

## Agenda

- 1) what is recursion
- 2) some easy questions

## Why Recursion is Important:

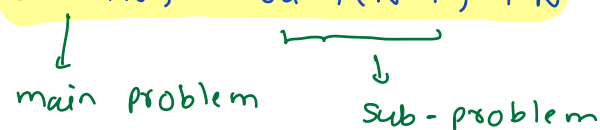
- i) Trees, graphs
- ii) Backtracking, DP

**Recursion** : function calling itself.

$$\text{sum}(N) = 1 + 2 + 3 + \dots + N$$

$$\text{sum}(N-1) = 1 + 2 + 3 + \dots + N-1$$

$$\text{sum}(N) = \text{sum}(N-1) + N$$



How to apply recursion?

- 1) **Assumption** : what the function is going to do
- 2) **Main logic** : solving **main problem** using <sup>just smaller</sup> **sub-problem** of same type.
- 3) **Base condition** : when the recursion should stop.

```
int sum(int n) {
```

```
    if (n == 1) {
```

```
        return 1;
```

```
    }
```

```
    int temp = sum(n-1);
```

```
    return temp + n;
```

```
}
```

```
void main() {
```

```
    int n = sc.nextInt();
```

```
    sopln(sum(n));
```

```
}
```

**Assumption** : Given  $n$ , returning  
sum of  $n$  natural no.

**Main logic** :

$Sum(n) = Sum(n-1) + n$

**Base condition** : smallest problem  
whose ans is known.

$if(n == 1) \rightarrow 1$

```
int sum (int n) {
```

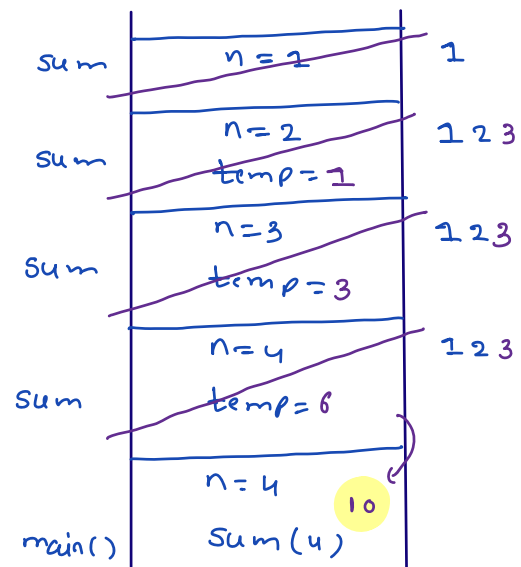
```
    if (n == 1) {
```

```
        1 | return 1;
```

```
    2 int temp = sum(n-1);
```

```
    3 return temp+n;
```

```
}
```



```
void main() {
```

```
    int n = scn.nextInt();
```

```
    sopln (sum(n));
```

```
}
```

```
int sum (int n) {
```

```
    if (n == 1) {
```

```
        1 | return 1;
```

```
    2 int temp = sum(n-1);
```

```
    3 return temp+n;
```

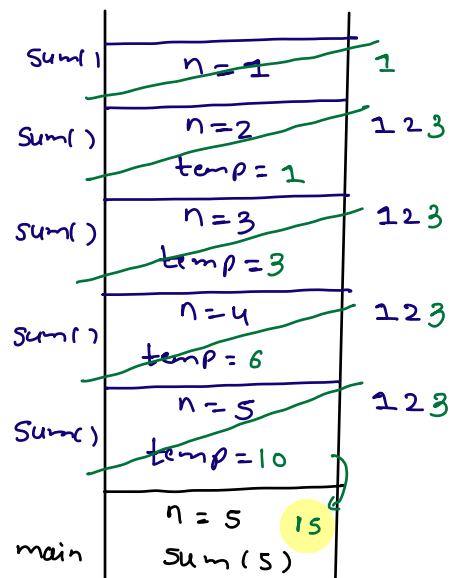
```
}
```

```
void main() {
```

```
    int n = scn.nextInt();
```

```
    sopln (sum(n));
```

```
}
```



Q.2 Given  $n$ , find factorial of  $n$ .

```
int factorial (int n) {  
    if (n == 0) {  
        return 1;  
    }  
  
    int temp = factorial (n-1);  
    return temp * n;  
}
```

**Assumption** : Given  $n$ , return factorial of  $n$ .

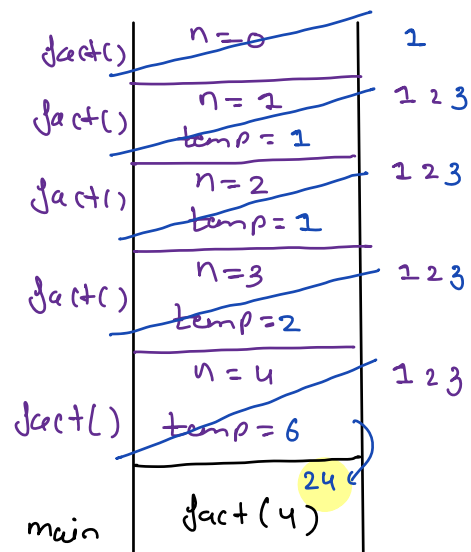
**Main logic** :

$\text{factorial}(N) = \text{factorial}(N-1) * N$

**Base condition** :

$\text{if } (n == 1) \{$	$\text{if } (n == 0) \{$
$\text{return } 1;$	$\text{return } 1;$
$\}$	$\}$

```
int factorial (int n) {  
    1 | if (n == 0) {  
        return 1;  
    }  
  
    2 | int temp = factorial (n-1);  
  
    3 | return temp * n;  
}
```



Q.3 Given  $n$ , find  $n^{\text{th}}$  fibonacci number (using recursion)

0	1	2	3	4	5	6	7	8	9
0	1	1	2	3	5	8	13	21	34

```
int fib(int n) {  
    if (n == 0 || n == 1) {  
        return n;  
    }  
}
```

**Assumption** : Given  $n$ , find  $n^{\text{th}}$  fibonacci term.

```
int temp1 = fib(n-1);  
int temp2 = fib(n-2);  
return temp1 + temp2;  
}
```

**Main logic :**

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$

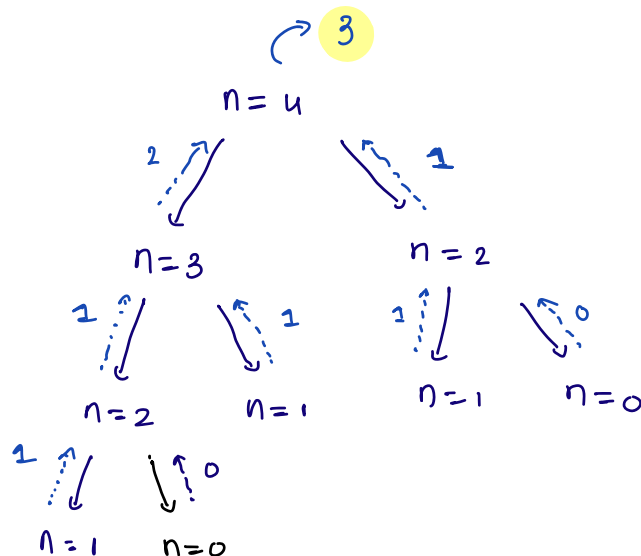
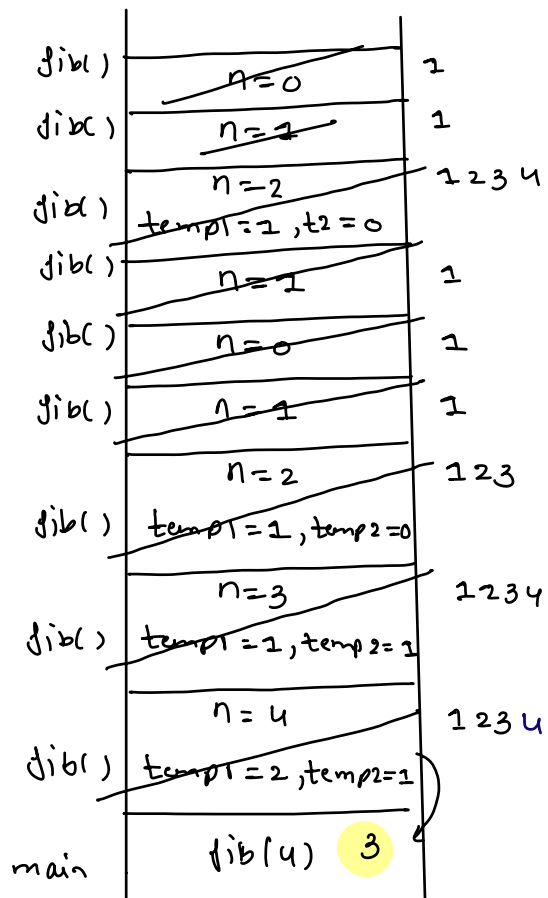
**Base condition :**

```
if (n == 0 || n == 1) {  
    return n;  
}
```

```

int fib(int n) {
    1 | if (n == 0 || n == 1) {
        return n;
    }
    2 | int temp1 = fib(n-1);
    3 | int temp2 = fib(n-2);
    4 | return temp1 + temp2;
}

```



euler  
diagram

Q.4 Given n, print values from 1 to n (Increasing)

n=5, o/p: 1 2 3 4 5

n=4, o/p: 1 2 3 4

```
void Inc(int n) {  
    if(n==0) {  
        return;  
    }  
    Inc(n-1);  
    sopdn(n);  
}
```

Assumption: given n, print  
1 to n.

Main logic:

$Inc(n) \Rightarrow Inc(n-1) + sopdn(n)$

Base condition:

<pre>if(n==1) {     print(1);     return; }</pre>	<pre>if(n==0) {     return; }</pre>
---------------------------------------------------------------	---------------------------------------------

```
void Jnc (int n) {
```

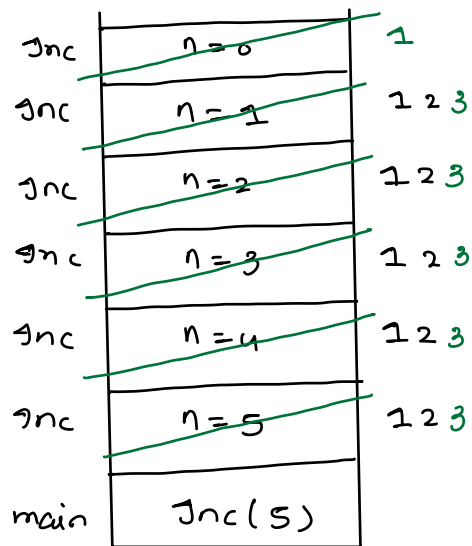
```
    if (n == 0) {
```

```
        1 | return;
```

```
        2 Jnc (n-1);
```

```
        3 sOPdn(n);
```

```
    }
```



o/p: 1 2 3 4 5

```
void Jnc (int n) {
```

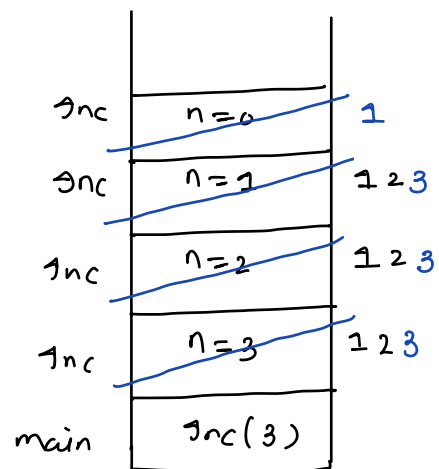
```
    if (n == 0) {
```

```
        1 | return;
```

```
        2 Jnc (n-1);
```

```
        3 sOPdn(n);
```

```
    }
```



o/p: 1 2 3



3

todo: to are and  
confirm your  
answer.

Answer.

$$\text{SOPdn}(n);$$

3

$s$  to  $e$  is palindromic or not.

1 4  $\rightarrow$  true

2 5  $\rightarrow$  false

A  $\rightarrow$  

$$\text{if } (A[s] == A[e]) \{$$

```

    Pal(A, s+1, e-1);
    ans

```

Σ

a b c b t b c d c b  
 0 1 2 3 4 5 6 7 8 9

$s = 3$

$e = 6$

```
boolean palindrome (char [] A, int s, int e) {
```

```
    if (s == e || s > e) {
```

```
        return true;
```

```
    }
```

```
    if (A[s] != A[e]) {
```

```
        return false;
```

```
    }
```

```
    else {
```

```
        boolean ans = palindrome(A, s+1, e-1);
```

```
        return ans;
```

```
    }
```

```
}
```

a b c b a

s

e

$s == e$

a b b a

e s

$s > e$

```
boolean palindrome (char [] A, int s, int e) {
```

S=1, e=5

```
    if (s==e || s>e) {
```

```
        1 | return true;
        3
```

```
    if (A[s] != A[e]) {
```

```
        2 | return false;
```

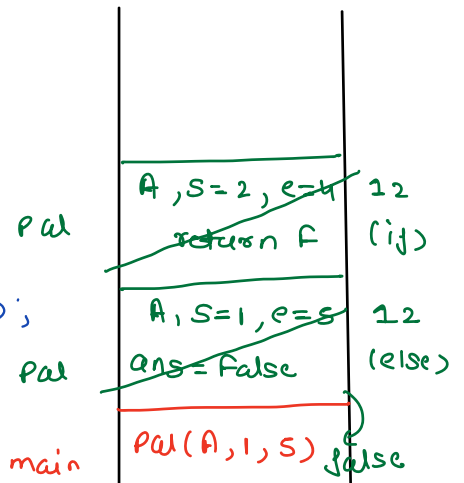
```
    }
    else {
```

```
        boolean ans = palindrome(A, s+1, e-1);
```

```
        return ans;
```

```
    }
    3
```

A = a b d c b b t  
0 1 2 3 4 5 6



```
boolean palindrome (char [] A, int s, int e) {
```

S=2, e=8

```
    if (s==e || s>e) {
```

```
        1 | return true;
        3
```

```
    if (A[s] != A[e]) {
```

```
        2 | return false;
```

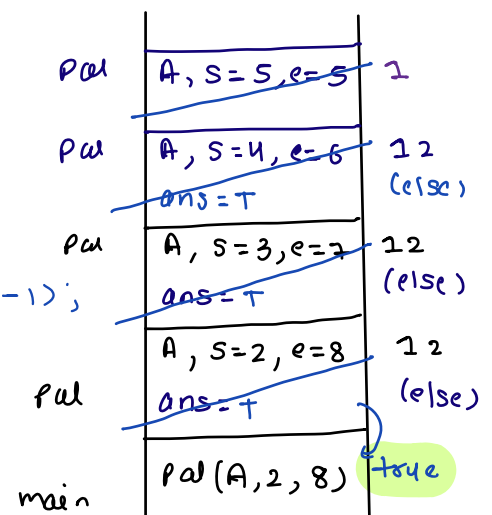
```
    }
    else {
```

```
        boolean ans = palindrome(A, s+1, e-1);
```

```
        return ans;
```

```
    }
    3
```

A = a b c n i t i n c a  
0 1 2 3 4 5 6 7 8 9



Doubts

Colorful number

3245  $\Rightarrow$  [3, 2, 4, 5]

3	=	3	2	=	2	4	=	4	5	=	5
3	2	=	6	2	4	=	8	4	5	=	20
3	2	4	=	24	2	4	5	=	40		
<del>3 2 4 5</del>											

Subarrays product with hashset.

$O(n^2)$  : TC

```
for (s=0; s<n; s++) {  
    int prod=1;  
    for (e=s; e<n; e++) {  
        if (s==0 && e==A.length-1) { continue; }  
        prod = prod*A[e];  
        if (hs.contains(prod)) {  
            return false;  
        }  
        hs.add(prod);  
    }  
}
```

return true;

## longest consecutive sequence

100 2 4 99 5 3 98 1

cons. seq: 1 2 3 4 5

98 99 100



100 2 4 99 5 3 98 1

100 → ~~T~~ F      5 → ~~T~~ F

2 → ~~T~~ F      3 → ~~T~~ F

4 → ~~T~~ F      98 → T

99 → ~~T~~ F      1 → T

HashMap

ele vs can this  
ele be  
a starting  
pt. of  
any cons. seq.

- i) travel the array and put true in front of every ele in your map.
- ii) travel the array and put false in front of every invalid start pt.  
if  $A[i]-1$  is present then  $A[i]$  can't be a start pt.
- iii) now travel map and the sp in front of which true is present create cons. seq. using that sp.

$100 \rightarrow \cancel{T} F$        $5 \rightarrow \cancel{T} F$   
 $2 \rightarrow \cancel{T} F$        $3 \rightarrow \cancel{T} F$   
 $4 \rightarrow \cancel{T} F$        $* 98 \rightarrow T$   
 $99 \rightarrow \cancel{T} F$        $* 1 \rightarrow T$

$ans = \cancel{\emptyset} \cancel{5}$

$SP = 98$      $den = \emptyset$   
 $\cancel{1}$   
 $\cancel{2}$   
 $3$

```
for (int sp : map.keySet()) {
```

```
    if (map.get(sp) == true) {
```

```
        int den = 0;
```

```
        while (map.containsKey(sp + den)) {
```

```
            den++;
```

```
        }
```

```
        ans = Math.max(den, ans);
```

```
    }
```

```
}
```

$SP = 1$      $den = \emptyset$   
 $\cancel{1}$   
 $\cancel{2}$   
 $\cancel{3}$   
 $4$   
 $5$

$\Rightarrow O(n)$

2    14    3    5    9    14    5    9    14

$max = 14$

i) find max

freq of max = 3

ii) find freq of max

$ans = n - \text{freq of max}$

$= 9 - 3 = 6$