Example – Weighted MAX-SAT with EC methods

The Boolean satisfiability problem (SAT) is the problem of deciding, given a Boolean expression in variables $x_1, \ldots, x_n$, whether some assignment of the variables makes the expression true. SAT is historically notable because it was the first problem proven to be NP-complete. (Before this point, the idea of NP-completeness had been formulated, but no one had proven that there actually existed any NP-complete problems.)

We will consider not arbitrary Boolean expressions but only expressions in conjunctive normal form (CNF), i.e. of the form

$$((A_{11} \vee A_{12} \vee \ldots) \wedge (A_{21} \vee A_{22} \vee \ldots) \wedge \ldots)$$

where each literal A_{ij} is either a single variable or its negation, and each clause does not contain more than one literal associated with a single variable. An example with $x_1, \ldots, x_4$ might look like this:

$$(\neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2) \wedge (x_1 \vee \neg x_3) \wedge (x_2 \vee x_4),$$

which is satisfied for $x = [0, 0, 0, 1]$.

In the MAX-SAT variation of SAT, we do not require that all the disjunctive clauses be satisfied. Instead, we want find an assignment which maximizes the number of satisfied clauses. For example, the CNF formula:

$$(x_1 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2)$$

is not satisfiable: no matter which truth values are assigned to its two variables, at least one of its four clauses will be false. However, it is possible to assign truth values in such a way as to make three out of four clauses true; indeed, every truth assignment will do this.

Weighted MAX-SAT extends MAX-SAT by assigning a weight (or cost) to each clause. Each clause C_i has an associated weight $w_i > 0$. The goal is to find a truth assignment that maximizes the sum of weights of satisfied clauses:

$$\max \sum_{i:\text{Clause } C_i \text{ is satisfied}} w_i.$$

Suppose we have the CNF:

$$(w_1)(x_1 \vee x_2), \quad (w_2)(\neg x_1 \vee x_3), \quad (w_3)(\neg x_2)$$

with weights $w_1 = 2$, $w_2 = 5$, $w_3 = 1$. Then, the assignment $x = [0, 0, 1]$ would get an objective (weighted sum of satisfied clauses) of 6.

In the file `ex11.mat` you will find an instance of weighted MAX-SAT with 20 binary variables. The best possible (globally optimal) objective value for this instance is 1035 (with all clauses satisfied, you can verify this by brute-forcing all 2^{20} solutions).

Assignment: Implement a simple Hill-Climbing method and the (μ, λ) and $(\mu + \lambda)$ Evolution Strategy for this problem. Try to find the values for μ and λ that achieve the best performance with a budget of 10^4 evaluations of the objective function.