CS & IT ENGINEERING

THEORY OF COMPUTATION

Regular Languages

Lecture No.- 02

By- Venkat sir

Recap of Previous Lecture

Topic Regular Expression

Topic Construction of Regular Expression

Topic DFA States

Topics to be Covered

Topic Conversion from $\in$ NFA to NFA

Topic ??

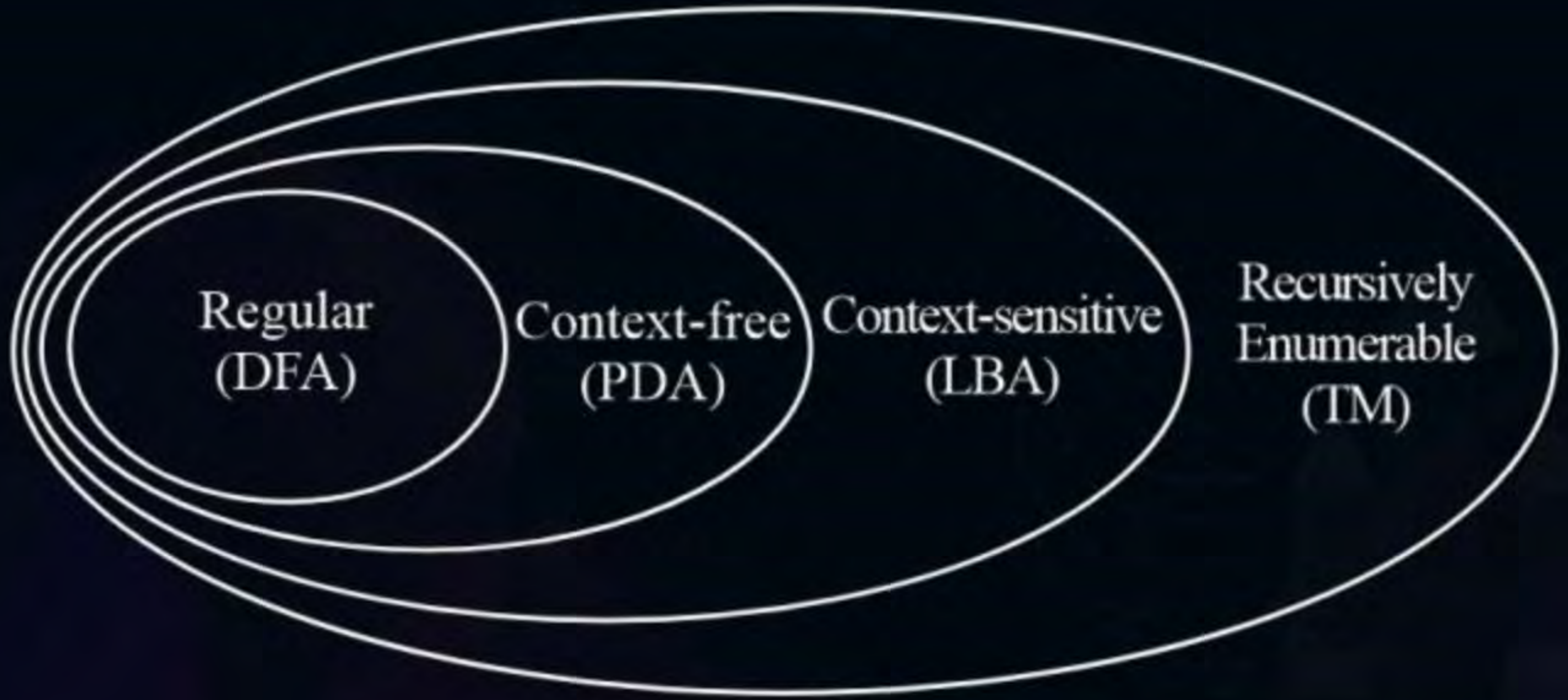
Topic ??

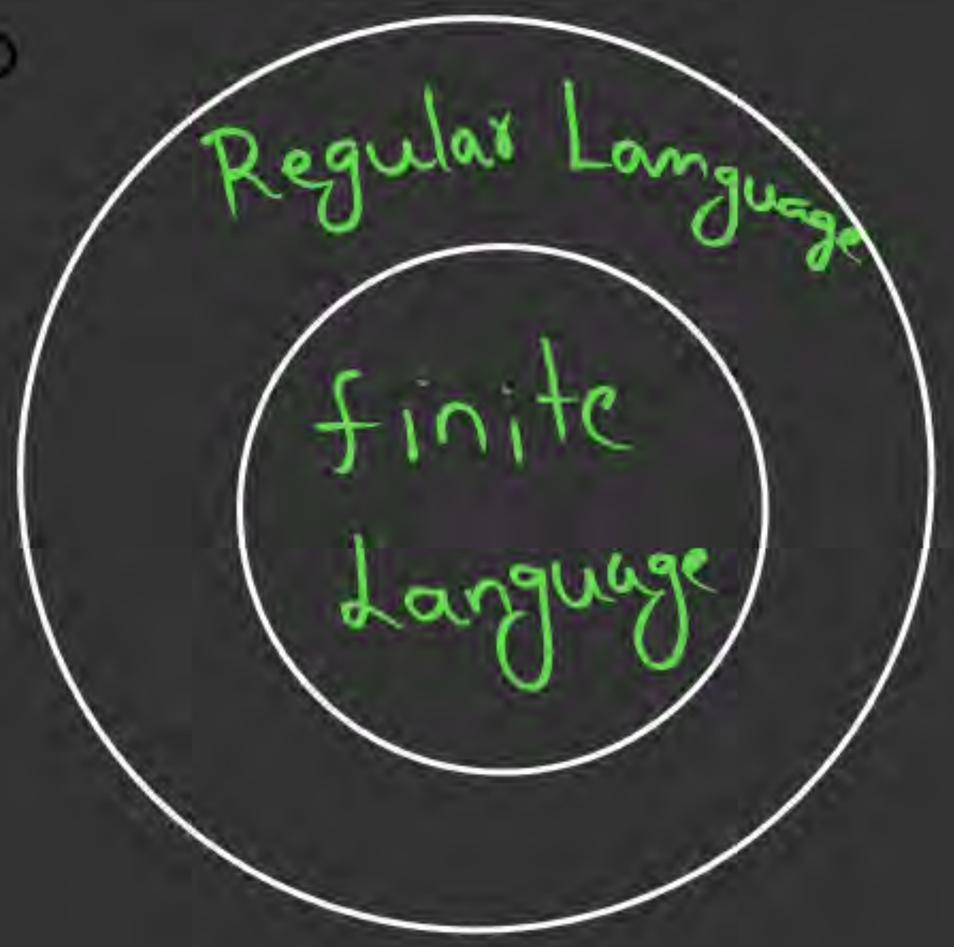
1 Finite language

Every finite is Regular

Every Infinite language → may be Regular

→ may not be Regular


Regular
(DFA)
Context-free
(PDA)
Context-sensitive
(LBA)
Recursively
Enumerable
(TM)


Regular Language
finite
Language

Topic : Regular Language Detection

Which of these Languages are Regular

1. $L = \{a^n b^n c^n \mid 1 \le n \le 100\ c\}$

2. $L = \{a^n b^m \mid n + m = 10\}$

3. $L = \{a^n b^m \mid n - m = 5\} \xrightarrow{\text{infinite}}$ Non Regular.

4. $L = \{a^n b^m \mid n * m = 100\} \xrightarrow{\text{finite}}$ Regular. ✓

5. $L = \{a^n b^m \mid n = 2m + 1\}$

6. $L = \{a^n b^m \mid n > m\}$

7. $L = \{a^n b^m \mid n > 1\cancel{\text{and}}\}$

Infinite Language

Dependency exist

non Regular

Dependency not exist

Regular

Topic : Regular Language Detection

8. $L = \{a^n b^m \mid n > m \text{ (or) } n < m\}$ $= \{a^n b^m \mid n \neq m\}$ = Non Regular

9. $L = \{a^n b^m c^{n+m} \mid n, m \geq 1\}$ $\xrightarrow[\text{infinite}]{\text{Dependency}}$ Non Regular

10. $L = \{a^n b^n c^{n+m} \mid n, m \geq 1\}$ $\xrightarrow[\text{Dependency}]{}$ Non Regular

11. $L = \{a^n b^{2m} c^{3k} \mid n, m, k \geq 0\}$ $\longrightarrow$ Regular

12. $L = \{a^n b^{m^2} c^{k^3} \mid n, m, k \geq 1\}$ $\xrightarrow[\text{NO Dependency}]{}$ Non Regular

13. $L = \{a^n b^{m^2} c^{k^3} \mid n, m, k \geq 1\}$ $\longrightarrow$ Non Regular

14. $L = \left\{ \begin{matrix} a^{2^n} \\ a^{3^n} \end{matrix} \middle| n \geq 0 \right\}$

(Q) Which of the following is Regular?

(a) $L = \{ a^{m^2} b^{2^n} \mid m,n \geq 1 \} \xrightarrow{\text{No Common diff}}$ Non Regular

a^1, a^2, a^6, a^{24}

(b) $L = \{ a^{n!} b^{m!} c^{k!} \mid m,n,k \geq 1 \} \longrightarrow$ Non Regular

(c) $L = \{ a^n b^m c^{m+n} \mid m,n \geq 1 \} \xrightarrow{\text{Dependency}}$ Non Regular

(d) $L = \{ a^{n^n} b^{m^n} c^{k^k} \mid n,m,k \leq 1000 \} \xrightarrow{\text{finite}}$ Regular

$\{a^m b^{n^2} c^{k^3} \mid m, n, k \geq 0\} \rightarrow$ No Common diffence in symbol } Non Regular

Common diff

$L = \{a^n b^{2m} c^{3k}\}$

Common diff

(Q) which of the following is Regular

(a) $L = \{a^n b^m \mid n \geq m \text{ and } n \leq m\} = \{a^n b^n \mid n \geq 1\}$

(b) $L = \{a^n b^m \mid n > m \text{ (or) } n < m\} = \{a^n b^m \mid n \neq m\}$

(c) $L = \{a^n b^m \mid n > m \text{ (and) } n < m\} = \{\ \} \Rightarrow \varnothing^{a,b}$

(d) none

(Q) Which of the following is Non Regular

(a) $L = \{a^n b^m \mid (n+m) \text{ is even}\} = (aa)^*(bb)^* + a(aa)^* b(bb)^*$

(b) $L = \{a^n b^m \mid (n+m) \text{ is odd}\} = (aa)^* b(bb)^* + a(aa)^*(bb)^*$

(c) $L = \{a^n b^m \mid n = m^2\} = \{a^{m^2} b^m\}$ Dependency } Non Regular

(d) $L = \{a^{2m} b^{3n} \mid n, m \geq 1\} \longrightarrow$ no dependency } Regular

Topic : Regular Language Detection

1 Symbol

15. $L = \{a^{n^3} \mid n \geq 1\} \rightarrow \{a^1, a^{2^3}, a^{3^3}, a^{4^3} \ldots\} \rightarrow$ Non Regular

16. $L = \{a^{n^n} \mid n \geq 1\} \rightarrow \{a^{1^1}, a^{2^2}, a^{3^3} \ldots\} \rightarrow$ Non Regular

17. $L = \{a^{2^n} \mid n \geq 0\} \rightarrow \{a^{2^0}, a^{2^1}, a^{2^2}, a^{2^3} \ldots\} \rightarrow$ Non Regular

18. $L = \{a^{100^{100^{100}}} \mid\} \rightarrow \{\ 1\ \} = \text{finite}\} \rightarrow$ Regular

19. $L = \{(a^P)^* \mid$ p is prime number$\}$ → Regular

20. $L = \{a^p \mid$ p is prime number$\}$ → Non Regular

(Q) Which of the following is Non Regular?

$\{a^0, a^2, a^4, a^6 \ldots\}$

(a) $L = \{a^k \mid k \text{ is even number}\} \longrightarrow$ Regular

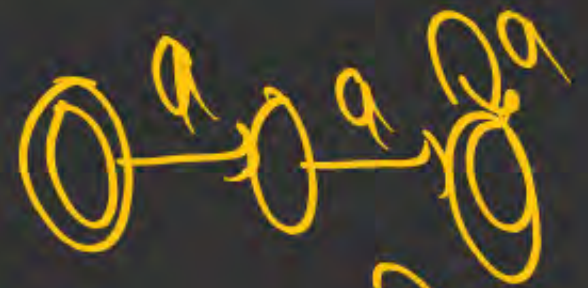
$\{a^1, a^3, a^5 \ldots\}$

(b) $L = \{a^m \mid m \text{ is odd number}\} \longrightarrow$ Regular

(c) $L = \{(a^p)^* \mid p \text{ is prime number}\}$

$\{(a^2)^*, (a^3)^*, (a^5)^*, (a^7)^* \ldots\}$

$\{\epsilon, a^2, a^3, a^4, a^5, a^6, a^7, a^8, a^9 \ldots\} \longrightarrow$ Regular



(d) none

$$\{a^2 \underbrace{,}_{1} a^3 \underbrace{,}_{2} a^5 \underbrace{,}_{2} a^7 \underbrace{,}_{4} a^{11} \underbrace{,}_{2} a^{13} \ldots\}$$

$$L = \{a^p \mid p \text{ is prime number}\} \longrightarrow \text{Non Regular}$$

If a language is formed over 1 symbol

If Common difference exist (A·P) → Regular

(Q) Which of the following is Regular?

(a) $L = \{ a^{n!} \mid n \geq 1 \} = \{ a^1, a^2, a^6, a^{24} \dots \} \longrightarrow$ Non Regular

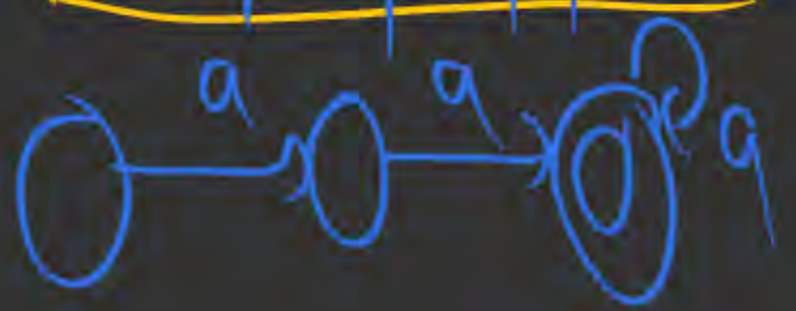
(b) $L = \{ a^{n^{n^2}} \mid n \geq 1 \} \to \{ a^1, a^{2^{2^2}}, a^{3^{3^2}}, a^{4^{4^2}} \dots \} \to$ Non Regular

(c) Regular $L = \{ a^{m^n} \mid m > n,\ \boxed{n \geq 1} \} = \{ a^{m^1} \} \cup \{ a^{m^2} \} \cup \{ a^{m^3} \} \dots$

$a \cdot a^+$

$\{ a^2, a^3, a^4, \dots \} \cup \{ a^{3^2}, a^{4^2}, a^{5^2}, \dots \} \cup \{ a^{4^3}, a^{5^3}, a, \dots \}$

(d) none $\{ a^2, a^3, a^4, a^5 \dots \}$



(Q) which of the following is Regular?

(a) $L = \{1, 2, 4, 8, 16 \ldots 2^n \ldots\} = \{1^{2^n} \mid n \geq 0\} \rightarrow$ Non Regular

$1, 11, 1^4, 1^8, 1^{16}$

all these numbers written in Unary

(b) $L = \{1, 2, 4, 8, 16 \ldots 2^n \ldots\}$

$1, 10, 100, 1000, 10000$

all these numbers written in binary $\} \; 0^* 1 0^* \rightarrow$ Regular ✓

(c) $L = \{1^{2^n} \mid n \geq 0\} \longrightarrow$ Non Regular.

(d) none

Unary $\{1, 11, 1111, 1^8 - - - \}$ = No Common diff $\}$ Non Reg

binary $\{1, 2, 4, 8 - - - \}$

10000

00010

100000

$\boxed{0^* 1 0^*}$

Topic : Regular Language Detection

21. $L = \{a^k \mid k \text{ is even number}\} \longrightarrow$ Regular

22. $L = \{ww^R \mid w\} \in (a+b)^* \} \longrightarrow$ Non Regular

23. $L = \left\{ ww^R \middle| \begin{matrix} w \in \{a, b\}^* \\ \cancel{w \in \{a, b\}^*} \end{matrix} \right\}$

24. $L = \{wbw^R \mid w \in \{a\}^*\}$

25. $L = \{x \mid x \in \{a, b\}^* \; n_a(x) \bmod 3 = n_b(x) \bmod 2\}$

26. $L = \{x \mid x \in \{a, b\}^* \; n_a(x) \bmod 2 > n_b(x) \bmod 3\}$

27. $L = \{x \mid x \in \{a, b\}^* \; n_a(x) \bmod 3 \neq n_b(x) \bmod 3\}\}$

28. $L = \{x \mid x \in \{a, b, c\}^* \; n_a(x) \neq n_b(x)\}$

Palindrome Languages

$$L = \{ \underline{W}\,\underline{W}^R \}_{w \in (}$$

nitin

$W \quad W^R$

malayalam

$W \quad W^R$

$L = \{ \underline{W}\, \underline{W}^R \}$ $\quad W \in (a)^* \rightarrow$ Regular

$W \in (a+b)^* \rightarrow$ Non Regular.

Dependency.

① $L = \{ w w^R \mid w \in (b)^* \} = \{ \epsilon, bb, b^4, b^6 \ldots \} = (bb)^* \rightarrow$ Regular

② $L = \{ w x w^R \mid w \in \{a\}^* \} = \{ x, axa, aaxaa, a^3 x a^3, a^4 x a^4 \ldots \}$ Non Regular

③ $L = \{ w w^R \mid w \in (a+b)^* \} = \{ \epsilon, aa, ab, abba, baab \ldots \}$ Non Regular

④ $L = \{ w \cancel{x} w^R \mid w \in (a+b)^* \} = \{ x, abxba, abbxbba \ldots \}$ Non Regular

⑤ $L = \{ w x w^R \mid w \in (a+b), x \in (a+b)^* \} = \boxed{a(a+b)^* a + b(a+b)^* b}$ Regular

$w\,x\,w^R$

$a(a+b)^* a$

$b(a+b)^* b$

1. $L = \{ w w^R x \mid w, x \in (a+b)^* \} \rightarrow (a+b)^* \rightarrow$ Regular.

2. $L = \{ w x w^R \mid w, x \in (a+b)^* \} \rightarrow (a+b)^* \rightarrow$ Regular.

3. $L = \{ x w w^R \mid w, x \in (a+b)^* \} \rightarrow (a+b)^* \rightarrow$ Regular

   $(a+b)^* \cup \ldots$

4. $L = \{ w w \mid w \in (a+b)^* \} = \{ \epsilon, aa, bb, abab, baba \ldots \} =$ Non Regular

5. $L = \{ w x w \mid w, x \in (a+b)^* \} = (a+b)^* \}$ Regular ✓

6. $L = \{ w \cancel{x} w \mid w \in (a+b)^* \} \rightarrow$ Dependency $\}$ Non Regular

Home Work

1. $L = \{ ww^R x \mid w, x \in (a+b)^+ \}$

2. $L = \{ x w w^R \mid w, x \in (a+b)^+ \}$

3. $L = \{ w w x \mid w, x \in (a+b)^+ \}$

4. $L = \{ w x w^R \mid w, x \in (a+b)^+ \}$

5. $L = \{ w w^R w w^R \mid w \in (a+b)^+ \}$

$w, x \in (a+b)^{*}$

$$L = \{ w x w \} = (a+b)^{*}$$

$\underline{w = \epsilon}$

$$\epsilon\,(a+b)^{*}\,\epsilon = (a+b)^{*}$$

$w = a$

$w = b$

$$\cup \; \ldots \; \cup \; \ldots \; \cup \; \ldots \;\; (a+b)^{*}$$

$$\left\{ \underline{w}\,\underline{x}\,w^R \right\} = (a+b)^*$$

$\underline{w = \epsilon}$ $\quad \epsilon\,(a+b)^*\,\epsilon = (a+b)^*$

$\cup$

$w = a$

$w = b$

$\cup$

$(a+b)^*$

$$\left\{ w w^R x \;\middle|\; w, x \in (a+b)^* \right\} = \boxed{(a+b)^*}$$

$w = \epsilon$ $\quad (a+b)^*$ Super

$\cup$

$w = a$ $\quad a\,a\,(a+b)^*$ Sub

$\cup$

$w = b$ $\quad b\,b\,(a+b)^*$

$\cup$

$w = ab$ $\quad abba(a+b)^*$

$\cup$

$\vdots$

$\rightarrow (a+b)^*$

$$(a+b)^* \cup a^* b^* = (a+b)^*$$

Super

$$a^* + a = a^*$$

$$a^* \cdot a = a^+$$

$$a^* \cup a^5 = a^*$$

2 mins Summary

Topic One

Topic Two

Topic Three

Topic Four

Topic Five

THANK - YOU