COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 02

BOOLEAN THEOREMS AND GATES

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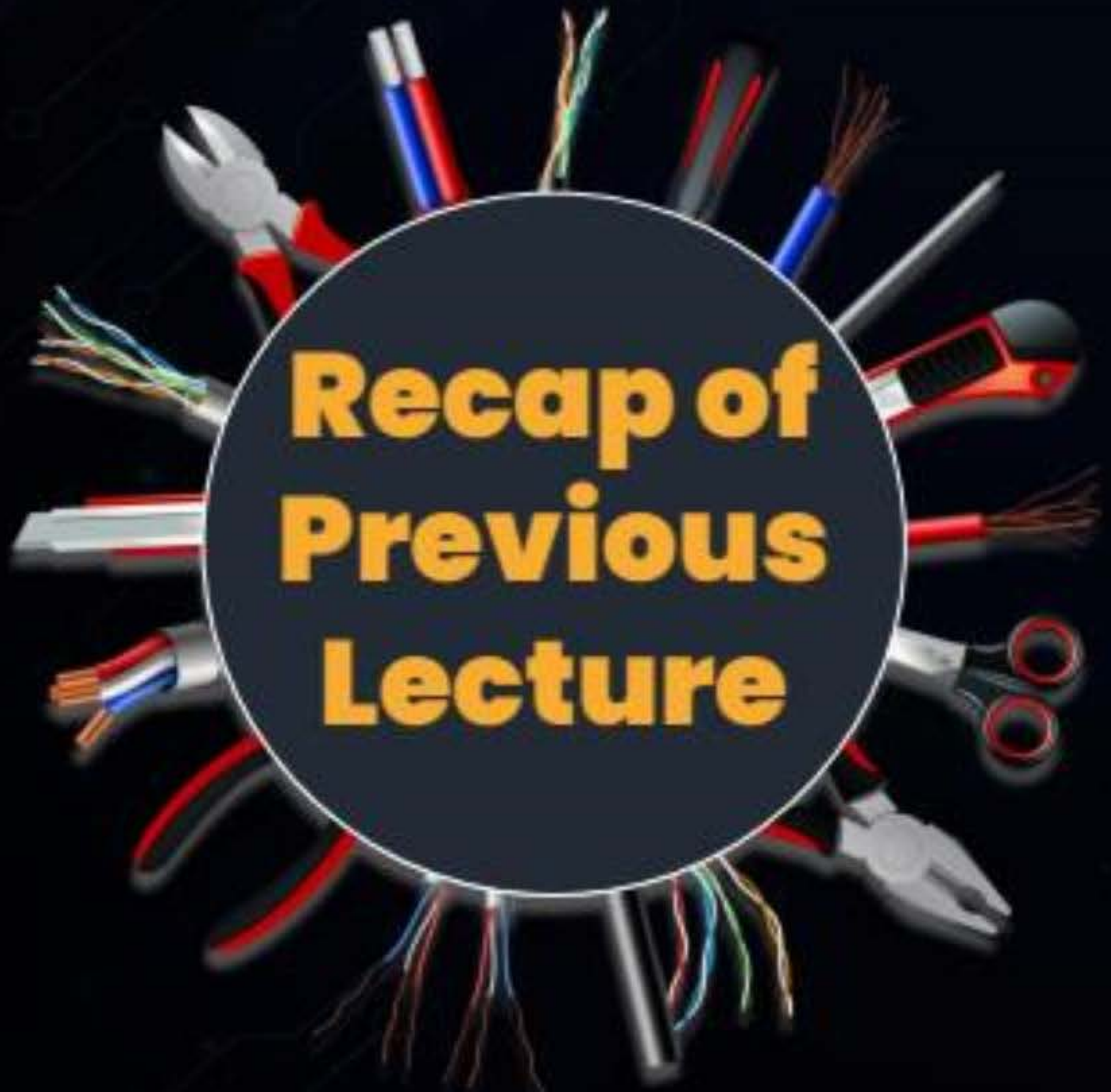


Boolean Theorems → 'OR' & AND related

Topics to be Covered

Boolean Theorems

Question Discussion

Q. $(P+Q+R)(P+Q+\bar{R})(\bar{P}+Q+R)$

$= [(P+Q)+R\cdot\bar{R}][\bar{P}+Q+R] = (P+Q)(\bar{P}+Q+R) = Q + P\cdot(\bar{P}+R)$

$= Q + PR$

Q. $(\bar{A}+\bar{B}+C)(A+B+C)(A+\bar{B}+C) = (\bar{B}+C)(A+B+C) = C + \bar{B}(A+B)$

$= C + A\bar{B}$

$= (\bar{A}+\bar{B}+C)[(A+C)+B\cdot\bar{B}] = (\bar{A}+\bar{B}+C)(A+C) = C + A\cdot(\bar{A}+\bar{B})$

$= C + A\bar{B}$

$= [C + (\bar{A}+\bar{B})(A+B)][(A+\bar{B}+C)]$

$= [C + (\bar{A}+\bar{B})(A+B)(A+\bar{B})] = C + (\bar{A}+\bar{B})A = C + A\bar{B}$

3. Very imp Boolean theorems

- Redundancy Theorem :→

$$A B + \bar{A} C + \underbrace{BC}_{\text{redundant term}} = AB + \bar{A}C$$

$$A B + \bar{A} C + \underbrace{\bar{B}C}_{\text{non redundant}} \longrightarrow$$

$$A B + \bar{A} \bar{C} + \underbrace{B\bar{C}}_{\text{redundant}} = AB + \bar{A}\bar{C}$$

- $\underbrace{\bar{A}B}_{\text{redundant}} + \bar{A}C + B\bar{C} = \bar{A}C + B\bar{C}$

- $\bar{A}\bar{C} + \overbrace{\bar{B}\bar{C}}^{\text{redundant}} + A\bar{B} = \bar{A}\bar{C} + A\bar{B}$

[DeMorgan Theorem]

- $\overline{A+B+C} = \bar{A}\cdot\bar{B}\cdot\bar{C}$
- $\overline{AB+C} = \overline{A\cdot B}\cdot\bar{C} = (\bar{A}+\bar{B})\cdot\bar{C}$
- $\overline{A\cdot B\cdot C} = \bar{A}+\bar{B}+\bar{C}$
- $\overline{(A+B)\cdot C} = \overline{(A+B)}+\bar{C}$

$= \bar{A}\cdot\bar{B}+\bar{C}$

$\overline{(AB+C)\cdot A\cdot D}$

$= \overline{(AB+C)}+\bar{A}+\bar{D}$

$= \overline{AB}\cdot\bar{C}+\bar{A}+\bar{D}$

$= (\bar{A}+\bar{B})\cdot\bar{C}+\bar{A}+\bar{D}$

- $\overline{(\bar{A}+B+C)} = A\cdot\bar{B}\cdot\bar{C}$

[**Question**]

$A \cdot 1 + BC + \bar{A}C$ is equal to

(a) $(A + C)$

(b) $(A + B)(B + C)$

(c) $(A + B)(\bar{A} + C)$

(d) $C(\bar{A} + B)$

$A + (\bar{A} \cdot C) + BC$

$= (A + \bar{A}) \cdot (A + C) + BC$

$= A + C + BC$

$= A + C(1 + B)$

$= A + C$

Question

$\bar{A}B + AC + \bar{B}C$ is equivalent to

(a) $\bar{A}B + AC$

(b) $\bar{A}B + C$

(c) $AC + \bar{B}C$

(d) $\bar{A}B + \bar{B}C$

$\bar{A}B + AC + \bar{B}C \longrightarrow$ not applicable

$A + (B \cdot C)$

$= (A+B) \cdot (A+C)$

$\bar{A}B + AC + \bar{B}C \longrightarrow$ not applicable

$\Rightarrow \bar{A} \cdot B + (A + \bar{B}) \cdot C$

$P + (\bar{P} \cdot C)$

$= (P + \bar{P}) \cdot (P + C)$

$= (P + C) = \bar{A}B + C$

$\bar{A}B = P$

$\overline{\bar{A} \cdot B} = \bar{P}$

$A + \bar{B} = \bar{P}$

Question

$\overline{A\bar{B}(\bar{A}B + \bar{B}C + A\bar{B}D + A\bar{B}\bar{D})}$ is equal to

(a) $(\bar{A} + B)$

(b) $(\bar{A} + B)(B + \bar{C})$

(c) $(\bar{A} + B)(A + \bar{B})$

(d) $\bar{A}B + \bar{B}D$

$A\bar{B}(D+\bar{D}) = A\bar{B}$

$\overline{P \cdot [\bar{A}B + \bar{B}C + P]}$

$= \overline{P} = \overline{A\bar{B}} = (\bar{A} + B)$

$\boxed{P \cdot [P + \text{anything}] = P}$

$= P \cdot P + P \cdot \text{anything}$

$= P + P \cdot \text{anything}$

$= P(1 + \text{anything})$

$= P$

$$\# \quad \overline{\bar{A}BC\left[AB + \bar{B}C + \bar{A}B + BCD + \bar{A}CD\right]}$$

$$= \overline{\bar{A}BC\left[B + \bar{B}C + BCD + \bar{A}CD\right]}$$

$$= \overline{\bar{A}C \cdot B \cdot \left[B + \bar{B}C + BCD + \bar{A}CD\right]}$$

$$= \overline{\bar{A}CB}$$

$$= A + \bar{C} + \bar{B} = A + \bar{B} + \bar{C}$$

$$= \overline{C \cdot \bar{A}B\left[\bar{A}B + \text{anything}\right]}$$

$$= \overline{C\bar{A}B} = A + \bar{B} + \bar{C}$$

Question

$\overline{(\bar{A}+\bar{B})\cdot(\bar{B}+\bar{C})}$ is equal to $= \overline{\bar{B} + \bar{A}\,\bar{C}} = B\cdot\overline{\bar{A}\cdot\bar{C}} = B\cdot(A+C)$

(a) $\bar{B}(A+C)$

(b) $A(B+C)$

(c) $B(A+C)$

(d) $C(A+B)$

$= \overline{(\bar{A}+\bar{B})} + \overline{(\bar{B}+\bar{C})}$

$= A\cdot B + B\cdot C$

$= B(A+C)$

Question

$\bar{A}\bar{B} + AC + \underline{\underline{\bar{B}C}}$ is equivalent to

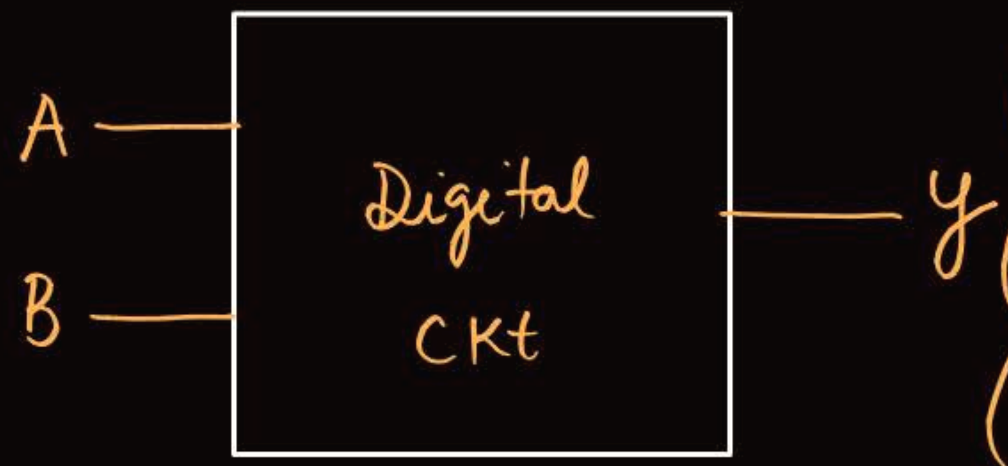
redundant

(a) $(A + \bar{B}) \cdot (\bar{A}\bar{B} + C)$

(b) $\bar{A}\bar{B} + AC$

(c) $AC + \bar{B}C$

(d) $\bar{A}\bar{B} + \bar{B}C$



POS	SOP	A	B		Y
$(A+B)$	$\bar{A}\cdot\bar{B}$	0	0	0	0
$(A+\bar{B})$	$\bar{A}\cdot B$	0	1	1	1
$(\bar{A}+B)$	$A\cdot\bar{B}$	1	0	2	0
$(\bar{A}+\bar{B})$	$A\cdot B$	1	1	3	0

$$y = \Sigma(1) = \bar{A}B$$

$$y = \pi(0,2,3)$$

$$= (A+B)(\bar{A}+B)$$

$$(\bar{A}+\bar{B})$$

$$= B(\bar{A}+\bar{B})$$

$$= \bar{A}B$$

SOP $\rightarrow$ sum of product term

$0 \longrightarrow \bar{A}$

$1 \longrightarrow A$

$$y = \Sigma 1 = \pi(0,2,3)$$

POS $\longrightarrow$ Product of sum term

$0 \longrightarrow A$

$1 \longrightarrow \bar{A}$

- $y(\underline{\underline{A}},\underline{\underline{B}}) = \sum(\underline{\underline{1}},\underline{\underline{2}}) = \pi(0,3) = (A+B)\cdot(\bar{A}+\bar{B})$

$$= A\bar{B} + \bar{A}B = \bar{A}B + A\bar{B}$$

$$= \bar{A}\cdot B + A\bar{B}$$

- $y(A,B) = \sum(0,1,2) = \pi(3) = (\bar{A}+\bar{B})$

$$= \bar{A}\cdot\bar{B} + \bar{A}\cdot B + A\cdot\bar{B}$$

$$= \underline{\underline{\bar{A}}} + (\underline{\underline{A}}\cdot\bar{B})$$

$$= (\bar{A}+A)\cdot(\bar{A}+\bar{B})$$

$$= (\bar{A}+\bar{B})$$

- $f(A,B) = \sum(1,3) = \Pi(0,2)$

  $\bar{f}(A,B) = f_1(A,B) = \sum(0,2) = \Pi(1,3)$

- $f(A,B,C) = \sum(1,2,3,5)$

  $\bar{f}(A,B,C) = \Pi(1,2,3,5) = \sum(0,4,6,7)$

- $f(A,B,C) = \sum(0,4,6)$

  $\bar{f}(A,B,C) = \sum(1,2,3,5,7) = \Pi(0,4,6)$

H.W.

Q. $AB + A\bar{B}C$ is simplified to

a. $AB + BC$

b. $AB + AC$

c. $AB + \bar{B}C$

d. None of them

Q. $AB + B\bar{C} + \bar{A}C$ simplifies to

a. $(\bar{A}+B)(B+C)$

b. $AB + B\bar{C}$

c. $AB + \bar{A}C$

d. $B\bar{C} + \bar{A}C$

Q. $(A+B+CD)(\bar{A}+B+\bar{C}D)(\bar{A}+B+C) \longrightarrow$ simplify it.

Q. $f(A,B,C) = \overline{\Sigma(1,2,4,6,7)}$

then $f(A,B,C)$ is

a. $\pi(0,3,5)$

b. $\Sigma(0,3,5)$

c. $\Sigma(1,2,4,6,7)$

d. $\pi(1,2,4,6,7)$

2 Minute Summary

→ Boolean Theorems:

Thank you

GW Soldiers !