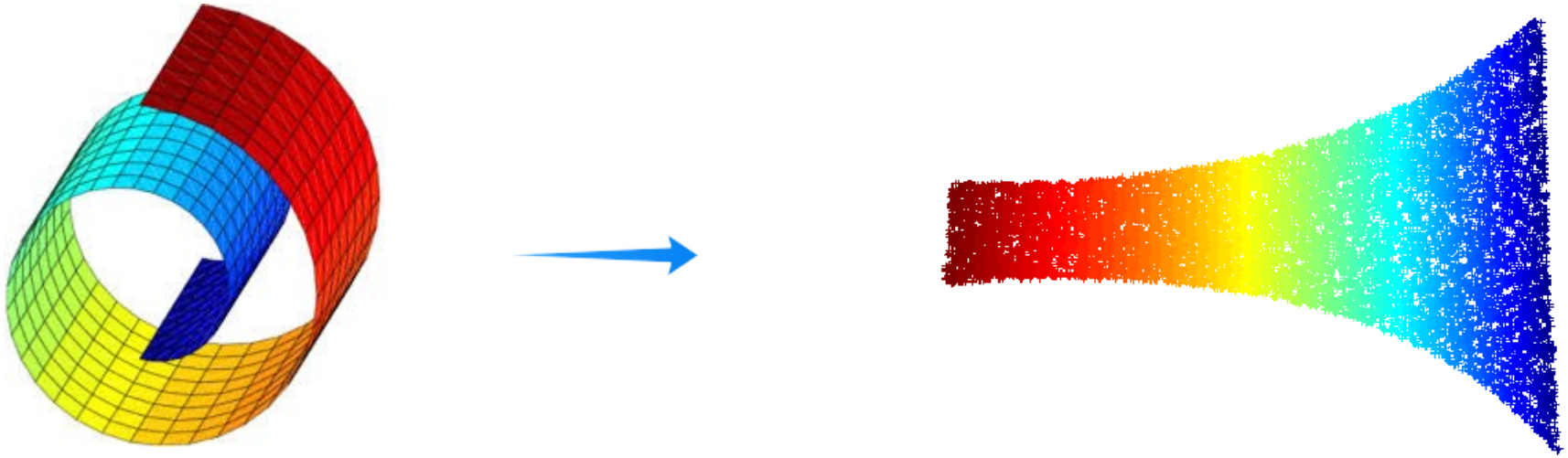
NonLinear Dimensionality Reduction in Pattern Recognition Applications



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Introduction

- Introduction about Dimensionality Reduction

- Presentation sections
  - Why we need information compression
  - Dimensionality Reduction Algorithms (PCA, SVD, ISOMAP, Laplassian Eigenmaps, LLE)
  - The Locally Linear Embeddings Algorithm (LLE)
  - **Surpass LLE limitations with two new variations of LLE algorithm**
  - Experiments using native LLE and the two new methods
    - Datasets: MNIST, SVHN, Arcene
  - **Results visualization and explanation**
  - Future work
  - Conclusion

Why we need information compression

- Example: Image with size [480,640]
  - That is equal to a vector with size: 480x640 = 307200.
  - For a “small” dataset, N = 100K we have a matrix of size: 100.000 x 307200

- Very large Computational complexity
- Very large memory requirements
- A large amount of these pixels are just Noise, affecting negative machine learning or image processing algorithms.

- Why dont we just simulate the Human brain …
- How? Using keypoints into each image
  - Features like SIFT, HoG, PFH etc …
  - Find out which pixels have meaningful information

- Dimensionality reduction techniques are able to extract d (ex. 8<d<256, from 480x640) meaningful key points

Dimensionality Reduction Algorithms

- Linear:
  - **Principal Component Analysis (PCA)**
    - Minimize a mean square error equation (sum of eigenvalues)
    - Produces statistical independent features
  - Multi Dimensional Scaling (MDS)
  - **Singular Value Decomposition (SVD)**
    - Solving the equation: $X = U_r \Lambda^{(\frac{1}{2})} V_r^H$

- Non-Linear:
  - Isometric Mapping (ISOMAP)
  - Laplassian-Eigenmaps
  - Locally Linear Embeddings

NonLinear Dimensionality Reduction – Locally Linear Embeddings

- Step-1: Find K-Nearest Neighbors for each data (Adjacency matrix)
  - Executed in CUDA
- Step-2: Find Weighs matrix (W)

$$arg\,min\,E_w = \sum_{i=1}^{N} \| X_i - \sum_{j=1}^{N} W(i,j) X_j \|^2$$

  - Minimize cost function:

$$|\vec{x} - \sum_j w_j \vec{x}_j|^2 = |\sum_j w_j (\vec{x} - \vec{x}_j)|^2 = \sum_{jk} w_j w_k C_{jk}$$

  - E =

$$C_{jk} = (\vec{x} - \vec{x}_j) \cdot (\vec{x} - \vec{x}_k)$$

  - Gram matrix:

$$\sum_j Cjk \cdot w_k = 1$$

  - Solve the system:

NonLinear Dimensionality Reduction – Locally Linear Embeddings

- Step-3: Find the embedding coordinates, using the weights matrix W
  - Minimize cost function: $arg\,min\, E_y = \sum_{i=1}^{N} \| Y_i - \sum_{j=1}^{N} W(i,j) Y_j \|^2$
    - $E_y = |(I - W)Y|^2 = Y_T M Y$
  - Find eigen-values of the square [NxN] **sparse** matrix
    - $M = (I - W)^T (I - W)$
- Step-4: Keep the final embedded coordinates
  - Discard the λ_0 eigenvalue (equal to zero)
  - Keep the rest d eigen-values. The eigen-vectors are the final embedded d coordinates

LLE Limitations

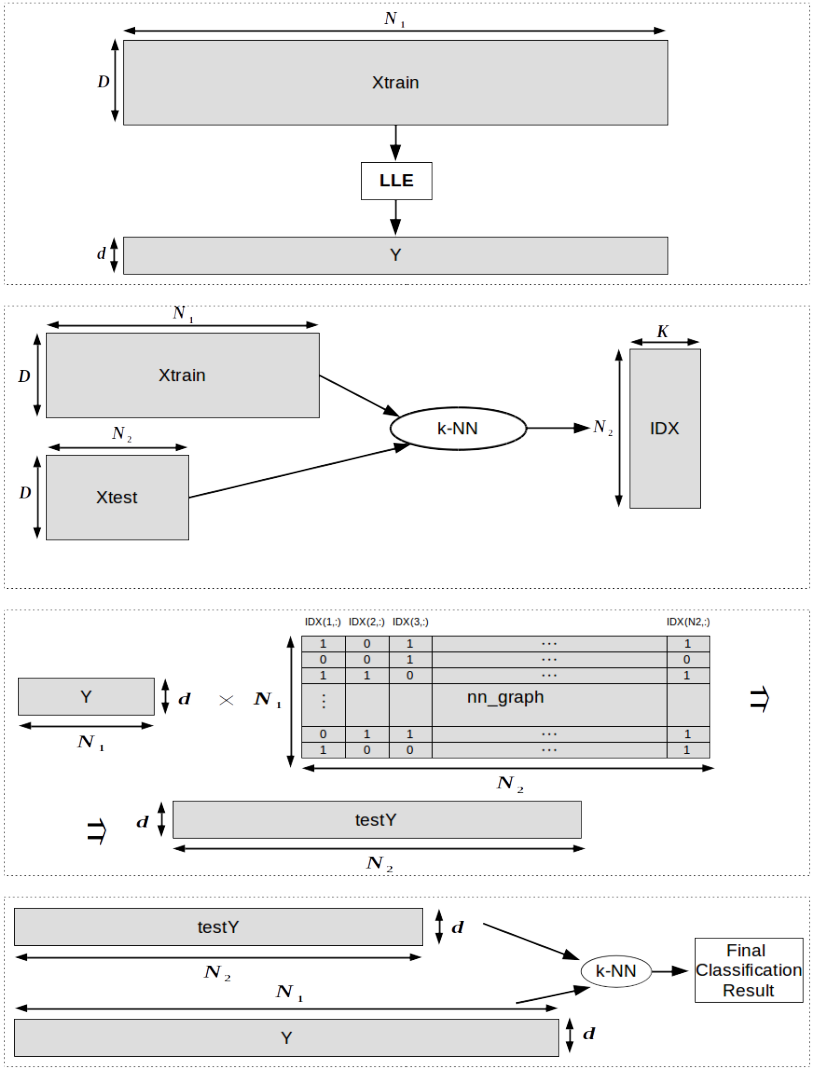
- Eigen-value decomposition step has complexity $k \cdot O(N^2)$
  - For sparse matrix M, solving with Lanczos algorithm

- We must run LLE algorithm with both train and test datasets as input data
  - Dimensionality reduction on train and separate on test produces spaces with different base vectors
  - We cannot find any realation between the low dimensional test and train data

Method-1: LLE with Test-Projection

- Apply LLE dimensionality reduction on Train dataset [DxN1]
  - Result: matrix Y of size [dxN1]
- K-NN between Test dataset [DxN2] and Train dataset
  - Result: matrix IDX of size [N2xK]
- Create the adjacency zero matrix nn_graph of size [N1xN2]
- For every column in nn_graph set IDX(column,1:K) elements equal to ones
- Execute the matrix multiplication: Y[dxN1] x nn_graph[N1xN2]
  - Result: matrix testY of size [dxN2]
- Final step, execute classification algorithm (k-NN) between dimansional reduced test (testY) and train (Y) datasets

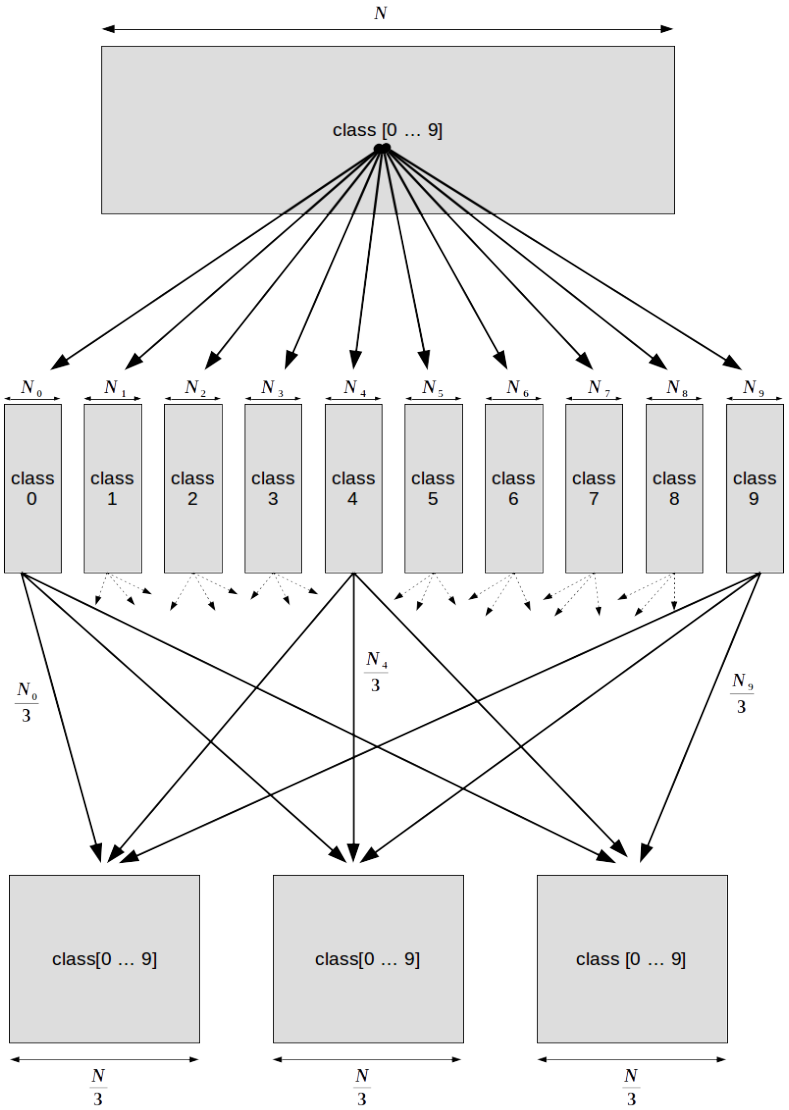
Method-1 : LLE with Test-Projection



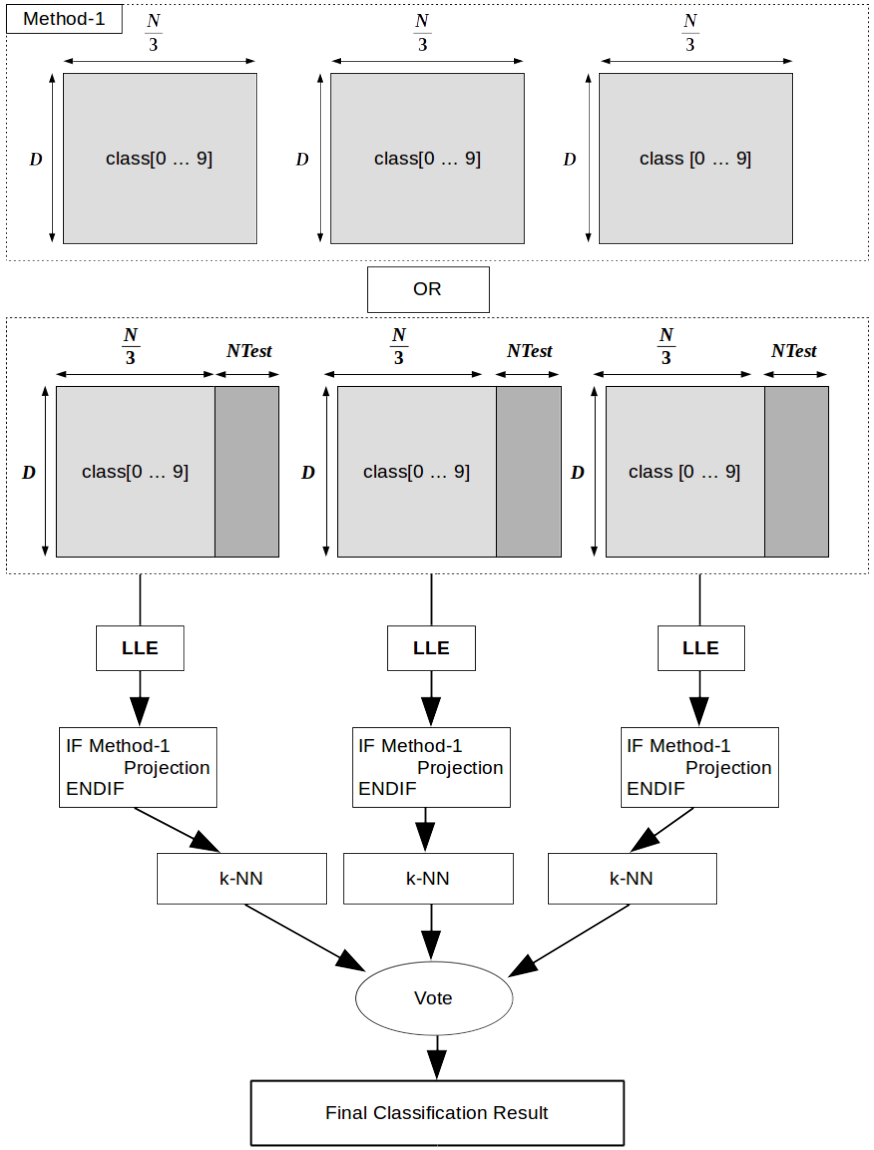
Method-2: LLE with SubSpace Voting

- Split Train dataset into sub-sets
  - Sub-sets MUST contain equal information for every class
- Execute dimensions reduction using LLE on every sub-set
  - Method-1 can be used, in order to avoid including Test data into each sub-dataset
- Execute classification process (k-NN) for every Test data
- The final classification result depends from the majority class after each sub-space voting

Method-2: LLE with SubSpace Voting



Method-2: LLE with SubSpace Voting



Experiments

- Datasets: MNIST, SVHN, ARCENE
  - MNIST: Gray scale images of handwritten digits.
    - 60K Train data, 10K Test data, D=[28x28]
  - SVHN: Google street view house numbers [32x32] RGB images.
    - 73257 Train data, 26032 Test data, D=[32x32] (531131 additional, somewhat less difficult Train data)
  - Arcene: Cancer dataset with a large number of predefined features for each patient.
    - 200 Train data, 700 Test data, D = 10K.
- Classification algorithm: k-NearestNeighbors
- Classification metric: Mean average % error

MNIST Experiments

- LLE parameters k, d. Method-2 parameter: batch_size
  - 1st-Exp: K = [6, 7, 8, 9, 10, 12, 16, 20, 24, 32, 64], d = [10, 16, 20, 24, 32, 40, 52, 64, 96, 128, 256], subSet_size=[60000,30000,20000] of Train dataset
    - Classification error using k-NN (k=2): **K=12, d=128, subSet_size=60K,** qual to **3.06%**
    - Classification error using k-NN (k=2) without dimensionality reduction: **3.5%**
    - **This is also a VERY important result: K=8, d=10, subSet_size=60K** equal to **3.31%**
    - From the above results, we can say that LLE algorithm can be used as a feature extraction process with great data compression ability.

MNIST Experiments

- Method-2 results are very impressive:
  - K=16, d=256, batch_size=20K. Best classification error equal to **3.27%.**
  - K=10, d=128, batch_size=10K. Best classification error equal to **3.31%.**
  - **Huge reduction both in time and space.** (Step-3 of LLE has $O(N^2)$ complexity)

- Method-1: Not impressive results, but this is the only realistic use of LLE dimensionality reduction in “Real time” application
  - K=8, d=256, batch_size=60K. Best classification error equal to 3.85%.

SVHN Experiments

- Extract HoG features due to noise and light distortions.
  - Split image into sub sections and calculate the gradient of pixel intensity.
- 1st-Exp: K = [8, 10, 12], d = [16, 20, 32, 64, 96, 128, 164, 196, 256], 30K of SVHN Train data-set
  - HoG features parameters: Kernels size: [2x2], [4x4], [8x8] produce features of length: 8100, 1764, 324 .
  - **Best parameters for SVHN dataset: K=12, d=32, kernel=[4x4].**
- 2nd-Exp: K=12, d=32, kernel=[4x4]. Full SVHN dataset (73257 Train data, 26032 Test data)
  - Classification error with LLE dimensionality reduction is equal to: **16.67%.**
  - Classification without dimensionality reduction (D=1024) is equal to: **17.00%.**
  - The above classification results came from k-NN Classification algorithm and k=8.

SVHN Experiments

- 3rd-Exp Method-1: LLE parameters: K=12, d=32, kernel=[4x4]. 42K of SVHN Train dataset
  - Classification error with LLE dimensionality reduction on Train data and projection for Test data is equal to 18.34%
  - Classification error with LLE dimensionality reduction on Train+Test data is equal to 16.67%
  - Classification error without dimensionality reduction is equal to 17.00%
- 4th-Exp Method-2: LLE parameters: K=12, d=32, kernel=[4x4]. 30K of SVHN Train dataset.

  Method-2 parameters: 3 subspaces
  - Classification results for each of 3 subspaces: 20.19%, 19.05%, 19.97%
  - Classification after subspace voting: **18.30%**

SVHN Experiments

- 5th-Exp Method-2: LLE parameters: K=12, d=32, kernel=[4x4]. 30K of SVHN Train dataset.

  Method-2 parameters: 5 subspaces

  - Classification results for each of 5 subspaces: 20.28%, 21.11%, 20.93%, 20.92%, 21.11%
  - Classification error after subspace voting: **18.37%**
  - Classification error for 3 subspaces (4th-Exp) is equal to **18.30%**
- 6th-Exp Method-2: LLE parameters: K=12, d=32, kernel=[4x4]. 30K of SVHN Train dataset.

  Method-2 parameters: 10 subspaces

  - Classification error after subspace voting: **18.58%**
- 7th-Exp Method-2: LLE parameters: K=12, d=32, kernel=[4x4]. 30K of SVHN Train dataset.

  Method-2 parameters: 20 subspaces

  - Classification error after subspace voting: **19.13%**

ARCENE Experiments

- 1st-Exp: K = [10, 12, 16, 20, 24, 32, 64], d = [10,16, 20, 24, 32, 40, 52, 64, 96, 128]
  - Lack of test labels. 150 Train patients and 50 Test patients and 10K features for each one.
  - For every K and d (<=96) combinations, classification error after LLE dimensionality reduction is by far better than the D=10K dimensional space
  - Best classification error for parameters (K=10, d=10), (K=10, d=52), (K=12, d=128) equal to **10%**
  - Classification error without dimensionality reduction (D=10K) equal to **76%.**
  - **With 10 of 10K features, we gain a boost of 14% classification accuracy.**
  - **Accurate classification result with 90% probability**

Future Work

- Parallelization: Even though k-NN algorithm both in LLE and in Classification process is executed in CUDA, further parallelization can be achieved from sub-set splitting into threads.
- Extend to large datasets and invest the behaviour of subspace voting
- Find the cutting edge between number of subsets-Train dataset size and Test data size
- Maybe a very accurate dimensionality reduction algorithm for Medical image retreival.

Conclusion

- LLE produces impressive results after huge dimensionality reduction.
- Huge space and time savings at Final Classification Step
- Method-1 can give as real time Classification into the low dimensional Train and Test spaces
- Method-2 achieves dramatically reduction on execution sources. Both space and time.
- Method-1 and Method-2 makes the execution of LLE algorithm available for normal PCs.
- LLE algorithm can be used both as feature extraction and feature selection process.
- LLE has the ability to remove noise-data, producing very accurate classification results

Thank You