Case Study 1 AKSTA Statistical Computing

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Task 1

a) Using for loop

We will add the check if the given number is positive integer and equals at least 2.

Then we create numeric vector filled with NA of length n + 1 to store numbers of Fibonacci sequence there and initialize first two Fibonacci numbers.

In the case n is 2, we return ratio of 2 first numbers, else we calculate in a loop all numbers of sequence up to n + 1 inclusively.

We initialize numeric vector of length n to store ratios and calculate in a loop final results.

```
for_impl_fib_ratio <- function(n) {
  if (!is.integer(n) || n < 2) {
    stop("n must be a positive integer greater than or equal to 2")
  }

  fib <- numeric(n + 1)
  fib[1:2] <- 1

  for (i in 3:(n + 1)) {
    fib[i] <- fib[i - 1] + fib[i - 2]
  }

  r <- numeric(n)
  for (i in 1:n) {
    r[i] <- fib[i + 1] / fib[i]
  }

  return(r)
}

print(for_impl_fib_ratio(4L))
```

```
## [1] 1.000000 2.000000 1.500000 1.666667
```

a) Using while loop

Rewriting the function to use while loop by setting conditions for 2 while loops and updating i variable in each loop.

```r
while_impl_fib_ratio <- function(n) {
  if (!is.integer(n) || n < 2) {
    stop("n must be a positive integer greater than or equal to 2")
  }

  fib <- numeric(n + 1)
  fib[1:2] <- 1

  if (n == 2) {
    return(fib[2] / fib[1])
  }

  i <- 3
  while(i <= (n + 1)) {
    fib[i] <- fib[i - 1] + fib[i - 2]
    i <- i + 1
  }

  r <- numeric(n)

  i <- 1
  while(i <= n) {
    r[i] <- fib[i + 1] / fib[i]
    i <- i + 1
  }

  return(r)
}

print(for_impl_fib_ratio(4L))
```

```
## [1] 1.000000 2.000000 1.500000 1.666667
```

b) Using microbenchmark function

It was decided to run evaluation for each function 1000 times

```r
library(microbenchmark)
```

```
## Warning: package 'microbenchmark' was built under R version 4.3.3
```

```r
benchmark_200 <- microbenchmark(
  for_loop = for_impl_fib_ratio(200L),
  while_loop = while_impl_fib_ratio(200L),
  times = 1000L
)

benchmark_2000 <- microbenchmark(
  for_loop = for_impl_fib_ratio(2000L),
  while_loop = while_impl_fib_ratio(2000L),
  times = 1000L
```

```
)

print(benchmark_200)
```

```
## Unit: microseconds
##        expr  min   lq     mean median     uq     max neval
##    for_loop 55.7 57.5  85.8560  61.30 116.10   354.9  1000
##  while_loop 79.4 81.4 144.4874  85.65 168.55 21675.1  1000
```

```
print(benchmark_2000)
```

```
## Unit: microseconds
##        expr   min     lq     mean median      uq     max neval
##    for_loop 441.9 503.45 658.7190  543.5  707.15 11658.9  1000
##  while_loop 645.8 724.35 952.4394  780.0 1092.55  2792.8  1000
```

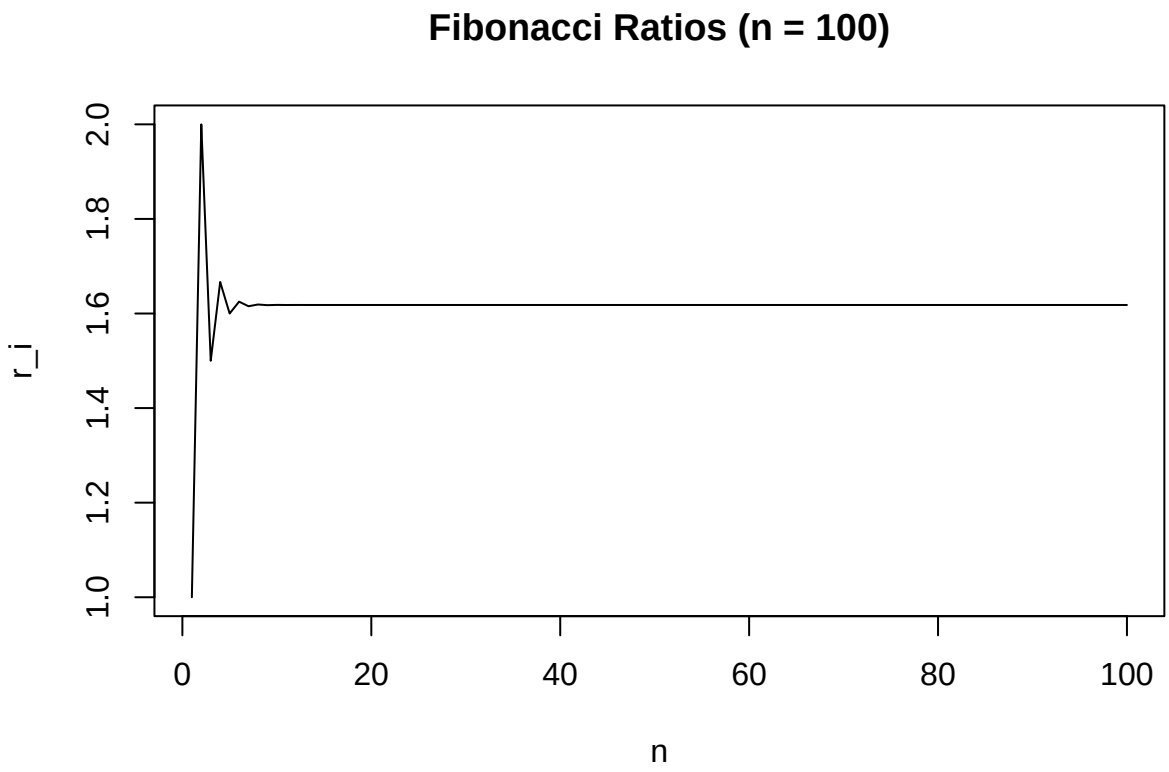
Conclusion: we can summarize that for loop is more efficient, while the mean and median execution time are lower than these indicators for while loop. It may be because in for loop, we don't need to initialize counter variable and update it manually. However, we noticed, that in case n = 2000 the max execution time of for loop is greater than while loop's time. We think, that potential reason might be some external factors, for example system load or CPU usage. But mean and median are more indicative for execution time distribution analysis, so we conclude the better efficiency of for loop.

c)

We will plot sequence for n = 100 to see, where is stabilizes

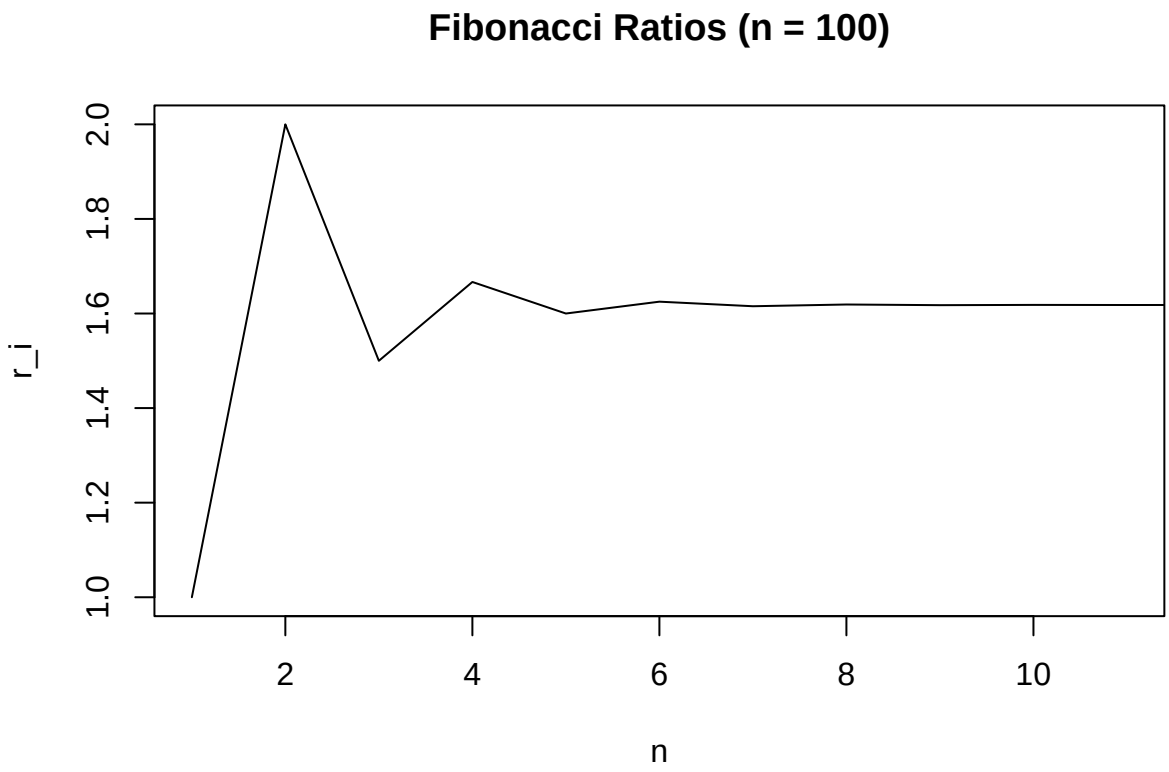
```
# Call the function for n = 100
seq <- for_impl_fib_ratio(100L)

# Plot the sequence
plot(seq, type = "l", xlab = "n", ylab = "r_i", main = "Fibonacci Ratios (n = 100)")
```



We see, that sequence stabilizes roughly at n = 10, we can zoom in this sector of the plot to detect the preciser number.

```
plot(seq, type = "l", xlab = "n", ylab = "r_i", main = "Fibonacci Ratios (n = 100)", xlim = c(1, 11))
```



As we can see, ratios converge to the value ≈ 1.6, starting from the n ≈ 5 and stabilizing more from n ≈ 8.

Task 3

a) Using recursion

First attempt to implement the function with the usage of recursion wasn't successful, because for n values greater than 100, the execution time turned out to be hours. Due to that, we decided to use memoization technique to optimize our code.

We define the helper function inside of main function, which is recursive and calculates the Fibonacci numbers with memoization using the environment. We define base cases for n $= 0$ (calculate ϕ^1) and n $= 1$ (calculate ϕ^2), which is $\frac{\sqrt{5}+1}{2}$ and $(\frac{\sqrt{5}+1}{2})^2 = \frac{\sqrt{3}+1}{2}$ correspondingly. Then we calculate recursively all following element of the sequence. Also, we check if n is integer that equals at least to 1.

Our both functions will return $\phi^(n+1)$ (if n $= 1$, we calculate ϕ^2).

```
recursion_golden_ratio <- function(n = 1000L) {
  if (!is.integer(n) || n < 0) {
    stop("n must be a non-negative integer")
  }

  memo_env <- new.env(parent = emptyenv())

  compute_golden_ratio <- function(n, memo) {
    if (n == 0) {
      return((sqrt(5) + 1) / 2)
```

```
    } else if (n == 1) {
      return((sqrt(5) + 3) / 2)
    }

    if (!is.null(memo[[as.character(n)]])) {
      return(memo[[as.character(n)]])
    }

    res <- compute_golden_ratio(n - 1L, memo) + compute_golden_ratio(n - 2L, memo)
    memo[[as.character(n)]] <- res

    return(res)
  }

  compute_golden_ratio(n, memo_env)
}

recursion_golden_ratio(100L)
```

```
## [1] 1.281598e+21
```

b) Using power operator

Add the same check condition and simply return power n + 1 of ϕ

```
power_golden_ratio <- function(n) {
  if (!is.integer(n) || n < 0) {
    stop("n must be a non-negative integer")
  }

  phi <- (sqrt(5) + 1) / 2
  return(phi ^ (n + 1L))
}

power_golden_ratio(100L)
```

```
## [1] 1.281598e+21
```

c)

We will use sapply function to apply defined functions to 4 specified values and store them in vectors.

While comparing values with ==, we create logical vector, where TRUE indicates that the values are exactly equal, FALSE otherwise.

To compare values with all.equal, we use the sapply again to iterate over each pair of recursion_vals and power_vals and apply all.equal() for comparison.

```
recursion_vals <- sapply(as.integer(c(12, 60, 120, 300)), recursion_golden_ratio)
power_vals <- sapply(as.integer(c(12, 60, 120, 300)), power_golden_ratio)

comparison_eq <- recursion_vals == power_vals
```

```r
comparison_all_equal <- sapply(seq_along(recursion_vals), function(i) {
  all.equal(recursion_vals[i], power_vals[i])
})

cat("Comparison with == operator:\n", comparison_eq, "\n")
```

```
## Comparison with == operator:
##  FALSE FALSE FALSE FALSE
```

```r
cat("Comparison with all.equal function:\n", comparison_all_equal, "\n")
```

```
## Comparison with all.equal function:
##  TRUE TRUE TRUE TRUE
```

In the result, we observe that comparison with == outputs inequality, while comparison with all.equal - visa versa. The reason is likely due to floating-point precision. Since == operator performs an exact comparison, small differences in representation of numbers may be influence the output. On the other hand, the all.equal compares numeric vectors with some tolerance to differences in floating-point values.