

Computer Science & IT



Data Structure & Programming

Tree

Lecture No. 08



By- Abhishek Sir

Recap of Previous Lecture



Topic

AVL Tree

Topic

Single Rotation

Topic

Double Rotation

Topic

Topic

Topics to be Covered



Topic

Heap

Topic

Max heap

Topic

Min Heap

Topic

Insertion

Topic



Topic : Tree



Skewed tree. 3 Node



if with n key value

No. of bst with

$$\text{height } n-1 = 2^{n-1}$$





Topic : Tree



if BST constructed with 4 Node

How many BST will have height

at most 2. 6

Total BST - total BST of height 3 with 4 Node

$$C_4 = 14 - 2^{n-1} = 14 - 8 = \textcircled{6}$$

if with n key value

No. of bst with

$$\text{height } n-1 = \underline{2^{n-1}}$$



Topic : Heap



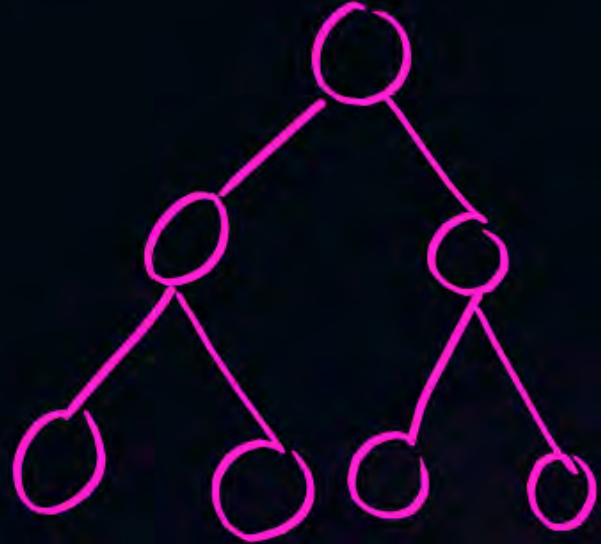
Heap is Complete Binay tree (CBT)

Complete Binay tree is tree in which each level (Height) is full except possibly the last. (Last may or may not be full)

In last level element will strictly filled from left to right



Topic : Heap



CBT
yes



CBT
yes



CBT?
yes



Topic : Heap



empty Not CBT



empty Not CBT

← FBT



Topic : Heap

Sequential representation

3.



1	2	3
---	---	---



1		2				3
1	2	3	4	5	6	7

CBT takes minimum space in Sequential representation



Topic : Heap



Height
3

Height formula

$$\lfloor \log_2 n \rfloor$$

$$\lfloor \log_2 15 \rfloor$$

$$\lfloor 3.9 \rfloor$$

$$= 3$$



Topic : Heap



Height
3

Height formula
 $\lfloor \log_2 n \rfloor$

$$\lfloor \log_2 13 \rfloor$$

$$= \lfloor 3.7 \rfloor$$

$$= 3$$



Topic : Heap



Height
3

Height formula

$$\lfloor \log_2 n \rfloor$$

$$\lfloor \log_2 10 \rfloor$$

$$= \lfloor 3. _ \rfloor$$

$$= 3$$



Topic : Heap



$$CBT \text{ Height} = \lfloor \log_2 n \rfloor$$

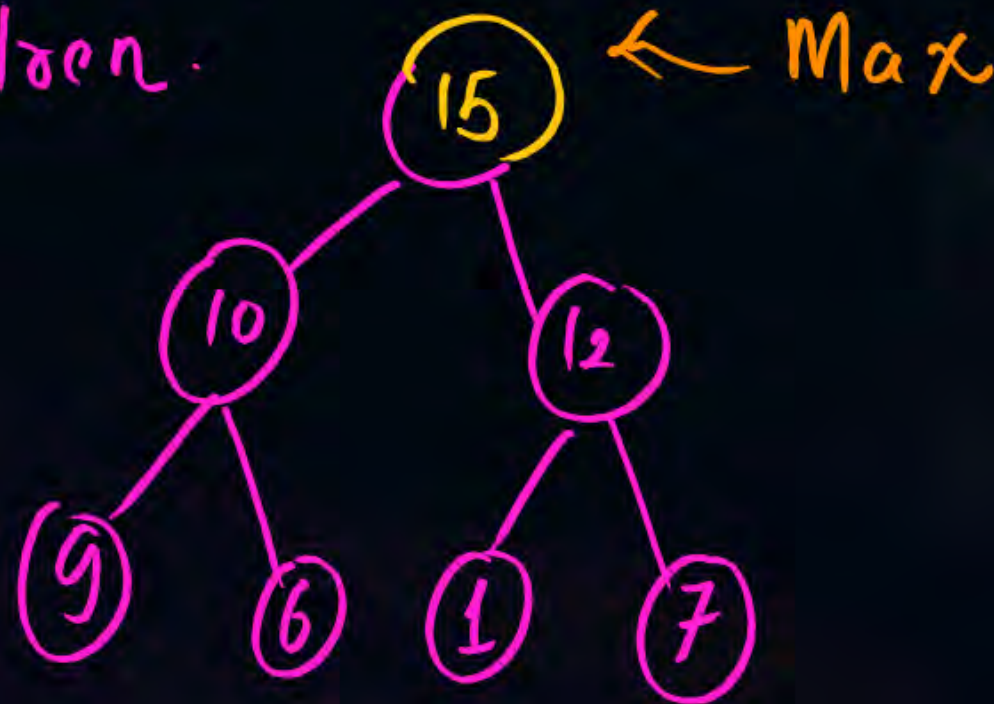


Topic : Heap



Heap is Not BST

priority queue : Max Heap! A max Heap is a CBT in which the key value at parent Node is greater than the key value at its children.





Topic : Heap



Max heap = ?

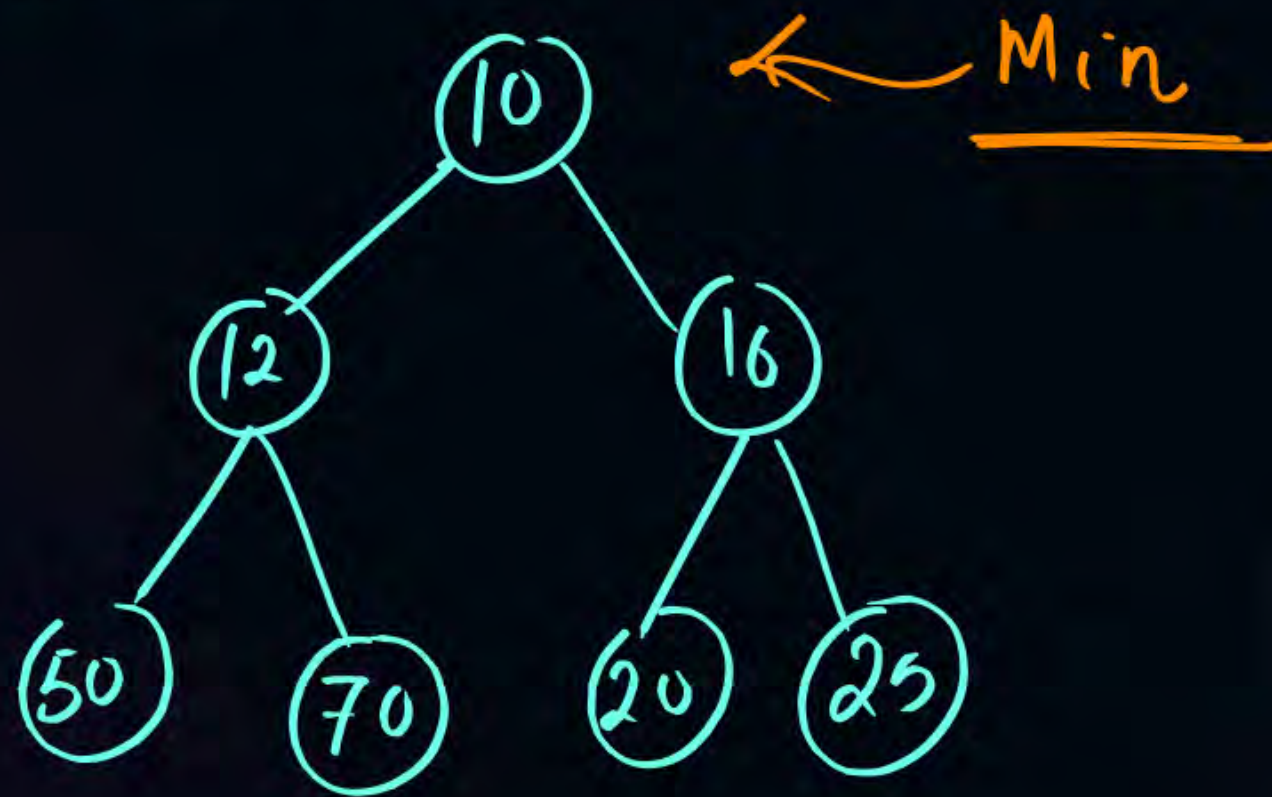
No

Not CBT



Topic : Heap

Min Heap: The key value at parent Node will be lesser than the key value at its children.





Topic : Heap



Heap construction 2 ways

1. By inserting element one after another
2. By converting entire array to Heap (Build Heap)



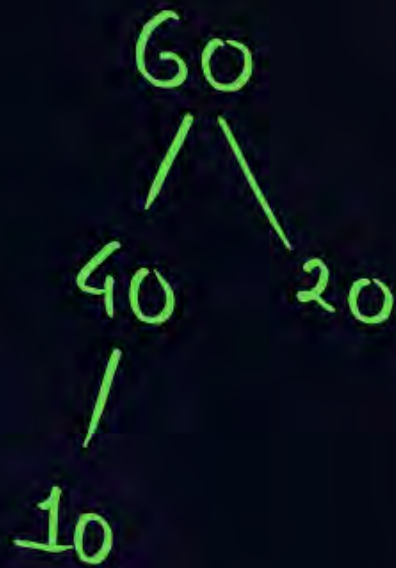
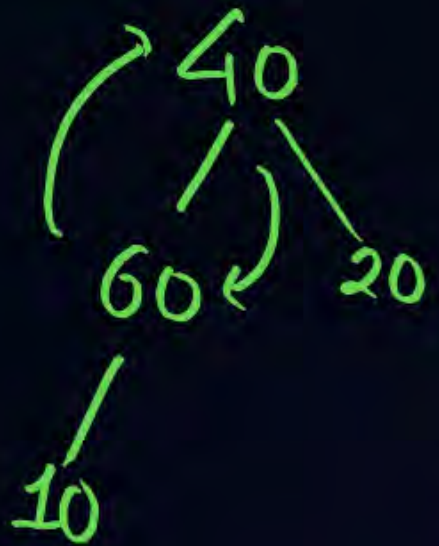
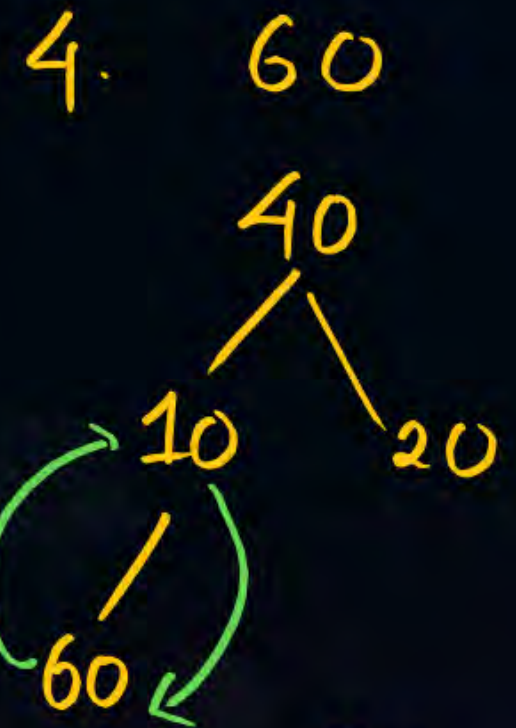
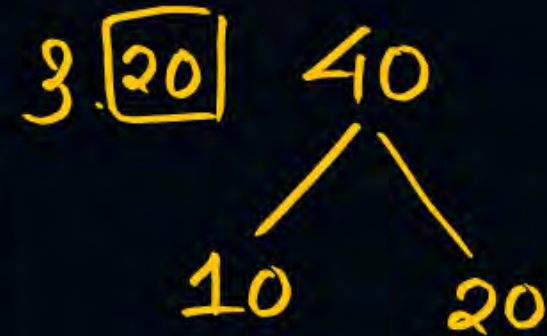
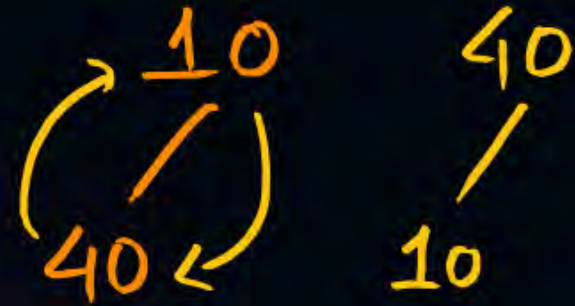
Topic : Heap



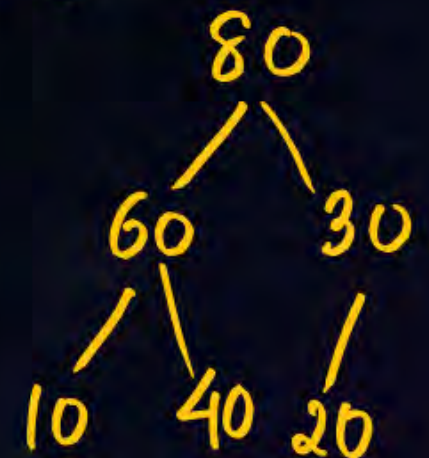
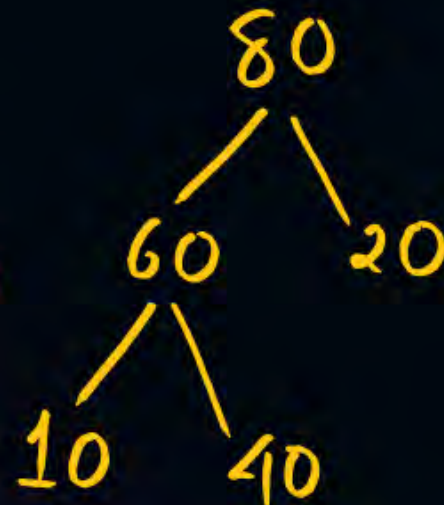
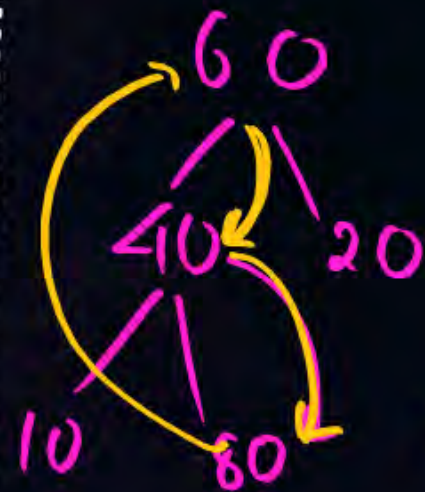
10, 40, 20, 60, 80, 30, 90

1. 10

2. 40



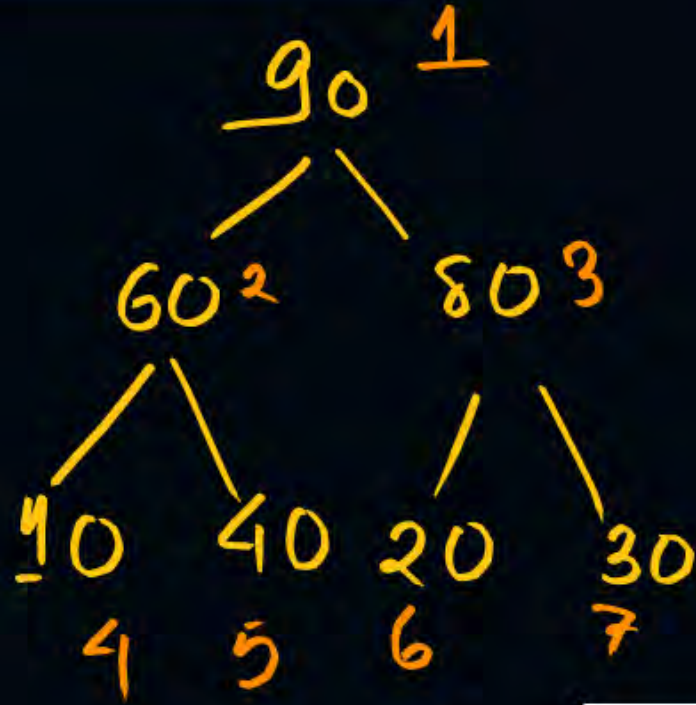
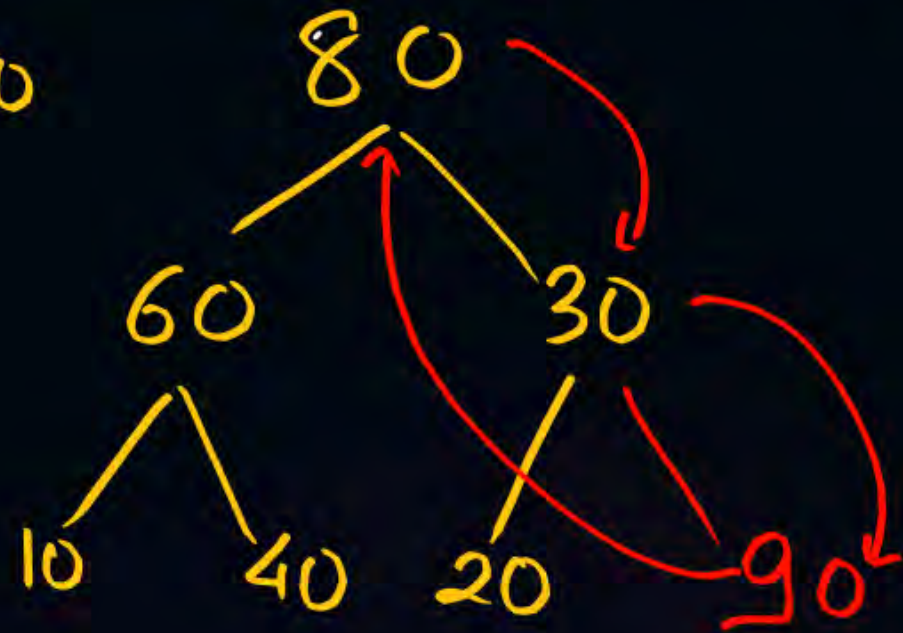
5. 80





Topic : Heap

7. 90



Sequential Representation

90	60	80	10	40	20	30
1	2	3	4	5	6	7



Topic : Heap

Algorithm Insert (a, n, x) { $O(\log n)$

$a[n] = x; \quad i = n;$

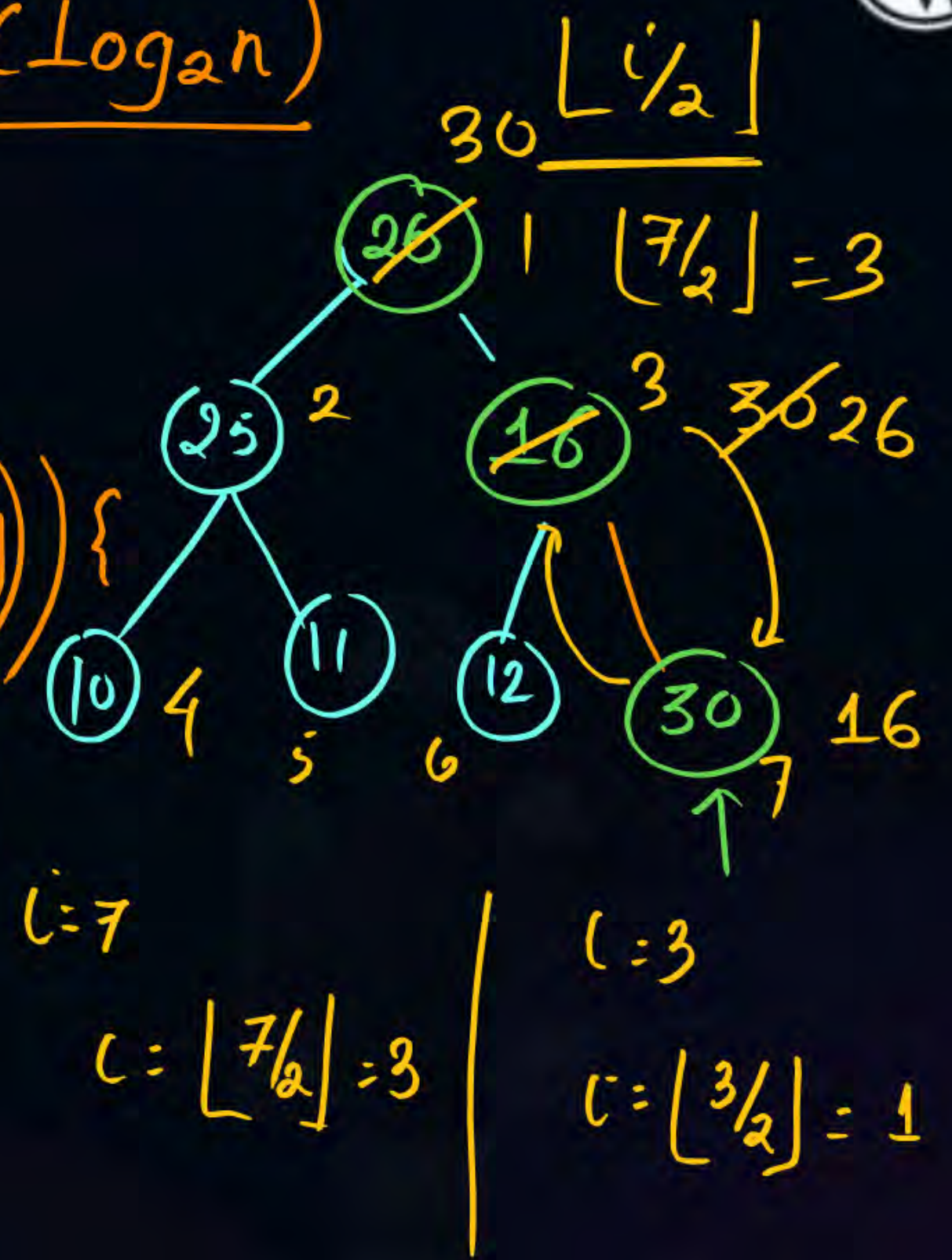
while ($i > 1$ and $(a[\lfloor i/2 \rfloor] < a[i])$) {

 swap($a[i], a[\lfloor i/2 \rfloor]$)

$i = \lfloor i/2 \rfloor;$

$i = 1$

}





Topic : Heap

Algorithm Insert (a, n, x) { $O(\log n)$

$a[n] = x; \quad i = n;$

while($i > 1$ and $(a[\lfloor i/2 \rfloor] > a[i])$) {

 swap($a[i], a[\lfloor i/2 \rfloor]$)

$i = \lfloor i/2 \rfloor;$

}

}

Min Heap



Topic : Question



3

The number of nodes a heap of height k can hold is

(A) 2^k to $2^{k+1} - 1$ ✓

(B) 2^{k+1} to $2^{k+1} - 1$

(C) 2^{k-1} to $2^{k+1} - 1$

(D) $2^k - 1$ to $2^{k+1} - 1$



$$2^{3+1} - 1 = 15$$



Insertion in Binary Heaps

Inserts $a[n]$ into the heap which is stored in $a[l...n-1]$

```
Algorithm Insert(a,n,x) {  
    i := n;  
    while ((i > 1) and (a[ $\lfloor i/2 \rfloor$ ] < item)) {  
        swap( a[i], a[ $\lfloor i/2 \rfloor$ ] );  
        i :=  $\lfloor i/2 \rfloor$   
    }  
}
```

Insertion in Binary Heaps

Inserts $a[n]$ into the heap which is stored in $a[l...n-1]$

```

Algorithm Insert(a,n,x) {
    i := n;
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        swap( a[i], a[ $\lfloor i/2 \rfloor$ ] );
        i :=  $\lfloor i/2 \rfloor$ 
    }
}

```




Topic : Heap



Consider a binary max-heap implemented using an array.

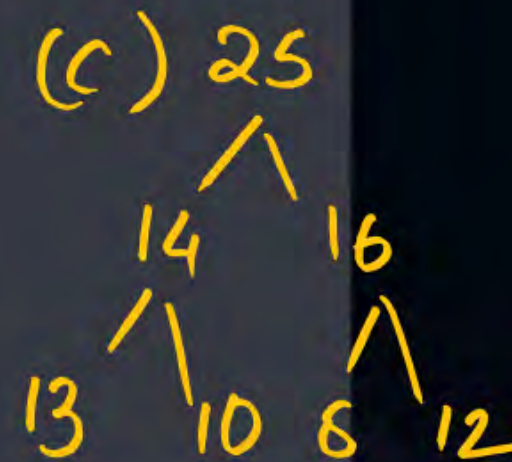
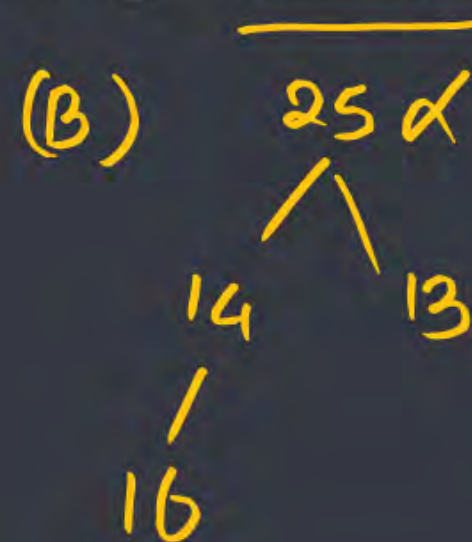
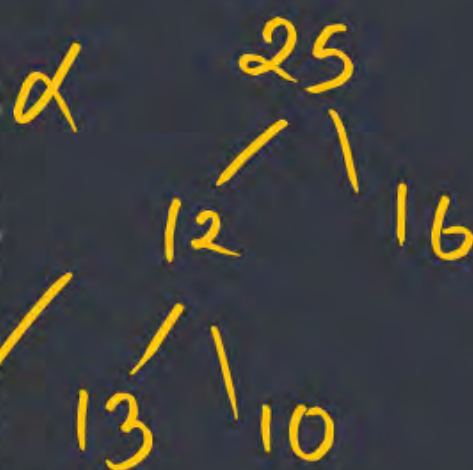
Which one of the following array represents a binary max-heap?

A. {25, 12, 16, 13, 10, 8, 14}

B. {25, 14, 13, 16, 10, 8, 12}

C. {25, 14, 16, 13, 10, 8, 12}

D. {25, 14, 12, 13, 10, 8, 16}



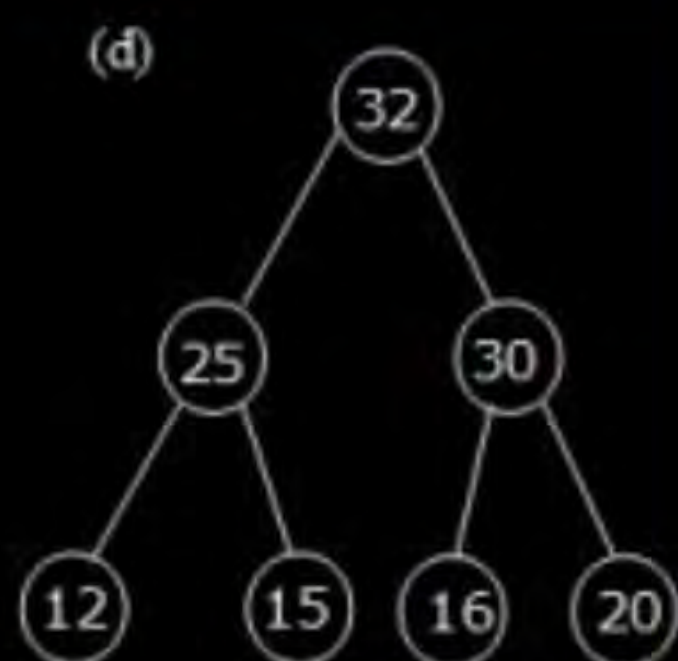
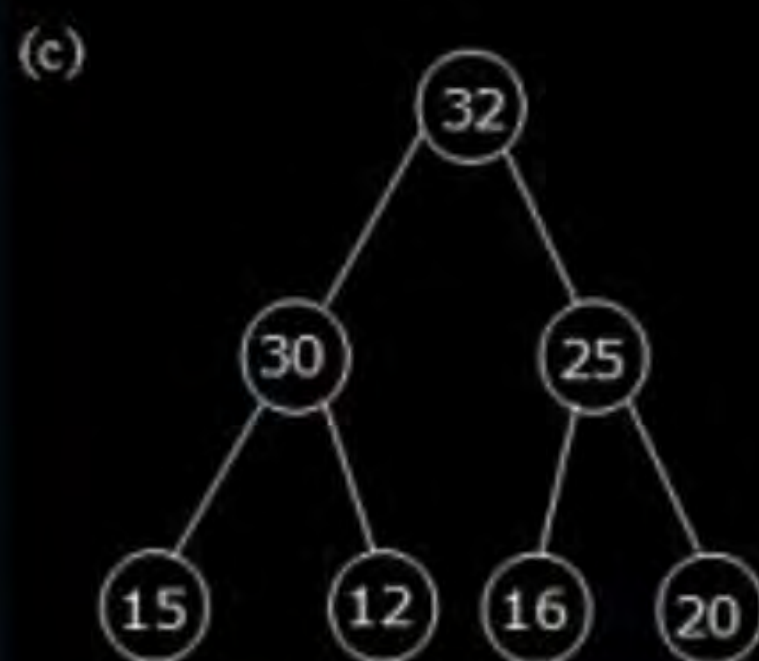
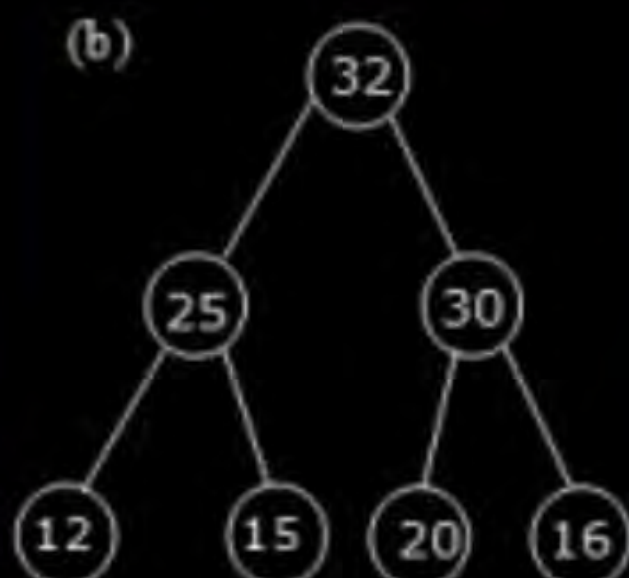
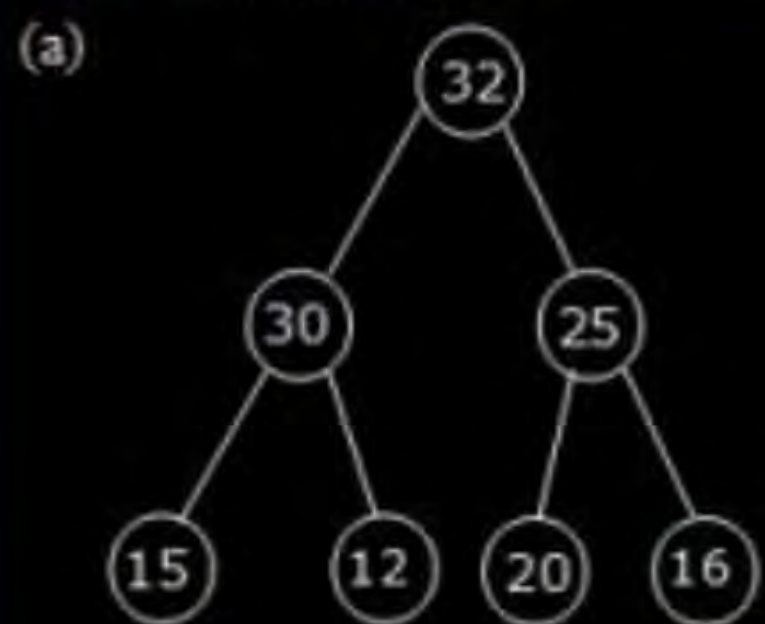


Topic : Heap

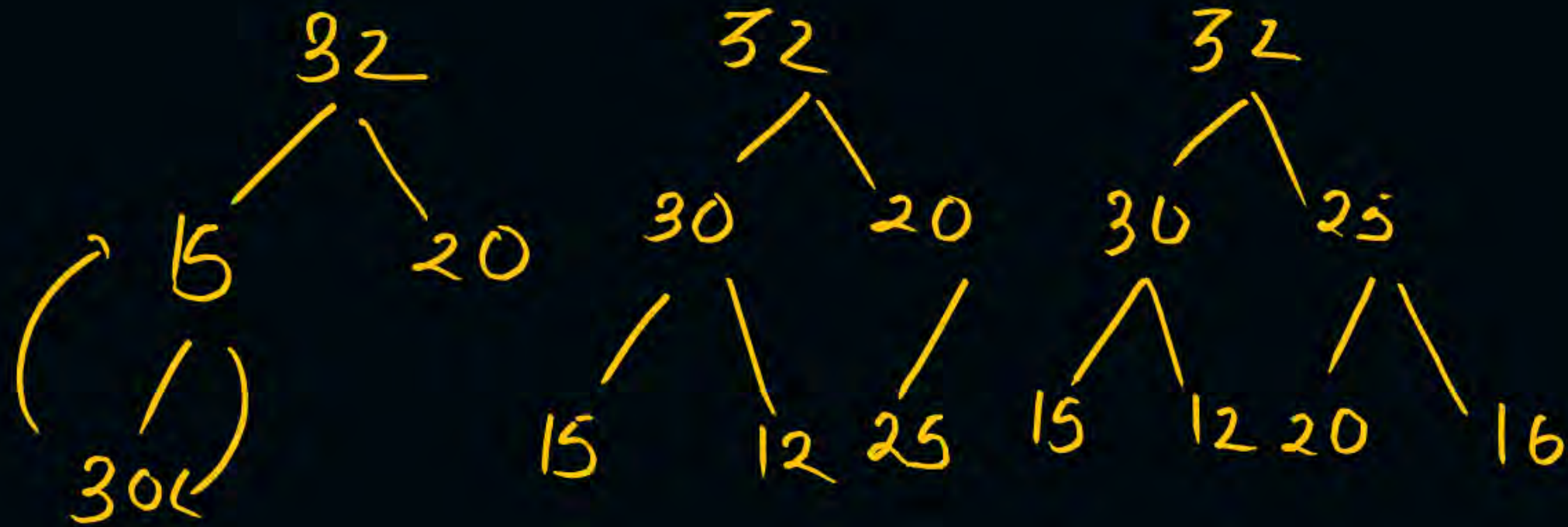
GATE 2004, Question Number 37, 2-Marks,

The elements 32, 15, 20, 30, 12, 25, 16, are inserted one by one in the given order into a maxHeap. The resultant

maxHeap is



32, 15, 20, 30, 12, 25, 16





Topic : Heap

GATE 2015 Set-I, Question Number 27, 2-Marks

Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is



Topic : Heap



- (A) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
- (B) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
- (C) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
- (D) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

- (A) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
- (B) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
- (C) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
- (D) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30



Topic : Heap

if heap is represented by {25,14, 16, 13, 10,8 ,12}

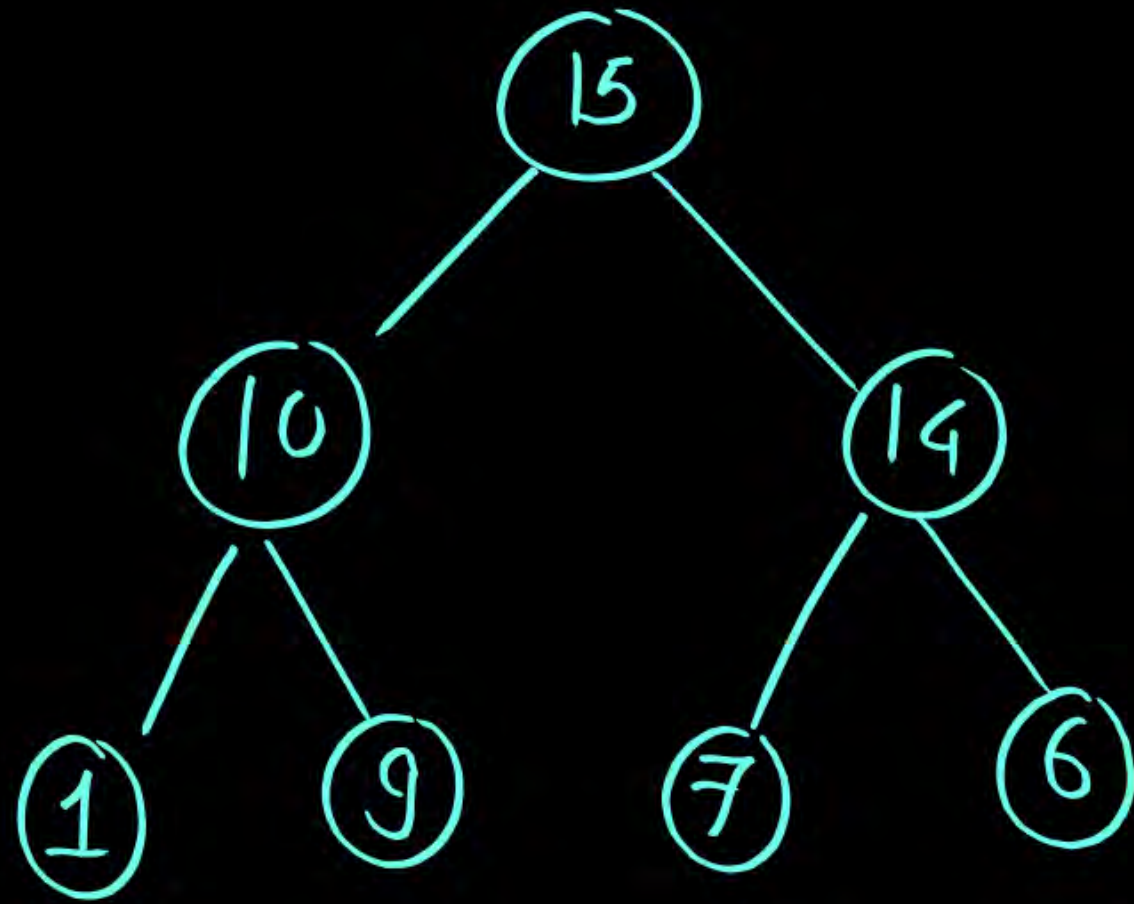
What is the content of the array after two delete operations on the correct answer to the previous question:

- (a) {14, 13, 12, 10, 8}
- (b) {14, 12, 13, 8, 10}
- (c) {14, 13, 8, 12, 10}
- (d) {14, 13, 12, 8, 10}



Topic : Heap

Deletion from Max heap

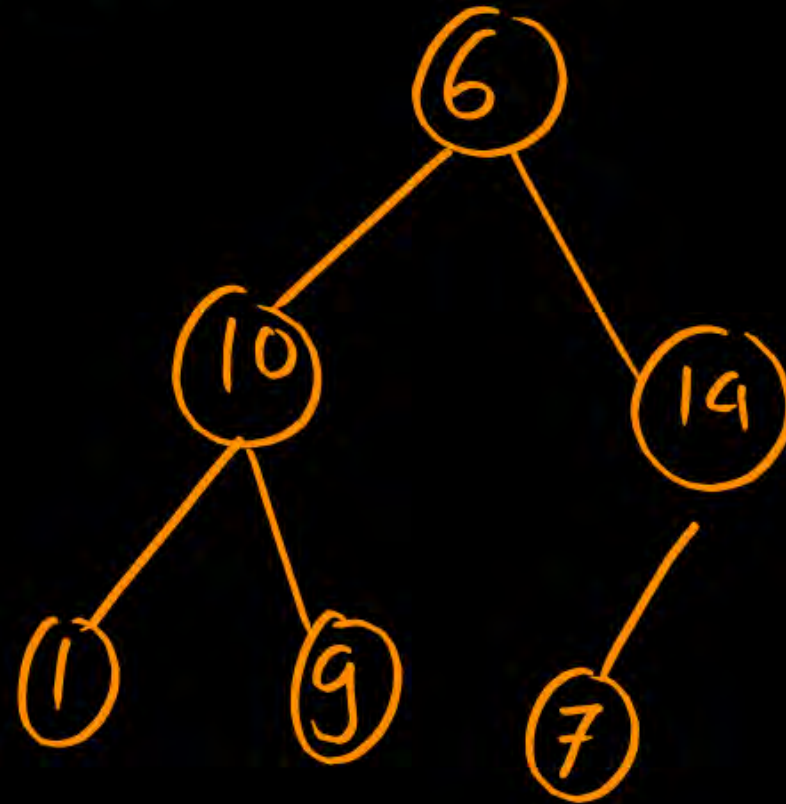
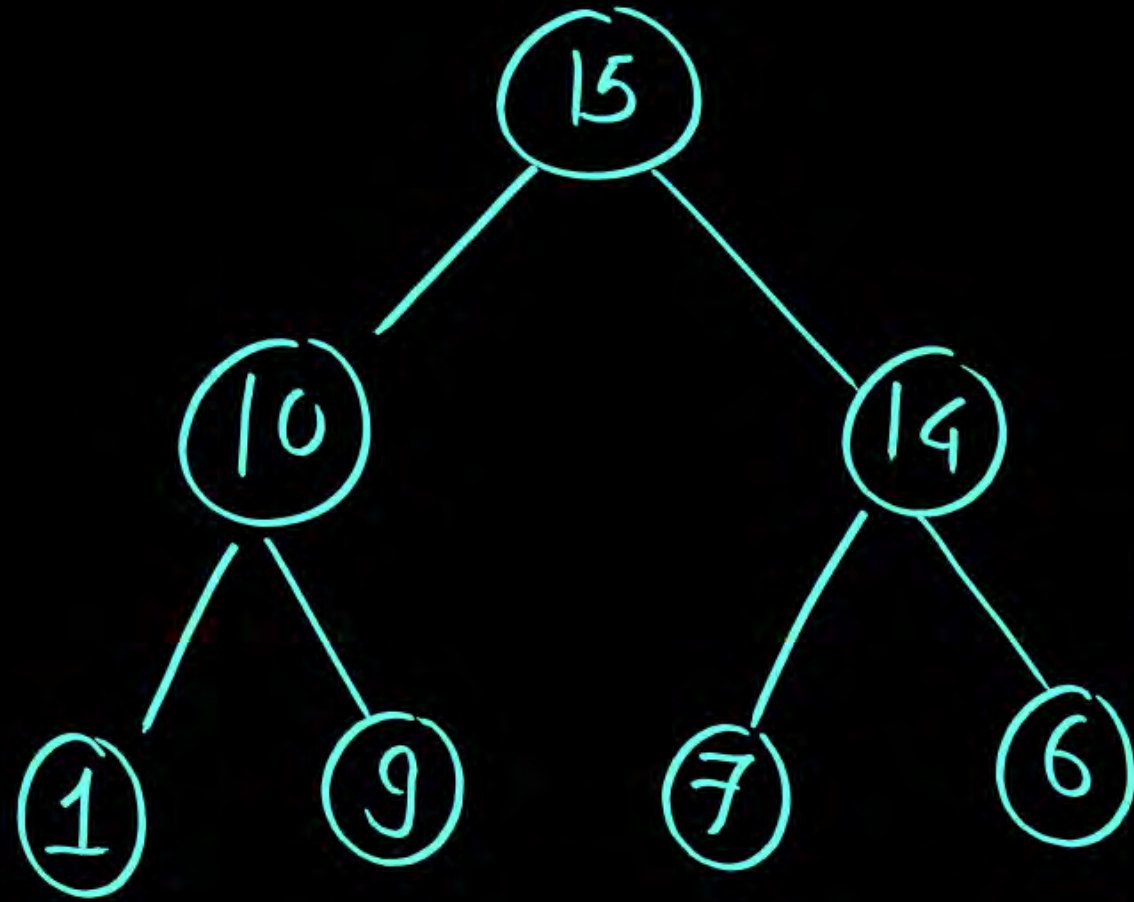


1. Delete only root element of Max heap
2. Copy last element as root
3. Heapify or Adjust the Heap



Topic : Heap

Deletion from Max heap



Heapify or Adjust

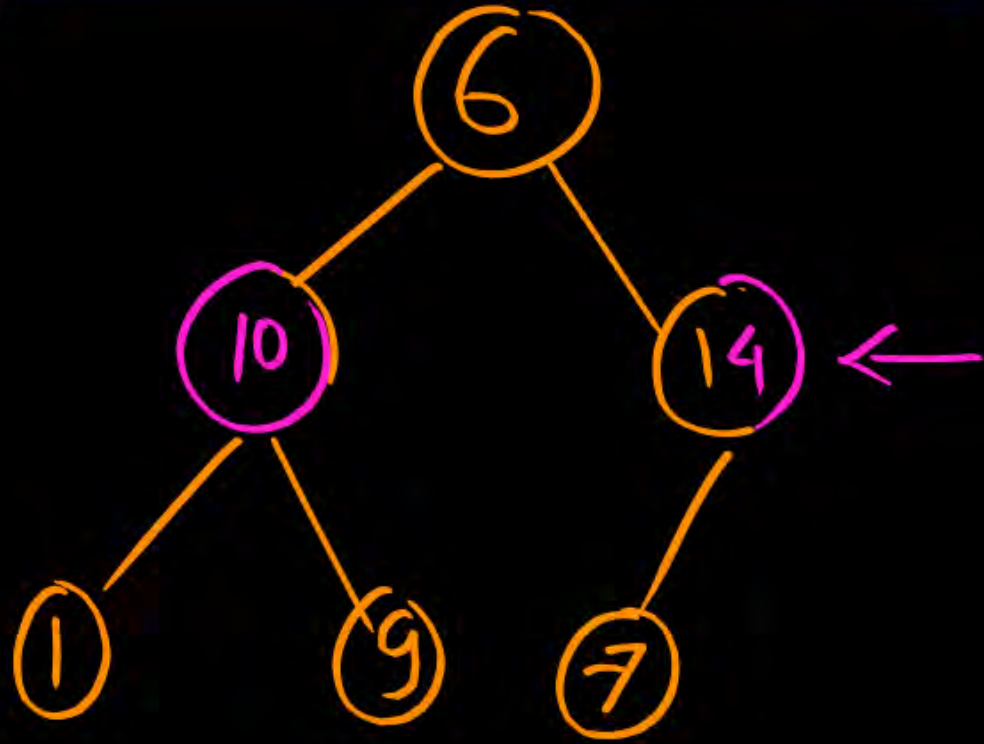


Topic : Heap

Adjust algorithm applied on index i which violates Heap property & rest of element satisfy Heap property.



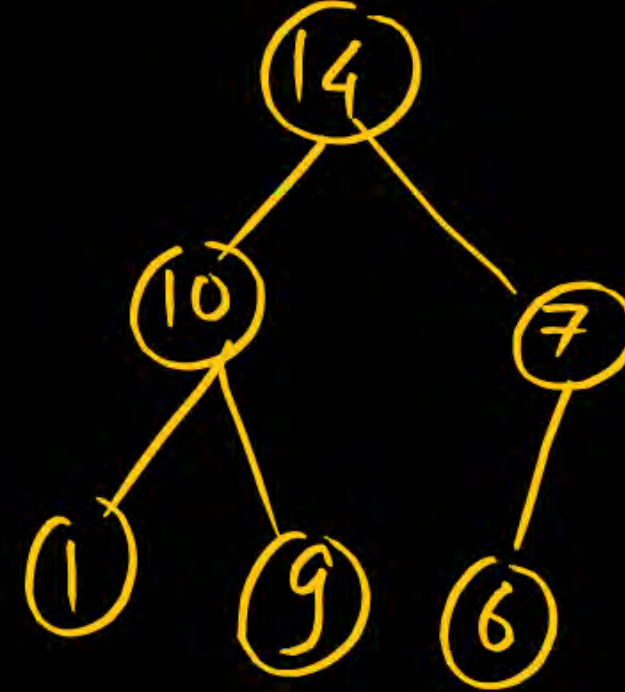
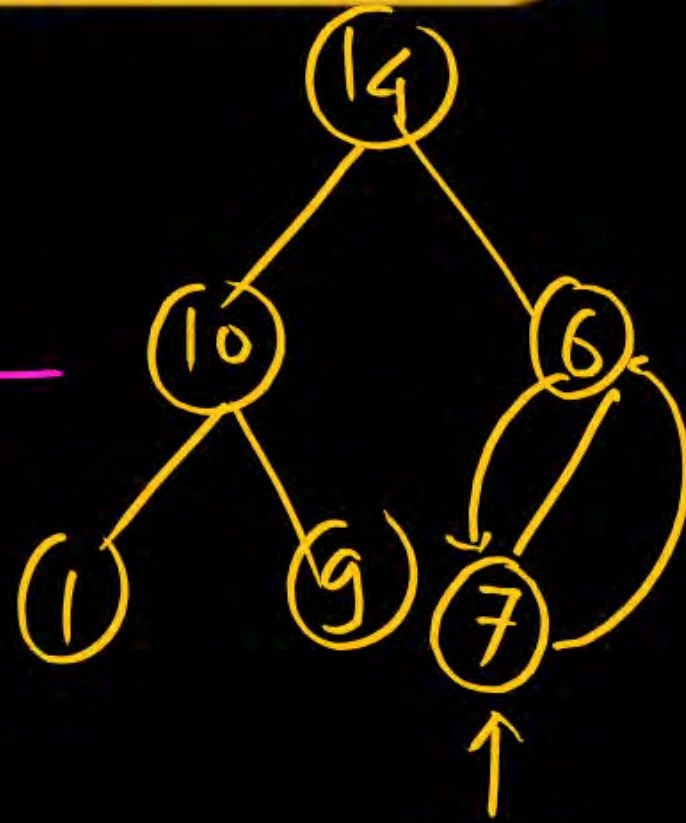
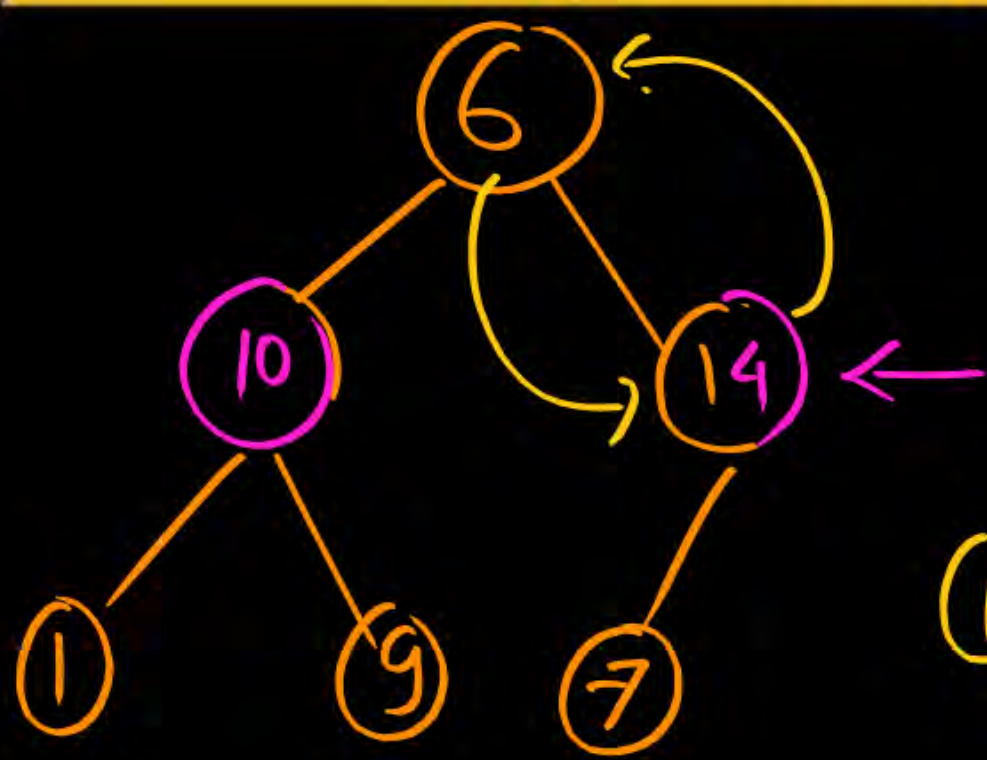
Topic : Heap



1. Compare left & right child & find max(max heap) min(min heap)
2. if element rooted at i is greater (max heap) then stop
3. else swap i and max child
repeat above process



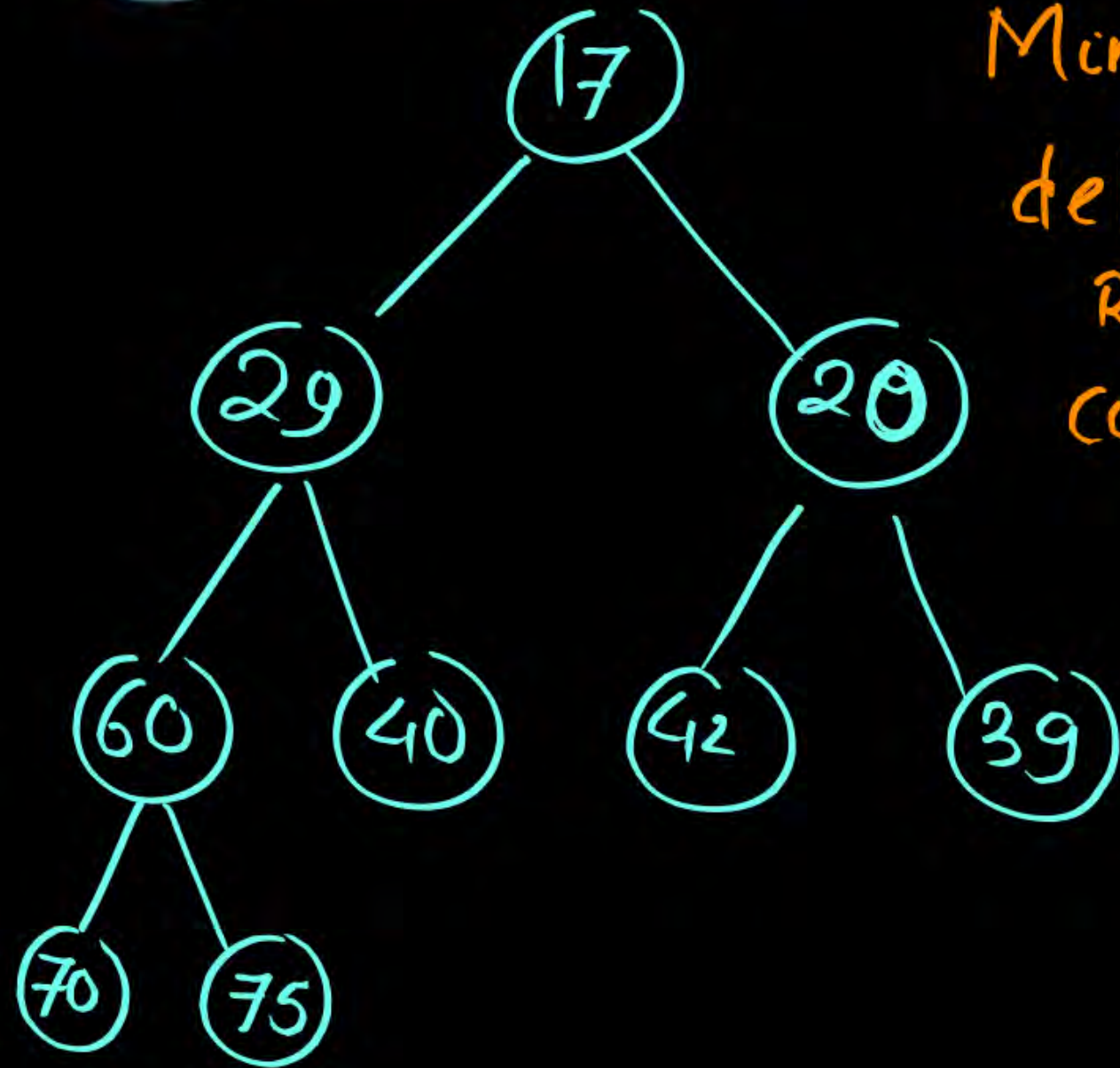
Topic : Heap



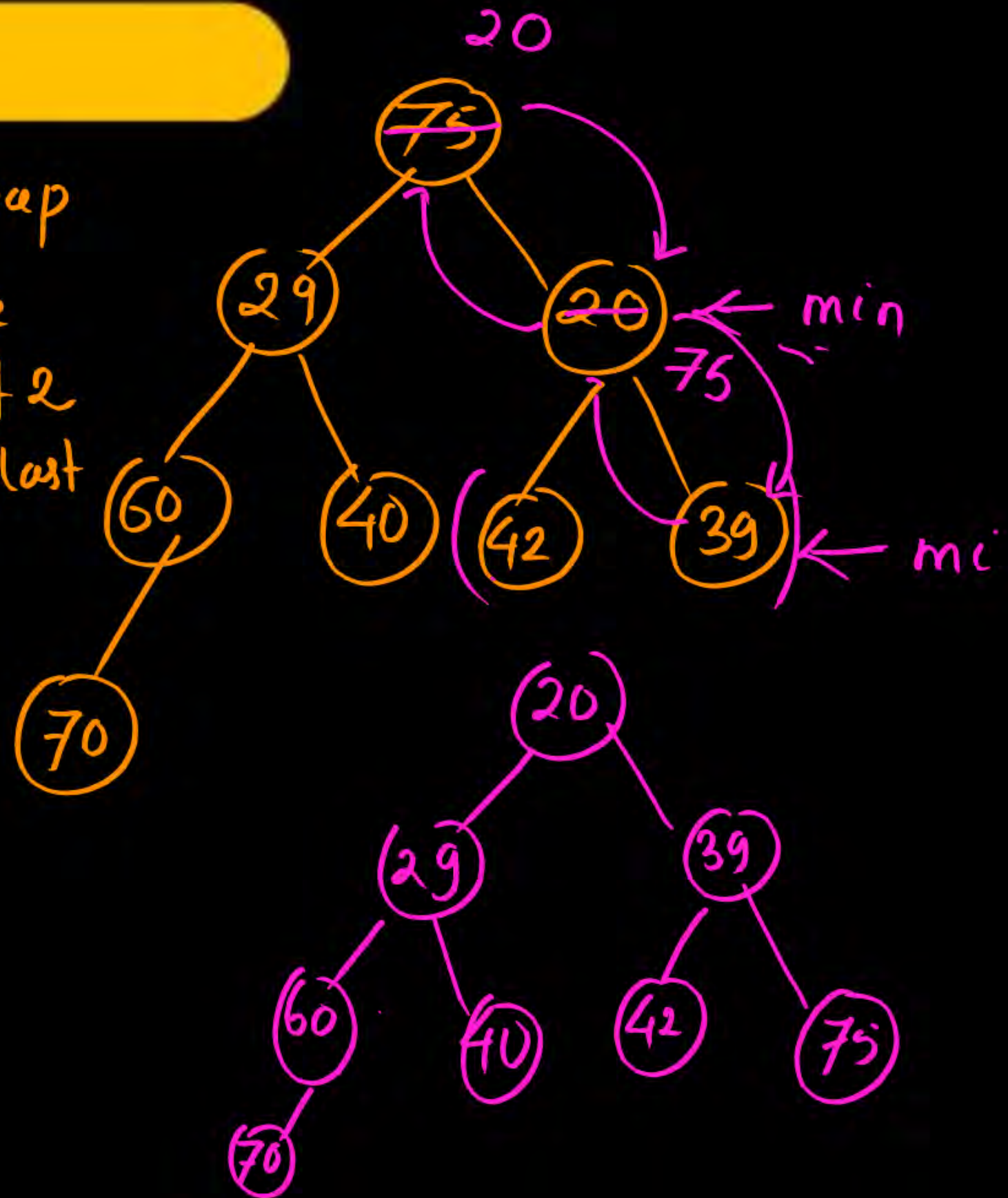
← Max Heap



Topic : Heap



Min Heap
delete
Root 2
Copy last





Topic : Heap



Consider a binary max-heap implemented using an array.
Which one of the following array represents a binary max-heap?

- A. {25, 12, 16, 13, 10, 8, 14}
- B. {25, 14, 13, 16, 10, 8, 12}
- C. {25, 14, 16, 13, 10, 8, 12}
- D. {25, 14, 12, 13, 10, 8, 16}

Suppose max-heap represented by

25, 14, 16, 13, 10, 8, 12

what is content of array after 2 delete operation

A. 14, 13, 12, 10, 8

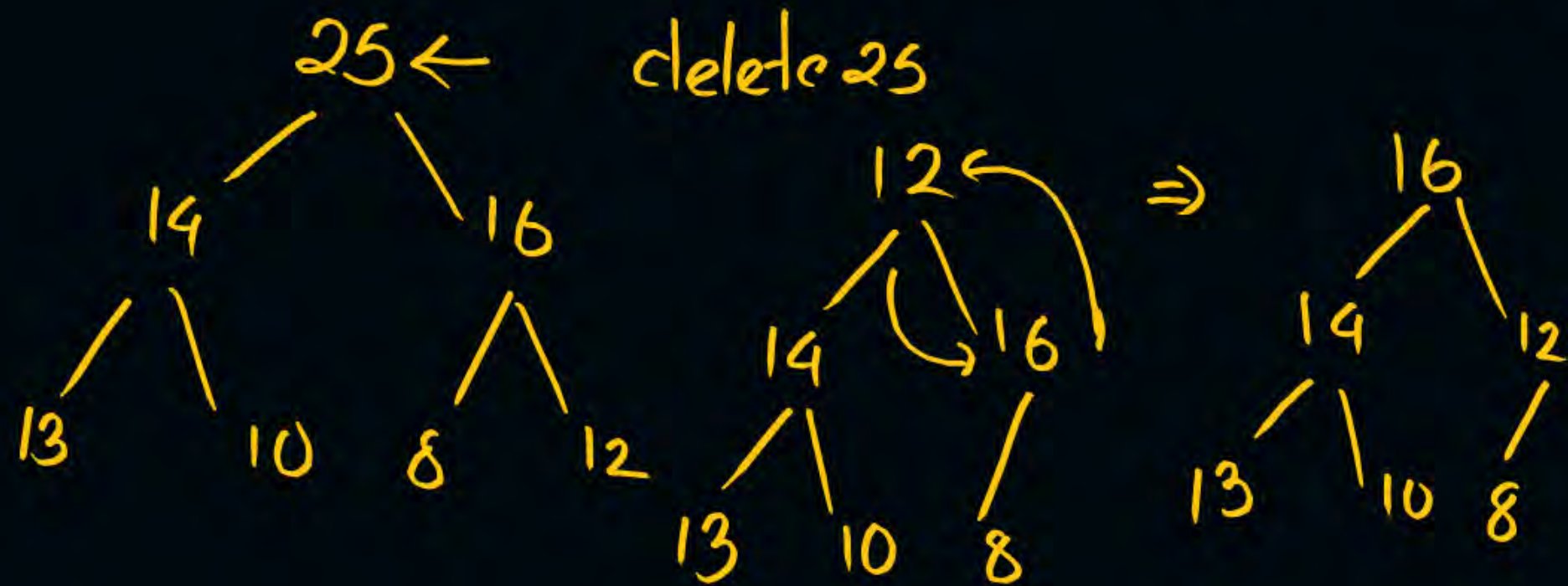
B. 14, 12, 13, 8, 10

C. 14, 13, 8, 12, 10

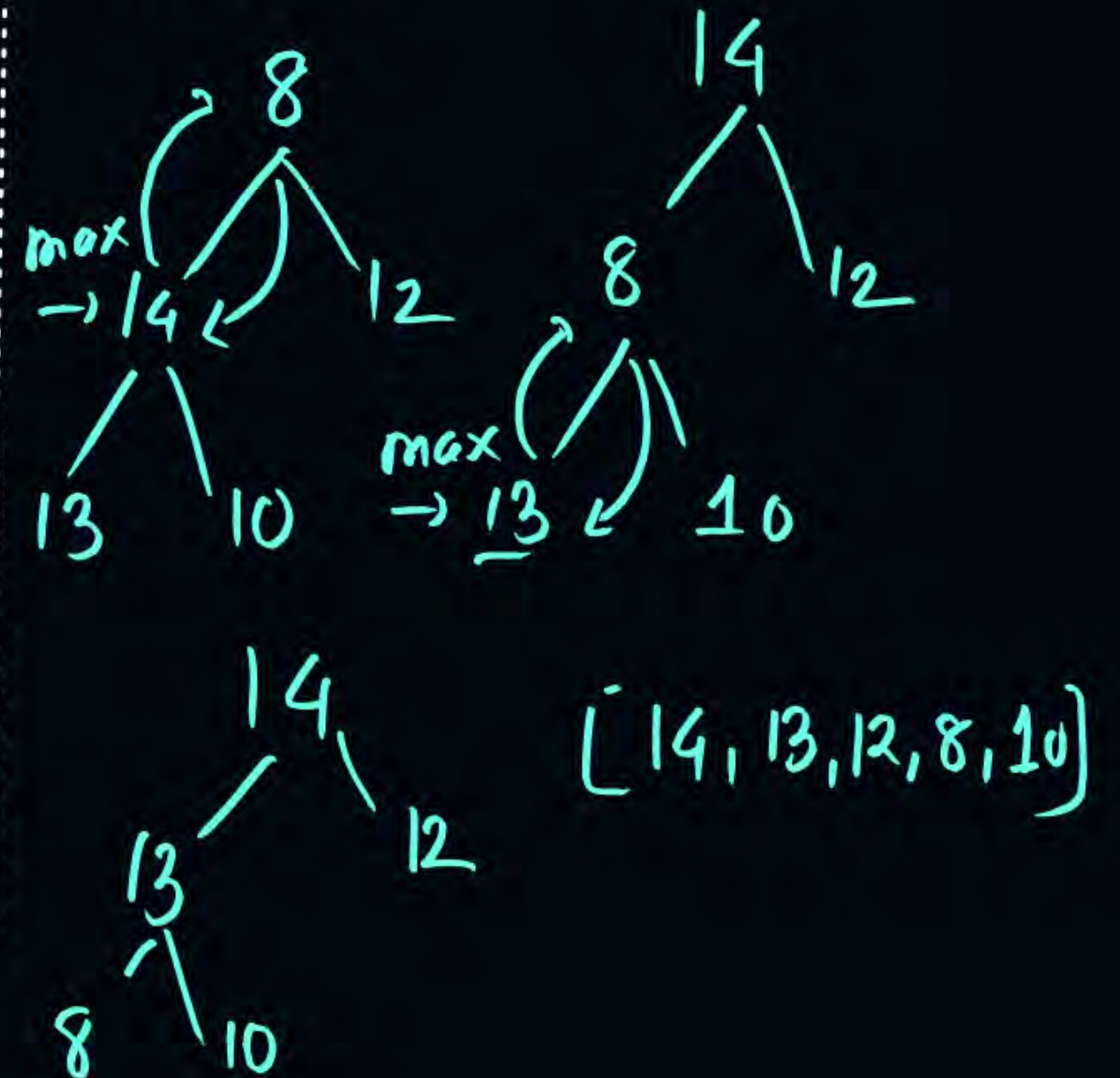
D. 14, 13, 12, 8, 10

Suppose max-heap represented by

25, 14, 16, 13, 10, 8, 12



delete 16





2 mins Summary



Topic

Heap

Topic

CBT

Topic

Max - Min Heap

Topic

Insert

Topic

Delete.

THANK - YOU