

1. Is the following a proper distance function? Why? Explain your answer. Measure the distance b/w $(0, 0, 0)$ & $(0, 1, 0)$.

$$d(x, y) = \sum ((x_i - y_i)^5)$$

→ Distance function can only be proper function if it follows the following properties—

i) $d(x, y) \geq 0$ & $d(x, y) = 0$ if $(x = y)$

ii) $d(x, y) = d(y, x)$

A term can be negative, but the distance can never be negative.

$$d(x, y) \text{ for } (0, 0, 0) \text{ \& } (0, 1, 0) = (0-0)^5 + (0-1)^5 + (0-0)^5 = \underline{\underline{-1}} \quad \textcircled{\text{I}}$$

$$d(y, x) \text{ for } (0, 0, 0) \text{ \& } (0, 1, 0) = (0-0)^5 + (1-0)^5 + (0-0)^5 = \underline{\underline{1}} \quad \textcircled{\text{II}}$$

① violates property (i) &

① \neq ②; violates property (ii).

Hence $d(x, y) = \sum ((x_i - y_i)^5)$ is not a proper distance function.

$$\begin{aligned} \text{Distance b/w } (0, 0, 0) \text{ \& } (0, 1, 0) &= \sqrt{(0-0)^2 + (0-1)^2 + (0-0)^2} \\ &= \sqrt{0 + 1 + 0} \\ &= \sqrt{1} \\ &= 1 \text{ unit} \end{aligned}$$