Study of Series LCR Resonant Circuit

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In this experiment, we study the behaviour of a series LCR circuit by measuring voltage across different junctions against different input frequncies. Hence we estimate the resonant frequency of the ciruit — frequency at which the output voltage attains its maximum value. We also determine the Quality factor of the circuit, which represents the efficiency and selectivity of the circuit at resonance. This study contributes to our understanding of LCR circuits and their application in modern electronics.

I. THEORY

Let an alternating voltage $V_i = V_o \sin \omega t$ be applied to an inductor L, a resistor R and a capacitor C all in series as shown in the circuit diagram. If I is the instantaneous current flowing through the circuit, then the applied voltage is given by,

$$\begin{aligned} \boldsymbol{V_i} &= \boldsymbol{V_{R_{dc}}} + \boldsymbol{V}_L + \boldsymbol{V_C} \\ &= I\left[R_{dc} + j\left(\omega L - \frac{1}{\omega C}\right)\right] \end{aligned} \tag{1}$$

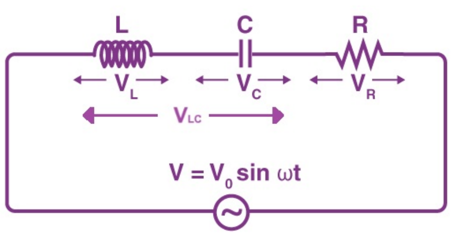

FIG. 1: Circuit diagram for the setup.

Here, R_{dc} is the total d.c. resistance of the circuit that includes the resistance of the pure resistor, inductor and the internal resistance of the source, since the resistance of the inductor and source are not negligible as compared to the load resistance.

Also, $j\omega C$ and $1/j\omega C$ are the capacitive and inductive reactances of the capacitor and inductor respectively.

One can form a second order differential equation to describe the behaviour of a LCR circuit,

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_o \sin \omega t \tag{2}$$

By solving Eqn. (2) we can find the total impedance of the circuit,

$$I_o = \frac{V_o}{\boldsymbol{Z}} \implies \boldsymbol{Z} = \left[R_{dc} + j\left(\omega L - \frac{1}{\omega C}\right)\right] \tag{3}$$

Where its magnitude and phase are given by,

$$|\boldsymbol{Z}| = \left[R_{dc}^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2} \tag{4}$$

$$\tan \phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R_{dc}} \tag{5}$$

Where ϕ is the phase difference between the voltage across the source and the current in the circuit. From Eq. (5), we can arrive at 3 possible cases:

Case I: $\omega L > 1/\omega C$, tan ϕ is positive and applied voltage leads current by phase angle ϕ

Case II: $\omega L < 1/\omega C$, tan ϕ is negative and applied voltage lags current by phase angle ϕ

Case III: $\omega L = 1/\omega C$, tan ϕ is zero and applied voltage and current are in phase. Here, since $V_L = V_C$, the impedance, Z is minimum, which means that the circuit is purely resistive. Also, current I is maximum and V_{LC} is minimum. This condition is known as **resonance**.

If ω_o is the frequency at which resonance occurs,

$$\begin{aligned} \omega_o L = 1/\omega_o C &\implies \omega_o = \frac{1}{\sqrt{LC}} \\ \text{or, } f_o &= \frac{1}{2\pi\sqrt{LC}} \end{aligned} \tag{6}$$

At resonant frequency, since the impedance is minimum, hence frequencies near f_o are passed more readily than the other frequencies by the circuit. Due to this reason LCR series circuit is called **acceptor circuit**. The band of frequencies which is allowed to pass readily is called **pass-band**.

Conventionally, the band is chosen to be the range of frequencies between which the current is equal to or greater than $I_o/\sqrt{2}$. If f_1 and f_2 are the limiting values of frequency as defined, then the width of the band (**bandwidth**) is $f_2 - f_1$.

The **selectivity** of a tuned circuit is its ability to select a signal at the resonant frequency and reject other signals that are close to this frequency. A measure of the selectivity is the quality factor (Q), which is defined as,

$$Q = \frac{f_o}{f_2 - f_1} \quad (7)$$

$$= \frac{\omega_o L}{R_{dc}} = \frac{1}{R_{dc}\omega_o C} \quad (8)$$

In this experiment, we will probe the magnitude and phase of V_R, V_{LC}, V_L, V_C in the vicinity of the resonant frequency of a LCR circuit. The working formulae for these quantities are,

$$\left|\frac{V_R}{V_i}\right| = \frac{R_{dc}}{|Z|} \text{ and, } \phi_R = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R_{dc}}\right) \quad (9)$$

$$\left|\frac{V_{LC}}{V_i}\right| = \frac{\omega L - \frac{1}{\omega C}}{|Z|}$$

$$\text{and, } \phi_{LC} = \tan^{-1}\left(\frac{R_{dc}}{\omega L - \frac{1}{\omega C}}\right) \quad (10)$$

$$\left|\frac{V_L}{V_i}\right| = \frac{\omega L}{|Z|} \quad (11)$$

$$\left|\frac{V_C}{V_i}\right| = \frac{1/\omega C}{|Z|} \quad (12)$$

Applications

Radio receivers, television sets, and oscillator circuits use LCR circuits for tuning purposes. These circuits mainly deal with the communication system and signal processing. The series LCR is used for voltage magnification. They are also used in induction heating.

II. EXPERIMENTAL SETUP

Circuit components

1. Inductor
2. Capacitor
3. Resistors
4. Function generator
5. Oscilloscope
6. Multimeter/LCR meter
7. Connecting wires
8. Breadboard

Circuit Diagram

Given in Fig. 1.

III. DATA ANALYSIS AND CALCULATIONS

- $L = 952.4\,\mu$H
- $C = 75.41\,$nF
- $f_o = \frac{1}{2\pi\sqrt{LC}} = 18.78$ kHz
- Internal resistance of inductor $= 11.8\,\Omega$
- Output impedance of Function generator $= 50\,\Omega$
- $R_1 = 98.4\,\Omega$ and $R_2 = 46.1\,\Omega$

A. V_R/V_i vs frequency plot

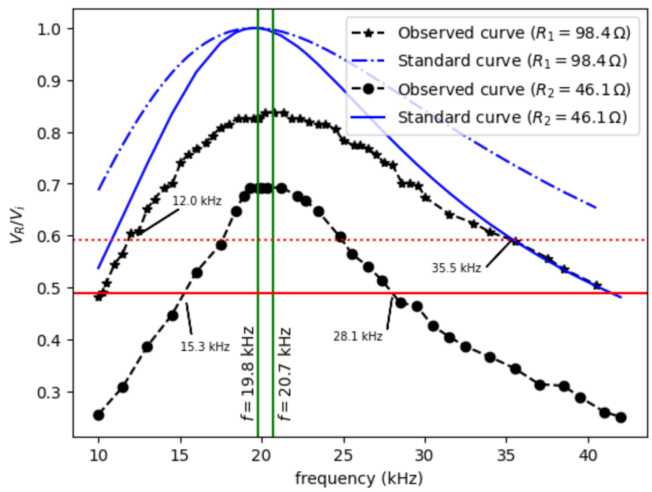

FIG. 2: Observed and computed values of V_R/V_i vs input frequency plotted for two values of resistances

From Fig. 2 we can graphically estimate the resonant frequency of the circuit (as when the curve reaches the maxima):

- f_o (when $R_1 = 98.4\,\Omega$) = 20.7 kHz
- f_o (when $R_2 = 46.1\,\Omega$) = 19.8 kHz

Hence the average value of resonant frequency would be $f_o = 20.05$ kHz.

Q-factor for both resistances can be calculated using Eqn. (7),

- Q_1 (for R_1) = 0.773
- Q_2 (for R_2) = 1.098

Q-factor can also be estimated from the plot by finding f_2 and f_1 as the values where V_R/V_i becomes $1/\sqrt{2} = 0.707$ of the maximum value.

- For R_1, these points come out to be $f_1 = 12.0$ kHz and $f_2 = 35.5$ kHz. Hence using Eqn. (8), we can calculate Q factor to be $Q_1 = 0.881$.
- For R_2, $f_1 = 15.3$ kHz and $f_2 = 28.1$ kHz. Hence using Eqn. (8), $Q_2 = 1.547$.

Here we can see that for both methods, Q-factor of R_1 is lower that that of R_2, which means resistive energy loss will be higher in the first circuit (with higer R_{dc}). Higher Q-factor means better selectivity and thus a narrower bandwidth.

B. ϕ_R vs frequency plot

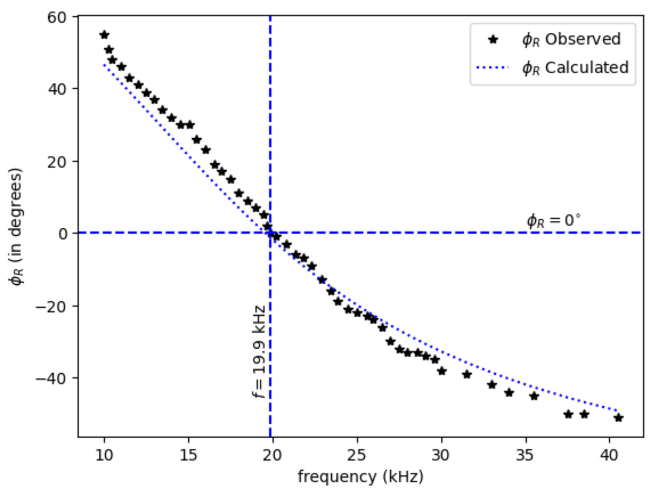

FIG. 3: Observed and computed values of ϕ_R vs input frequency plotted for $R_1 = 98.4\,\Omega$

At resonance, $\phi_R = 0$ and hence we can est the resonance frequency as $f_o = 19.9$ kHz

C. V_{LC}/V_i and ϕ_{LC} vs frequency plots

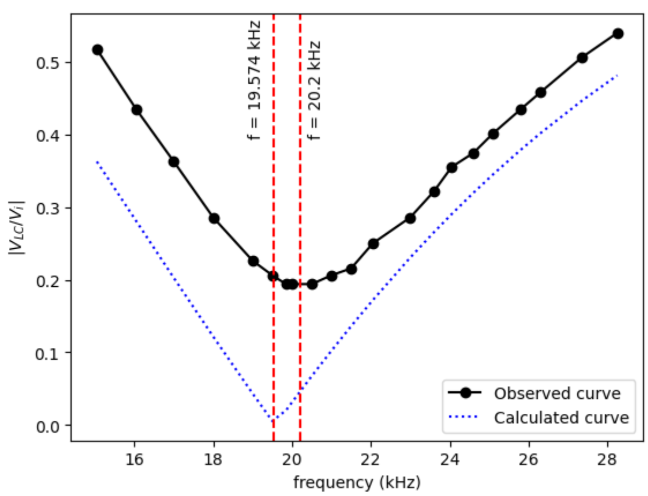

FIG. 4: Observed and computed values of V_{LC}/V_i plotted against input frequency $(R = R_1)$

At resonance, $|V_{LC}/V_i|$ should be minimum (ideally 0). From the observed values as shown in Fig. 4, we can estimate the resonance frequency to be $f_o = 20.2$ kHz.

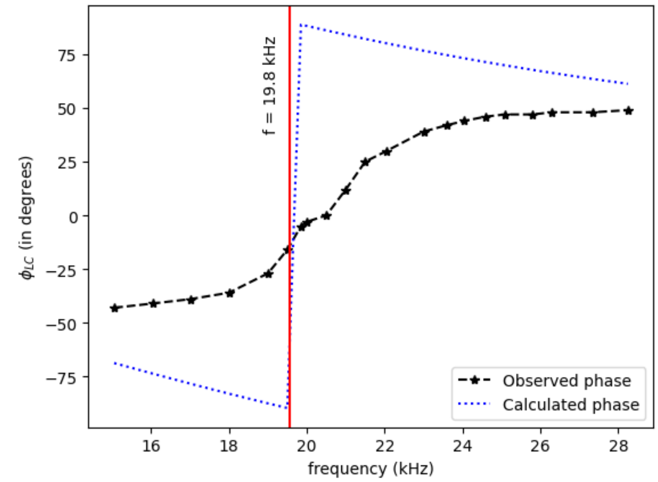

FIG. 5: Observed and computed values of ϕ_{LC}/V_i plotted against input frequency $(R = R_1)$

At resonance, $\phi_{LC} = 90°$, ie. the slope of ϕ_{LC} vs f curve changes drastically. Hence from Fig. 5, we can estimate the resonance frequency to be $f_o = 19.8$ kHz.

D. V_C/V_i and V_L/V_i vs frequency plots

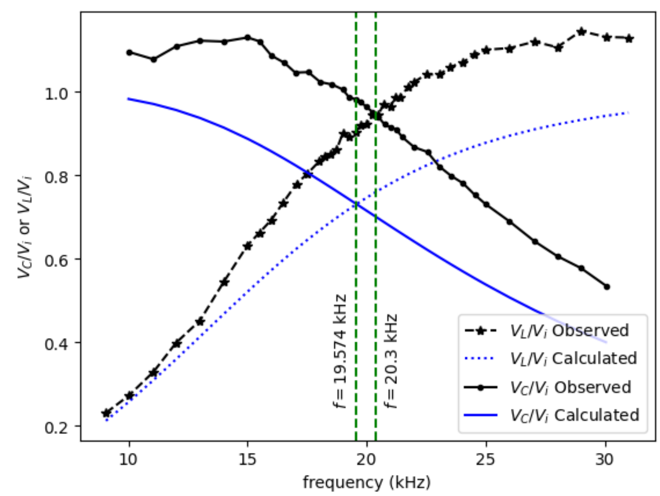

FIG. 6: Observed and computed values of V_L/V_i and V_L/V_i plotted against input frequency $(R = R_1)$

At resonance, $V_L = V_C$. Hence, the point of intersection if the two curves in Fig. 6 indicates resonance. For the observed values, the corresponding resonance frequency comes out to be $f_o = 20.30$ kHz.

IV. ERROR ANALYSIS

From Eqn. (6), uncertainity in f_o would be,

$$
\begin{aligned}
\Delta f_o &= f_o\sqrt{\left(\frac{\Delta L}{2L}\right)^2 + \left(\frac{\Delta C}{2C}\right)^2} \\
&= 19.574\cdot\sqrt{\left(\frac{0.1}{2\cdot 952.4}\right)^2 + \left(\frac{0.01}{2\cdot 75.41}\right)^2} \\
&= 0.001
\end{aligned}
$$

And, from Eqn. (8), uncertainity in Q will be,

$$
\Delta Q = Q\sqrt{\left(\frac{\Delta f_o}{f_o}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta R_{dc}}{R_{dc}}\right)^2}
$$
$$
\implies \Delta Q_1 = 0.0005 \text{ and } \Delta Q_1 = 0.003
$$

where R_{dc} is the sum of resistance of the resistor, inductor and the output impedance of the function generator.

V. RESULTS AND DISCUSSION

We have studied the properties of a series LCR circuit by measuring voltage across different junctions against various input frequencies. The calculated values of resonance frequency of the circuit is (19.574 ± 0.001) kHz.

At resonance, we have found out that,

- V_R/V_i is maximum, i.e. the circuit is purely resistive. And therefore $\phi_R = 0°$. (Hence, $f_o = 19.78$ kHz).
- V_{LC} is minimum and ϕ_{LC} is close to 90°. (Hence, $f_o = 20.00$ kHz).
- since the capacitor attenuates high frequency signal and the inductor attenuates low frequency signal, at resonance $V_L = V_C$. (Hence, $f_o = 20.30$ kHz).

The observed values of the resonance frequency from the plots obtained closely match the calculated value, and the average observed value of the resonance frequency is $f_o = 20.03$.

We have also found out the Quality factor of the circuit, which comes out to be $Q = (0.773 \pm 0.0005)$ when $R = 98.4\,\Omega$ and $Q = (1.098 \pm 0.003)$ when $R = 46.2\,\Omega$. Graphically, the corresponding values of Q-factor are 0.881 and 1.547 respectively.

So, one can conclude that resistive energy loss will be lower in the second cicuit, and it will have better selectivity (narrow bandwidth or a sharper resonance).

VI. PRECAUTIONS

1. Make the ground connections carefully.
2. Take readings only when the values on the oscilloscope stabilize.
3. Do not change any component of the circuit while it is switched on.

[1] SPS. Lab manual. *Website*, 2023. `https://www.niser.ac.in/sps/sites/default/files/5-study%20of%20lcr%20resonant%20circuit.pdf`.