CS & IT ENGINEERING

THEORY OF COMPUTATION

Pushdown Automata

Lecture - 04

By- Venkat sir

Recap of Previous Lecture

→ DPDA & NPDA

Topic ?????

→ $L = \{ww^R\}$ → CFL but not DCFL

NPDA but not DPDA

→ CFL Detection

→ DCFL → Dection

Topics to be Covered

CFG

$\Downarrow$

Topic Push down automat

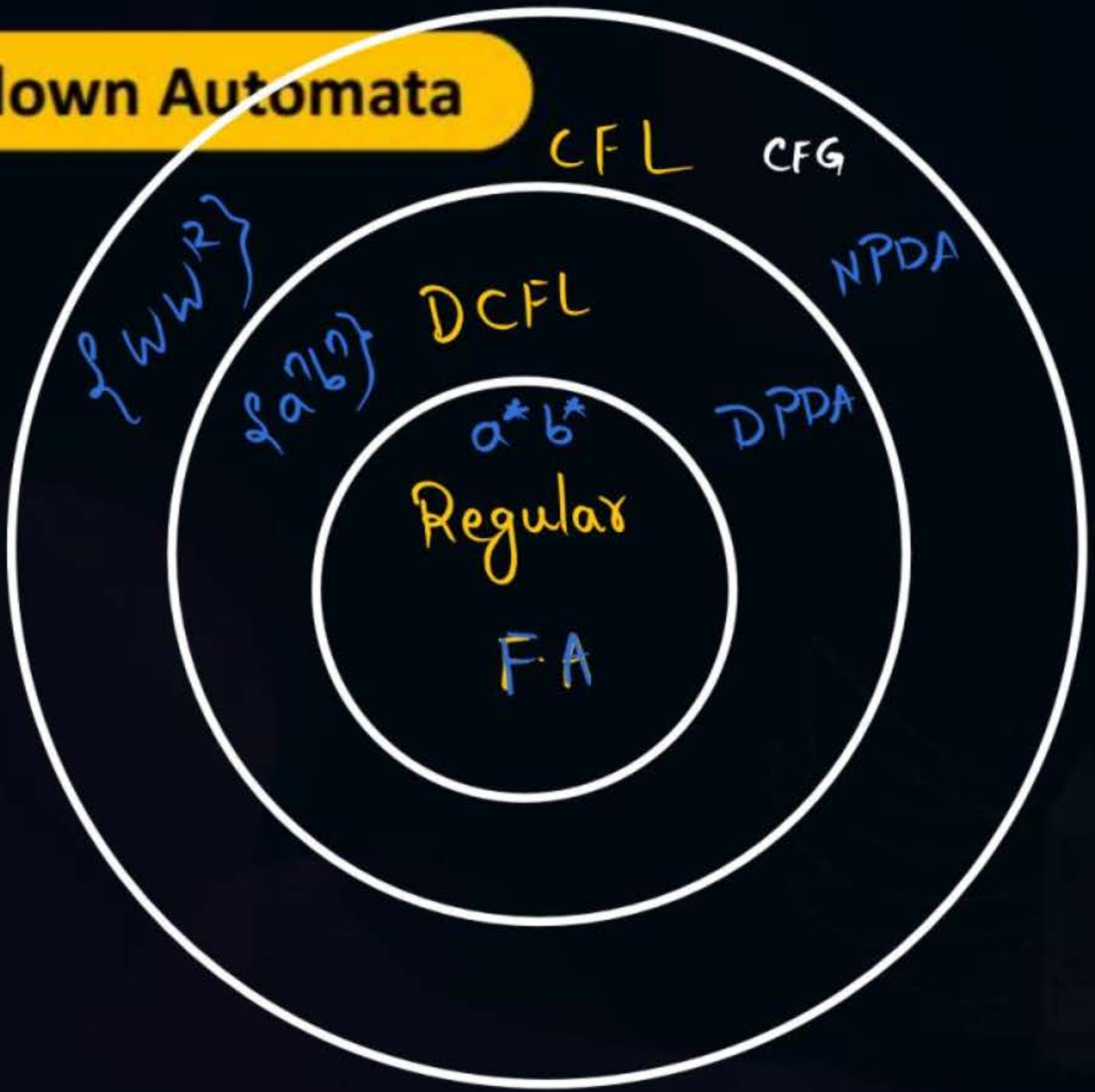
Topic ?? CFL Detection

Topic ?? DCFL Detection

Topic ?? closure properties of CFL & DCFL not closed {Closure properties of Regular.}

1. Subset op
2. Infinite Union
3. Infinite Intersection

Topic : Pushdown Automata



CFL

$L = \{ww^R \mid w \in (a+b)^*\}$

$\Downarrow$ CFG

$S \to aSa \mid bSb \mid \epsilon$

even length palindrome

{DPDA not possible
only NPDA possible}

Topic : Pushdown Automata

NOTE

$L_1 = \{ww \mid w \in (a+b)^*\}$

$L_2 = \{wcw \mid w \in (a+b)^*\}$

These two are non CFL

But Complement of

$\Sigma^* - \{ww\}$

$\Sigma^* - \{wcw\}$

$\rightarrow$ CFL ✓

L_1^c

L_2^c

is CFL

1 Every finite language $\longrightarrow$ Regular $\longrightarrow$ CFL

Q

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\} \longrightarrow$ CFL

$L_2 = \{a^n b^n c^{2n} : n \geq 1\} \longrightarrow$ Non CFL.

Which one of the following is TRUE?

abbbcccc

[2016(Set-2): 2 Marks]

(A) Both L_1 and L_2 are context-free.

(B) L_1 is context-free while L_2 is not context-free

(C) L_2 is context-free while L_1 is not context-free

(D) Neither L_1 nor L_2 is context-free

Q

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ ~~and~~ $\rightarrow$ CFL

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$. $\rightarrow$ CFL

Which of the following are context-free languages?

I. $L_1 \cup L_2$ $\rightarrow$ CFL

II. $L_1 \cap L_2$ $\rightarrow \boxed{\{a^n b^n c^n\}}$ non CFL

[2017(Set-1): 2 Marks]

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II

Q

Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context-free → ✓

II. $\overline{L_1}$ is context-free — ✗

III. $L_1 - R$ is context-free — ✓

IV. $L_1 \cap L_2$ is context-free — ✗

[2017(Set-2): 1 Marks]

(A) I, II and IV only

(B) I and III only

(C) II and IV only

(D) I only

Q

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\}$ → non CFL

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$ → CFL

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$ → non CFL.

$L_4 = \{a^n b^n \mid n \geq 1\}$ → CFL & DCFL

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

[2017(Set-2): 2 Marks]

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only

Q

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

CFL.
Reg. CFL → CFL.

(A) $L_1 \cdot L_2$

Reg ∪ CFL → always CFL

(B) $L_1 \cup L_2$

(C) $L_1 - L_2$

(D) $L_1 \cap L_2$

Reg − CFL

Reg ∩ CFL → always CFL

$Reg \cap CFL^c$ = may (or) may not

closure properties

of CFL & DCFL

$$L = \{a^n b^{2n}\} \cup \{a^n b^{3n}\}$$

DCFL $\cup$ DCFL

not DCFL

Union of two DCFL may (or) may not DCFL

Hence DCFL not closed for Union op.

$L_1 \to CFL \Rightarrow CFG \to S_1$

$L_2 \to CFL \Rightarrow CFG \to S_2$

$$L_1 \cup L_2 \Rightarrow S \to S_1 | S_2 \Rightarrow PDA$$

$$S \to S_1 | S_2 \to CFG \to L_1 \cup L_2$$

Union of two CFL's is always CFL

Intersection op

aabbcc $\quad$ aabbcc $\quad$ Non CFL.

$$\{a^n b^n c^m\} \wedge \{a^n b^m c^m\} = \{a^n b^n c^n\}$$

DCFL $\quad \wedge \quad$ DCFL

$$\{abc, a^2b^2c^2, a^3b^3c^3, a^4b^4c^4 \ldots\}$$

CFL not closed for Intersection op.

DCFL " " " "

Intersection with Regular

Always CFL.

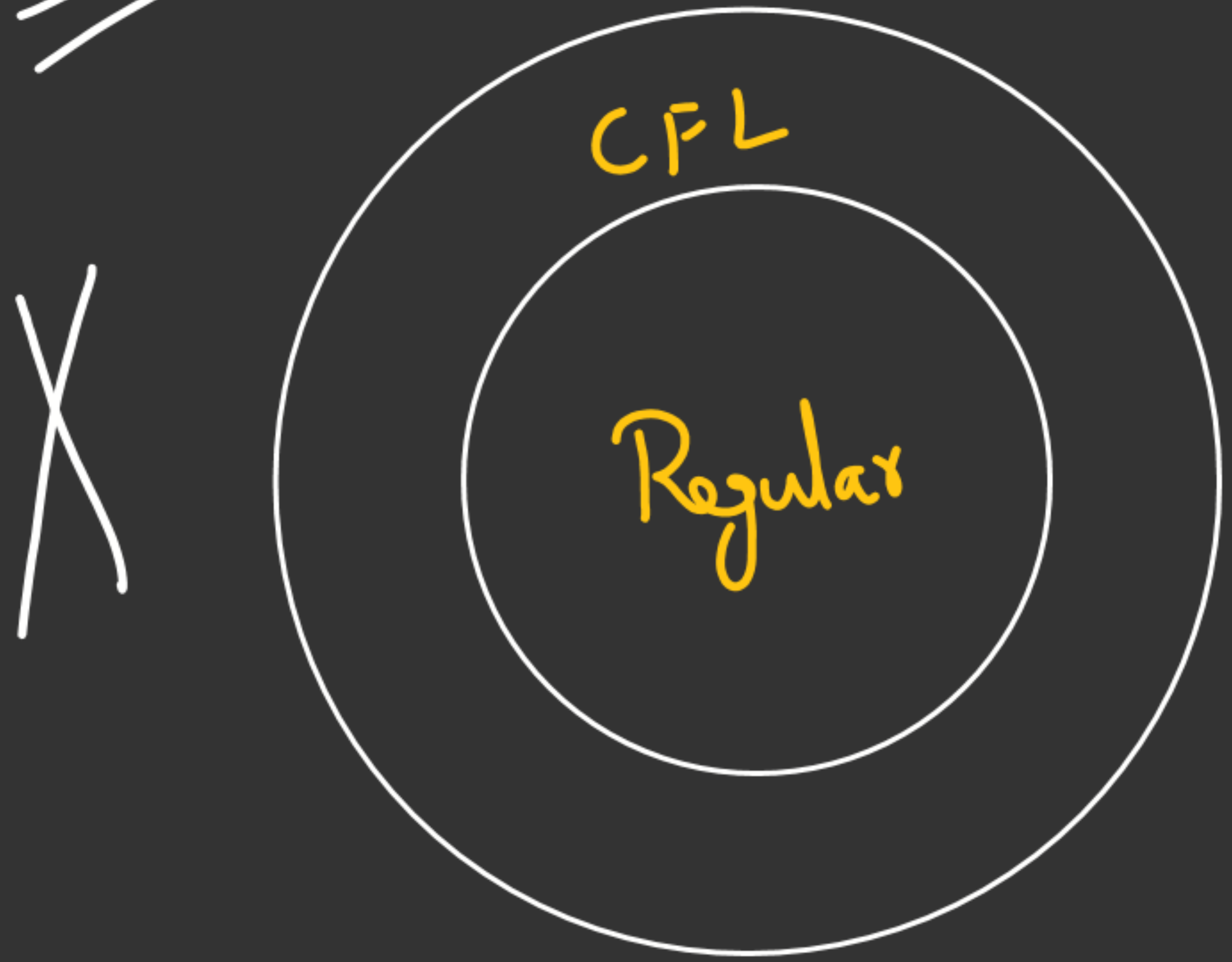
$CFL \wedge Regular$ → may (or) may not Regular

$\{a^n b^n\} \wedge (a+b)^*$ → $\{a^n b^n\}$

CFL $\wedge$ 1

CFL
DCFL
CSL $\wedge$ regular → always X
REL

Chomsky Hierarchy



$\underline{CFL} \cap \underline{Reg} \neq Regu$

$S \cap sub$

→ Complement of CFL is may (or) may not CFL.

→ Complement of DCFL is always DCFL.

(Q) $P = CFL$ } Q = Regular. which of the following is always Regular

(a) $P \wedge Q$ $\rightarrow$ always CFL.

(b) $\Sigma^* - P$ $\xrightarrow{\text{Complement}}$ may (or) may not CFL

(c) $\Sigma^* - Q$ $\xrightarrow{\text{Complement}}$ always Regular.

(d) none

L_1 & L_2 } DCFL

L_3 & L_4 } CFL

$L_5 \rightarrow$ Regular

Which of the following is false.

(B, C)

CFL $\cup$ CFL

(a) $L_3 \cup L_4 = CFL \rightarrow$ true.

(b) $L_3^c \longrightarrow DCFL$ } $\rightarrow$ false

(c) $L_3^c \cap L_4^c \longrightarrow DCFL$ } $\rightarrow$ false

(d) none

Operation	CFL	DCFL
① Union op	✓ (Always CFL)	✗ (not closed) (may (or) may not)
② Concatenation	✓	✗
③ Complement	✗	✓
④ Intersection	✗	✗
⑤ Kleene closure	✓	✗
⑥ positive closure	✓	✗

4 pm

	CFL	DCFL
(7) Intersection with Regular	✓	✓
(8) Difference	×	×
(9) L − Regular	✓	✓
(10) Regular − L	× $Reg \wedge CFL^c =$	✓ $Reg \wedge DCFL^c = Reg \wedge DCFL = DCFL$
(11) Reversal (L^R)	✓	×
(12) Quotient	×	×

$$\boxed{L_1 - L_2 = L_1 \cap L_2^c}$$

$$L - Reg = L \wedge Reg^c = L \wedge Reg = L$$

	CFL	DCFL
(13) Substitution	✓	X
(14) Homomorphism	✓	X
(15) Inverse Homomorphism	✓	✓
(16) L U Regular	✓ CFL U Reg = CFL	DCFL U Reg = DCFL ✓
(17) Prefix	✓	✓
(18) Suffix	✓	X
(19) Subset	X	X

Let $\boxed{L_1 \& L_2}$ } CFL, $\boxed{L_3 \& L_4}$ } Regular, which of the following is always CFL.

① $\underbrace{(L_1 \cup L_2)}_{\text{CFL} \vee \text{CFL}}^{*} \wedge (L_3^R \wedge L_4^c)$ ⟹ CFL ∧ Reg ⟹ always CFL.

CFL, Reg, Reg ∧ Reg

② $(L_1 \cup L_3)^R \cup (\boxed{L_2 \wedge L_4})^R$ ⟹ always CFL.

CFL, ∪, CFL

③ $(L_1^R \wedge L_4)^* \wedge (L_3^* \wedge L_2^R)$ ⟹ may (or) may not CFL.

CFL, ∧, CFL, (CFL ∧ reg), Reg ∧ CFL

④ $(\boxed{L_1 \cup L_2})^R - (\boxed{L_3 - L_4})^R$ ⟹ CFL − Reg = CFL ∧ Reg^c = CFL ∧ Reg → always CFL

CFL − Reg

$\underline{\delta(q_0, a, a)}$

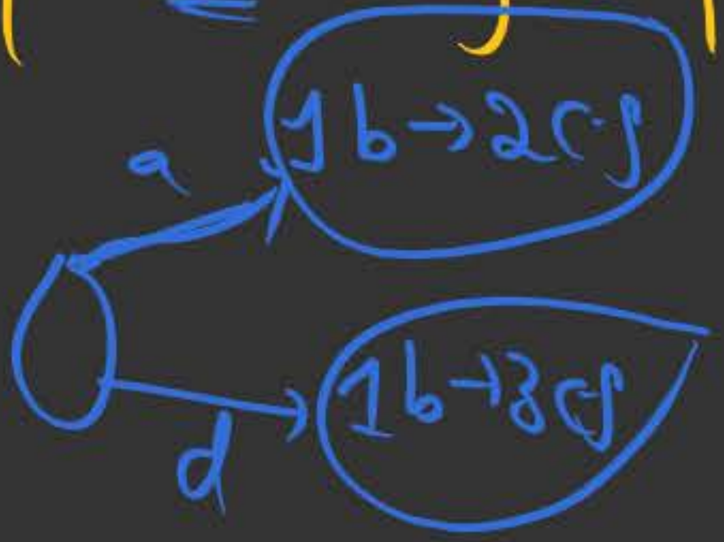
$L_1 = \{ww^R\} \rightarrow$ CFL but not DCFL

$L_2 = \{a^n b^{2n}\} \cup \{a^n b^{3n}\} =$ CFL but not DCFL.

$L_3 = \{a^i b^j c^k \mid i=j \text{ (or) } j=k\} \Rightarrow$ CFL but not DCFL.

$\{a^n b^n c^m\} \cup \{a^m b^n c^n\}$

$L_4 = \{a b^n c^{2n}\} \cup \{d b^n c^{3n}\} = \underline{\text{DCFL}} \rightarrow$ CFL



$\{a^n b^{2n}\} \longrightarrow$ 1 a $\rightarrow$ push 2 a's

(a)

$\cup$

$\{a^n b^{3n}\} \longrightarrow$ 1 a $\longrightarrow$ push 3 a's.

a a a a - - - - -

$w \cdot w^R$

$\delta(q_0, a, a) = (q_0, aa)$ (or) (q_1, ϵ)

$\delta(q_0, b, b) =$

$\left.\begin{matrix} L_1 \\ L_2 \end{matrix}\right\}$ CFL $\quad$ $L_3, L_4\}$ regular $\quad$ always CFL.

⑤ $(L_1 \cdot L_2)^R \cap (L_3 \cdot L_4)^* \Longrightarrow$ always CFL.

CFL $\cap$ Reg

CFL $\cdot$ CFL

CFL $\cdot$ Reg $\rightarrow$ CFL

⑥ $(L_1 \cdot L_3)^R \cap (L_2 \cdot L_4)^R \Longrightarrow$ may (or) may not CFL.

CFL $\quad$ CFL

CFL $\quad$ CFL

⑦ $(L_1 \cap L_3) \,/\, (L_2 \cap L_4) \Longrightarrow$ may (or) may not CFL.

⑧ $(L_1 L_3)^R - (L_2 \cdot L_4)^* \Longrightarrow$ May (or) may not CFL

CFL $-$ CFL

$$L_1 = \{a^n b^n c\, a^m b^m \mid n, m \geq 0\}$$ ← CFL

$$L_2 = \{a^i b^j c^k \mid i, j, k \geq 1\}$$ ← Reg

$L_1 \cap L_2$ is ?

$\boxed{a^n b^n c}$

(a) CFL and Regular

(b) CFL but not Regular

~~(c) non CFL~~

~~(d) finite Language~~

CFL ∩ Regular → always CFL

$\{a^n b^n\} \cap \{ab\} \rightarrow ab$

Let $L_1 = \{c\, a^n b^{2n}\}, n \geq 1$ → DCFL

$L_2 = \{d\, a^n b^{3n}\}, n \geq 1$ → DCFL

} Which of the following is false

DCFL ∪ DCFL → may (or) may not DCFL

$\{L_1 \cup L_2\}$ → DCFL.

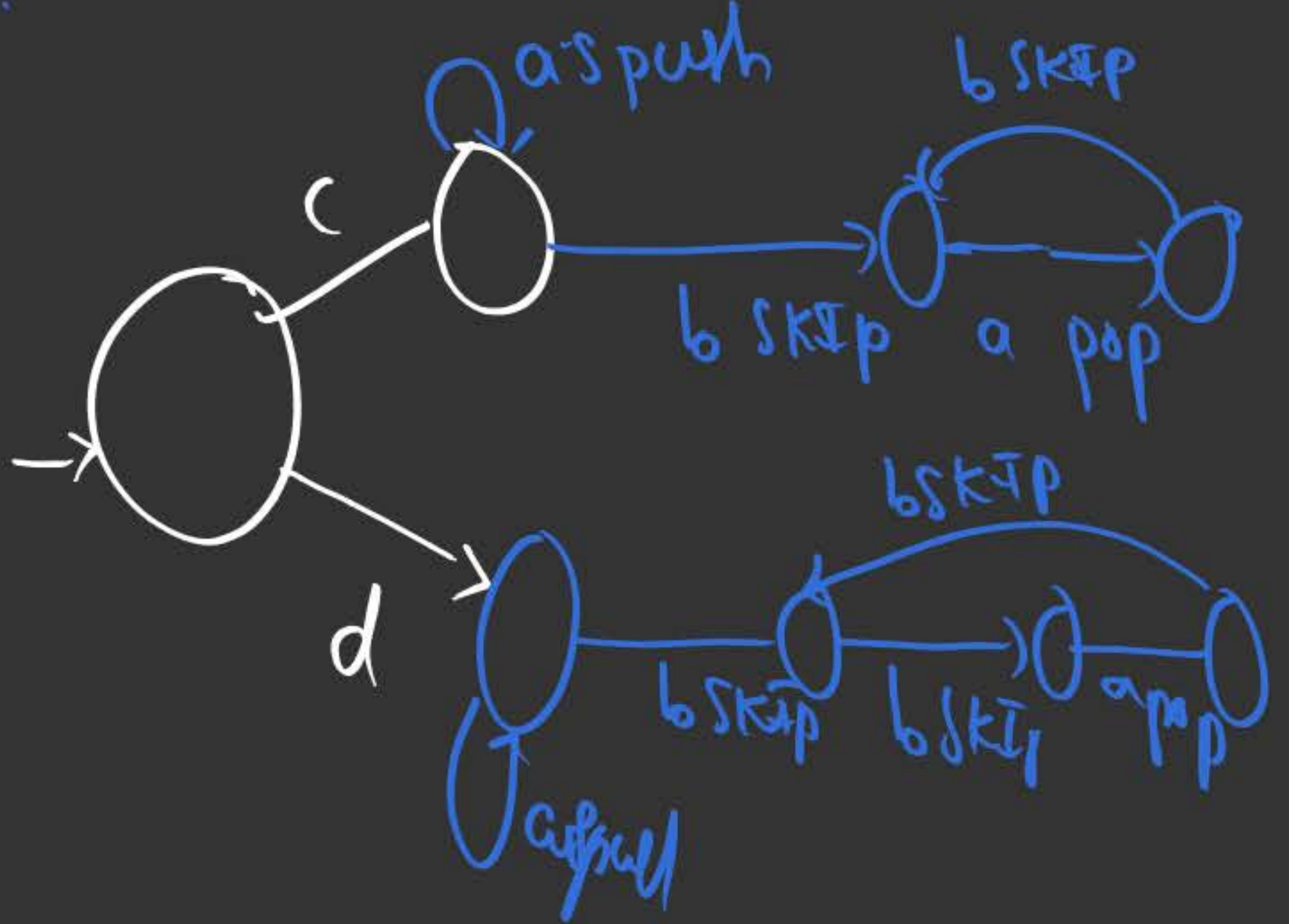
ⓐ $L_1 \cup L_2$ is DCFL → true.

ϕ → regular → DCFL.

ⓑ $L_1 \cap L_2 = \phi$ is DCFL } true

ⓒ L_1 & L_2 are DCFL } true

ⓓ ~~none~~



2 mins Summary

Topic One → CFG Ambiguity checking

Topic Two → Simplification of CFG.

Topic Three → Normal form of CFG → CNF, → GNF

Topic Four → PDA Construction

Topic Five → PDA → Language

→ CFL, DCFL & Detection

→ closure properties

THANK - YOU