CS & IT ENGINEERING

THEORY OF COMPUTATION

REGULAR EXPRESSIONS

Lecture No.- 06

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Recap of Previous Lecture

Regular Expression

Topic ?????

- → Construction of Reg Expression
- → DFA states.

Topics to be Covered

Topic Conversion from $\in$-NFA to NFA

Topic ??

Topic ??

Topic ??

1. $R + \phi = R$

2. $R \cdot \phi = \phi$

3. $R \cdot \epsilon = R$

4. $R + \epsilon \neq R$?

$\{R, \epsilon\} \neq R$

Assume

$\{\phi = 0\}$

$\{\epsilon = 1\}$

⑤ $R^* = (R^*)^*$ ✓

$$R^* = \{\epsilon, R, R^2, R^3 \dots\}$$

$$(R^*)^* = R^* + (R^*)^2 + (R^*)^3 + \dots$$

$$\{\epsilon, R, R^2, R^3 \dots\}$$

(Q) Which of the following are not equal?

(a) r^* & $(r^*)^+$ $\longrightarrow$ equal

r^* $\quad$ r^*

(b) $r^* (r^+)^*$ & $(r^*)^*$ $\longrightarrow r^*$

(c) $(r^*)^*$ & r^*

(d) none

$$(r^+)^* = (r^+)^0 + r^+ + (r^+)^2 + \cdots$$

$$= \epsilon + r^+ + \cdots$$

$$= r^* \checkmark$$

1. $\epsilon^* = \epsilon^0 + \epsilon^1 + \epsilon^2 + \cdots = \underline{\{\epsilon\}}$ ✓

2. $\epsilon^+ = \epsilon^1 + \epsilon^2 + \cdots = \{\epsilon\}$ ✓

3. $\phi^+ = \phi$

4. $\phi^* = \epsilon$

				DFA		NFA
1	$(a+b)(a+b)(a+b)$	$\Rightarrow$ exactly 3	$\Rightarrow$	5	$\rightarrow$	4
2	$[(a+b)(a+b)]^*$	$\Longrightarrow$ div by 2	$\Rightarrow$	2	$\rightarrow$	2
3	$\underbrace{(a+b)[(a+b)(a+b)]^*}_{o}$	$\Longrightarrow$ length odd	$\Rightarrow$	2	$\rightarrow$	2.

Topic : Regular Expression

DFA → Expression

$(a+b)^*$

- The simplest way of representing a regular language is known as Regular expression.
- For every regular language regular expression can be constructed.
- To construct regular expression following 3 operators are used.
- + is known as union operator
- . is known as concatenation operator
- * is known as Kleene closure operator

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atmost 4.

$$\epsilon + (a+b) + (a+b)^2 + (a+b)^3 + (a+b)^4$$

(or)

$$\underbrace{(a+b+\epsilon)}_{1}\underbrace{(a+b+\epsilon)}_{1}(a+b+\epsilon)(a+b+\epsilon)$$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is divisible by 4.

$$\{0, 4, 8, 12, 16 ---\}$$

$$\checkmark \left[(a+b)(a+b)(a+b)(a+b)\right]^*$$

$$(a+b)^* = \epsilon + (a+b)^1 + (a+b)^2 + (a+b)^3 + (a+b)^4 + \cdots$$

$$r^* = r^0 + r^1 + r^2 + r^3 + r^4 + \cdots$$

$$r^+ = r^1 + r^2 + r^3 + r^4$$

OR

$$\gamma_1 + \gamma_2 = \{\gamma_1, \gamma_2\}$$

$$\gamma_1 \cdot \gamma_2 = \{\gamma_1\gamma_2\}$$

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are exactly 4.

$$b^* a b^* a b^* a b^* a b^*$$

× $aaaab^*$

× b^*aaaab^*

× b^*aaaa

$$\Sigma = \{a, b\}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

$$(a+b)^{*}\, a\, (a+b)^{*}\, a\, (a+b)^{*}\, a\, (a+b)^{\infty}$$

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atmost 3.

$\{0, 1, 2, 3\}$ a's

$$b^* (a+\epsilon)\, b^* (a+\epsilon)\, b^* (a+\epsilon)\, b^*$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

$$(a+b)^* a (a+b)^* a (a+b)^* a (a+b)^*$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are divisible by 3. MCQ

$$\left(\left(b^* a b^* a b^* a b^*\right)^* + b^*\right)$$

bbbb

bbbb

(a) $(b^* a a a b^*)^*$

(b) $(b^* a b^* a b^* a b^*)^*$ X

(c) $(aaa)^* + b^*$

(d) none

exactly

#Q. How many states are there in minimal DFA that accept following regular expression.

length

$n+2$

1. $(a+b)(a+b)(a+b) \Longrightarrow 5$

2. $(a+b+\epsilon)(a+b+\epsilon)(a+b+\epsilon) \Longrightarrow 5$

3. $b^* a b^* a b^* a b^* a b^* \Longrightarrow 6$

$n+1$

1. $$\underbrace{(a+b)^{*} \underline{a} (a+b)^{*} \underline{a} (a+b)^{*} \underline{a} (a+b)^{*}}_{\text{no. of a's atleast 3}} \Rightarrow 4$$

2. $$\underbrace{[(a+b)(a+b)]}_{\text{Div by 2}}^{*} \Rightarrow 2$$

3. $(a+b)$

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where each string starting and ending with different symbol.

$a \left(\begin{matrix} a\text{'s} \\ b\text{'s} \end{matrix} \right) b \;\Big|\Big|\; b \left(\begin{matrix} a\text{'s} \\ b\text{'s} \end{matrix} \right) a$

$$\underline{\underline{a(a+b)^* b}} + \underline{\underline{b(a+b)^* a}}$$

every string

$\begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix} aab \begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix}$

#Q. Construct regular expression that generates set of all strings of a's and b's where having substring aab.

$$\begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix} \underline{aab} \begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix}$$

$$\boxed{(a+b)^* aab (a+b)^*}$$

every string

#Q. Construct regular expression that generates set of all strings of a's and b's where having substring aba (or) bab.

$$\begin{pmatrix} a\cdot s \\ b\cdot s \end{pmatrix} (\underline{aba} + \underline{bab}) \begin{pmatrix} a\cdot s \\ b\cdot s \end{pmatrix}$$

$$\downarrow$$

$$(a+b)^*(aba+bab)(a+b)^*$$

$\begin{pmatrix} a \cdot S \\ b \cdot S \end{pmatrix}$ b a/b a/b a/b $2^4 = 16$

#Q. Construct regular expression that generates set of all strings of a's and b's where 4th input symbol is b from end.

$(a+b)^* \, b \, (a+b) \, (a+b) \, (a+b)$

$2^n \rightarrow$ DFA

$n+1 \rightarrow$ NFA

5

#Q. Construct regular expression that generates set of all odd length palindrome strings over {a}.

$$\{a, a^3, a^5, a^7 \text{---} \} \longrightarrow \text{Regular}$$

2 2 2

#Q. Construct regular expression that generates set of all odd length palindrome strings over {a, b}.

not possible

Topic : NOTE

$\{ww^R\}$

- Palindrome languages over more than one symbol are not regular .Hence regular expression not possible.
- Palindrome languages over one symbol are regular.

Odd length Palindrome

$$L_1 = \{ W c W^R \mid W \in (a+b)^* \}$$

$$L_2 = \{ \underline{\overset{\epsilon}{w}} \; \underline{\overset{\epsilon}{w}}^R \mid \underline{w} \in (a)^* \}$$

$$\{ \epsilon, aa, aaaa --- \}$$

$$\{ \epsilon, a^2, a^4 --- \} \equiv \underline{(aa)^*}$$

What is the Regular Expression?

$$L_3 = \{ \underline{W}\, b\, \underline{W}^R \mid W \in (a)^* \}$$

$$L_3 = \{ b, aba, a^2ba^2, a^3\underline{b}a^3 \ldots \}$$

not Regular

Dependency

$$L_4 = \{ \underline{w}\,\underline{w} \mid w \in (a)^* \}$$

$$\{ \epsilon, aa, aaaa, aaaaaa \ldots \}$$

$$= \underline{(aa)^*} \checkmark$$

$$L_5 = \left\{ w\,x\,w^R \;\middle|\; \begin{array}{l} w \in (a+b)^* \\ x \in (a+b) \end{array} \right\}$$

not possible

$(a+b)^*$

Possible

$$L_6 = \{ \underset{\text{ababa}}{w x w^R} \mid w, x \in (a+b)^* \} = (a+b)^*$$

$\{\epsilon, a, b, aa, ab \ldots\}$

$\Downarrow$

$(a+b)^*$

(a) Yes

(b) NO

$a \; \underline{ab} \; a$

$$\{ W X W^R \mid W, X \in (a+b)^* \} = (a+b)^*$$

$$\{ \epsilon, a, b, aa, ab, ba, bb \dots \}$$

$$(a+b)^*$$

For which of the following Regular Expression is possible?

Lang

$\{\epsilon, aa, bb, abab, baba ---\}$

non Regular

$L_1 = \{ WW \mid W \in (a+b)^* \}$

$(a+b)^*$

$L_2 = \{ WxW \mid W, x \in (a+b)^* \}$

$\{\epsilon, a, b, aa, ab, bb ---\}$

$\Downarrow$

$(a+b)^*$

(a) L_1 only

(b) L_2 only

(c) L_1 and L_2

(d) none

$$\{WXW^R\}$$

$$\{\underline{\underline{W}}\_\underline{\underline{W}}^{R}\}$$

nitin

liril

madam

$W \mid W^R$

$\{\phi = 0\}$ $R \rightarrow$ regular Expr

$\{\epsilon = 1\}$

(1) $R + \phi = \phi + R = R$

(2) $R \cdot \phi = \phi \cdot R = \phi$

(3) $R + \epsilon = \underbrace{\epsilon + R}_{}^{\{\epsilon, R\}} \neq R$

(4) $R \cdot \epsilon = \epsilon \cdot R = R$

(5) $(R^*)^* = (R^*)^+ = (R^+)^* = R^*$

(6) $R \cdot R^* = R^+ = R^* R$

(7) $\epsilon^* = \epsilon$

(8) $\epsilon^+ = \epsilon$

(9) $\phi^* = \epsilon$

(10) $\phi^+ = \phi$

(11) $(a + b)^* \neq a^* b^*$

(12) $(a + b)^* \neq a^* b^*$

(13) $a(ab)^* \cancel{\neq} (ab)^* a$

$\{a, aab, aabab, aababab - \}$

$\{a, aba, ababa, abababa, \ldots\}$

$$R \cdot R^* = R^+$$

$$R^+ + \epsilon = R^*$$

$$R \left\{ \begin{matrix} \cancel{\epsilon} \\ R \\ R^2 \\ \vdots \end{matrix} \right\} = \left\{ \begin{matrix} R \\ R^2 \\ R^3 \\ \vdots \end{matrix} \right\}$$

13 $a(ab)^* \neq (ba)^* a$

$\{a, aab, aabab \ldots\} = \{a, baa$

14. $(ab)^* a = a(ba)^*$ $\{a, aba, ababa, abababa \ldots\}$

$\{a, aba, ababa, \ldots\}$

abababa

$$(a+b)^* \neq a^* b^*$$

$$ba$$

$$(a+b)^* \neq (ab)^*$$

$$(a+b)^* \neq a^* + b^*$$

ba

ab

(14) $(a+b)^* = (a+b^*)^*$ 1

$= (a^*+b^*)$ 2

$= (a^*+b^*)^*$ 3

$= (a^*b^*)^*$ 4 Imp.

$$(r_1+r_2)^* = (r_1^* r_2^*)^*$$

(15) $a^*+a^* = a^* = a^*a^*$

(16) $a+b = b+a = \{a,b\}$

(17) $a\cdot b \neq ba$

$(ab)^2 = ab\,ab$

④ $(a+b)^* = (a^* b^*)^*$

$$= (a^* b^*)^0 + a^* b^* + (a^* b^*)^2 + (a^* b^*)^3 + \cdots$$

$a^* b^*$

$b^* a^*$

$a^* b^* a^* b^* a^*$

$$= \underbrace{\epsilon}_{1} + \underbrace{a^* b^*}_{2} + \underbrace{a^* b^* a^* b^*}_{3} + \underbrace{a^* b^* a^* b^* a^* b^*}_{4} + \cdots$$

$$= \{\epsilon, a, b, aa, bb, ab, ba$$

$$(a^*b^*)^* = \left\{ \begin{array}{l} \epsilon \\ a^* \\ b^* \\ a^*b^* \\ b^*a^* \\ a^*b^*a^*b^*a^* \cdots \end{array} \right\} = \underline{\underline{(a+b)^*}}$$

$$(r_1 + r_2)^* = \left\{ \begin{array}{l} r_1^* \\ r_2^* \\ (r_1 r_2)^* \\ (r_2 r_1)^* \\ (r_1 r_2 r_1 r_2)^* \; - \; - \end{array} \right\}$$

$$(a+b)^* =$$

1 2

$$(a+b)^* = (a^* + b^*)^*$$

$$= (a' + b')^*$$

$$= (a+b)^* \text{ equal}$$

$$(a+b)^* = (a^* + b)^*$$

$$= (a' + b)^*$$

$$= \underline{\underline{(a+b)^*}} \text{ equal}$$

$$(a+b)^* = (a+b^*)^*$$

$$= (a+b')^*$$

$$= \underline{(a+b)^*} \quad \text{equal}$$

$$\boxed{(a+b)}^{*} = (a+b)^{0} + (a+b)^{1} + \underline{(a+b)^{2}} + \cdots$$

$$\Sigma^{*} = \{\epsilon, a, b, aa, ab, ba, bb, \cdots\}$$

<u>Complete language</u>

3

$$R + \epsilon \neq R$$

$$\{R, \epsilon\} \neq R$$

$$R^* = (R^*)^* = (R^*)^+ = (R^+)^* = R^*$$

$$= (R^+)^*$$

$$= (R^+)^0$$

$$\{\epsilon, R^+\} = R^+ + \epsilon = R^*$$

$$R \cdot R^* = R^+$$

$$R \cdot \{\epsilon, \underline{R}, R^2, R^3, R^4 \ldots\}$$

$$\Downarrow$$

$$\{\underbrace{R \cdot \epsilon}_{R}, R^2, R^3 \ldots\} = R^+$$

| * | * | * | * | * | * | * | * / * | * | * | * | * \ * |

[MCQ]

$\phi = 0$

$R + \phi = R$

#Q. Identify language accepted by following regular expression

$b^*(a^*.\phi . b + ab + a\phi^*b^*)(b + \phi)^*$

ϕ

$\rightarrow b^*(\phi + ab + ab^*)b^*$

b^*ab^*

b^*ab^*

A. Exactly one a

B. At least one a ✗ $\rightarrow b^*(ab + ab^*)b^*$

C. At most one a ✗ $\rightarrow b^*(a(b + b^*))b^*$

D. None

$b^*ab^*b^* \rightarrow b^*ab^*$

$\rightarrow b^*(ab^*)b^* \longrightarrow$

$$b^* + b = b^*$$

[MCQ]

$0(00)^* \rightarrow$ odd

#Q. Which of the following regular expressions are equivalent?

I. $(00)^*(\varepsilon+0) \longrightarrow$ all

II. $(00)^* \longrightarrow$ even

III. $0^* \longrightarrow$ all

IV. $0(00)^* \longrightarrow$ odd

A (I) And (II)

B (ii) and (iii)

C (i) And (iii)

D (iii) and (iv)

$$(00)^*(\epsilon+0)$$

$$\left[(00)^* + (00)^* \cdot 0\right]$$

$\Downarrow$

even + odd

$\Downarrow$

all 0's

[MCQ]

#Q. Which of the following pair of regular expressions are not equal

A $(r^*)^*$ and $(r^+)^*$ → equal

B $(r + \varepsilon)^*$ and r^* → equal

C $(rr + \varepsilon)^*$ and r^* → not equal

D None of the above

r^* r^*

r^*

$(r+\epsilon)^* = r^*$

$(a+b)^*$

$(a+b)^*$

$$(\underline{rr}+\epsilon)^* = \{\epsilon, rr, (rr)^2, (rr)^3, (rr)^4 \cdots\}$$

$\neq$

r^*

(Q) Which of the following is true? Home work

$$L_1 = 11(0+1)^*$$

$$L_2 = (0+1)^* 11$$

$$L_3 = 11(0+1)^* 11 + 111 + 11$$

(a) ~~$L_1 = L_2 = L_3$~~

(b) ~~$L_1 \cup L_2 = L_3$~~

(c) $L_1 \cap L_2 = L_3$

(d) none

$L_1 = 11(0+1)^* = \{110 - - - \}$

$L_2 = (0+1)^* 11 = \{011, - - - \}$

$L_1 \cup L_2 = \{110, 011 - - - \}$

X X

$L_1 \cap L_2 = \{11, 111, 11011 - - - \}$

$\checkmark$ $\underline{11(0+1)^* 11} + 111 + 11$

$\neq L_3$

L_3

(Q) Which of the following Regular Expressions are equal ?

1. $(a+ba)^*(b+\epsilon) \rightarrow \epsilon$

2. $(a^*(ba)^*)^*(b+\epsilon) + a^*(b+\epsilon) + (ba)^*(b+\epsilon) \rightarrow \epsilon$

3. $(a+ba)(a+ba)^*(b+\epsilon)$

(a) 1 & 2 (b) 1 & 3 (c) 1, 2, 3

(d) 2 & 3

$(r_1 + r_2)^*$ $(r_1^* r_2^*)^*$

$(r_1^* r_2^*)^*$ $\left(a^*(ba)^*\right)^*(b+\epsilon) + a^*(b+\epsilon) + (ba)^*(b+\epsilon)$

$\quad r_1 \quad r_2$

Super

$(a+ba)^*(b+\epsilon) + a^*(b+\epsilon) + (ba)^*(b+\epsilon)$

Sub $\quad$ Sub

$\times \quad \times$

$(a+ba)^*(b+\epsilon)$

$a^* + a = a^*$

$\{\epsilon, a^1, a^2, a^3 \cdots\}$

$r_1^* + r_1 = r_1^*$

Super Sub

(Q) Which of the following is equal to $(a^*b^*)^*a^*b^*$

$(a+b)^*$

(a) $(a+ba)^* \neq (a+b)^*$

(b) $(ab+ba)^* \neq (a+b)^*$

(c) $(a+b)^* = (a+b)^*$ ✓

(d) $a^*b^* \neq (a+b)^*$

$ba \neq ba$

$$(a^*b^*)^*a^*b^*$$

$$\downarrow$$

$$(a+b)^*a^*b^*$$

$$\downarrow$$

$$(a+b)^*$$

$$(a+b)^*$$

(Q) Which of the following regular expressions are not equal?

$(a+b)^*$ $(a+b)^*$

ⓐ $(a^* b^*)^*$ & $(a+b)^*$ ⇒ equal

ⓑ $(a+b)^*$ & $(a\boxed{b^*}+b\boxed{a^*})^* = (a+b)^*$ ⇒ equal

$(a+b)^* \neq (ab)^* = \{(ab)^0, ab, abab, (ab)^3 \ldots\}$

ⓒ $(a^* b^*)^*$ ≠ & $(ab)^*$

ⓓ $(a^*\boxed{b^*}\boxed{a^*}b^*)^*$ & $(a^* b^*)^*$ ⇒ Equal

F.A = Reg Expression

Regular Language

FINITE AUTOMATA TO REGULAR EXPRESSION

F.A

⇓

Regular Expression

ε-NFA

⇓

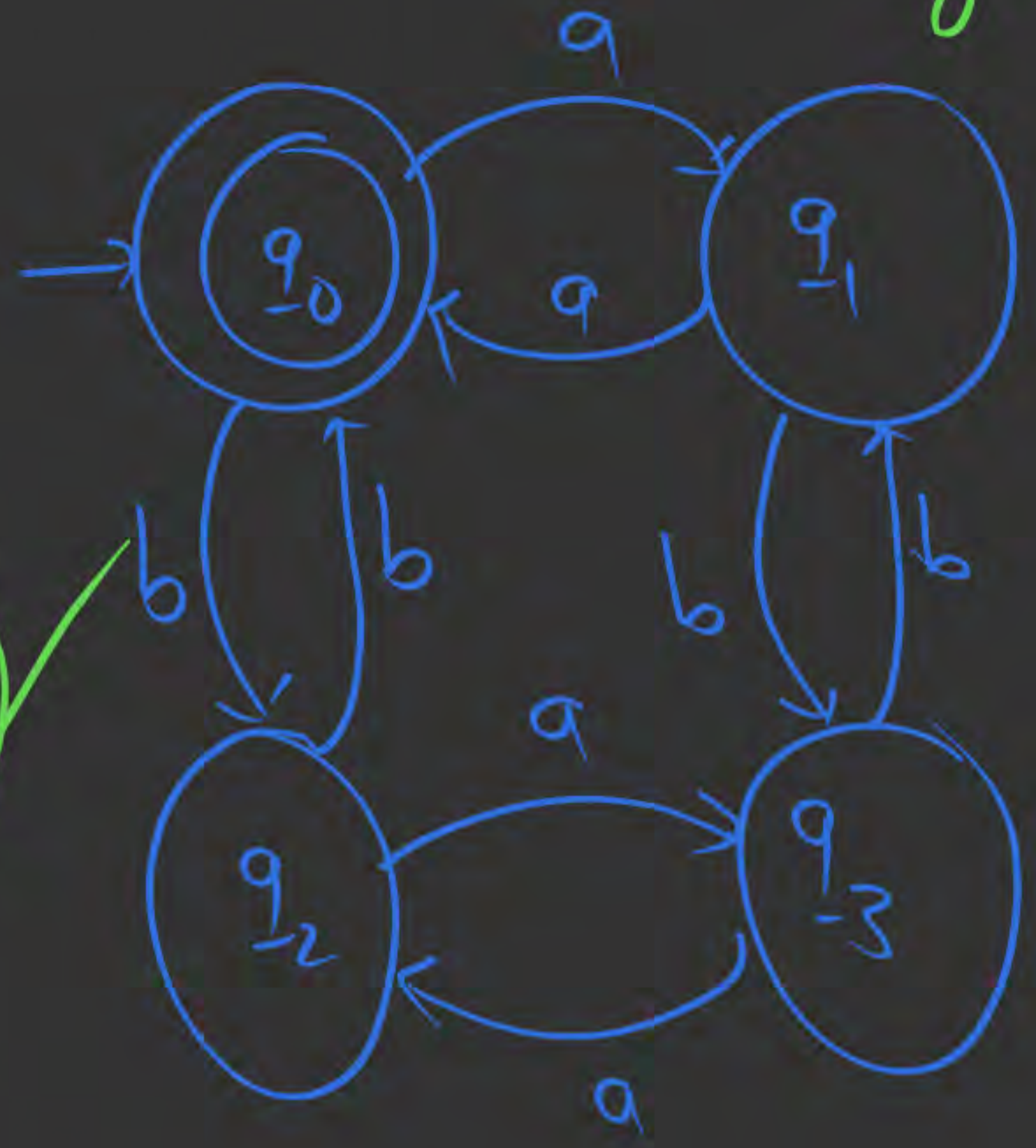
Reg Expr

2024

DFA

↓

Reg Expr

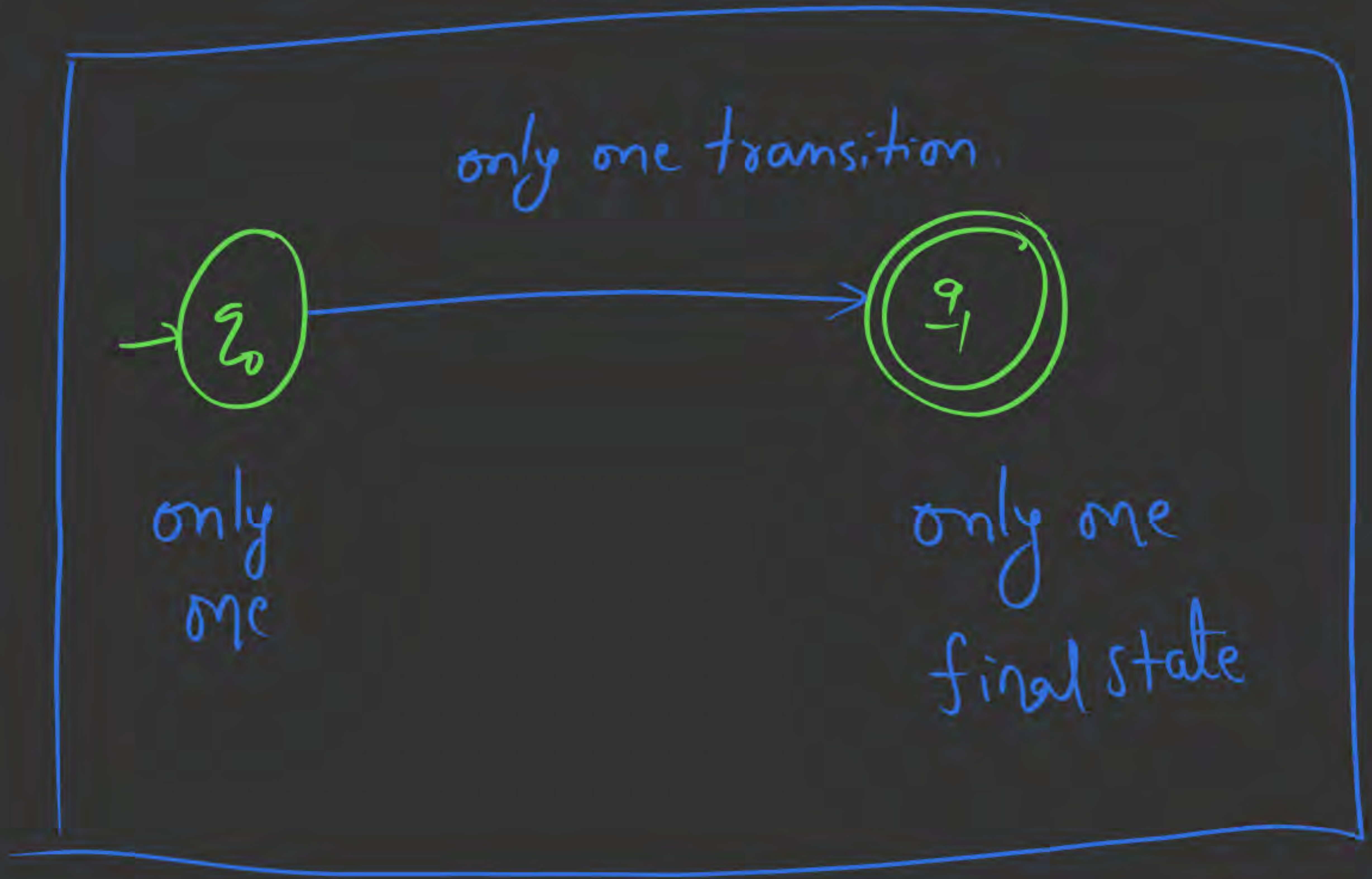


Reg

NFA

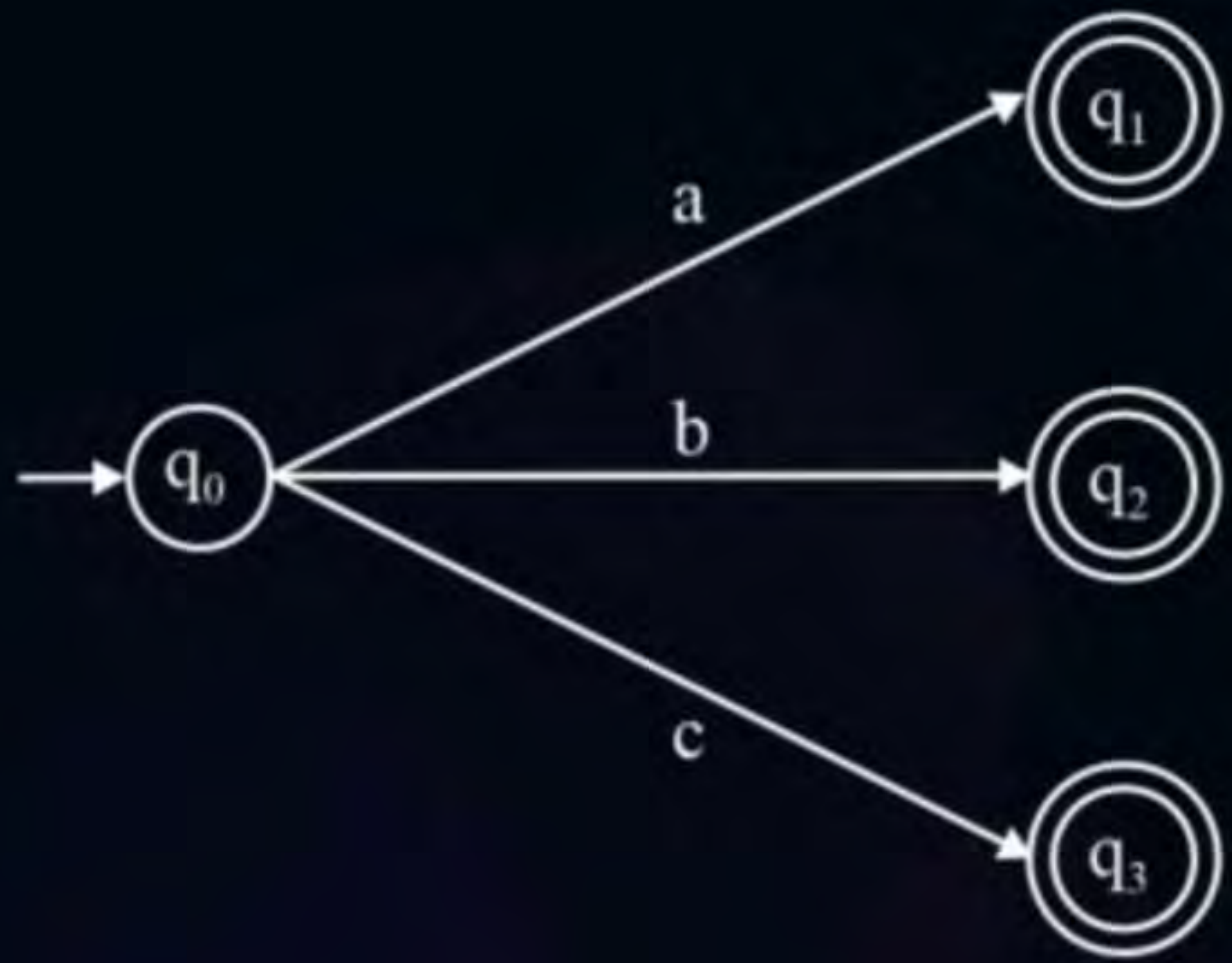
↓

Reg Expr


only one transition
q_0
q_1
only one
only one final state

1.

① State elimination
② Ardens method
③ R_{ij}^{K} method.



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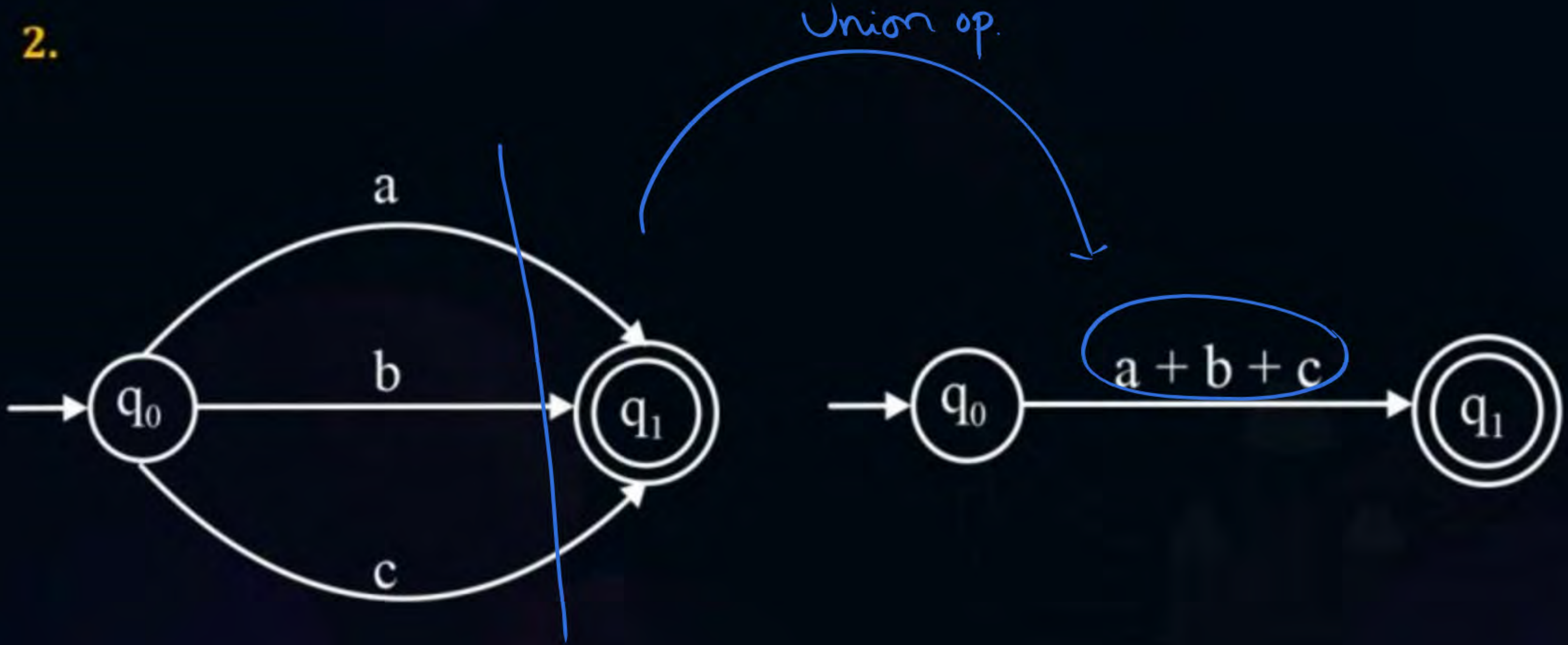

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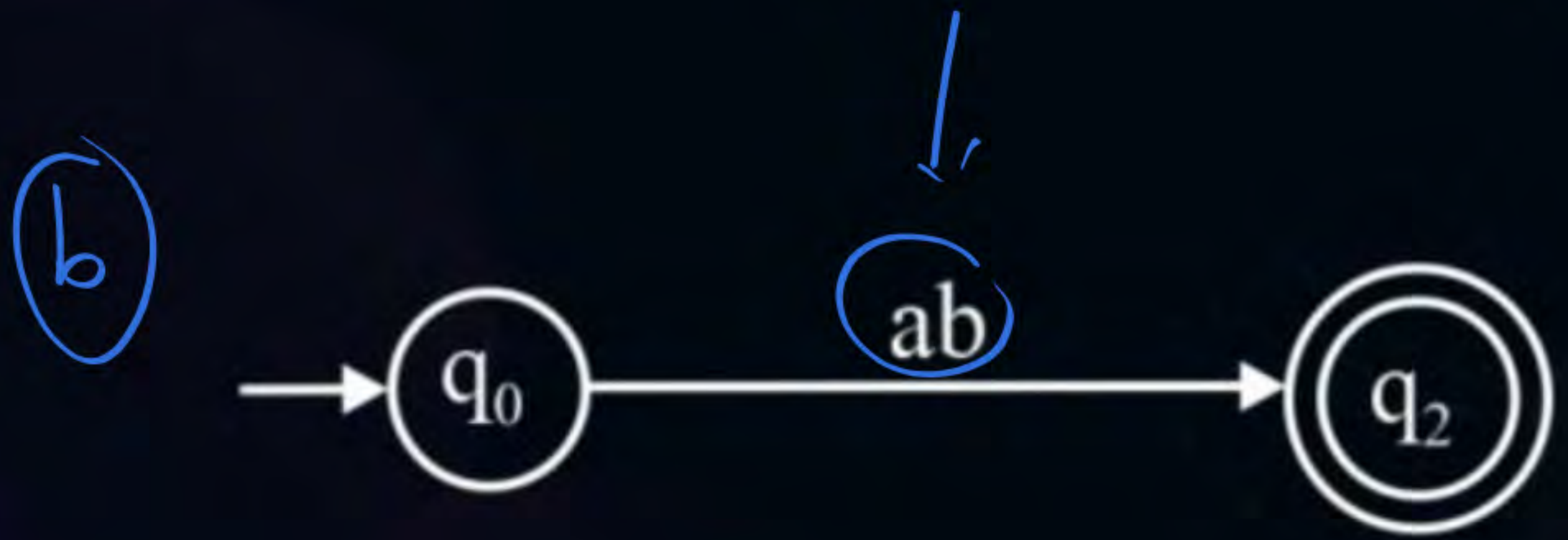
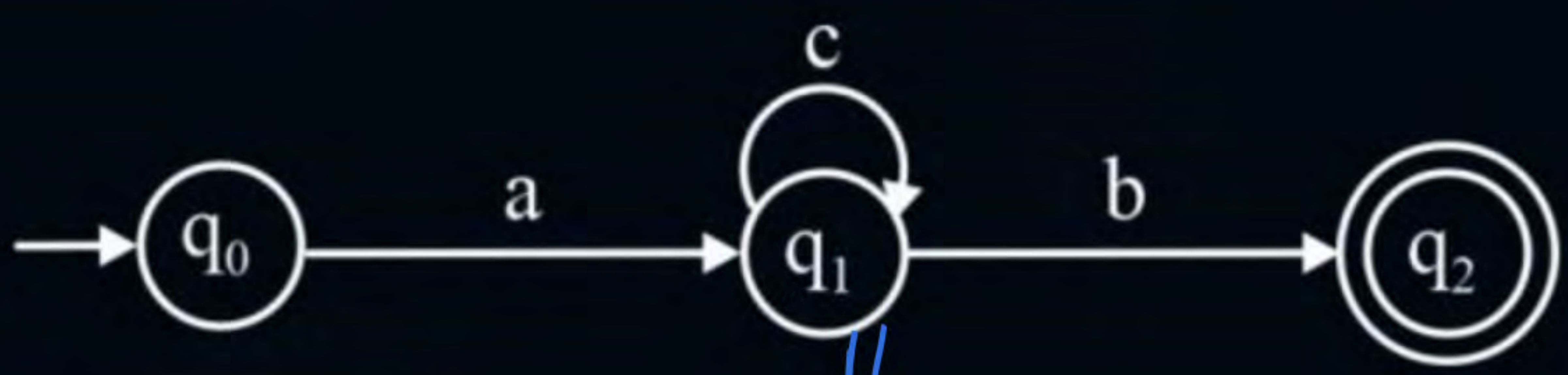
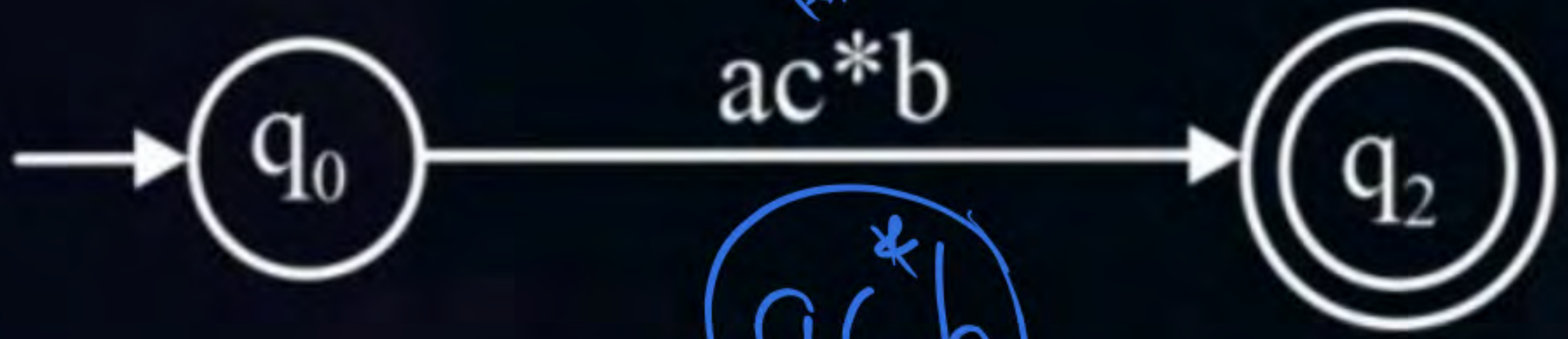
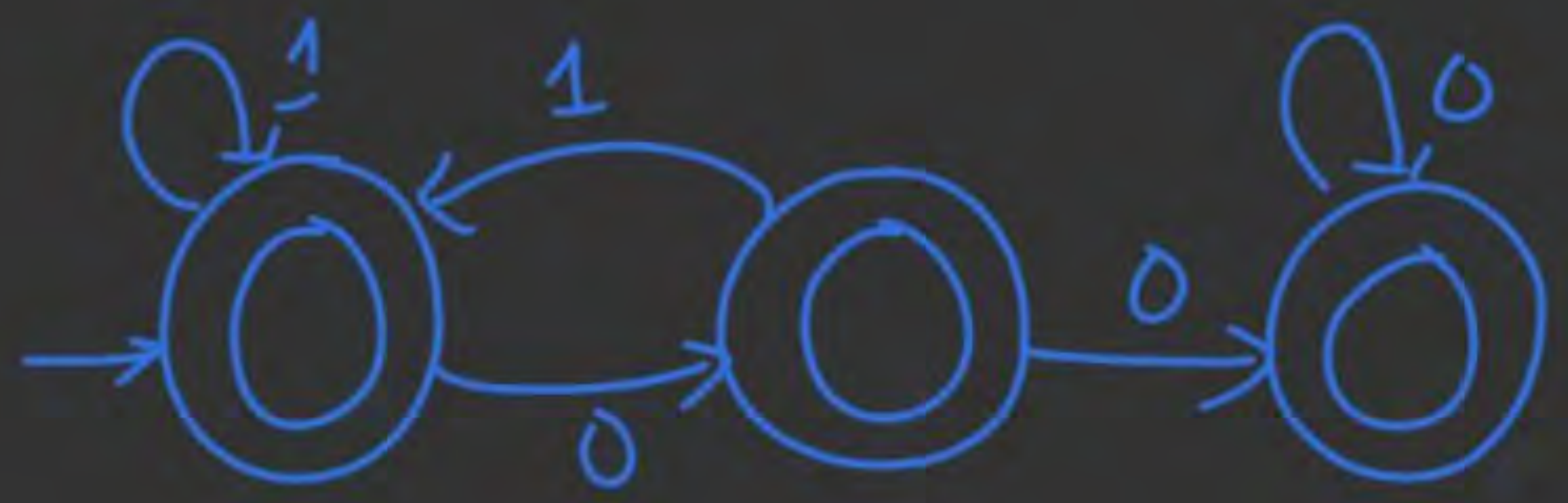


4.

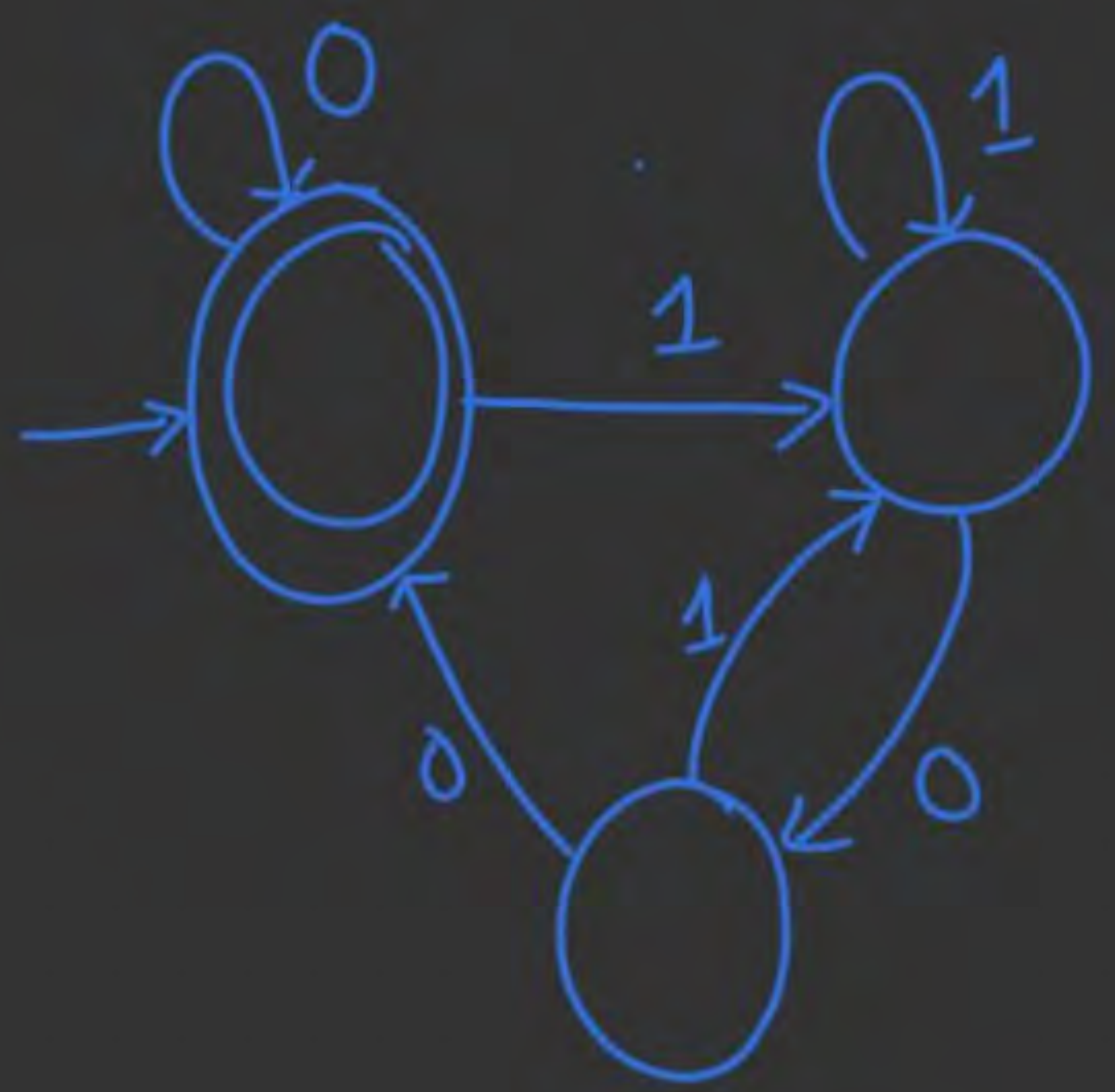




ac*b


1
1
0
0
0

7



$(00)^*$

THANK - YOU