GATE

Computer Science & Information Technology

Theory of Computation

DPP: 2

Regular Expression and Closure Properties

Q1 Let

$L_1 = a^* b^*$

$L_2 = b^* a^*$

$L_3 = (a + b)^*$

$L_4 = a^* b^* a$

$L = (L_1 \cap L_2) - (L_3 \cup L_4)$

Number of strings in above language L will be _____.

Q2 Consider a regular expression (R):

$R = (a + b)^* (a + b)^2 (a + b)^*$.

How many equivalences classes are existing for above regular expression R?

(A) 2 (B) 3

(C) 4 (D) None

Q3 Let L be any formal language. If L^* is regular language then what is L?

(A) L is regular.

(B) L is non-regular.

(C) L is CFL.

(D) None of these.

Q4 Consider the following two statements:

[I]: There exist a regular language L_1, such that for all language L_2, $L_1 \cup L_2$ is always regular.

[II]: If all states of deterministic finite automata (DFA) except start state are final states then language accepted by DFA is $\sum^+$.

Which of the following is correct?

(A) S_1 only.

(B) S_2 only.

(C) Both S_1 and S_2 are true.

(D) None of these.

Q5 Consider the language L given by the regular expression $(a + b)^* ab(a + b)^*$ over the alphabet {a, b}. What is the correct regular expression of $\overline{L}$?

(A) $(a + b)^* (ab + ba + bb + aa) + \in$

(B) $(a^* b^*)^* (ba + bb + aa) (a^*b^*)^* + a + b$

(C) $(a + b)^* ba (a + b)^* + a + b$

(D) b^*a^*

Q6 For language L = {Every odd bit is a}

On alphabet $\sum = \{a, b\}$. Which of the following is/are correct regular expression?

(A) $(aa + ab)^* (\in + a)$

(B) $(aa + ab + ba + b)^*a$

(C) $(aa + ba)^*(\in + a + b)$

(D) $(a(a + b))^* + (a(a + b))^* a$

Q7 Let us consider the following regular expression

$R = a^*b^* + b^*a^*$.

How many equivalence classes of expression that represent language are equivalent to regular expression R?

Q8 Consider the following languages:

$L_1 = \{a^m b^n c^p \mid m, n, p \geq 0\}$.

$L_2 = \{a^m b^m c^p \mid m, p \geq 0\}$.

$L_3 = \{a^{2m} b^{2m} c^p \mid m, p \geq 0\}$.

Which of the following is/are correct?

(A) $L_1 \subseteq L_2$ and $L_2 \subseteq L_1$.

(B) $L_2 \subseteq L_1$ and $L_3 \subseteq L_1$.

(C) $L_3 \subseteq L_2$ and $L_2 \subseteq L_1$.

(D) $L_2 \subseteq L_3$ and $L_3 \subseteq L_1$

Answer Key

Q1	0
Q2	(B)
Q3	(D)
Q4	(A)
Q5	(D)
Q6	(A, D)
Q7	6
Q8	(B, C)

Hints & Solutions

Q1 Text Solution:

$L_1 \cap L_2 = a^* + b^*$

$L_3 \cup L_4 = (a + b)^*$

$L = (a^* + b^*) - (a + b)^*$

$= \phi$

Number of strings = 0

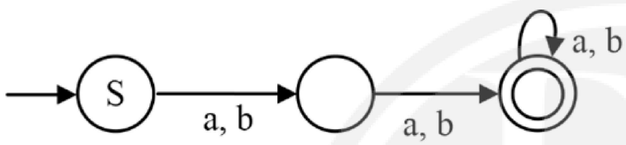
Q2 Text Solution:

$R = (a + b)^* (a + b)^2 (a + b)^*$

Number of equivalence classes in my hill Nerode

= Number of states in minimal DFA

DFA for R:



Number of states = 3

Number of equivalence classes = 3

Q3 Text Solution:

If L^* is regular, L may or may not be a regular.

Example 1: $L^* = (a + b)^*$ is regular, $L = (a + b)$ is regular.

Example 2: $L^* = \{(a^P)^* \mid P \text{ is prime}\}$ is regular but $L = \{a^P \mid P \text{ is prime}\}$ is non-regular.

$\therefore$ Option (d) is correct.

Q4 Text Solution:

S_1 True:

$L_1 = \sum^*$

$L_1 \cup L_2 = \sum^* \cup L_2 = \sum^*$ (Regular)

S_2 False:

May or may not be $\sum^+$

For example: DFA for language ending with "a" on alphabet {a, b}.

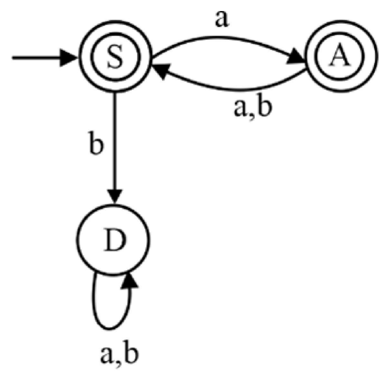
Q5 Text Solution:

$L = (a + b)^* ab (a + b)^*$

L = {containing 'ab' as a substring}

$\overline{L} = \{b^*a^*\}$

Q6 Text Solution:



F A:

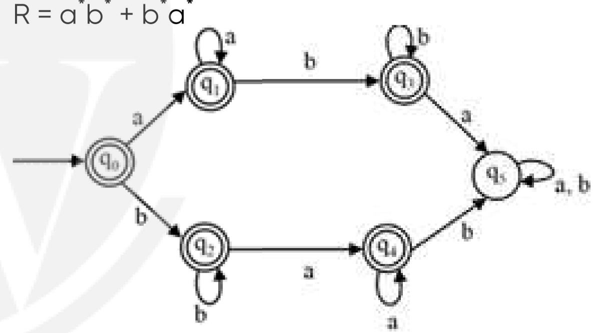
Regular expression = $(a(a + b))^* a + (a (a + b))^* a$

$= (a(a + b))^* (\in + a) : (aa + ab)^*(\in + a)$

Hence, (a, d) are correct.

Q7 Text Solution:

$R = a^*b^* + b^*a^*$



$R = a^*b^* + b^*a^*$

$= (\in + aa^*) = (\in + bb^*) + (\in + bb^*) (\in + aa^*)$

$= \in + aa^* + bb^* + aa^* bb^* + bb^* aa^*$

$[\because a^* = (\in + aa^*)]$

Number of equivalence classes are equivalent to minimum number of states in DFA.

Regular expression for each state represents each equivalence class.

So,

$[q_0] = \in$

$[q_1] = aa^*$

$[q_2] = bb^*$

$[q_3] = aa^* + bb^*$

$[q_4] = bb^* aa^*$

$[q_5] = (aa^*bb^*a + bb^*aa^*b)(a + b)^*$

Q8 Text Solution:

- $L_3 \subseteq L_1$ True
- $L_2 \subseteq L_1$ True
- $L_3 \subseteq L_2$ True

(a) False (b) True

(c) True (d) False