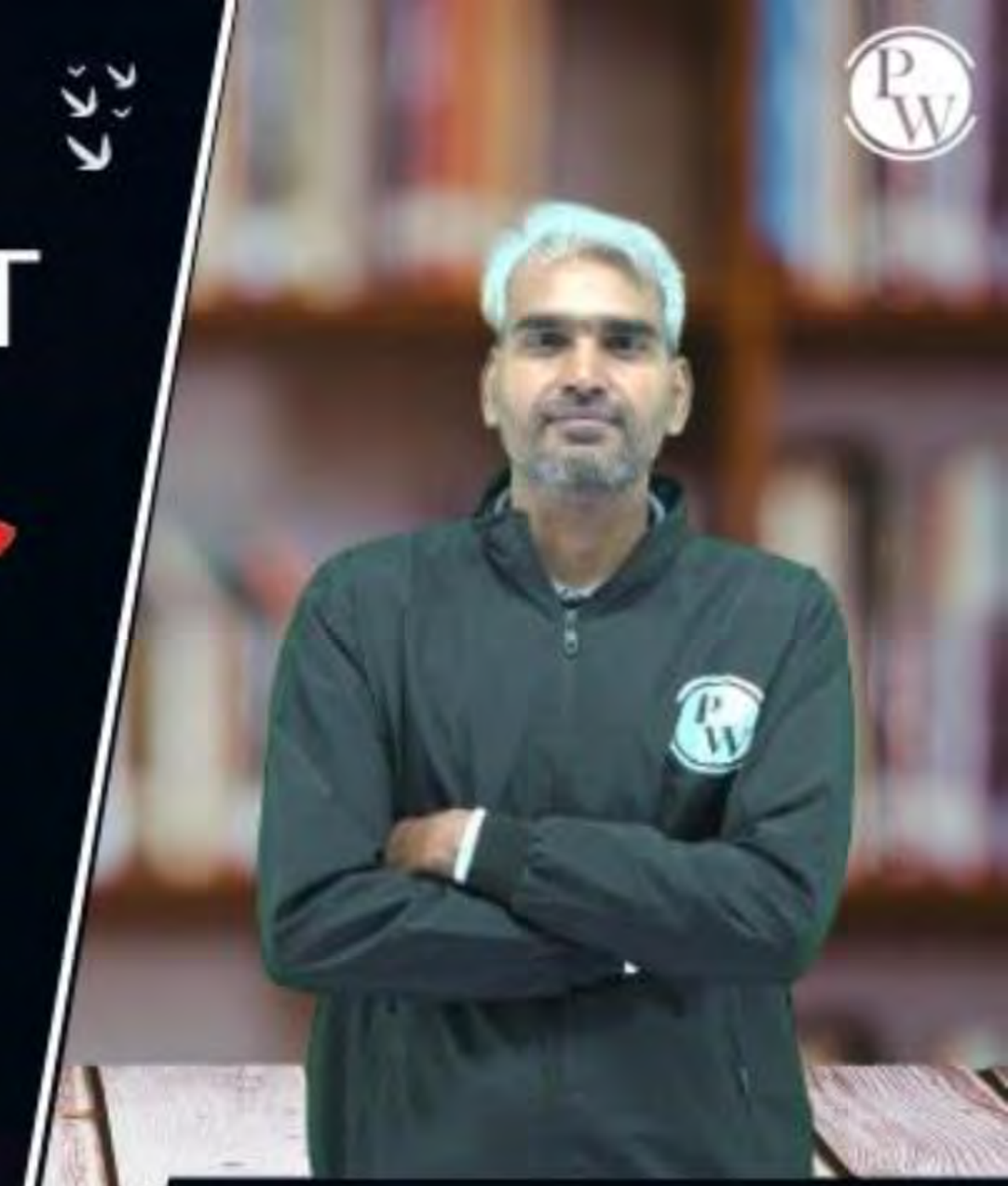
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 11



By- Vishal Sir

Recap of Previous Lecture

- **Topic** Minimal / Maximal elements in a POSET
- **Topic** Minimum(Least) element of POSET
- **Topic** Maximum(Greatest) element of POSET
- **Topic** Lattice

Slide

Topics to be Covered

Topic ~~Minimal / Maximal elements in a POSET~~ Lattice

Topic ~~Minimum(Least) element of POSET~~ Hasse diagram

Topic ~~Maximum(Greatest) element of POSET~~

Topic ~~Lattice~~

Topic ~~Types of lattice~~

Slide

Topic : Lattice

Note:- In a lattice lub as well as glb exists for every pair of element and it is unique.

{ i.e In a lattice for a pair of elements we can not have two or more lub (or) two or more glb }

Slide

Note: If n is any non-zero positive integer, then D_n denotes the set of all divisors of 'n'

i.e. $D_2 = \{1, 2\}$

$D_3 = \{1, 3\}$

$D_4 = \{1, 2, 4\}$

$D_5 = \{1, 5\}$

$D_6 = \{1, 2, 3, 6\}$

$D_{12} = \{1, 2, 3, 4, 6, 12\}$

etc.

Note: If D_n is a set of all divisors of 'n', then $(D_n, \div)$ is a lattice

Note: If A is any finite set, and P(A) is Power set of set A, then $(P(A), \subseteq)$ is a lattice

Topic : Lattice

A lattice is denoted by

$[L, \vee, \wedge]$

Where L is the underlying set.

'$\vee$' denote the least upper bound

'$\wedge$' denote the greatest lower bound

Slide

Topic : Lattice

Following Properties always hold true in any lattice.

① $a \vee a = a, \forall a \in L$
$a \wedge a = a, \forall a \in L$
} Idempotent law

② $a \vee b = b \vee a, \forall a,b \in L$
$a \wedge b = b \wedge a, \forall a,b \in L$
} Commutative Property

③ $(a \vee b) \vee c = a \vee (b \vee c), \forall a,b,c \in L$
$(a \wedge b) \wedge c = a \wedge (b \wedge c), \forall a,b,c \in L$
} Associativity Property

Slide

Topic : Lattice

Following Properties always hold true in any lattice.

④ $a \vee (a \wedge b) = a, \quad \forall a, b \in L$

$a \wedge (a \vee b) = a, \quad \forall a, b \in L$

} Absorption Law

Above four properties will always hold true in any lattice

Slide

Note: Distributive property need not hold true in all the lattices.

If distributive property holds true in a lattice, then that lattice is called a distributive lattice

otherwise not a distributive lattice

Topic : Hasse Diagram / POSET Diagram

In a Hasse diagram of a POSET, → let (A, R)

1. There is a vertex corresponding to every element of set.

2. There is an edge from vertex a to vertex b only if a is related to b and there is no element x in the set such that a relates to x, and x relates to b (Transitivity is implied in the Hasse diagram not represented explicitly)

   i.e. $(a,b) \in R$

   relates to

   $(a,x) \in R$ and $(x,b) \in R$

3. No self-loop on the vertices (i.e. reflexivity is implied in the Hasse diagram not represented explicitly).

4. It is not directed but it uses implied upward orientation.

   if $(a,b) \in R \Rightarrow$ then $\Rightarrow$

   b ← upper side

   a ← lower side

   it is not same as

   it will be $(b,a) \in R$

   a

   b

   (some author may define directed Hasse diagram)

Slide

Topic : Hasse Diagram / POSET Diagram

H.W.

Draw the hasse diagram for the following POSETs

1) $(\{-1,0,2.5,4,6\}, \leq)$
2) $(D_6, \div)$
3) $(D_{12}, \div)$
4) $(\{2,3,4,6\}, \div)$
5) $(\{2,3,6,12\}, \div)$
6) $(\{1,2,3,4,6,9\}, \div)$

Slide

Topic : Hasse Diagram / POSET Diagram

Draw the hasse diagram for the following POSET

$(\{-1,0,2.5,4,6\}, \leq)$

Topic : Hasse Diagram / POSET Diagram

Draw the hasse diagram for the following POSET

$(D6, \div)$

Topic : Hasse Diagram / POSET Diagram

Draw the hasse diagram for the following POSET

$(D12, \div)$

Topic : Hasse Diagram / POSET Diagram

Draw the hasse diagram for the following POSET

$(\{2,3,4,6\},\div)$

Topic : Hasse Diagram / POSET Diagram

Draw the hasse diagram for the following POSET

$(\{2,3,6,12\},\div)$

Topic : Hasse Diagram / POSET Diagram

Draw the hasse diagram for the following POSET

$(\{1,2,3,4,6,9\},\div)$

2 mins Summary

Topic — Lattic Revision

Topic — Properties w.r.t. lattice

Topic — Hasse diagram

Topic

Topic

Slide

THANK - YOU

Slide