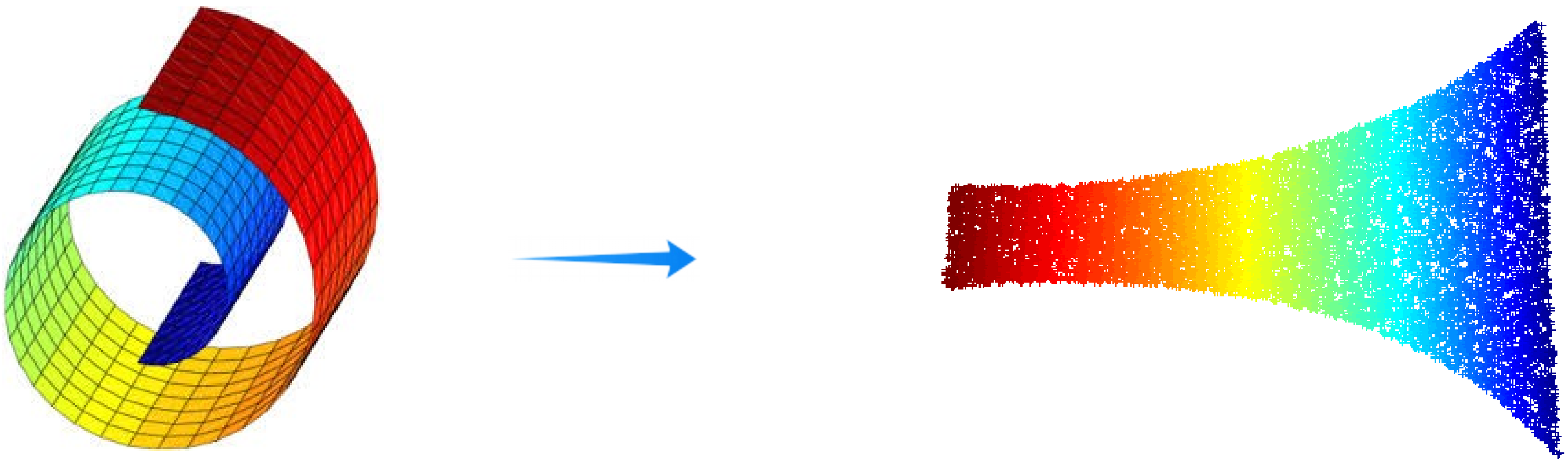
Locally Linear Embeddings in Pattern Recognition



- Elaboration: Katsileros Petros
- Supervision: Nikolaos Pitsianis

Introduction

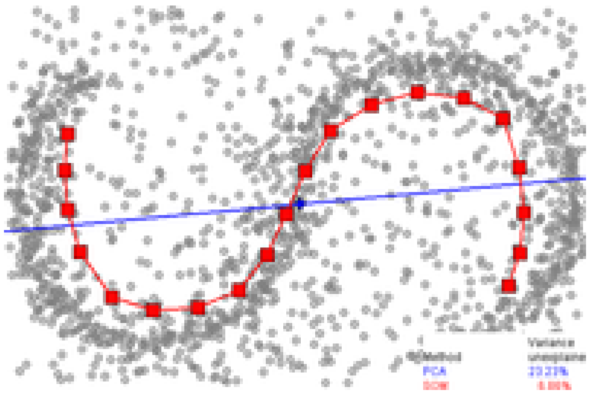
- Why we need information compression
- Dimensionality Reduction Algorithms (PCA, SVD, ISOMAP, Laplassian Eigenmaps, LLE)
- The Locally Linear Embeddings Algorithm (LLE)
- **Surpass LLE limitations with two new variations of LLE algorithm**
- Experiments using native LLE and the two new methods
  - Datasets: MNIST, SVHN, Arcene
- **Results analysis**
- Future work
- Conclusion

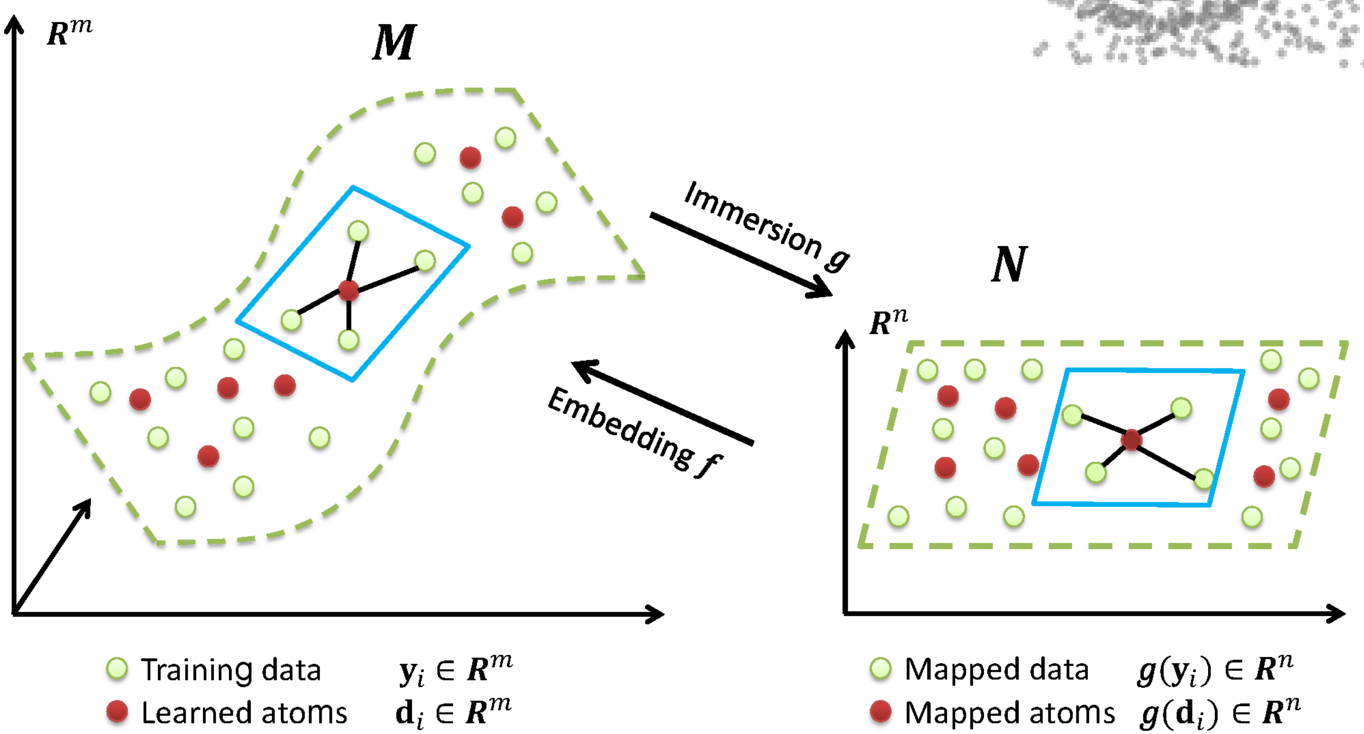
Dimensionality Reduction Algorithms

- Linear:
  - **Principal Component Analysis (PCA)**
  - Multi Dimensional Scaling (MDS)
  - **Singular Value Decomposition (SVD)**

Dimensionality Reduction Algorithms

- Non-Linear:
  - Isometric Mapping (ISOMAP)
  - Laplassian-Eigenmaps
  - Locally Linear Embeddings





Locally Linear Embeddings

- Step-1: Find K-Nearest Neighbors for each data (Adjacency matrix)
  - Executed in CUDA
- Step-2: Linear prediction for every point using it’s neighbors. Find Weight matrix (W).
  - Minimize cost function: $arg\,min\, E_w = \sum_{i=1}^{N} \| X_i - \sum_{j=1}^{N} W(i,j) X_j \|^2$
  - Weights W must follow these two restrictions:
    - W(i,j) = 0, if j and i are not neighbors
    - $\sum_{i=1}^{N} W(i,j) = 1$
    - Following these restrictions we have Translation, Rotation and Scaling independence.

Locally Linear Embeddings

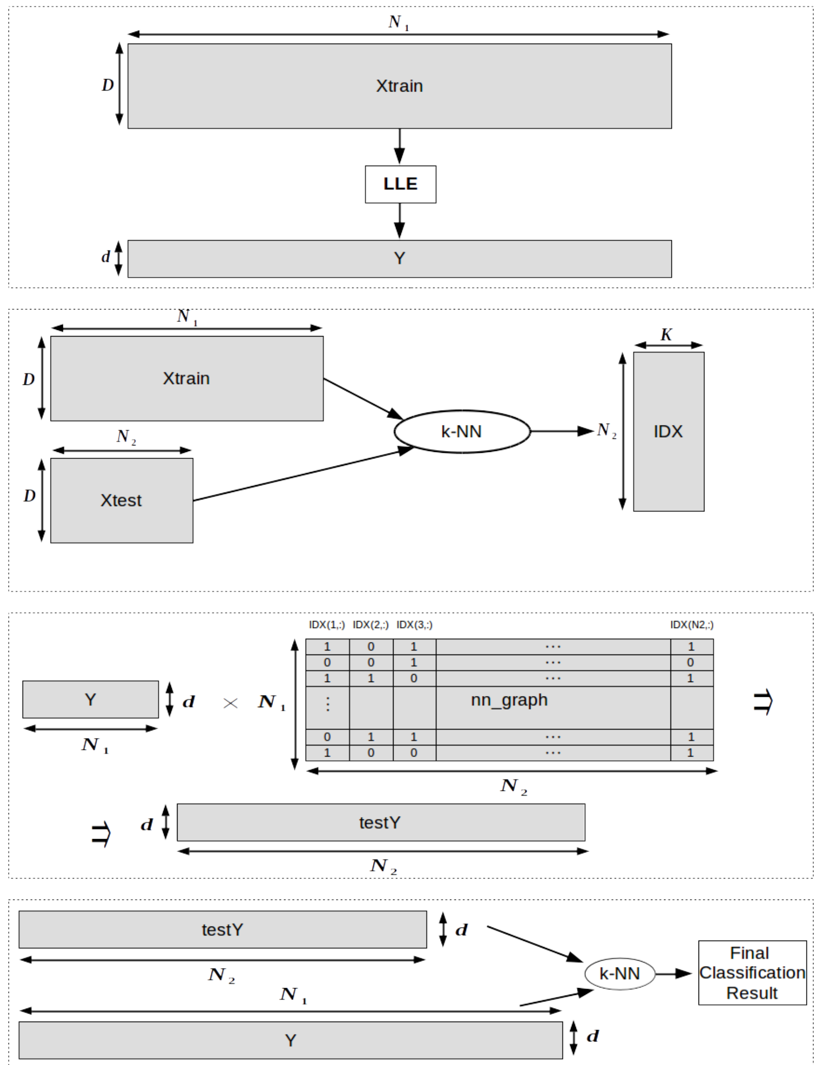
- Step-3: Find the embedding coordinates, using the weights matrix W
  - Minimize cost function: $arg\,min\, E_y = \sum_{i=1}^{N} \|Y_i - \sum_{j=1}^{N} W(i,j) Y_j\|^2$
  - $E_y = |(I-W)Y|^2 = Y_T M Y$
  - Find eigen-values of the square [NxN] **sparse** matrix
  - $M = (I-W)^T (I-W)$
- Step-4: Keep the final embedded coordinates
  - Discard the λ_0 eigenvalue (equal to zero)
  - Keep the rest d eigen-values. Their eigen-vectors are the final d embedded coordinates

LLE Limitations

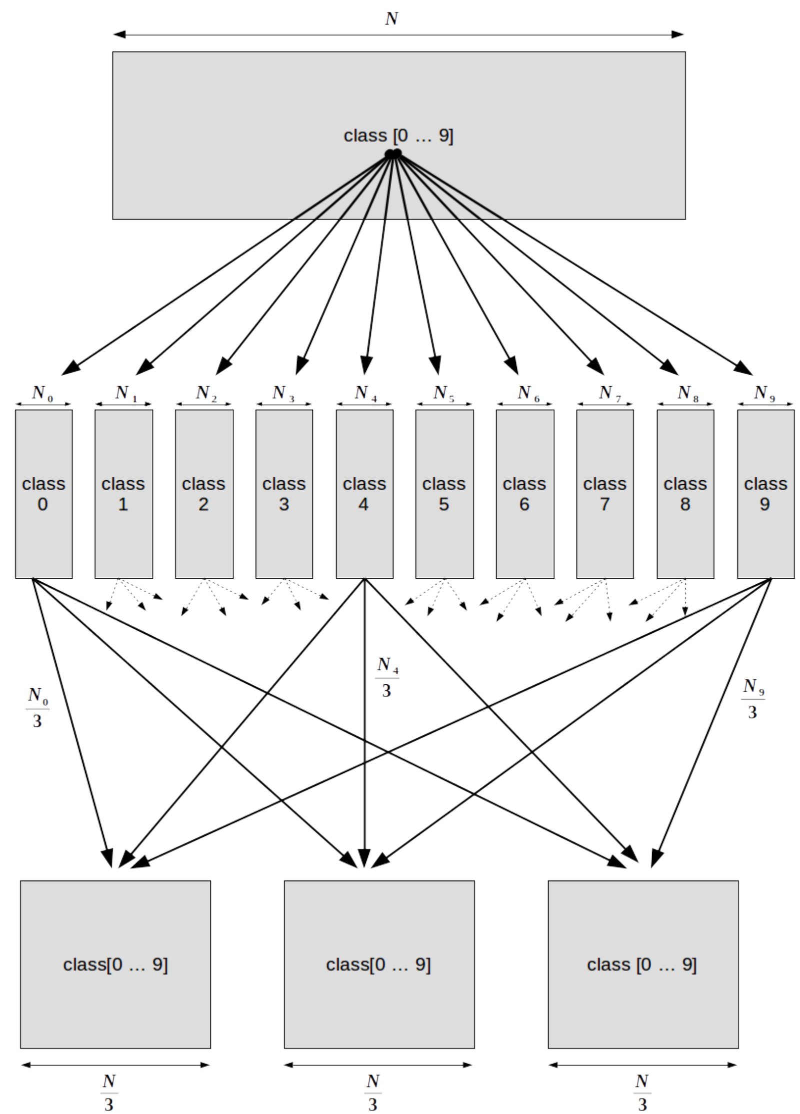
- Eigen-value decomposition step has complexity $k \cdot O\left(N^2\right)$
  - For sparse matrix M, solving with Lanczos algorithm
- We must run LLE algorithm with both train and test datasets as input data
  - Dimensionality reduction on train and separate on test produces spaces with different base vectors
  - We cannot find any realation between the low dimensional test and train data

- **Solutions**:
  - Method-1 will solve 2^{nd} limitation
    - Execute LLE on Train+Test data
  - Method-2 will solve 1^{st} limitation
    - Runtime algorithm complexity

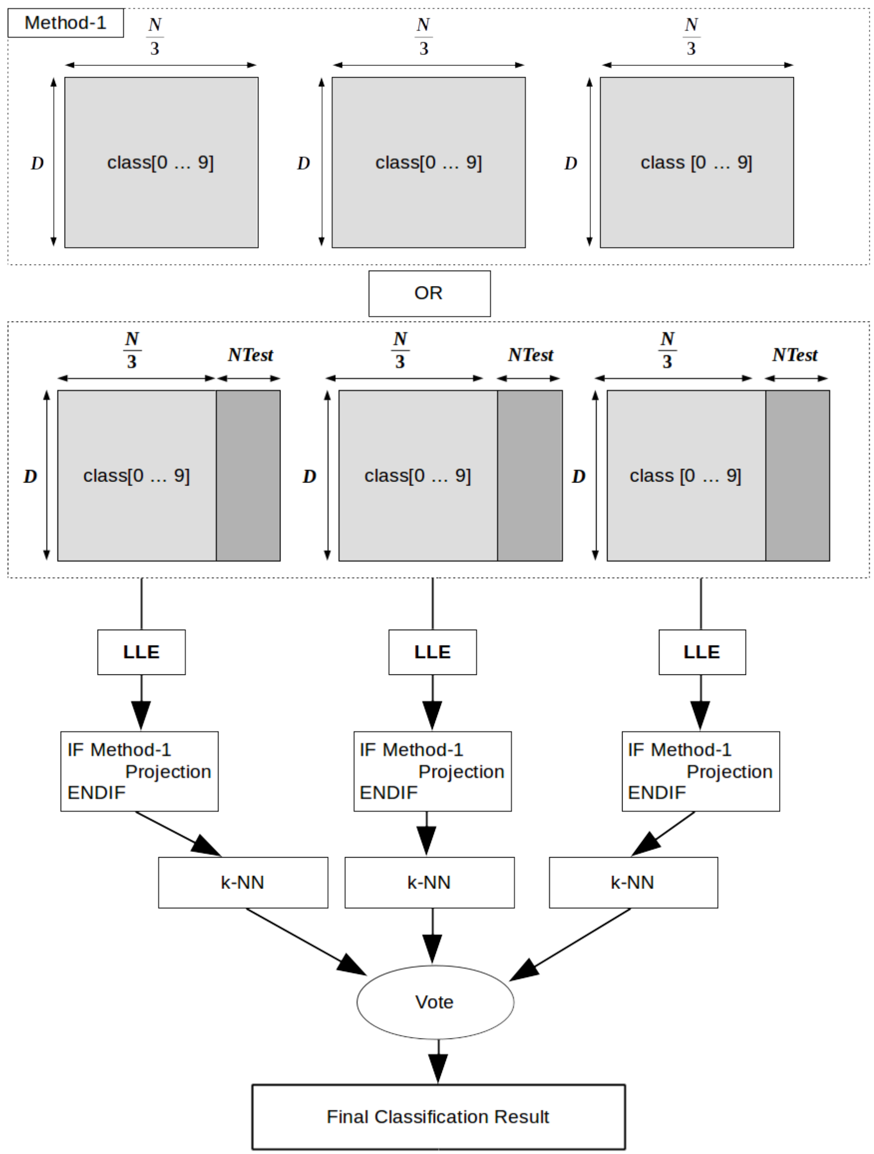
Method-1: LLE with Test-Projection



Method-2: Sub-set Majority Voting LLE



Method-2: Sub-set Majority Voting LLE



Experiments

- <u>Datasets</u>: MNIST, SVHN, ARCENE
  - MNIST: Gray scale images of handwritten digits.
    - 60K Train data, 10K Test data, D=[28x28]
  - SVHN: Google street view house numbers RGB images.
    - 73257 Train data, 26032 Test data, D=[32x32]
  - Arcene: Cancer dataset with a large number of predefined features for each patient.
    - 200 Train data, 700 Test data, D = 10K.
- <u>Classification algorithm</u>: k-NearestNeighbors
- <u>Classification metric</u>: Mean average % error
  - Acuuracy from every class / number of classes

MNIST Experiments

- Invest how LLE parameters (k, d) affect the classification process. Also find out if sub-sampling (Method-2) can lead to acceptable results.
  - K = [6, 7, 8, 9, 10, 12, 16, 20, 24, 32, 64], d = [10, 16, 20, 24, 32, 40, 52, 64, 96, 128, 256], subSet_size=[60000,20000,10000] of Train dataset
    - Classification error using k-NN (k=2) without dimensionality reduction (D=784): **3.5%**
    - Classification error using k-NN (k=2): **K=12, d=128, subSet_size=60K,** qual to **3.06%**
    - Classification error using k-NN (k=2): **K=8, d=10, subSet_size=60K** equal to **3.31%**
    - From the above results, we can say that LLE algorithm can be used as a feature extraction process with a great data compression ability.

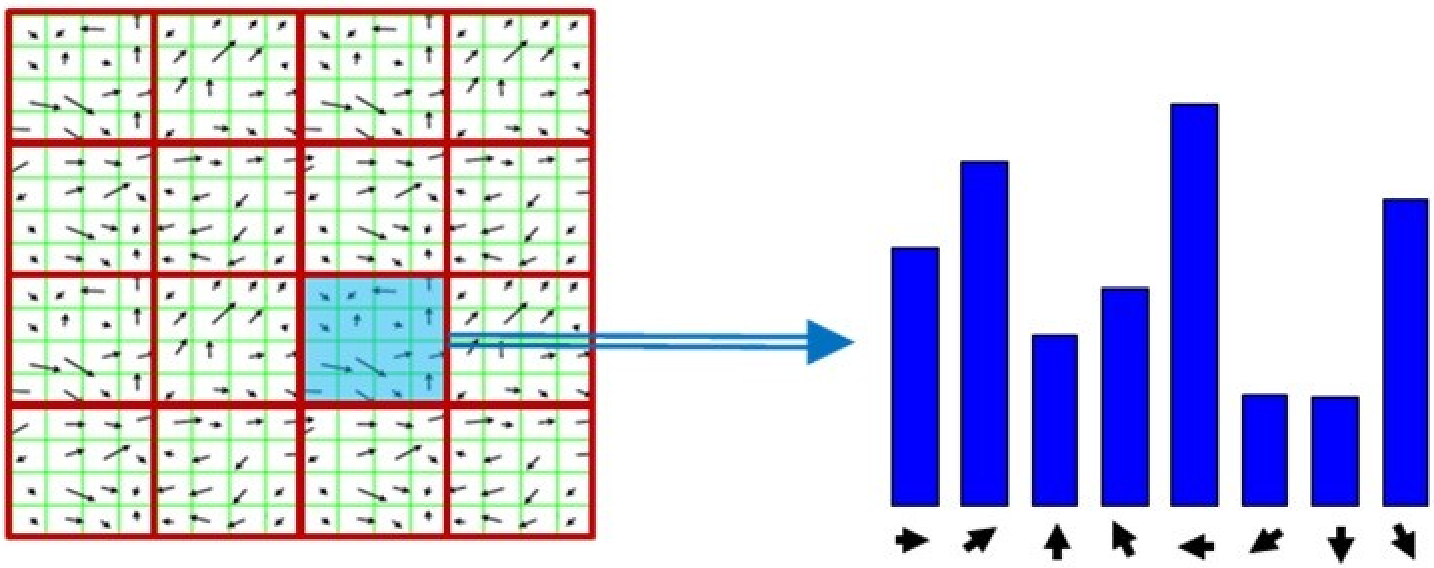
MNIST Experiments

- Method-2 results:
  - K=16, d=256, batch_size=20K. Best classification error equal to **3.27%.**
  - K=10, d=128, batch_size=10K. Best classification error equal to **3.31%.**
  - **Huge reduction both in time and space.** (Step-3 of LLE has $O(N^2)$ complexity)

- Method-1: Use of LLE dimensionality reduction in "Real time" applications
  - K=8, d=256, batch_size=60K. Best classification error equal to 3.85%.

SVHN Experiments

- Extract HoG features due to noise and light distortions.
  - Split image into sub sections and calculate the gradient of pixel intensity.

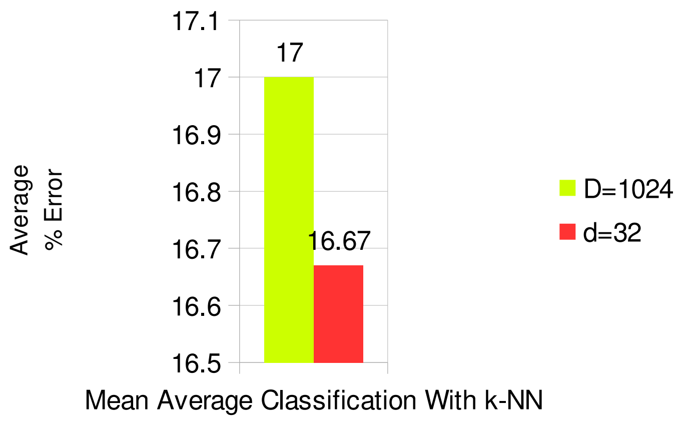


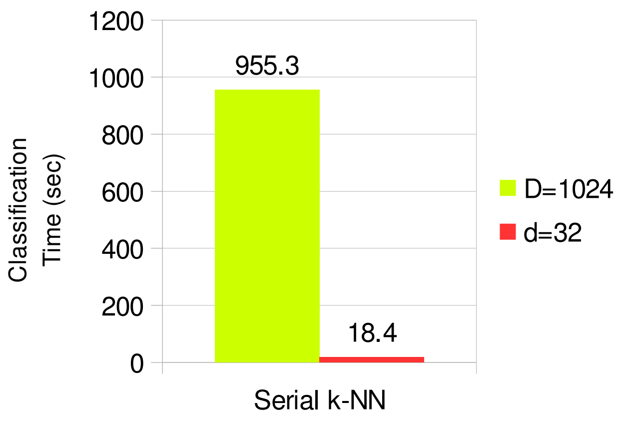
SVHN Experiments

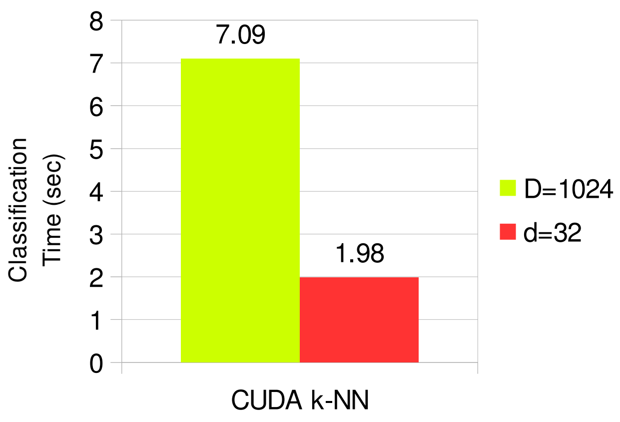
- **1st-Exp: Investigate LLE behaviour to this dataset. Also find out the best HoG kernel size**.
  - K = [8, 10, 12], d = [16, 20, 32, 64, 96, 128, 164, 196, 256], 30K of SVHN Train data-set
  - HoG Kernel size: [2x2], [4x4], [8x8] produce features of length: 8100, 1764, 324 .
  - **Best parameters for SVHN dataset: K=12, d=32, kernel=[4x4].**
- **2nd -Exp Method-1: Find out if Method-1 can lead to acceptable results.**
  - K=12, d=32, kernel=[4x4]. 42K of SVHN Train dataset
  - Classification error with LLE dimensionality reduction on Train data and projection for Test data is equal to **18.34%**
  - Classification error without dimensionality reduction is equal to **18.07%**

SVHN Full Dataset Experiment

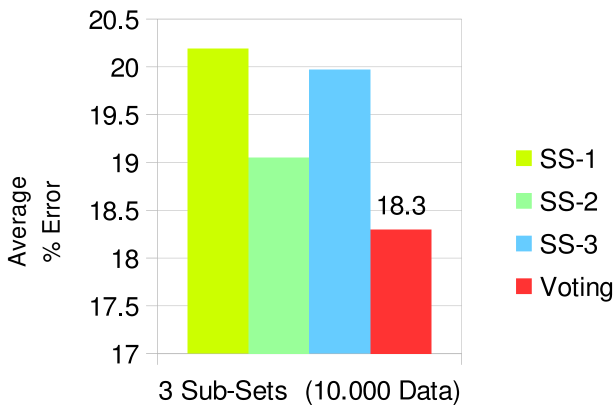
- **3rd-Exp: Classification result with vs without LLE dimensionality reduction.**
  - K=12, **d=32**, kernel=[4x4]. Full SVHN dataset (73257 Train data, 26032 Test data)

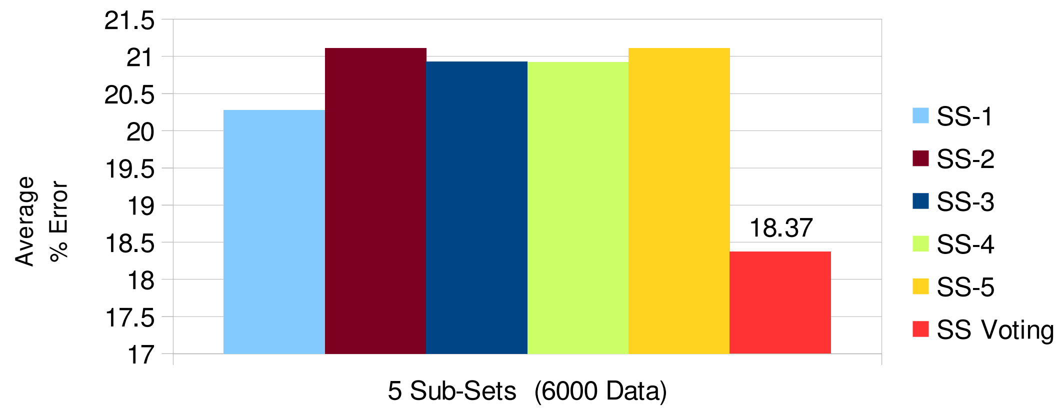


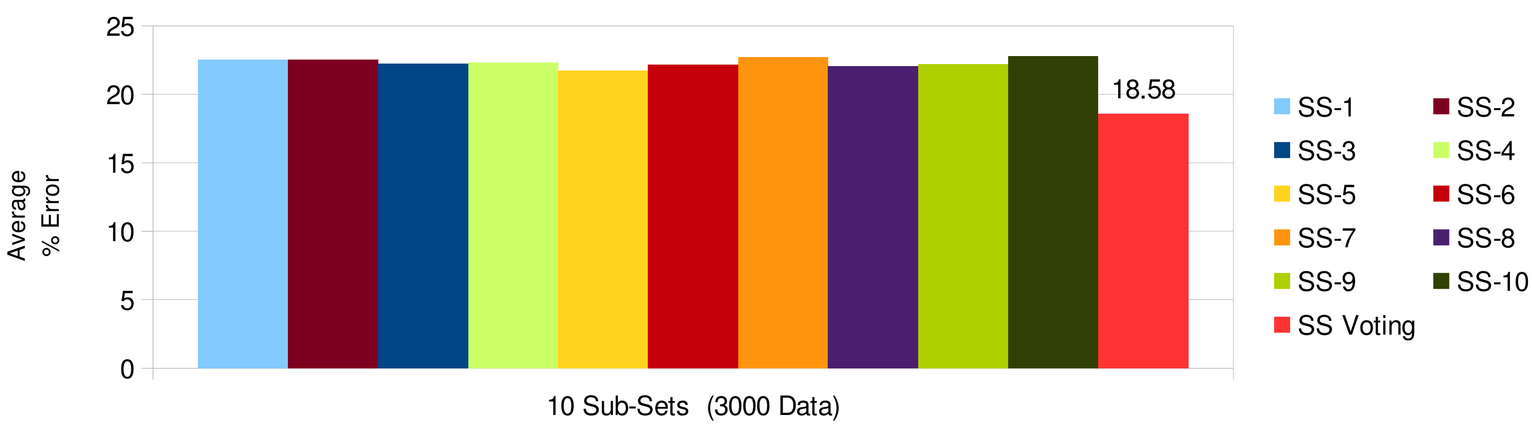




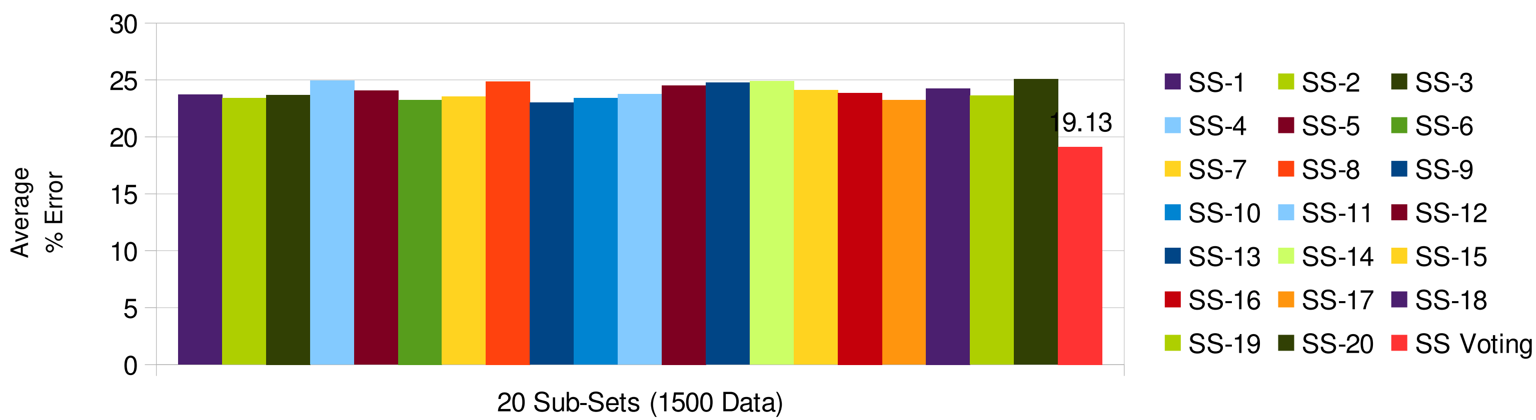
SVHN Method-2 Experiments

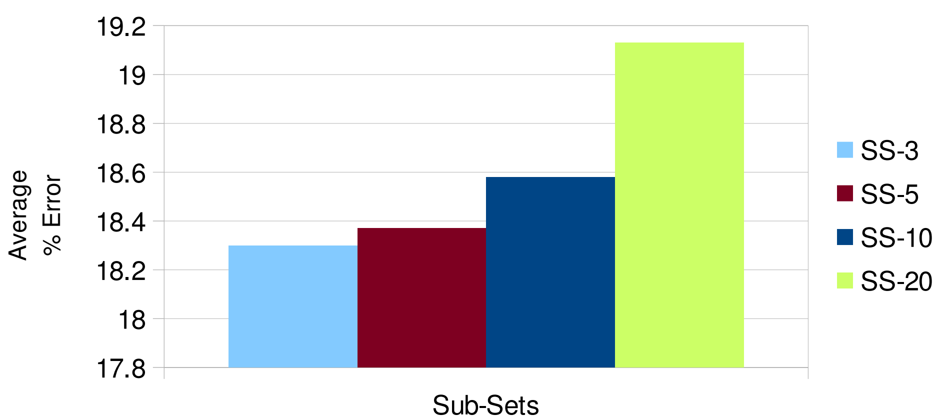


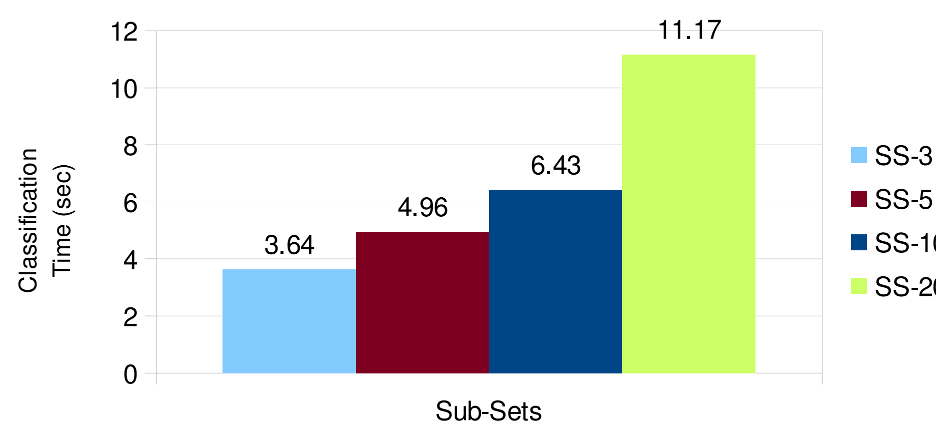




SVHN Method-2 Experiments







ARCENE Experiments

- **Investigate LLE dimensionality reduction as feature selection/extraction algorithm.**

	K=10	K=12	K=16	K=20	K=24	K=32	K=64
d=10	10	12	18	18	20	18	18
d=16	14	22	16	22	18	18	18
d=20	16	18	16	16	22	16	16
d=24	14	14	18	20	18	18	16
d=32	14	20	24	18	20	18	18
d=40	16	14	14	16	16	22	18
d=52	10	16	14	14	16	16	14
d=64	14	16	22	20	22	18	14
d=96	22	22	22	18	12	16	22
d=128	24	10	26	14	28	28	26
D=10.000				24			

Classification Accuracy Error using k-NN Algorithm

Future Work

- Parallelization: Even though k-NN algorithm both in LLE and in Classification process is executed in CUDA, further parallelization can be achieved espessially in Method-2.
- Extend to large datasets and invest the behaviour of subspace average voting. (SVHN extra data)
- Find the best fit between size of Train dataset subsets and Test data.

Conclusion

- LLE produces very accurate results after huge dimensionality reduction.
- LLE algorithm can be used both as feature extraction and feature selection algorithm.
- LLE has the ability to remove noise-data, producing very accurate classification results.
- Huge space and time savings at Final Classification Step ($d << D$).
- Method-1 can lead to real time Classification results. Using LLE on Train Data, Classification process can be executed for low dimensional data.
- Method-2 achieves huge reduction both in space and time, without or with minimal information loss.
- Method-1 and Method-2 make the execution of LLE algorithm available for normal PCs.

Thank You