COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 01

BOOLEAN THEOREMS AND GATES

By- Chandan Gupta Sir

Boolean Theorems

$$\overset{2^2}{1}\ \overset{2^1}{1}\ \overset{2^0}{0}$$

$$(1\ 1\ 0)_2 = 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 0$$
$$= (6)_{10}$$

$\times\ (12)_2 \longrightarrow$

Binary no. system $\rightarrow$ base-(2)

$\downarrow$

$[0 - 1]$

Decimal no. system $\rightarrow$ base-(10)

$\downarrow$

$(0 - 9)$

$$\overset{10^2}{\boxed{7}}\ \overset{10^1}{\boxed{3}}\ \overset{10^0}{\boxed{9}} = 7 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$$

$$\overset{10^1}{1}\ \overset{10^0}{9} = 1 \times 10 + 10^0 \times 9 = 19$$

$(10)_{10} \longrightarrow$ is a number not a digit.

$$\begin{array}{r} 248 \\ 129 \\ \hline 119 \\ \hline \end{array}$$

$2^3\,2^2\,2^1\,2^0$

$(1101)_2 = 8+4+0+1 = (13)_{10}$

$(1110)_2 = 8+4+2+0 = (14)_{10}$

$(10101)_2 = 16+0+4+0+1 = (21)_{10}$

2^1 A	2^0 B		
0	0	$\longrightarrow$	0
0	1	$\longrightarrow$	1
1	0	$\longrightarrow$	2
1	1	$\longrightarrow$	3 ✓

$$(1111)_2 = (15)_{10} = (\underline{\underline{2}}^4 - 1)$$

$$(111)_2 = (2^3 - 1) = (7)_{10}$$

$$(10^2 - 1) = (99)_{10}$$

$$(10^3 - 1) = (999)_{10}$$

$$(10^4 - 1) = (9999)_{10}$$

2^2	2^1	2^0		
A	B	C		
0	0	0	=	0
0	0	1	=	1
0	1	0	=	2
0	1	1	=	3
1	0	0	=	4
1	0	1	=	5
1	1	0	=	6
1	1	1	=	7

2^3	2^2	2^1	2^0
A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0

A	B	C	D
1	1	0	1
1	1	1	0
1	1	1	1

$\hookrightarrow (15)$

$\boxed{n\text{-variables} \longrightarrow 2^n \rightarrow \text{combinations} \longrightarrow 0 - (2^n - 1)}$

[Boolean Theorems]

1. Related to 'OR' operation

- $A + 0 = A$ $\quad (+) \rightarrow$ logical 'OR', $\bar{A} + 0 = \bar{A}$

A	$\longleftrightarrow$	$\bar{A}$
0		1
1		0

***
- $A + 1 = 1 + A = 1$

$1 + \text{anything} = 1$ $\quad 1 + [AB + \bar{B}C + CD] = 1 + f = 1$

$1 + \overline{\bar{A}B + \bar{B}C} + CD = 1$

- $A + A = A$, $\quad A + A + A = A + A = A$, $\quad \bar{A} + \bar{A} = \bar{A}$

- $A + \bar{A} = \bar{A} + A = 1$

- $$\boxed{\begin{aligned} 0+0 &= 0 \\ 1+1 &= 1 \\ 1+0 &= 1 \\ 0+1 &= 1 \end{aligned}}$$

$$\boxed{\begin{aligned} AB + \overline{AB} &= 1 \\ P + \overline{P} &= 1 \end{aligned}}$$

$$ABC + ABC = ABC$$

$$1+1 = (10)_2$$

addition

$$\boxed{\begin{aligned} \bar{A}B + C + 0 &= (\bar{A}B + C) \\ \bar{A}B + C + 1 &= 1 \end{aligned}}$$

$$\bar{A}\bar{B}CD + \bar{A}\bar{B}CD = \bar{A}\bar{B}CD$$

2. Related to 'AND' operation

- $A \cdot 0 = 0 \Rightarrow 0 \cdot A = 0$

  $0 \cdot \text{anything} = 0$

  $0 \cdot (\bar{A}B + CD) = 0$

- $A \cdot 1 = A$

  $\bar{A} \cdot 1 = \bar{A}$

- $A \cdot A = A$, $A \cdot A \cdot A = A$, $\bar{A} \cdot \bar{A} = \bar{A}$, $\overline{AB} \cdot \overline{AB} = \overline{AB}$

- $A \cdot \bar{A} = \bar{A} \cdot A = 0$, $AB \cdot \overline{AB} = 0$

  $P \cdot \bar{P} = 0$

- $0 \cdot 0 = 0$
- $0 \cdot 1 = 0$
- $1 \cdot 0 = 0$
- $1 \cdot 1 = 1$

3. Very imp Boolean theorems

- $A\cdot(B+C) = A\cdot B + A\cdot C$ → 'AND' is distributive over 'OR'.

- $A+(B\cdot C) = (A+B)\cdot(A+C)$ → 'OR' is distributive over 'AND'

$A+(\bar{A}\cdot B) = (A+\bar{A})\cdot(A+B) = 1\cdot(A+B) = (A+B)$

$\bar{A}+(A\cdot B) = (\bar{A}+A)\cdot(\bar{A}+B) = 1\cdot(\bar{A}+B) = (\bar{A}+B)$

$[A + B\cdot C\cdot D] = (A+B)\cdot(A+C)\cdot(A+D)$

$$\cdot\ (A+\overline{B})(\overline{A}+C)(\overline{A}+\overline{B}) = (\overline{B}+A\cdot\overline{A})(\overline{A}+C) = \overline{B}\cdot(\overline{A}+C)$$

$$= \overline{A}\cdot\overline{B}+\overline{B}\cdot C$$

$$= (A+\overline{B})(\overline{A}+\overline{B}C) = A\cdot\overline{A}+A\overline{B}C+\overline{A}\cdot\overline{B}+\overline{B}\cdot\overline{B}C$$

$$= 0 + A\overline{B}C+\overline{A}\,\overline{B}+\overline{B}C$$

$$= \overline{B}C(A+1)+\overline{A}\,\overline{B} = \overline{B}C+\overline{A}\,\overline{B} = \overline{A}\,\overline{B}+\overline{B}C$$

2 Minute Summary

→ Basics & Boolean Theorems.

Chandangupta Sir PW

Thank you

GW Soldiers !