COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 09

BOOLEAN THEOREMS AND GATES

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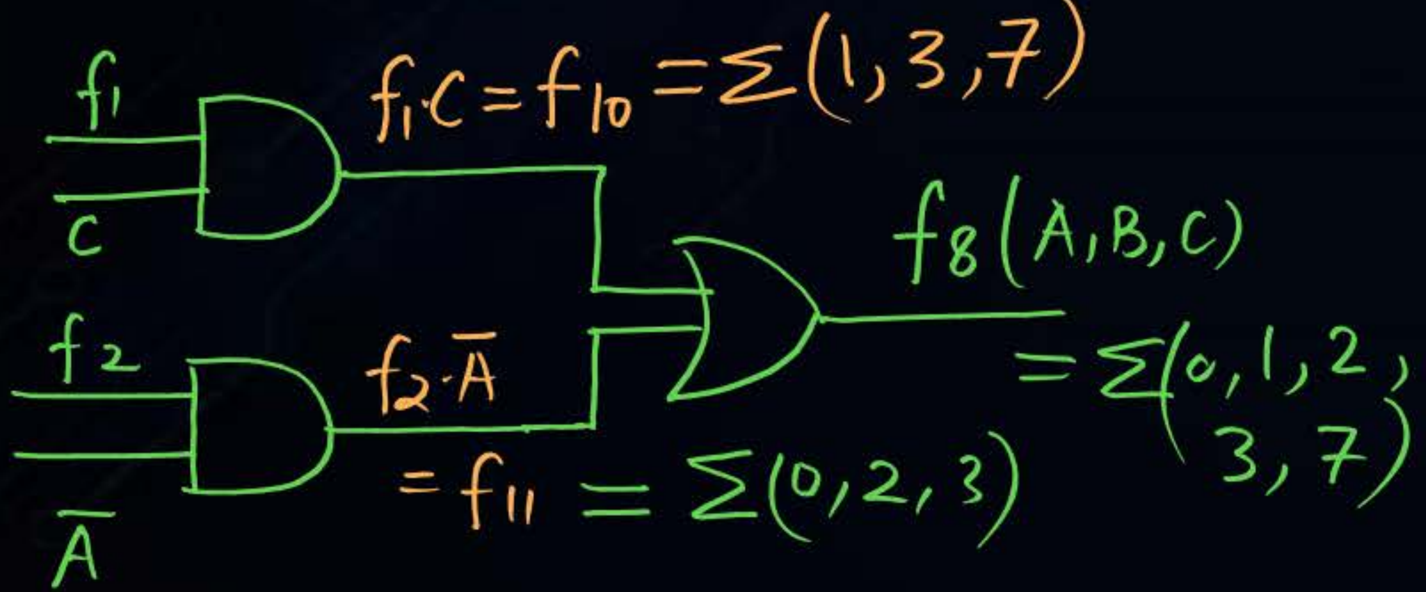
Universal gates

Topics to be Covered

Duality

Some imp. basics.

Q.

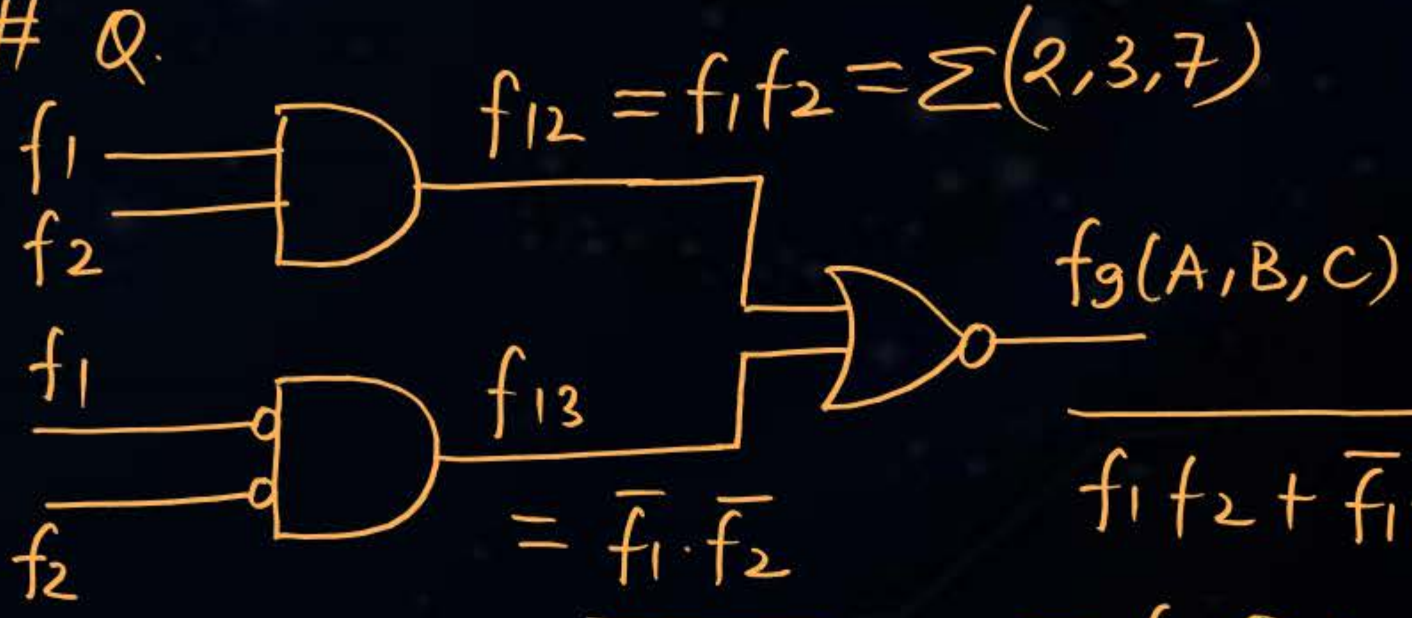


$f_1(A,B,C) = \Sigma(1,2,3,6,7)$, $f_2(A,B,C) = \Sigma(0,2,3,5,7)$

$$f_{10} = \left(\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC\right)C$$

$=$

Q.



$f_{13} = \bar{f_1} \cdot \bar{f_2} = \overline{f_1 + f_2} = \Sigma(4)$

2^2 A	2^1 B	2^0 C	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

$$f_9 = \overline{f_1 f_2 + \bar{f_1}\bar{f_2}}$$
$$= f_1 \oplus f_2$$
$$= \Sigma(0,1,5,6)$$
$$= \Sigma(0,1,5,6)$$
$$= \Pi(2,3,4,7)$$

→ Duality :→

AND $(\cdot) \longleftrightarrow$ OR $(+)$, OR $(+) \longleftrightarrow$ AND $(\cdot)$, NOT $\longleftrightarrow$ NOT

Replacing AND by OR & OR by AND & NOT by NOT is called as finding dual.

AND $\longleftrightarrow$ OR , $A+B \longleftrightarrow A\cdot B$

$A\cdot B$ $\quad (A+B)$, OR $\quad$ AND

$\bar{A} \longleftrightarrow \bar{A}$

$AB+BC \longleftrightarrow (A+B)(B+C)$

$(A+B)\cdot(B+C) \longleftrightarrow (A\cdot B)+(B\cdot C)$

- $f(A,B,C) = AB+BC$

$f^D(A,B,C) = (A+B)\cdot(B+C)$

$[f^D(A,B,C)]^D = (AB+BC)$

$f(A,B,C) = A\overline{B} + B\overline{C}$

$$f^D(A,B,C) = (A+\overline{B})\cdot(B+\overline{C}) = AB + \underline{\underline{A\overline{C}}} + \overline{B}\,\overline{C} = AB + \overline{B}\,\overline{C}$$

$$\left[f^D(A,B,C)\right]^D = (A+B)(\overline{B}+\overline{C}) = A\overline{B} + A\overline{C} + B\overline{C} = A\overline{B} + B\overline{C}$$

$$\Rightarrow f(A,B,C) \neq f^D(A,B,C)$$

$f(A,B,C) = AB + BC + CA$

$$f^D(A,B,C) = (A+B)(B+C)\cdot(C+A) = (B+AC)(C+A) = BC + A\cdot B + AC + AC$$

$$= AB + BC + CA$$

$$f^D(A,B,C) = f(A,B,C)$$

$\hookrightarrow$ self dual boolean function.

$f(A,B) = \bar{A}B + A\bar{B} = A \oplus B = \Sigma(1,2) = \Pi(0,3)$

$$f^D(A,B) = (\overset{1}{\bar{A}} + \overset{0}{B}) \cdot (\overset{0}{A} + \overset{1}{\bar{B}}) = A \odot B = \Pi(1,2) = \Sigma(0,3)$$

$$f^D(A,B) \neq f(A,B)$$

$\hookrightarrow$ non self dual

$f(A,B,C) = AB + BC + CA$

$$= AB(\bar{C}+C) + (\bar{A}+A)BC + A(\bar{B}+B)C$$

$$= AB\bar{C} + ABC + \bar{A}BC + ABC + A\bar{B}C + ABC$$

$$= \underset{110}{AB\bar{C}} + \underset{111}{ABC} + \underset{011}{\bar{A}BC} + \underset{101}{A\bar{B}C}$$

$$= \Sigma(3,5,6,7)$$

$$f(A,B,C) = \Sigma(1,3)$$

$$= \bar{A}\bar{B}C + \bar{A}BC$$

$$= \bar{A}C(\bar{B}+B)$$

$$= \bar{A}C$$

$$f(A,B,C,D) = BD + \bar{A}\bar{C}$$

$$= (\bar{A}+A)(\bar{C}+C)(BD) + \bar{A}(\bar{B}+B)\bar{C}(\bar{D}+D)$$

$$= (\bar{A}\bar{C}+\bar{A}C+A\bar{C}+AC)\,BD + \bar{A}\bar{C}\left[\bar{B}\bar{D}+\bar{B}D+B\bar{D}+BD\right]$$

$$= \underbrace{\bar{A}B\bar{C}D}_{\substack{0101 \\ 5}} + \underbrace{\bar{A}BCD}_{\substack{0111 \\ 7}} + \underbrace{AB\bar{C}D}_{\substack{1101 \\ 13}} + \underbrace{ABCD}_{\substack{1111 \\ 15}} + \underbrace{\bar{A}\bar{B}\bar{C}\bar{D}}_{\substack{0000 \\ 0}} + \underbrace{\bar{A}\bar{B}\bar{C}D}_{\substack{0001 \\ 1}} + \underbrace{\bar{A}B\bar{C}\bar{D}}_{\substack{0100 \\ 4}} + \underbrace{\bar{A}B\bar{C}D}_{\substack{0101 \\ 5}}$$

$$=$$

$$= \sum(0,1,4,5,7,13,15)$$

2^3	2^2	2^1	2^0	
A	B	C	D	
0	1	0	1	$\rightarrow 5$
0	1	1	1	$\rightarrow 7$
1	1	0	1	$\rightarrow 13$
1	1	1	1	$\rightarrow 15$

$\bar{A}$		$\bar{C}$		
0	0	0	0	$\rightarrow 0$
0	0	0	1	$\rightarrow 1$
0	1	0	0	$\rightarrow 4$
0	1	0	1	$\rightarrow 5$

$f(A,B,C) = AB + BC + CA \quad = \Sigma(3,5,6,7)$

$$\begin{array}{ccccc} A & B & & & \\ 1 & 1 & 0 & \longrightarrow & 6 \\ 1 & 1 & 1 & \longrightarrow & 7 \end{array}$$

$$\begin{array}{ccccc} & B & C & & \\ 0 & 1 & 1 & \longrightarrow & 3 \\ 1 & 1 & 1 & \longrightarrow & 7 \end{array}$$

$$\begin{array}{ccccc} A & & C & & \\ 1 & 0 & 1 & \longrightarrow & 5 \\ 1 & 1 & 1 & \longrightarrow & 7 \end{array}$$

$f(A,B,C,D) = A\bar{B}D + \bar{C}D + \bar{A}C + A\bar{D} = \Sigma(1,2,3,5,6,7,8,9,10,11,12,13,14)$

$A\bar{B}D$

$1\ 0\ 0\ 1 \rightarrow 9$

$1\ 0\ 1\ 1 \rightarrow 11$

$\bar{A}C$

$0\ 0\ 1\ 0 \rightarrow 2$

$0\ 0\ 1\ 1 \rightarrow 3$

$0\ 1\ 1\ 0 \rightarrow 6$

$0\ 1\ 1\ 1 \rightarrow 7$

$\bar{C}D$

$0\ 0\ 0\ 1 \rightarrow 1$

$0\ 1\ 0\ 1 \rightarrow 5$

$1\ 0\ 0\ 1 \rightarrow 9$

$1\ 1\ 0\ 1 \rightarrow 13$

$A\bar{D}$

$1\ 0\ 0\ 0 \rightarrow 8$

$1\ 0\ 1\ 0 \rightarrow 10$

$1\ 1\ 0\ 0 \rightarrow 12$

$1\ 1\ 1\ 0 \rightarrow 14$

$f(A,B,C) = AB + BC + CA = \Sigma(3,5,6,7) = \pi(0,1,2,4)$

$$= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$f^D(A,B,C) = (\overset{1}{\bar{A}} + \overset{0}{B} + \overset{0}{C})(\overset{0}{A} + \overset{1}{\bar{B}} + \overset{0}{C}) \cdot (\overset{0}{A} + \overset{0}{B} + \overset{1}{\bar{C}})(\overset{0}{A} + \overset{0}{B} + \overset{0}{C}) = \pi(0,1,2,4)$$

$$= \Sigma(3,5,6,7)$$

$f^D(A,B,C) = f(A,B,C)$

$\hookrightarrow$ self dual boolean function.

$\Rightarrow$ A function will be self dual only if it contains exactly half of the total terms in its SOP or POS expression $\longrightarrow$ necessary but not sufficient condition

$\Rightarrow (0, 1, 2, 3, 4, 5, 6, 7) \longrightarrow (0,7)\ (1,6)\ (2,5)\ (3,4)$

$f(A,B,C) = \Sigma(1,2,4,7) \longrightarrow$ self dual

$= A \oplus B \oplus C$

NAND $\longleftrightarrow$ NOR

$\overline{A \cdot B}$ $\qquad$ $\overline{A + B}$

XOR $\longleftrightarrow$ XNOR

$A\bar{B} + \bar{A}B$ $\qquad$ $(A+\bar{B})(\bar{A}+B)$

$(XOR)^D = XNOR$

non self dual $\leftarrow$ $(A \oplus B)^D = A \odot B$

$A \oplus B \oplus C \xleftrightarrow{\text{Dual}} A \odot B \odot C$

$\hookrightarrow$ self dual because $(A \oplus B \oplus C)^D = (A \oplus B \oplus C)$

$A \odot B \odot C = A \oplus B \oplus C$

$f(A,B,C) = \sum(0,2,4,6) \longrightarrow$ self dual

$f(A,B,C) = \sum(1,3,5,7) \longrightarrow$ self dual

$f(A,B,C) = \sum(1,2,3,7) \longrightarrow$ self dual

$f(A,B,C) = \sum(3,5,6,7) \longrightarrow$ self dual

$f(A,B,C) = \sum(1,2,3) \longrightarrow$ non self dual

$f(A,B,C) = \sum(1,5,6,7) \longrightarrow$ non self dual

$f(A,B,C,D) = \sum(0,1,2,4,5,8,9,12)$

$\boxed{\text{self dual}}$

$\rightarrow$ n-variable $\longrightarrow$ $N = 2^n \longrightarrow$ terms or combinations

$AB \longrightarrow$

0 0	$\rightarrow$	0
0 1	$\rightarrow$	1
1 0	$\rightarrow$	2
1 1	$\rightarrow$	3

Total boolean function $= {}^{N}C_0 + {}^{N}C_1 + {}^{N}C_2 + {}^{N}C_3 + {}^{N}C_4 + {}^{N}C_5 + \cdots {}^{N}C_N$

$$M_1 = 2^N = 2^{2^n}$$

No. of self dual boolean functions $M_2 = 2^{N/2} = 2^{2^{(n-1)}}$

$\left(\frac{N}{2}\right) \longrightarrow$ groups

$f(A,B) = \Sigma(0)$

$f(A,B) = \Sigma(1)$

$f(A,B) = \Sigma(2)$

$f(A,B) = \Sigma(3)$

$f(A,B) = \Sigma(0,1)$

$f(A,B) = \Sigma(1,2)$

$\Rightarrow$ Only terms $\bar{A}\bar{B}$, $\bar{A}B$, AB are available of two variables then how many boolean functions are possible using these terms $\underline{2^N = 2^3}$.

$N = 3$

Q. 3-variables $\longrightarrow$

Total boolean function $M_1 = 2^{2^3} = 2^8 = 256$

self dual function $M_2 = 2^{2^2} = 2^4 = 16$

Q $\rightarrow$ 4-variables

Total boolean function $= 2^{2^4} = 2^{16} = 2^8 \times 2^8 = 256 \times 256$

Self dual boolean function $= 2^{2^3} = 2^8 = 256$

Non self dual $= \left(2^8 \times 2^8 - 2^8\right) = 2^8(2^8 - 1) = 256 \times 255$

2 Minute Summary

- Duality
- Imp basics.

Thank you

GW

Soldiers !