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Tele-Experiment

Sensor Measurement and Processing of the MERLIN Robot

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1 Encoder Measurement and Error Analysis

1.1 Encoder Recordings 1 to 10

The following table states the sums of all the encoder information from the recordings 1 - 10 provided in `encoder_recordings.zip` of a straight line motion.

Run no.	$\sum ecr\_l$	$\sum ecr\_r$
1	8329	8382
2	8007	8078
3	7918	8000
4	7962	8023
5	8313	8378
6	8002	8096
7	8028	8111
8	8073	8121
9	8234	8300
10	8205	8281

Table 1.1: Cumulative encoder counts for left and right wheels (Runs 1-10).

1.1.1 Mean

The mean value for encoder of the left wheel is calculated as

$$\bar{N_L} = \frac{1}{n}\sum_{i=1}^{n} N_{L,i}$$

$$= \frac{1}{10}\Big(8329 + 8007 + 7918 + 7962 + 8313 + 8002 + 8028 + 8073 + 8234 + 8205\Big)$$

$$= 8107.1$$

The mean value for encoder of the right wheel is calculated as

$$\bar{N_R} = \frac{1}{n}\sum_{i=1}^{n} N_{R,i}$$

$$= \frac{1}{10}\Big(8382 + 8078 + 8000 + 8023 + 8378 + 8096 + 8111 + 8121 + 8300 + 8281\Big)$$

$$= 8177.0$$

1.1.2 Standard Deviation

The standard deviation for encoder of the left wheel is calculated as

$$s_{N_L} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(s_{N_{L,i}} - s_{\bar{N_L}})^2}$$

$$= \Big(\frac{1}{10-1}\big((8329 - 8107.1)^2 + (8007 - 8107.1)^2 + (7918 - 8107.1)^2 + (7962 - 8107.1)^2 + (8313 - 8107.1)^2 + (8002 - 8107.1)^2 + (8028 - 8107.1)^2 + (8073 - 8107.1)^2 + (8234 - 8107.1)^2 + (8205 - 8107.1)^2\big)\Big)^{1/2}$$

$$= 150.04$$

The standard deviation for encoder of the right wheel is calculated as

$$s_{N_R} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(N_{R,i} - \bar{N_R})^2}$$

$$= \Big(\frac{1}{10-1}\big((8382 - 8177.0)^2 + (8078 - 8177.0)^2 + (8000 - 8177.0)^2 + (8023 - 8177.0)^2 + (8378 - 8177.0)^2 + (8096 - 8177.0)^2 + (8111 - 8177.0)^2 + (8121 - 8177.0)^2 + (8300 - 8177.0)^2 + (8281 - 8177.0)^2\big)\Big)^{1/2}$$

$$= 144.25$$

1.2 Encoder Recordings 11 to 20

The following table states the sums of all the encoder information from the recordings 1 - 10 provided in `encoder_recordings.zip` of a straight line motion.

Run no.	$\sum ecr\_l$	$\sum ecr\_r$
11	8156	8230
12	8120	8201
13	8015	8094
14	8255	8285
15	8093	8161
16	8166	8257
17	8013	8096
18	7988	8049
19	8080	8167
20	8052	8120

Table 1.2: Cumulative encoder counts for left and right wheels (Runs 11-20).

1.2.1 Mean

The mean value for the encoder of the left wheel is calculated as

$$\begin{aligned}\bar{N_L} &= \frac{1}{n}\sum_{i=11}^{20} N_{L,i} \\ &= \frac{1}{10}\Big(8156 + 8120 + 8015 + 8255 + 8093 + 8166 + 8013 + 7988 + 8080 + 8052\Big) \\ &= 8093.8\end{aligned}$$

The mean value for the encoder of the right wheel is calculated as

$$\begin{aligned}\bar{N_R} &= \frac{1}{n}\sum_{i=11}^{20} N_{R,i} \\ &= \frac{1}{10}\Big(8230 + 8201 + 8094 + 8285 + 8161 + 8257 + 8096 + 8049 + 8167 + 8120\Big) \\ &= 8166.0\end{aligned}$$

1.2.2 Standard Deviation

The standard deviation for the encoder of the left wheel is calculated as

$$
\begin{aligned}
s_{N_L} &= \sqrt{\frac{1}{n-1}\sum_{i=11}^{20}(N_{L,i} - \bar{N_L})^2} \\
&= \Big(\frac{1}{10-1}\big((8156 - 8093.8)^2 + (8120 - 8093.8)^2 + (8015 - 8093.8)^2 + \\
&\qquad (8255 - 8093.8)^2 + (8093 - 8093.8)^2 + (8166 - 8093.8)^2 + \\
&\qquad (8013 - 8093.8)^2 + (7988 - 8093.8)^2 + (8080 - 8093.8)^2 + \\
&\qquad (8052 - 8093.8)^2\big)\Big)^{1/2} \\
&= 82.75
\end{aligned}
$$

The standard deviation for the encoder of the right wheel is calculated as

$$
\begin{aligned}
s_{N_R} &= \sqrt{\frac{1}{n-1}\sum_{i=11}^{20}(N_{R,i} - \bar{N_R})^2} \\
&= \Big(\frac{1}{10-1}\big((8230 - 8166.0)^2 + (8201 - 8166.0)^2 + (8094 - 8166.0)^2 + \\
&\qquad (8285 - 8166.0)^2 + (8161 - 8166.0)^2 + (8257 - 8166.0)^2 + \\
&\qquad (8096 - 8166.0)^2 + (8049 - 8166.0)^2 + (8167 - 8166.0)^2 + \\
&\qquad (8120 - 8166.0)^2\big)\Big)^{1/2} \\
&= 77.23
\end{aligned}
$$

1.3 Error Analysis

From the calculate of mean and standard deviation of 20 encoder records, both systematic errors and non-systematic errors can be observed.

1.3.1 Systematic Errors

The mean encoder values for right wheel is observed to be greater than left wheel for runs 1 to 10 and runs 11 to 20. From the runs, the encoder values of the right wheel is also consistently greater than then encoder of the left wheel. This indicates the existance of the systematic errors. The potential causes of this error can be due to the following.

- the right wheel has bigger diameter compared to the left wheel
- the right moter is spinning faster than the left motor or
- the wheels could be misaligned

1.3.2 Non-systematic Errors

The calculated standard deviation indicates that the there could also be in non-systematic errors present. The potential causes for standard deviation in our case can be due to the following.

- the wheel slipage
- the surface of experiment could be uneven or
- the encoder noise

1.4 Fixing Systematic Errors

The robot is observed to have a systematic error by consistently drifting slightly to the left. Optimally, measurements could be made to identify the root cause of the systematic error and adjustments can be made. The systematic error can also be compensated during the simulation by having a small positive steering angle to the right to force the robot to drive a straight line.

2 Positioning and Error Propagation

2.1 Deriving x, y, and θ as functions from N_L and N_R

The derivation will start by substituting ΔU_L and ΔU_R into equations for ΔU_i and $\Delta\theta_i$.

$$\begin{aligned}
\Delta U_i &= \frac{\Delta U_R + \Delta U_L}{2} \\
&= \frac{\left(\frac{\pi \cdot d_{wheel}}{c_R} \cdot N_R\right) + \left(\frac{\pi \cdot d_{wheel}}{c_L} \cdot N_L\right)}{2} \\
&= \frac{\pi \cdot d_{wheel}}{2}\left(\frac{N_R}{c_R} + \frac{N_L}{c_L}\right)
\end{aligned}$$

$$\begin{aligned}
\Delta\theta_i &= \frac{\Delta U_R - \Delta U_L}{D} \\
&= \frac{\left(\frac{\pi \cdot d_{wheel}}{c_R} \cdot N_R\right) - \left(\frac{\pi \cdot d_{wheel}}{c_L} \cdot N_L\right)}{D} \\
&= \frac{\pi \cdot d_{wheel}}{D}\left(\frac{N_R}{c_R} - \frac{N_L}{c_L}\right)
\end{aligned}$$

Since the encoder resolutions of both wheels are identical, i.e.

$$c_L = c_R = c = 1024,$$

the expressions simplifies to

$$\Delta U_i = \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big)$$

$$\Delta\theta_i = \frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)$$

Substituting the values into x_i, y_i, and θ_i.

$$x_i = x_{i-1} + \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \cdot \cos\theta_i$$

$$y_i = y_{i-1} + \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \cdot \sin\theta_i$$

$$\theta_i = \theta_{i-1} + \frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)$$

Since the $x_{i-1} = 0$, $y_{i-1} = 0$, and $\theta_{i-1} = 0$, and substituting θ_i in x_i and y_i, the functions for x_i, y_i, and θ_i can be re-expressed as

$$x_i = \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \cdot \cos\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right)$$

$$y_i = \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \cdot \sin\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right)$$

$$\theta_i = \frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)$$

2.2 Calculating mean value of x, y, and θ

2.2.1 Runs 1 to 10

$$\bar{N_R} = 8177.0,\ \bar{N_L} = 8107.1$$

$$\begin{aligned}
\bar{\theta} &= \frac{\pi \cdot d_{wheel}}{cD}\big(\bar{N_R} - \bar{N_L}\big) \\
&= \frac{100\,mm \cdot \pi}{1024 \cdot 230\,mm}\big(8177.0 - 8107.1\big) \\
&= 0.0932\,rad \\
&= 5.34^{\circ}
\end{aligned}$$

$$
\begin{aligned}
\bar{x} &= \frac{\pi \cdot d_{wheel}}{2c}\left(\bar{N_R} + \bar{N_L}\right) \cdot \cos\bar{\theta} \\
&= \frac{100\,mm \cdot \pi}{2 \cdot 1024}\left(8177.0 + 8107.1\right)\cos(0.0932) \\
&= 2492.3\,mm \\
&= 2.49\,m
\end{aligned}
$$

$$
\begin{aligned}
\bar{y} &= \frac{\pi \cdot d_{wheel}}{2c}\left(\bar{N_R} + \bar{N_L}\right) \cdot \sin\bar{\theta} \\
&= \frac{100\,mm \cdot \pi}{2 \cdot 1024}\left(8177.0 + 8107.1\right)\sin(0.0932) \\
&= 232\,mm \\
&= 0.24\,m
\end{aligned}
$$

2.2.2 Runs 11 to 20

$$\bar{N_R} = 8166.0,\ \bar{N_L} = 8093.8$$

$$
\begin{aligned}
\bar{\theta} &= \frac{\pi \cdot d_{wheel}}{cD}\left(\bar{N_R} - \bar{N_L}\right) \\
&= \frac{100\,mm \cdot \pi}{1024 \cdot 230\,mm}\left(8166.0 - 8093.8\right) \\
&= 0.0963\,rad \\
&= 5.51^{\circ}
\end{aligned}
$$

$$\begin{aligned}
\bar{x} &= \frac{\pi \cdot d_{wheel}}{2c}\left(\bar{N_R} + \bar{N_L}\right) \cdot \cos\bar{\theta} \\
&= \frac{100\,mm \cdot \pi}{2 \cdot 1024}\left(8166.0 + 8093.8\right)\cos(0.0963) \\
&= 2489.7\,mm \\
&= 2.49\,m
\end{aligned}$$

$$\begin{aligned}
\bar{y} &= \frac{\pi \cdot d_{wheel}}{2c}\left(\bar{N_R} + \bar{N_L}\right) \cdot \sin\bar{\theta} \\
&= \frac{100\,mm \cdot \pi}{2 \cdot 1024}\left(8166.0 + 8093.8\right)\sin(0.0963) \\
&= 239\,mm \\
&= 0.24\,m
\end{aligned}$$

2.3 Deriving standard deviation of x, y, and θ

The Gaussian law of error propagation will be used to determine the standard deviation of x, y, and *theta*. The generic Gaussian law of error propagation states that

$$s_{\bar{z}} = \sqrt{\left(f_x(\bar{x},\bar{y}) \cdot s_{\bar{x}}\right)^2 + \left(f_y(\bar{x},\bar{y}) \cdot s_{\bar{y}}\right)^2}$$

where f_x and f_y are partial derivatives with respect to x and y,

$$f_x = \frac{\partial f}{\partial x},\ f_y = \frac{\partial f}{\partial y}$$

given that $\bar{z}$ is a function of $\bar{x}$ and $\bar{y}$

$$\bar{z} = f(\bar{x},\bar{y})$$

Since the robot coordinates x, y, and θ depends on encoders on the right and left wheels N_R and N_L, the Gaussian law of error propagation can be rewritten as

$$s_{\bar{z}} = \sqrt{\left(f_{N_R}(\bar{N_R},\bar{N_L}) \cdot s_{\bar{N_R}}\right)^2 + \left(f_{N_L}(\bar{N_R},\bar{N_L}) \cdot s_{\bar{N_L}}\right)^2}$$

with f_{N_R} and f_{N_L} being partial derivatives with respect to N_R and N_L,

$$f_{N_R} = \frac{\partial f}{\partial N_R},\ f_{N_L} = \frac{\partial f}{\partial N_L}$$

and z being the robot's coordinates x, y, or θ.

2.3.1 Standard deviation of x

From section 2.1, x can be expressed in term of N_R and N_L as

$$x = f(N_R, N_L) = \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \cdot \cos\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right)$$

The standard deviation for x can be expressed as

$$s_{\bar{x}} = \sqrt{\big(f_{N_R}(\bar{N_R}, \bar{N_L}) \cdot s_{\bar{N_R}}\big)^2 + \big(f_{N_L}(\bar{N_R}, \bar{N_L}) \cdot s_{\bar{N_L}}\big)^2}$$

$$s_{\bar{x}} = \sqrt{\left(\frac{\partial x}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial x}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

Calculating $\frac{\partial x}{\partial N_R}$

Let

$$u = N_R + N_L$$

$$v = \cos\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right)$$

The derivative can therefore be calculated by the use of the product rule

$$\frac{\partial x}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\left(\frac{\partial u}{\partial N_R} \cdot v + u \cdot \frac{\partial v}{\partial N_R}\right)$$

$$\frac{\partial u}{\partial N_R} = 1$$

To calculate $\frac{\partial v}{\partial N_R}$, we let

$$a = \cos(b)$$

$$b = \frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)$$

The derivative can be calculated by the use of the chain rule

$$\frac{\partial v}{\partial N_R} = \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial N_R}$$

$$= -\sin(b) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= -\sin\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

Substituting $\frac{\partial u}{\partial N_R}$ and $\frac{\partial v}{\partial N_R}$ back into $\frac{\partial x}{\partial N_R}$, we get

$$\frac{\partial x}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\left[\cos\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) - (N_R + N_L) \cdot \right.$$
$$\left. \sin\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}\right]$$

Using the expressions for θ and ΔU from section 2.1, the expression can be simplified to

$$\frac{\partial x}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\cos(\theta) - \Delta U \sin(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

Calculating $\frac{\partial x}{\partial N_L}$

The derivation for $\frac{\partial x}{\partial N_L}$ is similar to derivation for $\frac{\partial x}{\partial N_R}$.

However, since N_L is negative for cos, $\frac{\partial v}{\partial N_L}$ is

$$\frac{\partial v}{\partial N_L} = \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial N_L}$$

$$= -\sin(b) \cdot -\left(\frac{\pi \cdot d_{wheel}}{cD}\right)$$

$$= \sin\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

Therefore

$$\frac{\partial x}{\partial N_L} = \frac{\pi \cdot d_{wheel}}{2c} \Bigg[\cos\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) + (N_R + N_L) \cdot$$

$$\sin\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}\Bigg]$$

$$= \frac{\pi \cdot d_{wheel}}{2c} \cos(\theta) + \Delta U \sin(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

2.3.2 Standard deviation of y

From section 2.1, y can be expressed in term of N_R and N_L as

$$y = f(N_R, N_L) = \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \cdot \sin\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right)$$

The standard deviation for y can be expressed as

$$s_{\bar{y}} = \sqrt{\big(f_{N_R}(\bar{N_R}, \bar{N_L}) \cdot s_{\bar{N_R}}\big)^2 + \big(f_{N_L}(\bar{N_R}, \bar{N_L}) \cdot s_{\bar{N_L}}\big)^2}$$

$$s_{\bar{y}} = \sqrt{\left(\frac{\partial y}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial y}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

Calculating $\frac{\partial y}{\partial N_R}$

Let

$$u = N_R + N_L$$

$$v = \sin\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right)$$

The derivative can therefore be calculated by the use of the product rule

$$\frac{\partial y}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\left(\frac{\partial u}{\partial N_R} \cdot v + u \cdot \frac{\partial v}{\partial N_R}\right)$$

$$\frac{\partial u}{\partial N_R} = 1$$

To calculate $\frac{\partial v}{\partial N_R}$, we let

$$a = \sin(b)$$

$$b = \frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)$$

The derivative can be calculated by the use of the chain rule

$$\frac{\partial v}{\partial N_R} = \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial N_R}$$

$$= \cos(b) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \cos\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

Substituting $\frac{\partial u}{\partial N_R}$ and $\frac{\partial v}{\partial N_R}$ back into $\frac{\partial y}{\partial N_R}$, we get

$$\frac{\partial y}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\Bigg[\sin\left(\frac{\pi \cdot d_{wheel}}{cD}\left(N_R - N_L\right)\right) + (N_R + N_L) \cdot$$

$$\cos\left(\frac{\pi \cdot d_{wheel}}{cD}\left(N_R - N_L\right)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}\Bigg]$$

Using the expressions for θ and ΔU from section 2.1, the expression can be simplified to

$$\frac{\partial y}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\sin(\theta) + \Delta U\cos(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

Calculating $\frac{\partial y}{\partial N_L}$

The derivation for $\frac{\partial y}{\partial N_L}$ is similar to derivation for $\frac{\partial y}{\partial N_R}$.

However, since N_L is negative for sin, $\dfrac{\partial v}{\partial N_L}$ is

$$\frac{\partial v}{\partial N_L} = \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial N_L}$$

$$= \cos(b) \cdot -\left(\frac{\pi \cdot d_{wheel}}{cD}\right)$$

$$= -\cos\left(\frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

Therefore,

$$\frac{\partial y}{\partial N_L} = \frac{\pi \cdot d_{wheel}}{2c}\left[\sin\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) - (N_R + N_L) \cdot \right.$$

$$\left. \cos\left(\frac{\pi \cdot d_{wheel}}{cD}(N_R - N_L)\right) \cdot \frac{\pi \cdot d_{wheel}}{cD}\right]$$

$$= \frac{\pi \cdot d_{wheel}}{2c}\sin(\theta) - \Delta U\cos(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

2.3.3 Standard deviation of θ

From section 2.1, θ can be expressed in term of N_R and N_L as

$$\theta = f(N_R, N_L) = \frac{\pi \cdot d_{wheel}}{cD}\big(N_R - N_L\big)$$

The standard deviation for θ can be expressed as

$$s_{\bar{\theta}} = \sqrt{\big(f_{N_R}(\bar{N_R}, \bar{N_L}) \cdot s_{\bar{N_R}}\big)^2 + \big(f_{N_L}(\bar{N_R}, \bar{N_L}) \cdot s_{\bar{N_L}}\big)^2}$$

$$s_{\bar{\theta}} = \sqrt{\left(\frac{\partial \theta}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial \theta}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

$$\frac{\partial \theta}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{cD}$$

$$\frac{\partial \theta}{\partial N_L} = -\frac{\pi \cdot d_{wheel}}{cD}$$

Therefore,

$$s_{\bar{\theta}} = \sqrt{\left(\frac{\pi \cdot d_{wheel}}{cD} \cdot s_{\bar{N_R}}\right)^2 + \left(-\frac{\pi \cdot d_{wheel}}{cD} \cdot s_{\bar{N_L}}\right)^2}$$

Since the magnitude of the coefficients of both $s_{\bar{N_R}}$ and $s_{\bar{N_L}}$ are the same and since the negative sign is made invalid by squaring, the expression can be simplified as

$$s_{\bar{\theta}} = \frac{\pi \cdot d_{wheel}}{cD} \sqrt{s_{\bar{N_R}}^2 + s_{\bar{N_L}}^2}$$

2.4 Calculating standard deviation of x, y, and $theta$

2.4.1 Runs 1 to 10

$$\bar{N_R} = 8177.0,\ \bar{N_L} = 8107.1,\ s_{\bar{N_R}} = 144.25,\ s_{\bar{N_L}} = 150.04$$

$$\begin{aligned}\Delta U &= \frac{\pi \cdot d_{wheel}}{2c}\big(N_R + N_L\big) \\ &= \frac{\pi \cdot 100\,mm}{2 \cdot 1024}\big(8177.0 + 8107.1\big) \\ &= 2495 mm = 2.495\,m\end{aligned}$$

$$\begin{aligned}\bar{\theta} &= \frac{\pi \cdot d_{wheel}}{cD}\big(\bar{N_R} - \bar{N_L}\big) \\ &= \frac{100\,mm \cdot \pi}{1024 \cdot 230\,mm}\big(8177.0 - 8107.1\big) \\ &= 0.0932\,rad\end{aligned}$$

Calculating $s_{\bar{x}}$

$$\begin{aligned}\frac{\partial x}{\partial N_R} &= \frac{\pi \cdot d_{wheel}}{2c}\cos(\theta) - \Delta U \sin(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD} \\ &= \frac{\pi \cdot 100\,mm}{2 \cdot 1024}\cos(0.0932) - 2495\,mm \cdot \sin(0.0932) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm} \\ &= -0.1569\end{aligned}$$

$$\frac{\partial x}{\partial N_L} = \frac{\pi \cdot d_{wheel}}{2c}\cos(\theta) + \Delta U \sin(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024}\cos(0.0932) + 2495\,mm \cdot \sin(0.0932) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= 0.4621$$

$$s_{\bar{x}} = \sqrt{\left(\frac{\partial x}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial x}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

$$= \sqrt{(-0.1569 \cdot 144.25)^2 + (0.4621 \cdot 150.04)^2}$$

$$= 72.95\,mm = 0.073\,m$$

Calculating $s_{\bar{y}}$

$$\frac{\partial y}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c}\sin(\theta) + \Delta U \cos(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024}\sin(0.0932) + 2495\,mm \cdot \cos(0.0932) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= 3.331$$

$$\frac{\partial y}{\partial N_L} = \frac{\pi \cdot d_{wheel}}{2c}\sin(\theta) - \Delta U \cos(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024}\sin(0.0932) - 2495\,mm \cdot \cos(0.0932) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= -3.303$$

$$s_{\bar{y}} = \sqrt{\left(\frac{\partial y}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial y}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

$$= \sqrt{(3.331 \cdot 144.25)^2 + (-3.303 \cdot 150.04)^2}$$

$$= 689.6\,mm = 0.69\,m$$

Calculating $s_{\bar{\theta}}$

$$s_{\bar{\theta}} = \frac{\pi \cdot d_{wheel}}{cD} \sqrt{s^2_{\bar{N_R}} + s^2_{\bar{N_L}}}$$

$$= \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm} \sqrt{144.25^2 + 150.04^2}$$

$$= 0.2775\,rad = 15.9°$$

Therefore, for run 1 to 10, $s_{\bar{x}} = 0.073\,m$, $s_{\bar{y}} = 0.69\,m$, and $s_{\bar{\theta}} = 15.9°$.

2.4.2 Runs 11 to 20

$$\bar{N_R} = 8166.0,\ \bar{N_L} = 8093.8,\ s_{\bar{N_R}} = 77.23,\ s_{\bar{N_L}} = 82.75$$

$$\Delta U = \frac{\pi \cdot d_{wheel}}{2c} \big(N_R + N_L\big)$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024} \big(8166.0 + 8093.8\big)$$

$$= 2494mm = 2.494\,m$$

$$\bar{\theta} = \frac{\pi \cdot d_{wheel}}{cD} \big(\bar{N_R} - \bar{N_L}\big)$$

$$= \frac{100\,mm \cdot \pi}{1024 \cdot 230\,mm} \big(8166.0 - 8093.8\big)$$

$$= 0.0963\,rad$$

Calculating $s_{\bar{x}}$

$$\frac{\partial x}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c} \cos(\theta) - \Delta U \sin(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024} \cos(0.0963) - 2494\,mm \cdot \sin(0.0963) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= -0.168$$

$$\frac{\partial x}{\partial N_L} = \frac{\pi \cdot d_{wheel}}{2c} \cos(\theta) + \Delta U \sin(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024} \cos(0.0963) + 2494\,mm \cdot \sin(0.0963) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= 0.473$$

$$s_{\bar{x}} = \sqrt{\left(\frac{\partial x}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial x}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

$$= \sqrt{(-0.168 \cdot 77.23)^2 + (0.473 \cdot 82.75)^2}$$

$$= 41.3\,mm = 0.041\,m$$

Calculating $s_{\bar{y}}$

$$\frac{\partial y}{\partial N_R} = \frac{\pi \cdot d_{wheel}}{2c} \sin(\theta) + \Delta U \cos(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024} \sin(0.0963) + 2494\,mm \cdot \cos(0.0963) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= 3.312$$

$$\frac{\partial y}{\partial N_L} = \frac{\pi \cdot d_{wheel}}{2c}\sin(\theta) - \Delta U \cos(\theta) \cdot \frac{\pi \cdot d_{wheel}}{cD}$$

$$= \frac{\pi \cdot 100\,mm}{2 \cdot 1024}\sin(0.0963) - 2494\,mm \cdot \cos(0.0963) \cdot \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}$$

$$= -3.282$$

$$s_{\bar{y}} = \sqrt{\left(\frac{\partial y}{\partial N_R} \cdot s_{\bar{N_R}}\right)^2 + \left(\frac{\partial y}{\partial N_L} \cdot s_{\bar{N_L}}\right)^2}$$

$$= \sqrt{(3.312 \cdot 77.23)^2 + (-3.282 \cdot 82.75)^2}$$

$$= 373.2\,mm = 0.373\,m$$

Calculating $s_{\bar{\theta}}$

$$s_{\bar{\theta}} = \frac{\pi \cdot d_{wheel}}{cD}\sqrt{s_{\bar{N_R}}^2 + s_{\bar{N_L}}^2}$$

$$= \frac{\pi \cdot 100\,mm}{1024 \cdot 230\,mm}\sqrt{77.23^2 + 82.75^2}$$

$$= 0.1510\,rad = 8.65^\circ$$

Therefore, for run 11 to 20, $s_{\bar{x}} = 0.041\,m$, $s_{\bar{y}} = 0.373\,m$, and $s_{\bar{\theta}} = 8.65^\circ$.

2.5 Discussions of finding

From both of the calculations, the standard deviation for y, $s_{\bar{y}}$ is much greater than the standard deviation for x, $s_{\bar{x}}$. The standar deviation for θ, $s_{\bar{\theta}}$ is also significant (15.9° and 8.65°). This indicates that the estimated pose has significant uncertainty in heading, which leads to large uncertainty in lateral position.

3 Odometry Implementation

Using the MERLIN Odometry equations provided in Section 2.5 of the tutorial, the odometry function can be written as follows.

```
function odometry(encoder_left, encoder_right, steering_angle,
   current_pose)
{
    // constant parameters for MERLIN robot
    // WHEEL_DIAMETER and WHEEL_BASE are in mm

    const WHEEL_DIAMETER = 100;
    const ENCODER_RES = 1024;
    const WHEEL_BASE = 230;

    let X = current_pose[0];
    let Y = current_pose[1];
    let theta = current_pose[2];

    const wheel_circumference = Math.PI * WHEEL_DIAMETER;

    // incremental distance for each wheeel [2.9]

    let delta_U_left  = (encoder_left  / ENCODER_RES) * wheel_
       circumference;
    let delta_U_right = (encoder_right / ENCODER_RES) * wheel_
       circumference;

    // distance traveled [2.10]

    let delta_U = (delta_U_right + delta_U_left) / 2.0;

    // incremental change of robot orientation [2.11]

    let delta_theta = (delta_U_right - delta_U_left) / WHEEL_
       BASE;

    // update robot pose [2.12]

    let X_new = X + delta_U * Math.cos(theta);
    let Y_new = Y + delta_U * Math.sin(theta);
```

```
    let theta_new = theta + delta_theta;

    // return updated robot pose

    return [X_new, Y_new, theta_new];
}
```