CS & IT ENGINEERING

THEORY OF COMPUTATION

REGULAR EXPRESSION

Lecture No.- 05

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Recap of Previous Lecture

Regular Expression

Topic ?????

- → Construction of Reg Expression
- → DFA states.

		DFA	NFA
1	$(a+b)(a+b)(a+b) \Rightarrow$ exactly 3 $\Rightarrow$	5 $\rightarrow$	4
2	$[(a+b)(a+b)]^* \Longrightarrow$ div by 2 $\Rightarrow$	2 $\rightarrow$	2
3	$\underbrace{(a+b)[(a+b)(a+b)]^*} \Rightarrow$ length odd $\Rightarrow$	2 $\rightarrow$	2.

Topics to be Covered

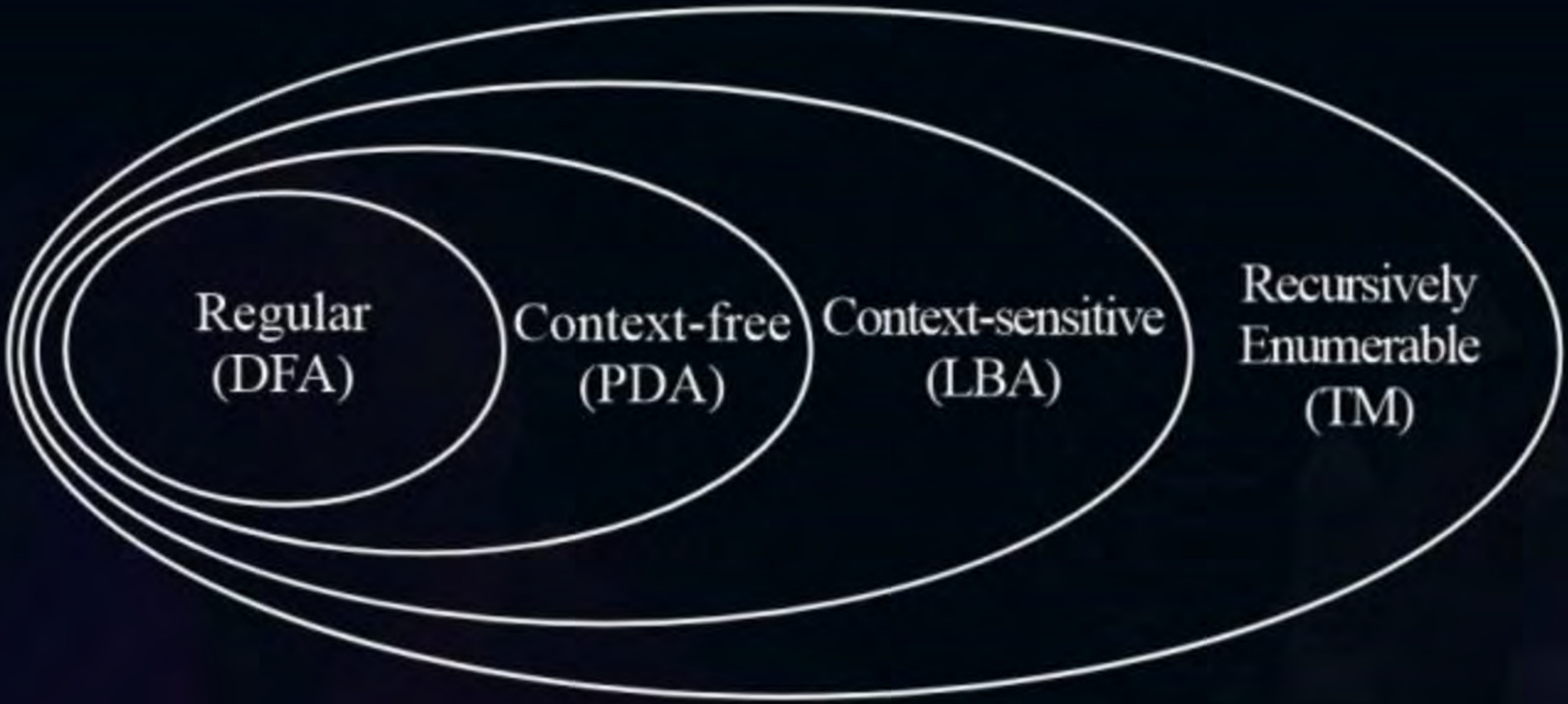
Topic Conversion from $\in$-NFA to NFA

Topic ??

Topic ??

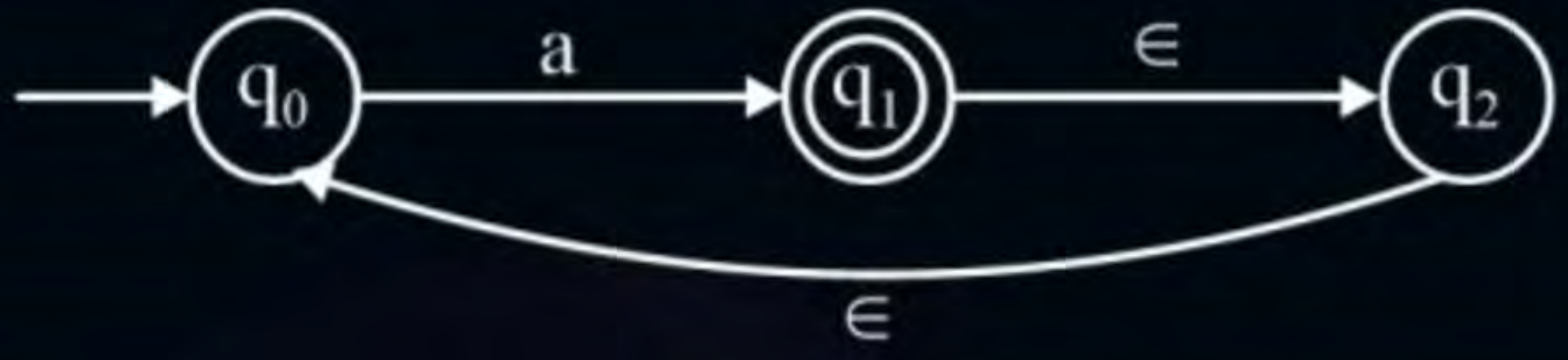
Topic ??

Topic : Theory of Computation



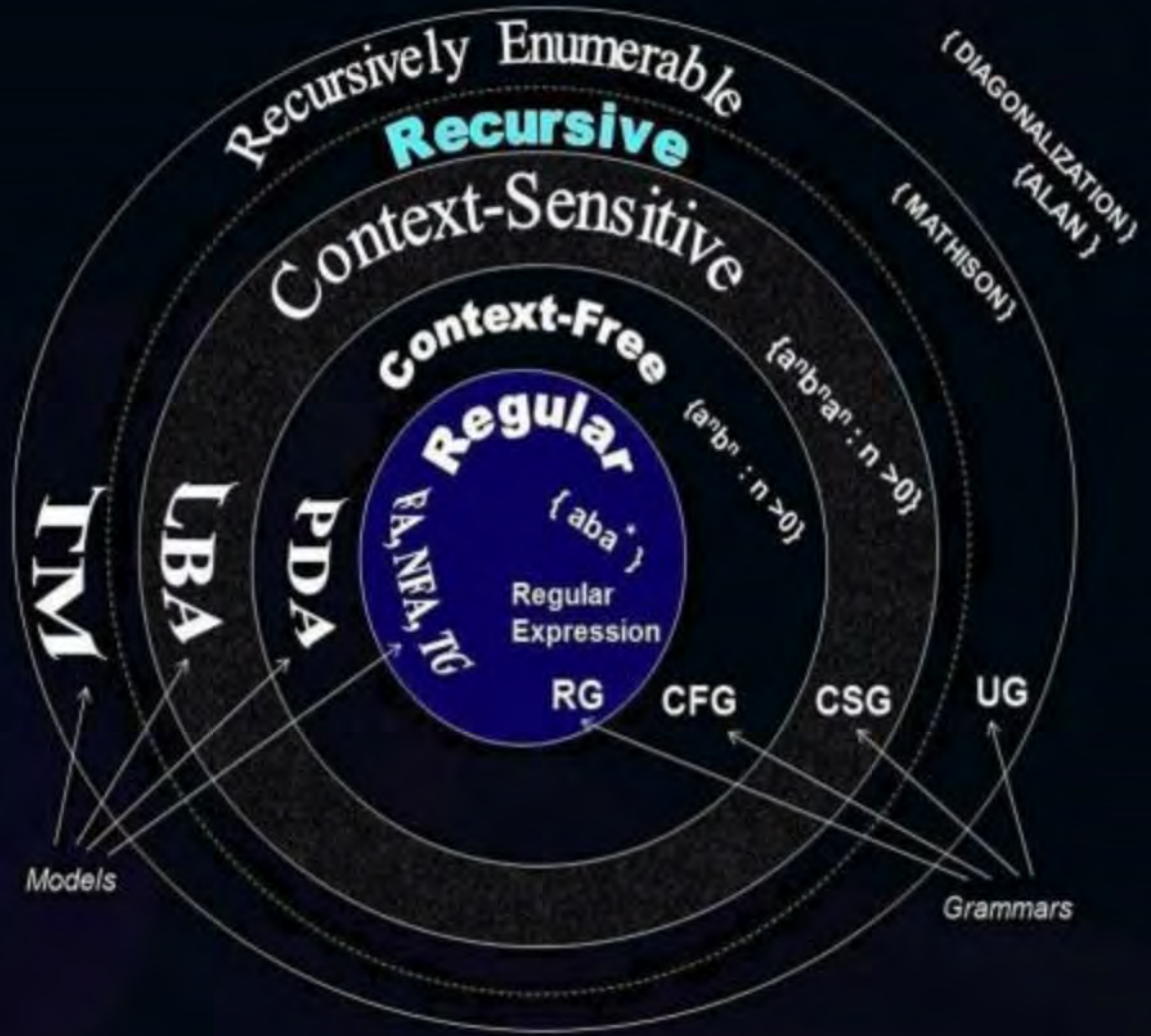
$\delta^1 (q_1, a) = \in\text{-closure} (\delta(\in\text{-closure} (q),a)$

#Q. Construct an equivalent NFA for the following Ɛ-NFA



#Q. Construct an equivalent NFA for the following Ɛ-NFA

Topic : Theory of Computation



Topic : Expressive Power

Number of languages accepted by particular automata is knowns as expressive power.

(TM > LBA > PDA > FA)

1. Expressive power of NFA and DFA same. Hence every NFA is converted into DFA.
2. Expressive power of NPDA is more than DPDA. Hence conversion not possible
3. Expressive power of DTM and NTM is same.

MCQ

#Q. Let D_f, D_p are number of languages accepted by DFA and DPDA respectively. Let N_f, N_P are number of languages accepted NFA and NPDA respectively. Which of the following is true.

A. $N_f = D_f$
$N_p = D_p$

B. $N_f \supset D_f$
$N_p \supset D_p$

C. $N_f = D_f$
$N_p \subset D_p$

D. None

MCQ

#Q. In which of the cases stated below the following statement is false?
"Every nondeterministic machine M_1 there exists an equivalent deterministic machine M_2 recognizing the same language"

A. M_1 is non deterministic FA

B. M_1 is non deterministic turing machine

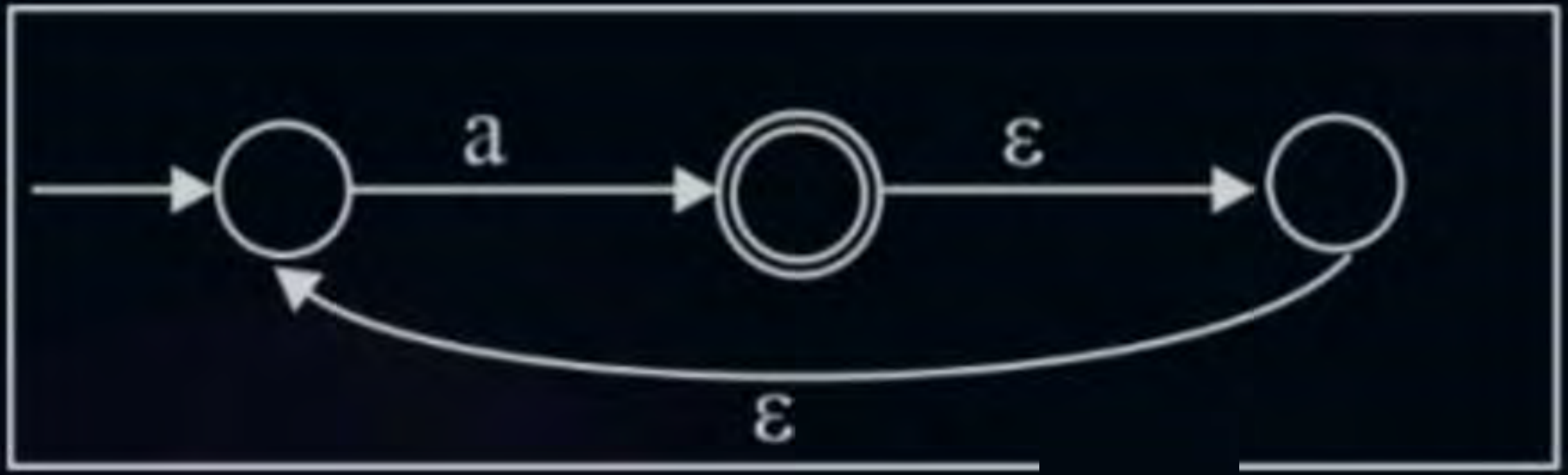
C. M_1 Is non deterministic PDA

D. None

Q

What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ε is the empty string.

[2012: 1 Mark]



A. $\varnothing$

B. $\{\varepsilon\}$

C. a^*

D. $\{a, \varepsilon\}$

Topic : Regular Expression

DFA → Expression

$(a+b)^*$

- The simplest way of representing a regular language is known as Regular expression.
- For every regular language regular expression can be constructed.
- To construct regular expression following 3 operators are used.
- \+ is known as union operator
- . is known as concatenation operator
- * is known as Kleene closure operator

$$L = \{\epsilon, a, b, aa, ab, ba \ldots\}$$

$$\Downarrow$$

$$(a+b)^*$$

$$\Downarrow$$

$$\Sigma^*$$

$L = \{a^n b^n \mid n \geq 0\}$

DFA not possible

Regular Expression also not possible.

Topic : NOTE

- For one regular language many number of regular expressions can be possible.

- One regular expression can generate only one regular language.

Which of the following Languages can give Regular Expression

ab, a^2b^2, a^3b^3

$L_1 = \{a^n b^m \mid n \geq m \text{ and } n \leq m\} = \{a^n b^n\}$

$\cancel{L_2} = \{a^n b^m \mid n > m \text{ and } n < m\} = \{\ \} = \phi$

(a) L_1 only (b) ~~L_2 only~~ (c) both L_1 & L_2 (d) none

$(n+2)$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is exactly 4.

$$\{\underset{2}{\overline{a|b}}\ \underset{2}{\overline{a|b}}\ \underset{2}{\overline{a|b}}\ \underset{2}{\overline{a|b}}\}$$

$$\{(a+b)\cdot(a+b)\cdot(a+b)\cdot(a+b)\}$$

length exactly 4

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atleast 4

$$(a+b)(a+b)(a+b)(a+b)(a+b)^*$$

$$(a+b)(a+b)(a+b)(a+b)^+$$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atmost 4.

$$\epsilon + (a+b) + (a+b)^2 + (a+b)^3 + (a+b)^4$$

(or)

$$(a+b+\epsilon)(a+b+\epsilon)(a+b+\epsilon)(a+b+\epsilon)$$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is divisible by 4.

$$\{0, 4, 8, 12, 16 \text{ ---}\}$$

$$\checkmark \; \left[(a+b)(a+b)(a+b)(a+b)\right]^*$$

$$(a+b)^* = \epsilon + (a+b)^1 + (a+b)^2 + (a+b)^3 + (a+b)^4 + \cdots$$

$$r^* = r^0 + r^1 + r^2 + r^3 + r^4 + \cdots$$

$$r^+ = r^1 + r^2 + r^3 + r^4$$

$$\underset{\text{OR}}{r_1 + r_2} = \{r_1, r_2\}$$

$$r_1 \cdot r_2 = \{r_1 r_2\}$$

$+$

$\cdot$

$*$

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are exactly 4.

$$b^* a b^* a b^* a b^* a b^*$$

× $aaaab^*$

× b^*aaaab^*

× b^*aaaa

$$\Sigma = \{a, b\}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

$$(a+b)^* \, a \, (a+b)^* \, a \, (a+b)^* \, a \, (a+b)^*$$

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atmost 3.

$\{0, 1, 2, 3\}$ a's

$$b^*(a+\epsilon)b^*(a+\epsilon)b^*(a+\epsilon)b^*$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

$$(a+b)^* a (a+b)^* a (a+b)^* a (a+b)^*$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are divisible by 3. MCQ

$$\left(b^*ab^*ab^*ab^*\right)^* + b^*$$

bbbb

bbbb

(a) $(b^*aaab^*)^*$

(b) $(b^*ab^*ab^*ab^*)^*$ X

(c) $(aaa)^* + b^*$

(d) nme

exactly

#Q. How many states are there in minimal DFA that accept following regular expression.

length $\quad$ $n+2$

1. $(a+b)(a+b)(a+b) \Longrightarrow 5$

2. $(a+b+\epsilon)(a+b+\epsilon)(a+b+\epsilon) \Longrightarrow 5$

3. $b^* a b^* a b^* a b^* a b^* \Longrightarrow 6$

$$n+1$$

① $\underbrace{(a+b)^* a (a+b)^* a (a+b)^* a (a+b)^*}_{\text{no. of a's atleast 3}} \Rightarrow 4$

② $\underbrace{[(a+b)(a+b)]^*}_{\text{Div by 2}} \Rightarrow 2$

③ $(a+b)$

#Q. How many states are there in minimal DFA that accept following regular expression.

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where each string starting and ending with different symbol.

$a\left(\begin{matrix} a \cdot s \\ b \cdot s \end{matrix}\right)b \;\|\; b\left(\begin{matrix} a \cdot s \\ b \cdot s \end{matrix}\right)a$

$$a(a+b)^*b + b(a+b)^*a$$

every string

$$\begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix} aab \begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where having substring aab.

$$\begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix} \underline{aab} \begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix}$$

$$\boxed{(a+b)^* aab (a+b)^*}$$

every string

#Q. Construct regular expression that generates set of all strings of a's and b's where having substring aba or bab.

$$\begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix}(\underline{aba}+\underline{bab})\begin{pmatrix} a\text{'s} \\ b\text{'s} \end{pmatrix}$$

$$\downarrow$$

$$(a+b)^*(aba+bab)(a+b)^*$$

#Q. Construct regular expression that generates set of all strings of a's and b's where 4^{th} input symbol is a from left side.

#Q. Construct regular expression that generates set of all strings of a's and b's where 4^{th} input symbol is b from end.

$\begin{pmatrix} a\cdot S \\ b\cdot S \end{pmatrix}$ $\underline{b}$ $\underline{a/b}$ $\underline{a/b}$ $\underline{a/b}$

$(a+b)^* b \; (a+b)(a+b)(a+b)$

$2^4 = 16$

$2^n \rightarrow$ DFA

$n+1 \rightarrow$ NFA

5

#Q. Construct regular expression that generates set of all even length palindrome strings over $\{a\}$.

#Q. Construct regular expression that generates set of all odd length palindrome strings over {a}.

$$\{a, \underbrace{a^3}_{2}, \underbrace{a^5}_{2}, \underbrace{a^7}_{2} - - - \} \rightarrow \text{Regular}$$

#Q. Construct regular expression that generates set of all even length palindrome strings over {a, b}.

#Q. Construct regular expression that generates set of all odd length palindrome strings over $\{a, b\}$.

not possible

#Q. Construct regular expression that generates set of all odd length palindrome strings of English language.

Topic : NOTE

$\{WW^R\}$

- Palindrome languages over more than one symbol are not regular .Hence regular expression not possible.

- Palindrome languages over one symbol are regular.

Odd length Palindrome

$$L_1 = \{ \underline{w} \, c \, \underline{w}^R \mid w \in (a+b)^* \}$$

$$L_2 = \{ \underline{\overset{\epsilon}{W}}\ \underline{\overset{\epsilon}{W}}^R \mid \underline{W} \in \underline{(a)}^* \}$$

$$\{\epsilon, aa, aaaa \ldots\}$$

$$\{\epsilon, a^2, a^4 \ldots\} \equiv \underline{(aa)^*}$$

What is the Regular Expression?

$$L_3 = \{ \underline{w}\, b\, \underline{w}^R \mid w \in (a)^* \}$$

$$L_3 = \{ b, aba, a^2ba^2, a^3\underline{b}a^3 \ldots \}$$

not Regular

Dependency

$$L_4 = \{ \underline{w}\,\underline{w} \mid w \in (a)^* \}$$

$$\{ \epsilon, aa, aaaa, aaaaaa \ldots \}$$

$$= \underline{(aa)^*} \checkmark$$

$$L_5 = \left\{ w\,x\,w^R \;\middle|\; \begin{array}{l} w \in (a+b)^* \\ x \in (a+b) \end{array} \right\}$$

not possible

Possible

$$(a+b)^*$$

$$L_6 = \{ \underset{abab}{W} X \overset{R}{W} \mid W, X \in (a+b)^* \} = (a+b)^*$$

ab a ba

$\{\epsilon, a, b, aa, ab - - -\}$

$(a+b)^*$

ⓐ Yes

ⓑ NO

$$\{\overset{a}{W}\,\overset{\underline{ab}}{X}\,\overset{a}{W}^R \mid W, X \in (a+b)^*\} = \underline{(a+b)^*}$$

$$\{\epsilon, a, b, aa, ab, ba, bb \ldots\}$$

$$(a+b)^*$$

for which of the following Regular Expression is possible?

Lang

$\{\epsilon, aa, bb, abab, baba, ---\}$

non Regular

$L_1 = \{ WW \mid W \in (a+b)^* \}$

$(a+b)^*$

$L_2 = \{ WxW \mid W, x \in (a+b)^* \}$

$\{\epsilon, a, b, aa, ab, bb ---\}$

$\Downarrow$

$(a+b)^*$

(a) L_1 only

(b) L_2 only

(c) L_1 and L_2

(d) none

$$\underbrace{\{ W X W^R \}}$$

$$\{ \underline{\underline{W}}\_\underline{\underline{W}}^{R} \}$$

nitin

liril

madam

$W \mid W^R$

$\{\phi = 0\}$ R → regular Expr

$\{\epsilon = 1\}$

(1) $R + \phi = \phi + R = R$

(2) $R \cdot \phi = \phi \cdot R = \phi$

$\{\epsilon, R\}$

(3) $R + \epsilon = \epsilon + R \neq R$

(4) $R \cdot \epsilon = \epsilon \cdot R = R$

(5) $(R^*)^* = (R^*)^+ = (R^+)^* = R^*$

(6) $R \cdot R^* = R^+ = R^* R$

(7) $\epsilon^* = \epsilon$

(8) $\epsilon^+ = \epsilon$

(9) $\phi^* = \epsilon$

(10) $\phi^+ = \phi$

(11) $(a + b)^* \neq a^* b^*$

(12) $(a + b)^* \neq a^* b^*$

(13) $a(ab)^* = (ab)^* a$

(14) $(a + b)^* = (a + b^*)^*$

$= (a^* + b^*)$

$= (a^* + b^*)^*$

$= (a^* b^*)^*$

(15) $a^* + a^* = a^* = a^*a^*$

(16) $a + b = b + a$

(17) $a \cdot b \neq ba$

3

$R + \epsilon \neq R$

$\{R, \epsilon\} \neq R$

$$R^* = (R^*)^* = (R^*)^+ = (R^+)^* = R^*$$

$$= (R^+)^*$$

$$= (R^+)^0$$

$$\{\epsilon, R^+\} = R^+ + \epsilon = R^*$$

$$R \cdot R^* = R^+$$

$$R \cdot \{\epsilon, \underline{R}, R^2, R^3, R^4 ---\}$$

$$\Downarrow$$

$$\{\underbrace{R \cdot \epsilon}_{R}, R^2, R^3 ---\} = R^+$$

| * | * | * | * | * | * | * | * / * | * | * | * \ * |

[MCQ]

$\phi = 0$ $\qquad R + \phi = R$

#Q. Identify language accepted by following regular expression
$b^*(a^*.\phi . b + ab + a\phi^*b^*)(b + \phi)^*$

ϕ

A. Exactly one a

B. At least one a

C. At most one a

D. None

$\rightarrow b^*(\phi + ab + ab^*)b^*$

$\rightarrow b^*(ab + ab^*)b^*$

$\rightarrow b^*(a(b + b^*))b^*$

$\rightarrow b^*(ab^*)b^* \longrightarrow b^*ab^*b^* \rightarrow b^*ab^*$

b^*ab^*

b^*ab^*

$$b^* + b = b^*$$

[MCQ]

$0(00)^* \rightarrow$ odd

#Q. Which of the following regular expressions are equivalent?

I. $(00)^*(\varepsilon+0) \rightarrow$ all

II. $(00)^* \rightarrow$ even

III. $0^* \rightarrow$ all

IV. $0(00)^* \rightarrow$ odd

A (I) And (II)

B (ii) and (iii)

C (i) And (iii)

D (iii) and (iv)

$$(00)^*(\epsilon + 0)$$

$$\left[(00)^* + (00)^* \cdot 0\right]$$

$$\downarrow$$

even + odd

$$\Downarrow$$

all 0's

[MCQ]

PW

#Q. Which of the following pair of regular expressions are not equal

A $(r^*)^*$ and $(r^+)^*$ → equal.

r^* r^*

B $(r + \varepsilon)^*$ and r^* → equal.

r^*

C $(rr + \varepsilon)^*$ and r^* → not equal.

D None of the above

$(r + \epsilon)^* = r^*$

$(a + b)^*$

$$(a+b)^*$$

$$(\underline{rr}+\epsilon)^* = \{\epsilon, rr, (rr)^2, (rr)^3, (rr)^4 \cdots\}$$

$$\neq r^*$$

(Q) Which of the following is true? Home work

$$\left\{ \begin{array}{l} L_1 = 11(0+1)^* \\ L_2 = (0+1)^* 11 \\ L_3 = 11(0+1)^* 11 + 111 + 11 \end{array} \right\}$$

(a) $L_1 = L_2 = L_3$

(b) $L_1 \cup L_2 = L_3$

(c) $L_1 \cap L_2 = L_3$

(d) none

(Q) Which of the following Regular Expressions are equal ?

1. $(a+ba)^*(b+\epsilon)$
2. $(a^*(ba)^*)^*(b+\epsilon) + a^*(b+\epsilon) + (ba)^*(b+\epsilon)$
3. $(a+ba)(a+ba)^*(b+\epsilon)$

(a) 1 & 2 (b) 1 & 3 (c) 1, 2, 3

(d) 2 & 3

THANK - YOU