

Agenda

- i) Linear search to find K in $A[]$
- ii) Binary search to find K in $A[]$
- iii) First occurrence of K in array
- iv) Floor of K in array
- v) Local minima

Q-1 Given $A[]$, find if K is present or not.

$A =$ 2 9 8 17 42 1
 0 1 2 3 4 5

$K = 19$ $ans = -1$

$K = 17$ $ans = 3$

Linear search

```
int search (int []A, int K) {  
    int n = A.length;  
    for (int i = 0; i < n; i++) {  
        if (A[i] == K) {  
            return i;  
        }  
    }  
    return -1;  
}
```

TC: $O(n)$

Organised vs unorganised data:

Searching in organised data takes less efforts.

(Organised clothes section, dictionary,)

Q.2 Given a sorted $A[]$, find if K is present or not.

$A =$

2	9	13	15	19	24	31	48	52
0	1	2	3	4	5	6	7	8

$K = 13$ ans = 2

$K = 48$ ans = 7

$K = 10$ ans = -1



Binary Search

- if $A[mid] == K$: got the answer, return mid
- $A[mid] < K$: discard left side
 $\Rightarrow lo = mid + 1$
- $A[mid] > K$: discard right side
 $\Rightarrow hi = mid - 1$

```
int search ( int [] A, int K) {
```

K = 8

```
int n = A.length;
```

```
int lo = 0, hi = n-1;
```

```
while ( lo <= hi) {
```

```
int mid = (lo+hi)/2;
```

```
if ( A[mid] == K) {
```

```
return mid;
```

```
5
```

```
else if ( A[mid] < K) {
```

```
// discard left side
```

```
lo = mid + 1;
```

```
3
```

```
else if ( A[mid] > K) {
```

```
// discard right side
```

```
hi = mid - 1;
```

```
5
```

```
}
```

```
return -1;
```

```
}
```

A = 2 4 8 13 19 29 38 42 49
0 1 2 3 4 5 6 7 8

lo hi
m

ans = 2

K = 42

A = 2 4 8 13 19 29 38 42 49
0 1 2 3 4 5 6 7 8

lo hi
m

ans = 7

K = 13

A = 2 4 8 13 19 29 38 42 49
0 1 2 3 4 5 6 7 8

hi
lo
m

ans = 3

K = 10

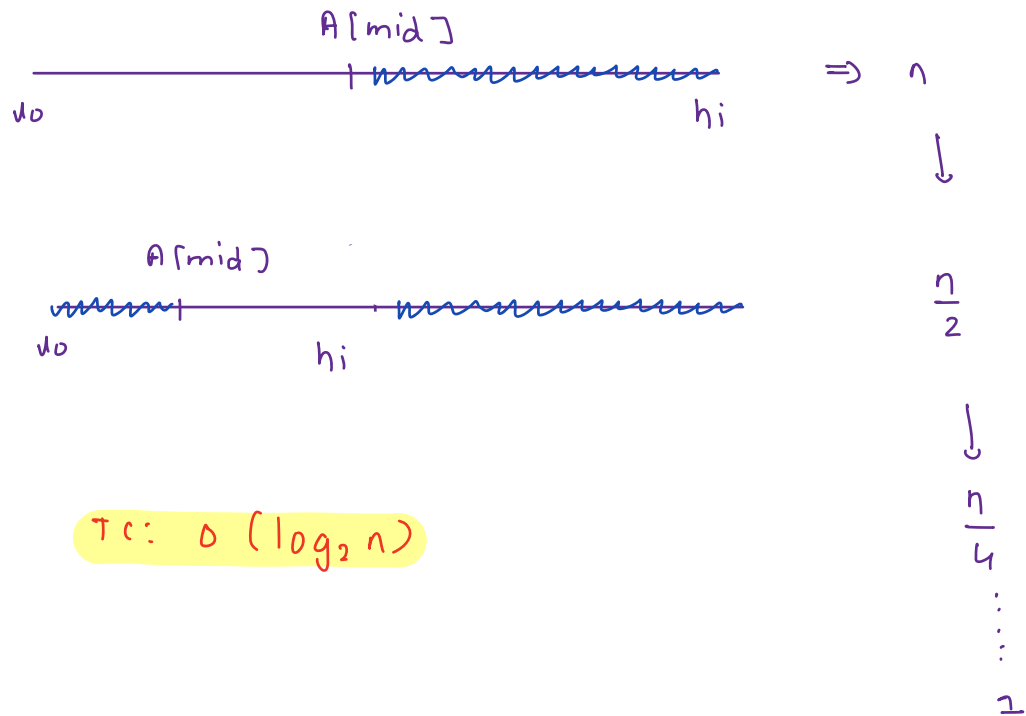
A = 2 4 8 13 19 29 38 42 49
0 1 2 3 4 5 6 7 8

hi

lo
m

get out of loop

ans = -1



$$Tc: O(\log_2 n)$$

Array is sorted

example: $n = 10^5$ ($10^5 = \sim 2^{16}$)

Linear search : $n \Rightarrow 10^5$ steps

Binary search : $\log_2 n \Rightarrow \log_2(10^5)$
 $= \log_2(2^{16}) = 16$ steps

Q.3 Given a sorted A[], find first occurrence of K.

A = 2 2 3 4 5 5 5 7 7 8 9 12 19 20
 0 1 2 3 4 5 6 7 8 9 10 11 12 13

Expected TC: $O(\log_2 n)$

K = 3 ans = 2

K = 5 ans = 4

K = 10 ans = -1

Slight modification of Binary search

↳ $A[mid] == K$

⇒ keep searching on left

```
int search (int [] A, int K) {
```

```
    int n = A.length;
```

K = 5

```
    int lo = 0, hi = n-1, ans = -1;
```

```
    while (lo <= hi) {
```

```
        int mid = (lo+hi)/2;
```

```
        if (A[mid] == K) {
```

```
            ans = mid;
```

```
            hi = mid-1;
```

```
        }
```

```
        else if (A[mid] < K) {
```

```
            // discard left side
```

```
            lo = mid+1;
```

```
        }
```

```
        else if (A[mid] > K) {
```

```
            // discard right side
```

```
            hi = mid-1;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

A = 2 2 5 5 5 5 7 8 9 9
 0 1 2 3 4 5 6 7 8 9
 hi lo
 m

ans = ~~7~~ 2

K = 9

A = 2 2 5 5 5 5 7 9 9 12
 0 1 2 3 4 5 6 7 8 9
 hi lo
 m

ans = ~~7~~

Q-4 Given a sorted array, find floor of k in the array.

floor(k) \Rightarrow max of all the values which are $\leq k$

A = 12 14 21 25 28 32 35 38 42 51
 0 1 2 3 4 5 6 7 8 9

k	floor
24	21
28	28
40	38
55	51

floor(k) = $\begin{cases} k & \text{when } k \text{ is present in } A[] \\ \text{just small than } k & \text{when } k \text{ is absent in } A[] \end{cases}$

A = 12 14 21 25 28 32 35 38 42 51
 0 1 2 3 4 5 6 7 8 9

if ($A[\text{mid}] \leq k$) {
 ans = $A[\text{mid}]$;
 lo = $\text{mid} + 1$;

}

else {

 hi = $\text{mid} - 1$;

}

```
int floor (int [] A, int k ) {
```

```
    int n = A.length;
```

```
    int lo = 0, hi = n-1, ans = -1;
```

```
    while (lo <= hi) {
```

```
        int mid = (lo+hi) / 2;
```

```
        if (A[mid] <= k) {
```

```
            ans = A[mid];
```

```
            lo = mid + 1;
```

```
        } else {
```

```
            hi = mid - 1;
```

```
        }
```

```
    }
```

```
    return ans;
```

```
}
```

k = 10

A =	2	8	13	14	24	31	48	51
	0	1	2	3	4	5	6	7

hi

lo
m

ans = ~~8~~

k = 10

A =	2	7	9	13	14	24	31	48	51
	0	1	2	3	4	5	6	7	8

hi

lo
m

ans = ~~7~~⁹

Q.5 Local minima

Given an array, find any local minima. Local minima is the element smaller than both of its neighbours. Corner elements will have only one neighbour. {Array contains distinct values}

$$A = 12 > \boxed{10} < 15 \quad 20 \quad \text{ans: } 10$$

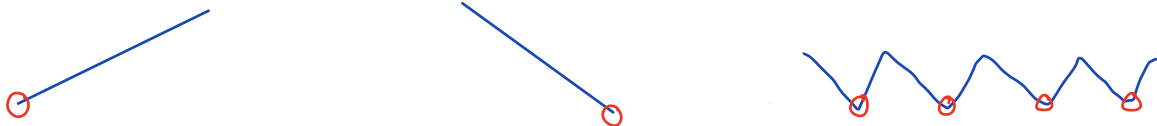
$$A = 12 \quad 10 \quad 9 > \boxed{7} < 15 \quad \text{ans: } 7$$

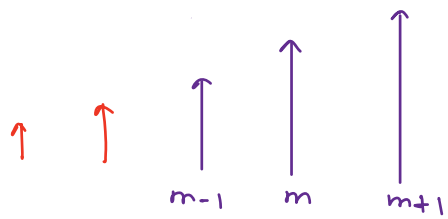
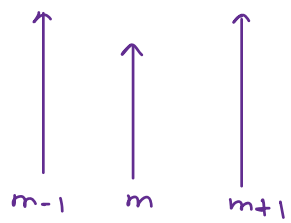
$$A = 12 \quad 15 \quad 17 \quad 14 \quad 8 \quad 20 \quad \text{ans: } 8 \text{ or } 12$$

→ Simple idea : $O(n)$

go on every element and check if it smaller than its neighbours.

→ local minima will always be there





```

int local_minima(int [] A) {
    int n = A.length;

    // corner cases
    if (A[0] < A[1]) {
        return A[0];
    }
    else if (A[n-1] < A[n-2]) {
        return A[n-1];
    }

    int lo = 1, hi = n-2;
    while (lo <= hi) {
        int m = (lo+hi)/2;

        if (A[m] < A[m-1] && A[m] < A[m+1]) {
            return A[m];
        }
        else if (A[m-1] < A[m]) {
            hi = m-1;
        }
        else if (A[m+1] < A[m]) {
            lo = m+1;
        }
    }

    return -1; // to satisfy compiler
}

```

3

Java

do-while

while (do <= hi) {

int m = (do+hi)/2;

if (A[m] < A[m-1] || A[m] < A[m+1]) {

return A[m];

}

else if (A[m-1] < A[m]) {

hi = m-1;

}

else if (A[m+1] < A[m]) {

do = m+1;

}

}

20	18	22	25	28	16	50
0	1	2	3	4	5	6
	do	hi				
	m					

20	19	30	25	23	16	50
0	1	2	3	4	5	6
				do	m	hi

T.C: $O(\log_2 n)$