Data Structures & Programming

CS & IT

Graph & Hashing

DPP: 1

Q1 Insert the characters of the string K R P C S N Y T J M into a hash table of size 10.

Use the hash function H(x) = (ord (x) – ord ('a') + 1) mod 10 and linear probing to resolve collisions.

Which insertions cause collisions?

(A) J
(B) C
(C) M
(D) P

Q2 Consider a hash table with n buckets, where external (overflow) chaining is used to resolve collisions. The hash function is such that the probability that a key value is hashed to a particular bucket is $\frac{1}{n}$.The hash table is initially empty and K distinct values are inserted in the table. What's the probability that bucket number 1 is empty after the k insertion?

(A) $\frac{(n-1)^k}{n^k}$
(B) $\frac{(n-1)^{k-1}}{n^{k-1}}$
(C) $\frac{(n-1)}{n}$
(D) $\frac{1}{n^k}$

Q3 Consider a hash table of size seven, with starting index zero, and a hash function (3x + 4) mod 7. Assuming the hash table is initially empty, which of the following is the contents of the table when the sequence 8, 10, 15, 17 is inserted into the table using closed hashing? Note that - denotes an empty location in the table.

(A) 15, –, –, –, –, –, 17
(B) 8, 15, 17, –, –, –, 10
(C) 8, –, –, –, –, –, 10
(D) 8, 17, 15, –, –, –, 10

Q4 Consider a hash table with 50 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

(A) $(47 \times 47 \times 47)/50^3$
(B) $(49 \times 48 \times 47)/50^3$
(C) $(47 \times 46 \times 45)/50^3$
(D) $(47 \times 46 \times 45)/(3! \times 50^3)$

Q5 Figure out true and false statements from below.

(A) Clusters formed by quadratic probing [open addressing] is called primary cluster.
(B) The worst case timing for successful and unsuccessful search in separate chaining is $\theta(1+\alpha)$ (where α is average chain length)
(C) Primary clusters degrade the hash table performance.
(D) Some insertion may cycle through the list in quadratic probing

Q6 A hash table of length 10 uses open addressing with hash function h(k)=k mod 10, and linear probing. After inserting 8 values into an empty hash table, the table is as shown below

0	10
1	21
2	32
3	43

4	54
5	65
6	76
7	77
8	
9	

How many different insertion sequences possible using the same hash function and linear probing will result in the hash table shown above?

(A) 7!(factorial) (B) 8!(factorial)
(C) 8 (D) 1

Q7 A hash table contains 9 buckets and uses linear probing to resolve collisions. The key values are integers and the hash function used is key % 9. If the values 41, 157, 72, 76, 31 are inserted in the table, in what location would the last key be inserted? _________________.

Q8 Consider a hash table with 11 slots. The hash function is h(k) = k mod 11. The collisions are resolved by chaining. The following 11 keys are inserted in the order: 28, 19, 15, 20, 33, 30, 42, 63, 60, 32, 43. The maximum, minimum, and average chain lengths in the hash table, respectively, are-

(A) 3, 0, 1 (B) 3, 3, 3
(C) 3, 0, 2 (D) 4, 0, 1

Answer Key

Q1	(A, C)
Q2	(A)
Q3	(B)
Q4	(A)
Q5	(B, C, D)
Q6	(B)
Q7	7~7
Q8	(A)

Hints & Solutions

Q1 Text Solution:

h(x) = ((ord(x) – ord(A)) + 1) mod 10

h(K) = ((ord(K) – ord(A)) + 1) mod 10 = ((11 – 1) + 1) mod 10 = 1

h(R) = ((ord(R) – ord(A)) + 1) mod 10 = ((18 – 1) + 1) mod 10 = 8

h(P) = ((ord(P) – ord(A)) + 1) mod 10 = ((16 – 1) + 1) mod 10 = 6

h(C) = ((ord(C) – ord(A)) + 1) mod 10 = ((3 – 1) + 1) mod 10 = 3

h(S) = ((ord(S) – ord(A)) + 1) mod 10 = ((19 – 1) + 1) mod 10 = 9

h(N) = ((ord(N) – ord(A)) + 1) mod 10 = ((14 – 1) + 1) mod 10 = 4

h(Y) = ((ord(Y) – ord(A)) + 1) mod 10 = ((25 – 1) + 1) mod 10 = 5

h(T) = ((ord(T) – ord(A)) + 1) mod 10 = ((20 – 1) + 1) mod 10 = 0

h(J) = ((ord(J) – ord(A)) + 1) mod 10 = ((10 – 1) + 1) mod 10 = 0

h(M) = ((ord(M) – ord(A)) + 1) mod 10 = ((13 – 1) + 1) mod 10 = 3

0	T
1	K
2	J
3	C
4	N
5	Y
6	P
7	M
8	R
9	S

J and M are causing the collision.

Q2 Text Solution:

Probability that buckets other than 1 are selected $= \frac{n-1}{n}$

this should happen k times and each of the k events are independent so $\frac{(n-1)^k}{n^k}$.

Q3 Text Solution:

8 will occupy location 0, 10 will occupy location 6, 15 hashed to location 0 which is already occupied so, it will be hashed to one location next to it. i.e. to location 1.

Since 17 also clashes, so it will be hashed to location 2.

Q4 Text Solution:

We have 100 slots each of which are picked with equal probability by the hash function (since hashing is uniform). So, to avoid first 3 slots, the hash function has to pick from the remaining 47 slots. And repetition is allowed, since chaining is used- meaning a list of elements are stored in a slot and not a single element.

So, required probability = 47/50×47/50×47/50

=(47×47×47)/40^3

Q5 Text Solution:

A is false they are called secondary cluster

B is TRUE

NOTATION: α= n/m = load factor

(A) MATHEMATICAL PROOF

(1) Assume that a new element is inserted at the end of the linked list

(2) Upon insertion of the i-th element, the expected length of the list is (i –1)/m

(3) In case of a successful search, the expected number of elements examined is 1 more that the number of elements examined when the sought-for element was inserted.

$$\frac{1}{n}\sum_{i=1}^{n}\left(1+\frac{i-1}{m}\right)=1+\frac{1}{nm}\sum_{i=1}^{n}(i-1)$$

$$=1+\frac{1}{nm}\cdot\frac{(n-1)n}{2}$$

$$=1+\frac{n-1}{2m}$$

$$=1+\frac{n}{2m}-\frac{1}{2m}$$

$$1+\frac{\alpha}{2}-\frac{1}{2m}$$

C is TRUE

D is TRUE

Q6 Text Solution:

Since all elements are in correct position you can insert them in any order

Hence we can insert them in any order 8! (factorial).

Q7 Text Solution:

41 mod 9 =5

157 mod 9 = 4

72 mod 9 = 0

76 mod 9 = 4

Since 157 occupies 4 and 41 occupies 5, 76 is hashed to 6

31 mod 9 = 4

Since 157 occupies 4, 41 occupies 5, 76 occupies 6, 31 is hashed to 7

Q8 Text Solution:

28 mod 11 = 6

19 mod 11 = 8

15 mod 11 = 4

20 mod 11 = 9

33 mod 11 = 0

30 mod 11 = 8

42 mod 11 = 9

63 mod 11 = 8

60 mod 11 = 5

32 mod 11 = 10

43 mod 11 = 10

Maximum chain length = 3

Minimum chain length = 0

Average chain length = 11/11 = 1