COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 05

BOOLEAN THEOREMS AND GATES

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Arithumatic gates

XOR & XNOR gates

- Properties of XNOR gate :

- $A \odot A = \bar{A} \cdot \bar{A} + A \cdot A = \bar{A} + A = 1$

- $A \odot \bar{A} = \bar{A} \cdot \bar{\bar{A}} + A \cdot \bar{A} = 0$

- $A \odot A = 1 \Rightarrow A \odot 1 = A = \bar{A} \cdot \bar{1} + A \cdot 1 = A \quad , \quad \bar{A} \odot 1 = \bar{A}$

- $A \odot \bar{A} = 0 \Rightarrow A \odot 0 = \bar{A} \Rightarrow \bar{A} \cdot \bar{0} + A \cdot 0 = \bar{A}$

  $\bar{A} \odot 0 = A$

  $AB \odot 0 = \overline{AB}$

- Exchange properties of XNOR gate :

- $A \odot B = C \longrightarrow$ Given $\Rightarrow B \odot A = C$

then $A \odot C = B \longrightarrow$ true

$$A \odot C = A \odot A \odot B = 1 \odot B = B$$

then also $B \odot C = A$

$$\Rightarrow B \odot C = B \odot A \odot B = B \odot B \odot A = 1 \odot A = A$$

- $A \odot A \odot A \odot A$ ....n times [n - represents no. of A]

  then o/p $= 1$ for n-even

  $= A$ for n-odd

$A \odot A = 1$

$A \odot A \odot A \odot A =$

$1 \odot 1 = 1$

$A \odot A \odot A = 1 \odot A$
$= A$

- Buffer and inverting buffer using XNOR :

$A \odot 0 = \bar{A}$

$A \odot 1 = A$

$$\left.\begin{array}{l}\text{Control i/p} = 0 \\ \qquad\qquad\quad = 1\end{array}\right\} \text{enable i/ps}$$

Combined IMP Properties of XOR and XNOR

- $\bar{A}\oplus B = \bar{\bar{A}}\cdot B + \bar{A}\cdot\bar{B} = A\odot B$

  $A\oplus\bar{B} = \bar{A}\cdot\bar{B} + A\cdot\bar{\bar{B}} = A\odot B$

  $\boxed{\overline{A\oplus B} = \bar{A}\oplus B = A\oplus\bar{B} = A\odot B}$

  $\bar{A}\oplus\bar{B} = \bar{\bar{A}}\cdot\bar{B} + \bar{A}\cdot\bar{\bar{B}} = A\cdot\bar{B}+\bar{A}\cdot B = A\oplus B$

- $\bar{A}\odot B = \bar{\bar{A}}\cdot\bar{B} + \bar{A}\cdot B = A\oplus B$
- $A\odot\bar{B} = \bar{A}\cdot\bar{\bar{B}} + A\cdot\bar{B} = A\oplus B$

  $\boxed{\overline{A\odot B} = \bar{A}\odot B = A\odot\bar{B} = A\oplus B}$

- $\bar{A}\odot\bar{B} = \bar{\bar{A}}\cdot\bar{\bar{B}} + \bar{A}\cdot\bar{B} = A\odot B$

$\overline{A\oplus B} = A\odot B$

$\bar{A}\oplus\bar{B} = A\oplus B$

- $A \oplus B \oplus C = \overline{A \odot B \odot C} \rightarrow$ true or false $\longrightarrow$ false

	$\overset{2^2}{A}$	$\overset{2^1}{B}$	$\overset{2^0}{C}$	$y_1 = A \oplus B \oplus C$	$y_2 = A \odot B \odot C$
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	0	0
4	1	0	0	1	1
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

$f(A,B,C)$
$= \sum(1,2,4,7)$
$= A \oplus B \oplus C$
$= A \odot B \odot C$

$\boxed{A \oplus B \oplus C = A \odot B \odot C}$

- $1 \oplus 0 \oplus 1 = 0$
- $1 \oplus 0 \oplus 1 \oplus 1 = 1$
- $1 \oplus 1 \oplus 0 \oplus 0 = 0$
- $1 \oplus 0 \oplus 0 \oplus 1 \oplus 1 = 1$
- $0 \oplus 0 \oplus 0 \oplus 0 = 0$
- $0 \oplus 1 = 1$
- $0 \oplus 0 \oplus 0 = 0$

$1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 0 = 0$

- $1 \odot 1 = 1$
- $1 \odot 1 \odot 1 = 1$, $1 \odot 1 \odot 0 = 0$
- $0 \odot 1 \odot 1 \odot 1 = 0$
- $1 \odot 0 \odot 1 \odot 0 = 1$
- $1 \odot 1 \odot 0 \odot 1 \odot 1 = 0$
- $1 \odot 1 \odot 0 \odot 1 \odot 0 = 1$
- $0 \odot 0 = 1$
- $0 \odot 0 \odot 0 = 0$
- $1 \odot 0 \odot 1 \odot 1 \odot 1 \odot 0 \odot 1 = 1$

$$\overline{A \oplus B} = A \odot B = \overline{A\bar{B} + \bar{A}B}$$
$$= \overline{A\bar{B}} \cdot \overline{\bar{A}B}$$
$$= (\bar{A} + B) \cdot (A + \bar{B})$$

From above we conclude that :

- XOR gate search for → odd no. of 1's for any no. of inputs.

  ↳ i.e. when total no. of 1's are odd then O/P will be '1' and if total no. of 1's are even or zero (not odd) then O/P will be '0'.

- XNOR gate search for → for even no. of 1's or zero no. of 1's for even no. of i/p lines

→ for odd no. of 1's for odd no. of input lines.

and thats why for odd no. of input XOR & XNOR are same and for even no. of i/p lines they are compliment of each other.

- $A \oplus B \oplus C = A \odot B \odot C$

$A \oplus B \oplus C \oplus D = \overline{A \odot B \odot C \odot D}$

- $A \oplus B \oplus C \oplus D \oplus E = A \odot B \odot C \odot D \odot E$

$1 \oplus 1 \oplus 0 \oplus 1 = 1$

$\overline{1 \odot 1 \odot 0 \odot 0 \odot 1} = \overline{0}$

$0 \odot 1 \odot 0 \odot 0 \odot 1 = 1$

$1 \odot 0 \odot 0 \odot 0 \odot 1 = 1$

$1 \odot 1 \odot 1 \odot 1 = 1$

$1 \odot 1 \odot 0 \odot 0 \odot 0 \odot 0 = 1$

$$1 \oplus 0 \odot 1 = 1$$

$$\overline{1 \oplus 0 \odot 1} = 0$$

$$0 \oplus 0 \odot 1 = 0$$

$$1 \oplus 1 \odot 1 = 0$$

$$1 \oplus 0 \odot 0 = 0$$

$$\bar{1} \oplus 0 \odot 1 = 1 \odot 0 \odot 1 = 1 \oplus 0 \oplus 1 = 0$$

$$1 \oplus \bar{0} \odot 1 = 1 \oplus 0 \oplus 1 = 1 \odot 0 \odot 1 = 0$$

$$1 \oplus 0 \odot \bar{1} = 1 \oplus 0 \oplus 1 = 1 \odot 0 \odot 1 = 0$$

- Compliment property of XOR and XNOR GATE :

- $$\overline{A \oplus B} = \bar{A} \oplus B = A \oplus \bar{B} = A \odot B$$

$$\overline{A \odot B} = \bar{A} \odot B = A \odot \bar{B} = A \oplus B$$

- $$\overline{A \oplus B \oplus C} = \bar{A} \oplus B \oplus C = A \oplus \bar{B} \oplus C = A \oplus B \oplus \bar{C}$$
$$= A \odot B \oplus C = A \oplus B \odot C$$

$$\overline{\overline{A \oplus B \oplus C}} = \overline{A \odot B \oplus C} = \overline{A \oplus B \odot C} = A \oplus B \oplus C = A \odot B \odot C$$

- $$\bar{A} \oplus \bar{B} = \bar{A} \odot B = A \oplus B$$

$$\boxed{\begin{aligned} & A \oplus B \odot C \oplus D \\ & = A \oplus B \oplus C \odot D \end{aligned}}$$

- $$\overline{\overline{A\oplus B}\odot\overline{C}\odot\overline{D}} = \overline{A\odot B\odot C\odot D} = A\oplus \underline{B\odot C\odot D} = A\odot B\oplus C\odot D$$
$$= A\odot B\odot C\oplus D$$
$$= A\oplus B\oplus C\oplus D$$

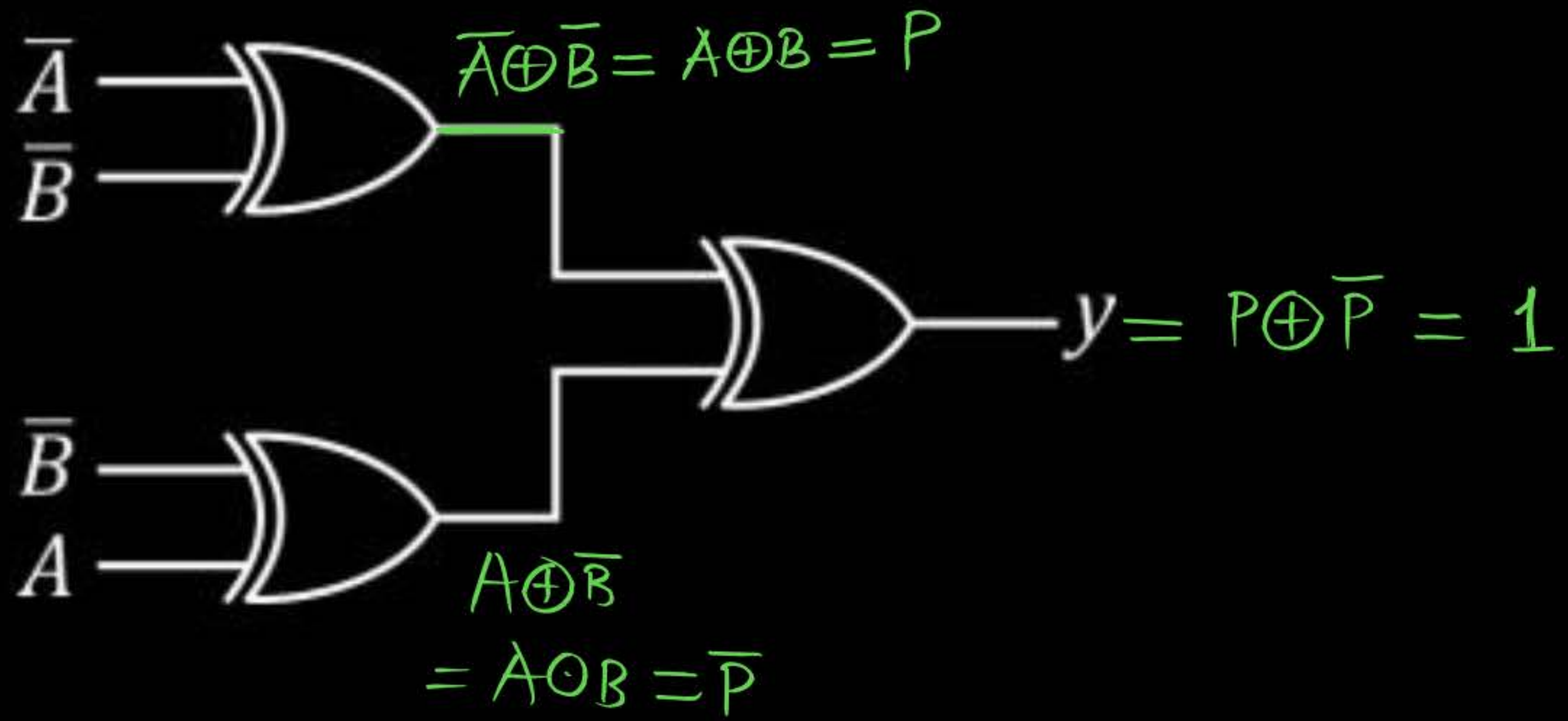
$$= A\oplus \underline{B\odot C\odot D}$$
$$= A\oplus B\oplus C\oplus D$$

- $$\overline{\overline{A\odot\overline{B}}\odot\overline{C}\odot\overline{D}} = \overline{A\odot B\odot C\odot D} = A\oplus B\odot C\odot D$$
$$= A\odot B\oplus C\odot D$$

- $$\overline{\overline{A\odot\overline{\overline{B}\odot\overline{C}}}\odot\overline{D\odot\overline{E}}} = \overline{A\odot B\odot C\odot D\odot E} = \overline{A\oplus B\oplus C\oplus D\oplus E}$$

- $$\overline{\overline{A\oplus\overline{B}}\odot\overline{\overline{C}\oplus D}} = \overline{A\oplus B\odot C\oplus D} = A\oplus B\oplus C\oplus D$$
$$= A\odot B\odot C\oplus D$$

Question

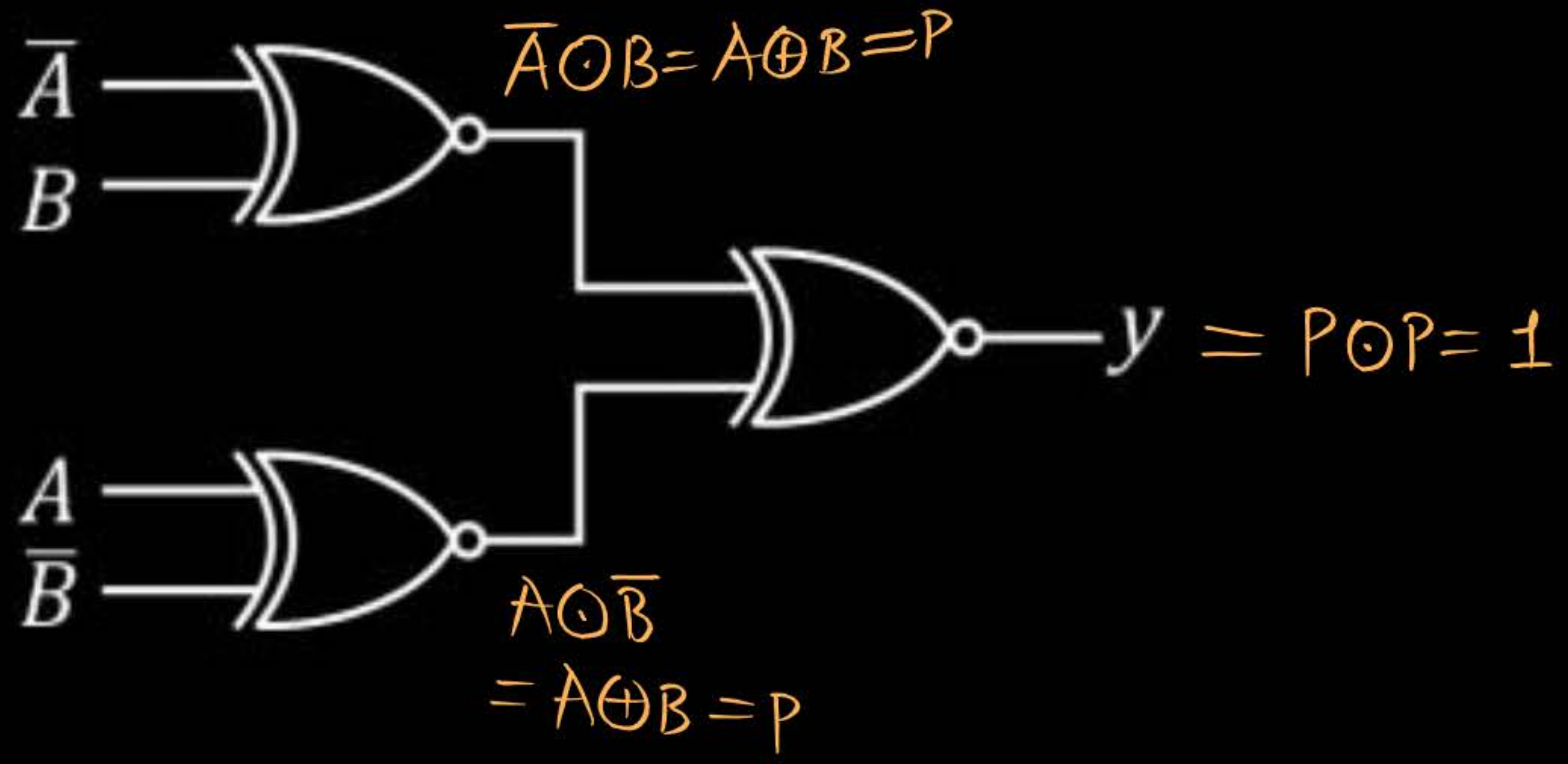


Output y is

(a) $A \odot B$ (b) $A \oplus B$

(c) Always '1' (d) Always '0'

Question



Output y is

(a) $A \oplus B$

(b) $A \odot B$

(c) Always '1'

(d) Always '0'

Question

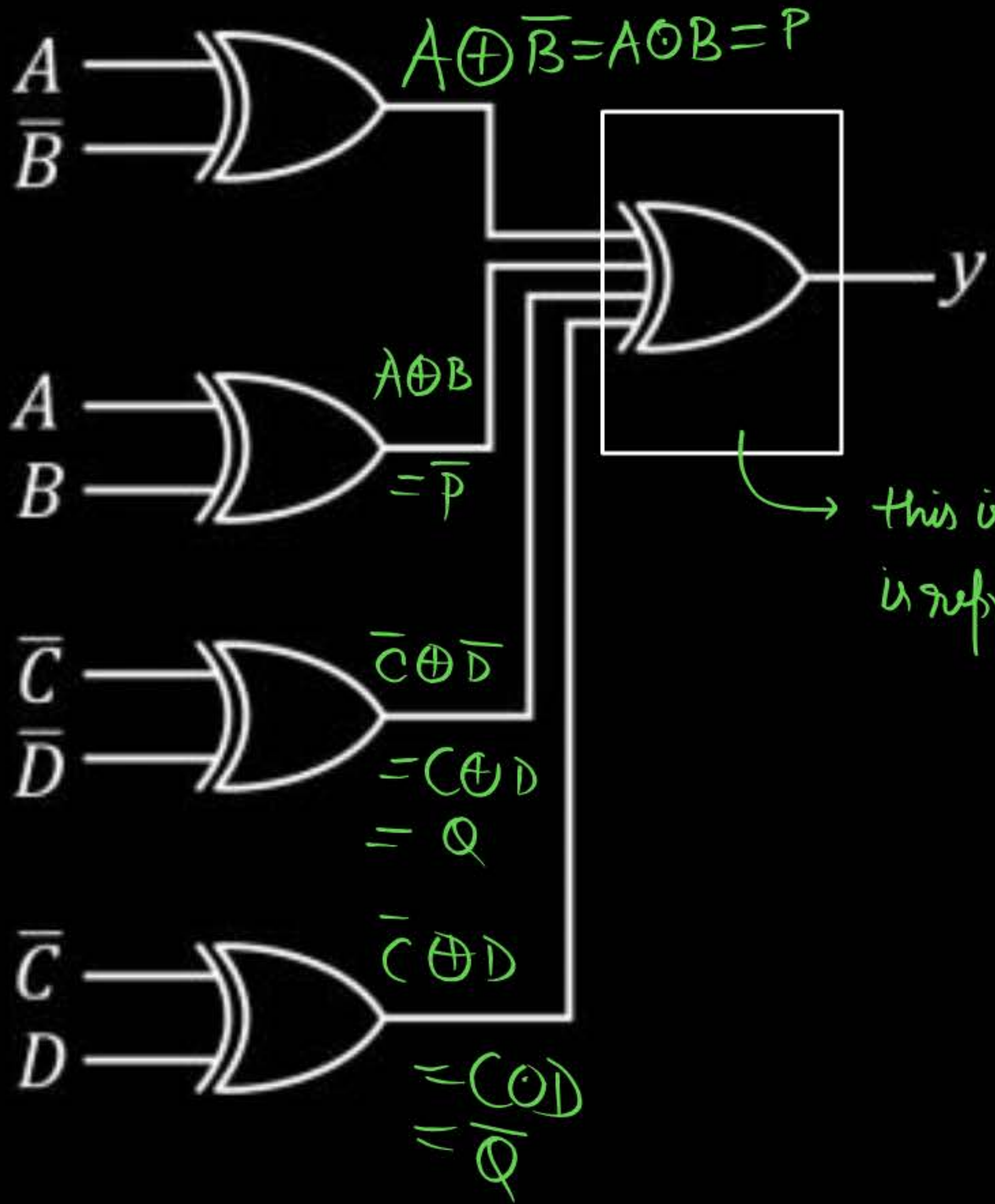
Output y is

(a) $A \odot B \oplus C \oplus D$

(b) $A \oplus B \oplus C \oplus D$

(c) Always '0'

(d) $A \odot B \odot C \odot D$



$= \underbrace{P \oplus \overline{P}} \oplus \underbrace{Q \oplus \overline{Q}}$

$= 1 \oplus 1$

$= 0$

this is 4 i/p XOR is represented.

Question

$y = \overline{\overline{A \oplus B} \oplus \bar{C}}$ will be equal to $= \overline{A \oplus B \oplus C} = A \odot B \oplus C = A \oplus B \odot C$

(a) $A \odot B \oplus C$

(b) $y = \overline{\bar{A} \oplus \overline{B \odot C}} = \overline{A \oplus B \odot C}$
$= A \oplus B \oplus C$
$= A \odot B \odot C$

(c) $y = \overline{A \oplus B \oplus \bar{C}}$
$= A \oplus B \oplus C$

(d) None of these

Question

$y = A \oplus [A + B]$ equal to

H.W.

(a) $A \oplus B$

(b) $y = A \odot B$

(c) $\bar{A}B$

(d) $A\bar{B}$

Question

$y = A \oplus B \oplus AB$ equal to H.W.

(a) AB

(b) $(A + B)$

(c) $\overline{AB}$

(d) $\overline{A + B}$

Q. $\overline{A \odot \overline{B} \oplus \overline{C}}$ is/are equal to

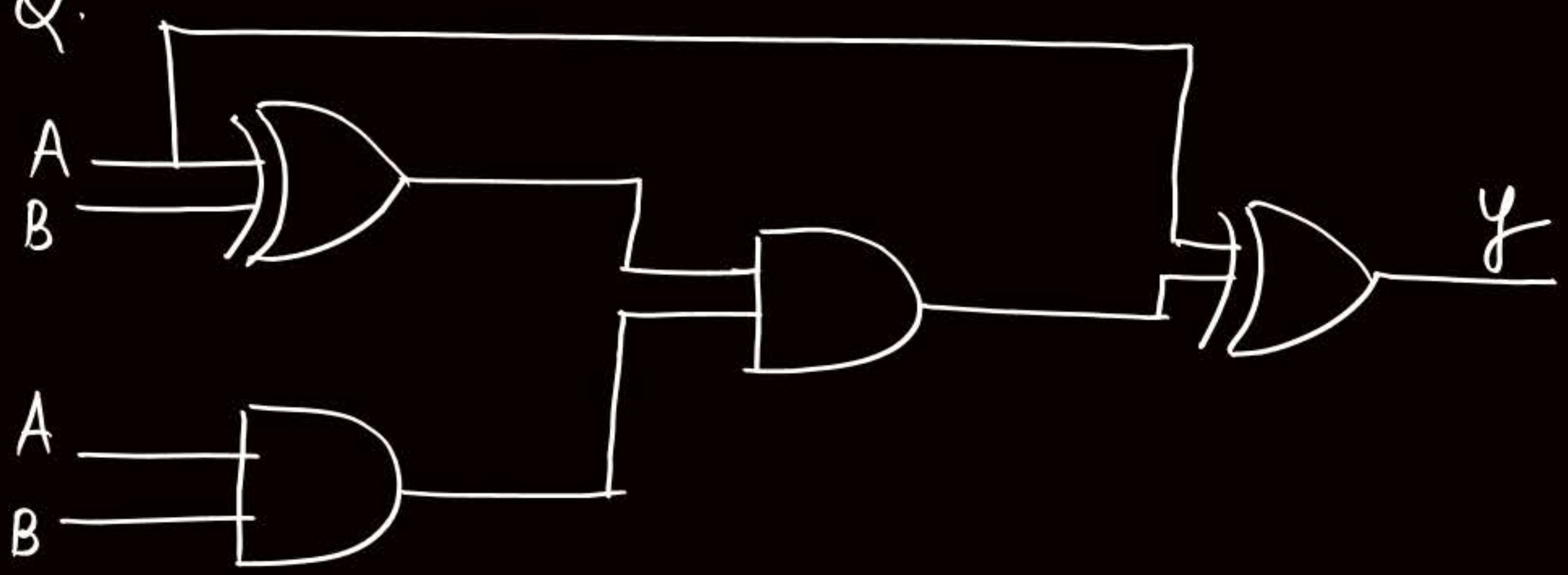
a. $A \oplus B \oplus C$

b. $\bar{A} \odot B \odot C$

c. $A \odot \bar{B} \odot \bar{C}$

d. $\bar{A} \oplus B \odot \bar{C}$

Q.



y is

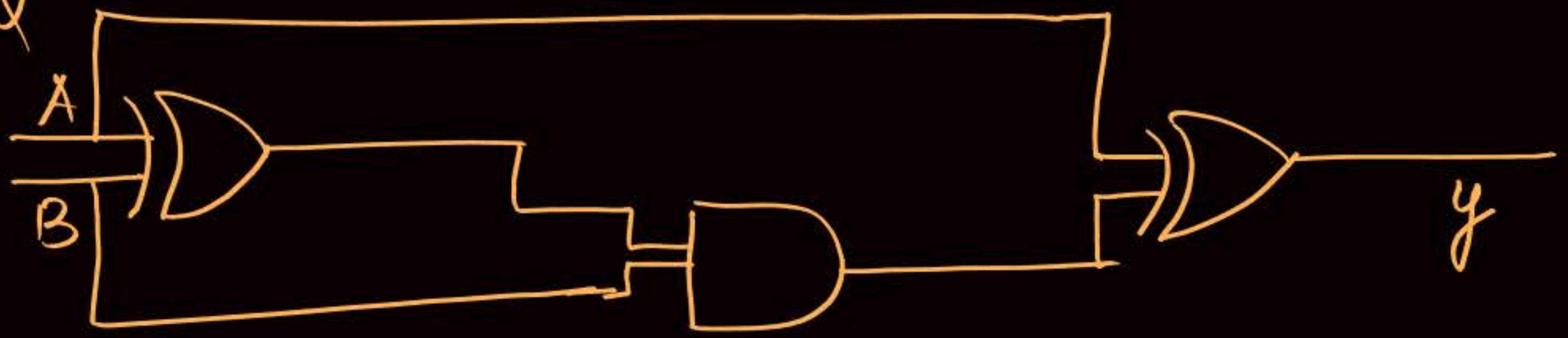
a. A

b. $A+B$

c. $A \odot B$

d. None of these.

#Q



y is

a. $A+B$

b. $A\ B$

c. $\overline{A}B$

d. $A \odot B$

2 Minute Summary

XOR & XNOR

Thank you

GW

Soldiers !