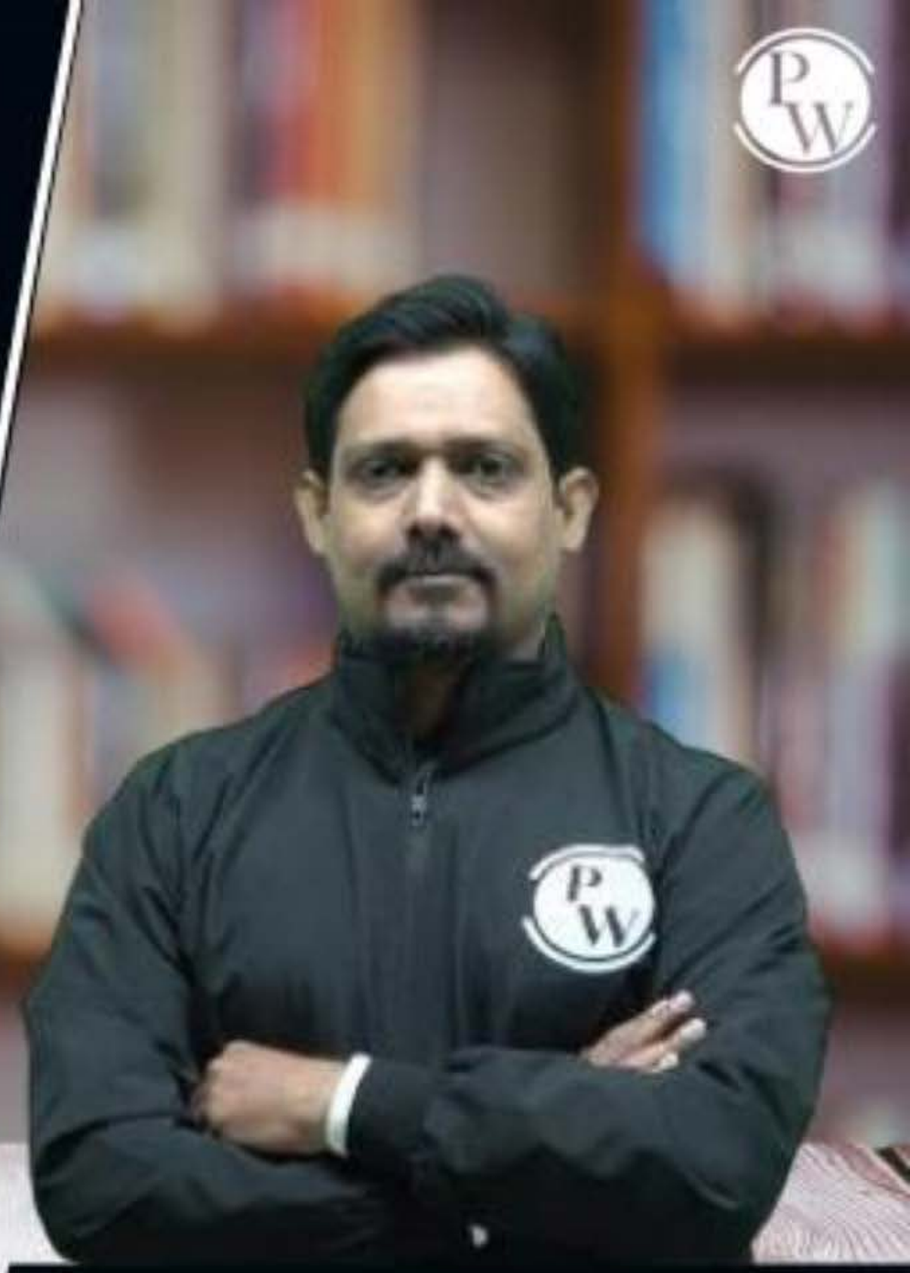
Computer Science & IT

Data Structure & programming

Tree

Lecture No. 01



By- Abhishek Sir

Recap of Previous Lecture

Topic

Topic

Topic

Topic

Topic

Circular Linked List

Queue using Linked List

Slide

Topics to be Covered

Topic — Tree fundamental

Topic — Tree Terminology

Topic — forest , Theorem

Topic — K-ary tree Theorem

Topic — Binary tree

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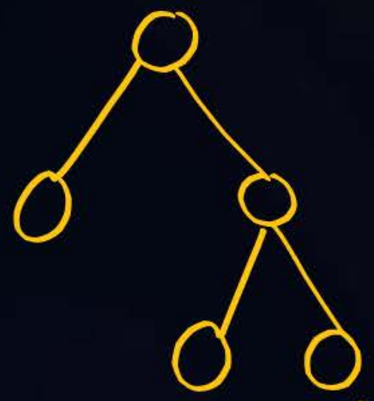
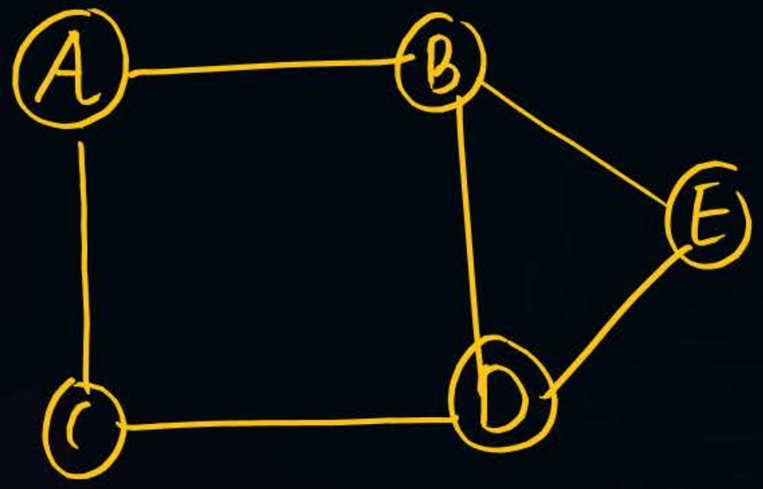
Topic : Tree

1. Tree is a Non Linear Data structure

2. Graph is represented by G(V,E)

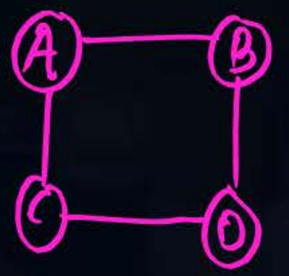


Tree is also set of vertex & edges



$E = \{ (A,B), (B,E), (B,D), (D,E), (A,C), (C,D) \}$

$V = \{ A, B, C, D, E \}$



Disconnected graph

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Topic : Tree

Tree is Acyclic graph (Does not contain cycle)

Tree is always connected

Tree with n vertices has n-1 edges.

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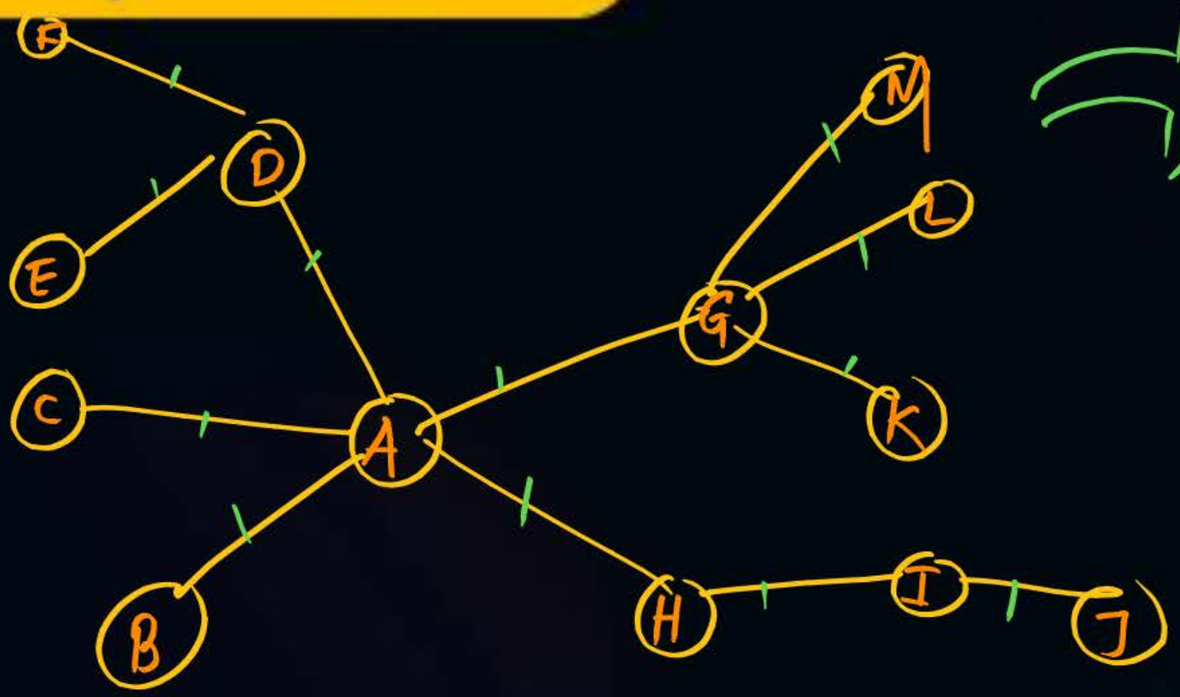
Topic : Tree



Spanning tree

rooted tree

2 parent 2 children

one distinguished Node

Selected as root of tree

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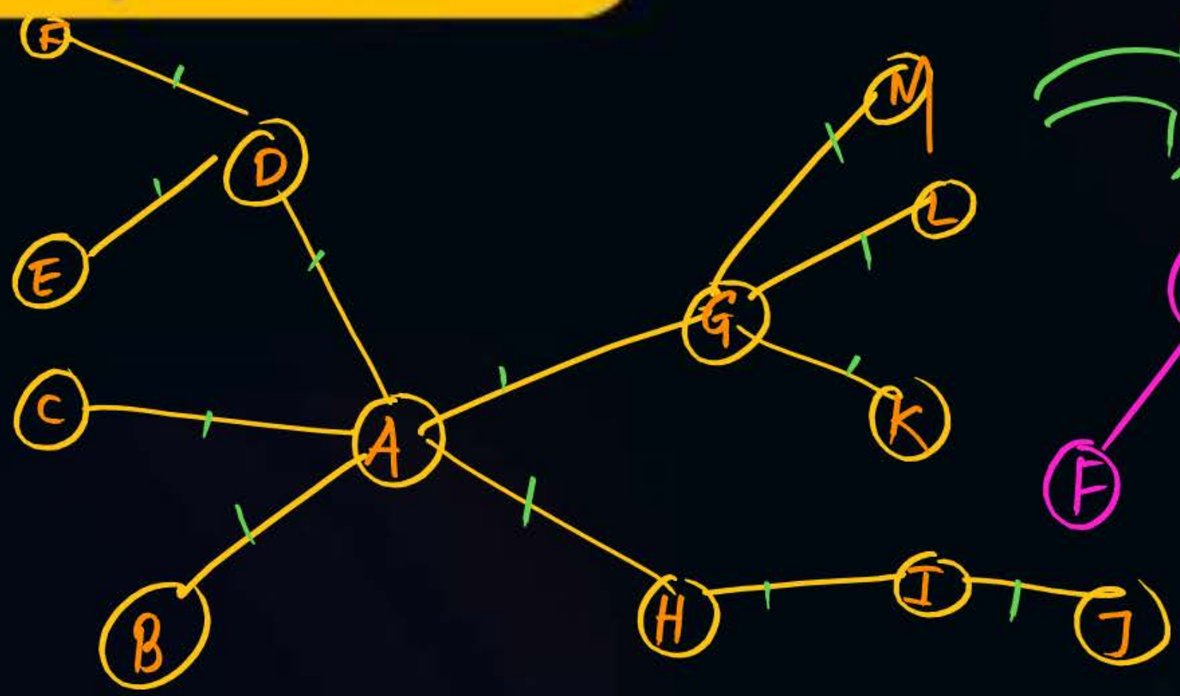
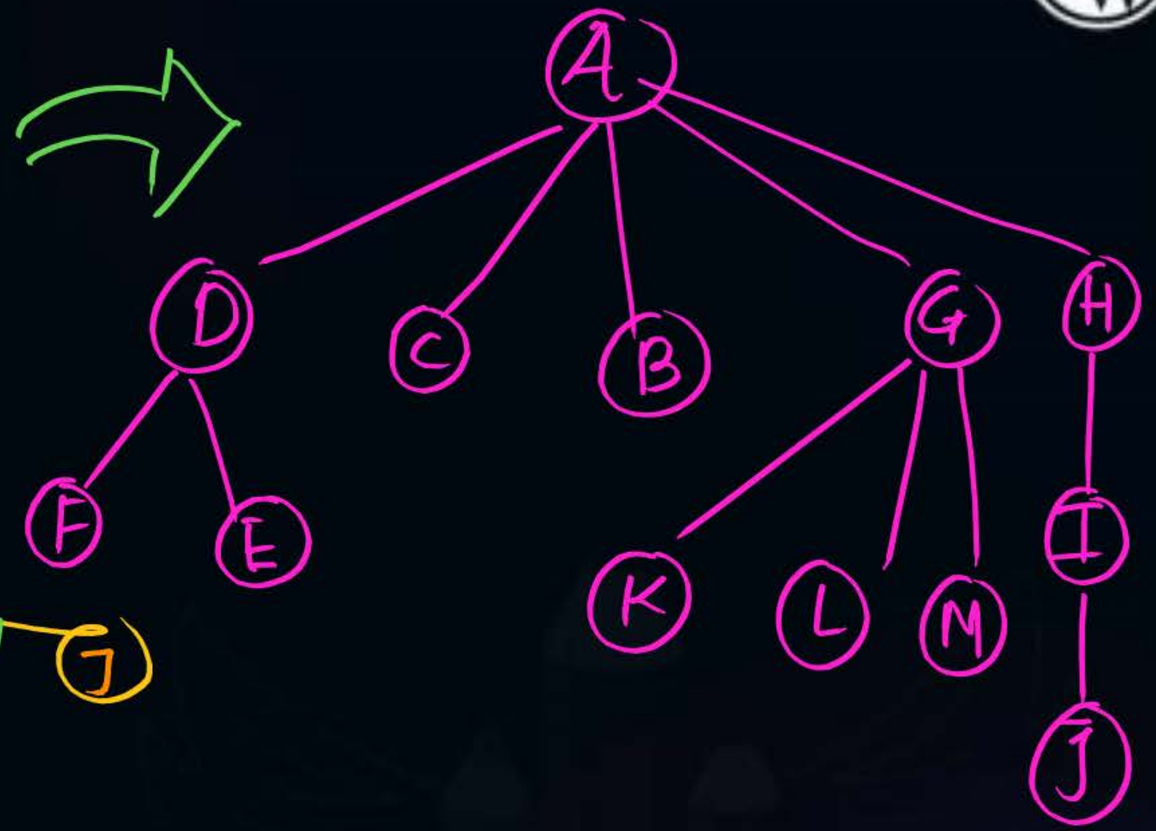
Topic : Tree



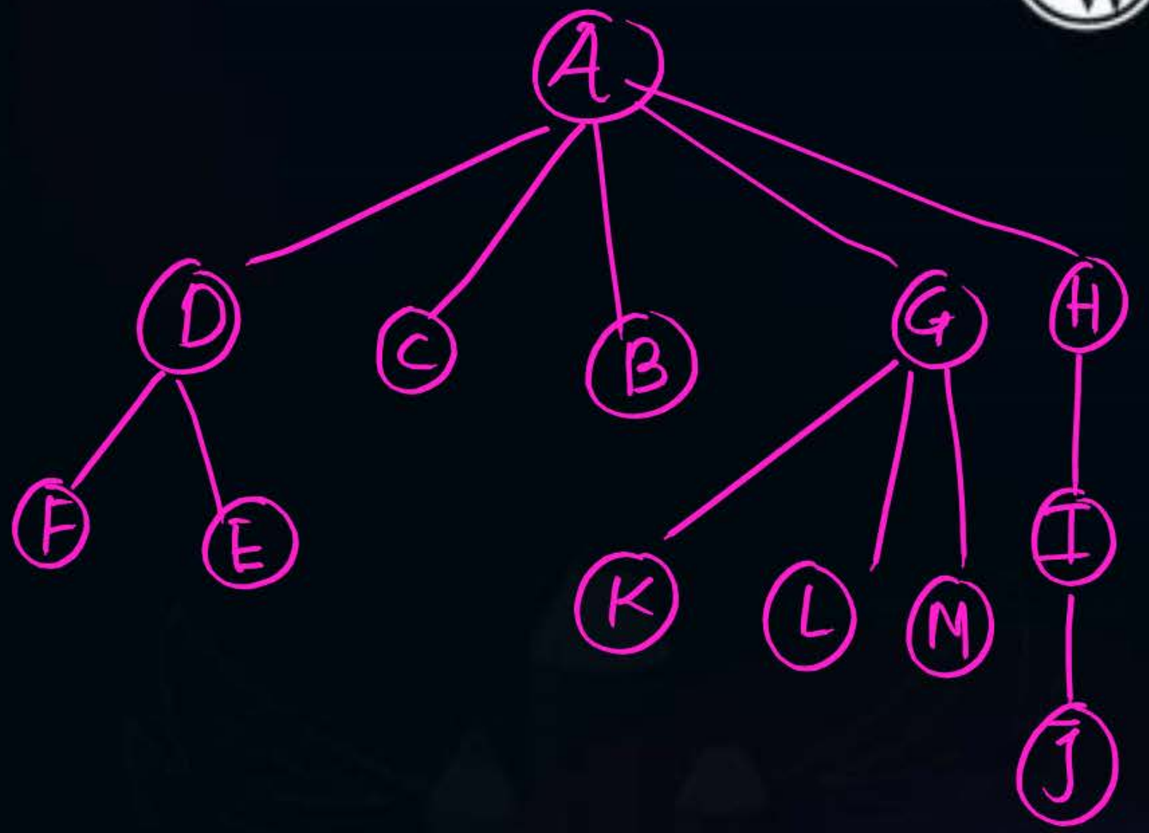


Spanning tree

Slide

Topic : Tree

Rooted Tree



1. A is root of tree

   Root does not have any parent.

2. parent children

   * K, L, M are children of G
   * D is parent of F, E
   * D, C, B, G, H are Sibling (children of same parent)

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Topic : Tree

3. Ancestor & Descendant

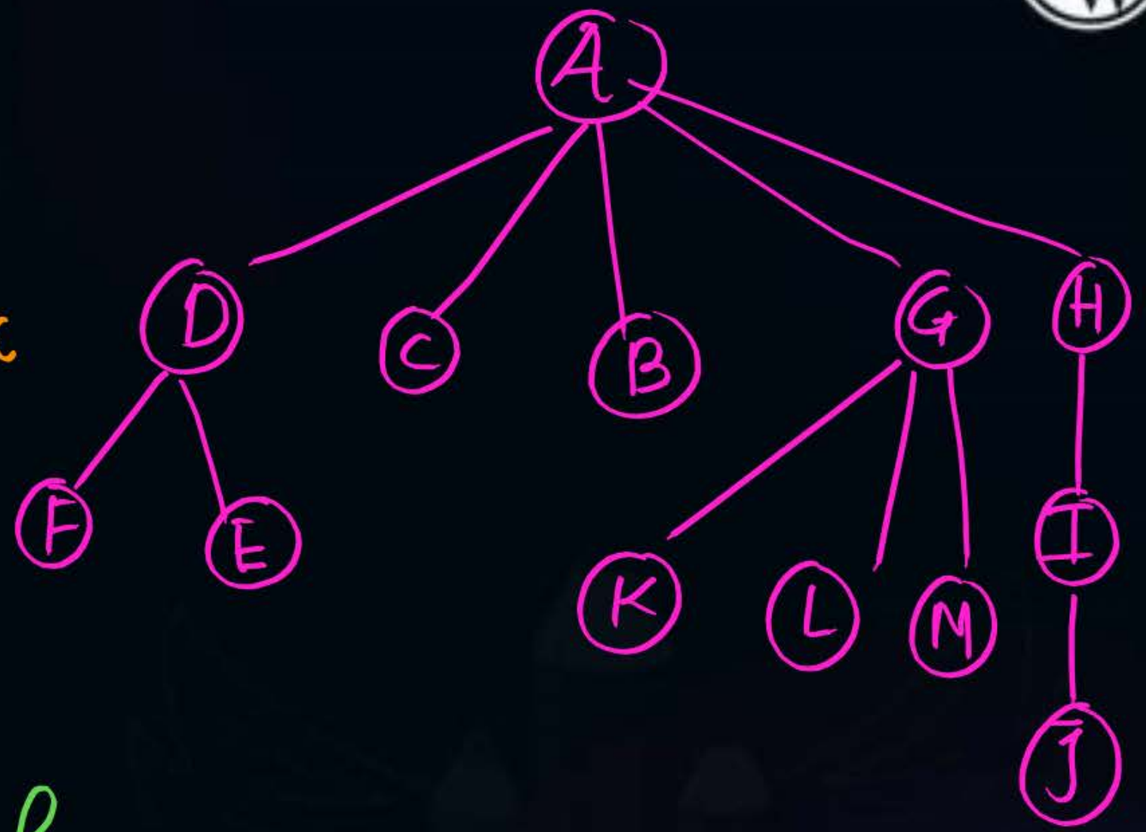
Ancestor : for a given Node x

All node in Node x to root path is ancestor of x.

G, A are ancestor of K

I, HA are ancestor of J

Rooted Tree



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Topic : Tree

Descendant :

for a given node x

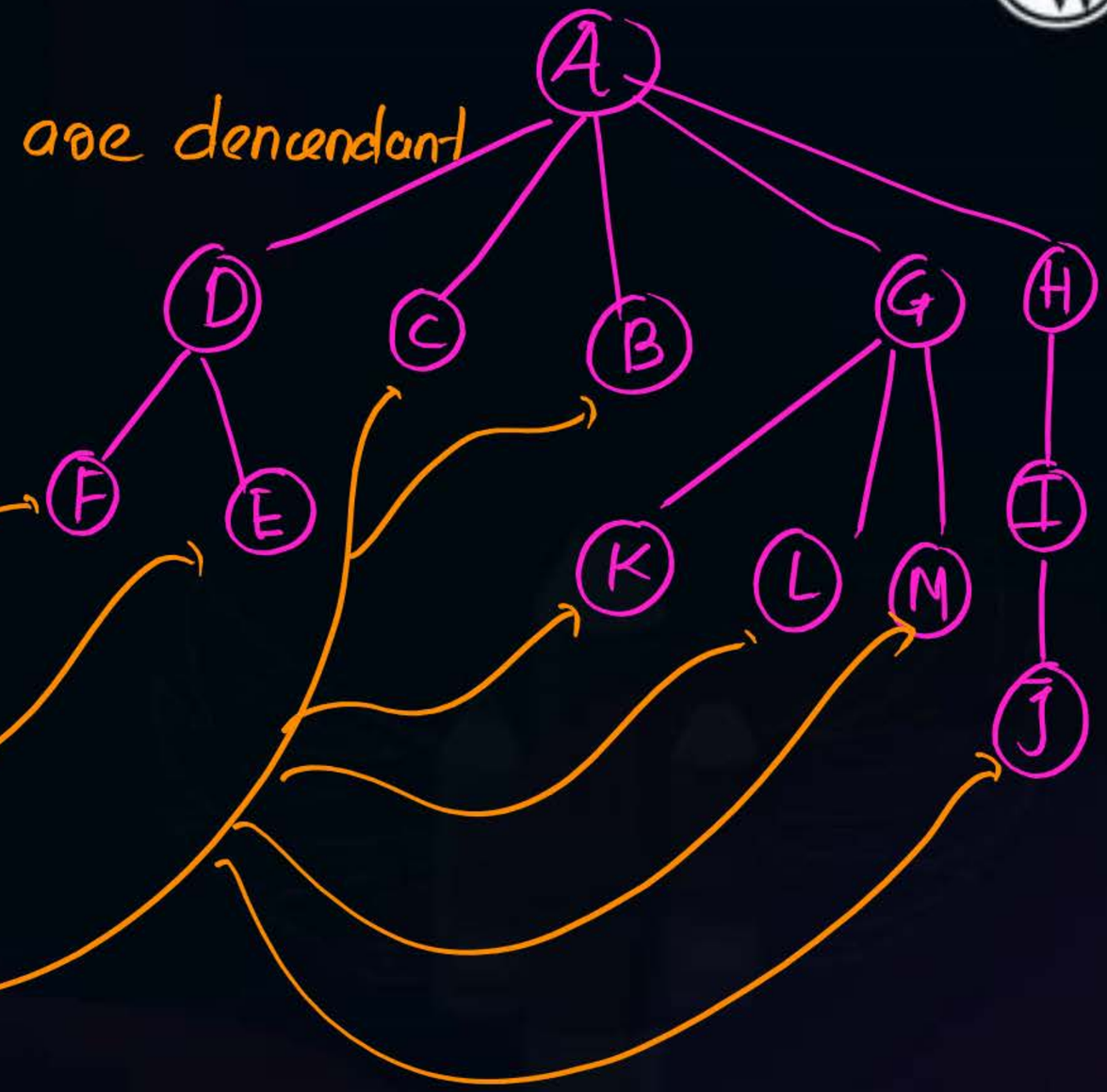
all node that encounter Node x to leaf Node path

are descendant of x

All Node are dencendant of A

I, J are descendant of H

Rooted Tree



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Topic : Tree

Internal Node

Not Leaf Node

Leaf Node

external Node

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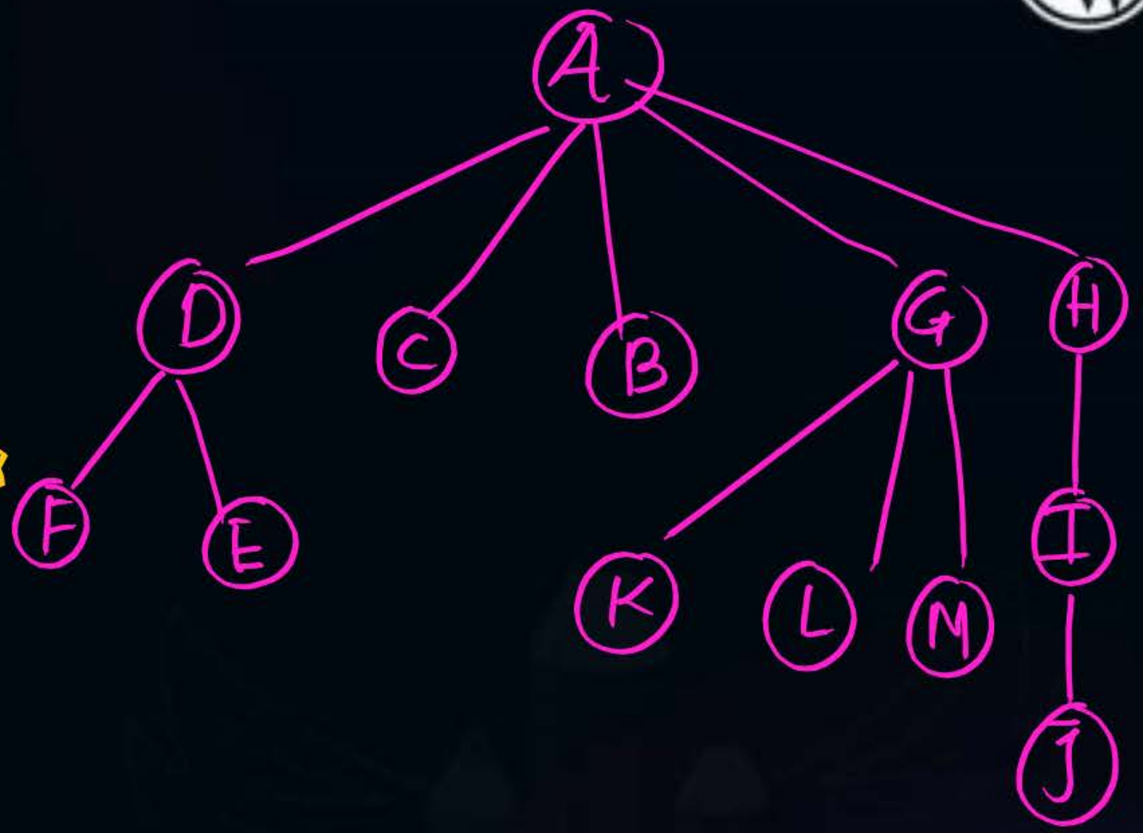
Topic : Tree

Total No. of Nodes

= No. of internal Nodes

+ No. of leaf Node

Rooted Tree



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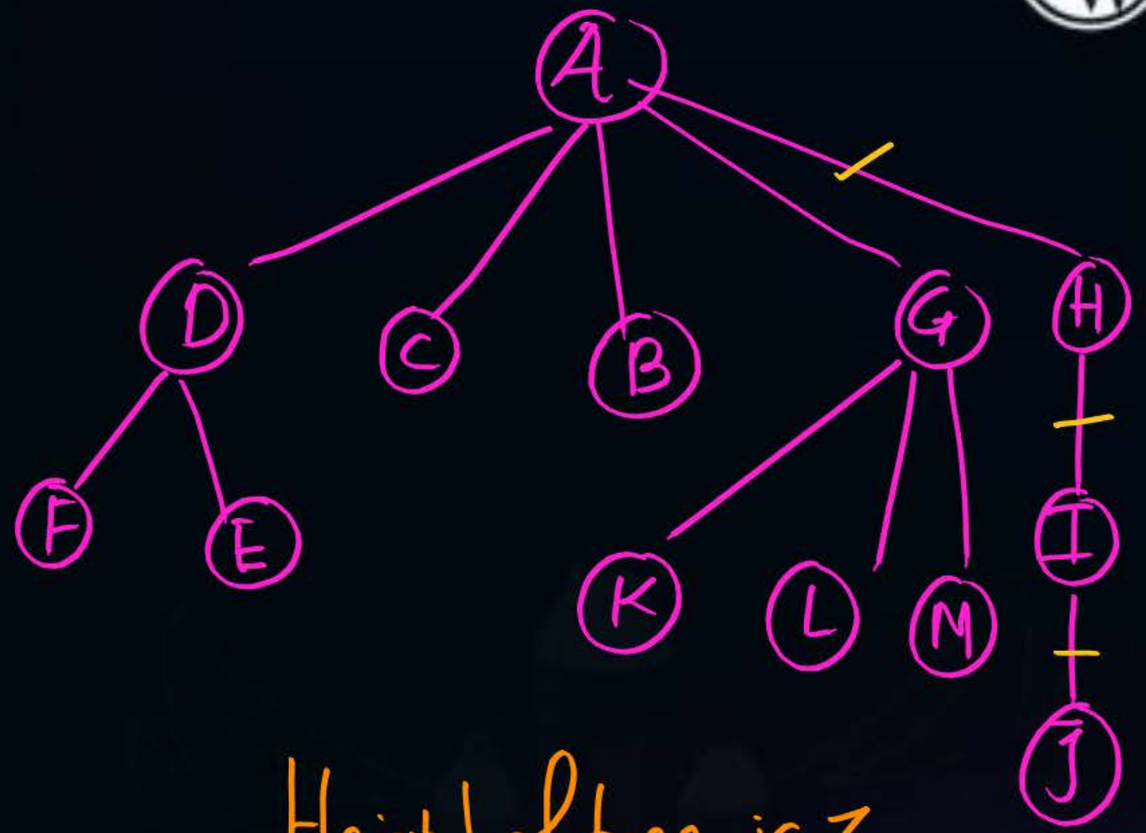
Topic : Tree

Height of Tree

The longest root to leaf path.

path length : No. of edges

Rooted Tree



Height of tree is 3

Node d is at depth 1 ✓

Node F is at depth 2 ✓

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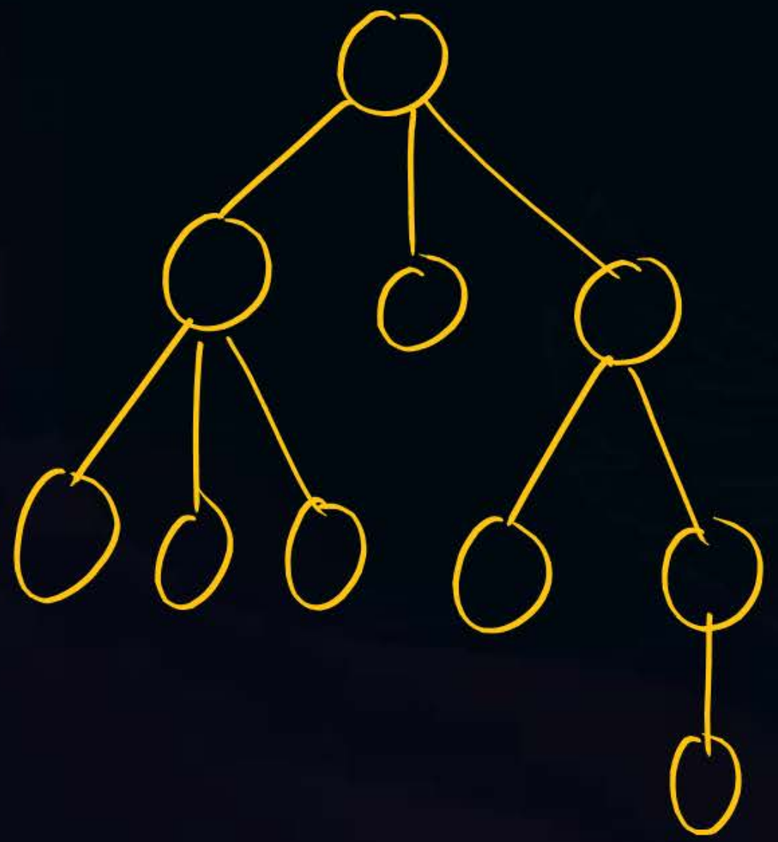
Topic : Tree

General Tree : A general Tree is a tree in which a node can have 0 or more children.



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Topic : Tree

Forest : Forest is collection of 1 or more general tree

Each general tree is called component.

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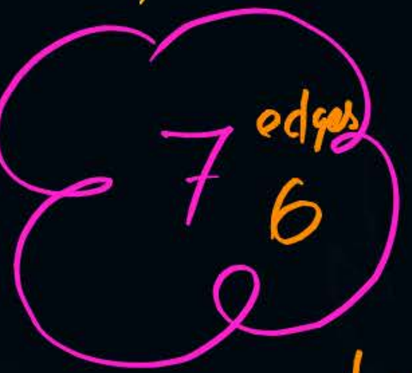
Topic : Tree

n vertex n-1 edges

Question : Suppose a forest consists of 15 vertex (total) 3 component then No. of edges in forest is 12 (15-3)

1 0



8 7 edges

6 5 edge = 12

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Topic : Tree

n vertices

No. of component	No. of edge
1	n - 1
2	n - 2
3	n - 3
⋮	⋮
p	n - p

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Topic : Tree

1. # of Internal Node + # of leaves = Total No. of Nodes
2. a Tree with n vertex has n-1 edges
3. A forest with n vertex and p component has n-p edges

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Topic : Tree

k-ary Tree

a k-ary tree is a Tree in which each Node is having 0 or k children.

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Topic : Tree

3-ary Tree

$I = 3$

$L = 3 \times (3-1) + 1$

$3 \times 2 + 1 = 7$

← degree $k+1$

$2+1 = 3$

2-ary tree

$I = 3$

$L = 3 \times (2-1) + 1$

$3 + 1 = 4$

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Topic : Tree

Theorem In a given k-ary tree if L is No. of leaves and I is No. of internal Nodes then

$$L = I(K-1) + 1$$

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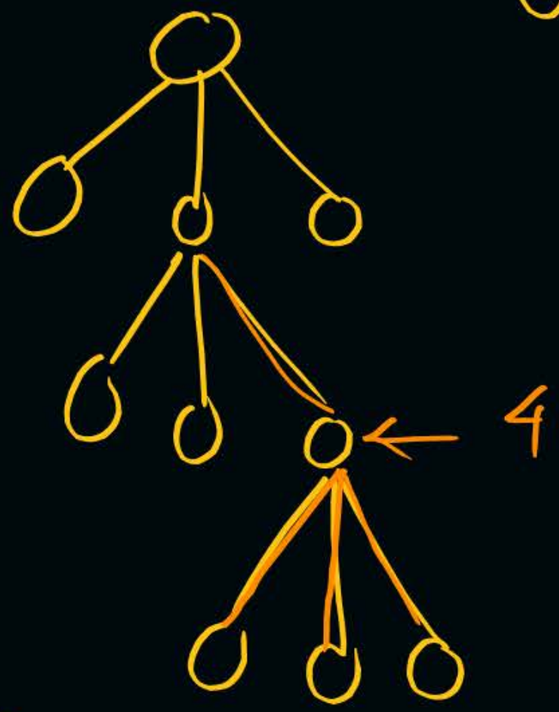
k-ary tree



3-ary tree

K children

1. L is No. of leaves, I, No. of Internal Node
2. Degree of L is 1
2. Degree of I is k+1 (Except 1 Root)
3. Total No. of Nodes L+I
4. Total No. of edges L+I-1

3+1

K+1

Handshaking Theorem

Sum of degree $= 2|E|$

$$L \times 1 + I(k+1) - 1 = 2 \times (1 + I - 1)$$

(1 ↑ Root)

$$L + Ik + I - 1 = 2L + 2I - 2$$

$$\Rightarrow 2L - L = Ik + I - 1 - 2I + 2$$

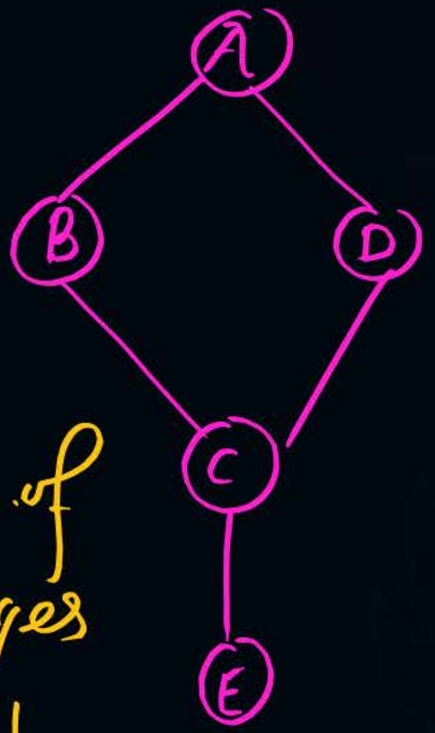
$$\Rightarrow L = Ik - I + 1$$

$$\boxed{L = I(k-1) + 1}$$

Topic : Tree

Handshaking Algorithm

graph



degree of Node is

No. of edges a Node is connected to

degree (A) = 2

degree (B) = 2

degree (D) = 2

degree (C) = 3

degree (E) = 1

⇓

Sum of degree : 2 * No. of edges

$2+2+2+3+1 = 2|E|$

$\Rightarrow 2|E| = 10$

$|E| = 5$

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Topic : Tree

$$\sum_{i=1}^{n} \text{degree}(v_i) = 2|E|$$

Topic : Question

A 2-3 tree is tree such that

(A) all internal nodes have either 2 or 3 children ✓

(B) all paths from root to the leaves have the same length

The minimum and maximum number of internal nodes of a 2-3 tree having 9 leaves could be

2-3 tree with 9 leaves.

If minimum No of Internal Node is x and maximum No. of internal Node is y then x+y is 4+7 = 11

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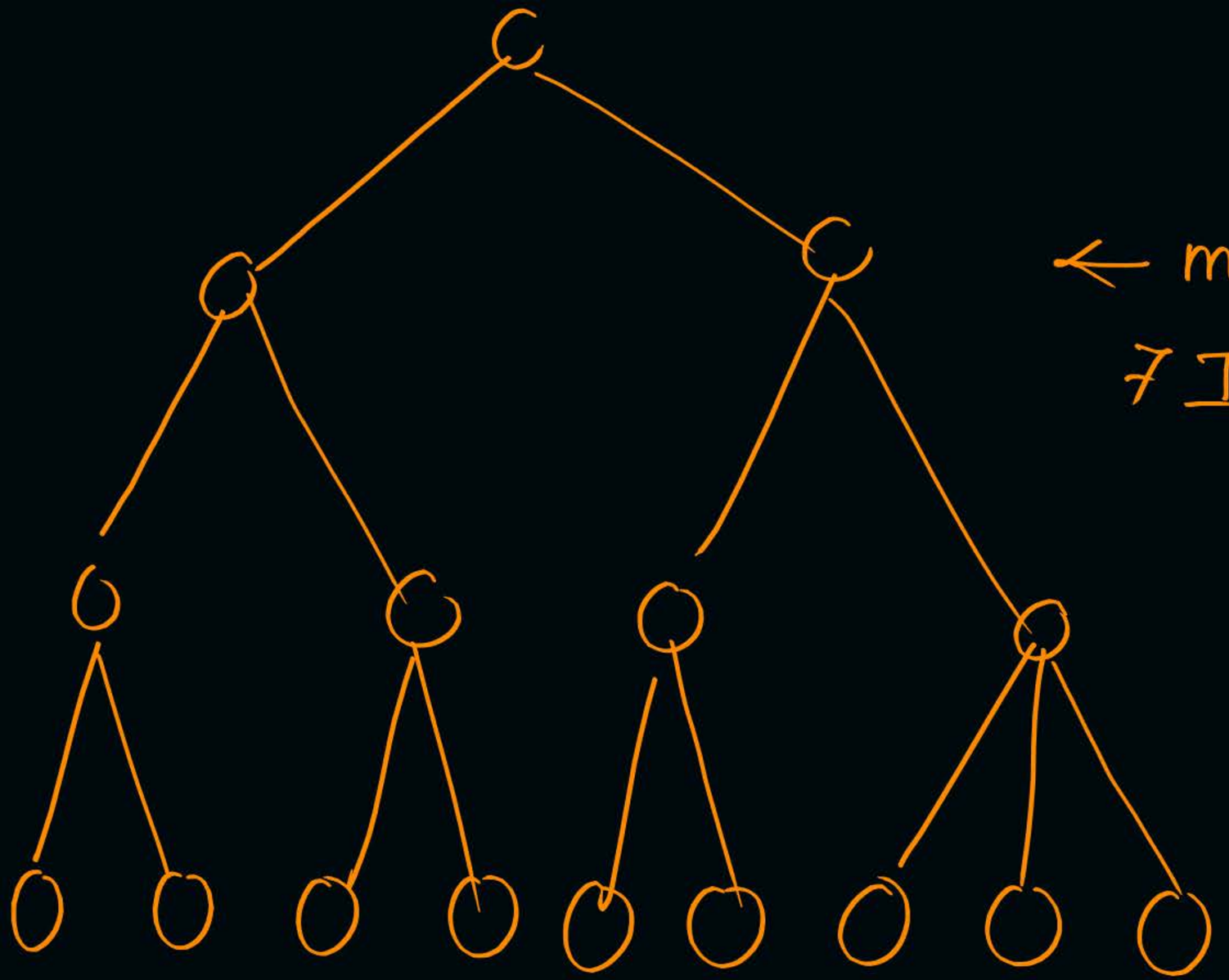
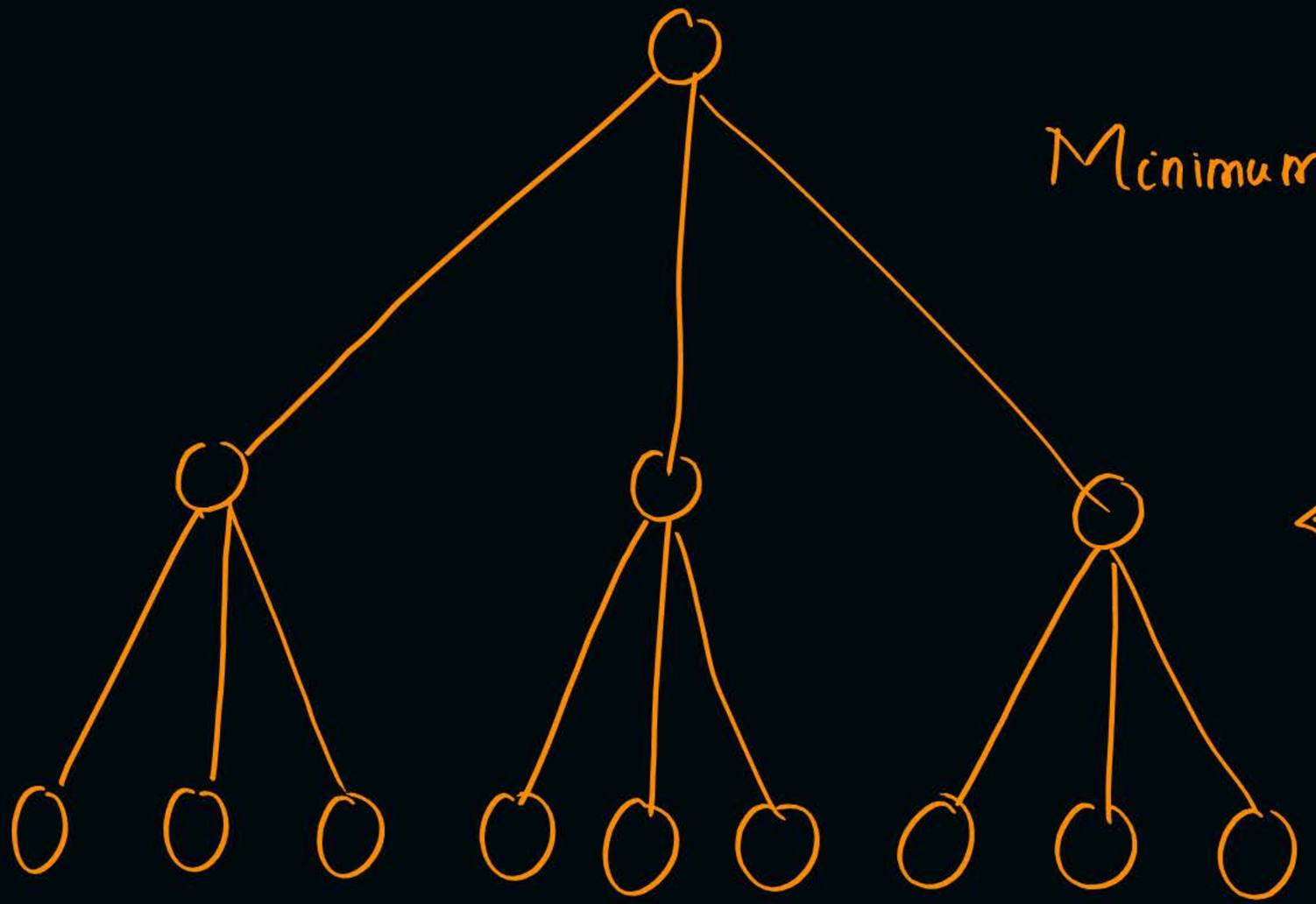

← maximum
7 Internal Node
maximum



Minimum No. of Node = 4

Minimum

Topic : Question

2007

A complete n-ary tree is a tree in which each node has n children. Let l be the number of internal nodes and L be the number of leaves in a complete n-ary tree. If L = 41, and I = 10, what is the value of n?

(A) 3

(B) 4

(C) 5

(D) 6

$$L = I(k-1)+1$$

$$41 = 10(k-1)+1$$

$$10(k-1) = 40$$

$$k-1 = 4$$

$$k = 5$$

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Topic : Question

Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is _____.

Sum of degree $\therefore$ $2|E|$

$= 2 \times 9$

$= 18$

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2 mins Summary

Topic — Tree Terminology

Topic — Forest Theorem

Topic — k-ary tree

Topic — Handshaking Algo.

Topic

Slide

THANK - YOU