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Tele-Experiment

Kinematics of Mobile Robots

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Contents

1 Task 3.1 Simplified Car-Like Robot Model

Proving the relationship between the steering angle of the virtual wheel φ and the steering angles of the two front wheels φ_l and φ_r.

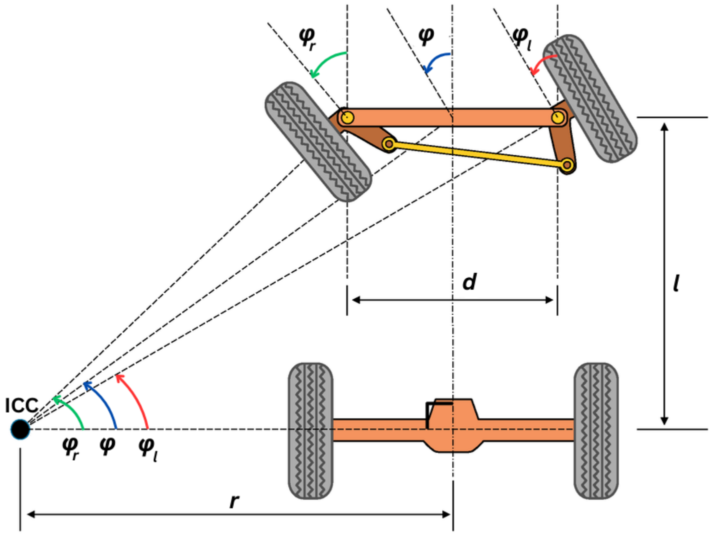

Figure 1.1: Simplified Car-Like Robot Model.

Since the track width d, the wheel base l and the radius r from the Instaneous Ceter of Curvature ICC forms a right angle triangle, the steering angles can be written as:

$$\tan \varphi = \frac{l}{r}$$

$$\tan \varphi_r = \frac{l}{(r + \frac{d}{2})}$$

$$\tan \varphi_l = \frac{l}{(r - \frac{d}{2})}$$

The equation for right steering angle can be rearranged by isolating for radius r.

$$\tan\varphi_r = \frac{l}{(r + \frac{d}{2})}$$

$$r + \frac{d}{2} = \frac{l}{\tan\varphi_r}$$

$$r = \frac{l}{\tan\varphi_r} - \frac{d}{2}$$

Substituding the radius into equation for steering angle of the virtual wheel,

$$\tan\varphi = \frac{l}{\dfrac{l}{\tan\varphi_r} - \dfrac{d}{2}}$$

Dividing numerator and denominator of right hand side by l,

$$\tan\varphi = \frac{1}{\dfrac{1}{\tan\varphi_r} - \dfrac{d}{2l}}$$

$$\tan\varphi = \frac{1}{\left(\dfrac{2l - d \cdot \tan\varphi_r}{2l \cdot \tan\varphi_r}\right)}$$

$$\tan\varphi = \frac{2l \cdot \tan\varphi_r}{2l - d \cdot \tan\varphi_r}$$

Dividing numerator and denominator of right hand side by $2l$,

$$\tan\varphi = \frac{\tan\varphi_r}{1 - \dfrac{d}{2l} \cdot \tan\varphi_r}$$

$$\therefore \varphi = \arctan\left(\frac{\tan\varphi_r}{1 - \dfrac{d}{2l} \cdot \tan\varphi_r}\right)$$

The equation for left steering angle can be rearranged by isolating for radius r.

$$\tan\varphi_l = \frac{l}{(r - \frac{d}{2})}$$

$$r - \frac{d}{2} = \frac{l}{\tan\varphi_l}$$

$$r = \frac{l}{\tan\varphi_l} + \frac{d}{2}$$

Substituding the radius into equation for steering angle of the virtual wheel,

$$\tan\varphi = \frac{l}{\frac{l}{\tan\varphi_l} + \frac{d}{2}}$$

Dividing numerator and denominator of right hand side by l,

$$\tan\varphi = \frac{1}{\frac{1}{\tan\varphi_l} + \frac{d}{2l}}$$

$$\tan\varphi = \frac{1}{\left(\frac{2l + d\cdot\tan\varphi_l}{2l\cdot\tan\varphi_l}\right)}$$

$$\tan\varphi = \frac{2l\cdot\tan\varphi_l}{2l + d\cdot\tan\varphi_l}$$

Dividing numerator and denominator of right hand side by $2l$,

$$\tan\varphi = \frac{\tan\varphi_l}{1 + \frac{d}{2l}\cdot\tan\varphi_l}$$

$$\therefore \varphi = \arctan\left(\frac{\tan\varphi_l}{1 + \frac{d}{2l}\cdot\tan\varphi_l}\right)$$

2 Task 3.2 Pose Determination from Kinematics Model

2.1 Differential Drive Robot

Case 1: Non-Straight-Line Motion

Since linear velocity v_l and v_r are constant, the angular velocity ω is also assumed to be constnat. Therefore, instead of notating them as a function of time, they will be notated as constants v and ω.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\Delta\theta = \int_{t_1}^{t_2} \omega \, dt$$

$$= \omega[t]_{t_1}^{t_2}$$

$$= \omega(t_2 - t_1)$$

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t) \, dt$$

$$= v \int_{t_1}^{t_2} \cos(\theta(t_1) + \omega(t - t_1)) \, dt$$

Let $u = \theta(t_1) + \omega(t - t_1)$.

$$du = \omega \, dt$$

$$dt = \frac{du}{\omega}$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta x = v\int_{\theta(t_1)}^{\theta(t_2)} \cos(u)\,\frac{du}{\omega}$$

$$= \frac{v}{\omega}\int_{\theta(t_1)}^{\theta(t_2)} \cos(u)\,du$$

$$= \frac{v}{\omega}[\sin(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{v}{\omega}[\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

Finding Δy with respect to time t_1 and t_2..

$$\Delta y = \int_{t_1}^{t_2} v\cdot\sin\theta(t)\,dt$$

$$= v\int_{t_1}^{t_2} \sin(\theta(t_1) + \omega(t - t_1))\,dt$$

Let $u = \theta(t_1) + \omega(t - t_1)$.

$$du = \omega\,dt$$

$$dt = \frac{du}{\omega}$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta y = v\int_{\theta(t_1)}^{\theta(t_2)} \sin(u)\,\frac{du}{\omega}$$

$$= \frac{v}{\omega}\int_{\theta(t_1)}^{\theta(t_2)} \sin(u)\,du$$

$$= \frac{v}{\omega}[-\cos(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{v}{\omega}[\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Therefore, for a differential-drive robot that is not moving in a straight line motion,

$$\Delta\theta = \omega(t_2 - t_1)$$

$$\Delta x = \frac{v}{\omega}[\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

$$\Delta y = \frac{v}{\omega}[\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Case 2. Straight Line Motion

In case of a straight line motion, the equations would not be the same. For the robot to move in a straight line motion, linear velocity v_l and v_r must be equal and angular velocity ω must be 0.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\begin{aligned}\Delta\theta &= \int_{t_1}^{t_2} \omega \, dt \\ &= 0[t]_{t_1}^{t_2} \\ &= 0\end{aligned}$$

Since $\Delta\theta$ is 0, $\theta(t_1)$ remains constant.

Finding Δx with respect to time t_1 and t_2.

$$\begin{aligned}\Delta x &= \int_{t_1}^{t_2} v \cdot \cos\theta(t_1) \, dt \\ &= v \cdot \cos\theta(t_1) \int_{t_1}^{t_2} dt \\ &= v \cdot \cos\theta(t_1)[t]_{t_1}^{t_2} \\ &= v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)\end{aligned}$$

Finding Δy with respect to time t_1 and t_2.

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin\theta(t_1)\, dt$$

$$= v \cdot \sin\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \sin\theta(t_1)[t]_{t_1}^{t_2}$$

$$= v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

Therefore, for a differential-drive robot that is moving in a straight line motion,

$$\Delta\theta = 0$$

$$\Delta x = v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

$$\Delta y = v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

2.2 Car-Like Robot

Case 1: Non-Straight-Line Motion

Steering angle φ and linear velocity v are constant.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\Delta\theta = \int_{t_1}^{t_2} v \cdot \frac{\tan\varphi}{l}\, dt$$

$$= v \cdot \frac{\tan\varphi}{l} \int_{t_1}^{t_2} dt$$

$$= v \cdot \frac{\tan\varphi}{l}[t]_{t_1}^{t_2}$$

$$= v \cdot \frac{\tan\varphi}{l}\,(t_2 - t_1)$$

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t)\, dt$$

$$= v \int_{t_1}^{t_2} \cos\left(\theta(t_1) + v \cdot \frac{\tan\varphi}{l}\,(t - t_1)\right) dt$$

Let $u = \theta(t_1) + v \cdot \dfrac{\tan\varphi}{l}(t - t_1)$.

$$du = \frac{v \cdot \tan\varphi}{l}\,dt$$

$$dt = \frac{l}{v \cdot \tan\varphi}\,du$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta x = v \int_{\theta(t_1)}^{\theta(t_2)} \cos(u) \cdot \frac{l}{v \cdot \tan\varphi}\,du$$

$$= \frac{l}{\tan\varphi} \int_{\theta(t_1)}^{\theta(t_2)} \cos(u)\,du$$

$$= \frac{l}{\tan\varphi}[\sin(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{l}{\tan\varphi}[\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

Finding Δy with respect to time t_1 and t_2.

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin\theta(t)\,dt$$

$$= v \int_{t_1}^{t_2} \sin\left(\theta(t_1) + v \cdot \frac{\tan\varphi}{l}(t - t_1)\right)\,dt$$

Let $u = \theta(t_1) + v \cdot \dfrac{\tan\varphi}{l}(t - t_1)$.

$$du = \frac{v \cdot \tan\varphi}{l}\,dt$$

$$dt = \frac{l}{v \cdot \tan\varphi}\,du$$

Using $\theta(t_1)$ and $\theta(t_2)$ as lower and upper bounds and substituting u and du,

$$\Delta x = v \int_{\theta(t_1)}^{\theta(t_2)} \sin(u) \cdot \frac{l}{v \cdot \tan\varphi}\, du$$

$$= \frac{l}{\tan\varphi} \int_{\theta(t_1)}^{\theta(t_2)} \sin(u)\, du$$

$$= \frac{l}{\tan\varphi} [-\cos(u)]_{\theta(t_1)}^{\theta(t_2)}$$

$$= \frac{l}{\tan\varphi} [\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Therefore, for a differential-drive robot that is moving in a straight line motion,

$$\Delta\theta = v \cdot \frac{\tan\varphi}{l}\,(t_2 - t_1)$$

$$\Delta x = \frac{l}{\tan\varphi} [\sin(\theta(t_2)) - \sin(\theta(t_1))]$$

$$\Delta y = \frac{l}{\tan\varphi} [\cos(\theta(t_1)) - \cos(\theta(t_2))]$$

Case 2. Straight Line Motion

The equations for Straight line motion for the car-like robot will also be different from the non-straight-line motion. For the robot moving in straight line motion, the steering angle φ is 0.

Finding $\Delta\theta$ with respect to time t_1 and t_2.

$$\Delta\theta = \int_{t_1}^{t_2} v \cdot \frac{\tan\varphi}{l}\, dt$$

$$= v \cdot \frac{\tan\varphi}{l} \int_{t_1}^{t_2} dt$$

$$= 0[t]_{t_1}^{t_2}$$

$$= 0$$

Since $\Delta\theta$ is 0, $\theta(t_1)$ remains constant.

Finding Δx with respect to time t_1 and t_2.

$$\Delta x = \int_{t_1}^{t_2} v \cdot \cos\theta(t_1)\, dt$$

$$= v \cdot \cos\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \cos\theta(t_1)[t]_{t_1}^{t_2}$$

$$= v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

Finding Δy with respect to time t_1 and t_2.

$$\Delta y = \int_{t_1}^{t_2} v \cdot \sin\theta(t_1)\, dt$$

$$= v \cdot \sin\theta(t_1) \int_{t_1}^{t_2} dt$$

$$= v \cdot \sin\theta(t_1)[t]_{t_1}^{t_2}$$

$$= v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$

Therefore, for a car-like robot that is moving in a straight line motion,

$$\Delta\theta = 0$$

$$\Delta x = v \cdot \cos\theta(t_1) \cdot (t_2 - t_1)$$

$$\Delta y = v \cdot \sin\theta(t_1) \cdot (t_2 - t_1)$$