PC334 B.Tech Research Internship

American Option Pricing Methods

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Introduction

American Options are quite unique in terms of behaviour as well as pricing. Pricing of discrete version of American Options cannot be done directly by mathematical formulation due to the fact that option price for American Options depends on exercise time of the option. So, Several Simulation and Computation based methods are developed and implemented for the pricing of American Options.

What are Options?

In finance, an option is a contract which gives the buyer the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price on a specified date, depending on the form of the option. An American option is an option that can be exercised anytime during its life.

What are American Options?

American options allow option holders to exercise the option at any time prior to and including its maturity date, thus increasing the value of the option to the holder relative to European options, which can only be exercised at maturity.

Pricing American Options

Idea behind American Options is to allow the owner the freedom of exercising the option any time before or at the expiration time. This makes these types of options different from standard European Options as they allow exercising only at expiration time. Due to Finite Time of Exercise, one can have various mathematical formulas to price European Options. While, American Options can be priced using Simulation/Computation based methods.

Pricing American Option Theoretically

In an N-period binomial model with up factor u, down factor d, and interest rate r such that $0 < d < 1 + r < u$, let derivative security pay off equals to $g(S_N)$ for some function g at time N. Value V_n of this derivative at each time n as a function V_n of the stock price at that time i.e., $V_n = v_n(S_n)$, $n = 0, 1..., N$. For, the function v_n, $0 \le n \le N$ can be expressed as,

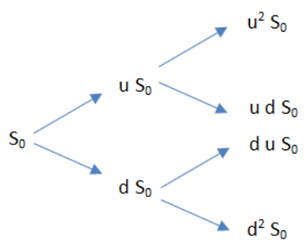
$$v_N(s) = max\{g(s), 0\}$$

$$v_n(s) = max\{ g(s), \frac{1}{1+r}[pv_{n+1}(us) + qv_{n+1}(ds)] \}, n = N-1, N-2, \dots, 0,$$

Simulation Methods for Pricing American Options

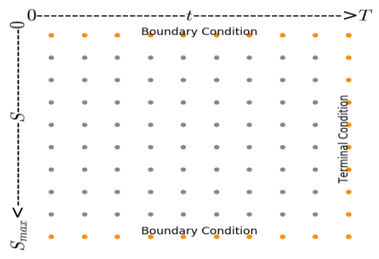
1. Binomial Tree

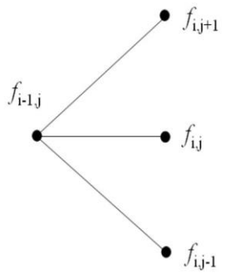
- Tree Generation based method.
- Option valuation using this method is, as described, a three-step process:
  - Price tree generation,
  - Calculation of option value at each final node,
  - Sequential calculation of the option value at each preceding node



2. Finite Difference

- This Method Uses Matrix Evaluation to price American Option.
- Mathematically, problem of solving PDE is converted into Dynamic programming based Matrix Evaluation.





3. Least Squared

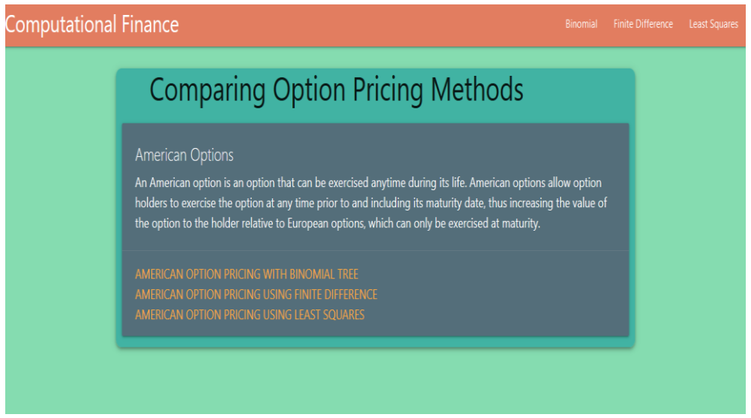
- This method is based on Monte-Carlo Simulation.
- Method uses least square based regression. Neural Network can be used as an alternative form of the regression.
- At each epoch this method generates stock price path dynamically for underlying assets.

Results

- APIs have been developed for each of the three approaches after obtaining a theoretical understanding of these pricing methods.
- APIs developed are quite efficient, though comparison among them is not quite sensible due to the fact that each of them have different concepts and computation technique at its core, like Least Squared is based on Regression and generating Stock Paths using Brownian Motion, while Binomial Tree Method makes an computation heavy attempt at solving the problem by generating whole Binomial to evaluate the whole tree, and Finite Difference Method uses Dynamic Programming as an approach for pricing.
- Also a demo website has also been developed to demonstrate working of these APIs. APIs are developed using Django REST Framework and frontend is developed using Angular JS hosted on AWS and Heroku.

Demo Website

www.sri.hulkbuster.tech



References

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2. Options, Futures and Other Derivatives, JC Hull