Date. 19/02/25

* SEMESTER - 2nd *

* DISCRETE - MATHEMATICAL STRUCTURES *

By reasoning we draw valid inferences from some well known facts. in mathematical language there are two kinds of reasoning namely:-
i) INDUCTIVE & ii) DEDUCTIVE mathematical induction is som short of inductive reasoning,

- SENTENCES:- a group of words which makes, complete sense. is called a sentence.
  - sentences helps us to communicate our ideas with other.
  - sentences are classified as:-
    i) declarative (assertive sentence)
    ii) imperative sentence,
    iii) exclamatory sentence.
    iv) Interrogative sentence.

i) Declarative sentence:- a sentence that makes an assertion is called a declarative sentence.
- The earth is oval.
- The sum of all angles of triangle is 180°,
- 2 is only even prime number.

ii) Imperative sentence:- a sentence that express a request, or a command is called a imperative sentence.

Ex:- Switch off the fan,
Give me a glass of water.
Keep quiet. (request / command).

iii) Interrogative sentence:- a sentence that ask some question is called an interrogative sentence.

Ex:- where are you coming from?

is every set finite.

iv) exclamatory sentence:- a sentence that express some strong feeling called an exclamatory sentence.

Ex:- Hurrah! we have won the match.

* In mathematical terms, we shall interested in particular types of declarative sentences called statements.

↳ Statement:- a declarative sentence which can be judge to be true or false but not both is called a statement.

Ex:- whether the following sentences are statement

1) The earth is a planet. (True statement)
2) 10 is a prime no. (false statement)
3) Sandip university is the best university (Not a statement)
4) There are 35 days in a month. (false statement)
5) Mathematics is difficult. → Not statement.
6) Every set is a finite set. → false statement.
7) Every real no. is a complex no. → True statement
8) The square of a natural no. is an even no. → false
9) 9 is less than 7. → false statement.
10) Every square is a rectangle. → True statement.

↳ Sentences involving variable time such as yesterday, today, & tomorrow are not statements.

Ex:- Today is sunday.

Yesterday was a cold day.

Tomorrow is a holiday.

↳ Sentences involving variables places such as here, There, e.t.c. are not statement.

Ex:- Sijoul is far from here.

↳ Sentences involving variable pronounce such as he, she, They, you, e.t.c are not statements.

Ex:- She is a Teacher.

he is a doctor.

↳ EXERCISE - 1A.

1. Which of the following sentences are statements? In case of a statement mention whether it is True or false.

(i) The sun is a star. → True statement.

(ii) $\sqrt{7}$ is an irrational number. → True statement.

(iii) The sum of 5 and 6 is less than 10. → false statement.

(iv) Go to your class. → Not a statement.

(v) Ice is always cold. → True statement.

(vi) Have you ever seen the red fort? → Not a statement.

(vii) Every relation is a function → false statement.

(viii) The sum of any two sides of a triangle is always

Greater than the third side. → True Statement.

(ix) May God bless you! → Not a statement.

2. Which of the following sentences are statements? In case of a statement, mention whether it is true or false.

(i) Paris is in France. → True Statement.
(ii) Each prime number has exactly two factors. → True Statement
(iii) The equation $x^2+5|x|+6=0$ has no real roots. → False
(iv) $(2+\sqrt{3})$ is a complex number → False
(v) Is 6 a positive integer? → Not statement
(vi) The product of -3 and -2 is -6. → False
(vii) The angles opposite to the equal sides of an isosceles triangle are equal. → True.
(viii) Oh! It's too hot. → Not statement.
(ix) Monika is a beautiful girl. → Not a Statement.
(x) Every quadratic equation has at least one real root. → False.

* → NEGATION OF A STATEMENT :- The deny of an assertion content in a statement is called its negation. It $(\sim p)$ denotes the negation of denial of p.

The deny of a statement can be expressed in various ways.
Let p: Kolkata is a city, express the denial of p in 3 different ways.

i) p: It is false that kolkata is a city.
ii) p: kolkata is not a city.
iii) p: It is not the case that kolkata is a city.

H.W.: 2) write the negation in 2 different ways.
let p: africa is a continaint.
let p: all aintegers are rational numbers.
let p: sum of prime numbers are odd number.
let p: $\sqrt{5}$ is a rational number.
let p: everyone at sandip university speaks english.

i) p: africa is a continent.
p: It is false that africa is a continent.
p: africa is ~~a~~ not a continent.
p: It is not the case that africa is a continent.

ii) p: It is false that all integers are rational numbers.
p: all integers are not rational numbers.
p: It is not the case that all integers are rational numbers.

iii) p: It is false that sum of prime numbers are odd number.
p: sum of prime numbers are not odd number.
p: It is not the case that sum of prime no. are odd no.

iv) p: It is false that $\sqrt{5}$ is a rational number.
p: $\sqrt{5}$ is not a rational number.
p: It is not the case that $\sqrt{5}$ is a rational number

Q3) Which statements are true and which are false? and Give me reason.

(i) p: $\sqrt{11}$ is an irrational number. (True statement).
(ii) p: circle is a particular case of an ellipse. (True).
→ Reason: A circle is an ellipse in which ellipse can be defined as set of all points.

*negation:-
(i) → There exist a natural number which is not greater than 0.

(iii) r: Each radius of a circle is a chord of the circle.
↳ false statement. (chord join two points on circle), radius join center to a point on circle

(iv) The center of a circle bisects each chord of the circle.
↳ False. (The center only bisects a chord if chord pass through the center.

(v) If a & b are integers such that $a < b$ then $-a > -b$.
└ True. (multiplying by -1 reverses the inequality sign).

(vi) The Quadratic equation $x^2 + x + 1 = 0$ has no real roots.
└ True.

*negation:-
(ii) Both the diagonals of a rectangle are equal".
∴ At least one diagonal of a rectangle is not equal to the other.
(iii) The sum of 4 and 5 is 8" → The sum of 4 and 5 is not 8.

↳ Logical Connectives:-

★ simple statement:- A statement which does not contain any other statement on its component part is called a simple statement.

Ex:- Hyderabad is the capital of Andhra pradesh.

Q. The set of real number is an infinite set.

- The sum of 7 & 4 is greater than 10.

★ Compound statement:- a compound statement is a combination of Two or more simple sentences is called a compound statement.

- The method of combining Two or more simple sentences called compounding.

★ Logical connectives:- The word of phrases which connect two or more simple sentences are called logical connectives or simply connectives.

Generally, there are four types of connectives.
i) And , ii) or, iii) If ---then --, iv) If and only if

Connectives	Symbol	nature of compound statement
and	$\wedge$	conjuction
or	$\vee$	disjunction
If --- then --	$\Rightarrow$	implication/ conditional
If and only if	$\Leftrightarrow$	biconditional / equivalence

- Some example of compound statement:

Ex:- 1) 2 is an even no. & 3 is an odd number.

2) 5 is an rational no. & $\sqrt{5}$ is an irrational no.

3) The sum of 4 & 5 is 8 or 9.

4) If it rains then university may be closed.

5) A triangle is equiangular if and only if it is equilateral.

Given examples of connective :- 1, 2, 3, 4 & 5 are respectively, and, and, or, If -- then --, If and only if.

Compounding of sentences:-

1) Conjunction:- Two simple sentences connected by the word and are said to be a form of conjunction. & we denoted it by "and" the symbol for it is "$\wedge$".

Truth table for $p \wedge Q$:-

- Truth table for $p \wedge Q$ is true only when each one of the statement $p \wedge Q$ is true.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

- Some points to remember:-

I) The compound statement with 'and' is true if all its component statements are true.

II) The compound statement with 'and' is false if any of its component statement is false or all of its component statement are false.

Q. Split each of the following compound statements into simple sentences and determine whether it is true or false.

1) The Grass is Green & the sky is blue. P: True & Q: True So both statements are true.
2) Agra is in UP & Shimla is in pujab. P: True & Q: false
3) all rational numbers are real and all real numbers are not complex numbers. P: True Q: falses.
4) $x=5$ & $x=2$ are the roots of the equation $3x^2-x-10=0$. P: False Q: True.

negation:

IV) The number 6 is greater than 4 :→ The number 6 is not greater than 4.

V) Every natural number is an integer :→ There exist at least one natural number that is not an integer.

VI) Negation of "The number -5 is a rational number": The number -5 is not a rational number.

VII) Negation of "All cats scratch": It is false that all

Cats Scratch.

viii) There exists a rational number x such that $x^2 = 3$":
⇒ It is false that there exists a rational number x such that $x^2 = 3$.

ix) All students study mathematics at the elementary level": ⇒ All students not study mathematics at the elementary.

x) Every student has paid the fees. ⇒ It is false that Every student has paid the fees.

- **Disjunction:-** Two simple sentences connected by the word 'or' are said to be Disjunction. and it is denoted by. ($\vee$)

- Truth Table p or q :-

p or q true only when either p is True or q is True or both are True.

[$p \vee Q$]

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

* A compound with 'or' is true when atleast one of them is True. otherwise, inclusive 'or' is used

Short type Question:- Imp

(1) Define conjunction & disjunction with example and also write Truth table for it.

- For each of the following statement determine whether an inclusive 'or' exclusive, and give reason for your answer:

1) For identification ~~or~~ you need a passport 'or' an aadhar card.

Ans Here 'or' is inclusive since a person can have both passport and aadhar card for identification.

2) The university is closed if it is a holiday or sunday.

Ans Here 'or' is inclusive since if it is a holiday or sunday the university is closed.

3) $\sqrt{3}$ is a rational number or an irrational number.

Ans Here, p: $\sqrt{3}$ is a rational number.
Q: $\sqrt{3}$ is an irrational number.
Here, The statement p is false and Q is True, so p ~~and~~ or Q is True. but or is exclusive.

4) Two lines intersect at a point or are parallel.

Ans Here, 'or' is exclusive because it is not possible for Two lines to intersect and parallel together.

5) Students can take sanskrit or french as their Third language.

Ans Here, 'or' is exclusive because we can't take both sanskrit & french as their third language.

Open sentences and their truth sets:-
Let p(x) be a declarative sentence involving a variable x and let a be a given set. such that for small a belongs to A. ($a \in A$).
P(a) is true or false. then p(x) is called an open sentence defined on a.

The subset of a consisting all those values of x. which convert p(x) into a true statement is called a Truth set of the open sentence p(x).

Ex:-
Q. Write down the truth set of each of the following open sentences.

1) $p(x) = x+5<9, x \in N$
Truth set = $\{x \in N, x+5<9\}$ $x<4$.
$\{x \in \{1,2,3\}\}$

2) $p(x) = x+3<3, x \in N$
Truth set = $\{x \in 0, x = \phi\}$

3) $p(x) = x+5>7, x \in R$
Truth set = $\{x \in (3, \infty)\}$.

4) $p(x) = 2x^2+5x-3 = 0, x \in I$
Truth Set = $\{x = -3\}$.

~~$2x^2+3x+2x-3=0$~~

x ~~$(2x+3)+1$~~ $(2x+3)$

$2x^2+5x-3=0$

$2x^2+6x-x-3=0$

$2x(x+3)-1(x+3)=0$

$2x-1=0$ or $x+3=0$

$x=\frac{1}{2}$ or $x=-3$.

• Quantifiers & Quantify statements:

- Two important symbols:- The symbol stands for '$\forall$' for all values" this is also known as universal Quantifiers. The symbol stands for '$\exists$' there exists" this symbol is also known as existential Quantifier.

○ Quantified statement:- a statement an open sentence with a quantifier between a statement called a Quantified statement.

- Use Quantifiers to convert each of the following open sentences defined on 'N', into a true statement.

i) $x+5=9$,

Ans $\exists\ x$ (belongs to) $\in N$
$x \in N$,
such that $4 \in N$, so
It is a true statement.
& satisfies the given equation
$x+5=9$.

ii) $x+7<11$.

Ans $\exists\ x \in N$.
such that $x<4$, so,
It is a true statement.

iii) $x+3>8$.

Ans $\exists\ x \in N$.
such that $x>5$,
so It is a true statement.

iv) $x^2 \geq 0$.

Ans $\exists\ x \in N$.
& we know that square of every natural number is ~~greater~~ positive & greater than 0 & satisfies the eqn.

v) Let $A=\{1,2,3,4\}$.
examine whether the statements given below are true or false.

Ans $\exists\ x \in A$, such that
$x+3=9$.
The given statement is false because no number in A satisfies $[x+3=9]$.

Q. $\forall\ x \in A$, such that $x+2<7$.

Ans $x<5$;
The given statement is true because every number in A satisfies $[x+2<7]$

Q. $x \in A$, such that $x+1<3$.

Ans $x<2$,
the given statement is true because $x \in A$ satisfies the eq^n.
$x+1<3$.

Q. $\forall\ x \in A$, such that $x+3>=5$.

Ans $x>=2$.
The given statement is false because $x \in A$ satisfies the eq^n $x+3>=5$.

EXERCISE 1(B)

1) Split each of the following into simple sentences and determine whether it is true or false.

(i) A line is straight and extends indefinitely in both the directions.

Ans P: A line is straight. (True)
Q: indefinitely in both the directions (True).
$P \cap Q$ = True.

(ii) A point occupies a position and its location can be determined.

Ans P: A point occupies a position (True)
Q: its location can be determined (True)
$\therefore P \cap Q$ = True.

(iii) The sand heats up quickly in the sun and does not cool down fast at night.

Ans P: The sand heats up quickly in the sun (True)
Q: does not cool down fast at night. (True)
$\therefore P \cap Q$ = True.

(iv) 32 is divisible by 8 and 12.

Ans P: 32 is divisible by 8 (True) $\therefore P \cap Q$ = false.
Q: 32 is divisible by 12. (false)

(v) $x=1$ and $x=2$ are the roots of the equation $x^2-x-2=0$
P: $x=1$ are the roots of equation $x^2-x-2=0$ (false)
Ans Q: $x=2$ are the roots of equation $x^2-x-2=0$ (True)
$\therefore P \cap Q$ = false

(vi) 3 is rational and $\sqrt{3}$ is irrational
Ans P: 3 is rational (True)
Q: $\sqrt{3}$ is irrational (True).
$P \cap Q$ = True.

(vii) All integers are rational numbers and all rational numbers are not real numbers.

Ans P: All integers are rational numbers. (False)
Q: all rational numbers are not real numbers. (True)
$P \cap Q$ = false.

(viii) Lucknow is in Uttar pradesh and Kanpur is in Uttrakhand.

Ans P: Lucknow is in Uttar pradesh (True)
Q: Kanpur is in Uttrakhand. (false)
$P \cap Q = f$.

L Conditional & biconditional statements

OBJ:-

1) Conditional statements:- Two sentences connected by "If --- then" is called an conditional statements.

*2) ~~biconditional st~~ for any statements p & Q, the statement If 'p' ~~&~~ then 'q' is denoted by $\boxed{p' \Rightarrow Q'}$ which means p' implies Q' where ~~as~~ p is called antecedent & Q is called the consequent.

* We can write if 'p' then 'q' by different ways:-
1) p' implies Q' written as $[p \Rightarrow Q]$.
2) p' only ~~than~~ if Q', Q is a necessary condition for p.
3) p is a sufficient condition for Q. $[\sim p = \sim Q]$.

IMP SHORT QUE ~~Re~~ ~~write~~ write the following statement with if then in 5 different ways conveying the same meaning:-
1) If a natural number is even, then its square is even. Ans a natural number is even implies that its

square is even.

- a natural is even if only if its square is even.
- It is necessary that square of natural number is even if natural number is even.
- for the square of a natural number to be even, it is sufficient that the natural number is even.
- If a square of a natural number is not even, then the natural number is not even.

Q.2) Splitting into simple sentences and determining the type of 'or':

1) The sum of 3 and 7 is 10 or 11.

Ans p: The sum of 3 and 7 is 10. (True).
Q: The sum of 3 and 7 is 11. (false).
This sentence is exclusive.

2) (1+i) is a real or a complex number.

Ans p: (1+i) is a real number (false)
Q: (1+i) is a complex number. (True)
This sentence is exclusive.

3) Every quadratic equation has one or two real roots.

Ans p: Every quadratic equation has one real roots. (false)
Q: Every quadratic equation has two real roots. (~~false~~) True
This sentence is exclusive.

Q.4) You are wet when it rains or you are in a river.

Ans p: You are wet when it rains (True)

Q: You are wet when you are in a river. (True)

This sentence is inclusive.

Q.5) 24 is a multiple of 5 or 8.

Ans p: 24 is a multiple of 5. (false)

Q: 24 is a multiple of 8. (True).

This sentence is exclusive.

Q.6) Every integer is rational or irrational.

Ans p: Every integer is rational (True)

Q: Every integer is irrational. (false).

This sentence is exclusive.

Q.7) For getting a driving license, you should have a ration card or a passport.

Ans p: You should have a ration card for getting a driving license (false)

Q: for getting a driving license, you should have a passport (false).

Q.8) 100 is a multiple of 6 or 8.

Ans p: 100 is a multiple of 6 (false)

Q: 100 is a multiple of 8. (false).

Q.9) Square of an integer is positive or negative.

Ans Square of an integer is positive (True).

Square of an integer is negative (false).

The sentence is Exclusive.

Q.10) Sun rises or Moon sets.

Ans p: Sun rises (True)
q: moon sets (True).
The sentence is inclusive.

Q.3) find truth sets for given open sentences:-

1) $x+2<10$.

Ans $x<8$, so truth set $\{1,2,3,4,5,6,7\}$.

2) $x+5<4$.

Ans $x<-1$, but since $x \in N$,
Truth set: $\{\phi\}$.

3) $x+3>2$

Ans $x>-1$, Truth set: $\{N\}$.

Q.4) Examining statements for set $A=\{2,3,5,7\}$.
1) $\exists x \in A$, such that $x+3>9$.
- checking values are:-
  - $2+3=5$ {false}.
  - $3+3=6$ {false}
  - $5+3=8$ {false}
  - $7+3=10$ {True}.
- True statements.

ii) $\exists\, x \in A$, such that x is even.

Ans Only $x = 2$ is even.

True.

iii) $\exists\, x \in A$ such that $x + 2 = 6$.

Ans
- $2+2=4$
- $3+2=5$
- $5+2=7$
- $7+2=9$.

false.

iv) $\forall\, x \in A$, x is prime.

Ans · $A = \{2, 3, 5, 7\}$ are prime.

True.

v) $\forall\, x \in A$, $x + 2 < 10$.

Ans
- $2+2=4<10$.
- $3+2=5<10$.
- $5+2=7<10$.
- $7+2=9<10$.

True.

vi) $\forall\, x \in A$, $x+4 \geq 11$.

Ans
- $2+4=6 \geq 11$.
- $3+4=7 \geq 11$.
- $5+4=9 \geq 11$.
- $7+4=11 \geq 11$.

false.

Q. Write each of the following statements in the form of if then.

(i) A quadrilateral is a parallelogram if its diagonals bisect each other.

Ans If the diagonals of the quadrilateral bisect each other then it is a parallelogram.

(ii) The banana tree will bloom if its stays warm for a month.

Ans if the banana tree stay warm, for a month then it will bloom.

(iii) There is traffic jam whenever it rains.

Ans If it rains, then there is traffic jam.

(iv) it is necessary, to have a password to logon to the server.

Ans If you have a password then you can logon to the server.

(v) you can access the website. if you pay a subscription fee,

Ans If you pay a subscription fee then you can access the website.

→ Converse and Contrapositive of statements:-

let p implies q then. $(p \Rightarrow q)$.

i) Converse of $(p \Rightarrow q)$ is $(q \Rightarrow p)$

ii) Contrapositive of $p \Rightarrow q$ is $[\sim q \Rightarrow \sim p]$

Ex:- write the converse & contrapositive of each of the following statements.

i) if n is an even no., then n^2 is even.

Sol^n: Its converse is: if n^2 is even, then n is an even no.
its contrapositive is: if n^2 is not even, then n is not even

ii) if Two integers A & B are such that $A > B$, then $A-B$ is always a positive integer

Sol^n: Its converse is: If $A-B$ is always a positive integer, then the two integers A & B are $A > B$,
Its contrapositive is: If $A-B$ is not a positive integer, then A & B are not $A > B$.

iii) if a triangle ABC is right angle at ~~the~~ B, then $(AB)^2 + (BC)^2 = (AC)^2$.

Sol^n: Its converse is: if $(AB)^2 + (BC)^2 = (AC)^2$, then a $\triangle ABC$ is right angle at B.

Its contrapositive is: If $(AB)^2 + (BC)^2 \neq (AC)^2$, then a $\triangle ABC$ is not a right angle at B.

iv) if $\triangle ABC$ & $\triangle BEF$ are congurent then they are equiangular.

Sol^n: Its converse:- if they are equiangular, then $\triangle ABC$ & $\triangle BEF$ are congurent.
Its contrapositive: if they are not equiangular, then $\triangle ABC$ & $\triangle BEF$ are not congurent.

v) you cannot comprehent geometry if you do not know how to region deductively.

Sol^n: Its converse:- If you donot know how to region deductively, then you cannot comprehent geometry.

Its contrapositive:- If you know how to region deductively, then you can comprehent geometry.

vi) Something is cold implies that it has low temperature

Sol:- Its converse:- If it has low temperature then there is something cold.

- Its contrapositive:- If it has high temperature then something hot.

Biconditional statement:-

Two simple sentences connected by "if & only if" form a biconditional statement. & it is denoted by "$\Leftrightarrow$"

$p \Leftrightarrow q$ is same as $(p \Rightarrow q)$ & $(q \Rightarrow p)$

OBJ:-

Q. $p \Leftrightarrow q = ?$

Ex:- In $\triangle ABC$, $\angle B = \angle C \Leftrightarrow AC = AB$. this statement is same as In $\triangle ABC$, $\angle B = \angle C \Rightarrow AC = AB$ & $AC = AB \Rightarrow \angle B = \angle C$

Q. Rewrite the following statements in the form:- $p \Leftrightarrow q$.

p:- if a quadrilateral is equiangular, then it is a rectangle
q:- if a quadrilateral is a rectangle, then it is equiangular

Sol:- The required statement is:-

- a quadrilateral is equiangular $\Leftrightarrow$ it is a rectangle.

Various cases showing various conditions:-

Q. Statement p is $\triangle ABC$ is equilateral,
Statement q is $\triangle ABC$ is equiangular,