DERIVATE

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Definizione: $f(x)$ é DERIVABILE in x_0 se il limite $\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$ esiste finito

Rapporto incrementale $\longrightarrow$ $\frac{f(x_0+h) - f(x_0)}{h}$ = $y - f(x_0) = f'(x_0)(x - x_0)$

Coefficiente angolare di retta tangente $\longrightarrow$ $\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0)$

Funzioni elementari

$f(x)$	$f'(x)$
c	0
x	1
x^α	$\alpha x^{\alpha-1}$
e^x	e^x
$\log x$	$\frac{1}{x}$
$\operatorname{sen} x$	$\cos x$
$\cos x$	$-\operatorname{sen} x$
a^x	$a^x \ln a$

$f(x)$	$f'(x)$	
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$	!
$\operatorname{cotg} x$	$\frac{1}{\operatorname{sen}^2 x}$	
$\operatorname{arcsen} x$	$\frac{1}{\sqrt{1-x^2}}$	
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$	!
$\operatorname{arccotg} x$	$-\frac{1}{1+x^2}$	

Proprietà delle derivate

- $f(x) \pm g(x) = f'(x) \pm g(x)$
- $f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
- $f(x) / g(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$
- $f(x)^{g(x)} = e^{g(x) \cdot \log f(x)} = f(x)^{g(x)} \left(g'(x) \log f(x) + g(x) \cdot \frac{f'(x)}{f(x)} \right)$
- $f(g(x)) = f'(gx) \cdot g'(x)$
- $\left(\frac{1}{f(x)} \right)' = \frac{-f'(x)}{[f(x)]^2}$
- $(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$

Hopital

- $\lim_{x \to x_0} \frac{f(x)}{g(x)} \overset{F.I.}{=} \left[\frac{0}{0}\right] \text{ o } \left[\frac{\pm\infty}{\pm\infty}\right]$

  se non esiste, allora FAIL

- $\lim_{x \to x_0} \frac{f'(x)}{g'(x)} = \ell \in \overline{\mathbb{R}}$

Taylor

con centro in 0 $\quad P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + f^{(n)}(0)$

Resto $\quad$ Differenza $f(x) - P_n(x) \longrightarrow f(x) = P_n(x) + o(x^n)$ per $x \to 0$

Sviluppi di Taylor

$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots + \frac{1}{n!}x^n + o(x^n)$ $\longrightarrow$ tutti positivi

$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots + \frac{(-1)^k}{(2k+1)!}x^{2k+1} + o(x^{2k+1})$ $\quad$ x+ dispari neg pos

$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots + \frac{(-1)^k}{(2k)!}x^{2k} + o(x^{2k})$ $\quad$ 1+ pari neg pos

$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots + (-1)^{n+1}\frac{x^n}{n} + o(x^n)$ $\quad$ no fatt, neg pos

$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2k+1})$ $\quad$ come sen, no fatt

$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots \frac{\alpha(\alpha-1)(\dots)(\alpha-n+1)}{n!}x^n + o(x^n)$

$\quad \hookrightarrow \alpha = -1 \quad \frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + o(x^n)$

con centro qualsiasi $\quad P_n(x - x_0) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2}(x - x_0)^2 + \dots + \frac{f^{n}(x_0)}{n!}(x - x_0)^n$