Tele-Experiment

Kinematics and Navigation of the MERLIN Robot

Name:	Kaung Sett Thu
Matriculation Number:	12503548
Insitute:	Technische Hochschule Deggendorf
Submission Date:	February 14, 2026

Contents

1 The Imaginary Wheel

The relationship between the steering angle of the imaginary wheel ϕ, the wheelbase L, and the radius of the robot R can be described as follow.

$$\tan \phi = \frac{L}{R}$$

1.1 Imaginary Wheel ϕ in Relation to Outer Wheel ϕ_o

The turning radius of the outer wheel from `ICR` is $R + \frac{D}{2}$. Therefore

$$\tan \phi_o = \frac{L}{R + \dfrac{D}{2}}$$

$$R + \frac{D}{2} = \frac{L}{\tan \phi_o}$$

$$R = \frac{L}{\tan \phi_o} - \frac{D}{2}$$

Substituting formula for R in terms of steering angle of the outer wheel to formula for the steering angle of the imaginary wheel

$$\tan \phi = \frac{L}{\dfrac{L}{\tan \phi_o} - \dfrac{D}{2}}$$

$$\tan \phi = \frac{1}{\dfrac{1}{\tan \phi_o} - \dfrac{D}{2L}}$$

$$\tan \phi = \frac{\tan \phi_o}{1 - \dfrac{D}{2L} \cdot \tan \phi_o}$$

$$\phi = \arctan \left(\frac{\tan \phi_o}{1 - \dfrac{D}{2L} \cdot \tan \phi_o} \right)$$

1.2 Imaginary Wheel ϕ in Relation to Inner Wheel ϕ_i

The turning radius of the inner wheel from `ICR` is $R - \frac{D}{2}$. Therefore

$$\tan \phi_i = \frac{L}{R - \frac{D}{2}}$$

$$R - \frac{D}{2} = \frac{L}{\tan \phi_i}$$

$$R = \frac{L}{\tan \phi_i} + \frac{D}{2}$$

Substituting formula for R in terms of steering angle of the inner wheel to formula for the steering angle of the imaginary wheel

$$\tan \phi = \frac{L}{\frac{L}{\tan \phi_i} + \frac{D}{2}}$$

$$\tan \phi = \frac{1}{\frac{1}{\tan \phi_i} + \frac{D}{2L}}$$

$$\tan \phi = \frac{\tan \phi_i}{1 + \frac{D}{2L} \cdot \tan \phi_i}$$

$$\phi = \arctan \left(\frac{\tan \phi_i}{1 + \frac{D}{2L} \cdot \tan \phi_i} \right)$$

2 Calculating the Robot Pose using the Odometry (Dead Reckoning)

2.1 Pose Estimation in relation to the Velocity v_{CR}

The angular velocity of the robot $\omega(t)$ in relation to the velocity of the point CR $v_{CR}(t)$ can be expressed as

$$\omega(t) = \frac{v_{CR}(t)}{R_{CR}}$$

Since v_{CR} is constant, angular velocity ω can be expressed as a constant.

$$\omega = \frac{v_{CR}}{R_{CR}}$$

The relation of the steering angle ϕ to the radius from Instantenous Center of Rotation (ICR) R_{CR} can be rearranged to express it in terms of the radius.

$$\tan \phi = \frac{L}{R_{CR}}$$

$$R_{CR} = \frac{L}{\tan \phi}$$

Replacing the formula for radius R_{CR} into the formula for angular velocity ω,

$$\omega = \frac{v_{CR}}{\dfrac{L}{\tan \phi}}$$

$$= \frac{v_{CR}}{L} \cdot \tan \phi$$

The pose estimation for the orientation of the robot θ can be expressed as

$$\theta(t) = \theta_0 + \int_0^t \omega \, d\tau$$

$$= \theta_0 + \omega \big[\tau\big]_0^t$$

$$= \theta_0 + \omega \cdot t$$

The pose estimation for x coordinate of the robot can be expressed as

$$x(t) = x_0 + \int_0^t v_{CR} \cdot \cos(\theta(\tau)) \, d\tau$$

$$= x_0 + v_{CR} \int_0^t \cos(\theta_0 + \omega \cdot \tau) \, d\tau$$

Integrating $\cos(\theta_0 + \omega \cdot \tau)$ using U-Substitution

Let

$$u = \theta_0 + \omega \cdot \tau$$

$$\frac{du}{d\tau} = \omega$$

$$d\tau = \frac{1}{\omega} \cdot du$$

When $\tau = 0$,

$$u = \theta_0$$

When $\tau = t$,

$$u = \theta_0 + \omega \cdot t$$

$$x(t) = x_0 + v_{CR} \int_{\theta_0}^{\theta_0 + \omega \cdot t} \cos(u) \frac{1}{\omega} \cdot du$$

$$= x_0 + \frac{v_{CR}}{\omega} \int_{\theta_0}^{\theta_0 + \omega \cdot t} \cos(u) \cdot du$$

$$= x_0 + \frac{v_{CR}}{\omega} \Big[\sin(u) \Big]_{\theta_0}^{\theta_0 + \omega \cdot t}$$

$$= x_0 + \frac{v_{CR}}{\omega} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

Replacing the formula for the angular velocity ω into the equation,

$$x(t) = x_0 + v_{CR} \cdot \frac{R_{CR}}{v_{CR}} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

$$= x_0 + R_{CR} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

6

The pose estimation for y coordinate of the robot can be expressed as

$$y(t) = y_0 + \int_0^t v_{CR} \cdot \sin(\theta(\tau)) \, d\tau$$

$$= y_0 + v_{CR} \int_0^t \sin(\theta_0 + \omega \cdot \tau) \, d\tau$$

Integrating $\sin(\theta_0 + \omega \cdot \tau)$ *using U-Substitution*

Let

$$u = \theta_0 + \omega \cdot \tau$$

$$\frac{du}{d\tau} = \omega$$

$$d\tau = \frac{1}{\omega} \cdot du$$

When $\tau = 0$,

$$u = \theta_0$$

When $\tau = t$,

$$u = \theta_0 + \omega \cdot t$$

$$y(t) = y_0 + v_{CR} \int_{\theta_0}^{\theta_0 + \omega \cdot t} \sin(u) \frac{1}{\omega} \cdot du$$

$$= y_0 + \frac{v_{CR}}{\omega} \int_{\theta_0}^{\theta_0 + \omega \cdot t} \sin(u) \cdot du$$

$$= y_0 + \frac{v_{CR}}{\omega} \Big[-\cos(u) \Big]_{\theta_0}^{\theta_0 + \omega \cdot t}$$

$$= y_0 + \frac{v_{CR}}{\omega} \big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t) \big)$$

Replacing the formula for the angular velocity ω into the equation,

$$y(t) = y_0 + v_{CR} \cdot \frac{R_{CR}}{v_{CR}} \big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t) \big)$$

$$= y_0 + R_{CR} \big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t) \big)$$

The pose estimation in relation to the velocity v_{CR} can collectively be expressed in a matrix form.

$$p(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} R_{CR}\big(\sin(\theta_0 + \omega t) - \sin\theta_0\big) \\ R_{CR}\big(\cos\theta_0 - \cos(\theta_0 + \omega t)\big) \\ \omega \cdot t \end{bmatrix}$$

2.2 Pose Estimation in relation to Velocity at Imaginary Wheel v_{IM}

In the right-angled triangle formed by the rear axle center, the imaginary front wheel, and the instantaneous center of rotation (ICR), the distance from the ICR to the imaginary wheel, R_{IM} is the hypotenuse of this triangle, while the wheelbase L corresponds to the side opposite the steering angle ϕ. The steering angle ϕ can therefore be expressed by the use of the *sine* ratio.

$$\sin\phi = \frac{L}{R_{IM}}$$

The equation can be rearranged to express it in terms of the radius R_{IM}.

$$R_{IM} = \frac{L}{\sin\phi}$$

An angular velocity ω can be expressed as follows.

$$\omega = \frac{v_{IM}}{R_{IM}}$$

$$= \frac{v_{IM}}{\dfrac{L}{\sin\phi}}$$

$$= \frac{v_{IM} \cdot \sin\phi}{L}$$

Since the angular velocity ω remains the same for both the rear axle and the front wheel, the relationship between velocity at the rear axis v_{CR} and the imaginary wheel v_{IM} can

be calculated.

$$\frac{v_{CR}}{R_{CR}} = \frac{v_{IM}}{R_{IM}}$$

$$v_{CR} = \frac{v_{IM}}{R_{IM}} \cdot R_{CR}$$

$$v_{CR} = \frac{v_{IM} \cdot \sin\phi}{L} \cdot \frac{L}{\tan\phi}$$

$$v_{CR} = v_{IM} \cdot \cos\phi$$

Using this relationship between v_{CR} and v_{IM}, the pose estimation for $x(t)$ and $y(t)$ can be expressed in terms of v_{IM} as

$$x(t) = x_0 + \int_0^t (v_{IM} \cdot \cos\phi) \cdot \cos(\theta(\tau)) \, d\tau$$

$$y(t) = y_0 + \int_0^t (v_{IM} \cdot \cos\phi) \cdot \sin(\theta(\tau)) \, d\tau$$

Since angular velocity ω is constant the pose estimation for orientation $\theta(t)$ remains the same.

Following the same integration steps as section 2.1 for $x(t)$ and $y(t)$, we get

$$x(t) = x_0 + \frac{v_{IM} \cdot \cos\phi}{\omega} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

$$= x_0 + (v_{IM} \cdot \cos\phi) \cdot \frac{L}{v_{IM} \cdot \sin\phi} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

$$= x_0 + \frac{L \cdot \cos\phi}{\sin\phi} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

$$= x_0 + \frac{L}{\tan\phi} \Big(\sin(\theta_0 + \omega \cdot t) - \sin(\theta_0) \Big)$$

$$y(t) = y_0 + \frac{v_{IM} \cdot \cos\phi}{\omega}\big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t)\big)$$

$$= y_0 + (v_{IM} \cdot \cos\phi) \cdot \frac{L}{v_{IM} \cdot \sin\phi}\big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t)\big)$$

$$= y_0 + \frac{L \cdot \cos\phi}{\sin\phi}\big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t)\big)$$

$$= y_0 + \frac{L}{\tan\phi}\big(\cos(\theta_0) - \cos(\theta_0 + \omega \cdot t)\big)$$

Since $\frac{L}{tan\phi}$ is radius from rear axle R_{CR}, we can see that pose estimation in relation to velocity at imaginary wheel v_{IM} has a similar matrix to pose estimation in relation to velocity at rear axle v_{CR}

$$p(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} R_{CR}\big(\sin(\theta_0 + \omega t) - \sin\theta_0\big) \\ R_{CR}\big(\cos\theta_0 - \cos(\theta_0 + \omega t)\big) \\ \omega \cdot t \end{bmatrix}$$

with angular velocity ω being expressed in terms of velocity at imaginary wheel v_{IM} as

$$\omega = \frac{v_{IM} \cdot \sin\phi}{L}$$

2.3 Discrete Form of the Robot's Pose in relation to v_{CR}

Based on the matrix derived in section 2.1, assuming constant velocity v_{CR} and constant steering angle ϕ over the sampling interval Δt, the exact discrete-time update of the robot pose can be expressed as

$$p(k+1) = \begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} R_{CR}\big(\sin(\theta_k + \omega\Delta t) - \sin\theta_k\big) \\ R_{CR}\big(\cos\theta_k - \cos(\theta_k + \omega\Delta t)\big) \\ \omega \cdot \Delta t \end{bmatrix}$$

2.4 Recurrent Form of Equations

For recurrent odometry updates under constant steering angle and velocity over the time Δt, the pose can be updated recurrently by following pipeline of equations

1. Update the orientation

$$\theta_{k+1} = \theta_k + \frac{v_{CR} \cdot \tan\phi}{L} \cdot \Delta t$$

2. Update the x coordinate

$$x_{k+1} = x_k + \frac{L}{\tan \phi} \left(\sin \theta_{k+1} - \sin \theta_k \right)$$

3. Update the y coordinate

$$y_{k+1} = y_k + \frac{L}{\tan \phi} \left(\cos \theta_k - \cos \theta_{k+1} \right)$$

In case of a straight line motion where ϕ is 0, the equations simplify to

1. Update the orientation

$$\theta_{k+1} = \theta_k$$

2. Update the x coordinate

$$x_{k+1} = x_k + v_{CR} \cdot \cos \theta_k \cdot \Delta t$$

3. Update the y coordinate

$$y_{k+1} = y_k + v_{CR} \cdot \sin \theta_k \cdot \Delta t$$

3 Shifting and Rotation