CS & IT ENGINEERING

THEORY OF COMPUTATION

Regular languages

Lecture No.- 03

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Recap of Previous Lecture

- **Topic** Regular Expression
- **Topic** Construction of Regular Expression
- **Topic** DFA States

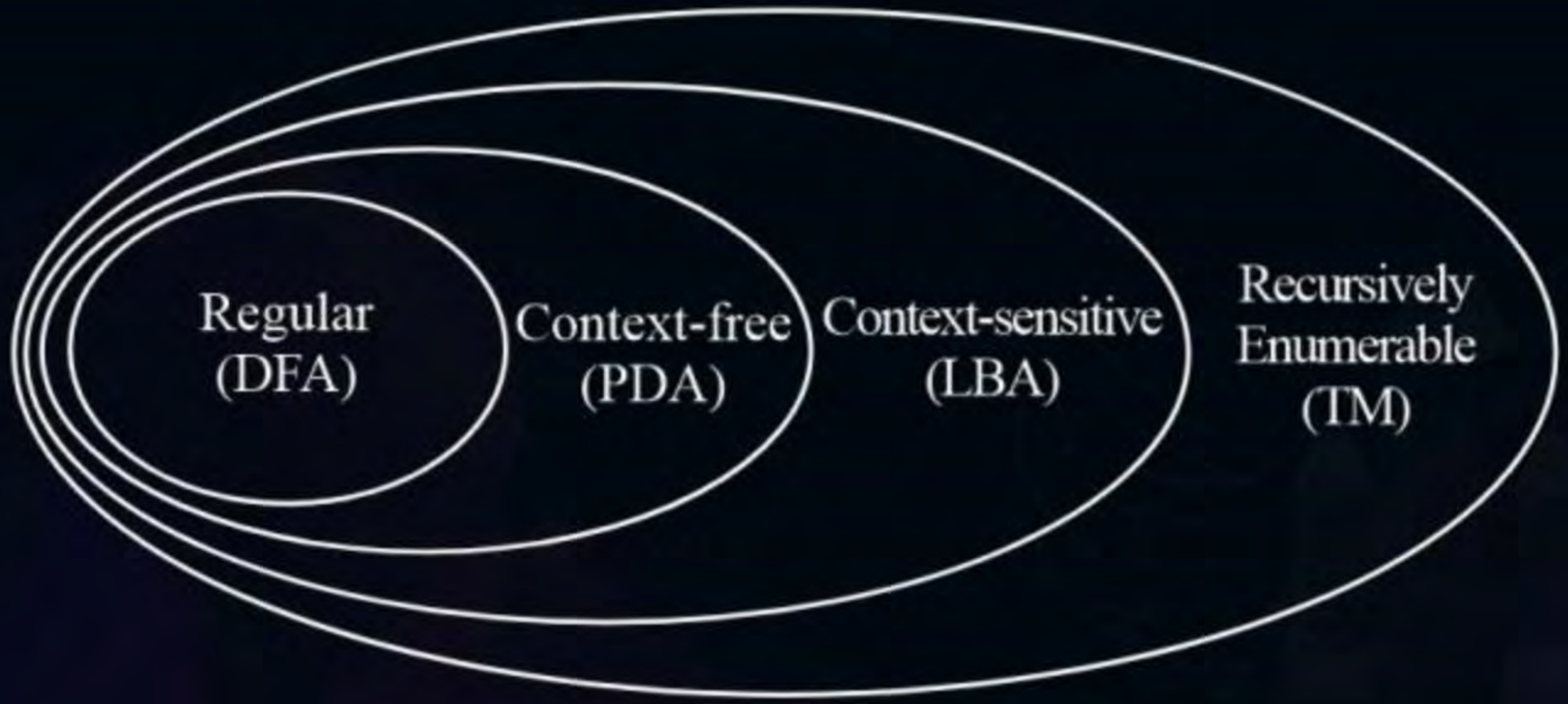
Topics to be Covered

Topic Conversion from $\in$ NFA to NFA

Topic ??

Topic ??

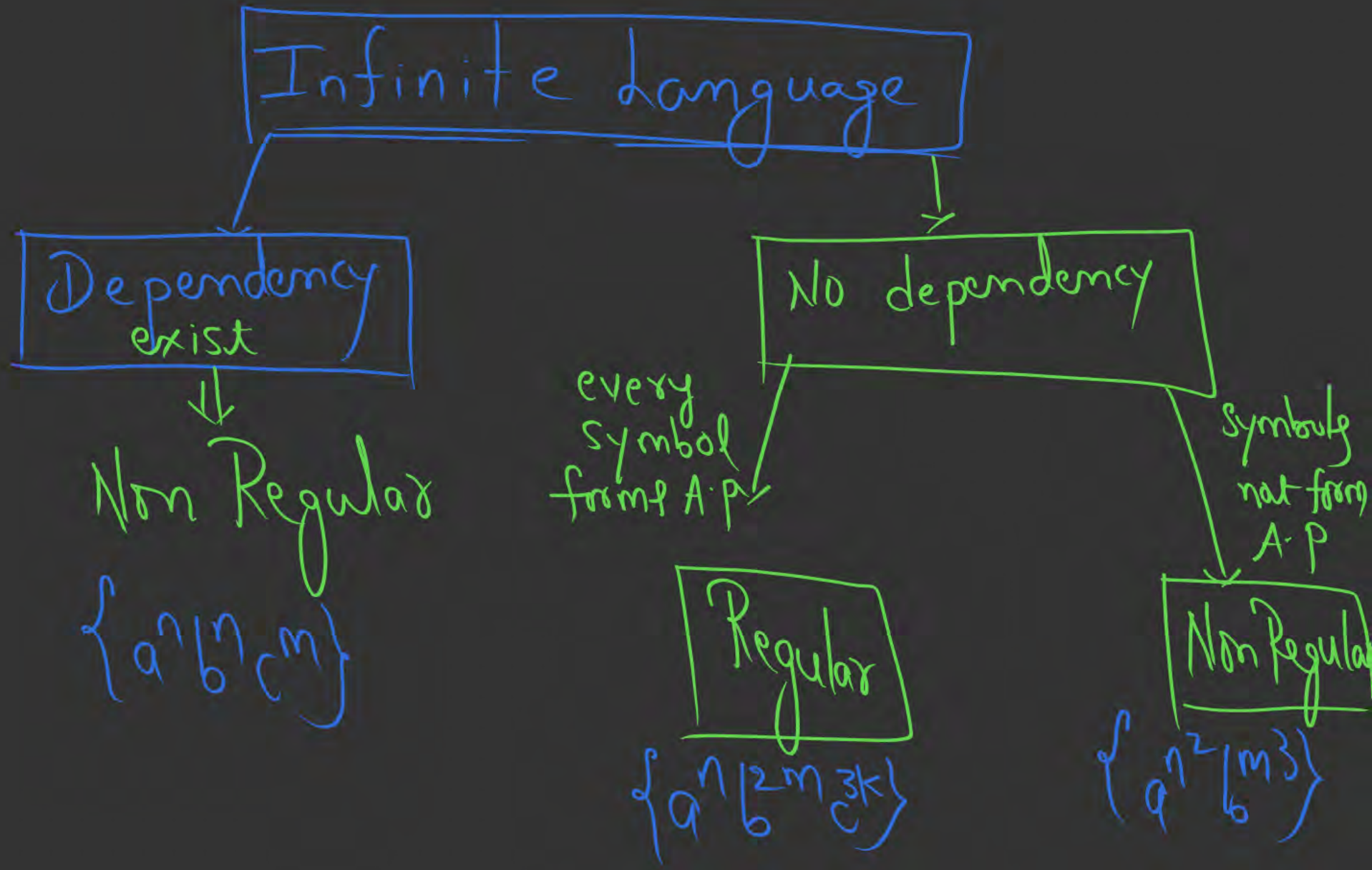
Topic : Theory of Computation



Regular Language Detection

1 symbol languages

- Common difference (A-P) } Regular
- no Common diff } Non Regular


Infinite language
Dependency exist
Non Regular
$\{a^n b^n c^m\}$
No dependency
every symbol from A.P
Regular
$\{a^n b^{2m} c^{3k}\}$
Symbols not from A.P
Non Regular
$\{a^{n^2} b^{m^3}\}$

Topic : Arden's Theorem

In order to find out a regular expression of a Finite Automaton, we use Arden's Theorem .

Statement-

Let P and Q be two regular expressions.

If P does not contain null string, R = Q + RP has a unique solution that is R = QP*

Topic : Regular Language Detection

Which of these Languages are Regular

1. $L = \{a^n b^n c^n \mid 1 \leq n \leq 100\ c\}$
2. $L = \{a^n b^m \mid n + m = 10\}$
3. $L = \{a^n b^m \mid n - m = 5\}$
4. $L = \{a^n b^m \mid n \times m = 100\}$
5. $L = \{a^n b^m \mid n = 2m + 1\}$
6. $L = \{a^n b^m \mid n > m\}$
7. $L = \{a^n b^m \mid n > and\}$

Topic : Regular Language Detection

8. $L = \{a^n b^m \mid n > m \text{ (or) } n < m\}$

9. $L = \{a^n b^m c^{n+m} \mid n, m \geq 1\}$

10. $L = \{a^n b^n c^{n+m} \mid n, m \geq 1\}$

11. $L = \{a^n b^{2m} c^{3k} \mid n, mm\ k \geq 0\}$

12. $L = \{a^n b^{m^2} c^{k^3} \mid n, mk \geq 1\}$

13. $L = \{a^n b^{m^2} c^{k^3} \mid n, mk \geq 1\}$

14. $L = \left\{ \begin{matrix} a^{2^n} \\ a^{3^n} \end{matrix} \middle| \, n \geq 0 \right\}$

Topic : Regular Language Detection

15. $L = \{a^{n^3} | n \geq 1\}$

16. $L = \{a^{n^n} | n \geq 1\}$

17. $L = \left\{ \begin{matrix} a^{2^n} \\ a^{3^n} \end{matrix} \middle| n \geq 0 \right\}$

18. $L = \{a^{100^{100^{100}}} | \}$

19. $L = \{(a^p)^* |$ p is prime number$\}$

20. $L = \{a^p |$ p is prime number$\}$

Topic : Regular Language Detection

21. $L = \{a^k | \ k \text{ is even number}\}$

22. $L = \{ww^R \mid w\}$

23. $L = \left\{ww^R \mid \begin{matrix} w \in \{a,b\}^* \\ w \in \{a,b\}^+ \end{matrix}\right\}$

24. $L = \{wbw^R | \ w \in \{a\}^*\}$

25. $L = \{x | x \in \{a, b\}^* \ n_a(x) \bmod 3 = n_b(x) \bmod 2\}$

26. $L = \{x | x \in \{a, b\}^* \ n_a(x) \bmod 2 > n_b(x) \bmod 3\}$

27. $L = \{x | x \in \{a, b\}^* \ n_a(x) \bmod 3 \neq n_b(x) \bmod 3\}\}$

28. $L = \{x | x \in \{a, b, c\}^* \ n_a(x) \neq n_b(x)\}$

(Q) Which of the following is Regular? [a']

(a) $L = \{ x \mid x \in (a+b)^* \mid n_a(x) = n_b(x) \}$ Non Regular

(b) $L = \{ x \mid x \in (a+b)^* \mid n_a(x) > n_b(x) \}$ Non Regular

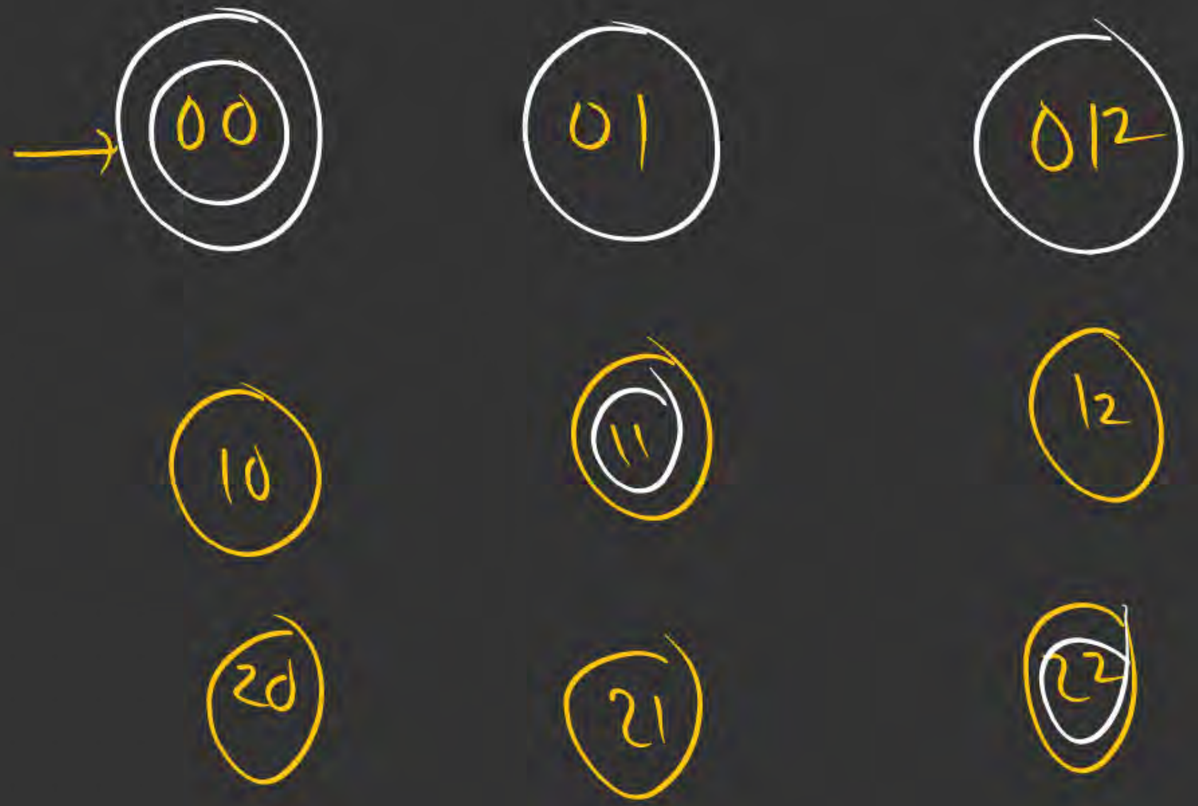
(c) $L = \{ x \mid x \in (a+b)^* \quad n_a(x) \bmod 3 = n_b(x) \bmod 3 \}$ Regular

DFA.

$\{0,1,2\}$ $\{0,1,2\}$

(d) none.


b
b
b
a
a
q_0
q_1
q_2
a
and
a
a
a
b
b
b_0
b_1
b_2
b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$x \in (a+b)^*$

$L_1 = \{ x \mid n_a(x) = n_b(x) \} = \{ \epsilon, ab, ba, aabb, bbaa \ldots \}$

Dependency

Non Regular

$L_2 = \{ a^n b^n \}$ =

Home work

① $L = \{ W W^R x \mid W, x \in (a+b)^+ \}$

② $L = \{ x \; W W^R \mid W, x \in (a+b)^+ \}$

③ $L = \{ W x W^R \mid W, x \in (a+b)^+ \}$

WWX

④ $L = \{ W x W \mid W, x \in (a+b)^+ \}$

⑤ $L = \{ WW \mid W \in (a+b)^* \}$

$$L = \{ ww x \mid w, x \in (a+b)^+ \}$$ Non Regular

$w = a$

$w = b$

$w = ab$

$aa(a+b)^+ + bb(a+b)^+$

not covered

$abab(a+b)^+$

(Q) Which of the following is Regular?

Non Regular

(a) $L = \{ ww \mid w \in (a+b)^* \} = \{ \epsilon, aa, bb, abab, baba, \ldots \}$

Non Regular

(b) $L = \{ xww \mid x, w \in (a+b)^+ \}$

$(a+b)^+aa + (a+b)^+bb$

$(a+b)^+abab$

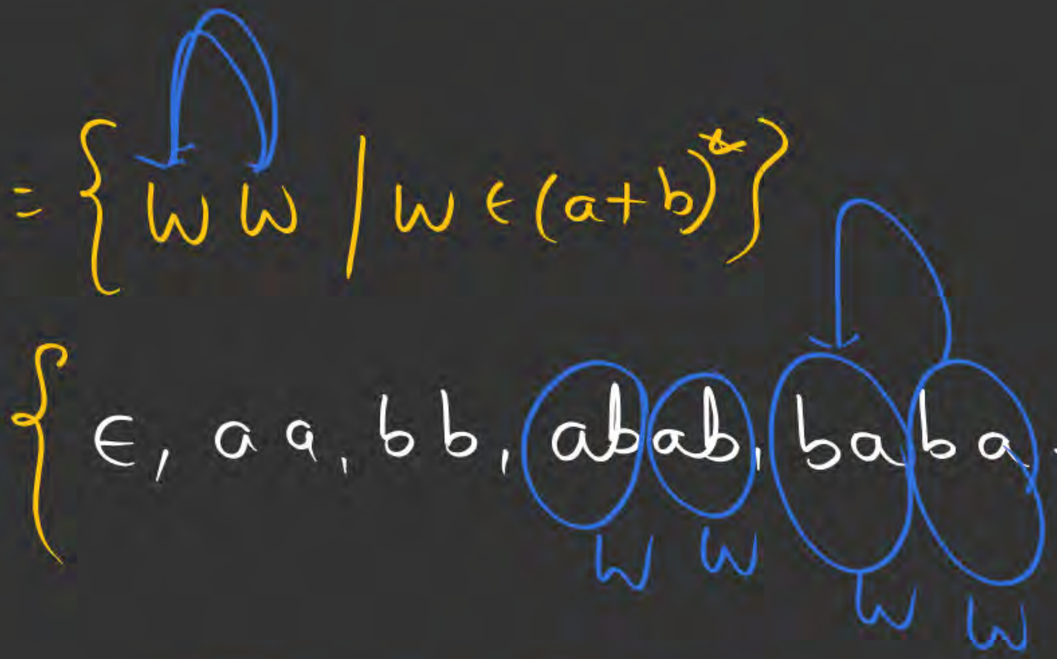
(c) $L = \{ wxw \mid x, w \in (a+b)^+ \}$

$a(a+b)^+a + a(a+b)^+b + b(a+b)^+a + b(b+a)^+a$

(d) none.

$L = \{ ww \mid w \in (a+b)^* \}$

$\{ \in, aa, bb, abab, baba \ldots \}$ $\neq (a+b)^*$



No Regular Expression possible

$L = \{ \underline{w} \times w \mid w, x \in (a+b)^+ \}$ regular

$w = min$

$a(a+b)^+a + b(a+b)^+b + a(a+b)^+b + b(a+b)^+a$

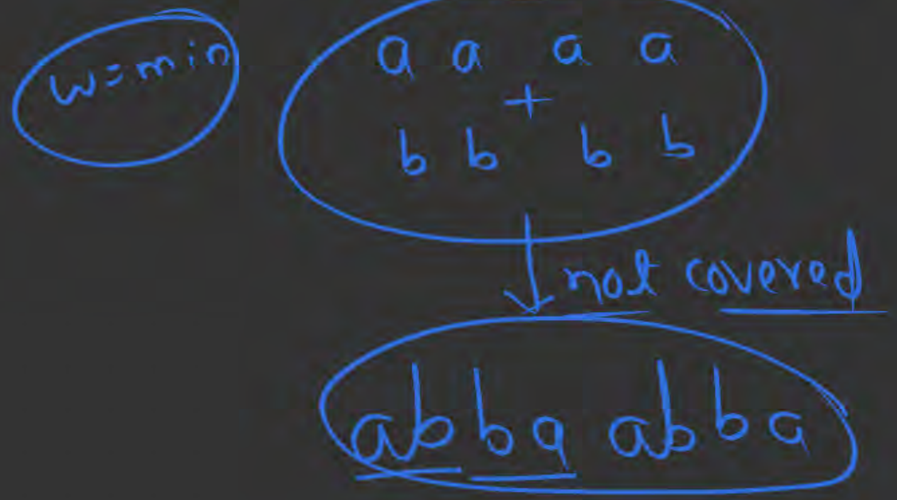
$ab(a+b)^+ab$

$ba(a+b)^+ba$ not covered

not covered

$ababab(a+b)^+abab$

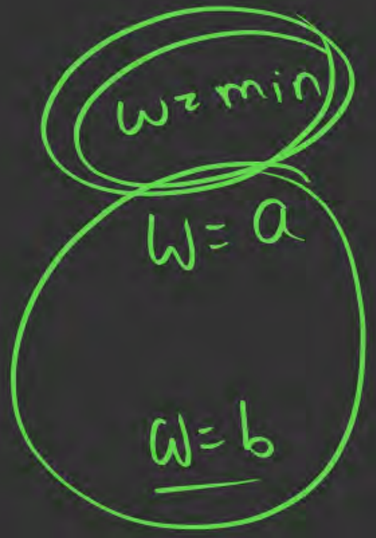
$$L = \{ W W^R W W^R \mid w \in (a+b)^+ \}$$ Non Regular



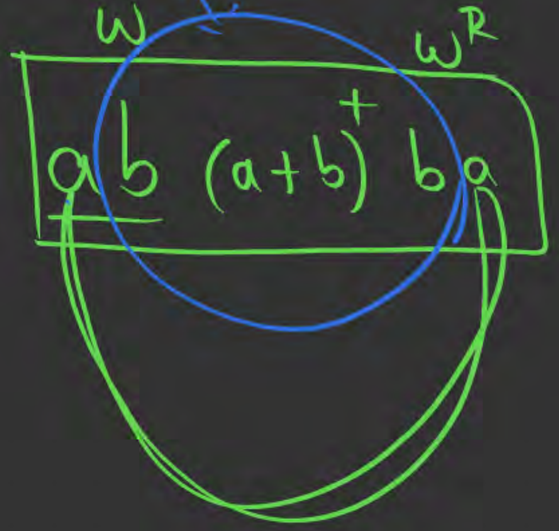
$\{ w x w^R \mid w, x \in (a+b)^+ \}$

Regular

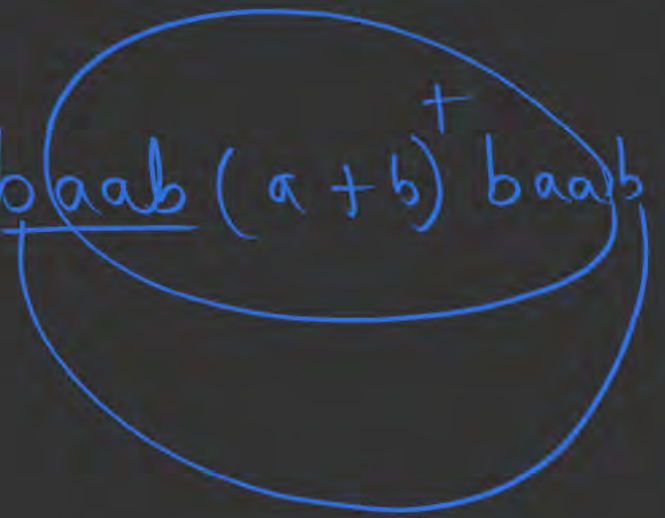
$a\,(a+b)^+\,a \;+\; b\,(a+b)^+\,b$



$w = ab$



$w =$



$$L = \{ X \underbrace{W W^R} \mid W, X \in (a+b)^+ \}$$

Non Regular

$W = a$ } mid $(a+b)^+ a a + (a+b)^+ b b$

$W = b$ }

not covered

$(a+b)^+ ab \boxed{ba}$ new

$L = \{ W X W^R \mid W, X \in (a+b)^+ \} \Rightarrow$ Regular ✓

w = min

W = a

W = b

$a(a+b)^+a + b(a+b)^+b$

$a(a+b)^+a$ + $b(a+b)^+b$

W = ab

$ab(a+b)^+ba$

W = abab

$ababa(a+b)^+bab$

$aaba(a+b)^+bbba$

$\epsilon \cdot \epsilon$

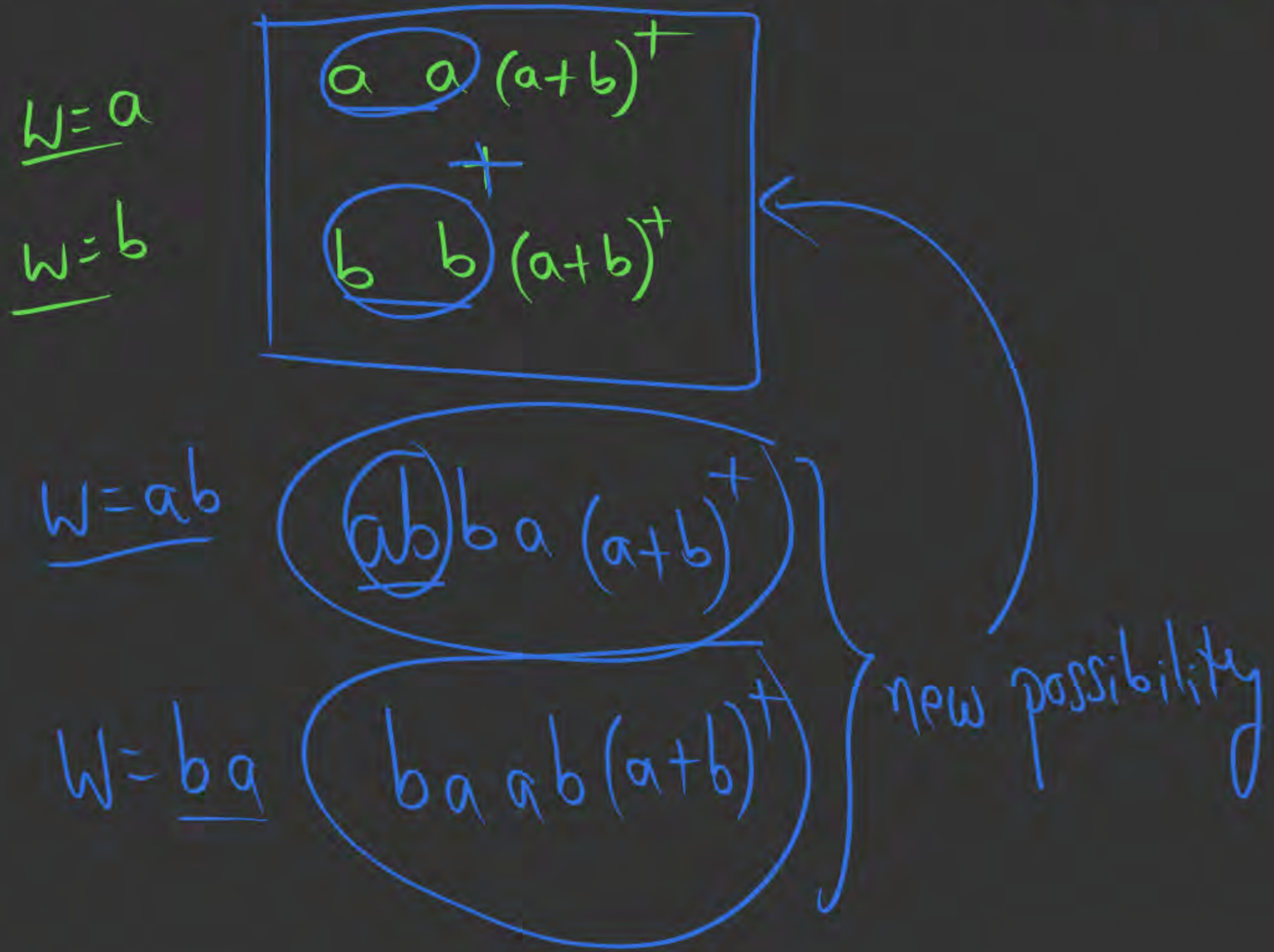
$$L = \{ w w^R x \mid w, x \in (a+b)^* \}$$

$w = \epsilon$ $\{ \epsilon \cdot \epsilon \, x \}$ $(a+b)^*$

$\{ (a+b)^* \}$

ab

$L = \{ W W^R X \mid W, X \in (a+b)^+ \}$ Non Regular



Topic : Regular Language Detection

- Set of all balanced parathesis
- Equal no.of open and close paranthesis
- Even length palindrome strings of English Language.
- Odd length palindrome strings of Hindi Language

[MCQ]

#Q. Consider the following languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$ → Non Regular

$L_2 = \{ww^R \mid w \in \{a, b\}^*, w^R \text{ is the reverse of } w\}$ → Non Regular.

$L_3 = \{0^{2i} \mid i \text{ is an integer}\}$ → Regular

$L_4 = \{0^{i^2} \mid i \text{ is an integer}\}$ → Non Regular

Which of the languages are regular?

A. Only L_1 and L_2

B. Only L_2, L_3 and L_4

C. Only L_3 and L_4

D. Only L_3

[MCQ]

$\{x c y \mid x, y \in (a+b)^*\}$

$(a+b)^* c (a+b)^*$

#Q. Which of the following are regular sets?

1. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$ → Regular
2. $\{a^n b^m \mid n = 2m\}$ → Non Regular
3. $\{a^n b^m \mid n \neq m\}$ → Non Regular
4. $\{xcy \mid x, y \in \{a, b\}^*\}$ → Regular

$(a+b)^* c (a+b)^*$

A 1 and 4 only

B 1 and 3 only

C 1 only

D 4 only

[MCQ] $\{wxw^R / w, x \in (a+b)^+\}$ $a(a+b)^+a + b(a+b)^+b$

#Q. Which of the following languages is/are regular?

$L_1 : \{wxw^R | w, x \in \{a, b\}^*\}$ and $|w|, |x| > 0$ w^R is the reverse of string w

$L_2 : \{a^n b^m \mid m \neq n \text{ and } m, n \geq 0\}$ ⟶ Non Regular

$L_3 = \{a^p b^q c^r | p, q, r \geq 0\}$ ⟶ Regular.

L_1 & L_3

A L_1 and L_2 only

B L_2 only

C L_1 and L_3 only

D L_3 only

(Q) Which of the following is Regular?

(a) $L = \{ ww \mid w \in (a+b)^* \}$ ⟹ Non Regular

(b) $L = \{ w \$ w^R \mid w \in (a+b)^* \}$ → odd length palindrome } Non Regular

(c) $L = \{ xy \mid x, y \in (a+b)^*, n_a(x) = n_b(y) \}$ = $(a+b)^*$ = Regular

(d) $L = \{ x \$ y \mid n_a(x) = n_b(y), x, y \in (a+b)^* \}$ = Non Regular

Dependency

$$\{x \$ y \mid n_a(x) = n_b(y)\}$$

$\{a\$b,$  $---\}$ Non Regular

$\{ xy \mid n_a(x) = n_b(y) \} = (a+b)^* =$ Complete Language

$\{ \epsilon, a, b, aa, ab, ba, bb \cdots \}$

[MCQ]

#Q. Which one of the following is True?

A The languages $L = \{a^n b^n \mid n \geq 0\}$ is regular

C The language $L = \{a^n \mid n \text{ is prime}\}$ is regular

B The language $L = \{w \mid w \text{ has } 3k + 1 \text{ b's for some } k \in N \text{ with } \Sigma = \{a, b\}\}$ is regular

D The language $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular.

Topic : Pumping Lemma

To Prove a Language L is Non –Regular

1. Assume L is Regular
2. There exist F.A for L and n is number of states in that F.A
3. Select some string W from L such that |W|>n.
4. Divide W into XYZ such that |xy|<=n and |y|>0.
5. Find a suitable integer i such that xy^iz is not belongs to L.

Then L is not Regular.

y is loop string

$xy^iz \notin L$

① If there exist a F.A with n states for L.

All strings of L having length less than n

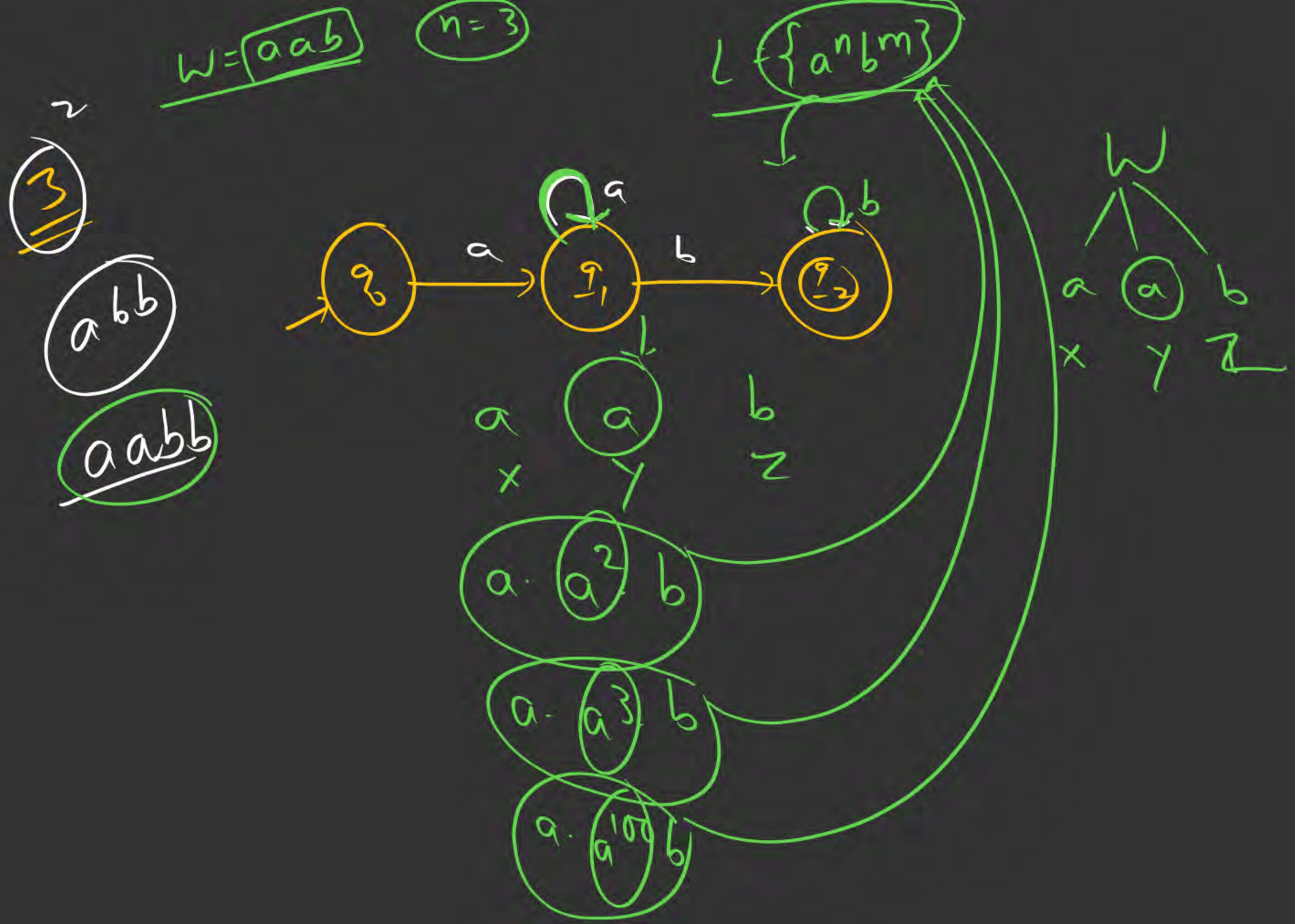
then L is known as finite language

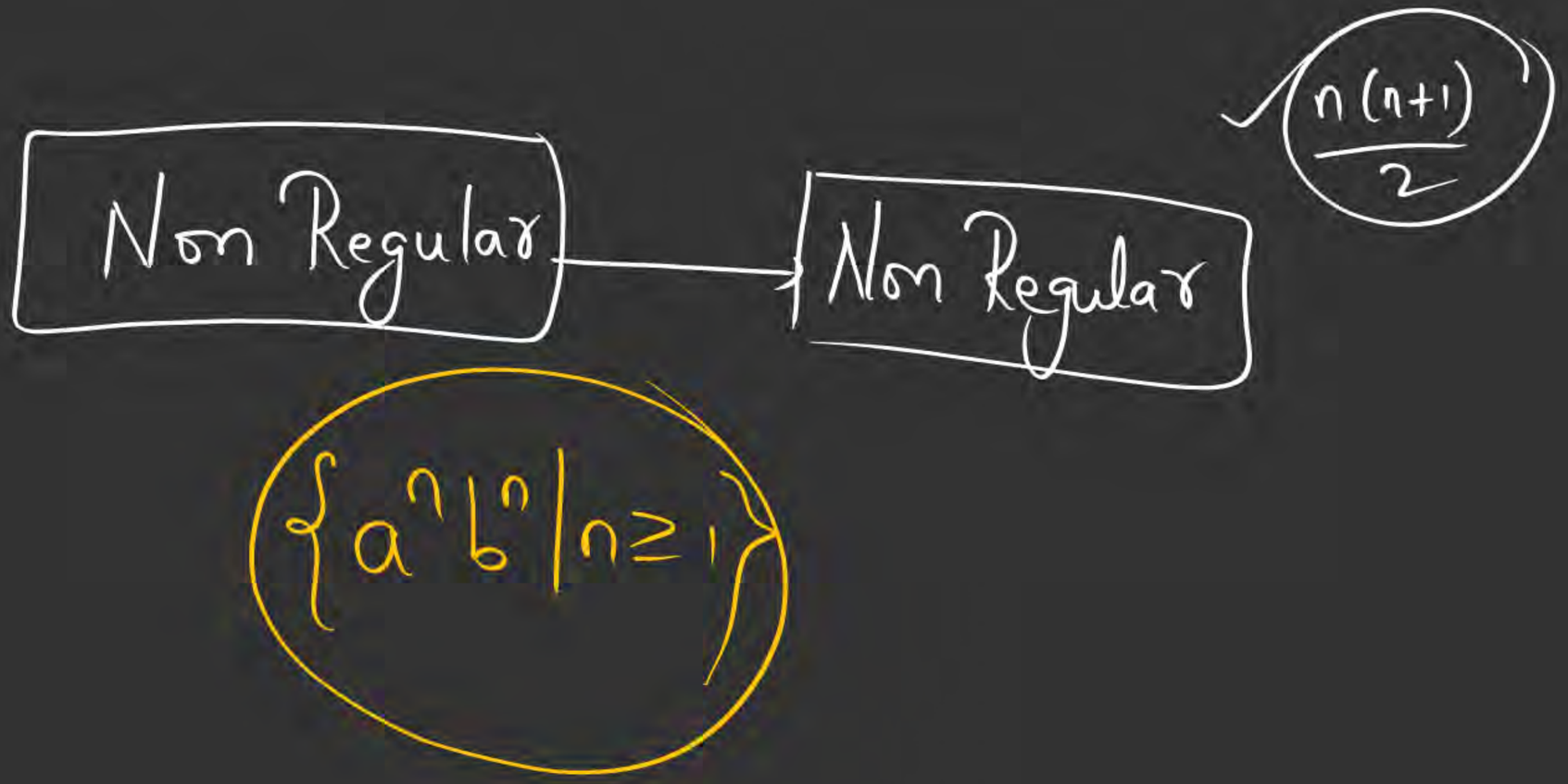
length < n

n

Non Regular } atleast 1 Case.

Regular } $xy^iz \in L \quad \forall \; i \geq 0$ } { Infinite times }


W = aab
n = 3
L {a^n b^m}
3
abb
aabb
q0
q1
q2
a
b
W
x
y
z


Non Regular
Non Regular
$\frac{n(n+1)}{2}$
$\{a^n b^n | n \geq 1\}$

THANK - YOU