Unit 3 Lecture 4: Lasso regression

October 18, 2022

In this R demo, we will learn about the `glmnetUtils` package and how to run cross-validated lasso and elastic net regressions using the `cv.glmnet()` and `cva.glmnet()` functions, respectively.

```
library(tidyverse)
library(glmnetUtils)  # for cv.glmnet(), cva.glmnet()
library(stat471)      # for plot_glmnet(), plot_cva_glmnet(), coef_tidy()
```

We will be applying lasso and elastic net regressions to study the crime data from the previous lecture

```
crime_data <- read_csv("CrimeData_FL.csv")
crime_data
```

```
## # A tibble: 90 x 98
##    population househol~1 race.~2 race.~3 race.~4 race.~5 age.p~6 age.p~7 age.p~8
##         <dbl>      <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>
##  1      16023       2.63   13.8     83.9    1.42    2.4    11.9     23.1   10.3
##  2      29721       2.34    3.52    95.1    1.03    1.7    10.5     18.8    9.27
##  3      10205       2.46    1.06    97.4    1.04    1.79    9.62    18.0    7.64
##  4     124773       2.47   29.1     68.2    1.75    3      23.7     42.7   28.8
##  5      13024       2.25   31.3     67.2    0.5     6.09    9.62    21.0    9.8
##  6     280015       2.44   25.0     70.9    1.35   15      13       27.5   13.4
##  7      79443       2.94    3.48    93.1    2.12    7.12   16.7     27.4   12.9
##  8      16444       2.57    5.38    91.2    1.96    8.65   14.4     28.1   14.2
##  9      46194       2.28   20.1     77.7    0.63    6.76    9.11    20.1    9.05
## 10      14044       2.17    0.48    98.3    0.58    2.03    8.94    19.8    9.42
## # ... with 80 more rows, 89 more variables: age.pct65up <dbl>, pct.urban <dbl>,
## #   med.income <dbl>, pct.wage.inc <dbl>, pct.farmself.inc <dbl>,
## #   pct.inv.inc <dbl>, pct.socsec.inc <dbl>, pct.pubasst.inc <dbl>,
## #   pct.retire <dbl>, med.family.inc <dbl>, percap.inc <dbl>,
## #   white.percap <dbl>, black.percap <dbl>, indian.percap <dbl>,
## #   asian.percap <dbl>, hisp.percap <dbl>, pct.pop.underpov <dbl>,
## #   pct.less9thgrade <dbl>, pct.not.hsgrad <dbl>, pct.bs.ormore <dbl>, ...
```

Let's split the data into training and testing:

```
set.seed(471)
train_samples <- sample(1:nrow(crime_data), 0.8 * nrow(crime_data))
crime_train <- crime_data %>% filter(row_number() %in% train_samples)
crime_test <- crime_data %>% filter(!(row_number() %in% train_samples))
```

Running a cross-validated lasso regression

We call `cv.glmnet` on `crime_train`:

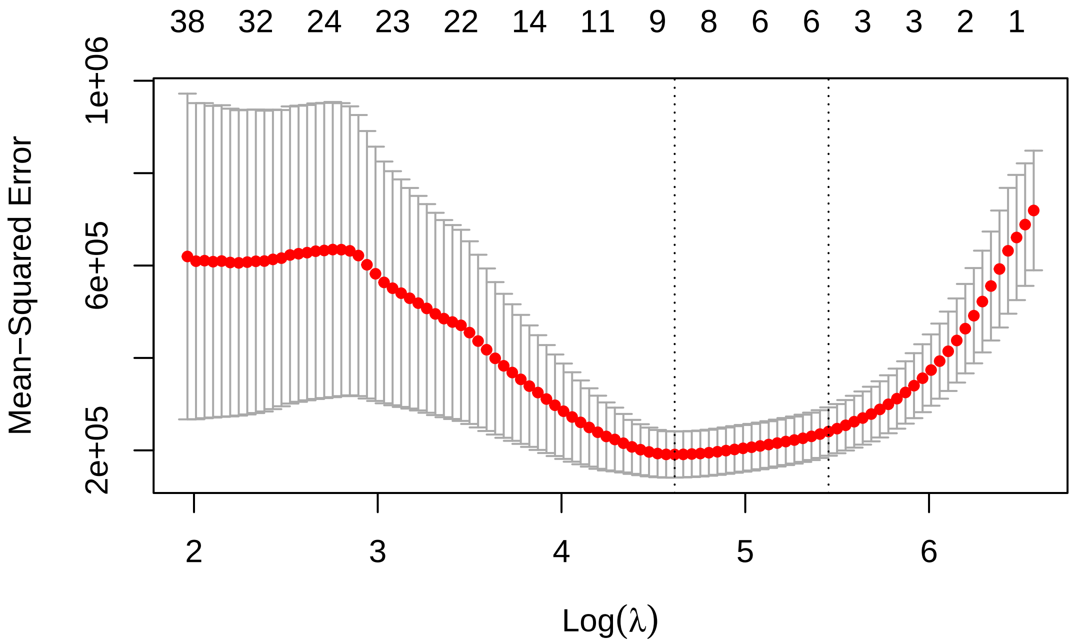
```
lasso_fit <- cv.glmnet(
  violentcrimes.perpop ~ ., # formula notation, as usual
  alpha = 1,                # alpha = 1 for lasso
```

```
  nfolds = 10,                # number of folds
  data = crime_train          # data to run lasso on
)
```

Inspecting the results

The `glmnet` package has a very nice `plot` function to produce the CV plot:

```
plot(lasso_fit)
```



The `lasso_fit` object has several fields with information about the fit:

```
# lambda sequence
head(lasso_fit$lambda)
```

```
## [1] 713.0629 680.6531 649.7163 620.1857 591.9973 565.0901
```

```
# number of nonzero coefficients
head(lasso_fit$nzero)
```

```
## s0 s1 s2 s3 s4 s5
##  0  1  1  2  2  2
```

```
# CV estimates
head(lasso_fit$cvm)
```

```
## [1] 719195.4 688634.8 660698.6 631908.0 592507.2 555672.5
```

```
# CV standard errors
head(lasso_fit$cvsd)
```

```
## [1] 129486.1 132612.6 135495.0 136165.2 126567.6 117928.5
```

```
# lambda achieving minimum CV error
lasso_fit$lambda.min
```

```
## [1] 101.0748
```

```r
# lambda based on one-standard-error rule
lasso_fit$lambda.1se
```

```
## [1] 233.4959
```

To get the fitted coefficients at the selected value of lambda, we can use the `coef_tidy()` function from the `stat471` package:

```r
coef_tidy(lasso_fit, s = "lambda.1se")
```

```
## # A tibble: 98 x 2
##    feature        coefficient
##    <chr>                <dbl>
##  1 (Intercept)          1139.
##  2 population              0
##  3 household.size          0
##  4 race.pctblack           0
##  5 race.pctwhite           0
##  6 race.pctasian           0
##  7 race.pcthisp            0
##  8 age.pct12to21           0
##  9 age.pct12to29           0
## 10 age.pct16to24           0
## # ... with 88 more rows
```

```r
coef_tidy(lasso_fit, s = "lambda.min")
```

```
## # A tibble: 98 x 2
##    feature        coefficient
##    <chr>                <dbl>
##  1 (Intercept)          7696.
##  2 population              0
##  3 household.size          0
##  4 race.pctblack           0
##  5 race.pctwhite           0
##  6 race.pctasian           0
##  7 race.pcthisp            0
##  8 age.pct12to21           0
##  9 age.pct12to29           0
## 10 age.pct16to24           0
## # ... with 88 more rows
```

If `s` is not specified then `s = lambda.1se` will be chosen by default:

```r
coef_tidy(lasso_fit)
```

```
## # A tibble: 98 x 2
##    feature        coefficient
##    <chr>                <dbl>
##  1 (Intercept)          1139.
##  2 population              0
##  3 household.size          0
##  4 race.pctblack           0
##  5 race.pctwhite           0
##  6 race.pctasian           0
##  7 race.pcthisp            0
##  8 age.pct12to21           0
```

```
##   9 age.pct12to29           0
## 10 age.pct16to24           0
## # ... with 88 more rows
```

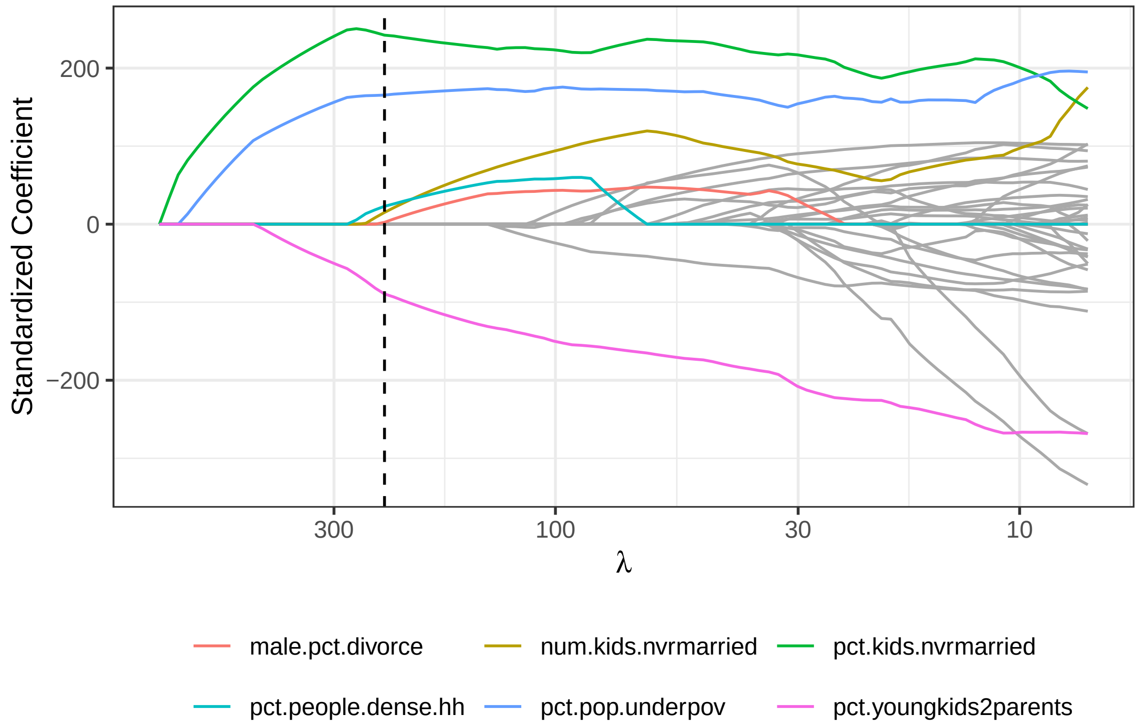
Note that these coefficient vectors are sparse. We can get a list of the nonzero standardized coefficients as follows:

```
coef_tidy(lasso_fit) %>%
  filter(coefficient != 0)
```

```
## # A tibble: 7 x 2
##   feature               coefficient
##   <chr>                       <dbl>
## 1 (Intercept)               1139.
## 2 pct.pop.underpov            22.7
## 3 male.pct.divorce             0.912
## 4 pct.youngkids2parents       -7.27
## 5 num.kids.nvrmarried          0.00301
## 6 pct.kids.nvrmarried         72.7
## 7 pct.people.dense.hh          5.38
```

To visualize the fitted coefficients as a function of lambda, we can make a plot of the coefficients like we saw in class. To do this, we can use the `plot_glmnet` function, which by default shows a dashed line at the lambda value chosen using the one-standard-error rule:

```
plot_glmnet(lasso_fit, crime_train)
```



By default, `plot_glmnet` annotates the features with nonzero coefficients. To interpret these coefficient estimates, recall that they are for the *standardized* features.

Making predictions

To make predictions on the test data, we can use the `predict` function (which we've seen before):

```
lasso_predictions <- predict(lasso_fit,
  newdata = crime_test,
  s = "lambda.1se"
) %>% as.numeric()
lasso_predictions
```

```
##  [1] 1901.3904 1331.1849  950.1509  809.1830  757.2346  756.9001  864.4810
##  [8]  786.1132  820.8253 1063.3772  647.6317 1343.7530 1163.0144 1078.2869
## [15] 1366.4746  741.4528  883.7059  820.3576
```

We can evaluate the root-mean-squared-error as before:

```
RMSE <- sqrt(mean((lasso_predictions - crime_test$violentcrimes.perpop)^2))
RMSE
```

```
## [1] 400.7576
```

Elastic net regression

Next, let's run an elastic net regression. We can do this via the `cva.glmnet()` function:

```
elnet_fit <- cva.glmnet(
  violentcrimes.perpop ~ ., # formula notation, as usual
  nfolds = 10,              # number of folds
  data = crime_train        # data to run elastic net on
)
```

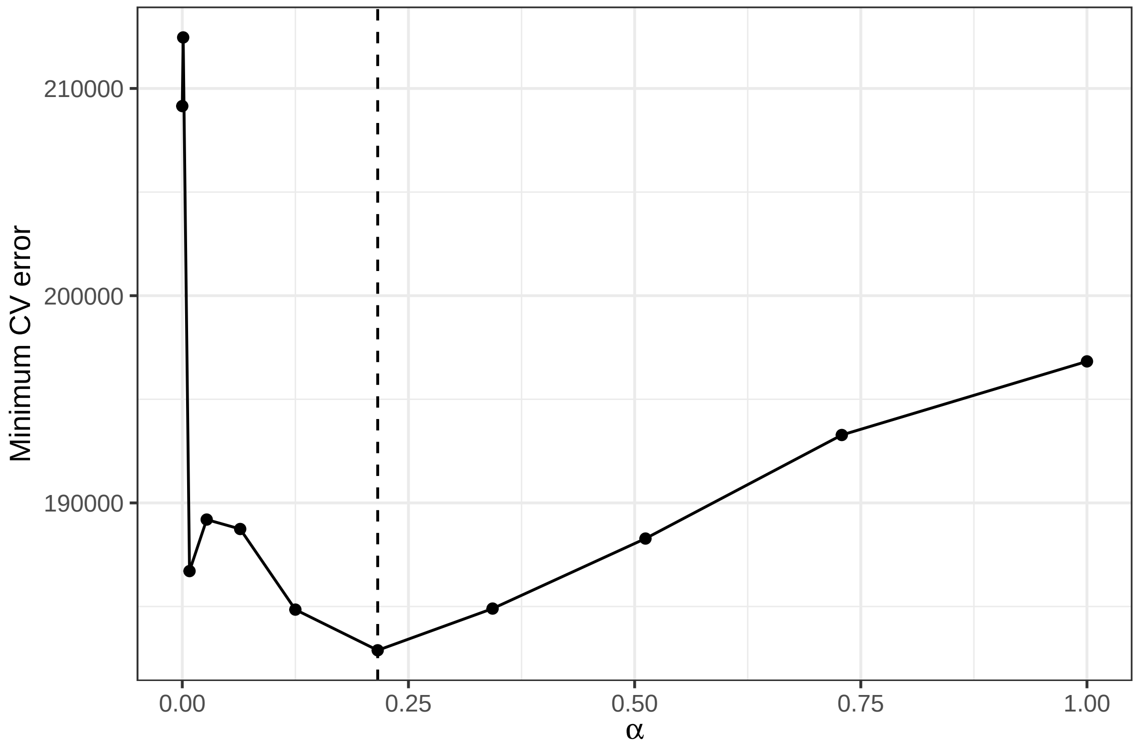
The following are the values of `alpha` that were used:

```
elnet_fit$alpha
```

```
##  [1] 0.000 0.001 0.008 0.027 0.064 0.125 0.216 0.343 0.512 0.729 1.000
```

We can plot the minimum CV error for each value of alpha using the helper function `plot_cva_glmnet()` from `plot_glmnet.R`:

```
plot_cva_glmnet(elnet_fit)
```



We can then extract the `cv.glmnet` fit object based on the optimal `alpha` using `extract_best_elnet` from `plot_glmnet.R`:

```
elnet_fit_best <- extract_best_elnet(elnet_fit)
```

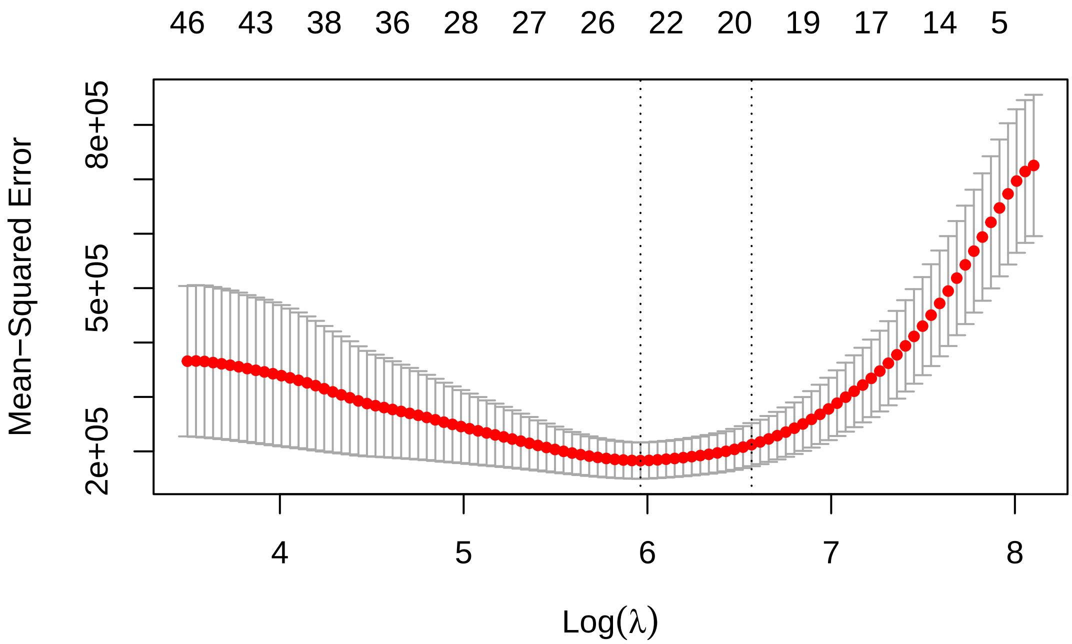
The `elnet_fit_best` object is a usual `glmnet` fit object, with an additional field called `alpha` specifying which value of `alpha` was used:

```
elnet_fit_best$alpha
```

```
## [1] 0.216
```

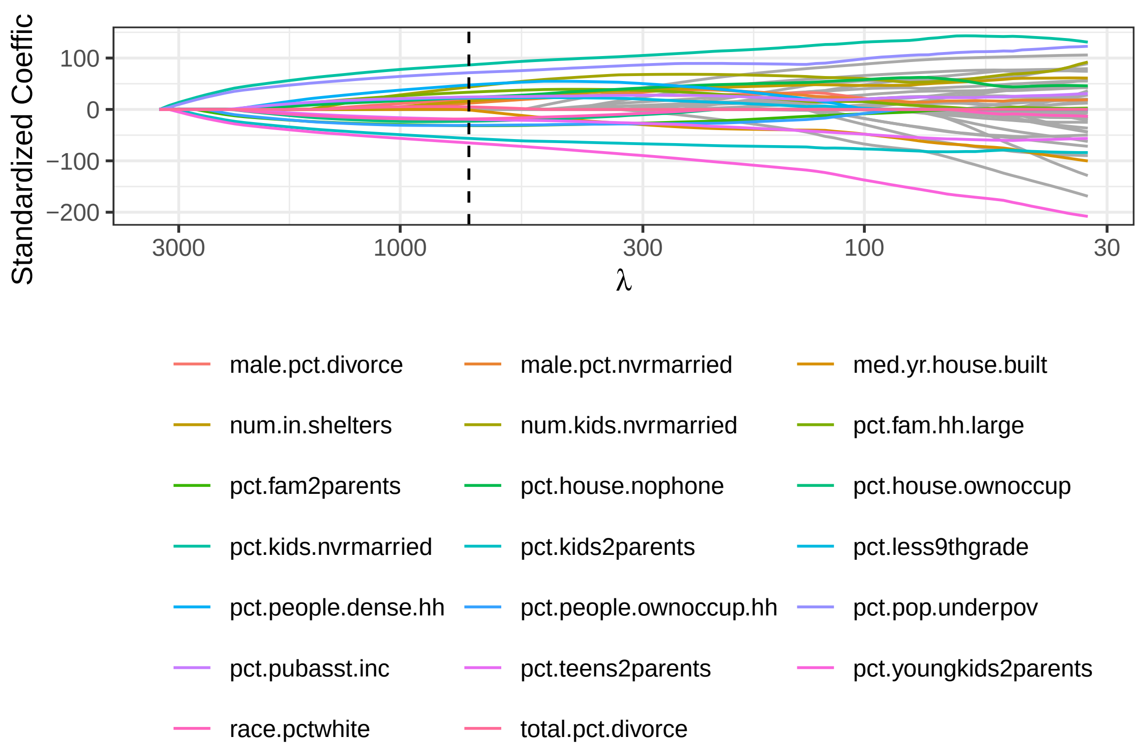
We can make a CV plot to select `lambda` as usual:

```
plot(elnet_fit_best)
```



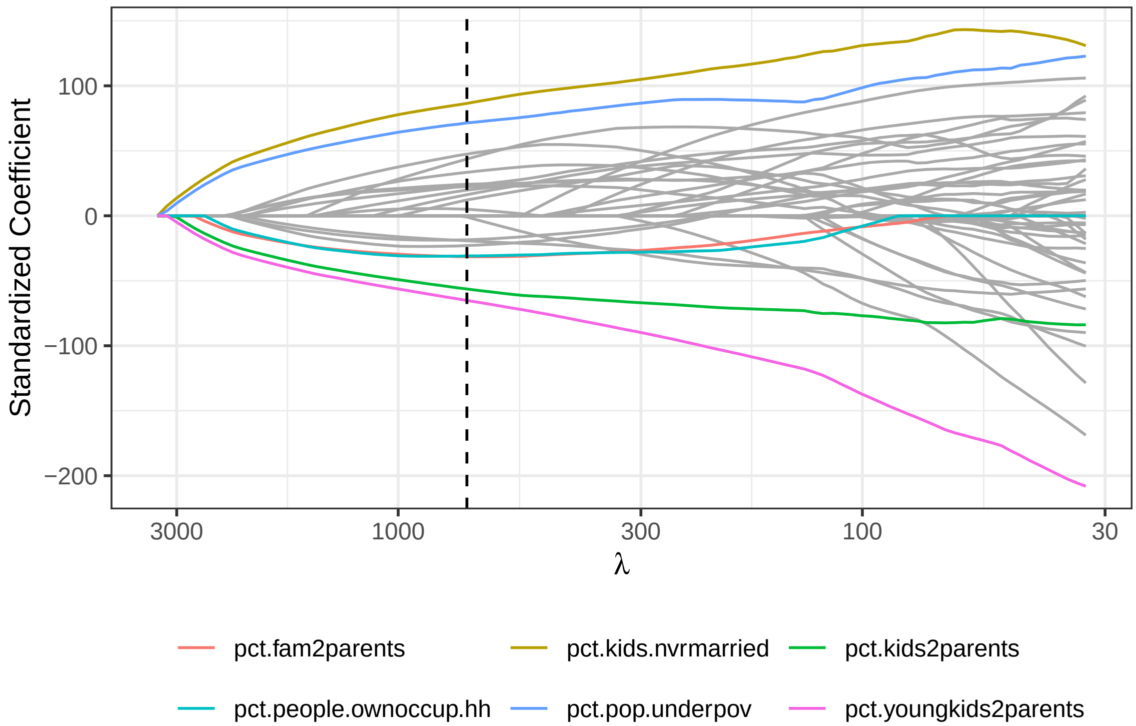
And we can make a trace plot for this optimal value of `alpha`:

```
plot_glmnet(elnet_fit_best, crime_train)
```



This is too many features to highlight, so let's choose a smaller number:

```
plot_glmnet(elnet_fit_best, crime_train, features_to_plot = 6)
```



We can make predictions and evaluate test error using the `elnet_fit_best` object:

```
elnet_predictions <- predict(elnet_fit,
  alpha = elnet_fit$alpha,
  newdata = crime_test,
  s = "lambda.1se"
) %>% as.numeric()
elnet_predictions
```

```
##  [1] 1737.2357 1344.0327 1036.0706  762.1868  671.2751  690.3775  975.6859
##  [8]  807.5988  833.6900  744.3694  545.3223 1384.0474 1250.4805 1258.0862
## [15] 1445.2206  702.8391  693.6531  701.8896
```

```
RMSE <- sqrt(mean((elnet_predictions - crime_test$violentcrimes.perpop)^2))
RMSE
```

```
## [1] 392.1493
```

Lasso logistic regression

We can also run a lasso-penalized logistic regression. Let's try it out on a binarized version of the crime data:

```
# redefine response based on whether violentcrimes.perpop is above the median
crime_binary_train <- crime_train %>%
  mutate(
    violentcrimes.perpop =
      as.numeric(violentcrimes.perpop > median(violentcrimes.perpop))
  )
crime_binary_test <- crime_test %>%
  mutate(
    violentcrimes.perpop =
      as.numeric(violentcrimes.perpop > median(violentcrimes.perpop))
```

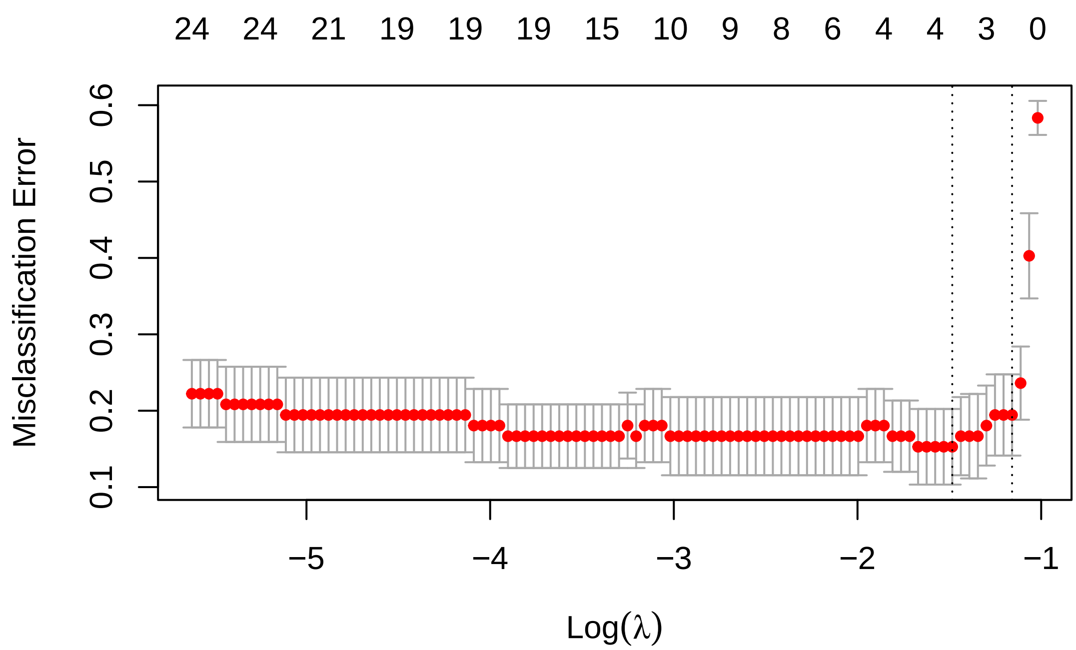
```
)
```

To run the logistic lasso regression, we call `cv.glmnet` as before, adding the argument `family = binomial` to specify that we want to do a logistic regression and the argument `type.measure = "class`: to specify that we want to use the misclassification error during cross-validation.

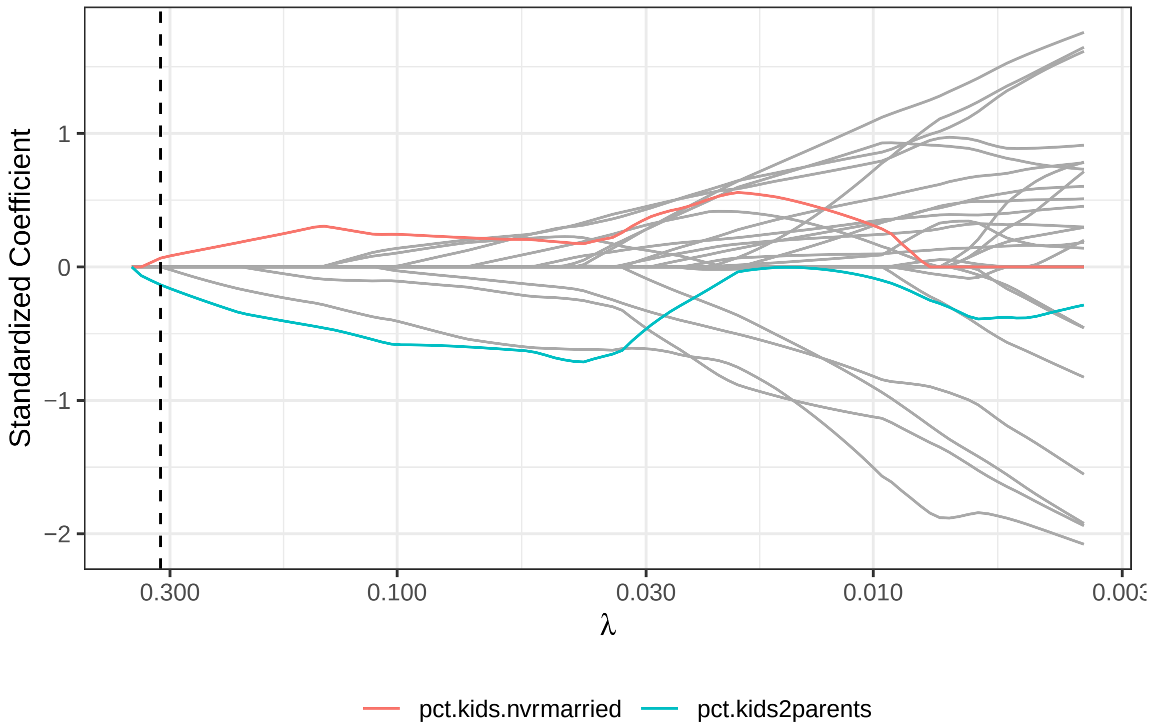
```
lasso_fit <- cv.glmnet(
  violentcrimes.perpop ~ .,          # formula notation, as usual
  alpha = 1,                         # alpha = 0 means lasso
  nfolds = 10,                       # number of CV folds
  family = "binomial",               # logistic regression
  type.measure = "class",            # use misclassification error
  data = crime_binary_train          # train on crime_binary_train data
)
```

We can then take a look at the CV plot and the trace plot as before:

```
plot(lasso_fit)
```




```
plot_glmnet(lasso_fit, crime_binary_train)
```



To predict using the fitted model, we can use the `predict` function again, this time specifying `type = "response"` to get the predictions on the probability scale (as opposed to the log-odds scale).

```
probabilities <- predict(lasso_fit, # fit object
  newdata = crime_binary_test,      # new data to test on
  s = "lambda.1se",                 # value of lambda to use
  type = "response"                 # output probabilities
) %>%
  as.numeric() # convert to vector
head(probabilities)
```

```
## [1] 0.5750745 0.5179763 0.4617836 0.4392386 0.4409267 0.4460029
```

We can threshold the probabilities to get binary predictions as we did with regular logistic regression.