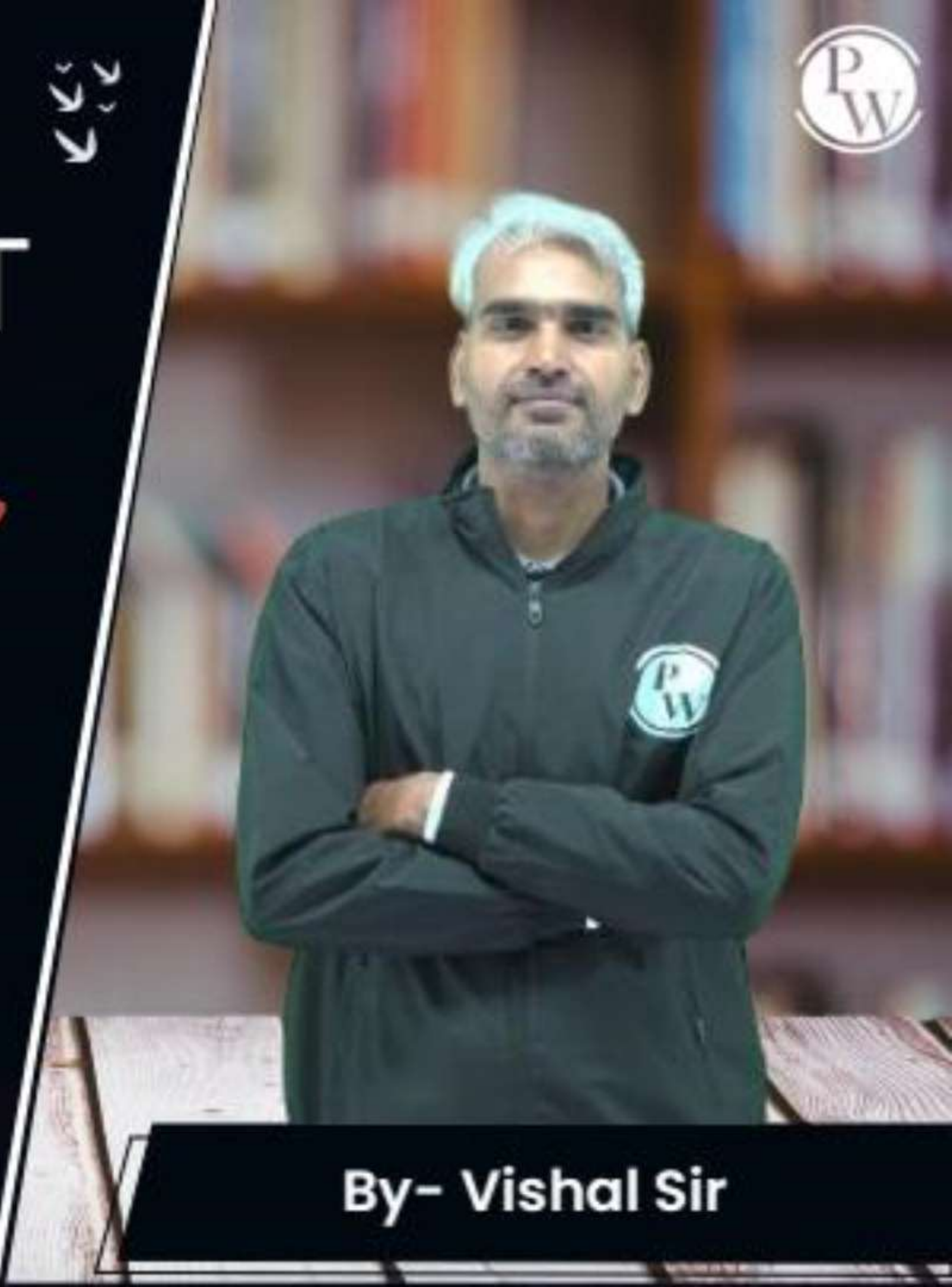
Computer Science & IT

Discrete Mathematics

Graph Theory

Lecture No. 10



Recap of Previous Lecture

Topic Matching

A subgraph in which $deg(v) \le 1,\ \forall v \in G$

Topic Maximal matching & maximum matching

No edge can be added

Matching with Maximum No. of edges

Topic Perfect matching

$deg(v) = 1,\ \forall v \in G$

Topic Matching number

No. of edges in Maximum Matching

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Topics to be Covered

- **Topic** Line / Edge Covering
- **Topic** Minimal & Minimum line covering
- **Topic** Line independent set
- **Topic** Maximal & Maximum line independent set

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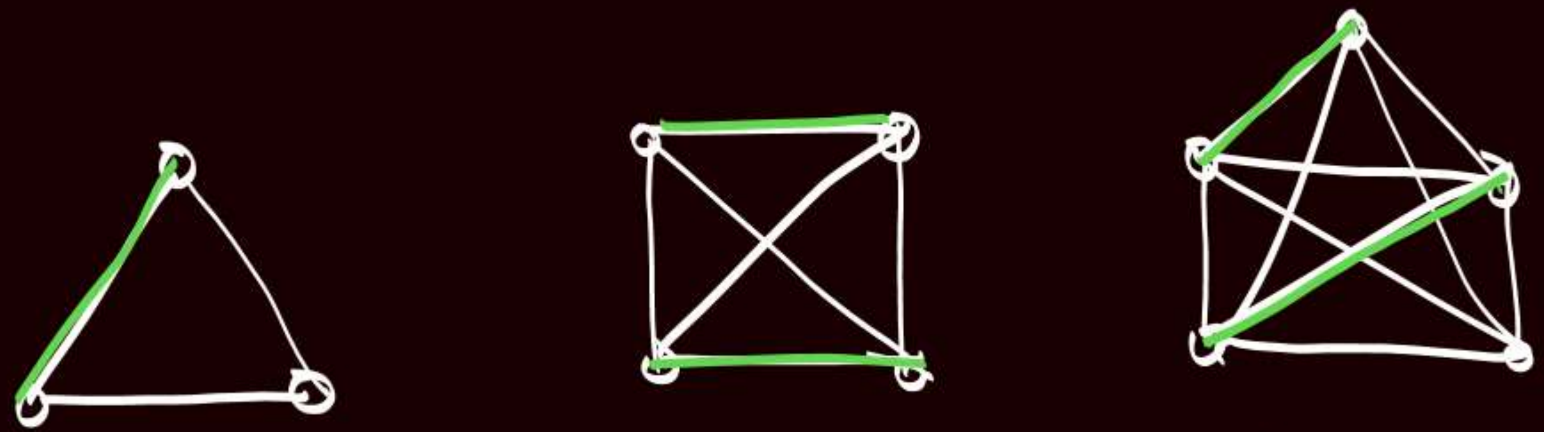
H.W. Find matching number of following graphs.

1. Complete graph K_n

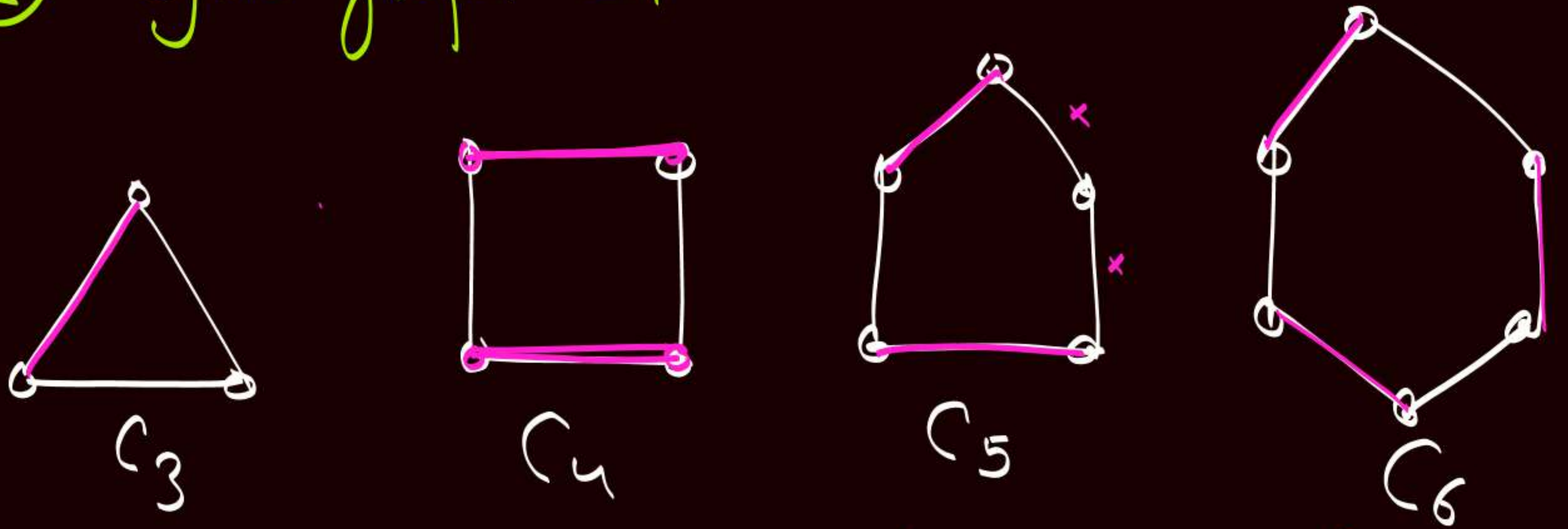


M.No. = 1   M.No. = 2   M.No. = 2

Matching No. for Complete graph $K_n = \left\lfloor \frac{n}{2} \right\rfloor$

H.W. Find matching number of following graphs.

(2) Cycle graph C_n

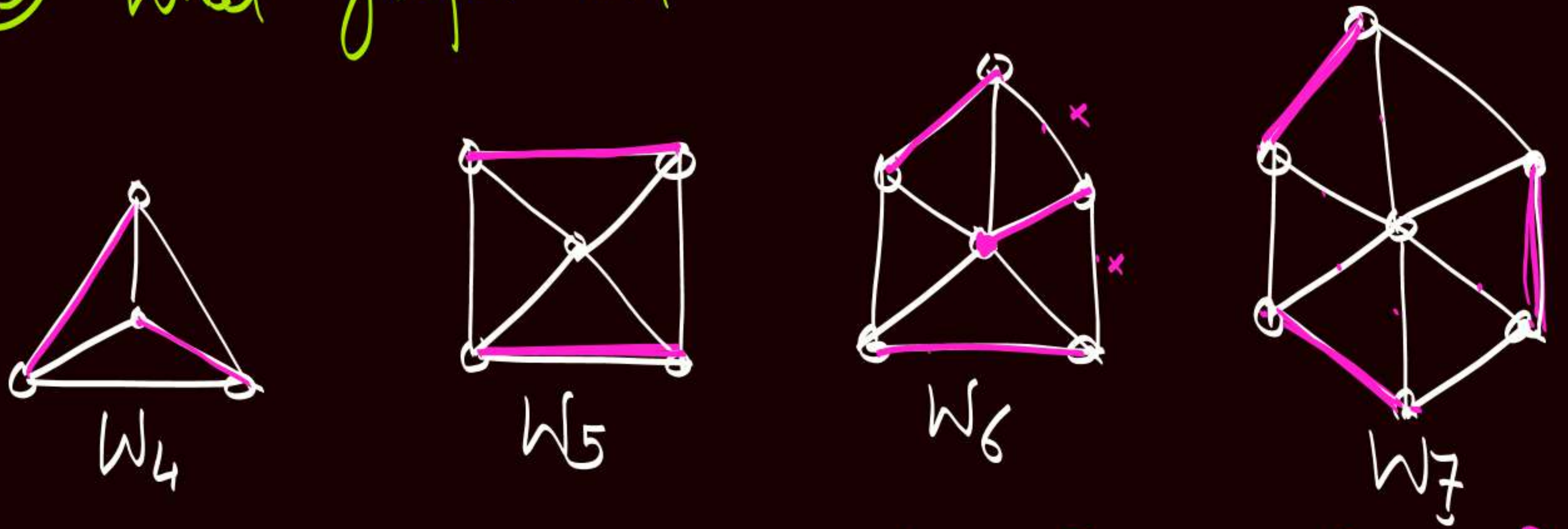


M.No = 1   M.No = 2   M.No = 2   M.No = 3

Matching No. of $C_n = \left\lfloor \frac{n}{2} \right\rfloor$

H.W. Find matching number of following graphs.

(3) Wheel graph W_n



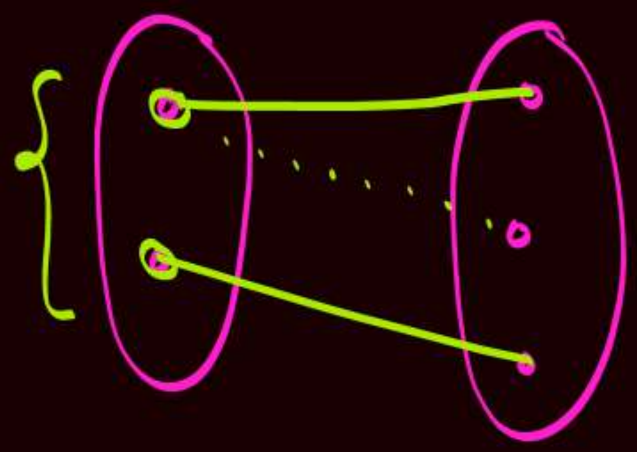
M.No = 2   M.No = 2   M.No = 3   M.No = 3

Matching No. of W_n = $\left\lfloor \frac{n}{2} \right\rfloor$

H.W. Find matching number of following graphs.

(4) Complete bipartite graph $K_{m,n}$

$K_{2,3}$

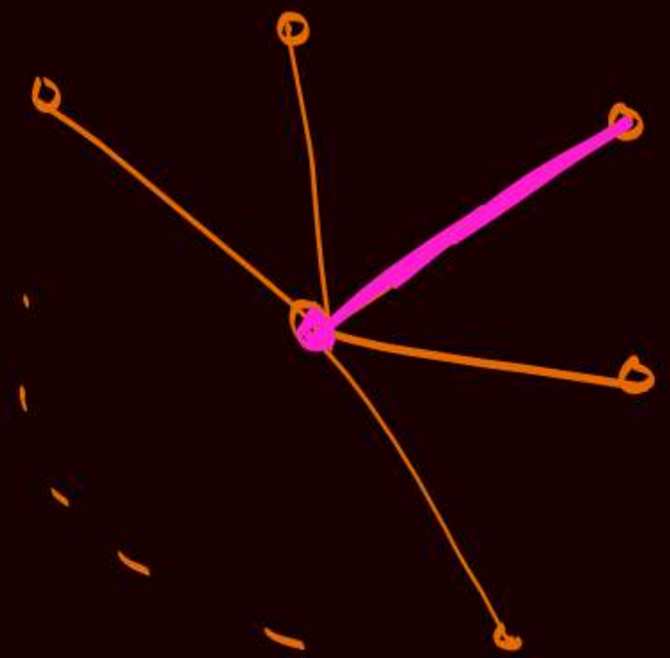


M. No = 2

Matching No. of $K_{m,n} = \text{Min}(m,n)$

H.W. Find matching number of following graphs.

(5) Star graph with n-vertices $\equiv K_{1,n-1} = K_{n-1,1}$



$\therefore$ Matching No. of Star graph with n-vertices $= \text{Min}(1, n-1) = 1$

M. No = 1

{ if we choose any other edge then degree of hub > 1 }

Covering

Line Covering (or) **Edge Covering**

We will try to cover all vertices of the graph using a subset of Edges (lines)

Vertex Covering

We will try to cover all the edges of the graph using a subset of Vertices

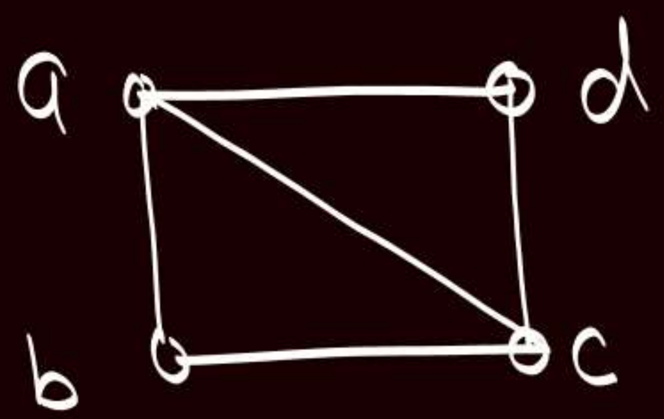
Topic : Line/Edge Covering

Let $G=(V,E)$ be a graph.

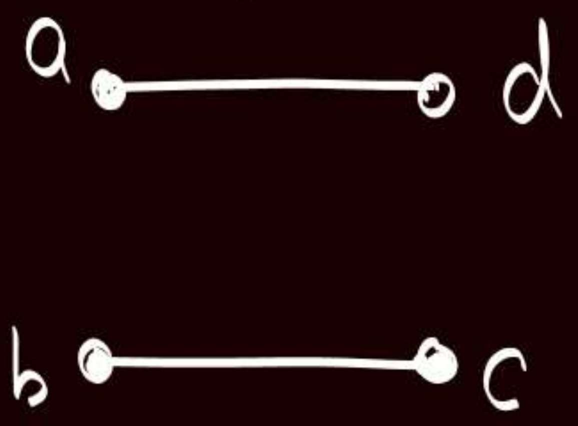
A subset C of set of edges E is called line covering of graph G if every vertex of the graph is incident with at least one edge of subset C.

i.e, in a line covering of graph G, $deg(v) \geq 1 \ \forall v \in G$.

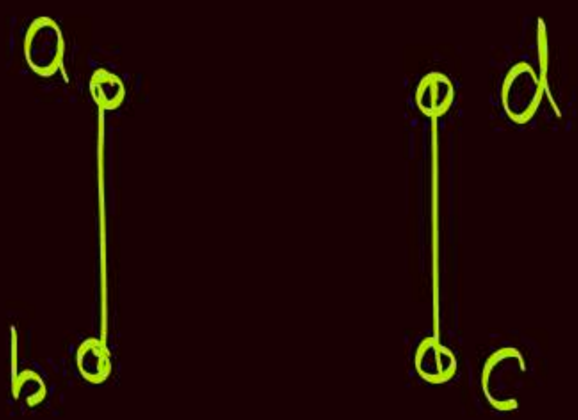
eg:



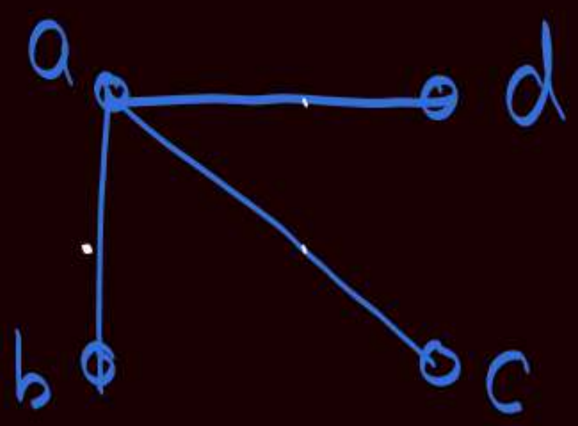
G



C_1



C_2



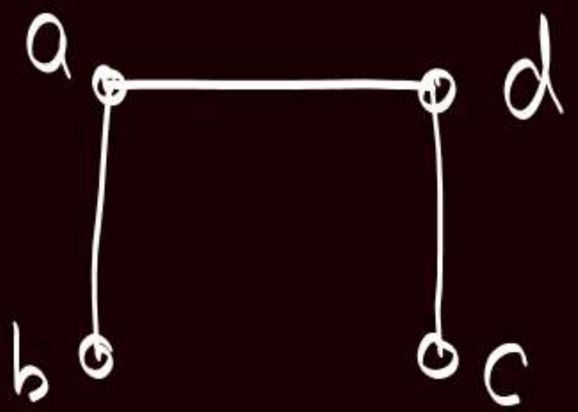
C_3

$C_1 = \{\{a, d\}, \{b, c\}\}$

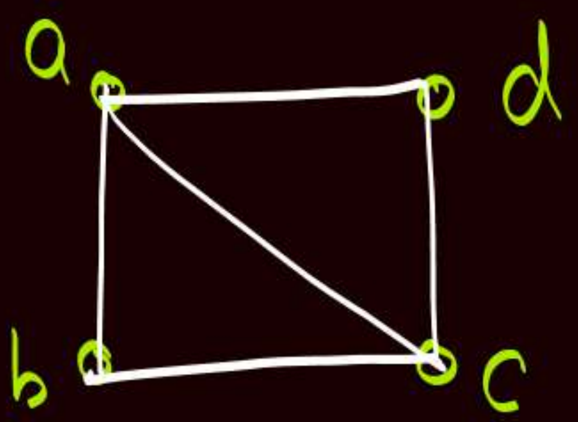
$C_2 = \{\{a, b\}, \{c, d\}\}$

$C_3 = \{\{a, b\}, \{a, c\}, \{a, d\}\}$

All C_1, C_2, C_3, C_4, C_5 etc. are line covering of graph G

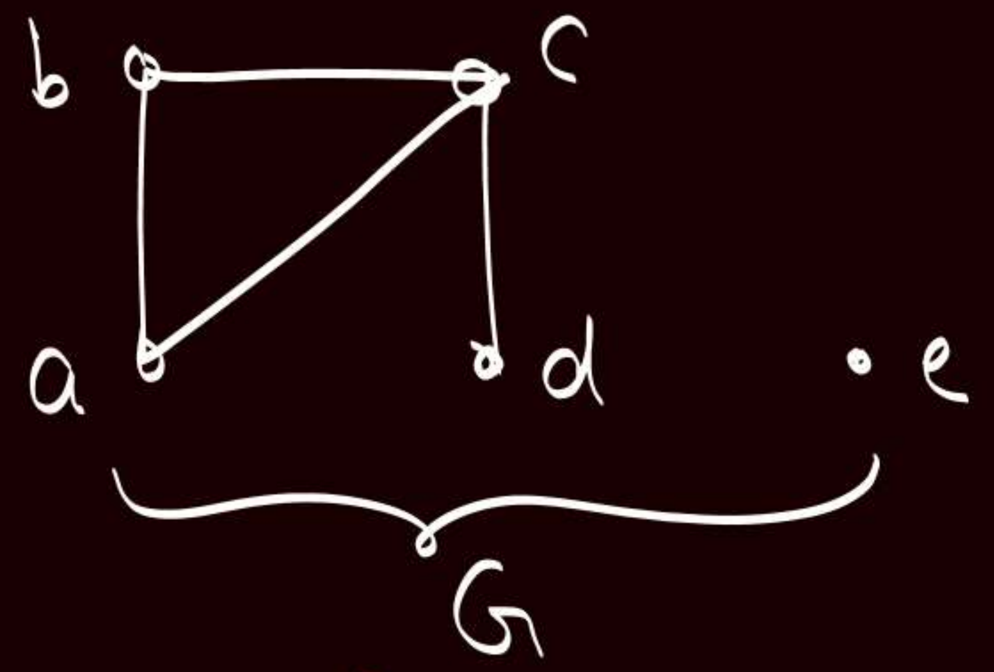


C_4



C_5

Note: Consider the following graph G,



{Do we have any line Covering of graph G}

No line Covering exists for above graph

Note: Line Covering does not exist for graph G if and only if isolated vertex exist in graph G.

Topic : Minimal line covering

A line covering of graph G is said to be minimal if no edge can be removed from the subset of edges without destroying its ability to cover all vertices.

In the above example C_1, C_2 & C_3 are minimal line covering of graph G_2

Topic : Minimum line covering

Smallest Minimal line Covering

Line coverings of graph G with minimum number of edges are called minimum line coverings.

* In the above example C_1 & C_2 are minimum line coverings

Topic : Line covering number (α_1)

Line covering number of graph G = No. of edges in any one of the minimum line covering

α_1 = No. of edges in minimum line covering

In the above eg.,
line covering number of graph $G = 2$

For any graph G with n vertices,
Line covering no. of $G \geq \left\lceil \frac{n}{2} \right\rceil$

Note:-

1. Minimal line covering can never contain cycles.

2. In the minimal line covering of graph G, there exists no path of length 3 or more.

3. Every minimum line covering is a minimal line covering, but minimal line covering need not be minimum line covering.

Topic : Line independent set

Let $G = (V, E)$ be a graph.

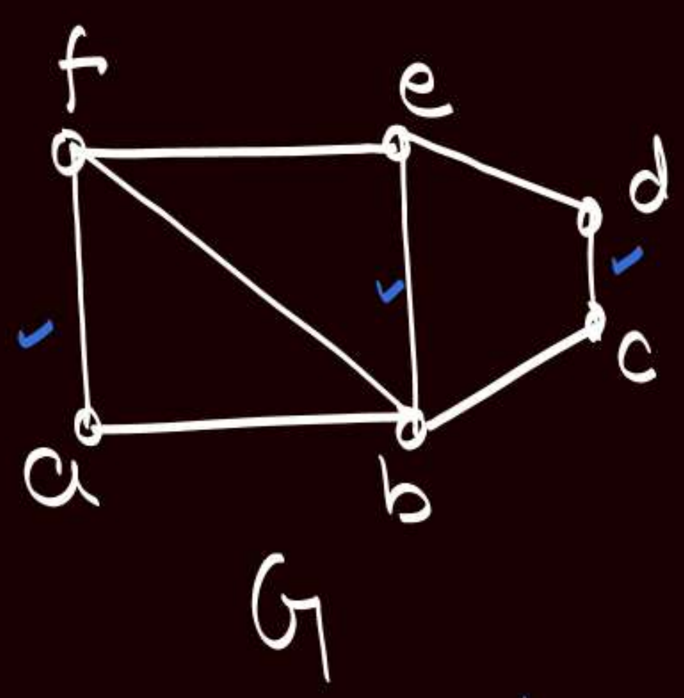
A subset L of E is called a line independent set of graph G, if no two edges of set L are adjacent to each other.

If no two edges are adjacent to each other then degree of every vertex will be ≤ 1

$\deg(v) \leq 1, \forall v \in G$ is also the condition for matching

$\therefore$ Every line independent set of graph G is also a matching of graph G. and vice-versa.

eg:



$L_1 = \{\{a,b\}\}$

A set of edges with only one edge will always be a line independent set

(Maximal) $L_2 = \{\{b,f\}, \{c,d\}\}$

(Maximal) $L_3 = \{\{a,b\}, \{e,f\}, \{c,d\}\}$

(Maximal) $L_4 = \{\{a,f\}, \{b,e\}, \{c,d\}\}$

(Maximal) $L_5 = \{\{a,b\}, \{d,e\}\}$

Maximal line independent set of G

L_3 & L_4 are Maximum line independent set.

$L_1, L_2, L_3, L_4, L_5, L_6$, etc. are line independent set of graph G.

$L_6 = \{\{a,b\}, \{f,e\}\}$

$\{c,d\}$ Can be added

$L_7 = \{\{b,f\}, \{e,f\}\}$ is not a line independent set

Topic : Maximal Line independent set

A line independent set of graph G is said to be maximal if no other edge can be added to the set without destroying its property of being an independent set.

In the above eg. L_2, L_3, L_4 & L_5 are maximal line independent set.

Topic : Maximum Line independent set

Largest maximal line independent set.

A line independent set with maximum number of edges is called a maximum line independent set

In the above example, L_3 & L_4 are maximum line independent set

* Every maximum line independent set is a maximal line independent set, but not vice-versa

Topic : Line independence number (β_1)

Line independence number of graph G is defined as number of edges in any one of the maximum line independent set

Line independence No. (β_1) of graph G = Matching No. of graph G

* There is no difference b/w Line independent set of graph G & Matching of graph G

$\therefore$ Line independence No. of graph G with n-vertices $\leq \left\lfloor \frac{n}{2} \right\rfloor$

In the above example,

Line independence No. = 3

Note: For any graph $G = (V, E)$

if, α_1 = Line covering No. of graph G

& β_1 = Line independence No. of graph G.

then,

$$\boxed{\alpha_1 + \beta_1 = |V(G)|}$$

H.W.

* Find α_1 & β_1 for the following graphs.

		α_1	β_1 {it will be same as Matching No.}
1	K_n	$\lceil \frac{n}{2} \rceil$	$\lfloor \frac{n}{2} \rfloor$
2	C_n	$\lceil \frac{n}{2} \rceil$	$\lfloor \frac{n}{2} \rfloor$
3	W_n	$\lceil \frac{n}{2} \rceil$	$\lfloor \frac{n}{2} \rfloor$
4	$K_{m,n}$	$Max(m,n)$	$Min(m,n)$
5	Star graph with n-vertices	$Max(1,n-1) = (n-1)$	$Min(1,n-1) = 1$

* We already know Matching No.,
  $\therefore$ We know β_1,
* And $\alpha_1 + \beta_1 = |V(G)| = n$
  $\therefore$ We can Compute α_1 directly.

2 mins Summary

- **Topic** Line / Edge Covering
- **Topic** Minimal & Minimum line covering
- **Topic** Line independent set
- **Topic** Maximal & Maximum line independent set

THANK - YOU

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