School of Engineering and Applied Science (SEAS), Ahmedabad University

B.Tech(ICT) Semester IV: Probability and Random Processes (MAT 202)

- Group No : SM1
- Roll no: Kaushal Patil (AU1841040)
- Roll no: Arpitsinh Vaghela (AU1841034)
- Project Title: Modelling probabilistic fatigue crack propogation data and predicting model parameters

1 Introduction

1.1 Background

Mechanical modelling tends to be a very complex mathematical task.One of the most important problems related to modelling probability in mechanics is to find the reliability of materials that are used to build objects. A lot of research has taken place to compute fatigue effects on materials and structures.There have been a lot of developments in this area after the proposal Paris' Law. Most of the models proposed related to fatigue crack propagation are deterministic and have some limitations such as that they arise from arbitrary empirical assumptions.A proper estimation of fatigue crack propagation rates with respect to residual stress and stress ratio helps us to estimate optimal materials for mechanical tasks. For e.g. : To determine the type of steel to be used when we are building a bridge. Proper estimation of fatigue crack propagation and finding its probabilistic curves also gives great insights to the properties of material and can be useful when the material is being improved, i.e. at the time of development on newer types of alloys.

The First breakthrough in the field of fatigue propagation estimation was the Paris' Law which was followed by various approaches[8].Initially Local strain-based approaches were proposed to model fatigue crack propagation on notch based components further a link was established between local strain-based approached to fatigue and Fracture mechanics based fatigue crack propagation models.[6,9-14] As research continued models were proposed that using residual stress concepts explained stress ratio effects as well as interaction effects on crack growth rates.Several approaches were proposed which were analytical and or numerical.[15-18]The underlying concept behind the proposed local approaches for fatigue crack propagation modelling consists of assuming fatigue crack propagation as a process of continuous failure of consecutive representative material elements (continuous re-initialization). Such a kind of approaches has been demonstrated to correlate fatigue crack propagation data from several sources, including the stress ratio effects[14-18]. The crack tip stress-strain fields are computed using elastoplastic analysis, which are applied together a fatigue damage law to predict the failure of the representative material elements. The simplified method of Neuber[19] or Moftakhar[20] et al. may be used to compute the elastoplastic stress field at the crack tip vicinity using the elastic stress distribution given by the Fracture Mechanics.[6,20-21] One Such model (given by Noorzi)[6,14-15] the unigrow model, modelled the fatigue crack growth based on elastic–plastic crack tip stress–strain history regarded the process of fatigue crack propagation as a process of successive crack re-initiation in the crack tip region. [1,6]

We take the unigrow model and extend it to relate it with a probabilistic construct to find probabilistic fatigue crack propagation rates various materials. The Unigrow model to derive probabilistic fatigue crack propagation $da/dN - \triangle K$ fields for a particular selected material ((S355) structural mild steel)[1], for distinct stress ratios.The Deterministic model uses Morrows equation, Strain life relation along with the SWT relation to model parameters deterministically.For Probabilistic approach the strain-life field proposed by Castillo and Fernandez-Canteli [7] and Shane-Watson-Topper-life field [1] which are based on Weibull Distribution are used to generalize the results to account for mean stress effects using percentile curves. The simulation was modelled using the data acquired[9,1] and was further extended where we tried to make a prediction model for Threshold value of life time[N0], Endurance limit of strain [E0], Fatigue Limit of SWT parameter[SWT] and the Weibull parameters(lambda(Position),delta(Scale),Beta(Shape)) by Running Batch Gradient Descent[3,23] on Loss function from [23] and further using probability-weighted moments from [4] to predict N0,E0,SWT and Weibull parameters. The Probabilistic Life time fields are combined with Unigrow model [1-6] to finally compute the probabilistic fatigue crack propagation field for distinct stress ratios.

1.2 Motivation

The current deterministic works take into account a lot of parameters and some of these parameters cannot be determined easily or at all and hence these works take arbitrary assumptions in the process of modelling. To solve this (Our Base Article) proposes a probabilistic model that not only arises from sound statistical and physical assumptions, it also manages to provide a probabilistic definition of the whole strain-life field. Further, since mechanical modelling is a complex task and takes into account a lot of parameters, we have tried to simplify the process of finding probabilistic fatigue crack propagation fields.Not only this we have tried to use Gradient Descent Regression and Probability-weighted moments method to estimate certain parameters required for the task.

1.3 Problem Statement/ Case Study

- To determine probabilistic fields of fatigue crack propagation rates wrt residual stress and stress ratio.
  - Performance metric : how closely the computed probabilistic field data (Strain Life) and plots matches the computed data and plots given in the base article
- To propose a probabilistic Shane-Watson-Topper Field using the mathematical modelling of the Strain-Life Field proposed by Cantelli[7].
  - Performance metric : how closely the shape of p-SWT-n curve match the experimental/deterministic data at a given parameter value (probability)

2 Data Acquisition

- Yes, this Special Assignment is Data Dependent.
- We have gathered data from Two Types of sources:
  - Data Mentioned in reference papers [1,5].
  - Data synthesised from using simple linear equations to model plots of data sets given in paper using theoretical formulas supplied.
- Postulated experimental data has been synthesised in such a way
  - Strain Life (Morrow equation):

$$\frac{\Delta\epsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \epsilon'_f(2N_f)^c$$

- Reversal Stress Ratio:

$$\frac{\Delta\sigma}{2} = \sigma_{max} = \sigma'_f(2N_f)^b$$

- SWT(Smith,Watson and Topper fatigue damage parameter):

$$SWT = \sigma_{max}\frac{\Delta\epsilon}{2} = \frac{(\sigma'^2_f)(2N_f)^{2b}}{E} + \sigma'_f\epsilon'_f(2N_f)^{b+c}$$

where, σ^*_f: fatigue strenght coefficient, b: fatigue strength exponent, ϵ^*_f: fatigue ductility coefficient, b: fatigue ductility exponent and E: Young modulus

The values of the constants for S355 Mild Steel are:

$EGPa$	$\sigma'_f MPa$	b	ϵ'_f	c
211.60	952.20	-0.0890	0.7371	-0.6640

3 Probabilistic Model Used/ PRP Concept Used

We extend the Unigrow Model for Fatigue crack propagation using probabilistic fields and work on the following model represented by:

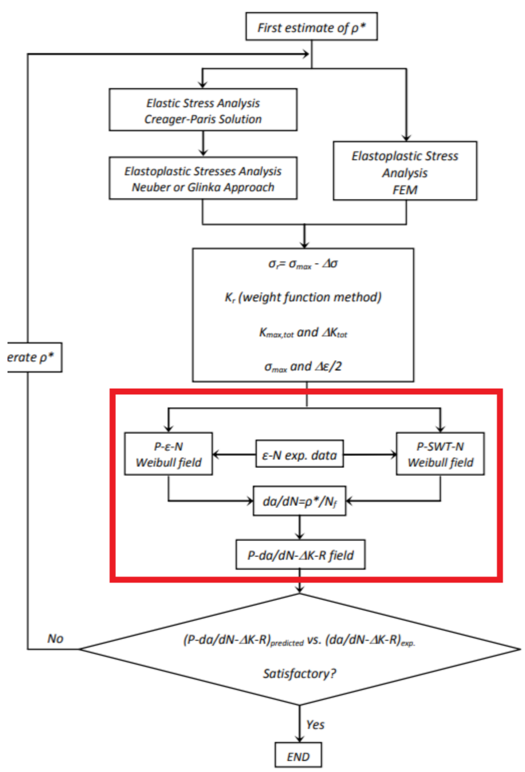


Figure 4: Procedure to generate probabilistic fatigue crack propagation fields.

The fatigue crack propagation modelling based on local approaches requires a fatigue damage relation to compute the number of cycles to fail the elementary material blocks.

Here Our main focus(Highlighted in red bounding box) is to model the generation of probabilistic Strain-Life and SWT-Life fields. The model assumes that the fatigue life, N_f, and the total strain amplitude, Ea, are random variables.

If X is a random variable denoting the *time to failure*, the **Weibull distribution** gives a distribution for which the *failure rate* is proportional to a power of time.

$$f_X(x) = \frac{\beta}{\delta}(\frac{x-\lambda}{\delta})^{\beta-1}e^{-(\frac{x-\lambda}{\delta})^{\beta}}$$
$$F_X(x;\lambda,\delta,\sigma) = 1 - e^{-(\frac{x-\lambda}{\delta})^{\beta}}$$

where $\beta > 0$ is the **shape parameter**,

$\delta > 0$ is the **scale parameter**,

$\lambda > x$ is the **location parameter** (the minimum value of X).

Percentile points,

$$x_p = \lambda + \delta(-log(1-p))^{\frac{1}{\beta}}$$

where $0 \leq p \leq 1$

Important Properties of Weibull Distribution

- Stable with respect to location and scale

$$X \sim W(\lambda,\delta,\beta) \iff \frac{X-a}{b} \sim W(\frac{\lambda-a}{b},\frac{\delta}{b},\beta)$$

- It is stable with respect to Minimum Operations.i.e., if $X_1, X_2, X_3, ...X_m$ are independent and identical distribution,then

$$X_i \sim W(\lambda,\delta,\beta) \iff min(X_1, X_2, ..., X_m) \sim W(\lambda, \delta m^{\frac{1}{\beta}}, \beta)$$

  if a set of independent and identical distribution is weibull then its minimum is also a Weibull Random Variable

P:Probability of fatigue failure N:Number of stress cycles to failure N_0:Threshold value of N (min lifetime) Ea: Strain Ea_0: Endurance limit of Ea; SWT:Smith,Watson and Topper fatigue damage parameter SWT_0:Fatigue limit

Putting Related variables together we have three variables(based on II Theorem)

For Strain Life Field

$$\frac{N}{N_0}, \frac{Ea}{Ea_0}, PP = q(\frac{N}{N_0}, \frac{Ea}{Ea_0})$$

where $q()$ is a function we are to determine so P can be any monotone function of $\frac{N}{N_0}$, $\frac{Ea}{Ea_0}$, as $h(\frac{N}{N_0})$ & $g(\frac{Ea}{Ea_0})$

We denote them as

$$N^* = h(\frac{N}{N_0})SWT^* = g(\frac{Ea}{Ea_0})$$

For SWT Life Field

$$\frac{N}{N_0}, \frac{SWT}{SWT_0}, PP = q(\frac{N}{N_0}, \frac{SWT}{SWT_0})$$

where $q()$ is a function we are to determine so P can be any monotone function of $\frac{N}{N_0}, \frac{SWT}{SWT_0}$, as $h(\frac{N}{N_0})$ & $g(\frac{SWT}{SWT_0})$

We denote them as

$$N^* = h(\frac{N}{N_0})SWT^* = g(\frac{SWT}{SWT_0})$$

Strain-Life Field

$$p = F(N_f^*; E_a^*) = 1 - exp(-(\frac{log\frac{N_f}{N_0} log\frac{Ea}{Ea_0} - \lambda}{\delta})^\beta)$$

here $log(\frac{N_f}{N_0}) loglog\frac{Ea}{Ea_0} \geq \lambda$

p is the probability of failure, N0 and a0 are normalizing values, and λ, δ, σ are the non-dimensional Weibull model parameters.

$$N^*Ea^* \sim W(\lambda, \delta, \beta)N_f^* \sim W(\frac{\lambda}{Ea^*}, \frac{\delta}{Ea^*}, \beta)$$

Proposed SWT-N or S-N Field We have proposed SWT field as it has advantages over the normal strain life field as it uses the SWT fatigue damage parameter.Using this Damage Parameter we can account for mean stress effects on fatigue life prediction.

$$p = F(N_f^*; E_a^*) = 1 - exp(-(\frac{log\frac{N_f}{N_0} log\frac{SWT}{SWT_0} - \lambda}{\delta})^\beta)$$

here $log(\frac{N_f}{N_0}) loglog\frac{SWT}{SWT_0} \geq \lambda$

p is the probability of failure, N0 and SWT_0 are normalizing values, and λ, δ, σ are the non-dimensional Weibull model parameters.

$$N^*Ea^* \sim W(\lambda, \delta, \beta)N_f^* \sim W(\frac{\lambda}{SWT^*}, \frac{\delta}{SWT^*}, \beta)$$

Considerations:

- **Weakest Link**: Fatigue lifetime of a longitudinal element is the minimum of its constituting particles.Thus we need minimum model for a longitudinal element $L = ml$
- **Stability**: The distribution function must hold for different lengths.
- **Limit Behaviour**: Need Asymptotic family of Distribution

- **Limited Range**: N^* & SWT^* has finite lower bound,coincide with theoretical end of CDF

$$N \geq N_0 SWT \geq SWT_0$$

- **Compatibility**:

$$E(N^*; SWT^*) = F(SWT^*; N^*)$$

  i.e., Distribution of N^* can be determined based on given SWT^* and similarly SWT^* from N^*.

All these are Statisfied by Weibull Distribution

p-curves

$$log(\frac{SWT}{SWT^*}) = \frac{\lambda + \delta[-log(1-p)]^{\frac{1}{\beta}}}{log(\frac{N}{N_0})}$$

Final Distribution

$$N^* SWT^* \sim W(\lambda, \delta, \beta) log(\frac{N}{N_0}) log(\frac{SWT}{SWT_0}) \sim W(\lambda, \delta, \beta) log(\frac{N}{N_0}) \sim W(\frac{\lambda}{log(\frac{SWT}{SWT_0})}, \frac{\delta}{log(\frac{SWT}{SWT_0})}, \beta)$$

—-

The values for this model are:

$logN_0$	$logSWT_0$	λ	δ	β
-4.1079	-4.4317	53.8423	7.2698	3.6226

$logN_0$	$log\epsilon_{a0}$	λ	δ	β
-3.2593	-9.1053	36.6676	5.8941	4.6952

4 Pseudo Code/ Algorithm

Algorithm for:

4.1 PDF of Weibull distribution

$$f_X(x) = \frac{\beta}{\delta}(\frac{x-\lambda}{\delta})^{\beta-1} e^{-(\frac{x-\lambda}{\delta})^{\beta}}$$

where $\beta > 0$ is the **shape parameter**,

$\delta > 0$ is the **scale parameter**,

$\lambda > x$ is the **location parameter** (the minimum value of X).

4.2 CDF of Weibull distribution

$$F_X(x;\lambda,\delta,\sigma) = 1 - e^{-(\frac{x-\lambda}{\delta})^\beta}$$

where $\beta > 0$ is the **shape parameter**,

$\delta > 0$ is the **scale parameter**,

$\lambda > x$ is the **location parameter** (the minimum value of X).

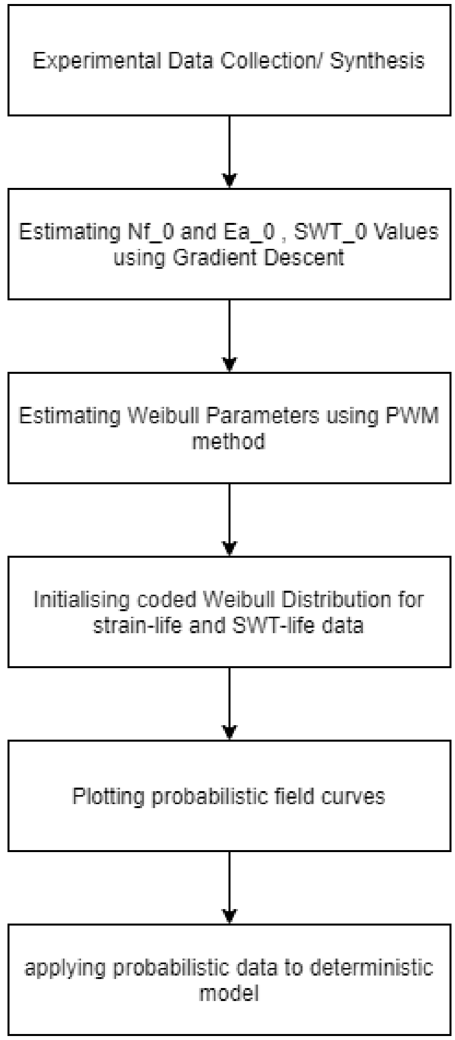
4.3 Percentile Curve of Weibull distribution

$$x_p = \lambda + \delta(-log(1-p))^{\frac{1}{\beta}}$$

where $0 \leq p \leq 1$

4.4 Simulation of Model

- Gathering Data from Base Article and Associated Article, Generating Unspecified Data using simple Linear Equation and Matrix Operations.
  - Data Gathering information is already given.
  - Matrix and Mathematical operations applied using numpy.
- Estimating/Predciting values for weibull distribution using any estimation/prediction method on the gathered data.
  - Estimated values taken from paper
  - Estimation using one selected method shown in 'New Analysis'.
- Initialising the coded Weibull Distribution class on required data.
  - Weibull distribution class contains methods the calculate pdf,cdf and percentile curves based on the Mathematical Formulaes stated above.
  - The initialisation of random variable is done using inbuilt weibull distribution function in numpy.
- Plotting the p-SWT-N curves for various values of p along with the plot of postulated data (assumed to be experimental).
  - Matplotlib is used to generate all the plots.
- Applying probabilistic field data to deterministic Unigrow Mode.



5 Coding and Simulation

5.1 Simulation Framework

The values for this model are:

- p-SWT-N field

$logN_0$	$logSWT_0$	λ	δ	β
-4.1079	-4.4317	53.8423	7.2698	3.6226

- p-Ea-N field

$logN_0$	$log\epsilon_{a0}$	λ	δ	β
-3.2593	-9.1053	36.6676	5.8941	4.6952

5.2 Reproduced Figures

- Used tools:
  - Python
  - Matplotlib
  - Numpy

Comparison of results Probabilistic Strain Life Field:

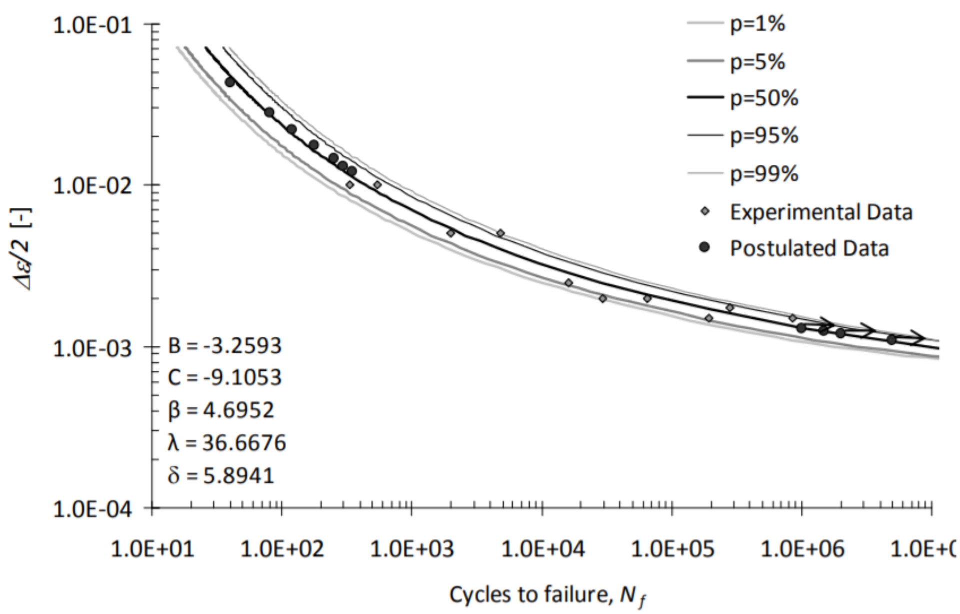
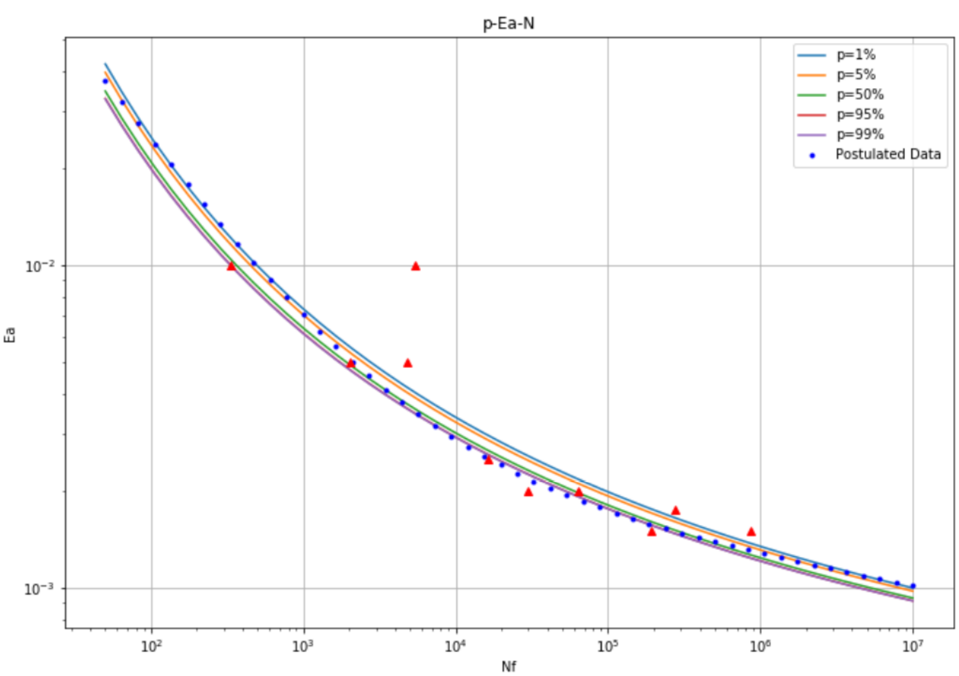


Figure 6: p-ε-N field for the S355 steel.



The Above plot (for Strain vs Number of Cycles to Failure) shows percent of probability curves along with postulated data (Experimental for 0.5 Percent).

Comparison of results Probabilistic Shane Watson Damage Life Field:

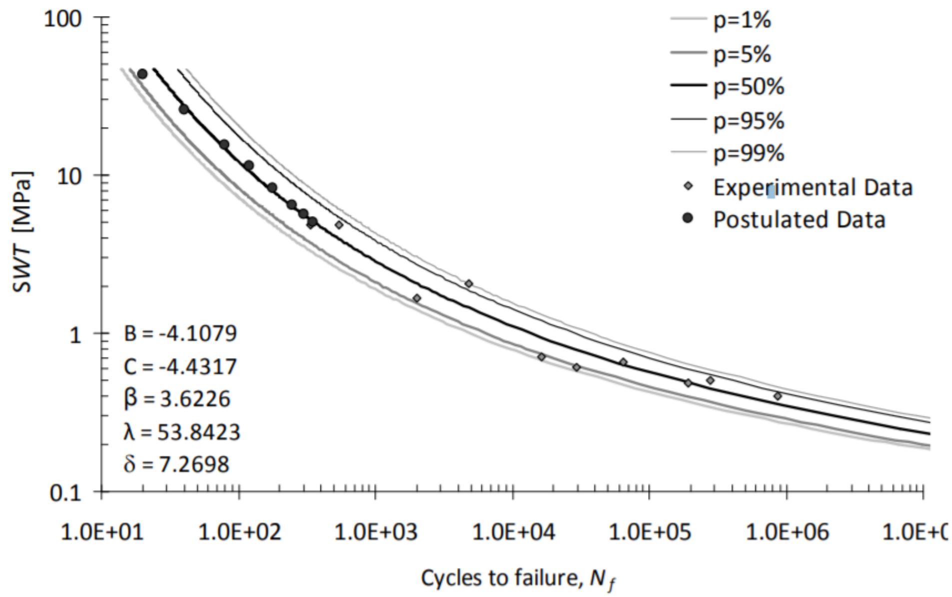
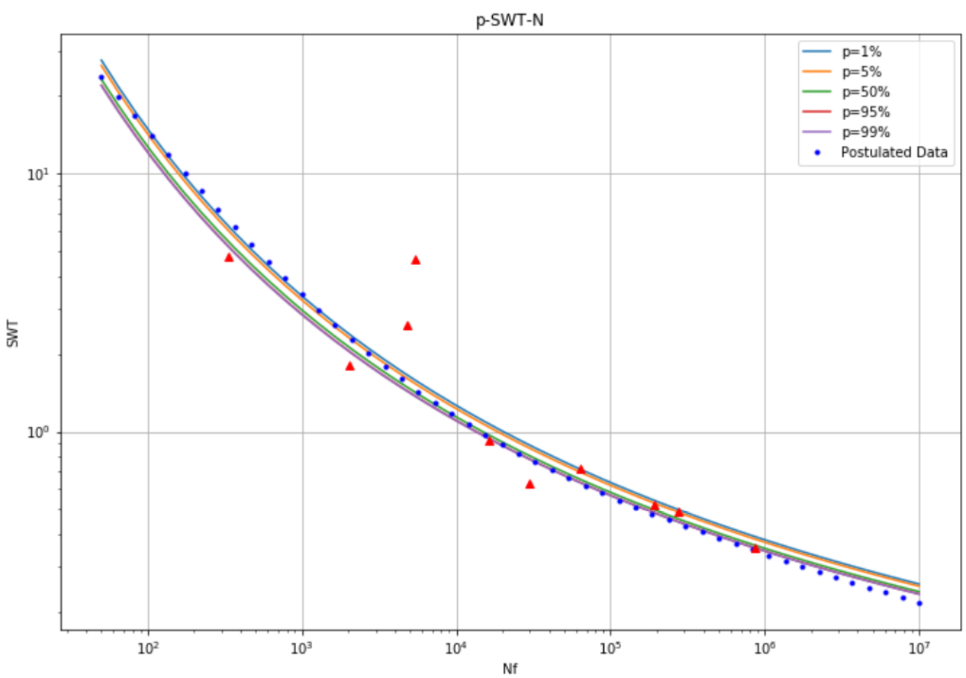


Figure 7: *p-SWT-N* field for the S355 steel.



The Above plot (for SWT Damage Parameter vs Number of Cycles to Failure) shows percent of probability curves along with postulated data (Experimental for 0.5 Percent).

5.3 New Work Done

Estimating and supplying analysis of parameters assumed/supplied externally and creating a closed form model with no external dependance that computes probabilistic fatigue crack propogation data using only the natural required input paramters.

5.3.1 New Analysis

- To Estimate and Check if the SWT-N parameters can be predicted using gradient descent based regression.
  - Performance metric : To see if the Loss is reduced by Gradient descent or not and if the gradient descent works, if the predicted values match the supplied data.

Using the analysis done by Castillo, E. and Galambos, J. [20] for lifetime regression models we formulate the following to predict the $SWT_0 and N_0$

$$E[log(\frac{N}{N_0})|log(\frac{\Delta SWT}{\Delta SWT_0})] = \frac{E[N^* \Delta SWT^*]}{log(\frac{\Delta SWT}{\Delta SWT_0})}$$

$$E[log(N)|log(\frac{\Delta SWT}{\Delta SWT_0})] = log(N_0) + \frac{K}{log(\frac{\Delta SWT}{\Delta SWT_0})} where, K = \lambda + \delta\Gamma(1 + \frac{1}{\beta})$$

Minimize Error Function Q

$$Q = \sum_{i=0}^{m} \sum_{j=1}^{n_i} (logN_{ij} - logN_0 - \frac{K}{log\Delta SWT_i - log\Delta SWT_0})^2$$

to get $logN_0$ and $log\Delta SWT_0$

We further minimise this cost function using a Gradient Descent algorithm to find optimal values of N_0 and SWT_0

The intial Estimation of Threshold - The starting parameters provided to gradient descent are calculated mathematically using,

$$\mu_i = \frac{1}{n_i} \sum_{j=1}^{n_i} (logN_{ij} = logN_0 - \frac{K}{log\Delta SWT_i - log\Delta SWT_0})^2$$

Gradient Descent is used to find the minimun of a function known as cost function, here in our case is the error function.Gradient Descent is a iterative learning algorithm and with each step it moves closer to the minimum based on the gradient of the cost function and learning rate. [3-23]

$$Costfunction = J(\theta) Gradient = \frac{\partial J(\theta)}{\partial \theta_i} Newparameter = \theta_i = \theta_i - \alpha \frac{\partial J(\theta)}{\partial \theta_i}$$

where θ are the parameters and α is the learning rate.Here,

$$Costfunction = Q = \sum_{i=0}^{m} \sum_{j=1}^{n_i} (logN_{ij} - logN_0 - \frac{K}{log\Delta SWT_i - log\Delta SWT_0})^2$$

$$\frac{\partial Q}{\partial logN_0} = \frac{1}{M}\sum -2(logN - logN_0 - \frac{K}{log\Delta SWT_i - log\Delta SWT_0})$$

$$\frac{\partial Q}{\partial K} = \frac{1}{M}\sum \frac{-2}{log\Delta SWT_i - log\Delta SWT_0}(logN - logN_0 - \frac{K}{log\Delta SWT_i - log\Delta SWT_0})$$

$$\frac{\partial Q}{\partial logSWT_0} = \frac{1}{M}\sum \frac{-2b}{(log\Delta SWT_i - log\Delta SWT_0)^2}(logN - logN_0 - \frac{K}{log\Delta SWT_i - log\Delta SWT_0})$$

- Estimate the parameters of Weibull Distribution using PWM estimation on real and predicted data.
  - Performance metric : To compare and see if the computed Weibull parameters using PWM match those supplied with the paper and if the predicted vs deterministic parameters match.

Probability Weighted Moments(PWM) method to determine Weibull parameters

The final equation based on paper [2] are,

$$\frac{3M_2 - M_0}{2M1 - M_0} = \frac{2 - 3.2^{\frac{-1}{\beta}} + 3^{\frac{-1}{\beta}}}{1 - 2^{\frac{-1}{\beta}}}\delta = \frac{2M_1 - M_0}{(1 - -2^{\frac{-1}{\beta}})\Gamma_\beta}\lambda = M_0 - \delta\Gamma_\beta$$

where $M_r = M_{1,r,0}, r = 0,1,2$ and $\Gamma_\beta = \Gamma(1 + \frac{1}{\beta})$ and

$$\hat{M_0} = \frac{1}{n}\sum_{i=1}^{n} x_i \hat{M_1} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(i-1)x_i \hat{M_2} = \frac{1}{n(n-1)(n-2)}\sum_{i=1}^{n}(i-1)(i-2)x_i$$

Since β cannot be determined metamatically from the equation Numerical Method is used,

- Newton's Method for Mathematical Approximation

  $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

  where,

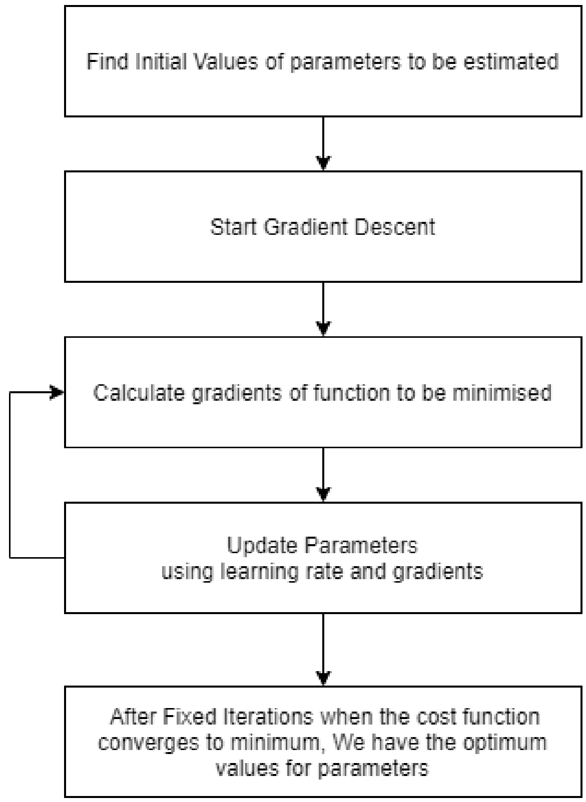
  $$f(x_n) = 2^x(C-3) + 3^x - (C-2)f'(x_n) = (log2)(2^x)(C-3) + log3(3^x)$$

  where $x = \frac{-1}{\beta}$ and $C = \frac{3M_2 - M_0}{2M1 - M_0}$

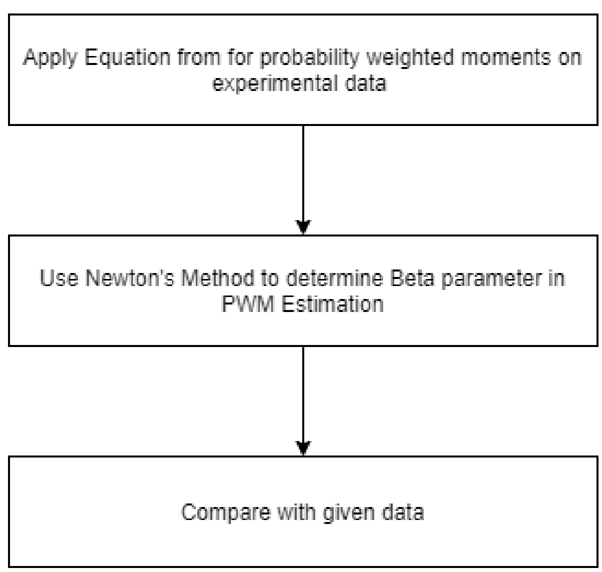
- Determining the p-Nf Weibull distribution given a fixed Value Of SWT (Shane Watson Damage Parameter)
  - Performance metric : To compare and observe distribution of similar shape with difference in probability of failure for different damage parameter values

$$N^*SWT^* \sim W(\lambda,\delta,\beta)log(\frac{N}{N_0})log(\frac{SWT}{SWT_0}) \sim W(\lambda,\delta,\beta)log(\frac{N}{N_0}) \sim W(\frac{\lambda}{log(\frac{SWT}{SWT_0})}, \frac{\delta}{log(\frac{SWT}{SWT_0})}, \beta)$$

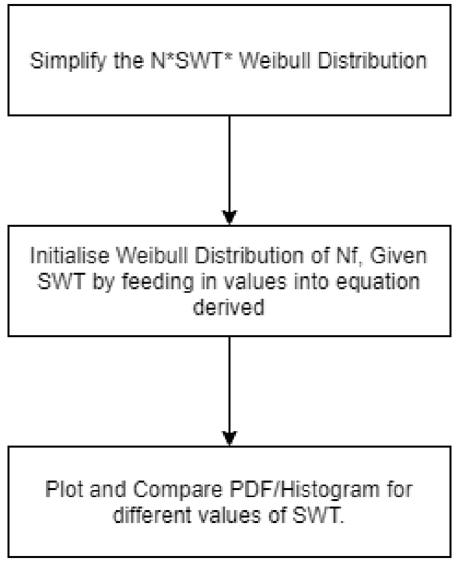
5.3.2 New Coding / Algorithm



Estimation of Threshold Value $(N_0, \Delta\sigma_0)$



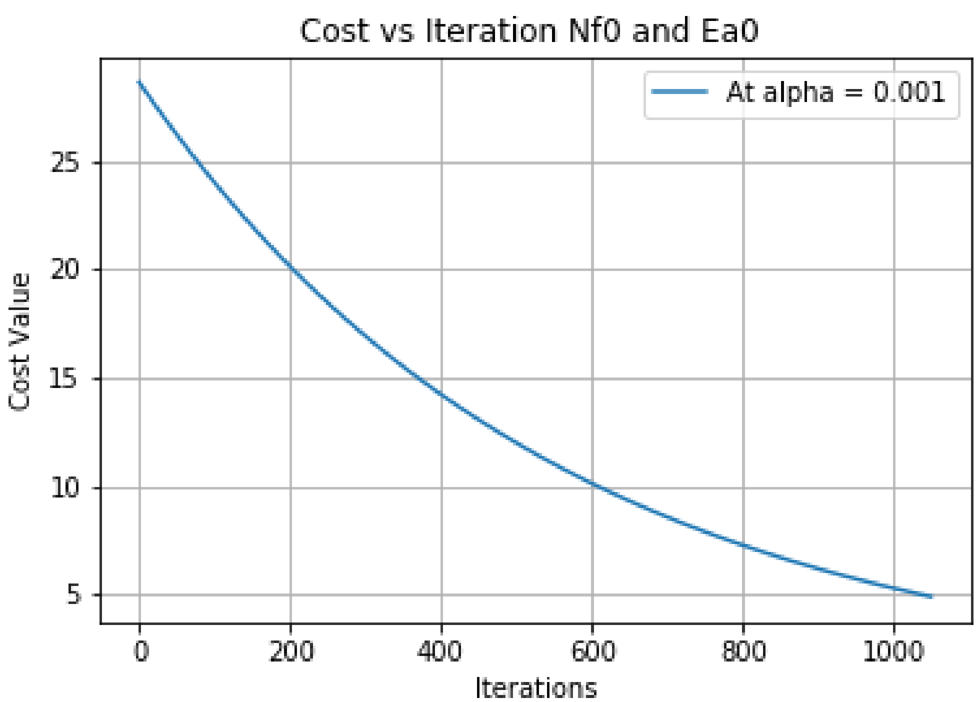
Probability Weighted Moments(PWM) method to determine Weibull parameters



p-Nf given SWT

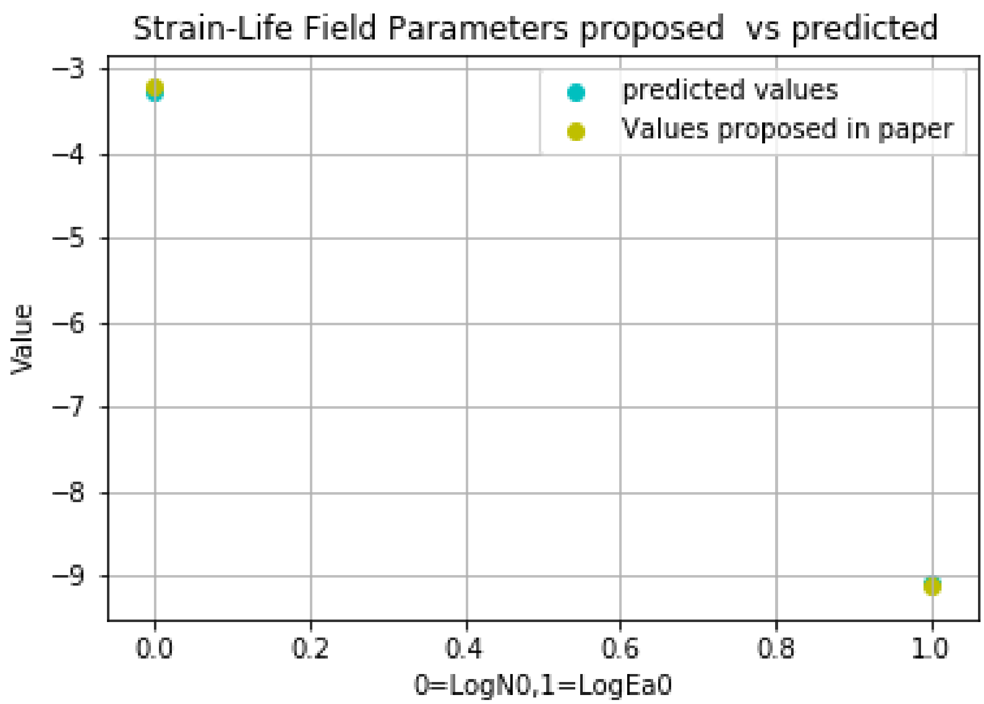
5.3.3 New Results

- To Estimate and Check if the Ea-N and SWT-N parameters can be predicted using gradient descent based regression.



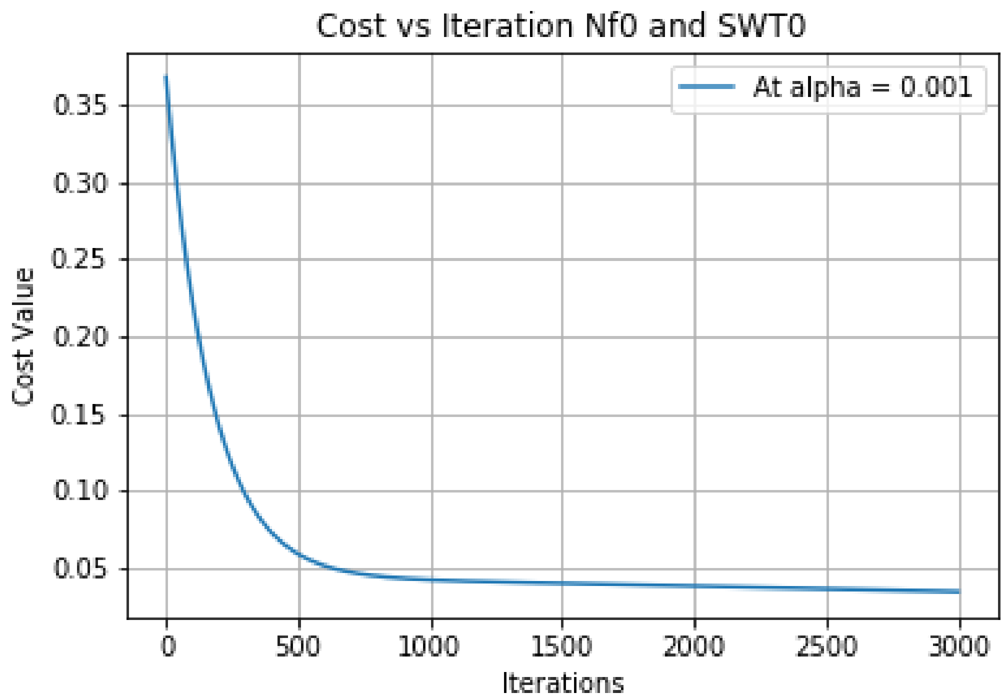
This is plot of Cost vs Iteration, it shows how cost function changes with iterations.It shows that the cost function taken from [2] converges using gradient descent algorithm on data generated

Strain-Life Field.

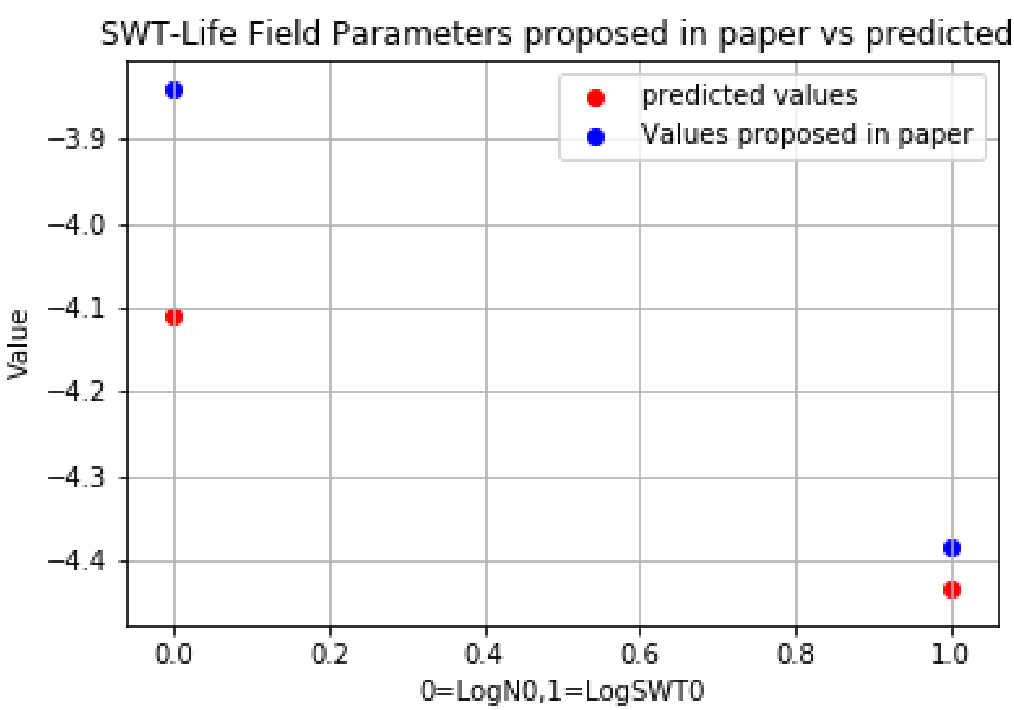


This is a plot of the proposed and predicted values and can be used to compare them.

Proposed $logN_0$	Proposed $log\epsilon_{a0}$	Predicted $logN_0$	Predicted $log\epsilon_{a0}$
-3.2593	-9.1053	-3.2169	-9.1285



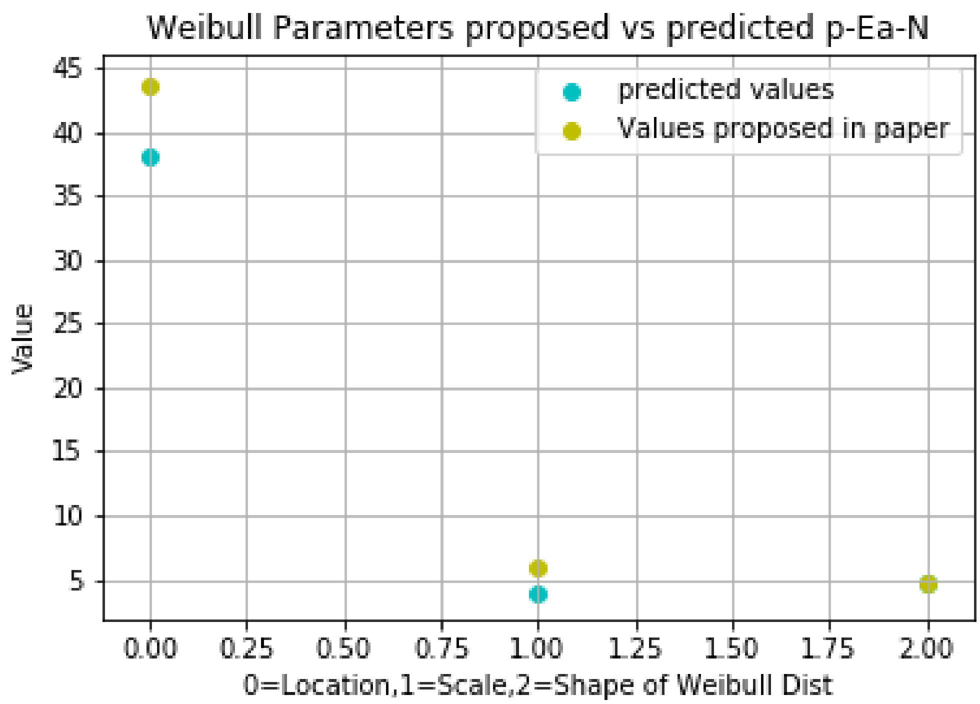
This is plot of Cost vs Iteration, it shows how cost function changes with iterations.It shows that the cost function taken from [2] converges using gradient descent algorithm on data generated for SWT-Life Field.



This is a plot of the proposed and predicted values and can be used to compare them.

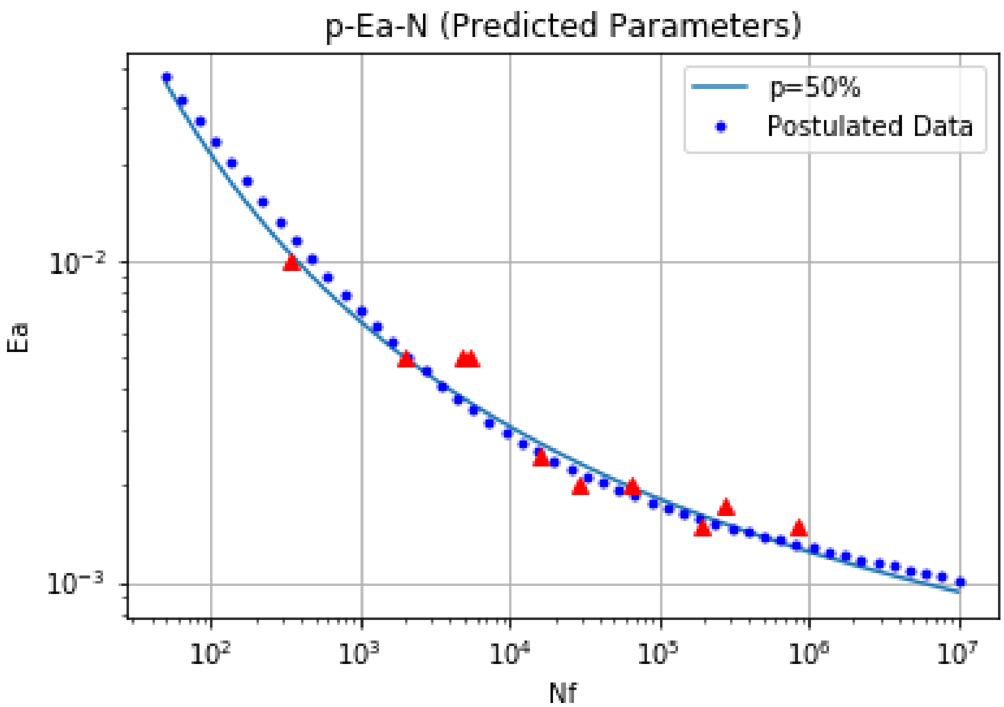
Proposed $logN_0$	Proposed $logSWT_0$	Predicted $logN_0$	Predicted $logSWT_0$
-4.1079	-4.4317	-3.8413	-4.383809

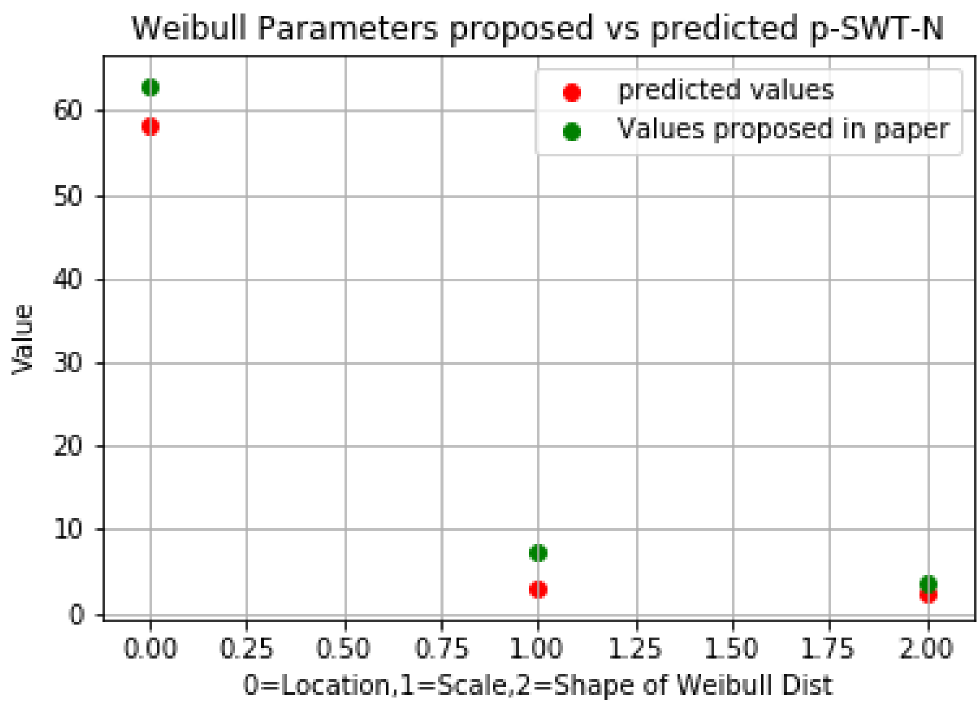
- Estimate the parameters of Weibull Distribution using PWM estimation on real and predicted data.



This is a plot of the proposed and predicted values for Location, Scale and Shape parametesrs of the Weibull Distribution of the Strain Life Field

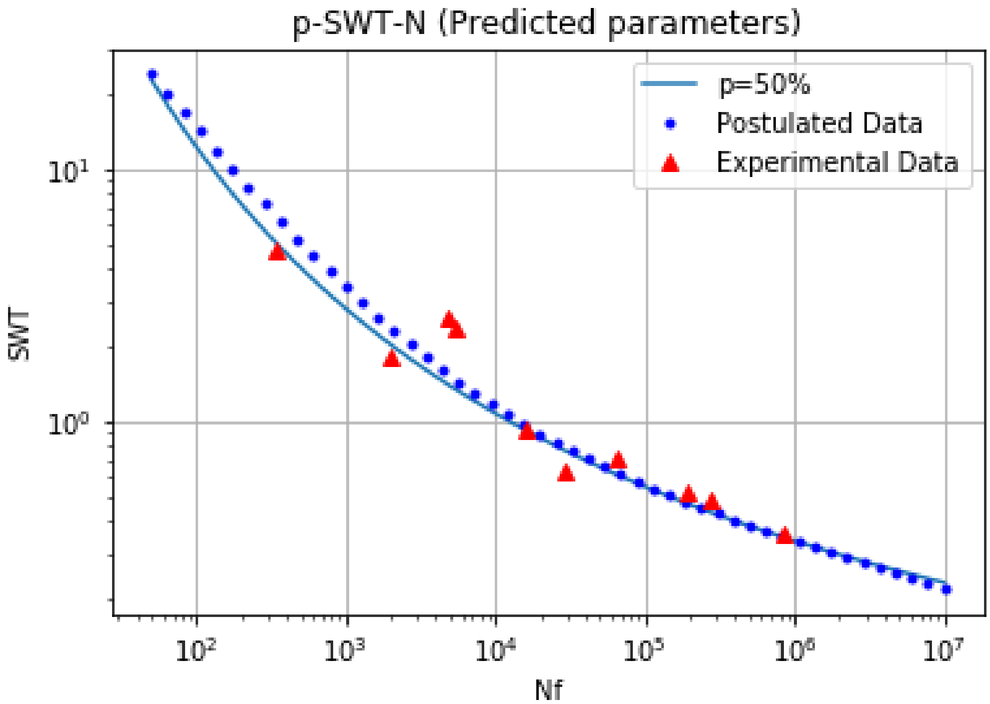
Proposed Location	Proposed Scale	Proposed Shape	Predicted	Predicted Scale	Predicted Shape
62.8423	7.2698	3.6226	58.31011	2.80965	2.30816



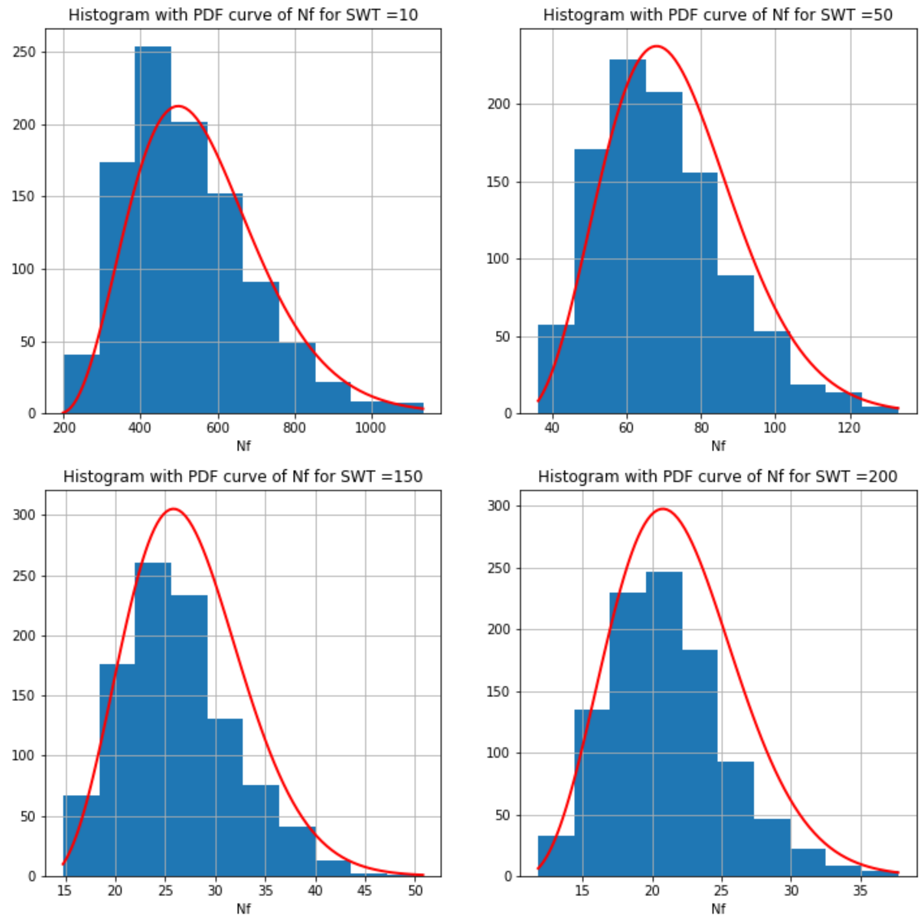


This is a plot of the proposed and predicted values for Location, Scale and Shape parametesrs of the Weibull Distribution of the SWT Life Field

Proposed Location	Proposed Scale	Proposed Shape	Predicted Location	Predicted Scale	Predicted Shape
43.6676	5.8941	4.6952	38.11514	3.8694	4.6747



- Determining the p-Nf Weibull distribution given a fixed Value Of SWT (Shane Watson Damage Parameter)



These Plots show the PDF of Number of Cycles to failure given one particular value of Shane Watson Damage parameter.

5.3.4 New Inferences

- To Estimate and Check if the SWT-N parameters can be predicted using gradient descent based regression.
  - By observing the cost vs iterations plot for both the fields we can confidently say that the cost function converges to a minimum and that the values predicted by gradient descent are approximately same as those proposed under the error assumption that the postulated data here can differ from that given in paper as it was generated synthetically.
  - We can infer that creating a closed form model on experimental data is possible given we combine the model with our proposed estimation model for SWT-N or Ea-N parameters

- Estimate the parameters of Weibull Distribution using PWM estimation on real and predicted data.
  - The values predicted by PWM method match the proposed values in the paper for both the probabilistic models under the error assumption that the postulated data here can differ from that given in paper as it was generated synthetically.
  - We can infer that creating a closed form model on experimental data is possible given we combine the model with our proposed estimation model for SWT-N or Ea-N Weibull parameters

6 Inference Analysis/ Comparison

- To Determine the $p - SWT - N_f$ and $p - E_a - N_f$ fields for given Experimental Date and Theoratical Data
  - Since there is a large variation in the value of N_f, a Deterministic Model fails to provides a complete analytical description of the statistical properties of the physical problem. While $p - SWT - N_f$ showed a satisfactory agreement with the experimental data.
- Determining the $p - N_f$ Weibull distribution given a fixed Value Of SWT (Shane Watson Damage Parameter)
  - The Plots show the PDF of Number of Cycles to failure given one particular value of Shane Watson Damage parameter, observing and comparing these we see that number of loading cycles to failure reduces as SWT increases showing that the distribution is practical in nature.

7 Contribution of team members

7.1 Technical contribution of all team members

Task	Arpit	Kaushal
Main Analysis	✓	✓
New Analysis	✓	✓
Coding	✓	✓

7.2 Non-Technical contribution of all team members

Task	Arpit	Kaushal
Abstract	✓	✓
Concept Map 1	✓	✓
Concept Map 2	✓	✓
Report	✓	✓

8 Submission checklist for uploading on Google Drive

This section provides the submission checklist for smooth and efficient submission process. (This is for your reference and please remove this while writing your report).

- Soft copy of this project Report
- Soft copy of Abstract
- Soft copy of Concept Map 1 and 2
- Soft copy of base article
- Soft copy of analysis (hand written)(jupyter notebooks)
- Folder of matlab(python) codes (with proper naming)
- Folder of reproduced results in .fig and .jpg format
- latex (.tex) file of the project report.

9 References

- [1] J.A.F.O. Correia, A.M.P. de Jesus, A. Fernández-Canteli, R.A.B. Calçada, Modelling probabilistic fatigue crack propagation rates for a mild structural steel, Frattura ed Integrità Strutturale, 31(2015) 80-96.
- [2] Castillo, E., Galambos, J., Lifetime Regression Models Based on a Functional Equation of Physical Nature", Journal of Applied Probability, 24 (1987) 160-169.
- [3] "Gradient Descent," Gradient Descent - ML Glossary documentation. [Online]. Available: https://ml-cheatsheet.readthedocs.io/en/latest/gradient_descent.html. Accessed: 01-Apr-2020.
- [4]Paul Dario Toasa Caizaa, Thomas Ummenhofera, General probability weighted moments for the three-parameter Weibull Distribution and their application in S–N curves modelling, International Journal of Fatigue 33(2011) 1533-1538.
- [5] De Jesus, A.M.P., Matos, R., Fontoura, B.F.C., Rebelo, C., Simões da Silva, L., Veljkovic, M., A comparison of the fatigue behavior between S355 and S690 steel grades, Journal of Constructional Steel Research, 79 (2012) 140–150.
- [6] Noroozi, A.H., Glinka, G., Lambert, S., A two parameter driving force for fatigue crack growth analysis, International Journal of Fatigue, 27 (2005)1277-1296.
- [7] Castillo, E., Fernández-Canteli, A., A Unified Statistical Methodology for Modeling Fatigue Damage, Springer, (2009).
- [8] Paris, P.C., Gomez, M., Anderson, W.E., A rational analytic theory of fatigue, Trend Engineering, 13 (1961) 9-14.
- [9] Coffin, L.F., A study of the effects of the cyclic thermal stresses on a ductile metal, Translations of the ASME, 76 (1954) 931-950.
- [10] Manson, S.S., Behaviour of materials under conditions of thermal stress, NACA TN-2933, National Advisory Committee for Aeronautics, (1954).
- [11] Morrow, J.D., Cyclic plastic strain energy and fatigue of metals, Int. Friction, Damping and Cyclic Plasticity, ASTM STP 378, (1965) 45-87.
- [12] Smith, K.N., Watson, P., Topper, T.H., A Stress-Strain Function for the Fatigue of Metals, Journal of Materials, 5(4) (1970) 767-778.
- [13] Shang, D.-G., Wang, D.-K., Li, M., Yao, W.-X., Local stress–strain field intensity approach to fatigue life prediction under random cyclic loading, International Journal of Fatigue, 23 (2001) 903–910.
- [14] Noroozi, A.H., Glinka, G., Lambert, S., A study of the stress ratio effects on fatigue crack growth using the unified two-parameter fatigue crack growth driving force, International Journal of Fatigue, 29 (2007) 1616-1633.
- [15] Noroozi, A.H., Glinka, G., Lambert, S., Prediction of fatigue crack growth under constant amplitude loading and a single overload based on elasto-plastic crack tip stresses and

strains, Engineering Fracture Mechanics, 75 (2008) 188-206.
- [16] Peeker, E., Niemi, E., Fatigue crack propagation model based on a local strain approach, Journal of Constructional Steel Research, 49 (1999) 139–155.
- [17] Glinka, G., A notch stress-strain analysis approach to fatigue crack growth, Engineering Fracture Mechanics, 21 (1985) 245-261.
- [18] Hurley, P.J., Evans, W.J., A methodology for predicting fatigue crack propagation rates in titanium based on damage accumulation, Scripta Materialia, 56 (2007) 681–684.
- [19] Neuber, H., Theory of stress concentration for shear-strained prismatic bodies with arbitrary nonlinear stress–strain law, Trans. ASME Journal of Applied Mechanics, 28 (1961) 544–551.
- [20] Moftakhar, A., Buczynski, A., Glinka, G., Calculation of elasto-plastic strains and stresses in notches under multiaxial loading, International Journal of Fracture, 70 (1995) 357-373.
- [21] Reinhard, W., Moftakhar, A., Glinka, G., An Efficient Method for Calculating Multiaxial Elasto-Plastic Notch Tip Strains and Stresses under Proportional Loading, Fatigue and Fracture Mechanics, ASTM STP 1296, R.S. Piascik, J.C. Newman, N.E. Dowling, Eds., American Society for Testing and Materials, 27 (1997) 613-629.
- [22] Mikheevskiy, S., Glinka, G., Elastic–plastic fatigue crack growth analysis under variable amplitude loading spectra," International Journal of Fatigue, 31 (2009) 1828–1836.
- [23] Sebastian Ruder, An overview of gradient descent optimization algorithms, arXiv:1609.04747v2 [cs.LG] 15 Jun 2017