Determination of Dielectric Constant of different materials

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In this experiment, we aim to determine the dielectric constants of different materials using a capacitance-based method. Dielectric constant, also known as relative permittivity, play a crucial role in the behaviour of electric fields within materials. The experiment involves constructing a parallel-plate capacitor with known dimensions and inserting various dielectric materials between the plates, and studying the variation of charge accumulated as a function of external voltage.

I. THEORY

The dielectric constant, also known as the relative permittivity, plays a vital role in understanding how a material responds to an applied electric field. This property is crucial in the design and optimisation of electronic devices, capacitors, and insulating materials, making it an essential parameter in modern technology.

Electrostatic processes in vacuum can be described by Maxwell's equations as,

$$\oiint \boldsymbol{E} \cdot \boldsymbol{dA} = \frac{Q}{\varepsilon_o} \tag{1}$$

$$\oint \boldsymbol{E} \cdot \boldsymbol{dr} = 0 \tag{2}$$

where $\boldsymbol{E}$ is the electric field intensity, Q is the charge enclosed by surface A and ε_o is the permittivity of free space. For a capacitor with two parallel plates of surface area A, if a potential difference U_c is applied, an electric field $\boldsymbol{E}$ is formed between them, given by,

$$U_c = \int_1^2 \boldsymbol{E} \cdot \boldsymbol{dr} \tag{3}$$

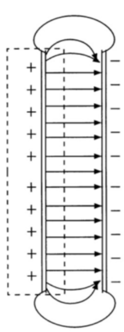

FIG. 1: Electric field lines between the capacitor plates

Due to this electric field, an equal amount of positive and negative charge is formed on both of the capacitor plates respectively. Assuming field lines between the plates to be perpendicular to the surface, using Eqn. (1) and (3),

$$\frac{Q}{\varepsilon_o} = \frac{U_c}{d} A \tag{4}$$

for small distances d. Since the charge on a capacitor is directly proportional to the potential difference between the plates, one can write,

$$Q = CU_c = \varepsilon_o \frac{U_c A}{d} \tag{5}$$

where C — the constant of proportionality — is the capacitance of the capacitor. From Eq (5), we can infer that,

$$C = \frac{\varepsilon_o A}{d} \tag{6}$$

which shows that the capacitance is inversely proportional to the distance between the plates and directly proportional to the area of the plates. These are the intrinsic properties of a particular capacitor.

Dielectric between the Capacitor Plates

Eq. (6) is derived with the assumption that there is vaccum between the plates, which has to be modified with the introduction of a dielectric (insulating material) between the plates.

Dielectrics have no free moving charge carriers, as metals have, but they do have positive nuclei and negative electrons, which can arrange itself along the lines of an applied electric field $\boldsymbol{E_0}$. Formerly non-polar molecules thus get polarized and behave as locally stationary dipoles. The effects of the single dipoles cancel each other macroscopically inside the dielectric, but not on the surface. Hence the surfaces will possess a stationary charge, called a free charge. The free charges in turn weaken the effective electric field $\boldsymbol{E}$ as given below,

$$\boldsymbol{E} = \frac{\boldsymbol{E_0}}{\varepsilon_r} \tag{7}$$

where ε_r is the relative permittivity (or dielectric constant) of the medium.

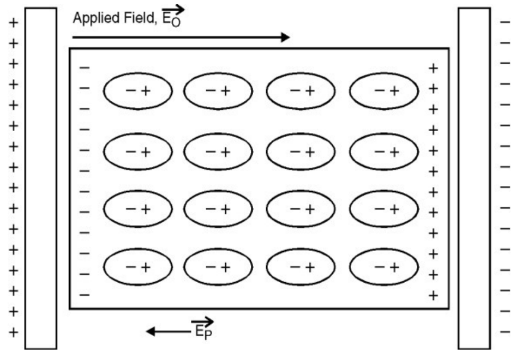

FIG. 2: Electric field lines between capacitor plates with a dielectric in between. Notice the dipoles aligned in the directtion of the external electric field.

The induced electric field $\boldsymbol{E_P}$ due to these charges will be in opposite direction to the applied electric field. If $\boldsymbol{P}$ is the polarization vector of the medium,

$$\begin{aligned}\boldsymbol{E_P} &= \boldsymbol{E_0} - \boldsymbol{E} = \boldsymbol{E_0}\left(1 - \frac{1}{\varepsilon_r}\right) \\ &= \frac{\boldsymbol{P}}{\varepsilon_o}\end{aligned} \tag{8}$$

The electric displacement vector for an isotropic medium is defined as,

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_o \varepsilon_r \boldsymbol{E} = \varepsilon_o \boldsymbol{E} + \boldsymbol{P} \tag{9}$$

where ε is the electrical permittivity of the dielectric medium. When a dielectric is inserted between the capacitor plates, according to Eq. (3), voltage U_c between the plates is reduced by the dielectric constant, ε_r, as compared to voltage in vacuum. Since the charge stored is constant, the capacitance will increase by a factor of ε_r.

$$C_{\text{dielectric}} = \varepsilon_r \varepsilon_o \frac{A}{d} \tag{10}$$

$$\implies Q = \varepsilon_r \varepsilon_o \frac{A U_c}{d} \tag{11}$$

Thus if one knows all the parameters, they can determine the dielectric constant of the medium by rearranging the above equation,

$$\varepsilon_r = \frac{d}{\varepsilon_o A} \cdot \frac{Q}{U_c} \tag{12}$$

II. EXPERIMENTAL SETUP

The schematic for the setup is given in Fig (3). Initially the plate capacitors are charged using a high voltage supply. Then the charge is transferred to a known capacitor C_{ref} (220nF). The voltage across C_{ref}, V_o is measured by the voltmeter. From this, the total stored on the capacitor is calculated by $Q = V_o C_{\text{ref}}$. Subsequently, using Eq. (12), we can find the value of ε_r for different media.

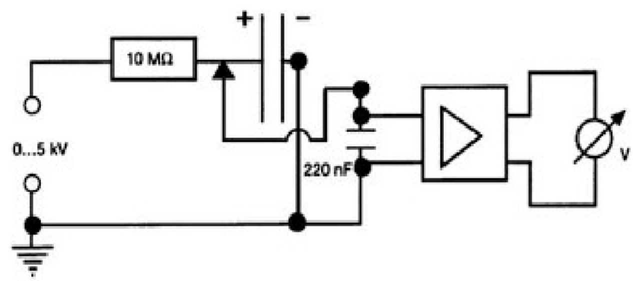

FIG. 3: Experimental setup schematic

Apparatus

1. Set of parallel plate capacitors (Diameter = 26 cm)
2. High voltage power supply (0-10 kV)
3. A 10 MΩ resistor
4. Reference capacitor (220nF)
5. Universal measuring amplifier
6. Voltmeter
7. Dielectric materials (Plastic and glass plates)
8. Connecting cables, adapters, T-connectors

III. OBSERVATIONS

Observational Data

U_c (kV)	V_o (V)	Q (nAs)
0.5	0.68	149.6
1.0	1.28	281.6
1.5	1.73	379.5
2.0	2.13	467.5
2.5	2.68	588.5
3.0	3.50	770.0
3.5	3.95	869.0
4.0	4.55	1001.0

TABLE I: Q vs U_C data with air as the dielectric medium

d (mm)	V_o (V)	Q (nAs)
1.0	3.8	836
1.5	2.6	572
2.0	2.0	440
2.5	1.5	330
3.0	1.3	286
3.5	1.1	242
4.0	1.0	220

TABLE II: Q vs d data with air as the dielectric medium

U_c (kV)	V_o (V)	Q (nAs)
0.5	0.136	30.066
1.0	0.246	54.266
1.5	0.373	82.133
2.0	0.503	110.733
2.5	0.633	139.333
3.0	0.736	162.067
3.5	0.873	192.133
4.0	0.973	214.133

TABLE III: Q vs U_c data with styrofoam as the dielectric medium

U_c (kV)	V_o (V)	Q (nAs)
0.5	1.4	308
1.0	1.9	425
1.5	2.4	542
2.0	2.9	652
2.5	3.5	770
3.0	3.9	872
3.5	4.3	946
4.0	4.6	1012

TABLE IV: Q vs U_C data with wood as the dielectric medium

Calculations

For all the calculations, we take $\varepsilon_o = 8.854 \times 10^{-12}$ F/m and $A = \pi(13)^2\text{cm}^2 = 5.31 \times 10^{-2}$ m^2

1. Measurement of ε_{air} from $Q \sim U_c$ data

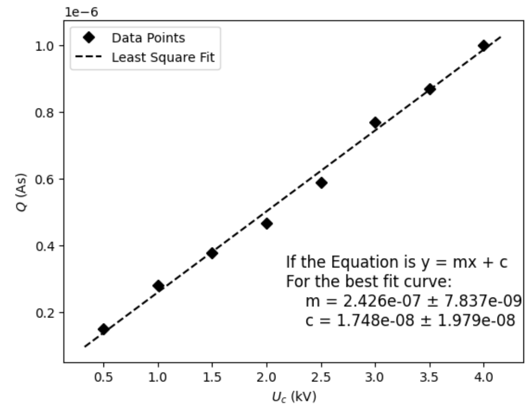

FIG. 4: Q vs U_C plot with air as the dielectric medium

From Fig. 4, slope $= 2.426 \times 10^{-10}$ As/V. From eq. (12), we get,

$$\varepsilon^a_{\text{air}} = \frac{d}{\varepsilon_o A} \cdot \text{slope} = 1.032$$

where $d = 2$ mm.

2. Measurement of ε_{air} from $Q \sim d^{-1}$ data

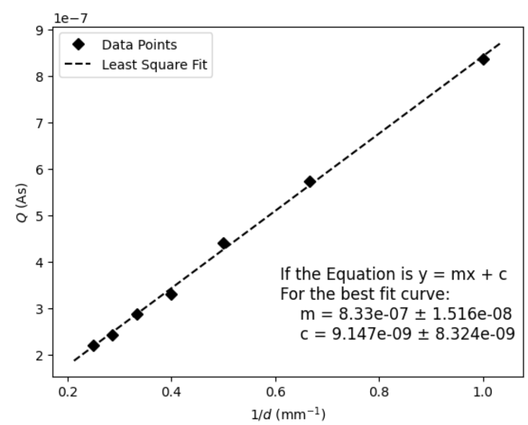

FIG. 5: Q vs d data with air as the dielectric medium

From Fig. 5, slope $= 8.33 \times 10^{-10}$ As/V. Rearranging eq. (12) to keep U_c constant and vary d, we get,

$$\varepsilon^b_{\text{air}} = \frac{\text{slope}}{\varepsilon_o A U_c} \cdot = 1.181$$

where $U_c = 1.5$ kV.
From $\varepsilon^a_{\text{air}}$ and $\varepsilon^b_{\text{air}}$, we can calculate the mean value of $\varepsilon_{\text{air}} = 1.106$.

3. Measurement of $\varepsilon_{\text{styrofoam}}$ from $Q \sim U_c$ data

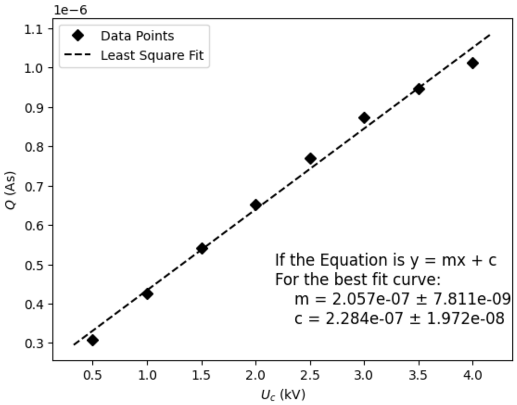

FIG. 6: Q vs U_c data with styrofoam as the dielectric medium

From Fig. 6, slope $= 5.348 \times 10^{-11}$ As/V. From eq. (12), we get,

$$\varepsilon_{\text{styrofoam}} = \frac{d}{\varepsilon_o A} \cdot \text{slope} = 2.184$$

where $d = 19.2$ mm.

4. **Measurement of $\varepsilon_{\textbf{wood}}$ from $Q \sim U_c$ data**

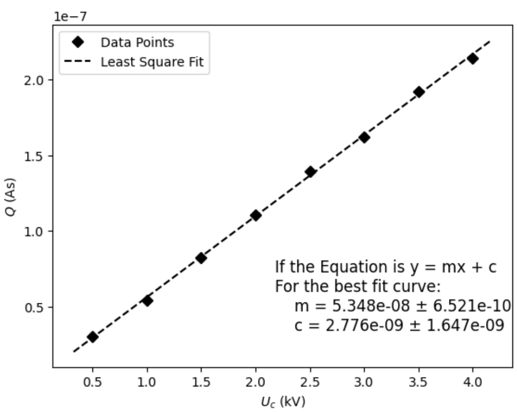

FIG. 7: Q vs U_c data with wood as the dielectric medium

From Fig. 7, $= 2.057 \times 10^{-10}$ As/V. From eq. (12), we get,

$$\varepsilon_{\text{wood}} = \frac{d}{\varepsilon_o A} \cdot \text{slope} = 3.588$$

where $d = 8.2$ mm.

IV. ERROR ANALYSIS

For a general case,

$$\varepsilon_r = \frac{d}{\varepsilon_o A} \cdot \text{slope}$$

There error in ε_r, $\Delta\varepsilon_r$ would be,

$$\frac{\Delta\varepsilon_r}{\varepsilon_r} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2} \tag{13}$$

where $\Delta d = 0.1$ mm. Similarly for

$$\varepsilon_{\text{r}} = \frac{\text{slope}}{\varepsilon_o A U_c},$$
$$\frac{\Delta\varepsilon_r}{\varepsilon_r} = \sqrt{\left(\frac{\Delta U_c}{U_c}\right)^2 + \left(\frac{\Delta \text{slope}}{\text{slope}}\right)^2} \tag{14}$$

where $\Delta U_c = 0.1$ kV.

1. **For $\Delta\varepsilon^a_{\textbf{air}}$**
From Fig. (4), Δ slope $= 0.078 \times 10^{-10}$ As/V. Using eq. (13), we get,

$$\Delta\varepsilon^a_{\text{air}} = \varepsilon^a_{\text{air}} \sqrt{\left(\frac{0.1}{2}\right)^2 + \left(\frac{0.078}{2.426}\right)^2} = 0.061$$

2. **For $\Delta\varepsilon^b_{\textbf{air}}$**
From Fig. (5), Δ slope $= 0.151 \times 10^{-11}$ As/V. Using eq. (14), we get,

$$\Delta\varepsilon^b_{\text{air}} = \varepsilon^b_{\text{air}} \sqrt{\left(\frac{0.1}{1.5}\right)^2 + \left(\frac{0.078}{2.426}\right)^2} = 0.082$$

3. **For $\Delta\varepsilon_{\textbf{air}}$**, the error in the mean of $\varepsilon^a_{\text{air}}$ and $\varepsilon^b_{\text{air}}$ can be calculated as,

$$\Delta\varepsilon_{\text{air}} = \frac{1}{2}\sqrt{(\Delta\varepsilon^a_{\text{air}})^2 + \Delta\varepsilon^b_{\text{air}})^2} = 0.051$$

4. **For $\Delta\varepsilon_{\textbf{styrofoam}}$**
From Fig. (6), Δ slope $= 0.065 \times 10^{-10}$ As/V. Using eq. (13), we get,

$$\Delta\varepsilon_{\text{styrofoam}} = \varepsilon_{\text{styrofoam}} \sqrt{\left(\frac{0.1}{19.2}\right)^2 + \left(\frac{0.065}{5.348}\right)^2} = 0.011$$

5. **For $\Delta\varepsilon_{\textbf{wood}}$**
From Fig. (4), Δ slope $= 0.078 \times 10^{-10}$ As/V. Using eq. (13), we get,

$$\Delta\varepsilon_{\text{wood}} = \varepsilon_{\text{wood}} \sqrt{\left(\frac{0.1}{8.2}\right)^2 + \left(\frac{0.078}{2.057}\right)^2} = 0.044$$

V. RESULTS

In this experiment, we have measured the relative permittivity, or the dielectric constant of different mediums by using two capacitor plates and varying the voltage or distance parameters. The values calculated are as follows:

- By varying the charge on the capacitor plates, keeping their distance constant, $\varepsilon_{\text{air}} = (1.032 \pm 0.061)$
- By varying the distance between the plates, keeping the voltage constant, $\varepsilon_{\text{air}} = (1.181 \pm 0.082)$
- Average dielectric constant of air, $\varepsilon_{\text{air}} = (1.106 \pm 0.051)$
- Dielectric constant of styrofoam, $\varepsilon_{\text{styrofoam}} = (2.184 \pm 0.051)$
- Dielectric constant of wood, $\varepsilon_{\text{wood}} = (3.588 \pm 0.011)$

VI. CONCLUSION

The literature value of the dielectric constant of air is 1 and that of styrofoam is 1.03. The dielectric constant of dry wood can range from 1.4 to 2.9. While the value of ε_{air} is quite close to 1, there is significant deviation in the other values.This deviation could be due to various reasons, majorly the moisture. Water is a polar molecule and has a high dielectric constant, which may affect our mesurement. This is especially true of wood, which can possess high moisture content. Other than that, variations in temperature, improper calibration of the measuring instruments or errors in the setupor charge leakage due to imperfect insulation of the capacitor plates could also contribute to the error.

If the dielectric constant is known, one can also of find the *electric susceptibility* of a material (χ), which is directly related to the polarizability of the molecules in the material. Hence, this helps us know more abour the internal structure of any solid.

VII. PRECAUTIONS

1. The capacitor plates must be dried at regular intervals using a blow dryer, to prevent moisture from collecting.
2. The high voltage supply must be handled carefully and should be turned off once not in use.
3. Use short cables as much as possible. Avoid loose connections.
4. The capacitor plates should not be touched when charged, to prevent any electric shock.

[1] C. Kittel. Introduction to solid state physics. *8th edition*, 2005.
[2] SPS. Lab manual. *Website*, 2023. `https://www.niser.ac.in/sps/sites/default/files/4_Dielectric%20Constant%20of%20different%20materials.pdf`.