Exercise 7 – Algorithms for Convex Problems

In folder for this seminar you will find a script for generating three convex problems (`cv8.m`). Your job is to write an algorithm to solve each of these. If you want, you can utilize the line search routines (`golden_section_search.m` and `bracket_minimum.m`) that are also in the folder (these routines were slightly modified to handle the logarithmic barriers).

- Problem 1 is equality constrained entropy maximization (or minimization of the negative netropy), i.e., finding the optimal $x \in \mathbb{R}^n$ such that

  $$\begin{aligned} \underset{x}{\text{minimize}} \quad & \sum_{i=1}^{n} x_i \log(x_i) \\ \text{subject to} \quad & Ax = b. \end{aligned}$$

  For this problem, write the infeasible start Newton's method with, starting from the (infeasible) $x^{(0)}$ supplied in the script. Set both primal and dual feasibility tolerances to $\varepsilon_1 = \varepsilon_2 =$ 1e-6. Find the biggest n, for which you can solve the problem in 1 second (on your machine).

- Problem 2 is linear programming problem in inequality form, i.e., finding the optimal $x \in \mathbb{R}^n$ such that

  $$\begin{aligned} \underset{x}{\text{minimize}} \quad & c^T x \\ \text{subject to} \quad & A_{\text{ineq}} x \leq b_{\text{ineq}}, \end{aligned}$$

  where $A \in \mathbb{R}^{m \times n}$. For this problem, write the barrier method, starting from the (feasible) $x^{(0)}$ supplied in the script. Set the starting $t = 0.1$, $\beta = 20$, and tolerance $\varepsilon =$ 1e-3. Find the biggest pair n, m (where $m = 3n$), for which you can solve the problem in 1 second (on your machine).

- Problem 3 is quadratic programming problem in inequality form (this particular one is a support vector machine, but more on that will be said in the seminar), i.e., finding the optimal $x \in \mathbb{R}^n$ such that

  $$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{2} x^T P x + f^T x \\ \text{subject to} \quad & A_{\text{ineq}} x \leq b_{\text{ineq}}. \end{aligned}$$

  For this problem, write the barrier method, starting from the (feasible) $x^{(0)}$ supplied in the script. Set the starting $t = 0.1$, $\beta = 20$, and tolerance $\varepsilon =$ 1e-3. Find the biggest n, for which you can solve the problem in 1 second (on your machine).