Proceedings of the 29th Annual International Conference of the IEEE EMBS
Cité Internationale, Lyon, France
August 23-26, 2007.

Pendulum Analysis of an Integrated Accelerometer to assess its suitability to measure Dynamic Acceleration for Gait Applications

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Abstract— **Advances in surface micro-machining technology have led to the production of miniature, inexpensive, integrated accelerometers suitable for use in human movement analysis. The objective of this paper was to assess the ability of the bi-axial ADXL202 integrated accelerometer to measure dynamic acceleration for gait applications. Mounting the integrated accelerometer at the centre of oscillation of a custom-designed pendulum and subjected it to a known repeatable, varying acceleration signal similar to that experienced in gait. This allowed direct comparison of the predicted pendulum acceleration (derived from a goniometer) with the acceleration measured by the integrated accelerometer. The predicted pendulum acceleration and the acceleration measured by the integrated accelerometer matched to a high degree with errors of less than 2% for radial and 7% for tangential acceleration.**

I. Introduction

The idea of using accelerometers to assess human body movement was first proposed as early as the 1970's [1-2]. The accelerometer devices used in these original studies of human movements had low sensitivity, were relatively expensive, bulky, unreliable and subject to drift. However, in the past decade a revolution has taken place in the fabrication of accelerometers [3] due to major advances in integrated surface micro-machining technology, allowing for the miniaturization and integration of the acceleration sensor, with all of the required signal conditioning electronics, onto a single integrated circuit. We will refer to these new devices as integrated accelerometers.

Integrated accelerometers can measure both static (e.g. gravity) and dynamic (e.g. vibration) acceleration and have numerous applications in tilt, orientation, vibration and shock measurements. The integrated accelerometer under consideration in this paper is the ADXL202 bi-axial accelerometer from Analog Devices. In the ADXL202, the sensor element is a moveable beam suspended on polysilicon springs that provide a resistance against acceleration. Deflection of the beam due to applied acceleration is measured using a differential capacitor arrangement.

However to use these devices with confidence in human movement, it is necessary to confirm their ability to accurately measure the acceleration experienced in human movement applications.

Testing the performance of an integrated capacitive accelerometer in measuring static acceleration is very straight-forward, the device can be tested under a variety of static acceleration conditions using gravity, by moving the accelerometer device through a variety of inclinations and measuring the output voltage under these conditions.

To evaluate the dynamic response of an accelerometer under the acceleration conditions experienced in gait, a test rig capable of subjecting the accelerometer to a known, repeatable, varying acceleration signal similar to that experienced in gait, is required. This paper proposes that a pendulum could provide such an acceleration signal, and that the magnitude of this varying acceleration signal could be accurately determined independent of the accelerometer device under test.

II. Method

The motion of a body is classified as having simple harmonic motion if its acceleration towards a particular point is directly proportional to its displacement from that point [4]. It is from this fundamental theory of circular motion that one derives the necessary requirements to classify the fundamentals of the swinging pendulum.

Background theory

The net acceleration acting on the mass of a moving pendulum is composed of two acceleration contributions.

- The gravitational acceleration contribution.
- The inertial acceleration contribution.

Both of these accelerations have components parallel to the radius (shaft) of the pendulum, the radial acceleration, and perpendicular to the axis of the pendulum, the tangential acceleration. By combining their respective gravitational and inertial contributions it is possible to determine the overall tangential and radial accelerations (Equations 1 and 2).

$$a_t = \alpha r + gSin\theta \quad \text{Overall tangential acceleration} \quad (1)$$

$$a_r = \omega^2 + gCos\theta \quad \text{Overall radial acceleration} \quad (2)$$

Where α is the angular acceleration, ω is the angular velocity of the pendulum mass, r is the length of the pendulum and θ is the angular displacement of the swinging pendulum.

If the angular displacement of the pendulum from the vertical, θ, is known, then a_r and a_t can be determined through single differentiation (to determine ω) and double differentiation (to determine α) using Equations 1 and 2.

It is proposed that if a pendulum is fitted with a biaxial accelerometer mounted at its centre of mass, one axis of the

Manuscript received March 30, 2007.

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accelerometer orientated in the tangential and one in the radial direction, the accelerations recorded at these sites could then be compared to those predicted by Equations 1 and 2, obtained by measuring θ using a potentiometer-based electro-goniometer.

Thus a determination of the accuracy of the ADLX202 in measuring the gait-like dynamic acceleration associated with the pendulum can be made.

Pendulum design

For a simple pendulum, its' period of oscillation (T) is given by Equation 3.

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{Period of a simple pendulum} \qquad (3)$$

An ideal simple pendulum is *"a particle of mass m suspended from an inextendable, massless string"* [6]. While it was not possible to realise this ideal pendulum considerable efforts were made to make the designed physical pendulum as close as possible to this ideal. A physical pendulum, taking the example of a meter stick in Figure 1, has a point P, referred to as the *Centre of Oscillation,* which is the length (OP) of an equivalent simple pendulum [6].

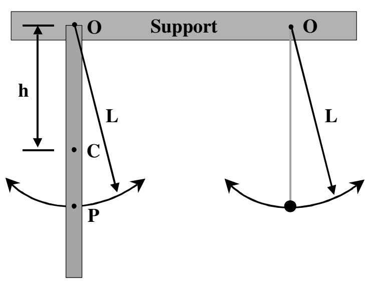

Fig. 1. Meter stick physical pendulum vs. Simple pendulum. C is the centre of mass of the beam and P is the Centre of Oscillation

If the *Centre of Oscillation*, P, of the constructed pendulum is determined and the accelerometers are placed at this location, the mathematical analysis derived using the Simple Pendulum model can be applied to our physical pendulum.

Several pendulum designs were considered and the following approach pendulum design was adopted. To enable the accelerometer devices be fitted at the Centre of Oscillation of the pendulum, the spherical mass was constructed in two sections, with a hollowed space large enough to accommodate a printed circuit board (PCB) used to mount the accelerometer device and having sufficient space to allow an adjustment in the positioning of the accelerometer PCB.

Plastic nylon was chosen as the material from which the sphere was to be constructed as it is relatively easy to machine, has an excellent surface finish, which along with the spherical shape provided good aerodynamic characteristics to the pendulum mass during it's oscillation. Once both sections were brought together, the tight specifications insured that no other bonding or fixation was necessary to maintain the two halve spheres together, which facilitated inspection of the accelerometer devices *in-situ*.

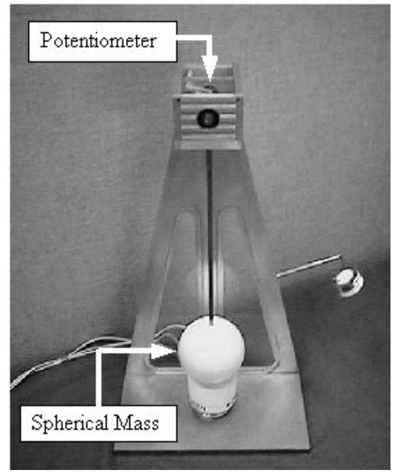

Fig. 1. Completed pendulum and rigid structure, showing spherical mass and location of potentiometer

III. SYSTEM DESIGN

A. *Integrated accelerometer*

As discussed, the dual axis accelerometer under test in the pendulum mass is the ADXL202 integrated accelerometer. The ADXL202 was mounted on a PCB with a buffering op-amp and miscellaneous passive components. Apart from a supply decoupling capacitor C_{DC}, two low pass capacitors, C_X and C_Y (one for each channel), were to be fitted to analogue outputs X_{Filt} and Y_{Filt}. The capacitor values (minimal value of 1000pF, which equates to 5kHz, the bandwidth of the ADXL202) were determined using Equation 4 [10], where f is the desired cut-off frequency in Hz.

$$C_{XorY} = \frac{1}{2\pi f.\left(32\times10^{3}\right)} \text{ (F)} \qquad (4)$$

The final configuration of the ADXL202 was to use the direct analogue output with a dual op-amp to buffer the output. The following were the values used for each of the capacitors:

C_{DC} = 100nF, C_X = 1000pF, C_Y = 1000pF

B *Electro-goniometer*

The pendulum angular displacement was measured using an electro-goniometer, where the pendulum angular displacement was measured by attaching the shaft of a potentiometer to the pendulum axis (via a flexible coupling) and the potentiometer body was attached to the pendulum frame. Thus the rotational movement of potentiometer shaft corresponded to the rotational movement of the pendulum.

To protect the potentiometer shaft from shear loading from the pendulum, which would interfere with its operation as an angle transducer, a Bellow's Flexible coupling was used to connect the potentiometer shaft to an axis, which supported the pendulum beam. The axis, which lay on bearings, took the load of the pendulum and rotated with the pendulum mass. Thus only the rotational forces experienced by the pendulum were transmitted through the Bellow's Flexible coupling to the potentiometer.

The calibration of the potentiometer, used to determine the sensitivity (V/°) of the device, was carried out by measuring the potentiometer output at several known angles, to a maximum of approximately 30° clockwise and counterclockwise of the potentiometer's resting point. The potentiometer was positioned at the known angles of rotation and the resulting signal was sampled by the data-logger for a period of 5 seconds.

C. The Pendulum Beam

The pendulum beam was constructed from a lightweight tubular bar of mild steel, with a centre hollow, which allowed the sensor cabling to run internally in the tubing from the centre of the pendulum mass to the top of the beam.

The tubing was treaded at each end so that once assembled the accelerometer PCB could be positioned parallel to the pendulum's plane of motion and exactly at the pendulum's *Centre of Oscillation*.

As the mass of the pendulum was over 700 grams a triangular aluminium construction with a heavy base was designed and constructed to provide support and to house the pendulum. This prevented motion of the pendulum base during testing, which would result in motion artifact noise being introduced. An arm was also added to the frame to hold the pendulum in a set angle before releasing it. This arrangement facilitated the execution of repeatable measurements so as to avoid ambiguity.

IV. TESTING

Following a five seconds calibration period, the pendulum was manually set into motion. After oscillating for approximately 25 seconds, the pendulum came to rest. During the full 30-second measurement period both the buffered analogue outputs voltages corresponding to the X and Y-axes of the ADXL202 device (X_{Filt} and Y_{Filt}) and the potentiometer output voltage were sampled at 100 Hz at a resolution of 12 bits using an ambulatory data-logger, the Biomedical Monitoring BM42 (Biomedical Monitoring Ltd, Glasgow, Scotland).

Calibration

For calibration, the pendulum sensors were data-logged while the pendulum was at rest, in other words the pendulum arm was resting in the vertical position. In this position, the radial acceleration of the pendulum mass would be +1g, the tangential acceleration 0g and the pendulum angular displacement was 0°. The sensitivity (in V/g) for each axis of the ADXL202 was obtained, using bench measurement, by measuring the output voltage for each axis at +1g and -1g (5).

$$\text{Accelerometer Sensitivity} = \frac{(\text{Output} + 1\text{g} - \text{Output} - 1\text{g})}{2} \qquad (5)$$

Similarly, the electro-goniometer voltage output was converted to pendulum displacement using the electro-goniometer calibration equation (6).

$$\text{Pendulum Angle} = \left(\frac{\text{Potentiometer V.} - \text{Potentiometer Calibration V.}}{\text{Goniometer Sensitivity}}\right) \qquad (6)$$

The data processing was carried out using MATLAB® and had the following flow:

1. Numerical low-pass filtering of the accelerometer and electro-goniometer signals.
2. Computation of θ from the electro-goniometer output using the goniometer calibration equation.
3. Conversion of the accelerometer voltage outputs to acceleration using the accelerometer calibration equations.
4. Single differentiation of the pendulum angular displacement, θ, to obtain ω, using polynomial methods.
5. Double differentiation of ω, to obtain α, again by polynomial methods.

Processing of the electro-goniometer data enabled a predication of a_{radial} and $a_{tangential}$ using Equations 1 and 2 and then a comparison made with the measured a_{radial} and $a_{tangential}$ from the accelerometer outputs.

Filtering

Filtering was implemented using a numerical 2nd order Butterworth low-pass digital filter similar to that discussed in Winter *et al.* [6]. The low-pass cut-off frequency was determined using Winter's Residual Method [7]. The filter used was a two-stage filter, where the data was passed through the filter firstly in a forward direction and then in the reverse direction, where the reverse direction simply involves rotating the data and passing it through the same filter.

Differentiation

Initially differentiation was carried out using the MATLAB® *diff* command, which finds the difference between two consecutive points. When *diff* is used to differentiate an experimental signal it will magnify any noise [6], [8]. It is possible to reduce the noise before differentiation by increased numerical filtering however this was seen to distort the signal excessively, as was also indicated by the errors seen by Ladin et al. [9]. Yet some form smoothing of displacement data is necessary before differentiating to obtain acceleration [2].

An alternative to using *diff* was to create a polynomial of the data and to then differentiate the polynomial. This was made feasible by using *polyfit* function from MATLAB®, which fits the data to a polynomial approximation using least-squares method, and *polyder,* which returns the derivative of the polynomial, created by *polyfit*. It was then possible, using *polyval* to create coefficients of the polynomial. Once a high order polynomial is used, this method returns a much cleaner double differentiation than that obtained using *diff*. Thus for the analysis described in this paper, when the angular acceleration and angular velocity were derived from angular displacement, the polynomial methods described were used.

V. Results

Radial Acceleration

Visual analysis of the radial acceleration derived from the potentiometer and that measured by the radial sensor in the ADXL202 reveal almost identical signals (Figure 2). The resulting root mean squared (RMS) error value and coefficient of multiple correlation (CMC) were 0.1211 and 0.9960 respectively. The percentage difference is approximately 1.69%.

Tangential Acceleration

Visual analysis of the tangential acceleration derived from the potentiometer and that measured by the tangential sensor in the ADXL202 also reveal similar signals (Figure 3). The RMS and CMC values were calculated to be 0.9889 and 0.3966. The resulting percentage difference is approximately 7.04%.

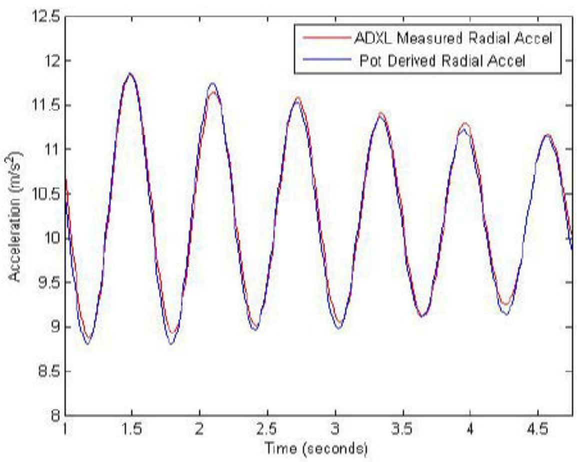

Fig. 3. Pendulum radial acceleration: potentiometer derived (blue) vs. ADXL202 measured (red)

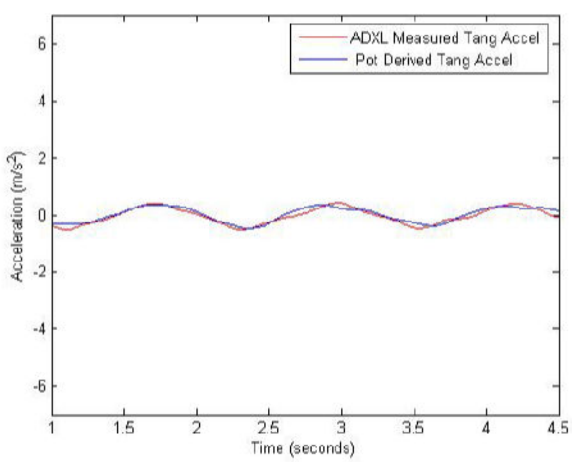

Fig. 4. Tangential acceleration: Potentiometer (blue) vs. ADXL202 measured (red)

VI. Conclusion

The objective of this technical paper was to assess the ability of the bi-axial ADXL202 integrated accelerometer to measure dynamic acceleration for gait applications. Mounting the bi-axial integrated accelerometer at the centre of oscillation of the pendulum, one axis oriented in the tangential and one oriented in the radial direction, allowed direct comparison of the predicted acceleration with the acceleration measured by the accelerometer.

The offset of the resulting radial and tangential accelerations to the derived goniometer accelerations may be attributed to the inaccuracy of sensor alignment that may have arisen during construction. It was also occasionally observed that under oscillating motion the heavy pendulum mass would deviate about its intended directional plane of motion. A more lightweight pendulum mass or heavier pendulum axis (radius) may overcome this problem.

However, the predicted pendulum acceleration derived from the goniometer and the acceleration measured by the ADXL202 accelerometer showed a close match with errors of less than 2% for radial acceleration. These findings indicate that the ADXL202 integrated accelerometer is a suitable means for measuring dynamic acceleration in human movement applications particularly in the radial direction.

Future work will involve repeated testing of multiple accelerometers at various oscillating frequencies by varying the length of the pendulum to better replicate that expected in gait for different subjects.

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