CS & IT ENGINEERING

THEORY OF COMPUTATION

Pushdown Automata

Lecture - 03

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Recap of Previous Lecture

Topic

→ DPDA & NPDA

?????

→ $L = \{ww^R\}$ → CFL but not DCFL

NPDA but not DPDA

→ CFL

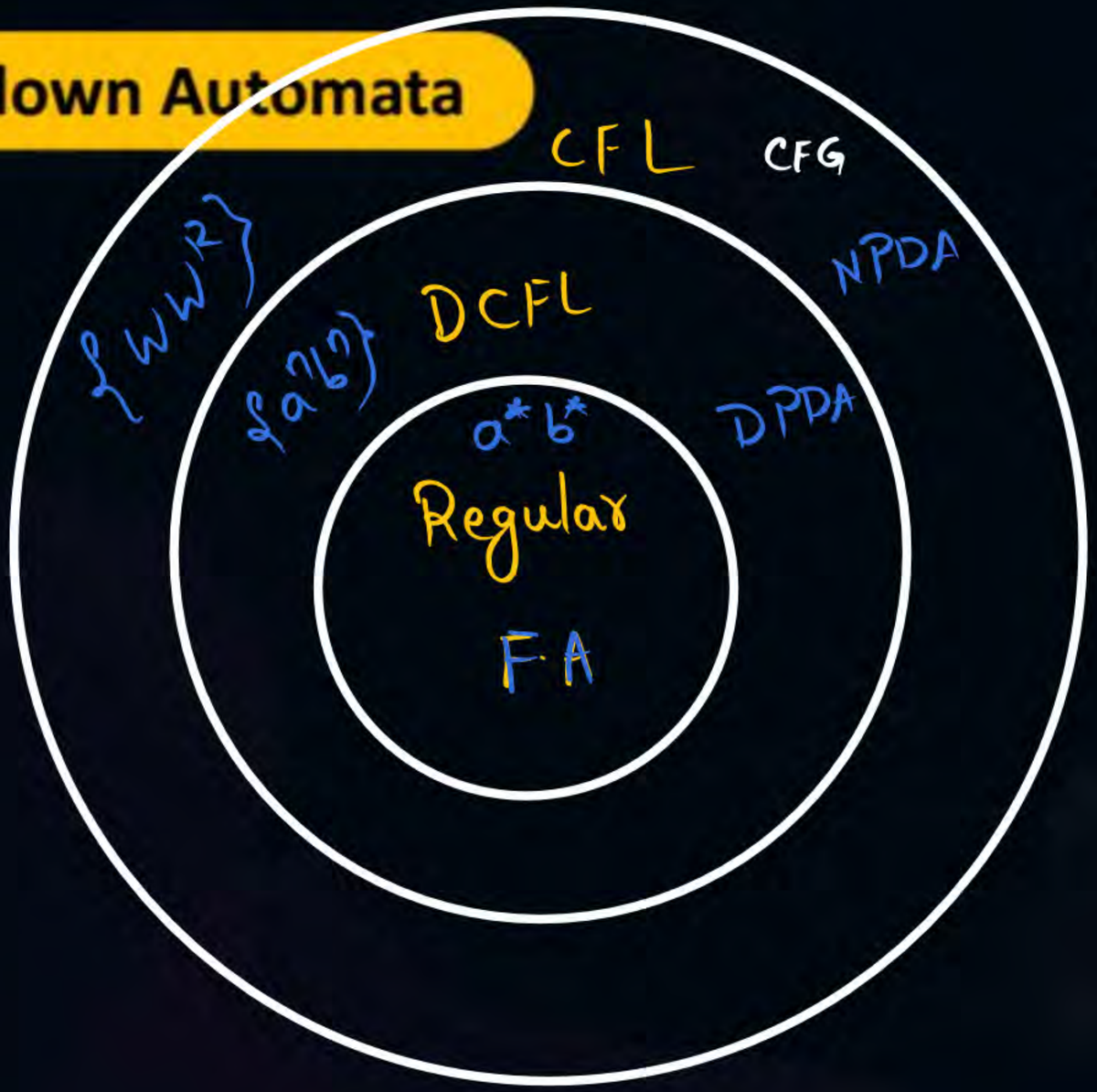
→ DCFL

Topics to be Covered

CFG
⇓

- Topic — Push down automat
- Topic — ?? CFL Detection
- Topic — ?? DCFL Detection
- Topic — ?? closure properties of CFL & DCFL

Topic : Pushdown Automata



CFL

$L = \{ WW^R \mid W \in (a+b)^* \}$

$\Downarrow$ CFG

$S \to aSa \mid bSb \mid \in$

even length palindrome

{DPDA not possible
only NPDA possible}

Notations:-

- Transition diagram
- Transitions

PDA (Acceptor)

- DPDA
- NPDA

NAT

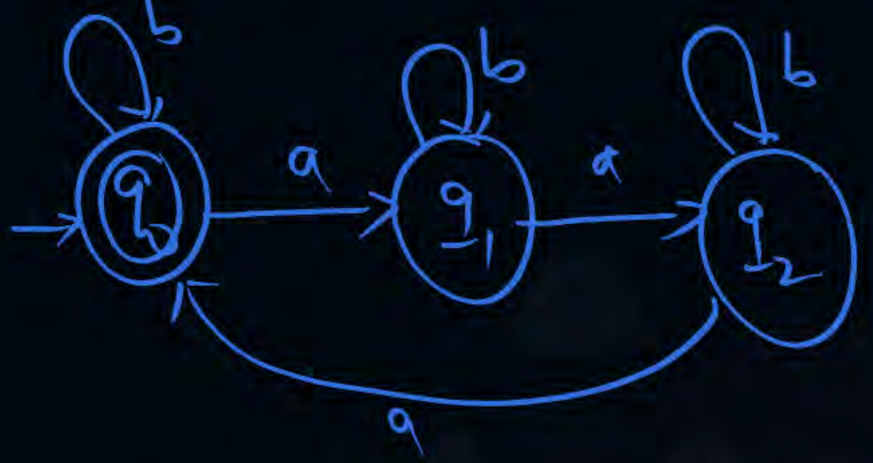
Topic : Pushdown Automata

Identify Language of following PDA?

F.A + stack = PDA

DFA

no. of as div by 3



$$\left(b^* a b^* a b^* a b^*\right)^* + b^*$$

Langular ⫋ Regular

$$\delta(q_0, a, z_0) = (q_1, z_0)$$
$$\delta(q_0, b, z_0) = (q_0, z_0)$$
$$\delta(q_1, a, z_0) = (q_2, z_0)$$
$$\delta(q_1, b, z_0) = (q_1, z_0)$$
$$\delta(q_2, a, z_0) = (q_0, z_0)$$
$$\delta(q_2, b, z_0) = (q_2, z_0)$$
$$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$$

Topic : Pushdown Automata

Construct PDA for the following Lang

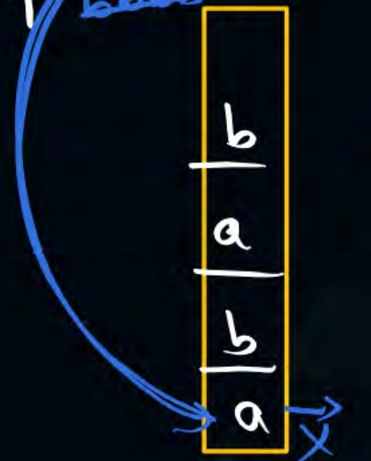
$L = \{ w c w^R \mid w \in (a+b)^* \}$

PDA not possible

{ non CFL }

ababcabab

Logic :-



no logic

Topic : Pushdown Automata

Construct PDA for

$L = \{ \underbrace{ww}_{\overleftarrow{\ }\ \overrightarrow{\ }} \mid w \in (a+b)^* \}$

PDA not possible

non CFL.

Topic : Pushdown Automata

NOTE

$L_1 = \{ww \mid w \in (a+b)^*\}$

$L_2 = \{wcw \mid w \in (a+b)^*\}$

These two are non CFL

But Complement of

$\Sigma^* - \{ww\}$

$\Sigma^* - \{wcw\}$

→ CFL

L_1^c

L_2^c

is CFL

Topic : Pushdown Automata

CFG → PDA

① $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$

② $\delta(q_1, \epsilon, A) = (q_1, \alpha) \quad \forall\ A \to \alpha$

$\delta(q_1, a, a) = (q_1, \epsilon) \quad \forall\ a \in \text{Terminal}$

③ $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

LL(1)

Topic : Pushdown Automata

CFG → PDA

① $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$

② $\delta(q_1, \epsilon, A) = (q_1, \alpha) \quad \forall\ A \to \alpha$

$\delta(q_1, a, a) = (q_1, \epsilon) \quad \forall\ a \in \text{Terminal}$

③ $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

① $S \rightarrow aSb \mid ab$

PDA

NPDA

$T = \{a, b\}$

① $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$

② $\delta(q_1, \epsilon, S) = (q_1, aSb)$ (or) (q_1, ab)

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

③ $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

Topic : Pushdown Automata

② $S \to aAB$

$A \to aA \mid b$

$B \to bB \mid b$

$\Longrightarrow$ PDA ✓

$N.T. = \{S, A, B\}$

$T = \{a, b\}$

① $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$

② $\delta(q_1, \epsilon, S) = (q_1, aAB)$

$\delta(q_1, \epsilon, A) = (q_1, aA)$ (or) (q_1, b)

$\delta(q_1, \epsilon, B) = (q_1, bB)$ (or) (q_1, b)

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

③ $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

CFL Detection

Topic : Context Free Language

Which of the following language are

1. CLF and Regular
2. CFL but not regular
3. Non CFL

① Every finite language → Regular → CFL

Topic : Context Free Language

1. $L = \{a^n b^n c^n / n \leq 10\}$ → finite → Regular → CFL

2. $L = \{a^n b^n c^m / n \neq m\}$ → Default infinite → more than 1 Dependency → Non CFL

3. $L = \{a^n b^m c^n / n > m, n, m \leq 1000\}$ → finite → Regular → CFL

4. $\{L = a^n b^m / n - m = 4\} = \{a^{m+4} b^m\}$ ($n = m+4$) → infinite, 1 Dep → CFL but not Regular.

$\{a^{4m} b^m\}$

$n = 4m$

5. $L = \{a^n b^m / \ n/m = 4\}$ → infinite } 1 Dep → CFL but not Regular.

6. $L = \{a^n b^m / n = 2m + 1\} = \{a^{2m+1} b^m\}$ → 1 Dep → CFL but not Regular.

7. $L = \{a^n b^m / n \neq m\}$ → 1 Dep → CFL but not Regular.

8. $L = \{a^n b^m / n \neq 2m\}$ → 1 Dep → CFL but not Regular.

9. $L = \{a^n b^m / n = m^2\}$ → 1 Dep, Strings not in A.P } Non CFL

$\{a^{m^2} b^m\}$

Topic : Context Free Language

a^{n^2} $a^{n!}$ $a^{n!}$

10. $\{a^{n!}\, b^{n!} / n \geq 13\}$ — 1 Dep, as not in A.P → Non CFL.

11. $L = \{a^n b^m / n \leq m\}$ — 1 Dep → CFL but not Regular.

12. $\{a^n b^m c^{n+m} / n, m \geq 1\}$ — 1 Dep (a's → push, b's → push, c → a,b pop) → CFL but not Regular.

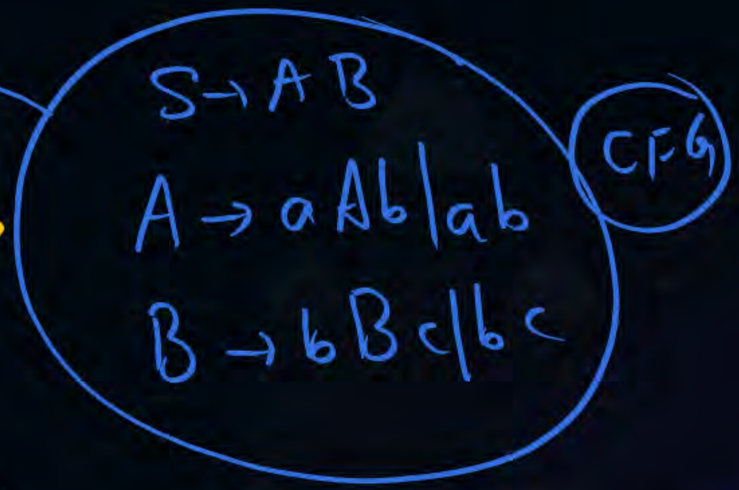
13. $L = \{L = a^n b^{n+m} / n, m = 1\}$ — finite → Regular → CFL.

14. $L = \{a^{m^2} b^{n^3} c^{k^5} / n, n, k > 1\}$ → not in A.P → non CFL.

15. $L = \{a^{3^n} b^{5^k} c^{2\ell} / n, k, \ell \geq 1\}$ → not in A.P → non CFL.

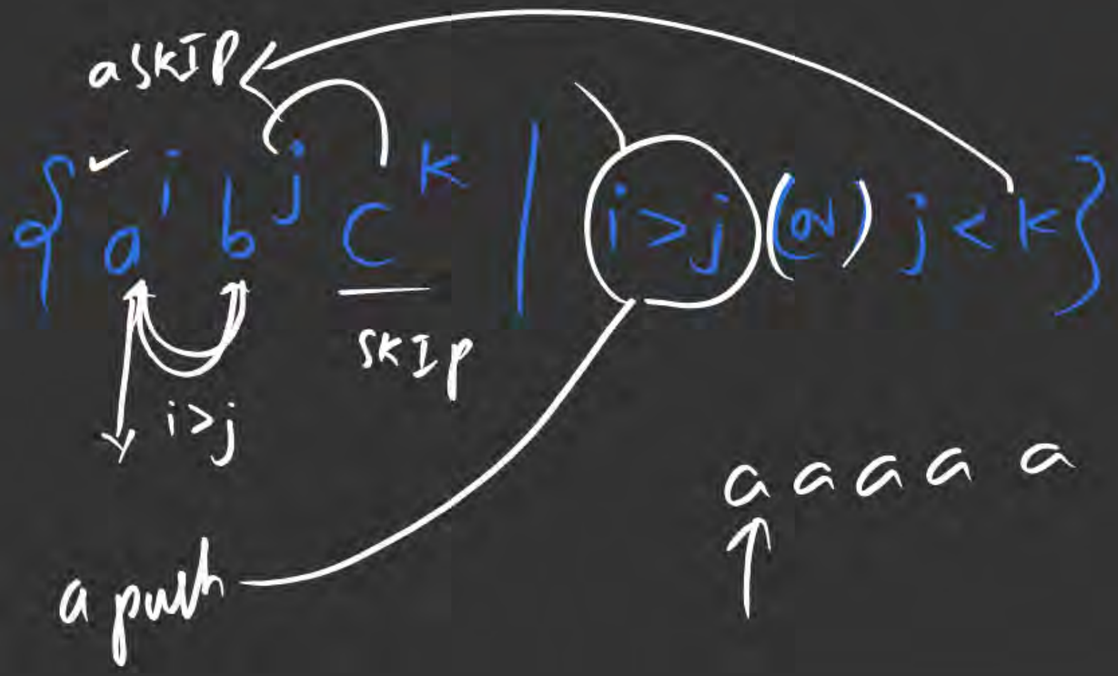
$\{a^i b^{i+k} c^k\} = \{a^i b^i b^k c^k\}$ CFL.

push push pop pop



16. $L = \{a^i b^j c^k / j = i + k\}$

17. $L = \{a^i b^j c^k / i > j \text{ (or) } j < k\}$ → CFL → not DCFL → not Regular


a skip
q_0 a^i b^j c^k / skip
i>j
i>j (or) j<k
a push
a a a a a a

Topic : Context Free Language

All these languages are infinite

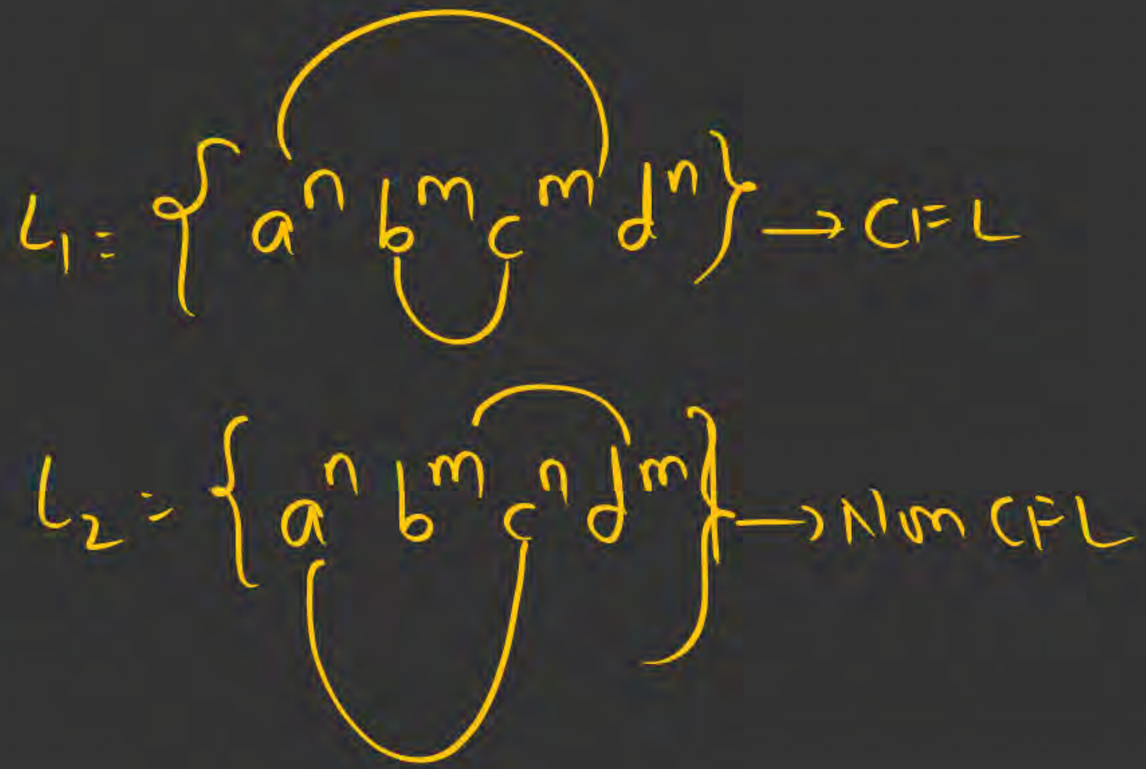
18. $L = \{a^i b^j c^k / i > j > k\}$ more than 1 Dep → non CFL

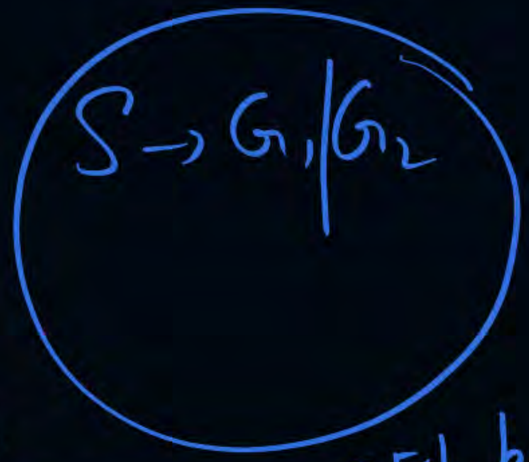
19. $L = \{a^i b^j c^k / j = \max(i, k)\}$ → Non CFL

20. $L = \{a^i b^j c^k / j = i^2 + k^2\}$ → Non CFL

21. $L = \{a^i b^j c^k d^\ell / i = \ell \text{ and } j = k\}$ → CFL

22. $L = \{a^i b^j c^k d^\ell / i = k \text{ and } i = \ell\}$ → Non CFL


$L_1 = \{a^n b^m c^m d^n\} \rightarrow CFL$
$L_2 = \{a^n b^m c^n d^m\} \rightarrow$ Not CFL



G_1 G_1

23. $L = \{a^i b^j c^k / d^\ell / i = k \text{ (or) } j = \ell\}$ → CFL but not DCFL

24. $L = \{a^i b^j c^k / d^\ell / i = 2k \text{ (or) } j \neq 5\ \ell\}$ → CFL but not DCFL.

25. $L = \{a^i b^j c^k / d^\ell / i + j = k + \ell\}$ → CFL but not Regular.

26. $L = \{a^i b^j c^k / d^\ell / i = 4\ \ell \text{ (and) } j = 3\ \ell\}$ → CFL but not Regular

Topic : Context Free Language

(27) $L=\{(a^p)^* \mid p \text{ is prime number}\}$ → reg → CFL

28. $\{a^{2^n} / n \geq 1\}$ not in A·P → non CFL

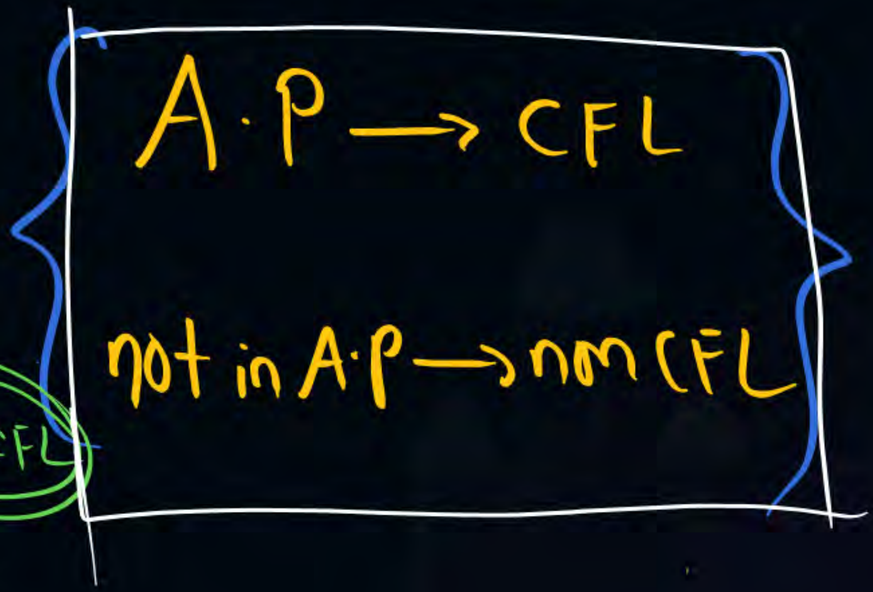
29. $\{a^{n^2} / n \geq 1\}$ not in A·P → non CFL

30. $L = \{1^{2n+1} / n \geq 1\}$ A·P → CFL.

31. $L = \{a^P / P \text{ is prime number}\}$ → non CFL

32. $L = \{a^k / k \text{ is odd number}\}$ → CFL.

If language formed over 1 symbol then no difference in CFL & Regular. Hence



Topic : Context Free Language

$\{wxw\} \longrightarrow$ Non CFL

33. $L = \{wxw / w \in \{a, b\}^*\}$

34. $L = \{wxw / w, x \in \{a, b\}^*\}$ $\xrightarrow{(a+b)^*}$ regular $\longrightarrow$ CFL.

35. $L = \{ww^R x / \{w, x, \in \{a, b\}^+\} = \{(ww^R)x / w, x \in (a+b)^+\} \longrightarrow$ CFL but not Regular.

(CFL* (By))

36. $L = \{\Sigma^* - \{ww / w \in \{a, b\}^+\}$ $\xrightarrow{(ww)^c}$ CFL

37. $L = \{wvw^R w / w \in \{a, b\}^+\}$ $\longrightarrow$ Non CFL

38. $L = \{ww^R ww^R / w \in \{a, b\}^+\}$ $\longrightarrow$ Non CFL

(ww) (ww)

abbabbbb

WW^R

39. $L = \{\overset{a\,a\,a\,a}{w}\overset{b\,b\,b\,b}{w}ww/w \in \{a, b\}\}$ → finite → Regular → CFL

40. $\{L = \{x/x \in \{a, b, c\}^* \; n_a(x) = n_b(x) = n_c(x)\}\}$ → Non CFL.

41. $L = \{x/x \in \{a, b, c\}^* \; n_a(x) = n_b(x) + n_c(x)\}$ → CFL.

42. $L = \{x/x \in \{a, b, c\}^* \; n_a(x) = n_b^2(x) + n_c^2(x)\}$ → Non CFL.

Topic : Context Free Language

aaabbbccccc

Regular → DCFL → CFL.

as 5 * 4 = 20

43. $L = \{x / x \in \{a, b\}^* \; n_a(x) \bmod 5 = 0 \text{ and } n_b(x) \bmod 4 = 0\}$

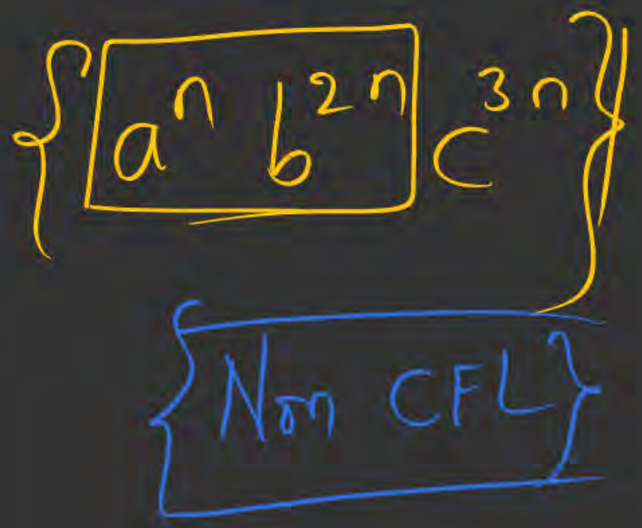
44. $L = \{a^n b^{2n} c^{3n} / \; n \geq 1\}$ → Non CFL

45. $L = \{a^n b^n c a^m b^m / n, m \geq 0\}$ → CFL

46. $L = \{a^n b^m c^k / n \neq m \text{ (or) } m \neq k\}$ → CFL but not DCFL

47. $L = \{a^n b^n c d^n e^n \mid n \geq 1\}$ → Non CFL

aa bb ddeee


$\{a^n b^{2n} c^{3n}\}$
{Non CFL}

Topic : Context Free Language

48 Set of all odd length palindrome string of Hindi language → CFL.

49 Set of all even length palindrome string of [redacted] language (Tamil) → CFL.

50 Set of all balanced parenthesis → CFL

((((()))(()))())

Q

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$ → CFL

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$ → Non CFL.

Which one of the following is TRUE?

a bbb cccc

[2016(Set-2): 2 Marks]

A. Both L_1 and L_2 are context-free.

B. L_1 is context-free while L_2 is not context-free

C. L_2 is context-free while L_1 is not context-free

D. Neither L_1 nor L_2 is context-free

Q

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$.

Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

[2017(Set-1): 2 Marks]

A. I only

B. II only

C. I and II

D. Neither I nor II

Q

Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context-free

II. $\overline{L_1}$ is context-free

III. $L_1 - R$ is context-free

IV. $L_1 \cap L_2$ is context-free

[2017(Set-2): 1 Marks]

A. I, II and IV only

B. I and III only

C. II and IV only

D. I only

Q

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\}$ → non CFL

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$ → CFL

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$ → non CFL.

$L_4 = \{a^n b^n \mid n \geq 1\}$ → CFL & DCFL

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

[2017(Set-2): 2 Marks]

A. I, II and IV only

B. II and III only

C. I and IV only

D. III and IV only

Q

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

A $L_1 \cdot L_2$

B $L_1 \cup L_2$

C $L_1 - L_2$

D $L_1 \cap L_2$

Topic : CONTEXTFREE LANGUAGE

closure properties

Operation	CFL	DCFL
① Union op	✓	X
② Concatenation	✓	X
③ Complement	X	✓
④ Intersection	X	X
⑤ Kleene closure	✓	X
⑥ Positive closure	✓	X

	CFL	DCFL
⑦ Intersection with Regular	✓	✓
⑧ Difference	×	×
⑨ ∂-Regular	✓	✓
⑩ Regular - L	×	✓
⑪ Reversal (L^R)	✓	×
⑫ Quotient	×	×

	CFL	DCFL
(13) Substitution	✓	X
(14) Homomorphism	✓	X
(15) Inverse Homomorphism	✓	✓
(16) $L \cup$ Regular	✓	✓
(17) Prefix	✓	✓
(18) Suffix	✓	X
(19) Subset	X	X

2 mins Summary

- Topic One
- Topic Two
- Topic Three
- Topic Four
- Topic Five

THANK - YOU