

CS & IT ENGINEERING



Theory of Computation

DFA

Lecture No.- 06



By- Venkat sir

Recap of Previous Lecture



Topic

Complement

Topic

minimization of DFA



Topics to be Covered



Topic

Finite Automaton & Regular Languages.

Topic

Pushdown Automata & Context free Languages.

Topic

Turing Machine & Recursive Enumerable Languages.

Topic

Undecidability.



Topic : Deterministic Finite Automata

FORMAL DFA :

DFA is defined as

$$\text{DFA} = (Q, \Sigma, q_0, F, \delta)$$

Q : Finite set of states

Σ : Input alphabet

q_0 : Initial state

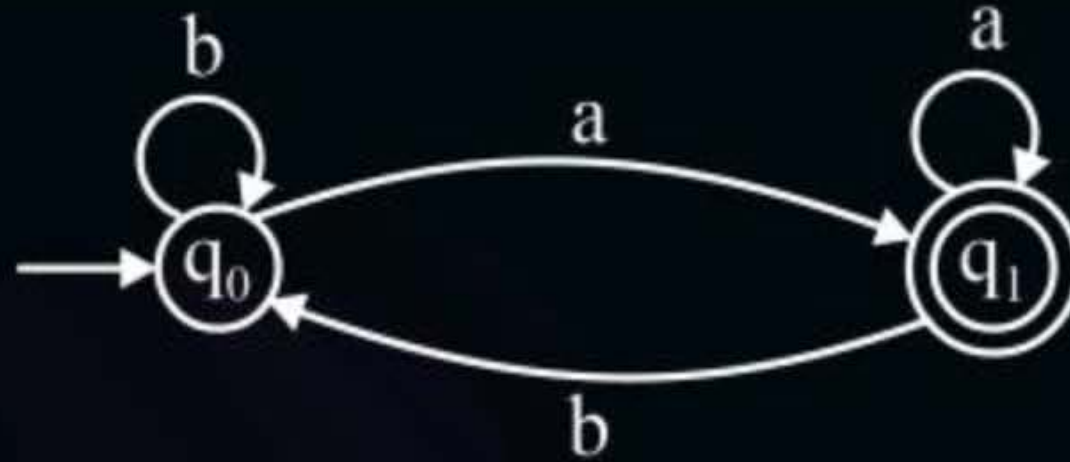
F : Set of final states

δ : Transition function $Q^* \Sigma \rightarrow Q$

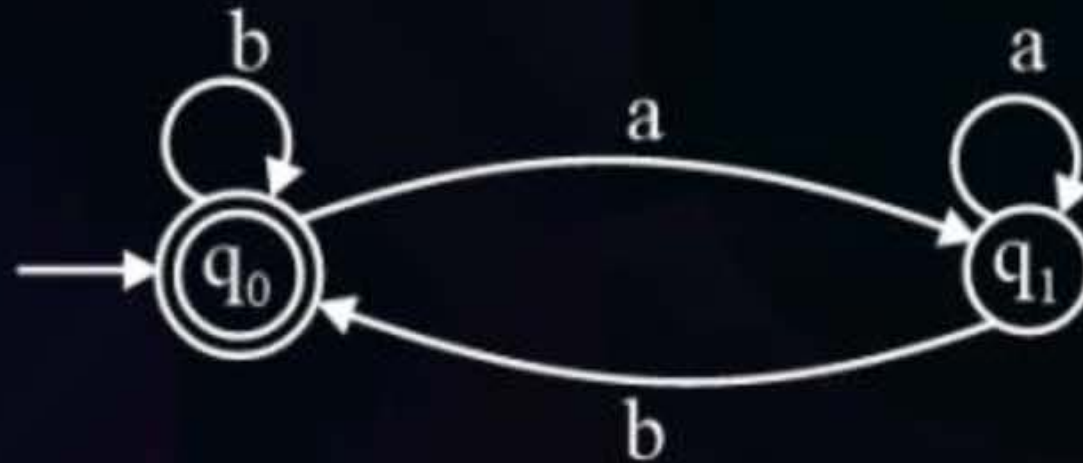


Topic : Complement of DFA

By interchanging final and non final states we can convert into complement DFA.



Set of all strings ending with a after complement



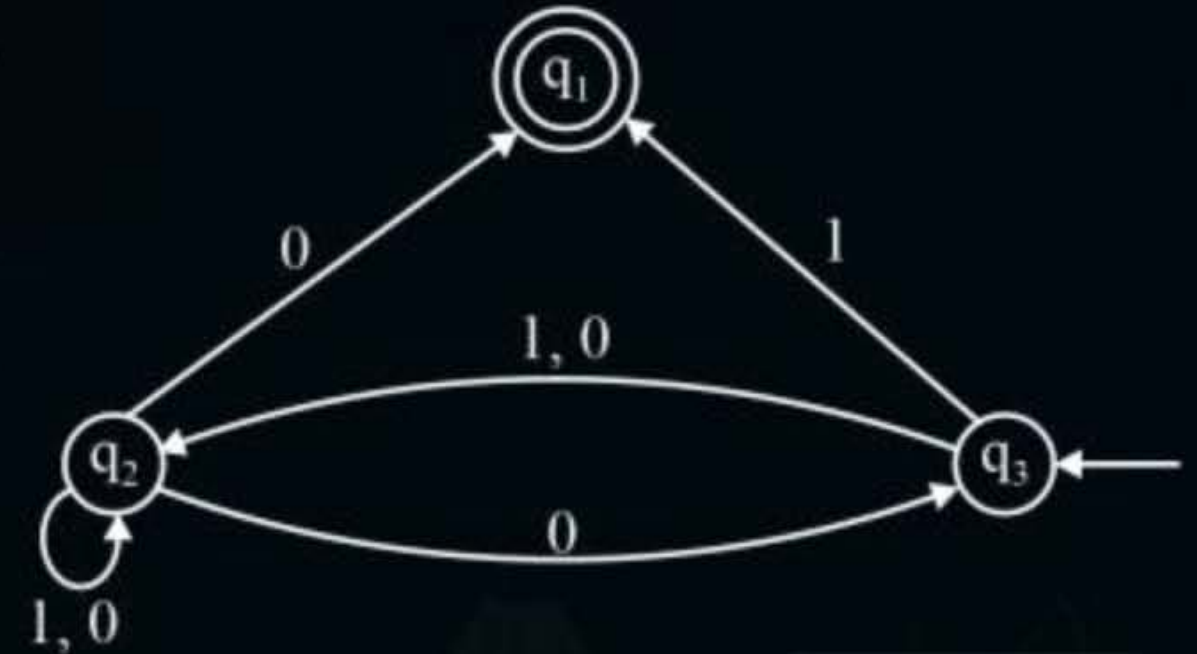
Set of all string Ending with

MCQ



#Q. Consider the NFA M shown below.

Let the language accepted by M be L . Let L_1 be the language accepted by the NFA M_1 , obtained by changing the accepting state of M to a non-accepting state and by changing the non-accepting state of M to accepting states. Which of the following statements is true?



A

$$L_1 = \{0, 1\}^* - L$$

B

$$L_1 = \{0, 1\}^*$$

C

$$L_1 \subseteq L$$

D

$$L_1 = L$$



Topic : DFA Construction



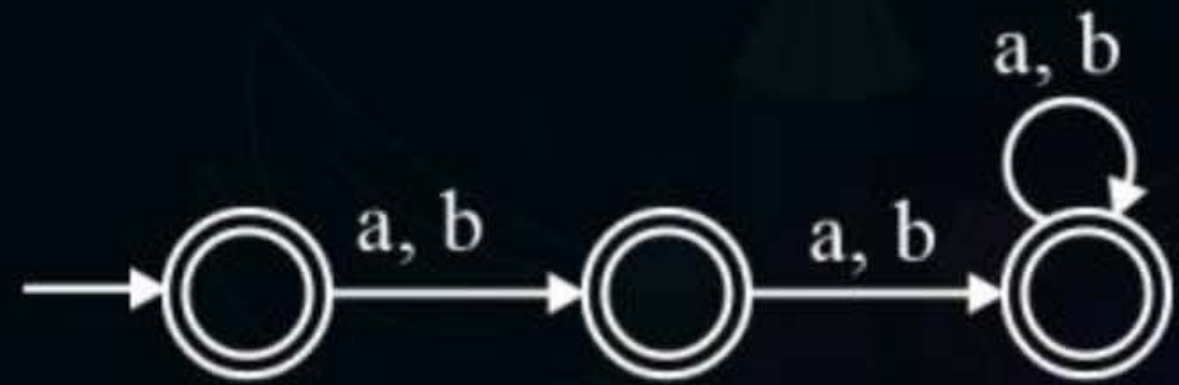
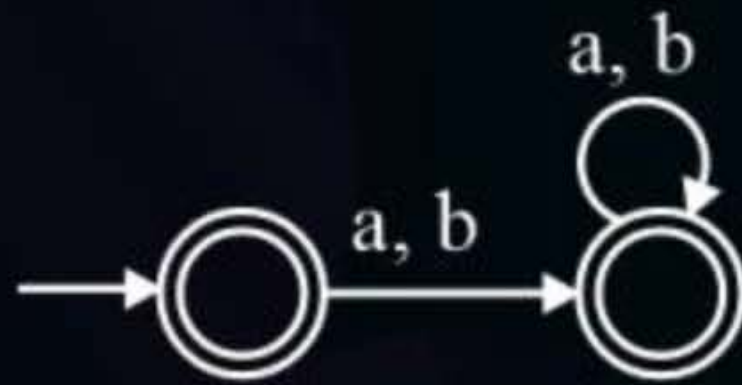
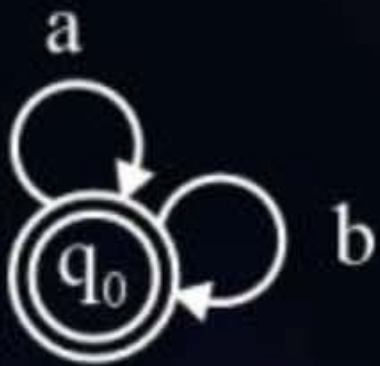
Construct minimal state DFA that accerpts all strings os 0's and 1's where each string ending with 00.



Topic : Minimization of DFA

→ For a given regular language even though many DFA exist but minimal state DFA is unique.

Ex: Complete Language: Σ^*





Topic : Minimization Algorithm

1. State equivalence algorithm
2. Table filling algorithm

Equivalent States:

Two states q_0, q_1 are said to be equivalent both $\delta(q_0, x)$ and $\delta(q_2, x), \forall x \in \Sigma^*$ should result either final state or non final state.





Topic : Procedure of minimization

1. Elimination inaccessible states.

inaccessible state:

Any State which is not reachable from dead state is inaccessible state.

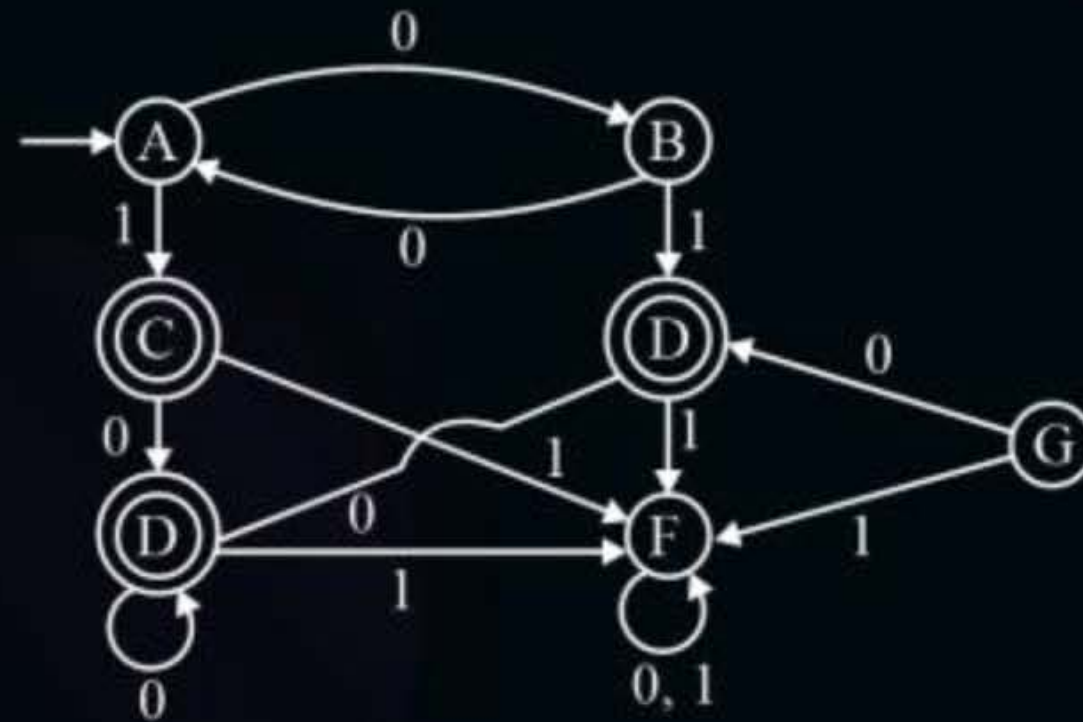
2. Apply algorithm steps
3. Merge single group into one state
4. Construct new minimized DFA



Topic : Procedure of minimization



1. Reduce states of following DFA



Step-1: Elimination inaccessible state.

Note: Dead state is different from inaccessible state.



Topic : Procedure of minimization



Step:2

State	0	1
A	B	C
B	A	D
F	F	F
Ⓒ	E	F
Ⓓ	E	F
Ⓔ	E	F

Algorithm:

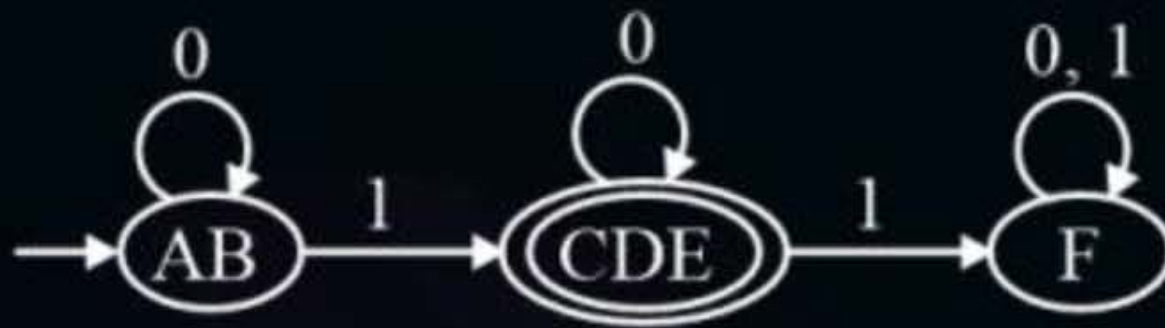
1. {A, B, F} {C, D, E}
- 2.
- 3.



Topic : Procedure of minimization



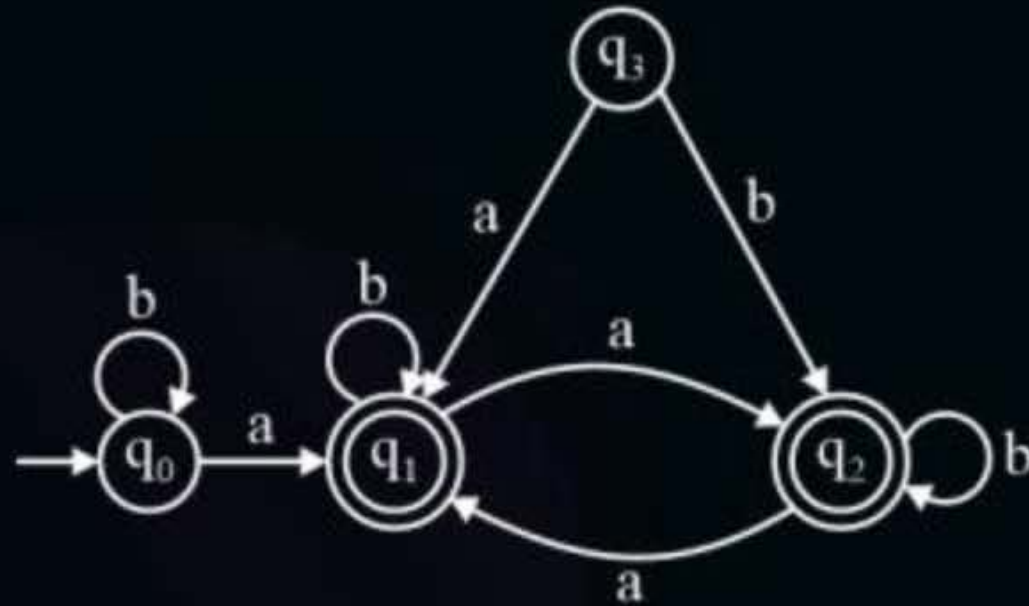
Minimized DFA





Topic : Procedure of minimization

Consider the following Finite State Automation





Topic : Procedure of minimization



Step 1: Eliminate q_3

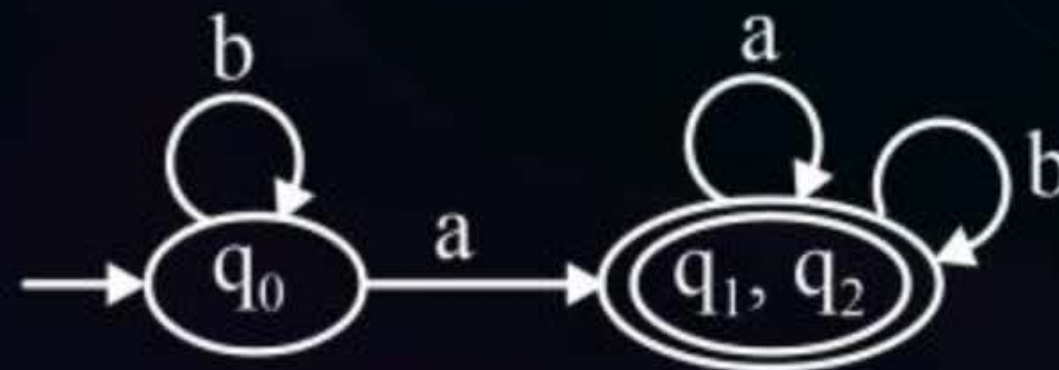
Step 2:

	a	b
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_1	q_2

Algorithm step

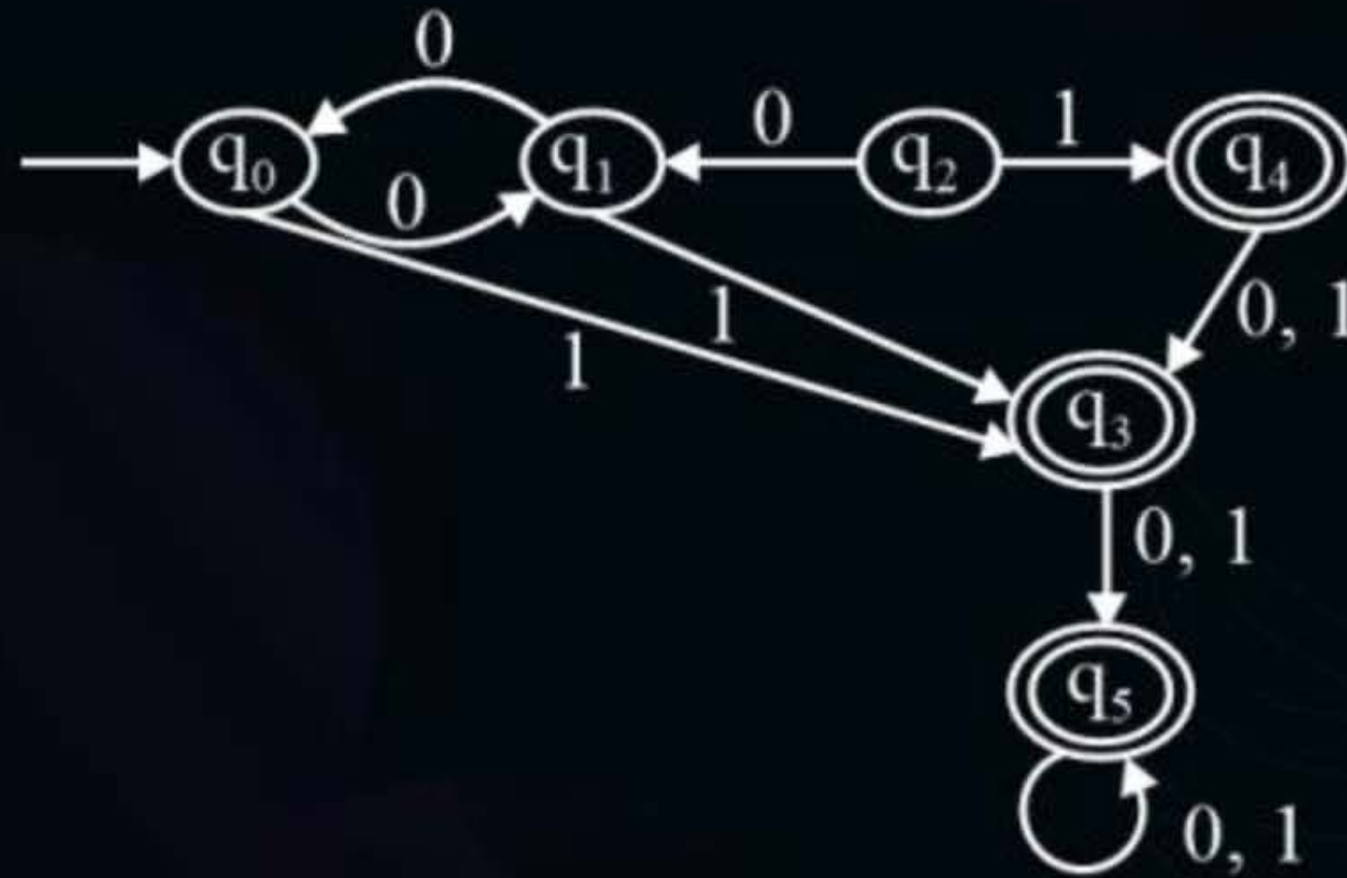
1. $\{q_0\} \{q_1, q_2\}$
2. $\{q_0\} \{q_1, q_2\}$

Minimum DFA





Topic : Procedure of minimization



Minimize given DFA



Topic : Procedure

Step 1: Eliminate

Step 2:

	a	b
q_1	q_1	q_3
q_2	q_0	q_3
q_3	q_5	q_5
q_5	q_5	q_5

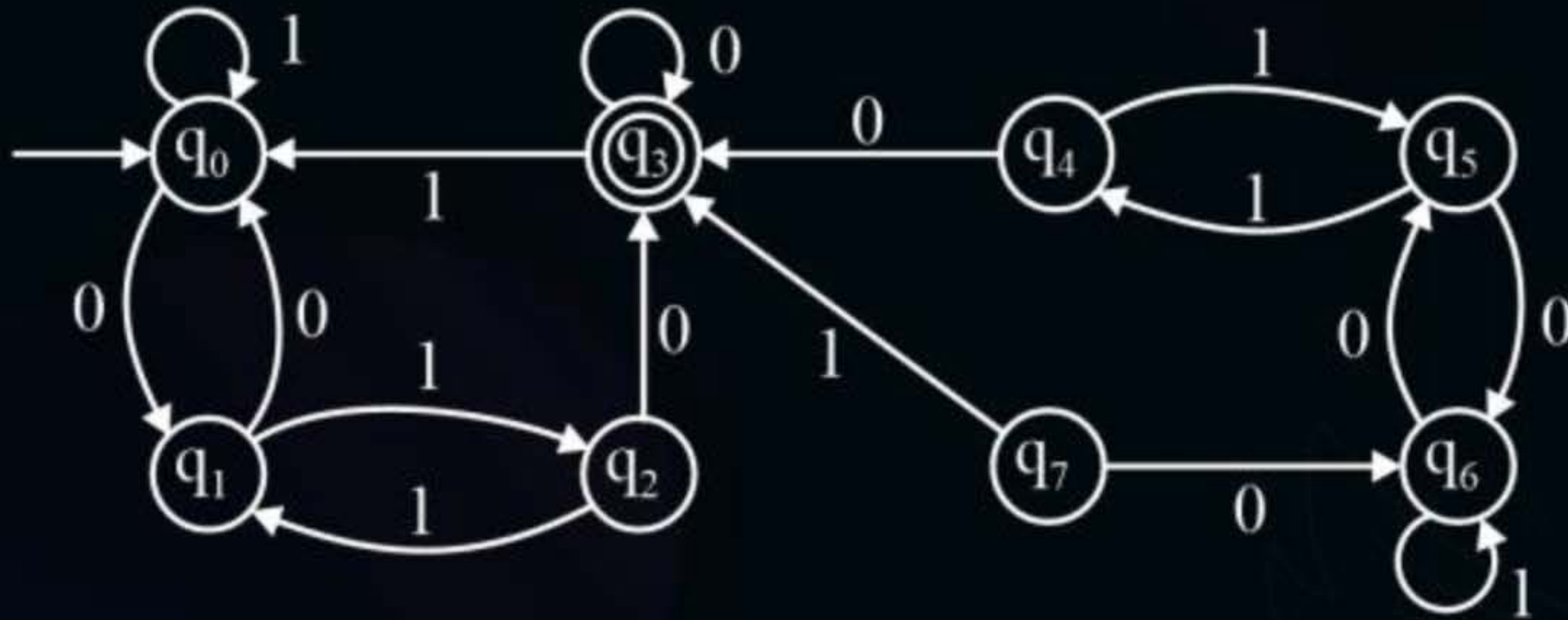
Algorithm

1. $\{q_0, q_1\} \{q_3, q_5\}$
- 2.

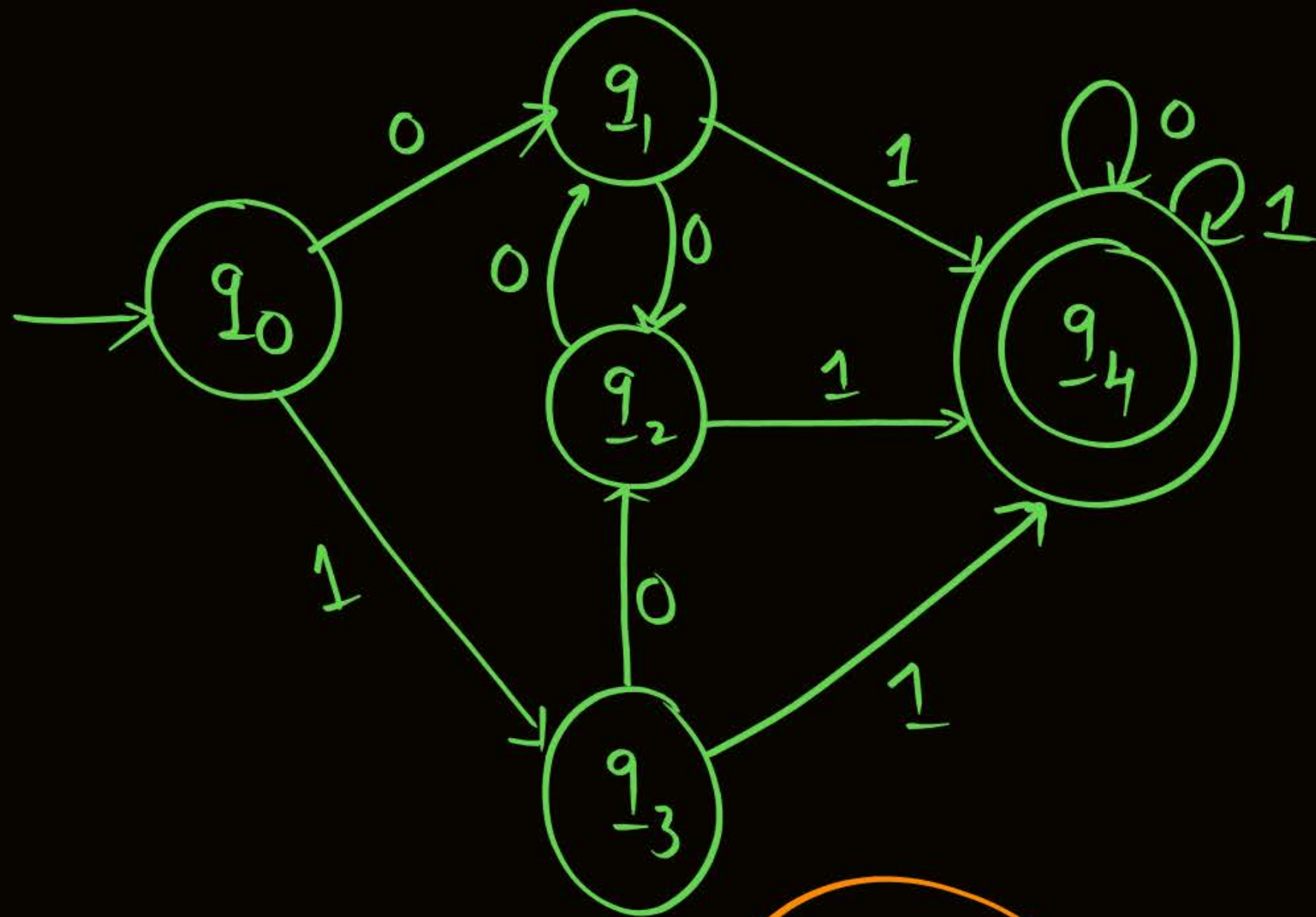
Minimum DFA



Topic : Procedure of minimization



How many inaccessible states present in given DFA



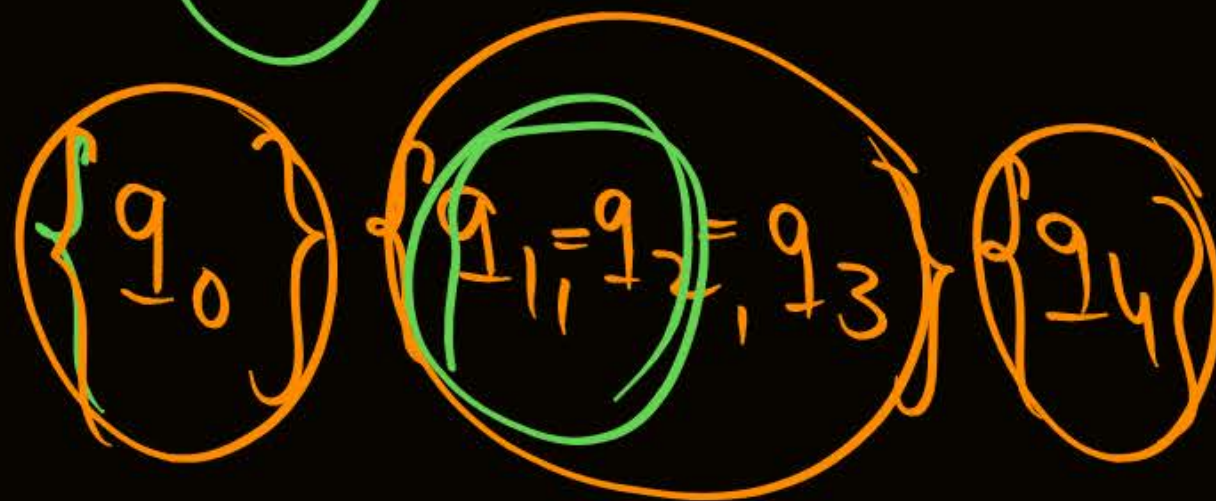
{How many no. of states
in min DFA for this DFA?}

(a) 2

☒ (b) 3

(c) 4

(d) 1



Q which of the following statement is false?

(a) 2 & 3 are equivalent states

(b) 1 & 2 are distinguishable ✓

(c) 3 & 4 are distinguishable "

(d) none.

[MSQ]

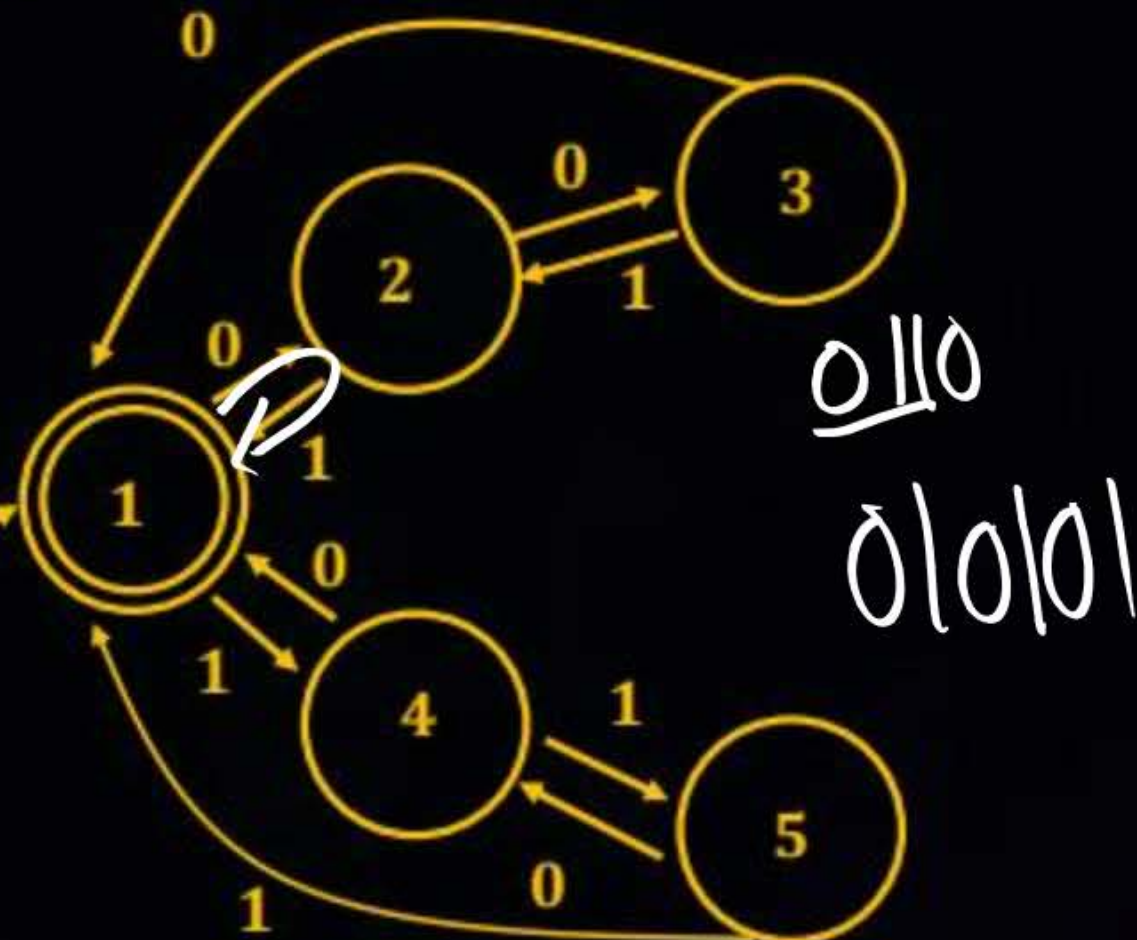
$\{1\}$ $\{2, 5\}$ $\{3, 4\}$



#Q. Consider the 5-state DFA M accepting the language $L(M) \subset \text{subset } (0 + 1)^*$ shown below. For any string $w \in (0 + 1)^*$ let $n_0(w)$ be the number of 0's in w and $n_1(w)$ be the number of 1's in w . Which of the following statements is/are FALSE?

[GATE-CS-shift-I-24: 2M]

- A** States 2 and 4 are distinguishable in M
- B** States 2 and 5 are distinguishable in $M \rightarrow \text{false}$
- C** Any string w with $n_0(w) = n_1(w)$ is in $L(M) \rightarrow \text{true}$
- D** States 3 and 4 are distinguishable in $M \rightarrow \text{false}$





Topic : DFA

$\{0, 4, 8, 12, 16, \dots\}$

numbers $4 \mid \begin{array}{r} 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \begin{array}{r} 2 \\ 2 \end{array} \begin{array}{r} 3 \\ 3 \end{array}$

#Q. Construct the minimal DFA that accept all binary ~~no~~ divisible by 4.



$4 \mid \begin{array}{r} 2 \\ 2 \end{array}$
 $4 \mid \begin{array}{r} 3 \\ 3 \end{array}$
 $4 \mid \begin{array}{r} 4 \\ 0 \end{array}$

Table

Reminders →

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3



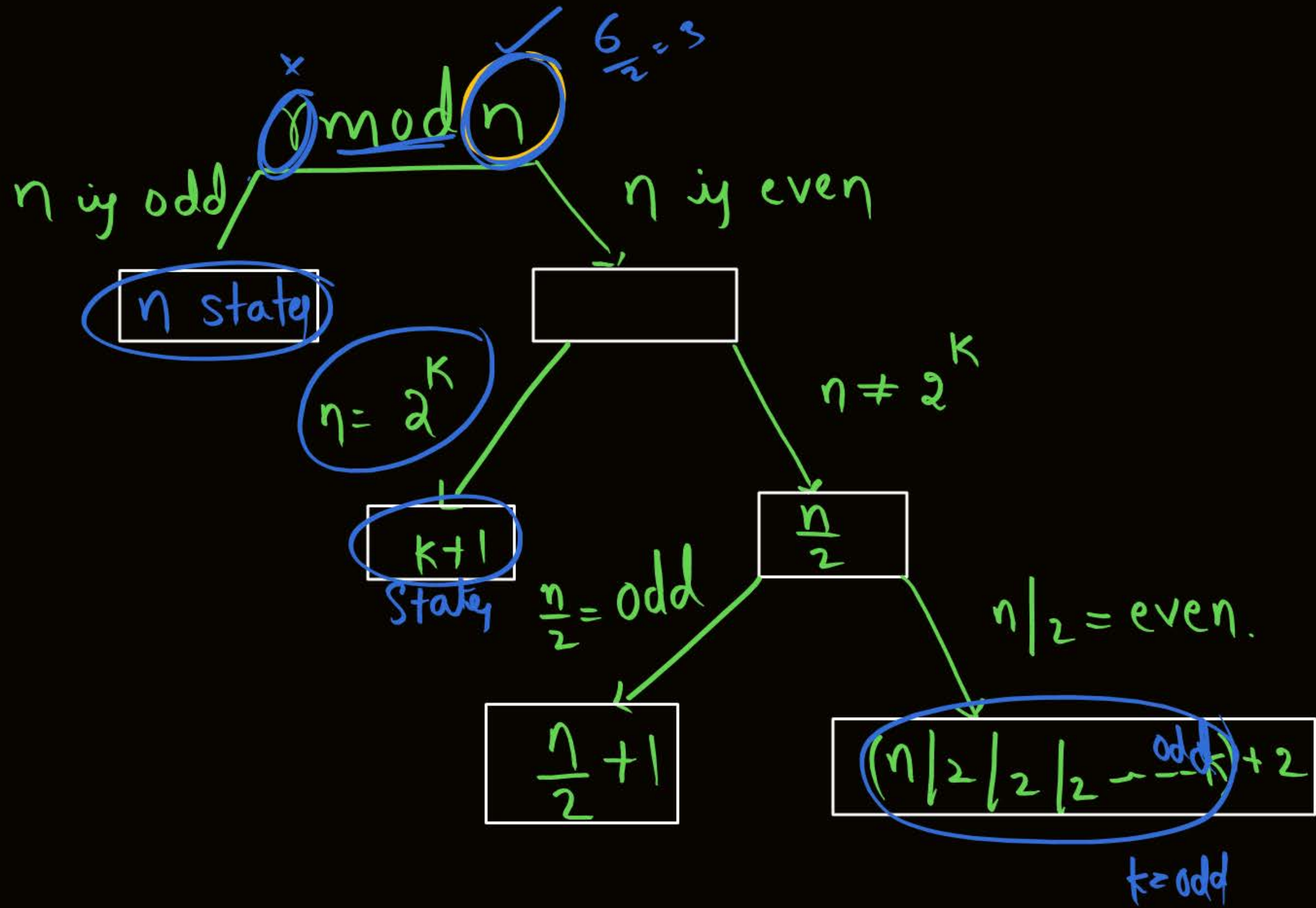
Topic : DFA

#Q. Construct the minimal DFA that accept all binary no divisible by 5.

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

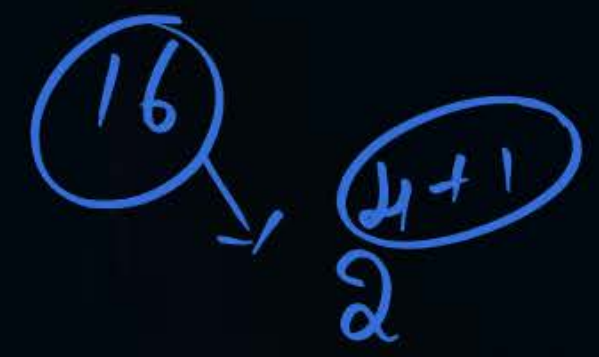
minimization
① DFA

5 states



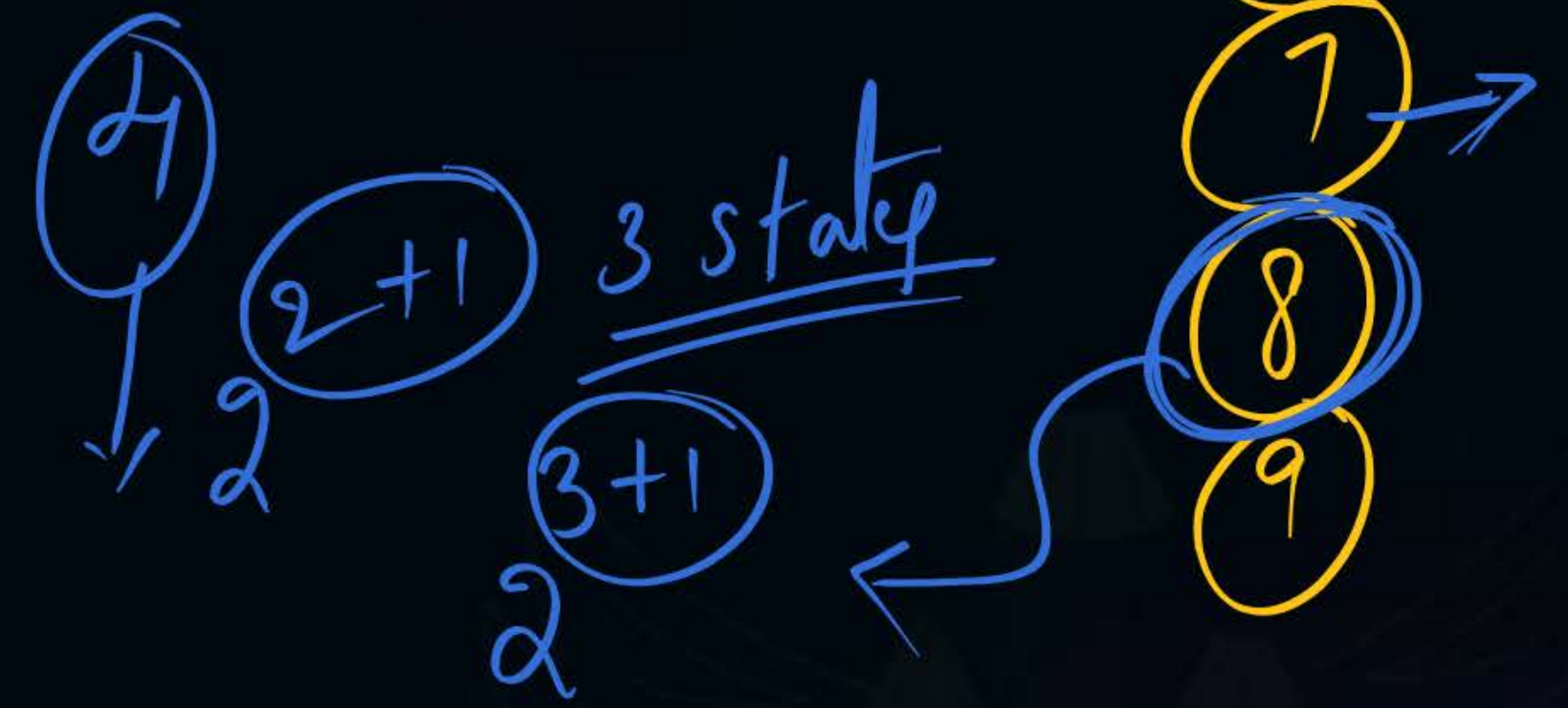


Topic : DFA



5 → 5
3 → 5

#Q. Construct the minimal DFA that accept all binary no divisible by 6.





Topic : DFA



#Q. Construct the minimal DFA that accept all binary no divisible by

$$3 \rightarrow 3$$

$$4 \rightarrow 3$$

$$5 \rightarrow 5$$

$$6 \rightarrow 4$$

$$8 \rightarrow 2^3 \rightarrow (3+1) \rightarrow 4$$

$$10 \rightarrow \frac{10}{2} = (5+1) = 6$$



Topic : DFA



#Q. Construct the minimal DFA that accept all binary no divisible by

$$12 \rightarrow \frac{12}{2} = \frac{6}{2} = \textcircled{3+2} = 5$$

$$\textcircled{24}$$

$$\frac{24}{2} \rightarrow \frac{12}{2} \rightarrow \frac{6}{2} = \textcircled{3+2} = 5$$



Topic : DFA



#Q. Construct the minimal DFA that accept all binary no divisible by

$$\frac{14}{2} = (7+1) \rightarrow 8 \quad \left. \vphantom{\frac{14}{2}} \right\} 7 \rightarrow 7$$



Topic : DFA



Construct the minimal DFA that accept all strings of a's and b's where

1. Each string ending with b.
2. Each string start with a and end with b.
3. Each string starting and ending with different symbol.
4. Each string starting and ending with same symbol.



Topic : DFA

$b^* a b^* a b^*$



Construct the minimal DFA that accept all string a's and b's where

1. Length of string exactly 4.
2. Number of a's length of string atleast 4.
3. Length of string atmost 4.
4. Length of string divisible by 4.
5. Number of a's exactly 5.
6. Number of b's exactly 2.
7. Number of a's divisible by 3.
8. Number of b's not divisible by 4.
9. Length of the string even.



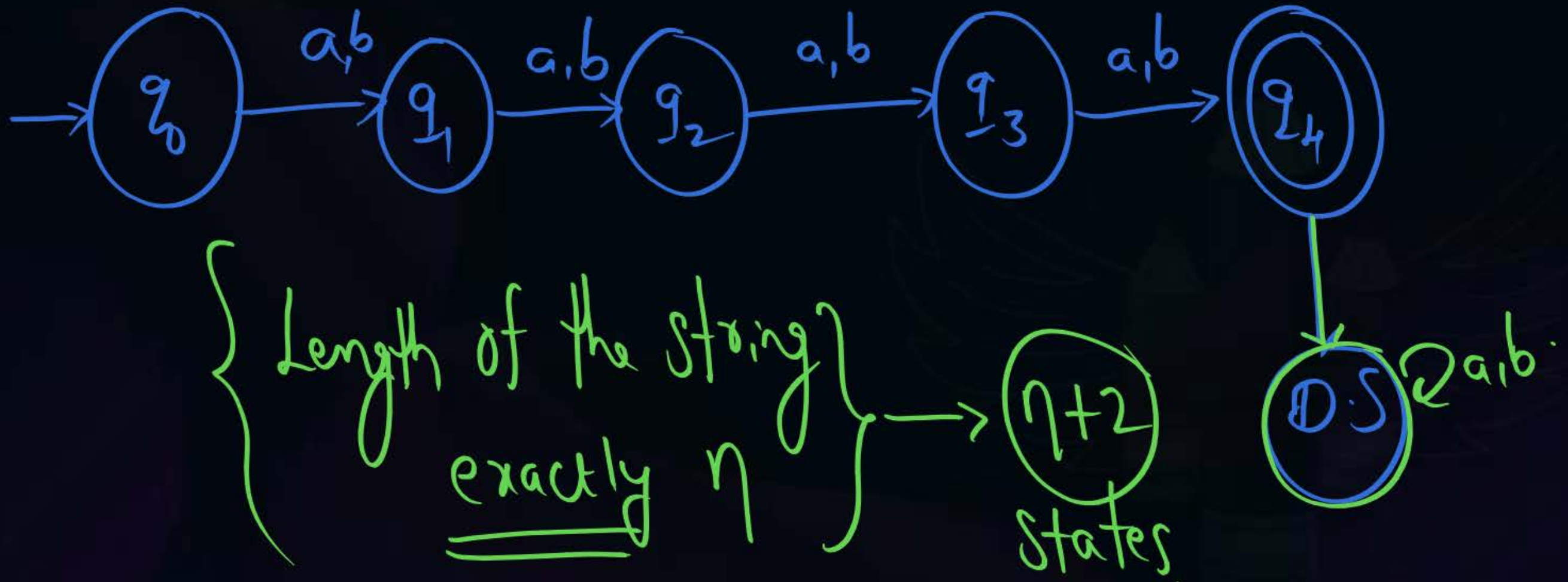


Topic : DFA

$$\Sigma = \{a, b\} \quad \underline{a/b} \quad \underline{a/b} \quad \underline{a/b} \quad \underline{a/b}.$$

#Q. Length of string exactly 4.

6 states





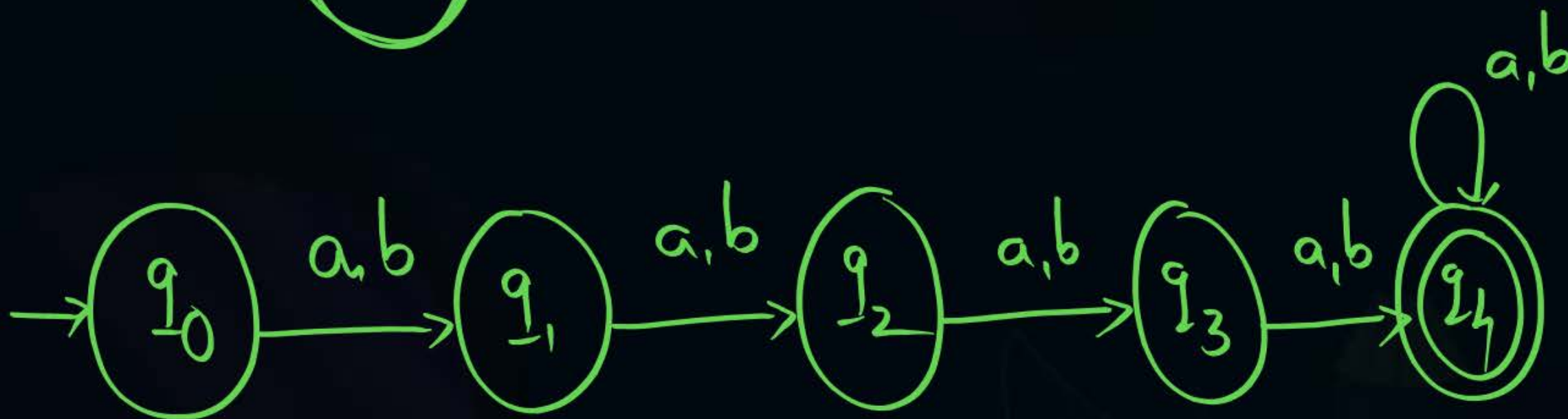
Topic : DFA



$4 \rightarrow 5$

$5 \rightarrow 6$

#Q. Length of string atleast 4. *at least $n \rightarrow (n+1)$ states*





Topic : DFA

#Q. Length of string atmost 4.

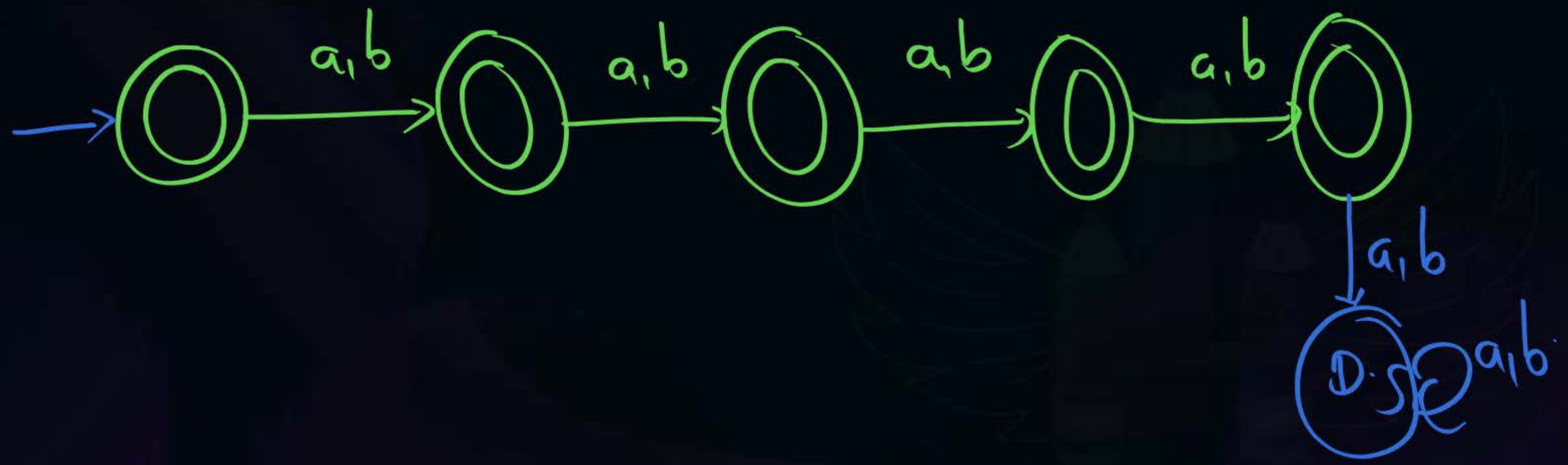
$\Sigma = \{a, b\}$

$4 \rightarrow 6$
 $5 \rightarrow 7$

$\{0, 1, 2, 3, 4\}$
 $1 + 2 + 4 + 8 + 16$

atmost $n \rightarrow (n+2)$ states

(31)





Sunday

@ 10 am

THANK - YOU