Representing Additive Models as Mixed Models

Notations Cheat Sheet

Truncated Power Basis

$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + ... + \theta_d x^d + \sum_{k=1}^{K} \theta_{dk} (x - \kappa_k)_+^d$$

θ_l	coefficients
κ_k	kth knot
K	number of knots

Additive Models

$$y = \sum_{j=1}^{p} V_j \xi_j + U\gamma$$

p	number of covariates
V_j	design matrices for non-linear effects
ξ_j	non-linear parameters
U	design matrix for linear effects
γ	linear parameter vectors
$b_k(.)$	kth basis function
λ_j	roughness penalty
K_j	penalty matrix

Mixed Models

$$y_i = \underbrace{X_i \beta}_{\text{fixed effects}} + \overbrace{Z_i b_i}^{\text{random effects}} + \epsilon_i$$

β_j	parameter for the fixed effects
b_j	parameter for the random effects
X	design matrix for the fixed effects
Z	design matrix for the random effects

Representation

$$\xi_j = \tilde{X}_j \beta_j + \tilde{Z}_j b_j$$

Σ_j	covariance matrix of prior
τ_j^2	variance of prior
β_j	non-penalized part of ξ_j
b_j	penalized part of ξ_j

Inference

W	diagonal matrix of σ^2
Q	block diagonal matrix of $\tau_j^2 I_{kj}$