COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 06

Combinational Circuit

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K- Map

Questions Practice

[Question]

The Boolean expression $XY + (X' + Y')\,Z$ is equivalent to

(a) $XYZ' + X'Y'Z$

(b) $X'Y'Z' + XYZ$

(c) $(X + Z)(Y + Z)$

(d) $(X' + Z)(Y' + Z)$

$$P + (\bar{P} \cdot Z) = (P + \bar{P}) \cdot (P + Z)$$
$$= (P + Z)$$
$$= (XY + Z)$$
$$= Z + XY$$
$$= (Z + X) \cdot (Z + Y)$$
$$= (X + Z)(Y + Z)$$

[MSQ]

Consider a Boolean expression given by $F(X, Y, Z) = \Sigma(3,5,6,7)$. $= \pi(0,1,2,4)$

Which of the following statements is/are CORRECT? [GATE-2024-CS: 1M]

$F(x,y,z) = xy + yz + zx$

↳ self dual

(A) F(X, Y, Z) is independent of input X

(B) $F(X, Y, Z) = XY + YZ + XZ$

(C) F(X, Y, Z) is independent of input Y

(D) $F(X, Y, Z) = \pi(0, 1, 2, 4)$

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{x}$			1	
x		1	1	1

yz

xy

xz

$=$

$F^D(x,y,z) = (x+y)(y+z)(z+x) = F(x,y,z)$

[MCQ]

The dual of a Boolean function $F(x_1, x_2, \ldots, x, +, \cdot, ')$, written as F^D, is the same expression as that of F with + and $\cdot$ swapped. F is said to be self-dual if $F = F^D$. The number of self-dual functions with n Boolean variables is

[GATE-2022-CS: 1M]

$n \longrightarrow 2^n \longrightarrow$ Combination (terms) $= N$

Total f^n $M_1 = 2^N = 2^{2^n}$

Self dual f^n $M = 2^{2^{(n-1)}}$

(A) $2n$

(B) 2^{n-1}

(C) 2^{2^n}

(D) $2^{2^{n-1}}$

[MCQ]

If w, x, y, z are Boolean variables, then which one of the following is INCORRECT?

[GATE-2017-CS: 1M]

(A) $wx + w(x+y) + x(x+y) = x + wy$ $\Rightarrow wx + wx + wy + x = wx + wy + x$

$= x(w+1) + wy$

$= x + wy$

(B) $\overline{w\bar{x}(y+\bar{z})} + \bar{w}x = \bar{w} + x + \bar{y}z$ $\Rightarrow \bar{w} + x + \overline{(y+\bar{z})} + \bar{w}x$

$= \bar{w} + x + \bar{y}z + \bar{w}x$

$= \bar{w} + x + \bar{y}z$

(C) $(w\bar{x}(y + x\bar{z}) + \bar{w}\bar{x})y = x\bar{y}$

(D) $(w+y)(wxy + wyz) = wxy + wyz$

$\Rightarrow wy(w+y)(x+z) = wy(x+z) = wxy + wyz$

$= [w\bar{x}y + 0 + \bar{w}\bar{x}]y$

$= \bar{x}y[wy + \bar{w}] = \bar{x}y[(y+\bar{w})] = \bar{x}y[y+\bar{w}] = \bar{x}\cdot y\cdot(y+\bar{w}) = \bar{x}\cdot y$

Question

If $X = 1$ in the logic equation $[X + Z\{\bar{Y} + (\bar{Z} + X\bar{Y})\}]\{\bar{X} + \bar{Z}(X+Y)\} = 1$ then

(a) $Y = Z$

(b) $\bar{Y} = Z$

(c) $Z = 1$

(d) $Z = 0$

$1 \cdot [0 + \bar{Z} \cdot 1] = 1$

$\bar{Z} = 1$

$Z = 0$

$\bar{y}\bar{z} + yz = 1$

$y \odot z = 1$

$\boxed{y = z}$

$yz = 1$

$y = 1$ and $z = 1$

$\bar{y}z + y\bar{z} = 1$

$y \oplus z = 1$

$y = \bar{z}$

$y + z = 1$

$y = 1$ or $z = 1$

$\bar{y}\bar{z} = 1 \Rightarrow \overline{y+z} = 1$

$\bar{y} = 1$ and $\bar{z} = 1$

$y = 0$ and $z = 0$

$\bar{y} + \bar{z} = 1$

$\overline{y \cdot z} = 1 \longrightarrow y = 0$ or $z = 0 \Rightarrow yz = 0$

[**Question**]

The Boolean expression $\underline{AB} + A\overline{C} + BC$ simplifies to

redundant

(a) $BC + A\overline{C}$

(b) $AB + A\overline{C} + B$

(c) $AB + A\overline{C}$

(d) $AB + BC$

[**Question**]

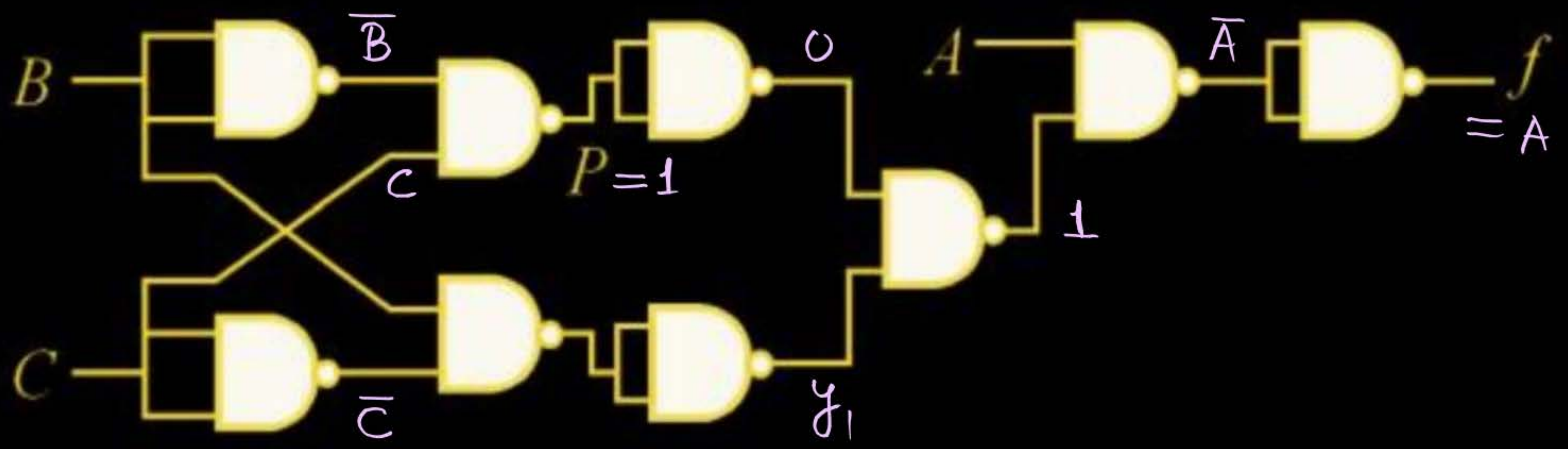
The point P in the following figure is stuck at 1. The output f will be

(a) $\overline{AB\overline{C}}$

(b) $\overline{A}$

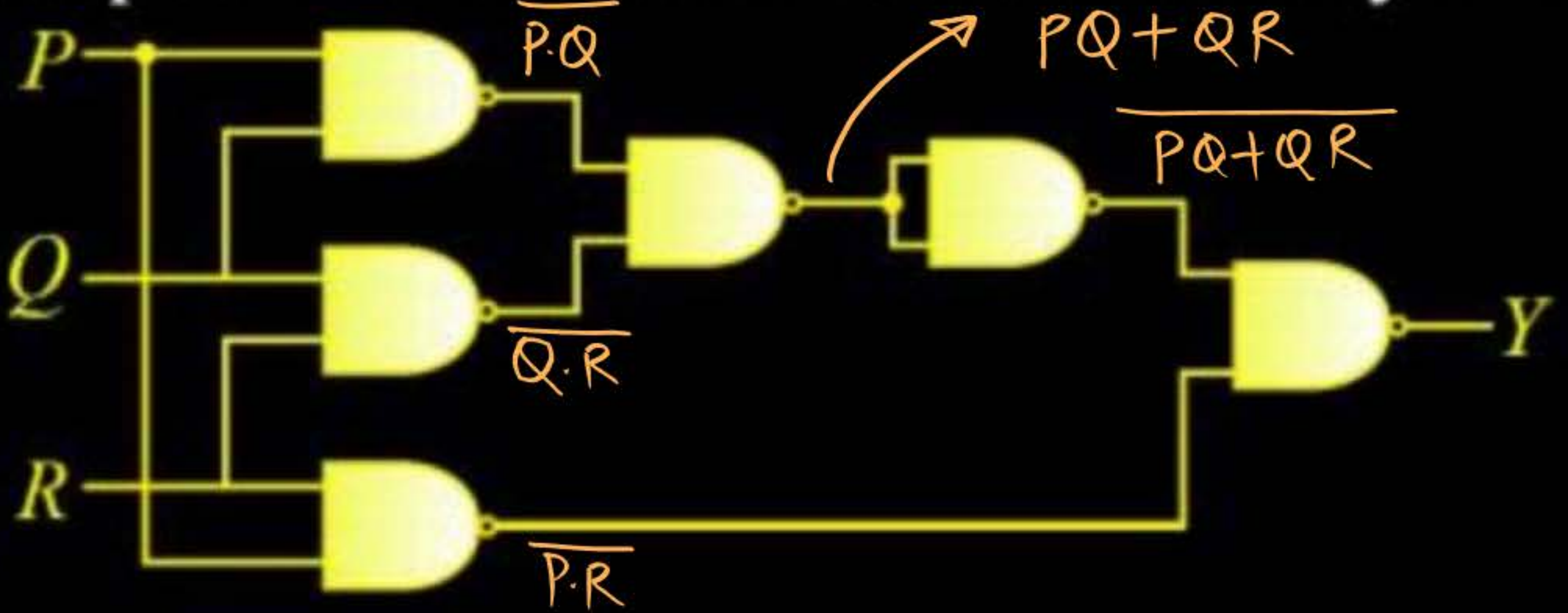
(c) $AB\overline{C}$

(d) A



[Question]

The output Y is the circuit below is always "1" when



$= PQ + QR + PR$

Carry output of F.A.

↳ majority i/p ckt

(a) Two or more of the input P, Q, R are "0". → $y = 0$

(b) Two or more of the inputs P, Q, R are "1".

(c) Any ~~odd~~ number of the inputs P, Q, R are "0".

$1 \to 0 \to 11 \to 1$

$3 \to 0 \longrightarrow \longrightarrow 0$

(d) Any odd number of the inputs P, Q, R are "1".

$1 \to 00 \to 0$

$111 \to \longrightarrow 1$

[MCQ]

The binary operator ≠ is defined by the following truth table

P	Q	p≠q
0	0	0
0	1	1
1	0	1
1	1	0

XOR = #

Which one of the following is true about the binary operator ≠?

[GATE-2015-CS: 1M]

(A) Both commutative and associative

(B) Commutative but not associative

(C) Not commutative but associative

(D) Neither commutative nor associative

[MCQ]

The number of min-terms after minimizing the following Boolean expression is ______. $[D' + AB' + A'C + AC'D + A'C'D]'$ $= \Sigma(15)$

[GATE-2015-CS: 1M]

$= [\bar{D} + A\bar{B} + \bar{A}C + \bar{C}\cdot D]'$

(A) 1

(B) 2

(C) 3

(D) 4

$= [(\bar{D} + \bar{C})\cdot(\bar{D} + D) + A\bar{B} + \bar{A}C]'$

$= [\bar{D} + \bar{C} + A\bar{B} + \bar{A}C]'$

$= [\bar{D} + (\bar{C} + \bar{A})\cdot(\bar{C} + C) + A\bar{B}]'$

$= [\bar{D} + \bar{C} + \bar{A} + A\bar{B}]' = [\bar{D} + \bar{C} + \bar{A} + \bar{B}]' = D\cdot C\cdot A\cdot B = \underline{\underline{ABCD}} \rightarrow (15)$

Question

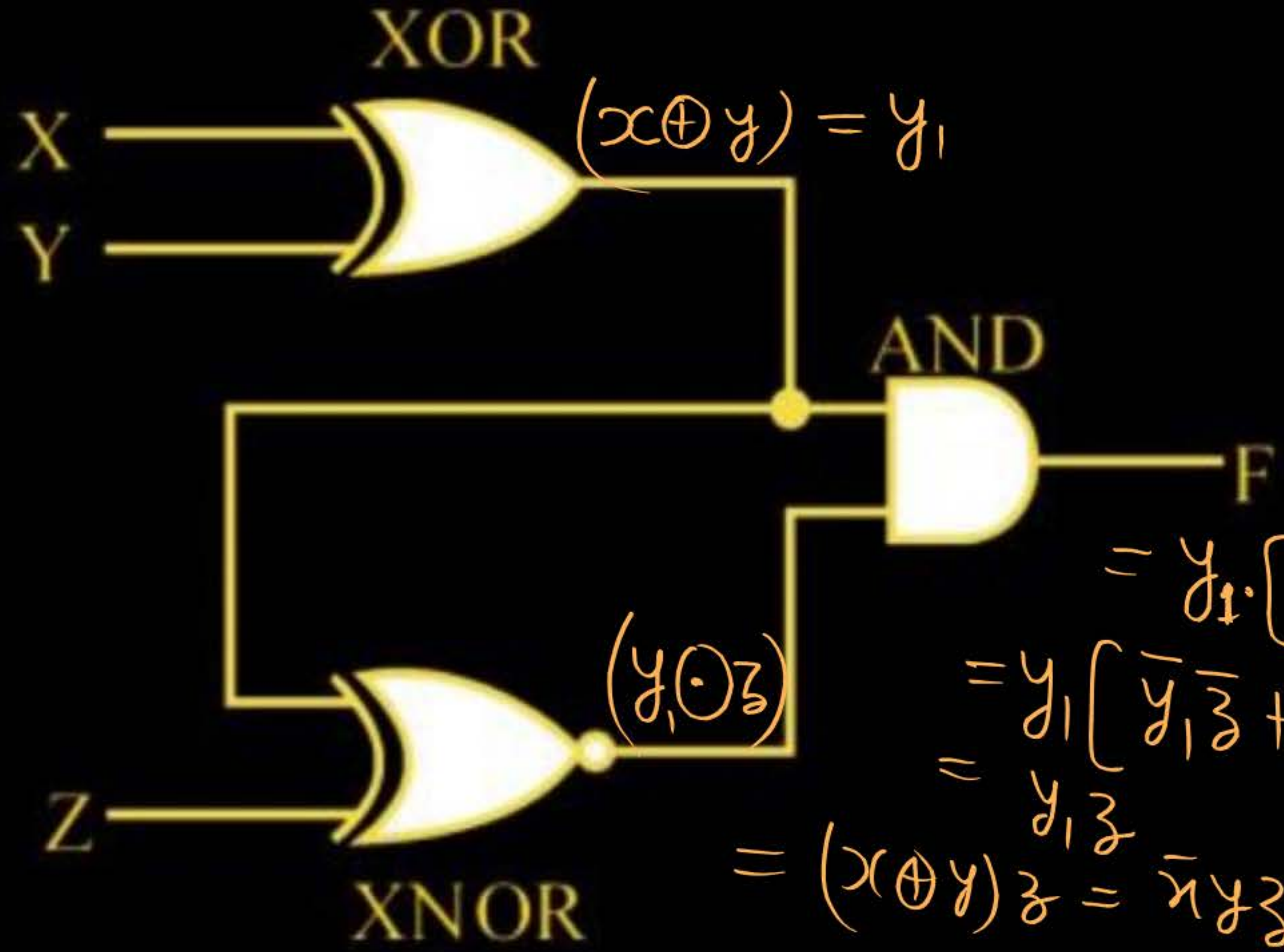
The output F in the digital logic circuit shown in the figure is

(a) $F = \bar{X}YZ + X\bar{Y}Z$

(b) $F = \bar{X}\bar{Y}Z + X\bar{Y}\bar{Z}$

(c) $F = \bar{X}\bar{Y}Z + XYZ$

(d) $F = \bar{X}\bar{Y}\bar{Z} + XYZ$



$= y_1 \cdot [y_1 \odot z]$

$= y_1 [\bar{y}_1 \bar{z} + y_1 z]$

$= y_1 z$

$= (x \oplus y) z = \bar{x} y z + x \bar{y} \cdot z$

[**Question**]

The Boolean expression for the truth table shown below is

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(a) $B(A+C)(\bar{A}+\bar{C})$

(b) $B=(A+\bar{C})(\bar{A}+C)$

(c) $\bar{B}(A+C)(\bar{A}+C)$

(d) $\bar{B}(A+C)(\bar{A}+\bar{C})$

$= \bar{A}BC + AB\bar{C}$

$= B[\underline{\bar{A}C + A\bar{C}}]$

$= B[\bar{A}+\bar{C}][A+$

$\Sigma(3,6)$

[MCQ]

Given the function F = P' + QR, wehre F is a function in three Boolean variables P,Q and R and P' = !P, consider the following statements.

(S1) $F = \sum(4, 5, 6)$

(S2) $F = \sum(0,1,2,3,7)$

(S3) $F = \prod(4, 5, 6)$

(S4) $F = \pi(0,1,2,3,7)$

$= \bar{P} + QR$

	Q	R	
0	1	1	$\to 3$
1	1	1	$\to 7$

$\bar{P}$			
0	0	0	$\to 0$
0	0	1	$\to 1$
0	1	0	$\to 2$
0	1	1	$\to 3$

Which of the following is true?

[GATE-2015-CS: 2M]

A. (S1) – False, (S2) – True, (S3) – True, (S4) -False

B. (S1) – True, (S2) – False, (S3) – False, (S4) -True

C. (S1) – False, (S2) – False, (S3) – True, (S4) –True

D. (S1) – True, (S2) – True, (S3) – False, (S4) -False

[MCQ]

The truth table

X	Y	F(X, Y)
0	0	0
0	1	0
1	0	1
1	1	1

H.W.

represents the Boolean function

[GATE-2012-CS: 1M]

(A) x

(B) x+y

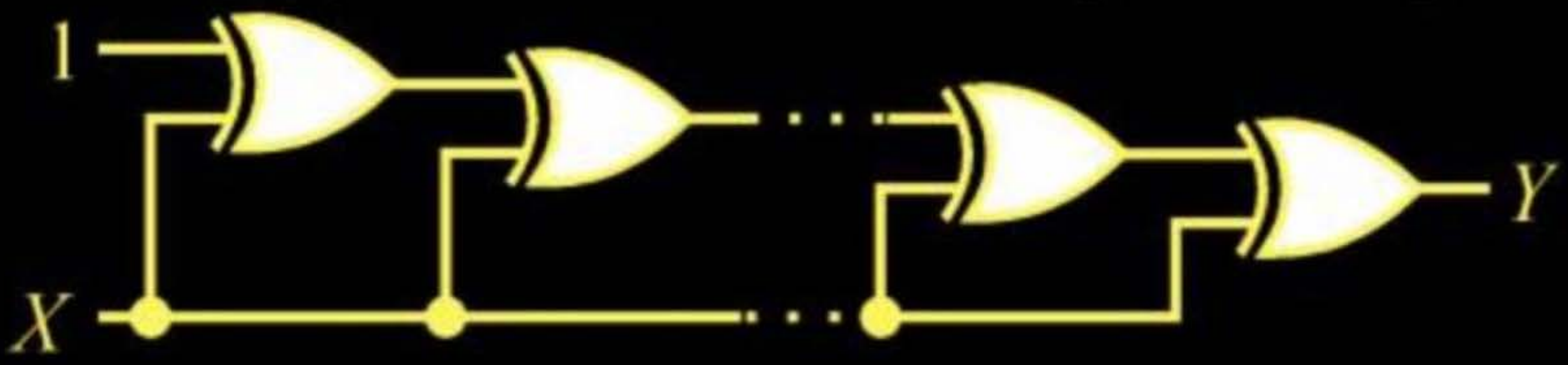
(C) $x \oplus y$

(D) y

[**Question**]

If the input to the digital circuit consisting of a cascade of 20 X-OR gates is X, then the output Y is equal to

H.W.



(a) 0

(b) 1

(c) $\overline{X}$

(d) X

[MCQ]

The simplified SOP (sum of product) form of the Boolean expression

$(P+\bar{Q}+\bar{R})\cdot(P+\bar{Q}+R)\cdot(P+Q+\bar{R})$ [GATE-2011-CS: 1M]

$0 \quad 1 \quad 1 \qquad 0 \quad 1 \quad 0 \qquad 0 \quad 0 \quad 1$

$f(P,Q,R) = \pi(1,2,3) = \Sigma(0,4,5,6,7)$

(A) $(\bar{P}.Q+\bar{R})$

(B) (P + Q .‘R’)

(C) (P’.Q + R)

(D) (P.Q + R)

$= [(P+\bar{Q}) + \bar{R}\cdot R][P+Q+\bar{R}]$

$= (P+\bar{Q})(P+Q+\bar{R})$

$= P + \bar{Q}(Q+\bar{R})$

$= P + \bar{Q}\bar{R}$

	$\bar{Q}\bar{R}$	$\bar{Q}R$	QR	$Q\bar{R}$
$\bar{P}$	1			
P	1	1	1	1

$= P + \bar{Q}\bar{R}$

Question

A Boolean function f of two variable x and y is defined as follows

$$f(0, 0) = f(0,1) = f(1,1)=1; f(1,0) = 0 \quad \boxed{\text{H.W}}$$

Assuming complements of x and y are not availed, the minimum cost solution for realizing f using only 2-input NOR gates and 2-input OR gates (each having unit cost) would have a total cost of

(a) 1 unit
(b) 4 unit
(c) 3 unit
(d) 2 unit

[MCQ]

The minterm expansion of $f(P, Q, R) = PQ + Q\overline{R} + P\overline{R}$ is [GATE-2011-CS: 1M]

H.W.

(A) $m_2 + m_4 + m_6 + m_7$

(B) $m_0 + m_1 + m_3 + m_5$

(C) $m_0 + m_1 + m_6 + m_7$

(D) $m_2 + m_3 + m_4 + m_5$

Question

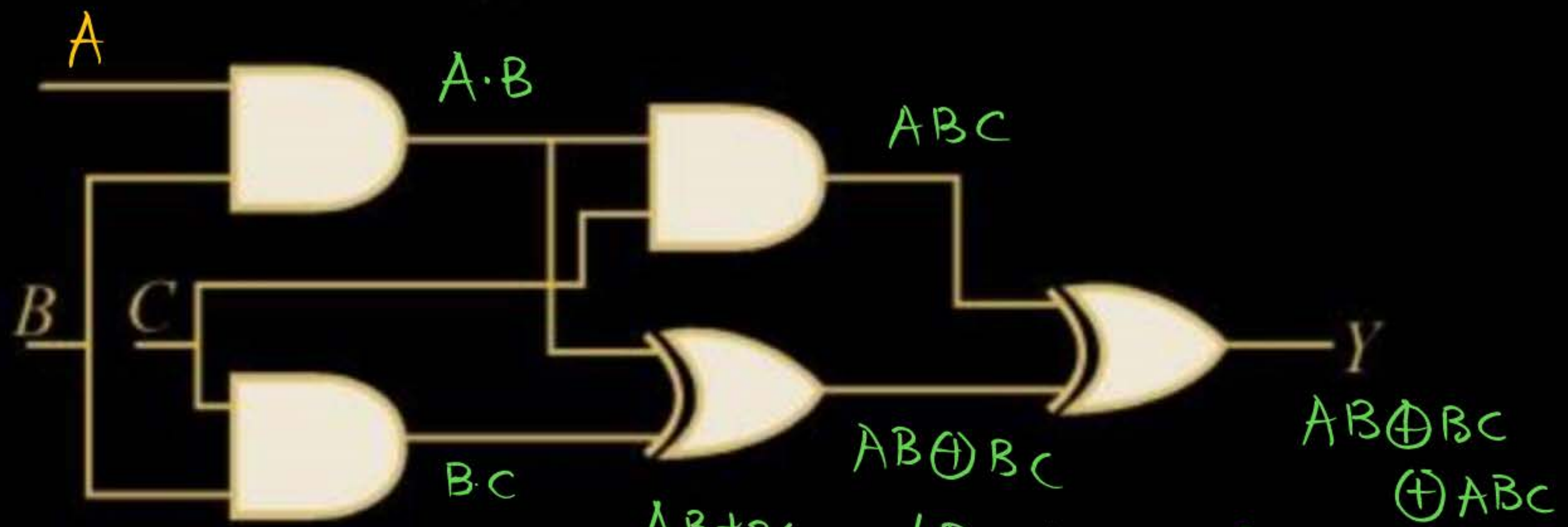
The input of the combinational circuit given below is

(a) A + B + C

(b) A(B + C)

(c) B(C + A)

(d) C(A + B)



$AB \oplus BC \oplus ABC$

$= B[A \oplus C \oplus AC]$

$= B[A+C]$

$AB+BC = (P+Q) = P \oplus Q \oplus P \cdot Q$

$B(A+C)$

$P \cdot Q = AB \cdot BC = ABC$

$$AB \oplus BC \odot ABC$$

$$= \overline{AB \oplus BC \oplus ABC}$$

$$= \overline{B(A \oplus C \oplus AC)}$$

$$= \overline{B \cdot (A + C)}$$

$$= \overline{B} + \overline{(A + C)}$$

$$= \overline{B} + \overline{A}\,\overline{C}$$

[MCQ]

If P, Q, R are Boolean variables, then (P + Q′) (P.Q′ + P.R) (P′.R′ + Q′) Simplifies to

[GATE-2008-CS: 1M]

(A) $P.\overline{Q}$

(B) $P.\overline{R}$

(C) $P.\overline{Q}+R$

(D) $P.\overline{\overline{R}}+Q$

$$= \left[\overline{Q}+P(\overline{P}\,\overline{R})\right]\left[P\overline{Q}+PR\right]$$

$$= \overline{Q}\left[P\overline{Q}+PR\right]$$

$$= P\,\overline{Q}\left[\overline{Q}+R\right]$$

$$= P\overline{Q}$$

$$P\left[P+\overline{Q}\right]\left[\overline{Q}+R\right]\left[\overline{P}\,\overline{R}+\overline{Q}\right]$$

$$P\left[\overline{Q}+(R)(\overline{P}\,\overline{R})\right]$$

$$= P\overline{Q}$$

[MSQ]

Let, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator. Which one of the following $\boxed{\text{must always be TRUE?}}$

[GATE-2022-CS: 1M]

$\rightarrow 0, 2, 4$

$\boxed{0, 1, 2, 3, 4}$

(A) $x_1x_2x_3x_4 = 0$ (0, 2, 4)

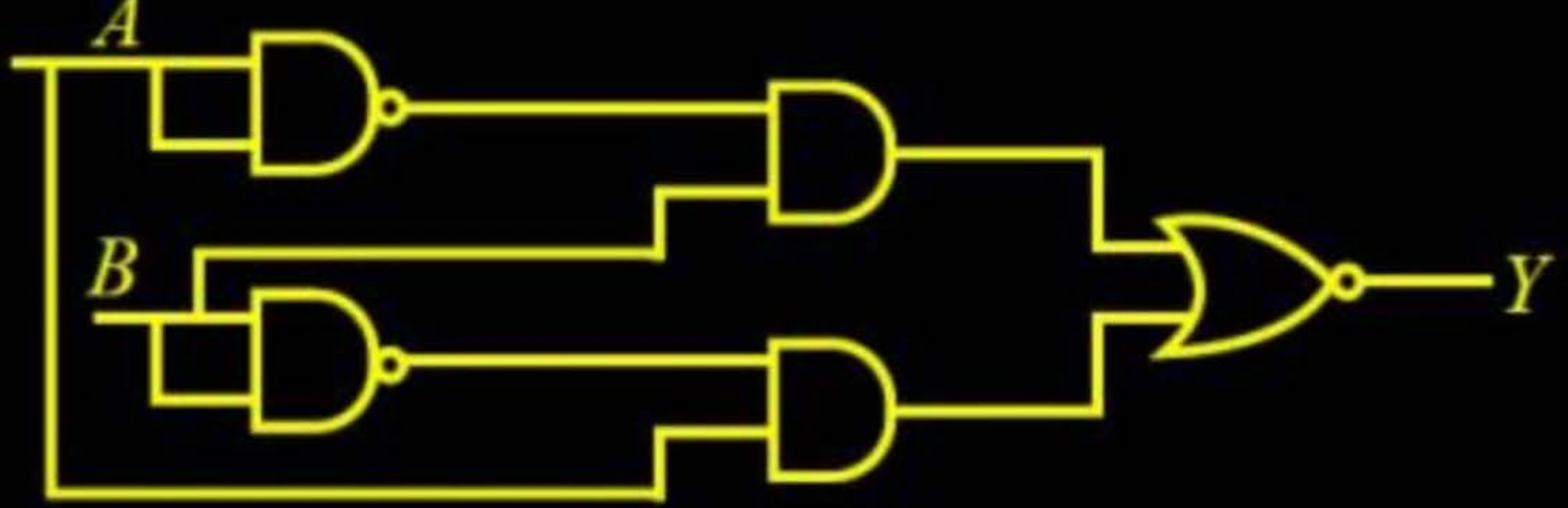
(B) $x_1x_3 + x_2x_4 = 0$ (0, 2, 4)

(C) $\overline{x_1} \oplus \overline{x_3} = \overline{x_2} \oplus \overline{x_4}$ $\Rightarrow x_1 \oplus x_3 = x_2 \oplus x_4 \longrightarrow 0, 2, 4$

(D) $x_1 + x_2 + x_3 + x_4 = 0$ (0, 2, 4)

[**Question**]

A circuit is as given below:



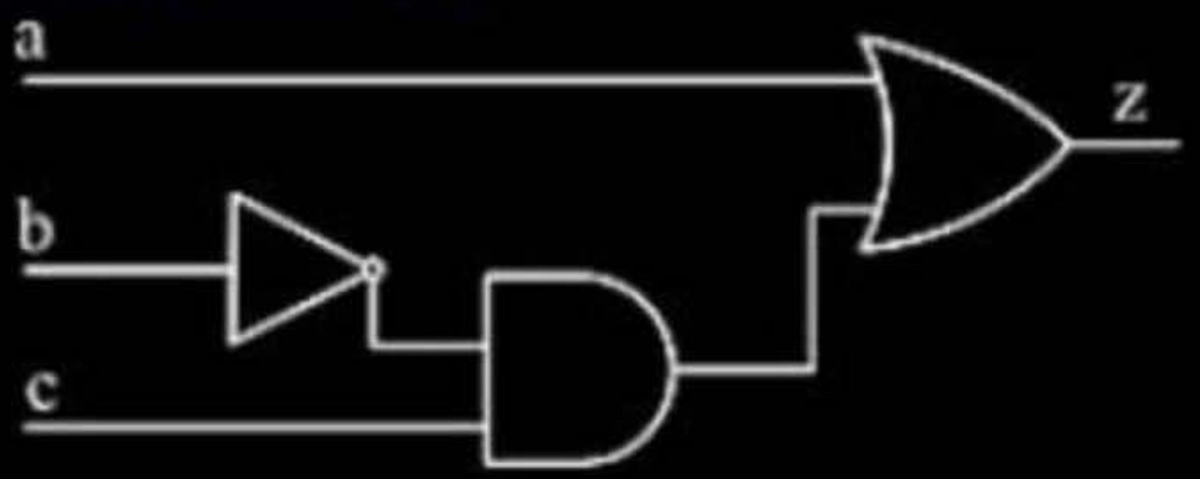
H.W.

If all the gates are replaced by its dual gates in above circuit then output Y will changes to Y_1 and to implement Y_1 minimum no. of two input NAND gate required is ______.

[MCQ]

Consider the Boolean function z(a,b,c). Which one of the following minterm lists represents the circuit given above?

[GATE-2020-CS: 2M]



H.W.

(A) $Z = \Sigma(0, 1, 3, 7)$

(B) $Z = \Sigma(2, 4, 5, 6, 7)$

(C) $Z = \Sigma(1, 4, 5, 6, 7)$

(D) $Z = \Sigma(2, 3, 5)$

[MCQ]

Which one of the following is NOT a valid identity?

[GATE-2019-CS: 1M]

(A) $(x + y) \oplus z = x \oplus (y + z)$ ✗

(B) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ ✓

(C) $x \oplus y = x + y$, if $xy = 0$ ✓

(D) $x \oplus y = (xy + x'y')'$ ✓

$= \overline{x \odot y}$

$$(x \oplus y) \oplus z = x \oplus (y \oplus z)$$

$$(x+y) \oplus z \neq x \oplus (y+z)$$

$x \oplus y = x+y$, if $\boxed{x \cdot y = 0}$

x	y	$x \oplus y$	$x+y$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	1

0	0
0	1
1	0

[NAT]

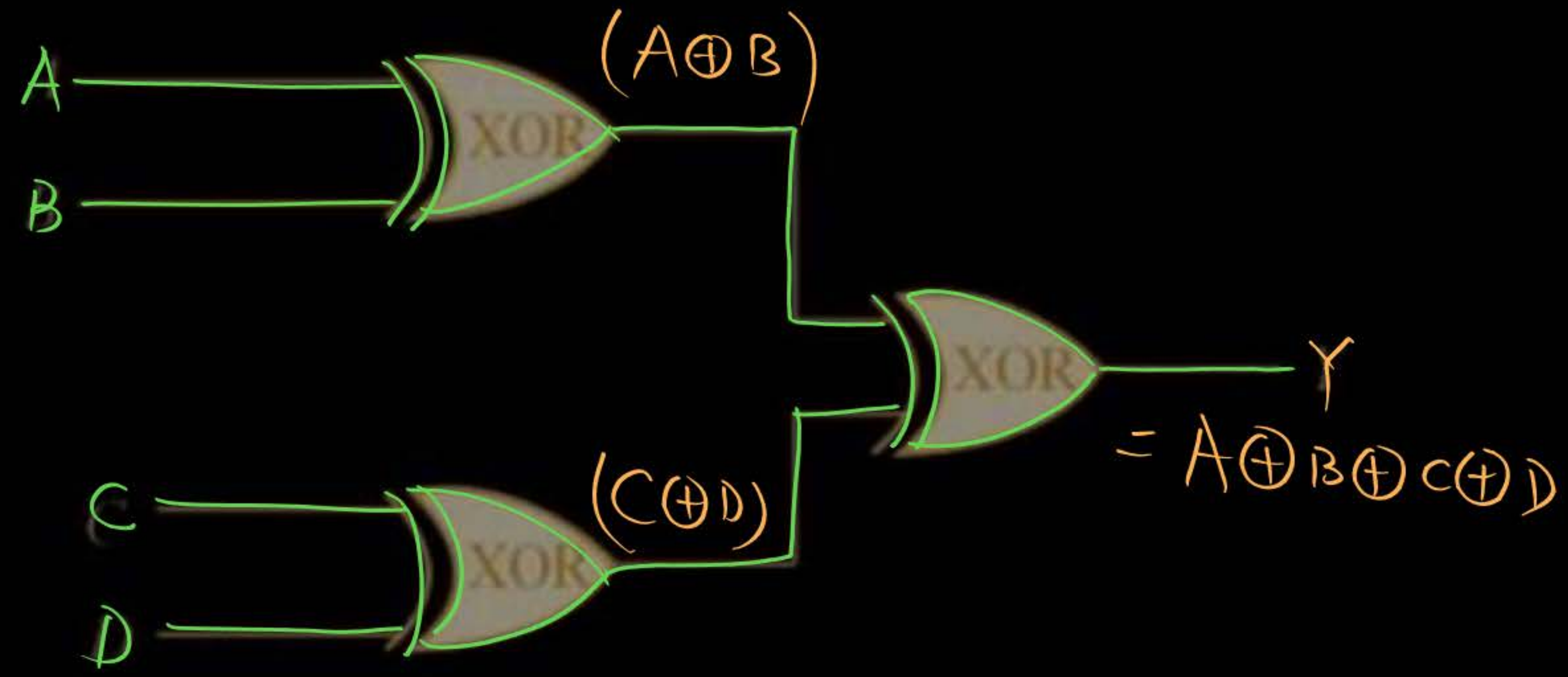
What is the minimum number of 2-input NOR gates required to implement a 4-variable function expressed in sum-of-minterms form as $f = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$? Assume that all the inputs and their complements are available.______

[GATE-2019-CS: 2M]

H.W.

Question

A, B, C and D are input bits and Y is the output bit into the XOR gate circuit of the figure below. Which of the following statements about the sum S of A, B, C, D and Y is correct?



(a) S is always either zero or odd.

(b) S is always either zero or even.

(c) S = 1 only if the sum of A, B, C and D is even.

(d) S = 1 only if the sum of A, B, C and D is odd.

$S = A+B+C+D+\underline{\underline{Y}}$

$Y = A \oplus B \oplus C \oplus D$

$\underline{\underline{0}}, 1, 2, 3, 4$

$\Rightarrow S = 0 \quad (0)$

$\Rightarrow S = 2 \quad (1)$

$\Rightarrow S = 2 \quad (2)$

$\Rightarrow S = 4 \quad (3)$

$\Rightarrow S = 4 \quad (4)$

$f(A, B, C, D) = A+B+C+D+Y$

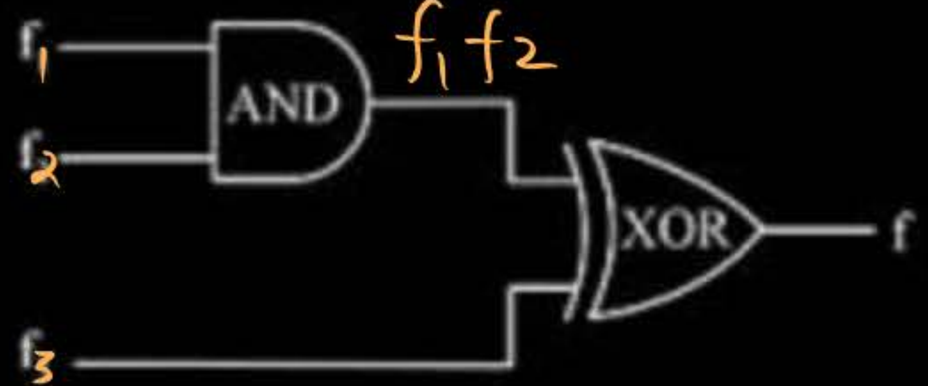
[MCQ]

Consider three 4-variable functions f1, f2 and f3, which are expressed in sum-of-minterms as

$f_1 = \Sigma(0, 2, 5, 8, 14)$, $f_2 = \Sigma(2, 3, 6, 8, 14, 15)$, $f_3 = \Sigma(2, 7, 11, 14)$

For the following circuit with one AND gate and one XOR gate, the output function f can be expressed as:

[GATE-2019-CS: 2M]



$= f_1 f_2 \oplus f_3 = f_4 \oplus f_3$

(A) $\Sigma(2, 14)$

(B) $\Sigma(7, 8, 11)$

(C) $\Sigma(2, 7, 8, 11, 14)$

(D) $\Sigma(0, 2, 3, 5, 6, 7, 8, 11, 14, 15)$

$f_4 = \Sigma(2, 8, 14)$

$f = f_4 \oplus f_3 = \Sigma(7, 8, 11)$

[MCQ]

Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is $\boxed{\text{NOT CORRECT?}}$ [GATE-2018-CS: 1M]

(A) $\overline{P \oplus Q} = P \odot Q$

(B) $\overline{P} \oplus Q = P \odot Q$

(C) $\overline{P} \oplus \overline{Q} = P \oplus Q$

(D) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

$= \overline{P \oplus P \oplus Q}$

$= P \odot P \odot Q$

$= P \oplus P \oplus Q$

[MCQ]

Let $\oplus$ denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q:

$$F(P, Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$

The equivalent expression for F is

[GATE-2014-CS: 1M]

H.W.

(A) $P + Q$

(B) $(P + Q)'$

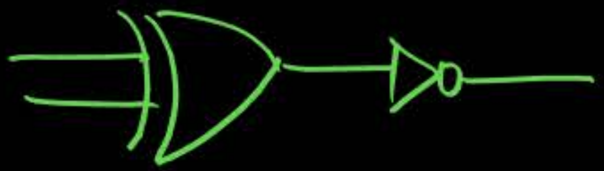
(C) $P \oplus Q$

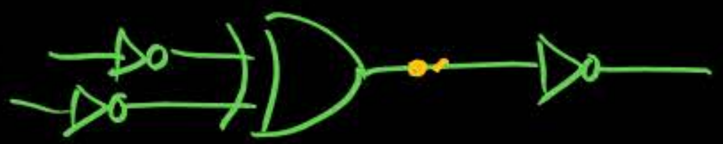
(D) $(P \oplus Q)'$

[MCQ]

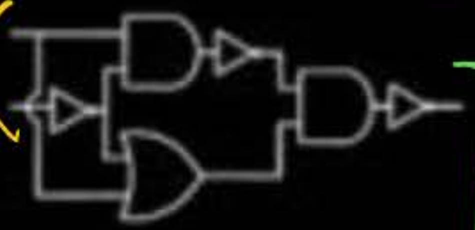
Which one of the following circuits is NOT equivalent to a 2-input X-NOR (exclusive NOR) gate?

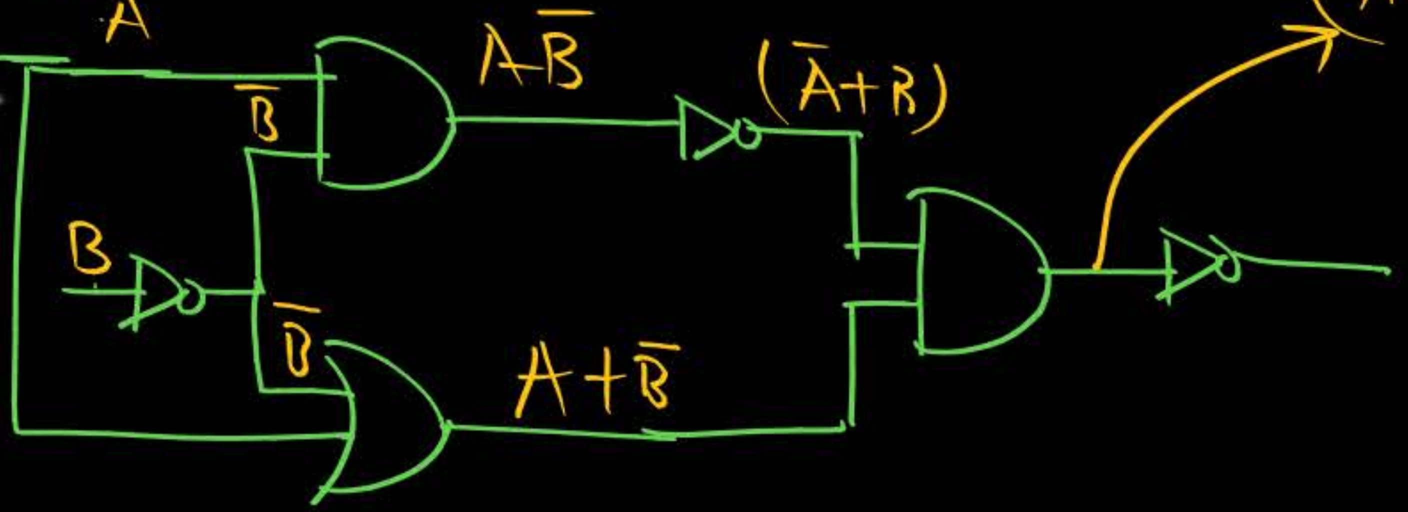
[GATE-2011-CS: 1M]

(A)  

(B)  

(C)  

(D) 



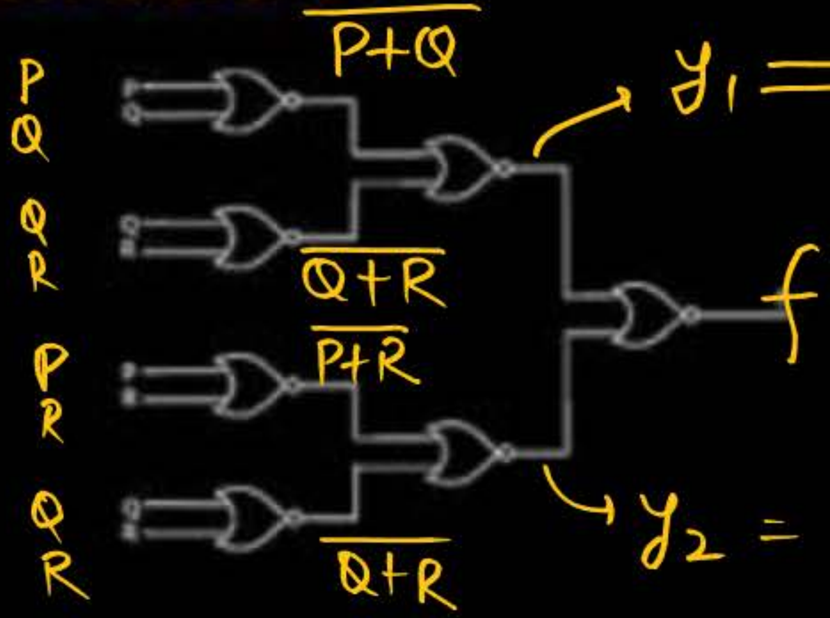
$(\bar{A}+B)(A+\bar{B})$

$= A \odot B$

$A \oplus B$

[MCQ]

What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below? [GATE-2010-CS: 1M]



$y_1 = (P+Q)(Q+R) = (Q+PR)$

$y_2 = (P+R)(Q+R)$

$= (R+PQ)$

$f = \overline{[(P+Q)(Q+R) + (P+R)(Q+R)]}$

$= \overline{(Q+R)[(P+Q) + P+R]}$

$= \overline{(Q+R)[P+Q+R]}$

$= \overline{X[X+P]}$

$= \overline{Q+R}$

$\overline{Q+PR+R+PQ}$

$= \overline{Q+R}$

(A) $\overline{Q+R}$ ✓

(B) $\overline{P+Q}$

(C) $\overline{P+R}$

(D) $\overline{P+Q+R}$

[MCQ]

What is the minimum number of gates required to implement the Boolean function (AB + C) if we have to use only 2-input NOR gates? [GATE-2009-CS: 1M]

H.W

(A) 2

(B) 3

(C) 4

(D) 5

[MCQ]

Given f1,f3 and f in canonical sum of products from (in decimal) for the circuit.

[GATE-2008-CS: 1M]

H.W.

$F_1 = \sum m\,(4,5,6,7,8)$

$F_3 = \sum m\,(1, 6, 15)$

$F = \sum m\,(1, 6, 8, 15)$

Then f_2 is

(A) $\sum m\,(4, 6)$

(B) $\sum m\,(4, 8)$

(C) $\sum m\,(6, 8)$

(D) $\sum m\,(4, 6, 8)$

[NAT]

Consider the Karnaugh map given below, where X represents "don't care" and blank represents 0.

dc \ ba	00	01	11	10
00		x	x	
01	1			x
11	1			1
10		x	x	

H.W.

Assume for all inputs(a, b, c, d) , the respective complements $(\bar{a}, \bar{b}, \bar{c}, \bar{d})$ are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is ______.

[GATE-2017-CS: 1M]

[MCQ]

Given $f(w, x, y, z) = \Sigma m(0, 1, 2, 3, 7, 8, 10) + \Sigma d(5, 6, 11, 15)$, where d represents the don't-care condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of $f(w, x, y, z)$? [GATE-2017-CS: 1M]

(A) $f = (\overline{w} + \bar{z})(\bar{x} + z)$

H.W.

(B) $f = (\overline{w} + z)(x + z)$

(C) $f = (w + z)(\bar{x} + z)$

(D) $f = (w + \bar{z})(\bar{x} + z)$

[MCQ]

Consider the following Boolean expression for

F: $F(P, Q, R, S) = PQ + P'QR + P'QR'S$

The minimal sum-of-products form of F is [GATE-2014-CS: 1M]

H.W.

(A) $PQ + QR + QS$

(B) $P + Q + R + S$

(C) $\overline{P} + \overline{Q} + \overline{R} + \overline{S}$

(D) $\overline{P}R + \overline{P}\overline{R}S + P$

Question

A logical function f(A, B, C) is given as $f(A,B,C)=\overline{A\oplus B\oplus C}+(A\oplus B)\overline{C}$

Then minimum no. of 2-input NAND gate required to implement f(A, B, C) will be ______.

$\boxed{\text{H.W}}$

2 Minute Summary

→ Practic session.

Thank you

GW Soldiers !