Determination of Magnetic susceptibility of a paramagnetic material by Quincke's method

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Different materials react differently in presence of external magnetic fields. Paramagnetic materials are a group of materials which are weakly attracted by an external magnetic field, due to the microscopic structure. This property of a material can be quantified to a physical quantity known as magnetic susceptibility. In this experiment we try to measure the susceptibility of a paramagnetic liquid using mechanical phenomena, in a process called Quincke's method, along with certain other related parameters.

I. THEORY

When a material is placed within a magnetic field, the magnetic forces of the material's electrons will be affected. This effect is known as Faraday's Law of Magnetic Induction. However, materials can react quite differently to the presence of an external magnetic field, depending on the atomic and molecular structure of the material, and the net magnetic field associated with the atoms. The magnetic moment associated with atoms has 3 origins — the electron motion, the change in motion caused by an external magnetic field, and the spin of the electrons.

Most materials can be classified as diamagnetic, paramagnetic or ferromagnetic. In most atoms, electrons occur in pairs with spins in opposite directions, which cause their magnetic fields to cancel each other. These materials are **diamagnetic**. Alternately, materials with some unpaired electrons will have a net magnetic field and will react more to an external field, these are **paramagnetic**. **Ferromagnetic** materials have some unpaired electrons so their atoms have a net magnetic moment. But they exhibit a strong attraction to magnetic fields due to the presence of magnetic domains, where large numbers of atom's moments are aligned parallel so that the magnetic force within the domain is strong.

a. ***Magnetic Susceptibility*** of a material tells us how susceptible it is to becoming temporarily magnetised by an applied magnetic field and defined as the magnetization, M (magnetic moment per unit volume) produced per unit magnetic field (H).

$$\chi = \frac{M}{H} \tag{1}$$

In this experiment, we use mechanical phenomena to find the susceptibility of a paramagnetic liquid, in this case $FeCl_3$. Specifically, we use **Quincke's method**, which determines χ of the solution by observing how the liquid rises up between the two pole pieces of an electromagnet, when a direct current is passed through the electromagnet coil windings.

A. Experimental Principle

Consider a paramagnetic medium in the presence of a uniform applied flux density B_o.They will contain microscopic magnetic dipoles of dipole moment m which are randomly oriented. However, in the presence of a uniform field B, each dipole possesses a magnetic potential energy

$$U = -m \cdot B \tag{2}$$

So they all tend to align up parallel to B, which is the orientation in which their potential energy is minimum. Consequently, the liquid, containing many such dipoles, will tend to be drawn into the region of maximum field to minimize its total magnetic potential energy. In other words, the liquid experiences an attractive magnetic force F_m pulling it into the region of strongest field. The dipoles in the liquid, $FeCl_3$ solution for this experiment, are due to Fe^{3+} ions which are paramagnetic in their ground-state. The spins of several outer electrons are aligned parallel to each other to gives rise to a net magnetic moment m which is not compensated by other electrons.

A region of space permeated by a magnetic field $\boldsymbol{H}$ possesses an energy density

$$u = \frac{1}{2}\mu H^2 \tag{3}$$

where $\mu = \mu_o(1 + \chi)$ is the magnetic permeability of the medium, and μ_o is the magnetic permeability of free space. μ can be considered constant for small enough magnetic fields. The tangential component of vector $\boldsymbol{H}$ is continuous across a boundary, so the value of H in the air above the meniscus is equal to that in the liquid. This is in contrast to the flux density, where B_o in air is different from the value B in the liquid:

$$H = \frac{B_o}{\mu_o} = \frac{B}{\mu} = \frac{B}{\mu_o(1 + \chi)} \tag{4}$$

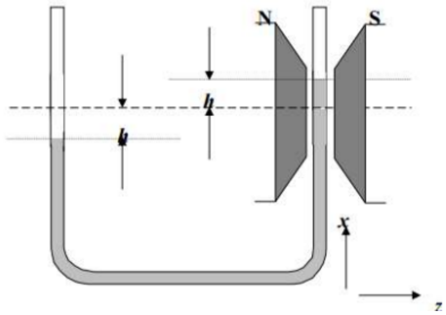

FIG. 1: Schematic of the experimental setup (Quincke's tube)

Suppose that, when the field is turned on, the meniscus in the narrow tube rises by an amount h, relative to its zero-field position. A volume $\pi r^2 h$ of air in the narrow tube is, therefore, replaced by liquid. Hence, the magnetic potential energy of this volume of space increases by

$$\Delta U = \frac{1}{2}(\mu - \mu_{\text{air}})H^2 \cdot \pi r^2 h \tag{5}$$

Which is equal to the work done by the magnetic force $\boldsymbol{F}_m$ in raising the liquid by an amount h.

$$F_m = \frac{\Delta U}{h} = \frac{1}{2}(\mu - \mu_{\text{air}})H^2 \pi r^2 \tag{6}$$

When the liquid in one arm of the tube rises by h, it falls on the other arm by h. It continues to rise till the upward magnetic force is balanced by the weight of the head of liquid. The downward gravitational force on the head of liquid, of mass m, is given by

$$F_g = mg = 2\pi r^2 \rho h g \tag{7}$$

where ρ is the density of the liquid. There is also a very small additional upwards force on the liquid due to the buoyancy of the air, displaced by the liquid, given by

$$F_b = 2\pi r^2 \rho_{\text{air}} h g \tag{8}$$

When the forces are balanced, we have $F_m + F_b = F_g$,

$$\frac{1}{2}(\mu - \mu_{\text{air}})H^2 \pi r^2 = 2\pi r^2 (\rho - \rho_{\text{air}}) h g \tag{9}$$

Substituting H from Eq.(4) in Eq.(9), we rearrange to get

$$\chi = \frac{h}{B^2}[4g\mu_o(\rho - \rho_{\text{air}})] - \chi_{\text{air}} \tag{10}$$

where χ is the total susceptibility of the solution $\chi = \chi_{\text{Fe}} + \chi_{\text{water}}$. From this, we can calculate certain other parameters like,

- mass susceptibility, $\chi' = \chi/\rho$
- molar susceptibility, $\chi'' = \chi' M$
- Curie constant, $C = \chi'' T$
- Magnetic moment of dipole of the specimen, $\mu = 2.8241\sqrt{C}$

where M = Molecular Weight, T = Temperature of the specimen.

Everything here assumes that the magnetic field acting on each ion is just the applied field B, and field and contributions due to neighboring magnetic ions are neglected. For dilute paramagnetic materials these other contributions are very small and the approximation is valid. This is not so for concentrated magnetic materials and ferromagnets.

II. EXPERIMENTAL SETUP

Quincke's tube is U-shaped glass tube (Fig.1). One arm of the tube is placed between the pole-pieces of an electromagnet shown as N-S such that the meniscus of the liquid lies symmetrically between N-S. The length of the limb is sufficient as to keep the other lower extreme end of this limb well outside the field H of the magnet. The rise or fall h is measured by means of a traveling microscope.

FIG. 2: Experimental setup

A. Apparatus

1. Adjustable electromagnet with pole pieces
2. Constant power supply (0-16 V, 5A DC)
3. Digital Gauss meter
4. Hall probe for magnetic strength measurement
5. Travelling Microscope
6. Quincke's tube (an U tube)
7. Measuring cylinder (100ml), dropper, Wash bottle
8. Specific gravity bottle
9. $FeCl_3$ for making solutions
10. Electronic balance
11. Connecting cords

III. OBSERVATIONS AND CALCULATIONS

A. Observational Data

1. Data for Calibration

I(A)	B (Gauss)
0.0	0
0.2	188
0.4	389
0.6	568
0.8	780
1.0	988
1.2	1170
1.4	1350
1.6	1580
1.8	1770
2.0	1980
2.2	2170
2.4	2360
2.6	2540
2.8	2740
3.0	2940
3.2	3130
3.4	3340
3.5	3420
3.6	3500
3.7	3600
3.8	3690
3.9	3790
4.0	3870

TABLE I: Data for calibration

2. Measurement of ρ

The $FeCl_3$ solution was prepared by adding 20g of $FeCl_3$ to 50ml of water, with a concentration of 2.37 mol/L.

- Weight of empty specific gravity bottle $(w_1) = 21.4$g
- Weight of gravity bottle filled with distilled water $(w_2) = 46.3$g
- Weight of gravity bottle filled with test liquid $(w_3) = 53.6$g

Hence,

$$\rho = \rho_{\text{water}} \frac{w_3 - w_1}{w_2 - w_1} = 1293.2\, kg/m^3$$

where $\rho_{\text{water}} = 1000\, kg/m^3$

3. Measurement of $h \sim B$

Least count of the travelling microscope = 0.001cm

I (A)	B (T)	B^2 (T^2)	Meniscus Reading (cm)	Difference 'h' (cm)
0.5	0.4925	0.2425	7.904	0.008
1.0	0.9795	0.9594	7.910	0.014
1.5	1.4665	2.1507	7.916	0.020
2.0	1.9536	3.8164	7.924	0.028
2.5	2.4406	5.9564	7.939	0.043
3.0	2.9276	8.5709	7.950	0.054
3.5	3.4146	11.6597	7.974	0.078
4.0	3.9017	15.2230	7.997	0.101

TABLE II: Rise of liquid (h) as a function of applied magnetic field (B) plot

B. Calculations

We can fit a straight line to the calibration table as follows.

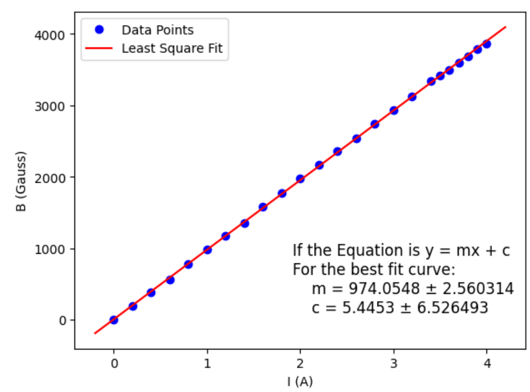

FIG. 3: Magnetic field (B) vs Current (I) from the calilbration data fitted to a straight line

From this, B for an arbitrary value of I can be found as:

$$B = (974.0548 \cdot I + 5.4453) \times 10^{-4}\, T \tag{11}$$

From Eq.(10), if we ignore the effects of χ_{air}, we can write

$$h = \frac{\chi}{4g\mu_o(\rho - \rho_{\text{air}})} B^2 \tag{12}$$

Plotting h vs. B^2, we get

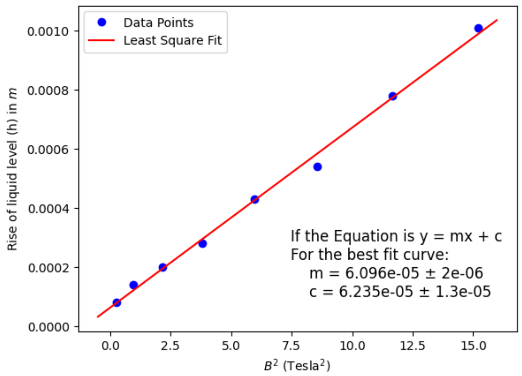

FIG. 4: h vs B^2 plot

where the slope is $m = 6.096 \times 10^{-5}$. Rearranging Eq.(12),

$$\chi = m \cdot 4g\mu_o(\rho - \rho_{\text{air}})$$

Using $g = 9.81\, m/s^2$, $\mu_o = 4\pi \times 10^{-7}\, H/m$, $\rho = 1293.2\, kg/m^3$, $\rho_{\text{air}} = 1.29\, kg/m^3$, we get

$$\chi = 3.88 \times 10^{-5} = \chi_{\text{Fe}} + \chi_{\text{water}}$$
$$\text{taking } \chi_{\text{water}} = -9.04 \times 10^{-6},$$
$$\chi_{\text{Fe}} = 4.79 \times 10^{-5}$$

is the volume susceptibility of Fe^{3+} solution. The mass susceptibility can be found by,

$$\chi'_{\text{Fe}} = \chi_{\text{Fe}}/\rho = 3.70 \times 10^{-8}\, m^2/kg$$

Since $M = 0.169$ kg/mol, the molar susceptibility is,

$$\chi''_{\text{Fe}} = \chi'_{\text{Fe}} \times M = 0.63 \times 10^{-8}\, m^3/mol$$

The Curie constant can be calculated as

$$C = \chi''_{\text{Fe}} T, \quad T = 299K$$
$$\implies C = 1.89 \times 10^{-6}\, m^3 K/mol$$

And magnetic moment (μ) of the dipole of the specimen,

$$\mu = 2.8241\sqrt{C} = 3.89 \times 10^{-3}\, A/m^2$$

IV. ERROR ANALYSIS

1. Error in ρ:

$$\Delta\rho = \sqrt{\left(\frac{\partial\rho}{\partial w_1}\Delta w_1\right)^2 + \left(\frac{\partial\rho}{\partial w_2}\Delta w_2\right)^2 + \left(\frac{\partial\rho}{\partial w_3}\Delta w_3\right)^2}$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\frac{(\Delta w_1)^2(w_3 - w_2)^2}{(w_2 - w_1)^2(w_3 - w_1)^2} + \left(\frac{\Delta w_2}{w_2 - w_1}\right)^2 + \left(\frac{\Delta w_3}{w_3 - w_1}\right)^2}$$

$$\therefore \Delta\rho = 6.7\, kg/m^3$$

2. Error in χ_{Fe}:

$$\frac{\Delta\chi_{\text{Fe}}}{\chi_{\text{Fe}}} = \sqrt{\left(\frac{\Delta\text{slope}}{\text{slope}}\right)^2 + \left(\frac{\Delta\rho}{\rho - \rho_{\text{air}}}\right)^2}$$
$$\Delta\chi_{\text{Fe}} = 0.03 \times 10^{-5}$$

3. Error in χ'_{Fe}:

$$\frac{\Delta\chi'_{\text{Fe}}}{\chi'_{\text{Fe}}} = \sqrt{\left(\frac{\Delta\chi_{\text{Fe}}}{\chi_{\text{Fe}}}\right)^2 + \left(\frac{\Delta\rho}{\rho}\right)^2}$$
$$\Delta\chi'_{\text{Fe}} = 0.03 \times 10^{-8}\, m^3/kg$$

4. Error in χ''_{Fe}:

$$\frac{\Delta\chi''_{\text{Fe}}}{\chi''_{\text{Fe}}} = \sqrt{\left(\frac{\Delta\chi'_{\text{Fe}}}{\chi'_{\text{Fe}}}\right)^2}$$
$$\Delta\chi''_{\text{Fe}} = 0.005 \times 10^{-8}\, m^3/mol$$

5. Error in C:

$$\frac{\Delta C}{C} = \sqrt{\left(\frac{\Delta\chi''_{\text{Fe}}}{\chi''_{\text{Fe}}}\right)^2 + \left(\frac{\Delta T}{T}\right)^2}$$
$$\Delta C = 0.02 \times 10^{-6}\, m^3\, K/mol$$

6. Error in μ:

$$\frac{\Delta\mu}{\mu} = \sqrt{\left(\frac{\Delta C}{2C}\right)^2}$$
$$\Delta\mu = 0.02 \times 10^{-3}\, A/m^2$$

V. RESULTS

In this experiment, using $FeCl_3$ solution of concentration 2.37 mol L^{-1}, the following parameters were determined:

- Magnetic Susceptibility of Fe,
$$\chi_{\text{Fe}} = (4.79 \pm 0.03) \times 10^{-5}$$
- Mass Susceptibility of Fe,
$$\chi'_{\text{Fe}} = (3.70 \pm 0.03) \times 10^{-8}\, m^3/kg$$
- Molar Susceptibility of Fe,
$$\chi''_{\text{Fe}} = (0.63 \pm 0.005) \times 10^{-8}\, m^3/mol$$
- Curie constant,
$$C = (1.89 \pm 0.02) \times 10^{-6}\, m^3\, K/mol$$
- Magnetic moment of the dipoles,
$$\mu = (3.89 \pm 0.02) \times 10^{-3}\, A/m^2$$

VI. CONCLUSION

The literature value of molar susceptibility of Fe is 1.69×10^{-8} m^3/mol. Hence there is a -62.7% deviation of the actual value from the expected value. This could be due to various reasons, such as error in calibration or instrumental error.

Here χ comes out to be positive, since $FeCl_3$ is a paramagnetic material. In this case, the magnetic field in the material is strengthened by the induced magnetization. Alternatively, if χ is negative, the material is diamagnetic. In this case, the magnetic field in the material is weakened by the induced magnetization.

VII. PRECAUTIONS AND SOURCES OF ERROR

1. Increase the current slowly and carefully. Make sure to limit the current at 4A, or the instrument might heat up.
2. To avoid backlash error, try to move the travelling microscope in only one direction
3. The circuit should be connected properly and must be verified before switching on

[1] Aranyo Mitra Shubham Dutta. Determination of magnetic susceptibility by quincke's method. *Website*, 2013. `https://www.researchgate.net/profile/Shubham_Dutta3/publication/308983019_Determination_of_Magnetic_Susceptibility_by_Quincke's_Method/links/580e47de08aef766ef10e2dc/Determination-of-Magnetic-Susceptibility-by-Quinckes-Method.pdf`.

[2] SPS. Lab manual. *Website*, 2023. `https://www.niser.ac.in/sps/sites/default/files/3_Magnetic%20susceptibility%20of%20a%20paramagnetic%20material%20by%20Quincke's%20method.pdf`.