Computer Science & IT

Data Structure & Programming

Graph & Hashing

Lecture No. 03

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Recap of Previous Lecture

Topic	Hashing
Topic	Collision
Topic	Chaining
Topic	Linear probing
Topic	

Slide

Topics to be Covered

- Topic
- Topic
- Topic
- Topic
- Topic

quadratic probing

Double probing

Slide

Topic : Hashing

How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(A) 10

(B) 20

(C) 30

(D) 40

4 element

0	
1	
2	42
3	23
4	34
5	52
6	46
7	
8	
9	

_ _ _ _ | 52 33

4!

=24

Slide

Topic : Hashing

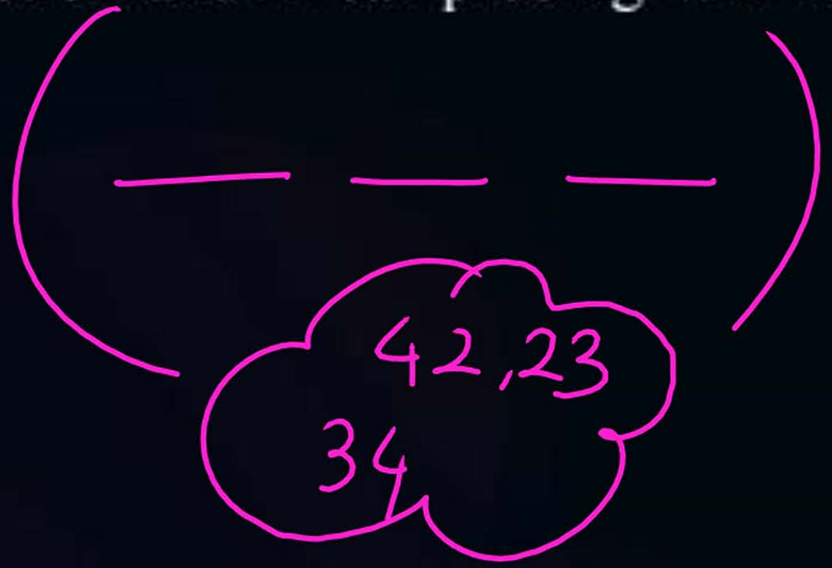
How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(A) 10

(B) 20

(C) 30

(D) 40



52, 46, 33

31 :=6

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

Slide

Topic : *Primary Clustering*

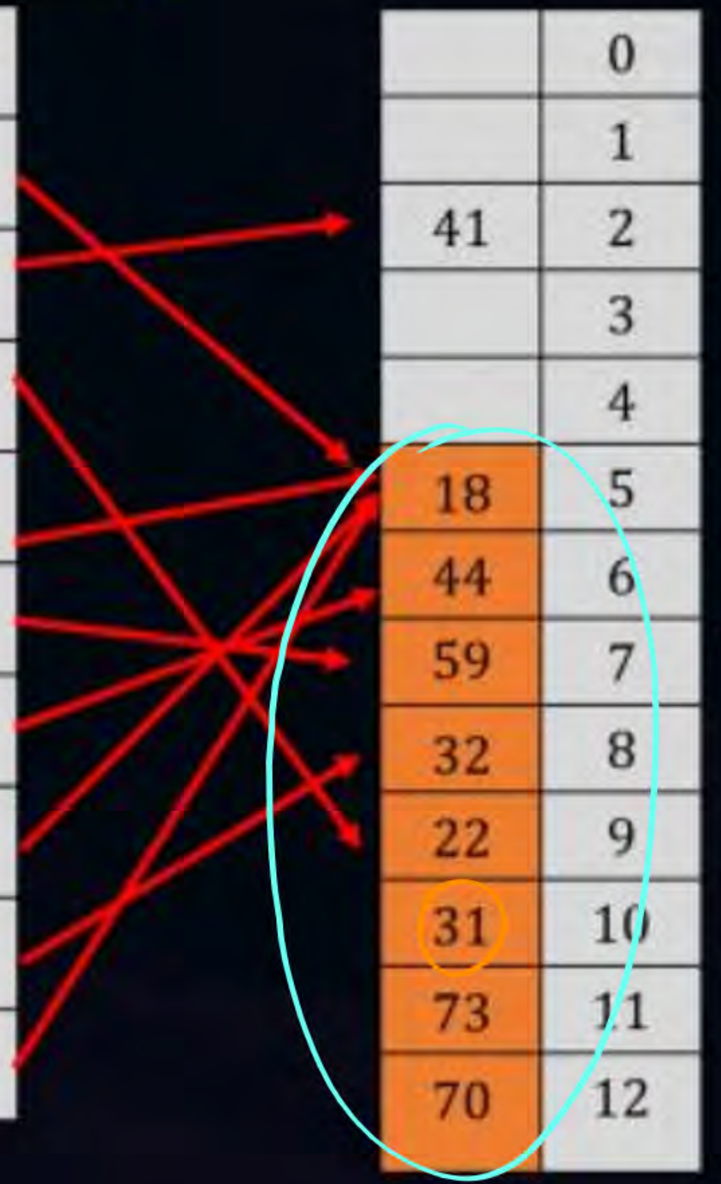
Primary clustering is the tendency for a collision resolution scheme such as linear probing to create *long runs of filled slots* near the hash position of keys.

If the primary hash index is x, subsequent probes go to $x + 1, x + 2, x + 3$ and so on, this results in Primary Clustering.

Once the *primary cluster forms*, the bigger the cluster gets, the *faster it grows*. And it *reduces the performance*.

Topic : Hashing

$Key - x$	$h(x) = x \bmod 13$
18	5
41	2
22	9
44	5
59	7
32	6
31	5
73	8
70	5



Linear probing

→ cluster

keys are accumulated in single place.

Slide

Topic : Hashing

Linear probing suffers problem of primary cluster

Cluster is long run of filled slot

once the cluster from bigger it gets faster it grows

performance of Hashing : Search time & Insertion time Increases

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Topic : Hashing

h(k) - probe0 = 7

Quadratic probing

probe0 = h(k)

$+4^2$

$+3^2$

$+2^2$

$+1^2$

$probe_1 = h(k) + 1^2$

$probe_2 = h(k) + 2^2$

$probe_3 = h(k) + 3^2$

$probe_4 = h(k) + 4^2$

Slide

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2) \ mod\ 20$
96	96 mod 20 = 16
48	48 mod 20 = 8
63	63 mod 20 = 3
29	29 mod 20 = 9
87	87 mod 20 = 7
77	77 mod 20 = 17
47	47 mod 20 = 7
73	
70	
86	
166	

probe$_1$
$= h(k) + 1^2$
$7 + 1^2 = 8$
occupied

probe$_2$: $h(k) + 4$
$= 7 + 2^2$
$= 7 + 4 = 11$

	0
	1
	2
63	3
	4
	5
	6
87	7
48	8
29	9
	10
47	11
	12
	13
	14
	15
96	16
77	17
	18
	19

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2) \ mod\ 20$
96	96 mod 20 = 16
48	48 mod 20 = 8
63	63 mod 20 = 3
29	29 mod 20 = 9
87	87 mod 20 = 7
77	77 mod 20 = 17
47	47 mod 20 = 7
73	73 mod 20 = 13
70	70 mod 20 = 10
86	86 mod 20 = 6
166	166 mod 20 = 6

find the index in which 166 will be mapped

$probe_1 = 6 + 1^2 = 7$ occupied

$probe_2 = 6 + 2^2 = 10$ occupied

$probe_3 = 6 + 3^2 = 15$

	0
	1
	2
63	3
	4
	5
86	6
87	7
48	8
29	9
70	10
47	11
	12
73	13
	14
166	15
96	16
77	17
	18
	19

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2) \ mod\ 20$
96	96 mod 20 = 16
48	48 mod 20 = 8
63	63 mod 20 = 3
29	29 mod 20 = 9
87	87 mod 20 = 7
77	77 mod 20 = 17
47	47 mod 20 = 7
73	73 mod 20 = 13
70	70 mod 20 = 10
86	86 mod 20 = 6
166	166 mod 20 = 6

Insert 146 & find the slot in which 146 will be inserted?

	0
	1
	2
63	3
	4
	5
86	6
87	7
48	8
29	9
70	10
47	11
	12
73	13
	14
166	15
96	16
77	17
	18
	19

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2) \ mod\ 20$
96	96 mod 20 = 16
48	48 mod 20 = 8
63	63 mod 20 = 3
29	29 mod 20 = 9
87	87 mod 20 = 7
77	77 mod 20 = 17
47	47 mod 20 = 7
73	73 mod 20 = 13
70	70 mod 20 = 10
86	86 mod 20 = 6
166	166 mod 20 = 6

146 mod 20
= 6

probe$_1$ = $6+1^2 = 7$

probe$_2$ = $6+2^2 = 10$

probe$_3$ = $6+3^2 = 15$

probe$_4$ = $6+4^2$
22
= (22 mod 20)
= (2)

	0
	1
146	2
63	3
	4
	5
86	6
87	7
48	8
29	9
70	10
47	11
	12
73	13
	14
166	15
96	16
77	17
	18
	19

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2) \ mod\ 20$
96	96 mod 20 = 16
48	48 mod 20 = 8
63	63 mod 20 = 3
29	29 mod 20 = 9
87	87 mod 20 = 7
77	77 mod 20 = 17
47	47 mod 20 = 7
73	73 mod 20 = 13
70	70 mod 20 = 10
86	86 mod 20 = 6
166	166 mod 20 = 6

Insert 67

find the slot No. in which it will be inserted

probe$_0$ = 7

probe$_1$ = $7+1^2 = 8$

probe$_2$ = $7+2^2 = 11$

probe$_3$ = $7+3^2 = 16$

probe$_4$ = $7+4^2 = 23\%20$

probe$_5$ = $7+25 = (12) = 3$

	0
	1
146	2
63	3
	4
	5
86	6
87	7
48	8
29	9
70	10
47	11
67	12
73	13
	14
166	15
96	16
77	17
	18
	19

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2) \ mod \ 20$
96	
48	
63	
29	
87	
77	
47	
73	
70	
86	
166	

	0
	1
	2
	3
	4
	5
	6
	7
	8
	9
	10
	11
	12
	13
	14
	15
	16
	17
	18
	19

Example

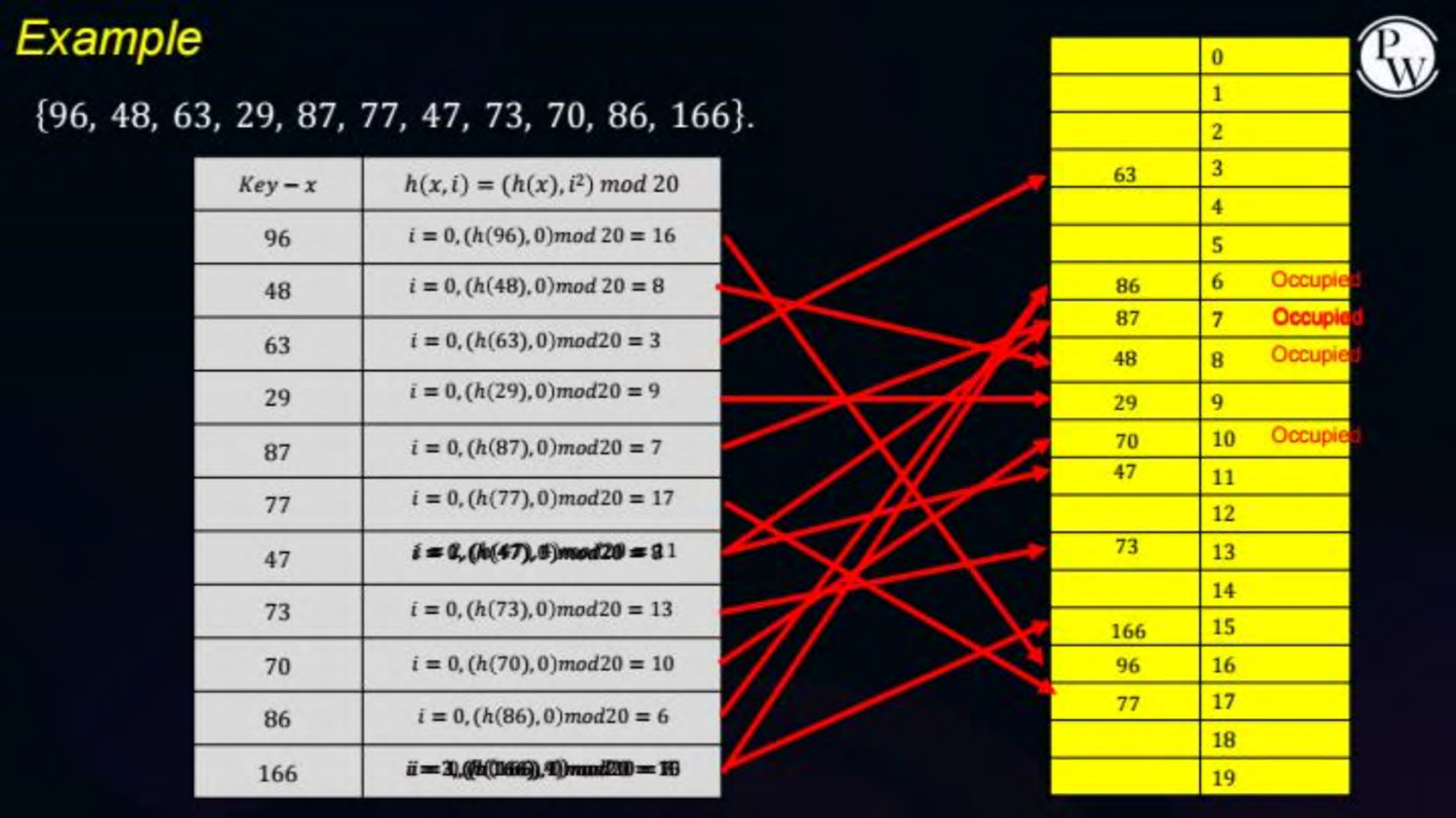
{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

	0
	1
	2
	3
	4
	5
	6
	7
	8
	9
	10
	11
	12
	13
	14
	15
	16
	17
	18
	19

Example

{96, 48, 63, 29, 87, 77, 47, 73, 70, 86, 166}.

$Key - x$	$h(x,i) = (h(x), i^2)\ mod\ 20$
96	$i = 0, (h(96), 0) mod\ 20 = 16$
48	$i = 0, (h(48), 0) mod\ 20 = 8$
63	$i = 0, (h(63), 0) mod 20 = 3$
29	$i = 0, (h(29), 0) mod 20 = 9$
87	$i = 0, (h(87), 0) mod 20 = 7$
77	$i = 0, (h(77), 0) mod 20 = 17$
47	$i = 0, (h(47), 0) mod 20 = 1$
73	$i = 0, (h(73), 0) mod 20 = 13$
70	$i = 0, (h(70), 0) mod 20 = 10$
86	$i = 0, (h(86), 0) mod 20 = 6$
166	$i = 2, (h(166), 4) mod 20 = 16$



Topic : Hashing

Cycle through the list

Key	$h(x,i) = (h(x), i^2) \bmod 10$
2	2 mod 10 =
3	3 mod 10 = 3
6	6 mod 10 = 6
8	8 mod 10 = 8
7	7 mod 10 = 7
11	11 mod 10 = 1
12	12 mod 10 = 2 (h(k))

probe$_1$ = h(k)+1^2 = 3

probe$_2$ = 2+2^2 = 6

probe$_3$ = 2+3^2 = 11

11 mod 10 = 1

probe$_4$ = 2+4^2 = 18

18%10 = 8

probe$_5$ = 2+5^2 = 27

27%10 = 7

probe$_6$ = 2+6^2 = 38

38%10 = 8

	0
11	1
2	2
3	3
	4
	5
6	6
7	7
8	8
	9

Slide

Topic : Hashing

Some element may cycle through the list

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Double Hashing

Two hash functions are present

primary hash function, $h_1(k)$: probe

Secondary hash function, $h_2(k)$: offset

Topic : Hashing

$h(k) + 1$ Linear probing

↑

offset

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Topic : Hashing

```
Algorithm DoubleHashing (k) {
    if table is full then error;
    probe = h1(k);
    offset = h2(k);
    while (table[probe] is occupied)
        probe = (probe + offset) % m
    table[probe] = k;
}
```

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Topic : Hashing

$h_1(k) = x \bmod 13$ $h_2 = 8 - (k \bmod 8)$

18	$18 \bmod 13 = 5$	
41	$41 \bmod 13 = 2$	
22	$22 \bmod 13 = 9$	
44	$44 \bmod 13 = 5$	$8 - (44 \bmod 8) = 8 - 4 = 4$
59		
32		
31		
73		

probe 0 = 5 occupied

probe 1 = $5 + 4 = 9$ occupied

probe 2 = $9 + 4 = 13$

$13 \% 13 = 0$

0	44
1	
2	41
3	
4	
5	18
6	
7	
8	
9	22
10	
11	
12	

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Topic : Hashing

$h_1(k) = x \bmod 13$ $h_2 = 8 - (k \bmod 8)$

18	$18 \bmod 13 = 5$	
41	$41 \bmod 13 = 2$	
22	$22 \bmod 13 = 9$	
44	$44 \bmod 13 = 5$	$8 - (44 \bmod 8) = 8 - 4 = 4$
59	$59 \bmod 13 = 7$	
32	$32 \bmod 13 = 6$	
31	$31 \bmod 13 = 5$	$8 - (31 \bmod 8) = 8 - 7 = 1$
73	$73 \bmod 13 = 8$	$8 - (73 \bmod 8) = 8 - 1 = 7$

probe 0 = 5 occupied

probe 1 = $5 + 4 = 9$ occupied

probe 2 $9 + 4 = 13$

$13 \% 13 = 0$

0	44
1	
2	41
3	
4	
5	18
6	32
7	59
8	31
9	22
10	
11	
12	

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Topic : Hashing

$h_1(k) = x \bmod 13$ $h_2 = 8 - (k \bmod 8)$

18	$18 \bmod 13 = 5$	
41	$41 \bmod 13 = 2$	
22	$22 \bmod 13 = 9$	
44	$44 \bmod 13 = 5$	$8 - (44 \bmod 8) = 8 - 4 = 4$
59	$59 \bmod 13 = 7$	
32	$32 \bmod 13 = 6$	
31	$31 \bmod 13 = 5$	$8 - (31 \bmod 8) = 8 - 7 = 1$
73	$73 \bmod 13 = 8$	$8 - (73 \bmod 8) = 8 - 1 = 7$

probe$_0$ = 8

probe$_1$ = 8+7 = 15 % 13 = 2 occupied

probe$_2$ = 2+7 = 9 occupied

probe$_3$ = 9+7 = 16

16 % 13 = 3

0	44
1	
2	41
3	73
4	
5	18
6	32
7	59
8	31
9	22
10	
11	
12	

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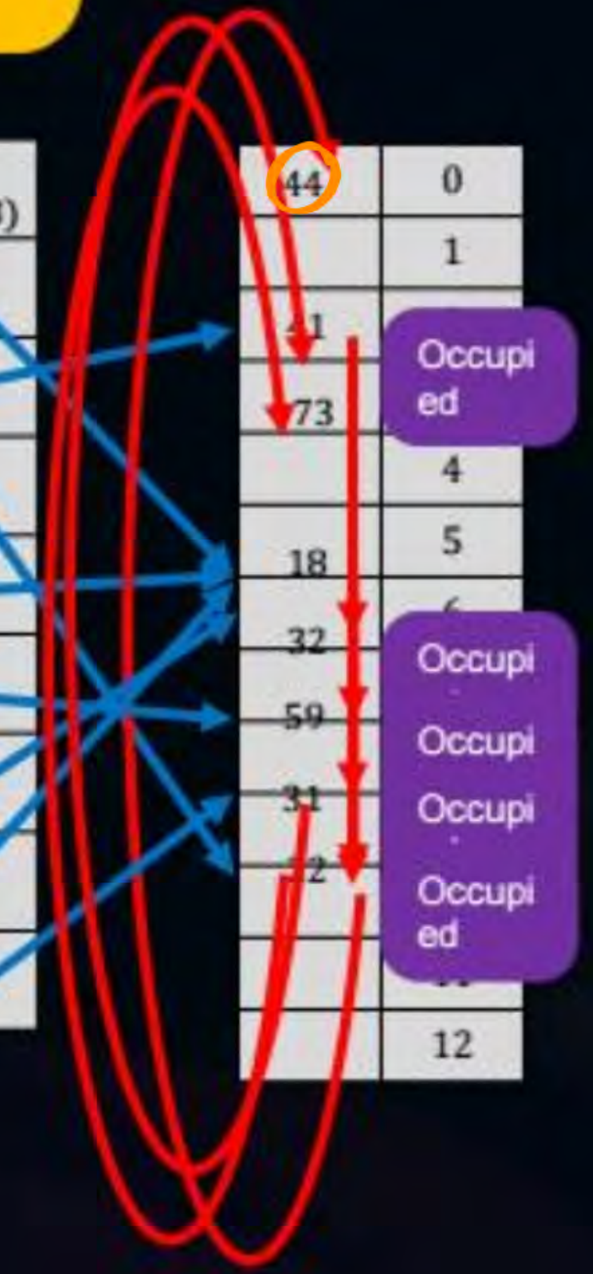
Topic : Hashing

$Key - x$	$h_1(x) = x \bmod 13$	$h_2(x) = 8 - (x \bmod 8)$
18	5	6
41	2	7
22	9	2
44	5	4
59	7	5
32	6	8
31	5	1
73	8	7



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Topic : Hashing

Consider a double hashing scheme in which the primary has function is h, (k) = k mod 23, and the secondary hash function is $h_2(k) = 1+(k \bmod 19)$. Assume that the table size is 23. Then the address returned by probe 1 in the probe sequence (assume that the probe sequence begins at probe 0) for key value k = 90 is 13

probe$_0$ = $h_1(90) = 90 \bmod 23 = 21$

$h_2(90) = 1 + 90 \bmod 19$

$= 1 + 14 = (15)$

probe$_1$ = $21 + 15 = 36$

$36 \% 23 = (13)$

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Topic : Hashing

Consider a double hashing scheme in which the primary hash function is $h_1(k) = k \bmod 10$, and the secondary hash function is $h_2(k) = 7 - (k \bmod 7)$. Assume that the table size is 10. Then the address returned by probe 1 in the probe sequence (assume that the probe sequence begins at probe 0) for key value $k = 49$ is

probe 0 : $h_1(k) = 49 \bmod 10 = 9$

probe 1 = 9 + 7 = 16

16 % 10 = 6

$h_2(k) = 7 - 49 \bmod 7$

$= 7$

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Topic : Hashing

Consider a double hashing scheme in which the primary hash function is $h_1(k) = k \bmod 10$, and the secondary hash function is $h_2(k) = 7 - (k \bmod 7)$. Assume that the table size is 10. Then the address returned by probe 1 in the probe sequence (assume that the probe sequence begins at probe 0) for key value $k = 49$ is **6**

2 mins Summary

Topic

Topic

Topic

Topic

Topic

quadratic probing

Double Hashing

Slide

THANK - YOU

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