Report

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1 School of Engineering and Applied Science (SEAS), Ahmedabad University

2 B.Tech(ICT) Semester IV: Probability and Random Processes (MAT 202)

- Group No : S_M1
- Kaushal Patil (AU1841040), Arpitsingh Vaghela(AU1841034)
- Project Title : Modelling probabilistic fatigue crack propogation data and predicting model parameters

2.1 1 Introduction

2.1.1 1.1 Background

Mechanical modelling tends to be a very complex mathematical task.One of the most important problems realted to modelling probability in mechanics is to find the reliabity of materials that are used to build objects. A lot of research has taken place to compute fatigue effects on materials and structures.There have been a lot of developments in this area after the proposal Paris' Law. Most of the models proposed realted to fatigue crack propogation are deterministic and have some limitations such as that they arise from arbitary empirical assumptions.A proper estimation of fatigue crack porpogation rates with respect to residual stress and stress ratio helps us to estimate optimal materials for mechanical tasks. For e.g. : To determine the type of steel to be used when we are building a bridge. Proper estimation of fatigue crack proporgation and finding its probabilisitic curves also gives great insights to the propertiees of material and can be useful when the material is being improved, i.e. at the time of development on newer types of alloys.

The First breakthrough in the field of fatigue propogation estimation was the Paris' Law which was followed by various approaches[3].Initially Local strain-based approches were proposed to model fatigue crack propogation on notch based components further a link was established between local strain-based approached to fatigue and Frature mechanics based fatigue crack propogation models.[1,5-10] As research continued models were proposed that using residual stress concepts ecplained stress ratio effects as well as interaction effects on crack growth rates.Several approches were proposed which were analytical and or numerical.[11-14]The underlying concept behind the proposed local approaches for fatigue crack propagation modelling consists of assuming fatigue crack propagation as a process of continuous failure of consecutive representative material elements (continuous re-initializations). Such a kind of approaches has been demonstrated to correlate fatigue crack propagation data from several sources, including the stress ratio effects[10-14]. The crack tip stress-strain fields are computed using elastoplastic analysis, which are applied together a fatigue damage law to predict the failure of the representative material elements. The simplified method of Neuber[15] or Moftakhar[16] et al. may be used to compute the elastoplastic stress field at the crack tip vicinity using the elastic stress distribution given by the Fracture Mechanics.[1,16-17] One Such model (given by Noorzi)[1,10-11] the unigrow model, modelled the fatigue crack growth based on elastic–plastic crack tip stress–strain history regarded the process of fatigue crack propogation as a process of successive crack re-initiation in the crack tip region. [1][0]

We take the unigrow model and extend it to relate it with a probabilistic contruct to find probabilistic fatigue crack propagation rates various materials. The Unigrow model to derive probabilistic fatigue crack propogation $da/dN - \triangle K$ fields for a particular selected material ((S355) structural mild steel)[0], for distinct stress ratios.The Deterministic model uses Morrows equation, Strain life relation along with the SWT relation to model parameters deterministically.For Probabilistic approach the strain-life field proposed by Castillo and Fernandez-Canteli [20] and Shane-Watson-Topper-life field [0] which are based on Weibull Distribution are used to generalize the results to account for mean stress effects using percentile curves. The simulation was modelled using the data acquired[19,0] and was further extended where we tried to make a prediction model for Threshold value of life time[N0], Endurance limit of strain [E0], Fatigue Limit of SWT parameter[SWT] and the Weibull parameters(lambda(Position),delta(Scale),Beta(Shape)) by Running Batch Gradient Descent[Reference to ML Glossary] on Loss function from [27] and further using probability-weighted moments from [27] to predict N0,E0,SWT and Weibull parameters. The Probabilistic Life time fields are combined with Unigrow model [0-1] to finally compute the probabilistic fatigue crack proppogation field for distinct stress ratios.

2.1.2 1.2 Motivation

The current deterministic works take into account a lot of parameters and some of these parameters cannot be determined easily or at all and hence these works take arbitary assumptions in the process of modelling. To solve this (Our Base Article) proposes a probabilistic model that not only arises from sound statistical and physical assumtions, it also manages to provide a probabilistic definiton of the whole strain-life field. Further, since mechanical modelling is a complex task and takes into account a lot of parameters, we have tried to simplify the process of finding probabilistic fatigue crack propogation fields.Not only this we have tried to use Gradient Descent Regression and Probability-weighted moments method to estimate certain parameters required for the task.

2.1.3 1.3 Problem Statement/ Case Study

- To determine probabilistic fields of fatigue crack propogation rates wrt residual stress and stress ratio.
  - Perfomance metric : how closely the computed probabilistic field data and plots matches the computed data and plots given in the base article
- To propose a probabilistic Shane-Watson-Topper Field using the mathematical modelling of the Strain-Life Field proposed by Cantelli[20].
  - Performance metric : how closely the shape of p-SWT-n curve match the experimental/deteriminstic data at a given parameter value (probability)

2.2 2 Data Acusition

- Yes, this Special Assignment is Data Dependent.

- We have gathered data from Two Types of sources:
  - Data Mentioned in reference papers [0,19].
  - Data synthesised from using simple linear equations to model plots of datasets given in paper using theortical formulaes supplied.
    - Postulated experimental data has been synthesised in such a way

2.3 3 Probabilistic Model Used/ PRP Concept Used

We extend the Unigrow Model for Fatigue crack propogation using probabilistic fields and work on the following model represented by:

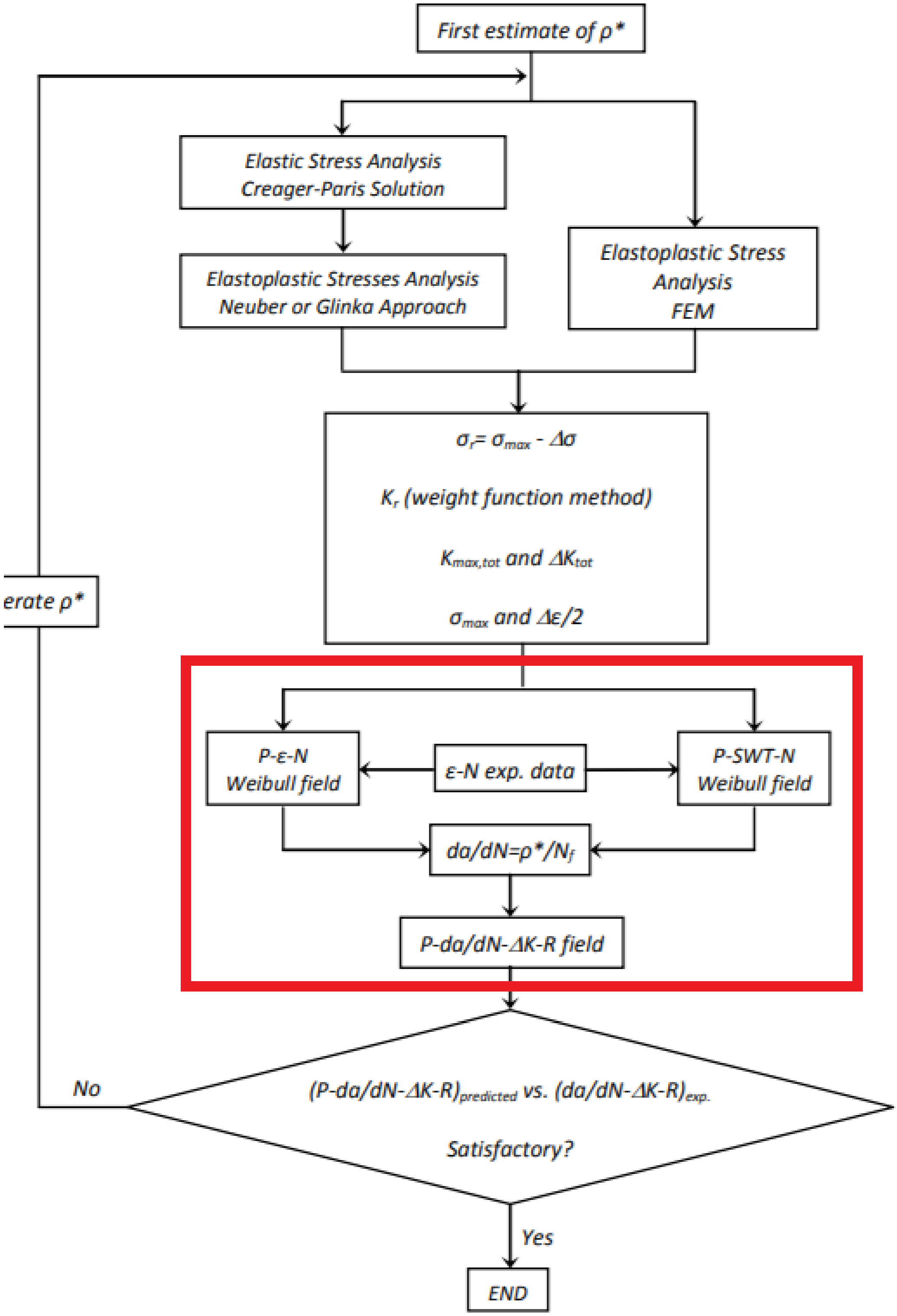


Figure 4: Procedure to generate probabilistic fatigue crack propagation fields.

The fatigue crack propagation modelling based on local approaches requires a fatigue damage relation to compute the number of cycles to fail the elementary material blocks.

Here Our main focus(Highlighted in red bounding box) is to model the generation of probabilistic Strain-Life and SWT-Life fields. The model assumes that the fatigue life, N_f, and the total strain amplitude, Ea, are random variables.

If X is a random variable denoting the *time to failure*, the **Weibull distribution** gives a distribution for which the *failure rate* is proportional to a power of time.

$$f_X(x) = \frac{\beta}{\delta}(\frac{x-\lambda}{\delta})^{\beta-1}e^{-(\frac{x-\lambda}{\delta})^{\beta}}$$

$$F_X(x;\lambda,\delta,\sigma) = 1 - e^{-(\frac{x-\lambda}{\delta})^{\beta}}$$

where $\beta > 0$ is the **shape parameter**,

$\delta > 0$ is the **scale parameter**,

$\lambda > x$ is the **location parameter** (the minimum value of X).

Percentile points,

$$x_p = \lambda + \delta(-log(1-p))^{\frac{1}{\beta}}$$

where $0 \leq p \leq 1$

Important Properties of Weibull Distribution

- Stable with respect to location and scale

$$X \sim W(\lambda,\delta,\beta) \iff \frac{X-a}{b} \sim W(\frac{\lambda-a}{b},\frac{\delta}{b},\beta)$$

- It is stable with respect to Minimum Operations.i.e., if $X_1, X_2, X_3, ...X_m$ are independent and identical distribution,then

$$X_i \sim W(\lambda,\delta,\beta) \iff min(X_1, X_2, ..., X_m) \sim W(\lambda, \delta m^{\frac{1}{\beta}}, \beta)$$

  if a set of independent and identical distribution is weibull then its minimum is also a Weibull Random Variable — #### Relevant Variable involved for modeling: — P:Probability of fatigue faliure N:Number of stress cycles to failure N_0:Threshold value of N (min lifetime) Ea: Strain Ea_0: Endurance limit of Ea; SWT:Smith,Watson and Topper fatigue damage parameter SWT_0:Fatigue limit

Putting Related variables together we have three varaibles(based on II Theorem) ##### For Strain Life Field

$$\frac{N}{N_0}, \frac{Ea}{Ea_0}, PP = q(\frac{N}{N_0}, \frac{Ea}{Ea_0})$$

where $q()$ is a function we are to determine so P can be any monotone function of $\frac{N}{N_0}, \frac{Ea}{Ea_0}$, as $h(\frac{N}{N_0})$ & $g(\frac{Ea}{Ea_0})$

We denote them as

$$N^* = h(\frac{N}{N_0})SWT^* = g(\frac{Ea}{Ea_0})$$

For SWT Life Field

$$\frac{N}{N_0}, \frac{SWT}{SWT_0}, PP = q(\frac{N}{N_0}, \frac{SWT}{SWT_0})$$

where $q()$ is a function we are to determine so P can be any monotone function of $\frac{N}{N_0}, \frac{SWT}{SWT_0}$, as $h(\frac{N}{N_0})$ & $g(\frac{SWT}{SWT_0})$

We denote them as

$$N^* = h(\frac{N}{N_0})SWT^* = g(\frac{SWT}{SWT_0})$$

Strain-Life Field

$$p = F(N_f^*; E_a^*) = 1 - exp(-(\frac{log\frac{N_f}{N_0}log\frac{Ea}{Ea_0} - \lambda}{\delta})^\beta)$$

here $log(\frac{N_f}{N_0})loglog\frac{Ea}{Ea_0} \geq \lambda$

p is the probability of failure, N0 and a0 are normalizing values, and , and are the non-dimensional Weibull model parameters.

$$N^*Ea^* \sim W(\lambda, \delta, \beta)N_f^* \sim W(\frac{\lambda}{Ea^*}, \frac{\delta}{Ea^*}, \beta)$$

Proposed SWT-N or S-N Field We have proposed SWT field as it has advantages over the normal strain life field as it uses the SWT fatigue damage parameter.Using this Damage Parameter we can account for mean stress effects on fatigue life prediction.

$$p = F(N_f^*; E_a^*) = 1 - exp(-(\frac{log\frac{N_f}{N_0}log\frac{SWT}{SWT_0} - \lambda}{\delta})^\beta)$$

here $log(\frac{N_f}{N_0})loglog\frac{SWT}{SWT_0} \geq \lambda$

p is the probability of failure, N0 and SWT_0 are normalizing values, and , and are the non-dimensional Weibull model parameters.

$$N^*Ea^* \sim W(\lambda, \delta, \beta)N_f^* \sim W(\frac{\lambda}{SWT^*}, \frac{\delta}{SWT^*}, \beta)$$

Considerations:

- **Weakest Link**: Fatigue lifetime of a longitudinal element is the minimum of its constituting particles.Thus we need minimum model for a longitudinal element $L = ml$
- **Stability**: The distribution function must hold for different lengths.
- **Limit Behaviour**: Need Asymptotic family of Distribution
- **Limited Range**: N^* & SWT^* has finite lower bound,coincide with theoretical end of CDF

$$N \geq N_0SWT \geq SWT_0$$

- **Compatibility**:

$$E(N^*; SWT^*) = F(SWT^*; N^*)$$

i.e., Distribution of N^* can be determined based on given SWT^* and similarly SWT^* from N^*.

All these are Statisfied by Weibull Distribution

p-curves

$$log(\frac{SWT}{SWT^*}) = \frac{\lambda + \delta[-log(1-p)]^{\frac{1}{\beta}}}{log(\frac{N}{N_0})}$$

Final Distribution

$$N^*SWT^* \sim W(\lambda, \delta, \beta) log(\frac{N}{N_0}) log(\frac{SWT}{SWT_0}) \sim W(\lambda, \delta, \beta) log(\frac{N}{N_0}) \sim W(\frac{\lambda}{log(\frac{SWT}{SWT_0})}, \frac{\delta}{log(\frac{SWT}{SWT_0})}, \beta)$$

—-

The values for this model are:

$logN_0$	$logSWT_0$	λ	δ	β
-4.1079	-4.4317	53.8423	7.2698	3.6226

$logN_0$	$log\epsilon_{a0}$	λ	δ	β
-3.2593	-9.1053	36.6676	5.8941	4.6952

2.4 4 Pseudo Code/ Algorithm

Algorithm for:

2.4.1 PDF of Weibull distribution

$$f_X(x) = \frac{\beta}{\delta}(\frac{x-\lambda}{\delta})^{\beta-1} e^{-(\frac{x-\lambda}{\delta})^\beta}$$

where $\beta > 0$ is the **shape parameter**,

$\delta > 0$ is the **scale parameter**,

$\lambda > x$ is the **location parameter** (the minimum value of X).

2.4.2 CDF of Weibull distribution

$$F_X(x; \lambda, \delta, \sigma) = 1 - e^{-(\frac{x-\lambda}{\delta})^\beta}$$

where $\beta > 0$ is the **shape parameter**,

$\delta > 0$ is the **scale parameter**,

$\lambda > x$ is the **location parameter** (the minimum value of X).

2.4.3 Percentile Curve of Weibull distribution

$$x_p = \lambda + \delta(-log(1-p))^{\frac{1}{\beta}}$$

where $0 \leq p \leq 1$

2.4.4 Simulation of Model

- Gathering Data from Base Article and Associated Article, Generating Unspecified Data using simple Linear Equation and Matrix Operations.
  - Data Gathering information is already given.
  - Matrix and Mathematical operations applied using numpy.
- Initialising the coded Weibull Distribution class on required data.
  - Weibull distribution class contains methods the calculate pdf,cdf and percentile curves based on the Mathematical Formulaes stated above.
  - The initialisation of random variable is done using inbuilt weibull distribution function in numpy.
- Plotting the p-SWT-N curves for various values of p along with the plot of postulated data (assumed to be experimental).
  - Matplotlib is used to generate all the plots.
- Add alogrithm for Deterministic contruct from here on.

Add a Draw.io Flowchart here just in case.

2.5 5 Coding and Simulation

2.5.1 5.1 Simulation Framework

The values for this model are:

- p-SWT-N field

$logN_0$	$logSWT_0$	λ	δ	β
-4.1079	-4.4317	53.8423	7.2698	3.6226

- p-Ea-N field

$logN_0$	$log\epsilon_{a0}$	λ	δ	β
-3.2593	-9.1053	36.6676	5.8941	4.6952

2.5.2 5.2 Reproduced Figures

- Used tools:
  - Python
  - Matplotlib
  - Numpy

Comparison of results Probabilistic Strain Life Field:

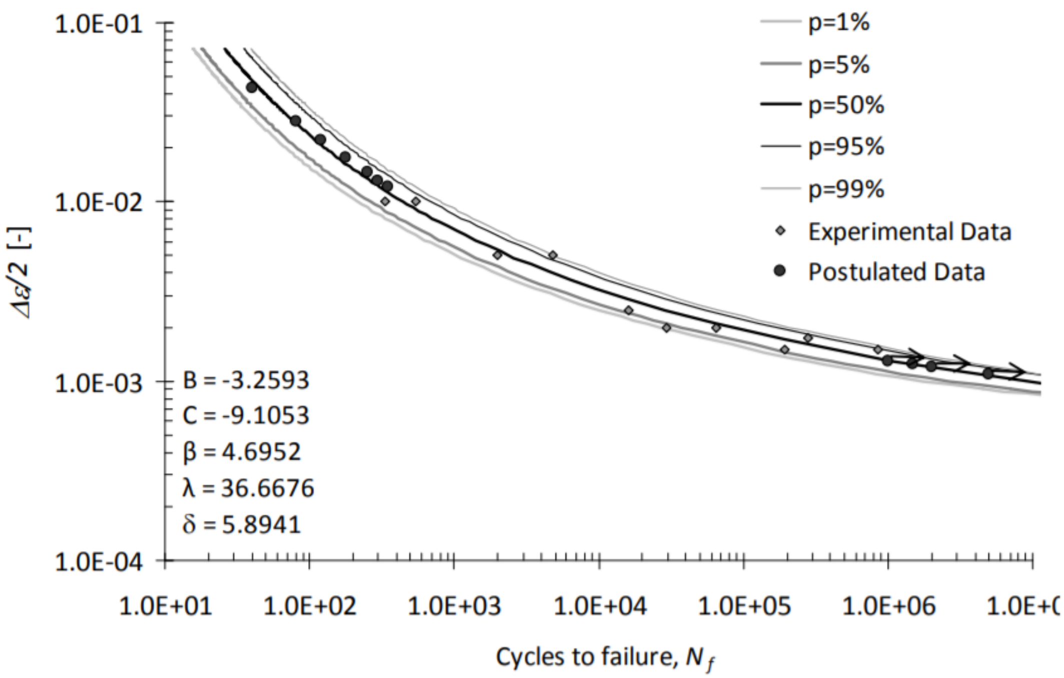
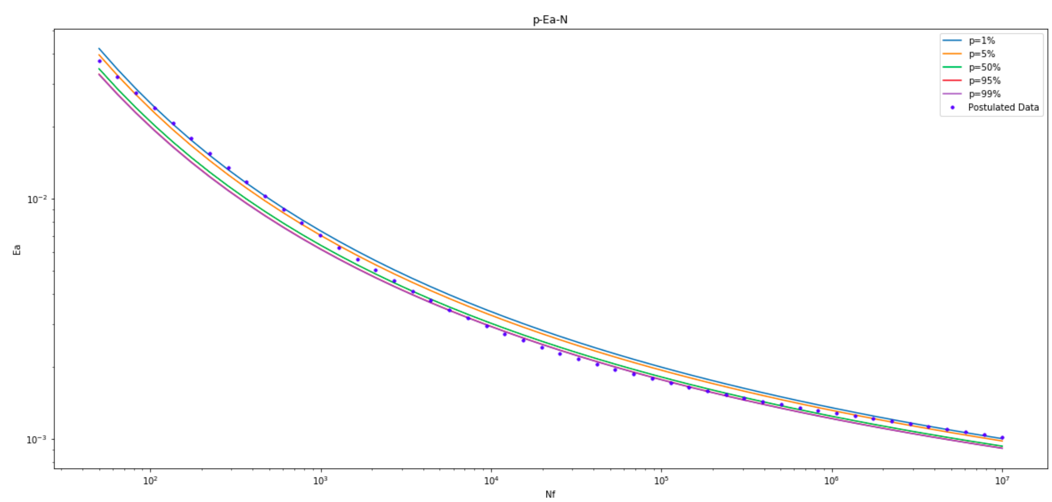


Figure 6: p-ε-N field for the S355 steel.



The Above plots are for Strain vs Number of Cycles to Failure for percent of probability along with postulated data (Experimental for 0.5 Percent).

Comparison of results Probabilistic Shane Watson Damage Life Field:

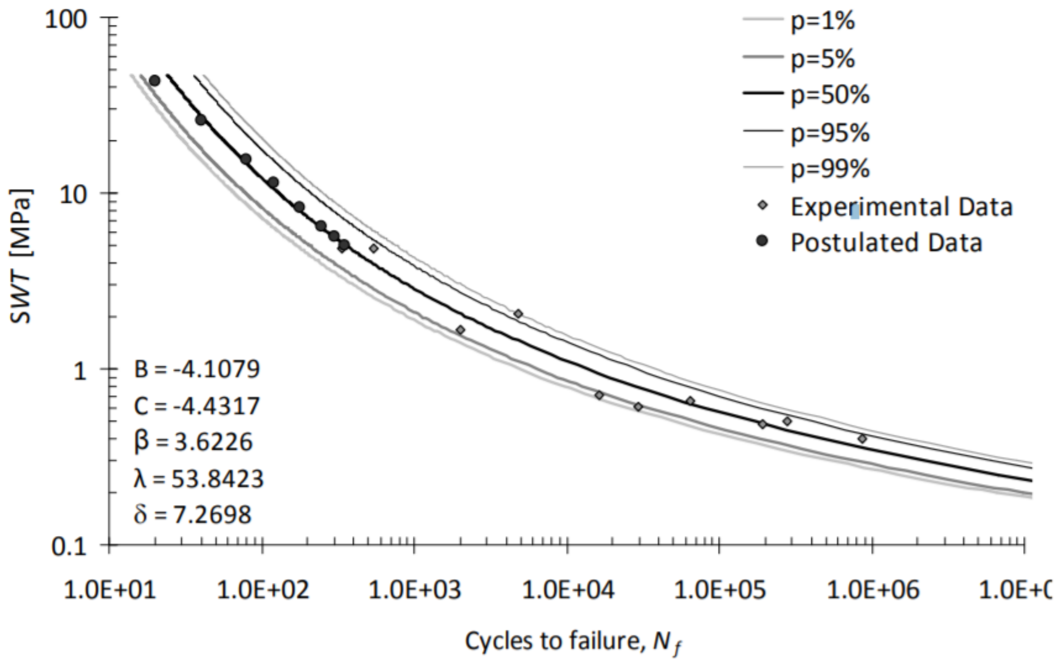
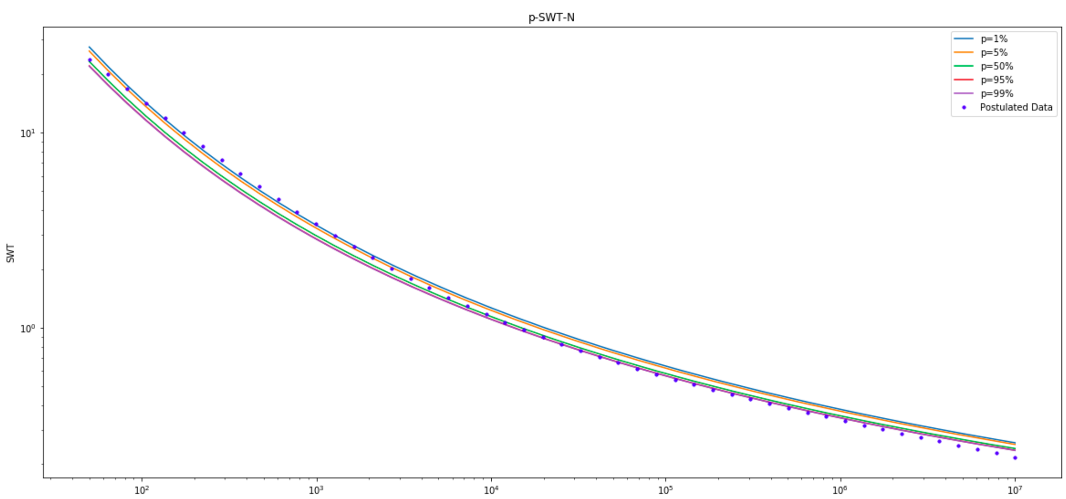


Figure 7: *p-SWT-N* field for the S355 steel.



The Above plots are for SWT Damage Parameter vs Number of Cycles to Failure for percent of probability along with postulated data (Experimental for 0.5 Percent).

2.5.3 5.3 New Work Done

Estimating and supplying analysis of parameters assumed/supplied externally and creating a closed form model with no external dependance that computes probabilistic fatigue crack propogation data using only the natural required input paramters.

5.3.1 New Analysis

- To Estimate and Check if the SWT-N parameters can be predicted using gradient descent based regression.

  - Performance metric : To see if the Loss is reduced by Gradient descent or not and if the gradient descent works, if the predicted values match the supplied data.
- Estimate the parameters of Weibull Distribution using PWM estimation on real and predicted data.
  - Performance metric : To compare and see if the computed Weibull parameters using PWM match those supplied with the paper and if the predicted vs deterministic parameters match.

5.3.2 New Coding / Algorithm

- Coding and algorithm details here

5.3.3 New Results

- Comparision of computed and given parameters.

5.3.4 New Inferences

- Something like: parameter can be estimated using simple closed form coded solutions and shouldn't be estimated and or computed externally.

Students are advised to share the new derivations with results in correlation with the reproduce results. Write clear inference for the new results. You are also advised to add new analysis along with the codes.

2.6 6 Inference Analysis/ Comparison

Add Stuff Here From Sir Arpitsinhs's Notebook.

2.7 7 Contribution of team members

2.7.1 7.1 Technical contribution of all team members

2.7.2 7.2 Non-Technical contribution of all team members

2.8 8 Submission checklist for uploading on Google Drive

This section provides the submission checklist for smooth and efficient submission process. (This is for your reference and please remove this while writing your report). - Soft copy of this project Report - Soft copy of Abstract - Soft copy of Concept Map 1 and 2 - Soft copy of base article - Soft copy of analysis (hand written)(jupyter notebooks) - Folder of matlab(python) codes (with proper naming) - Folder of reproduced results in .fig and .jpg format - latex (.tex) file of the project report.

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