

# Representing Additive Models as Mixed Models

## Notations Cheat Sheet

### Truncated Power Basis

$$y = \theta_0 + \theta_1 x + \theta_2 x^2 + \dots + \theta_d x^d + \sum_{k=1}^K \theta_{dk} (x - \kappa_k)_+^d$$

$\theta_l$	coefficients
$\kappa_k$	$k$ th knot
$K$	number of knots

### Additive Models

$$y = \sum_{j=1}^p V_j \xi_j + U \gamma$$

$p$	number of covariates
$V_j$	design matrices for non-linear effects
$\xi_j$	non-linear parameters
$U$	design matrix for linear effects
$\gamma$	linear parameter vectors
$b_k(\cdot)$	basis function for $k$ th knot
$\lambda_j$	roughness penalty
$K_j$	penalty matrix

### Mixed Models

$$y_i = \underbrace{X_i \beta}_{\text{fixed effects}} + \underbrace{\sum_{i=1}^K Z_i b_i}_{\text{random effects}} + \epsilon_i$$

$\beta_j$	parameter for the fixed effects
$b_j$	parameter for the random effects
$X$	design matrix for the fixed effects
$Z$	design matrix for the random effects

### Representation

$$\xi_j = \tilde{X}_j \beta_j + \tilde{Z}_j b_j$$

$\Sigma_j$	covariance matrix of prior
$\tau_j^2$	variance of prior
$\beta_j$	non-penalized part of $\xi_j$
$b_j$	penalized part of $\xi_j$

### Inference

$W$	diagonal matrix of $\sigma^2$
$Q$	block diagonal matrix of $\tau_j^2 I_{K_j}$