

Computer Science & IT

Data Structure & Programming



Array

Lecture No. 02

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Recap of Previous Lecture



Topic

1-D array

Topic

generalized formula

Topic

2D array

Topic

Row major order

Topic

Topics to be Covered



Topic

Column major order

Topic

Sparse matrix

Topic

Lower Triangular Matrix.

Topic

Topic



Question : 1-D Array

Consider the above array $A[-5 \dots 100]$.

Base address of the array is 1000 and each element occupies 2-Bytes of space. What is the address of

$A[70]$?

$A[70]$

(A) 1150 ✓

(B) 1250 ✓

(C) 1200

(D) 1350

$$BA + (i - LB) \times \text{Size}$$

$$1000 + (70 - (-5)) \times 2$$

$$1000 + 75 \times 2 = 1000 + 150 = 1150$$



Topic : 2-D Row major order:

A_{11} 100	A_{12} 104	A_{13} 108
A_{21} 112	A_{22} 116	A_{23} 120
A_{31} 124	A_{32} 128	A_{33} 132

1

2-16

$A[1..3][1..3]$

Two part calculation

$A[3][2]$

↑

(1) Before 3rd Row

No. of Rows arranged

(2) on Third Row.



Question : 2-D Array, MSQ

Consider the following array $A[-23 \dots 93] [-7 \dots 31]$. Base address is 1000 and each element occupies 2-Bytes of space. Which of the following is TRUE ?

- (A) The number of rows is 117 ~~elements in a row~~ ✓ (A)
- (B) The Array $B[0 \dots 116] [2 \dots 40]$ have same number of elements as the array declared in question
- (C) Address of $A[0][5]$ is 3538 row major order
- (D) Address of $A[0][0]$ is 3538 row major order

Rows : $Ub_1 - Lb_1 + 1$
 $93 - (-23) + 1$
 $93 + 23 + 1$
117



Question : 2-D Array, MSQ



Consider the following array $A[-23 \dots 93] [-7 \dots 31]$. Base address is 1000 and each element occupies 2-Bytes of space. Which of the following is TRUE ?

(A) The number of rows is 117 elements in a row

(B) The Array $B[0 \dots 116] [2 \dots 40]$ have same number of elements as the array declared in question

(C) Address of $A[0][5]$ is 3538 row major order

(D) Address of $A[0][0]$ is 3538 row major order

$$U_{b_2} - L_{b_2} + 1$$

$$\frac{31 - (-7) + 1}{= 32 + 7 = 39}$$

$$A[117][39]$$

$$(U_{b_1} - L_{b_1} + 1)$$

$$93 - (-23) + 1 = 117$$

$$B[117][39]$$



Question : 2-D Array, MSQ

Consider the following array $A[-23 \dots 93] [-7 \dots 31]$. Base address is 1000 and each element occupies 2-Bytes of space. Which of the following is TRUE ?

(A) The number of rows is 117 ~~elements in a row~~ ✓

(B) The Array $B[0 \dots 116] [2 \dots 40]$ have same number of elements as the array declared in question ✓

(C) Address of $A[0][5]$ is 3538 row major order ✗

(D) Address of $A[0][0]$ is 3538 row major order ✗

$$1000 + [(0 - (-23)) \times (31 - (-7) + 1)]$$

$$+ (5 - (-7)) \times 2$$

$$1000 [23 \times 39 + 12] \times 2$$

$$= 2818 \checkmark$$



Question : 2-D Array, MSQ

Consider the following array $A[-23 \dots 93] [-7 \dots 31]$. Base address is 1000 and each element occupies 2-Bytes of space. Which of the following is TRUE ?

(C) Address of $A[0][5]$ is 3538 row major order

$A[0][0]$

$$1000 + [(0 - (-23)) \times 39 + (0 - (-7))] \times 2$$
$$1000 + [23 \times 39 + 7] \times 2$$
$$2808$$



Question : 2-D Array, MSQ

Consider the following array $A[-23 \dots 93] [-7 \dots 31]$. Base address is 1000 and each element occupies 2-Bytes of space. Which of the following is TRUE ?

(D) Address of $A[0][0]$ is 3538 column major order


$$A[Lb_1 \dots Ub_1][Lb_2 \dots \underline{Ub_2}]$$

↑ column

No. of element in each Row:

i^{th} row if we are in j^{th} column
No. of columns arranged
 $(j - Lb_2)$



Topic : 2-D Row major order

Address of $A[i][j]$

BA - Base Address

S - Size

$$BA + \left[\underbrace{(i - Lb_1)}_{\text{No of Rows arranged}} \times \underbrace{(Ub_2 - Lb_2 + 1)}_{\text{No. of element in each Row}} + \underbrace{(j - Lb_2)}_{\text{in } i^{\text{th}} \text{ row we are in } j^{\text{th}} \text{ No. of element column arranged}} \right] \times S$$

No of Rows
arranged

No. of element
in each Row ✓

in i^{th} row we are in j^{th}
No. of element column
arranged



Topic : 2-D Row major order

Address of $A[i][j]$

BA - Base Address

S - Size

$$BA + \left[\underbrace{(i - Lb_1)}_{\substack{\uparrow \\ \text{No of Rows} \\ \text{arranged}}} \times \underbrace{(Ub_2 - Lb_2 + 1)}_{\substack{\text{No. of element} \\ \text{in each Row}}} + \underbrace{(j - Lb_2)}_{\substack{\text{in ith column} \\ \text{No. of element} \\ \text{arranged}}} \right] \times S$$

No of Rows
arranged

No. of element
in each Row

in ith column
No. of element
arranged



Topic : 2-D Column Major Order

$A[1..3][1..3]$

Row-1	a_{11}	a_{12}	a_{13}
Row-2	a_{21}	a_{22}	a_{23}
Row-3	a_{31}	a_{32}	a_{33}

Column 1 Column 2 Column 3

Column-1			Column-2		Column-3			
a_{11}	a_{21}	a_{31}	a_{12}	a_{22}	a_{32}	a_{13}	a_{23}	a_{33}
100	104	108	112	116	120	124	128	132

$A[3][2]$ Before 2nd column No. of column arranged $(2 - 1) = 1$

No. of element in a column =
Size of Row = 3
total = $1 \times 3 = 3$



Topic : 2-D Column Major Order

$A[1..3][1..3]$

Row-1	a_{11}	a_{12} ✓	a_{13}
Row-2	a_{21}	a_{22} ✓	a_{23}
Row-3	a_{31}	a_{32}	a_{33}

↑ ↑
Column 1 Column 2 Column 3

Column-1			Column-2			Column-3		
a_{11}	a_{21}	a_{31}	a_{12}	a_{22}	a_{32}	a_{13}	a_{23}	a_{33}

100 104 108 112 116 120 124 128 132

$A[3][2]$ in 2nd column we are in
↑ 3rd row

No. of element arranged = $(3 - Lb_1)$
= $(3 - 1) = 2$ ✓



Topic : 2-D Column Major Order

$A[1..3][1..3]$

Row-1 →	a_{11}	a_{12} ✓	a_{13}
Row-2 →	a_{21}	a_{22} ✓	a_{23}
Row-3 →	a_{31}	a_{32}	a_{33}
	↑ Column 1	↑ Column 2	↑ Column 3

Column-1			Column-2			Column-3		
a_{11}	a_{21}	a_{31}	a_{12}	a_{22}	a_{32}	a_{13}	a_{23}	a_{33}
100	104	108	112	116	120	124	128	132

$$\text{Total} = 3 + 2 = 5$$

Address of $A[3][2]$

$$\begin{aligned} &= 100 + 5 \times 4 = 100 + 20 \\ &= 120 \end{aligned}$$



Topic : 2-D Column Major Order

$A[Lb_1 \dots Ub_1][Lb_2 \dots Ub_2]$ generalized formula.

Address of $A[i][j]$ =

$$BA + [(j - Lb_2) \times (Ub_1 - Lb_1 + 1) + (i - Lb_1)] \times S$$



Topic : 2-D Column Major Order



Formula/Trick

Consider the following array $A[-4 \dots 100]$
 $[3 \dots 100]$. Base address is 1000 and each
element occupies 2-Bytes of space. What is the
address of $A[50][48]$ in column major order ?

$$A[-4 \dots 100][3 \dots 100]$$

$$A[50][48]$$

$$\begin{aligned} & \text{1 Before } 48^{\text{th}} \text{ column No.} \\ & \text{of column arranged} = (48 - 3) \\ & = 45 \end{aligned}$$

No. of element in each column:

$$\text{Size of Row} = (100 - (-4) + 1) = 105$$



Topic : 2-D Column Major Order

Consider the following array $A[-4 \dots 100][3 \dots 100]$. Base address is 1000 and each element occupies 2-Bytes of space. What is the address of $A[50][48]$ in column major order ?

In 48th column we are in 50th Row

No. of elements arranged
 $(50 - (-4)) = 54$

Address of $A[50][48] =$

$$1000 + [45 \times 105 + 54] \times 2 =$$

10558

Q Suppose array is declared as $A[\underline{10..19}][\underline{-3..20}]$

BA = 1000 Size = 2B

Ans find Address of $A[15][13]$ in column major order.

$$BA + [(j - lb_2) \times (ub_1 - lb_1 + 1) + (i - lb_1)] \times \text{Size}$$

$$1000 + [(13 - (-3)) \times (19 - 10 + 1) + (15 - 10)] \times 2B$$

$$1000 + [16 \times 10 + 5] \times 2 = 1000 + 165 \times 2$$

$$1000 + 330 = 1330$$



Topic : 2-D Column Major Order

Consider the following array $A[-4 \dots 100][3 \dots 100]$. Base address is 1000 and each element occupies 2-Bytes of space. What is the address of $A[50][48]$ in column major order ?



Topic : 2-D Column Major Order

We need to find the address of $A[50][48]$. Before reaching to 48th column 3...47 columns i.e. $47-3+1=45$ columns are already been arranged (Second way is $(48 - \text{lower bound})$) and each column consists of 105 elements. So total $45 \times 1005 = 4725$ elements are arranged.

For the 50th row we have already arranged -4...49 i.e. 54 $(49 - (-4) + 1)$ elements (look at the first dimension) are already arranged.

So total number of elements arranged are $= 4725 + 55 = 4779$.

The address is $1000 + 4779 \times 2 = 10558$



Topic: Sparse Matrix



Sparse Matrix are Those matrix in which selectively few entries are Non Zero.

Some special sparse Matrix

1. Lower Triangular Matrix



Topic: Lower Triangular Matrix

A square matrix in which elements above principle diagonals are zero. for example a_{44}

	a_{12}	a_{13}	a_{14}
a_{11}	0	0	0
a_{21}	a_{22}	0 ^{a_{23}}	0 ^{a_{24}}
a_{31}	a_{32}	a_{33}	0 ^{a_{34}}
a_{41}	a_{42}	a_{43}	a_{44}

$a[i][j] \leftarrow$

for what condition on i & j

$$a[i][j] = 0 \quad (j > i)$$



Topic: Lower Triangular Matrix

A square matrix in which elements above principle diagonals are zero. for example a_{44}

a_{11}	0	0	0
a_{21}	a_{22}	0	0
a_{31}	a_{32}	a_{33}	0
a_{41}	a_{42}	a_{43}	a_{44}

No. of NonZero elements : $\frac{4 \times 4}{2}$
 $1 + 2 + 3 + 4 = 10$ LTM

No of Non zero element in $n \times n$ LTM
 $\frac{1 + 2 + 3 + \dots + n}{2} = \frac{n(n+1)}{2}$



Topic: Lower Triangular Matrix

A square matrix in which elements above principle diagonals are zero. for example a_{44}

a_{11}	0	0	0
a_{21}	a_{22}	0	0
a_{31}	a_{32}	a_{33}	0
a_{41}	a_{42}	a_{43}	a_{44}

Non Zero elements of LTM
Stored in
* Row major order
* Column major order



Topic: Lower Triangular Matrix

Row major order

$A[1..4][1..4]$

a_{11} 100	a_{21} 104	a_{22} 108	a_{31} 112	a_{32} 116	a_{33} 120	a_{41} 124	a_{42} 128	a_{43} 132	a_{44} 136
Row-1	Row-2		Row-3			Row-4			

a_{11}	0	0	0
a_{21}	a_{22}	0	0
a_{31}	a_{32}	a_{33}	0
a_{41}	a_{42}	a_{43}	a_{44}

$A[4][3]$

if we are in 4th row, No.
of Rows completed = $(4 - Lb_1)$
= $(4 - 1) = 3$
No. of elements arranged in 3 rows.
 $1 + 2 + 3 = 6$



Topic: Lower Triangular Matrix

Before i th Row No. of Rows completed (for $LB = 1$)
 $(i-1)$

No. of elements among $(i-1)$ rows

$$1 + 2 + 3 + \dots + (i-1) =$$



2 mins Summary



Topic

2-D column major

Topic

practice

Topic

Lower Triangular Matrices

Topic

Topic

THANK - YOU