GATE

Computer Science & Information Technology

Theory Of Computation

DPP: 1

Regular Language and Grammar

Q1 Consider alphabet $\Sigma = \{a, b\}$, the empty string $\in$ and the set of strings S, P, Q and R generated by the corresponding non-terminals of a regular grammar. S, P, Q and R related as follows (S is a start symbol):

$S \to aP \mid bQ \mid \in$

$P \to bR \mid aS$

$Q \to aR \mid bS$

$R \to aQ \mid bP$

(A) $L = \{w: n_a(w)$ and $n_b(w)$ both are even$\}$.

(B) $L = \{w: n_a(w)$ and $n_b(w)$ both are odd$\}$.

(C) $L = \{w: n_a(w)$ or $n_b(w)$ are even$\}$.

(D) None of these.

Q2 Consider the following language L on alphabet eod re$\Sigma = \{a, b\}$

$L = \{wxw^R \mid w, x \in \{a, b\}^+\}$

The correct regular grammar of above language is/are possible?

(A) $S \to aAa \mid bAb$

$A \to aA \mid bA \mid a \mid b$

$B \to aA \mid bA \mid a \mid b$

(B) $S \to aAa \mid bAb \mid \in$

$A \to ab$

(C) $S \to aA \mid bB$

$A \to aA \mid bA \mid a$

$B \to bB \mid aB \mid b$

(D) $S \to Aa \mid Bb$

$A \to Aa \mid Ab \mid a$

$B \to Bb \mid Ba \mid b$

Q3 Consider the following grammar G:

G:

$S \to A\,B\,C$

$A \to aA \mid a$

$B \to bc$

$C \to cC \mid \in$

The language generated by above grammar is?

(A) $L = \{a^* \, bc \, c^*\}$

(B) $L = \{a^+ \, b \, c^+\}$

(C) $L = \{a^* \, b \, c^*\}$

(D) None of these

Q4 Consider the following two language L_1 and L_2.

$L_1 = \{www \mid w \in \{a\}^*\}$

$L_2 = \{\{a^{n^n}\}^* \mid n \geq 1\}$

Which of the following is correct?

(A) L_1 is regular.

(B) L_2 is regular.

(C) Both L_1 and L_2 are regular.

(D) None of these.

Q5 Which of the following language is non-regular?

(A) $L = \{wxw^R \mid x, w \in \{a, b\}^*\}$.

(B) $L = \{wxw \mid w, x \in \{a, b\}^*\}$.

(C) $L = \{wxwx \mid w, x \in \{a, b\}^*\}$.

(D) None of these

Q6 Consider the following grammars G_1 and G_2:

G_1:

$S \to aAb$

$A \to aB \mid \in$

$B \to Ab$

G_2:

$S \to aABb$

$A \to aA \mid \in$

$B \to bB \mid \in$

Which of the following grammar is/are regular?

(A) G_1 only

(B) G_2 only

(C) Both G_1 only G_2

(D) None of these

Q7 Consider the following three languages:

(1) $L = \{a^{n^n} | n \geq 1\}$

(2) $L = \{a^{m^n} | m = n^2, n \geq 1\}$

(3) $L = \{a^{m^n} | n \geq 1, m > n\}$

Total number of regular languages is/are______.

Q8 Which of the following language is non-regular?

(A) L = $\{a^{2m} b^n b^n \mid m, n \geq 1\}$

(B) L = $\{a^m b^n X \mid m, n \geq 1, X \in \{a,b\}^*\}$

(C) $L = \left\{\left\{a^{n^2}\right\}^* \middle| n \geq 0\right\}$

(D) None of these

Q9 Consider following statements:

S_1: Kleene Closure (*)of infinite set is always finite.

S_2: Kleene Closure (*) of finite set is always infinite.

Which of the following is correct?

(A) S_1 only (B) S_2 only

(C) Both S_1 and S_2 (D) None of these

Q10 Consider the following statements:

[I] If L is regular, then $\overline{L}$ is regular.

[II] If $\overline{L}$ is regular, then L is regular.

[III] Union of L and its complement is Σ^*

Number of correct statement is/are______.

Q11 Consider a regular language L, which of the following statements are true regarding L.

(A) Prefix(L) = $\{w \mid ww_1 \in L, w_1 \in \Sigma^*\}$ is regular.

(B) Suffix(L) = $\{w \mid w_1 w \in L, w_1 \in \Sigma^*\}$ is regular.

(C) Quotient(L) is regular.

(D) L is closed under infinite intersection.

Q12 Consider a regular language L over the alphabet

$\Sigma = \{a, b\}$. L is defined as L= (a + b*) (bab*).

If homomorphism h is defined over T = {c, d, e} and

h(a) = cd

h(b) = cddec

Then the regular language h(L) is given as

(A) (cd + cddec) (cddec cd cddec)

(B) (cddec) (cd + cddec*)

(C) (cd + (cddec)*) ((cddec) (cd) (cddec)*)

(D) None of these

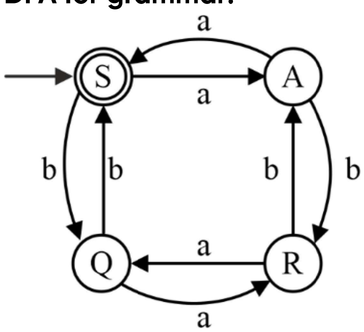
Answer Key

Q1	(A)
Q2	(A)
Q3	(B)
Q4	(C)
Q5	(D)
Q6	(B)
Q7	1
Q8	(D)
Q9	(D)
Q10	3
Q11	(A, B, C)
Q12	(C)

Hints & Solutions

Q1 Text Solution:

DFA for grammar:



L = $(aa + ab + ba + bb)^*$

Hence, option (a) is correct.

Q2 Text Solution:

L = $\{wxw^R \mid w, x \in \{a, b\}^+\}$

$a(a + b)^+ a \mid b(a + b)^+ b$

↓ ↓

$ab(a + b)^+ba \quad ba(a + b)^+ ba$

$aa(a + b)^+aa \quad bb(a + b)^+ bb$

L = Regular

Regular expression = $a(a + b)^+ a + b(a + b)^+b$

$S \to aAa \mid bAb$

$A \to aA \mid bA \mid a \mid b$

$B \to aA \mid bA \mid a \mid b$

Regular expression for above grammar is $a(a + b)^+ a + b(a + b)^+b$

Hence, only (a) is correct.

Q3 Text Solution:

$S \to ABC = aa^*bcc^*$

$A \to aA \mid a = aa^*$

$B \to bc = bc$

$C \to cC \mid \in = c^*$

Regular expression = aa^*bcc^*

$= a^+ bc^+$

Hence, option (b) is correct.

Q4 Text Solution:

$L_1 = \{wxw \mid w \in \{a\}^*\}$

$L_1 = (aaa)^*$

Regular language

$$L_2 = \{\{a^{n^n}\}^* \mid n \geq 1\}$$

$L_2 = \{a^1\}^*$

$= a^*$

= Regular.

Q5 Text Solution:

(a) $L = \{wxw^R \mid x, w \in \{a, b\}^*\}$

Minimal string = $\in \cdot (a + b)^* \in$

$= (a + b)^*$

Regular

(b) $L = \{wx\,w \mid w, x\,\{a, b\}^*\}$

$L = \in \cdot (a + b)^* \cdot \in$

$= (a + b)^*$

Regular

(c) $L = \{wxwx \mid w, x \in \{a, b\}^*\}$

regular

Hence option (d) is correct.

Q6 Text Solution:

Only G2 is regular.

Q7 Text Solution:

(1) $L = \{a^{n^n} \mid n \geq 1\}$

$L = \{a, a^4, a^{27}, \ldots\}$ Non-regular

(2) $L = \{a^{m^n} \mid m = n^2, n \geq 1\}$

$$L = \left\{a^{1^1}, a^{4^2}, a^{9^3} \ldots\right\}$$

$$= \{a, a^{16}, a^{43} \ldots\}$$

Non-regular

(3) $L = \{a^{m^n} \mid n \geq 1, m > n\}$

$$L = \left\{a^{2^1}, a^{3^1}, a^{4^1} \ldots\right\}$$

$$= \{a^2, a^3, a^4 \ldots\}$$

$= aa(a)^*$

Regular

Q8 Text Solution:

(a) $L = \{a^{2m} b^n b^n \mid m, n \geq 1\}$

$= (aa)^+ b^{2n}$

$= (aa)^+ (bb)^+$ Regular

(b) $L = \{a^m b^n X \mid X \in \{a, b\}^* m, n \geq 1\}$

$= (a)^+ (b)^+ (a + b)^*$

= Regular

(c) $L = \left\{\left\{a^{n^2}\right\}^* \middle| n \geq 0\right\}$

$L = \{\in, a, aa, aaa \ldots\}$

$= a^*$

= Regular

Hence, option (d) is correct.

Q9 Text Solution:

S_1: False

Set = $\{\in\} = \{\in\}^* = \in$ only (Finite)

S_2: Set = $\{a\} = \{a\}^* = \in, a, aa, aaa, .. = (a^*)$ (Infinite)

So, both statements are false.

Hence, option (d) is correct.

Q10 Text Solution:

- L is regular if and only if Complement of L is regular.
- $L \cup \overline{L} = \Sigma^*$

Hence, all are correct statements.

Q11 Text Solution:

Regular language is closed under Prefix, Suffix and quotient of the language. But regular language are not closed under infinite intersection.

So, a, b, c are correct.

Q12 Text Solution:

Homomorphism is a function from strings to string which is based on concatenation.

for any a and b

L is defined as

$x = (a + b)^* (bab^*)$

then,

$h(L) = (h(a) + h(b)^*(h(b)h(a)h(b)^*)$

$= (cd + (cddec)^*)((cddec)(cd)(cddec)^*)$.