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Tele-Experiment

Kinematics of Mobile Robots

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1 Simplified Car-Like Robot Model

Proving the relationship between the steering angle of the virtual wheel φ and the steering angles of the two front wheels φ_l and φ_r.

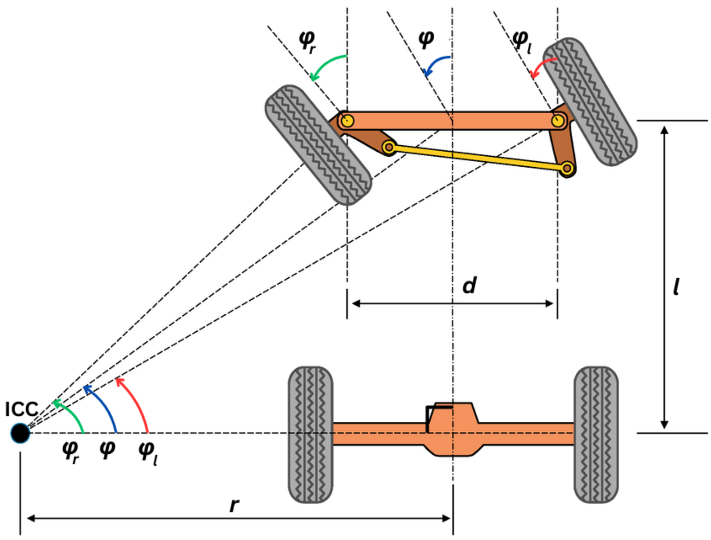


Figure 1.1: Simplified Car-Like Robot Model.

Since the track width d, the wheel base l and the radius r from the Instaneous Ceter of Curvature ICC forms a right angle triangle, the steering angles can be written as:

$$\tan \varphi = \frac{l}{r}$$

$$\tan \varphi_r = \frac{l}{(r + \frac{d}{2})}$$

$$\tan \varphi_l = \frac{l}{(r - \frac{d}{2})}$$

The equation for right steering angle can be rearranged by isolating for radius r.

$$\tan\varphi_r = \frac{l}{(r + \frac{d}{2})}$$

$$r + \frac{d}{2} = \frac{l}{\tan\varphi_r}$$

$$r = \frac{l}{\tan\varphi_r} - \frac{d}{2}$$

Substituting the radius into equation for steering angle of the virtual wheel,

$$\tan\varphi = \frac{l}{\frac{l}{\tan\varphi_r} - \frac{d}{2}}$$

Dividing numerator and denominator of right hand side by l,

$$\tan\varphi = \frac{1}{\frac{1}{\tan\varphi_r} - \frac{d}{2l}}$$

$$\tan\varphi = \frac{1}{\left(\frac{2l - d\cdot\tan\varphi_r}{2l\cdot\tan\varphi_r}\right)}$$

$$\tan\varphi = \frac{2l\cdot\tan\varphi_r}{2l - d\cdot\tan\varphi_r}$$

Dividing numerator and denominator of right hand side by $2l$,

$$\tan\varphi = \frac{\tan\varphi_r}{1 - \frac{d}{2l}\cdot\tan\varphi_r}$$

$$\therefore \varphi = \arctan\left(\frac{\tan\varphi_r}{1 - \frac{d}{2l}\cdot\tan\varphi_r}\right)$$

The equation for left steering angle can be rearranged by isolating for radius r.

$$\tan\varphi_l = \frac{l}{(r - \frac{d}{2})}$$

$$r - \frac{d}{2} = \frac{l}{\tan\varphi_l}$$

$$r = \frac{l}{\tan\varphi_l} + \frac{d}{2}$$

Substituting the radius into equation for steering angle of the virtual wheel,

$$\tan\varphi = \frac{l}{\frac{l}{\tan\varphi_l} + \frac{d}{2}}$$

Dividing numerator and denominator of right hand side by l,

$$\tan\varphi = \frac{1}{\frac{1}{\tan\varphi_l} + \frac{d}{2l}}$$

$$\tan\varphi = \frac{1}{\left(\frac{2l + d\cdot\tan\varphi_l}{2l\cdot\tan\varphi_l}\right)}$$

$$\tan\varphi = \frac{2l\cdot\tan\varphi_l}{2l + d\cdot\tan\varphi_l}$$

Dividing numerator and denominator of right hand side by $2l$,

$$\tan\varphi = \frac{\tan\varphi_l}{1 + \frac{d}{2l}\cdot\tan\varphi_l}$$

$$\therefore \varphi = \arctan\left(\frac{\tan\varphi_l}{1 + \frac{d}{2l}\cdot\tan\varphi_l}\right)$$

2 Pose Determination from Kinematics Model

2.1 Differential Drive Robot

Case 1: Non-Straight-Line Motion

Since linear velocity v_l and v_r are constant, the angular velocity ω is also assumed to be constnat. Therefore, instead of notating them as a function of time, they will be notated as constants v and ω.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\Delta\theta = \int_{t_1}^{t_2} \omega \, dt$$

$$= \omega[t]_{t_1}^{t_2}$$

$$= \omega(t_2 - t_1)$$

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t) \, dt$$

$$= v \int_{t_1}^{t_2} \cos(\theta(t_1) + \omega(t - t_1)) \, dt$$

Let $u = \theta(t_1) + \omega(t - t_1)$.

$$du = \omega \, dt$$

$$dt = \frac{du}{\omega}$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta x = v \int_{\theta(t_1)}^{\theta(t_2)} \cos(u) \, \frac{du}{\omega}$$

$$= \frac{v}{\omega} \int_{\theta(t_1)}^{\theta(t_2)} \cos(u) \, du$$

$$= \frac{v}{\omega} [\sin(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{v}{\omega} [\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

Finding Δy with respect to time t_1 and t_2..

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin \theta(t) \, dt$$

$$= v \int_{t_1}^{t_2} \sin(\theta(t_1) + \omega(t - t_1)) \, dt$$

Let $u = \theta(t_1) + \omega(t - t_1)$.

$$du = \omega \, dt$$

$$dt = \frac{du}{\omega}$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta y = v \int_{\theta(t_1)}^{\theta(t_2)} \sin(u) \, \frac{du}{\omega}$$

$$= \frac{v}{\omega} \int_{\theta(t_1)}^{\theta(t_2)} \sin(u) \, du$$

$$= \frac{v}{\omega} [-\cos(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{v}{\omega} [\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Therefore, for a differential-drive robot that is not moving in a straight line motion,

$$\Delta\theta = \omega(t_2 - t_1)$$

$$\Delta x = \frac{v}{\omega}[\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

$$\Delta y = \frac{v}{\omega}[\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Case 2. Straight Line Motion

In case of a straight line motion, the equations would not be the same. For the robot to move in a straight line motion, linear velocity v_l and v_r must be equal and angular velocity ω must be 0.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\Delta\theta = \int_{t_1}^{t_2} \omega \, dt$$

$$= 0[t]_{t_1}^{t_2}$$

$$= 0$$

Since $\Delta\theta$ is 0, $\theta(t_1)$ remains constant.

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t_1) \, dt$$

$$= v \cdot \cos\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \cos\theta(t_1)[t]_{t_1}^{t_2}$$

$$= v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

Finding Δy with respect to time t_1 and t_2.

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin\theta(t_1)\, dt$$

$$= v \cdot \sin\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \sin\theta(t_1) [t]_{t_1}^{t_2}$$

$$= v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

Therefore, for a differential-drive robot that is moving in a straight line motion,

$$\Delta\theta = 0$$

$$\Delta x = v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

$$\Delta y = v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

2.2 Car-Like Robot

Case 1: Non-Straight-Line Motion

Steering angle φ and linear velocity v are constant.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\Delta\theta = \int_{t_1}^{t_2} v \cdot \frac{\tan\varphi}{l}\, dt$$

$$= v \cdot \frac{\tan\varphi}{l} \int_{t_1}^{t_2} dt$$

$$= v \cdot \frac{\tan\varphi}{l} [t]_{t_1}^{t_2}$$

$$= v \cdot \frac{\tan\varphi}{l} (t_2 - t_1)$$

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t)\, dt$$

$$= v \int_{t_1}^{t_2} \cos\left(\theta(t_1) + v \cdot \frac{\tan\varphi}{l} (t - t_1)\right) dt$$

Let $u = \theta(t_1) + v \cdot \dfrac{\tan\varphi}{l}\,(t - t_1)$.

$$du = \frac{v \cdot \tan\varphi}{l}\,dt$$

$$dt = \frac{l}{v \cdot \tan\varphi}\,du$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta x = v \int_{\theta(t_1)}^{\theta(t_2)} \cos(u) \cdot \frac{l}{v \cdot \tan\varphi}\,du$$

$$= \frac{l}{\tan\varphi} \int_{\theta(t_1)}^{\theta(t_2)} \cos(u)\,du$$

$$= \frac{l}{\tan\varphi} [\sin(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{l}{\tan\varphi} [\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

Finding Δy with respect to time t_1 and t_2.

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin\theta(t)\,dt$$

$$= v \int_{t_1}^{t_2} \sin\left(\theta(t_1) + v \cdot \frac{\tan\varphi}{l}\,(t - t_1)\right)\,dt$$

Let $u = \theta(t_1) + v \cdot \dfrac{\tan\varphi}{l}\,(t - t_1)$.

$$du = \frac{v \cdot \tan\varphi}{l}\,dt$$

$$dt = \frac{l}{v \cdot \tan\varphi}\,du$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\begin{aligned}
\Delta x &= v \int_{\theta(t_1)}^{\theta(t_2)} \sin(u) \cdot \frac{l}{v \cdot \tan\varphi}\, du \\
&= \frac{l}{\tan\varphi} \int_{\theta(t_1)}^{\theta(t_2)} \sin(u)\, du \\
&= \frac{l}{\tan\varphi} [-\cos(u)]_{\theta(t_1)}^{\theta(t_2)} \\
&= \frac{l}{\tan\varphi} [\cos(\theta(t_1)) - \cos(\theta(t_2))]
\end{aligned}$$

Therefore, for a differential-drive robot that is moving in a straight line motion,

$$\Delta\theta = v \cdot \frac{\tan\varphi}{l}\,(t_2 - t_1)$$

$$\Delta x = \frac{l}{\tan\varphi} [\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

$$\Delta y = \frac{l}{\tan\varphi} [\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Case 2. Straight Line Motion

The equations for Straight line motion for the car-like robot will also be different from the non-straight-line motion. For the robot moving in straight line motion, the steering angle φ is 0.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\begin{aligned}
\Delta\theta &= \int_{t_1}^{t_2} v \cdot \frac{\tan\varphi}{l}\, dt \\
&= v \cdot \frac{\tan\varphi}{l} \int_{t_1}^{t_2} dt \\
&= 0[t]_{t_1}^{t_2} \\
&= 0
\end{aligned}$$

Since $\Delta\theta$ is 0, $\theta(t_1)$ remains constant.

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t_1)\, dt$$

$$= v \cdot \cos\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \cos\theta(t_1)[t]_{t_1}^{t_2}$$

$$= v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

Finding Δy with respect to time t_1 and t_2.

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin\theta(t_1)\, dt$$

$$= v \cdot \sin\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \sin\theta(t_1)[t]_{t_1}^{t_2}$$

$$= v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

Therefore, for a car-like robot that is moving in a straight line motion,

$$\Delta\theta = 0$$

$$\Delta x = v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

$$\Delta y = v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

3 Kinematics of Bicycles and Tricycles

The kinematic differences between the Ackermann steering robot and the standard bicycle or tricycle model can be seen the in the following table.

Ackermann Steering	**Bicycles and Tricycles**
two forward steering wheels	only one single steering front wheel
two independent steering angles φ_l and φ_r and a virtual steering angle φ	only one steering angle φ
the orientation of both front wheels must be perpendicular to ICC	the orientation of front wheel must be perpendicular to ICC φ

Table 3.2: Differences between Ackermann steering and standard bicycles or tricycles

When the ICC falls between the rear wheels in a rear wheel driven tricycle, assuming that the steering angle is not zero ($\varphi \neq 0$) the robot will move in circles around the ICC.

4 Inverse Kinematics

The angular velocity ω is the change in the orientation $\Delta\theta$ over the period between times t_1 and t_2. Therefore it can be described as

$$\omega = \frac{\Delta\theta}{\Delta t}$$

The linear velocity is the distance the robot has travelled over the arc length s over the period between times t_1 and t_2 and therefore can be described as

$$v = \frac{s}{\Delta t}$$

The arc length can be calculate by the use of the radius and the angle difference using the forumla $s = r \cdot \theta$. Therefore, the linear velocity v can be rewritten as

$$v = \frac{r \cdot \Delta\theta}{\Delta t}$$

$$= r \cdot \omega$$

The angular velocity w and the linear velocity v of a differential drive robot can also be calculated from the linear velocity of the left and the right wheels of the robot, v_l and v_r using the formulas

$$\omega = \frac{1}{d}(v_r - v_l)$$

$$v = \frac{1}{2}(v_r + v_l)$$

where d is the track width of the robot.

The equation for linear velocity v can be rearranged to isolate v_l.

$$2v = v_r + v_l$$

$$v_l = 2v - v_r$$

Substituting the equation into the equation for angular velocity w.

$$\omega = \frac{1}{d}(v_r - (2v - v_r))$$

$$d \cdot \omega = v_r - (2v - v_r)$$

$$d \cdot \omega = 2v_r - 2v$$

$$2v_r = 2v + d \cdot \omega$$

$$v_r = v + \frac{d \cdot \omega}{2}$$

Substituting it back to the equation for v_l.

$$v_l = 2v - \left(v + \frac{d \cdot \omega}{2}\right)$$

$$v_l = v - \frac{d \cdot \omega}{2}$$

4.1 Car-Like Robot

The linear velocity v of the imaginary wheel of the car-like robot is the distance it travelled over the period between times t_1 and t_2 and therefore can also be described as

$$v = r \cdot \omega$$

The relationship between the virtual steering angle φ, wheel base l and track width d can be described as

$$\tan \varphi = \frac{l}{r}$$

Therefore the value of the virtual steering angle can be calculated by simply rearranging the formula.

$$\varphi = \arctan \left(\frac{l}{r}\right)$$

5 Simulation of Kinematics

5.1 Ackermann steering robot

Before determing a set of maneuvers for the Ackermann steered robot, the minimum radius that is physically possible from the ICC needs to be calculated to ensure that the maneuver follows the kinematic constants. Since the maximum steering angle is 25°, the minimum radius can be calculated assumed

$$\begin{aligned} r_{min} &= \frac{l}{\tan\varphi} \\ &= \frac{2}{\tan 25^\circ} \\ &= 4.29 \end{aligned}$$

5.1.1 Scence 1

Initial pose: $p_i = \begin{bmatrix} 5 & 5 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Final pose: $p_g = \begin{bmatrix} 17 & 5 & -\frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Set of maneuvers

1. Turn right along a circular arc of radius 6 sq through π rad for 2 s.

Commands

For maneuver 1,

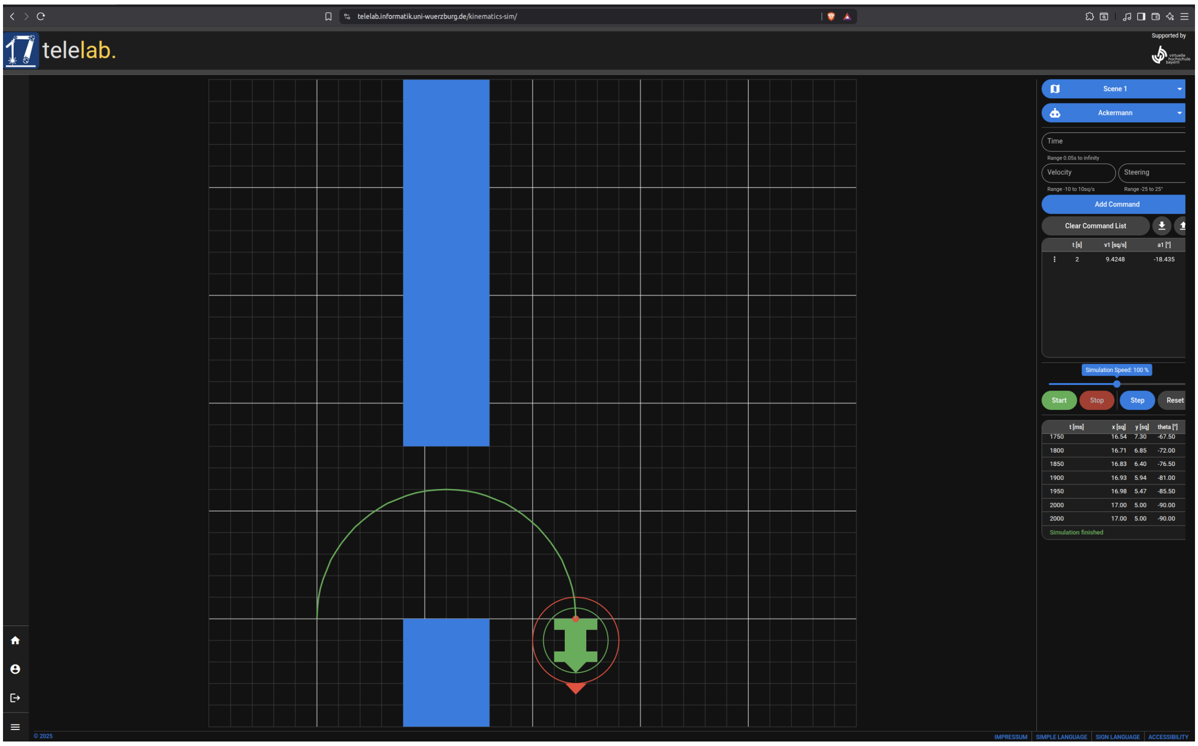
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi}{2}$$

$$v = r \cdot \omega = 6 \cdot \frac{\pi}{2} = 9.4248\, sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{6}\right) = 18.43^\circ$$

Since the robot is turning right, the steering angle φ is -18.43°.

Simulation



5.1.2 Scence 2

Initial pose: $p_i = \begin{bmatrix} 5 & 5 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Final pose: $p_g = \begin{bmatrix} 17 & 3 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Set of maneuvers

1. Turn right along a circular arc of radius 5 sq through $\frac{\pi}{2}$ rad for 1 s.
2. Travel in a straight line of 10 sq for 2 s.
3. Turn right along a circular arc of radius 7 sq through $\frac{\pi}{2}$ rad for 2 s.
4. Turn left backwards along a circular arc of radius 5 sq through π rad for 2 s.

Commands

For maneuver 1,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{1} = \frac{\pi}{2}\, rad/s$$

$$v = r \cdot \omega = 5 \cdot \frac{\pi}{2} = 7.854\, sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{5}\right) = 21.8^\circ$$

Since the robot is turning right, the steering angle φ is -21.8°.

For maneuver 2,

$$v = \frac{s}{\Delta t} = \frac{10}{2} = 5\, sq/s$$

$$\varphi = 0^\circ$$

For maneuver 3,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{2} = \frac{\pi}{4}\, rad/s$$

$$v = r \cdot \omega = 7 \cdot \frac{\pi}{4} = 5.4978\, sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{7}\right) = 15.95^\circ$$

Since the robot is turning right, the steering angle φ is -15.95°.

For maneuver 4,

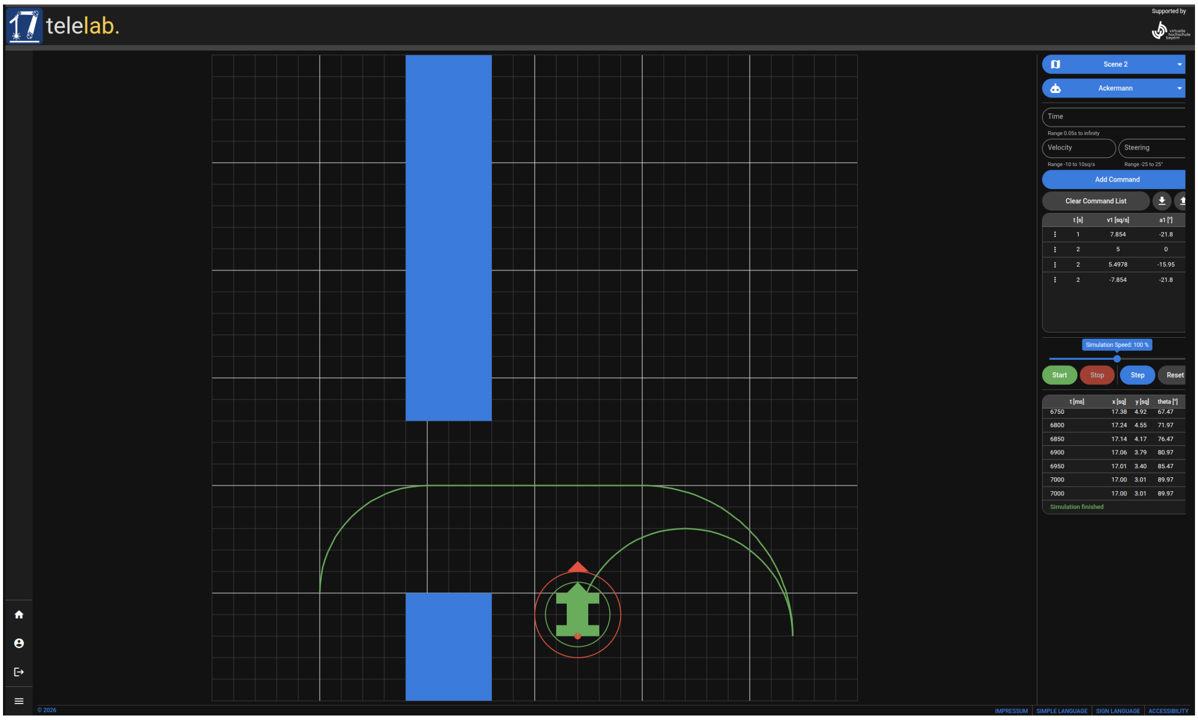
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi}{2}\,rad/s$$

$$v = r \cdot \omega = 5 \cdot \frac{\pi}{2} = 7.854\,sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{5}\right) = 21.8°$$

Since the robot is turning left backwards, the linear velocity v is -7.854 sq/s and the steering angle φ is $-21.8°$.

Simulation



5.1.3 Scence 3

Initial pose: $p_i = \begin{bmatrix} 5 & 5 & \frac{\pi}{4} \end{bmatrix}^\mathsf{T}$

Final pose: $p_g = \begin{bmatrix} 17 & 3 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Set of maneuvers

1. Turn right along a circular arc of radius 5 sq through $\frac{\pi}{4}$ rad for 1 s.

2. Travel in a straight line of 5.5 sq for 1 s.
3. Turn left along a circular arc of radius 5 sq through $\dfrac{\pi}{2}$ rad for 1 s.
4. Travel in a straight line of 4.5 sq for 1 s.

Commands

For maneuver 1,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/4}{1} = \frac{\pi}{4}\, rad/s$$

$$v = r \cdot \omega = 5 \cdot \frac{\pi}{4} = 3.927\, sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{5}\right) = 21.8^\circ$$

Since the robot is turning right, the steering angle φ is -21.8°.

For maneuver 2,

$$v = \frac{s}{\Delta t} = \frac{5.5}{1} = 5.5\, sq/s$$

$$\varphi = 0^\circ$$

For maneuver 3,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{1} = \frac{\pi}{2}\, rad/s$$

$$v = r \cdot \omega = 5 \cdot \frac{\pi}{2} = 7.854\, sq/s$$

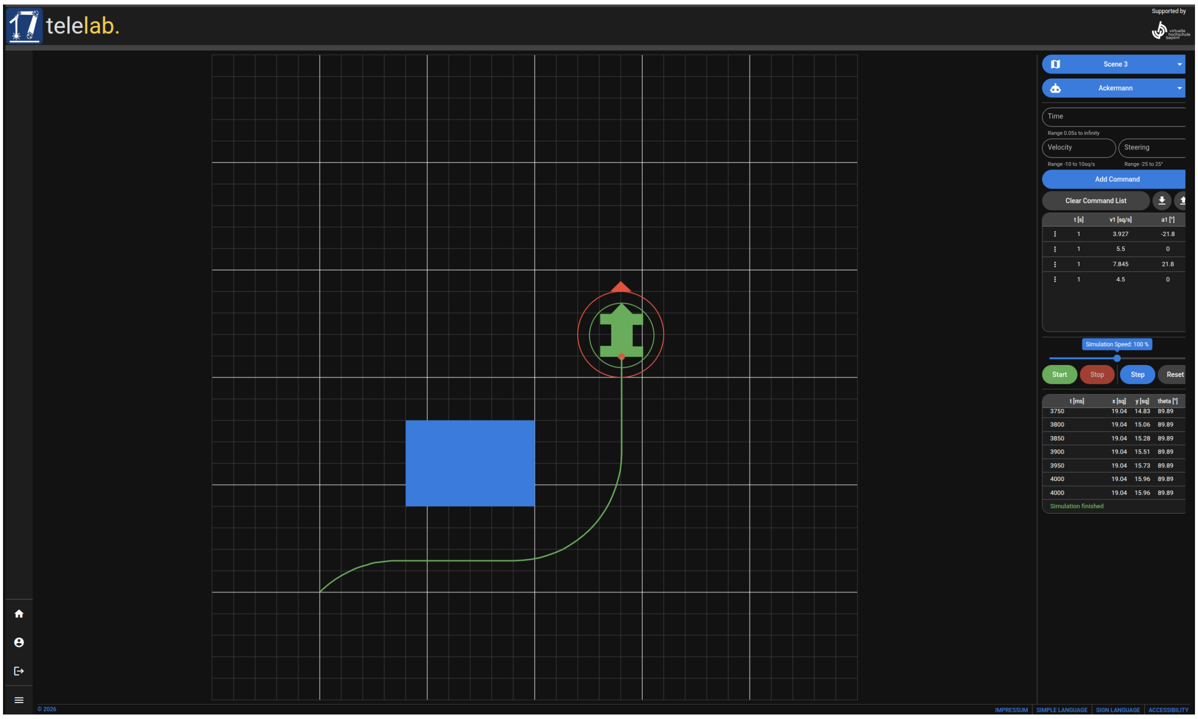
$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{5}\right) = 21.8^\circ$$

For maneuver 4,

$$v = \frac{s}{\Delta t} = \frac{5.5}{1} = 4.5\, sq/s$$

$$\varphi = 0^\circ$$

Simulation



5.1.4 Scence 4

Initial pose: $p_i = \begin{bmatrix} 7 & 14 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Final pose: $p_g = \begin{bmatrix} 11 & 8 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Set of maneuvers

1. Travel backwards in a straight line of 7.5 sq for 1 s.
2. Turn left along a circular arc of radius 4.5 sq through $\frac{\pi}{2}$ rad for 1 s.
3. Travel backwards in a straight line of 4 sq for 0.5 s.
4. Turn right backwards along a circular arc of radius 4.5 sq through $\frac{\pi}{2}$ rad for 1 s.
5. Travel in a straight line of 1.5 sq for 0.5 s.

Commands

For maneuver 1,

$$v = \frac{s}{\Delta t} = \frac{7.5}{1} = 7.5\, sq/s$$

$$\varphi = 0^\circ$$

Since the robot is moving backwards, the linear velocity v is -7.5 sq/s.

For maneuver 2,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{1} = \frac{\pi}{2}\, rad/s$$

$$v = r \cdot \omega = 4.5 \cdot \frac{\pi}{4} = 7.0686\, sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{4.5}\right) = 23.96^\circ$$

For maneuver 3,

$$v = \frac{s}{\Delta t} = \frac{4}{0.5} = 8.0\, sq/s$$

$$\varphi = 0^\circ$$

Since the robot is moving backwards, the linear velocity v is -8.0 sq/s.

For maneuver 4,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{1} = \frac{\pi}{2}\, rad/s$$

$$v = r \cdot \omega = 4.5 \cdot \frac{\pi}{4} = 7.0686\, sq/s$$

$$\varphi = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{2}{4.5}\right) = 23.96^\circ$$

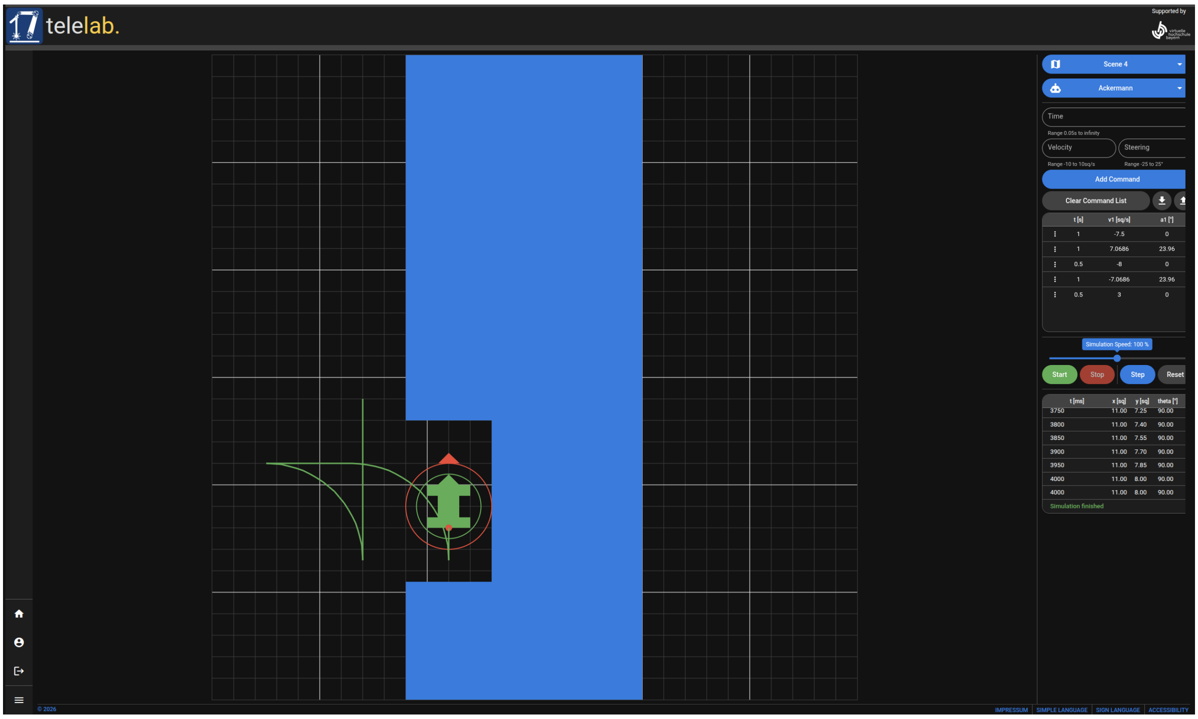
Since the robot is moving backwards, the linear velocity v is -7.0686 sq/s.

For maneuver 5,

$$v = \frac{s}{\Delta t} = \frac{1.5}{0.5} = 3.0\, sq/s$$

$$\varphi = 0^\circ$$

Simulation



5.2 Differential-drive robot

5.2.1 Scence 3

Initial pose: $p_i = \begin{bmatrix} 5 & 5 & \frac{\pi}{4} \end{bmatrix}^\mathsf{T}$

Final pose: $p_g = \begin{bmatrix} 17 & 3 & \frac{\pi}{2} \end{bmatrix}^\mathsf{T}$

Set of maneuvers

1. Rotate right through $\dfrac{\pi}{4}$ rad for 0.5 s.
2. Travel in a straight line of 14 sq for 2 s.
3. Rotate left through $\dfrac{\pi}{2}$ rad for 1 s.
4. Travel in a straight line of 11 sq for 2 s.

Commands

For maneuver 1,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{-\pi/4}{0.5} = -\frac{\pi}{2}\, rad/s$$

$$v = 0\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 0 + \frac{1 \cdot (-\pi/2)}{2} = -1.5708\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 0 - \frac{1 \cdot (-\pi/2)}{2} = 1.5708\, sq/s$$

For maneuver 2,

$$\omega = 0\, rad/s$$

$$v = \frac{s}{\Delta t} = \frac{14}{2} = 7\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 7 + \frac{1 \cdot 0}{2} = 7\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 7 - \frac{1 \cdot 0}{2} = 7\, sq/s$$

For maneuver 3,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2}{1} = \frac{\pi}{2}\, rad/s$$

$$v = 0\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 0 + \frac{1 \cdot \pi/2}{2} = 1.5708\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 0 - \frac{1 \cdot \pi/2}{2} = -1.5708\, sq/s$$

For maneuver 4,

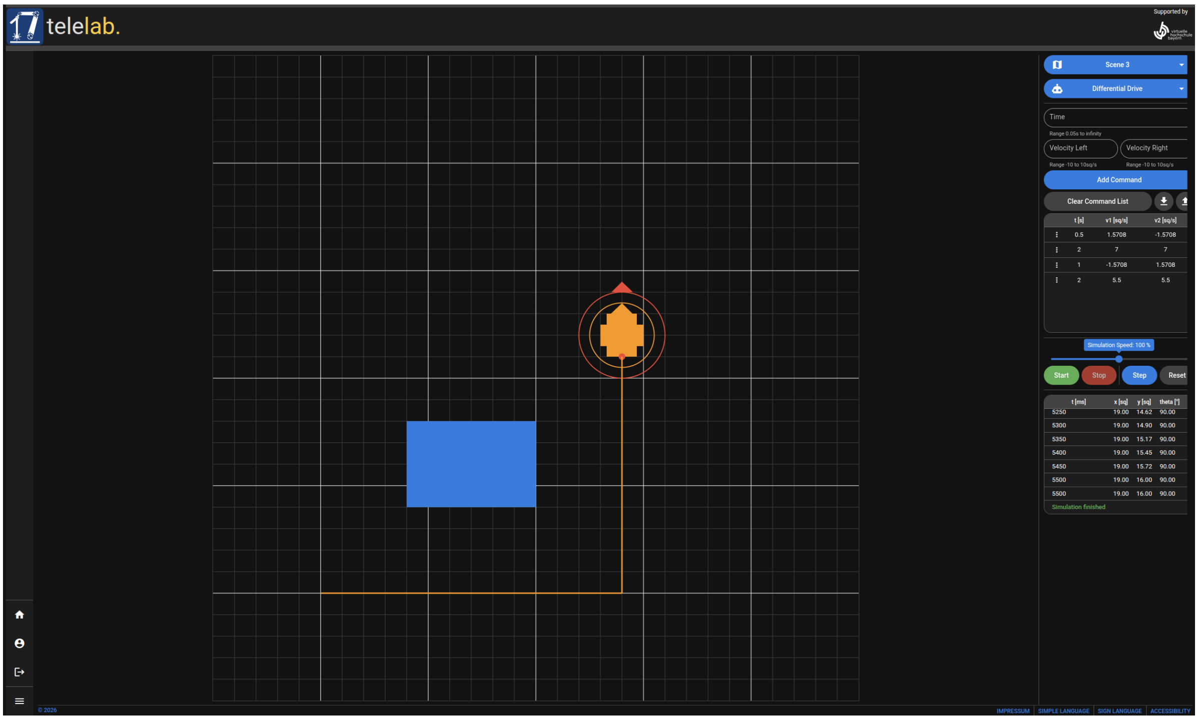
$$\omega = 0\, rad/s$$

$$v = \frac{s}{\Delta t} = \frac{11}{2} = 5.5\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 5.5 + \frac{1 \cdot 0}{2} = 5.5\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 5.5 - \frac{1 \cdot 0}{2} = 5.5\, sq/s$$

Simulation



5.2.2 Scence 4

Initial pose: $p_i = \begin{bmatrix} 7 & 14 & \frac{\pi}{2} \end{bmatrix}^\top$

Final pose: $p_g = \begin{bmatrix} 11 & 8 & \frac{\pi}{2} \end{bmatrix}^\top$

Set of maneuvers

1. Travel backwards in a straight line of 2 sq for 0.5 s.

2. Rotate left through $\frac{\pi}{4}$ rad for 0.5 s.
3. Travel backwards in a straight line of $\left(\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{4^2 + 4^2}\right)$ 5.66 sq for 1 s.
4. Rotate left through $\frac{\pi}{2}$ rad for 1 s.

Commands

For maneuver 1,

$$\omega = 0\, rad/s$$

$$v = \frac{s}{\Delta t} = \frac{-2}{0.5} = -4\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = -4 + \frac{1 \cdot 0}{2} = -4\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = -4 - \frac{1 \cdot 0}{2} = -4\, sq/s$$

For maneuver 2,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/4}{0.5} = \frac{\pi}{2}\, rad/s$$

$$v = 0\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 0 + \frac{1 \cdot \pi/2}{2} = 1.5708\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 0 - \frac{1 \cdot \pi/2}{2} = -1.5708\, sq/s$$

For maneuver 3,

$$\omega = 0\, rad/s$$

$$v = \frac{s}{\Delta t} = \frac{-6.4}{1} = -6.4\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 8 + \frac{1 \cdot 0}{2} = 8\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 8 - \frac{1 \cdot 0}{2} = 8\, sq/s$$

For maneuver 4,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{-\pi/4}{0.5} = -\frac{\pi}{2}\, rad/s$$

$$v = 0\, sq/s$$

$$v_r = v + \frac{d \cdot \omega}{2} = 0 + \frac{1 \cdot (-\pi/2)}{2} = -1.5708\, sq/s$$

$$v_l = v - \frac{d \cdot \omega}{2} = 0 - \frac{1 \cdot (-\pi/2)}{2} = 1.5708\, sq/s$$

Simulation