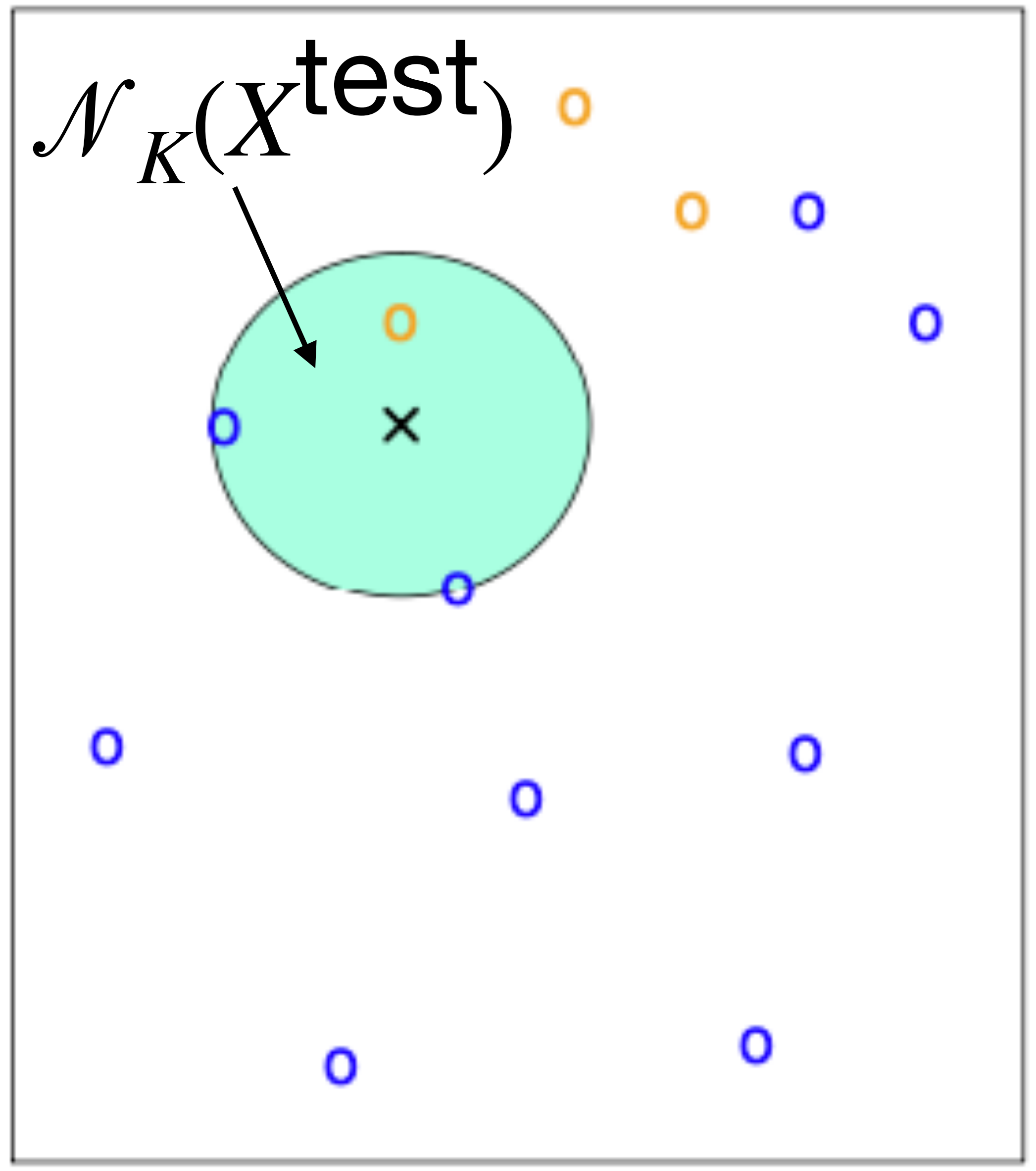
Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:

Positive (rare)

Negative (common)

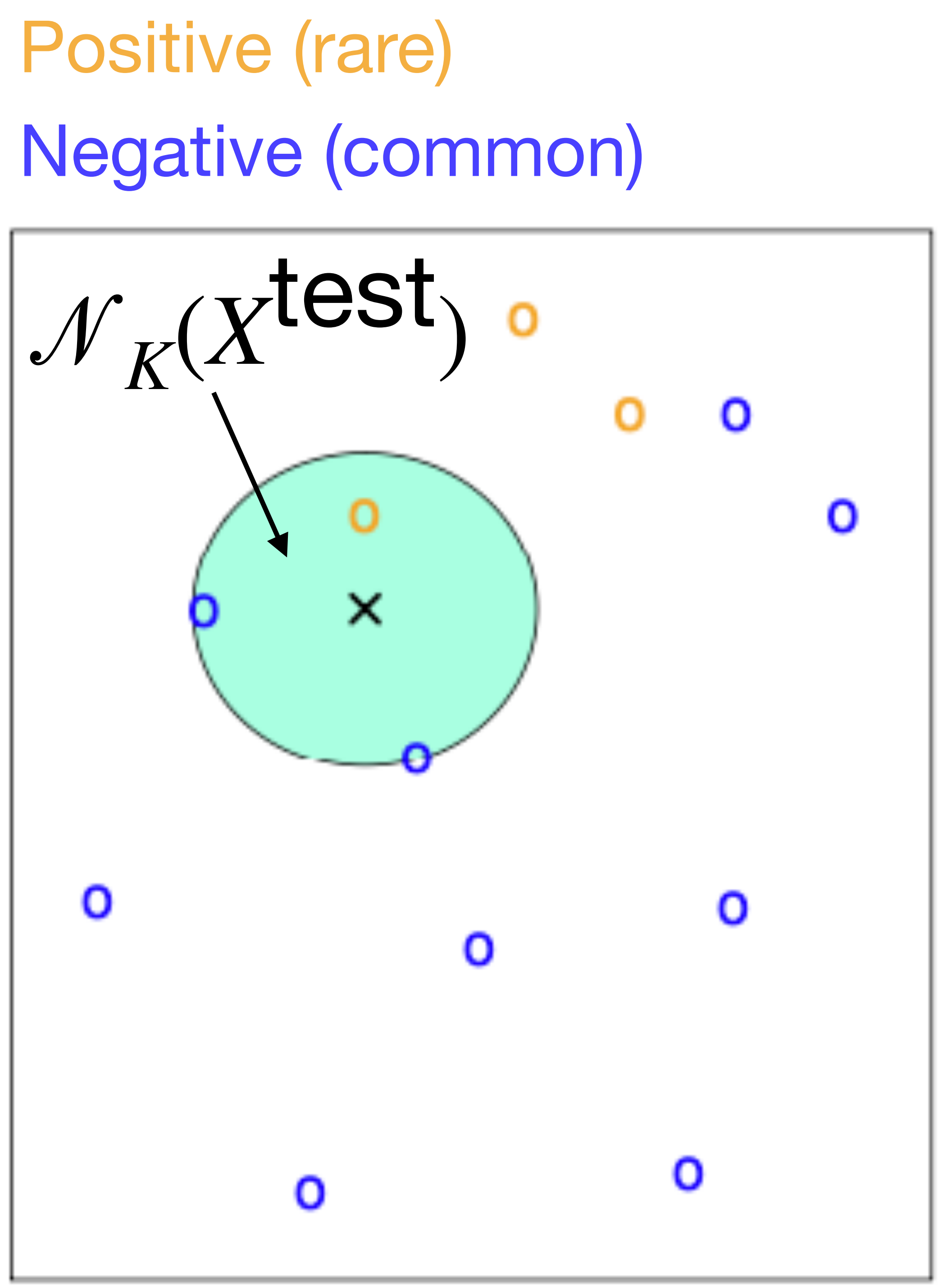


$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:



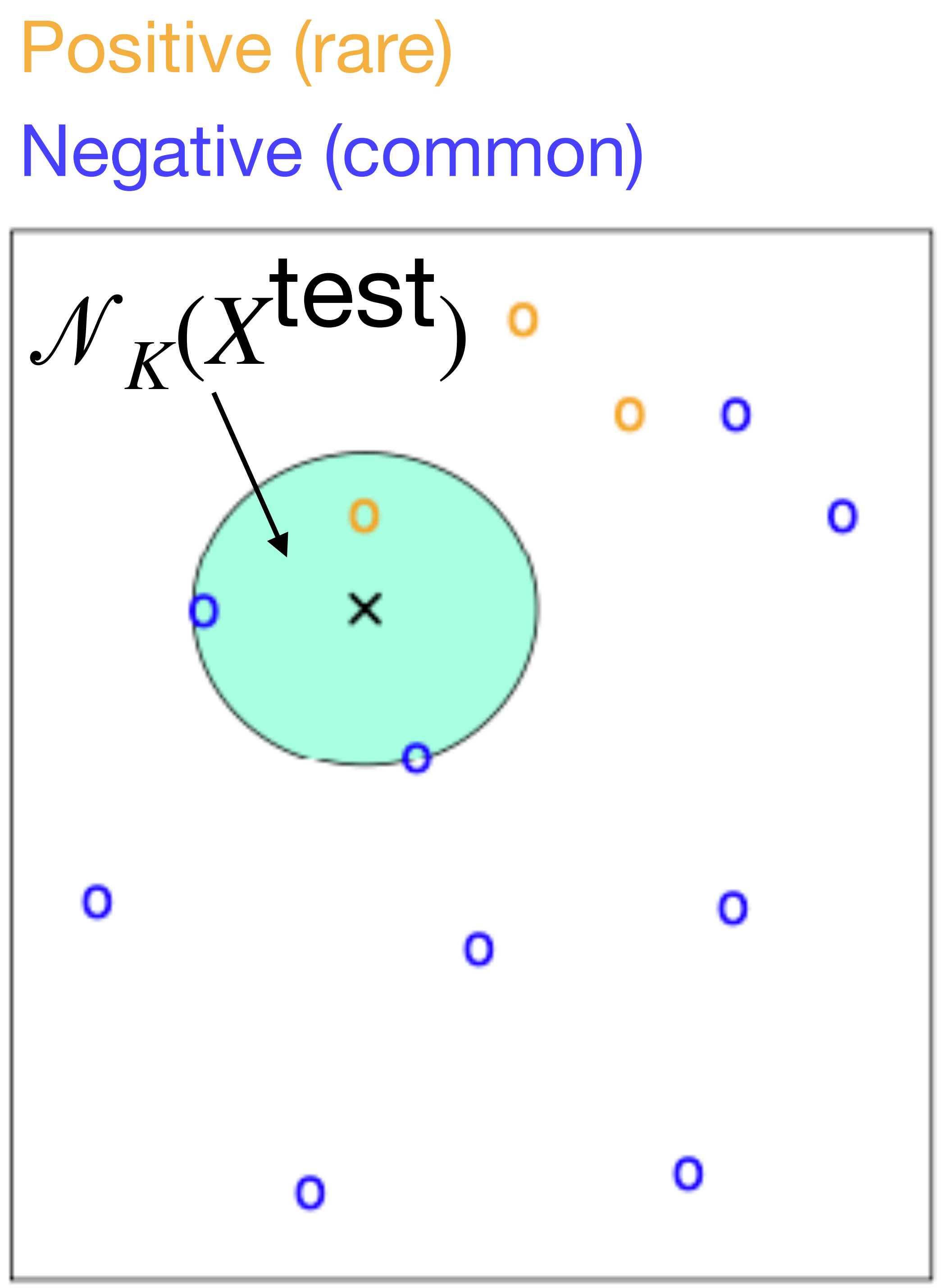
$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

w_{blue}	w_{yellow}	$\hat{p}(X^{\text{test}})$	Predicted class
1	1	1/3	Blue

Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:



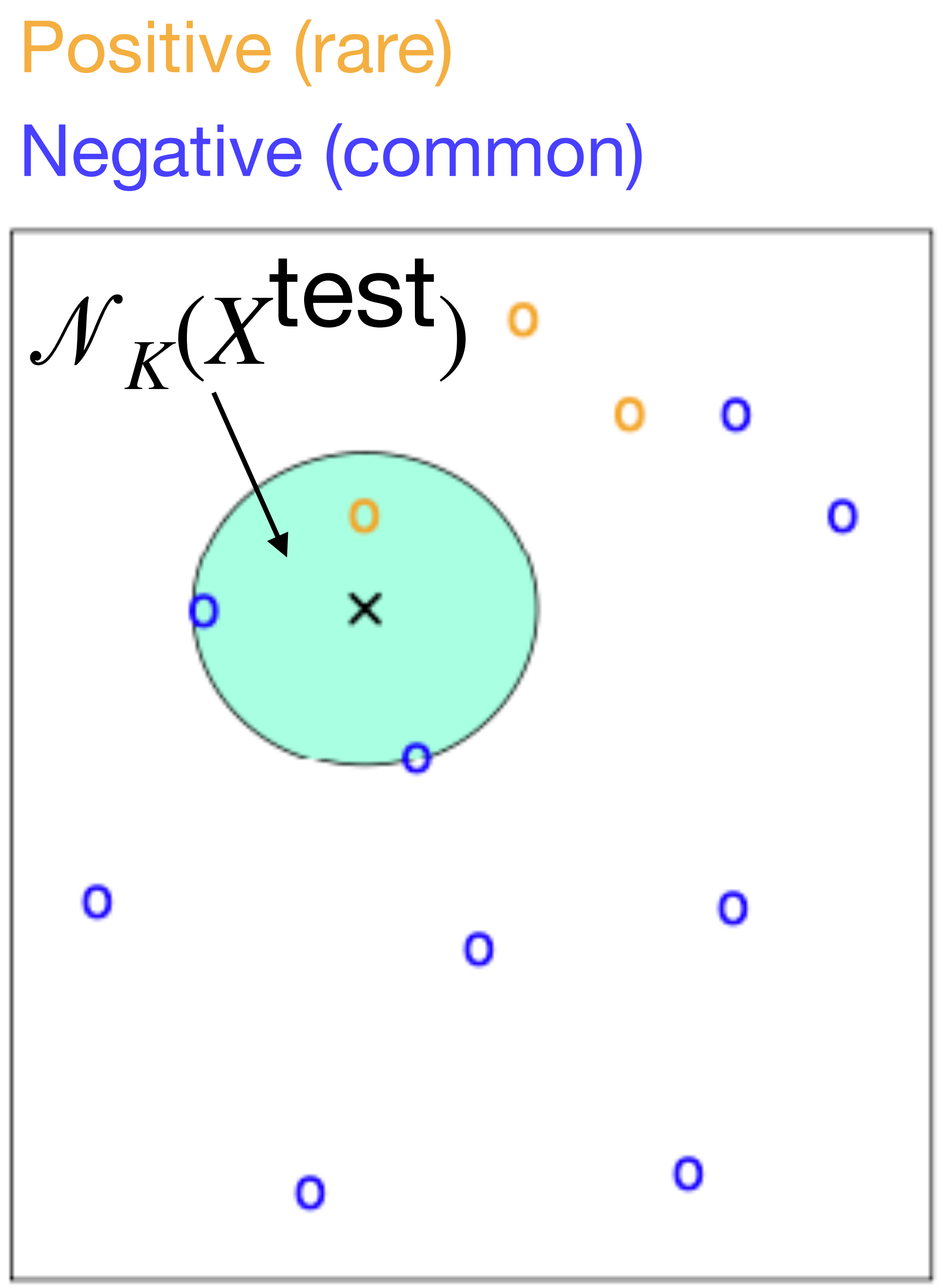
$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

w_{blue}	w_{yellow}	$\hat{p}(X^{\text{test}})$	Predicted class
1	1	1/3	Blue
1	2	1/2	Yellow

Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:



$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

w_{blue}	w_{yellow}	$\hat{p}(X^{\text{test}})$	Predicted class
1	1	1/3	Blue
1	2	1/2	Yellow

(As though 2 yellows in circle)

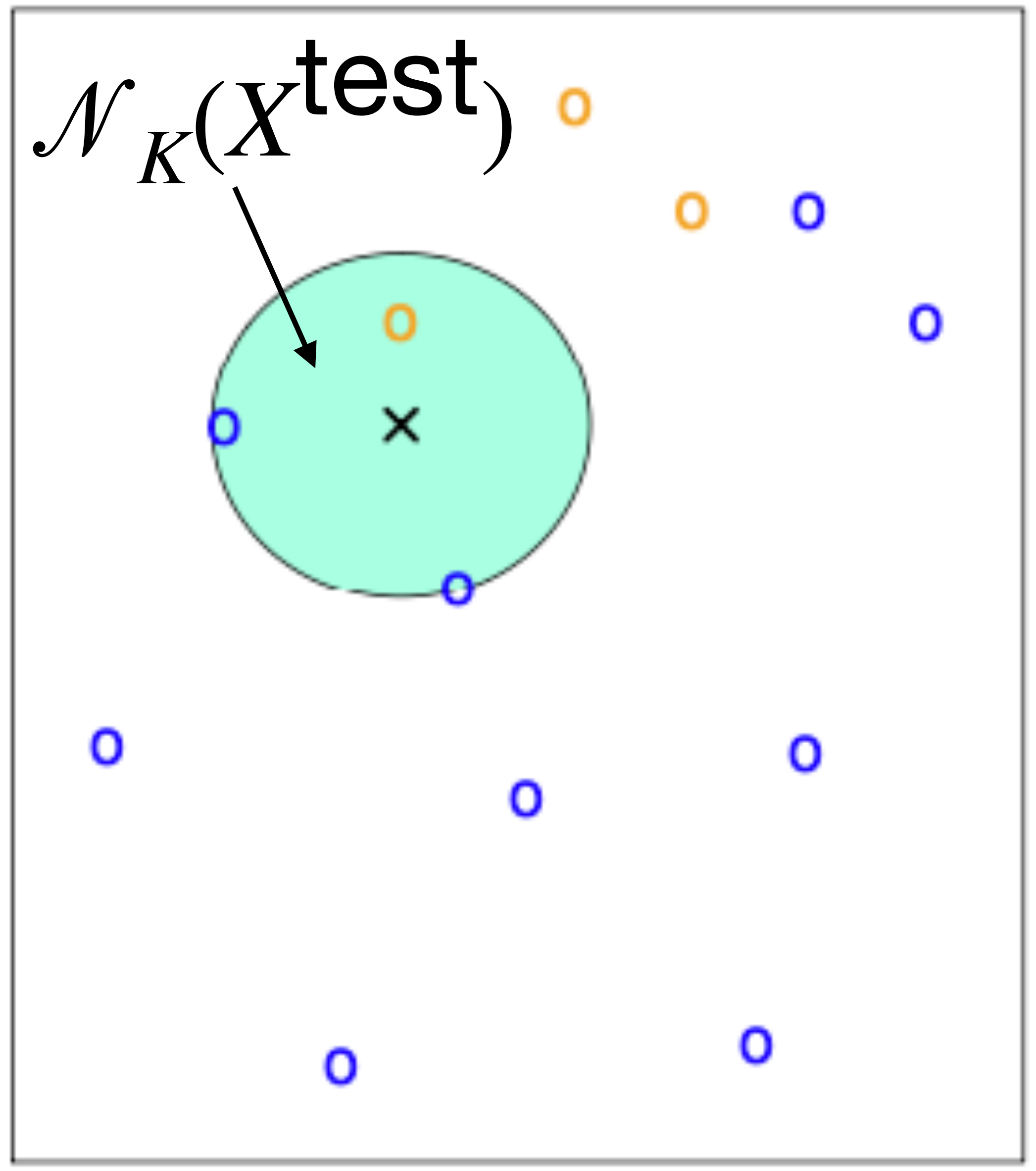
Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:

Positive (rare)

Negative (common)



$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

w_{blue}	w_{yellow}	$\hat{p}(X^{\text{test}})$	Predicted class
1	1	1/3	Blue
1	2	1/2	Yellow
1	3	3/5	Yellow

(As though 2 yellows in circle)

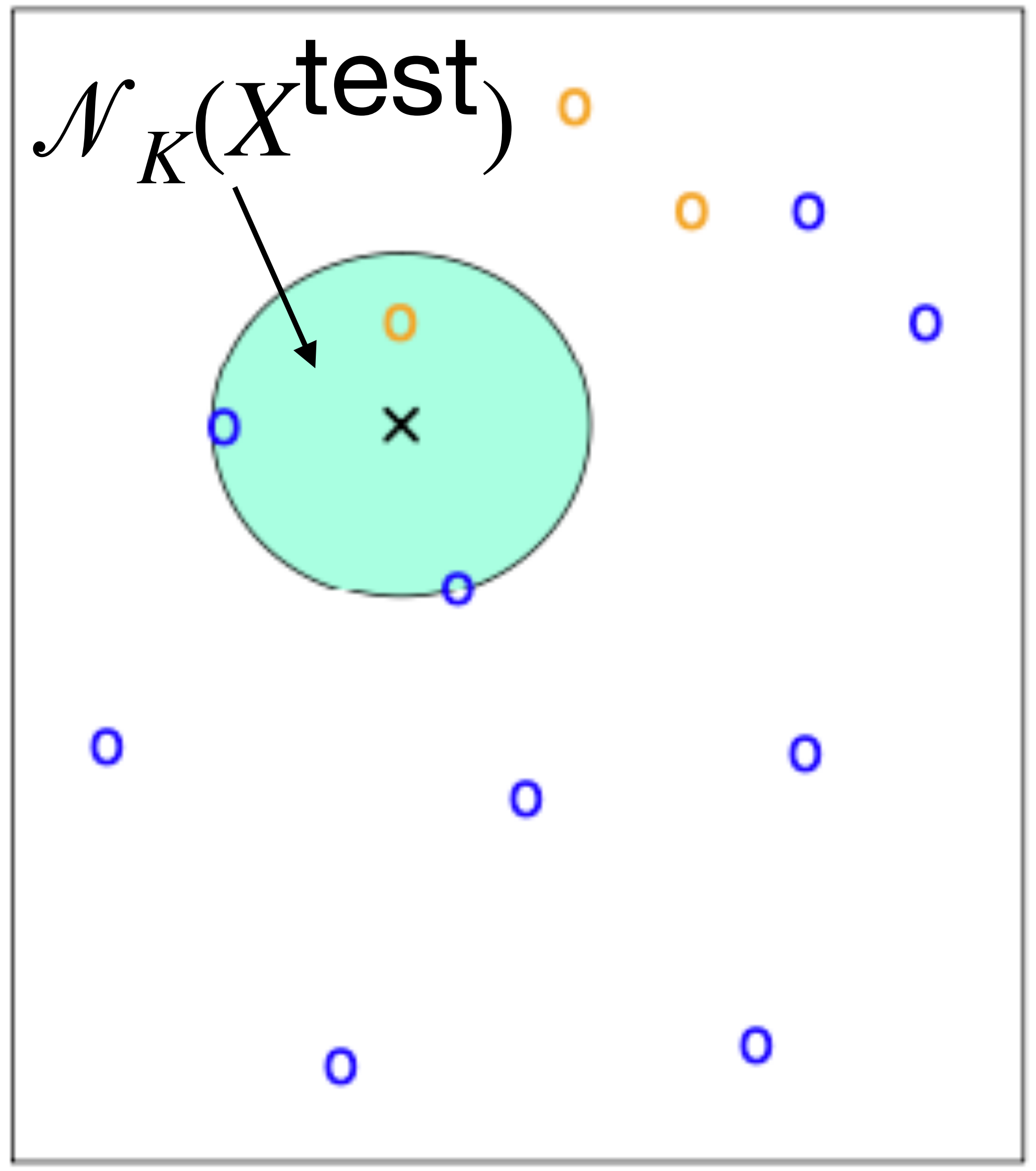
Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:

Positive (rare)

Negative (common)



$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

w_{blue}	w_{yellow}	$\hat{p}(X^{\text{test}})$	Predicted class
1	1	1/3	Blue
1	2	1/2	Yellow
1	3	3/5	Yellow

(As though 2 yellows in circle)

(As though 3 yellows in circle)

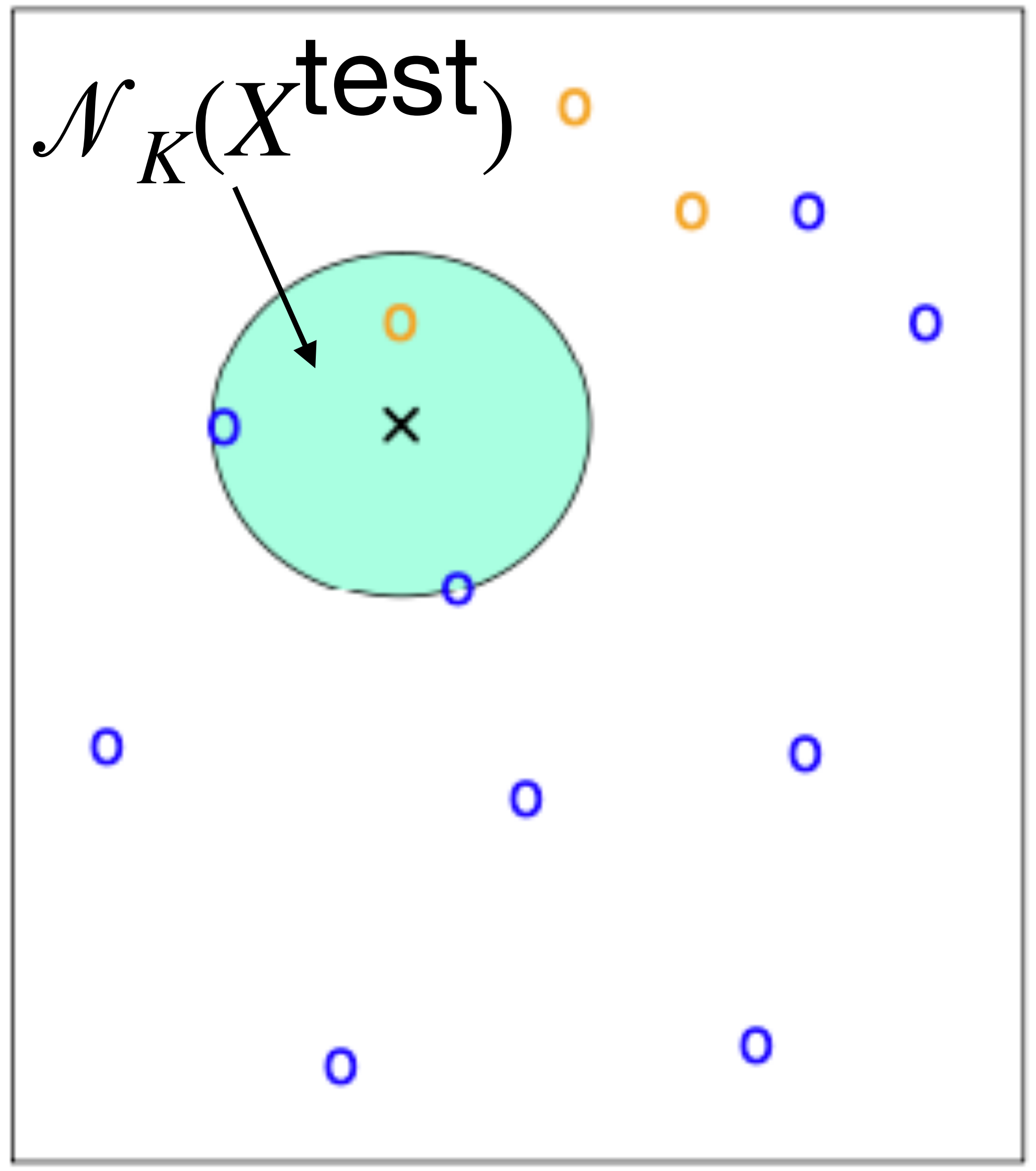
Building observation weights into training

Many machine learning algorithms accommodate observation weights, i.e. seek to optimize the weighted misclassification error.

For example, consider weighted K-nearest neighbors:

Positive (rare)

Negative (common)



$$\hat{p}(X^{\text{test}}) = \frac{\sum_{i \in \mathcal{N}_K} w_i \cdot I(Y_i^{\text{train}} = 1)}{\sum_{i \in \mathcal{N}_k} w_i}.$$

w_{blue}	w_{yellow}	$\hat{p}(X^{\text{test}})$	Predicted class	
1	1	1/3	Blue	
1	2	1/2	Yellow	(As though 2 yellows in circle)
1	3	3/5	Yellow	(As though 3 yellows in circle)

Note: Only relative values of weights matter, e.g. only $w_{\text{yellow}}/w_{\text{blue}}$ matters.

Evaluating classification errors on a test set

Evaluating classification errors on a test set

Given C_{FN} and C_{FP}, best single number to summarize classification performance is the weighted misclassification error on the test set.

Evaluating classification errors on a test set

Given C_{FN} and C_{FP}, best single number to summarize classification performance is the weighted misclassification error on the test set.

Another way of assessing classification performance—without quantifying costs—is the confusion matrix and associated metrics (e.g. false positive rate).

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	**Actually Negative**
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

The confusion matrix and associated metrics

Confusion matrix

Metrics based on confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{FP}{N}$$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{FP}{N} = \frac{20}{50}$$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

False positive rate (FPR) $= \dfrac{FP}{N} = \dfrac{20}{50}$

True negative rate (TNR) $= \dfrac{TN}{N}$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

False positive rate (FPR) $= \dfrac{FP}{N} = \dfrac{20}{50}$

True negative rate (TNR) $= \dfrac{TN}{N} = \dfrac{30}{50}$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

False positive rate (FPR) $= \frac{FP}{N} = \frac{20}{50}$

True negative rate (TNR) $= \frac{TN}{N} = \frac{30}{50}$

False negative rate (FNR) $= \frac{FN}{P}$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

False positive rate (FPR) $= \frac{FP}{N} = \frac{20}{50}$

True negative rate (TNR) $= \frac{TN}{N} = \frac{30}{50}$

False negative rate (FNR) $= \frac{FN}{P} = \frac{40}{50}$

True positive rate (TPR) $= \frac{TP}{P}$

The confusion matrix and associated metrics

Confusion matrix

	Actually Positive	Actually Negative
Predicted Positive	10 True Positives (TP) (E.g. Sick people testing positive)	20 False Positives (FP) (E.g. Healthy people testing positive)
Predicted Negative	40 False negatives (FN) (E.g. Sick people testing negative)	30 True Negative (TN) (E.g. Healthy people testing negative)
Total	50 positives (P)	50 negatives (N)

Metrics based on confusion matrix

False positive rate (FPR) $= \dfrac{FP}{N} = \dfrac{20}{50}$

True negative rate (TNR) $= \dfrac{TN}{N} = \dfrac{30}{50}$

False negative rate (FNR) $= \dfrac{FN}{P} = \dfrac{40}{50}$

True positive rate (TPR) $= \dfrac{TP}{P} = \dfrac{10}{50}$

The confusion matrix and associated metrics

Metrics based on confusion matrix

$$= \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$= \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$= \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$= \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

The confusion matrix and associated metrics

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

The confusion matrix and associated metrics

- FPR and TNR measure how reliably we detect negatives; FPR + TNR = 1.

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

The confusion matrix and associated metrics

- FPR and TNR measure how reliably we detect negatives; FPR + TNR = 1.
- FNR and TPR measure how reliably we detect positives; FNR + TPR = 1.

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

The confusion matrix and associated metrics

- FPR and TNR measure how reliably we detect negatives; FPR + TNR = 1.
- FNR and TPR measure how reliably we detect positives; FNR + TPR = 1.

Neither TPR nor TNR, taken individually, tells the whole story of classifier.

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

The confusion matrix and associated metrics

- FPR and TNR measure how reliably we detect negatives; FPR + TNR = 1.
- FNR and TPR measure how reliably we detect positives; FNR + TPR = 1.

Neither TPR nor TNR, taken individually, tells the whole story of classifier.

F-score, the geometric mean of TPR and TNR, summarizes performance in single number:

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

The confusion matrix and associated metrics

- FPR and TNR measure how reliably we detect negatives; FPR + TNR = 1.
- FNR and TPR measure how reliably we detect positives; FNR + TPR = 1.

Neither TPR nor TNR, taken individually, tells the whole story of classifier.

F-score, the geometric mean of TPR and TNR, summarizes performance in single number:

Metrics based on confusion matrix

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

$$\text{F-score} = \left(\frac{1}{2}\left(\frac{1}{\text{TPR}} + \frac{1}{\text{TNR}}\right)\right)^{-1}$$

The confusion matrix and associated metrics

Metrics based on confusion matrix

- FPR and TNR measure how reliably we detect negatives; FPR + TNR = 1.
- FNR and TPR measure how reliably we detect positives; FNR + TPR = 1.

Neither TPR nor TNR, taken individually, tells the whole story of classifier.

F-score, the geometric mean of TPR and TNR, summarizes performance in single number:

$$\text{False positive rate (FPR)} = \frac{\text{FP}}{\text{N}} = \frac{20}{50}$$

$$\text{True negative rate (TNR)} = \frac{\text{TN}}{\text{N}} = \frac{30}{50}$$

$$\text{False negative rate (FNR)} = \frac{\text{FN}}{\text{P}} = \frac{40}{50}$$

$$\text{True positive rate (TPR)} = \frac{\text{TP}}{\text{P}} = \frac{10}{50}$$

$$\text{F-score} = \left(\frac{1}{2}\left(\frac{1}{\text{TPR}} + \frac{1}{\text{TNR}}\right)\right)^{-1} = 0.3$$

Summary

Summary

- Classification problem is similar in some ways to regression; different in others.

Summary

- Classification problem is similar in some ways to regression; different in others.
- Classification done by estimating $\mathbb{P}[Y = 1 \,|\, X]$, thresholding at 0.5 (e.g. KNN).

Summary

- Classification problem is similar in some ways to regression; different in others.
- Classification done by estimating $\mathbb{P}[Y = 1 \,|\, X]$, thresholding at 0.5 (e.g. KNN).
- Bias-variance tradeoff carries over intuitively, but not mathematically.

Summary

- Classification problem is similar in some ways to regression; different in others.
- Classification done by estimating $\mathbb{P}[Y = 1 \mid X]$, thresholding at 0.5 (e.g. KNN).
- Bias-variance tradeoff carries over intuitively, but not mathematically.
- Misclassification error not a good metric for problems when different misclassifications have different costs; often the case with imbalanced classes.

Summary

- Classification problem is similar in some ways to regression; different in others.
- Classification done by estimating $\mathbb{P}[Y = 1 \,|\, X]$, thresholding at 0.5 (e.g. KNN).
- Bias-variance tradeoff carries over intuitively, but not mathematically.
- Misclassification error not a good metric for problems when different misclassifications have different costs; often the case with imbalanced classes.
- Differential misclassification costs can be remedied by building observation weights into training.

Summary

- Classification problem is similar in some ways to regression; different in others.
- Classification done by estimating $\mathbb{P}[Y = 1 \,|\, X]$, thresholding at 0.5 (e.g. KNN).
- Bias-variance tradeoff carries over intuitively, but not mathematically.
- Misclassification error not a good metric for problems when different misclassifications have different costs; often the case with imbalanced classes.
- Differential misclassification costs can be remedied by building observation weights into training.
- Performance metrics for classifiers include the weighted misclassification error and confusion matrix based metrics like false positive and false negative rates.