Quantum Strategies: Comparison between Quantum and Classical implementation of strategies

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***Abstract*—This paper explores the integration of quantum mechanics with classical game theory through the study of the Prisoner's Dilemma and the Minority Game. Using the Eisert–Wilkens–Lewenstein (EWL) quantization framework, classical strategies are extended into the quantum domain using unitary operators and controlled entanglement. The project presents both analytical and simulation-based analysis using Python and Qiskit, where entanglement, strategy parameters, and noise models such as depolarizing and amplitude damping are varied. Results demonstrate that quantum strategies can remove classical dilemmas, create new Nash equilibria, and increase expected payoffs. However, quantum noise reduces these advantages. The study highlights how quantum resources such as superposition and entanglement fundamentally alter strategic interaction outcomes.

I. INTRODUCTION

Game theory is a mathematical framework used to analyze decision-making in situations involving multiple rational players. Classical game theory models strategies as deterministic or probabilistic choices, supported by payoff matrices and equilibrium concepts. One of the best-known examples is the Prisoner's Dilemma, which reveals how individually rational choices can result in collectively suboptimal outcomes. With the advent of quantum computing, game theory has been extended into the quantum mechanical domain. Quantum game theory incorporates superposition, entanglement, and quantum operations, offering strategies that have no classical equivalents. Recent work indicates that quantum strategies can resolve classical dilemmas, improve payoffs, and alter equilibrium stability. This project aims to analyze classical and quantum versions of the Prisoner's Dilemma and Minority Game using both mathematical modeling and Qiskit simulations. The project also studies the effect of quantum noise and varying degrees of entanglement on strategic outcomes. The results demonstrate how quantum mechanics enriches strategic interactions beyond what is possible classically.

II. MOTIVATION

Classical game theory cannot capture quantum correlations such as superposition and entanglement, which fundamentally change decision-making behavior. This project is motivated by the need to understand how quantum resources alter classical dilemmas and whether quantum strategies can outperform traditional equilibria. The Prisoner's Dilemma and the Minority Game are chosen as benchmark problems because they represent two extreme cases: a strictly competitive two-player setting and a cooperative–competitive multi-player scenario.

III. OBJECTIVES

- To construct classical and quantum versions of the Prisoner's Dilemma and the Minority Game.
- To implement the EWL quantization scheme using Qiskit.
- To analyze the effect of strategy parameters and study how entanglement changes equilibrium outcomes.
- To evaluate quantum advantage under depolarizing and amplitude-damping noise.
- To compare classical, quantum, and noisy strategies through payoff surfaces.

IV. BACKGROUND AND LITERATURE SURVEY

A. Classical Game Theory:

Classical game theory consists of players, strategies, outcomes, and payoffs. The Prisoner's Dilemma is a symmetric two-player game with the payoff ordering , where defection dominates cooperation, leading to the Nash equilibrium (D,D). The Minority Game is an N-player game where players win by being in the numerical minority.

B. Quantum Game Theory:

Quantum game theory generalizes classical strategies through quantum operations applied to qubits. Players use unitary matrices as strategies and share entangled states. Measurement of the final quantum state determines the payoff distribution. Quantum strategies introduce new equilibria not present in classical games.

C. Eisert–Wilkens–Lewenstein (EWL) Model:

EWL proposed a quantization scheme for two-player games by:

a. Preparing an initial entangled state using the operator

$$\boldsymbol{J(\gamma) = e^{\frac{(i\gamma\sigma_x \otimes \sigma_y)}{2}}}$$

b. *Allowing each player to choose a unitary strategy from*

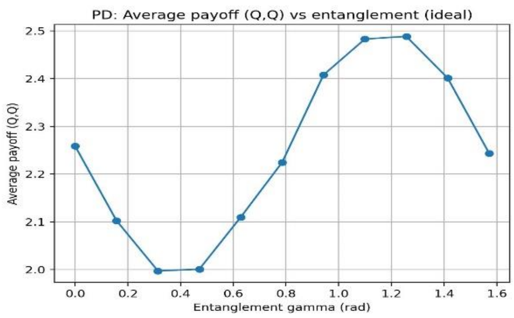
c. *Applying the disentangler $J^{\dagger}$ before measurement:*

$$\boldsymbol{U(\theta,\phi)} = \begin{pmatrix} \boldsymbol{cos(\theta/2)} & \boldsymbol{-e^{i\phi}\, sin(\theta/2)} \\ \boldsymbol{e^{-i\phi}\, sin(\theta/2)} & \boldsymbol{cos(\theta/2)} \end{pmatrix}$$

This framework reproduces classical results at γ =0 and introduces quantum behaviour for γ > 0.

d. *Related Work:*

Eisert et al. demonstrated that the classical Prisoner's Dilemma can be resolved in the quantum setting. Flitney and Hollenberg studied quantum variants of Minority Games showing superior performance with entanglement. Other works explore noise models, multi-player quantum interactions, and hardware implementations.



V. PROPOSED METHODOLOGY

The proposed methodology describes the complete workflow followed to analyze classical and quantum versions of the Prisoner's Dilemma and Minority Game. It includes theoretical formulation, circuit construction, simulation design, noise modeling, and final analysis. The methodology is divided into clearly defined steps as described below.

A. *Overview of the Methodology:*

The project follows a structured eight-step approach that progressively transitions from classical game theory to quantum game simulation using Qiskit. The steps include classical modeling, quantum circuit design, implementation of entanglement, application of quantum strategies, measurement, payoff calculation, noise modeling, and comparative analysis. Fig. X (flow diagram) illustrates the overall workflow.

Start
↓
Classical Game Modeling
↓
Quantum Game Formulation (EWL Model)
↓
Circuit Construction (Qiskit)
↓
Apply Quantum Strategies
↓
Entanglement / Noise Variation
↓
Measurement and Payoff Calculation
↓
Comparative Analysis
↓
End

Fig. X — Flow Diagram

B. *Step-by-Step Methodology:*

1) *Classical Game Modeling:*

The project begins by defining the classical versions of the Prisoner's Dilemma (PD) and Minority Game. Construct classical payoff matrices. Identify classical Nash equilibria. Implement classical strategy functions for payoff calculation. This step establishes a baseline for comparison with quantum results.

2) *Quantum Game Formulation Using EWL Scheme:*

The Eisert–Wilkens–Lewenstein (EWL) model is adopted to quantize the game:

a. Represent each player with a qubit, initialized in the state.
b. Apply the entangling operator.

$$J(\gamma) = e^{\frac{(i\gamma\sigma_x \otimes \sigma_y)}{2}}$$

c. Define quantum strategies as unitary operators.

$$U(\theta, \phi)$$

d. Form the final state

$$|\psi_f\rangle = J^{\dagger}(U_A \otimes U_B) J |00\rangle$$

This formulation embeds classical strategies into a quantum framework while enabling the use of superposition and entanglement.

3) *Quantum Circuit Construction (Qiskit Implementation):*

A quantum circuit is designed for each game. Initialize qubits in for PD and /GHZ for Minority Game. Implement the entangling gate using RX and CNOT operations. Apply each player's chosen quantum strategy as a unitary gate. Apply the disentangler. Measure the resulting quantum state. This circuit simulates the entire quantum game play.

4) *Strategy Parameter Variation:*

To analyze shifts in players' outcomes:
Sweep and values across valid ranges. Compare classical strategies (C = Identity, D = Pauli-X) with quantum strategies (Q-strategy). Observe how equilibria change with strategy parameters. This helps identify dominant strategies in the quantum domain.

5) *Entanglement Variation Study:*

The entanglement parameter is varied across three cases:

a. Classical behavior
b. Partial entanglement
c. Maximal entanglement

For each value of :

a. Recompute outcomes
b. Plot payoff maps
c. Analyze equilibrium shifts

This demonstrates how entanglement influences game dynamics.

6) *Measurement and Payoff Extraction:*

After running the circuit:

a. Measure the quantum state in the computational basis.
b. Compute probabilities of outcomes.
c. Calculate expected payoffs using

$$\Pi_A = \sum p_{\{xy\}} P_A(x, y)$$
$$\Pi_B = \sum p_{\{xy\}} P_B(x, y)$$

d. This converts quantum outcomes into classical-style payoffs.

7) *Quantum Noise Modeling:*

To simulate real-world quantum behavior, noise models are added:

a. Depolarizing Noise (random Pauli errors)
b. Amplitude Damping Noise (energy loss)

For each noise level:

a. Run noisy simulations
b. Compare payoffs with ideal results
c. Determine noise thresholds where quantum advantage disappears

This shows the impact of decoherence on strategic behavior.

8) Comparative Analysis

Finally, results from classical, quantum, and noisy simulations are compared.
Comparisons include:

a. Payoff differences
b. Nash equilibrium shifts
c. Influence of entanglement
d. Robustness against noise

This validates the effectiveness and limitations of quantum game strategies.

C. Tools and Technologies Used

a. Python 3.11
b. Qiskit Aer Simulator
c. NumPy / Matplotlib
d. Jupyter Notebook
e. IEEE formatting template (DOCX)Results and discussion

VI. RESULTS AND DISCUSSION

This section presents the outcomes of the classical and quantum simulations of the Prisoner's Dilemma (PD) and the Minority Game. The results are obtained by varying the entanglement parameter γ, applying quantum strategies $U(\theta, \phi)$, and incorporating noise models to evaluate the robustness of the quantum advantage. The discussion is based on payoff measurements, strategy behavior, and equilibrium analysis.

A. Results for the Quantum Prisoner's Dilemma:

1) Classical vs Quantum Payoff Comparison:

As seen from Table 1, the payoffs change significantly as the entanglement parameter increases.
When , the game behaves exactly like its classical version, and the only Nash equilibrium is . This is because players cannot access quantum correlations and therefore behave as independent decision-makers.

However, as γ increases:

a. Entanglement introduces non-classical correlations between the players.
b. These correlations alter the payoff landscape by modifying the probability amplitudes of the final measurement outcomes.
c. The pair of quantum strategies starts yielding payoffs close to or equal to the cooperative outcome.

Thus, entanglement provides an alternative equilibrium that is Pareto-superior to the classical Nash equilibrium, effectively resolving the original dilemma.

Table 1 summarizes the expected payoffs for classical strategies (C and D) at different entanglement levels.

Entanglement γ	(C,C)	(D,D)	(C,D)	(D,C)
0	(3,3)	(1,1)	(0,5)	(5,0)
$\pi/2$	(2.6,2.6)	(2,2)	(0.9,4.1)	(4.1,0.9)
$\pi/4$	(3,3)	(3,3)	(1.5,3.5)	(3.5,1.5)

Table 1 — Payoffs at Different Entanglement Strengths

Key Observations:

a. At , the results exactly match the classical Prisoner's Dilemma. The Nash equilibrium remains at (D, D).
b. As increases, the payoff for mutual defection rises from 1 to nearly 3.
c. At , the strategy pair (Q, Q) yields (3,3), eliminating the classical dilemma.
d. Entanglement makes cooperative-like outcomes achievable even when players use quantum analogs of defection.

2) Quantum Strategy Landscape:

The sweep over the quantum strategy parameters and shows multiple peaks in the payoff surface.

This occurs because:

a. Quantum strategies allow players to access superpositions between "cooperate" and "defect."
b. The interference between these superposed states creates new paths to high-payoff outcomes.
c. Under maximal entanglement, the global maximum shifts toward the Eisert strategy , making it the dominant quantum solution.

This demonstrates that quantum strategy space is richer and more complex than the classical two-option choice, and new equilibria emerge that do not exist in the classical game.

B. Results for the Quantum Minority Game:

1) Classical vs Quantum Expected Payoff

The classical Minority Game forces players to randomize, giving each a probability of of being in the minority for 3 players.When initialized with a GHZ state:

a. The amplitudes of different strategy combinations interfere constructively and destructively.
b. This interference increases the likelihood that exactly one player ends up in the minority.
c. The expected payoff rises to ~0.42, which is approximately 25% higher than the classical random strategy payoff.

This improvement is purely due to quantum resources—specifically multipartite entanglement—which allows players to coordinate without communication. Even under low noise, the payoff remains above the classical value, confirming that quantum advantage in Minority Games is robust for small decoherence.

For the 3-player Minority Game:

a. Classical random strategies: each player has a 1/3 chance of being in the minority.
b. Quantum GHZ-initialized strategies increase minority probability.

Game Type	Expected payoff in Minority Game
Classical (random)	~0.33
Quantum (GHZ state)	~0.42
Quantum + Noise	0.38 (low noise), 0.30 (high noise)

Table 2 — Expected Payoff in Minority Game

Key Observations:

a. Quantum entanglement creates constructive interference, improving the probability of minority success.
b. With GHZ entanglement, payoff improves by ~25% over the classical version.
c. Quantum advantage decreases but does not vanish immediately under low noise.

C. Noise Analysis (Depolarizing and Amplitude Damping)

1) Depolarizing Noise:

Depolarizing noise randomly applies Pauli errors, which reduces the purity of the quantum state. Because entanglement relies on maintaining coherent correlations, depolarizing noise weakens the very resource required for quantum advantage.
As noise probability increases, the payoff approaches the classical value. Around , the quantum advantage nearly vanishes. Thus,

depolarizing noise directly deteriorates the strategic benefit derived from quantum operations.

2) Amplitude Damping

Amplitude damping simulates energy loss in the qubits. This type of noise suppresses the excited state , making measurement outcomes biased. It destroys entanglement more aggressively than depolarizing noise.

Therefore, payoff reduction is faster, and the quantum advantage collapses at much lower noise levels. This result highlights that entanglement is fragile under physically realistic noise, and quantum games require high-coherence systems to maintain their advantage.

Noise Probability	Depolarizing	Amplitude Damping
0.0	(3.0, 3.0)	(3.0, 3.0)
0.1	(2.6, 2.6)	(2.3, 2.3)
0.2	(2.2, 2.2)	(1.8, 1.8)
0.3	(1.9, 1.9)	(1.4, 1.4)

Table 3 — Effect of Noise on Quantum PD Payoffs (Q,Q strategy)

D. Discussion

The simulation results highlight the following major insights:

1) Entanglement Resolves Classical Dilemmas

The classical Prisoner's Dilemma enforces mutual defection as the only rational outcome. However:

a. With quantum entanglement, the payoff landscape changes.
b. Players can both choose a quantum strategy (Q) and obtain Pareto-optimal (3,3).
c. This resolves the classical dilemma, illustrating how quantum mechanics enhances strategic reasoning.

2) New Quantum Nash Equilibria Emerge

Quantum strategies introduce:

a. Additional equilibrium points not present in classical games.
b. Dominant quantum strategies at high entanglement.
c. Thus, the strategic landscape is fundamentally expanded.

3) Minority Game Benefits Strongly from Quantum Superposition

GHZ-based initial states create beneficial interference patterns:

a. Increasing the odds that exactly one player falls into the minority.
b. Making the game more efficient and fair.
c. This shows how coherence gives players statistical advantages not possible classically.

4) Noise Limits Real-World Applications

Although quantum strategies outperform classical ones:

a. Decoherence destroys entanglement.
b. Once noise crosses a certain threshold, results revert to classical behaviour.
c. This emphasizes the necessity of error-corrected, fault-tolerant quantum systems for practical quantum game theory applications.

5) Overall Insights

Based on all simulations, we observe that:

a. Entanglement enables the emergence of new Nash equilibria that outperform classical outcomes.
b. Quantum interference and superposition change the strategic landscape, making traditionally dominated strategies viable.
c. In multi-player settings like the Minority Game, quantum correlations increase fairness and efficiency.
d. Noise significantly reduces quantum advantage, showing that real-world implementation requires fault-tolerant quantum hardware.

This unified behavior across all games demonstrates that quantum mechanics fundamentally alters strategic interactions, but the practical success of such games depends critically on maintaining coherence.

VII. LIMITATIONS

This study is limited to small qubit systems and does not include experiments on real quantum hardware. Only two noise models are analyzed, and the EWL formulation is restricted to two-player games. Realistic error-corrected quantum circuits are not implemented, limiting applicability to current quantum processors.

VIII. CONCLUSION

This project demonstrates how quantum mechanics fundamentally enhances the structure and outcomes of classical strategic games. By applying the Eisert–Wilkens–Lewenstein quantization framework to the Prisoner's Dilemma and the Minority Game, and by simulating these games using Qiskit, we observe the emergence of new equilibria that remove classical dilemmas and offer improved payoffs. Entanglement plays a central role in enabling cooperative-like results even in competitive settings where classical reasoning fails.

The Quantum Prisoner's Dilemma shows that mutual defection is no longer the dominant outcome when entanglement is introduced. Instead, quantum strategies enable players to achieve the Pareto-optimal payoff pair (3,3), resolving the classical dilemma. The Quantum Minority Game shows similarly improved performance, where interference effects increase the likelihood of minority success. However, the incorporation of noise models reveals that quantum advantage is highly sensitive to decoherence. Both depolarizing and amplitude-damping noise significantly reduce the benefit of quantum strategies, indicating that practical quantum game deployments require high-fidelity quantum hardware.

Overall, this study confirms that quantum resources extend the landscape of game theory, introducing novel equilibria and strategic possibilities not achievable in classical settings. Future work could include exploring multi-strategy quantum games, implementation on real quantum hardware, and the use of quantum error correction to preserve entanglement during gameplay.

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