

1. Decimal No. System

base: 10 , digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$$3849 = 3 \times 10^3 + 8 \times 10^2 + 4 \times 10^1 + 9 \times 10^0$$

2. Binary No. System

base: 2 , digits: 0, 1

$$1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$8 + 4 + 0 + 1 = 13$$

$$11001 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 16 + 8 + 0 + 0 + 1 = 25$$

} binary to decimal

Decimal to binary

$$(25)_{10} = (11001)_2$$

2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

$$(45)_{10} = (101101)_2$$

2	45	
2	22	1
2	11	0
2	5	1
2	2	1
2	1	0
	0	1

$$(101101)_2 = (45)_{10}$$

$$1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$32 + 0 + 8 + 4 + 0 + 1 = 45$$

$$(30)_{10} = (11110)_2$$

2	30	
2	15	0
2	7	1
2	3	1
2	1	1
	0	1

$$(11110)_2 = (30)_{10}$$

$$16 + 8 + 4 + 2 + 0 = 30$$

Addition of binary numbers

$$\begin{array}{r}
 0121 \\
 3849 \\
 + 275 \\
 \hline
 4124
 \end{array}$$

Sum.

$$d = \text{sum} \cdot 10$$

$$c = \text{sum} / 10$$

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 1 \\
 \downarrow \\
 \begin{array}{r}
 1111 \\
 1011 \\
 + \quad 111 \\
 \hline
 2232
 \end{array} \\
 \text{Sum} \\
 d = \text{sum} \cdot 10 \\
 c = \text{sum} / 2
 \end{array}$$

$$d = \text{sum} \cdot 10$$

$$c = \text{sum} / 2$$

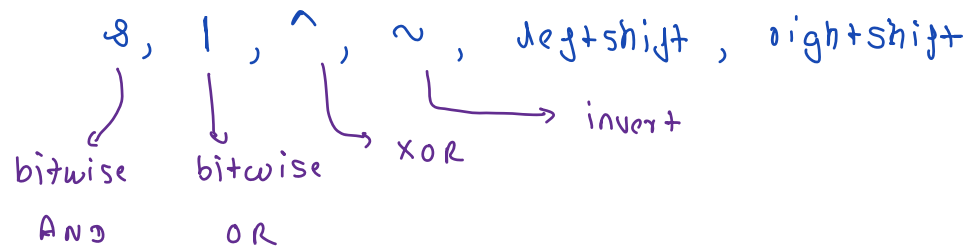
$$\begin{array}{r}
 1011 \rightarrow 11 \\
 111 \rightarrow 7 \\
 \hline
 10010 \rightarrow 18
 \end{array}$$

eg-2

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 1 \\
 \downarrow \\
 \begin{array}{r}
 1101 \\
 0111 \\
 + \quad 111 \\
 \hline
 2322
 \end{array} \\
 \text{Sum} \\
 d = \text{sum} \cdot 10 \\
 c = \text{sum} / 2
 \end{array}$$

$$\begin{array}{r}
 1101 \rightarrow 13 \\
 111 \rightarrow 7 \\
 \hline
 10100 \rightarrow 20
 \end{array}$$

Bitwise operators



$\& \rightarrow 0$ is dominating

$| \rightarrow 1$ is dominating

$\wedge \rightarrow$ Same same puppy shame

a	b	$a \& b$	$a b$	$a \wedge b$	$\sim a$
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

1. `int A = 13;`
`int B = 10;`
`SOPDN (A & B);`

8

$$\begin{array}{r} 1101 \rightarrow 13 \\ \& 1010 \rightarrow 10 \\ \hline 1000 \rightarrow 8 \end{array}$$

2. `int A = 13;`
`int B = 10;`
`SOPDN (A | B);`

15

$$\begin{array}{r} 1101 \rightarrow 13 \\ \text{OR } 1010 \rightarrow 10 \\ \hline 1111 \rightarrow 15 \end{array}$$

3. `int A = 13;`
`int B = 10;`
`SOPDN (A ^ B);`

7

$$\begin{array}{r} 1101 \rightarrow 13 \\ \wedge 1010 \rightarrow 10 \\ \hline 0111 \rightarrow 7 \end{array}$$

Important facts

$$n \& 0 = 0$$

$$n \& n = n$$

$$n | 0 = n$$

$$n | n = n$$

$$n \wedge 0 = n$$

$$n \wedge n = 0$$

n is an int no.

brainstorming exercise

$$n \& 1 =$$

$$n = 20$$

$$\begin{array}{r} 10100 \\ \& 00001 \\ \hline 00000 \end{array}$$

$$n = 13$$

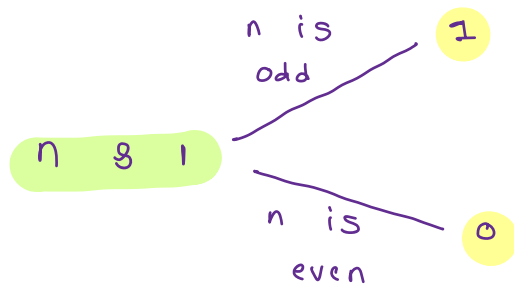
$$\begin{array}{r} 1101 \\ \& 0001 \\ \hline 0001 \end{array}$$

$$n = 10$$

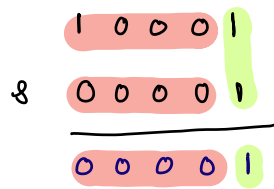
$$\begin{array}{r} 1010 \\ \& 0001 \\ \hline 0000 \end{array}$$

$$n = 9$$

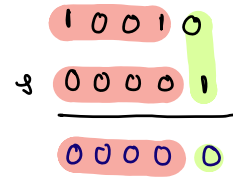
$$\begin{array}{r} 1001 \\ \& 0001 \\ \hline 0001 \end{array}$$



$n = 17$



$n = 18$



Commutative

$A, B, C \rightarrow \text{int no.}$

$$1. A \& B = B \& A$$

$$2. A | B = B | A$$

$$3. A \wedge B = B \wedge A$$

$$A \& B \& C = A \& C \& B = B \& A \& C = B \& C \& A = C \& A \& B = C \& B \& A$$

$$A | B | C = A | C | B = B | A | C = B | C | A = C | A | B = C | B | A$$

$$A \wedge B \wedge C = A \wedge C \wedge B = B \wedge A \wedge C = B \wedge C \wedge A = C \wedge A \wedge B = C \wedge B \wedge A$$

$$5 \wedge 3 \wedge 5 \wedge 8 \wedge 3 = \underbrace{5 \wedge 5} \wedge \underbrace{3 \wedge 3} \wedge 8$$

$$\underbrace{0 \wedge 0} \wedge 8$$

$$0 \wedge 8 = 8$$

$$\begin{array}{r} 101 \\ \wedge 011 \\ \hline 110 \\ \wedge 101 \\ \hline 0011 \\ \wedge 1000 \\ \hline 1011 \\ \wedge 0011 \\ \hline 1000 \end{array}$$

Q.2 Single element

Given an array in which all no. are coming twice except for a single no. find the single element.

A = [5 4 4 4 8 5 4]

Tc: $O(n)$

Sc: $O(1)$

```
int solve (int [] A) {
```

```
    int ans = 0;
```

```
    for (int i = 0; i < A.length; i++) {
```

```
        | ans = ans ^ A[i];
```

```
    }
```

```
    return ans;
```

```
}
```

$$ans = 5^4 \cdot 4^4 \cdot 4^4 \cdot 8^1 \cdot 5^4 \cdot 4^4$$

$$= \frac{5^4}{0} \cdot \frac{4^4}{0} \cdot \frac{4^4}{0} \cdot 8^1$$

$$= 8$$

left shift

<<

with every left
shift the no.
is twiced.

$a = 13$	0 0 1 1 0 1	$\Rightarrow 13$
$a << 1$	0 1 1 0 1 0	$\Rightarrow 26$
$a << 2$	1 1 0 1 0 0	$\Rightarrow 52$

$$a << 1 = 2a$$

$$a << 2 = 4a$$

$$a << 3 = 8a$$

⋮

$$a << N = (2^N) a$$

$$1 << N = 2^N$$

↓

$O(1)$

Right shift

>>

with every right
shift no. is
getting halved

$a = 13$	0 0 1 1 0 1	→ 13
$a >> 1$	0 0 1 1 0	→ 6
$a >> 2$	0 0 1 1	→ 3

$$a >> 1 = a / 2$$

$$a >> 2 = a / 4$$

$$a >> 3 = a / 8$$

⋮

$$a >> n = a / 2^n$$

Doubts
=

$$\begin{aligned} a &= 25 & 11001 &= 25 \\ a \ll 1 & & 110010 &= 50 \\ a \ll 2 & & 1100100 &= 100 \end{aligned}$$

$$a \ll 1 = 2a$$

$$a \ll 2 = 4a$$

$$a \ll 3 = 8a$$

⋮

$$a \ll N = (2^N) a$$

$$40 \ll 3 = 320$$

$$80 \rightarrow 160 \rightarrow 320$$

$$A = 3$$

temp = 1 to 9

1	2	3
8	9	4
7	6	5

i	j	n
0	0	3
1	1	1

cf

$$A = \begin{bmatrix} 9 & 5 & -2 & 4 & 3 & 1 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$b = 4$$

left

right

4

0

3

1

2

2

1

3

0

4

$$A = \begin{bmatrix} 9 & 5 & -2 & 4 & 3 & 1 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$\text{sum} = 9 + 5 + -2 + 4$$

$$\text{ans} = 18 \quad 18 \quad 22$$

$$i = 0$$

$$j = n-1$$

sum

i

j

$$9 + 5 + -2 + 4 - 1 + 7$$

3

6

$$9 + 5 - 2 + 7 - (-2) + 1$$

2

5

$$9 + 5 + 7 + 1 - 5 + 3$$

1

4

$$9 + 7 + 1 + 3 - 9 + 4$$

0

3

-1

2

$$i \geq 0 \left\{ \begin{array}{l} \text{sum} -= A[i] \\ \text{sum} += A[j] \\ i--; \\ j--; \end{array} \right.$$

Q. No. of minimum swaps to make all $\text{ele} \leq B$ together

A: [1 10 15 2 4 13 4] $B = 5$
 0 1 2 3 4 5 6
 ans = 2

A: [25 30 2 18 7 6 9 50 3] $B = 10$
 0 1 2 3 4 5 6 7 8
 ans = 1

A: [19 11 3 9 7 25 6 20 4] $B = 10$
 0 1 2 3 4 5 6 7 8
 ans = 1

A: [19 11 3 9 7 25 6 20 4] $B = 10$
 0 1 2 3 4 5 6 7 8
 K = 5

i	j	swaps
0	4	2 $\hookrightarrow \text{val} > B$
1	5	2 - 1 + 1
2	6	2 - 1 \rightarrow 1
3	7	1 + 1
4	8	2

if ($A[s-1] > B$) {
 swaps--;
 }
 if ($A[e] > B$) {
 swaps++
 }