Bayesian Epistemology

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1 Introduction

This article will discuss Bayesian confirmation theory and make a case for it. It goes over a famous example in Bayesian confirmation theory which is Eddington's experiment, followed by a philosophical justification for Bayesian confirmation theory via the dutch book arguments, and then a discussion on the problems of induction.

2 Case Study: Eddington's experiment

British astrophysicist Arthur Eddington proposed an experiment to test Einstein's theory of general relativity, the experiment involved observing the bending of starlight by the gravitational field of the sun during a solar eclipse. According to Einstein's theory of general relativity, the gravity of the sun would make light follow geodesic lines such that they would appear to be bent when their origin point is from behind the sun.

Eddington's experiment was not only a test of Einstein's theory but also a test of the competing theory of Newtonian gravity. According to Newtonian gravity, starlight should not bend as it passes near the sun. Eddington's experiment could therefore be crucial in the confirmation or disconfirmation of their respective theories depending on the results of the experiment.

The confirmation and disconfirmation of the theories T_N, T_E (for Newtonian and Einstenian respectively) based on some empirical consequence E can be modelled as a probability update $P(T_E|E) > P(T_E)$. And the priors can be set very high for Newtonian mechanics, since it had been so well tested up until that point (with some exceptions like Mercury's orbit) so $P(T_E)$ is set to very low and $P(T_N)$ is set to very high.

But how should we update our probabilities after the experiment, depending on the results? For Bayesian confirmation theory, this relies on two main norms:

Probabilism, which are roughly the three axioms of classical probability theory:

$$P(C) \geq 0$$

$$P(C_1) + \ldots + P(C_n) = 1$$
$$P(C_1 \cup C_2) = P(C_1) + P(C_2)$$

And conditionalisation: If evidence E is found, then every theory incompatible with E is set to probability 0 (or something very low).

One can then define conditional probability as

$$P(E|T) = P(E \cap T)/P(E)$$

With the above definition one can derive Bayes's theorem.

$$P(T|E) = P(E|T)P(T)/P(E)$$

Which now gives us a procedure to update our beliefs in theories T given evidence E.

In the case of Eddington's experiment, the probability of the evidence given Einstein's theory $P(E|T_E)$ was high, and the prior probability of the theory $P(T_E)$ was low. Therefore, the posterior probability of Einstein's theory was much higher than the prior probability, while the posterior probability of Newtonian gravity was much lower than the prior probability. This is what allowed Eddington's experiment to strongly confirm Einstein's theory over Newtonian gravity. Conditionalisation had set Newtons theory to something ver low since it's incompatible with the evidence. [1]

3 Justifying Bayesian confirmation theory

But how do you justify probabilism and conditionalism? One way is with the Dutch Book arguments.

The dutch book arguments generally argue that our credence on our beliefs regard our desire to bet money (or some other cost) that a certain belief is true. If someone claims to have a credence of say, $P(X) = 0.4$, then we should expect that person to bet \$40 where the win is greater than \$100 since this would yield an higher expected value. This class of arguments are powerful, since they would reveal that some agent's belief set is inconsistent if their betting would be inconsistent. [2]

However this presupposes a certain type of rationality which claims that one ought to bet on every bet with positive expected return. An objection to this view is whether or not an individual could be risk-averse, like for example having large anxiety regarding placing bets on beliefs regardless of conviction. Someone may be strongly convinced of a certain belief yet still decides not to bet even a little bit on it. This has led some Bayesian's to revise their theory in pursuit of non-pragmatic foundations for dutch-book arguments.

4 Problems of induction

Bayesian epistemology is argued to have solved Hume's problem of induction.[1] By consistently updating our belief over and over again, we can eventually approach probability 1 of our beliefs over time. While we cannot logically derive a universal quantified statement from this finite set of observations, our credence can go arbitrarily high.

But can Bayesian confirmation theory handle Raven's Paradox? As an example, assume we are doing a field experiment by trying to determine the color of ravens.

Then $P(\textbf{ All ravens are black } | N+1 \textbf{ previous ravens were black }) >$ $P(\textbf{ All ravens are black } | N \textbf{ previous ravens were black })$

Which is very nice. However there is a problem. Depending on how priors were set previously, the updating can sometimes increase and sometimes decrease. This is known as the problem of the priors.

There is one other problem with the Bayesian approach is that it, atleast in the given formulation, can't solve the new riddle of induction. Assume an emerald is grue, that is to say, green until 2050 and blue all time after that. Then whenever we update our credence for H_1 that the emerald is grue and H_2 that it is green, then $P(H_1|E) = P(H_2|E)$ since the same evidence confirmed and updates their beliefs. This means they still make the same prediction

Future developments in this field or similar fields will either need to deal with the riddle of induction by getting rid of it, or accepting that certain classes of predictions cannot be made with this model.

5 conclusion

In general, I believe that Bayesian confirmation theory to be a good model for credence and belief updating. It has some hurdles to overcome if it wants to be absolutely dominant, and replace other types of confirmation such as AGN and HD methods, as-well as replacing classical interpretations of statistics within science can be a tough hurdle to overcome.

References

[1] Hanti Lin. "Bayesian Epistemology". In: *Stanford Encyclopedia of philosophy* (2022).

[2] Susan Vineberg. "Dutch Book Arguments". In: *Stanford Encyclopedia of philosophy* (2022).