COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 07

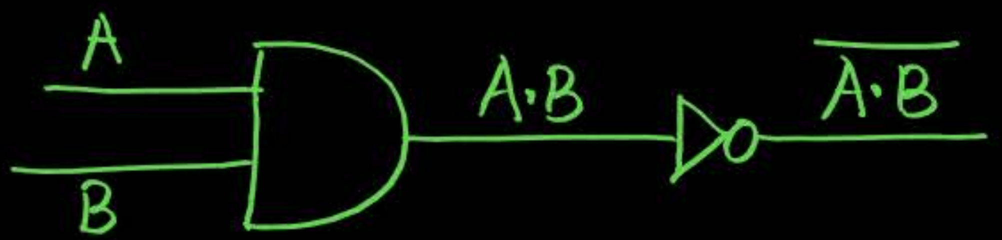
BOOLEAN THEOREMS AND GATES

By- Chandan Gupta Sir

Arithematic gates

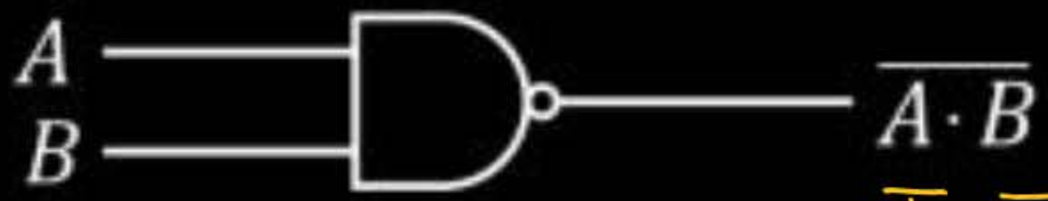
- Universal gates

[NAND GATE]



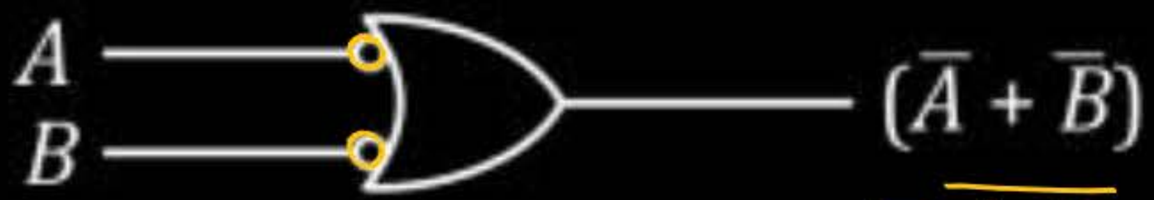
$\equiv$ $\boxed{\text{AND + NOT}}$

Representation :

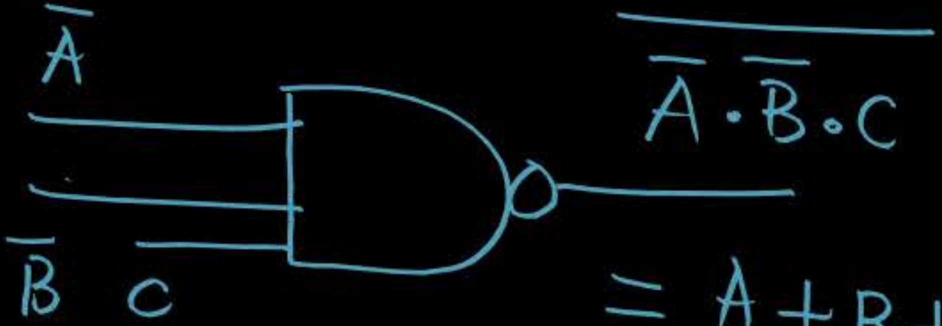


$= \bar{A} + \bar{B}$

$\equiv$



$= \overline{A \cdot B}$



$= A + B + \bar{C}$

	A	B	$y = \overline{A \cdot B}$
0	0	0	1
1	0	1	1
2	1	0	1
3	1	1	0

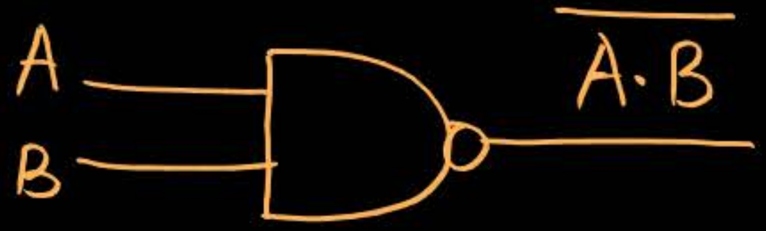
$y(A, B) = \Sigma(0, 1, 2)$

$= \pi(3)$

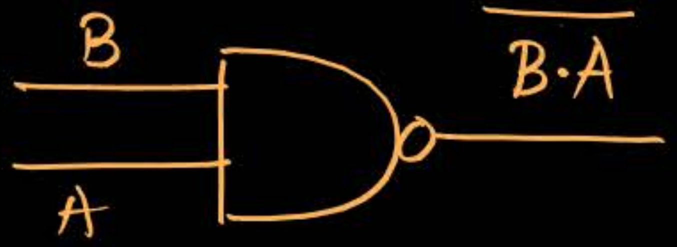
$= (\bar{A} + \bar{B})$

$= \overline{A \cdot B}$

- Commutative Law :



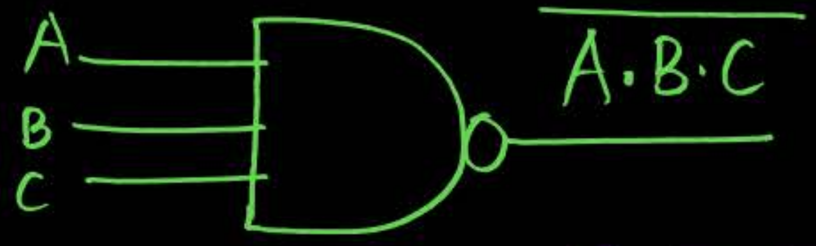
$\equiv$



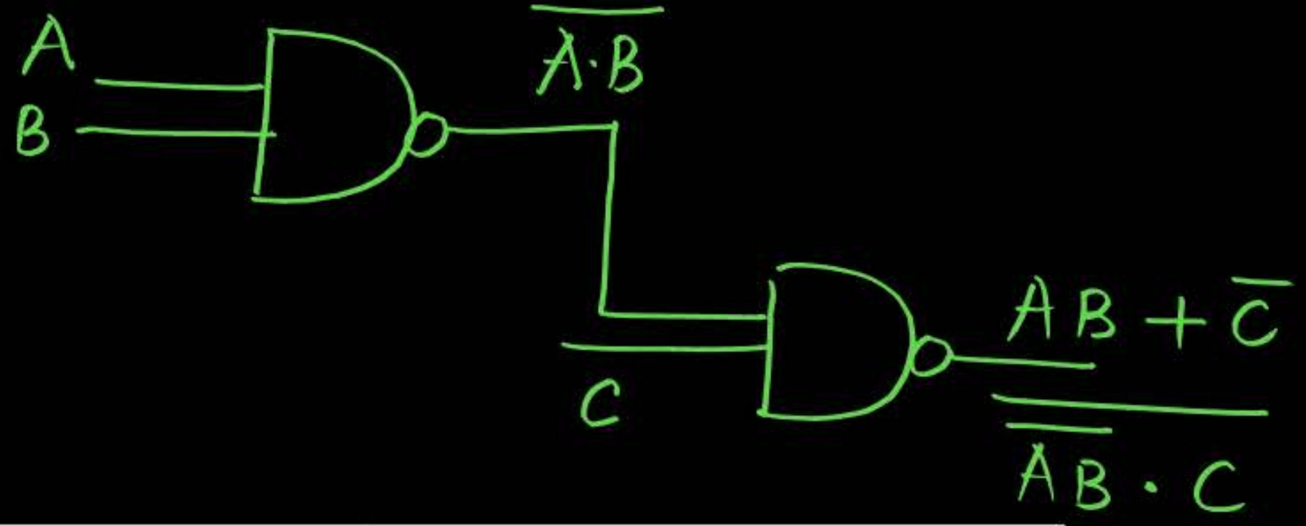
$= \overline{A\cdot B}$

$\boxed{\text{It holds commutative law} \longrightarrow \text{position of variables is irrelevant.}}$

- Associative Law :



$= \bar{A}+\bar{B}+\bar{C} \neq$



$\boxed{\text{It does not hold associative law.}}$

- IMP Points :

- If any one of the i/p line is at logic '0' then irrespective of other i/p lines, o/p will be at logic '1' for sure.

- O/p will be logic '0' only in one case when all the i/p lines will be at logic '1'.

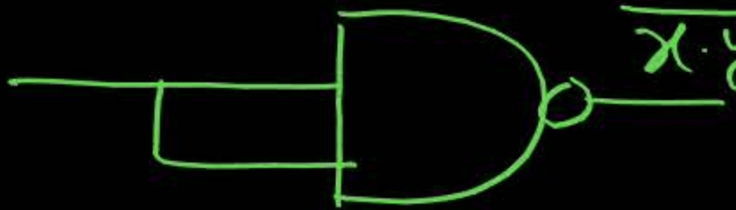
NAND As an Universal Gate

- NOT GATE :

A

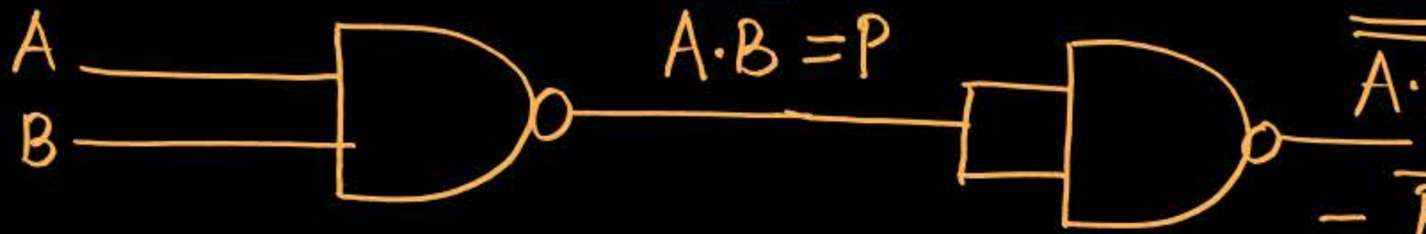
$$\overline{A\cdot A} = \overline{A}$$

$\overline{x\cdot y}$



$$\overline{\overline{x\cdot y}} = x\cdot y$$

- AND GATE :

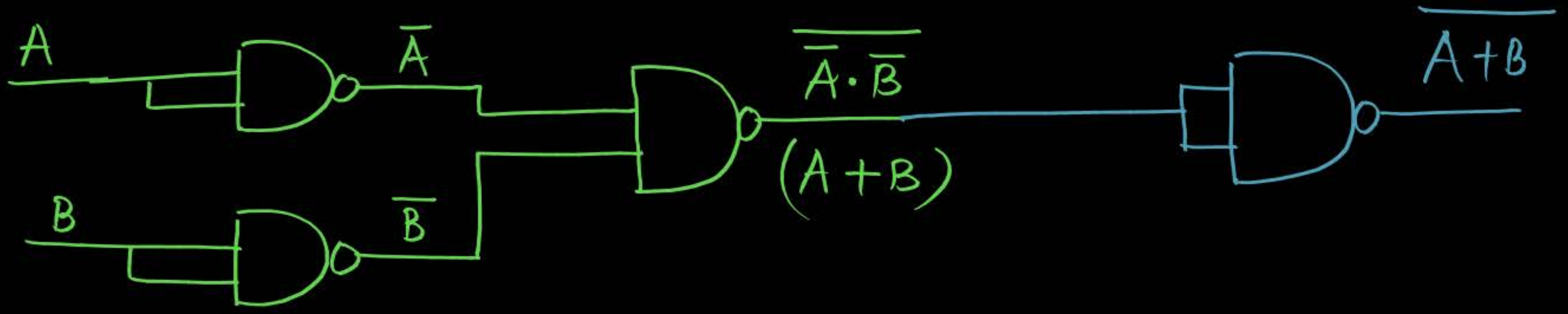


$\overline{A\cdot B} = P$

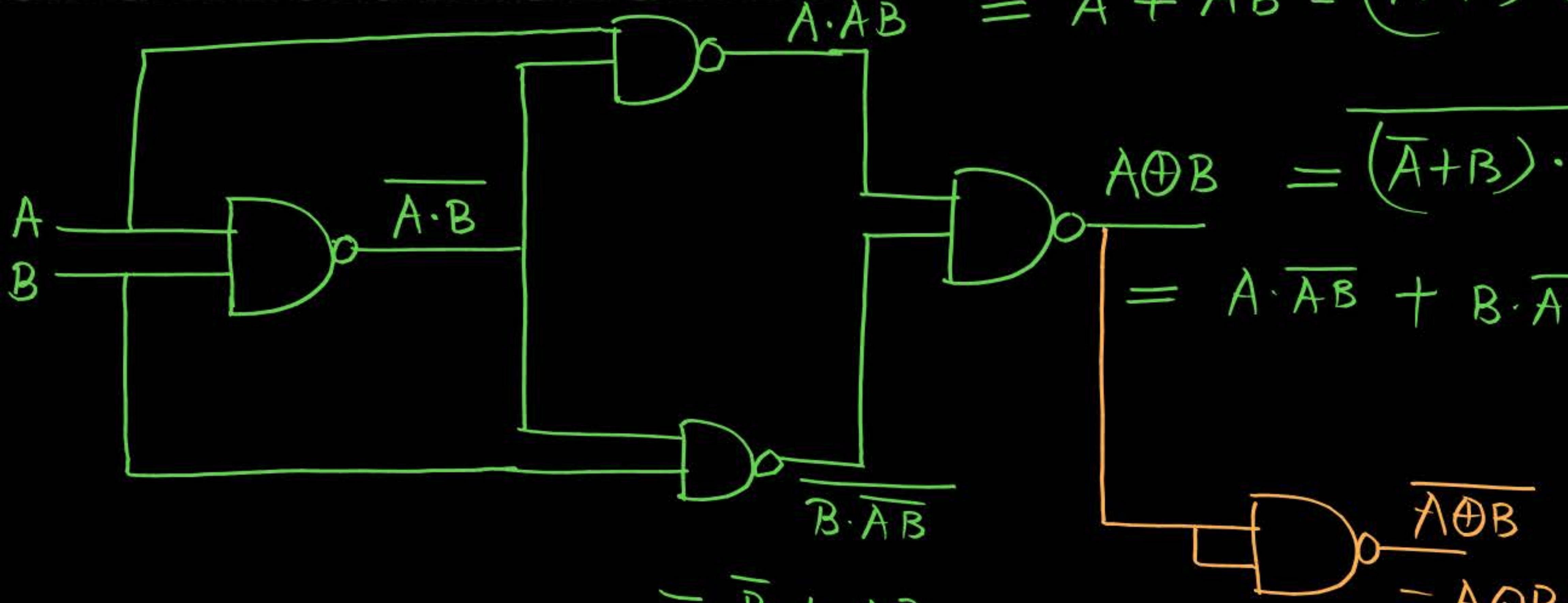
$\overline{\overline{A\cdot B}} = A\cdot B$

$= \overline{P\cdot P} = \overline{P} = \overline{\overline{AB}} = A\cdot B$

- OR GATE :



- XOR GATE and XNOR GATE :



$\overline{A\cdot\overline{AB}} = \bar{A} + AB = (\bar{A}+A)(\bar{A}+B) = (\bar{A}+B)$

$A\oplus B = \overline{(\bar{A}+B)\cdot(A+\bar{B})} = \overline{A\odot B} = (A\oplus B)$

$= A\cdot\overline{AB} + B\cdot\overline{AB} = A(\bar{A}+\bar{B}) + B(\bar{A}+\bar{B})$

$= A\bar{B} + \bar{A}B$

$= \bar{A}B + A\bar{B} = A\oplus B$

$\overline{B\cdot\overline{AB}} = \bar{B} + AB$

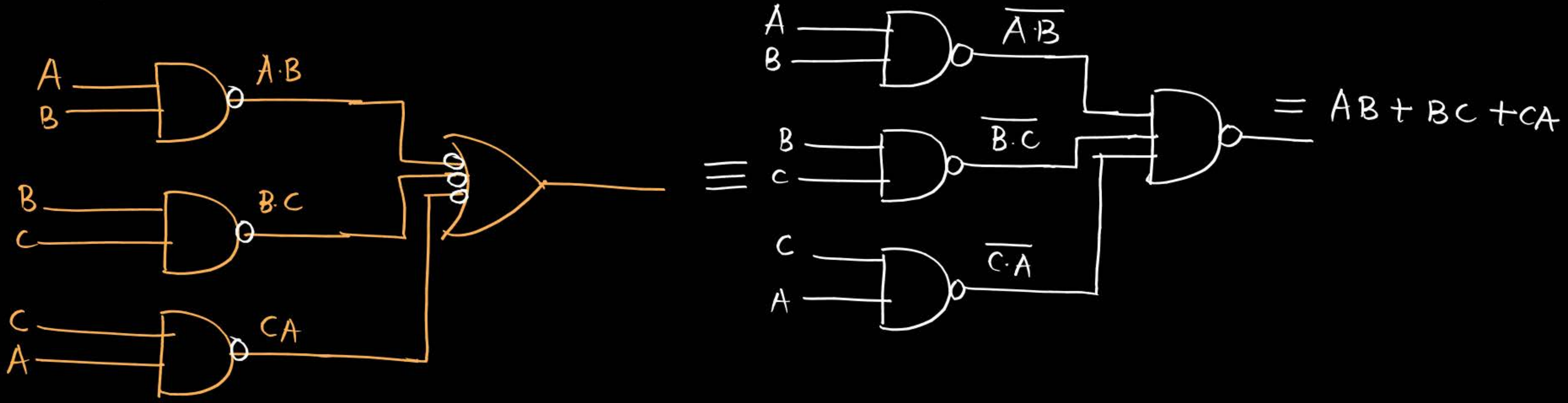
$= (\bar{B}+A)(\bar{B}+B)$

$= (A+\bar{B})$

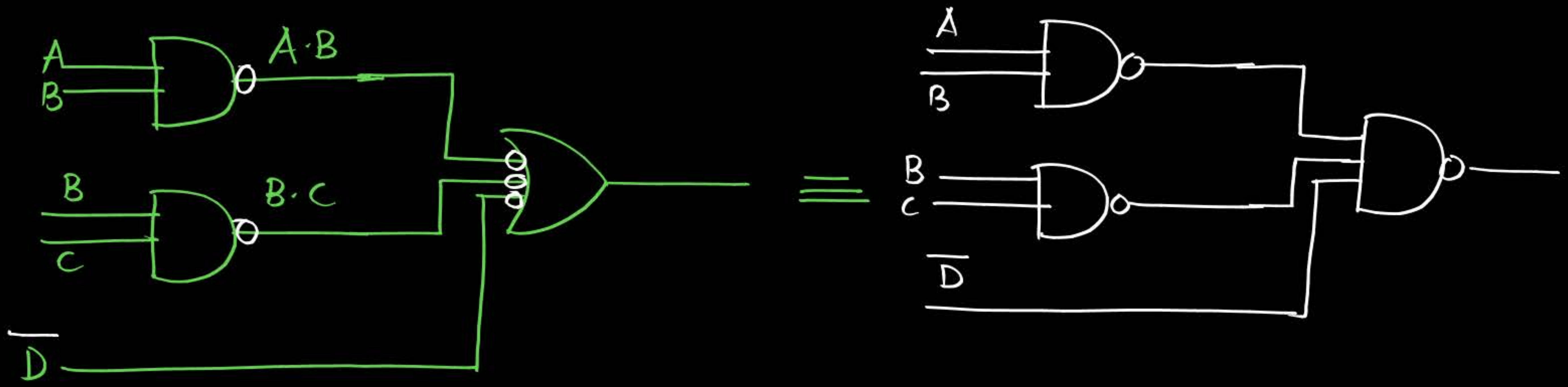
$\overline{A\oplus B} = A\odot B$

2-Stage Implementation of SOP using NAND Gate

- $y = AB + BC + CA$



- $y = AB + BC + D$

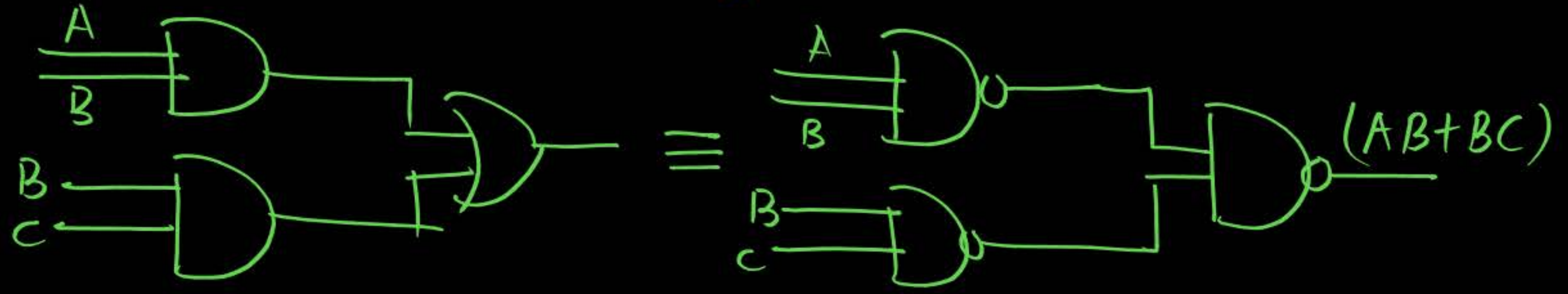


Question on minimum no. of NAND Gates

Question

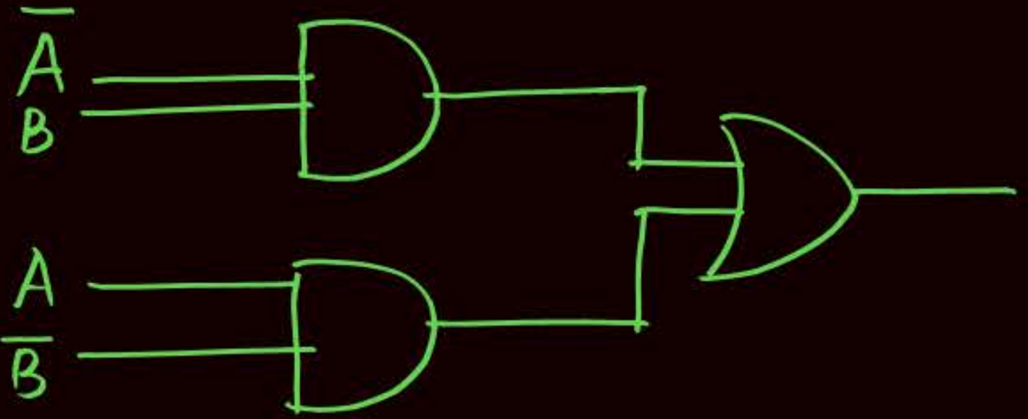
$y = AB + BC$, to implement y, minimum no. of 2-input NAND gate required is ______.

$$\left.\begin{array}{l} A\cdot B \longrightarrow 2\text{ NAND} \longrightarrow P \\ B\cdot C \longrightarrow 2\text{ NAND} \longrightarrow Q \\ P+Q \longrightarrow 3\text{ NAND} \longrightarrow y \end{array}\right\} 7\text{ NAND}$$

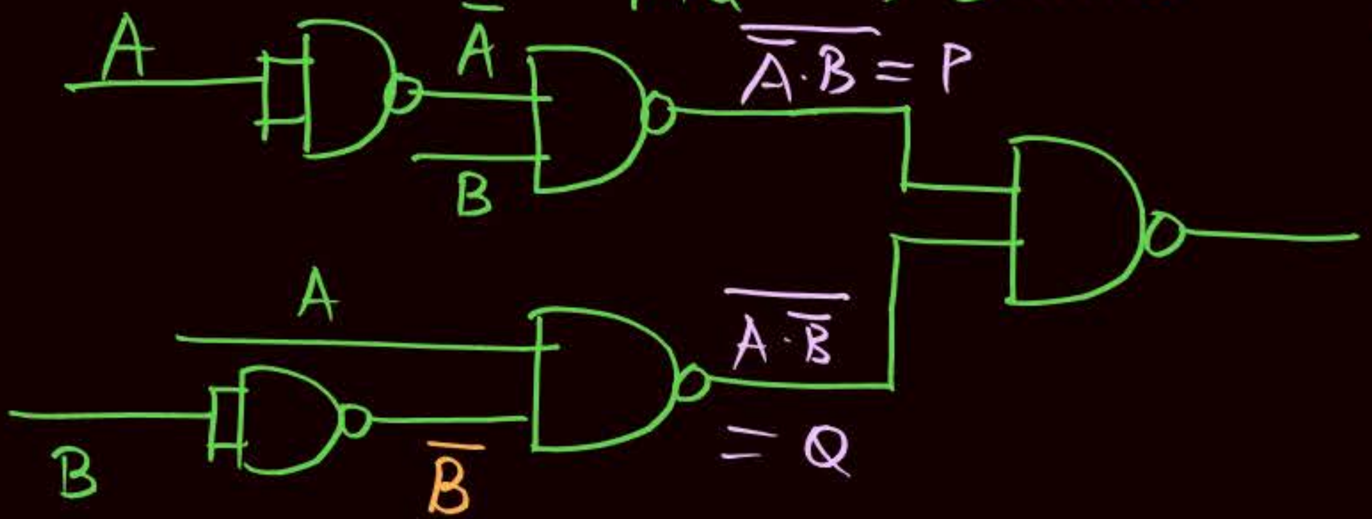


$y(A,B) = \bar{A}B + A\bar{B} = A \oplus B$,

$$\left.\begin{array}{l} \bar{A}\cdot B \longrightarrow 3\,NAND \longrightarrow P \\ A\cdot\bar{B} \longrightarrow 3\,NAND \longrightarrow Q \\ P+Q \longrightarrow 3\,NAND \longrightarrow y \end{array}\right\} 9\,NAND$$



$\equiv$

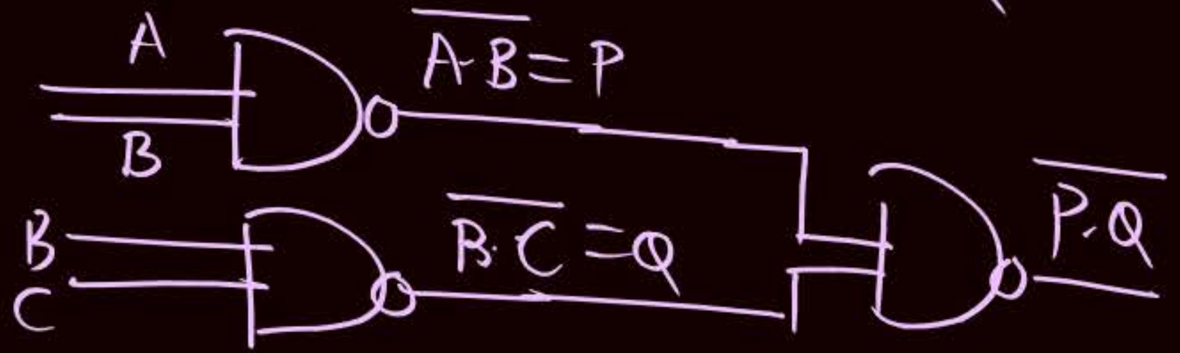


$\Longrightarrow$ 5 NAND

- $y = AB + BC$

$\bar{y} = \overline{AB + BC} = \overline{AB}\cdot\overline{BC}$

$y = \overline{\overline{AB}\cdot\overline{BC}} = \overline{P\cdot Q}$



$A\cdot B \to 2\,NAND$

$\overline{A\cdot B} \to 1\,NAND$

$A+B \to 3\,NAND$

$$\bullet \quad y = \bar{A}B + A\bar{B}$$

$$\bar{y} = \overline{\bar{A}B + A\bar{B}} = \overline{\bar{A}B} \cdot \overline{A\bar{B}}$$

$$y = \overline{\overline{\bar{A}\cdot B} \cdot \overline{A\bar{B}}} = \overline{P\cdot Q}$$

$$\left.\begin{array}{l} P = \overline{\bar{A}\cdot B} \rightarrow 2\,\text{NAND} \\ Q = \overline{A\cdot \bar{B}} \rightarrow 2\,\text{NAND} \\ y \rightarrow \overline{P\cdot Q} \rightarrow 1\,\text{NAND} \end{array}\right\} \underline{5\,\text{NAND}}$$

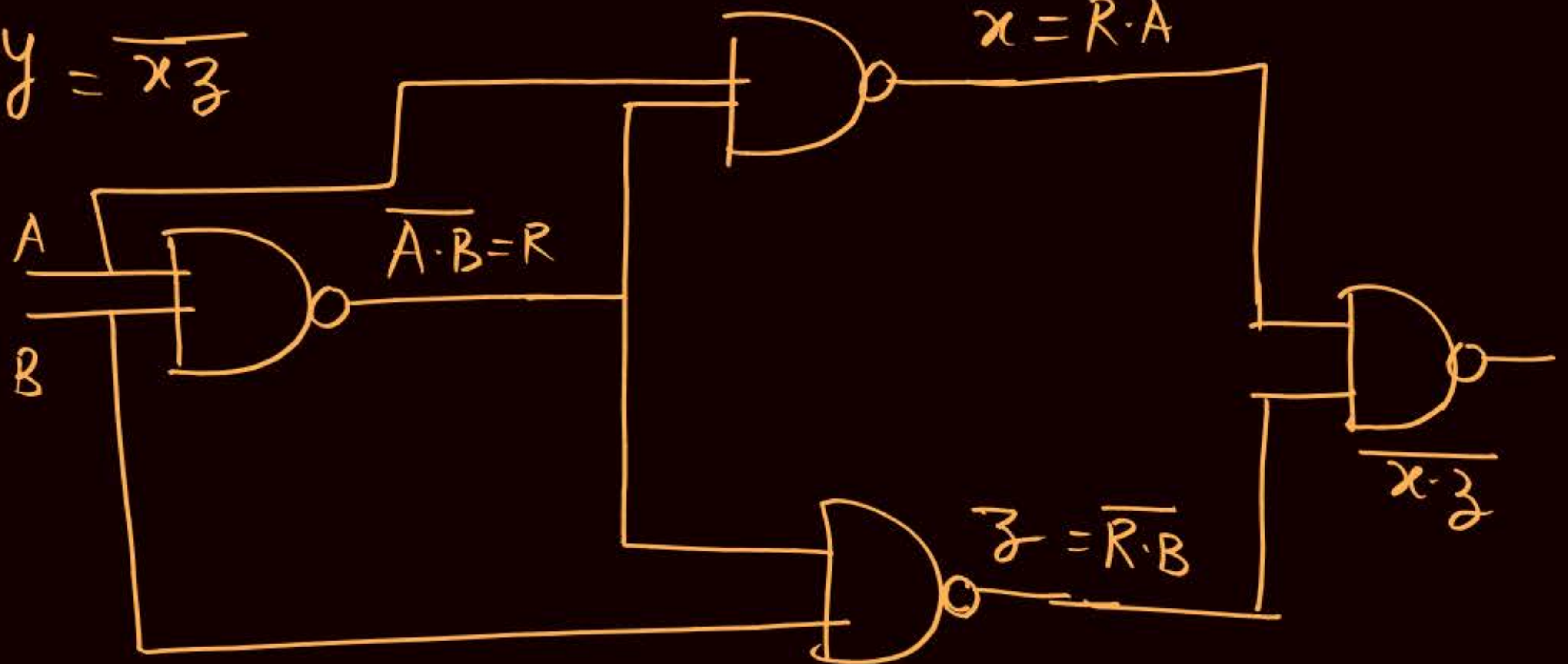
$$y = \bar{A}B + A\bar{B} = (\bar{A}+\bar{B})(A+B)$$

$$= \overline{A\cdot B}\,(A+B) = R\,(A+B)$$

$$y = R\cdot A + R\cdot B$$

$$\bar{y} = \overline{R\cdot A + R\cdot B} = \overline{R\cdot A} \cdot \overline{R\cdot B} = x\cdot z$$

$$y = \overline{x z}$$



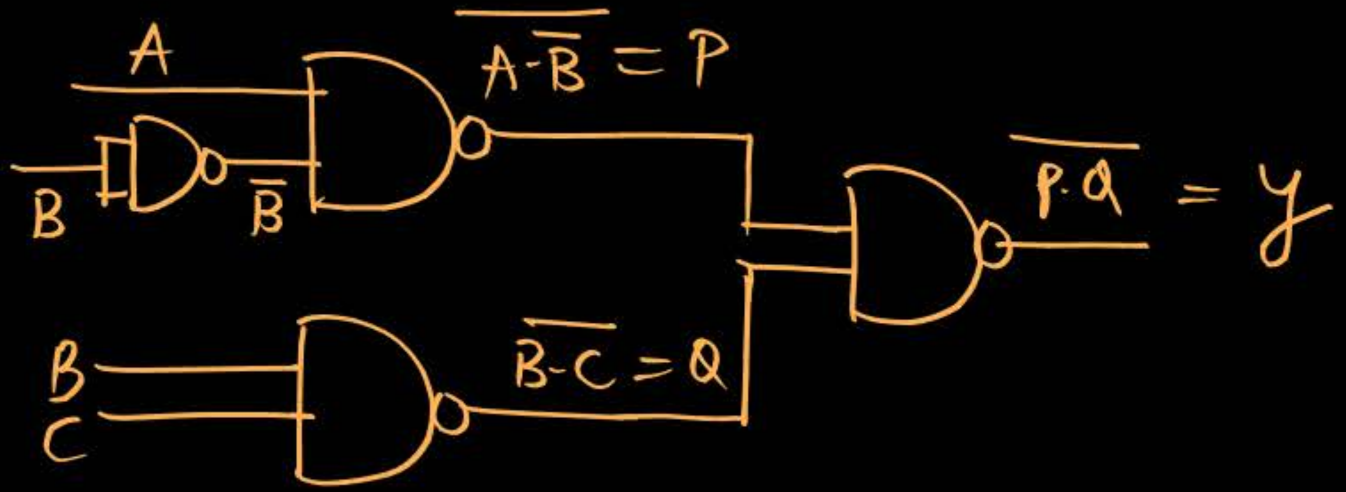
[Question]

$y = A\bar{B} + BC$

Minimum no. of 2-input NAND gates required to implement y ___4___

$\bar{y} = \overline{A\bar{B}+BC} = \overline{A\cdot\bar{B}}\cdot\overline{BC} = P\,Q$

$y = \overline{P\cdot Q}$



[**Question**]

$f(A, B, C) = \bar{A} + \bar{A}B + \bar{A}B\bar{C}$ $= \bar{A}(1 + B + B\bar{C}) = \bar{A} \cdot 1 = \bar{A} \longrightarrow$ 1NAND

Minimum no. of 2-input NAND gates required to implement y __1__

\# Q. $y = \overline{AB} \underline{\underline{+}} \overline{B\cdot C} = \bar{A} + \bar{B} + \bar{B} + \bar{C} = \bar{A} + \bar{B} + \bar{C} = \overline{A\cdot B\cdot C}$

Minimum no. of 2-i/p NAND gate required $\underline{\ \ 3\ \ }$.

$$\left.\begin{aligned} \overline{A\cdot B} &\longrightarrow 1\text{NAND} \rightarrow P \\ \overline{B\cdot C} &\longrightarrow 1\text{NAND} \rightarrow Q \\ P+Q &\longrightarrow 3\text{NAND} \rightarrow y \end{aligned}\right\} 5\text{NAND}$$

$\bar{y} = ABBC = ABC$

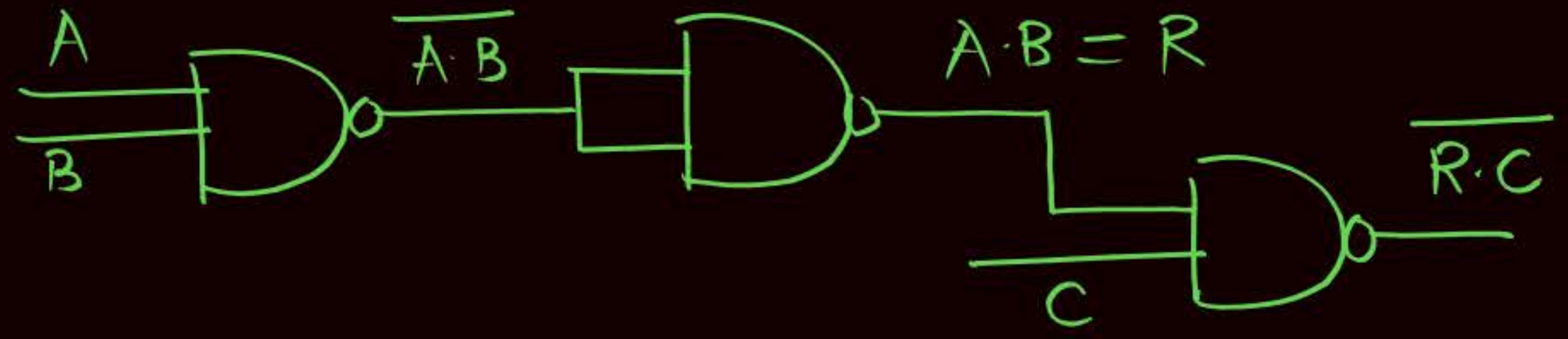
$y = \overline{ABC}$

$\overline{A\cdot B\cdot C} = \overline{R\cdot C}$

$A\cdot B \longrightarrow 2\text{NAND} \rightarrow \underline{\underline{R}}$

$R\cdot C \longrightarrow 2\text{NAND}$

$\overline{R\cdot C} \longrightarrow 1\text{NAND}$



Q. $y = \overline{AB} + \overline{CD} = \bar{A} + \bar{B} + \bar{C} + \bar{D} = \overline{A \cdot B \cdot C \cdot D}$

Minimum no. of 2-i/p NAND gate required ________.

$$\left.\begin{array}{l} \overline{A \cdot B} \longrightarrow 1\,\text{NAND} \Rightarrow P \\ \overline{CD} \longrightarrow 1\,\text{NAND} \Rightarrow Q \\ P + Q \longrightarrow 3\,\text{NAND} \end{array}\right\} 5\,\text{NAND}$$

$\overline{A \cdot B \cdot C \cdot D}$

$$\left.\begin{array}{l} A \cdot B \longrightarrow R \longrightarrow 2\,\text{NAND} \\ R \cdot C \longrightarrow S \longrightarrow 2\,\text{NAND} \\ \overline{S \cdot D} \longrightarrow 1\,\text{NAND} \end{array}\right\} 5\,\text{NAND}$$

$\bar{y} = \overline{\overline{AB} + \overline{CD}} = AB \cdot CD$

$y = \overline{ABCD}$

Question

$y = \bar{A}BC$

To implement y, minimum no. of 2-input NAND gates required is 5

$y = \bar{A}\,\underline{BC}$

$B \cdot C \longrightarrow 2\,\text{NAND} \longrightarrow P$

$\bar{A} \cdot P \longrightarrow \bar{A} \rightarrow 1\,\text{NAND} \longrightarrow \bar{A} \cdot P \rightarrow 2\,\text{NAND}$

Question

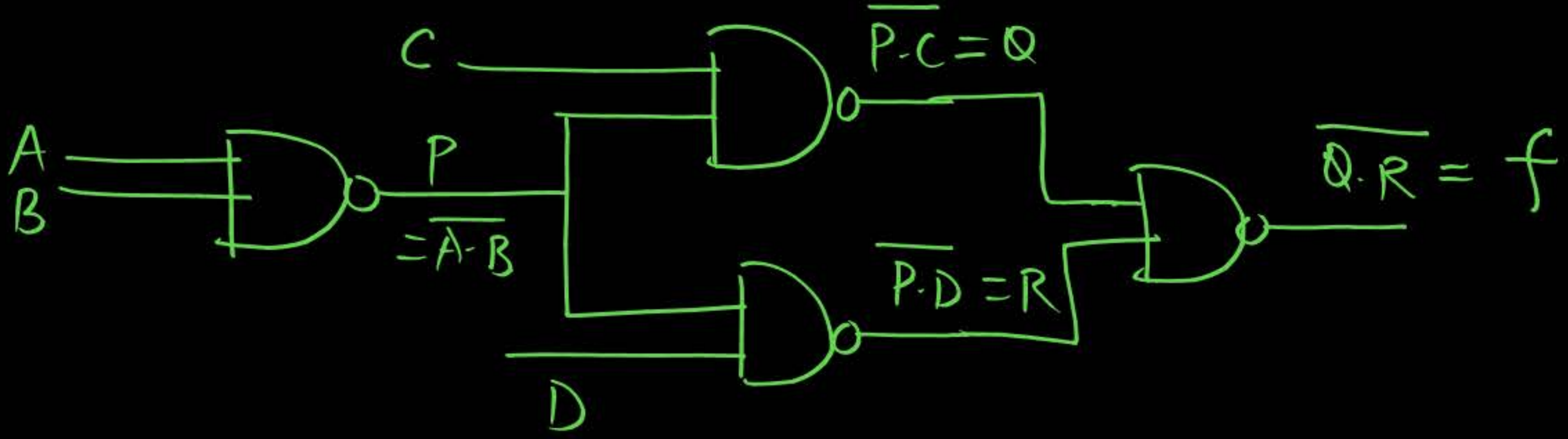
$f(A, B, C, D) = (\bar{A} + \bar{B})(C + D)$ $= \overline{A \cdot B}\,(C+D) = P(C+D) = P \cdot C + PD$

Minimum no. of 2-input NAND gates required to implement $f(A, B, C, D)$ ___4___

$f = PC + PD$

$\bar{f} = \overline{PC + PD} = \overline{PC} \cdot \overline{PD} = Q \cdot R$

$f = \overline{Q \cdot R}$



Implement following function with minimum no. of 2-i/p NAND gates:

#Q. $y = \bar{A}B + B\bar{C}$

#Q. $y = (\bar{A}+\bar{B})(\bar{C}+D)$

#Q. $y = \overline{AB} + \bar{C}D$

#Q. $y = \overline{\bar{A}BC}$

#Q. $y = AB + BC + CA$

#Q. $y = \Sigma(1,2,4,7)$

2 Minute Summary

$\hookrightarrow$ universal gates

Thank you

GW
Soldiers !