

CS & IT ENGINEERING



THEORY OF COMPUTATION

REGULAR EXPRESSION

Lecture No.- 04



By- Venkat sir

Recap of Previous Lecture

Regular Expression

Topic

?????



Topics to be Covered



Topic

Conversion from ϵ -NFA to NFA

Topic

??

Topic

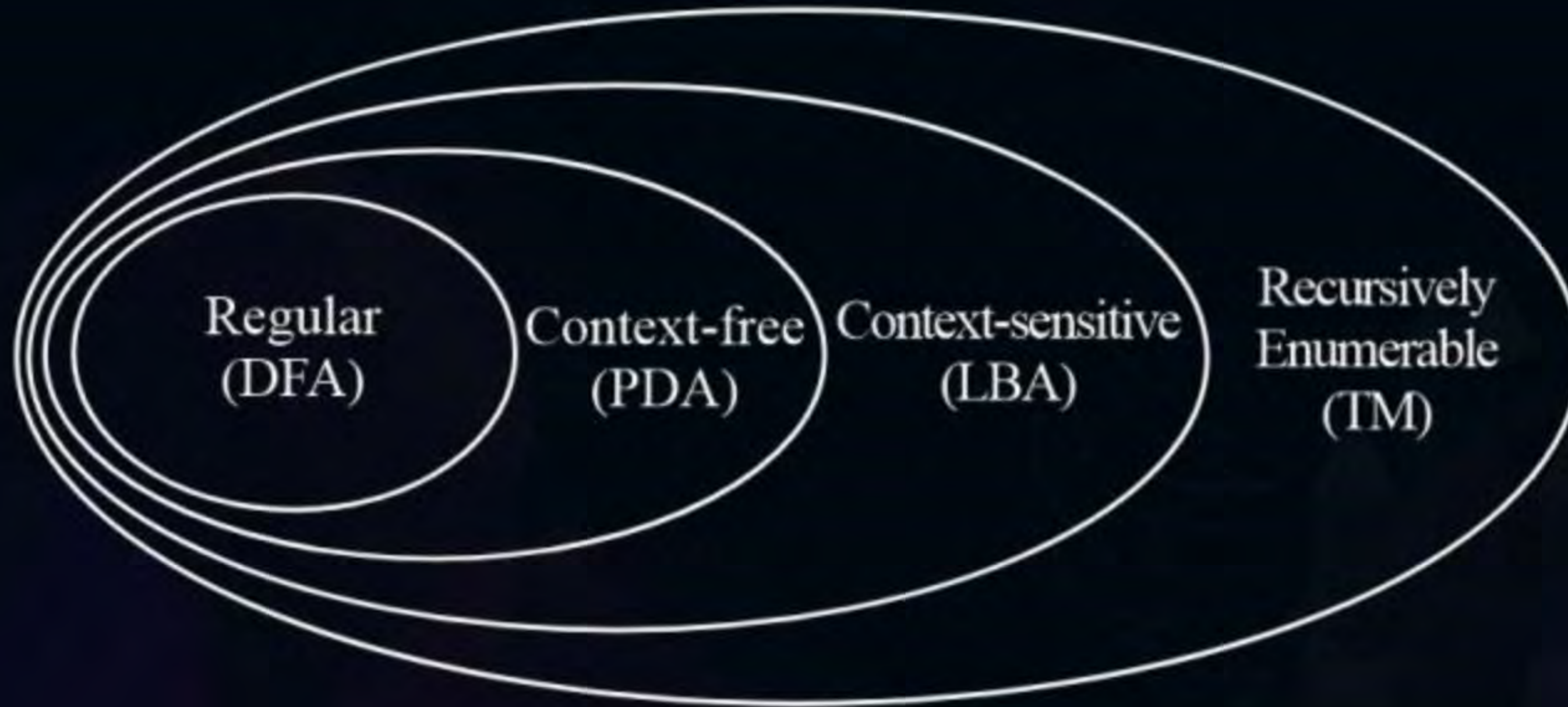
??

Topic

??



Topic : Theory of Computation



$$\delta^1(q_1, a) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q), a))$$

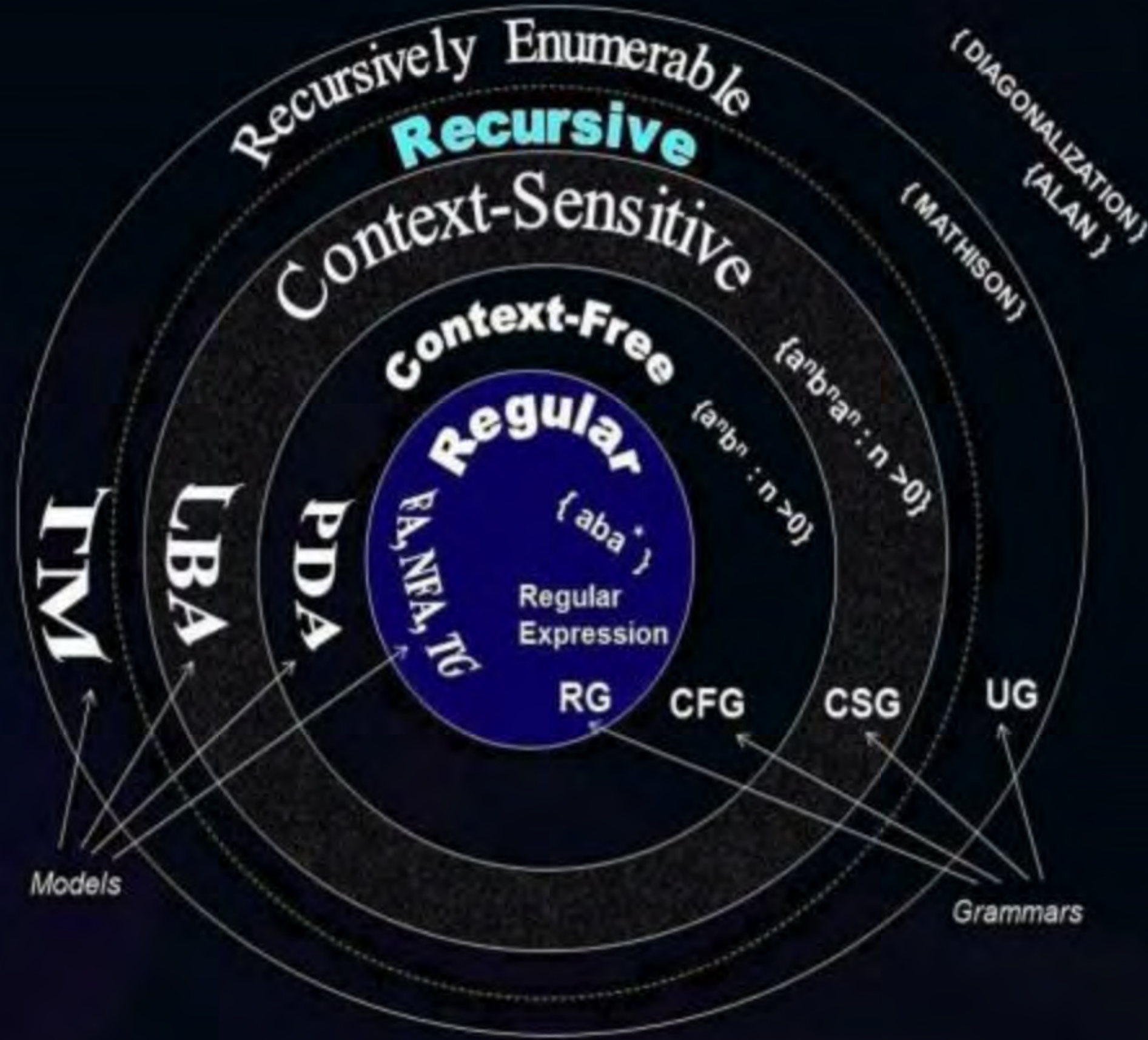


#Q. Construct an equivalent NFA for the following ϵ -NFA





Topic : Theory of Computation





Topic : Expressive Power

Number of languages accepted by particular automata is known as expressive power.

$(TM > LBA > PDA > FA)$

1. Expressive power of NFA and DFA same. Hence every NFA is converted into DFA.
2. Expressive power of NPDA is more than DPDA. Hence conversion not possible
3. Expressive power of DTM and NTM is same.

MCQ



#Q. Let D_f, D_p are number of languages accepted by DFA and DPDA respectively.
Let N_f, N_p are number of languages accepted NFA and NPDA respectively.
Which of the following is true.

A

$$N_f = D_f$$
$$N_p = D_p$$

B

$$N_f \supset D_f$$
$$N_p \supset D_p$$

C

$$N_f = D_f$$
$$N_p \subset D_p$$

D

None

#Q. In which of the cases stated below the following statement is false?
“Every nondeterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language”

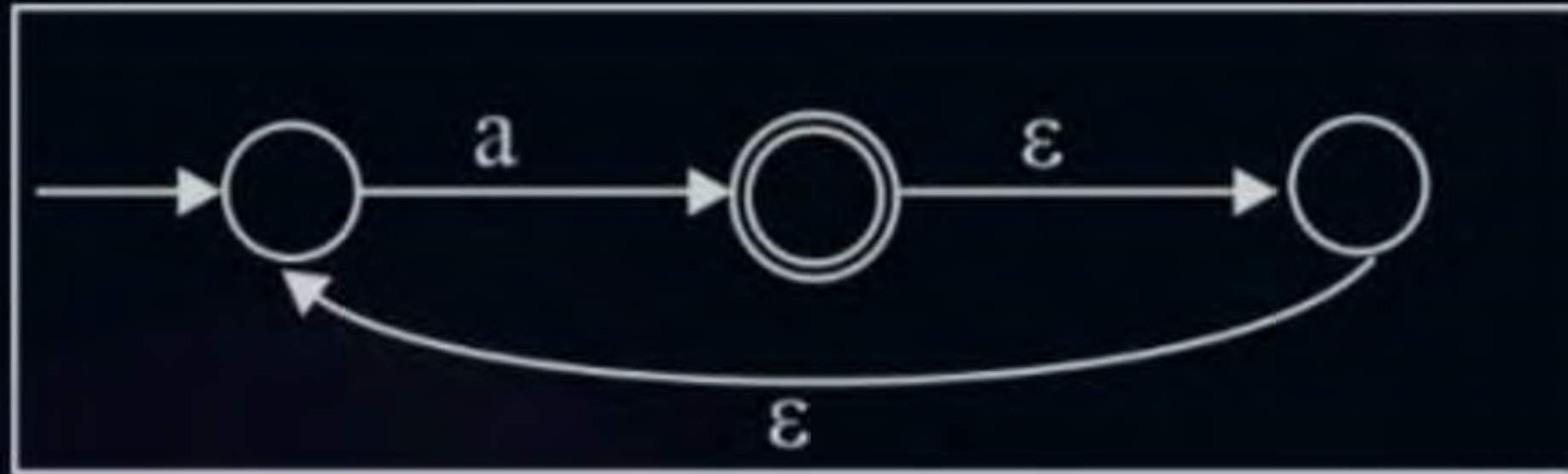
- A** M_1 is non deterministic FA
- B** M_1 is non deterministic turing machine
- C** M_1 Is non deterministic PDA
- D** None



What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ϵ is the empty string.



[2012: 1 Mark]



- A \emptyset
- B $\{\epsilon\}$
- C a^*
- D $\{a, \epsilon\}$



Topic : Regular Expression



DFA \rightarrow Expression
 $(a+b)^*$

- The simplest way of representing a regular language is known as Regular expression.
- For every regular language regular expression can be constructed.
- To construct regular expression following 3 operators are used.
- + is known as union operator
- . is known as concatenation operator
- * is known as Kleene closure operator

$$L = \{\epsilon, a, b, aa, ab, ba, \dots\}$$

$$\Downarrow'$$

$$(a+b)^*$$

$$\Downarrow^*$$

$$\Sigma^*$$

$$L = \{a^n b^n / n \geq 0\}$$

{ DFA not possible

Regular Expression also
not possible.



Topic : NOTE



- For one regular language many number of regular expressions can be possible.
- One regular expression can generate only one regular language.

Which of the following Languages can give Regular Expression

$$L_1 = \{a^n b^m \mid \overset{ab, a^2b^2, a^3b^3}{n \geq m \text{ and } n \leq m}\} = \{a^n b^n\}$$

$$\cancel{L_2 = \{a^n b^m \mid n > m \text{ and } n < m\} = \{\} = \emptyset}$$

(a) L_1 only (b) L_2 only (c) both L_1 & L_2 (d) none

(11+2)

#Q. Construct ~~regular expression~~ that generates set of all strings of a's and b's where length of each string is exactly 4.

$$\left\{ \frac{a/b}{2} \cdot \frac{a/b}{2} \cdot \frac{a/b}{2} \cdot \frac{a/b}{2} \right\}$$

length exactly 4

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atleast 4

$$\begin{array}{ccccccccc} \underline{(a+b)} & \underline{(a+b)} & \underline{(a+b)} & \underline{(a+b)} & \underline{(a+b)}^* & & & & \\ & & & & \swarrow + \searrow & & & & \\ (a+b) & (a+b) & (a+b) & (a+b) & & & & & \end{array}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atmost 4.

$$\epsilon + (a+b) + (a+b)^2 + (a+b)^3 + (a+b)^4$$

(or)

$$\epsilon + (a+b) + (a+b)^2 + (a+b)^3 + (a+b)^4$$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is divisible by 4.

$\{0, 4, 8, 12, 16, \dots\}$

✓ $\left[(a+b)(a+b)(a+b)(a+b) \right]^*$

$$\textcircled{(a+b)}^* = \epsilon + (a+b)^1 + (a+b)^2 + (a+b)^3 + (a+b)^4 + \dots$$

$$\gamma^* = \gamma^0 + \gamma^1 + \gamma^2 + \gamma^3 + \gamma^4 + \dots$$

$$\gamma^+ = \gamma^1 + \gamma^2 + \gamma^3 + \gamma^4$$

$$\text{OR} \\ \textcircled{\gamma_1} + \textcircled{\gamma_2} = \{\textcircled{\gamma_1}, \textcircled{\gamma_2}\}$$

$$\gamma_1 \cdot \gamma_2 = \{\textcircled{\gamma_1 \gamma_2}\}$$

+

.

*

$$\Sigma = \{a, b\}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are exactly 4.



x $aaaa b^*$

x $b^* aaaa b^*$

x $b^* aaaaaa$

$$\Sigma = \{a, b\}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

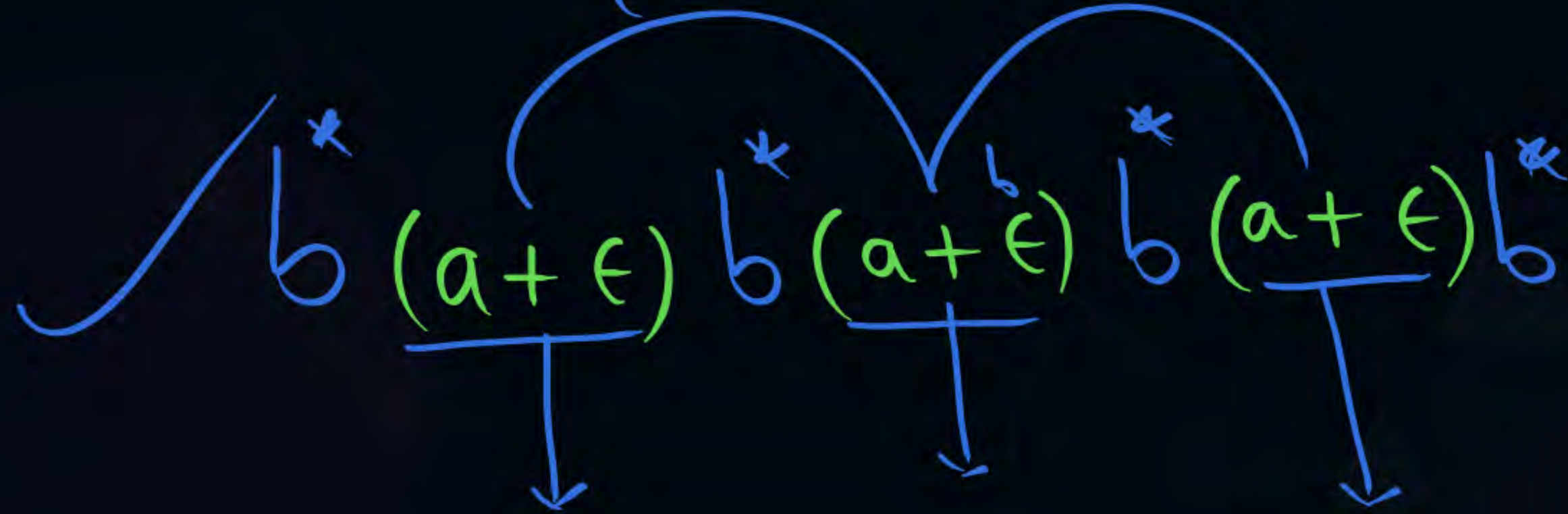
$$(a+b)^* a (a+b)^* a (a+b)^* a (a+b)^*$$

The diagram illustrates the construction of a regular expression for strings with at least 3 'a's. It shows the expression $(a+b)^* a (a+b)^* a (a+b)^* a (a+b)^*$. The first $(a+b)^*$ is circled in blue. The three 'a's are highlighted in green. Arrows point from the 'a's to the $(a+b)^*$ terms, indicating that each 'a' is followed by any string of 'a's and 'b's. The final $(a+b)^*$ has a blue arrow pointing to it, indicating it can follow the entire string.

$$\Sigma = \{a, b\}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atmost 3.

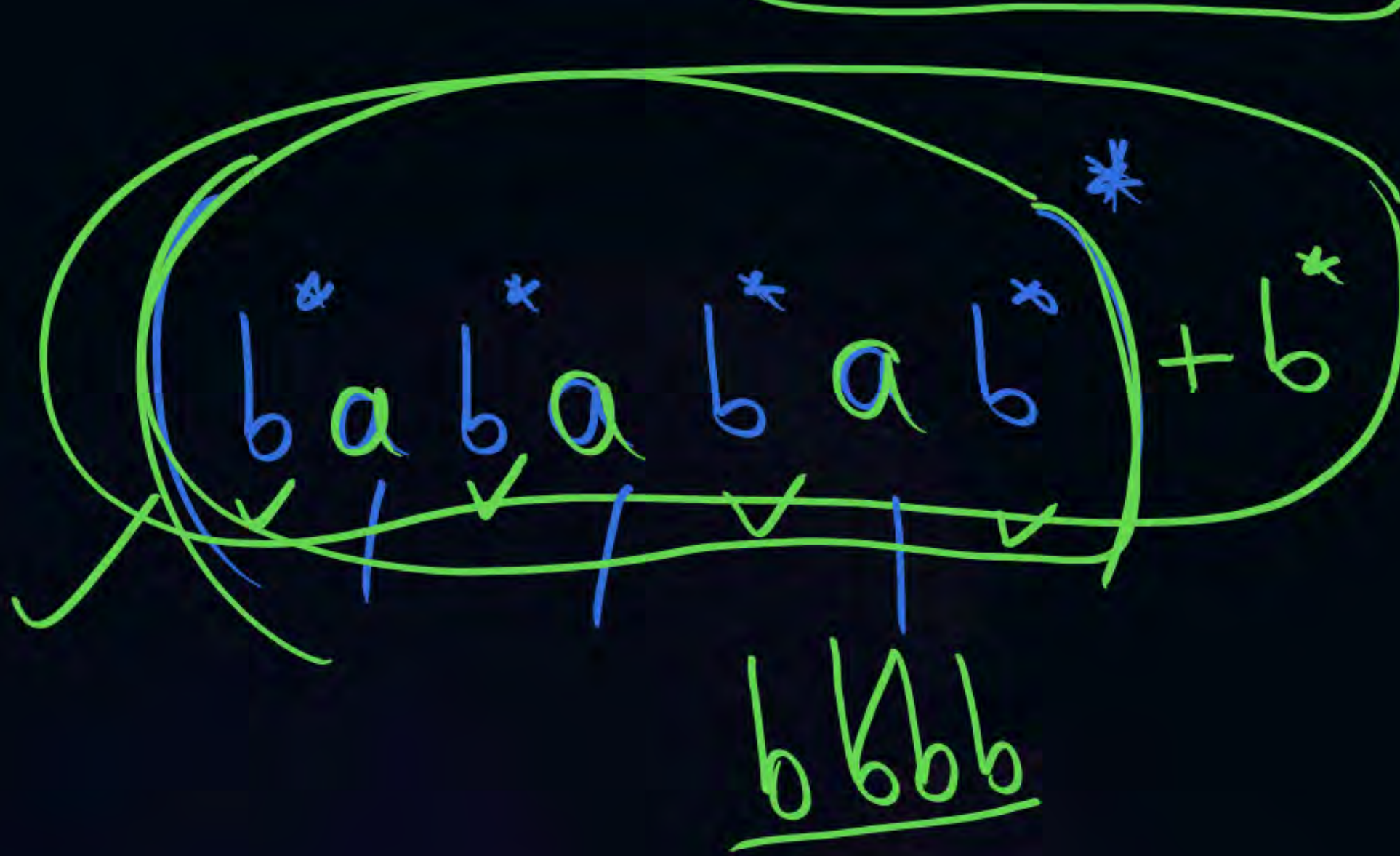
$\{0, 1, 2, 3\} \text{ a's}$

$$b^* (a + \epsilon) b^* (a + \epsilon) b^* (a + \epsilon) b^*$$


#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

$$(a+b)^* a (a+b)^* a (a+b)^* a (a+b)^*$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are divisible by 3. MCQ



- (a) $(b^*ab^*ab^*ab^*)^*$
- (b) $(b^*ab^*ab^*ab^*)^* \times$
- (c) $(aaa)^* + b^*$
- (d) none

exactly

#Q. How many states are there in minimal DFA that accept following regular expression.

length

$n+2$

$$(1) \quad \underline{(a+b)} \underline{(a+b)} \underline{(a+b)} \Rightarrow 5$$

$$(2) \quad \underline{(a+b+\epsilon)} \underline{(a+b+\epsilon)} \underline{(a+b+\epsilon)} \Rightarrow (5)$$

$$(3) \quad b^* \underline{a} b^* \underline{a} b^* \underline{a} b^* \underline{a} b^* \Rightarrow (6)$$

$$(1) \quad (a+b)^* \underline{a} (a+b)^* \underline{a} (a+b)^* \underline{a} (a+b)^* \Rightarrow \textcircled{n+1}$$

no. of a's atleast 3

$$(2) \quad \left[(a+b) (a+b) \right]^* \Rightarrow \textcircled{2}$$

Div by 2

$$(3) \quad (a+b)$$



Topic : NOTE

- Palindrome languages over more than one symbol are not regular .Hence regular expression not possible.
- Palindrome languages over one symbol are regular.

$$(1) \quad R + \phi = \phi + R = R$$

$$(2) \quad R \cdot \phi = \phi \cdot R = \phi$$

$$(3) \quad R + \epsilon = \epsilon + R \neq R$$

$$(4) \quad R \cdot \epsilon = \epsilon \cdot R = R$$

$$(5) \quad (R^*)^* = (R^*)^+ = (R)^* = R^*$$

$$(6) \quad R \cdot R^* = R^+ = R^*R$$

$$(7) \quad \epsilon^* = \epsilon$$

$$(8) \quad \epsilon^+ = \epsilon$$

$$(9) \quad \phi^* = \epsilon$$

$$(10) \quad \phi^+ = \phi$$

$$(11) \quad (a + b)^* \neq a^* b^*$$

$$(12) \quad (a + b)^* \neq a^* b^*$$

$$(13) \quad a(ab)^* = (ab)^*a$$

$$\begin{aligned}
 (14) \quad (a + b)^* &= (a + b^*)^* \\
 &= (a^* + b^*)^* \\
 &= (a^* b^*)^*
 \end{aligned}$$

$$(15) \quad a^* + a^* = a^* = a^* a^*$$

$$(16) \quad a + b = b + a$$

$$(17) \quad a \cdot b \neq ba$$

[MCQ]

#Q. Identify language accepted by following regular expression
 $b^*(a^*.\phi . b + ab + a\phi^*b^*)(b + \phi)^*$

- A** Exactly one a
- B** At least one a
- C** At most one a
- D** None

[MCQ]

#Q. Which of the following regular expressions are equivalent?

I. $(00)^* (\epsilon + 0)$

II. $(00)^*$

III. 0^*

IV. $0(00)^*$

A (I) And (II)

B (ii) and (iii)

C (i) And (iii)

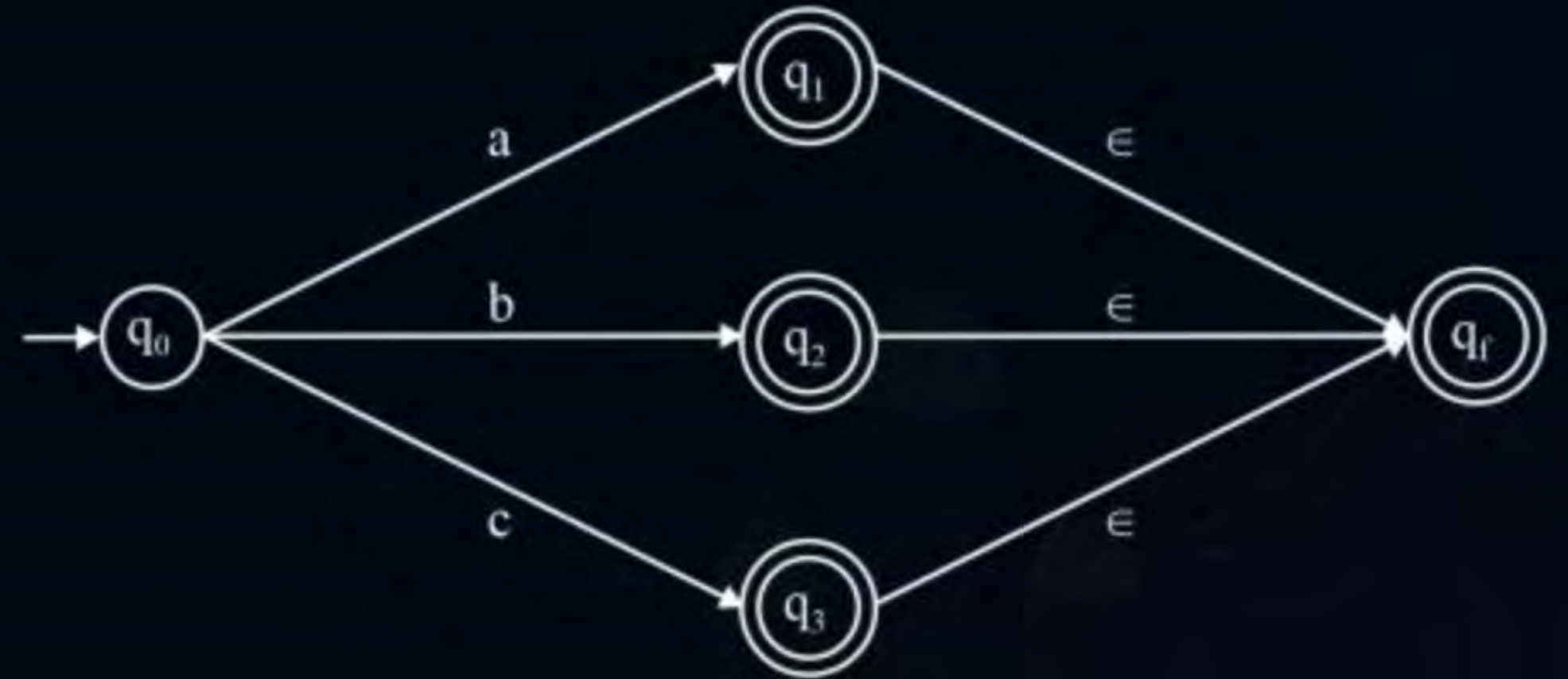
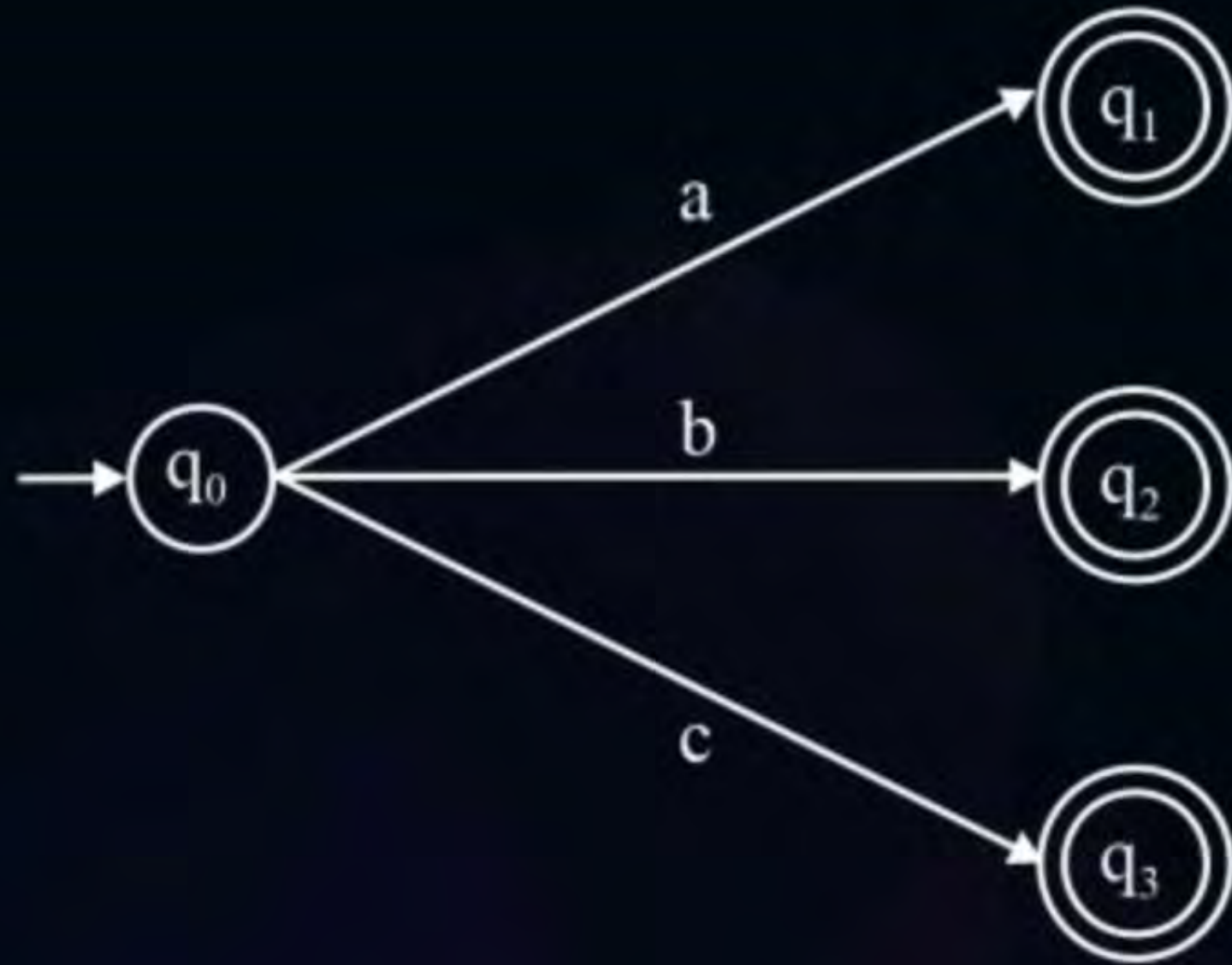
D (iii) and (iv)

[MCQ]

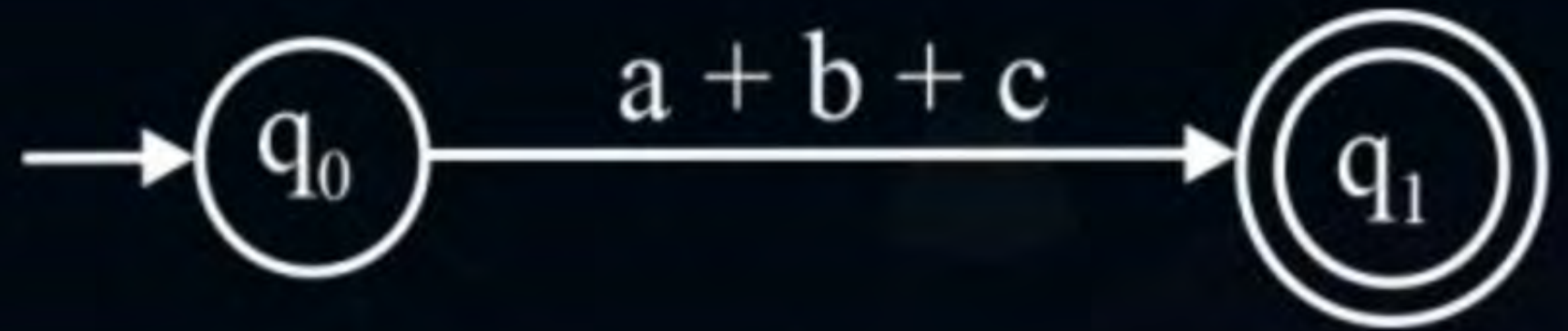
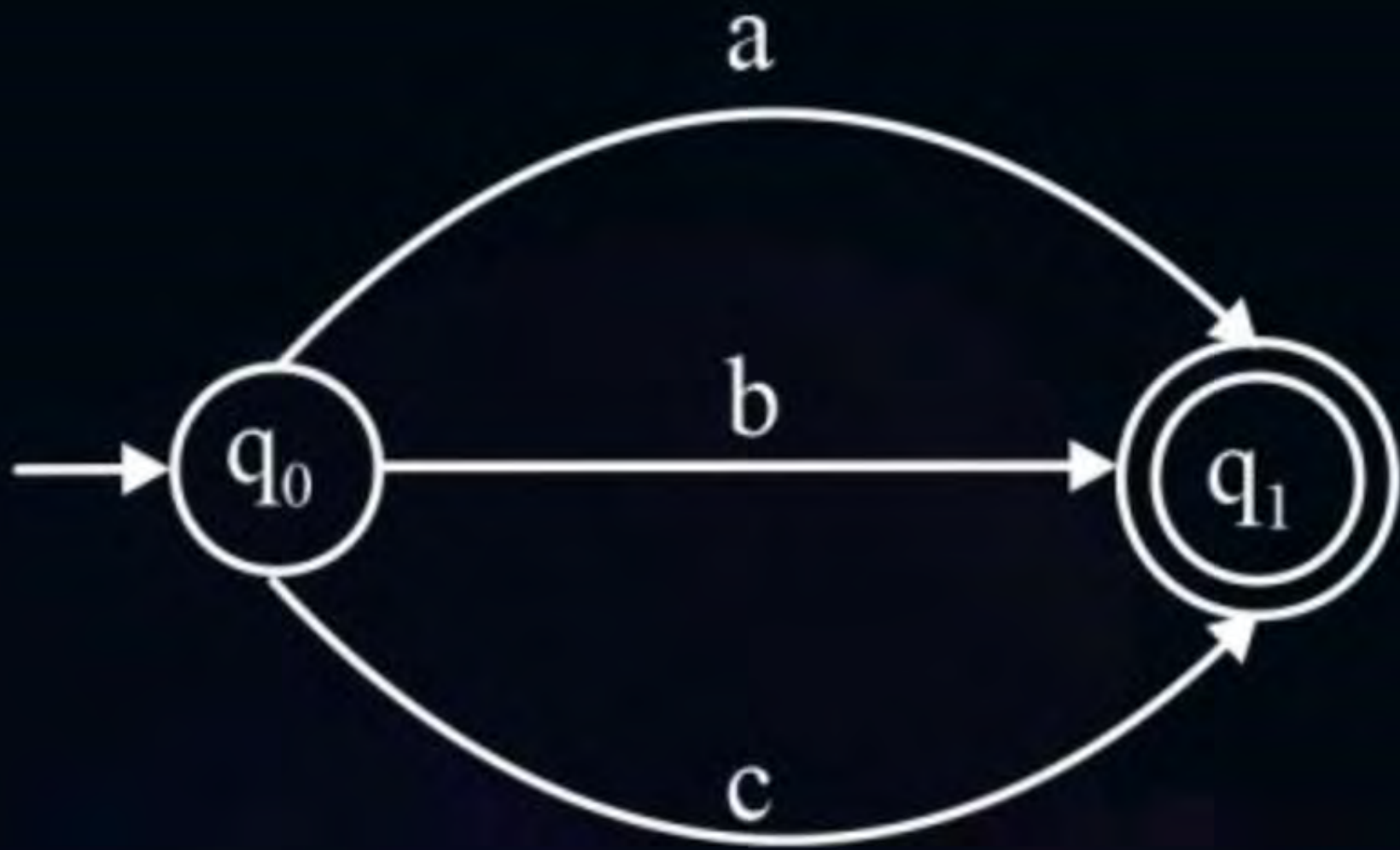
#Q. Which of the following pair of regular expressions are not equal

- A** $(r^*)^*$ and $(r+)^*$
- B** $(r + \epsilon)^*$ and r^*
- C** $(rr + \epsilon)^*$ and r^*
- D** None of the above

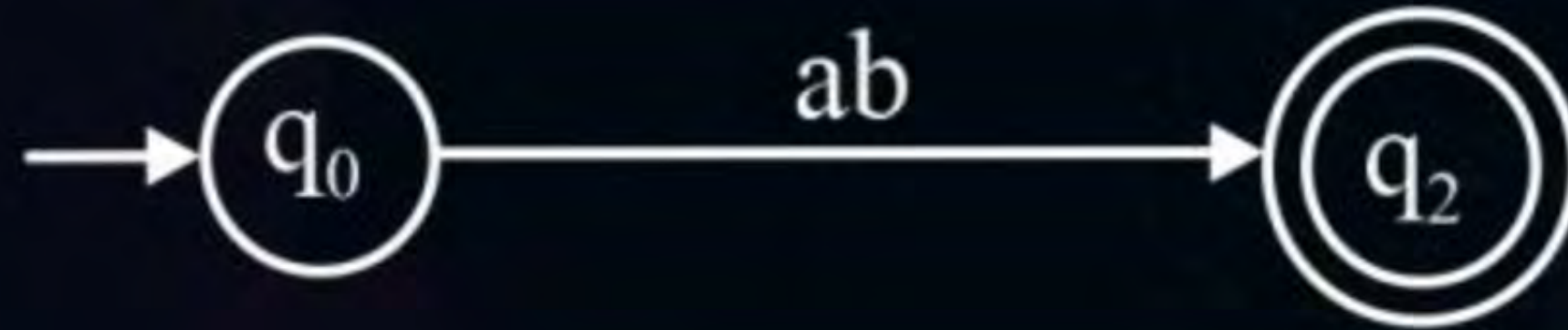
1.



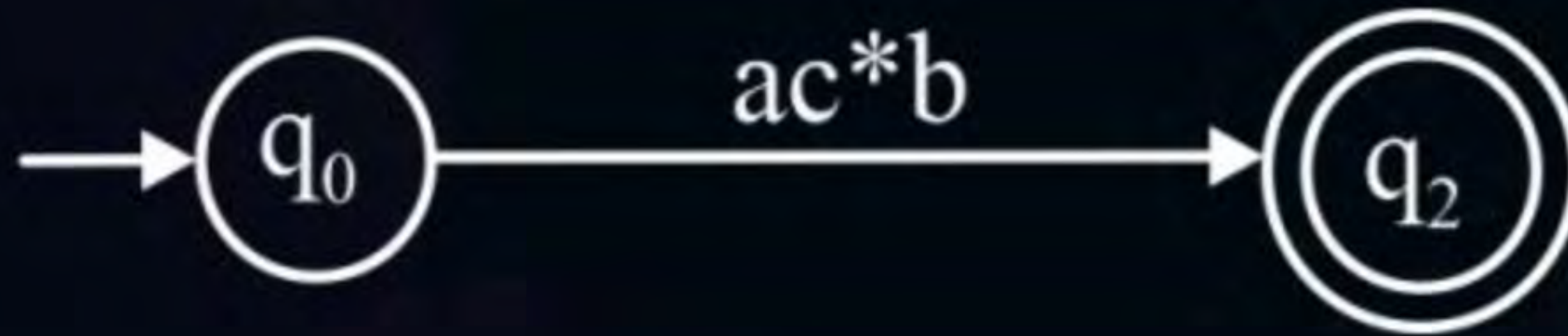
2.



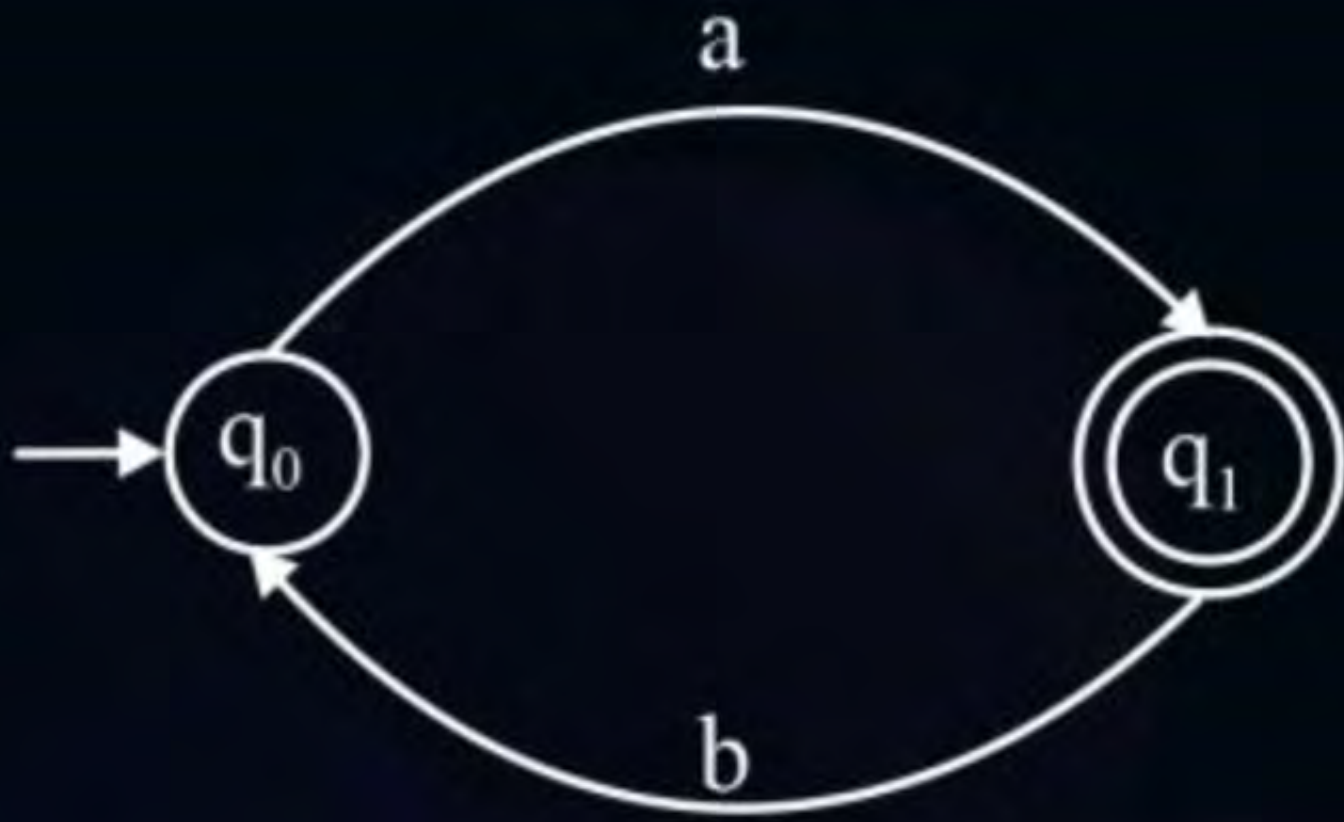
3.



4.

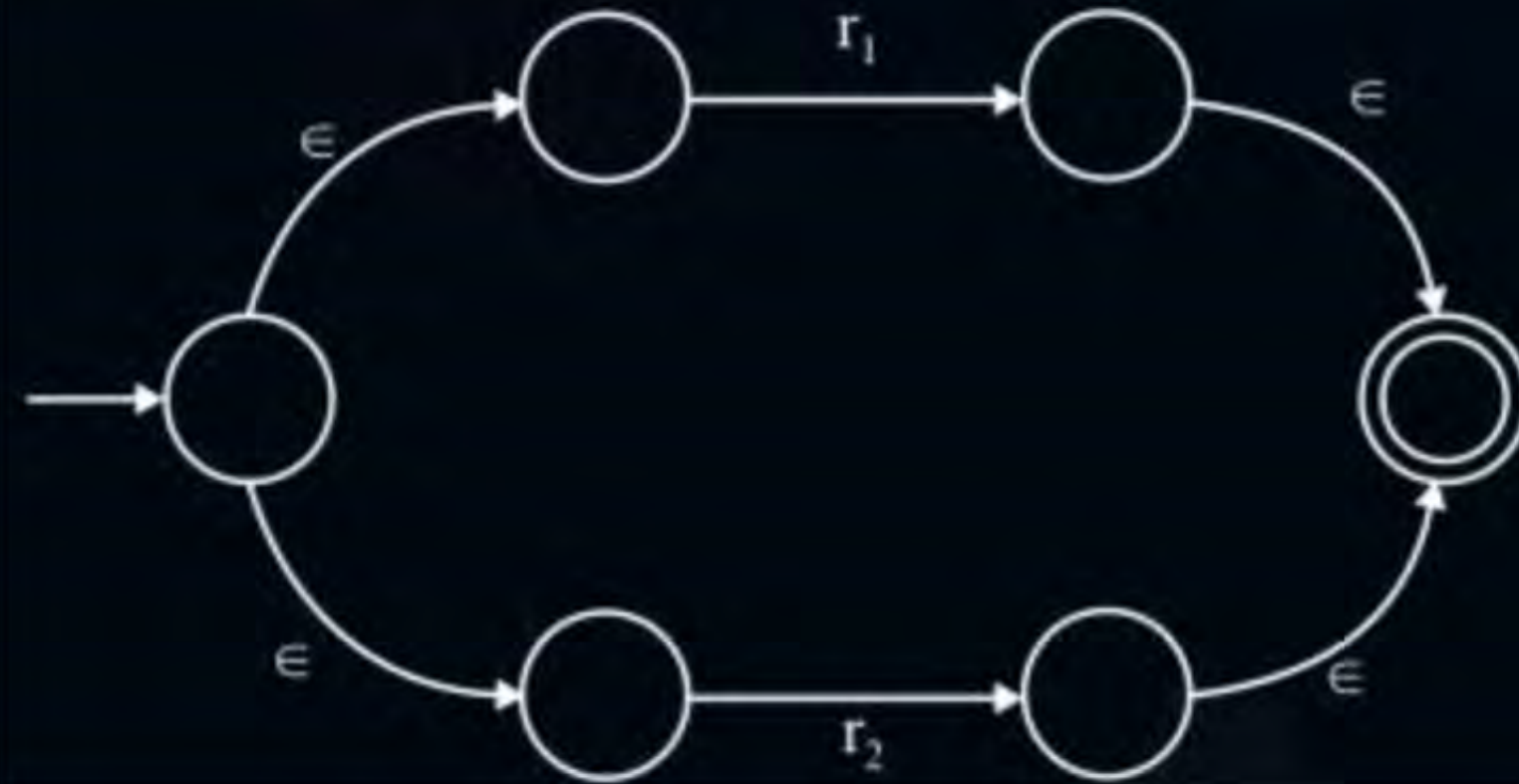


5.



Req. Expr	ϵ NFA
1. ϕ	
2. ϵ	
3. α	

4. $r_1 + r_2$



5. $r_1 \cdot r_2$



6. r^*





THANK - YOU