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Idioms and Phrases



The rules for Idioms and phrases are frequently the same.

Idioms, in general, demonstrate human behaviour, reaction to certain things, habits, and traditions. An idiom is a group of words that, when used together, produces a meaning that is entirely different from the meaning of each word when used individually.

To explain phrases, think of them as the inverse of idioms. This means that idioms and phrases can be used interchangeably and in conjunction with one another.

Idioms and phrases are literal terms that imply a meaning that would otherwise be hidden behind the words themselves.

- They have a cultural value that is attached to them.
- They are metaphorical in the sense that extracting the literal meaning of all the words stated would be too unusual to believe, if not farcical.
- Idioms and phrases can add flavour to boring prose or book, and authors use them to add something interesting or remove monotones from their writing.

DIFFERENCE BETWEEN IDIOMS AND PHRASES

The difference is that in an idiom, the meaning is derived from common usage. A phrase, on the other hand, is a small group of words that, when taken together, has some meaning.

Idioms

Understand With Example



- **Being comfortable in one's skin:**

If you take the literal meaning, it would be impossible to explain.

However, the actual meaning here is: 'the feeling of being at ease with yourself.'

One's skin refers to one's own skin, or, to put it another way, 'yourself'.

• Beat around the bush:

This idiom does not mean to take a bat and beat the bush, but rather to discuss everything surrounding the topic while attempting to ignore or avoid the main topic, the person, or a specific thing

Phrases

Understand With Example



- Deep down—deeper feelings
- Group of men and women—a group of men and group of women
- Challenge of imagination—challenging to be imaginative
- Latin and Greek—some strange meaning

Tips and tricks and shortcuts for idioms and phrases

We have some tips and tricks for idioms and phrases that will help you with the questions, but first, let's start with the definition.

Idioms are the collections of words and phrases that have a figurative meaning. They are mostly used in the text's linguistic sections. A phrase is defined as a small group of words that form



one unit and are spoken or written as such. Phrases are not like idioms in that they are to the point and provide direct meaning.

Tip 1:

Understand the idioms and phrases and do not limit yourself to meaning alone

Tip 2:

Try to relate the idiom and phrase with visual and real image

Tip 3:

Do not try to merge it at once or learn many idioms and phrases.

Some key points to improvise your cache of idioms and phrases for their more flowing use are:

1. Continuous reading

Reading as many books as possible is the only way to gain expertise and to acclimatise with the use of idioms and phrases in daily life. Reading not only entertains us but is also a great way to exercise our brain.

2. Understand the meaning of each idiom and phrase

The next step is to not only learn new idioms and phrases, but also to comprehend their

List of idioms and phrases

1. A blessing in disguise: When a misfortune turns into a blessing
2. A dime a dozen: A very common, regular thing which is not unique
3. A piece of cake: Something elementary in nature which is easy to do
4. Above water: Free of trouble or difficulty, generally used for financial terms
5. Accident of birth: When one gets lucky because of the family or place one was born in
6. Accident waiting to happen: When any action is obvious to create trouble or mishap
7. According to Hoyle: Doing something by sticking to the procedures/or the certain way it was supposed to happen
8. Ace in the hole: An advantage which is hidden
9. Ace up one's sleeve: A surprising advantage that nobody is aware of
10. Achilles' heel: One weak spot of a powerful person, that can be lethal

meaning by investigating their origin and the context in which they are commonly used.

3. Avoid cramming

Avoid learning too many idioms at once by focusing on understanding the basic concept behind that particular idiom or word.

One should limit oneself to 6-7 phrases per day. However, one can categorise idioms with similar meanings/or themes and memorise them section or theme-wise.

4. Using visual imagination

The final and most important tip is to use our visual imagination because the origins of the majority of the idioms are fascinating. One should not only investigate the origins of these items but also relate them to specific images and link them to specific stores. This is the most effective method for memorising a wide range of idioms and phrases.

Jot down the ready reckoners (most commonly used Idioms and words) for revision:

Another simple way to gain expertise in understanding such questions is to make a list of commonly used idioms, words, or phrases, along with their respective meanings, and revise them on a regular basis.



11. Acid test: A way of testing the worthiness of something
12. Acknowledge the corn: To admit one's mistake, even if it's not of major significance
13. Acquired taste: When one learns to appreciate something, on a frequent/regular use of it
14. Across the board: It denotes all the categories, types or people
15. Across the pond: On or to the other side of the Atlantic Ocean
16. Act high and mighty: To believe oneself to be better than the rest
17. Act of Congress: Hard to get, said of authorisation
18. Act one's age: Used for a mature person acting childishly to act maturely
19. Actions speak louder than words: The intentions of a person are more expressed through his actions, not just words
20. Add fuel to the fire: To worsen the existing tension
21. Add insult to injury: Ridiculing a person who is already injured.
22. After one's own heart: To like or dislike similar to one's own
23. After the fact: After something is finalised, when it's too late
24. After the lord mayor's show (UK): Anticlimactic, occurring after something impressive
25. At daggers drawn: To be extremely hostile to each other
26. Bark up the wrong tree: To have a wrong idea about something
27. Be in touch: To communicate via call or text each other
28. Beat around the bush: Avoid sharing your true viewpoint or feelings because it is uncomfortable
29. Beating a dead horse: Giving time or energy to something that has ended or over
30. Best of both worlds: To enjoy the advantages of two contrasting things at the same moment
31. Between the devil and the deep blue sea: In a very tough situation
32. Bite the bullet: To get an unfavourable situation or chore over with now because it will need to get finished eventually
33. Biting off more than you can chew: To take over an assignment or a task which is out of your capacity
34. Break a leg: is to wish luck (often said to actors before they go on stage)
35. Burn the candle at both ends: To overwork yourself until you're exhausted
36. By the skin of your teeth: To barely make it through
37. Call a spade a spade: To express frankly about what you see
38. Call it a day: Stop working on something
39. Can't make an omelette without breaking some eggs: You can't make everyone happy
40. Caught between a rock and a hard place: Making a choice between two unpleasant choices.
41. Clouds on the horizon: Trouble is coming or is on its way
42. Costs an arm and a leg: Something that is overpriced or very expensive
43. Cry for the moon: Ask for something impossible
44. Devil's advocate: To take the side of the counter-argument, or offer an alternative point of view
45. Doing something at the drop of a hat: Doing something at the moment of being asked
46. Don't count your chickens before they hatch: Not to count on something happening until after it has already happened
47. Don't judge a book by its cover: Not judging something by its initial appearance
48. Fall on your sword: To resign or get out of a project after failing
49. Feeling under the weather: To feel sick
50. Fit as a fiddle: To be fit and healthy
51. Get off of work: To get done or finish your work/shift
52. Getting a second wind: To become energetic again after a tiring phase
53. Getting a taste of your own medicine: To get treated the same way you treated others
54. Gift of the gab: Remarkable oratory skill.



55. Giving someone the cold shoulder: Snubbing someone
56. Giving the benefit of the doubt: To believe a baseless story even though it seems unbelievable
57. Going on a wild goose chase: Doing a pointless task
58. Harbinger of doom: A sign that something bad is about to happen
59. Hat in hand: To beg for favour from someone
60. Have your heart in your mouth: To be very frightened and scared
61. Heard it on the Grapevine: To hear a rumour
62. Hit the road: To leave
63. Hitting the nail on the head: To perform a task to its exactness
64. In hot water: In serious difficulty
65. Killing two birds with one stone: To accomplish two tasks by one action
66. Laugh like a drain: To laugh coarsely
67. Let someone off the hook: To allow someone, who has been caught, to not be punished.
68. Let the cat out of the bag: To accidentally reveal a secret.
69. Letting someone off the hook: To not hold someone responsible for something
70. Make a long story short: To explain something in brevity
71. No pain, no gain: Means that you have to do the hard work to achieve something worthwhile
72. On the ball: To do a great job, being prompt, or being responsible
73. Once in a blue moon: An event that happens not very frequently
74. Out of the woods: To be safeguarded from
75. Pulling someone's leg: Joking with someone
76. See eye to eye: To accept something
77. Sitting on the fence: To avoid decision-making
78. Speak of the Devil: The person who you were talking about arrives
79. Spitting image: A perfect resemblance
80. Stealing someone's thunder: Stealing someone else's credit for work
81. Straight from the horse's mouth: Hearing or reading the content from the original source.
82. Take a leaf out of one's book: To emulate someone
83. Take a powder: To leave quickly
84. The best of both worlds: It means you can enjoy two different opportunities at the same time.
"By working part-time and looking after her kids two days a week she managed to get the best of both worlds."
85. The elephant in the room: An issue, person, or problem that someone is trying to avoid
86. The last straw: The last difficulty or annoyance that makes the entire situation unbearable
87. Throwing caution to the wind: Being reckless or taking a risk
88. To cut corners: To do something badly or cheaply.
89. To hide one's light under a bushel: To be modest about your achievements
90. When pigs fly: Something that will never happen.
91. Wrap your head around something: Understand something that is complicated
92. You can lead a horse to water, but you can't make him drink: You can't force someone to make what is seemingly the right decision
93. You can't judge a book by its cover: To not judge someone or something based solely on their appearance
94. Your guess is as good as mine: To not know something



SOLVED EXAMPLES

One proverb or idiom is mentioned below along with its meaning. Choose the correct meaning of that particular idiom from the given options.

1. All in all
 - A. Each person
 - B. Every one
 - C. Call everyone at the same time
 - D. Most important

Answer: D

Explanation: All in all, signifies the crux, the minutes of any meeting, or the most important points of any discussion. Hence, option D is the correct one.

2. Add fuel to the flames
 - A. To burn something
 - B. To do or say something that makes someone react more strongly
 - C. To consider something
 - D. None of the above

Answer: B

Explanation: To add fuel to flames means to over-exaggerate anything that worsens a situation.

3. To smell a rat
 - A. To see a rat
 - B. To suspect foul dealings

- C. To find plague in a specific area
- D. None of the above

Answer: B

Explanation: To smell a rat means to suspect a nasty dealing or sensing something wrong in a particular situation. This can be better explained with the help of an example given below:

If I don't go to school today, then the teacher will smell a rat.

4. Strike while the iron is hot
 - A. To beat the hot iron
 - B. To act promptly when the situation is in favour
 - C. To alleviate an already tense situation
 - D. To raise objections

Answer: B

Explanation: It means to take advantage of any opportunity as and when it arises without waiting. It can be better explained with the help of an example:

Sam told Martha, You've got such a great job opportunity. If I were you, then I'd rather strike while the iron is hot.



PRACTICE QUESTIONS

1. The shareholder was in news for ‘tipping off’ the creditors and investors.
 - A. Bribed
 - B. Kill someone
 - C. Given advance information
 - D. Threatened
2. Alia ‘set her face against’ the idea of participating in the quiz.
 - A. To oppose with determination
 - B. To criticise others
 - C. To be very angry
 - D. To victimise someone
3. The psychiatrist could finally ‘hit the nail right on the head’, in her session today with Arjun on identifying his difficulty understanding his own emotions because of people-pleasing habits.
 - A. To teach someone a lesson
 - B. To destroy one’s reputation
 - C. To forget about past
 - D. To do the right thing
4. He will ‘smell a rat’ if I keep meeting you post-work hours.
 - A. To suspect foul dealings
 - B. To see signs of plague epidemic
 - C. To get bad smell of a bad dead rat
 - D. To get something of bad quality
5. It’s difficult for Sam to achieve his dreams, ‘he is a man of straw’.
 - A. A very active person
 - B. A worthy fellow
 - C. A lonely person
 - D. A man of no substance
6. He left the band to collaborate with other artists, ‘leaving his band in the lurch’.
 - A. To put someone at ease
 - B. To come to a compromise with someone
 - C. To desert someone in his difficulties
 - D. A constant source of annoyance to someone
7. It is the common practice of the team head ‘to pick holes’ in the pitch prepared by the Interns in this company.
 - A. To find some reason to quarrel
 - B. To cut some part of item
 - C. To criticise someone
 - D. To destroy something
8. The best way to start a conversation with a stranger is to ask a general question about their day, or something of common knowledge, even if you have to cry wolf.
 - A. To turn pale
 - B. To keep off starvation
 - C. To give a false alarm
 - D. None of the above
9. The goon thought I didn’t shoot him because I was scared, but he doesn’t know yet that he is ‘caught in a tartar’!
 - A. To deal with a person who is more than one’s match
 - B. To catch a dangerous person
 - C. To meet with disaster
 - D. None of the above
10. The strategy to get hold of the don is to first expose the illegal business of ‘his right-hand man’.
 - A. An honest person
 - B. Most efficient assistant
 - C. A competent and hardworking person
 - D. A foolish person
11. The director ‘fell flat on his face’ on receiving such harsh reviews for his film.
 - A. Very important
 - B. Fails completely
 - C. Quarrel
 - D. Retreat
12. With the discrepancies in the group, it is expected that the ‘heads will roll’.
 - A. Appraising situation
 - B. Punishment
 - C. People will die
 - D. Transfers will take place



- 13.** To avoid future confusion on the specifics of a deal, one must deal ‘above board’.
- To have a high opinion of oneself
 - Hate others
 - To be honest in any deal
 - Indulges in dream
- 14.** The student felt like ‘a fish out of the water’ for a week in the new school.
- Homeless
 - Frustrated
 - A sad thing that appears suddenly
 - Painful
- 15.** Samuel’s invitation to Tanya at his wedding, after their breakup, felt like ‘heaping coals of fire on her head’.
- To sort out the old from the new
 - To agitate someone
 - To annoy someone very much
 - Put someone to shame
- 16.** In the professional world, everyone must not be gullible to share their own work, people often use it to ‘feather their own nest’.
- To enrich oneself by using one’s influence or position or with other people’s money.
 - To win a race by a short distance
 - To be shouted at for something one has done
 - To give oneself the courage or strength to do something
- 17.** She demanded a raise because she knew the ‘worth of her salt’ in the company.
- To constantly worry about something
 - Competent and deserve
 - To feel physical discomfort because of nervousness
 - To have an edge over other contenders in a competition
- 18.** They ‘burnt their fingers’ by giving out their bank details to a stranger on the call.
- To be guarded in your speech
 - To think clearly before taking a decision
 - Got himself into trouble
 - To obstinately stick to your viewpoint.
- 19.** The miserable demeanour of ducks during thunder has been proverbial since the late 18th century. You did look for all the world like ‘a dying duck in a thunderstorm’.
- Something stupid or mad
 - A distressing situation
 - Confused or puzzled
 - Spontaneous
- 20.** Sid ‘got cold feet’ before checking his all-India Rank in NEET.
- Very clear about something
 - To lose confidence
 - Wasting of time and money
 - Paying attention to less important issues

SOLUTIONS

- (C)** Given advance information. Tipped off means to give someone a warning or secret information about something.
- (A)** To oppose with determination. The idiomatic expression ‘to set one’s face against something’ means to strongly disapprove of something.
- (D)** To do the right thing means to hit the nail right on the head: to describe exactly what is causing a situation or problem.
- (A)** To suspect foul dealings means to suspect or realise that something is wrong in a particular situation.
- (D)** A man of no substance means a man whose character is weak and who lacks definite beliefs.
- (C)** To desert someone in his difficulties means to abandon or desert someone in difficult straits.
- (C)** To criticise someone means to make



- an effort to find flaws or negative aspects in something through excessive analysis or criticism.
8. **(C)** To give a false alarm means to cry wolf means to ask for assistance when you don't need it.
9. **(B)** To catch a dangerous person means to deal with someone or something that proves unexpectedly troublesome or powerful.
10. **(B)** Most efficient assistant means someone's right-hand man or right-hand woman is their close assistant and the person they trust to help and support them in everything they do.
11. **(B)** Fails completely, 'fall flat on one's face' means to fail completely, especially in an embarrassing way.
12. **(B)** Punishment, if you say that heads will roll as a result of something bad that has happened, you mean that people will be punished for it, especially by losing their jobs.
13. **(C)** To be honest in any deal in a straightforward manner: openly
14. **(A)** Homeless, if you feel like a fish out of water, you do not feel comfortable or relaxed because you are in an unusual or unfamiliar situation.
15. **(D)** Put someone to shame, if someone felt bad because they forgot to get you a Christmas gift, for you to buy them an especially nice gift is heaping coals on their head.
16. **(A)** To enrich oneself by using one's influence or position or with other people's money.
17. **(B)** Competent and deserved. The expression 'to be worth one's salt' means you're competent and deserve what you're earning.
18. **(C)** Got himself into trouble. The phrase 'to burn one's fingers' means to suffer unpleasant results of an action
19. **(B)** A dying duck in a thunderstorm. It means a distressing situation where you are dejected and pessimistic.
20. **(B)** Getting cold feet means losing confidence.



WHAT IS A SYNONYM?

It may be referred to as a word that has a similar or identical meaning as a particular word. If joyful and happy have the same meaning, then 'joyful' can be called the synonym of happy and vice versa.

Following are some important synonyms

WORD	MEANING	SYNONYMS
Ability	A skill to do something well	Skill, capability, competence, talent
Anger	A feeling of annoyance or displeasure	Annoyance, irritation, fury, rage, resentment
Annoy	To make someone feel slightly irritated	Irritate, bother, bug, displease
Answer	To reply to a question	Reply, response, acknowledgement
Ask	To question someone or get them to do something	Ask, demand, beg, request
Aware	To know information about something	Aware, alert, informed, mindful, conscious
Beginner	A person who is new at something	Apprentice, novice, learner
Break	To divide something into pieces, usually by force	Burst, crack, smash
Careful	Showing attention to all details	Thorough, conscientious, painstaking
Cautious	Careful and not willing to take any risk	Careful, vigilant, wary, secretive, prudent, cagey
Change	To bring about a difference in something	Convert, alter, modify, shift, transform, vary
Collect	To bring together things	Assemble, gather, hoard
Complain	To express annoyance or dissatisfaction about something	Grumble, protest, object, whine



WORD	MEANING	SYNONYMS
Correct	Accurate, true, without any mistake	True, exact, accurate
Courage	Ability to deal with a dangerous situation without fear	Nerve, guts, bravery
Defeat	To succeed against someone	Thrash, conquer, beat, overcome, triumph
Dirty	Not clear or untidy	Squalid, untidy, grubby, filthy, grimy, soiled
Disagree	To not have the same opinion as someone else	Dispute, contradict, differ, argue
Dislike	Not liking someone or something	Hatred, disgust, hate
Easy	The quality of not being difficult	Uncomplicated, simple, straightforward
Famous	Known by many people	Renowned, notorious, infamous, well-known
Fatal	Causing death	Deadly, mortal, terminal, lethal
Follow	To come after something	Pursue, stalk, chase
Fragile	Easily breakable	Breakable, flimsy, delicate
Friend	A person known and liked	Mate, pal, companion, acquaintance
Funny	Something that makes people laugh	Humorous, hilarious, comical, witty
Give	To pass to someone else	Present, donate, grant, confer
Good	The quality of being of suitable standard	Satisfactory, excellent, acceptable, wonderful
Habit	Something that is done regularly by someone	Custom, practice, routine, tradition
Hard	Not easy	Tough, tricky, awkward, difficult
Harm	Causing someone or something injury	Wound, damage, hurt, injure



WORD	MEANING	SYNONYMS
Increase	Become higher in number or amount	Extend, enlarge, expand
Intelligent	Able to understand things well	Talented, gifted, bright, able, clever
Kill	To cause something to die	Put down, slaughter, assassinate, murder, kill, execute
Lack	Be short of something	Deficiency, deficit, shortage
Leader	In charge of something	Captain, supervisor, incharge, manager, boss
Like	To have good feelings about something	Appreciate, love, enjoy
Love	To have a strong feeling of liking for someone or something	Infatuation, passion, affection, fondness, liking
Mistake	Something that is wrong	Slip, blunder, error
Moving	Something that causes strong emotions	Touching, stirring, emotional
Necessary	Important	Required, essential, vital
New	Something made recently	Original, innovative, novel
Noisy	Making a lot of noise	Piercing, rowdy, deafening, loud
Now	At this point in time	Immediately, instantly, presently, promptly
Occasional	Something that happens sometimes	Intermittent, odd, periodic
Often	On many occasions	Regularly, frequently, repeatedly, again-and-again
Old	Dating back to a long time ago	Antique, old-fashioned, ancient, elderly
Protect	Keep something safe	Shield, guard, defend, shelter
Pull	To move something towards yourself	Haul, tug, jerk, tow, drag, draw



WORD	MEANING	SYNONYMS
Quiet	Not making any noise	Inaudible, non-communicative, noiseless, silent, peaceful
Raise	To bring something to a higher level	Elevate, hoist, lift, pick up
Sad	Not feeling cheerful	Miserable, depressed, fed up, unhappy
Shine	To be bright	Glitter, glow, dazzle, blaze
Smell	A sense through the nose	Odour, aroma, stink, scent
Strong	Having a lot of strength or power	Mighty, fit, sturdy, powerful
Stupid	Not sensible	Silly, senseless, irresponsible, foolish
Suggest	To make a recommendation	Recommend, propose, advise
Teach	To show to instruct someone how to do something	Instruct, educate, train, coach, tutor
Temporary	Lasting only a short period of time	Short-lived, passing, fleeting
Think	To have a thought or opinion about something	Work out, figure out, conclude, assume, reason
Type	A group of things with similar characteristics	Category, genre, species, sort, kind
Unattractive	Not attractive	Plain, hideous, ugly, unsightly
Uncertain	Not sure about something	In doubt, sceptical, dubious, doubtful, unsure
Usual	Common in most occasions	Customary, normal, routine, traditional
Very	Signifies the intensity of an adjective or adverb	Remarkably, extremely, exceptionally
Watch	To notice something	Look, stare, observe
Weak	Not strong	Unhealthy, feeble, frail
Wet	Covered in or absorbed liquid	Soaking, damp, waterlogged, moist
Worry	A cause of consistent thought	Responsibility, anxiety, burden, problem



PRACTICE QUESTIONS

Write the synonym of the given words. (The words can have more than one synonym.)

1. Old	
2. Ambitious	
3. Effortless	
4. Depraved	
5. Dauntless	
6. Interesting	
7. Kill	
8. Predicament	
9. Strange	

10. Ugly	
11. Scared	
12. Factual	
13. Natty	
14. Impish	
15. Miniature	
16. Corpulent	
17. Droll	
18. Abhor	
19. Succour	
20. Afflict	

SOLUTIONS

1. Aged/worn/used/dilapidated
2. Hankering/purposeful/aggressive
3. Easy
4. Corrupt/spoiled
5. Brave/intrepid/courageous/fearless
6. Fascinating/intelligent
7. Slay/demolish/ruin
8. Quandary/dilemma/plight
9. Peculiar/unusual/unfamiliar
10. Hideous/horrible/monstrous
11. Fearful/unnerved/insecure
12. Correct/accurate/true
13. Neat/trip/smooth
14. Mischievous/naughty/prankish/sportive/waggish
15. Tiny/little/diminutive
16. Fat/plump/thick
17. Humorous/amusing/laughable
18. Hate/despise/loathe
19. Help/assist/encourage/aid
20. Distress/pain/damage/hurt



WHAT DOES ANTONYM MEAN?

Words that have a contrast or opposite meaning to a word are called their antonyms. Antonym has been a more recent addition to the English language in comparison with synonyms.

Following are some important antonyms

WORD	MEANING	ANTONYMS
Abandon	To leave someone or something	Keep
Abbreviate	To shorten something	Lengthen, increase, expand
Ability	The skill to do something	Inability, incompetence
Able	Having the skill to do something	Incapable, incompetent
Abundant	Being enough in amount or number	Insufficient, scarce, scanty
Accurate	The quality of being right	Inaccurate, wrong
Achieve	To get something	Fail
Active	Having energy	Sluggish, idle, lethargic
Adamant	Unwilling to change an opinion	Yielding, manoeuvrable
Add	To increase	Subtract
Adequate	Enough for the need	Sparse, insufficient, inadequate
Adjourn	To stop something for some time	Recommence, continue
Advocate	To show support for something	Oppose
Afraid	To be apprehensive of something	Brave, courageous
Aggressive	Showing angry tendencies	Peaceful, passive
Aid	To help someone or something	Hinder
Amateur	To be new at something	Professional
Ambitious	To be aspiring about something	Indifferent, lazy, indolent
Antagonise	To anger someone	Tranquillise, soothe

What is the importance of antonyms?

Similar to the importance of synonyms, learning antonyms help us expand our vocabulary. They also help us understand any text better. Questions based on ‘antonyms’ are asked in GATE and other engineering entrance exams.



WORD	MEANING	ANTONYMS
Apparent	Clear from what is seen	Obscure, subtle, hidden
Approve	To ratify something	Disapprove, censure
Arrive	To reach somewhere	Leave, depart
Arrogant	To think high about oneself	Modest, humble
Artificial	Not natural	Real, authentic, unnatural
Beautiful	Something or someone that looks good	Ugly
Beginning	The start of something	Finish
Believe	To accept something or someone without doubt	Distrust
Below	Under something	Above
Beneficial	Helpful in something	Harmful, adverse
Brave	Courageous	Cowardly, timid
Busy	Occupied with something	Idle, inactive
Calm	Quiet and peaceful	Excited, turbulent
Careful	Marked by watchfulness	Careless, reckless
Cease	To stop	Continue, recommence
Charming	Delightful, attractive	Obnoxious, gross, vulgar
Chubby	Thick, plump	Thin, skinny
Clarify	To explain something	Confuse
Coarse	Marked by bumpiness	Fine, smooth
Colossal	Very big or major	Tiny, insignificant, trivial
Comical	Funny and inducing laughter	Tragic, sorrowful
Comprehend	Understand properly	Confuse, misinterpret
Condense	To compress something	Expand, enlarge
Confess	To admit something	Deny



WORD	MEANING	ANTONYMS
Dangerous	Not safe	Safe
Daring	Bold, brave	Cautious
Dawn	Start of the day	Evening
Dead	Without life	Alive, active
Deduct	Remove something	Add
Defend	Protect something	Attack, assault
Defy	To resist	Obey, comply
Delicate	Very weak	Sturdy
Denounce	Blame something	Commend
Dense	Very thick	Sparse, empty
Depart	Leave a place	Arrive
Deposit	To store	Withdraw
Desolate	Barren	Dense, verdant
Despise	Hate someone/something	Love
Eccentric	Not usual or common	Normal
Ecstasy	The feeling of happiness	Sadness, depression
Encourage	To support something	Discourage
Enemy	One's ill-wisher, contender	Ally, friend
Enjoy	To like something	Dislike, hate
Enlarge	To increase the size of something	Reduce, shrink
Enormous	Very big in size	Tiny, microscopic
Enough	Sufficient in number or amount	Insufficient
Fancy	Elaborate in style	Simple, plain
Fantastic	Amazing	Ordinary, usual
Fast	Quick	Slow



WORD	MEANING	ANTONYMS
Feasible	Possible to do	Impossible
Feeble	Weak	Strong
Ferocious	Brutal	Tame, gentle
Gallant	Heroic, brave	Coward
Gather	Collect some things or people	Scatter, disperse
Gaudy	Showy	Tasteful, refined
Gaunt	Thin	Overweight, plump
Generous	Giving	Selfish, stingy
Gentle	Tender	Rough, harsh
Help	Aid	Hinder, thwart
High	Elevated	Low
Hold	Grasp something	Release, discharge
Honest	Without any lies or pretences	Untruthful, insincere
Hospitable	Welcoming	Rude, unfriendly
Hostile	Angry and defensive in character	Friendly, cordial
Imperative	Required to be done	Unnecessary, optional
Imperfect	With flaws	Perfect, flawless
Impetuous	Likely to be rash and careless	Restrained, careful
Important	Meaningful	Unimportant, meaningless
Independent	Dependent on oneself	Dependent, unsure
Inferior	Of a lesser standard	Superior
Infuriate	To anger	Soothe, calm
Ingenious	Clever	Unoriginal, dull
Innocent	Without blame	Guilty
Least	The minimum	Most, maximum



WORD	MEANING	ANTONYMS
Legible	Something that can be read	Illegible, unreadable
Lenient	Merciful, not harsh	Harsh, strict
Listless	Without any energy	Active, energetic
Logical	Something that can be rationally understood, rational	Illogical, unreasonable
Mediocre	Moderate in character	Outstanding
Mend	To fix something	Break
Migrant	Something or someone who travels	Stationary, immovable
Militant	Aggressive	Peaceful
Minor	Lesser in standard	Major
Mirth	Fun, enjoyment	Gloom, sadness
Mischiefous	Naughty	Well-behaved, angelic
Nonchalant	Carefree, unconcerned	Concerned, apprehensive
Normal	Ordinary	Abnormal, unusual
Numerous	Several	Few, scanty
Opponent	Enemy, rival, competitor	Ally, friend
Optimistic	Positive	Pessimistic
Optional	Not mandatory	Required
Permanent	Lasting, fixed, perennial	Temporary, changing
Perpetual	Unending	Transient, fleeting
Persuade	Convince	Dissuade, deter
Plausible	Believable	Unbelievable
Plentiful	Enough, adequate	Scarce, insufficient
Rational	Logical	Irrational, crazy
Ravage	To ruin something	Restore, revitalise



WORD	MEANING	ANTONYMS
Raze	To destroy	Build, construct
Recreation	Something that is done for pleasure	Work, labour
Reduce	To lessen the amount or quantity	Increase, enlarge, amplify
Serious	Of grave nature	Flighty, fickle
Shrewd	Of clever character	Unthinking, careless
Shy	Timid in nature	Bold, aggressive
Sick	Unhealthy, ill, unwell	Well, healthy
Trivial	Insignificant	Important, crucial
Turbulent	Violent	Calm, peaceful
Turmoil	Commotion	Quiet, tranquillity
Wrong	Not correct	Correct, right
Yield	To produce	Keep, retain
Zenith	The peak of something	Bottom, base

PRACTICE QUESTIONS

Write the antonyms of the given words. (There can be more than one antonym to a single word.)

1. Answer	
2. Admit	
3. Bravery	
4. Captivity	
5. Dismal	
6. Generous	
7. Healthy	
8. Knowledge	
9. Rapid	



10. Sorrow	
11. Possible	
12. Interesting	
13. Horizontal	
14. Giant	
15. Strong	
16. Scatter	
17. End	
18. Cunning	
19. Beautiful	
20. Artificial	

SOLUTIONS

- | | |
|-------------------------------------|-----------------------------------|
| 1. Question/query | 13. Vertical |
| 2. Deny/oust | 14. Dwarf/pygmy |
| 3. Cowardice/timidity | 15. Weak/idle/frail |
| 4. Freedom | 16. Gather/collect |
| 5. Cheerful/luminous | 17. Beginning/start/origin |
| 6. Greedy/miserly | 18. Ignorant/simple |
| 7. Unhealthy/ill | 19. Ugly/hideous/monstrous |
| 8. Ignorance/impotence | 20. Natural/real/genuine |
| 9. Slow/sluggish/languishing | |
| 10. Joy/cheer | |
| 11. Impossible/unlikely | |
| 12. Dull/uninteresting | |



WHAT ARE THE PARTS OF SPEECH?

Every word in the English language is a part of speech. The role of a word in a sentence is referred to as “part of speech.” In traditional grammar, it is one of the nine major categories into which words are grouped according to their roles in sentences, such as nouns or verbs. These are the building blocks of grammar, sometimes known as word classes. The part of speech identifies how a word behaves in a sentence, both grammatically and in terms of meaning. A single word can act as more than one component of speech when used in different contexts. Knowing the parts of speech is essential when using a dictionary to determine the correct definition of a word. Every phrase written or spoken in English comprises words from one of the nine parts of speech. These include words like nouns, pronouns, verbs, adjectives, adverbs, prepositions, conjunctions, articles/determiners, and interjections.

Open classes (nouns, verbs, adjectives, and adverbs) and closed classes (pronouns, prepositions, conjunctions, articles/determiners, and interjections) are the most prevalent divisions of speech. Open classes, on the other hand, can be changed and added to as the language evolves, whereas closed classes are pretty much set in stone. Every day, for example, new nouns are formed, but conjunctions do not change.

Nine parts of speech

The nine parts of speech in the English language are as follows: noun, pronoun, verb, adjective, adverb, preposition, conjunction, interjection, and articles.

1. Noun (Naming word):

A noun is a word that refers to a specific person, place, thing, or idea. A noun is something that refers to a “thing.” Nouns are the most basic

of the nine components of speech. Depending on the situation, nouns can serve as the subject, direct object, indirect object, subject complement, object complement, appositive, or adjective in a phrase. Five types of nouns can be found in the English language:

- a. Proper Noun:** A proper noun, which is a term that identifies a specific person, place, or thing. Proper nouns in written English begin with capital letters. e.g., John, India, Africa, Friday.
- b. Common Noun:** Nouns that refer to people or things, in general, are known as common nouns, e.g., boy, country, bridge, city, birth, day, happiness.
- c. Collective Noun:** Nouns that refer to groups of people or things are collective nouns, e.g., audience, family, government, team, jury.
- d. Abstract Noun:** A noun that refers to concepts, traits, and conditions that cannot be seen or touched, as well as things that have no physical reality is known as an abstract noun is, e.g., truth, danger, happiness, time, friendship, humour.
- e. Concrete Noun:** A noun that refers to persons and things that can be seen, touched, smelled, heard, or tasted and exists physically is known as a concrete noun, e.g., dog, building, coffee, tree, rain, beach, tune.

For example:

- **Stacy** lives in **Singapore**.
- **James** uses **pen** and **paper** to write **letters**.
- We live in **London**.
- Always speak the **truth**.
- A young **woman** brought me a chocolate **cake**.

2. Pronoun (Replaces a noun):

To eliminate redundancy, a pronoun is substituted for a noun or noun phrase. They’re



more generalised nouns that exclusively refer to people. A pronoun is frequently used to substitute an antecedent, which is a specific noun. They're used to avoid sounding unnatural by repeating the same noun in a sentence. Pronouns are further defined by type:

- a. *Personal pronouns*: Pronouns that are used in place of a specific person or thing are known as personal pronouns, e.g., I, you, she, he, it, we, they, me, us, them.
- b. *Relative pronouns*: Pronouns that are used to link one phrase or clause to another phrase or clause are known as relative pronouns. Relative pronouns introduce a subordinate clause, e.g., who, whom, which, what, that.
- c. *Reflexive pronouns*: Pronouns that refer back to the subject of the clause or sentence are known as reflexive pronouns. They are used when the subject of a sentence is also the object of the sentence. Reflexive pronoun ends with the suffix- self or selves, e.g., myself, yourself, itself, herself, himself, ourselves, themselves.
- d. *Demonstrative pronouns*: Pronoun that point to and identify a noun or a pronoun is known as a demonstrative pronoun, e.g., this, that, these, those.
- e. *Indefinite pronouns*: Indefinite pronouns refer to an identifiable, but not specified, person or thing. Indefinite pronouns convey the idea of all, any, none, or some, e.g., some, somebody, anyone, anywhere, nothing, everybody.
- f. *Reciprocal pronouns*: Reciprocal pronouns are used to express reciprocal relationships or acts., e.g., each other, one another.
- g. *Interrogative pronouns*: Pronouns that are used to ask questions about unknown people or things are known as interrogative pronouns, e.g., who, whom, what, which, whose

For example:

- **Someone**, please call the ambulance.
- **His** son has been kidnapped.

- I had forgotten **my** umbrella in the room.
- The person sitting by **your** side is **my** uncle.
- **She** wants to sleep.

3. Adjective (Describing word):

An adjective is a word that describes, modifies, or adds to the meaning of a noun or pronoun. They indicate which one, how much, and what type, among other things. Adjectives let readers and listeners visualise something more clearly by allowing them to engage their senses. Because the adjectives appear directly before the nouns they modify, they are simple to notice. They are frequently placed before the noun or pronoun that they modify to identify or quantify individual persons and distinct things. Multiple adjectives can also be seen in some sentences. Adjectives can be classified into the following types:

- a. *Descriptive adjective*: An adjective describing a noun by expressing a quality or attribute is known as a descriptive adjective. They are the most common of the various types of adjectives.
- b. *Quantitative adjective*: A quantitative adjective provides information about the quantity of the nouns/pronouns.
- c. *Indefinite adjective*: An adjective that describes or modifies a noun unspecifically is known as an indefinite adjective. They give general/indefinite information about the noun.
- d. *Demonstrative adjective*: An adjective that directly refers to something or someone is known as a demonstrative adjective. They are always positioned directly in front of the noun they are modifying.
- e. *Interrogative adjective*: An interrogative adjective asks a question. A noun or a pronoun must come after an interrogative adjective. These words will not be considered adjectives unless they are followed by a noun.
- f. *Possessive adjective*: A possessive adjective indicates possession or ownership. It



suggests the belongingness of something to someone/something.

For example:

- I gave **some** candy to her.
- He is a **good** boy.
- They have **three** children.
- The **hungry** cats are crying.
- I have **ten** candies in my pocket.

4. Verb (Action word):

Verbs are parts of speech used to describe people's or things' activities, processes, situations, or states of being. They are verbs that describe what happens in a sentence. A verb expresses the action of someone or something. It is considered the soul of language since it is a vital aspect of a phrase. The following are various types of verbs:

- a. *Finite verbs*: Finite verbs are ones that can take on different forms depending on the subject. It defines the time (past or present) as well as the subject. It also informs you of who is performing the action and when it will be completed.
- b. *Infinitive verbs*: A verb's infinitive form is the verb in its most basic form. A verb's infinitive form is frequently preceded by the word "to." The "to" is a part of the verb when you use an infinitive verb. In this situation, it is not operating as a preposition.
- c. *Transitive verbs*: In a sentence, a transitive verb is one that is accompanied by a direct object. The noun, pronoun, or noun phrase that is having something done to it by the sentence's subject is called the direct object. A transitive verb is one that requires an object to complete itself.
- d. *Intransitive verbs*: A verb is an intransitive verb if it is not used with a direct object. Hence, a verb that doesn't need any object to complete itself is called an intransitive verb. Verbs that express action but do not take an object are known as intransitive verbs. Without an object, the subject and verb express a complete notion. The verb

can, however, be followed by an adverb or other modifier.

- e. *Regular verbs*: If the past tense form and past participle of a verb ending in -ed, -d, or the verb is a -t variant verb, it is termed a regular verb.
- f. *Irregular verbs*: An irregular verb is one that does not end in -ed, -d, or uses the -t variation in the past tense or past participle form.

For example:

- I **work** at a factory.
- The mansion **has** five bathrooms.
- Leonardo **ate** a delicious pepperoni pizza.
- Aeroplanes **fly**.
- The writing in this book is too small **to see**.

5. Adverb (Describes a verb):

An adverb is a word that is used to amend or qualify the meaning of an adjective, a verb, a clause, another adverb, or any other kind of word or phrase, with the exception of determiners and adjectives that directly modify nouns. Adverbs describe or modify verbs, adjectives, and other adverbs, but never nouns. They describe when, when, how, and why something happened, as well as the amount and frequency of the occurrence. Adverbs are broadly classified as follows:

- a. *Adverbs of time*: Adverbs of time describe when the action of the verb occurred. These adverbs are typically found at the beginning or end of a sentence.
- b. *Adverbs of manner*: Adverbs of manner describe how or in what way something was done. They usually modify verbs and appear at the end of a clause or immediately before the word they modify. This category comprises the most common adverbs—the ones that end in -ly.
- c. *Adverbs of degree*: Adverbs of degree provide additional information about the verb's intensity in the sentence. They describe the extent to which something occurred. Adverbs of degree are frequently



placed before the word they modify, but in some cases, they can also be placed after the word.

- d. *Adverbs of frequency:* Frequency adverbs tell us how frequently something happens. These adverbs usually appear just before or after the main verb in the sentence.
- e. *Adverbs of place:* Adverbs of place tell us about where something happens or where something is. They come after the main verbs of the clauses that they modify.

For example:

- He trimmed the white roses **neatly**.
- This short essay is **hardly** sufficient.
- We **never** stay up past ten o'clock.
- You can park **anywhere**.
- They **recently** relocated to Santa Fe.

6. Preposition (Shows relationship):

A preposition illustrates how a noun or pronoun is related to another word. It's a word that comes before a noun or pronoun in a sentence to produce a phrase that modifies another word. The noun or pronoun that the preposition connects is the object of the preposition. The following are the five parts of preposition:

- a. *Simple prepositions:* Simple prepositions are common prepositions that can be used to describe a location, time, or place. These include words like at, for, in, off, on, over, under.
- b. *Compound prepositions:* To convey location, compound prepositions (also known as complicated prepositions) are made up of two or more words, usually a simple preposition and another word. These include words like in addition to, on behalf of, and in the middle of.
- c. *Double prepositions:* Double prepositions are two prepositions that are used together, usually to indicate direction. These include words like into, upon, onto, out of, from within.
- d. *Participle prepositions:* Participle prepositions have -ed and -ing endings.

These include words like considering, during, concerning, provided.

- e. *Phrase prepositions:* A preposition, an object, and the object's modifier make up phrase prepositions (or prepositional phrases). These include phrases like on time, at home, before class, and on the floor.

For example:

- He sat **on** the chair.
- The loud noise came **from within** the room.
- She has art class **in addition to** his regular classes today.
- The dog kept **following** him home.
- She lives **near** her workplace.

7. Conjunction (Joining word):

A conjunction joins two words, ideas, phrases, or clauses together in a sentence and shows how they are connected. We can build complex sentences with conjunctions that show a link between actions and ideas. The following are the three types of conjunctions:

- a. *Coordinating conjunctions:* Coordinating conjunctions link words or phrases in a sentence that have the same grammatical function. They join words or ideas together. The following are the seven main coordinating conjunctions in English that make up the acronym FANBOYS:
 - **For**—To explain the reason or purpose
 - **And**—To add one thing to another
 - **Nor**—To present an alternate negative idea
 - **But**—To show contrast
 - **Or**—To present a choice or an alternative.
 - **Yet**—To introduce a contrasting idea that follows the preceding idea logically
 - **So**—To indicate effect, result, or consequence
- b. *Subordinating conjunctions:* Subordinating conjunctions join a subordinate clause to a main clause and establish a relationship between the two. They also demonstrate that the dependent clause (or subordinate



clause) contains the less essential notion in the sentence, as opposed to the independent clause. There are many different types of subordinating clauses, but here are a handful of the most common ones: even if, even though, if, in order that, in case, in the event that, lest, now that, once, only, only if, provided that, since, so, supposing, that, till, unless, until, when, whenever, where, whereas, wherever, whether or not, while.

- c. **Correlative conjunctions:** Correlative conjunctions are pairs of conjunctions that work together. These conjunctions correlate, working in pairs to join phrases or words that carry equal importance within a sentence. Some common pairs include both/and, whether/or, either/or, neither/nor, not/but, not only/but also.

For example:

- Her favourite colours were purple **and** red.
- Lia plays **not only** the violin **but also** the drums.
- She was late for school **so** she took a shortcut.
- **Because** it was snowing, we had to cancel the plan.
- Jason went to get a drink of water **before** his exam started.

8. Interjection (Expressive word):

A word or phrase that displays a strong sentiment or emotion is called an interjection. It is a short exclamation. These are expressions that can stand alone or be used as part of a sentence. These words and phrases are frequently used to indicate strong emotions and reactions. They are frequently followed by an exclamation mark. Usually, an interjection is used in an informal language. Interjections communicate a wide range of emotions such as joy, grief, surprise, disgust, and so on, even though they are not grammatically related to the other parts of the sentence or help the reader grasp the link between words and

phrases in the sentence. The following are the different types of interjections:

- a. **The interjection of greeting:** The Interjection of greeting indicates the emotion of warmth to meet with anyone, such as hello!, hi!, hey!, and so on.
- b. **The interjection of joy:** The interjection of joy is used to communicate immediate delight and satisfaction in response to a specific event, such as wow!, hurrah!, congratulations!, good!, and so on.
- c. **The interjection of attention:** The interjection of attention is used to draw someone's attention to us, such as listen!, look!, shh!, behold!, hush!, and so on.
- d. **The interjection of approval:** The interjection of approval is used to convey approval or agreement in a sentence, such as bravo!, well done!, brilliant!, and so on.
- e. **The interjection of surprise:** The interjection of surprise is used to indicate surprise at any event that has occurred, for example, oh!, what!, ah!, ha!, etc.
- f. **The interjection of sorrow:** The interjection for sorrow is used to describe the sense of grief in a sentence, such as ah!, alas!, ouch!, oops!, and so on.

For example:

- **Hello!** I'm Adam, your new professor.
- **Wow!** You look so beautiful!
- **Shh!** keep the mouth shut.
- **What!** Are you serious?
- **Oops!** Sorry. It was my fault.

9. Article (Defining word):

An article is used before a noun. An article is a word that modifies a noun (a person, place, thing, or idea). An article is a type of adjective, which refers to any word that modifies a noun. Adjectives usually alter nouns by describing them, but articles are employed to point out or refer to nouns. There are two sorts of articles that we employ to point out or refer to a noun or group of nouns in writing and conversation:



- a. **Definite article:** This article is the word ‘the’, which refers directly to a specific noun or groups of nouns. The definite article (the) comes before a noun to emphasise that the reader understands the word’s identity. The definite article ‘the’ is used when the reader knows the specific identification of a noun (whether singular or plural, count or noncount).
- b. **Indefinite article:** The words ‘a’ and ‘an’ are indefinite articles. Each of these articles refers to a noun, however, the noun in question isn’t a specific person, place, thing, or concept. Any noun from a set of nouns

can be used. When the identity of a noun is unclear, it is preceded with the indefinite article (a, an). When referring to a singular count noun whose identity is unknown to the reader, the indefinite article ‘a’ or ‘an’ should be used. Use ‘a’ before nouns that begin with a consonant sound, and ‘an’ before nouns that begin with a vowel sound.

For example:

- I ate **an** apple yesterday.
- **The** apple was juicy and delicious.
- I own **a** cat and two dogs.
- Thank you for **the** advice.
- I do not want **a** gun in my house.

PRACTICE QUESTIONS

Identify which part of speech the highlighted word is.

1. I bought a **shiny** watch at the mall.
2. She thinks **we** will arrive at roughly 3 pm.
3. The **music** is very loud.
4. She **quickly** packed her bag and left.
5. **Wow!** You have got a great score.
6. I went **to** Madras and met my friend.
7. Lia knocked on the door **but** nobody answered.
8. The baby crawled **under** the bed.
9. She was wearing a really **pretty** dress.
10. **Take** the second left then go over the bridge.
11. Today I wore my **blue** skirt.
12. I am **extremely** excited about our trip.
13. Let’s make **cookies**.
14. Your suitcase is **in** the closet.
15. **Ouch!** That hurts.
16. **Hi**, it’s good to see you.
17. He is the **best** in the area.
18. He probably hates **himself** now.
19. She handled it very **smoothly**.
20. **Reading** is important.

Fill in the blanks using the correct form of the word.

21. Today was a very ___ day.
 - A. Cold
 - B. Colder
 - C. Coldly
22. We will have to ___ if we do not want to be late.
 - A. Ran
 - B. Run
 - C. Running
23. ___ was that man you were talking to?
 - A. What
 - B. Why
 - C. Who
24. I forgot my umbrella ___ I got wet.
 - A. Because
 - B. So
 - C. Although
25. ___ is a global problem.
 - A. Poorness
 - B. Poor
 - C. Poverty



26. It's OK to use that phone, it's ____.
A. Mine
B. Me
C. I
27. When was the last time you ____?
A. Cried
B. Cry
C. Cries
28. Ask ____ what she thinks.
A. She
B. Her
C. Hers
29. I ____ sleepy.
A. Are
B. Is
C. Am
30. I sat ____ the chair.
A. In
B. On
C. At
31. This song is ____ than that song.
A. Better
B. More better
C. Best
32. Today, I ____ for three hours.
A. Played
B. Plays
C. Playing
33. My brother and ____ got into a fight.
A. Me
B. I
C. Mine
34. Your room is ____ than my room.
A. More organized
B. Most organized
C. Best organized
35. ____ is my bag?
A. Who
B. Where
C. What

Fill in the blanks with the appropriate article:

36. Ravi made ____ error of judgement.
37. She gave me ____ call in the evening.
38. Suresh is ____ tallest boy in my class.
39. ____ Earth moves around ____ Sun.
40. We saw ____ tiger in ____ zoo.

Solution

- Adjective** An adjective adds to the meaning of a noun. Here, shiny adds to the meaning of a watch.
- Pronoun** Pronouns replace nouns. Here, “we” is replacing the people the speaker is referring to.
- Noun** A noun is a word that refers to persons, places, things, or ideas. The noun here is music, which is a thing.
- Adverb** Adverbs modify verbs, nouns, and other adverbs. This is modifying the verb packed.
- Interjection** Interjections are unrelated to the rest of the sentence and express enthusiasm or emotion. Wow implies amazement or surprise in this context.
- Preposition** A preposition shows the relationship of a noun or pronoun to another word. Here, ‘to’ shows the relationship of ‘I’ to and where ‘I’ went.
- Conjunction** Conjunctions join two or more words, phrases, or clauses together. This conjunction “and” is joining “Andy knocked on the door” and “nobody answered.”



- 8. Preposition** Prepositions indicate the relationship between nouns or pronouns and other words or elements in the sentence. Here, it shows the relationship between the baby and where it crawled to.
- 9. Adjective** An adjective adds to the meaning of a noun. Here, pretty adds to the meaning of dress.
- 10. Verb** Verbs show an action or state of being. In the given sentence, there is no subject as it is a command/instruction, so the first word is the verb.
- 11. Adjective** An adjective modifies a noun or pronoun. Here, blue modifies the skirt.
- 12. Adverb** Adverbs modify verbs, nouns, and other adverbs. This is modifying the verb excited.
- 13. Noun** A noun specifies people, places, things, or ideas. This noun is a thing, cookies.
- 14. Preposition** Prepositions show the relationship between nouns or pronouns and other words or elements in the sentence. Here, in the given sentence, it shows the relationship between the suitcase and where it is.
- 15. Interjection** Interjections show excitement or emotion and are not connected to the rest of the sentence. Here, no is showing emotion of sadness.
- 16. Interjection** Interjections show excitement or emotion and are not connected to the rest of the sentence. Here, hi is an interjection of greeting.
- 17. Adjective** An adjective modifies a noun or pronoun. Here, best modifies him.
- 18. Pronoun** Pronouns replace nouns. Here, "himself" is replacing the person the speaker is referring to.
- 19. Adverb** Adverbs modify verbs, nouns, and other adverbs. This is modifying the verb handled.
- 20. Noun** A noun specifies people, places, things, or ideas. This noun is an idea, reading.
- 21. (A)** Today was a very *cold* day.
- 22. (B)** We will have to *run* if we do not want to be late.
- 23. (C)** Who was that man you were talking to?
- 24. (B)** I forgot my umbrella so I got wet.
- 25. (C)** Poverty is a global problem.
- 26. (A)** It's OK to use that phone, it's *mine*.
- 27. (A)** When was the last time you *cried*?
- 28. (B)** Ask *her* what she thinks.
- 29. (C)** I am sleepy.
- 30. (B)** I sat on the chair.
- 31. (A)** This song is *better* than that song.
- 32. (A)** Today I *played* for three hours.
- 33. (B)** My brother and *I* got into flight.
- 34. (A)** Your room is *more organized* than my room.
- 35. (B)** Where is my bag?
- 36. An** Here, the identification of a noun is unknown and since the noun starts with a vowel sound, the indefinite article, 'a' is used.
- 37. A** Here, the identification of a noun is unknown and since the noun starts with a consonant sound, the indefinite article, 'a' is used.
- 38. The** Here, the exact identity of the noun is known. Hence the definite article 'the' is used.
- 39. The** Here, the exact identity of both the nouns are known. Hence the definite article 'the' is used in both places.
- 40. A, the** Here, the identification of the first noun is unknown and since the noun starts with a consonant sound, the indefinite article, 'a' is used. The exact identity of the second noun is known and hence the definite article 'the' is used.

5

Pronouns



WHAT IS A PRONOUN?

A pronoun may be called a word that replaces a noun or a noun phrase. It can help you remove redundancy or unnecessary repetitive usage of nouns in writing.

Look at the following examples.

Riya went to the grocery store, but Riya did not find anything there.

Martin told me to come to his office and Martin also told me to get the files to Martin.

Mom made some food and mom gave it to us in a lunchbox.

Janet wrote the letter but Janet did not send the letter on time.

Does it sound repetitive?

Look at the repetitive usage of the following nouns:

1. Riya
2. Martin
3. Mom
4. Janet
5. letter

Does the repetitive usage of these nouns make the sentences sound boring or difficult to read?

This is where pronouns come in. A pronoun takes the place of a noun to prevent its repetitive usage. The noun, in this case, may be called the antecedent. The antecedent specifies the nature of a pronoun.

For example:

The girl went to sleep because she was very exhausted.

Here, ‘the girl’ is the antecedent, and ‘she’ is the pronoun. Look at how a change in the antecedent can also cause a change in the pronoun.

The boy went to the store because he was out of supplies.

Here, a change in the antecedent causes a change in the nature of the pronoun.

TYPES OF PRONOUN

There are different types of pronoun.

Personal pronouns

Personal pronouns are the pronouns that refer to a specific person or a thing. They can be divided into two types.

Normative personal pronouns—They act as the subject!

For example:

I came home after the work was finished.

You should skip the show for today.

Objective personal pronouns—They act as objects!

Riya came to the store with Michael and me.

Michael hates him.

Possessive pronouns

Possessive pronouns show ownership. They show that an object belongs to someone.

For example:

The pencil box was mine.

Their problems are not mine to handle.

The piece of cake is yours.

The duty is hers.

This tent is ours.

Reflexive pronouns

Reflexive pronouns show that the subject in a sentence is receiving the action of the verb.

For example:

I can handle the project by myself.

They can write the paper themselves.

She can get home by herself.

He will tell them the instructions himself.

The situation can handle itself.



PERSONAL (NORMATIVE)	PERSONAL (OBJECTIVE)	POSSESSIVE	REFLEXIVE
I	You	My/mine	Myself
You	Me	Your/yours	Yourself
He	Him	His	Himself
She	Her	Her/hers	Herself
It	It	Its	Itself
We	Us	Our/ours	Ourselves
They	Them	Their/theirs	Themselves

Intensive pronouns

Intensive pronouns are only used to place emphasis on the subject and are not important *per se* to the meaning of the sentence. They may look similar to reflexive pronouns but have different usage and are also used immediately next to the subject that they are emphasising.

For example:

You yourself must complete the work.

She herself must inform the authorities.

They themselves should approach me regarding this.

We ourselves must achieve this task.

Some intensive pronouns:

Myself, himself, herself, itself, ourselves, themselves

Demonstrative pronouns

Demonstrative pronouns are used to identify nouns and answer the question “which?”.

For example:

These are the people that we selected for the play.

They are the participants.

She is the girl Ray was talking about.

These are the pens I could find.

Some demonstrative pronouns:

This, that, these, those

Interrogative pronouns

Interrogative pronouns are pronouns that are used only in reference to a question regarding who, which, whose, what, whom.

For example:

Which one of the notebooks is hers?

Who is that person?

Which one of these phones is yours?

Who is that boy?

Some relative pronouns:

Who, what, which, Whom, whose

Relative pronouns

Relative pronouns are used to connect clarifying information to nouns or other pronouns in a sentence.

For example:

Who wrote this letter? ('who' is used for a subject)

Who went to the store without telling me?

With whom did you do this project? ('whom' is used for a subject)

This notice should be addressed to whom?

Jennie, who choreographed the play, has received a contract offer from Hollywood.

The things that I bought from the market were not helpful to me. ('that' is used when referring to things)

The community gymnasium that is down the street is very clean.

Our luggage was stolen, which really upset my mother. ('which' is used in nonessential clauses)

Some relative pronouns:

Who, that, which, whom, Whose, whoever, whichever, Whomever, whatever



Indefinite pronouns

Indefinite pronouns are used while referring to a person or a thing not specific or known. They may also be used to identify a general group of people or things.

For example:

Everyone has to follow the instructions sent yesterday.

Some were excited to go on the cruise.

All of the students submitted the assignment on time.

Most of the girls were against the proposal.

Some indefinite pronouns:

both, many, several, few, one, some, none, all, most, somebody, anybody, anyone, everybody, everyone, each, every

Reciprocal pronouns

Reciprocal pronouns are used when referring to a mutual set of people.

For example:

They needed each other's addresses.

We have to help one other to survive.

You have to work with each other to complete the work before the assigned deadline.

Some reciprocal pronouns:

each other, one another, each other's, one another's

Collective nouns

In the case of collective nouns, like family, class, panel, college, society, group, school, committee, company, the pronouns referring to them are singular.

For example:

The jury delivered its verdict yesterday.

The school told its students to submit their forms as soon as possible.

However, when a collective noun is used to refer to a group of individuals with different goals, the pronoun would be plural.

For example:

The jury discussed their verdict among themselves.

The class submitted their assignments.

TIPS TO WRITE/IDENTIFY PRONOUNS CORRECTLY

Avoid making ambiguous uses of pronouns.

For example:

Jenna told Maria that she had to go to the office early.

The sentence poses an ambiguity regarding who actually had to go to the office early, Jenna or Maria.

Therefore, you must make sure that it is clear who the pronoun refers to in a sentence.

Jenna told Maria that she, Maria, had to go to the office early.

The given sentence makes it clear who the pronoun refers to. Therefore, it is not ambiguous.

Use the correct form of the pronoun when comparing persons or things.

For example:

Jennie is shorter than me [am]. (incorrect)

Jannie is shorter than I [am]. (correct)

I walked four miles with she. [incorrect]

I walked four miles with her. [correct]

PRACTICE QUESTIONS

Fill in the blanks with appropriate pronouns.

1. Each person thanked _____ parents.
2. Raymond injured _____ left arm.
3. Her neighbours sometimes leave _____ house door open.
4. The committee works for the upliftment of _____ members.
5. The council members discussed _____ views about the changing situation.



6. Sara went out with some people yesterday and _____ discussed at length about Maria.
7. The professor called Nathan and _____.
8. We have to help _____ if we wish to reach on time.
9. Mr Michaels and _____ will present in the meeting.
10. _____ wanted our results quicker than they were willing to give.
11. _____ would like my cardigan back.
12. The team's problem is that _____ doesn't have good leadership.
13. Jake, Winnie, and I submitted the work on time but _____ didn't receive any acknowledgement.
14. I don't think this is any of _____ business.
15. We aim at upliftment of all, so _____ request everyone to register accordingly.
16. _____ of us agreed to the proposal.
17. _____ of these packet's is his?
18. You _____ must come to me if something like this happens the next time.
19. _____ one of you didn't inform me?
20. _____ are the people Jay shortlisted for the auditions.

SOLUTIONS

- | | |
|---------------------------|--------------------|
| 1. His or her/their | 11. I |
| 2. His | 12. It |
| 3. Their | 13. We |
| 4. Its | 14. My/our/your |
| 5. Their | 15. We |
| 6. They | 16. Some/all/a few |
| 7. Me | 17. Which |
| 8. Each other/one another | 18. Yourself |
| 9. I | 19. Which |
| 10. We | 20. These/those |

6

Prepositions



WHAT ARE PREPOSITIONS?

It is a term signifying the link between two objects indicating their position. For example: The pen is on the table. Here, on shows the relationship between the pen and the table. It is a word that indicates direction (to in “a letter to you”), location (at in “at her house”), or time (by in “by midnight”), or that introduces an object (of in “a basket of mangoes”). Prepositions are usually followed by an object, which can be a noun (midnight) or a pronoun (you).

The most commonly used prepositions have been penned down here as follows: about, above, across, after, against, along, among, around, at, because of, before, behind, below, beneath, beside, between, by, close to, down, during, except, for, from, inside, in, instead of, into, like, near, of, off, on, on top of, onto, out of, outside, over, past, since, through, to, towards, under, until, up, upon, with, within, without.

Types of preposition

The different types of prepositions are as follows:

Prepositions of direction: In order to refer to a direction, the prepositions used are “to,” “in,” “into,” “on,” and “onto.”

- She rode to the school.
 - Don’t knock during an ongoing lecture.
- Come right in(to) the classroom.

Prepositions of time: In order to refer to one point in time, the prepositions used are “in,” “at,” and “on.”

- He was born in 1999.
- I go to gym at 6:00.
- We get off on second Saturdays of each month.

In order to refer to extended time, the prepositions used are “since,” “for,” “by,” “during,” “from...to,” “from...until,” “with,” and “within.”

- I have lived in Singapore since 2010.

- He will be in Bahrain for 2 weeks.
- He will finish his work by 8:00.
- She works part time during the autumn.
- I will take care of the car from July to December.
- They are in school from January until June.
- She will graduate within 3 years.
- Prepositions of place: The prepositions used to refer to a place are as follows: “in”, “at”, “on”, and “inside”.
- They will meet in the lunchroom.
- Lia was waiting at the corner.
- She left her wallet on the bed.
- Keep the pen inside the pouch.

In order to refer to an object higher than a point, the prepositions used are “over” and “above.” In order to refer to an object lower than a point, the prepositions used are “below,” “beneath,” “under,” and “underneath.”

- There is hard wood beneath the carpet.
- The bird flew over the house.
- He hid under the table.
- The mouse is hiding underneath the bed.

In order to refer to an object close to a point, the prepositions used are “by,” “near,” “next to,” “between,” “among,” and “opposite.”

- The post office is by the grocery store.
- My house is near the post office.
- Park your car next to the garage.
- The garage is opposite the house.

Prepositions of location: In order to refer to a location, the prepositions used are “in”, “at”, and “on”.

- He will find her at the library.
- They live in the country.
- He sat on the chair.

Prepositions of spatial relationships: To refer to a spatial relationship, use the prepositions “above,” “across,” “against,” “ahead of,” “along,” “among,” “around,” “behind,” “below,” “beneath,” “beside,” “between,” “from,” “in front



of,” “inside,” “near,” “off,” “out of,” “through,” “toward,” “under,” and “within.”

- His house is across the street from the grocery store.
- We will stop at many attractions along the way.
- The children are hiding behind the tree.
- Take your shoes off.
- Walk toward the library and then turn right.
- Place a check mark within the box.

SOME RULES WHILE USING PREPOSITIONS

1. A preposition must have an object.
2. A preposition is followed by a “noun”. It is never followed by a verb.
3. A preposition (usually) comes before its object. For example: I put it in the bag. However, at times when the preposition does not come before its object, it is still closely related to its object: Who did you talk to? / I talked to Lia.

PRACTICE QUESTIONS

Fill in the blanks with correct prepositions.

1. We took the decision ____ the telephone.
2. I met Lia ____ December.
3. We were paralysed ____ fear.
4. Jacob left ____ Singapore.
5. She turned ____ and looked at me.
6. The boy ____ the blue shirt is my brother.
7. The cat sprang ____ the table.
8. I shall return ____ an hour.
9. He will join school ____ tomorrow.
10. She went back home ____ lunchtime.
11. Do you take pride ____ your appearance?
12. I tripped ____ the box and fell down the stairs.
13. I am looking forward ____ seeing you at the party.
14. He trembled ____ fear when he got caught.
15. I think the shop should be open ____ now.
16. People used to waste a lot of time ____ queuing up to pay bills.
17. Our shop remains closed ____ Sundays.
18. We visited Manali ____ the summer vacations.
19. She is proud ____ her achievements.
20. I am no good ____ singing.
21. My sister works ____ the bank.
22. He hung a calendar ____ the fireplace.
23. Children ____ the age of three are not allowed here.
24. There is a shop ____ the house.
25. He is leaning ____ the wall.
26. Come to my office ____ half-past ten.
27. He is available ____ 10 am and 12 noon.
28. The old lady is walking ____ a stick.
29. We pay our bills ____ cash.
30. The house is ____ fire.
31. The peaches are sold ____ 150 rupees a kilo.
32. The spoon is made ____ gold.
33. You are eligible ____ the post.
34. I was shocked ____ her behaviour.
35. I am short ____ money.
36. I have a good relationship ____ my classmates.
37. Concentrate ____ your work.
38. She is ill ____ fever.
39. The situation is ____ control.
40. She congratulated me ____ winning the game.
41. It rained ____ days together.
42. They were freed ____ the danger.
43. The meeting starts ____ three o'clock.
44. He warned me ____ the danger.
45. Be kind ____ your peers.
46. She invited me ____ her house.
47. They went ____ a picnic.
48. Ram lost ____ Lia in the finals.
49. James died ____ overeating.
50. There is some dispute ____ the property.



SOLUTIONS

- 1.** Over
- 2.** In
- 3.** With
- 4.** For
- 5.** Around
- 6.** In
- 7.** Upon
- 8.** Within
- 9.** From
- 10.** At
- 11.** In
- 12.** Over
- 13.** To
- 14.** With
- 15.** By
- 16.** In
- 17.** On
- 18.** During
- 19.** Of
- 20.** At
- 21.** At
- 22.** Over
- 23.** Under
- 24.** Across
- 25.** Against
- 26.** At
- 27.** Between
- 28.** With
- 29.** In
- 30.** On
- 31.** At
- 32.** Of
- 33.** For
- 34.** By
- 35.** Of
- 36.** With
- 37.** On
- 38.** With
- 39.** Under
- 40.** On
- 41.** For
- 42.** From
- 43.** At
- 44.** Of
- 45.** To
- 46.** To
- 47.** On
- 48.** To
- 49.** Of
- 50.** Over

7

Conjunctions



WHAT ARE CONJUNCTIONS?

They are parts of speech that connect words, sentences, phrases, or clauses. It is the glue that binds together words, phrases, and clauses. Conjunctions allow you to construct complex, elegant sentences without the choppiness that several short sentences might cause. Conjunctions are of three types.

They are: coordinating, correlative, and subordinating. Each of which has a different purpose, but all facilitate the word connection.

TYPES OF CONJUNCTION

Coordinating conjunctions

Coordinating conjunctions link words or phrases in a sentence that have the same grammatical function. They join words or ideas together. For example: I like carrom *and* chess. Here, *and* acts as a coordinating conjunction that joins the two nouns mentioned here: carrom *and* chess.

The seven main coordinating conjunctions in English, which form the acronym (FANBOYS), are as follows:

- **For**—To explain the reason or purpose
- **And**—To add one thing to another
- **Nor**—To present an alternate negative idea
- **But**—To show contrast
- **Or**—To present a choice or an alternative.
- **Yet**—To introduce a differing idea that follows the preceding idea logically
- **So**—To indicate any result or inference

Understand With Example



- I love cricket *and* football.
- You may take this book *or* that one.
- She is kind *so* she helps people.

Subordinating conjunctions

Subordinating conjunctions join a subordinate clause to a main clause and establish a relationship between the two. It also demonstrates that the dependent clause (or subordinate clause) contains the less essential notion in the sentence, as opposed to the independent clause.

Understand With Example



Lia went to bed *after* she finished her work. Here, the subordinate clause is “*after* she finished her work” which is not of the same significance as the main clause, which is “Lia went to bed”.

Below are a few of the most common subordinating clauses:

after	once
although	provided
as	provided that
as if	rather than
as long as	since
as much as	so that
as soon as	supposing



as though	than
because	that
before	though
even	till
even if	unless
even though	until
if	when
if only	whenever
if when	where
if then	whereas
inasmuch as	where if
in order that	wherever
just as	whether
lest	which
now	while
now since	who
now that	whoever
now when	why

A subordinating conjunction can be used in two ways to form a sentence:

1. Main clause + subordinate clause
 - The student submitted the test after completing all the questions.
2. Subordinate clause + , + main clause
 - After completing all the questions, student submitted the test.

Correlative conjunctions

Correlative conjunctions are pairs of conjunctions that work together. These conjunctions correlate, working in pairs to join phrases or words that carry equal importance within a sentence.

Some common pairs include:

as ... as	no sooner ... than
both ... and	not only ... but also
either ... or	rather ... than
if ... then	scarcely ... when
neither ... nor	such ... that

Understand With Example



- She studies *both* English Literature and History.
- She seems undecided *whether* to go or stay.
- She can have *either* tea or coffee.
- She likes *neither* apples *nor* oranges.

Some rules while using correlative conjunctions are given below:

1. **Maintain a parallel structure.** Since correlative conjunctions come in pairs, one must use the same type of word after each one in the pair.
2. **Don't use commas with correlative conjunctions.** However, if the second conjunction sits before an independent clause (i.e., words that could be a standalone sentence), then use a comma.

Important points to be noted while using conjunctions:

1. So...as and as.....as is used to show a comparison between two things, people, etc.
2. Although and though are followed by yet or comma (,)
3. Lest is followed either by *should* or the first form of the verb. Lest is a negative word thus, it should never be used with not, never, to avoid redundancy.



4. *Unless* is an action-oriented term, whereas *Until* is a time-oriented word. They should not be used with no, not, never, etc. since they are negative terms.
5. In positive statements, the words *doubt* and *doubtful* are followed by the words *if* or *whether*. In negative phrases, however, they are followed by *that*.
6. The word *between* is followed by the word *and*. The word *to* comes after the word *from*.
7. *Neither* of means none of the two, *none* of is used when there are more than two, *either* of means one of the two, *one* of is used when there are more than two.
8. Subordinating conjunctions should be used after *rather/other*.

PRACTICE QUESTIONS

Fill in the blanks with appropriate conjunction.

1. Many things have happened ____ I left this place.
2. He bled so profusely ____ he died.
3. Lia ____ her brother attended the party.
4. The principal ____ the teacher had signed the papers.
5. He likes cricket ____ hates football.
6. ____ his bike is old, it still runs well.
7. Is it Monday ____ Tuesday today?
8. Neither Varun ____ Tarun owns a car.
9. Write this down ____ you forget.
10. You will fail the test ____ you study.

SOLUTIONS

1. Since
2. That
3. And
4. As well as
5. But
6. Although
7. Or
8. Nor
9. Lest
10. Unless

8

Subject–Verb Agreement



I runs. She run. You runs.

Do the given options sound correct? Or does something feel out of place?

If you look at the sentences carefully, you will find what is wrong. If you have not found what is wrong yet, worry not, because in this chapter, we will study exactly why these sentences are grammatically incorrect and how to make them correct. The correct subject, with the right verb makes it easier to comprehend the sentence improving the flow of the sentence. The numerical representation a verb holds in a sentence is of great importance to apprehend the person/ group of people we are talking about.

In the given sentences, I is the subject. **Run/runs** is the verb. It must always be made sure that the subjects and verbs, in a sentence, agree with each other. What does agreeing exactly mean here? Agreeing refers to concord between the number of the subjects and the verb. A verb changes according to what time it needs to specify. It also changes its form according to the number of subjects being referred to in the sentence.

This is exactly what the subject-verb agreement means. The subjects and verbs must always agree in number.

Read the following out loud:

	SINGULAR SUBJECTS	PLURAL SUBJECTS
First Person	I walk.	We walk.
Second Person	You walk.	You all walk.
Third Person	He walks. She walks.	They walk.

Do the given sentences sound correct? You may have noticed that the sentences in the third person singular have ‘s’ added to the verbs. Think about the following verbs: **run, walk, eat, study, work, cry**. In the first person singular, all of them may be written like this: **I run, I walk, I eat, I study, I work, I cry**. It is similar for the second person singular: **you run, you walk, you eat, you study, you work, you cry**.

The situation changes when we talk about third-person singulars, like **he, she, or the dog**. Here, an ‘s’ must be added to each verb. It can be written like this: **he runs, he walks, he eats, he studies, he works, he cries**. Or **she runs, she walks, she eats, she studies, she works, she cries**. Or **the dog runs, the dog walks, the dog eats, the dog studies, the dog works, the dog cries**.

Hotkeys

If you cannot decide whether a sentence is grammatically right or wrong, try reading it out loud! It is a tried and tested tip that many writers and speakers use.

Reading out can help you detect mistakes better!

The importance of a correct subject-verb agreement rests in the fact that the absence of one may lead to a serious grammatical error. These are called subject-verb agreement Errors and they point at a serious absence of grammatical knowledge. **Therefore, we must make sure that we abide by the following rules to ensure an error-free sentence.**



Rules for subject-verb agreement

1. A singular subject, like Jamie, park, bus, has a singular verb and a plural subject has a plural verb.

Following is an example:

- **Jamie walks to the park.**
- **Jamie and his friends walk to the park.**

2. In sentences having ‘of’, the subject is the one that comes ‘of’. Identification of the subject is important to prevent mistakes.

Following is an example:

- **The swarm of bees lives in the tree.**
- **The school of dolphins swims in the sea.**

3. Two subjects joined by or, either/or, or neither/nor will have a singular verb.

Following is an example:

- **Either my mom or my dad is accompanying me to the fair.**
- **Neither of my friends has the same opinion.**

4. Some sentences begin with ‘here’ or ‘there’. In such sentences, the subject would follow the verb.

Following is an example:

- **Here are the clothes.**
- **There is the shoe.**

5. Two or more subjects, when joined together, are considered plural and therefore require a verb without an ‘s’.

Following is an example:

- **Ram, Shyam, and Emily run to the park.**
- **Kanika and her friends run to the park.**

6. If a subject is preceded by words like ‘every’ and ‘each’, then that subject is considered singular and therefore the verb must end with an ‘s’.

Following is an example:

- **Each woman and man leaves the room.**

7. Infinite pronouns (like everyone, everything, neither, somebody, anything, either, anybody, anyone, each, etc.) are usually

considered singular and thus the verb ends with an ‘s’.

Following is an example:

- **Everyone runs to the park.**
- **Somebody leaves the room.**

8. In plural subjects joined by ‘or’, ‘but’ or ‘nor’, the verb has to agree with the subject closest to it.

Following is an example:

- **Either Ram or Shyam leaves the room.**
- **Neither Ram nor his brothers run to the park.**

9. There are some indefinite pronouns (like some, most, none, part, etc.) in which the verb may be singular or plural depending on the object.

Following is an example:

- **All of the carpenters are gone.**
- **All of the cake is gone.**

10. A collective noun is considered a singular subject: therefore, the verb will end with an ‘s’.

Following is an example:

- **The committee is considering his expulsion.**
- **The class is unusually quiet today.**

11. If a subject is a measurement of time, money, distance, weight, etc., then it is considered singular and therefore the verb will end in an ‘s’.

Following is an example:

- **Ten thousand euros is too much to spend in just a day.**
- **Seventeen kilos of wheat is enough for us.**

12. It is important for the verb to agree only with the subject and not with any other part of the sentence.

Following is an example:

- **The problem we face in this meeting is all of the participants have brought**



their children with themselves and we don't have enough space.

What is the subject in this sentence? Is it the problem, this meeting or the participants? Identification of the subject is important. In this sentence, ‘the problem’ is the subject. It is singular. So, the verb will also be singular.

Following are more examples:

There are many queries.

There is a query.

What is the subject in the given sentences? It is ‘queries’ in the first sentence and ‘query’ in the second sentence.

The news was devastating.

Civics is her favourite subject.

The sentences given above have subjects with nouns that end with an ‘s’. But that does not necessarily indicate its plurality. ‘The news’ and ‘civics’ indicate a singular subject: therefore, the verb would be plural.

Study the following table and compare the correct and incorrect sentences.

INCORRECT	CORRECT
Half of the task are finished.	Half of the task is finished.
All of the data are uploaded.	All of the data is uploaded.
Much of the cake are eaten.	Much of the cake is eaten.
Ten euros are a lot of money here.	Ten euros is a lot of money here.
The number of people in this room do not matter.	The number of people in this room does not matter.
One of the mobile phones are stolen.	One of the mobile phones is stolen.
The lady, along with her friends, exit the room hastily.	The lady, along with her friends, exits the room hastily.
Jill and her friends walks to the park.	Jill and her friends walk to the park.
Neither Jill nor her friends stays at home.	Neither Jill nor her friends stay at home.
Everyone walk to the store.	Everyone walks to the store.
Most of the students is gone.	Most of the students are gone.
Some of the people stays inside.	Some of the people stay inside.
The group dance with perfection.	The group dances with perfection.
Physics are the most difficult subject.	Physics is the most difficult subject.



INCORRECT	CORRECT
Seventeen metres are a lot of distance.	Seventeen metres is a lot of distance.
The aim of the meeting, among other things, are to achieve a proper and equitable distribution of resources among the members of the committee.	The aim of the meeting, among other things, is to achieve a proper and equitable distribution of resources among the members of the committee.
Reading with my friends are my favourite pastime activity.	Reading with my friends is my favourite pastime activity.

Hotkeys

While reading the text out loud may help, it is not always possible in examination environments. In such a case, try subvocalising (uttering

words to yourself, in your mouth, with little or no audible noise) the text! Subvocalising can help you detect mistakes!

PRACTICE QUESTIONS

Select the collect option and fill in the blanks.

1. Some of the bundles _____ misplaced. (is/are)
2. The main criterion I will judge this meeting on ____ the reactions and responses it will attract from the mainstream media houses. (is/are)
3. I told him yesterday that seventy pounds of chocolate _____ more than enough for this batch of cakes. (was/were)
4. Most of the problems _____ solved now that Jill has arrived. (are/is)
5. His family _____ eager to let me in, even though I did not make quite the impression the last time. (was/were)
6. I don't want to see _____ trousers lying around anymore. Make sure you fold them and keep them neatly in your backpack. (these/this)
7. This phenomenon rarely _____ nowadays, climate change has had adverse effects on this. (occur/occurs)
8. Ananda, along with her friends, _____ made a nerve-wracking short horror film. (has/have)
9. I saw a bizarre-looking peahen walking towards me in the park the other morning. Its steps _____ like that of a crooked man. (was/were)
10. If the supervisor _____ me doing this, she will most certainly dismiss me from my job. (notice/notices)
11. Those dogs _____ at all the strangers. (bark/barks)
12. There _____ no news available regarding this incident. (is/are)
13. The issue with the assignments _____ that I can't understand their meanings. (is/are)
14. What _____ she trying to tell me? (is/are)
15. If the team _____ the authorities, we will have no other option but to withdraw our support. (sue/sues)
16. They will _____ me soon whether I am accepted or not. (inform/informs)
17. She is not in the mood to _____ your complaints. (entertain/entertains)
18. Radha shall _____ this presentation along with her dear friends. (deliver/delivers)
19. A few of the people _____ out of the race already. (is/are)
20. Half of the cake _____ eaten by the students. (was/were)
21. _____ I even allowed to enter the studio? (Was/Were)



22. A million dollars _____ a huge amount of money to spend in a year. (is/are)
23. I can see that he _____ at the end of this. (fail/fails)
24. The viceroy _____ the room in a hurry. (leave/leaves)
25. The committee _____ amongst themselves regarding this issue. (argues/argue)
26. Raymond, along with his friends, _____ the dark tunnel. (enter/enters)
27. Jeanie and her friends _____ leaving for Mexico tomorrow. (is/are)
28. How will they _____ the project while the company struggles with its finances? (complete/completes)
29. _____ you invited to the party? (Is/Are)
30. With whom _____ you planning to conduct research on this? (Is/Are)

SOLUTIONS

1. **Are**—The subject, some is plural.
2. **Is**—The subject, criterion is singular.
3. **Was**—The subject, seventy pounds of chocolate, is taken as a single unit of measurement, and therefore, it is considered singular.
4. **Are**—The subject, most of the problems is plural.
5. **Was**—The subject, family, is singular.
6. **These**—The subject, trousers, even though referring to a singular subject, is considered plural.
7. **Occurs**—The subject, this phenomenon is singular. Therefore, the verb will have an ‘s’ attached to it.
8. **Has**—Ananda is the subject of this sentence, not she along with her friends. Therefore, the subject is singular.
9. **Were**—The subject, its steps, is plural.
10. **Notices**—The subject, the supervisor, is singular. Therefore, the verb will have an ‘s’ attached to it.
11. **Bark**—The subject, dog, is singular.
12. **Is**—News is considered a singular subject.
13. **Is**—The subject, the issue, is singular.
14. **Is**—The subject, she, is singular.
15. **Sues**—The subject, the team, is considered a singular subject.
16. **Inform**—The subject, they, will have a singular verb according to the rules.
17. **Entertain**—The subject, she, will have a singular verb when attached with ‘to’.
18. **Deliver**—The future tense will have a singular verb even if the subject, Radha, is singular.
19. **Are**—The subject, a few of the people, are plural.
20. **Was**—Half of the cake is considered a singular subject.
21. **Was**—The subject, I, will have a singular verb.
22. **Is**—The subject, a million dollars, is a unit of measurement. Therefore, it is a singular subject.
23. **Fails**—The subject, he, is singular.
24. **Leaves**—The subject, the viceroy, is singular.
25. **Argues**—The subject, the committee, is not acting as a singular unit. It is arguing among themselves as individuals. Therefore, the subject is plural.
26. **Enters**—The subject, Raymond, is singular.
27. **Are**—The subject, Jeanie and her friends, are plural.
28. **Complete**—The subject, they, is plural.
29. **Are**—The subject, you, even though singular, will have a plural verb because it is second person.
30. **Are**—Similar explanations as above

9

Tenses



Jack went to the market.

Jack goes to the market.

Jack will go to the market.

The given sentences indicate ‘when’ Jack went to market through different forms of verbs: past, present, and future. As you can see the whole context and time frame of the sentence alters with the different forms of the verb ‘go’ used here. It is crucial to have a good hold on tenses, not just for writing purposes, but to also not be misinterpreted when explaining any event.

The first sentence indicates that Jack ‘went’ to the market sometime in the past. The second sentence indicates that Jack ‘goes’ to the market sometime in the present or he may go to the market regularly. The third sentence indicates that Jack ‘will go’ to the market sometime in the future. The three sentences essentially refer to the same act—that of going to the market—but they indicate different times of going to the market. For this, the sentences use different forms of verbs—first form, second form, and third form.

Can you now infer what tenses are?

Tenses indicate the time of action in a sentence. They are roughly of three forms—past tense, present tense, and future tense. They can be further divided into: Simple tense, perfect tense, continuous tense, perfect continuous tense.

But before that, read the following sentences and try to identify the differences in them.

Jack goes to the market.

Jack is going to the market.

Jack has gone to the market.

Jack has been going to the market.

Do the given sentences refer the action to sometime in the present? If yes, what are the differences between these three sentences? Read them out loud for better understanding. Before answering these questions, study the following sentences:

Jack went to the market.

Jack was going to the market.

Jack had gone to the market.

Jack had been going to the market.

Do the given sentences refer to the action to sometime in the past? If yes, what are the differences between these three sentences? Now study the following sentences by reading them out loud:

Jack will go to the market.

Jack will be going to the market.

Jack will have gone to the market.

Jack will have been going to the market.

Do the given sentences refer the action to sometime in the future? If yes, what are the differences between these three sentences? In this chapter, we will understand how these sentences denote different times in the past, present, and future.

Hotkeys

No need to learn the tense rules by heart! Study the sentences and understand the different parts of speech.

Reading the examples can help you understand and learn the rules more than learning the rules in isolation will!

PRESENT TENSE

Simple present

It may denote a universal truth, scientific facts, or tasks/actions done on a daily basis. The actions or acts happening at the moment are expressed in the present tense. Actions happening unceasingly and regularly are called present indefinite.

Rule: subject + verb 1 + s/es + object

Present continuous

It denotes an action happening in the present which will/ or might continue for a short while in the future too. It can be distinguished from



the simple present tense as the action here is temporary and is in progress in the present. However, the simple present tense is to express the actions in the immediate present.



Understand With Example

- Nancy goes to school every day. (Present indefinite)
- The sun rises in the east.
- Dogs hate water.
- I cook thrice a day for my family.
- She writes a letter to the authorities.
- They walk to their office daily.
- Water evaporates when heated.
- My father drops me to the destination.
- The earth revolves around the sun.
- The guests are in the dining hall.

Rule: subject + is/am/are + verb 1 + ing + object

Present perfect

It denotes something that started in the past and is just finished. Any action which took place in the past, at an indefinite time, is also expressed in Present perfect (Example: I presume we have met before).

Hotkey

Any certain time period if provided or the



Understand With Example

- Nancy is going to school.
- I am cooking for my family.
- Dua is writing a letter to the authorities regarding this situation.
- I am learning to read and write in Spanish.
- She and her friends are watching a comedy movie.
- I am residing in Moscow with my colleagues for the winter.

- We are renting an apartment.
- The sun is shining bright.
- They are listening to jazz.
- You are not participating in the tournament.

completion of the work in the past is not to be expressed in the present perfect tense. For example: We have briefed the team about the work at 11 this morning. (incorrect)
We have briefed the team about the work. (Correct)

Rule: subject + has/have + verb 3 + object



Understand With Example

- Nancy has gone to school.
- Dua has written to the authorities regarding this situation.
- She has not seen her mother in a while.
- Have you been to this place before?
- I have received seventy-eight responses till now.
- I have ironed all the clothes.
- We have lived in this city for twenty years.
- Her parents have gone to bed.
- Have you ever broken a bone?
- They have prepared some presentations for our referral.

Present perfect continuous

It signifies an action that started in the past and is still continuing in present.

Rule: subject + has/have + been + verb 1 + ing + object



PAST TENSE

Simple past

It denotes any action that already happened and has been completed in the past. Unlike Past continuous tense, Simple past tense highlights the finality of the action that we are talking about.

Hotkey

Whenever a verb follows did, the verb must be in the base form. Did + past form of a verb is grammatically incorrect. Here 'did' is an auxiliary verb, which if incorporated in a sentence, makes it necessary for the 'main verb' to be in the base form. For instance,
When did you sleep yesterday?
Did you sleep well last night?

Rule: subject + verb 2 + object

Past continuous

It indicates an action that was going on some time in the past. Such action was ongoing in the past, and is no longer continuing in the present.

Rule: subject + was/were + verb 1 + ing + object



Understand With Example

- Dua has been writing to the authorities regarding the current situation.
- You have been lying to me all this time.
- He has been working for seven hours without a break and I am concerned for his health.
- She has been spending a lot of time with her friends these days.
- They have been saving money for their next trip to the Maldives.
- Randy has been working in this firm for eleven years now.
- Has my son been attending the online classes?
- Have I been making myself sufficiently clear?

- James has been texting the wrong person for months.
- The committee has been reprimanding all the rule-breakers harshly.

Past perfect

It usually denotes an action done a long time ago in the past. It would usually denote an action that happened earlier than the other actions mentioned in the sentence.

Rule: subject + had + verb 3 + object



Understand With Example

- I was watching that movie last night.
- She was living in Paraguay with her husband before she moved to the United States.
- He was working at that time.
- You were not playing with your friends yesterday, were you?
- Were you running in the park when I called you?
- They were not washing the laundry.
- Was I cooking food in the kitchen when the doorbell rang?
- As I was reaching out for the book, someone else grabbed it.
- The boss was doing a great job handling the workplace conflict.
- They were not watching anything super scary.

Past perfect continuous

It denotes an action that began in the past and continued till sometime in the past. The point of difference between the Present Perfect Continuous tense and past perfect continuous is that the action here was continuous at some point in the past and is not happening right now.

Rule: subject + had been + verb 1 + ing + object



Understand With Example



- She had arrived late at the meeting.
- I had completed the assignment before the professor asked for it.
- She had lived in Paraguay with her husband before she moved to the United States.
- Jeanine had not called him on time, so he was hostile.
- Had they informed you before barging into your home like that?
- By the time someone came in to help, the patient had already passed away.
- Had I been clear in my wording?
- Until I knocked on the door, no one had come out of the apartment.
- I had never seen such beautiful mountains before I went to Wyoming.
- I had forgotten to turn off the washing machine, so I had to return home.

- Had they been working at all?
- It had been raining all day, so I had to take an umbrella with me.
- Had you been sending them the reports on time?
- This shop had been selling original, handmade souvenirs before the authorities ordered it to stop its business.

It denotes an ongoing action in the future. The action will start sometime in the future and will continue for a certain time period in the future. The action will not be finished until then but will be in motion.

Rule: subject + will/shall + be + verb 1 + ing + object

Future perfect

It denotes an action that will occur in the

FUTURE TENSE

Simple future

It indicates an action that will happen in the future. The helping verb ‘shall’ can be used with ‘we’ and ‘I’ unless denoting a promise or a commitment, in which case, ‘shall’ is used with we/I.

Rule: subject + will/shall + verb 1 + object

Future continuous

Understand With Example



- I had been working for seven hours.
- Jack had been staying in London in a rented apartment.
- She had been helping people find jobs for months.
- The team had been doing a good job until the new boss arrived.
- Had you been cooking when I called?
- When the team arrived, the cleaners had been finishing up their work.

Understand With Example



- I shall write a letter to the authorities regarding this situation.
- Mr. Jacobs will help you with your financial bills.
- I will not repeat this again, so please note down the information right now.
- Shall I send them a notice regarding your arrival?
- They will reside in Moscow, now that Sebastian has acquired a job there.
- We shall pay them once they have delivered all the shipments.
- Their team will win the match today.
- The missionaries will sail to the island.
- I will watch a documentary tomorrow.
- Shall we go to the salon tomorrow?

future and will be finished by a particular point of time in the future. Such an action can take place anytime between now and the particular moment in the future.



Rule: subject + will/shall + have + verb 3 + object



Understand With Example

- They will be walking to their school at seven in the morning tomorrow.
- I shall be calling him home if he does not behave.
- He will be studying during his finals.
- I will be teaching their class next semester.
- She will be out running at that time of the day.
- The professor will be teaching the class.
- I shall be having a talk with her regarding this.
- Will you be waiting for me?
- The train will be leaving the station on time.
- I will not be bothering you anymore.

Future perfect continuous

It indicates an action that will happen in the future and will continue for some time. We cast ourselves sometime in the future, and look back at the ongoing action (which is still a part of the future). There must be a time reference in such sentences.

Rule: subject + will/shall + have been + verb 1 + ing + object



Understand With Example

- They will have cleaned the rooms by the time the guests arrive.
- We will not have prepared for the presentation by seven if Janet does not arrive on time.
- Will the supervisor have written his feedback by the time the tournament is completed?
- The team will have come up with a plan by now.

- Mr. Johnson shall have given them the tender by this time tomorrow.
- By the end of the month, I will have earned a fortune.
- Will we have completed the task by next week?
- My mother will not have cleaned the house before my boss arrives.
- They will have won the match.
- She will have annoyed her friends all day.



Understand With Example

- She will have been working on that project for the rest of the day tomorrow.
- They will have been enjoying the party by tomorrow night.
- Will Mr. Chandran have been writing them a notice?
- The landlady will have been vacuuming the whole apartment tomorrow.
- I will have been studying Political Governance by next semester.
- They will have been sailing through the ocean at this time next month.
- Will they have been laughing at my Christmas performance on New Year's Eve?
- The students will have been working on their projects for three hours.
- The ministers will not have been delivering their speeches by next week.
- The maid will have been cleaning the kitchen for half an hour.

Hotkey

Understanding tenses rules can help you solve questions like identification of incorrect sentences in GATE and other engineering entrance exams. Plus, a proper understanding of tenses can help you understand the correct meanings of the passages.



Following are the rules you can apply to make your sentences better.

RULES

Simple Present

Assertive – SUBJECT + VERB 1 + s/es + OBJECT

Negative – SUBJECT + does/do not + VERB 1 + s/es + OBJECT

Interrogative – Does/Do + SUBJECT + VERB 1 + s/es + OBJECT

Present Continuous

Assertive – SUBJECT + is/am/are + VERB 1 + Ing + OBJECT

Negative – SUBJECT + is/am/are + not + VERB 1 + Ing + OBJECT

Interrogative – Is/Are + SUBJECT + VERB 1 + Ing + OBJECT

Present Perfect

Assertive – SUBJECT + has/have + VERB 3 + OBJECT

Negative – SUBJECT + has/have + not + VERB 3 + OBJECT

Interrogative – Has/Have + SUBJECT + VERB 3 + OBJECT

Present Perfect Continuous

Assertive – SUBJECT + has/have + been + VERB 1 + Ing + OBJECT

Negative – SUBJECT + has/have + not been + VERB 1 + Ing + OBJECT

Interrogative – Has/Have + SUBJECT + been + VERB 1 + Ing + OBJECT

Simple Past

Assertive – SUBJECT + VERB 2 + OBJECT

Negative – SUBJECT + did not + VERB 1 + OBJECT

Interrogative – Did + SUBJECT + VERB 1 + OBJECT

Past Continuous

Assertive – SUBJECT + was/were + VERB 1 + Ing + OBJECT

Negative – SUBJECT + was/were + not + VERB 1 + Ing + OBJECT

Interrogative – Was/were + SUBJECT + VERB 1 + Ing + OBJECT

Past Perfect

Assertive – SUBJECT + had + VERB 3 + OBJECT

Negative – SUBJECT + had not + VERB 3 + OBJECT

Interrogative – Had + SUBJECT + VERB 3 + OBJECT

Past Perfect Continuous

Assertive – SUBJECT + had been + VERB 1 + Ing + OBJECT

Negative – SUBJECT + had not been + VERB 1 + Ing + OBJECT

Interrogative – Had + SUBJECT + been + VERB 1 + Ing + OBJECT

Simple Future

Assertive – SUBJECT + will/shall + VERB 1 + OBJECT

Negative – SUBJECT + will/shall + not + VERB 1 + OBJECT

Interrogative – Will/Shall + SUBJECT + VERB 1 + OBJECT



Simple Future

Assertive – SUBJECT + will/shall + VERB 1 + OBJECT

Negative – SUBJECT + will/shall + not + VERB 1 + OBJECT

Interrogative – Will/shall + SUBJECT + VERB 1 + OBJECT

Future Continuous

Assertive – SUBJECT + will/shall + be + VERB 1 + Ing + OBJECT

Negative – SUBJECT + will/shall + not + be + VERB 1 + Ing + OBJECT

Interrogative – Will/Shall + SUBJECT + be + VERB 1 + Ing + OBJECT

Future Perfect

Assertive – SUBJECT + will/shall + have + VERB 3 + OBJECT

Negative – SUBJECT + will/shall + not + have + VERB 3 + OBJECT

Interrogative – Will/shall + SUBJECT + have + VERB 3 + OBJECT

PRACTICE QUESTIONS

Select the correct tense form and fill in the blanks.

Past Tense

1. We _____ the food we brought from the farmers' market. (Past perfect | cook)
2. He _____ that he was ill. (Simple past | inform)
3. Marionette _____ with her friends before we arrived. (Past perfect | leave)
4. The fireworks _____ the children. (Simple past | terrify)
5. The assistants _____ the rooms very neatly. (Past perfect | clean)
6. I _____ all of my friends at the party. (Simple past | invite)
7. I _____ in the office when my colleagues arrived and asked me to leave the room. (Past perfect continuous | work)
8. _____ you _____ cooking when we knocked at the door? (Past perfect continuous)
9. _____ they _____ you regarding the meeting? (Past perfect | notify)
10. Mr. Jamieson _____ me to preside this committee for a month until her arrival. (Past continuous | tell)

11. I _____ for work when I saw the mail on my porch. (Past continuous | leave)
12. She _____ in love with him when he broke her heart. (Past perfect | be)
13. _____ I _____ to everyone? (Simple past | audible)
14. He told me that Jeremy _____ another book when the publishers reached out to him. (Past perfect continuous | write)
15. I am of the view that she _____ not _____ to be our president. (Simple past | fit)
16. Mrs. Kennedy _____ in the fields yesterday. (Past continuous | work)
17. They _____ her skills when they found out that she had won the competition. (Past perfect continuous | appreciate)
18. Last month, Seema _____ to Delhi for her internship. (Simple past | travel)
19. She _____ the whole night before she presented her research in the class. (Past perfect continuous | study)
20. Meera and I _____ movie last night (simple past | watch)



Present Tense

1. Lilly _____ a letter to the authorities regarding this situation. (Simple present | write)
2. He _____ all through this semester. (Present perfect | study)
3. I _____ to the Elton John concert this weekend. (Present continuous | go)
4. He _____ home for the summer break. (Present continuous | return)
5. I _____ on the notice board every morning regarding the schedules. (Simple present | write)
6. Their family _____ a lot of festivals together recently. (Present perfect continuous | celebrate)
7. _____ I _____ myself sufficiently clear? (Present perfect continuous | make)
8. _____ the team _____ together properly? (Present continuous | work)
9. The problem _____ quite often these days. (Present perfect continuous | arise)
10. I _____ a witness in a criminal case. (Present perfect | serve)
11. They _____ not _____ whether they can accept such a behaviour. (Simple present | sure)
12. Her plane _____ in Moscow as we speak. (Present continuous | land)
13. I _____ on my mental health for the past few weeks. (Present perfect continuous | focus)
14. She _____ the work on time. (Present perfect | submit)
15. Mr. Tumnus _____ in Narnia. (Simple present | live)
16. Peter _____ her every day for the past few months. (Present perfect | call)
17. The kid's socks _____ on the floor. (Present continuous | lie)
18. Leela _____ in Malaysia for 3 years. (Present perfect continuous | live)
19. Lata is _____ the concert. (Present continuous | attend)
20. I want to _____ in the event this evening. (Simple present | sing)

Future Tense

1. They _____ for the station at seven in the morning tomorrow. (Future continuous | leave)
2. She _____ piano lessons from me next week. (Future continuous | take)
3. I _____ to her tomorrow. (Simple future | talk)
4. The team _____ the work by this time tomorrow. (Future perfect | complete)
5. Tokyo _____ Rangers _____ the game tomorrow. (Future perfect continuous | win)
6. _____ they _____ at the train station? (Simple future | be)
7. The landlord _____ for the rent tomorrow morning. (Future continuous | ask)
8. Maria _____ with the presentation by that time. (Future perfect continuous | finish)
9. I _____ the lessons today. (Future continuous | conclude)
10. _____ the Sharma's not _____ their portion of the work? (Future continuous | do)
11. They _____ her sooner than expected. (Future continuous | contact)
12. My plane _____ by this time tomorrow. (Future perfect | land)
13. Mr. Keller _____ his job as the new security guard of this building soon. (Simple future | begin)
14. I am not if they _____ the party or not. (Future continuous | attend)
15. Who _____ in-charge of this company by then? (Future perfect | made)
16. Radhika _____ as the new barmaid from this time tomorrow. (Future perfect continuous | start)
17. He _____ a better result after changing his routine. (Simple future | see)
18. Sarthak _____ his revision by



- tomorrow. (Future Perfect | finish)
19. The family _____ to Kolkata on Tuesday. (Future Continuous | travel)

- 20.** _____ (He) us for dinner tomorrow? (Simple future | join)

SOLUTIONS

Past Tense

1. had cooked
2. informed
3. had left
4. terrified
5. had cleaned
6. invited
7. had been working
8. had been
9. had notified
10. was telling
11. was leaving
12. had been
13. was audible
14. had been writing
15. was fit
16. was working
17. had been appreciating
18. travelled
19. had been studying
20. watched

Present Tense

1. writes
2. has studied
3. am going
4. is returning
5. write
6. has been celebrating
7. have been making
8. has worked
9. has been arising
10. have served

11. are sure
12. is landing
13. have been focusing
14. has submitted
15. lives
16. has called
17. are lying
18. has been living
19. attending
20. sing

Future Tense

1. will be leaving
2. will be taking
3. shall/will talk
4. will have completed
5. will have been winning
6. will be
7. will be asking
8. will have been finishing
9. will be concluding
10. will be doing
11. shall/will be contacting
12. will have landed
13. will/shall begin
14. will be attending
15. will have been made
16. will have been starting
17. will/shall see
18. will have finished
19. will/ shall be travelling
20. will he join



In this chapter, we will discuss key grammar concepts and what they mean. For a better hold of language, we must clarify our understanding of the basics of the grammar keywords of English. We will be touching on the different parts of speech and sketching out the point of difference between each of them in simple terms.

What is the meaning of parts of speech? The parts that make up a sentence are parts of speech. Sentences are made up of different parts. These parts may be:

- nouns
- pronouns
- adjectives
- verbs
- adverbs
- propositions
- conjunctions
- interjections

These parts are known as parts of speech.

PARTS OF SPEECH

Nouns

They are used to refer to names of people, things, or places. Even the names of concepts, which aren't physical, may have names. Therefore, they too are called nouns.

For example:

Jenna bought a Samsung from Seoul last November.

Jeremy does not like Dominos.

The Big Bang is widely considered to be the origin of the universe.

They don't plan to go to the Hawaii this Thanksgiving.

The Beatles performed here once.

Pronouns

These words are used as a replacement for nouns. They need antecedents. Antecedents are nouns previously used in a sentence.

The pronoun changes according to the antecedent.

For example:

Jenna went to the store, where she bought a few books.

We need to help each other to succeed.

The committee kept this within itself.

You need to go to the dentist.

Kenny, Lenny, and Jenny told me that they are not attending the ceremony.

Adjectives

These words add detail to a sentence. They give necessary details about the noun/pronoun.

For example:

A cool wind blew by the lake.

I wanted to buy the blue sandals.

The day was hot and humid.

I like the fuzzy ambience of this restaurant.

The sound was loud, so we left sooner than we wanted to.

Verbs

Verbs describe actions.

For example:

I walked to my office yesterday.

She told me that I was not invited.

Jenna came home yesterday.

I will go to their house.

They will work under my supervision.

Adverbs

An adverb is a word that modifies (describes) a verb, an adjective, another adverb, or even a whole sentence. They describe verbs. Most adverbs end with the suffix 'ly'.

For example:

She stepped towards me slowly.

The work was done badly, so I could not accept it.

The driver was driving rashly.

I could reach your house easily.



He chose out of the pool randomly.

Prepositions

They indicate where or when something is.

For example:

The photograph is above the notice.

You will find the cake on the counter.

He is hidden behind the tree.

I am stuck in traffic.

The club is located in Hollywood.

Conjunctions

These words connect parts of sentences.

For example:

I went to their house but no one was there.

They could not give us the instructions and guidelines.

She found it difficult to focus and complete her work on time.

Jenna was not talking to them so I had to intervene.

No one volunteered but she did.

Interjections

These words show emotions like awe, surprise, disappointment, etc.

For example:

Hey! How are you doing?

Yay! We won the match.

Wow! This place is beautiful.

PRACTICE QUESTIONS

Identify the part of speech underlined in the following sentences.

1. I went home yesterday to find that my mom was back.
2. She murmured sadly that he was not well.
3. The team is ready with its presentation.
4. The sugar box is beside the box of cookies.
5. Alas! There is no food left for us.
6. Everyone is ready to concede to the request.
7. Jake will hopefully recover from his recent injury before his wedding.
8. Their company wishes to be the biggest market entry of this year.
9. The Alps were heavenly to look at.
10. I thought the dish was disgusting in taste.

11. I believe that honesty should be practised everywhere, especially with the people you're close to.
12. She looked angelic in that outfit.
13. I am willing to approach her, however, her rude behaviour stops me from doing so.
14. The company wants to create a new blockbuster game now.
15. Their work was done properly this time.
16. The family is tired of her shenanigans every weekend.
17. Wow! That's beautiful scenery.
18. You will find the portrait on the mantle.
19. The overhead tanks burst due to the impact.
20. I am going to take a break from one this June.

SOLUTIONS

1. Verb
2. Adverb
3. Noun
4. Preposition
5. Interjection
6. Pronoun
7. Adverb
8. Verb
9. Noun
10. Adjective

11. Noun
12. Adjective
13. Conjunction
14. Verb
15. Adverb
16. Pronoun
17. Interjection
18. Preposition
19. Verb
20. Noun



WHAT ARE FIGURES OF SPEECH?

A figure of speech is an expression in which the words are not used literally. A figure of speech is a device that is frequently used to convey a message more clearly or creatively. It is essentially a figurative language made up of a single word or phrase. A figure of speech is a deviation from normal word usage in order to heighten the efficacy of the words. Because it has a rhetorical effect, it is also known as a rhetorical figure. These rhetorical constructions are to be taken non-literally.

The use of a figure of speech can readily capture the attention of the reader and highlight the purpose of the use. It is used to build a comparison and add drama to a piece of writing or speech. It is simply a single word or phrase of figurative language. A simile, metaphor, or personification can be used to communicate a meaning other than the literal one.

It adds to the beauty of the writing. It provides more depth to the text and a sense of amazement to the reader. It breathes life into the writer's words. The figure of speech demonstrates not just the writer's intent, but also his purpose for using such language. The most common types of figures of speech are metaphor, simile, idioms, personification, hyperbole, and euphemism.

TYPES OF FIGURES OF SPEECH

There are many different forms of figures of speech that can be used. The list of figures of speech is extensive, however, examples of some of the most commonly used forms are provided.

1. Personification

Personification is the process of ascribing human qualities or characteristics to inanimate or abstract objects. Non-living things, abstract ideas, or attributes are referred to as humans

or living things in personification. As well as making text interesting (by bringing it to life), personification can be an efficient way to describe inanimate things because readers will find it easy to identify with the human trait and its connotations. By choosing the right human trait, a writer can be descriptive and project their feelings about the object being personified.

For example:

- The door complained as it opened.
- The snowflakes danced at night.
- The Earth was thirsty for water.
- The sun glared down at me from the sky.
- At precisely 4:00 am, my alarm clock sprang to life.
- The river swallowed the earth as the tide continued to rise higher and higher.

2. Metaphor

Metaphors are figurative expressions that are not literally true. But they're neither lies nor mistakes because metaphors aren't meant to be taken literally. They are a sort of figurative language used to express a meaning other than the literal denotative meaning of the words or phrases used. A metaphor is a comparison of two dissimilar things or ideas. It is when two unlike or different items or thoughts are compared. The terms 'like' and 'as' are avoided in this informal or implied simile. They are illustrations that make a strong point by comparing two things you wouldn't necessarily pair together.

For example:

- You are the apple of my eye.
- His words cut deeper than a knife.
- I'm feeling blue.
- She's going through a rollercoaster of emotions.
- This is the icing on the cake.



3. Simile

A simile is a figure of speech that uses the words “like” or “as” to compare two different things in an entertaining way. The goal of a simile is to make an interesting link in the minds of the reader or listener. One of the most popular types of figurative language is the simile. They can help us express ourselves in a more descriptive and pleasant way. They can be funny, serious, mean, or creative.

For example:

- You were as brave as a lion.
- They fought like cats and dogs.
- Last night, I slept like a baby.
- They looked like peas in a pod.
- He eats like a pig.

4. Alliteration

A literary device in which a series of words begin with the same consonant sound is known as alliteration. It is used to emphasise an important point that a writer or speaker wants to make. Alliterative words do not have to begin with the same letter; they just have to have the same first sound. Alliteration is a frequently used stylistic device that can help you remember names and phrases by adding emphasis and interest to a sentence. It is the repetition of the same letter or syllable at the beginning of two or more words.

For example:

- **She sells seashells.**
- **Nick needed new notebooks.**
- **Seven sisters slept soundly on the sand.**
- **Kim's kid kept kicking like crazy.**
- **Walter wondered where Winnie was.**

5. Onomatopoeia

Onomatopoeia is a term that refers to a word that sounds like the thing it describes. It is a figure of speech when the term is used to describe a sound. Onomatopoeia is a term for turning sounds into words to describe an action. Onomatopoeia literally means “to make a name (or sound).” That is, apart from the sound it makes, the term has no meaning.

For example:

- The buzzing bee flew over my head.
- She fell into the water with a splash.
- Water plops into the pond.
- The soda fizzed over the top of my glass.
- The steak is sizzling on the barbecue.

6. Hyperbole

Hyperbole is a figure of speech that uses great exaggeration to emphasise a point. The word hyperbole is derived from a Greek term meaning “excess.” The use of exaggerated terminology to emphasise or heighten the impression of something is known as an exaggeration. Depending on how it’s used, hyperbole in writing and speech can give a message a dramatic or serious tone. It’s a technique for making something appear larger or more important than it really is.

For example:

- She has got a pea-sized brain.
- I have a million things to complete.
- The joke she cracked is so old, the last time I heard it, I was riding a dinosaur.
- I could do this forever.
- That bike went faster than the speed of light.
- I had a ton of homework.
- It has been ages since I have had a day to myself.

7. Euphemism

Euphemism is a polite language that replaces words or phrases that would otherwise be harsh or unpleasant. It’s a gentle or oblique term that frequently replaces a harsh, direct, or insulting term. Euphemisms can be used in a variety of situations. They can assist you in maintaining a courteous tone and avoiding being overly blunt. These expressions are commonly used, and there are several examples of euphemisms in ordinary speech.

For example:

- Using **passed away** instead of died
- Using **letting you** go instead of firing
- Using **well-off** instead of rich



- Using **running a little behind** instead of late
- Using **time of the month** instead of menstruating

8. Irony

Irony occurs when what actually happens turns out to be completely different from what would be expected. In writing or speaking, irony involves using words so the intended meaning is the opposite of the literal meaning. When there is a significant difference between what is stated and what is meant, or between appearance and truth, irony develops. These are frequently used in a light-hearted manner.

For example:

- A traffic cop gets suspended for not paying his parking tickets.
- “How nice!” she said when I told her I had to work all weekend.
- “That’s just perfect”—when the printer jams yet again.
- “Lovely weather today”—when it’s pouring with rain.
- “Oh, great!”—when there’s a huge line at the coffee shop.

9. Anaphora

Anaphora is a rhetorical device that adds rhythm to a paragraph while emphasising meaning. It adds emphasis or emotion by repeating a word or phrase from the beginning of a work in subsequent clauses or phrases. This strategy involves repeating a single word or phrase at the start of each line or paragraph. The recurrence of a word can heighten the piece’s overall meaning. Anaphora is a type of persuasion, a means of reinforcing a specific message, and an artistic aspect used by writers and public speakers. It is a strategy in which the same word or words appear in multiple phrases or verses.

For example:

- “**I** came, **I** saw, **I** conquered.”—Julius Caesar
- “**With** malice toward none; **with** charity for all; **with** firmness in the right.”—Abraham Lincoln

- **We shall** not stop. **We shall** go on and on. **We shall** move forward.
- “**Mad** world! **Mad** kings! **Mad** composition!” —King John II, William Shakespeare.

10. Oxymoron

A figure of speech containing words that appear to contradict each other is known as an oxymoron. An oxymoron is a phrase that uses two opposing terms together. It is defined as a combination of words or phrases that have opposing meanings. As a result, an oxymoron is frequently referred to as a contradiction in terms. This inconsistency conjures up a contradictory image in the mind of the reader or listener, resulting in a new concept or meaning for the whole. Oxymorons should never be taken literally. Instead, the context in which an oxymoron is employed should be used to determine its meaning.

For example:

- Close distance
- Bitter-sweet
- Love-hate
- Random order
- Pretty ugly
- Small crowd

11. Pun

A pun is generally used in plays where one word has two different meanings. It is used to create humour. It is the humorous use of words with various meanings or words with the same sound but different meanings.

For example:

- This vacuum sucks.
- If you stand by the window, I’ll help you out.
- I like archery, but it’s hard to see the point.
- Her cat is near the computer to keep an eye on the mouse.
- Now that I have graph paper, I guess it’s time to plot something.

12. Assonance

The recurrence of vowel sounds in nearby words is known as assonance. It is utilised



to emphasise a word's meaning or to create a mood. Assonance is a literary method in which vowel sounds are repeated in close proximity inside phrases or sentences in the text. It can even happen inside a single word. Assonance can be defined as the repeating of vowel sounds that are identical or very similar. Assonance produces an echoing effect. Basically, when we use repetition of vowel sounds, it is known as assonance.

For example:

- “Hear the mellow wedding bells”—“The Bells” by Edgar Allan Poe
- “The rain in Spain stays mainly on the plain.”—My Fair Lady by Alan Jay Lerner
- “When he was nearly thirteen”—To Kill a Mockingbird by Harper Lee

- “O Romeo, Romeo, wherefore art thou Romeo?”—Romeo and Juliet by William Shakespeare
- “Strips of tinfoil winking like people”—The Bee Meeting by Sylvia Plath

13. Idiom

An idiom is a group of words established by usage as having a meaning not deducible from those of the individual words. It is a common phrase with a meaning that has nothing to do with the literal meaning of its words.

For example:

- Bite off more than you can chew
- Raining cats and dogs
- If you scratch my back, I'll scratch yours
- Spill the beans
- Back to square one

PRACTICE QUESTIONS

Identify the subject of the personification and the human characteristic it was assigned.

1. I could hear Hawaii calling my name.
2. She did not realise that her last chance was walking out the door.
3. Her computer throws a fit every time she tries to use it.
4. The ocean danced in the moonlight.
5. My life came screeching to a halt.
6. This city never sleeps.
7. The party died as soon as Lia left.
8. The sunflowers nodded in the wind.
9. This advertisement speaks to me.
10. This article says that spinach vegetables are good for health.

Identify the figure of speech in the given sentences.

11. At last, they agreed to disagree.
12. A lie has no legs.
13. Words are easy like the wind.
14. Netaji Subash Chandra Bose made a fiery speech.

15. Even the sky shed tears when Gandhiji died.
16. The company has decided to let you go.
17. The lake water is lapping with low sounds.
18. Break, break, break big boulder beside the river.
19. She is between jobs.
20. James is trying to be a good samaritan.
21. The rain in Spain stays mainly on the plain.
22. Ram claims that he can devour mountains of food, and drink rivers of whisky.
23. There is kind cruelty in the surgeon's knife.
24. Necessity is the mother of invention.
25. The murmurous haunt of insects on summer eves.
26. Is life worth living? It depends upon the liver.
27. How high his honour holds his haughty head?
28. What a fine mess of things you have made!
29. My cousin passed away last year.
30. Oh, fantastic! Now I cannot attend the wedding I had been waiting for the past month.



SOLUTIONS

1. Subject of the personification: Hawaii
Human characteristic: calling
2. Subject of the personification: chance
Human characteristic: walking
3. Subject of the personification: computer
Human characteristic: throws a fit
4. Subject of the personification: ocean
Human characteristic: danced
5. Subject of the personification: life
Human characteristic: screeching
6. Subject of the personification: city
Human characteristic: sleeps
7. Subject of the personification: party
Human characteristic: died
8. Subject of the personification: sunflowers
Human characteristic: nodded
9. Subject of the personification:
advertisement
Human characteristic: speaks
10. Subject of the personification: article
Human characteristic: says
11. Oxymoron. In this sentence, both the words agree and disagree are used together. Both of them are opposite words. So, the figure of speech is Oxymoron.
12. Personification. The lie works here in the sentence like a person. It has no legs. It is personified. So, it is Personification.
13. Simile. In this sentence, Words are compared with the wind. And comparison word-like is used here in the sentence. Therefore, the figure of speech is Simile.
14. Metaphor. Here in this sentence, the speech is like fire. It is compared but the words like, so, as, etc are not used. Therefore, it is a Metaphor.
15. Hyperbole. In this sentence, it is an exaggeration to say that the sky weeps at the death of a person. Therefore, the figure of speech is Hyperbole.
16. Euphemism. In this sentence, "let you go" is used as an indirect term that substitutes the word 'fire'. Hence, it is a euphemism.
17. Onomatopoeia. Lapping shows the sound. So, this is Onomatopoeia.
18. Alliteration. In this sentence, a series of words begin with the same consonant sound. Break, break, break big boulder beside the river.
19. Euphemism. In this sentence, "between jobs" is used as an indirect term that substitutes the word unemployed. Hence, it is a euphemism.
20. Idiom. "A good samaritan" is a person who helps someone in need with no thought of a reward.
21. Assonance. The rain in Spain stays mainly on the plain. In this sentence, there is a repetition of vowel sounds in nearby words. Hence, it is assonance.
22. Hyperbole. This sentence is an exaggeration and hence, the figure of speech is Hyperbole.
23. Oxymoron. In this sentence, both the words kind and cruelty are used together. Both of them are opposite words. So, the figure of speech is Oxymoron.
24. Personification. In the given sentence, the word necessity is given the human characteristic of a mother. It is personified. So, it is Personification.
25. Onomatopoeia. Murmurous shows the sound. So, this is onomatopoeia.
26. Pun. In this sentence, the word 'liver' could have various meanings. A pun is the humorous use of words with various meanings. Hence, it is a pun.
27. Alliteration. In this sentence, a series of words begin with the same consonant sound. How high his honour holds his haughty head?



- 28.** Irony. Irony is a dryly humorous or light-hearted mode of speech, in which words are used to convey a meaning contrary to their literal sense, as in the given question.
- 29.** Euphemism. In this sentence, ‘pass away’ is used as an indirect term that substitutes the word ‘die’. Hence, it is a euphemism.
- 30.** Irony. In this sentence, ‘Oh, fantastic!’ is used to mean the opposite of its literal meaning. Hence, it is ironic.



To be successful in error spotting, a candidate must have a good understanding of English vocabulary and grammar. You must identify a mistake in the sentences provided in error spotting. Nouns, pronouns, adjectives, adverbs, or any other grammatical flaw in the sentence can be the source of the error.

The first step in resolving problems about recognising faults is to carefully examine the entire sentence. In most circumstances, you'll be able to spot the mistake on the first try. You must carefully check the subject-verb agreement while reading the complete sentence. The following step is to double-check all spellings. Very often a mistake can be detected in wrong spellings. If you still can't see the error or aren't sure what the correct answer is, read each component of the sentence again and carefully check which element contains an error. You should also go over the list of basic rules so that you can have a deeper understanding of the error spotting.

TYPICAL MISTAKES

- **Nouns**

A noun is a word that functions as the name of a place, person, thing, action or quality. It can be singular or plural, however, the verbs used for singular and plural are different.

Army, clergy, people, peasantry, animals, and other singular nouns represent plurality and take a plural verb in a sentence.

Clothes, scissors, trousers, amends, spectacles, and other nouns take the plural verb because of their plural form.

The noun form (singular/plural form) of a noun expressing weight, number, money, length, or measure that follows a number does not change as long as another noun or pronoun follows it, for example—meters,

quarter, dozen, yard, million, etc.

Few sample statements:

1. The APCs were placed at a hundred yards distance. (Plural form)
2. It took them 50 days to climb up the 8848 meter Mount Everest. (Singular form)

- **Pronouns**

In the instance of possession, a pronoun is a term that refers to the noun in the phrase. When using the pronoun 'one,' make sure to keep it throughout the phrase. 'Whose' is normally used for live persons, while 'which' is used for non-living things.

- **Adjectives**

An adjective is a word that describes a person, place, or thing's specialisation, merits, demerits, quality, or flaw. The relative pronoun 'that' is used after adjectives in the superlative degree instead of 'who' or 'which.' 'As' is used both before and after the adjective to show equality.

- **Adverbs**

An adverb is a word that changes the meaning of a verb or an adjective. Some adverbs have the same meaning, which causes people to become confused. Less and fewer are two words that come to mind. 'Fewer' denotes quantity whereas 'Less' denotes a number, and adjectives such as little, a little, and the little are employed in many contexts.

Other Rules

- Words beginning with the letter ‘H’ such as hour, honourable, heiress, and so on, are regarded as silent. As a result, the article accepts the vowel ‘An’ rather than ‘A’. As a result, proper usage is “an hour,” “an heiress,” “an honour,” and so on.
- Do a double-checking of subject-verb agreement. If the subject is singular, so should the verb be. Similarly, if the subject is plural, the verb must also be plural. Furthermore, when written in the present tense, both the noun and the verb take plural forms in opposing ways. In the singular form of a noun, for example, a “S” is added in the singular form of a verb, the “s” is removed.
- Look for issues with conjunction usage, remember that each phrase can only use one conjunction at a time. It is incorrect to use “As” and “So” in the same sentence, for example.
- The distinction between “Many” and “Much” is that “much” is used before uncountable nouns, whereas “many” is used before countable nouns. Uncountable nouns are substances that can no longer be broken down into smaller components. “Litres of water,” for example (the term “water” here cannot be split down further into smaller units). Countable nouns, on the other hand, are substances that can be fragmented into separate chunks. “Two birds,” for instance (the presence of the word “two” makes the verb countable).
- The distinction between “Whose” and “Which”—“whose” relates to live things, whereas “which” refers to inanimate objects. The phrase “whose mobile is lying there?” is incorrect, whereas the phrase “which mobile is lying there?” is correct.
- The phrase “one of” must always be followed by a plural noun. For example, the phrase “it is one of the loveliest weather” is inaccurate, but “it is one of the loveliest weathers” is correct.
- Nouns that refer to a group of people, such as jury, panel, squad, audience, government, and so on, can be used as collective nouns.
- Other collective nouns can be used singularly or plurally, depending on their meaning. The verb is singular when these terms refer to a single unit otherwise, it is plural.
- The words ‘scarcely’ and ‘hardly’ are followed by the word ‘when’.
- The word ‘unless’ is usually negative. It’s a conditional expression. With ‘unless,’ ‘not’ is never used.
- The format of the different words in the sentence that serve the same function should be the same. For example—All undergraduate students should learn word processing, accounting, and java programming instead of All undergraduate students should learn word processing, accounting, how to work with java programmes.

- Modifiers are used to change the subject and must be placed next to it. The sentence's meaning is altered as a result of their actions.
- The proper words should be used in the proper context. For example—Using the sentence—It will affect you instead of It will affect you.
- Avoid making the same point over and over again. For example—Please repeat the sentence again is incorrect. The correct way is—Please repeat the sentence.
- The question tag is positive, while the statement is negative, and vice-versa. For example, It a little late, is it? (Incorrect). It is a little late, isn't it? (Correct).
- 'Unless' conveys a condition that is always employed in the negative sense, 'not' is never used with it. For example—You will not be released unless you do not pay the bail bond (Incorrect). You will not be released unless you do pay the bail bond (Correct).

INCORRECT SENTENCE	CORRECT SENTENCE
1. I would like the host for the propose the toast.	1. I would like the host to propose the toast.
2. Rishi wants to hear the tone for one more time. Could you please repeat it for her again?	2. Rishi wants to hear the tone for one more time. Could you please repeat it for her?
3. You will not get the ice cream until you do not complete your work.	3. You will not get the ice cream until you complete your work.
4. Rita would love to plays her guitar.	4. Rita would love to play her guitar.
5. It was cruel of you to berating her in front of her parents.	5. It was cruel of you to berate her in front of her parents.
6. Although I am very busy, but I'll take out time for your work.	6. Although I am very busy, I'll take out time for your work.
7. She was having dance practice before she was called by her father.	7. She had dance practice before she was called by her father.
8. No sooner that I had finished my work when my mother called me into the kitchen.	8. No sooner had I finished my work than my mother called me into the kitchen.
9. He worked hardly to gain this position.	9. He worked hard to gain this position.
10. She was there in a market but she never told me about your fracture.	10. She was there in a market but she did not tell me about your fracture.

PRACTICE QUESTIONS

Direction—Some elements of the sentence in the following questions may be incorrect. Choose the correct option, after determining whether part of the phrase has an error. Select ‘No error’, if a sentence is devoid of errors.

1. The charm (A)/ prince is(B)/ wearing a(C)/ black suit. (D)/ No error (E).
2. The act of arson by Peter(A)/ was reported (B)/ to the near (C)/ fire station. (D)/ No error (E).
3. Seven quarts of honey were (A)/required to get the chocolaty consiseny (B)/and it was really difficult to (C)/acquire that consistency at the moment. (D)/ No error (E).
4. Turkey’s government (A)/ signed a deal with Space Origin (B)/ to launch three internet satellites into (C)/ the moon’s orbit, after a previous attempt end in disaster. (D)/ No error (E).
5. This goes beyond a simple client-attorney (A)/ relationship but I think our friend, logician (B)/ and guide need to pour out (C)/ everything that’s disturbing her. (D) / No error (E).
6. Tribal anguish over monetary issues (A)/ leading to the scapegoating of nontribal long-time residents (B)/reflects the continued failure (C)/to forge more comprehensive policies in Assam (E).
7. In spite their best efforts (A)/ they failed to retain (B)/ the agreement due to (C)/ undesirable political interference. (D)/ No error (E).
8. Due to his prolonged illness (A)/ he could not focus (B)/ on his studies even though (C)/ he was very much desired to do so. (D)/ No error (E).
9. The Supreme Court says Australian (A)/ judges has been too lenient in (B)/ punishment for 30 years in giving (C)/ offenders the lowest possible sentence (D)/No error (E).
10. The administration is (A)/ likely to issue a statement (B) /with regards to the investigation (C)/ of the press errors (D)/No error (E).
11. The company, Honda city, (A)/ has been producing twelve billion (B)/ units since it was (C)/ established in 1998 (D)/ No error (E).
12. Under no circumstances (A)/you would be allowed to remain (B)/ out of home after eleven o’clock, (C)/ said Rukmini to her daughter. (D)/ No error (E).
13. He held something (A) / at his side which (B) / was totally hiding (C) / by the folds of his pant. (D) / No error (E).
14. The mother forbade her daughter (A) / to walk in the sun (B) / and play with (C) / her friends in the park. (D) / No error (E).
15. New purchasing power will increasingly (A)/ come from Europe and America where (B)/ the demographics are still favourable (C)/ for high-income growth. (D)/ No error (E).
16. The old woman was (A)/ knocked down by a truck (B)/ upon crossing the street. (C)/ No error (D).
17. The angry wife (A)/ walked towards her (B)/ husband and shouted (C)/ loud at him. (D)/ No error (E).
18. I shall convey your message (A)/ to the president (B)/ when I shall reach his office (C)/, located in Kashmir. (D)/ No error (E).

19. I wonder (A) / what she has done (B) / with the music CD (C) I lend him. (D) / No error (E).
20. It had been a baffling (A) / discovery on

their (B) / anniversary that (C) he has been unsure of her at the time of their wedding. (D) / No error (E).

SOLUTIONS

1. **(A)** The error lies in the first part of the sentence. ‘Charm’ is a noun. ‘Charming’ is the adjective for ‘charm’. So, instead of ‘charm’, ‘charming’ should come as the **word, here, qualifies ‘prince’.**
2. **(C)** The answer is c. Here, the superlative degree of the adjective ‘near’ i.e., nearest should be used.
3. **(A)** Part A of the sentence contains the error as seven quarts of honey is used as a collective noun and hence ‘was’ should be used instead of ‘were’. Therefore, answer option A is correct here.
4. **(D)** The correct option is D. ‘Previous attempt’ implies that it happened in the past. Hence, ‘ended’ should come instead of end.
5. **(C)** The correct option is C. ‘Need’ should be replaced with ‘needs’
6. **(E)** No error in the sentence.
7. **(A)** When we use the phrase in spite, ‘of’ should also be used. Therefore, there is an error in part a.
8. **(D)** To qualify the subject “she,” replace “want” with “desirous.”
9. **(B)** The (B) part of the phrase contains a grammatical mistake. Because the subject of the phrase is “judges,” which is plural, the word “has” must be substituted with “had” to make the sentence grammatically correct. As a result, the answer would be option (B).
10. **(E)** There is no mistake here. Therefore, answer option E is correct.
11. **(B)** The (B) part of the phrase contains a grammatical mistake. The usage of “present perfect continuous” tense is improper here since “present perfect tense” is employed to show completion or to discuss the result. To make the line grammatically correct, “has been producing” must be changed with “has produced.”
12. **(B)** The (B) part of the phrase has a grammatical problem. Inversion is used whenever a statement begins with a negative adverb such as hardly, scarcely, barely, seldom, rarely, not only, under no circumstances, and in no manner. To make the statement grammatically correct, “you would be allowed” must be substituted with “would you be allowed.”
13. **(C)** Use “hidden” instead of “hiding,” as to Be + third form of a verb is used in passive voice.
14. **(E)** No error is there in the sentence.
15. **(E)** No error is there in the sentence.
16. **(C)** The (C) part of the question contains a grammatical error. “Upon” is used to depict a situation wherein we want to refer to something that is being followed by another so as to form a series. “While” is used to show that something happened at the same point of time. The old woman was knocked down by a truck “while” crossing the street.
17. **(D)** Part (D) of the sentence contains a grammatical error. In part D, the word loud has the wrong usage. It should be used as an adverb to describe the way of her shouting. Solution - The angry wife walked towards her husband and shouted loudly at him.
18. **(C)** Part (C) of the sentence contains a grammatical error. The use of shall in both places can be termed as redundant and therefore incorrect. Solution - I shall convey your message to the president when I (no shall) reach his office, located in Kashmir.
19. **(D)** Lent is the past tense of lend, which means to give someone money or goods with the expectation that it be given back. Herein the CD was given in the past and

hence lent should be used instead of lend.
Solution - I wonder what she has done with the music CD that I lent her.

20. (D) Has been is used to depict the fact that an action began at some time in the past and is still in progress. On the other hand, had been is used to depict that something

began in the past, lasted for some time, then ended. Herein the act of being unsure had already ended and therefore “had been” is to be used.

Solution—It had been a baffling discovery on their anniversary that he had been unsure of her at the time of their wedding.

13 Sentence Improvement



Sentence improvement or correction is part of verbal ability questions asked in competitive exams. It involves a sentence that requires some grammatical or vocabulary-related improvement in a phrase or a word or in several phrases or words. The sentence needs to be modified to make it grammatically correct and easy to understand. There are some basic grammar rules which are very important to understand sentence improvement questions. These are as follows:

1. Subject-Verb agreement

The verb in a sentence must always be in accordance with the subject.

Understand With Example



- They both should either be singular or plural.
For example—She is reading a novel (singular), Girls are reading the novels (plural)
- If the subject is a collective noun, the verb takes a singular form. There are exceptions to this rule.
For example—The group is doing a dance and (not) the group are doing dances.
- If the subjects are connected by “AND”: they require a plural verb.
For example—Gold and diamond are precious metals and (not) gold and diamond is precious metal.
- If subjects are connected by “OR”, a singular verb is used.
For example—You can give the parcel to my mother or my brother.
- All sentences that begin with EACH, EVERYONE, and ANYONE will have a

singular verb.

For example—Anyone can do this task.

- Whenever there is a comparison made using THAN or AS, the objective form of the pronoun is used.

For example—I am as intelligent as she is. She is faster than I am.

2. Numbering error

This is the error of writing the same thing twice which does not add to the meaning of the sentence but makes it difficult to understand. Such mistakes indicate a lack of knowledge of verbal ability.

Understand With Example



- He returned back from California.
- He lives in close proximity to my place. The correct way of writing these sentences is:
 - He returned from California. (here “back” is redundant).
 - He lives in proximity to my place. (here “close” is redundant).



Here is a list of common redundant errors to watch out for.

Absolutely certain	Cease and desist	Few in number	Manually by hand	Return back
Actual fact	Chase after	First and foremost	May possibly	Revert back
Added bonus	Collaborate together	First began	Merge together	Rough estimation
Adequate enough	Compete with each other	Follow after	New innovations	Same identical
Advance forward	Completely finished	Foreign imports	One and same	Sequential order
Advance warning	Continue on	Forever and ever	Orbit around	Since the time when
Ask a question	Crisis situation	Free gift	Past experience	Spell out in detail
At the present time	Current status quo	General public	Past history	Still remains
ATM machine	Current trend	Hence why	Past record	Sudden explosion
Bald headed	Definite decision	Hurry up	Personal opinion	Sum total
Basic necessities	Difficult dilemma	In my personal opinion	Plan ahead	Summarise briefly
Biography of her life	Direct confrontation	In spite of the fact that	Plus in addition	The reason is because
Black darkness	Drop down	In the event that	Postpone until later	The reason why
Blatantly obvious	During the course of	Interestingly enough	Progress forward	Therapeutic treatment
Blend together	Each and every	Invited guests	Protest against	Toxic poison



Brief moment	End result	Join together	Raise up	Unexpected surprise
Browse through	Enter in	Little baby	Regular routine	Unintentional mistake
Burning fire	Exactly the same	Longer in length	Repeat again	Usual custom
But yet	False pretence	Major breakthrough	Retreat back	Various different

3. Modifier error

Another common error is to leave a participle without a subject.

For example:

seating on the gate, a scorpion stung him.
Here the usage of “seating” for a scorpion is wrong. The correct formation should be:
While he was sitting on the gate, a scorpion stung him.

Some types of modifier errors are:

a. Misplaced modifier

For example—Nina wore the hat on her head, which she brought yesterday.

Here it seems that she bought the head yesterday (due to the formation of the sentence).

Thus, the correct formation would be: Nina wore the hat which she bought yesterday, on her head.

b. Dangling modifier

For example—Waiting for the bus guano fell on my shoe.

Here it seems that the guano was waiting for the bus!

Thus, the correct formation is—While I was waiting for the bus, guano fell on my shoe.

4. Comparisons

Comparisons should always be made between two similar things.

For example—The literacy rate in Kerala is greater than any other city in India.

The correct comparison here would be the literacy rate in both places. So, the correct formation would be—The literacy rate in Kerala is greater than that of any other city in India.

Some rules are as follows:

- a. When a comparative degree is used, the thing being compared must be excluded from the rest.

For example:

Correct—He is braver than any other man living is correct but the sentence.

Incorrect—He is braver than any man living.

- b. In a superlative degree, the thing being compared is included.

For example :

Correct—Mother Teresa is the kindest of all women.

Incorrect—Mother Teresa is kindest than all

5. Parallelism

Parallelism relates to the structure of the sentence. Different phrases or words performing the same function should be used in the same format.

For example:

other women.



INCORRECT SENTENCE	CORRECT SENTENCE
1. She likes football and running.	1. She likes playing football and running or She likes to play football and to run.
2. Helen likes to paint, draw, and sketching.	2. Helen likes to paint, draw and sketch or, Helen likes painting, drawing, or sketching.
3. The visit to the chapel was beautiful, moved, and inspired.	3. The visit to the chapel was beautiful, moving, and inspiring.
4. She likes listening but to talk.	4. She likes listening but not talking or She likes to listen but not to talk.

6. Error in diction

A diction error refers to an error in the choice of words or phrases. Some idioms

may also occur in a sentence in the wrong manner.

A common pair of words and phrases that are often confused are:

Accept It is a verb that means “to receive something willingly.” For example: I accept this proposal on behalf of my company.	Expect It is a verb that means “to leave out or exclude.” For example: All of you can leave except Alan.
Adapt It means to change something to suit your needs or change yourselves according to something. It means to change something to suit your needs or change yourselves according to something. For example: Meera struggled to adapt to the demanding work culture of the corporate world.	Adopt It means to take over something. For example: She adopted Christianity after moving to Spain.
Affect It is a verb that means “to produce an effect upon”. For example: His possessiveness affected my mental health and sense of self-worth.	Effect It is a noun that means “a change that results when something is done or something happens”. For example: Rising global temperatures had an effect on ice caps in the poles.



Argue against It means to speak in a manner to highlight the negative aspects of something. For example: She argued against the topic in the debate.	Argue with It means to speak in favour of something. For example: she argued with the topic in the debate.
Few It is equivalent to something negligible. For example: Few people understand Sanskrit in modern times.	A few It is equivalent to some. For example: A few people believe in the idea of community ownership.
Few Used for countable nouns. For example: there are few oranges in the basket and (not) there are less oranges in the basket.	Less Used for uncountable nouns. For example: there is less water in the well and (not) there is few waters in the well.
Imply It means to hint at something without directly saying it. For example: He implied that Elena was in trouble, but he would not tell why.	Infer It means to deduce something that has not been stated directly. For example: I inferred that she was shaken from the expression on her face.
Its It is a possessive pronoun that means "belonging to it". For example: Chester is obsessed with the painting because of its mysterious aura.	It's It is a contraction of "it is". For example: I should study for my test as it's just two days away.
Later It is an adverb that means after a particular time or any time after the present. For example: She said she would meet me later.	Latter It is an adjective that means occurring at or near the end of an activity. For example: Kate and Nia will come tomorrow. The latter will not have lunch as she has some errands to run.
Lay To lay means to put in place. For example: She laid out her outfit on the bed. The past tense of lay is "laid".	Lie To lie means to recline. For example: Perhaps I should lie down. The past tense of lie is "lay"
Little It refers to hardly any quantity. For example: there is little hope for recovery in his case.	A little It refers to a small quantity. For example: be a little wise and talk to him.
Lose It is always used as a verb. For example: Dahlia was careful not to lose her ticket.	Loose It is usually used as an adjective. For example: the tape was really loose.



Than

It is used for comparisons. For example: She is faster than shelly.

Then

It is used to indicate time or sequence. For example: Anna finished the test and then took a nap.

That

It is used to restrict the content of the sentence. For example: I really like that dress you wore.

Which

It is used to further specify, identify or distinguish a particular thing. For example: It is the empty space that makes this bowl useful.

Your

It is used to indicate possessiveness. It is not a contraction. For example: are these your glasses?

You're

It is a contraction of the phrase “you are”. For example: I think you're the perfect person for the job.

Hotkeys

Identify the concept applied in the sentence. In general, one or two rules usually apply in such questions.

- Look out for time indicators (before,during, after, etc.)

- The non-underlined part may help you to understand the error. Pay attention to it.
- Try reading the options in your mind once. The correct answer will sound correct when read, others will not.

SOLVED EXAMPLES

Directions: Choose the most appropriate option that would improve the underlined part of the sentence. If there is no improvement, choose option D.

1. The gripping tale had its beginning more than fifty years ago.
- A. Initiated
 - B. Was started
 - C. Began
 - D. No improvement needed

Here the correct answer is option B as it makes a grammatically correct sentence.

2. Sincere people do not rest until they have reached perfection in their work.
- A. They had achieved
 - B. They have achieved
 - C. They had reached
 - D. No improvement needed

Here the correct answer is option B as perfection is something which needs to be “achieved” not “reached”.

3. You are an engineer, aren't you?
- A. Are
 - B. Wasn't
 - C. Isn't
 - D. No improvement needed

Here the correct answer is option D as the word “aren't” is correct here because this is a rhetorical question where this word is usually used.



4. I usually did not take sugar in my tea.
- A. Do not takes
 - B. Do not take
 - C. Will not take
 - D. No improvement needed

Here the correct answer is option B as “do not take” is simple present tense and is appropriate to be used with the subject “I” which is singular.

5. The bike that I bought was difficult to use.
- A. Was inconvenient
 - B. Was convenient
 - C. Was manageable
 - D. No improvement needed

Here the correct answer is option A as “inconvenient” means something that is difficult to use.

PRACTICE QUESTIONS

Directions: Choose the most appropriate option that would improve the underlined part of the sentence. If there is no improvement, choose option D.

1. Unless some systematic changes is made, the future generations will bear the brunt of the inefficiencies of the system.
 - A. Unless some systematic changes are made
 - B. Unless some systematic changes will be made
 - C. Unless some systematic changes have been made
 - D. No improvement needed.
2. The winter season of the parliament will begin in a few days.
 - A. Mission
 - B. Recession
 - C. Session
 - D. No improvement needed
3. The movie which you recommended, was really very good.
 - A. Than
 - B. That
 - C. The one which
 - D. No improvement needed
4. There is generally agreed that inbred children are more prone to diseases than those who are not inbred.
 - A. It
 - B. Which
 - C. Than
 - D. No improvement needed
5. Thieves steal Hondas and Toyotas from the 1990s more than other models because they can chop them up and sell them for parts that are worth more than the car.
 - A. More than other models because they can chop them up and sell them for parts that are worth more than the car
 - B. More than they steal other models because they can chop them up and sell them for parts that are worth more than the car
 - C. More than they do other models because they can dismantle the cars and sell the parts that are worth more than the car
 - D. No improvement needed.
6. NASA reports that ancient observers took note of solar eclipses at least as far back as 2500 BC, as evidenced by surviving records from ancient Babylon and China.
 - A. As evidenced by survived records from
 - B. As is evident by surviving records from
 - C. As the surviving records show the evidence of
 - D. No improvement needed



7. When the hockey team reached the hotel, he dropped off his luggage there.
- She dropped off her luggage.
 - They dropped off there luggage
 - They dropped off their luggage
 - No improvement needed
8. I can't hardly believe what Anna told me.
- Can't believe
 - Can hardly believe
 - Either A or B is correct
 - No improvement needed.
9. The characters in this extremely modern version of Shakespeare's Macbeth shall no longer be believable.
- Are
 - Is
 - Will
 - No improvement required.
10. Each of the compartments contains several litres of water.
- Contained
 - Contain
 - Is Containing
 - No improvement required
11. You can take Harry with you. He enjoys gardening and play in the sun.
- Playing in the sun
 - Played in the sun
 - To play in the sun
 - No improvement needed.
12. For dinner, we like lamb chops and to boil Brussels sprouts.
- Like fried lamb chops
 - To fry lamb chops
 - Fried lamb chops.
 - No improvement needed.
13. Public transit such as buses or a train can help reduce air pollution.
- Or trains
 - Or many trains
 - Or several trains
 - No improvement needed
14. Cherry does nothing but read that book all day. She is very taken by the writing style of it's author
- It is
 - Its
 - The
 - No improvement needed
15. The existence of god is still a contentious question.
- Was still a contentious question.
 - Will still be a contentious question.
 - Shall still be a contentious question.
 - No improvement needed
16. Your salary being dependent on the quality of your work.
- Depends upon
 - Going to depend
 - Is dependable
 - No improvement needed
17. She did not report him as she was fear of losing her job.
- Fearful of
 - Afraid of
 - Had been afraid of
 - No improvement needed
18. After 15 years, Seema is finally stationed in Kolkata, who is her birthplace.
- That is
 - There is her
 - Which is
 - No improvement needed
19. We were nervous as we was the next to present.
- Who
 - Where
 - Were
 - No improvement needed
20. Occupied with a lot of work, he could not make it to his daughter's birthday on time.
- Occupying
 - While occupied
 - Occupation



- D. No improvement needed
- 21.** He is a wise man, which knows when to speak up.
A. Who
B. What
C. When
D. No improvement needed
- 22.** Every of the applications must be approved by the head of HR.
A. Each one of the
B. Each of the
C. Every one of the
D. No improvement needed
- 23.** The firm have asked for the names of all employees working on the project.
A. Is asking
- B. Was asked that
C. Has
D. No improvement needed
- 24.** Considerate the traffic, it is better to leave for the station an hour early.
A. Considerate of
B. Considering
C. While considerate of
D. No improvement needed
- 25.** He was determined for completing the task before the due date.
A. For completion
B. About completed
C. To complete
D. No improvement needed

SOLUTIONS

1. **(A)** is correct as the verb ARE refers to the subject SYSTEMATIC CHANGES. Both are plural. Option B is wrong as it uses the verb of future tense and C is wrong as it uses the verb of past tense. Option D is irrelevant.
2. **(C)** is correct as the word SESSION is the most logically appropriate word here as it refers to the meeting of the parliament. Options A and B are wrong as the words mentioned have different meanings from what is needed in the context.
3. **(B)** is correct as THAT is used to denote THE RECOMMENDED MOVIE. Option A is wrong as “than” is used for comparison and no comparison is being made here. Option C is wrong as “the one which” cannot be used with the words “you recommended”
4. **(A)** is correct as the word IT is appropriate to refer to the subject DISEASES AMONG INBRED CHILDREN (as per subject-verb agreement) Option C is wrong as “than” is used for comparison and option B is wrong as “which” refers to a particular thing and cannot be used at the start of a sentence.
5. **(C)** is correct as the phrase THEY DO refers to the subjects THIEVES doing the other models, the word DISMANTLE is a replacement for the word CHOP. Options A and B lack on the same grounds as they use inappropriate words.
6. **(B)** is correct as the phrase AS IS EVIDENT is in accordance with the verb REPORTS as both are in the present tense. Options A and C are grammatically incorrect as they mention verbs in the past tense and incorrect sequence respectively.
7. **(C)** is correct as the verbs THEY and THERE ARE used for the subject THE HOCKEY TEAM which is plural. Options A and B are grammatically incorrect as the words “she” and “they” are used wrongly.
8. **(C)** is correct as either option A or option B are grammatically correct. Both cannot be



used together. Can't and hardly are double negatives and it is grammatically incorrect to use them together.

9. **(A)** is correct as the verb ARE would come for the subject THE CHARACTERS as per subject-verb agreement. Options B and C are grammatically incorrect as the verb "is" singular is used in place of the plural verb "characters" and "will" is in future tense and cannot be used with "be" respectively.
10. **(B)** is correct as the verb CONTAIN would be appropriate with the subject EACH, as per subject-verb agreement. Options A and C are grammatically incorrect as they used verbs in past and present continuous tense respectively.
11. **(A)** is correct as the phrase PLAYING IN THE SUN is the only option that goes correctly with the word GARDENING. (According to the concept of parallelism). Options B and C are eliminated as they are not parallel with the word GARDENING. (As per parallelism).
12. **(B)** is correct as the phrase TO FRY LAMB CHOPS is the only option that goes correctly with the phrase TO BOIL BRUSSELS SPROUTS. (According to the concept of parallelism). Options A and C are eliminated as they are not parallel with the phrase TO BOIL BRUSSELS SPROUTS (as per parallelism).
13. **(A)** is correct as the word TRAINS is correct according to the concept of parallelism. Since the word BUSES is used thus TRAINS would be correct instead of TRAIN. Options B and C are eliminated as they are not parallel with the word BUSES (as per parallelism)
14. **(B)** is correct as the word IT'S will be replaced by ITS which denotes possessiveness.

After this, the phrase will mean "the writing style belonging to the author" which is both grammatically and factually correct. Options A and C are grammatically incorrect as "it is" is a pronoun and "the" cannot be used to refer to the author of the particular book she is reading respectively.

15. **(D)** is correct as there is no need for any improvement in the sentence. The verb IS is correct for the subject of the question of the existence of God. Options A, B, and C are grammatically incorrect as "will still" is not the proper usage, "shall still" is wrong in the context of the question, and "was still" is not correct in the context of the question.
16. **(A)** is the correct answer as it is in the present tense. Option B is wrong as the word "is" is absent in the option. Option C is wrong as "dependable" is in the past tense. Option D is irrelevant.
17. **(B)** is the correct answer as "afraid" is the action of being fearful. Option A is wrong as "fearful" is the feeling and not action. Option C is wrong as "had been" is in the past tense. Option D is irrelevant.
18. **(C)** is the correct answer as "which" is used to refer to the place Hyderabad. Option A is wrong as "that" is very restrictive in nature. Option B is wrong as "there is" cannot be used at the start of a clause. Option D is irrelevant.
19. **(C)** is the correct answer as "we" is a plural subject hence the verb "were" used is also plural. Option A is wrong as "who" is not a verb. Option B is wrong as "where" is not a verb. Option D is irrelevant.
20. **(D)** is the correct answer as "occupied with" is correct usage option A is wrong as "occupying" is in present continuous tense.



Option B is wrong as “while occupying” cannot be used the word “with”. Option C is wrong as “occupation” refers to a job and not occupation with work.

21. (A) is the correct answer as “who” is used to refer to a person. Option B is wrong as “what” refers to a thing or place and not a person. Option C is wrong as “when” refers to time not a person. Option D is irrelevant.

22. (B) is the correct answer as “applications” are countable so the word used must be “each”. Option A is wrong as the usage of “each one” is redundant. Option C is wrong as “every one: is redundant. Option D is irrelevant.

23. (C) is the correct answer as the “firm” is plural so the verb used will be singular. Option A is wrong as it is using the verb in the present continuous tense. Option B is wrong as the verb is in the past tense. Option D is irrelevant.

24. (B) is the correct answer as the “considering” in present continuous tense. Option A is wrong as it is in the past tense. Option C is wrong as it is past tense. Option D is irrelevant.

25. (C) is the correct answer as “to complete” is in present tense. Option A is wrong as it is in the past tense. Option B is wrong as it is past tense. Option D is irrelevant.



The process of reorganising a jumbled group of lines is known as sentence rearrangement. You must first put the sentences in the correct order before answering questions about them. Candidates must carefully study the statements to solve such questions, as the motive, topic matter, or part from which the jumbled phrases may be picked varies.

The following are examples of questions that could be asked:

- The correct order of all the statements
- Identifying the passage's first statement
- To find the passage's final statement, and so on.

TYPES OF SENTENCE REARRANGEMENT QUESTIONS

Para jumble questions can be asked in four different ways

The Simple Format: In this format, candidates are given all of the statements in a jumbled state and must arrange them in a logical order.

Static First Statement: In the second type of para jumble format, the first statement is given as a constant, and you must logically organise the other statements so that the initial statement remains constant.

The last statement can also be static, and candidates must arrange the other sentences to make the paragraph logical while keeping the last sentence constant.

The first and last statements can be fixed, and the remaining sentences must be arranged logically between them.

Typically, a set of sentence rearrangement problems contains 6 or 7 sentences. It's a set of five questions based on the same 6-7 sentences arranged in different ways. After rearrangement, the questions are for the correct set of first, second, third, fourth, fifth, and sixth sentences. The set of sentences is the same for all five questions.

There are three parts to every paragraph (after combining the sentences of the rearrangement):

- a. The beginning
- b. The main part
- c. The conclusion

How to solve sentence rearrangement questions correctly

You can make a lot of mistakes when answering questions about sentence rearranging. If you approach the problems appropriately, you will be able to answer the para jumbles without making any mistakes.

Hotkeys

Here are some tips on how to deal with sentence rearrangement issues:

- Before starting to rearrange the sentences, carefully read them all and evaluate the common point between them.
- Refer to a sequence discussed in the statement.
- Try pertaining the words given, such as yet, but, when, then, they, anyway, and so on. It will give a good idea of how the sentences can be reorganized.
- Candidates with a larger vocabulary are more likely to answer the question quickly since knowing the meanings of each word in each sentence helps in the arrangement of the statements.
- It's easier to comprehend paragraphs with a single static statement since you can see how the theme begins to evolve or ends.
- Reread the entire section to confirm that it is logically valid once you believe you have correctly organised the statements.



In a rearranged sentence, how do you find the first and last sentences?

- a) Find and pick sentences that use a noun as the subject. Such sentences may serve as the first sentences for our reorganisation.
- b) Remove sentences using a pronoun as the subject. Such sentences cannot be the initial sentence because a pronoun is always used after a noun.
- c) Get rid of sentences that start with a conjunction. When conjunction appears at the beginning of a sentence, it shows that there is clearly a sentence preceding it. As a result, sentences that start with a conjunction cannot also be the initial sentence.

The conclusion is usually the last sentence of a paragraph, and we frequently use terms like finally, lastly, Thus, Thus ultimately, hence, and so on when writing conclusions.

As a result, any sentence beginning with these words can serve as our last sentence.

Steps to find the middle sentences are as follows:

The key to identifying the middle sentences with the greatest accuracy is to choose our answers from the options.

Simply put, once the first and last sentences have been determined, we may look at the question choices to see which one will yield the correct answer for the remaining sentences.

There will be one question in the questions where the first and last sentences that we would have found by then will be in the options for finding the second/ third/ fourth/fifth sentence.

With this, you have two options for the first and last sentences. Because they have already been assigned to the first and last sentence, the options for the second, third, fourth, or fifth sentence will be eliminated in questions where we must find the second, third, fourth, or fifth sentence, because they cannot be the correct option for the remaining sentences.

After removing the first and last sentences from the five options, our fourth sentence (which would not be in the options) will be eliminated as well.

Now we can find the best options by matching the requirement at the beginning and end of the remaining sentences.

More options are eliminated as more questions are answered, resulting in maximum accuracy.

See example

Type 1:

Rearrange the sentences below and answer the questions.

- A. So, you spend the majority of your time on pitches.
 - B. Nowadays, most teams prefer to play on wickets.
 - C. If you want output in three days, it's not easy for batsmen.
 - D. You keep running into pitches that are difficult.
 - E. Our cricket is competitive.
 - F. When you're aiming for 20 wickets in three days, the game can sometimes be completed in two days.
1. After the rearrangement, which of the following will be the first sentence?
- A. A
 - B. C
 - C. B
 - D. D
 - E. E



2. After the rearrangement, which of the following will be the second sentence?
A. A B. C
C. B D. D
E. E
3. After the rearrangement, which of the following will be the third sentence?
A. A B. C
C. B D. D
E. E
4. After the rearrangement, which of the following will be the fourth sentence?
A. A B. C
C. B D. D
E. E
5. After the rearrangement, which of the following will be the fifth sentence?
A. A B. C
C. B D. D
E. F

EDBCFA is the correct rearrangement.

- E. Our cricket is competitive.
- D. You keep running into pitches that are difficult.
- B. Nowadays, most teams prefer to play on wickets.
- C. If you want output in three days, it's not easy for batsmen.
- F. When you're aiming for 20 wickets in three days, the game can sometimes be completed in two days.
- A. So, you spend the majority of your time on pitches.

Type 2:

Then there is a new set of rearrangement questions where one of the sentences from the 6/7 sentences is fixed to a place in the arrangement itself. For example, the D option could be in bold letters and fixed.

This means that the remaining options are all unarranged, whereas option d is fixed and

will remain in the same position even after rearrangement.

We can try this type of rearrangement in the same way that we learned in the trick mentioned above.

This fixed position will benefit us here because it will:

- a) assist us in judging the sequence of the sentences
- b) reduce our options for rearrangement

Type 3:

Rearrange the sentences below to form a meaningful paragraph. Sentence C, which is highlighted in bold letters, is correct and remains in place. Rearrange the remaining sentences and answer the following questions.

- A. One winter my father returned from Bangalore with a bag full of beans.
 - B. My mother said that the optimal time to harvest soybeans is after a period of rainfall when the bean is at its optimum size and flavour.
 - C. Excited, my parents, sister and I sat together to shell the beans.
 - D. The next few days were all about the soya bean, a recipe that she learnt from her mother.
 - E. The peculiar aroma of beans in a cotton bag is something I could never forget.
 - F. I kept looking at my fingers changing colour from the oily texture of the bean pod.
1. After the rearrangement, which of the following will be the fourth sentence?
A. A B. C
C. B D. D
E. F
 2. After the rearrangement, which of the following will be the third sentence?
A. A B. C
C. B D. D
E. E
 3. After the rearrangement, which of the following will be the fifth sentence?



- A. A B. C
C. B D. D
E. E

4. After the rearrangement, which of the following will be the first sentence?
A. A B. C
C. B D. D
E. E
5. After the rearrangement, which of the following will be the sixth sentence?
A. A B. C
C. B D. D
E. E

AECFB is the correct rearrangement.

- A. One winter my father returned from Bangalore with a bag full of beans.
E. The peculiar aroma of beans in a cotton bag is something I could never forget.
C. Excited, my parents, sister and I sat together to shell the beans.
F. I kept looking at my fingers changing colour from the oily texture of the bean pod.
B. My mother said that the optimal time to harvest soybeans is after a period of rainfall when the bean is at its optimum size and flavour.
D. The next few days were all about the soya bean, a recipe that she learnt from her mother.

Type 4:

This type of rearrangement is not a sentence rearrangement, but rather a rearrangement of unarranged phrases within a single sentence. The trick to determining the correct arrangement here is as follows:

- a. Look for the part of the sentence that contains the sentence's subject. The subject of a sentence is usually found at the beginning of the sentence.
- b. Now that we've found the first, we can examine the last words of it, as well as the first words of the other parts, to see which

part follows it and conveys the information needed in the first.

Seeing the end words of a phrase and connecting them to the first words of the following phrases can help significantly in selecting the proper sequence for constructing a coherent sentence.

Question: Rearrange the sentence's parts to create a grammatically correct and coherent sentence.

Time to time, come forward (a)/ India and China have been at loggerheads on (b)/ to assuage the exasperating element of hostility (c)/ political and diplomatic fronts since the inception of the (d)/ latter and both countries have from (e).

BDEAC is the correct rearrangement

Answer: India and China have been at loggerheads on political and diplomatic fronts since the inception of the latter and both countries have from time to time, come forward to assuage the exasperating element of hostility.

Type 5:

The set of sentences for each of the five-sentence rearrangement questions will be different from the set of sentences for the other questions.

We'll also be asked to select the correct choice, which will display the correct rearrangement pattern. One sentence will be set in place (the first of the five sentences), and we must choose the option that shows the correct rearranging pattern.

Because the first sentence has already been fixed, the trick with these types of questions is to just match the rearrangement from the possibilities already offered.

As we try to read the sentence for a meaningful paragraph, the possibilities will be removed, and we will be able to locate the correct answer here with maximum precision.

Question

- A. The virus has infected more than 60,000 people in China and has been spread to more than 36 countries.



- B. It has precipitated a rush to sanitise public spaces and limit human interaction.
 - C. China is cauterizing and cloistering used banknotes as part of ventures to end the spread of the deadly coronavirus that has killed over 2,000 people.
 - D. China's central bank stated at a press conference.
 - E. Banks sterilize notes with an ultraviolet light or high temperatures, then clinch and hold the currency for seven to 14 days before sending out again, depending on the severity of the outbreak in a given region.
- Q. Which of the following options shows the proper rearrangement to form a coherent paragraph?
- a) CEDAB
 - b) CEABD
 - c) CDAEB
 - d) CDEAB
 - E) CDBEA

Option A—CEDAB is the correct answer.

- C. China is cauterizing and cloistering used banknotes as part of ventures to end the spread of the deadly coronavirus that has killed over 2,000 people.
- E. Banks sterilize notes with an ultraviolet light or high temperatures, then clinch and hold the currency for seven to 14 days before sending out again, depending on the severity of the outbreak in a given region.
- D. China's central bank stated at a press conference.

- A. The virus has infected more than 60,000 people in China and has been spread to more than 36 countries.

- B. It has precipitated a rush to sanitise public spaces and limit human interaction.

The above-mentioned trick for finding the last sentence and the first sentence (if some other part of the rearrangement is fixed) may be helpful.

Type 6:

This type of sentence rearrangement combines the previously mentioned Type 3 and Type 1 rearrangements.

There will be a group of four sentences that are jumbled or unorganised. In addition, each sentence will be disorganised too.

In addition, each sentence will be disorganised too. First, we'll need to put the parts of each sentence together to make a coherent statement, and then we'll need to make a pair of three sentences that each carry the information from the same segment of the four sentences.

We'll need to rearrange the three sentences (with the same theme) to produce a comprehensible paragraph after rearranging the parts of each sentence and finding the pair of three sentences from the same paragraph. The remaining sentence will be omitted from constructing a paragraph since it does not reflect the theme of the other three sentences.

PRACTICE QUESTIONS

Question: Three sentences: A, B, and C are each divided into four parts: 1, 2, 3, and 4. Answer the questions that follow by rearranging the broken parts of each sentence.

(A) Maybe carrying this toxic plume (1)/ waste mingle with the clean water, each monsoon (2)/ but the pipes coming into some homes run right (3)/ through drains, and on rainy days, filth and (4).

(B) The president was emphatic in exhorting the 89 million (1)/ small, medium-sized firms, helping guarantee jobs for the youth (2)/ that would generate a movement for mass-financing (3)/ strong CPC machines to adopt concrete measures (4).

(C) With toxic chemicals, even persistent organic pollutants and heavy metals (1)/ ten studies by government and nongovernmental (2)/ around the abandoned factory is



contaminated (3)/ agencies show that the soil and groundwater in and (4).

1. Which of the following is the correct sequence of sentence rearrangement (A)?
A. 1234
B. 3421
C. 3214
D. 4312
E. None of the above

2. Which of the following is the correct sequence of sentence rearrangement (B)?
A. 4213

3. Which of the following is the correct sequence of sentence rearrangement (C)?
B. 4231
C. 3214
D. 1432
E. None of the above

SOLUTIONS

1. Option (B)—3421 is the correct answer.
2. Option (D)—1432 is the correct answer.

3. Option (A)—2431 is the correct answer.



An analogy is something that shows how two things are similar, but the ultimate goal is to make a point about this comparison.

An analogy's purpose is not only to demonstrate but also to explain. As a result, an analogy is more complex than a simile or metaphor. (An analogy can be made using similes or metaphors, but analogies usually include additional information to make their point.)

Examples of analogies

Some analogies are so useful that they are used in everyday conversation. These are frequently referred to as figures of speech. Each of the following analogies compares two things:

- Leaf: tree: petal: flower

This analogy emphasises the connection between the whole object (a tree and a flower) and its parts (a leaf and a petal). One part of the analogy is left blank, and students must select an answer that makes sense to complete the comparison.

- Hammer : Nail :: Comb:Hair
- White : Black :: Up : Down
- Mansion : Shack :: Yacht : Dinghy
- Short : Light :: Long : Heavy

- Bees : Hive :: Bears : Den
- Speak : Sing :: Walk : Dance
- Chef : Food :: Sculptor : Stone
- Like : Love :: Dislike : Hate
- Clay : Brick :: Glass : Bottle
- Spin : Dizzy :: Jump : Elevate
- Itch : Scratch :: Virus : Cold
- Read : Learn :: Try : Improve
- Funny : Humorous :: Hardworking : Diligent
- Lead : Guide :: Drawing : Illustration
- Night : Day :: Right : Left
- Wet : Dry : Hot : Cold
- Electron : Molecule :: Country : Continent
- Toe : Foot :: Finger : Hand
- Stars : Galaxy :: Molecules : Object
- Broom : Sweep :: Paintbrush : Paint
- Freezer : Freeze :: Towel : Wipe
- Democracy : Equality :: Monarchy : Potentate
- Water : Wet :: Concrete : Hard
- Mountain : Tall :: Desert : Dry
- Sugar : Sweet :: Cheetah : Fast
- Wall : Bricks :: Sonnet : Lines
- Cello : Symphony :: Creek : Forest
- Plane : Hangar :: Surfboard : Beach
- Field : Farm :: Audience : Arena
- Conflict : Story :: Emoji : Texting
- Orange : Fruit :: Hydrogen : Element
- Kangaroo : Marsupial : Dog :: Mammal

PRACTICE QUESTIONS

In each of the following questions, find out the alternative which will replace the question mark.

1. Cup : Lip :: Broom : ?
 - A. Break
 - B. Grass
 - C. Forest
 - D. Hand
2. Flow : River :: Waves : ?
 - A. Rain
 - B. Stream
 - C. Sea
 - D. Canal
3. Kitten : Cat :: Cub : ?
 - A. Lamb
 - B. Elephant
 - C. Horse
 - D. Lion
4. Ornithologist : Bird :: Cardiologist : ?
 - A. Islands
 - B. Mediators
 - C. Heart
 - D. Aquatic



5. Peacock : India :: Kangaroo : ?
A. Australia
B. America
C. Russia
D. England
6. Conference : Chairman :: Journalism: ?
A. Reporter
B. Distributor
C. Printer
D. Editor
7. Safe : Secure :: Bolt : ?
A. Lock
B. Sure
C. Guard
D. Conserve
8. Microphone : Loud :: Microscope : ?
A. Elongate
B. Investigate
C. Magnify
D. Examine
9. Dawn : Daybreak :: Brook : ?
A. Broom
B. Condense
C. River
D. Force
10. Meter :: Kilometer :: Cent : ?
A. Euro
B. Penny
C. Pound
D. Yen

SOLUTIONS

1. **(D)** Cup is used to drink something with the help of lips. Similarly, broom is used by hand to clean.
2. **(C)** As water of a river flows similarly waves are in the sea.
3. **(D)** Offspring of the following animals: Cat—kitten; Sheep—Lamb; Elephant—Baby elephant/calf; Horse- foal; Lion—Cub.
4. **(C)** As an ornithologist is a specialist of birds, cardiologist is a specialist of heart.
5. **(A)** As peacock is the national bird of India, similarly, kangaroo is the national animal of Australia.
6. **(D)** As Chairman is the highest authority in a conference. Similarly, the highest authority in journalism is an editor.
7. **(A)** Safe and secure have the same meaning. Bolt and lock have the same meaning.
8. **(C)** As Microphone makes the sound louder, microscope makes the object magnified.
9. **(C)** Dawn and daybreak are synonyms. Similarly, brook is synonymous with river.
10. **(A)** Meter is a smaller unit of kilometer. Similarly, cent is a smaller unit of euro.



Contrary to popular belief, critical reasoning is one of the most uncomplicated subjects in the GATE syllabus. It does not require maximum effort of the part of a candidate as compared to subjects like mathematics, etc. One can very easily solve critical reasoning questions with the help of common sense, lack of bias, and a little bit of practice.

The following steps are required to answer Critical Reasoning questions:

- Accuracy, precision, and discernment in language comprehension and use.
- Recognising implicit assumptions and values and using relevant information from the facts of the case.
- Interpreting data, assessing evidence, and weighing arguments.
- Determining whether logical relationships exist (or do not exist) between propositions.
- Making inferences and judgements based on your findings.

The process of judging a given statement and assuming the most feasible sentence among a set of assumptions is referred to as ‘statement and assumptions’. In most cases, two assumptions are given. Depending on the provided statement, one or more assumptions may become reasonable. Here, an assumption can be described as a thought or an idea that is assumed to be true to arrive at a stated conclusion, but does not have any supporting evidence in the premises. To put it another way, it’s an inferred premise from which the conclusion is derived. When we examine assumptions, we discover that when someone says or writes something, he doesn’t quite put everything into words and leaves some bits unsaid or unwritten. Why does he do this? He does so because he assumes that the unsaid part will be understood without saying anything and there is no need to put it into words. Hence,

close reading and rational evaluation of a given sentence are necessary.

Tips and Tricks

- Don’t look for absolute or flawless solutions. There will often be exceptions to the correct answer. Accept it as it is. Mark a certain choice as the correct answer if you believe it would be agreeable to a majority of individuals with reasonable standards.
- Consider things from the perspective of the examiner. Try guessing the correct answer without even reading the question. This will be possible after attempting a good number of practice questions. So keep practising!
- Common sense varies from person to person. So, avoid marking your response solely on the basis of your own common sense as it might differ from the examiner’s point of view.
- While marking the correct answer, keep your biases hidden.
- Do not make the assertions too logical. Analyse the facts provided, and make only assumptions based on the data provided given in the statement. Don’t make things too complex for yourself.
- An assumption does not follow if it directly opposes the given statement.
- If you can’t figure out the answer, use the elimination approach. When you read the statement in the question and then the given assumptions in the alternatives, you’ll find that some of them are definitely not going to occur. Eliminating them and then selecting from a smaller pool of possibilities may be more efficient for a candidate.



Let's look at the following *example* and see how to approach such questions in the right manner:

Statement: A vital role is played by the philosophers in shaping society's intellect.

Assumption (i): Philosophers are essential for people's intellectual growth.

(Incorrect—rephrasing a statement doesn't quite make it an assumption)

Assumption (ii): Society requires intellectual growth.

(Incorrect—An irrelevant remark that contradicts the conclusion is not an assumption.)

Assumption (iii): Society's intellectual progress may not occur at all.

(Incorrect—An argument that contradicts the statement cannot be an assumption.)

Assumption (iv): Society trusts what philosophers say.

(Correct—The Conclusion is acknowledged and verified by the assumption)

In the following 'examples', each of the given questions consists of one statement followed by two assumptions. You need to choose which of the two (if any) is implicit and can be followed.

Example 1. Statement: From January 1, 2016, telecom carriers must use TRAI mode to compensate for missed calls.

Assumptions:

- (i) The faith and confidence of TRAI's telecom subscribers will grow.
- (ii) In the future, there will be no frauds.
 - A. Only (i) is implicit.
 - B. Only (ii) is implicit.
 - C. Neither (i) nor (ii) is implicit.
 - D. Both (i) and (ii) are implicit.

Answer: A

Explanation: There can be no assurance that a scam will not occur in the future. As a result, assumption (ii) is not implicit.

Example 2. Statement: "Unfortunately, the court is unable to answer all questions.

However, it appears that the public is resorting to the Supreme Court as a last choice, fed up with an inert Executive.

Assumptions:

- (i) The Supreme Court is superior to the Executive Branch.
- (ii) The Supreme Court is more committed than the Executive Branch.
- (iii) The Executive branch lacks sufficient authority.
 - A. (i) and (ii) are implicit.
 - B. (ii) and (iii) are implicit.
 - C. (i) and (iii) are implicit.
 - D. Only (ii) is implicit.

Answer: C

Explanation: Just because the public is resorting to the Supreme Court as their last choice does not imply that it is more active than the Executive. So, (ii) is not implied. (i) and (iii) can be drawn from the statement given. Hence, (i) and (iii) are implicit.

Example 3. Statement: People's entertainment has grown inextricably linked to films.

Assumptions:

- (i) Film is the only form of entertainment available.
- (ii) Films are popular.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: The fact that movies are necessary does not imply that they are the only source of entertainment. As a result, (i) is not implied. (ii) is unmistakably implied by the statement. As a result, it is assumed.

Example 4. Statement: One should put his child in school by the age of 3.

Assumptions:

- (i) A three-year-old toddler is still not prepared to learn.



- (ii) If a person does so, his child's academic performance will improve.
- Only (i) follows.
 - Only (ii) follows.
 - Neither (i) nor (ii) follows.
 - Both (i) and (ii) follow.

Answer: C

Explanation: As per the given statement, one should enrol his child in school by the age of 3, but no information is provided as to whether the child is prepared to learn or whether doing so will help the child succeed in school. Therefore, neither (i) nor (ii) is implicit.

Example 5. Statement: “Smoking is injurious to health and can cause lung cancer. Hence, it should be avoided at all costs.”—Head of Oncology Department.

Assumptions:

- There are many other reasons responsible for cancer other than smoking.
 - Only oncologists recognise that lung cancer is caused solely by smoking.
- Only (i) follows.
 - Only (ii) follows.
 - Neither (i) nor (ii) follows.
 - Both (i) and (ii) follow.

Answer: C

Explanation: The speaker shows no interest in other causes of lung cancer, so assumption (i) is not implicit. Because it is a doctor's recommendation, we can assume that doctors were aware that smoking is harmful to one's health. However, it is not intended that only oncologists are aware that smoking causes lung cancer. Therefore, (ii) is also not implicit.

Practice Questions

Directions: A statement is presented in each of the questions below, followed by two or more assumptions. You must assume that everything in the statement is accurate based on the information provided and then determine which of the offered assumptions are implicit and can be followed.

1. Statement: Why don't you invite George to your son's birthday party this year?

Assumptions:

- George does not live in the same city.
 - George will only attend the birthday party if he is invited.
- Only (i) follows.
 - Only (ii) follows.
 - Neither (i) nor (ii) follows.
 - Both (i) and (ii) follow.

Answer: B

Explanation: The statement makes no mention of George's residence. As a result, (i) is not implied. Assumption (ii) is implicit because it follows quite directly from the statement given in the question.

2. Statement: “We must provide transportation allowance to our staff in order to improve tardiness in our office” — the personnel manager of a company suggests to the CEO.

Assumptions:

- Tardiness will not be improved by a transportation allowance.
 - Discipline should always be rewarded.
- Only (i) follows.
 - Only (ii) follows.
 - Neither (i) nor (ii) follows.
 - Both (i) and (ii) follow.

Answer: B

Explanation: Assumption (i) contradicts the statement. As a result, it is not implied. The allowance will serve as a motivator for employees to arrive on time. Hence, (ii) is implicit.

3. Statement: “You may refer to the book by Mary J. in case you face any difficulty in the subject.”—the teacher says to the students.

Assumptions:

- Mary J.'s book is accessible to the students.



- (ii) No other book is available on the subject.
- A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: The book's availability is implied by the teacher's suggestion. As a result, (i) is implied. Furthermore, while the book has been praised, this does not rule out the possibility of additional works on the subject. As a result, (ii) isn't true.

- 4. Statement:** Any country's economic prosperity is determined by the quality of its human resources.

Assumptions:

- (i) It is every country's dream to achieve economic prosperity.
 - (ii) Measuring the quality of a country's human resources is possible.
- A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: The statement does not address the status of economic prosperity as a national aim. As a result, (i) is not implied. (ii) is implied because it follows directly from the statement.

- 5. Statement:** The current state of education is a disaster, and the country is on the verge of collapse.

Assumptions:

- (i) For the country's well-being, a quality education system is necessary.
 - (ii) For the country's well-being, a quality education is sufficient.
- A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: Evidently, the statement refers to the country's decline as a result of the disruption of the educational system. As a result, (i) is implied. However, this does not imply that education alone is adequate and that no other element is responsible for the nation's well-being. As a result, (ii) is not implied.

- 6. Statement:** "Please make sure that you study for the test." — P tells Q.

Assumptions:

- (i) Q will give heed to P's advice.
 - (ii) Q will not give heed to P's advice.
- A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: When P tells Q to study for the test, he is just assuming that Q will follow P's advice; otherwise, he would not have advised Q in the first place. As a result, (i) is a reasonable assumption. Similarly, he would not have expected that Q would disregard his advice, as he would not have advised Q in such a case. As a result, (ii) is not implied.

- 7. Statement:** What an idiot I am to put my trust in a liar like Asha!

Assumptions:

- (i) Asha is untrustworthy.
 - (ii) I am an idiot.
- A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: Because one is condemned to rely on Asha, (i) is implied. It was a mistake to rely on Asha, according to the statement. Hence, the individual is a fool and as a result, (ii) is implicit.

- 8. Statement:** Global market trends are constantly changing, and as competition grows, consumers' demands for lower



prices and higher quality are constantly rising.

Assumptions:

- (i) Previously, consumers were unconcerned with quality and price.
- (ii) Consumers do not benefit from market competitiveness.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Consumer demands are referred to in the statement as 'growing.' This isn't to say that customers were previously unconcerned about pricing or quality. As a result, (i) isn't true. Furthermore, the statement states that rising competition has provided customers with a wider range of options, prompting them to compare and select the option that best meets their needs and budget. As a result, they get more 'value for the money.' As a result, (ii) isn't implicit.

9. Statement: In light of the institute's new financial priorities, no budgetary provision for the appointment of additional staff will be made.

Assumptions:

- (i) Funds are necessary for appointing additional staff.
- (ii) Other than staff appointments, there are certain areas that require extra financial resources.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: (i) is implied by the phrase 'budgetary provision for the recruitment of additional staff.' In addition, because no budgetary provision was made for staff appointments due to a shift in financial

priorities, several additional concerns will require extra financial attention. As a result, (ii) is also implied.

10. Statement: Asin applied for a loan of Rs.3,00,000 from the bank, mortgaging her home, and promising to repay it within seven years.

Assumptions:

- (i) A house is accepted as collateral for such loans by the bank.
- (ii) The bank has a policy of approving loans of Rs. 2,00,000 and up.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: (i) is implicit because it follows straight from the statement. Whereas because Asin has filed for a loan of Rs. 3,00,000, the bank is able to grant a loan of more than Rs. 2,00,000. As a result, (ii) is implied

11. Statement: The bus drivers' union has declared an indefinite strike in protest of the recent hike in the road tax.

Assumptions:

- (i) The strike will aid the bus drivers' union's case.
- (ii) The administration may change its mind about raising the tax.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: The sole purpose of the strike mentioned in the statement is to compel the administration to take back the decision of raising road taxes. As a result, both (i) and (ii) are implicit.

12. Statement: Whoever emerges from prayer a better man, has had his prayers answered.

**Assumptions:**

- (i) All of our sins are expiated via prayer.
- (ii) A man's compassion is enhanced by prayer.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Prayer does not always make man humane, as evidenced by the fact that only those who improve by praying are answered to. As a result, (i) is not implied. There is no discussion of the effectiveness of prayer. As a result, (ii) isn't implied.

13. Statement: Increased industrial and vehicular pollution have accompanied India's economic expansion.

Assumptions:

- (i) In today's modern society, pollution is inescapable.
- (ii) India's economic growth is solely driven by industrial expansion.
- (iii) Economic expansion with controlled adverse effects is what a country desires.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Only (iii) follows.
 - D. Only (i) and (iii) follow.

Answer: D

Explanation: India had to pay the price of higher pollution levels in order to achieve economic progress, as given in the statement. As a result, (i) and (iii) are both implied. This does not, however, imply that India's economic development is solely due to industrialisation. As a result, (ii) isn't true.

14. Statement: "To improve our examinations for admission to engineering schools, we must incorporate subjective type testing."
— The Chairperson of the Admission Committee suggests the Committee.

Assumptions:

- (i) The Admissions Committee is thinking about improving the admission process.
- (ii) Currently, admission is conducted only on the basis of an interview.
- (iii) The Chairperson himself is an engineering graduate.
 - A. Only (i) follows.
 - B. Only (ii) and (iii) follow.
 - C. Only (i) and (ii) follow.
 - D. None follows.

Answer: A

Explanation: Neither (ii) nor (iii) is implied as the statement makes no mention of the current admission system or the Chairman's qualification. (i) follows immediately from the statement and as a result, is implied.

15. Statement: The S&S Corporation has just introduced a series of bonuses for timely and conscientious professionals.

Assumptions:

- (i) As a result, the company's productivity may rise.
- (ii) The company's profit may be more than the amount to be spent on bonuses.
- (iii) Those who are currently not attentive may very well be encouraged by the announcement.
 - A. Only (i) and (iii) follow.
 - B. Only (ii) and (iii) follow.
 - C. Only (i) follows.
 - D. All follow.

Answer: A

Explanation: Announcement of bonuses for timely and conscientious professionals will undoubtedly encourage more colleagues to be punctual, resulting in increased efficiency. As a result, (i) and (iii) are both implied. The statement, on the other hand, contains no information on the company's earnings. As a result, (ii) is not implied.



16. Statement: “The easiest and most budget-friendly approach to update your lifestyle — Exchange your old electronic equipment for new electronics and save 30% to 55% on the new items.” — An electronic company’s advertisement.

Assumptions:

- (i) Electronic items are no longer in demand unless they come with a compelling promotional scheme.
- (ii) Most customers are always looking for the greatest quality and are unconcerned about price or accessibility.
- (iii) Many customers prefer to keep their electronics fully updated to keep up with the latest technology at a minimal cost and with minimal effort.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Only (iii) follows.
 - D. Only (i) and (ii) follow.

Answer: C

Explanation: The presence of an exchange offer in marketing does not mean that there is no demand for electronic devices without one. As a result, (i) is not implied. Because both ‘easiest’ and ‘most budget-friendly’ are highlighted in the commercial, (ii) is not implied. Customers can also buy new digital products at a discounted price without having to dispose of their old ones, according to the advertising. As a result, only (iii) is implied.

17. Statement: The powdered donuts from ‘Le Beignet’ are far superior to those of the other big famous donut chains. Isn’t it obvious that the size of the store makes little difference when it comes to the quality of the donuts served?

Assumptions:

- (i) Le Beignet is smaller in size than the other donut chains.
- (ii) Powdered donuts are a true representation of the donuts sold at any given donut franchise.

- (iii) The smaller the donut chain, the higher the quality of the donuts supplied in that specific donut chain.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Only (i) and (ii) follow.
 - D. All (i), (ii), and (iii) follow.

Answer: C

Explanation: (i) is inferred from the first statement (other big famous donut chains). Using the example of powdered donuts, the author has broadened his statement. As a result, he implies that the same is a good representation of the donuts provided in well-known donut chains. Hence, (ii) is implicit. Because the author claims that the size of the donut shop has little bearing on the quality of the donuts, (iii) is not implied.

18. Statement: A number of air-conditioned buses have been installed by the public transport corporation on several routes in the city, in an effort to entice people who drive to work and thereby minimise traffic jams on the roads.

Assumptions:

- (i) A vast majority of individuals may continue to prefer driving to work.
- (ii) Several individuals may then choose to take these buses to and from work.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: It is implausible to believe that a ‘vast majority’ of people still want to drive their own cars. Hence, (i) is not a valid assumption. (ii) is correct since it implies that the public transportation corporation has installed air-conditioned buses to entice car users, with the primary goal being to encourage people to take the bus instead of driving. As a result, it is reasonable to suppose that many car owners will now prefer to take the bus.



19. Statement: According to a notice posted at the entrance of the Drushti Cooperative Housing Society, no salespeople are allowed to enter the property.

Assumptions:

- (i) All salespeople will avoid entering the society.
- (ii) The security staff stationed at the gate might be able to prevent salespeople from accessing the property.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: Since both assumptions are implied by the provided statement, both (i) and (ii) are implicit.

20. Statement: “One can easily go to Sri Lanka through air or water.”—Aman tells Swati.

Assumptions:

- (i) Swati is planning to travel to Sri Lanka.
- (ii) Aman likes to give people advice about travelling.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: The statement does not mention anything about Swati’s plans to travel anywhere. As a result, (i) is not implicit. Similarly, assumption (ii) may also not be a fact as the given statement tells nothing about Aman’s nature. Hence, neither (i) nor (ii) follows.

21. Statement: Even if a novice does not have any access to a tutor, he can study history with this book.

Assumptions:

- (i) A novice aspires to learn history without the help of a tutor.
- (ii) A history tutor may not always be accessible.

- (iii) It is difficult for a novice to understand history by himself without a tutor.
 - A. Only (i) and (ii) follow.
 - B. Only (ii) and (iii) follow.
 - C. Only (i) and (iii) follow.
 - D. All (i), (ii), and (iii) follow.

Answer: B

Explanation: The statement is obviously intended to emphasise the book’s utility. The aspirations of a novice are not mentioned. As a result, (i) isn’t true. The book is also meant to serve as a guide when a tutor is unavailable. As a result, both (ii) and (iii) are true.

22. Statement: If it rains excessively, Udit’s father advised him to take a taxi back home.

Assumptions:

- (i) If it rains excessively, Udit may be unable to make a decision.
- (ii) Even when it is raining outdoors, taxis are available.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: Udit’s father has given him this advice out of concern and love for his child, not because Udit would be unable to make his own decision. As a result, (i) isn’t implied. In addition, Udit’s father advises him to take a cab home if it is raining badly, implying that taxis are available even when it is pouring. Therefore, (ii) is implicit.

23. Statement: “On Anuj and Anita’s anniversary, I would like to give them a book about Reiki and meditation practices.”—Shubham tells Gauri.

Assumptions:

- (i) Anuj and Anita will invite Shubham to their wedding anniversary celebrations.
- (ii) The couple to whom the book will be given is under a great deal of stress



- in their personal lives and is not experiencing ideal emotional stability.
- (iii) A book can be considered an acceptable present to give someone on their anniversary.
- A. Only (i) and (ii) follow.
 - B. Only (ii) and (iii) follow.
 - C. Only (i) and (iii) follow.
 - D. All (i), (ii), and (iii) follow

Answer: C

Explanation: Shubham's decision to give Anuj and Anita a book on their anniversary implies that he will be welcomed by the couple and that a book will suffice as a present. As a result, (i) and (iii) are both implied. Nothing can be inferred about the couple's mental health from the statement. As a result, (ii) is not implied.

24. Statement: These mangoes are far too inexpensive to be of decent quality.

Assumptions:

- (i) Whenever the mango harvest is plentiful, prices drop.
 - (ii) If a product has a very low selling price, there is a sure-shot guarantee of its quality not being decent enough.
 - (iii) Mangoes that are quite inexpensive are also delicious.
- A. Only (i) and (iii) follow,
 - B. Only (ii) follows,
 - C. Only (ii) and (iii) follow,
 - D. None follows,

Answer: B

Explanation: The mangoes are said to be so inexpensive that they cannot possibly be of decent quality. This means that good mangoes will never be too cheap, and very cheap mangoes will never be good. As a result, neither (i) nor (iii) is implied. The statement that mangoes are of questionable quality because they are inexpensive leads to assumption (ii). Therefore, it is implicit.

25. Statement: Sumedh chose to make a train reservation in November for a trip to Kolkata in February.

Assumptions:

- (i) Reservations are taken three months in advance on the railway.
 - (ii) There are several trains that run to Kolkata from the city in which Sumedh lives.
 - (iii) There will be a vacancy in the class that Sumedh desires to travel in.
- A. Only (i) follow.
 - B. Only (ii) and (iii) follow.
 - C. Only (i) and (iii) follow.
 - D. All (i), (ii), and (iii) follow.

Answer: A

Explanation: Sumedh makes the reservation in November for a February trip, therefore (i) is clearly implied. The number of trains to Kolkata or the number of vacancies in various classes cannot be calculated from the available information. As a result, neither (ii) nor (iii) can be assumed.



Almost all engineering entrance examinations have questions on the topic ‘Statement and Conclusion’ in the Reasoning section of the question paper. If a person is able to understand the right approach to answer these questions, it can be a very high-scoring topic.

In this topic, a statement will be provided, followed by a set of conclusions, where a statement is a set of words combined together to make a complete sentence and a conclusion is an opinion made or a result obtained upon analysis of the provided information. You must select the conclusion that most logically follows the statement. Sometimes the candidate can deduce the conclusions simply by reading the sentence, and other times the candidate must analyse it in order to deduce the indirect conclusion. Any of the conclusions may be followed at times, and all of them may be followed at other times. It is also possible that either one or none of them will follow. To arrive at the correct conclusion, the candidate’s cognitive power is required to analyse the statements. This reasoning portion covers a variety of problems, including one statement with two conclusions, several statements with multiple conclusions, and so on.

Understanding some common words, referred to as keywords, is one of the most significant components of analysing statements and concluding reasoning. The most important application of keywords is to differentiate between statements that are definitely true, possibly true, and definitely untrue. Verbs, adverbs, adjectives, and conjunctions included in the statement can be considered as the basic keywords that often convey important information about the quantity as well as the quality of the statement. None, some, must, sometimes, always, never, only, and so on are some examples.

Let us take the help of the given *example* below to understand the concept better.

Statement: India is often called the land of festivals.

Conclusion:

- (i) Every Indian should engage in a variety of festivities.
 - (ii) Various festivals are celebrated in every country.
- A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Solution: Although the statement claims that India is known as the ‘land of festivals,’ it does not highlight the necessity for each and every Indian to engage in a range of celebrations. Furthermore, the statement only contains information about one country. Both conclusions are meaningless because they are unrelated to the statement. As a result, neither conclusion (i) nor (ii) can be drawn.

WHAT IS A CONCLUSION?

The term ‘conclusion’ means a decision or verdict reached after a period of deliberation or inquiry into particular facts or a statement made by someone. Before arriving at the end outcome or conclusion of a given premise, a consequent effect must always be considered. This necessitates a methodical and logical approach.

There are two types of conclusions:

- **Direct Conclusion:** Certain conclusions can be drawn straight from the provided statement. All you have to do is pay attention when reading them. Direct conclusion statements are what these statements are referred to as. Let us look at a few examples to help you grasp it better.



Statement: Stuti is the only daughter of Anil and Sudha.

Conclusions:

- (i) Stuti has a brother.
- (ii) Anil and Sudha have one daughter.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: Take a look at the statement that states that Stuti is her parents' only daughter. This indicates that she does not have any sisters. As a result, the direct conclusion is (ii). On the other hand, we have no information regarding Stuti's brother. As a result, (i) may or may not be correct. As a result, option B is the correct response.

Statement: Sudip was insulted in front of the entire staff by the organisation's Senior Director.

Conclusions:

- (i) Sudip acted inappropriately in front of the Senior Director.
- (ii) Sudip was despised by the Senior Director.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: We cannot conclude that the Senior Director did not like Sudip or that he misbehaved with him because the statement does not specify why he was offended. As a result, neither conclusion (i) nor (ii) follows the statement.

- **Indirect Conclusion:** The candidate is expected to comprehend the given statement, and then evaluate the conclusion based on how well it matches the given statement. To tackle these kinds of questions, thorough reading and a

rational approach is required. Let us look at a few examples to help you grasp it better.

Statement: It has been decided by the panel of IPJ University to only shortlist individuals who have a career average of at least 70%. Himadri received 56% in 10th grade, 75% in 12th grade, and graduated with a percentage of 82%.

Conclusions:

- (i) Himadri has the required academic record and will be shortlisted.
- (ii) Himadri does not have the required academic record and will not be shortlisted.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: We must carefully reach a conclusion in this case. Himadri received over 70% in both graduation and 12th grade. Her tenth percentage, however, does not exceed 70%. As a result, she will not be considered for admission to the mentioned university. We arrived at this conclusion in a roundabout manner, but as you can see, it is not that difficult.

Statement: The profits of firm X in 2018 were Rs. 7,50,000. It had a profit of Rs. 10,00,000 in 2019. In 2020, the firm lost all the profits that it earned in both the years 2018 and 2019.

Conclusions:

- (i) Company X made a total profit of Rs. 17,50,000 in 2018 and 2019.
- (ii) In the year 2020, Company X lost Rs. 12,00,000.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: According to the statement, the company made a profit of Rs. 17,50,000



(7,50,000+10,00,000) in 2018 and 2019. In 2020, the company lost the same amount. As a result, the statement is followed by only conclusion (i). Candidates' most common mistake while solving statement-conclusion questions is to assume

Tips and Tricks

- Outside of the given statement, don't make any assumptions. That is, if we come across a statement like 'Daffodils are better than roses.' and one of the conclusions is 'Roses are better than sunflowers,' we cannot assume the same, even if we believe that it is true. This is due to the fact that the given sentence has no information about roses. As a result, we cannot take it as a truthful statement.
- Try not to judge a statement or a set of statements based on how long and incomprehensible they are. Before you get to a conclusion, be sure you read the statement thoroughly.
- When dealing with this type of topic, we must assume that everything stated in the statement is correct. For example, if the statement states, 'The Sun rises in the West and sets in the East,' this is the correct statement for us.
- Analyse the statement carefully for keywords that appear in both the statement and the conclusions.
- If a statement is formed by two or maybe more sentences, the sentences must be connected and mutually contradictory.
- If the statement allows for more than one conclusion, aspirants must make sure that the conclusions they choose are interrelated.
- Negative marking causes candidates to lose a lot of points in such questions. Therefore, avoid guessing the answers in this topic.

things while answering a question, which leads to them marking the wrong alternative as a conclusion for the provided statement.

In the following 'examples', each of the given questions consists of one statement followed by two conclusions. You have to decide which one of them, if any, can be followed.

Example 1. Statement: It is beneficial to one's health to drink warm water first thing in the morning.

Conclusions:

- (i) In the morning, everyone who is healthy drinks a glass of warm water.
- (ii) It is not a good idea to drink warm water late at night.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: The statement claims that drinking warm water first thing in the morning is beneficial to one's health. This does not, however, imply that all healthy people drink warm water first thing in the morning. As a result, (i) is not true. In addition, the statement makes no mention of drinking warm water late at night. As a result, (ii) is also untrue.

Example 2. Statement: Drinking is one of those personal weaknesses that tend to push an alcoholic's willpower to the limit.

Conclusions:

- (i) Even if alcoholic wishes to stop drinking, it is extremely difficult for him to do it.
- (ii) Other weaknesses also exist in humans.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: According to the statement, quitting drinking requires a lot of willpower



and is therefore quite difficult. So, (i) follows. It is also stated that drinking is one of the personal weaknesses. This implies that there are other flaws too that exist in a human being. Therefore, (ii) follows too.

Example 3. Statement: Mansi won an art competition held at the national level.

Conclusions:

- (i) Mansi is the greatest artist on the planet.
- (ii) Mansi is also a very good mathematician.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Winning a national painting competition does not imply that she is the greatest artist in the world, nor does the statement reveal anything about her mathematical abilities. So, neither (i) nor (ii) follows.

Example 4. Statement: A group of friends, Amalika, Anu, Ayushi, Likhita, and Srushti plan a social service campaign to benefit an NGO. They have the assistance of a few relatives and co-workers. They raise Rs. 50,000 in order to assist the NGO.

Conclusions:

- (i) The five women like working on social concerns on occasion.
- (ii) The five women are not social workers.
 - A. Only (i) follows.

- B. Only (ii) follows.
- C. Neither (i) nor (ii) follows.
- D. Both (i) and (ii) follow.

Answer: A

Explanation: It is likely that the five friends are social workers who volunteer on a regular basis, or that they contribute to such organisations on occasion. Therefore, only (i) follows but (ii) does not.

Example 5. Statement: By the end of the year, AS Industries has decided to acquire RM Industries.

Conclusions:

- (i) By taking over RM Industries, AS Industries will benefit.
- (ii) AS Industries has performed much better than RM Industries.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: Both conclusions are valid since AS Industries seeks to enrich itself by acquiring the other firm, and it is able to do so because it is more successful than RM Industries. Therefore, both (i) and (ii) follow.

Candidates should solve more and more relevant questions on the statement-conclusion topic to better comprehend the concept. This will not only make the topic more relatable to the aspirant, but it will also assist them in immediately determining the conclusions for the statements.

PRACTICE QUESTIONS

Directions: A statement is presented in each of the questions below, followed by two or more conclusions. You must assume that everything in the statement is accurately based on the information provided in the statement, and then determine which of the offered

conclusions logically follow the data given in the statement beyond a reasonable doubt.

1. **Statement:** Mr. Agarwal is one of the likely contenders for the post of Vice-Chancellor of R.J.P.V. Institute.



Conclusions:

- (i) R.J.P.V. Institute will appoint Mr. Agarwal as its Vice-Chancellor.
- (ii) R.J.P.V. Institute will not appoint Mr. Agarwal as its Vice-Chancellor.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: The fact that Mr. Agarwal is a likely contender for being the Vice-Chancellor of the institute, does not guarantee his appointment. So, neither (i) or (ii) follows.

- 2. Statement:** In politics, money is crucially significant.

Conclusions:

- (i) Politics is open to all wealthy men.
- (ii) It is impossible for the poor to become politicians.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: The statement does not mention the poor or the wealthy, but rather the influence of money in politics. Therefore, neither (i) nor (ii) follows.

- 3. Statement:** In the modern world, it is practically impossible to thrive and succeed without abandoning human values.

Conclusions:

- (i) Human values are inapplicable in everyday life.
- (ii) Some concepts are valued by society, yet they may not be sustained.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: (i) Directly follows from the given statement. However, (ii) is ambiguous and hence, does not follow.

- 4. Statement:** The Indian Cricket Team scored a total of 260 runs in a one-day cricket match. Pace bowlers accounted for 182 of the total runs.

Conclusions:

- (i) The pace bowlers make up 70% of the squad.
- (ii) Pace bowlers were the opening batsmen.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Pace bowlers were responsible for 70% of the total runs, according to the statement. As a result, (i) does not follow. The statement makes no reference to the opening batsmen. Hence, (ii) does not follow either.

- 5. Statement:** In the market, gold prices are skyrocketing these days.

Conclusions:

- (i) Gold is becoming an extremely scarce commodity.
- (ii) People are unable to purchase gold.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: The statement does not address the availability of gold. So, (i) does not follow. Also, (ii) is not directly related to the given statement and so it also does not follow.

- 6. Statement:** Only good dancers are invited to the event. No one can dance well without a flexible body.

Conclusions:

- (i) Those dancers who do not have a flexible body are not invited to the event.
- (ii) All invited dancers in the event have a flexible body.



- A. Only (i) follows.
- B. Only (ii) follows.
- C. Neither (i) nor (ii) follows.
- D. Both (i) and (ii) follow.

Answer: D

Explanation: A good dancer always has a flexible body, according to the statement, and only good dancers are invited to the event. This suggests that only good dancers who have a flexible body are invited to the event, and other dancers are not. As a result, both (i) and (ii) follow.

7. Statement: In urban areas, the average number of people per household is 4.1, whereas in rural areas, it is 4.5. Whereas, the national average is 4.3.

Conclusions:

- (i) Rural areas have a higher population density per unit area than urban ones.
- (ii) In rural areas, there are more people living in a single household than in urban areas.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: The statement refers to the population per household rather than the population per unit area. As a result, only (ii) follows, but (i) does not.

8. Statement: Both a church and a mosque are houses of worship.

Conclusions:

- (i) Christians and Muslims both pray at the same place.
- (ii) Every mosque is a church.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Churches and mosques may or may not intersect. Furthermore, no information regarding Muslims or Christians is provided. As a result, the conclusion is reached. Hence, neither (i) nor (ii) follows.

9. Statement: “Values, morality, and inner peace are all necessary for a meaningful existence,” Ram says.

Conclusions:

- (i) Ram feels that a beautiful life requires each of these qualities: values, morality, and inner peace.
- (ii) Ram thinks that an individual who is materialistic cannot live a meaningful life.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: (i) Follows directly from the statement. (ii) Follows since materialism is often anti-value and integrity. So, both the conclusions follow.

10. Statement: The students of Mrs. Sharma’s class have a high probability of passing their finals.

Conclusions:

- (i) Mrs. Sharma thoroughly covered the entire curriculum.
- (ii) Mrs. Sharma has bright students in her class.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Only if the teacher has properly covered the curriculum, will the students have a high chance of passing the examination. However, in order for the statement to be true, this assumption must be true (and is not a conclusion). As a result, conclusion (i) is invalid.



(ii) is equally invalid because the referred-to students are not described properly.

11. Statement: Nowadays, parents are willing to pay whatever it takes to provide their children with an excellent education.

Conclusions:

- (i) Parents are passionate about their children's optimal development through quality education.
- (ii) These days, all parents are wealthy.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: We can conclude from the given statement that because parents want their children to develop perfectly through quality schooling, they are willing to spend any price for a decent education, but the statement does not create the impression that the parents are wealthy. As a result, (i) follows but (ii) does not.

12. Statement: Because of mainline repairs, water supplies in zones 1 and 4 of the city will be cut by around 40% on Wednesday.

Conclusions:

- (i) On Wednesday, residents in these areas should reduce water consumption.
- (ii) On the preceding day, residents in these zones should preserve some water to use the next day.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: Since there is going to be a reduction in the water supply, it will be a wise decision to conserve water the previous day and reduce the water consumption on Wednesday, in order to not face a shortage. Hence, both (i) and (ii) follow.

13. Statement: Mutual funds are subject to market risk. Before you invest, talk to your financial counsellor or agent.

Conclusions:

- (i) The market risk is precisely calculated by the financial advisor.
- (ii) It is not advisable to invest in mutual funds.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: Because the statement recommends consulting an investment professional before making a decision, (i) follows. Mutual fund investing involves some risk. This is not to say that mutual funds are not worth investing in. As a result, (ii) does not follow.

14. Statement: Because Bhopal has a smaller population, it is a safer city to reside in.

Conclusions:

- (i) The number of crimes in Bhopal is lower than in Delhi.
- (ii) It is preferable to reside in a city with fewer residents.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Both conclusions are illogical. In the first conclusion, there is no information in the statement about crime rates in Delhi; therefore, conclusion (i) does not follow. Similarly, the statement in the second conclusion is actually a premise upon which the argument statement is built. It is not a conclusion that follows from the given statement; therefore, conclusion (ii) does not follow.

15. Statement: Everyone who is well-organised makes time for rest. Simran, despite her hectic schedule, finds time to relax.

**Conclusions:**

- (i) Simran is a well-organised individual.
- (ii) Simran is a diligent worker.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: Simran's schedule is jam-packed. This indicates that she is a hard worker. Nonetheless, she finds time to rest. This indicates that she is a very well-organised individual. As a result, both conclusions (i) and (ii) are valid.

16. Statement: Manu abused Khushi, so she slapped him in front of everyone.

Conclusions:

- (i) Manu has acted inappropriately with Khushi.
- (ii) Manu and Khushi are bitter rivals.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: Manu publicly humiliated Khushi, as stated in the statement above, and as a result, she slapped him. The fact that Manu and Khushi are adversaries is not implied anywhere in the preceding statement. Therefore, the statement is followed by only conclusion (i) but not (ii).

17. Statement: The market price of silver is increasing every day.

Conclusions:

- (i) Finding silver shops in the market has gotten more difficult.
- (ii) Nowadays, no one wears silver.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Silver prices are not rising as a result of either of the two statements. As a result, they cannot be inferred from the statement. Therefore, neither (i) nor (ii) follows.

18. Statement: In the coming years, many developing nations will face overpopulation combined with resource scarcity.

Conclusions:

- (i) In the foreseeable future, the population of developing nations will not keep growing.
- (ii) Governments in developing nations will have a hard time providing good living conditions for their citizens.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: B

Explanation: The fact presented in (i) is completely contradictory to the statement made. As a result, (i) is not following. Conclusion (ii) addresses the situation discussed in the statement and its immediate implications. Hence, (ii) follows.

19. Statement: All major footwear brands will be on sale at a shopping mall. The discount will run for a week and will include all products in the store. Customers who can present their membership cards will receive an extra 25% discount on all the items.

Conclusions:

- (i) The sale is open to everyone, not just members.
- (ii) There is a sale in the grocery section as well.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: Conclusion (i) follows because anybody can come in and shop; members can



only get a special discount. Conclusion (ii) does not follow because the statement clearly specifies that the offer is only on footwear brands and not groceries.

20. Statement: Students will be able to opt out of Mathematics and Computer for their school-leaving certificate exams commencing in the next academic year.

Conclusions:

- (i) Students who do not possess strong math or computer skills will also be given a fair chance to succeed.
- (ii) Students previously had no option, but to take these subjects if they wanted to continue their studies.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: D

Explanation: Because the new approach allows students to skip Mathematics and Computer, students who are poor in these courses can still be admitted. As a result, conclusion (i) follows. It is also stated that the new system will be implemented from the next academic session. This indicates that it did not previously exist. Therefore, conclusion (ii) also follows.

21. Statement: Company ABC has been fully committed to efficiently use, save, and manage energy for more than two decades.

Conclusions:

- (i) This is an area where the Company ABC has yet a lot of scope for improvement and learning necessary knowledge.
- (ii) Expertise and knowledge are less crucial than perseverance.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: C

Explanation: Because the organisation has been working in this field for more than two

decades, it must have the requisite experience and knowledge. As a result, conclusion (i) does not follow. The attributes that have made Company ABC effective in this industry, on the other hand, have still not been mentioned. Therefore, conclusion (ii) does not quite make logical sense either. So, neither (i) nor (ii) follows in the given question.

22. Statement: As WWI was going on, the XY government enacted the Official Secrets Act (OSA), which appears to be one of the greatest contributors to political corruption in country Z.

Conclusions:

- (i) To put a stop to political corruption in country Z, the OSA must be repealed immediately.
- (ii) The XY administration intended to promote political corruption in state offices.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.
 - D. Both (i) and (ii) follow.

Answer: A

Explanation: According to the statement, the OSA's implementation was the prime reason for the rise in political corruption. As a result, conclusion (i) follows. However, an administration's implementation of legislation is done for the welfare of the nation, not to facilitate political corruption in the future. Therefore, conclusion (ii) is incorrect.

23. Statement: Because of the heavy snowfall in Uttarakhand in February, some portions of the state have been without electricity.

Conclusions:

- (i) In the month of February, people in various parts of Uttarakhand experience a lot of challenges.
- (ii) Snowfall is heavier in some parts of Uttarakhand than in others.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (ii) follows.



- D. Both (i) and (ii) follow.

Answer: D

Explanation: Because it is stated in the statement that electricity has been shut off only in specific parts of Uttarakhand owing to significant snowfall, conclusion (i) is correct. As a result, we may reasonably assume that some sections of the country receive more snow than others and clearly, people will suffer without electricity. Therefore, both (i) and (ii) are true.

24. Statement: Running for at least an hour every day can add at least two years to a person's life expectancy.

Conclusions:

- (i) Regular exercise at a reasonable level is essential to a healthy lifestyle.
- (ii) Those who run daily, never die.
- (iii) People who work at a desk will almost certainly develop health problems.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Only (i) and (iii) follow.
 - D. All (i), (ii), and (iii) follow.

Answer: A

Explanation: The benefit of running every day is emphasised in the statement, but no information about the health of someone who works at a desk is provided. Furthermore, it is not plausible that a person will never die if he runs for an hour every day. As a result, only conclusion (i) follows.

25. Statement: In company ABC, 45% of employees are females, 55% are males, 72% are ethical, and 20% are from Gandhinagar.

Conclusions:

- (i) None of the male employees is from Gandhinagar.
- (ii) All the female employees are ethical.
- (iii) 30% of male employees are not ethical.
 - A. Only (i) follows.
 - B. Only (ii) follows.
 - C. Neither (i) nor (iii) follows.
 - D. None of these follows.

Answer: D

Explanation: None of the given conclusions can be derived from the data that is given in the statement. Therefore, none follows.



Statement and Arguments are very important sections of critical reasoning. In this, a statement about a *political*, *social*, or *economic* topic is presented, followed by arguments in support or opposition to the statement. The candidate must first assess the statement, then the arguments in the context of the statement, before deciding which of the arguments is the most compelling and is also assisting in the formulation of the best perspective on the topic.

WHAT IS AN ARGUMENT?

In simple terms, an argument is a viewpoint on a topic that is backed up by evidence. The aspirant must assess the persuasiveness of the provided argument, determining whether it is weak or strong. An argument, in other words, is a set of two or more phrases, clauses, or sentences that incorporates a proposition or conclusion.

Arguments can be of two types:

1. Strong Arguments: Strong arguments are those that are valid, strong, and factual, discuss strength, are clearly connected to the statement and meet the logic requirement. Following are the types of strong arguments-

- **Arguments that include already proven facts:** The provided argument is a well-known fact that is usually true in this form of a strong argument.
- **Arguments in which the outcome is expected based on previous experience:** Because of past events, the arguments in this form of a strong argument are accurate.
- **Arguments that state the universal truth:** The arguments in this form of a strong argument are certain or

universally accepted to be true and are often scientifically established.

2. Weak Arguments: Weak arguments are ones that are just vaguely related to the statement and do not meet the logic criteria; they are erroneous, weak, and incorrect. Following are the types of weak arguments-

- **Arguments that are ambiguous in nature:** This type of weak argument does not define how the argument relates to the course of action or what the author intends to communicate; hence, these arguments are weak.
- **Arguments that are redundant in nature:** Arguments are redundant in this type of weak argument, and they do not provide a thorough analysis of the issue at hand; hence, they are weak in nature.
- **Arguments that are interrogative in nature:** The argument in this form of weak argument is made up of the arguer's rebuttal to a question.

Now, let us look at the following *example* to understand the concept better.

Statement: Has the Indian economy suffered as a result of globalisation?

Arguments:

- (i) Yes. Indian businesses are being pushed out by multinational corporations.
- (ii) No. Because multinationals sell standard-quality goods at an affordable rate.

Now here, let us take a closer look at the issue. The question, here, is if the Indian economy has 'suffered' because of globalisation. As a result, we must determine if the arguments present any evidence or basis that would conclusively indicate that the economy is being affected or not.



As per argument (i), Indian businesses are unable to compete with their overseas rivals. As a result, we might claim that ‘Indian companies are being damaged.’ However, Indian businesses are not the only ones who make up the Indian economy. Foreign corporations also contribute to the economy of the country. As a result, the argument is weak. Argument (ii) claims that international corporations that have arrived in India in terms of globalisation are selling standard products at reasonable prices, because of which, the customer benefits. Is this, however, ‘no damage to the national economy?’ The declarations make no mention of whether Indian businesses were offering inferior items at higher costs. We can use our external information to create a

Tips and Tricks

- A strong argument should provide a genuine diagnostic of the circumstance represented in the statement, whereas the nature of weak arguments is that they are vague.
- Before answering the questions on statement and argument reasoning, conduct a basic assessment of the questions.
- A strong argument should include an in-depth examination of the topic at hand, whereas a weak argument is overly simplistic and often unnecessary.
- If an argument is based on a universal truth, it is always strong. But, a weak argument contradicts the known facts.
- If an argument is vague, it can be dismissed.
- A strong argument should be backed up by facts or well-established ideas and should also be related to the given statement, whereas arguments that incorporate the terms only, definitely, the best until, until, etc will most likely be weak.

generalisation that globalization is beneficial; however, this is disputed and dependent on the individual’s perspective. As a result, we can say that argument (ii) is also a weak argument. Therefore, both the arguments are weak.

In the following examples, each of the given questions is followed by a series of arguments. You must determine which of the arguments is ‘strong’ and which is ‘weak’.

Example 1. Statement: Should ‘Sanskrit’ be taught in India’s schools as a mandatory subject?

Arguments:

- (i) No. We do not have teachers that are qualified enough to teach such a difficult language.
- (ii) Yes. Sanskrit is an ancient language and there is a desperate need to protect it at all costs.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: C

Explanation: Argument (i) depends on a person’s perspective, as there are different teachers in every school and some of them may be qualified enough, while some maybe not. As a result, argument (i) is not particularly convincing. When we talk about the second argument, although it is true that we should be proud of our ancient language, making ‘Sanskrit’ compulsory in schools is not desirable solely for this reason. Hence, argument (ii) is not strong either.

Example 2. Statement: Should it be mandatory for female students to master self-defence?

Arguments:

- (i) Yes. In a world where instances of attacks on women are on the rise, self-defence training has become an important aspect of women’s safety.
- (ii) No. Even if girls do learn self-defence, they will never be as physically strong as the assailter.



- A. Only argument (i) is strong.
- B. Only argument (ii) is strong.
- C. Neither (i) nor (ii) is strong.
- D. Both (i) and (ii) are strong.

Answer: A

Explanation: Argument (i) is compelling because, with the rise in assault cases, girls need to be prepared to defend themselves. Argument (ii) does not hold strong simply because it varies from one person to another.

Example 3. Statement: Should India's public universities be free?

Arguments:

- (i) Yes, improving literacy levels is the only way to develop India.
- (ii) No, it would contribute to the exchequer's already tremendous load.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: B

Explanation: Argument (i) is weak since it contains the word "only," whereas argument (ii) is strong because such a step would definitely necessitate vast sums of money and result in a financial drain.

Example 4. Statement: Should all government agencies and departments implement a system of exclusively granting jobs to the children of government employees?

Arguments:

- (i) No. It denies many qualified people a chance to succeed, and the government might suffer as a result.
- (ii) No. The government owes its responsibilities to all of its citizens, as per the idea of equality.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.

- C. Neither (i) nor (ii) is strong.
- D. Both (i) and (ii) are strong.

Answer: D

Explanation: If taken care of; merit, impartial evaluation, and equal opportunity for all can enable the government to recruit qualified officials while also achieving the Constitution's goals. As a result, both arguments are valid.

Example 5. Statement: Should every university student in India be required to participate in military training?

Arguments:

- (i) Yes. Only in this manner can a strong and resilient nation be built.
- (ii) Yes. It is the only way to instil discipline in the youth.
- (iii) No. Repulsion follows compulsion in every case.
- (iv) No. This is a violation of an individual's fundamental democratic freedom to choose his or her courses.
 - A. Only arguments (i) and (ii) are strong.
 - B. Only arguments (ii) and (iv) are strong.
 - C. Only arguments (iii) and (iv) are strong.
 - D. None of the above.

Answer: C

Explanation: Making military training mandatory for all university students would undoubtedly improve India's troops and instill strong discipline among them, but it is not the only way to accomplish these goals. As a result, neither (i) nor (ii) holds up. Furthermore, students who are truly willing to join the armed services and serve the country will be able to defend the country better than those who are forced to do so. Every student has the fundamental right to pursue a vocation of his or her choosing. As a result, both (iii) and (iv) hold true.



PRACTICE QUESTIONS

Directions: Each question begins with a statement, followed by two or more arguments denoted by the numerals (i), (ii), and so on. You must assess which of the arguments are 'strong' arguments and which are 'weak' arguments, and then select an answer from the options presented below each question.

- 1. Statement:** Should it be compulsory for students in grades V and VI to take board exams?

Arguments:

- (i) Yes. Students must be trained from the start to tackle such challenging assessments in today's competitive environment.
- (ii) No. Students will be compelled to learn and will not embrace education if we put such additional strain on them.
 - A. Only (i) is strong.
 - B. Only (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: Exams will prepare students for competitions later in life. As a result, (i) is valid. On the other hand, the essential basics of subjects should be taught to the students in Classes V and VI in a gradual process using practical examples and practice in an entertaining manner. They do not need to be forced to study, and their age does not allow them to withstand the stress and load of exams. Therefore, both (i) and (ii) hold strong.

- 2. Statement:** Should the government open more engineering schools in the country?

Arguments:

- (i) Yes. The country is in desperate need of qualified engineers.
- (ii) No. The government should make sure that quality education is being

provided in the existing engineering schools, as most of them lack the required facilities.

- A. Only argument (i) is strong.
- B. Only argument (ii) is strong.
- C. Neither (i) nor (ii) is strong.
- D. Both (i) and (ii) are strong.

Answer: B

Explanation: Undoubtedly, offering modern and advanced infrastructure to existing engineering schools may enable them to generate more and more qualified engineers, since they can accommodate even more students and give them quality education. As a result, only argument (ii) is valid, whereas argument (i) is not.

- 3. Statement:** Should social networking sites completely prohibit people less than 18 years of age from signing up?

Arguments:

- (i) Yes. Such websites often mislead people under the age of 18.
- (ii) No. It enables young people to use their right to free speech and expression when they express themselves on such platforms.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: C

Explanation: (i) does not explain the situation in which such websites mislead people of age less than 18 years. Furthermore, these kinds of distractions are frequent among both minors and adults, which is one of the main reasons why many organisations prohibit the use of social media platforms. As a result, (i) is weak. (ii) is weak because there are other ways to exercise the legitimate right to freedom of expression, and allowing people under the age



of 18 access to social media platforms solely for that purpose would be incorrect.

4. Statement: Should there be a limit on the number of people who can run in parliamentary elections in each constituency?

Arguments:

- (i) Yes. The parliamentary elections will be more impactful as voters will be able to make an informed decision when casting their vote.
- (ii) No. In a democracy, anyone who meets the qualification requirements can run for parliament, so there should be no constraints on the same.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: Obviously, if there were fewer candidates, voters would have an easier time making a decision. As a result, argument (i) is valid. Furthermore, everyone who meets the Constitution's requirements should be given a chance and should not be refused just to reduce the number of candidates. As a result, argument (ii) is also valid.

5. Statement: Should all pharmaceuticals that are manufactured and patented in any Western country be first tested in India prior to being given a license to sell to the wider population?

Arguments:

- (i) Yes. Many of these drugs have varying doses and duration for the population of India, therefore it is crucial to keep track of them.
- (ii) No. This is not permitted, hence it cannot be achieved.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: A

Explanation: Evidently, the government is concerned about the citizens' health. Therefore, before the government grants any kind of a license to sell a product, such as pharmaceuticals, it is very necessary to first investigate it properly and see if it works in the Indian setting too. As a result, only argument (i) is valid.

6. Statement: Is there a way to control pollution in this modern world?

Arguments:

- (i) Yes. If people are able to recognise the dangers that pollution poses and decide to work together to eliminate it, pollution can be minimised eventually.
- (ii) No. Overcrowding on roads, in industries, and in organisations, and also an ever-expanding population hungry to capture more and more land for residential construction is inevitable.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: On one hand, pollution control appears to be unattainable due to the constantly increasing needs and demands of the public, but on the other hand, pollution control is possible through a cooperative effort of the masses. As a result, both of the arguments will withstand.

7. Statement: Should victory take precedence over participation?

Arguments:

- (i) Yes. Every sportsman is now a professional who survives primarily on prize money.
- (ii) No. In India, sportsmen make more money through sponsorships than match fees.



- A. Only argument (i) is strong.
- B. Only argument (ii) is strong.
- C. Neither (i) nor (ii) is strong.
- D. Both (i) and (ii) are strong.

Answer: C

Explanation: The primary statement is a statement that may be applied to a wide range of circumstances. Sports have been used to prove argument (i) and as a result, it is a weak argument. Argument (ii) discusses sponsorships that are not covered by the main statement. As a result, none of the arguments is valid.

8. Statement: Should the state place limitations on journalists' access to confidential or sensitive data in order to minimise media frenzy?

Arguments:

- (i) Yes. It can lead to bullying of those who have been impacted by the issue and are suspected of being engaged in it.
- (ii) Yes. The press occasionally creates frenzy and broadcasts misleading data.
- (iii) No. The media should have complete access to all the data because the media is the finest source for exposing societal flaws.
 - A. Only argument (i) is strong.
 - B. Only (ii) and (iii) are strong.
 - C. Only (i) and (ii) are strong.
 - D. All (i), (ii), and (iii) are strong.

Answer: D

Explanation: All the arguments are strong as they provide a legitimate reason to support the primary statement.

9. Statement: Would it be appropriate for a cricket team to have multiple captains during a match?

Arguments:

- (i) Yes. Having many heads come to a consensus before making a choice is always preferable.
- (ii) No. In such a circumstance, one must make judgments on the fly, and there

will be no time to settle opposing viewpoints among the captains on the field.

- A. Only argument (i) is strong.
- B. Only argument (ii) is strong.
- C. Neither (i) nor (ii) is strong.
- D. Both (i) and (ii) are strong.

Answer: B

Explanation: Every individual may hold a different viewpoint. As a result, making a decision on the spot may take longer. Hence, only argument (ii) is strong.

10. Statement: Is there a need to ban foreign films in India?

Arguments:

- (i) Yes. They show a foreign lifestyle that has a harmful impact on Indian people's beliefs.
- (ii) No. The artistic quality of foreign films is extraordinary and they deserve to be seen by the world.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: C

Explanation: Foreign films clearly represent a foreign lifestyle, but this only aids in learning more. As a result, argument (i) is inadequate. Furthermore, argument (ii) is not sufficient enough to reverse the prohibition. As a result, (ii) is not strong either.

11. Statement: Should university education be halted for a period of time?

Arguments:

- (i) Yes. It will help decrease the number of unemployed educated people.
- (ii) No. It will obstruct the nation's future development.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: B



Explanation: Clearly, unemployment is not caused by a lack of advanced education. Moreover, it has resulted in more career prospects. As a result, argument (i) is a little ambiguous. Furthermore, higher education aids in the development of a country. As a result, argument (ii) is valid.

12. Statement: Should students at Indian universities wear uniforms as they do in schools?

Arguments:

- (i) No. Undergrads should not be forced to wear uniforms and should be allowed to choose their attire for the day.
- (ii) Yes, because all of the students will be properly attired, the atmosphere of the colleges will improve.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: A

Explanation: Apparently, after years of severe discipline and adherence to the school's formal dress code, youngsters must be given some leeway in college life as they prepare to take on life's responsibilities. Furthermore, schools employ uniform to ensure the safety of students, a factor that is less important in universities. As a result, argument (i) is appealing. In addition, the college's atmosphere is determined by the students' commitment and decorum, not by their clothes. As a result, argument (ii) is ambiguous.

13. Statement: Is it reasonable for the government to spend so much money on defence?

Arguments:

- (i) Yes. The country's security is of paramount importance.
- (ii) No, this money can be used to help the country prosper during peacetime.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.

- D. Both (i) and (ii) are strong.

Answer: A

Explanation: Unquestionably, defence is required for the country's safety, which is of paramount importance. As a result, the argument (i) is valid. Furthermore, only when a country is safe from external provocations can it focus on internal progress and development. As a result, argument (ii) is invalid.

14. Statement: Should a university or institution have a student association?

Arguments:

- (i) Yes. It is extremely important. Students are the next generation of political figures.
- (ii) No. This will give the campus a political vibe.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: The creation of a students' association will be a good initiative to provide students with basic political education. It will, however, foster a similar political climate on campus. As a result, both arguments are valid.

15. Statement: Is sex education something that should be taught in schools?

Arguments:

- (i) Yes. This is something that all progressive nations practise.
- (ii) No. In co-educational schools, we will be unable to impart it as it might distract the students and make them feel uncomfortable around each other.
- (iii) Yes. It would undoubtedly contribute to the removal of existing misunderstandings and the physical and emotional health of the youth of today.
- (iv) No. It will obliterate the moral integrity and highly regarded value system that our ancestors instilled in us.



- A. Only (i), (iii), and (iv) are strong
- B. Only (ii), (iii), and (iv) are strong
- C. All (i), (ii), (iii), and (iv) are strong
- D. None of the above

Answer: D

Explanation: Clearly, a policy cannot be implemented in India just on the basis that it is implemented in other nations, because each country has its environment and circumstances. As a result, argument (i) is a dubious proposition. In addition, providing sex education in co-educational institutions, where girls and boys study together, may degrade the atmosphere and impede academic progress. As a result, argument (ii) is convincing. Sex education at schools, on the other hand, might assist kids to clear up their preconceptions and concerns at a time when they might otherwise be hesitant to address the subject with others. Furthermore, sex is an important element of students' future lives, and knowledge of it is neither regressive nor embarrassing. As a result, argument (iii) is persuasive, whereas argument (iv) is ineffective. Therefore, only arguments (ii) and (iii) are strong.

16. Statement: Should a censor board be appointed to monitor commercials produced by various organisations?

Arguments:

- (i) Yes. Many times, a commercial has sparked heated controversy by depicting scenarios that are harmful to our cultural structures.
- (ii) No. Having a censor board for a variety of reasons is unrealistic.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: A

Explanation: Argument (i) is persuasive because many commercials contain undesirable content, which has sparked national debate. Argument

(ii) is unconvincing because commercials can be supervised by a censor board.

17. Statement: Should the state provide an "unemployment allowance" to educated unemployed youth of the nation?

Arguments:

- (i) Yes. It would provide them with financial assistance to help them find work or create a 'self-employment' business.
- (ii) No. It will stifle their will to work in order to support themselves, promoting lethargy among the jobless people.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: Young individuals who are unable to find work due to a huge number of applications in all professions must undoubtedly be provided with a stipend to help them maintain themselves. As a result, argument (i) is correct. Allowances like these, on the other hand, would stifle their will to work and render them indolent. As a result, argument (ii) also holds true.

18. Statement: Should all new and major factories be based in Bangalore?

Arguments:

- (i) Yes. There will be more employment opportunities.
- (ii) No. The city's pollution will worsen.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: In this case, both of the arguments are viable. The establishment of new factories will create more employment



opportunities, but it will also raise pollution.

19. Statement: Should all university examination bodies allow calculators to be used during exams?

Arguments:

- (i) Yes. With the advent of information technology in all sectors, human computations are no longer necessary.
- (ii) No. Knowing how to do manual calculations is crucial for students to understand their concepts.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: D

Explanation: People require manual computation methods, and they must be encouraged. As a result, argument (ii) is strong. Argument (i) is also strong in its own way as with the extensive use of computers in all sectors, there is very little need for a person to do manual calculations. Therefore, both arguments (i) and (ii) withstand.

20. Statement: Should the legal drinking age in bars and nightclubs be lowered from 21 years?

Arguments:

- (i) Yes. Once a person reaches the age of 18-20 years, the age at which one can correctly vote, he or she is mature enough to distinguish between the good and the evil, the right and the wrong.
- (ii) No. A person experiences a variety of circumstances by the time they reach this age. Emotional maturity and financial independence are usually achieved after this age. If the age restriction is lowered, the youth will be susceptible to face consequences as a result of uninformed decisions.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.

- D. Both (i) and (ii) are strong.

Answer: D

Explanation: Argument (i) is compelling because it asserts that people reach maturity between the ages of 18 and 20. Argument (ii) is also compelling because it asserts that one does not reach social maturity until they are 21 years old. As a result, both are compelling arguments.

21. Statement: Should the government take ownership of all engineering schools in the country?

Arguments:

- (i) Yes. This is the only way to make sure that all engineering students receive a standard education.
- (ii) No. The government lacks necessary funding to efficiently administer such institutions.
- (iii) No. Each engineering school should be permitted to operate autonomously.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Only arguments (i) and (iii) are strong.
 - D. None of the above.

Answer: D

Explanation: Clearly, if such institutes benefit the young generation of the country, the government can pool resources to administer them. As a result, (ii) does not hold up. (iii) fails to present a compelling argument. Consequently, government ownership of the institutes is not required to deliver greater education than is now provided. As a result, (i) is also invalid.

22. Statement: Should all individuals found guilty of severe offences such as rape or murder be sentenced to death or given the death penalty beyond a possible suspicion?

Arguments:

- (i) Yes. These kinds of harsh penalties will only deter people from doing such horrible atrocities, making the world a safer place.



- (ii) No. Anyone who has confessed to their crimes must be granted the opportunity to improve and live normally.
- A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: A

Explanation: A person who commits a horrendous crime such as rape or murder should, without a doubt, be severely punished in order to deter others from attempting such atrocities in the future. As a result, argument (i) is persuasive. Furthermore, it is impossible to know whether an offender is truly sorry for his actions; he may do so simply to avoid imprisonment. Therefore, argument (ii) fails.

23. Statement: Should all of the city's illegitimate buildings be totally destroyed?

Arguments:

- (i) Yes. By doing so, a strong message will be sent to the public, and they will avoid building illegal constructions.
- (ii) No. What will happen to the people who live in these buildings?
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: A

Explanation: Unauthorised structures would be destroyed to send a strong message to deceitful constructors and to remind people not to engage in similar projects in the area. Illegal structures place an unnecessary strain on the city's infrastructure, therefore this is critical. As a result, only argument (i) is valid.

24. Statement: Should handloom enterprises in rural regions be encouraged?

Arguments:

- (i) Yes. Individuals living in rural areas are very creative.

- (ii) Yes. To a certain degree, this would assist in resolving the unemployment crisis.
- (iii) No. People who live in rural areas are not skilled.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Both (i) and (ii) are strong.
 - D. All (i), (ii), and (iii) are strong.

Answer: B

Explanation: Handloom industries, certainly, need to be pushed in order to provide more employment opportunities for rural populations in their own localities. Therefore, (ii) holds strong. There is no proof that individuals living in rural areas are very creative or very unskilled; hence, arguments (i) and (iii) are vague.

25. Statement: Should a candidate's performance in an interview really be considered the deciding factor in his selection process?

Arguments:

- (i) Yes. It is one of the only ways to assess a candidate's personality, aspirations, and motivations.
- (ii) No. In terms of evaluation, it is extremely subjective.
 - A. Only argument (i) is strong.
 - B. Only argument (ii) is strong.
 - C. Neither (i) nor (ii) is strong.
 - D. Both (i) and (ii) are strong.

Answer: B

Explanation: Apparently, aside from interviews, there are various forms of written assessment that can be used to assess a candidate's interests. As a result, argument (i) is insufficient. Without a doubt, the interview is a subjective evaluation of a candidate's personality. Therefore, argument (ii) is valid.



The word syllogism means ‘conclusion, inference’. Syllogisms are a type of logical argument that uses deductive reasoning to reach a conclusion.

Syllogism is an important topic, and we find question from this topic in various engineering entrance exams. This topic is often asked in GATE. This chapter will explain this concept in detail and also include some practice questions for you can practice. After reading this chapter and solving questions, you do not have to go to any source for practicing this topic.

Two or more statements follow each of the questions in this section, and two or more conclusions follow these statements. It is necessary to determine which of these conclusions logically follows from the given statements. Even if the statements appear to contradict commonly held facts, they must be considered true.

Syllogism questions can be solved in a number of ways. Among all the available methods, a Venn diagram is the most effective and efficient method. It is essential to draw all possible diagrams based on the given statements and then to solve each of these diagrams separately. Finally, the correct answer is the one that is common in all of the diagrams.

Usually, the questions are given in the following way:

Directions: In the following type of questions, two statements are provided followed by two conclusions A and B. You have to read the two statements and then decide whether from those statements,

- (a) Only A follows
- (b) Only B follows
- (c) Both A and B follow
- (d) Either A or B follows
- (e) Neither A nor B follows

1. Statement:

All pine trees are coniferous.

All coniferous trees are beautiful.

Conclusion:

- A. All pine trees are beautiful.
- B. Some beautiful are pine trees.

Some extra tips and tricks to solve the problems of this chapter

1. Proposition

A proposition is that part that makes up a statement and shows that two things are related in some or the other way. It has three parts: 1: subject; 2: predicate; and 3: relation between subject and predicate.

Examples included are as follows:

- All tigers are big.
- Some boys are handsome.
- The girls of this school are not disciplined.

Subject and predicate

The subject is the part of the proposition about which something is said. A predicate is that part of the proposition which is stated or related to the subject.

For Example: 1. Tigers and boys are all subjects mentioned in above examples, while big, handsome, and disciplined are predicates.

Categorical proposition

A categorical proposition makes a direct assertion. For example, ‘All S are P’, ‘No S is P’ and ‘Some S are P’. It has no categorical proposition. There are two types of categorical proposition:

1. Universal propositions: These either fully include the subject or fully exclude it.

Examples are as follows:

- a. All girls are cute.
- b. No boy is happy.

A proposition ‘All boys are bad’ is called a universal positive proposition. A proposition in the form of ‘No S is P’ is called a universal negative proposition.

2. Particular propositions: These either only partly include or only partly exclude



the subject while making a statement.
Examples are as follows:

1. Some rabbits are peacocks.
2. Some circles are not round.

They have clauses such as ‘some’, ‘not many’, ‘very little’, etc.

PRACTICE QUESTIONS

Questions 1-3: Two statements are given followed by two conclusions numbered I and II. You have to consider these statements to be true, even if they seem to vary from commonly known facts. Decide which of the given conclusions logically follows from the given statement.

1. Statements:

Some trains are trucks.

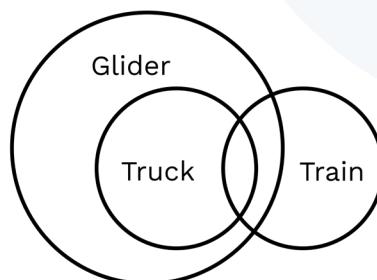
All trucks are gliders.

Conclusions:

- I. Some trucks are trains.
 - II. Some trains are gliders.
- A. If only conclusion I follows.
 - B. If only conclusion II follows.
 - C. If either conclusion I or conclusion II follows.
 - D. If both conclusion I and II follow.

Answer: D

Explanation: According to the diagram, both conclusions I and II follow.



2. Statements:

Some students are toppers.

No topper is a failure.

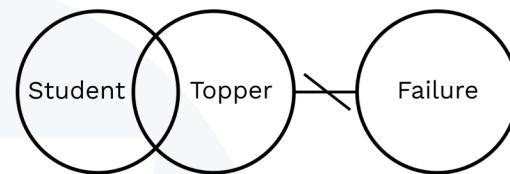
Conclusions:

- I. Some students are failure.
 - II. No student is a failure.
- A. Only conclusion I follows.
 - B. Only conclusion II follows.

- C. Either conclusion I or conclusion II follows.
- D. Neither conclusion I nor conclusion II follows.

Answer: C

Explanation: Either conclusion I or conclusion II follows.



From the diagram, we can infer that both conclusions I and II have the same elements and are individually wrong.

3. Statements:

No silver is a ring.

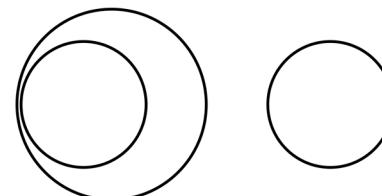
All golds are rings.

Conclusions:

- I. No gold is silver.
 - II. Some golds are rings.
- A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Both conclusions I and II follow.
 - D. Neither conclusions I nor conclusion II follows.

Answer: C

Explanation: Both conclusions I and II follow.



From the diagram, we can infer that both conclusions are true. Therefore, option C is correct.



Questions 4-10: Three statements are given, followed by three conclusions numbered I, II and III. You have to consider these statements to be true, even if they seem to vary from commonly known facts. Decide which of the given conclusions logically follows from the given statement.

4. Statements:

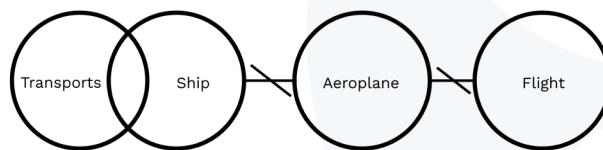
- Few transports are ships.
- No ship is an aeroplane.
- No aeroplanes are flight.

Conclusions:

- I. Some ships are transports.
- II. All transports are ship.
- III. No transports are aeroplane.
 - A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Only conclusion III follows.
 - D. None follows.

Answer: A

Explanation: Only conclusion I follows.



From the diagram, we can infer that some transports are ships, which is clearly stated in statement I.

5. Statements:

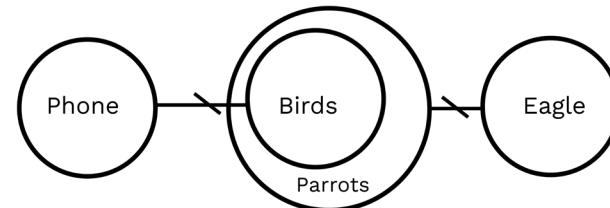
- No phone is a bird.
- All birds are parrots.
- No parrot is an eagle.

Conclusions:

- I. Some phones are birds.
- II. Some birds are parrots.
- III. No bird is a parrot.
 - A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Only conclusion III follows.
 - D. Only conclusions I and III follow.

Answer: B

Explanation: Only conclusion II follows.



From the diagram, we can infer that some birds are parrots, which is clearly stated in statement II. If all birds are parrots, then some birds are definitely parrots.

6. Statements:

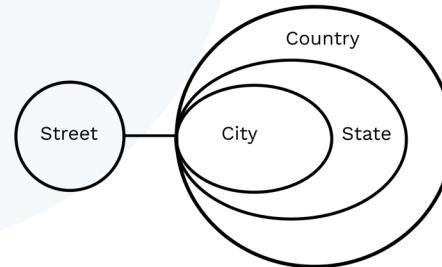
- No street is a city.
- All cities are states.
- All states are countries.

Conclusions:

- I. Some streets are states.
- II. Some cities are countries.
- III. All states are cities.
 - A. If only conclusion I follows.
 - B. If only conclusion II follows.
 - C. If either conclusion II or conclusion III follows.
 - D. If both conclusions I and II follow.

Answer: B

Explanation: Only conclusion II follows.



From the diagram, we can infer that all cities are states and all states are countries. Therefore, some cities are countries.

7. Statements:

- All actors are movies.
- Some movies are money.
- No money is rich.

Conclusions:

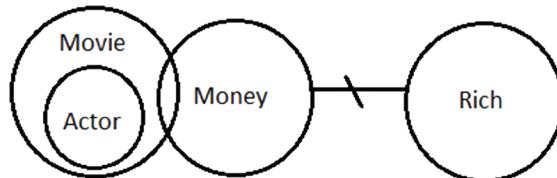
- I. Some actors are money.
- II. All actors are money.
- III. Some actors are movies.
 - A. Only conclusion I follows.
 - B. Only conclusion II follows.



- C. Only conclusion III follows.
- D. Only conclusions I and III follow.

Answer: C

Explanation: Only conclusion III follows.



From the diagram, we can infer that some actors are movies, which is clearly stated in statement I.

8. Statements:

Some rats are cats.

Some cats are bears.

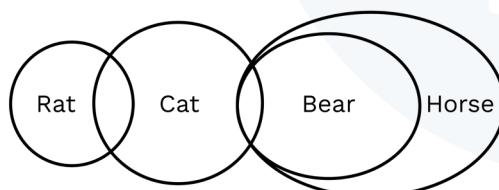
All bears are horses.

Conclusions:

- I. Some rats are horses.
- II. Some horses are cats.
- III. Some rats are bears.
 - A. If only conclusion I follows.
 - B. If only conclusion II follows.
 - C. If either conclusion II or conclusion III follows.
 - D. If both conclusions I and II follow.

Answer: B

Explanation: Only conclusion II follows.



From the diagram, we can infer that some cats are bears and all bears are horses. Therefore, some horses are cats.

9. Statements:

Few flowers are plants.

Some plants are not green.

All green are trees.

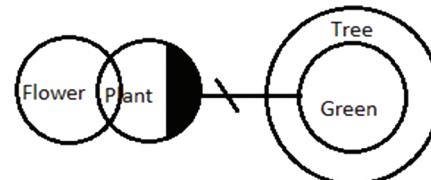
Conclusions:

- I. Some greens are not trees.
- II. Some greens are trees.
- III. No trees are plants.
 - A. Only conclusion I follows.
 - B. Only conclusion II follows.

- C. Only conclusion III follows.
- D. Either conclusion I or conclusion II follows.

Answer: B

Explanation: Only conclusion II follows.



From the diagram, we can infer that the second conclusion will follow; as, in syllogism, we know that, if in the statement we are given that all P are Q that means some P are Q conclusion will be true, so option B is correct.

10. Statements:

All pens are erasers.

All erasers are pencils.

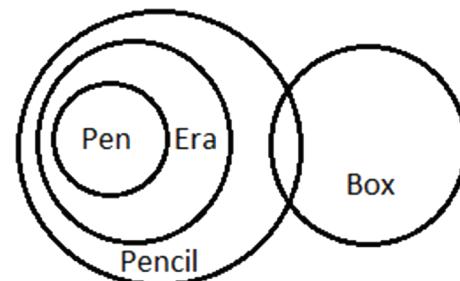
Some pencils are boxes.

Conclusions:

- I. No erasers are pencils.
- II. All pens are pencils.
- III. No pen is an eraser.
 - A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Only conclusion III follows.
 - D. Either conclusion II or conclusion III follows.

Answer: B

Explanation: Only conclusion II follows.



From the diagram, we can infer that all pens are pencils. Therefore, option B is correct.

Questions 11-20: Four statements are followed by four conclusions numbered I, II, III and IV.



You have to consider these statements to be true, even if they seem to vary with commonly known facts. Decide which of the given conclusions logically follows from the given statement.

11. Statements:

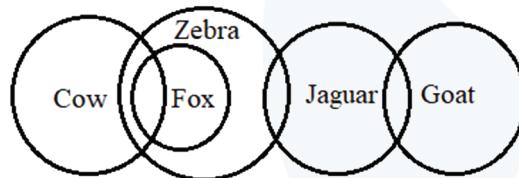
Some cows are foxes.
All foxes are zebras.
Some zebras are jaguars.
Some jaguars are goats.

Conclusions:

- I. Some foxes are jaguars.
 - II. Some foxes are zebras.
 - III. Some cows are zebras.
 - IV. Some jaguars are not goats.
- A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Only conclusions II and III follow.
 - D. Only conclusions III and IV follow.

Answer: C

Explanation: Only conclusions II and III follow.



From the diagram, we can infer that some foxes are zebras and some cows are zebras.

12. Statements:

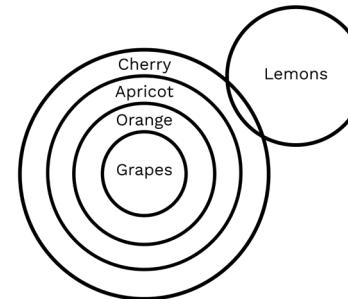
All grapes are oranges.
All oranges are apricots.
All apricots are cherries.
Some cherries are lemons.

Conclusions:

- I. Some grapes are cherries.
 - II. All oranges are cherries.
 - III. Some grapes are lemons.
 - IV. Some lemons are apricots
- A. If only conclusion I follows.
 - B. If only conclusion II follows.
 - C. If only conclusion III follows.
 - D. If both conclusions I and II follow.

Answer: A

Explanation:



According to the diagram, both conclusions I and II follow.

13. Statements:

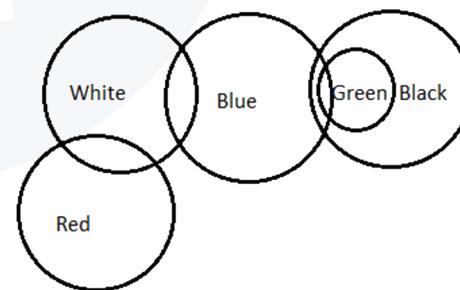
All greens are blacks.
Some greens are blues.
Some blues are whites.
Some whites are reds.

Conclusions:

- I. Some whites are greens.
 - II. Some greens are blacks.
 - III. No blue is black.
 - IV. All greens are blues.
- A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Only conclusion III follows.
 - D. Only conclusion IV follows.

Answer: B

Explanation: Only conclusion II follows.



From the diagram, we can infer that some greens are blacks, that is clearly stated in statement I. If all greens are blacks, then some greens are definitely blacks.

14. Statements:

Some oranges are grapes.
Some colours are trees.
All trees are sky.
No sky are grapes.

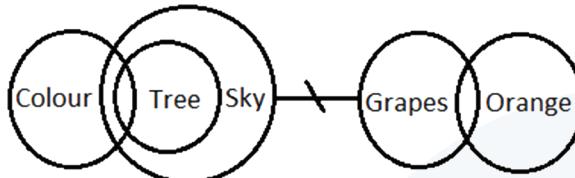


Conclusions:

- I. Some oranges are sky.
 - II. No oranges are trees.
 - III. Some grapes are trees.
 - IV. No trees are grapes.
- A. Only conclusion I follows.
B. Only conclusion II follows.
C. Only conclusion III follows.
D. Only conclusion IV follows.

Answer: D

Explanation: Only conclusion IV follows.



From the diagram, we can infer that no trees are grapes, which is already stated in statement IV.

15. Statements:

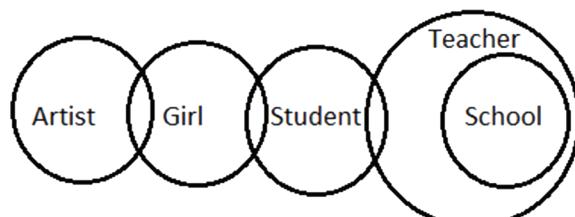
- Most artists are girls.
Few girls are students.
Some students are teachers.
All schools are teachers.

Conclusions:

- I. Some schools are teachers.
 - II. Few schools are students.
 - III. Some students are girls.
 - IV. No artists are teachers.
- A. Only conclusion I follows.
B. Only conclusion II follows.
C. Only conclusions I and III follow.
D. Only conclusions I and IV follow.

Answer: C

Explanation: Only conclusions I and III follow.



From the diagram, conclusion I and III follows.

16. Statements:

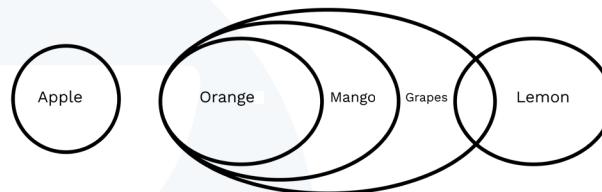
- No apple is an orange.
All oranges are mangoes.
All mangoes are grapes.
Some grapes are lemons.

Conclusions:

- I. All oranges are grapes.
 - II. Some apples are mangoes.
 - III. Some lemons are mangoes.
 - IV. Some apples are lemons.
- A. If only conclusion I follows.
B. If only conclusion II follows.
C. If only conclusion III follows.
D. If only conclusion IV follows.

Answer: A

Explanation: Only conclusion I follows.



From the diagram, we can infer that all oranges are mangoes and all mangoes are grapes. Therefore, all oranges are grapes.

17. Statements:

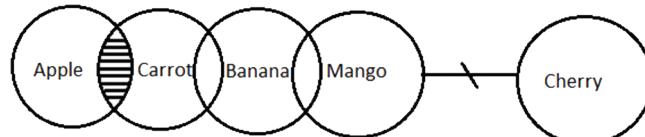
- Few apples are carrots.
Some carrots are bananas.
Some bananas are mangoes.
No mango is cherry.

Conclusions:

- I. Some apples are carrots.
 - II. Some carrots are mangoes.
 - III. Some bananas are not mangoes.
 - IV. No apple is banana.
- A. Only conclusion I follows.
B. Only conclusion II follows.
C. Only conclusion III follows.
D. Only conclusion IV follows.

Answer: A

Explanation: Only conclusion I follows.





From the diagram, we can infer that only conclusion I will follow.

18. Statements:

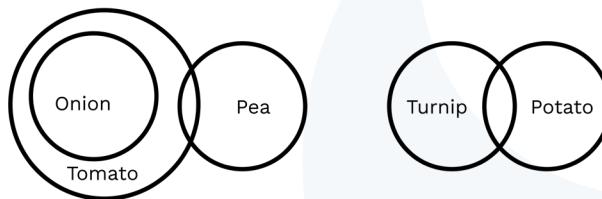
- All onions are tomatoes.
- Some tomatoes are peas.
- No pea is turnip.
- Some turnips are potatoes.

Conclusions:

- I. Some onions are peas.
- II. Some turnips are tomatoes.
- III. Some potatoes are onions.
- IV. Some turnips are onions.
 - A. If only conclusion I follows.
 - B. If only conclusion II follows.
 - C. None of the conclusions follow.
 - D. If both conclusions I and II follow.

Answer: C

Explanation: None of the conclusion follows.



According to the diagram, none of the conclusions follow.

19. Statements:

- Most crabs are shells.
- No shell is a mouse.
- All mice are fishes.
- Few fishes are boards.

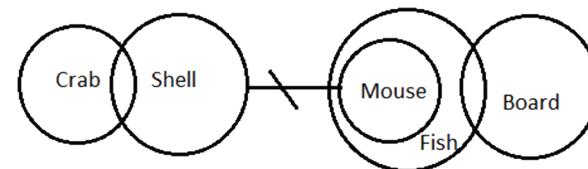
Conclusions:

- I. No shell is a fish.
- II. Some shells are fishes.
- III. Some crabs are boards.
- IV. Some mice are boards.
 - A. Only conclusion I follows.
 - B. Either conclusion I or conclusion III follows.

- C. Either conclusion I or conclusion II follows.
- D. Only conclusion IV follows.

Answer: C

Explanation: Either conclusion I or conclusion II follows.



From the diagram, we can infer that in conclusions I and II given elements are similar and individually wrong. Therefore, option C is correct.

20. Statement:

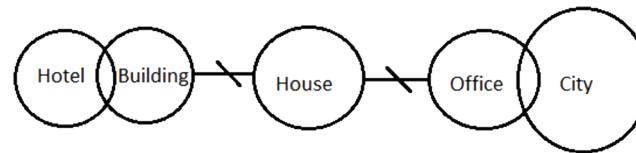
- Some hotels are buildings.
- No building is a house.
- No house is an office.
- Some offices are cities.

Conclusions:

- I. No office is a hotel.
- II. All offices are buildings.
- III. Some buildings are houses.
- IV. No office is a house.
 - A. Only conclusion I follows.
 - B. Only conclusion II follows.
 - C. Only conclusion III follows.
 - D. Only conclusion IV follows.

Answer: D

Explanation: Only conclusion IV follows.



From the diagram, we can infer that no office is a house, as stated in statement IV. Therefore, option D is correct.



Facts, Inferences, and Judgements or ‘FIJ’s’ is one of the simplest yet confusing critical reasoning topics that come in the entrance exams. For the past few years, Questions related to FIJs are important from the perspective of GATE and other engineering entrance exams. Questions related to FIJs have frequently been asked in engineering entrance exams. There are chances that questions related to this topic are asked in engineering entrance exams, including GATE.

INTRODUCTION

1. Facts are observable, verifiable information.

Facts are what you see, with no opinions or values added. They are learned through our senses: sight, smell, hearing, etc. They exist outside the observer, meaning they are a reality, not the observer’s impression of that reality.

You need to read the words carefully here. It is a piece of information that is known to people but may or may not be true.

Let us see an example:

‘The Earth is flat’ was a fact at that time, but not now.

All Reported Statements which are mechanisms for reporting the content of another utterance without directly quoting it are Facts.

Axioms that are a statement or claim that is considered to be proven, accepted, or self-evidently true, are Facts.

For example, Honesty is the best policy.

Here, we are stating what society has said or believes. And, since all reported statements are facts, it is a fact too.

Facts are:

- Universal Truths
- Someone else’s opinion stated by the author
- Verifiable
- Data without any opinion

- Reported statements
- Axioms
- Anything that is seen, heard or read

2. Inferences are based on analysis. They are unknowable propositions based on the known. They are drawn from reason, or, to put it another way, they are founded on our prior experience. They reside within the spectator and, as a result, might vary depending on the observer’s past experiences.

For example: If there is a tsunami, people will be killed.

(Expectations or possibilities of something happening as a result of something else.)

FACT + LOGICAL DEDUCTION → INFERENCE

An inference is:

- Based on reality
- Many implications can be drawn from a single fact.
- It usually follows a cause-and-effect pattern.
- An inference is unaffected by one’s point of view.
- If a fact is altered, it loses its validity and becomes invalid.

Let us look at an example to understand the difference between a fact and an inference:

1. Prateek attends all the lectures.
2. Prateek is very regular when it comes to attendance.

Statement 1 is a fact whereas Statement 2 is an inference.

3. Judgements are subjective observations.

When it comes to judgements, I prefer not to use the phrase Judgement because inferences are frequently opinions as well. However, judgements are declarations of acceptance or disapproval. They are derived from our ideals



and convey our thoughts and feelings about something.

A Judgement is

- Any comment expressing favour or disapproval is referred to as a judgement.
- Does not provide a rational explanation for the same.
- Changes depending on the point of view
- The author's viewpoint is his or her own.

Some tips to tackle FIJs (Fact Inference Judgement) :

- Determine the primary clause to determine the core of a statement.
- The sentences may appear to be related at times, but because they are numbered independently, you must treat them as separate sentences.
- Take a look at them from the author's perspective. It's not a matter of fact, inference, or judgement for us.

Some steps to follow:

1. Search for facts.

These are the statements that cannot be changed and hence, are facts. Look for statements that have a personality saying something / historical event / universal truth/ daily events etc.

2. Do not strike out any option because there is a possibility that you might have marked an inference as a fact.

3. Look for judgment.

Check the statements where someone is showing approval or disapproval or judging a situation as good or bad etc.

4. Search for inference

5. Check all the options with your obtained answer.

Now, let us work on some questions.

Example:

Consider the following statements and identify the facts, judgements and inferences among them.

1. The Swachh Bharat Abhiyaan, according to the minister, is a fantastic initiative.
 2. The idea has been adopted by the city's schools, and preparations to start a variety of programmes have been created.
 3. The Swachh Bharat Abhiyaan will be a success because many elements of society are contributing to it.
 4. The Swachh Bharat Abhiyaan will contribute significantly to a cleaner India.
- A. FIJF B. FIIJ
C. FFIJ D. JFFI

Answer: C

Explanation: Statement 1: According to the minister, the Swachh Bharat Abhiyaan is a fantastic effort. The keyword in this sentence is - Minister stated, which indicates that we are quoting someone's opinion and so making it verifiable. It is a FACT because it can be proven.

Statement 2: The idea has been adopted by the city's schools, and plans to launch a variety of programmes have been created. This entire sentence is verifiable, making it a FACT.

Statement 3: The Swachh Bharat Abhiyaan will be a success because many strata of society are contributing to it. By using the term success, this statement is expressing approval for the Swachh Bharat Abhiyaan. But consider the first part of the sentence: success is the result of diverse segments of society contributing to it. As a result, this is an INFERENCE because it is an unknown conclusion based on known data.

Statement 4: The Swachh Bharat Abhiyaan would help India become cleaner. This statement expresses approbation but does not explain why, making it a JUDGEMENT.



PRACTICE QUESTIONS

1. Statements

- I. Privacy is the protection of one's personal information by oneself and security is how your personal information is protected by the authorized ones.
 - II. The technological advancements humans are making are undoubtedly the need of the house, but they also come with some repercussions.
 - III. When someone's data gets into the wrong hands, it can prove to be dangerous.
 - IV. Like we try to maintain balance in our daily life, it is also necessary to maintain the balance between the risks and benefits of trusting someone.
- A. FJIJ B. FIJJ
C. JIFF D. JIIF

Answer: A

Explanation: Statement 1 is a fact as it provides definitions of the words like privacy and security which can be verified. Statement 2 is a judgement because it presents an opinion regarding technological advancements and further the positive or negative effects of these causes that are technological advancements can be challenged. Statement 3 is an inference because here lies a condition that if someone's data gets into the wrong hands, it 'can' prove to be dangerous, so it forms a cause-effect relationship that is verifiable as well. Statement 4 is again an opinion so it should be a judgement. Therefore, the correct answer is option A.

2. Statements

- I. Photoshop is a popular tool in the fashion photography field.
- II. Zendaya, a model in an interview said that "I believe that photo editors' primary goal is to promote a product; the alterations they make frequently

have negative consequences for viewers."

- III. The truth is that the retouching done by these editors has far-reaching consequences for our society.
 - IV. Photoshop editors and magazine photographers must recognise that they have a significant impact on how people perceive themselves and must accept that societal duty.
- A. FJJJ B. FFJJ
C. JIFF D. JIIF

Answer: B

Explanation: Statement 1 is a fact as it provides a verifiable and observable statement. Statement 2 is a fact because it is a stated or reported statement about what Zendaya, a model, feels about photoshopping and its effect on people and society at large. Statements 3 and 4 are judgements because here the person, who wrote this piece of article is presenting his opinion about photoshopping and editing and further opinionated that they (photo shippers) must recognise that they have a significant impact on how people perceive themselves and must accept that societal duty. Therefore, the correct answer is option B.

3. Statements

- I. The definition of ambition is motivation or a strong urge to achieve something. An example of someone who has ambition is an ice skater who practices for hours each day in hopes of competing in the Olympics
- II. Ambition is like choler; which is humour that makes men active, earnest, full of alacrity, and stirring, if it is not stopped.
- III. But if Ambition is stopped, and cannot have his way, it becometh adust, and thereby malign and venomous.



- IV. Good commanders in the wars must be taken, be they never so ambitious; for the use of their service, dispenses with the rest; and to take a soldier without ambition, is to pull off his spurs.
- A. FJJI B. JFJI
C. FIIJ D. FJIJ

Answer: D

Explanation: Statement 1 is a fact because it gives the general meaning of ambition which is the same for everyone. Statement 2 is a judgement because the writer here presents his view of what ambition is like or about, it's his opinion. Statement 3 is an inference because here the cause and effect relationship can be established like stooping an ambitious person can be detrimental as it will make him venomous. Statement 4 is a Judgement because the writer is presenting his view about an ambitious person via an example. Therefore, the correct answer is option D.

4. Statements

- I. The word Feminism is one of the most misused and misapprehended words of all time.
 - II. But it's imperative we realise that feminism is not just a women's movement, it's a 'movement for all humans', that is concerned with the liberation of both, men and women.
 - III. This discrimination is reflected in the statistics released by the government, which states that almost 50 per cent of teenage Indian girls are underweight and 52 per cent are anaemic.
 - IV. Working women in India earn only 66% of what their male counterparts earn for the same amount of work. This gender pay gap and disparity in opportunities discourage women from performing well in the professional domain.
- A. FJJI B. JIFI
C. FIIJ D. FJIJ

Answer: B

Explanation: Statement 1 is a Judgement because it tells the opinion of the author about feminism which might not be the same for everyone. Statement 2 is an inference because here the cause and effect relationship can be established like 'movement for all' leads to "liberation of all men and women". Statement 3 is a fact since it presents data that can be verified. Statement 4 is an inference because the writer is presenting his data and then substantiating it with his or her judgement leading to inference, in short, it is like less pay in jobs leads to a gender gap which ultimately leads to poor performance of women in the professional domain. Therefore, the correct answer is option B.

5. Statements

- I. A report suggests that India would be 27% richer if there were higher employment rates for women.
 - II. In India, females make up only 11.2% of board members in firms which is less than the global average.
 - III. India needs feminism because even professions and family roles have been stereotyped based on gender.
 - IV. According to Paul Marth, a sociologist "Stereotyping in family roles includes how men are expected to be the sole breadwinners of a family and females are presumed to single-handedly take up the responsibilities of managing the home."
- A. FJJI B. JIFI
C. FIIJ D. FJIJ

Answer: D

Explanation: Statement 1 is a fact because it presents data. Statement 2 is a judgement because it compares the data which can be verified. Statement 3 is an inference because here the cause and effect relationship can be established like India needs feminism because (why?) professions and family roles have been stereotyped based on gender. Statement 4 is a



fact because it is quoting someone's opinion. Therefore, the correct answer is option D.

6. Statements

- I. Over the last 20 years, food grain production in India has risen from 198 million tonnes to 269 million tonnes.
- II. Ideally, this should have ensured that nobody went without access to food.
- III. National Food Security Act of 2013 covers 75% of the rural population and 50% of the urban population under its Targeted Public Distribution System.
- IV. Lack of food production or disruption in food distribution systems is the reason why India still struggles to battle hunger.
A. FJFI B. JIFI
C. JIFJ D. FJIF

Answer: A

Explanation: Statement 1 is a fact because it is presenting data. Statement 2 is a judgement because it is presenting an opinion and comparing as well that the larger food production should ensure that India moves out of starvation. Statement 3 is a fact as it is presenting data. Statement 4 is an inference because here the cause and effect relationship can be established like India still struggles to battle hunger due to Lack of food production or disruption in food distribution systems. Therefore, the correct answer is option A.

7. Statements

- I. The abundance of food in the market is not sufficient to eradicate hunger unless people have the required purchasing power.
- II. Naturally, the poor have stopped consuming the more expensive food, which is relatively richer in nutrients. This is likely to further exacerbate the undernutrition crisis in India.
- III. Furthermore, out of the total funds allocated for the Mid-Day Meal Scheme

for children in 2018-'19, only 14 states in India utilised the funds entirely.

- IV. While the government has rejected the findings of the Global Hunger Index as "unscientific", we cannot ignore the dismal ground realities.
- A. FJFI B. JIFI
C. JIFJ D. FJIF

Answer: C

Explanation: Statement 1 is a Judgement because the writer is presenting his opinion that an abundance of food on the market will not be enough to eliminate hunger unless people have the financial means to buy it. Statement 2 is an inference because it is establishing a cause and effect relationship. The cause is that the impoverished have ceased eating the more expensive, nutrient-dense food and the effect will be that the undernutrition epidemic in India is likely to worsen. Statement 3 is a fact as it is presenting data. Statement 4 is a judgement as the writer is presenting his opinion that rejecting the findings of the Global Hunger Index is something through which we cannot ignore the dismal ground realities. Therefore, the correct answer is option C.

8. Statements

- I. The idea that some characteristics of an organism are explained by the organism's intrinsic nature, whilst others reflect the influence of the environment, is an ancient one.
- II. It has even been argued that this distinction is itself part of the evolved psychology of the human species.
- III. The idea that heritability scores measure the degree to which a characteristic is innate is a vulgar fallacy.
- IV. The belief that a trait is innate is today commonly expressed by saying it is 'in the genes' but genes play an essential role in the production of every trait.
A. FJFI B. JIFI
C. IIFJ D. FJJJ



Answer: D

Explanation: Statement 1 is a fact because it is presenting a piece of information that has been proved many times earlier as well and it is now a proven and accepted statement. It's a biological fact. Statement 2 is judgement because here the writer is presenting his opinion about the distinction between an organism's intrinsic nature and other extrinsic nature. Statement 3 is again a judgement because the writer has an opinion that it is a common mistake to believe that heritability ratings reflect the degree to which a trait is innate. Statement 4 is again a judgement because the writer has an opinion that it is an assumption that a trait is innate, which is now widely articulated as 'in the genes,' however genes play an important part in the development of every trait. Therefore, the correct answer is option D.

9. Statements

- I. The Indian government had promised 10% of the GDP as Atmanirbhar (self-reliant) package for Covid recovery.
- II. We believe if this amount is spent judiciously on what we call an Indian Green Deal (IGD), India can come out on top of the crisis and stay ahead of the climate change curve.
- III. Further, the 5% investment in Infrastructure helps in the employment generating capacity of these sectors which is quite high when compared to the other employment generation policies will lead to the success of the Indian Green Deal
- IV. The green energy programme would result in curbing India's total carbon emissions by 0.8 gigatonnes by 2030.
A. FJFI B. FJIJ
C. IIFJ D. FJJI

Answer: B

Explanation: Statement 1 is a fact because it is presenting data. Statement 2 is a judgement because it is presenting an opinion and further

predicting the belief that if this money is invested wisely in an Indian Green Deal (IGD), India would be able to weather the storm and stay ahead of the climate change curve. Statement 3 is an inference because it is trying to establish the connection between facts by using logical deduction to reach the conclusion. The fact that a 5% investment in Infrastructure will help in generating more employment opportunities than other policies leads us to the conclusion that the Indian Green Deal will succeed. Therefore, the correct answer is option B.

10. Statements

- I. India has refused to join an alliance to phase out coal on an urgent basis because it is aware that to phase out coal at this time is near an impossible task as 72% of India's electricity is generated by coal-fired power plants.
- II. Coal is inexpensive, and growing economies like India want low-cost fuels to meet their per capita demand.
- III. India contributes less to global pollution because India's carbon emissions are less as it accounts for 3% of total global emissions compared to 25% for the United States.
- IV. A green economy is a win-win proposition both on emissions (and pollution) and employment.
A. FJFI B. IIIJ
C. IIFJ D. FJJI

Answer: B

Explanation: Statement 1 is an inference because it is trying to establish the connection between facts by using logical deduction to reach the conclusion. India has refused to join an alliance to phase out coal on an urgent basis because it recognises that doing so at this time is nearly unachievable, as coal-fired power plants supply 72 per cent of India's electricity. Statement 2 is again an inference as it is trying to establish a cause and effect relationship.



Growing economies like India want low-cost fuels to meet its per capita demand that's why it is resorting to coal which is inexpensive. Statement 3 is trying to establish a comparison between India and US in terms of carbon emission to reach a conclusion that India contributes less to global pollution. Statement 4 is a judgement as it is presenting the writer's opinion. Therefore, the correct answer is option B.

11. Statements

- I. Nobody has the right to not be offended. That right doesn't exist in any declaration I have ever read.
 - II. Acclaimed novelist Salman Rushdie said, "If you are offended, it is your problem, and frankly, lots of things offend lots of people."
 - III. A speech is called hate speech when the speech is 'offensive' and projects the 'extreme' form of emotion.
 - IV. The fact that defining features of sentiments is subjective leads to the rampant misuse of this clause and ultimately to the suppression of free speech.
- A. FJFI B. IIJJ
C. JFFI D. FJJI

Answer: C

Explanation: Statement 1 is a judgement by the writer as he is presenting his opinion. Statement 2 is a fact because it is quoting something that someone else has said. Statement 3 is again a fact because it is underlying a simple definition of hate speech which might be defined somewhere like any statute or law and is the same for everyone. Statement 4 is an inference because it is trying to establish a relationship between two things to reach a conclusion. The fact that defining features of sentiments is subjective results in rampant misuse of this clause and ultimately resulted in the suppression of free speech. Therefore, the correct answer is option C.

12. Statements

- I. Climate is sometimes mistaken for the weather.
 - II. Climate is measured over a long period of time, whereas weather can change from day to day, or from year to year.
 - III. Different places can have different climates.
 - IV. In polar regions, the warming global temperatures associated with climate change have meant ice sheets and glaciers are melting at an accelerated rate from season to season.
- A. JFJI B. IIJJ
C. JFJI D. FJJI

Answer: C

Explanation: Statement 1 is a judgement because it is presenting an opinion and comparison. Statement 2 is a fact because it is presenting a simple definition that is verifiable as it stands the same for everyone. Statement 3 is a judgement because it is a possibility that different places 'can' have different climates. Statement 4 is an inference because here the cause and effect relationship can be established like in polar regions, the warming global temperatures associated with climate change which is a cause that results in ice sheets and glacial melting at an accelerated rate from season to season. Therefore, the correct answer is option C.

13. Statements

- I. The focus on maternal mental health has not become an integral part of India's health care, despite India's national mental health program enacted in 1982.
- II. National statistics on Postpartum depression (PPD) in Indian mothers are lacking as data is limited to specific geographic regions.
- III. The overall aggregated prevalence of PPD in Indian mothers is estimated to be 22%, with the highest prevalence in



- urban areas (24%) and the southern regions.
- IV. Southern areas have a higher prevalence of maternal mental health issues due to increased urban slums and domestic violence levels.
- A. JFJI B. IIJJ
C. JFFI D. JIFI

Answer: D

Explanation: Statement 1 is a judgement because here the writer is concerned about maternal mental health and wants it to become an issue of discussion by terming it as ‘an integral part of India’s health care’. But this maternal mental health concern might not hold the same importance for others, something which is non-verifiable. Statement 2 is an inference because it is establishing a cause and effect relation. The effect is that the National statistics on Postpartum depression (PPD) in Indian mothers are lacking and the cause is that the data is limited to specific geographic regions. Statement 3 is a fact as it is presenting data. Statement 4 is an interrogating statement because it is answering ‘why’. Here the issue is that the Southern areas have a higher prevalence of maternal mental health issues and “why” because of increased urban slums and domestic violence levels. So it is an inference. Therefore, the correct answer is option D.

14. Statements

- I. A girl’s birth is not preferred in India as she is typically considered a burden to her parents.
- II. There are expectations for a woman to deliver a male child.
- III. Discrimination against a female child may lead to foeticide or infanticide, a common practice along the ‘infanticide belt’ including Tamil Nadu.
- IV. Childbirth is labelled as the happiest time in a woman’s life; thus, feelings of anxiety or sorrow are considered taboo.

- A. JJIJ B. IIJJ
C. JFFI D. JIFI

Answer: A

Explanation: Statement 1 is a judgement because it is an opinion of the writer that a girl’s birth is not preferred in India as she is typically considered a burden to her parents which can or cannot be the same for all, so it can’t be verified. Statement 2 is again a judgement because it is an opinion of the writer that there are expectations for a woman to deliver a male child which can or cannot be the same for all, so it can’t be verified. Statement 3 is an inference because it is establishing a cause and effect relation. The cause is discrimination and its effect is infanticide and foeticide. Statement 4 is again a judgement that is non-verifiable cause it might be different for different peoples. Therefore, the correct answer is option A.

15. Statements

- I. The most notable distinction between living and inanimate things is that the former maintain themselves by renewal.
 - II. A stone when struck resists. If its resistance is greater than the force of the blow struck, it remains outwardly unchanged.
 - III. While the living thing may easily be crushed by superior force, it nonetheless tries to turn the energies that act upon it into means of its own further existence.
 - IV. The plant uses light, air, moisture, and the material of soil to survive.
- A. JJII B. IIJJ
C. JIJF D. JIFI

Answer: C

Explanation: Statement 1 is a judgement as it is an opinion of the writer. Further, the distinction between living and inanimate things that is the former maintains itself by renewal this “the most notable” for the writer but might not be



for others, so it is not verifiable. Statement 2 is an inference because here we can establish a cause and effect relationship. The cause is that when the stone's resistance is greater than the force of the blow struck and the effect is that it remains outwardly unchanged. Statement 3 is a judgement because it is an opinion that despite the fact that it is easily crushed by superior power, the living thing tries to transform the energies that act on it into means of its own continued life. Statement 4 is a fact because it is a universal truth. Therefore, the correct answer is option C.

16. Statements

- I. In July 2020, India's peak electricity demand witnessed a new high at 200.57 GW.
- II. This new peak demand is indicative of the consistently rapid growth of per capita electricity consumption because it has increased from 914 kWh in 2012-13 to 1208 kWh in the 2019-2020 fiscal year, an increase of 32 per cent.
- III. While demand has been witnessing exponential growth, India's electricity generation capacity has not been able to keep pace.
- IV. The per capita energy consumption in India is close to half of the global standard on average in 2020.
A. JJII B. IIJF
C. FIJF D. JIFI

Answer: C

Explanation: Statement I is a fact as it is presenting data. Statement 2 is an inference because it is not just presenting data but at the same time it is comparing these data to reach a conclusion that due to India's per capita energy demand is increasing 'consistently'. So, it has used a fact and applied a logical deduction to reach a conclusion therefore it's an Inference. Statement 3 is a judgement as it is merely presenting an opinion without substantiating

it with facts. Statement 4 is again a fact as it is presenting verifiable data. Therefore, the correct answer is option C.

17. Statements

- I. Owing to the rapidly increasing industrial base and an aspirational population with increasing purchasing power, India's energy demand in the next few years is set to grow manifold.
- II. At present, coal accounts for a larger portion of the energy generation capacity of India.
- III. India has already committed to net-zero emission by 2070 during the recent Cop26 event.
- IV. In order to meet net-zero emissions the country not only needs to reduce coal dependency but also rapidly expand renewable energy capacity to meet the expected rise in demand.
A. JJII B. IJFJ
C. FIJF D. JIFI

Answer: B

Explanation: Statement I is an inference because it is establishing a cause and effect relationship which is- India's energy demand is expected to skyrocket in the coming years as a result of its fast-expanding industrial base and aspirational populace with rising purchasing power. Statement 2 is judgement as the adjective 'larger' is not defined so it might be different for different peoples. Statement 3 is a fact that can be verified. Statement 4 is a judgement as it is suggesting that to achieve net-zero emissions, the government must not only cut its reliance on coal but also swiftly grow renewable energy production to satisfy anticipated demand, so it is a kind of opinion. Therefore, the correct answer is option B.

18. Statements

- I. Globally, Nuclear power contributed close to 10.4% of electricity production in 2019, just 0.2 per cent higher than the previous year, though it has



- been reported the first increase in production since 2015.
- II. USA, France, China, and Russia are leading in nuclear energy production and India is far behind these countries as they are developed nations.
- III. The lack of progress on the expansion of India's nuclear energy capacity is particularly puzzling given that the country is not exactly a late adopter of the technology.
- IV. India has been operating nuclear reactors since 1957.
- A. JJII B. IJFJ
C. FIJF D. JIFI

Answer: C

Explanation: Statement 1 is a fact because it is presenting data. Statement 2 is an inference because it is comparing India which is a developing nation with other developed nations in terms of nuclear energy capacity. Statement 3 is a judgement because it has used the adjective 'puzzling' in terms of India's performance in nuclear energy. This tells us that this fact about of India's performance might not be 'puzzling' for everyone. Statement 4 is a fact. Therefore, the correct answer is option C.

19. Statements

- I. With booming internet users on a daily basis, people are getting addicted to a new drug called the Internet.
- II. 4.66 billion people around the world used the internet in January 2021, up by 316 million (7.3 per cent) since this time last year.
- III. Global internet penetration now stands at 59.5% which is more than half of the world population, and with this rate within a decade, almost everyone on the planet will have internet access.
- IV. Marketers are using this for their benefit.
- A. JJII B. JFJJ
C. FIJF D. JIFI

Answer: B

Explanation: Statement 1 is a judgement as it is a mere opinion of the writer which can or cannot be true for others. Statement 2 is a fact as it is presenting data. Statement 3 is a judgement as it is predicting the future that within a decade almost everyone on the planet will have internet access which can or cannot happen. Statement 4 is again a judgement as it is the opinion of the writer which is not the same for everyone. Therefore, the correct answer is option B.

20. Statements

- I. When we say brand, it is not just the products we have to offer but it is a combination of vision, mission, goals, and beliefs of the company which makes it a brand.
- II. Geoffrey, a market analyst says "Brand is something that is able to attract the target audience and they can also explain to the customers that it will add value to their lives and this is how a brand builds identity."
- III. Sports brands are the best example of brand-building via content marketing.
- IV. In 2021, the Nike brand was valued at approximately 30.44 billion U.S. dollars, which was a decrease of over four billion U.S. dollars from 2020.
- A. JJII B. JFJJ
C. JIHF D. JFJF

Answer: D

Explanation: Statement 1 is a sort of suggestion that the writer is offering regarding brand therefore it's a judgement. Statement 2 is a fact because it is quoting someone's opinion and we have discussed earlier that reported or quoted statements are facts. Statement 3 is a judgement as we are using the adjective 'best' so, there can be cases like sports branding may be considered to be best for some persons and may not be by others. Therefore, it cannot be verified. Statement 4 is a fact as it is presenting data. Therefore, the correct answer is option D.



21. Statements

- I. Brazil has been a close multilateral partner in Indian foreign policy after the Cold War.
- II. It has been India's closest partner in BRICS (Brazil, Russia, India, China, and South Africa) therefore it has cooperated in dealing with the climate crisis, Security Council reform, and South-South cooperation.
- III. One reason for the non-fulfilment of desired goals is the lack of economic and strategic links between tier-two powers.
- IV. Mr Bolsonaro thus quotes Brazil and India's relationship to be 'very benevolent'.
 - A. JFII
 - B. FFJJ
 - C. JIIF
 - D. JFJI

Answer: C

Explanation: Statement I is a judgement because it's somebody's opinion that Brazil has been a close multilateral partner in Indian foreign policy after the Cold War, it can or can't be true for the public in general. Statement II is an inference because it is assigning reason to substantiate why India and Brazil have been good friends. Statement III is an inference because it is again substantiating the reason for a cause and statement IV is a fact because it's a quoted statement. Therefore the answer is C.

22. Statements

- I. In a world marked by increased geopolitical rivalry and reduced multilateral cohesion, many two-tier nations are strengthening relations.
- II. The most striking example in the Indian context has been the deep relationship that has emerged with Brazil.
- III. Brazil is a developing economy because it has successfully implemented universal health care, adopted genetically modified crops, and navigated the urbanisation process.

- IV. Mr Bolsonaro has made deeply objectionable statements about women and climate.
 - A. JJIF
 - B. FFJJ
 - C. JIIF
 - D. JFJI

Answer: A

Explanation: Statement I is a judgement because it is a mere opinion of the author, there could be the possibility that it is just a perception that is obviously different for different peoples. Statement II is a judgement because it is not assigning the reason for this declaration. Statement III is an inference because there is a cause-and-effect relationship, Brazil has successfully implemented universal health care, adopted genetically modified crops, and navigated the urbanisation process which is the cause for it being developed. Statement IV is a fact because it is a quoted statement or incident which can be verified. Therefore the answer is A.

23. Statements

- I. Human beings are naturally visual creatures.
- II. Our eyes, capable of counting single photons, have been optimized over evolutionary time to the very limits of the laws of physics.
- III. It's no surprise that we live in an era in which the visible competes for our attention ever more forcefully.
- IV. Our eyes and minds are bombarded by information, much of which we don't have time to process, let alone fully understand.
 - A. JJIF
 - B. FFJI
 - C. JIII
 - D. FFJI

Answer: D

Explanation: Statement I is a fact because it can be verified as humans can be seen. Statement II is a fact because it's a natural truth that evaluation is necessary and is evident in many things as well. Statement III is a judgement because it's an opinion of the



writer which can or can't be true. Statement IV is an inference because it is establishing a relationship as to why the human mind can't process all the information because it's a sort of impossible task. Therefore the answer is D.

24. Statements

- I. Forest fires or bushfires have been a part of "traditional" narratives in Australia, featuring prominently even in storytelling.
- II. However, the climate crisis has changed the traditional narrative of such 'normality' in current eco-fiction, exacerbating such events as disastrous and beyond the limits of the resilience of ecosystems.
- III. Starting in August 2019, this time, Australia's average temperature rise has been estimated to be about 1.4° Celsius above pre-industrial levels.
- IV. With Australia's average summer temperatures increasing, there has been an increase in the frequency and intensity of heatwaves and droughts
 - A. JIFJ
 - B. FIFI
 - C. JIIF
 - D. FIJF

Answer: B

Explanation: Statement I is a fact because it can be verified whether the Forest fires, often known as bushfires, have long been a component of 'traditional' Australian traditions, even appearing prominently in storytelling or not. Statement II is an inference because it is assigning the reason for the decreasing or vanishing of the traditional narrative.

Statement III is a fact because it is verifiable data. Statement IV is an inference because it has assigned the reason for the fact that the frequency and intensity of heatwaves and droughts have increased because Australia's average summer temperatures have risen. Therefore the answer is B.

25. Statements

- I. Air pollution is now a public health emergency.
- II. As per the World Health Organisation, 14 of the 15 most polluted cities in the world are in India.
- III. Many cities in the world have experienced high pollution levels, and have made substantial progress with a sustained effort to eradicate them.
- IV. Beijing is a case in point, which adopted an intensive air pollution control strategy in 2013 — by the end of 2017, levels of dangerous fine particulate matter (PM2.5) had reduced by 35 per cent.
 - A. JIFJ
 - B. FIFI
 - C. FFIF
 - D. JIJF

Answer: C

Explanation: Statement I is a FACT as it is something that is evident and can be verified. Statement II is a fact because it is presenting data. Statement III is an inference because it is establishing a cause and effect relationship. Many cities throughout the world have endured high levels of pollution that's why they have made significant progress with consistent efforts to eradicate it. Statement IV is a fact as it is presenting data. Therefore the answer is C.



SYNOPSIS

- Introduction
- Factorial
- Permutations
- Combinations

INTRODUCTION

The concept of permutation and combination is used to select or arrange some items out of a group according to certain predetermined conditions. This concept helps find the number

of ways to arrange, select, or reject these items. The concepts of permutations and combinations derive from the principles of factorials the fundamental principle of counting.

FACTORIAL

The factorial of a natural number n , denoted by $n!$ And is the product of all natural numbers from 1 up to n .

$$n! = 1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n$$

OR

$$n! = n \times \{(n-1) \times (n-2) \times (n-3) \times (n-4) \times \dots \times 1\}$$

The factorials of the first few natural numbers given as follows:

1!	1	1
2!	2×1	2
3!	$3 \times 2 \times 1$	6
4!	$4 \times 3 \times 2 \times 1$	24
5!	$5 \times 4 \times 3 \times 2 \times 1$	120
6!	$6 \times 5 \times 4 \times 3 \times 2 \times 1$	720
7!	$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	5040
8!	$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	40320
9!	$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	362880
10!	$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	3628800

Example: What is the value of $5! \times 3!$?

Solution: First expand both the factorials,

$$\begin{aligned} &= (5 \times 4 \times 3 \times 2 \times 1) \times (3 \times 2 \times 1) \\ &= 120 \times 6 = 720 \end{aligned}$$

Example: What is the value of $\frac{(12! \times 8!)}{(9! \times 10!)}?$

Solution: First of all, simplify the above given factorials,

$$\begin{aligned} &\frac{12 \times 11 \times 10! \times 8!}{9 \times 8! \times 10!} = \frac{12 \times 11}{9} \\ &= \frac{132}{9} = \frac{44}{3} \end{aligned}$$



PERMUTATIONS

If there are n objects and one is supposed to arrange r ($r \leq n$) objects out of these, then the number of ways in which this can be done is written as ${}^n P_r$ and is read as ‘the number of permutations of n objects taken r at a time’.

The number of permutations is given as:

$${}^n P_r = \frac{n!}{(n-r)!}$$

Example: In how many ways can you arrange two books out of three?

$$\text{Solution: } {}^3 P_2 = \frac{3!}{(3-2)!}$$

$$= \frac{3!}{1!} = 3 \times 2 = 6$$

Some specific cases related to permutations are:

If $r = 0$,

$${}^n P_0 = \frac{n!}{(n-0)!} = 1$$

If $r = 1$,

$${}^n P_1 = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

If $r = n$,

$${}^n P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

If $r = n - 1$,

$${}^n P_{n-1} = \frac{n!}{(n-n+1)!} = \frac{n!}{1!} = n!$$

Example: Find the value of ${}^5 P_5$.

Solution: As we know, ${}^n P_n = n!$

$$\text{So, } {}^5 P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Permutations of repeated objects

If n objects are to be arranged among themselves and these contain p identical objects of one kind, q identical objects of another kind, r identical objects of still another kind, and so on, the total number of ways in which they can be arranged is given by,

$$\frac{n!}{p!q!r!...}$$

Example: How many words can be formed by arranging the letters of the word ‘weeded’?

Solution: Total number of letters in the word ‘weeded’ = 6.

D occurs 2 times and E occurs 3 times. This is a case where n objects (6 in this case) are to be arranged among themselves and they contain 2 identical objects of one kind (D) and 3 identical objects of another kind (E).

$$\text{So, number of words} = \frac{6!}{2!3!} = 720/12 = 60.$$

Permutations with repetitions

The number of permutations of n different objects taken r at a time, when repetitions are allowed is n^r .

Example: How many four-digit numbers can be formed using the digits 4, 5, 6, 7, and 9 if repetition of digits is allowed?

Solution: Here, there are five different objects, i.e., the digits 4, 5, 6, 7, and 9; $n = 5$

Since repetition is allowed, the first digit can be taken in five different ways, the second can be taken in five different ways, and so on.

So, the number of four-digit numbers is $5 \times 5 \times 5 \times 5 = 625$.

COMBINATIONS

If there are n objects, and r out of them ($r \leq n$) are to be selected, then the number of ways in which this can be done is ${}^n C_r$ and is read as ‘the number of combinations of n objects taken r at a time’. The number of combinations is given as:

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

Example: In how many different ways can you select 2 students out of 3?

Solution: It can be written as ${}^3 C_2$.

$${}^3 C_2 = \frac{3!}{(3-2)!2!} = 3.$$

The specific cases of combinations are:

$${}^n P_r = {}^n C_r \times r!$$

If $r = 0$,

$${}^n C_0 = \frac{n!}{(n-0)! \times 0!} = 1$$



If $r = 1$,

$${}^nC_1 = \frac{n!}{(n-1)! \times 1!} = n$$

If $r = n$,

$${}^nC_n = \frac{n!}{(n-n)! \times n!} = 1$$

If $r = n - 1$,

$${}^nC_{n-1} = \frac{n!}{(n-n+1)! \times (n-1)!} = \frac{n!}{1! \times (n-1)!} = n$$

Example: Find the value of 7C_2 ?

$$\begin{aligned} \textbf{Solution: } {}^7C_2 &= \frac{7!}{(7-2)!2!} \\ &= \frac{7!}{5!2!} = \frac{5040}{120 \times 2} = 21. \end{aligned}$$



Chapter Summary

$$n! = 1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n$$

$$\text{Permutation} = {}^nP_r = \frac{n!}{(n-r)!}$$

Some specific cases related to permutations are:

If $r = 0$,

$${}^nP_0 = \frac{n!}{(n-0)!} = 1$$

If $r = 1$,

$${}^nP_1 = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

If $r = n$,

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

If $r = n - 1$,

$${}^nP_{n-1} = \frac{n!}{(n-n+1)!} = \frac{n!}{1!} = n!$$

$$\text{Combination} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

The specific cases of combinations are:

$${}^nP_r = {}^nC_r \times r!$$

If $r = 0$,

$${}^nC_0 = \frac{n!}{(n-0)! \times 0!} = 1$$

If $r = 1$,

$${}^nC_1 = \frac{n!}{(n-1)! \times 1!} = n$$

If $r = n$,

$${}^nC_n = \frac{n!}{(n-n)! \times n!} = 1$$

If $r = n - 1$,

$${}^nC_{n-1} = \frac{n!}{(n-n+1)! \times (n-1)!} = \frac{n!}{1! \times (n-1)!} = n$$



PRACTICE QUESTIONS

1. If the ratio between $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{!(n-4)!}$ is 2:1, then find the value of n .
A. 5
B. 4
C. 3
D. 2
2. Find the value of n in $(n+1)! = 12(n-1)!$
A. 2
B. 3
C. 4
D. 5
3. Find the value of n in $(n+2)! = 60(n-1)!$
A. 1
B. 2
C. 3
D. 4
4. If ${}^nP_3 = 120$, find the value of n .
A. 3
B. 4
C. 5
D. 6
5. If ${}^{10}P_r = 720$, then find the value of r .
A. 2
B. 3
C. 4
D. 5
6. How many numbers of four digits can be formed with digits 2, 4, 6, 7, and 8? (Repetition of digits is not allowed.)
A. 100
B. 110
C. 120
D. 130
7. How many numbers between 200 and 800 can be made with digits 0, 1, 2, 3, 4, and 5? (Repetition of digits not allowed.)
A. 50
B. 60
C. 70
D. 80
8. A gentleman has 5 friends to invite. In how many ways can he send invitation cards to them, if he has three servants to carry the cards?
A. 243
B. 247
C. 351
D. 498
9. In how many ways 3 prizes can be given away to 6 boys when each boy is eligible for any of the prizes?
A. 195
B. 200
C. 216
D. 248
10. In how many ways can 7 boys be seated at a round table so that 3 particular boys are next to each other?
A. 12
B. 121
C. 11
D. 144
11. In how many ways can 7 boys be seated at a round table so that 2 particular boys are separated.
A. 480
B. 490
C. 500
D. 510
12. How many different letter arrangements can be made from the letters of the word ARISE?
A. 100
B. 120
C. 140
D. 160
13. How many different letter arrangements can be made from the letters of the word RECOVER?
A. 1,230
B. 1,240
C. 1,250
D. 1,260



- 14.** How many permutations can be made out of the letters of the word TRIANGLE, which begin with *T* and end with *E*?
- A. 720
B. 740
C. 760
D. 800
- 15.** In how many ways can the letters of the word MOTHER be arranged so that all the vowels come together?
- A. 100
B. 120
C. 140
D. 160
- 16.** In how many ways can the letters of the word DIRECTOR be arranged so that the three vowels never come together?
- A. 12,000
B. 15,000
C. 18,000
D. 20,000
- 17.** There are 10 points in a plane out of which 4 are collinear. Find the number of straight lines formed by joining them.
- A. 40
B. 45
C. 50
D. 55
- 18.** In how many different ways can the letters of the word TRUST be arranged?
- A. 60
B. 70
C. 80
D. 90
- 19.** In how many different ways can the letters of the word ATTEND be arranged?
- A. 320
B. 360
C. 400
D. 420
- 20.** In how many different ways can the letters of the word BANKING be arranged?
- A. 2,500
B. 2,520
C. 2,550
D. 3,000
- 21.** In how many different ways can the letters of the word PEANUT be arranged?
- A. 700
B. 710
C. 720
D. 730
- 22.** A committee of five members is to be formed out of 4 students, 3 teachers, and 2 sports coaches. In how many ways can the committee be formed if the committee should consist of 2 students, 2 teachers, and 1 sports coach?
- A. 6
B. 18
C. 36
D. 48
- 23.** A committee of five members is to be formed out of 4 students, 3 teachers, and 2 sports coaches. In how many ways can the committee be formed if any five can be selected?
- A. 123
B. 124
C. 125
D. 126
- 24.** In how many different ways can the letters of the word REPLACE be arranged?
- A. 2,550
B. 2,540
C. 2,530
D. 2,520
- 25.** In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
- A. 63
B. 64
C. 65
D. 66
- 26.** How many different words can be formed with the letters of the word ALLAHABAD?
- A. 7,560
B. 7,550
C. 7,540
D. 7,530
- 27.** In how many different ways can the letters of the word SOFTWARE be arranged in



- such a way that the vowels always come together?
- A. 4,320
B. 4,330
C. 4,340
D. 4,350
- 28.** In how many different ways can a group of 4 men and 4 women be formed out of 7 men and 8 women?
- A. 2,400
B. 2,450
C. 2,500
D. 2,550
- 29.** In how many different ways can the letters of the word MIRACLE be arranged?
- A. 5,020
B. 5,040
C. 5,080
D. 5,160
- 30.** A committee of 12 persons is to be formed from 9 women and 8 men. In how many ways can this be done if atleast 5 women have to be included in a committee?
- A. 6,061
B. 6,062
C. 6,063
D. 6,064
- 31.** A committee of 12 persons is to be formed from 9 women and 8 men. In how many of these committees the women are in majority?
- A. 2,700
B. 2,701
C. 2,702
D. 2,703
- 32.** A committee of 5 members is to be formed out of 3 trainees, 4 professors, and 6 research associates. In how many different ways can this be done if the committee should have all 4 professors and 1 research associate or all 3 trainees and 2 professors?
- A. 8
B. 10
C. 12
D. 14
- 33.** A committee of 5 members is to be formed out of 3 trainees, 4 professors, and 6 research associates. In how many different ways can this be done if the committee should have 2 trainees and 3 research associates?
- A. 40
B. 50
C. 60
D. 70
- 34.** In how many different ways can the number 25,69,774 be arranged, using each digit only once in each arrangement, such that the digits 6 and 5 are at the extreme ends in each arrangement?
- A. 12
B. 24
C. 36
D. 48
- 35.** Two girls and 4 boys are to be seated in a row in such a way that the girls do not sit together. In how many different ways can it be done?
- A. 120
B. 240
C. 360
D. 480
- 36.** In how many different ways can the letters of the word DRASTIC be arranged in such a way that the vowels always come together?
- A. 660
B. 680
C. 700
D. 720
- 37.** In how many different ways can the letters of the word CASUAL be arranged?
- A. 360
B. 180
C. 90
D. 45
- 38.** A dinner party is to be fixed for a group of 100 persons. In this party 50 people do not prefer fish, 60 prefer chicken, and 10 do not prefer either chicken or fish. Find the number of people who prefer both fish and chicken?



- A. 20
B. 30
C. 40
D. 50
- 39.** In how many different ways the letters of the word RECTITUDE can be arranged so that vowels come together?
A. 4,320
B. 4,330
C. 4,340
D. 4,350
- 40.** A person can go from Mumbai to Delhi by rail, road, or air. There are five routes to reach Manali from Delhi. Only one road goes from Manali to Ladakh. If Alina wants to go to Ladakh again but one of the roads between Delhi and Manali is closed, in how many different ways can she go from Mumbai to Ladakh?
A. 12
B. 24
C. 36
D. 48
- 41.** Munim likes to have passwords of eight characters, the first four being different vowels and the last four being different digits. How many different passwords can he have?
A. 5,37,800
B. 6,04,800
C. 7,28,800
D. 8,94,500
- 42.** A family of five goes in a five-seater car. In how many different ways can they sit?
A. 100
B. 120
C. 140
D. 160
- 43.** How many words can be formed by arranging the letters of the word HELPING?
A. 5,000
B. 5,040
C. 5,080
D. 5,120
- 44.** Using all the letters of the word LINEAR, how many different words can be formed that start and end with a vowel?
A. 121
B. 144
C. 169
D. 181
- 45.** Using all the letters of the word LINEAR, how many different words can be formed that start with a vowel but end with a consonant?
A. 214
B. 215
C. 216
D. 217
- 46.** In how many different ways can the letters of the word READING be arranged in such a way that the vowels always come together?
A. 700
B. 710
C. 720
D. 730
- 47.** Lakhan has 15 DVDs out of which 3 are of the movie Mother India, 5 are of the movie Sholay and the rest are all different. In how many ways can he arrange the DVDs on a shelf?
A. 1234567890
B. 1244985690
C. 1816214400
D. 1327528930
- 48.** In how many ways can 5 people be seated around a circular table?
A. 20
B. 22
C. 24
D. 26
- 49.** In how many ways can 6 different beads be arranged to form a necklace?
A. 30
B. 40
C. 50
D. 60
- 50.** In how many ways can four cards be selected from a pack of cards such that exactly one of them is an ace?
A. ${}^4C_2 \times {}^{52}C_4$
B. ${}^4C_3 \times {}^{52}C_3$
C. ${}^4C_4 \times {}^{48}C_4$
D. ${}^4C_1 \times {}^{48}C_3$



SOLUTIONS

1. (A) $\frac{n!/2!(n-2)!}{n!/4!(n-4)!} = \frac{2}{1}$
 $\frac{n!}{2!(n-2)!} \times \frac{4!(n-4)!}{n!} = \frac{2}{1}$

$$(n-2)(n-3) = 3 \times 2 \\ n-2 = 3 \text{ and } n-3 = 2$$

So, $n = 5$

2. (B) $\frac{(n+1) \times n \times (n-1)!}{(n-1)!} = 12$

$$(n+1) \times n = 12 \text{ or } (n+1) \times n = 4 \times 3$$

Hence, $n = 3$

3. (C) $\frac{(n+2) \times (n+1) \times n \times (n-1)!}{(n-1)!} = 60$

$$(n+2)(n+1)n = 60 \text{ or } (n+2)(n+1)n = 5$$

$$\times 4 \times 3$$

Hence, $n = 3$

4. (D) $\frac{n!}{(n-3)!} = 120$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 120$$

$$n(n-1)(n-2) = 120 = 6 \times 5 \times 4$$

Hence, $n = 6$

5. (B) $\frac{10!}{(10-r)!} = 720 = 10 \times 9 \times 8$

$$\frac{10!}{(10-r)!} = \frac{10 \times 9 \times 8 \times 7!}{7!}$$

$$\frac{10!}{(10-r)!} = \frac{10!}{7!}$$

$$(10-r)! = 7!$$

So, $r = 3$

6. (C) There are five numbers and number of places to be filled up = 4

So, required number of numbers is 5P_4
 ${}^5P_4 = \frac{5!}{(5-4)!} = 120$

7. (D) Any number between 200 and 800 will be of three digits and the first number must be fulfilled by 2, 3, 4, or 5 because if we start from 0 it will be lower than 200.

So, required number of ways to fill first place = 4

And, required number of ways to fill remaining two places = 5P_2

$$\begin{aligned} \text{Required number of numbers} &= 4 \times {}^5P_2 \\ &= 4 \times \frac{5!}{(5-2)!} = 80 \end{aligned}$$

8. (A) Invitation cards may be sent to each of 5 friends by any one of the three servants in 3 ways.

$$\begin{aligned} \text{So, required ways} &= 3 \times 3 \times 3 \times 3 \times 3 = 3^5 \\ &= 243. \end{aligned}$$

9. (C) Each of the three prizes can be given away to any of the 6 boys in 6 ways.

$$\begin{aligned} \text{So, the required number of ways} &= 6^3 = 216 \\ \text{ways.} & \end{aligned}$$

10. (D) Let the 3 particular boys be taken together as one unit, then the number of units will be 5.

They can sit around the table in 4! Ways, and for each of this arrangement, 3 can be interchanged in 3! Ways.

$$\begin{aligned} \text{Hence, total number of arrangements} &= 4! \\ &\times 3! = 24 \times 6 = 144 \end{aligned}$$

11. (A) Total ways in which 7 boys can be seated at a round table = $(7-1)! = 6!$

Total ways in which 2 particular boys sit together while arranging 7 boys = $5! \times 2!$

The arrangements that the two persons are separated = $6! - 5! \times 2! = 480$ ways.

12. (B) In word ARISE all 5 letters are different.

$$\begin{aligned} \text{Hence, total number of permutations} &= {}^5P_5 \\ &= \frac{5!}{(5-5)!} = 120 \end{aligned}$$

13. (D) In word RECOVER, 2 letters E and R have come two times.

$$\begin{aligned} \text{Total number of permutation} &= \frac{7!}{2!2!} = \\ &1260 \text{ ways.} \end{aligned}$$

14. (A) Total letters in word TRIANGLE = 8

Two letters T and E have fixed positions, so remaining letters = 6

$$\text{Number of permutations} = 6! = 720.$$



- 15. (B)** Number of vowels in word MOTHER = (O, E) = 2
 Total letters in MOTHER after leaving vowels = M, T, H, R = 4
 As all vowels come together, so we will count them as 1, then we have only $4 + 1 = 5$ letters.
 Arrangements of these 5 letters = $5! = 120$.
- 16. (C)** Total number of letters = 8 and letter R occurring twice. Number of vowels = 3
 Total number of arrangements when there is no restriction = $\frac{8!}{2!}$
 When three vowels are together, taking them as one letter, we have only $5 + 1 = 6$ letters.
 These 6 letters can be arranged in $\frac{6!}{2!}$ ways, since R occurs twice.
 Number of arrangements when 3 vowels are together = $\frac{6!}{2!} \times 3!$
 Required number = $\frac{8!}{2!} - \frac{6!}{2!} \times 3! = 18000$.
- 17. (A)** Let us suppose that the 10 points are such that no three of them are collinear. Now a straight line will be formed by any two of these 10 points. Thus forming a straight line amounts to selecting two of the 10 points.
 Now out of 10 points 2 can be selected in ${}^{10}C_2$ ways.
 Number of straight lines formed by 10 points when now let the four points become collinear, then 4C_2 straight lines formed by them will reduce to only one straight line.
 Required number of lines formed = ${}^{10}C_2 - {}^4C_2 + 1 = 45 - 6 + 1 = 40$.
- 18. (A)** The word TRUST consists of five letters in which T comes twice.
 Number of arrangements = $\frac{5!}{2!} = 60$.
- 19. (B)** The word ATTEND consists of six letters in which T comes twice.
 Number of arrangements = $\frac{6!}{2!} = 360$.
- 20. (B)** The word BANKING consists of seven letters in which N comes twice.
 Number of arrangements = $\frac{7!}{2!} = 2,520$.
- 21. (C)** The word PEANUT consists of six distinct letters.
 Number of arrangements = $6! = 720$.
- 22. (C)** Required number of combinations = ${}^4C_2 \times {}^3C_2 \times {}^2C_1 = 36$.
- 23. (D)** Required number of combinations = ${}^9C_5 = 126$.
- 24. (D)** The word REPLACE consists of seven letters in which E comes twice.
 Number of arrangements = $\frac{7!}{2!} = 2520$.
- 25. (A)** There are 7 men and 3 women. We have to select 5 men out of 7 and 2 women out of 3. This can be done in ${}^7C_5 \times {}^3C_2$ ways.
 The number of ways of making the selection = ${}^7C_5 \times {}^3C_2 = 63$.
- 26. (A)** There are 9 letters in the word ALLAHABAD out of which 4 are As and 2 are L.
 So, required number of words = $\frac{9!}{4!2!} = 7,560$.
- 27. (A)** There are 8 letters in the word SOFTWARE, including 3 vowels (O, A, E) and 5 consonants (S, F, T, W, R). Considering three vowels as one letter, we have six letters which can be arranged in ${}^6P_6 = 6!$ ways.
 But corresponding to each way of these arrangements, the vowels can be put together in $3!$ ways.
 So, the required number of words = $6! \times 3! = 4320$.
- 28. (B)** 4 men out of 7 men and 4 women out of 8 women can be chosen in ${}^7C_4 \times {}^8C_4 = 2,450$.
- 29. (B)** The word MIRACLE has 7 distinct letters.
 So, the number of arrangements = $7! = 5,040$.
- 30. (B)** There are 9 women and 8 men. A committee of 12 consisting of at least 5 women can be formed by choosing.
 5 women and 7 men
 6 women and 6 men



7 women and 5 men
 8 women and 4 men
 9 women and 3 men
 So, total number of ways of forming the committee = ${}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$
 $= 126 \times 8 + 84 \times 28 + 36 \times 56 + 9 \times 70 + 1 \times 56 = 6,062.$

31. (C) There are 9 women and 8 men. A committee of 12, consisting of at least 5 women, can be formed by choosing.

5 women and 7 men

6 women and 6 men

7 women and 5 men

8 women and 4 men

9 women and 3 men

As, women are in majority in 3rd, 4th, and 5th case.

So, total number of such committees = ${}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$
 $= 36 \times 56 + 9 \times 70 + 1 \times 56 = 2,702.$

32. (C) Number of combinations =

$${}^4C_4 \times {}^6C_1 + {}^3C_3 \times {}^4C_2 \\ 1 \times 6 + 1 \times 6 = 12.$$

33. (C) We have to select 2 trainees out of 3 and 3 research associates out of 6.

So, the number of combinations = ${}^3C_2 \times {}^6C_3 = 60.$

34. (D) Case 1 when arrangement is 6----5

Four empty places can be filled by 2, 9, 7 and 4 in 4! = 24 ways.

Case 1 when arrangement is 5----6

Four empty places can be filled by 2, 9, 7 and 4 in 4! = 24 ways.

Required number of arrangements = 24 + 24 = 48 ways.

35. (D) 4 boys can be seated in row in ${}^4P_4 = 4!$ ways.

Now in the 5 gaps 2 girls can be arranged in 5P_2 ways.

Hence, the number of ways in which no two girls sit together = $4! \times {}^5P_2 = 480$ ways.

36. (D) There are 7 letters in the word DRASTIC including 2 vowels (A, I) and 5 consonants

(D, R, S, T, C), considering two vowels as one letter, we have 6 letters which can be arranged in 6! ways. But corresponding to each way of the arrangements, the vowels can be put together in 2! ways.

$$= 6! \times \frac{2!}{2!} = 720 \text{ ways.}$$

37. (A) The word CASUAL has 6 letters in which letter A comes twice.

So, number of arrangements = $\frac{6!}{2!} = 360$ ways.

38. (A) 50 out of 90 do not prefer fish.

So, the required answer = $60 - 40 = 20.$

39. (A) There are 9 letters in the word RECTITUDE including 4 vowels (E, I, U, E) and 5 consonants (R, C, T, T, D), and letters T and E come twice.

So, number of arrangements = $\frac{6! \times 4!}{2! \times 2!} = 4,320 \text{ ways.}$

40. (A) Mumbai to Delhi = 3 ways (rail, road, air)

Delhi to Manali = 5 ways (5 routes)

Manali to Ladakh = 1 way (one road)

So, total ways to reach Ladakh from Mumbai are $3 \times 5 \times 1 = 15$ ways.

Since one road between Delhi and Manali is closed, there are four possible roads.

Hence, total ways = $3 \times 4 \times 1 = 12$ ways.

41. (B) The first 4 characters of Munim's password are to be chosen from the 5 vowels. Since there are 5 vowels available, the first vowel can be chosen in 5 different ways. Now, as all four vowels need to be different, the second vowel can be chosen only in 4 ways, the third in 3 ways, and the last in 2 ways. Similarly, the four digits have to be different. The first digit can be chosen from the 10 digits in 10 ways, the second can be chosen from the remaining 9 digits in 9 ways. Similarly, the third and fourth digit can be chosen in 8 and 7 ways, respectively. Hence, the four numbers can be chosen from the 10 digits from 0 to 9 in 10, 9, 8, and 7 ways, respectively.



Thus the number of different passwords that Munim can have = $5 \times 4 \times 3 \times 2 \times 10 \times 9 \times 8 \times 7 = 604800$.

- 42. (B)** Number of seats = 5 and number of people = 5.

So, required arrangement = ${}^5P_5 = 5! = 120$ ways.

- 43. (B)** The word HELPING has 7 distinct letters. So, the number of arrangements = $7! = 5040$.

- 44. (B)** The word LINEAR has 3 vowels = I, E and A.

If a word starts and ends with a vowel, the two letters to occupy the first and the last positions can be selected and arranged in ${}^3P_2 = 6$ ways.

The remaining 4 letters can be arranged among themselves in ${}^4P_4 = 4! = 24$ ways.

Hence, the number of words that start and end with a vowel = $24 \times 6 = 144$.

- 45. (C)** The word LINEAR has 3 vowels (I, E, A) and 3 consonants (L, N, R).

If a word starts with a vowel but ends with a consonant, its first letter can be selected from I, E, and A in 3 ways. Its last letter can be selected from L, N, and R in 3 ways. The remaining four letters can be arranged in $4!$ ways.

Hence, the number of words that start with a vowel but end with a consonant = $3 \times 3 \times 4! = 9 \times 24 = 216$.

- 46. (C)** The word READING has 7 different letters.

The vowels (E, A, I) can be arranged among themselves in $3! = 6$ ways

Since E, A, and I are considered to be together, consider these as one letter.

Hence, letters to be arranged are R, D, N, G, vowels. These can be arranged in $5! = 120$ ways

So, the required number of ways = $120 \times 6 = 720$ ways.

- 47. (C)** There are 15 DVDs out of which 3 are of one kind and 5 are of another kind.

So, the total number of arrangements possible = $\frac{15!}{3! \times 5!} = 1816214400$.

- 48. (C)** n objects can be arranged in a circle in $(n - 1)!$ ways.

Similarly, 5 people can be seated around a circular table in $(5 - 1)! = 4! = 24$ ways.

- 49. (D)** n objects can be arranged in a necklace in $\frac{1}{2} \times (n - 1)!$ ways

Thus, 6 beads can form a necklace in $\frac{1}{2} \times (6 - 1)! = \frac{5!}{2} = 60$ ways.

- 50. (D)** An ace can be selected from 4 aces in 4C_1 ways.

Since exactly one card is an ace, the remaining three cards can be selected from the $52 - 4 = 48$ non-ace cards in ${}^{48}C_3$ ways.

So, total number of selections = ${}^4C_1 \times {}^{48}C_3$.



SYNOPSIS

- Introduction
- General terms of Probability
- Types of Events
- Probability
- Addition Theorem of Probability
- Conditional Probability
- Odds

Probability is a method of expressing the event of something occurring or not happening. The higher the probability, the greater the odds of an event occurring and vice versa.

SOME GENERAL TERMS OF PROBABILITY

Deterministic Experiment: The experiment that gives a definite result is called a deterministic experiment. For example, rolling a fair die: each number on a six-sided die has the same odds ($1/6$) of coming up.

Random Experiment: A random experiment is an action that gives one or more results. For example, we toss a coin three times and watch the heads/tails sequence. The sample space in this case could be described as follows:
 $S = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)\}$.

Outcome: The result of a random experiment is called an outcome.

Sample Space: The sample space (S) is the set of all possible outcome of an event. The number of elements in the sample space is denoted by $n(S)$. For example, when a single die is thrown, it has 6 outcomes since it has 6 faces. Therefore, the sample is given as $S = \{1, 2, 3, 4, 5, \text{ and } 6\}$.

Unbiased Experiment: A random experiment having equally likely outcomes is called an unbiased experiment.

TYPES OF EVENT

Simple Event: If an event contains only one sample point, then it is called a simple event. Consider the sample space of the experiment of rolling an unbiased die,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Any subset of S is called an event.

The event E of getting a perfect cube when an unbiased die is rolled once is an example of a simple event, i.e., $E = \{1\}$.

Impossible Event: An impossible event is one in which the sample set is empty, i.e., it contains no sample points. For example, getting 7 by throwing a die is an impossible event.

Certain Event: If the event is sure to happen, it is a certain event. For example, getting a value less than 7 on throwing an unbiased die.

Complementary Event: Let A be an event in the sample space S . Then A is a subset of S . The complement of A is $(S - A)$. It is represented as A' .

Combination of Events: The *union* of the events A and B of a sample space ($A \cup B$) is the event that either A or B or both take place.

The *intersection* of the events A and B of a sample space ($A \cap B$) is the event that both A and B take place.

Exhaustive Events: If two events A and B of a sample space S are such that $(A \cup B) = S$, then A and B are called exhaustive events.

If A and B are exhaustive, $(A \cup B) = S$

So, $n(A \cup B) = n(S)$

Independent Events: If the happening of one event, A has no effect on the other event B , then A and B are said to be independent of each other.

Example: When three unbiased dice are rolled, what type of events are these —‘getting a total greater than 2’ and ‘getting a total greater than 20’?

Solution: When three dice are rolled, the minimum total is $1 + 1 + 1 = 3$ and the maximum total is $6 + 6 + 6 = 18$.



Hence, the event getting a total greater than 3° degrees will always happen and is hence a ‘certain event’. On the other hand, the event ‘getting a total greater than 18’ will never happen and hence, an ‘impossible event’.

PROBABILITY

Probability refers to the occurrence of an event occurring. If S is an unbiased experiment’s sample space and E is an event, then the probability that it occurs is,

$$P(E) = \frac{n(E)}{n(S)}$$

Example: Consider tossing an unbiased coin. Let E be the event that the result is heads. Let E' be an event that the result is tails.

Then, $n(S) = 2$; $n(E) = 1$ and $n(E') = 1$

$$\text{So, } P(E) = \frac{1}{2} \text{ and } P(E') = \frac{1}{2}$$

So when a coin is tossed, the probability of occurrence of heads is $\frac{1}{2}$. Similarly, the probability of occurrence of tails is $\frac{1}{2}$.

Example: Two unbiased coins are tossed. What is the probability that both the tosses give different results?

Solution: When a coin is tossed the result is either heads (H) or tails (T).

Tossing two coins gives 4 outcomes, i.e., $S = \{\text{HH}, \text{HT}, \text{TH}, \text{TT}\}$

Hence, $n(S) = 4$

Now, let A be the event that both the tosses give different results. The first coin shows heads and the second shows tails or the first coin shows tails and the second shows heads.

So, $A = \{\text{HT}, \text{TH}\}$ and $n(A) = 2$

$$\text{Hence, } P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

ADDITION THEOREM OF PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B are mutually exclusive events, $n(A \cap B) = 0$.

So, $P(A \cap B) = 0$

Hence, $P(A \cup B) = P(A) + P(B)$

Example: A bag contains 20 coins numbered 1 to 20. At random, a coin is chosen. What is the probability that the coin will come up with a number that is a multiple of 3 or 5?

Solution: Multiples of 3 from 1 to 20 = 3, 6, 9, 12, 15, 18, i.e., 6 values.

Multiples of 5 from 1 to 20 = 5, 10, 15, 20, i.e., 4 values.

Multiples of both 3 and 5 = 15, i.e., only 1 value. So, number of multiples of 3 or 5 = $6 + 4 - 1 = 9$. There are 20 numbers in all.

Hence, required probability = $\frac{9}{20}$

CONDITIONAL PROBABILITY

Conditional probability is the probability of an event or outcome that depends on the occurrence of a preceding event or outcome.

Let A and B be two events defined on a sample space S .

Let $P(B) > 0$. Then the conditional probability of A given B , is denoted by $P(A|B)$ and is defined by,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example: There is a 0.03 probability that it is Friday and a student is absent. As only school is open only for 5 days, there is a 0.2 probability that it is Friday. Given that it is Friday, what is the probability that a student will be absent?

Solution:

$$P(\text{Absent} | \text{Friday}) = \frac{P(\text{Absent} \cap \text{Friday})}{P(\text{Friday})} = \frac{0.03}{0.2}, \\ \text{i.e., } 0.15 = 15\%$$

ODDS

$$\text{Odds in Favour} = \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

$$\text{Odds Against} = \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

Example: A boy randomly picks a marble from his bag containing 10 yellow, 18 orange, 6 red, and 20 blue marbles. What are the odds that his marble is blue or orange?



Solution: Total marbles $10 + 18 + 6 + 20 = 54$
So, total events = ways of picking 1 marble out of 54 = 54
Number of favourable events = ways of picking a blue or orange marble = $20 + 18 = 38$.

And, the number of unfavourable events = $54 - 38 = 16$.
Hence, odds of getting a blue or orange marble = $38:16 = 19:8$.

PRACTICE QUESTIONS

1. A bag contains seven balls, four of which are red and three of which are blue. How probable is it that a blue ball will be chosen?
 - A. $4/7$
 - B. $3/7$
 - C. $2/7$
 - D. $1/7$
2. A dice is tossed. Calculate the probability of getting a multiple of three.
 - A. $1/12$
 - B. $1/9$
 - C. $1/6$
 - D. $1/3$
3. What is the probability of throwing a number larger than 4 using regular dice with numbered faces ranging from 1 to 6?
 - A. $1/3$
 - B. $1/4$
 - C. $1/5$
 - D. $1/6$
4. Three balls are picked at random from a bag containing four white and five black balls. Calculate the probability that all three are black.
 - A. $1/14$
 - B. $2/21$
 - C. $5/42$
 - D. $1/42$
5. Two dice are thrown at the same time. What is the probability of having an even-numbered doublet?
 - A. $1/12$
 - B. $1/3$
 - C. $1/4$
 - D. $1/6$
6. Tickets with numbers ranging from 1 to 20 are mixed together and a ticket is selected at random. What is the probability that the ticket will contain a number that is a multiple of 3 or 7?
 - A. $1/5$
 - B. $2/5$
 - C. $3/5$
 - D. $4/5$
7. A word is made up of nine letters, five consonants, and four vowels. At random, three letters are picked. What is the probability that more than one vowel will be chosen?
 - A. $({}^3C_2 \times {}^4C_2 + {}^4C_3)/{}^9C_3$
 - B. $({}^2C_2 \times {}^3C_2 + {}^1C_2)/{}^9C_3$
 - C. $({}^4C_2 \times {}^5C_1 + {}^4C_3)/{}^9C_3$
 - D. $({}^1C_3 \times {}^3C_1 + {}^4C_3)/{}^9C_3$
8. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of choosing one man, one woman, and two children?
 - A. $1/7$
 - B. $3/7$
 - C. $5/7$
 - D. $2/7$
9. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of selecting exactly two children?
 - A. $8/21$
 - B. $3/7$
 - C. $10/21$
 - D. $11/21$



- 10.** From a group of three men, two women, and four children, four people will be picked at random. What is the probability of choosing two women?
- A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{2}{3}$
- 11.** What are the chances that a leap year with 53 Sundays is chosen at random?
- A. $\frac{1}{7}$
B. $\frac{4}{7}$
C. $\frac{6}{7}$
D. $\frac{2}{7}$
- 12.** If the year chosen is not a leap year, what are the chances of 53 Sundays?
- A. $\frac{2}{7}$
B. $\frac{1}{7}$
C. $\frac{4}{7}$
D. $\frac{3}{7}$
- 13.** From a typical deck of 52 playing cards, a single card is chosen at random. What are the chances of picking a king or a club?
- A. $\frac{1}{13}$
B. $\frac{2}{13}$
C. $\frac{3}{13}$
D. $\frac{4}{13}$
- 14.** There are 17 boys and 13 girls in a maths class of 30 students. Four boys and five girls received an A on a unit test. What are the chances of picking a female or an 'A grade student' at random from the class?
- A. $\frac{7}{15}$
B. $\frac{8}{15}$
C. $\frac{17}{30}$
D. $\frac{19}{30}$
- 15.** From a class of 50 males, a teacher selects one student at random. What is the probability that the chosen student is a boy?
- A. $\frac{1}{2}$
B. 1
- 16.** There are 5 white and 8 black balls in each bag. Two consecutive draws of three balls are made, with the balls not being replaced before the second draw. What is the probability that the first draw will provide three white balls and the second draw will yield three black balls?
- A. $\frac{7}{429}$
B. $\frac{5}{666}$
C. $\frac{8}{671}$
D. $\frac{13}{456}$
- 17.** A has a $\frac{1}{3}$ probability of hitting the target, but B has a $\frac{2}{5}$ probability of hitting it. If each of A and B shoots at the target, what is the probability that the target will be hit?
- A. $\frac{1}{5}$
B. $\frac{2}{5}$
C. $\frac{3}{5}$
D. $\frac{4}{5}$
- 18.** Six red, five green, and eight blue balls are contained in a basket. What is the probability that all four balls are red or that any two of the four are green if four balls are chosen at random?
- A. $\frac{437}{3474}$
B. $\frac{675}{3214}$
C. $\frac{925}{3876}$
D. $\frac{435}{3475}$
- 19.** Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are taken at random and both are green or both are yellow?
- A. $\frac{1}{35}$
B. $\frac{2}{35}$
C. $\frac{3}{35}$
D. $\frac{4}{35}$
- 20.** Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that at least one of the five balls chosen at random is blue?



- A. $\frac{137}{141}$
B. $\frac{37}{41}$
C. $\frac{137}{143}$
D. $\frac{37}{43}$
- 21.** Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are drawn at random and both turn out to be blue?
A. $\frac{1}{7}$
B. $\frac{1}{9}$
C. $\frac{1}{11}$
D. $\frac{1}{13}$
- 22.** Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are red and two are green if four balls are chosen at random?
A. $\frac{1}{455}$
B. $\frac{2}{455}$
C. $\frac{3}{455}$
D. $\frac{4}{455}$
- 23.** There are six blue, two red, four green, and three yellow balls in a basket. What is the probability that none of the three balls chosen at random are yellow?
A. $\frac{6}{13}$
B. $\frac{43}{91}$
C. $\frac{44}{91}$
D. $\frac{3}{13}$
- 24.** Three blue and four red balls are contained in a basket. What is the probability that three balls are picked at random from the basket and all three are blue or red?
A. $\frac{1}{3}$
B. $\frac{1}{5}$
C. $\frac{1}{6}$
D. $\frac{1}{7}$
- 25.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that both or at least one marble will be red if two marbles are picked at random?
A. $\frac{23}{91}$
B. $\frac{25}{91}$
C. $\frac{27}{91}$
D. $\frac{29}{91}$
- 26.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that at least one marble will be yellow if three marbles are picked at random?
A. $\frac{197}{364}$
B. $\frac{199}{364}$
C. $\frac{201}{364}$
D. $\frac{50}{91}$
- 27.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that there are equal numbers of marbles of each colour if 8 marbles are picked at random?
A. $\frac{50}{1001}$
B. $\frac{60}{1001}$
C. $\frac{70}{1001}$
D. $\frac{80}{1001}$
- 28.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that none of the three marbles picked at random are green?
A. $\frac{20}{91}$
B. $\frac{30}{91}$
C. $\frac{40}{91}$
D. $\frac{50}{91}$
- 29.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that two blue marbles and two red marbles are picked at random?
A. $\frac{10}{1001}$
B. $\frac{20}{1001}$
C. $\frac{30}{1001}$
D. $\frac{40}{1001}$
- 30.** There are 13 white and 7 black balls in each bag. At random, two balls are picked. What's the probability they're the same colour?
A. $\frac{1}{2}$
B. $\frac{99}{190}$
C. $\frac{99}{199}$
D. $\frac{95}{199}$



- 31.** What is the probability that one card picked at random from a well-shuffled pack of 52 playing cards will be a black king?
- A. $\frac{1}{13}$
B. $\frac{2}{13}$
C. $\frac{3}{13}$
D. $\frac{4}{13}$
- 32.** What is the probability of rolling two dice and obtaining the number 4 consecutively?
- A. $\frac{1}{9}$
B. $\frac{1}{18}$
C. $\frac{1}{27}$
D. $\frac{1}{36}$
- 33.** Three red balls, two blue balls, and one black ball are contained in a bag. What are the chances of getting a blue ball out of the bag?
- A. $\frac{1}{6}$
B. $\frac{1}{2}$
C. $\frac{1}{3}$
D. $\frac{1}{4}$
- 34.** What is the probability of getting an even number when a fair die is thrown? What's the probability you'll obtain 4 or a greater number?
- A. $\frac{1}{3}$
B. $\frac{1}{4}$
C. $\frac{1}{2}$
D. $\frac{1}{6}$
- 35.** Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is less than 2?
- A. $\frac{1}{9}$
B. $\frac{1}{36}$
C. 0
D. $\frac{1}{18}$
- 36.** Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is 9?
- A. $\frac{1}{18}$
B. $\frac{1}{3}$
C. $\frac{1}{9}$
D. $\frac{1}{12}$
- 37.** Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is even?
- A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$
- 38.** What is the probability of receiving the same number on both dice if two dice are thrown at the same time?
- A. $\frac{1}{3}$
B. $\frac{1}{6}$
C. $\frac{1}{9}$
D. $\frac{1}{12}$
- 39.** What is the probability of receiving the same face on a coin tossed twice?
- A. 1
B. $\frac{1}{4}$
C. $\frac{3}{4}$
D. $\frac{1}{2}$
- 40.** What is the probability of receiving the same face if three coins are tossed simultaneously?
- A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$
- 41.** In a class of 20, 13 students passed the statistics exam. What is the probability of a student passing statistics if they are chosen at random from this group?
- A. $\frac{7}{20}$
B. $\frac{11}{20}$
C. $\frac{12}{20}$
D. $\frac{13}{20}$
- 42.** What is the probability of getting two consecutive numbers on a pair of dice if both are rolled at the same time?
- A. $\frac{1}{9}$
B. $\frac{1}{6}$
C. $\frac{2}{9}$
D. $\frac{5}{18}$



- 43.** What is the probability of getting the same face on a dice if you roll it three times?
- 1/9
 - 1/18
 - 1/27
 - 1/36
- 44.** Two dice are rolled at the same time. What is the probability that the sum of the faces will be more than three?
- 4/9
 - 5/9
 - 7/12
 - 11/12
- 45.** What is the probability of getting at least one 6 if a fair die is thrown twice?
- 11/27
 - 11/36
 - 1/18
 - 7/36
- 46.** Ram tosses a die into the air. What are the chances that it won't land on the same side two times in a row?
- 1/6
 - 1/2
 - 5/7
 - 1/3
- 47.** What are the chances of getting a sum of 10 if a pair of dice is rolled?
- 1/12
 - 1/3
 - 1/6
 - 1/9
- 48.** Ravi will win the game if he gets a 4 on the die roll. What is the probability?
- 1/3
 - 1/4
 - 1/5
 - 1/6
- 49.** Calculate the probability of getting a multiple of two when you roll a die.
- 2/3
 - 1/2
 - 1/6
 - 1/3
- 50.** Three dice are thrown at the same time. What are the chances of all three faces being the same on the top?
- 1/36
 - 1/27
 - 1/18
 - 1/9

SOLUTIONS

- (B) Number of favourable outcomes = 3
Total number of outcomes = 7
Probability of one ball = $3/7$
- (D) $S = \{1, 2, 3, 4, 5, 6\}$
Total number of outcomes = 6
Total number of favourable event = $\{3, 6\} = 2$
Probability of getting a multiple of 3 = $2/6 = 1/3$
- (A) Total number of outcomes = 6
Total number of favourable event = $\{5, 6\} = 2$
Probability of getting a number greater than 4 = $2/6 = 1/3$
- (C) 3 balls can be selected from 9 balls in $n(S) = {}^9C_3$ ways

If A be the event of getting 3 black balls in $n(A) = {}^5C_3$ ways

$$\text{Required probability } P(A) = \frac{n(A)}{n(S)} = \frac{5}{42}$$

- (A) Here, $n(S) = 36$
And doublet of even number, $n(A) = \{(2, 2), (4, 4), (6, 6)\} = 3$
So, probability = $3/36 = 1/12$
- (B) Let A be the required event then, $A = \{3, 6, 7, 9, 12, 14, 15, 18\}$
and, $n(A) = 8$; $n(S) = 20$
Required probability = $8/20 = 2/5$
- (C) Three letters can be chosen out of 9 letters in 9C_3 ways



More than one vowel can be chosen in the following way. 2 vowels and one consonant or 3 vowels.

If A be the required event, then, $n(A) = {}^4C_2 \times {}^5C_1 + {}^4C_3 / {}^9C_3$

Hence, this is the required probability.

8. (D) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

Hence, $n(S) = 126$

Let A be the required event then, $n(A) = {}^3C_1 \times {}^2C_1 \times {}^4C_2 = 36$

Hence required probability = $36/126 = 2/7$

9. (C) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

Hence, $n(S) = 126$

Let A be the required event then, $n(A) = {}^4C_2 \times {}^5C_2 = 60$

Hence required probability = $60/126 = 10/21$

10. (A) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

Hence, $n(S) = 126$

Let A be the required event then, $n(A) = {}^2C_2 \times {}^7C_2 = 21$

Hence required probability = $21/126 = 1/6$

11. (D) A leap year has 366 days so it has 52 complete weeks and 2 more days. The two days can be Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday, i.e., $n(S) = 7$.

Out of these 7 cases, cases favourable for more Sundays are (Sunday and Monday, Saturday and Sunday), i.e., $n(E) = 2$

Hence, required probability = $2/7$

12. (B) When the year is not a leap year, it has 52 complete weeks and 1 more day that can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, $n(S) = 7$

Out of these 7 cases, cases favourable for one more Sunday is $n(E) = 1$.

Hence, required probability = $1/7$

13. (D) Here, $n(S) = 52$

So, $P(\text{King or Club}) = P(\text{King}) + P(\text{Club}) -$

$P(\text{King and Club}) = 4/52 + 13/52 - 1/52$

$= 16/52 = 4/13$

14. (C) $n(S) = 30$

$$\begin{aligned}P(\text{Girl or A}) &= P(\text{Girl}) + P(A) - P(\text{Girl and A}) \\&= 13/30 + 9/30 - 5/30 = 17/30\end{aligned}$$

15. (B) As all the students are boys, so required probability is,

$$= 50/50 = 1$$

16. (A) Let A be the event of drawing 3 white balls in the first draw and B be the event of drawing 3 black balls.

$$\begin{aligned}\text{Hence, required probability} &= P(A \cap B) = \\P(A) \times P(B/A) &= \\P(A) &= 5/143\end{aligned}$$

After drawing 3 white balls in the first draw 10 balls are left in the bag, out of which 8 are black balls.

$$\text{So, } P(B/A) = 7/15$$

$$\text{Hence, } P(A \cap B) = P(A) \times P(B/A) = 5/143 \times 7/15 = 7/429$$

17. (C) Let A and B are the events of hitting the target and A' and B' are not hitting the target.

$$\begin{aligned}\text{So, } P(A) &= 1/3; P(A') = 2/3; P(B) = 2/5; P(B') = \\3/5\end{aligned}$$

Required probability = A hits the target and B does not hit the target

Or

B hits the target and A does not hit the target

Or

$$\begin{aligned}\text{A hits the target and B hits the target} \\= (1/3 \times 3/5) + (2/5 \times 2/3) + (1/3 \times 2/5) = 3/5\end{aligned}$$

18. (C) Number of balls = 19

$$\begin{aligned}\text{Ways of selecting 4 balls out of 19} &= {}^{19}C_4 = \\3,876\end{aligned}$$

Selecting 4 red balls or any two green balls out of the four = $15 + 910 = 925$

Hence, required probability = $925/3,876$

19. (C) Total numbers of the balls in the basket = 15

$$\begin{aligned}\text{Number of ways of selecting 2 balls out of} \\15 \text{ balls} &= {}^{15}C_2 = 105\end{aligned}$$

$$\text{Favourable number of cases} = {}^4C_2 + {}^3C_2 = 9$$

Hence, required probability = $9/105 = 3/35$

20. (C) Total numbers of the balls in the basket = 15

$$\begin{aligned}\text{Number of ways of selecting 5 balls out of} \\15 \text{ balls} &= {}^{15}C_5 = 105\end{aligned}$$



Let no blue ball be selected

So, number of ways of selecting 5 balls out of 9 balls without blue balls = 9C_5

$$\text{Hence, required probability} = 1 - {}^9C_5 / {}^{15}C_5 \\ = 1 - 6/143 = 137/143$$

- 21. (A)** Total numbers of the balls in the basket = 15

Number of ways of selecting 2 balls out of 15 balls = ${}^{15}C_2 = 105$

Favourable number of cases = ${}^6C_2 = 15$

$$\text{Hence, required probability} = 15/105 = 1/7$$

- 22. (B)** Number of ways of selecting 4 balls out of 15 balls = ${}^{15}C_4 = 1365$

Favourable number of cases = ${}^2C_2 + {}^4C_2 = 6$

$$\text{Hence, required probability} = 6/1365 = 2/455$$

- 23. (C)** Number of ways of selecting 3 balls out of 15 balls = ${}^{15}C_3 = 455$

Favourable number of cases = ${}^{12}C_3 = 220$

$$\text{Hence, required probability} = 220/455 = 44/91$$

- 24. (D)** Number of possible outcomes = ${}^7C_3 = 35$

$$\text{Favourable number of cases} = {}^4C_3 + {}^3C_3 = 4 + 1 = 5$$

$$\text{Hence, required probability} = 5/35 = 1/7$$

- 25. (B)** Total number of marbles in the urn = $4 + 5 + 2 + 3 = 14$

Total possible outcome = Selection of 2 marbles out of 14 marbles = ${}^{14}C_2 = 91$

$$\text{Favourable number of cases} = {}^2C_2 + {}^2C_1 \times {}^{12}C_1 = 1 + 2 \times 12 = 25$$

$$\text{Hence, required probability} = 25/91$$

- 26. (B)** Total possible outcome = ${}^{14}C_3 = 364$

When no marble is yellow, favourable number of cases = ${}^{11}C_3 = 165$

$$\text{So, probability that no marble is yellow} = 165/364$$

$$\text{Hence, required probability} = 1 - 165/364 = 199/364$$

- 27. (B)** Total possible outcome = ${}^{14}C_6 = 3003$

$$\text{Favourable number of cases} = {}^4C_2 \times {}^5C_2 \times {}^2C_2 \times {}^3C_2 = 180$$

$$\text{Hence, required probability} = 180/3003 = 60/1001$$

- 28. (B)** Total possible outcome = ${}^{14}C_3 = 364$

No ball is green. So, selection of 3 marbles out of 5 blue, 2 red and 3 yellow marbles

$$= {}^{10}C_3 = 120$$

$$\text{Hence, required probability} = 120/364 = 30/91$$

- 29. (A)** Total possible outcome = ${}^{10}C_3 = 1,001$

$$\text{Favourable outcome} = {}^5C_2 \times {}^2C_2 = 10$$

$$\text{Hence, required probability} = 10/1,001$$

- 30. (B)** Total possible outcome = ${}^{20}C_2 = 190$

$$\text{Total favourable outcome} = {}^{13}C_2 + {}^7C_2 = 78 + 21 = 99$$

$$\text{Hence, required probability} = 99/190$$

- 31. (C)** Total possible outcome = ${}^{52}C_1 = 52$

$$\text{Favourable number of cases} = {}^{12}C_1 = 12$$

$$\text{Required probability} = 12/52 = 3/13$$

- 32. (D)** Total outcome for a pair of dice = 36

$$\text{Favourable outcome} = 1$$

$$\text{Hence, required probability} = 1/36$$

- 33. (C)** Total number of balls = 6

Favourable outcome = 2 blue balls

$$\text{Probability of blue ball} = 2/6 = 1/3$$

- 34. (C)** For even number,

$$\text{Total outcome} = 6$$

Favourable outcome (even numbers) = 3

$$\text{Hence, required probability} = 3/6 = 1/2$$

For 4 or higher number,

$$\text{Total outcome} = 6$$

Favourable outcome (4, 5, and 6) = 3

$$\text{Hence, required probability} = 3/6 = 1/2$$

- 35. (C)** Total outcome = 36

Favourable outcome (sum is less than 2) = 0

$$\text{Hence, required probability} = 0/6 = 0$$

- 36. (C)** Total outcome = 36

Favourable outcome, sum is 9 {(4, 5) (5, 4)}

$$(3, 6) (6, 3)} = 4$$

$$\text{Hence, required probability} = 4/36 = 1/9$$

- 37. (A)** Total outcome = 36

Favourable outcome (even sum) = 18

$$\text{Hence, required probability} = 18/36 = 1/2$$

- 38. (B)** Total outcome = 36

Favourable outcome (same number) = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)} = 6

$$\text{Hence, required probability} = 6/36 = 1/6$$



- 39. (D)** Total outcome (2^2) = 4
Favourable outcome (same face appearing)
= $\{(H, H) (T, T)\} = 2$
Hence, required probability = $2/4 = 1/2$
- 40. (C)** Total outcome (2^3) = 8
Favourable outcome (same face appearing)
= $\{(H, H, H) (T, T, T)\} = 2$
Hence, required probability = $2/8 = 1/4$
- 41. (D)** Total outcome (number of students) = 20
Favourable outcome (passed in statistics)
= 13
Hence, required probability = $13/20$
- 42. (D)** Total outcome = 36
Favourable outcome (two consecutive numbers) = $\{(1, 2) (2, 3) (3, 4) (4, 5) (5, 6) (2, 1) (3, 2) (4, 3) (5, 4) (6, 5)\} = 10$ ways
Hence, required probability = $10/36 = 5/18$
- 43. (D)** Total outcome (6^3) = 216
Favourable outcome (having same face) =
 $\{(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6)\} = 6$ ways
Hence, required probability = $6/216 = 1/36$
- 44. (D)** Total outcome = 36
Favourable outcome (sum less than or equal to 3) = $\{(1, 1) (2, 1) (1, 2)\} = 3$
Hence, required probability = $3/36 = 1/12$
- 45. (B)** Total outcome = 36
- Favourable outcome (at least one 6) = $\{(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5)\} = 11$
Hence, required probability = $11/36$
- 46. (C)** Total outcome = 36
Possible outcome with same side = $\{(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)\} = 6$ outcome
So, probability of same side = $6/36 = 1/6$
Hence, probability of not landing on same side = $1 - 1/6 = 5/6$
- 47. (A)** Total outcome = 36
Favourable outcome (getting a sum of 10) =
 $\{(4, 6) (5, 5) (6, 4)\} = 3$
Hence, required probability = $3/36 = 1/12$
- 48. (D)** Total outcome = 6
Favourable outcome (rolling a 4) = 1
Hence, required probability = $1/6$
- 49. (B)** Total outcome = 6
Favourable outcome (multiple of 2) =
 $\{2, 4, 6\} = 3$
Hence, required probability = $3/6 = 1/2$
- 50. (A)** Total outcome (6^3) = 216
Favourable outcome (having all the three same faces on the top) = $\{(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6)\} = 6$ outcome
Hence, required probability = $6/216 = 1/36$



SYNOPSIS

- Introduction
- 2D Shapes
- Quadrilaterals
- Area
- Perimeter
- Formulae Related to Two-Dimensional Figures
- Circles
- Triangle
- Polygons
- Regular Polygon
- Regular Hexagon

INTRODUCTION

This chapter deals with the concepts related to all the two-dimensional (2D) shapes that also includes the area and perimeter of all 2D figures. In this chapter, you will revise how to calculate the perimeter and area of squares, rectangles, triangles, and circles. The perimeter of a shape is the distance all the way around the sides of the shape. The area of a shape is the flat space inside the shape. You will also learn how to calculate the areas of different 2D figures, as well as investigate the effect on the perimeter and area of a shape when its dimensions are changed.

2D SHAPES

The 2D shapes are also known as flat shapes. These are the shapes having two dimensions only. It has length and breadth. It does not have thickness. The two different measures used for measuring the flat shapes are area and the perimeter. Two-dimensional shapes are the shapes that can be drawn on the piece of paper. Some of the examples of 2D shapes are square, rectangle, circle, and triangle.

QUADRILATERALS

Any four-sided polygon is called as a quadrilateral in general.

Different quadrilaterals and their properties

1. Rectangle

- Adjacent sides are at right angles to each other.
- Opposite sides are equal and parallel.
- Diagonals are equal.

2. Square

- All four sides are equal.
- All four angles are equal to 90° .
- The diagonals are equal.
- Diagonals are perpendicular bisector of each other.

3. Rhombus

- All sides are equal.
- All angles are not necessarily equal to 90° .
- All diagonals are perpendicular bisectors of each other but are not equal.

4. Kite

- The adjacent sides are equal.
- The diagonals are perpendicular to each other.
- The longer diagonal bisects the shorter one.

5. Trapezium

A quadrilateral in which one pair of lines is parallel.

6. Isosceles Trapezium

The non-parallel lines are equal.

7. Parallelogram

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Diagonals bisect each other.
- Sum of adjacent angles is 180° .

*Rectangle, Square, and Rhombus are special cases of parallelogram.



AREA

An area is a quantity that expresses the extent of a 2D figure or shape or planar lamina in the plane. Lamina shapes include 2D figures that can be drawn on a plane, e.g., circle, square, triangle, rectangle, trapezium, rhombus, and parallelogram. Area of shapes such as circle, triangle, square, rectangle, parallelogram, etc., are the region occupied by them in space.

Polygon shape: A polygon is a 2D shape that is formed by straight lines. The examples of polygons are triangles, hexagons, and pentagons. The names of shapes describe how many sides exist in the shape. For instance, a triangle consists of three sides and a rectangle has four sides. Hence, any shape that can be formed using three straight lines is known as a triangle and any shape that can be drawn by linking four lines is known as a quadrilateral. The area is the region inside the boundary/perimeter of the shapes which is to be considered.

PERIMETER

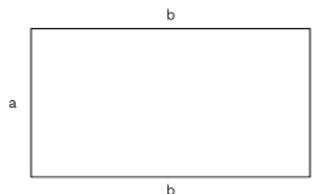
A perimeter is a closed path that surrounds a 2D shape. The Perimeter of a shape is defined as the total distance around the shape, it is the length of the outline or boundary of any 2D geometric shape. The word perimeter has been derived from the Greek word ‘peri’ meaning around, and ‘metron’ which means measure. Perimeter is the total length of the sides of a two-dimensional shape.

We often find perimeter when we have to put fencing around our field or when have to calculate the dimensions of a park. We use ruler to measure the length of the sides of a small regular shape. The perimeter is determined by adding the lengths of the sides/edges of the shape. For small irregular shapes, we can use a string of thread and place it exactly along the boundary of the shape, once. The total length of the string used along the boundary is the perimeter of the shape. The perimeter of all polygons can be determined by adding the lengths of their sides/edges.

FORMULAE RELATED TO TWO-DIMENSIONAL FIGURES

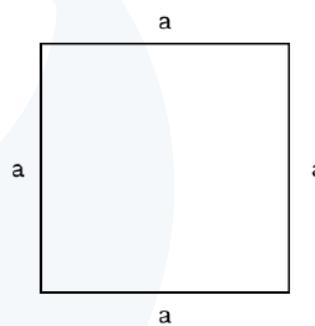
1. Rectangle

- Area of rectangle = $a \times b$
- Perimeter of a rectangle = $2(a + b)$
- Length of diagonal = $\sqrt{a^2 + b^2}$



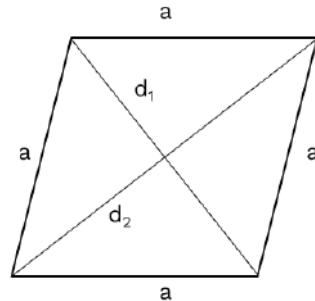
2. Square

- Area of square = a^2
- Perimeter of square = $4a$
- Length of diagonal = $a\sqrt{2}$



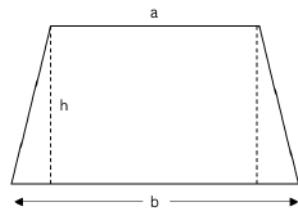
3. Rhombus

- Area = $\frac{1}{2} \times d_1 \times d_2$
- Side = $\frac{1}{2} \sqrt{d_1^2 + d_2^2}$



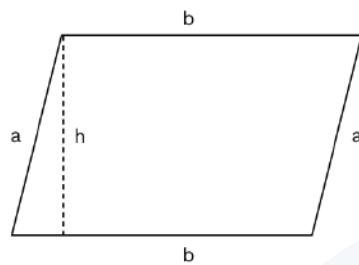
4. Trapezium

$$\text{Area of trapezium} = \frac{1}{2} \times (a + b) \times h$$



5. Parallelogram

Area of parallelogram = $b \times h$

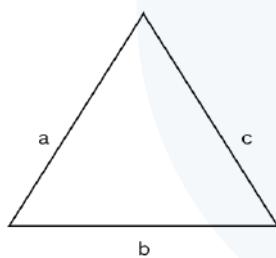


6. Triangle

a. Area of triangle = $\frac{1}{2} \times b \times h$

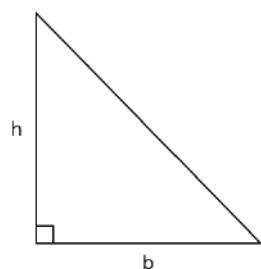
b. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$;

where $s = \frac{a+b+c}{2}$



7. Right triangle

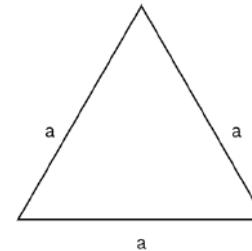
Area = $\frac{1}{2} \times b \times h$



8. Equilateral Triangle

a. Area = $\frac{\sqrt{3}}{4} a^2$

b. Perimeter = $3a$

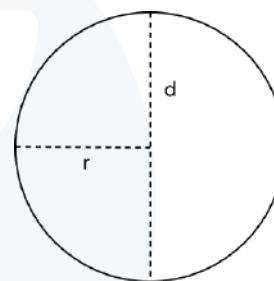


CIRCLES

A circle is a set of points in a plane, which are at a constant distance from a fixed point in the plane. The fixed point is then known as the centre and the fixed distance is called the radius of the circle.

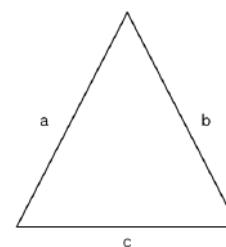
Circumference (perimeter) of a circle = $2\pi r = \pi d$, where r is the radius and d the diameter.

Area of circle = πr^2



TRIANGLE

A triangle is a plane figure bounded by three straight lines, or it is defined as a polygon of three sides. A vertex of a triangle is a point where two straight lines meet. Thus, in a triangle there are three vertices. The symbol $\triangle ABC$ is used to denote the triangle with vertices A, B, and C. The three straight lines AB, BC, and CA are called the sides and three angles $\angle BAC$, $\angle ABC$ and $\angle BCA$ are called the angles of the $\triangle ABC$.



Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

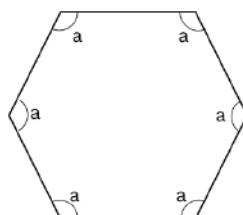


POLYGONS

A figure bounded by three or more sides is called a polygon. There are different polygons with different number of sides which are as follows:

NUMBER OF SIDES	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

A polygon is regular, if all its sides as well as angles are equal, else it is an irregular polygon.



Regular Polygon

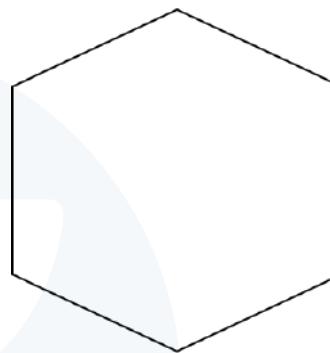


Irregular Polygon

REGULAR POLYGON

- Interior angle + exterior angle = 180°
- Perimeter = number of sides \times length of side(s)
- Each interior angle = $\frac{(n-2) \times 180}{n}$; where n is the number of sides
- Each exterior angle = $\frac{360^\circ}{n}$
- Sum of exterior angles = 360°

REGULAR HEXAGON



- Perimeter = $6 \times$ Side
- Area = $\frac{3\sqrt{3}}{2} \times (\text{side})^2$

PRACTICE QUESTIONS

- Find the area of rhombus whose diagonals are 8 cm and 10 cm.
 - 40 cm^2
 - 36 cm^2
 - 44 cm^2
 - 38 cm^2
- Find the area of an equilateral triangle whose side is 6 cm.
 - $6\sqrt{3} \text{ cm}^2$
 - $9\sqrt{3} \text{ cm}^2$
 - $9\sqrt{6} \text{ cm}^2$
 - $8\sqrt{3} \text{ cm}^2$
- In a four-sided field, the longer diagonal is 220 m. The perpendicular that is drawn from the opposite vertices to the longer diagonal are 40.4 m and 50.8 cm. Find the area of the field.
 - $10,032 \text{ m}^2$
 - $10,080 \text{ m}^2$
 - $10,400 \text{ m}^2$
 - $10,060 \text{ m}^2$
- If the perimeter of the square is 16 cm, find its area.
 - 10 cm^2
 - 16 cm^2
 - 14 cm^2
 - 18 cm^2



5. If the area of a circle is 314 cm^2 , find the circumference of the circle.
- 60 cm
 - 58.4 cm
 - 61.4 cm
 - 62.8 cm
6. The perimeter of a rectangle is 326 cm and its length is 98 cm. Find the area of the rectangle.
- 6,370
 - 6,820
 - 7,420
 - 6,680
7. The breadth of a rectangle is 12 cm and its perimeter is 50 cm. Find the length of the rectangle.
- 15 cm
 - 18 cm
 - 14 cm
 - 13 cm
8. The area of a rectangle is $1,029 \text{ cm}^2$ and if the sides of the rectangle are in the ratio $3 : 7$, then find its perimeter.
- 120
 - 180
 - 140
 - 160
9. The length and breadth of a garden is 10 m and 7 m, respectively. If the cost of fencing is Rs. 40/m, then what will be the cost for fencing the garden?
- 1,400
 - 1,360
 - 1,340
 - 1,260
10. The perimeter of a rectangle is 50 m and the sides of the rectangle are in the ratio $3 : 2$. Find the area of the rectangle.
- 600 m^2
 - 580 m^2
 - 640 m^2
 - 560 m^2
11. The perimeter of the square hall is 200 ft. If the cost of flooring is Rs. 50 per m^2 , then find the total cost of flooring the hall.
- 72,000
 - 68,000
 - 70,000
 - 75,000
12. Find the area of a square if the length of the diagonal of the square is 10 m.
- 40 m^2
 - 50 m^2
 - 54 m^2
 - 46 m^2
13. Find the area of the right-angled triangle whose hypotenuse is 15 cm and one of the sides is 12 cm.
- 60 cm^2
 - 48 cm^2
 - 50 cm^2
 - 54 cm^2
14. Find the area of a circle whose diameter is 20 cm.
- 300
 - 310
 - 314
 - 306
15. The area of the triangle is $3,125 \text{ cm}^2$, and the base and height are in the ratio $5 : 2$. What is the height of the triangle?
- 40
 - 48
 - 50
 - 46
16. The perimeter of a triangle is 96 cm. If two sides of the triangle are 40 cm and 24 cm, then find the area of the triangle.
- 384
 - 380
 - 428
 - 364
17. The base and height of a parallelogram field is 54 m and 24 m, respectively. Find the cost of levelling the field at the rate of Rs. 5 per m^2 .
- 6,110
 - 6,850
 - 7,420
 - 6,480



- 18.** The perimeter of one square is 24 m and that of another is 32 m. Perimeter of a square whose area is equal to the sum of the areas of the two squares will be:
- A. 40
 - B. 38
 - C. 44
 - D. 36
- 19.** Find the area of a triangle whose sides are 3 cm, 4 cm, and 5 cm, respectively.
- A. 8 cm^2
 - B. 4 cm^2
 - C. 5 cm^2
 - D. 6 cm^2
- 20.** The area of a parallelogram is 192 cm^2 , if the height of the parallelogram is one-third of the base, then find the height of the parallelogram.
- A. 6
 - B. 12
 - C. 8
 - D. 10
- 21.** The perimeter and area of rhombus are 180 cm and 315 cm^2 , respectively. Find the altitude of the rhombus.
- A. 10 cm
 - B. 7 cm
 - C. 8 cm
 - D. 6 cm
- 22.** The side of a square shaped pool is 10 m and if the cost of flooring is Rs. 100 per m^2 then, find the total cost of flooring the pool.
- A. 12,000
 - B. 10,000
 - C. 8,000
 - D. 6,000
- 23.** The area of a square is increased by 60 cm^2 when its side is increased by 4 cm. Find the side of the square.
- A. 5.5
 - B. 8
 - C. 4.5
 - D. 5
- 24.** The length and breadth of a field are 36 m and 21 m, respectively. Poles are required to be fixed all along the boundary at a distance of 3 m apart. The number of required poles will be?
- A. 40
 - B. 48
 - C. 38
 - D. 36
- 25.** If the length of the diagonal of the square is 5.2 cm , then find the area of the square.
- A. 14 cm^2
 - B. 13.50 cm^2
 - C. 14.42 cm^2
 - D. 13.52 cm^2
- 26.** The perpendicular and area of a right-angled triangle is 20 cm and 10 cm^2 . Find the base of the triangle.
- A. 1 cm
 - B. 3 cm
 - C. 4 cm
 - D. 5 cm
- 27.** The perimeter of the rectangle is 50 cm and its breadth is 10 cm . Find its area.
- A. 100 cm^2
 - B. 150 cm^2
 - C. 140 cm^2
 - D. 160 cm^2
- 28.** The perimeter of a square garden is 400 m and the cost of gardening is Rs. 3 per m^2 , then find the total cost for gardening the garden.
- A. Rs. 1,000
 - B. Rs. 2,800
 - C. Rs. 4,000
 - D. Rs. 3,000
- 29.** Find the area of the rhombus whose length of the diagonal is 10 cm .
- A. 40 cm^2
 - B. 38 cm^2
 - C. 50 cm^2
 - D. 46 cm^2
- 30.** Find the height of the triangle whose area is 100 cm^2 and length of the base is 20 cm .
- A. 10
 - B. 8
 - C. 14
 - D. 6



SOLUTIONS

1. **(A)** Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$
 $\text{Area} = \frac{1}{2} \times 8 \times 10$
 $= 40 \text{ cm}^2$, hence, the option (A) is correct.
2. **(B)** Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$
 $= \frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}$
 The area of equilateral triangle is $9\sqrt{3} \text{ cm}^2$, thus, option (B) is correct.
3. **(A)** Area of the field = $\frac{1}{2} \times \text{diagonal} \times \text{sum of the perpendicular from the opposite vertices on to this diagonal}$
 $= \frac{1}{2} \times 220 \times (40.4 + 50.8)$
 $= 10,032 \text{ m}^2$, thus, option (A) is correct.
4. **(B)** Perimeter of square = $4a$
 $4a = 16, a = 4$
 Area of square = a^2
 $= 4 \times 4 = 16 \text{ cm}^2$
5. **(D)** Area of a circle = πr^2
 $= 3.14 \times r^2 = 314$
 $= r^2 = 100, r = 10 \text{ cm}$
 Circumference of a circle = $2\pi r$
 $= 2 \times 3.14 \times 10 = 62.8 \text{ cm}$
 The circumference of the circle is 62.8 cm; hence, option D is correct.
6. **(A)** Given: Perimeter = 326 cm, Length = 98 cm
 Perimeter = $2(l + b)$
 $326 = 2(98 + b)$
 $326 = 196 + 2b$
 $2b = 130$
 Breadth = 65 cm
 Area = Length \times Breadth
 Area = $98 \times 65 = 6370 \text{ cm}^2$, hence, option (A) is correct.
7. **(D)** Given: Breadth = 12 cm, Perimeter = 50 cm
 Perimeter = $2(l + b)$
 $50 = 2(l + 12)$
 $50 = 2l + 24$
 $2l = 26, l = 13$
 The length of the rectangle is 13 cm, hence, option (D) is correct.
8. **(C)** Given: Area = 1,029 cm²
 $\text{Area} = l \times b$
 Let the length and breadth be $7x$ and $3x$
 $1029 = 3x \times 7x$
 $1029 = 21x^2$
 $x^2 = 49, x = 7$
 $\text{Length} = 7(7) = 49$
 $\text{Breadth} = 3(7) = 21$
 $\text{Perimeter} = 2(l + b)$
 $= 2(49 + 21)$
 $= 140 \text{ cm}$, hence, option (C) is correct.
9. **(B)** Given: Length = 10 m, Breadth = 7 m
 Perimeter = $2(l + b)$
 $= 2(10 + 7)$
 $= 34 \text{ m}$
 Cost of fencing = $34 \times 40 = \text{Rs. } 1,360$, hence, option (B) is correct.
10. **(A)** Let the length and breadth be $3x$, and $2x$ respectively
 Perimeter = $2(l + b)$
 $50 = 2(3x + 2x)$
 $50 = 6x + 4x$
 $50 = 10x, x = 10$
 $\text{Length} = 10(3) = 30 \text{ m}$, Breadth = $10(2) = 20 \text{ m}$
 $\text{Area} = l \times b$
 $= 30 \times 20 = 600 \text{ m}^2$, hence, option (A) is correct.
11. **(D)** Given: Perimeter = 200 ft
 Perimeter = $4a$
 $200 = 4a, a = 50 \text{ ft}$
 Area = side \times side
 $= 50 \times 50 = 2,500 \text{ ft}^2$
 Cost of flooring = $2,500 \times 50 = \text{Rs. } 75,000$, hence, option (D) is correct.
12. **(B)** Given: Diagonal = 10 m
 Diagonal = $a\sqrt{2}$
 $10 = a\sqrt{2}$
 $100 = 2a^2$
 $a^2 = 50$
 The area of the square is 50 m^2 , hence, option (B) is correct.
13. **(D)** $AB^2 = AC^2 - BC^2$
 $= (15)^2 - (12)^2$
 $= 225 - 144$



$$\begin{aligned}AB^2 &= 81 \\AB &= 9 \text{ cm} \\ \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\&= \frac{1}{2} \times 12 \times 9 \\&= 54 \text{ cm}^2, \text{ hence, option (D) is correct.}\end{aligned}$$

14. (C) Given: Diameter = 20 cm, radius = 10 cm
Area of circle = πr^2
= $3.14 \times 10 \times 10$
= 314 cm², hence, option (C) is correct.

15. (C) Let the base and height be 5x and 2x, respectively.
Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
 $3125 = \frac{1}{2} \times 5x \times 2x$
 $3125 = 5x^2$
 $625 = x^2$
 $x = 25$
Height of triangle = $2(25) = 50$ cm, thus, option (C) is correct.

16. (A) Given: Perimeter = 96 cm, a = 40 cm, b = 24 cm
96 = 40 + 24 + c
c = 32 cm
 $S = \frac{(a+b+c)}{2}$
 $= \frac{(32+24+40)}{2} = \frac{96}{2}$
= 48 cm
Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$
= $\sqrt{[48(48-40)(48-24)(48-32)]}$
= $\sqrt{(48 \times 8 \times 24 \times 16)}$
= 384 cm², hence, option (A) is correct.

17. (D) Area of parallelogram = base × height
= $54 \times 24 = 1296 \text{ m}^2$
Cost of levelling = $1,296 \times 5 = 6,480$
The total cost of levelling is Rs. 6,480, hence, option (D) is correct.

18. (A) Perimeter of first square = 24 m
Side of first square = $24/4 = 6$
Perimeter of second square = 32 m
Side of second square = $32/4 = 8$
Area of first square = $6 \times 6 = 36 \text{ m}^2$
Area of second square = $8 \times 8 = 64 \text{ m}^2$
Sum of areas = $36 + 64 = 100 \text{ m}^2$
Side of square will be = $\sqrt{100} = 10 \text{ m}$
Perimeter = $4 \times 10 = 40 \text{ m}$, hence, option (A) is correct.

19. (D) a = 3 cm, b = 4 cm, c = 5 cm
 $S = \frac{(a+b+c)}{2} = \frac{(3+4+5)}{2} = 6 \text{ cm}$
Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$
= $\sqrt{[6(6-3)(6-4)(6-5)]}$
= $\sqrt{36} = 6 \text{ cm}^2$, hence, option (D) is correct.

20. (C) Let the height and base of parallelogram be x cm and 3x cm respectively.
Area = 192 cm²
Area of parallelogram = base × height
 $192 = 3x(x)$
 $192 = 3x^2$
 $x^2 = 64, x = 8$
The height of the parallelogram is 8 cm; hence, option (C) is correct.

21. (B) Perimeter = 180 cm
Side = $180/4 = 45 \text{ cm}$
Area = $b \times h$
 $315 = 45 \times h$
 $h = 7 \text{ cm}$, hence, option (B) is correct.

22. (B) Area = side × side
= $10 \times 10 = 100 \text{ m}^2$
Total cost of flooring = $100 \times 100 = \text{Rs. } 10,000$
Total cost of flooring is Rs. 10,000, hence option (B) is correct.

23. (A) Let the side be x cm
Therefore, $(x+4)^2 - x^2 = 60$
 $= x^2 + 8x + 16 - x^2 = 60$
 $x = 5.5 \text{ cm}$, hence, option (A) is correct.

24. (C) Perimeter = $2(l \times b)$
= $2(36 + 21) = 144 \text{ m}$
Number of poles required = $144/3 = 38$, hence, option (C) is correct.

25. (D) Area = $\frac{1}{2} \times \text{diagonal} \times \text{diagonal}$
 $= \frac{1}{2} \times 5.2 \times 5.2$
= 13.52 cm², hence, option (D) is correct.

26. (A) Given: Perpendicular = 20 cm, area = 10 cm²
 $\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular}$
 $10 = \frac{1}{2} \times \text{base} \times 20$
Base = 1 cm, hence, option (A) is correct.

27. (B) Perimeter = $2(l + b)$
Given: Perimeter = 50 cm, Breadth = 10 cm



$$50 = 2(l + 10)$$

$$50 = 2l + 20$$

$$l = 15$$

$$\text{Area} = l \times b$$

$= 15 \times 10 = 150 \text{ cm}^2$, hence, option (B) is correct.

- 28. (D)** Perimeter = $4 \times \text{side}$

$$400 = 4 \times \text{side}$$

$$\text{Side} = 100 \text{ m}$$

$$\text{Area} = 100 \times 100 = 10,000 \text{ m}^2$$

Total cost for gardening = $10,000 \times 3 = \text{Rs. } 3,000$, hence, option (D) is correct.

- 29. (C)** Area = $\frac{1}{2} \times \text{diagonal}^2$

$$= \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ cm}^2$$
, hence, option (C) is correct.

- 30. (A)** Area = $\frac{1}{2} \times \text{base} \times \text{height}$

Given: Area = 100 cm^2 , Base = 20 cm

$$100 = \frac{1}{2} \times 20 \times h$$

$$200 = 20 h$$

$$h = 10 \text{ cm}$$
, hence, option (A) is correct.

24 Surface Area and Volume



SYNOPSIS

- Introduction
- Surface Area
- Total Surface Area
- Curved Surface Area
- Volume
- Formulae Related to Three-Dimensional Figures
- Summary

INTRODUCTION

This topic deals with the mensuration which means measurement of lengths in a geometric figure that deals with the relation between lengths, area and volume of two-dimensional (2D) and three-dimensional (3D) figures. Mensuration deals with relating lengths of 2D figures with their area and perimeter. For 3D figures or solids, how volume and surface area is related to the measure of its sides:

Area = Product of Sides; Volume = Base area \times side length (Height)

SURFACE AREA

The surface area and volume can be calculated for any 3D geometrical shape. The surface of any area is the region occupied by the surface of an object. The volume is the amount of space available in an object. We have different types of shapes such as a hemisphere, sphere, cube, cuboid, cylinder, etc. All 3D shapes have area and volume. But in 2D shapes such as square, rectangle, triangle, circle, etc., 2D, we can only measure the area. The area occupied by a 3D object by its outer surface is called the surface area. It is measured in square units.

The area is of two types:

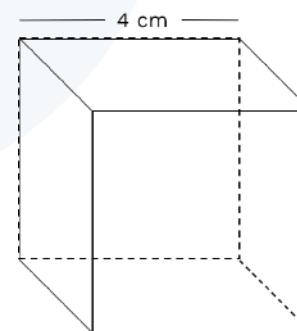
- Total surface area
- Curved surface area/lateral surface area

TOTAL SURFACE AREA

The area including the base(s) and the curved portion corresponds to the overall surface area. It is the amount of the area enclosed by the object's surface. If the form has a curved base and surface, so the sum of the two regions would be the total area. The total surface area can be defined as 'the total area covered by an object including its base as well as the curved part. If an object has both the base and curved area then the total surface area will be equal to the sum of a base and curved area'.

The total surface area is the total area occupied by an object. For example, take cuboid as an example the cuboid has 6 faces, 12 edges, and 8 vertices. The sum of all those 6 areas will be our total surface area of the particular shape.

Example: Given below figure is a cube whose side is 4 cm. Find the total surface area of the cube.



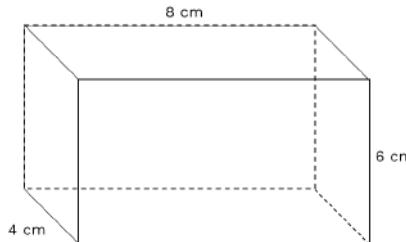
Solution: Given side = 4 cm

$$\text{Total surface area of cube} = 6a^2 \text{ sq. units}$$

$$= 6(4)^2 \text{ cm}^2$$

$$= 96 \text{ cm}^2$$

Example: Given below figure is a cuboid having its dimensions: Length = 4 cm, Height = 6 cm and Breadth = 8 cm. Find the total surface of the cuboid.



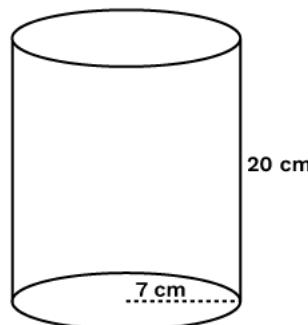
Solution: Given $L = 4 \text{ cm}$, $B = 8 \text{ cm}$, $H = 6 \text{ cm}$

$$\begin{aligned}\text{Total surface area} &= 2[(l \times b) + (l \times h) + (b \times h)] \\ &= 2[(4 \times 8) + (4 \times 6) + (8 \times 6)] \\ &= 2[(32) + (24) + (48)] \\ &= 2(104) \\ &= 208 \text{ cm}^2\end{aligned}$$

CURVED SURFACE AREA

Curved surface area, except its centre, corresponds to the area of only the curved portion of the shape (s). For shapes such as a cone, it is often called the lateral surface area. The lateral surface area can be defined as ‘the area which includes only the curved surface area of an object or lateral surface area of an object by excluding the base area of an object’. The lateral surface area is also known as the curved surface area. Most of the shapes or objects refer to the curved surface area, the shape or object-like cylinder refers to it as a lateral surface area. In simple, ‘The area which is visible to us is called a lateral surface area’.

Example: Given below figure is a cylinder and height and radius of the cylinder is 7 cm and 20 cm, respectively. Find the curved surface area of the cylinder.



Solution: Given, $r = 7 \text{ cm}$, $H = 20 \text{ cm}$

$$\begin{aligned}\text{Curved surface area} &= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 7 \times 20 \\ &= 880\end{aligned}$$

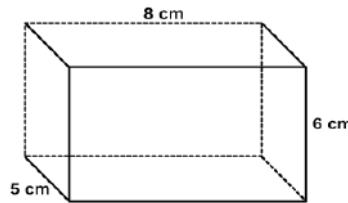
VOLUME

The volume is the amount of space in a certain 3D object. The total amount of space that an object or substance occupies is called volume. It is measured in cubic units. We have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the volume of the object.

If an object is solid, then the space occupied by such an object is measured and is termed the volume of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the capacity of the container. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid. Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.

Example: Given below figure is a cuboid, if length, breadth, and height of the cuboid is 8cm, 5 cm, 6 cm, respectively, then find the volume of the cuboid.

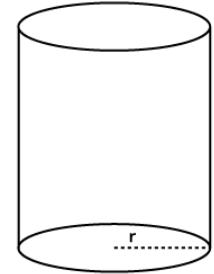


Solution: Given, $l = 8 \text{ cm}$, $h = 6 \text{ cm}$, $b = 4 \text{ cm}$

$$\text{Volume} = l \times b \times h$$

$$= 8 \times 4 \times 6$$

$$= 192 \text{ cm}^3$$



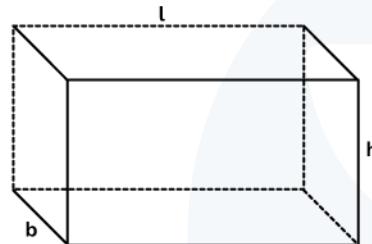
Formulae related to three-dimensional figures

1. Cuboid

a. Volume = $(l \times b \times h)$ cu. units

b. Surface area = $2(lb + bh + lh)$ sq. units

c. Longest diagonal = $\sqrt{l^2 + b^2 + h^2}$



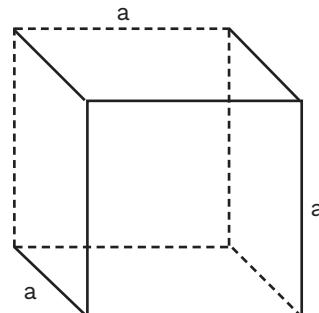
2. Cube

Each edge is 'a' unit.

a. Volume = a^3 cu. units

b. Total surface area = $6a^2$ sq. units

c. Longest diagonal = $\sqrt{3a^2} = \sqrt{3}a$



3. Right circular cylinder

a. Curved surface area = $2\pi rh$

b. Area of each circular face = πr^2

c. Total surface area = $2\pi r(r + h)$

d. Volume = $\pi r^2 h$

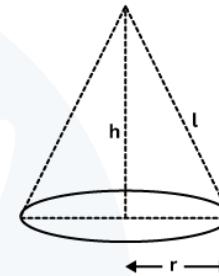
4. Right circular cone

a. Curved surface area = πrl ;

where $l = \sqrt{h^2 + r^2}$;

b. Total surface area = $\pi r^2 + \pi rl = \pi r(r + l)$

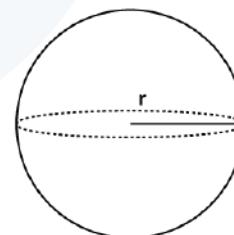
c. Volume = $\frac{1}{3} \pi r^2 h$



5. Sphere

a. Volume = $\frac{4}{3} \pi r^3$

b. Total surface area = $4\pi r^2$

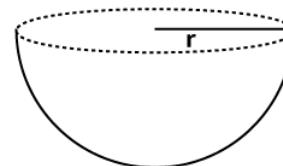


6. Hemisphere

a. Volume = $\frac{2}{3} \pi r^3$

b. Curved surface area = $2\pi r^2$

c. Total surface area = $2\pi r^2 + \pi r^2 = 3\pi r^2$





PRACTICE QUESTIONS

1. What is the total surface area of a cuboid whose dimensions are 25 cm, 10 cm, and 2 cm?
 - A. 250 cm^2
 - B. 400 cm^2
 - C. 500 cm^2
 - D. $1,000 \text{ cm}^2$
2. The diameter of the base of a right circular cylinder is 56 cm and its height is 20 cm. What is the curved surface of the cylinder?
 - A. $3,520 \text{ cm}^2$
 - B. $1,120 \text{ cm}^2$
 - C. $1,760 \text{ cm}^2$
 - D. $3,000 \text{ cm}^2$
3. A conical tent has floor area of 616 m^2 and a straight height of 48 m. If canvas costs Rs. $42/\text{m}^2$, find the cost of making the tent.
 - A. Rs. 46,200
 - B. Rs. 92,400
 - C. Rs. 88,704
 - D. Rs. 69,300
4. A solid sphere of radius 21 mm is melted to form 27 equal spherical balls. What is the surface area of each of the smaller spheres?
 - A. 204 mm^2
 - B. 308 mm^2
 - C. 412 mm^2
 - D. 616 mm^2
5. How much water can be stored in a cone with its base radius 7 m and height 12 m?
 - A. 204 m^3
 - B. 308 m^3
 - C. 412 m^3
 - D. 616 m^3
6. The volume of a cube is 343 cm^3 , find the edge of the cube.
 - A. 7 cm
 - B. 14 cm
 - C. 21 cm
 - D. 3.5 cm
7. Find the area of the cloth required to cover a cylindrical vessel of height 1 m and diameter 140 cm.
8. Two cubes each of volume 125 cm^3 are joined from end to end then, what is the total surface area of the resulting cuboid?
 - A. 125 cm^2
 - B. 200 cm^2
 - C. 250 cm^2
 - D. 300 cm^2
9. Find the total surface area of a cylinder whose base radius is 7 cm and height is 14 cm.
 - A. 564 cm^2
 - B. 231 cm^2
 - C. 462 cm^2
 - D. 924 cm^2
10. If the radius of a dome of a building is 7 m, then find the cost of whitewashing the dome from outside if cost of whitewashing is Rs. $5/\text{m}^2$.
 - A. Rs. 770
 - B. Rs. 1,540
 - C. Rs. 1,410
 - D. Rs. 1,630
11. How many spherical balls of 2 cm radius can be made out of a solid cube whose edge measures 44 cm?
 - A. 2,542
 - B. 5,084
 - C. 1,271
 - D. 7,626
12. Find the volume of a right circular cone whose diameter of base is 6 cm and height is 14 cm.
 - A. 33 cm^3
 - B. 66 cm^3
 - C. 132 cm^3
 - D. 44 cm^3
13. If the volume of a metallic cube is 125 m^3 , then find the cost of painting the cube if cost of painting is Rs. $10/\text{m}^2$.
 - A. Rs. 3000
 - B. Rs. 2250
 - C. Rs. 1800
 - D. Rs. 1500



14. What is the curved surface area of a hemisphere of radius 14 cm?
- 616 cm²
 - 924 cm²
 - 1,232 cm²
 - 1,848 cm²
15. The volume and radius of base of a right circular cylinder is 770 cm³ and 7 cm respectively. Find the height of the cylinder.
- 3 cm
 - 4 cm
 - 5 cm
 - 6 cm
16. If the surface area of a sphere is equal to its volume, then find the surface area of the sphere.
- 113
 - 85
 - 97
 - 109
17. The dimensions of a cuboid are 10 m, 12 m, and 8 m. Find the total cost of covering the cuboid with a sheet, if the cost of sheet is Rs. 12/m².
18. The dimensions of a box are 10 cm, 10 cm, and 5 cm. Calculate the length of the longest stick that can be put into the box.
- 14 cm
 - 15 cm
 - 16 cm
 - 18 cm
19. Find the water holding capacity of a conical flask having height 21 cm and radius of base 7 cm.
- 1,078 cm³
 - 1,144 cm³
 - 796 cm³
 - 1,012 cm³
20. The radius of a hemisphere is 14 m. Find the cost of painting it if the cost of painting is Rs. 10/m².
- Rs. 9,240
 - Rs. 12,320
 - Rs. 16,240
 - Rs. 18,480

SOLUTIONS

1. (C) Total surface area of cuboid = $l \times b \times h$
= $25 \times 10 \times 2$
= 500 cm², hence, option (C) is correct.
2. (A) Curved surface area of cylinder = $2\pi rh$
Given: Base = 56 cm, Height = 20 cm
= $2 \times \pi \times 28 \times 20$
= 3520 cm², hence, option (A) is correct.
3. (B) Floor area = $\pi r^2 = 616$
 $= r^2 = \frac{616 \times 7}{22} = 196 \text{ m}^2$
 $r = 14 \text{ m}$
Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2}$
= 50 m
Surface area = $\pi rl = \frac{22}{7} \times 14 \times 50 = 2,200 \text{ m}^2$

Cost of tent making = $2,200 \times 42 = 92,400$
Hence, option (B) is correct.

4. (D) Volume of the larger sphere = $\frac{4}{3} \times \pi \times (21)^3$
Volume of each smaller sphere = $\frac{4 \times \pi \times (21)^3}{3 \times 27}$
Let the radius of each small sphere be r then
 $= \frac{4}{3} \pi (r)^3 = \frac{4}{3} \pi (\frac{21}{3})^3$
 $r = 7 \text{ mm}$
Surface area of each smaller sphere = $4 \pi r^2$
 $= 4 \times \frac{22}{7} \times 7 \times 7 = 616 \text{ mm}^2$, hence, option (D) is correct.



5. (D) Given: Radius = 7 m, Height = 12 m
 Volume of a cone = $\frac{1}{3} \pi r^2 h$
 $= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12 = 616 \text{ m}^3$, hence,
 option (D) is correct.
6. (A) Volume of cube = side^3
 $\text{Side}^3 = 343 \text{ cm}$
 $\text{Side} = (7 \times 7 \times 7)^{\frac{1}{3}}$
 $\text{Side} = 7 \text{ cm}$, hence, option (A) is correct.
7. (B) Required cloth = Total surface area of cylinder
 Total surface area of cylinder = $2\pi r(h + r)$
 $= 2 \times \frac{22}{7} \times \frac{70}{100} (1 + \frac{70}{100})$
 $= 2 \times 22 \times 0.1 \times 1.7 = 7.48 \text{ m}^2$
 Hence, option (B) is correct.
8. (C) Volume = side^3
 $125 = \text{side}^3$
 $\text{Side} = 5 \text{ cm}$
 After joining 2 cubes, its length will be $5 + 5 = 10 \text{ cm}$
 Total surface area of a cube = $2(lb + bh + lh)$
 $= 2(10 \times 5 + 5 \times 5 + 10 \times 5)$
 $= 2(50 + 25 + 50)$
 $= 250 \text{ cm}^2$, hence, option (C) is correct.
9. (D) Total surface area of cylinder = $2\pi r(r + h)$
 Given: Radius = 7 cm, Height = 14 cm
 $= 2 \times \frac{22}{7} \times 7(7 + 14)$
 $= 44 \times 21 = 924 \text{ cm}^2$
 Hence, option (D) is correct.
10. (B) Given: Radius = 7 m
 Surface area of dome = $2\pi r^2$
 $= 2 \times \frac{22}{7} \times 7 \times 7 = 308 \text{ m}^2$
 Cost of whitewashing = $308 \times 5 = \text{Rs. } 1,540$,
 hence, option (B) is correct.
11. (A) Volume of cube = side^3
 $= (44)^3 = 85,184 \text{ cm}^3$
 Volume of sphere = $\frac{4}{3} \pi r^3$
12. (C) Volume of cone = $\frac{1}{3} \pi r^2 h$
 Given: Diameter = 6 cm, radius = 3 cm,
 Height = 14 cm
 $= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14$
 $= 132 \text{ cm}^3$, hence, option (C) is correct.
13. (D) Volume = side^3
 $125 = \text{side}^3$
 $\text{Side} = 5 \text{ m}$
 Surface area = $6(\text{side}^2)$
 $= 6 \times 25 = 150 \text{ m}^2$
 Cost of painting = $150 \times 10 = \text{Rs. } 1500$,
 hence, option (D) is correct.
14. (C) Curved surface area = $2\pi r^2$
 Given: Radius = 14 cm
 $= 2 \times \frac{22}{7} \times 14 \times 14$
 $= 1,232 \text{ cm}^2$, hence, option (C) is correct.
15. (C) Given: Volume = 770 cm^3 , Radius = 7 cm
 Volume = $\pi r^2 h$
 $770 = \frac{22}{7} \times 7 \times 7 \times h$
 $h = \frac{770}{154} = 5 \text{ cm}$
 The height of the cylinder is 5 cm, hence,
 option (C) is correct.
16. (A) Let 'r' be the radius of the sphere
 Surface area of sphere = volume of sphere
 $4\pi r^2 = \frac{4}{3}\pi r^3$
 $r = 3 \text{ cm}$
 Surface area = $4 \times \frac{22}{7} \times 3 \times 3$
 $= 113.04 \approx 113$, hence, option (A) is correct.
17. (A) Given: Dimensions = 10 m, 12 m and 8 m
 Total surface area = $l \times b \times h$



$$= 10 \times 12 \times 8$$

$$= 960$$

Total cost of covering = 960×12 = Rs. 11,520

Hence, option (A) is correct.

- 18. (B)** Given: Dimensions = 10 cm, 10 cm, 5 cm
Length of longest stick = Diagonal of cuboid

$$\text{Required length} = \sqrt{10^2 + 10^2 + 5^2}$$

$$= \sqrt{225} = 15 \text{ cm}$$

The longest stick that can be put into box is 15 cm, hence, option (B) is correct.

- 19. (A)** Given: Height = 21 cm, Radius = 7 cm

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 21$$

= 1,078 cm³, hence, option (A) is correct.

- 20. (D)** Surface area = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times 14 \times 14$$

$$= 1,848 \text{ m}^2$$

Total cost of painting = $1,848 \times 10$ =
Rs. 18,480, hence, option (D) is correct.



SYNOPSIS

- Definition of triangle
- Properties of a triangle
- Types of triangles
 - On the basis of length
 - Equilateral triangle
 - Isosceles triangle
 - Scalene triangle
 - On the basis of angle
 - Acute angle triangle
 - Right angle triangle
 - Obtuse angle triangle
- Area and perimeter of triangle and its types
- Some Important Theorems Related to Triangles

DEFINITION OF A TRIANGLE

A triangle can be defined as a polygon with three vertices and three edges.

In a triangle, the sum of all the three internal angles must be equal to 180° . This is called the *angle sum property of a triangle*.

A triangle is a two-dimensional closed shape. It's a polygon having three sides. Straight lines run along all three sides. The vertex is the intersection of two straight lines. As a result, there are three vertices in the triangle. An angle is formed by each vertex.

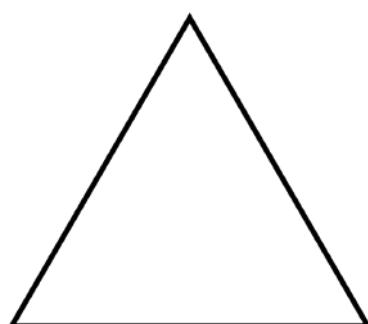


Fig. 5.1

PROPERTIES OF A TRIANGLE

- There are three edges and three sides of a triangle
- Sum of interior angles of a triangle is equal to 180° .
- Sum of exterior angles of a triangle is equal to 360° .
- The sum of two sides of a triangle will always be greater than the third side.
- The difference between any two sides of a triangle is always less than the length of the third side.
- The shortest side of a triangle will always be opposite to the smallest interior angle.
- The longest side of a triangle will always be opposite to the largest interior angle.

TYPES OF TRIANGLE

Triangles can be categorized on two bases, i.e., on the basis of length of sides and on the basis of angle.

On the basis of length of sides

There are three types of triangles based on the length of its sides. They are as:

1. **Equilateral triangle:** The triangle whose all the three sides are equal is called an equilateral triangle. Due to equal sides, it also has equal interior angles, each measuring 60° .

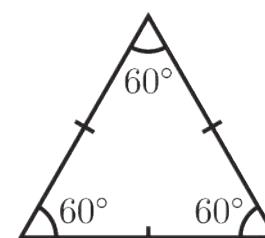


Fig. 5.2

2. **Isosceles Triangle:** The triangle that has two equal sides are called isosceles triangles.



The interior angles opposite to the two equal sides are also equal.

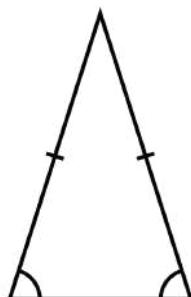


Fig. 5.3

- 3. Scalene Triangle:** A triangle that has all sides of different lengths and thus no equal interior angles, is called a scalene triangle.

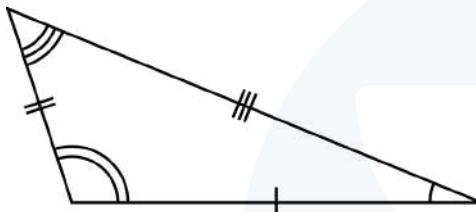


Fig. 5.4

On the basis of angles

There are three types of triangles on the basis of their angles. They are as follows:

- 1. Acute Angle Triangle:** The triangle which has all of its angle smaller than 90° is called an acute angle triangle.

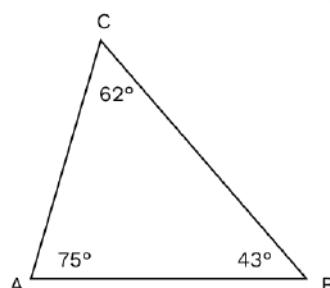


Fig. 5.5

- 2. Obtuse Angle Triangle:** The triangle that has one of its angles more than 90° is called an obtuse angle triangle.

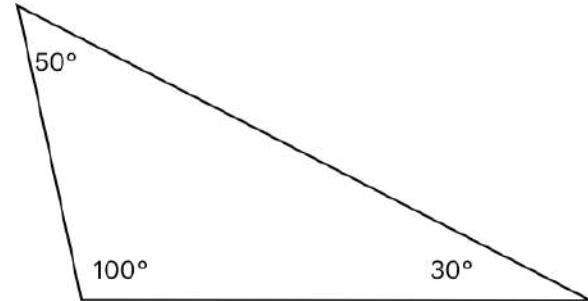


Fig. 5.6

- 3. Right-Angle Triangle:** The triangle whose one angle is 90° is called a right-angle triangle.

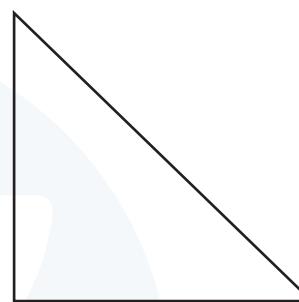


Fig. 5.7

AREA AND PERIMETER OF TRIANGLES AND ITS TYPES

Perimeter of a triangle = Sum of all sides of a triangle

Area of a triangle = Half of product of base and height
 $= \frac{1}{2} \times \text{Base} \times \text{Height}$

Area of triangle using Heron's formula

If the height of a triangle is not given, we use Heron's formula to calculate the area of the given triangle.

For this we need the semi perimeter of the triangle.

$$S = \frac{\text{Perimeter of the triangle}}{2}$$



Consider a triangle with sides a , b , and c . Then,

$$S = \frac{a + b + c}{2}$$

Then the area of the triangle is given by:

$$A = \left[(s s - - a s)(b s)(-c) \right]$$

Area of isosceles triangle using only sides

$$\frac{1}{2} \sqrt{\frac{a^2 - b^2}{4}} \times b$$

Here a = length of the equal side
 b = base of the triangle
 h = height of the triangle

PERIMETER OF ISOSCELES TRIANGLE

The perimeter of the isosceles triangle;

$$P = 2a + b$$

Here a = length of the equal side

b = length of the unequal side

AREA OF ISOSCELES RIGHT-ANGLE TRIANGLE

$$A = \frac{1}{2} \times a^2$$

SOME IMPORTANT THEOREMS RELATED TO TRIANGLE

1. Two triangles are similar, if
 - a. Their corresponding angles are equal and
 - b. Their corresponding sides are in the same ratio (or proportion).
2. The ratio of any two corresponding sides in two equiangular triangles is always the same.
3. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

4. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
5. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.
6. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
7. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

PYTHAGORAS THEOREM

In a right-angle triangle, sum of the square of the hypotenuse is equal to the sum of the square of the other two sides.

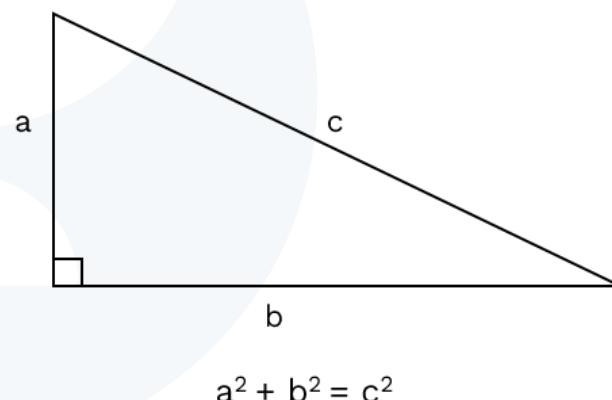


Fig. 5.8

Here, c is the hypotenuse and a and b are the other two sides of the triangle.

- If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.
- In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



PRACTICE QUESTIONS

1. Which of the three sides cannot be used to construct a triangle?
 - A. 2, 4, 7
 - B. 3, 4, 5
 - C. 5, 2, 5
 - D. 8, 6, 10
2. Calculate the perimeter of a triangle with 13 cm on each side.
 - A. 38 cm
 - B. 39 cm
 - C. 40 cm
 - D. 41 cm
3. In a triangle with a perimeter of 55 cm and two sides of 17 cm each, what is the missing side length?
 - A. 23 cm
 - B. 22 cm
 - C. 21 cm
 - D. 20 cm
4. What is the area of a triangle with a 26 cm base and a 12 cm height?
 - A. 153 cm^2
 - B. 154 cm^2
 - C. 155 cm^2
 - D. 156 cm^2
5. One side of an equilateral triangle is 15 cm long. What is the triangle's perimeter?
 - A. 45 cm
 - B. 47 cm
 - C. 43 cm
 - D. 41 cm
6. The lengths of the two short sides of a right triangle are 6 and 8. What is the triangle's perimeter?
 - A. 23 cm
 - B. 24 cm
 - C. 25 cm
 - D. 26 cm
7. The residences of Tuku, Bobbie, and Duggu form a triangle. Bobbie's residence is 9 feet distant from Tuku's. Bobbie's residence is 13 feet away from Duggu's. Tuku's residence is 15 feet distant from Duggu's. What is the perimeter of the triangle formed by their houses?
 - A. 35 feet
 - B. 36 feet
 - C. 37 feet
 - D. 38 feet
8. What is the perimeter of a right triangle with a height 56 and a 33 cm base?
 - A. 151 cm
 - B. 152 cm
 - C. 153 cm
 - D. 154 cm
9. The triangle's three sides are 12 cm, 15 cm, and 18 cm. Calculate the area of the triangle.
 - A. 75.9 cm^2
 - B. 85.9 cm^2
 - C. 95.9 cm^2
 - D. 105.9 cm^2
10. Find the base of the triangle with an area of 102 m^2 and a height of 12 m.
 - A. 14 cm
 - B. 15 cm
 - C. 16 cm
 - D. 17 cm
11. Find the height of the triangle with a 14-unit base and a 147-square-unit area.
 - A. 20 units
 - B. 21 units
 - C. 22 units
 - D. 23 units
12. A triangle's base and height are in the ratio 13:12 and its area is 312 m^2 . What is the sum of triangle's height and base?
 - A. 50
 - B. 54
 - C. 56
 - D. 58
13. Find the area of a triangle whose sides are 10 cm, 24 cm, and 26 cm.
 - A. 110 cm^2
 - B. 120 cm^2
 - C. 130 cm^2
 - D. 140 cm^2



- 14.** The area of triangle whose base is 8 cm is equal to the area of a circle of radius 7 cm. What is the height of the triangle?
- A. 18.5 cm
 - B. 28.5 cm
 - C. 38.5 cm
 - D. 48.5 cm
- 15.** A right angled triangle has its area equal to that of a square whose side is 6 cm. What is the altitude of the triangle, if it is twice its base?
- A. 42 cm
 - B. 32 cm
 - C. 22 cm
 - D. 12 cm
- 16.** What is the area of a triangle having sides 4 cm, 7 cm, and 9 cm?
- A. 13.41 cm^2
 - B. 15.43 cm^2
 - C. 17.45 cm^2
 - D. 19.47 cm^2
- 17.** A triangular field's base is three times its height. Find the base and height of the field if the cost of cultivating the field at Rs. 28 per hectare be Rs. 364.
- A. 249:90
 - B. 259:91
 - C. 269:92
 - D. 279:93
- 18.** The ratio of the areas of two triangles is 4:3, while the ratio of their heights is 3:4. Calculate their base ratio.
- A. 18:6
 - B. 17:7
 - C. 16:9
 - D. 15:8
- 19.** The base of a triangle is 17 cm and height is 14 cm. The height of another triangle of double the area having the base 20 cm is?
- A. 23.8 cm
 - B. 33.8 cm
 - C. 43.8 cm
 - D. 53.8 cm
- 20.** A triangle having the base AB is ABC . D is a point on AB where AB equals 8 cm and DB equals 6 cm. What is the ratio of area of triangle ADC to that of the area of triangle ABC ?
- A. 5 cm
 - B. 6 cm
 - C. 7 cm
 - D. 8 cm
- 21.** The area of right angled triangle is 20 times its base. What is the height?
- A. 40 cm
 - B. 41 cm
 - C. 42 cm
 - D. 43 cm
- 22.** If a triangle's area is 1176 cm^2 and the base-to-height ratio is 3:4, what is the triangle's altitude?
- A. 55 cm
 - B. 56 cm
 - C. 57 cm
 - D. 58 cm
- 23.** A triangle's three sides are 9 cm, 40 cm, and 41 cm, respectively. Then its area is?
- A. 160 cm^2
 - B. 170 cm^2
 - C. 180 cm^2
 - D. 190 cm^2
- 24.** The sides of a triangle are in the ratio of $1/4 : 1/6 : 1/8$. If the perimeter is 52 cm, then the length of the small side is?
- A. 15 cm
 - B. 14 cm
 - C. 13 cm
 - D. 12 cm
- 25.** A triangle has a surface area of 216 cm^2 and sides in the ratio 3:4:5. What is the triangle's perimeter?
- A. 71 cm
 - B. 72 cm
 - C. 73 cm
 - D. 74 cm
- 26.** The sides of a triangle are 6 cm, 8 cm, and 10 cm. The area of the triangle formed by joining the mid-points of the sides of this triangle is?



- A. 4 cm^2
B. 5 cm^2
C. 6 cm^2
D. 7 cm^2
27. One side of a right angle triangle is two times the other, and the hypotenuse is 10 cm. The area of the triangle is?
A. 50 cm^2
B. 40 cm^2
C. 30 cm^2
D. 20 cm^2
28. A right angled triangle has a perimeter of 60 cm. It has a 26-cm hypotenuse. What is the triangle's area?
A. 120 cm^2

- B. 122 cm^2
C. 124 cm^2
D. 126 cm^2
29. If an isosceles right triangle's perimeter is $(6+3\sqrt{2}) \text{ m}$, what is the triangle's area?
A. 2.5 m^2
B. 3.5 m^2
C. 4.5 m^2
D. 5.5 m^2
30. A triangle's perimeter is 30 cm and its area is 30 cm^2 . What is the length of the shortest side of the triangle if the greatest side is 13 cm?
A. 2 cm
B. 3 cm
C. 4 cm
D. 5 cm

SOLUTIONS

1. **(A)** We already know that the total of any two triangle sides must be bigger than the third side. Option (A) does not meet this requirement.
2. **(B)** The triangle is an equilateral triangle because all three sides are of the same length.
 $\text{Perimeter} = 3 \times \text{side}$
 $= 3 \times 13 = 39 \text{ cm.}$
3. **(C)** The triangle is isosceles triangle as only two sides are equal here.
 $\text{So, perimeter} = 2L + B$
 $55 = 2 \times 17 + B$
 $B = 21 \text{ cm}$
4. **(D)** Area of triangle = $\frac{1}{2} \times b \times h$
 $= \frac{1}{2} \times 26 \times 12$
 $= 156 \text{ cm}^2$
5. **(A)** Perimeter of triangle = $3 \times 15 = 45 \text{ cm.}$
6. **(B)** For perimeter, first we have to find the length of the third side.
Using Pythagoras theorem to find the third side,
 $a^2 + b^2 = c^2$
 $6^2 + 8^2 = c^2$
Hence, (C) or third side = 10
Now, perimeter = $6 + 8 + 10 = 24 \text{ cm.}$
7. **(C)** The formula for calculating the perimeter of a triangle is to add the lengths of the three sides together.
The three sides of this triangle are 9ft, 13ft, and 15ft in length. When you combine these three sides together, you get a perimeter of 37 feet.
8. **(D)** Using the Pythagoras theorem,
 $(56)^2 + (33)^2 = C^2$
 $3136 + 1089 = C^2$
So, third side = 65 cm
Perimeter = $56 + 33 + 65 = 154 \text{ cm.}$
9. **(C)** Perimeter = $13 + 15 + 18 = 46 \text{ cm}$
Semi perimeter = $\frac{46}{2} = 23 \text{ cm}$
 $\text{Area} = \sqrt{23(23-13)(23-15)(23-18)}$
 $= \sqrt{23 \times 10 \times 8 \times 5}$
 $= \sqrt{9200} = 95.9 \text{ cm}^2$
10. **(D)** Area = $\frac{1}{2} \times b \times h$
 $102 = \frac{1}{2} \times b \times 12$
Base = $\frac{102}{6} = 17 \text{ cm.}$
11. **(B)** Area = $\frac{1}{2} \times b \times h$
 $147 = \frac{1}{2} \times 14 \times h$
Height = $\frac{147}{7} = 21 \text{ units.}$
12. **(A)** Let's take height and base is $12x$ and $13x$



$$\text{Area} = \frac{1}{2} \times b \times h$$

$$312 = \frac{1}{2} \times 13x \times 12x$$

$$312 = 78x^2$$

$$x = 2$$

$$\text{So, height} = 12 \times 2 = 24$$

$$\text{and, base} = 13 \times 2 = 26$$

- 13. (B)** Firstly we have to find the semiperimeter of this triangle = $\frac{10+24+26}{2} = 30 \text{ cm}$
 Now, area of triangle
 $= \sqrt{30(30-10)(30-24)(30-26)}$
 $= \sqrt{30 \times 20 \times 6 \times 4} = 120 \text{ cm}^2$

- 14. (C)** Area of circle = πr^2
 $= 22/7 \times 7 \times 7 = 154 \text{ cm}^2$
 Area of triangle = $\frac{1}{2} \times b \times h$
 $154 = \frac{1}{2} \times 8 \times h$
 Height = 38.5 cm.

- 15. (D)** Area of square = $(6)^2 = 36 \text{ cm}^2$
 Since the altitude of the triangle is twice the base, let the base and altitude is b and $2b$, respectively.
 Area of triangle = $\frac{1}{2} \times b \times h$
 $36 = \frac{1}{2} \times b \times 2b$
 $b = 6$
 So, altitude = $2b = 12 \text{ cm.}$

- 16. (A)** Firstly we have to find the semi-perimeter of this triangle = $\frac{4+7+9}{2} = 10 \text{ cm}$
 Now, area of triangle
 $= \sqrt{10(10-4)(10-7)(10-9)}$
 $= \sqrt{10 \times 6 \times 3 \times 1} = 13.41 \text{ cm}^2$

- 17. (D)** Area of the field = Total cost/Rate = $364/28 = 13$ hectare = 1,30,000 m²
 Let altitude be x and base be $3x$
 Then, $\frac{1}{2} \times 3x \times x = 1,30,000$
 $x = 93$
 Hence, base = 279 and altitude = 93

- 18. (C)** Let the bases of two triangles be x and y and their heights be $3h$ and $4h$, respectively.
 Then, $(\frac{1}{2} \times x \times 3h) / (\frac{1}{2} \times y \times 4h) = 4/3$
 $x/y = 4/3 \times 4/3 = 16/9$
 Hence, required ratio = 16:9

- 19. (A)** Area of first triangle = $\frac{1}{2} \times 17 \times 14 = 119 \text{ cm}^2$
 So the area of second triangle = 238 cm²

$$\text{Hence, } 238 = \frac{1}{2} \times 20 \times h$$

$$\text{Height} = 23.8 \text{ cm.}$$

- 20. (B)** Here, semi-perimeter = $\frac{1}{2} (8 + 6 + 10) = 12 \text{ cm}$

$$\text{Area} = \sqrt{12 \times 4 \times 6 \times 2} = 24 \text{ cm}^2$$

$$\text{Now, } \frac{1}{2} \times 8 \times h = 24$$

$$\text{So, height} = 6 \text{ cm.}$$

- 21. (A)** Area = $\frac{1}{2} \times b \times h$
 $20 \times b = \frac{1}{2} \times b \times h$
 So, height = 40 cm.

- 22. (B)** Let base be $3x$ and height is $4x$
 Then, $\frac{1}{2} \times 3x \times 4x = 1176$
 $12x^2 = 2,352$
 $x = 14 \text{ cm}$
 So, altitude = $4 \times 14 = 56 \text{ cm}$

- 23. (C)** It is a right angle triangle. So,
 Area = $\frac{1}{2} \times 40 \times 9 = 180 \text{ cm}^2$

- 24. (D)** Ratio of sides = $\frac{1}{4} : \frac{1}{6} : \frac{1}{8} = 6:4:3$
 Perimeter = 52 cm. So, sides are 24 cm, 16 cm, and 12 cm
 Hence, length of the smallest side = 12 cm

- 25. (B)** Let sides be $3x$ cm, $4x$ cm, and $5x$ cm.
 So, semiperimeter = $6x$ cm
 $\text{Area} = \sqrt{6x \times 3x \times 2x \times x} = 6x^2 \text{ cm}^2$
 $216 = 6x^2$
 $\text{So, } x = 6$
 Hence, sides are 18 cm, 24 cm, and 30 cm.
 So, perimeter = $18 + 24 + 30 = 72 \text{ cm.}$

- 26. (C)** It is a right angle triangle with base 6 cm and height 8 cm.
 So, area = $\frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$
 Hence, area of required triangle = $\frac{1}{4} \times 24 = 6 \text{ cm}^2$

- 27. (D)** Let the sides be x and $2x$
 Then, $(x)^2 + (2x)^2 = (10)^2$
 $x^2 = 20$
 $\text{Area} = \frac{1}{2} \times x \times 2x = x^2 = 20 \text{ cm}^2$

- 28. (A)** Perimeter = $b + h + 26$
 $60 - 26 = b + h$ or $(b + h)^2 = 34^2$
 Also, $b^2 + h^2 = 26^2$
 $\text{So, } (b + h)^2 - (b^2 + h^2) = 34^2 - 26^2$
 $2bh = (34 + 26)(34 - 26)$
 $bh = 240$ or $\frac{1}{2} bh = 120$
 Hence, area = 120 cm^2



- 29. (C)** Let the sides be a , a and b meters.
Then, $2a + b = 6 + 3\sqrt{2}$ and $b^2 = a^2 + a^2 = 2a^2$
 $b = \sqrt{2}a$
So, $2a + \sqrt{2}a = 6 + 3\sqrt{2}$
 $a = 3$
Hence, area = $\frac{1}{2} \times 3 \times 3 = 4.5 \text{ m}^2$

- 30. (D)** Let the smallest side be x

Then, the other sides are 13 cm. and $(17 - x)$ cm
Here, $a = 13$, $b = x$ and $c = (17 - x)$ with semi-perimeter = 15
So, Area = $\sqrt{15 \times 2 \times (15 - x) \times (x - 2)}$
 $30^2 = 30 \times (15 - x) (x - 2)$
 $x = 5$
Hence, smallest side = 5 cm.

26 Circles



SYNOPSIS

- Definition of Circle
- Parts of Circle
 - Radius
 - Diameter
 - Sector
 - Segment
 - Arc
 - Secant
 - Tangent
 - Chord
 - Centre
- Circle Formulas
 - Areas and perimeter
- Properties of Circle
- Some Important Theorems

DEFINITION OF CIRCLE

Circle may be defined as a collection of points in a plane which are equidistant from a fixed point in the same plane.

A circle divides the plane it exists on into 3 parts. They are: (i) inside the circle, also known as the circle's interior; (ii) the circle; and (iii) outside the circle, also known as the circle's exterior. The circular region is made up of the circle and its inside.

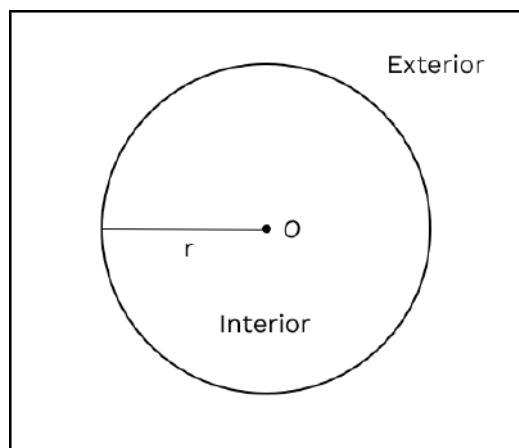


Fig. 6.1

Fig. 9.1 shows the interior of a circle, the exterior of a circle and the circle.

PARTS OF A CIRCLE

- **Centre:** The fixed point as shown in Fig. 6.1 (represented by O) is called as the centre of the circle.
- **Radius:** The fixed distance between the center of the circle and all the points forming the circle is defined as the radius of the circle. It is generally represented by r (see Fig. 6.1).
- **Chord:** A line segment whose both the endpoints lie on the circle is called a chord.
- **Diameter:** A line segment which has both its endpoints on the circle and passes through the center is called the diameter of the circle. It is generally denoted by d and is twice the size of the radius of the circle.

Diameter is the longest chord of the circle.

$$d = 2r$$
$$r = d/2$$

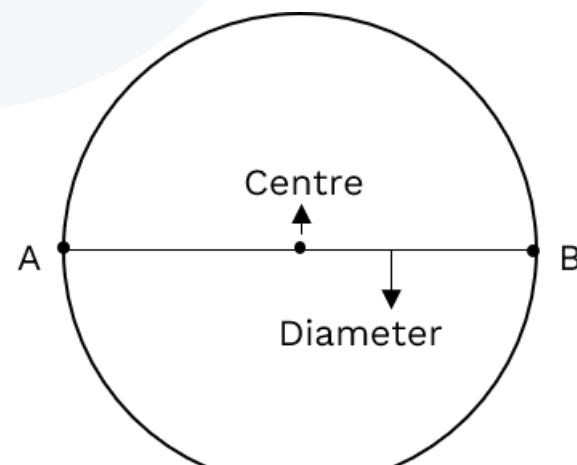


Fig. 6.2



- **Arc:** Arc is basically a part of the circumference of the circle. It is denoted by ‘ \cap ’ or ‘ $\widehat{ } \cdot$ ’.
- **For example:** In the Fig. 6.3, AB is the arc and is read as ‘arc AB’.

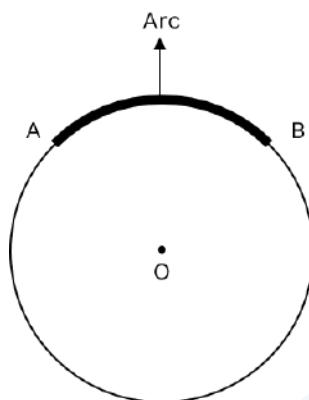


Fig. 6.3

- **Secant:** Secant can be defined as an extended chord which cuts the circle at two points.
- **Segment:** The region bounded by a chord and an arc between the two endpoints of the chord. It may be both in interior as well as exterior of the circle.
- **Sector:** The region in the interior of the circle bound by two radii and an arc formed between the two end points of the radii on the circle.
- **Tangent:** A line segment touching the circle at one and only point on the exterior is called a tangent.

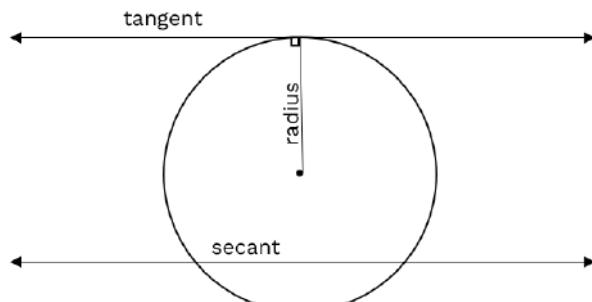


Fig. 6.4

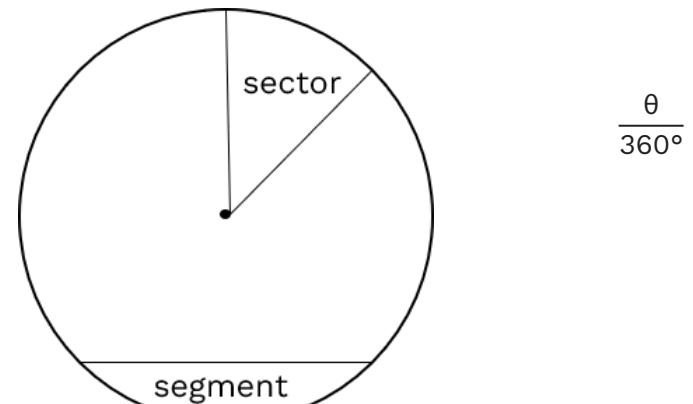
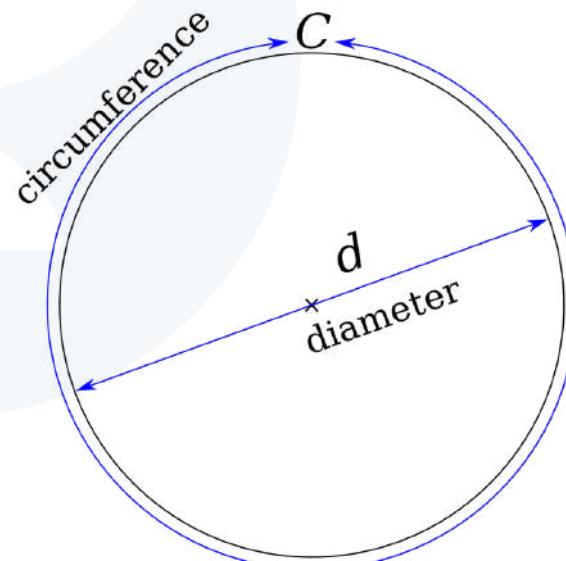


Fig. 6.5

Fig. 6.4 shows the sector and segment of a circle. Tangent and secant can be clearly seen in Fig. 6.5.

CIRCLE FORMULAS

- **Circumference of a circle:** Circumference of a circle is defined as the distance around the circle.



$$\text{Circumference of a circle (C)} = 2\pi r = \pi d$$

- **Area of a circle:** The total amount of space occupied by a circle is defined as the area of the circle.

$$\text{Area of the circle} = \pi r^2$$

- **Circumference of a semicircle:** Circumference of a semicircle is half the area of a circle.
Area of a semicircle = $\frac{1}{2}\pi r^2$



- **Area of sector:** $\frac{\theta}{360^\circ} \times \pi r^2$ (when θ is in degrees)
- **Perimeter of a sector:** $2 \text{Radius} + (\frac{\theta}{360^\circ} \times 2\pi r)$
- **Area of a segment:** Area of a segment in degrees = $\frac{1}{2} \times r^2 \times [(\frac{\pi}{180^\circ}) \theta - \sin \theta]$
Area of segment in radians = $\frac{1}{2} \times r^2 (\theta - \sin \theta)$
- **Perimeter of a segment:** The perimeter of a segment is the sum of chord length and arch length.
Perimeter of segment in degrees = $(\frac{\theta}{360^\circ} \times 2\pi r) + (2r \sin \frac{\theta}{2})$
Perimeter of segment in radians = $r\theta + (2r \sin \frac{\theta}{2})$

PROPERTIES OF A CIRCLE

- Two circles with equal radii are congruent to each other.
- Diameter is the longest chord of a circle and is double the radius in length.

- Diameter of a circle divides it into two equal parts.
- Two circles with equal radii are similar circles.

SOME IMPORTANT THEOREMS

- Angle made by equal chords at the centre are equal
- If two angles made by chords at the centre are equal the chords are equal.
- A perpendicular from the centre of a circle bisects the chord in two equal halves.
- There can be one and only one circle passing through three non-collinear points.
- Two equal chords of the circle are at equal distance from the centre.
- Two chords which are at equal distance from the centre are equal in length.
- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.

PRACTICE QUESTIONS

- Calculate the area of a circle with a radius of 6.
 - 28π
 - 36π
 - 42π
 - 46π
- Calculate the circumference of a circle with a radius of 49.
 - 306
 - 307
 - 308
 - 309
- Calculate the diameter of a circle with a circumference of 17π .
 - 14
 - 15
 - 16
 - 17
- Find the area of a sector of a 28-cm-radius circle with a 90-degree central angle.
 - 616 cm^2
 - 626 cm^2
 - 636 cm^2
 - 646 cm^2
- What is the area of the segment if the area of a sector is 128 sq. ft. and the area of the enclosed triangle is 67 square feet?
 - 61 sq. ft.
 - 71 sq. ft.
 - 81 sq. ft.
 - 91 sq. ft.
- AB subtends a 72-degree angle to the centre of a circle with a radius of 21 cm, given that arc. Determine the length of the arc AB.
 - 26 cm
 - 26.4 cm
 - 26.8 cm
 - 26.5 cm



7. If the area of the appropriate minor segment is 37 sq. units and the radius is 7 units, find the area of the major segment of a circle.
- 115 sq. units
 - 116 sq. units
 - 117 sq. units
 - 118 sq. units
8. If the central angle of a pizza slice is 60 degrees and the radius is 6 units, determine the area of the segment generated by removing the triangle part of the pizza slice.
- 3.19 sq. units
 - 4.27 sq. units
 - 3.67 sq. units
 - 3.27 sq. units
9. Calculate the length of an arc of a circle that forms a 150-degree angle with the centre of a circle with a radius of 27 cm.
- 70.45 cm
 - 70.55 cm
 - 70.65 cm
 - 70.75 cm
10. An arc is 70 meters long. Find the angle subtended by the arc if the radius of the circle is 21 m.
- 191 degrees
 - 191.08 degrees
 - 191.80 degrees
 - 192 degrees
11. Find the radius of an arc with a length of 127 cm and a 120-degree angle to the circle's centre.
- 6.44 cm
 - 6.54 cm
 - 6.64 cm
 - 6.74 cm
12. An equilateral triangle is the form of a wire. The triangle's sides are 14 cm long. A circle is formed by bending the wire. Calculate the circumference of the resulting circle.
- 44.63 square cm
 - 45.63 square cm
 - 46.63 square cm
 - 47.63 square cm
13. Calculate the length of an arc, having a radius of 17 m and an angle of 33.67 radians, in radians.
- 575.39 m
 - 574.39 m
 - 573.39 m
 - 572.39 m
14. If a wheel revolves 49 times to cover 343 meters, what is its radius?
- 3.83 m
 - 2.83 m
 - 0.83 m
 - 1.83 m
15. A ring's circumference and diameter differ by 28 cm. Calculate the ring's radius.
- 5.53 cm
 - 6.53 cm
 - 7.53 cm
 - 8.53 cm
16. 5:00 p.m. is displayed on a circular clock. The minute hand measures 14 units in length. When the time is 5:30 p.m., find the distance travelled by the tip of the minute hand.
- 44 units
 - 46 units
 - 48 units
 - 50 units
17. A circle's circumference is 576 yards. Calculate the circle's diameter.
- 183.43 yards
 - 185.43 yards
 - 187.43 yards
 - 189.43 yards
18. The shape of a race track is that of a circular ring. The track has a 47-yard inner radius and a 49-yard outside radius. Find the area of the race track.
- 601.42 sq. yards
 - 603.42 sq. yards
 - 605.42 sq. yards
 - 607.42 sq. yards
19. The area ratio of two circles is 16:25. Find the ratio of their radii using the area of circle formula.



- A. 1:5
B. 2:5
C. 4:5
D. 8:5
- 20.** If the radius is 28 cm and the angle is 108° , calculate the area of the sector.
A. 43.2 cm^2
B. 44.2 cm^2
C. 45.2 cm^2
D. 46.2 cm^2
- 21.** A man walks twice a day on a walking track that is shaped like a sector with an angle of 111° and a radius of 97 m. In a single day, determine the area traversed by the guy of the walking track.
A. 17642.68 m
B. 17742.68 m
C. 17852.68 m
D. 17967.68 m
- 22.** In a field, a horse is grazing. It's attached to a pole via a 7-m rope. The horse moves from point A to point B, forming a 72-degree arch. Find the horse's grazing place in the sector.
A. 26.8 sq. meter
B. 28.8 sq. meter
C. 30.8 sq. meter
D. 32.8 sq. meter
- 23.** If the area of the sector is 57 m and the part of a triangle in the sector is 23 m, find the area of the segment.
A. 31 sq. meter
B. 32 sq. meter
C. 33 sq. meter
D. 34 sq. meter
- 24.** If the angle subtended by a sector's arc at its centre is 180 degrees, the sector's area in square units is?
A. $\frac{\pi r^2}{2}$
B. $\frac{\pi r^2}{4}$
C. $\frac{3\pi r^2}{2}$
D. πr^2
- 25.** A bicycle's wheels have a diameter of 77 cm. To cover a distance of 147 meters, how many revolutions will each wheel make?
A. 41 revolutions
B. 51 revolutions
C. 61 revolutions
D. 71 revolutions
- 26.** Each wheel of a motorcycle has a radius of 0.280 m. If each wheel rotates 100 times, how far will the motorcycle travel? Assume that the motorcycle is on a straight path.
A. 146 m
B. 156 m
C. 166 m
D. 176 m
- 27.** To form a circle, a piece of wire in the shape of a rectangle with a length of 80 cm and a width of 35 cm is cut and folded. Calculate the radius and circumference of the completed circle.
A. 36.62 cm
B. 38.62 cm
C. 40.62 cm
D. 42.62 cm
- 28.** Calculate the perimeter of a 0.14-meter-radius circular flower garden.
A. 0.088 m
B. 0.88 m
C. 1.88 m
D. 2.88 m
- 29.** What is the area of a sector of a circle with a radius of 46 cm that is cut off by a 12.5 cm long arc?
A. 277.5 cm^2
B. 287.5 cm^2
C. 297.5 cm^2
D. 267.5 cm^2
- 30.** When a circular disc with a radius of 10 cm is partitioned into sectors with angles of 120 and 150, the ratio of the two sectors' areas is?
A. 1:5
B. 2:5
C. 3:5
D. 4:5



SOLUTIONS

1. **(B)** Area = πr^2
 $= \pi (6)^2$
 $= 36\pi$
2. **(C)** Circumference = $2\pi r$
 $= 2 \times \frac{22}{7} \times 49$
 $= 2 \times 22 \times 7 = 308.$
3. **(D)** Circumference = πd
 $17\pi = \pi \times d$
Hence, diameter = 17.
4. **(A)** Here, $r = 28$ cm and $\theta = 90$
 $l = \frac{\theta}{360} \times 2\pi r = \frac{90}{360} \times 2 \times \frac{22}{7} \times 28 = 44$ cm
Perimeter = $l + 2r = 44 + 2 \times 28 = 100$ cm
Area = $\frac{1}{2} \times 44 \times 28 = 616$ cm²
5. **(A)** Area of the segment = area of the sector - area of the triangle
 $= 128$ sq. ft. - 67 sq. ft.
 $= 61$ sq. ft.
Therefore, the area of the segment is 61 sq. ft.
6. **(B)** Here $r = 21$ cm, $\theta = 72$ degrees.
By substitution,
The length of an arc = $2\pi r (\frac{\theta}{360})$
Length of the arc = $2 \times \frac{22}{7} \times 21 \times \frac{72}{360}$
= 26.4 cm.
7. **(C)** Area of the major segment = area of the circle - area of the minor Segment
 $= \pi r^2 - 62$
 $= (\frac{22}{7}) \times 7 \times 7 - 37$
 $= 117$ sq. units
Therefore, the area of the major segment
= 117 sq. units.
8. **(D)** The radius of pizza is, $r = 6$ units.
The central angle is, $\theta = 60$ degrees.
The area of the segment is,
 $r^2 [\pi \frac{\theta}{360} - \sin \frac{\theta}{2}]$
 $= 6^2 [3.142 \times \frac{60}{360} - \sin \frac{60}{2}]$
 ≈ 3.27 square units.
Therefore, the area of the segment of the pizza = 3.27 square units.
9. **(C)** The length of an arc = $2\pi r (\frac{\theta}{360})$
 $= 2 \times 3.14 \times 27 \times 150/360$
 $= 70.65$ cm.
10. **(B)** The length of an arc = $2\pi r (\frac{\theta}{360})$
 $70 = 2 \times 3.14 \times 21 \times (\frac{\theta}{360})$
 $70 = 131.88 \times \frac{\theta}{360}$
Multiply both sides by 360 to remove the fraction.
 $25200 = 131.88\theta$
Divide both sides by 131.88
 $\theta = 191.08$ degrees.
11. **(D)** The length of an arc = $2\pi r (\frac{\theta}{360})$
 $127 = 2 \times 3.14 \times r \times \frac{120}{360}$
 $127 = 18.84 r$
Divide both sides by 18.84
 $r = 6.74$ cm.
So, the radius of the arc is 6.74 cm.
12. **(A)** Perimeter of the triangle = $3 \times$ side = $3 \times 14 = 42$ cm.
Perimeter of the triangle = circumference of the circle.
Thus, the perimeter of the triangle is also 42 cm.
Circumference of a circle = $2\pi r = 2 \times \frac{22}{7} \times r = 42$.
 $r = \frac{(42 \times 7)}{44} = 6.68$.
Therefore, the radius of the circle is 6.68 cm.
Area of a circle = $\pi r^2 = \frac{22}{7} \times (6.68)^2 = 44.63$ square cm.
Therefore, the area of a circle is 44.63 square cm.
13. **(D)** Arc length = $r\theta$
 $= 17 \times 33.67 = 572.39$ m.
14. **(C)** One rotation of the wheel = circumference of the wheel
49 rotations = 343 m
1 rotation = 7 m
So, circumference = 7 m
 $2\pi r = 7$ m
 $r = 7 \times \frac{7}{22} \times \frac{1}{2}$
 $r = 0.83$ m
15. **(B)** Circumference - Diameter = 28 cm
 $2\pi r - 2r = 28$ cm
 $2r(\pi - 1) = 28$ cm
 $2r(\frac{22}{7} - 1) = 28$ cm



$$2r \left(\frac{15}{7}\right) = 28 \text{ cm}$$

$$r = 28 \times \frac{7}{15} \times \frac{1}{2}$$

$$r = 6.53 \text{ cm}$$

- 16. (A)** At 5:30 p.m., the minute hand covers half of the circle. As a result, the minute hand's travel distance is actually half of the circumference.

Distance = πr (where r is the length of the minute hand).

As a result, the distance travelled is $\frac{22}{7} \times 14 = 22 \times 2 = 44$ units.

As a result, the total trip distance is 44 units.

- 17. (A)** Circumference = $2\pi R$

$$576 = 2 \times 3.14 \times R$$

$$576 = 6.28R$$

Divide both sides by 6.28 to get,

$$R = 91.71$$

Therefore, the radius of the circle is 91.71 yards. But, since the diameter is twice the radius of a circle, the diameter is equal to 183.43 yards.

- 18. (B)** $R = 49$ yd, $r = 47$ yd.

Let the area of outer circle be A_1 and the area of inner circle be A_2 .

Area of race track = $A_1 - A_2 = \pi R^2 - \pi r^2 = \pi (49^2 - 47^2) = \frac{22}{7} \times 192 = 603.42$ square yards.

Therefore, the area of the race track is 603.42 square yards.

- 19. (C)** It is given that $A_1 : A_2 = 16 : 25$

Area of a Circle = πr^2

$$\pi R_1^2 : \pi R_2^2 = 16 : 25$$

Taking square roots of both sides,

$$R_1 : R_2 = 4 : 5$$

Therefore, the ratio of the radii = 4:5

- 20. (D)** Area of sector = $\frac{108}{360} \times \frac{22}{7} \times 28 \times 28 = 46.2 \text{ cm}^2$

- 21. (B)** Area of the sector = $\frac{111}{360} \times \frac{22}{7} \times 97 \times 97 = 8871.34$ square meter

Area covered by the man of the walking track in a day = $8871.34 + 8871.34 = 17742.68$ m.

- 22. (C)** Area of sector = $\frac{72}{360} \times \frac{22}{7} \times 7 \times 7 = 30.8$ square meter

- 23. (D)** Area of segment = Area of sector – area of triangle
 $= 57 - 23 = 34$ square meter.

- 24. (A)** It's a semicircle, since the central angle is 180 degrees.

As a result, the area of this sector is $\frac{1}{2}$ the area of the circle = $\frac{\pi r^2}{2}$

- 25. (C)** Circumference = $\frac{22}{7} \times 77 = 242$ cm

To get the number of revolutions of the wheel, divide the distance covered by the circumference of the wheel.

We need to convert 147 meters to cm before dividing, so we multiply 147 by 100 to get 14,700 cm. Therefore,

$$\text{Number of revolutions} = \frac{14,700 \text{ cm}}{242 \text{ cm}}$$

$$= 61 \text{ revolutions (approx.)}$$

- 26. (D)** Circumference = $2 \times \frac{22}{7} \times 0.280 = 1.76$ m.

To find the distance travelled, multiply the circumference of the wheel by the number of revolutions taken.

$$\text{Distance} = 1.76 \times 100 = 176 \text{ m}$$

Therefore, the distance travelled is equal to 176 meters.

- 27. (A)** The circumference of the circle formed = the perimeter of the rectangular wire.

$$\text{Perimeter of a rectangle} = 2(80 + 35) \text{ cm} = 230 \text{ cm.}$$

Therefore, the circumference of the circle will be 230 cm.

$$\text{Circumference} = 2 \pi R$$

$$230 \text{ cm} = 2 \times \pi \times R$$

$$230 \text{ cm} = 2 \times 3.14 \times R$$

$$R = 36.62 \text{ cm}$$

So, the radius of the circle will be 36.62 cm.

- 28. (B)** Circumference = $2 \times \frac{22}{7} \times 0.14 = 0.88$ m.

- 29. (B)** Area = $\frac{1}{2} \times IR = \frac{1}{2} \times 12.5 \times 46 = 287.5 \text{ cm}^2$

- 30. (D)** Here, $\frac{120}{360} = \frac{1}{3}$

$$\text{and, } \frac{150}{360} = \frac{5}{12}$$

So, sector with angle 120 and 150 is part $\frac{1}{3}$ and $\frac{5}{12}$.

Hence, Ratio of area of two sectors = Ratio of central angle = 120:150 = 4:5.



SYNOPSIS

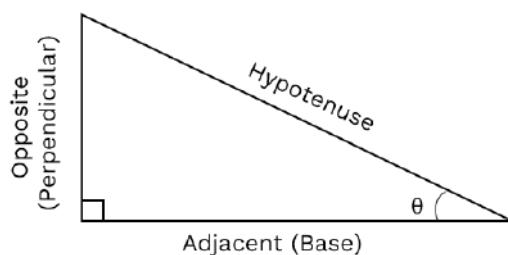
- Some Important Definitions
- Pythagoras Theorem
- Trigonometric Ratios
- Reciprocal Relationships
- Trigonometric Table
- Relation between Degree and Radian
- Table for Degree and Radian Relation
- Trigonometric Identities
- Pythagoras Identities
- Heights and Distances

INTRODUCTION

Trigonometry can be defined as the branch of mathematics which deals with the relation between the side lengths and angles of a right-angled triangle.

SOME IMPORTANT DEFINITIONS

- **Hypotenuse:** Hypotenuse is the largest side of the triangle and is opposite to the right angle of the triangle.
- **Adjacent:** The side of the triangle on which the right angle lies is called as the adjacent of the triangle. It is also referred as base of the triangle.
- **Opposite:** Side perpendicular to the base of the right-angled triangle. It is also referred as perpendicular for trigonometric calculations.



PHTYAGORAS THEOREM

Pythagoras theorem states that the square of the hypotenuse in a right-angled triangle is equal to the sum of squares of the other two sides.

$$\text{Hypotenuse}^2 = \text{Adjacent side}^2 + \text{Opposite side}^2$$

TRIGONOMETRIC RATIOS

$\sin\theta$	$\frac{\text{Opposite side}}{\text{Hypotenuse}}$
$\tan\theta$	$\frac{\text{Opposite side}}{\text{Adjacent side}}$
$\cos\theta$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$
$\operatorname{cosec}\theta$	$\frac{\text{Hypotenuse}}{\text{Opposite side}}$
$\sec\theta$	$\frac{\text{Hypotenuse}}{\text{Adjacent side}}$
$\cot\theta$	$\frac{\text{Adjacent side}}{\text{Opposite side}}$

RECIPROCAL RELATIONSHIP

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\operatorname{cosec}\theta \times \sin\theta = 1$$

$$\sec\theta \times \cos\theta = 1$$

$$\cot\theta \times \tan\theta = 1$$



TRIGONOMETRIC TABLE

Angles	0°	30°	45°	60°	90°	180°	270°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞

RELATION BETWEEN DEGREE AND RADIAN

$$2\pi \text{ radian} = 360^\circ$$

TABLE FOR DEGREE AND RADIAN RELATION

DEGREE	30°	45°	60°	90°	180°	270°	360°
RADIAN	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

TRIGONOMETRIC IDENTITIES

If an equation involves trigonometric ratios of an angle and is true for all the values of an angle, it is called trigonometric identity.

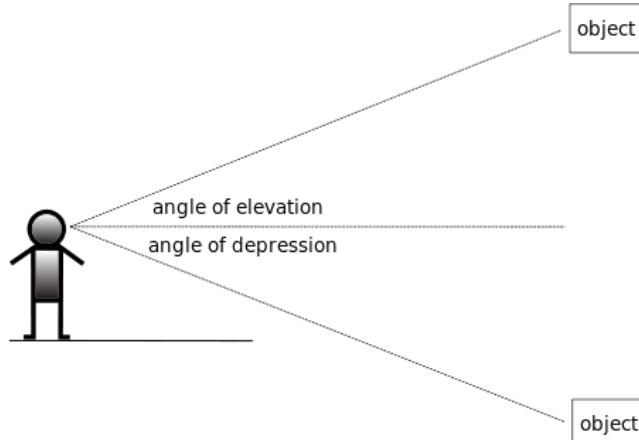
PYTHAGORAS IDENTITIES

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1; \\ \tan^2\theta + 1 &= \sec^2\theta; \\ \cot^2\theta + 1 &= \operatorname{cosec}^2\theta;\end{aligned}$$

HEIGHT AND DISTANCE

Trigonometry is best used in real-life for finding height and distance.

- **Angle of Elevation:** The angle formed by the line of sight with the horizontal, when the point being observed is above the horizontal level is called as the angle of elevation.
- **Angle of Depression:** The angle formed by the line of sight with the horizontal when the point being observed is below the horizontal level is called as the angle of depression.



Points to Remember

- In a right angled triangle, if the angles are in the ratio $45^\circ: 45^\circ: 90^\circ$, then the sides are in the ratio $1:1:2$.
- In a right angled triangle, if the angles are in the ratio $30^\circ: 60^\circ: 90^\circ$, then the sides are in the ratio $1:3:2$.

PRACTICE QUESTIONS

- The length of the shadow of a tree is equal to its height. What is the angle of the elevation of the light?
 - 30°
 - 60°
 - 90°
 - 45°
- During a cyclone, a tree in the coastal area broke in such a way that its top touched the ground making an angle of 60° . If the initial height of the tree is 15 m, at what height is the tree broken?
 - $15(1 + \sqrt{3})$ m
 - $\frac{15\sqrt{3}}{2 + \sqrt{3}}$ m
 - $\frac{30\sqrt{3}}{(2 + \sqrt{3})}$ m
 - 5 m
- The angle of elevation of a building from a point P is 45° . If P is 20 m away from the foot of the pole, then what is the height of the pole?
 - $20\sqrt{2}$ m
 - 10 m
 - 20 m
 - 15 m
- There are two points P and Q situated 18m and 32m away from the foot of a tower. The angles of elevations of the top of the tower from these two points are complementary. What is the height of the tower?
- The angle of elevation of the top of a tower from a point on the ground is 30° . If the tower is 40 m high, what is the distance between the base of the tower and the point of observation?
 - $13\sqrt{3}$ m
 - $40\sqrt{3}$ m
 - $15\sqrt{4}$ m
 - $24\sqrt{5}$ m
- There are 2 cars 200 m apart, the angles of depression of the cars from the top of a building are 45° and 30° towards east. What is the height of the building?
 - 100 m
 - 173 m
 - 200 m
 - 273 m
- The angle of elevation of an under construction building at a certain point 150 m from its base is 30° . If, as per the actual construction plan, the angle of elevation at the same point of the fully constructed building is to be 45° , then the building has to be raised by how many metres?
 - 59.4 m
 - 61.4 m
 - 62.4 m
 - 63.4 m



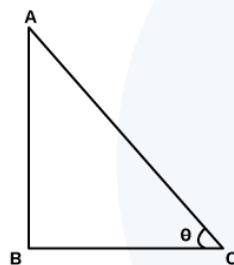
8. What is the height of the pillar when from the top of a large statue, which has the height of 90 metres, the angles of depression of the top and bottom of a pillar are 30° and 60° ?
- A. 30 m
B. 45 m
C. 60 m
D. 75 m
9. What is the height of a large plastic structure when the plastic structure got broken at a point and its top touches the ground at a distance 20 m from the base of the structure making an angle 30° with the ground?

- A. $\frac{40}{\sqrt{3}} \text{ m}$
B. $20\sqrt{3} \text{ m}$
C. $40\sqrt{3} \text{ m}$
D. 30 m

10. When the flood light's elevation is 30° , the shadow of a house is 15 m. How long must have been the shadow if the flood light's elevation would have been 60° ?
- A. 3 m
B. 4 m
C. 5 m
D. 6 m

SOLUTIONS

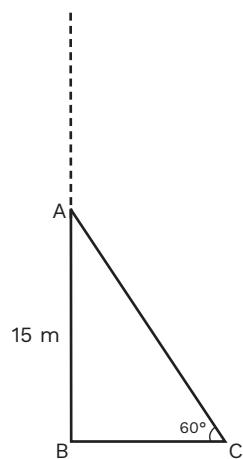
1. (D)



Let AB is the tree and BC be the shadow of the tree.

Given, AB = BC
 $\tan \theta = AB/BC = 1$
 $\theta = 45^\circ$

2. (B)



Let the tree be BAC after falling. Total length BAC = 15 m

$$BA + AC = 15$$

Let the height at which tree is broken is x .

$$BA = x; AC = 15 - x$$

$$\sin 60^\circ = AB/AC$$

$$\frac{\sqrt{3}}{2} = \frac{x}{15-x}$$

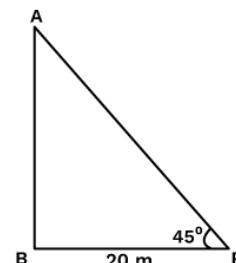
$$\sqrt{3}(15-x) = 2x$$

$$15\sqrt{3} - \sqrt{3}x = 2x$$

$$15\sqrt{3} = 2x + \sqrt{3}x$$

$$x = \frac{15\sqrt{3}}{2 + \sqrt{3}}$$

3. (C)



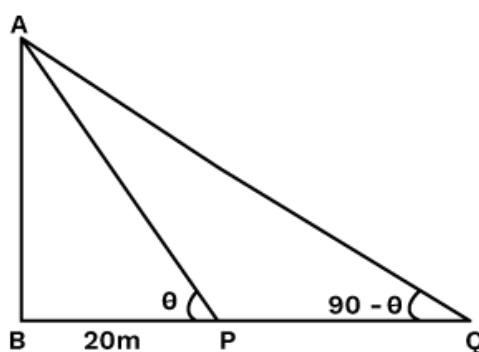
$$\tan 45^\circ = AB/BP$$

$$1 = \frac{AB}{20}$$

$$AB = 20 \text{ m}$$



4. (B)



Let angle at P = θ and height of the tower be h .

Then angle at P = $(90 - \theta)$

$$\tan \theta = \frac{AB}{BP} = \frac{h}{18}$$

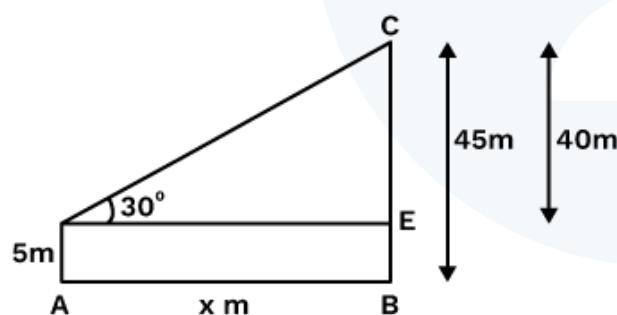
$$\tan(90 - \theta) = \frac{h}{32}; \cot \theta = \frac{h}{32}$$

$$\tan \theta \times \cot \theta = 1 = \frac{h}{18} \times \frac{h}{32}$$

$$18 \times 32 = h^2$$

$$h = 24 \text{ m}$$

5. (B)



In $\triangle DEC$,

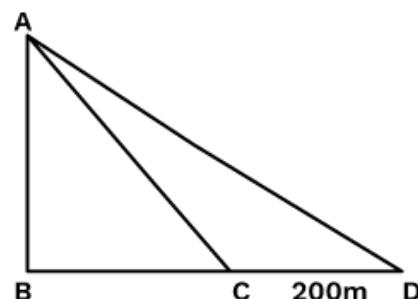
$$\tan 30^\circ = \frac{CE}{DE}$$

$$\tan 30^\circ = \frac{40}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$x = 40\sqrt{3} \text{ m}$$

6. (B)



Given,

$$\angle ACB = 45^\circ$$

$$\angle ADB = 30^\circ$$

Distance between two cars, i.e.,

$$CD = 200 \text{ m}$$

Then, AB = ?

Let BC = x m

In $\triangle ABC$,

$$\tan 45^\circ = AB/BC$$

$$(\because \tan 45^\circ = 1)$$

$$1 = \frac{AB}{x}$$

$$AB = x \text{ m}$$

....(i)

In $\triangle ABD$, $\tan 30^\circ = AB/BD$

$$\frac{1}{\sqrt{3}} = \frac{AB}{x + 200}$$

$$(\because \tan 30^\circ = \frac{1}{\sqrt{3}})$$

$$x = \sqrt{3}, AB = 200$$

....(ii)

From equations (i) and (ii),

$$AB = \sqrt{3} AB - 200$$

$$\sqrt{3} AB - AB = 200$$

$$0.732 AB = 200$$

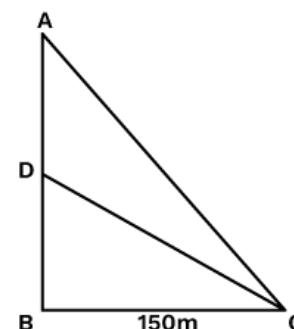
$$(\because \sqrt{3} = 1.732)$$

$$AB = \frac{200}{0.732} = 273.22$$

$$= 273 \text{ m}$$

Hence, option (D) is correct.

7. (D)





Given, BC = 150 m
 $\angle ACB = 30^\circ$
 and, $\angle DCB = 45^\circ$
 Then, AD = ?

In $\triangle ABC$, $\tan 30^\circ = AB/BC$

$$\frac{1}{\sqrt{3}} = \frac{AB}{150}$$

$$AB = \frac{150}{\sqrt{3}} = 86.6 \text{ m}$$

In $\triangle DBC$, $\tan 45^\circ = DB/BC$

$$1 = \frac{DB}{150}$$

$$DB = 150$$

$$AD + AB = 150$$

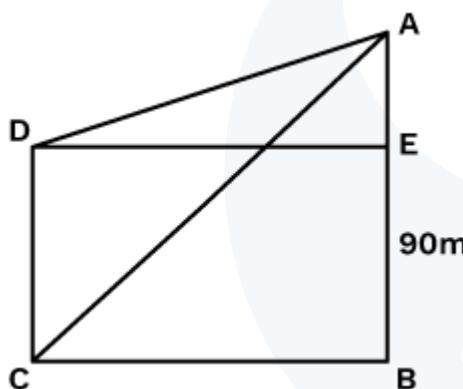
[$\because DB = AD + AB$]

$$AD = 150 - AB$$

$$= 150 - 86.6 = 63.4 \text{ m}$$

Hence, option (D) is correct.

8. (C)



Given, AB = 90 m

$\angle ADE = 30^\circ$

And $\angle ACB = 60^\circ$

Then, DC = ?

Ratio of angles,

$$\tan 30^\circ / \tan 60^\circ = (AE/ED)/(AB/BC)$$

[$\because ED = BC$]

$$\frac{1}{\sqrt{3}} = \frac{AE}{90}$$

$$\frac{1}{3} = \frac{AE}{90}$$

$$AE = 30 \text{ m}$$

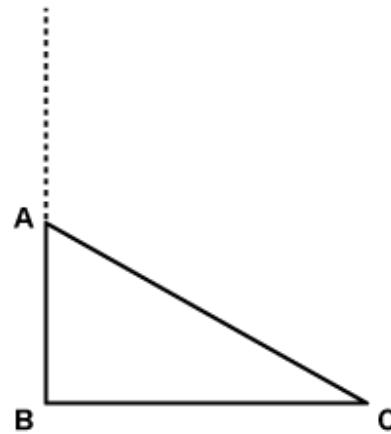
Now, DC = EB

$$= AB - AE$$

$$= 90 - 30 = 60 \text{ m}$$

Hence, option (C) is correct.

9. (B)



Given, BC = 20 m

$\angle ACB = 30^\circ$

Total height of the large plastic structure is
 $(AB + CA) = ?$

In $\triangle ABC$, $\tan 30^\circ = AB/BC$

$$\frac{1}{\sqrt{3}} = \frac{AB}{20}$$

$$AB = \frac{20}{\sqrt{3}} \text{ m}$$

Now, $\cos 30^\circ = BC/AC$

$$\frac{\sqrt{3}}{2} = \frac{20}{AC}$$

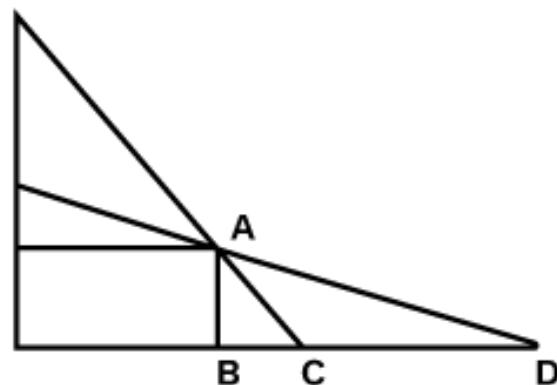
$$AC = \frac{40}{\sqrt{3}} \text{ m}$$

$$\text{So, } AB + CA = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

$$= 20\sqrt{3} \text{ m}$$

Hence, option (B) is correct.

10. (C)





Given, $\angle ADB = 30^\circ$ and $\angle ACB = 60^\circ$

When the flood light's elevation is 30° , the shadow of house is 'BD = 15 m' and when the flood light's elevation is 60° , the shadow of house is 'BC = ?'

Let, BC = x m

$$\text{In } \triangle ABD, \tan 30^\circ = \frac{AB}{BD}$$
$$\frac{1}{\sqrt{3}} = \frac{AB}{15}$$

$$AB = \frac{15}{\sqrt{3}} \quad \dots(i)$$

$$\text{In } \triangle ABC, \tan 60^\circ = AB/BC$$
$$\sqrt{3} = AB/x$$
$$AB = x\sqrt{3} \quad \dots(ii)$$

From equations (i) and (ii), we get

$$x\sqrt{3} = \frac{15}{\sqrt{3}}$$
$$x = 5 \text{ m}$$

Hence, option (C) is correct.

28 Data Interpretation



SYNOPSIS

- Table
- Pie chart
- Line graph
- Bar chart
- Mixed graph
- Caselet
- Most common types of questions and important formulae
- Some important tricks useful in data interpretation
- Summary

INTRODUCTION

Data interpretation is analysing the provided data and calculating the required value asked based upon that data. Data in competitive questions can be in various forms like tables, pie charts, bar graphs, line graphs, etc.

TABLES

In tabular data interpretations (DIs), data are divided systematically into horizontal rows and vertical columns followed by certain questions based on that data that are to be answered by students. For example, the table given below represents the loss of three companies in the period of 3 years from 2018 to 2020.

YEAR COMPANY	2018 (LOSS %)	2019 (LOSS %)	2020 (LOSS %)
A	18	11	7
B	13	17	11
C	7	14	15

Comprehending the Data Table

If a question asks to calculate the increase/decrease in loss of B's revenue from 2019 to 2020 when total revenue for the years is 2 crores:

1. Identify loss in 2019, i.e., the intersection of B row and 2019 column, 17%

2. Similarly, loss in 2020, 11%
3. Finally, calculate the change in loss = 17% of 2 crores – 11% of 2 crores

Note: Pay attention to the units.



Illustration:

Directions (Questions 1–4): Study the given table carefully and answer the questions that follow:

STATES	TOTAL NUMBER OF PEOPLE WHO VOTED	PERCENTAGE OF PEOPLE WHO VOTED FOR DIFFERENT PARTIES FROM RESPECTIVE STATES			
		A	B	C	D
P	80	25	20	15	40
Q	100	24	33	21	22
R	200	32	20	17	31
S	250	30	10	20	50

1. In state Q, the total number of people who voted for parties A and C together is what per cent less than the total number of people who voted for B and D?

$$\begin{array}{ll} \text{A. } 16\frac{2}{11} & \text{B. } 15\frac{2}{11} \\ \text{C. } 18\frac{2}{11} & \text{D. } 17\frac{3}{11} \end{array}$$

Solution: Total number of people who voted for parties A and C together in Q

$$\begin{aligned} &= 100 \times \frac{24}{100} + 100 \times \frac{21}{100} \\ &= 24 + 21 = 45 \end{aligned}$$

Total number of people who voted for parties B and D together in Q

$$\begin{aligned} &= 100 \times \frac{33}{100} + 100 \times \frac{22}{100} \\ &= 33 + 22 = 55 \end{aligned}$$

$$\text{Required percentage} = \frac{(55 - 45)}{55} \times 100 = 18\frac{2}{11}$$

2. What is the average number of people who voted in states P, Q, and S for party C?

$$\begin{array}{ll} \text{A. } 23 & \text{B. } 27 \\ \text{C. } 29 & \text{D. } 32 \end{array}$$

Solution: Number of people who voted in state

$$P \text{ for party C} = 80 \times \frac{15}{100} = 12$$

Number of people who voted in state R for party C = $200 \times \frac{17}{100} = 34$

Number of people who voted in state S for party C = $250 \times \frac{20}{100} = 50$

$$\text{Required average} = \frac{(12 + 34 + 50)}{3} = 32$$

3. What is the difference between the total number of people who voted in state P for parties A and D together and the total number of people who voted in state R for the same party together?

$$\begin{array}{ll} \text{A. } 63 & \text{B. } 74 \\ \text{C. } 85 & \text{D. } 96 \end{array}$$

Solution: Total number of people who have voted in P for parties A and D together

$$\begin{aligned} &= 80 \times \frac{25}{100} + 80 \times \frac{40}{100} \\ &= 20 + 32 = 52 \end{aligned}$$

Total number of people who have voted in R for parties A and D together

$$\begin{aligned} &= 200 \times \frac{32}{100} + 200 \times \frac{31}{100} \\ &= 64 + 62 = 126 \end{aligned}$$

$$\text{Required difference} = 126 - 52 = 74$$



4. It is given that in S, 42% are females. And if 20% of these total females voted for party B, what is the number of male people who voted for party B from state S?

- A. 145 B. 132
C. 129 D. 117

Solution: In S, females = 42%

Males = 58%

Total number of male people in S who voted for

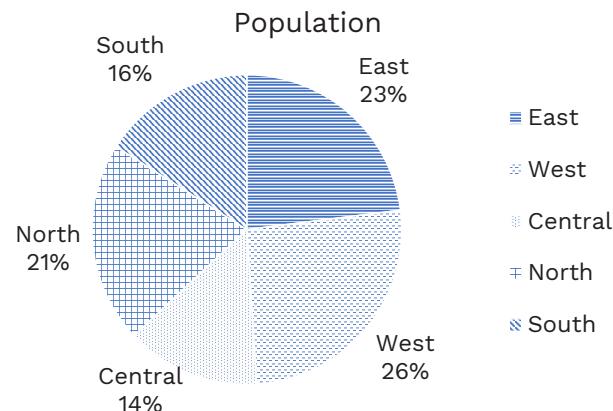
$$\text{party B} = \frac{58}{100} \times 250 = 145$$

PIE CHART

In pie charts, data are represented as sectors of circular charts, which are proportional to the quantities they represent. And the total quantity over the whole circle, i.e., 360° or 100%. For example, the following pie chart represents the distribution of population in various districts of Town X.

Illustration:

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:



COMPREHENDING PIE CHARTS

1. If data are represented in degrees:

Value of any sector =

$$\frac{\text{Angle of any sector}}{360^\circ} \times \text{Total value}$$

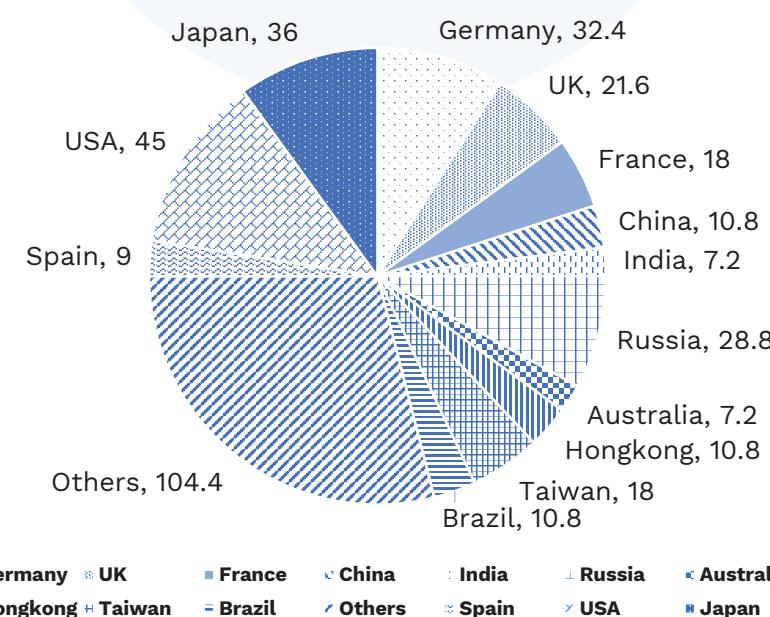
2. If data are represented in percentage:

Value of any sector

$$= \frac{\text{Per cent of the sector}}{100} \times \text{Total value}$$

Thus, from the above chart the population of any zone can be calculated using the later formula.

Country-wise Exports (in degree) Total = 72000 billion





- By how much does the value of the imports of the USA exceed that of Germany?
 - 2326 billion
 - 2789 billion
 - 2900 billion
 - 2520 billion

Solution: Difference in the angles subtended by the USA and Germany = $45^\circ - 32.4^\circ = 12.6^\circ$

The difference in the imports of the USA and

$$\text{Germany} = \frac{12.6^\circ}{360^\circ} \times 72000 \text{ billion} = 2520 \text{ billion}$$

- The difference in the values of the imports of Japan and France is how many times that of the UK and Taiwan?
 - 2 times
 - 3 times
 - 4 times
 - 5 times

Solution: The difference in the angles of the import of Japan and France

$$= 36^\circ - 18^\circ = 18^\circ$$

The difference in the angles subtended by the UK and Taiwan

$$= 21.6^\circ - 18^\circ = 3.6^\circ$$

Hence, the difference in the value of imports of Japan and France is 5 times that of the UK and Taiwan.

- The value of the imports of the Organization of the Petroleum Exporting Countries (OPECs) organisation is how much more than the value of the imports of India and Australia put together, given that OPEC has a 20% share in the value of the imports of others?
 - 2673 billion
 - 2569 billion
 - 1296 billion
 - 1325 billion

Solution: Value of imports of India and Australia

$$= \frac{(7.2^\circ + 7.2^\circ)}{360^\circ} \times 72000 = 2880 \text{ billion}$$

Value of imports of OPEC countries

$$= \frac{20}{100} \times \left(\frac{104.4^\circ}{360^\circ} \times 72000 \right) = 4176 \text{ billion}$$

Required difference = $4176 - 2880 = 1296$ billion

- If imports of developing countries accounted for 36% of the total worldwide imports, then what is the value of the imports of Japan as a percentage of the imports of the developing countries?
 - 23.3%
 - 27.7%
 - 29.9%
 - 33.3%

Solution: Imports of developing countries = 36% of total

$$\text{Imports of Japan} = \frac{36^\circ}{360^\circ} \times 100 = 10\% \text{ of total}$$

$$\text{Required per cent} = \frac{10}{36} \times 100 = 27.7\%$$

- What is the number of countries whose imports are more than the average imports per country?
 - 6138.5 billion
 - 5276.4 billion
 - 4829.3 billion
 - 5142.8 billion

Solution: Average imports

$$= \frac{\text{Total imports}}{\text{Number of countries}} = \frac{72000}{14} = 5142.8 \text{ billion}$$

After calculating imports of each country individually, only USA, Japan, Germany, Russia, and Others (i.e., 5) have imports greater than average.

LINE GRAPHS

In a line graph, data are distributed on X and Y-axes, which represent a variation of quantity with respect to these perimeters over axes.

Types of line graph

- Single-line graph:** Used for a single variable. For example, the following graph represents the sales of a company in the years 2011–2020.

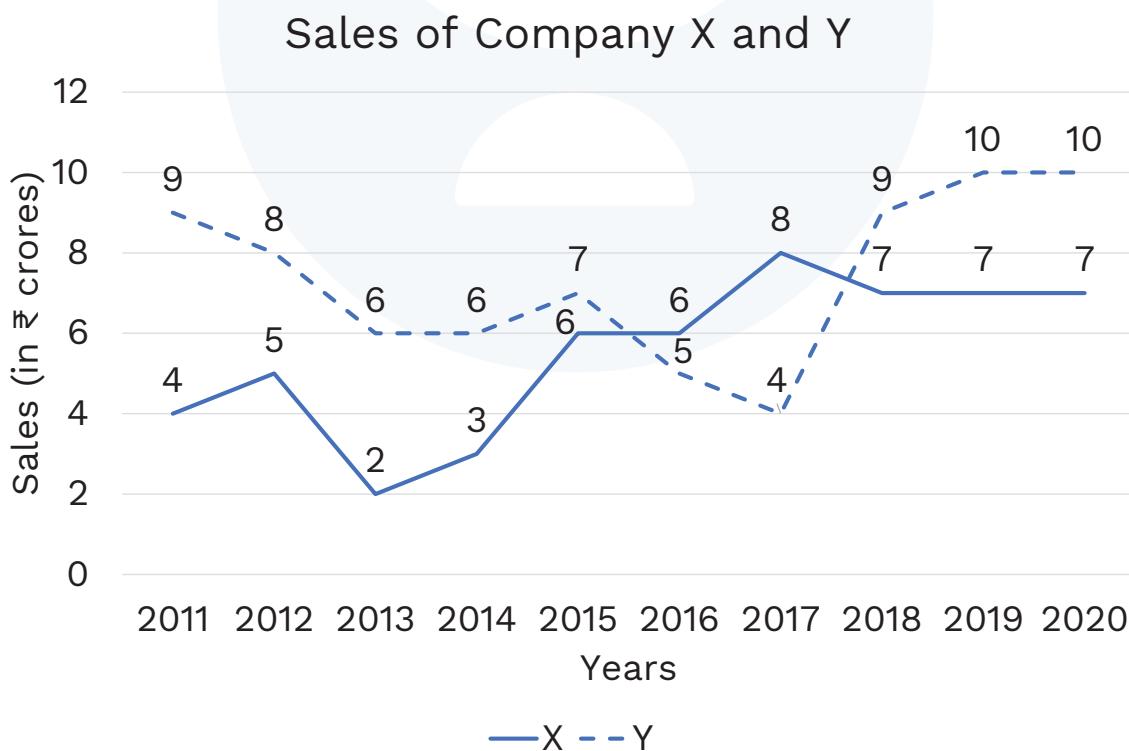


Sales of a Company



2. Two-variable graph:

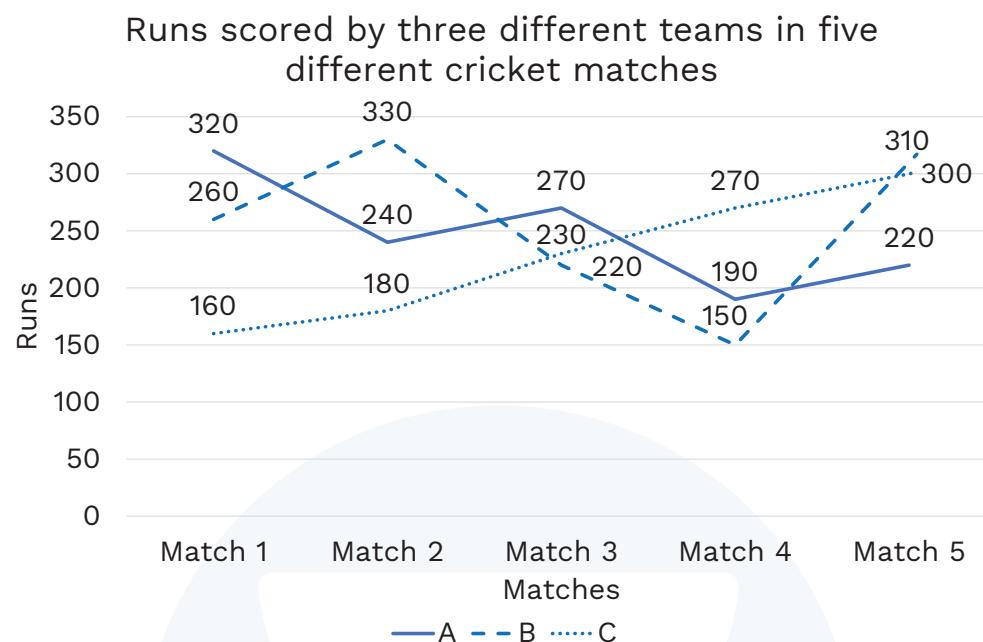
For example, the following graph represents the sales of two companies X and Y in the years 2011–2020.





3. Three-variable graph:

For example, the following graph represents the runs scored by three different countries in five different cricket matches.



COMPREHENDING LINE GRAPH

If a question asks, the sales of company X in 2015 is by how much per cent more than the sales of company Y in 2016:

1. Identify sales of company X in the year 2015 from the blue line
2. Similarly, sales of company Y in the year 2016 from the orange line

3. Finally, calculate increase = 6 crores – 5 crores

Illustration:

Directions (Questions 1–5): Study the given line graph carefully and answer the questions that follow:





1. What is the difference between the total number of articles sold by both the shops together on the 2nd day and that by both the shops together on the 4th day?

A. 230 B. 270
C. 290 D. 320

Solution: Total number of articles sold on 2nd day = $120 + 90 = 210$

And the total number of articles sold on the 4th day = $240 + 200 = 440$

$$\text{Required difference} = 440 - 210 = 230$$

2. The number of articles sold by shop Y on the 1st day is what per cent of the number of articles sold by the same shop on the 5th day?

A. $25\frac{1}{4}$ B. $22\frac{3}{4}$
C. $22\frac{1}{4}$ D. $25\frac{3}{4}$

Solution: Required percentage = $\frac{60}{270} \times 100 = \frac{600}{27} = 22.25$ or $22\frac{1}{4}$

3. What is the average number of articles sold by shop Y on the 1st and 4th day?

A. 130 B. 170
C. 190 D. 220

Solution: Average number of articles sold by shop Y on 1st and 4th day = $\frac{60 + 200}{2} = \frac{260}{2} = 130$

4. What is the respective ratio between the total number of articles sold by shop X on the 4th and 5th day together and by the same shop on the 2nd and 3rd day?

A. 3:2 B. 5:1
C. 7:3 D. 9:5

Solution: Total articles sold on 4th and 5th day = $240 + 300 = 540$

And total articles sold on 2nd and 3rd day = $120 + 180 = 300$

$$\text{Required ratio} = 540 : 300 = 9 : 5$$

5. The number of articles sold by shop X on the 3rd day is what per cent more than that sold by the same Y on the same day?

A. 30% B. 20%
C. 40% D. 10%

Solution: Required percentage

$$= \frac{180 - 150}{150} \times 100 = 20\%$$

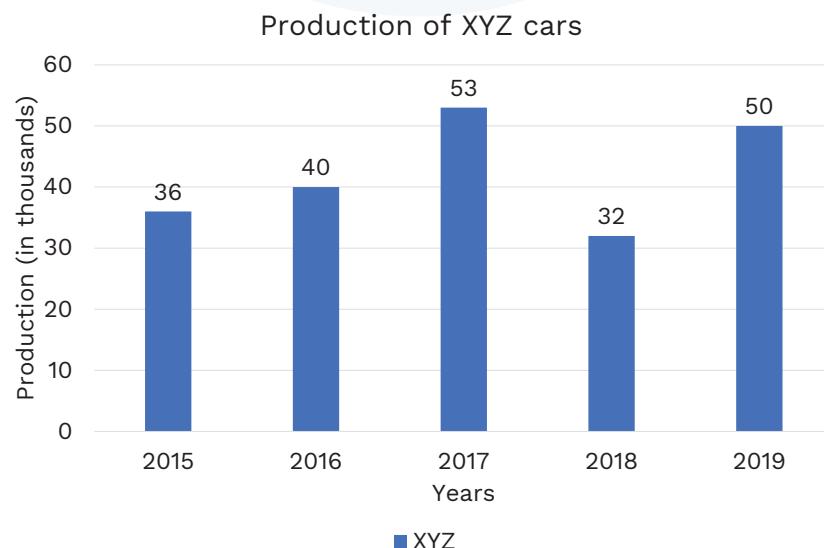
BAR GRAPH

A bar chart represents data in the form of rectangles whose lengths are proportional to the data they represent. And one axis represents variables while the other represents a parameter that is changing in the graph

Types of bar charts

1. Simple bar graph

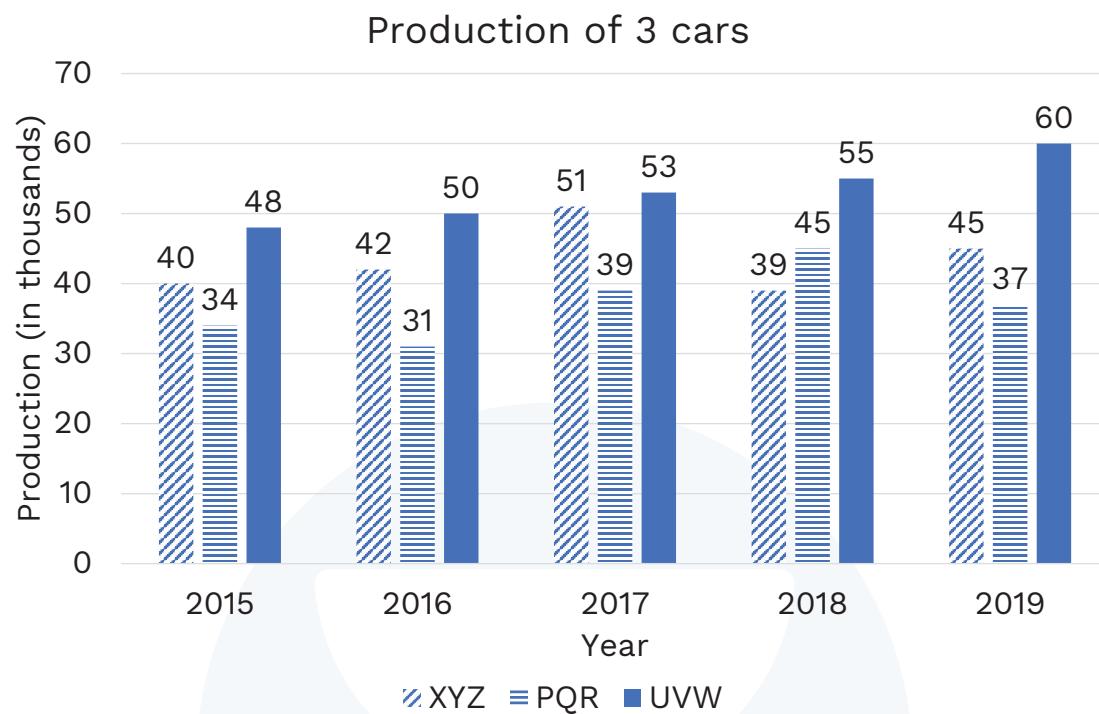
For example, the following bar graph represents the production of XYZ cars in five different years.





2. Grouped bar graph

For example, the following bar graph represents the production of XYZ, PQR, and UVW cars in five different years.



3. Stacked bar graph

For example, the following bar graph represents the production of various models of XYZ cars over three years 2019, 2020 and 2021.

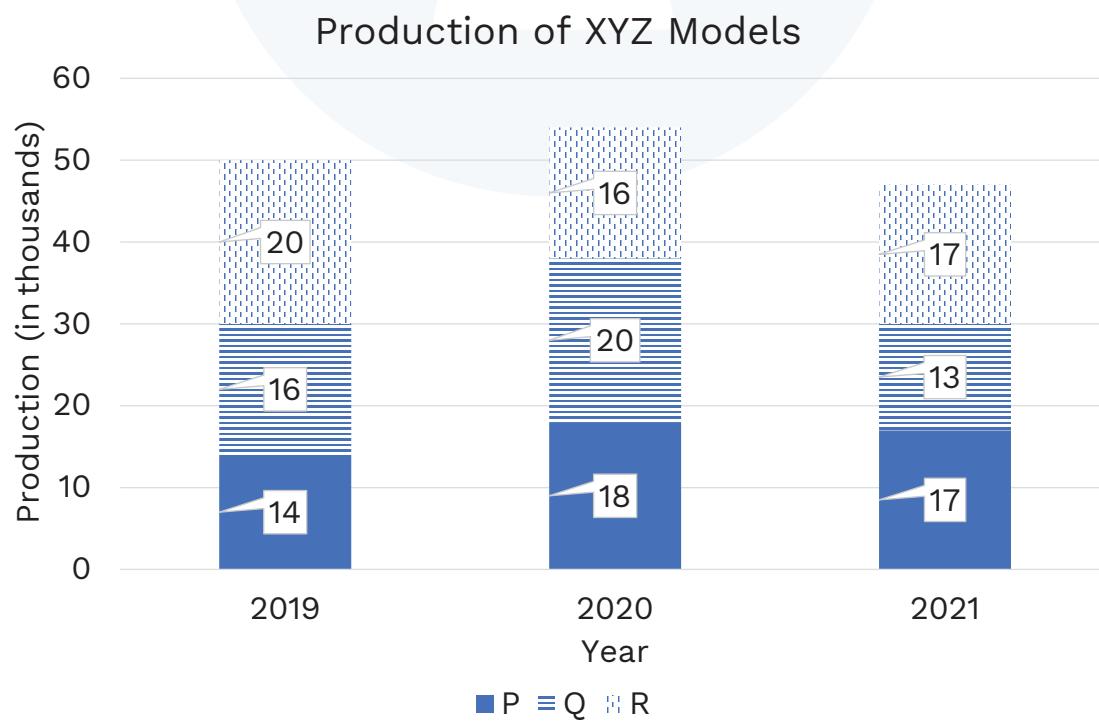
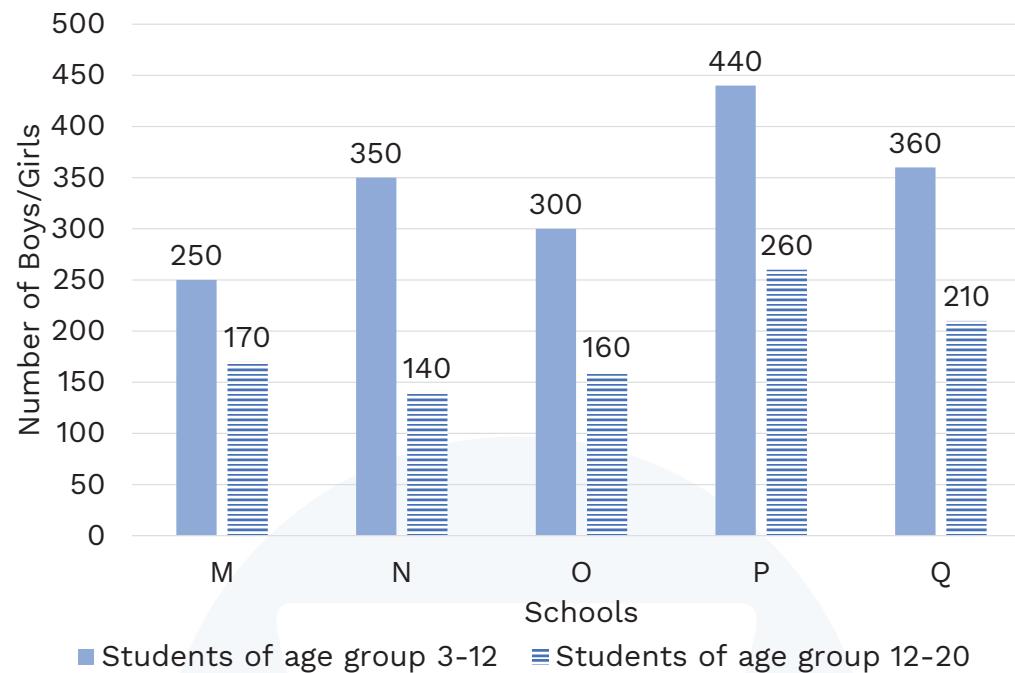




Illustration:

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:



1. What is the difference between the total number of students of age group 3-12 studying in N and O together and the total number of students of age group 12-20 in the same schools together?
- A. 210 B. 350
C. 490 D. 570

Solution: Total students of age group 3-12
 $= 350 + 300 = 650$

Total students of age group 12-20
 $= 140 + 160 = 300$

$$\text{Required difference} = 650 - 300 = 350$$

2. The number of students of age group 3-12 and students of age group 12-20 together in school N is what per cent less than that in school P?

- A. 30% B. 70%
C. 50% D. 20%

Solution: Total students in N $= 140 + 150 = 290$

Total students in P $= 260 + 440 = 700$

$$\text{Required per cent} = \frac{700 - 490}{700} \times 100 = 30\%$$

3. The number of students of age group 3-12 and students of age group 12-20 studying in class X of Q school is 20% less than those in class XII of the same school. How many students study in class X then?
- A. 456 B. 472
C. 489 D. 423

Solution: Total students of Q school in class XII
 $= 210 + 380 = 590$

$$\text{Number of students in class X} \\ = 590 - 590 \times \frac{20}{100} = 472$$

4. The number of students of age group 3-12 studying in Q school is what per cent more than the number of students of age group 3-12 studying in M?

- A. 48% B. 52%
C. 54% D. 56%

Solution: Students of age group 3-12 in Q $= 380$

Students of age group 3-12 in M $= 250$

$$\text{Required percentage} = \frac{380 - 250}{250} \times 100 = 52\%$$



5. What is the respective ratio between the number of students of age group 3-12 and students of age group 12-20 together studying in M and that in P?

A. 21:23 B. 23:27
C. 27:29 D. 29:31

Solution: Required ratio = $(170 + 250):(160 + 300)$
 $= 420 : 460 = 21:23$

MIXED GRAPH

Mixed graphs present data in a combination of two or more forms of data presentation. These

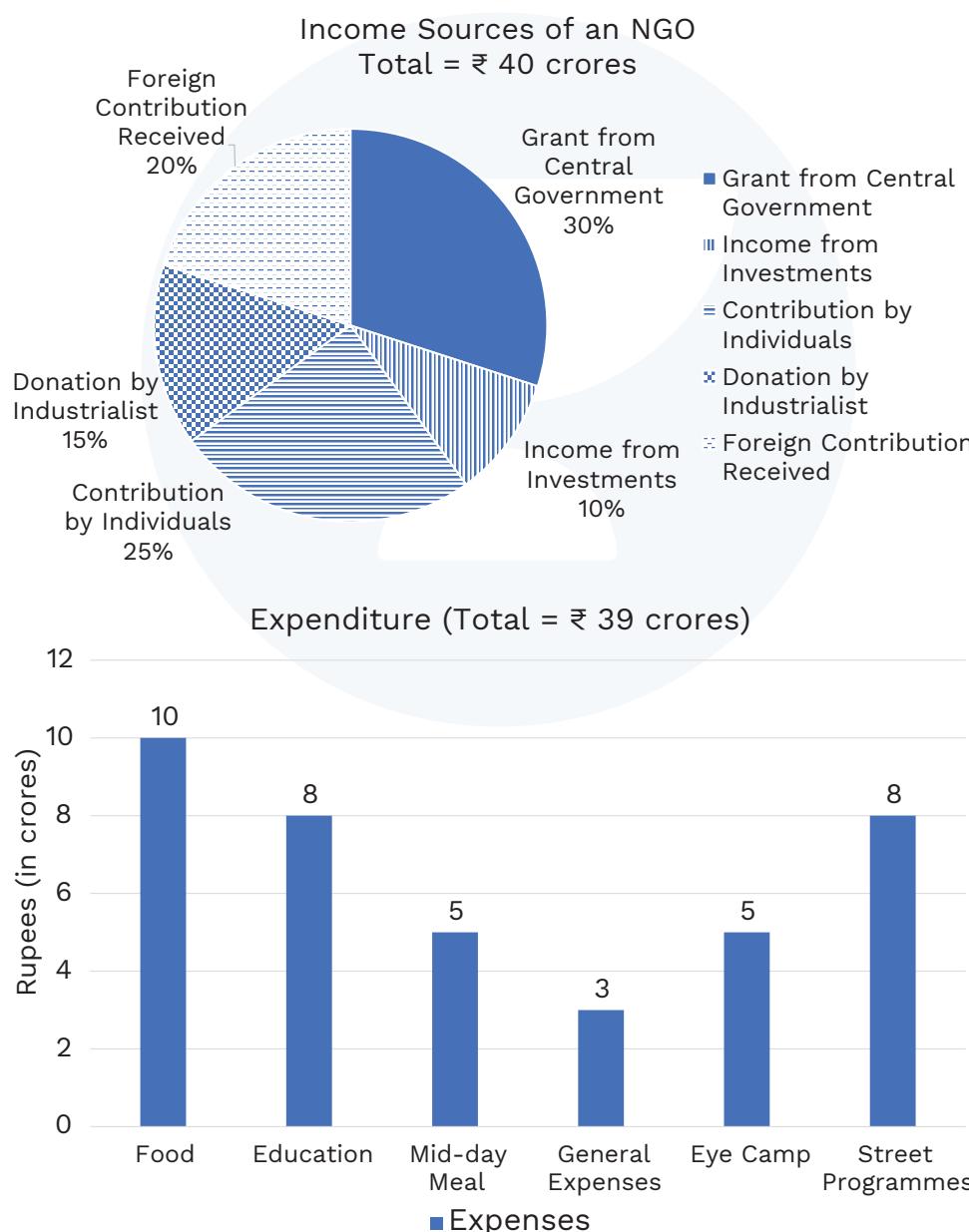
types of Data interpretation questions require the questions to be solved using both the interdependent data.

Types of mixed graphs

- Any two graphs/charts together

Illustration:

Directions (Questions 1–5): The following pie chart represents the different income sources of an NGO and the bar graph represents the expenditure of this NGO over different heads. Answer the questions that follow:





1. What percentage of money is saved?

A. 2.3% B. 2.5%
C. 2.7% D. 2.9%

Solution: Saving = Total income – total expenditure
= 40 – 39 crore = 1 crores

$$\text{Per cent of money saved} = \frac{1}{40} \times 100 = 2.5\%$$

2. If the industries stop donating and the expenditure pattern remains the same, then what will be the decrease in money spent on the mid-day meal programme?

A. 0.69 crores B. 0.73 crores
C. 0.76 crores D. 0.63 crores

Solution: Decreased income = 40 crores –
 $\frac{15}{100} \times 40 \text{ crores}$
= 34 crores

Percentage of expenditure on mid-day meal
 $= \frac{5}{39} \times 100 = 12.8\%$

New expenditure after a gradual decrease in expenditure with respect to income = $\frac{39}{40} \times 34$
= 33.15 crores

New expenditure on mid-day meal
 $= \frac{12.8}{100} \times 33.15 = ₹ 4.24 \text{ crores}$

Required decrease in expenditure on mid-day meal = 5 – 4.24 crores = 0.76 crores

3. What is the ratio of expenditure on food and mid-day meal programmes together to that of a grant from the central government?

A. 2:1 B. 3:2
C. 4:3 D. 5:4

Solution: Total expense on food and mid-day meal = (10 + 5) = 15 crores

Grant from central government

$$= \frac{30}{100} \times 40 = 12 \text{ crores}$$

Required ratio = 15:12 = 5:4

4. The general expenses are how many times income from the investment?

A. 0.25 times B. 0.50 times
C. 0.75 times D. 1 time

Solution: Income from investment =

$$\frac{10}{100} \times 40 = 4 \text{ crores}$$

4 crores is 0.75 times of 3 crores (i.e., general expense)

5. If in the next year, a grant from the central government increases by 10%, foreign contribution decreases by 10% and other income amounts and expense patterns remain the same. What is the per cent increase in expenditure on Food?

A. 1% B. 2%
C. 3% D. 4%

Solution: Initially, amount of grant from central government = $\frac{30}{100} \times 40 \text{ crores} = 12 \text{ crores}$

$$\text{After 10\% increase} = \frac{110}{100} \times 12 = 13.2 \text{ crores}$$

Similarly new foreign contribution
 $= \frac{8}{100} \times 90 = 7.2 \text{ crores}$

$$\text{New income} = 40 + (13.2 - 12) - (8 - 7.2) = 40.4 \text{ crores}$$

Gradual increase in expenditure

$$= \frac{39}{40} \times 40.4 = 39.9 \text{ crores}$$

Gradual increase in Food expense

$$= \frac{10}{39} \times 39.9 = 10.1 \text{ crores}$$

$$\text{Required per cent increase} \frac{10.1 - 10}{10} \times 100 = 1\%$$

CASELET

Caselets DI present information in the form of long paragraphs, instead of tables or graphs usually. And are followed by questions that are to be solved using that data. For example:

In data for 2020-2021 total minerals extracted in India were 10 lakh tonnes. The data recorded extraction of three major minerals: iron, aluminium, and sulphur. Of which Iron accounted for 70% of the total, while Aluminium and Sulphur were in the ratio of 2:1. In the 2018-2019 annual year these same were extracted 10% less than the total extraction



in 2019–2020 but were 10% more than the total for 2017–2018. While the total extraction in 2019–2020 was also 10% more than that in 2020–2021. And the data hypothesis is that the extraction of iron increases at a rate of 5% per annum.

Comprehending caselet

- Construct the data into a simple table, if it can be converted for reference:

YEAR	EXTRACTION (IN LAKH TONNES)
2020–2021	10
2019–2020	11
2018–2019	9.9
2017–2018	9.91

- Use data accordingly for each question.

Illustration:

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:

There are 1400 students in ABC University in the academic year 2021. The ratio of the boys to the girls in the University is 4 : 3. All the students are interested in different extracurricular activities: mooting, debating, dancing, research and arbitration and one student is interested in only one extra-curricular activity. The number of boys interested in research is 153. The ratio of the number of boys who are interested in mooting to the number of boys who are interested in dancing is 72 : 89. 32% of the students are interested in arbitration. The number of boys interested in debating is 12.5% to the total number of boys. The number of girls interested in arbitration is 142 which is 37 less than the number of girls enrolled in dancing. The number of girls interested in mooting is 46 more than the number of boys

interested in the same. The total number of students interested in research is 204.

- The difference in the number of boys and girls who are interested in research is what percentage of the total number of students who are interested in debating?
 - 51%
 - 53%
 - 57%
 - 59%
- Find the percentage of students of the university who are interested in mooting.
 - 16.25%
 - 17.5%
 - 18.75%
 - 19%
- Find the ratio of the number of boys interested in dancing to the number of girls interested in the same.
 - 67 : 70
 - 83 : 80
 - 67 : 90
 - 83 : 67
- The total number of girls in arbitration is what percentage of the total number of students in Arbitration?
 - 23%
 - 27%
 - 29%
 - 32%
- Find the number of girls who are enrolled in Debating.
 - 68
 - 72
 - 76
 - 80

Solution 1: Total number of students = 1400

$$\text{Total number of boys} = \frac{4}{7} \times 1400 = 800$$

$$\text{Total number of girls} = 1400 - 800 = 600$$

$$\text{Number of boys interested in research} = 153$$

$$\text{Number of students interested in arbitration} =$$

$$\frac{32}{100} \text{ of } 1400 = 448$$

$$\text{Number of girls interested in arbitration} = 144$$

$$\text{Number of boys interested in arbitration} = 448 - 144 = 304$$

$$\text{Number of boys interested in debating} = \frac{12.5}{100} \text{ of } 1400 = 100$$

$$\text{Number of girls interested in dancing} = 720$$

$$\text{Number of students interested in research} = 204$$

$$\text{Number of girls interested in research} = 204 - 153 = 51$$



Number of boys interested in mooting and dancing together = $800 - (100 + 153 + 304) = 242$
 Number of boys interested in Mooting =

$$= \frac{72}{(72 + 89)} \times 242 = 108$$

Number of girls interested in mooting = 154
 Number of boys interested in dancing = $242 - 108 = 134$
 Number of girls interested in debating = $600 - (154 + 180 + 51 + 144) = 72$

ACTIVITY	BOYS	GIRLS	TOTAL
Mooting	108	154	262
Dancing	134	180	314
Debating	100	72	172
Research	153	51	204
Arbitration	304	144	448

Difference in boys and girls interested in Research = $(153 - 51) = 102$

$$\text{Required percentage} = \frac{102}{172} \times 100 = 59\%$$

Solution 2: Required percentage

$$= \frac{262}{1400} \times 100 = 18.75\%$$

Solution 3: Required ratio = 134 : 180

$$= 67 : 90$$

Solution 4: Required percentage

$$= \frac{144}{448} \times 100 = 32\%$$

Solution 5: B

COMMON TYPE OF QUESTIONS AND IMPORTANT FORMULAE

1. Growth rate

Finding the growth rate of quantities is one of the most asked questions in DI. It is the percentage increase/decrease in a quantity over a period of time.

Illustration:

Following is the table of sales of total sales of almonds in different years.

YEAR	2010	2011	2012	2013
Sale (In Tonne)	90	95	94	101

1. By what per cent did sales grow over the years 2010–2013?

Solution: If quantity changes from A to B,

$$\text{Growth Rate} = \frac{B - A}{A} \times 100$$

Thus, here,

$$\text{Growth Rate} = \frac{101 - 90}{90} \times 10 = \frac{11}{9} \times 10 = 12.2\%$$

Note

- If B is greater than A, then growth is positive.
- If B is smaller than A, the growth rate is negative.

2. What was the average annual growth rate of sales over the years 2010–2013?

Solution: average annual/monthly growth rate is an average growth rate over the years/months.

Average growth rate

$$= \frac{\text{Total growth rate}}{\text{No. of years or months elapsed}}$$

Hence, here,

average annual growth rate of sales

$$= \frac{12.2\%}{3} = 4.07\% \text{ per year}$$

Note

Though the number of years given is 4 (2010, 2011, 2012, 2013) the growth happens over a year, i.e., as in 2002–2003. Thus, the number of years elapsed in the time period will be 3 (2010–2011, 2011–2012, 2012–2013)



2. Quantity X is how much of Quantity Y

This type of questions include calculating, how much is quantity 1 in respect of quantity 2 or quantity 1 in respect of total quantity or ratio of two quantities, etc.

Illustration:

Following is the table of the number of students appearing for an entrance exam in different cities.

CITY	A	B	C	D	E	F
Number of students (in thousand)	9.4	13	11	14.7	15.6	12

- The number of students appearing from city E is what per cent of the number of students appearing from city A?

Solution:

$$\text{Required percentage} = \frac{15600}{9400} \times 100 = 165.9\%$$

- If the number of students appearing from city G is 70% more than students appearing

from city E, what is the number of students appearing for the entrance from city G?

Solution:

$$\text{Number of students} = 15600 + \frac{70}{100} \times 15600$$

$$\text{Or } 170\% \text{ of } 15600 = 26520$$

3. Average of all or some units

Illustration:

Following is the table units sold (in hundred) by five companies over different years

YEAR	A	B	C	D	E
2001	1.3	2.2	1.7	2.2	1.4
2002	2.0	1.6	1.5	1.9	1.7
2003	0.9	1.6	1.0	1.5	1.1
2004	0.4	1.3	1.4	1.2	2.5
2005	1.5	1.2	2.1	1.1	2.6

- What is the average number of units sold by company D over all the years?

$$\text{Solution: Average} = \frac{\text{Sum of units}}{\text{Number of years}}$$

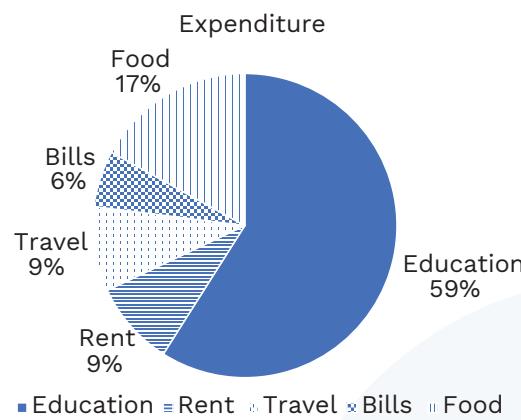
$$= \frac{220 + 190 + 150 + 120 + 110}{5} = 158$$



4. Calculating X if the total is given or vice versa

Illustration:

Following is the pie chart representing the percentage distribution of budget expenditure of a household



- If the house has a total expenditure of ₹ 30 thousand, then how much did they spend on Travel?

Solution:

$$\text{Expenditure on travel} = \frac{9}{100} \times 30000 = ₹ 2700$$

- If 9000 were spent on bills, then what would have been the total expenditure of the house?

Solution:

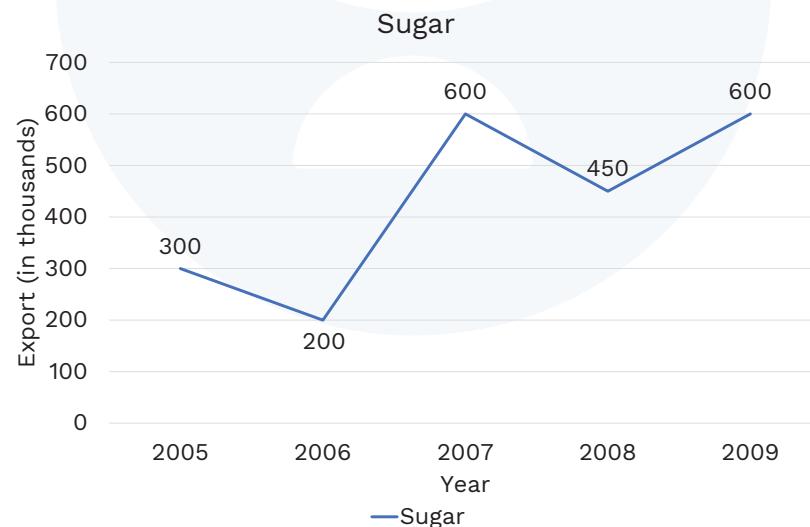
$$\text{Amount spent on bills} = \frac{6}{100} \times \text{total}$$

$$\text{Total} = \frac{9000}{6} \times 100 = 150000$$

Which year has the highest quantity

Illustration:

Following is the line graph of export of sugar in different years



- Which year has the highest per cent increase in exports as compared to the preceding year?

Solution: Percent change = $\frac{B - A}{A} \times 100$

$$2006 = \frac{200 - 300}{300} \times 100 = 33.33\% \text{ decrease}$$

$$2007 = \frac{600 - 200}{200} \times 100 = 200\% \text{ increase}$$

$$2008 = \frac{450 - 600}{600} \times 100 = 25\% \text{ decrease}$$

$$2009 = \frac{600 - 450}{450} \times 100 = 33.33\% \text{ increase}$$

Thus, the highest per cent increase is in the year 2007, i.e., 200%



SOME IMPORTANT TRICKS USEFUL IN DATA INTERPRETATION

- If the data are given in text form, converting the data first into a simple table saves a lot of time and is easy to refer to in a shortage of time.
- If the quantity of the item is not mentioned in bar graphs or line graphs, etc., label the quantity beforehand so that you don't look back again and again at both axes.
- Use approximation if options are not close. ex: in question, you do not need to calculate $\frac{11}{9}$ first and then multiply it by 100 or in any other longer way. You can take an approximation of $\frac{100}{9}$ as 11.1 or 11, and thus

the answer can be calculated in one step as 11^2 .

Or,

In question $\frac{15600}{9400}$ could be approximated

$$\text{as } \frac{15000}{9000} \times 100 = \frac{10}{6} \times 100$$

Next, you know $\frac{1}{6}$ is 0.166%. Thus, $\frac{1000}{6} =$

166.6%, and $\frac{15600}{9400}$ will be near to 166.6%;

hence, one could ultimately choose 165.9% from the options.

- Write the calculated values at a separate place so that if any question refers to them again, you do not have to spend time again in calculating.

Chapter Summary



- If data are represented in degrees:

$$\text{Value of any sector} = \frac{\text{Angle of any sector}}{360^\circ} \times \text{Total Value}$$

- If data are represented in percentage:

$$\text{Value of any sector} = \frac{\text{Percent of any sector}}{360^\circ} \times \text{Total Value}$$

- GrowthRate = $\frac{B - A}{A} \times 100$

- Average = $\frac{\text{Sum of units}}{\text{Number of years}}$

PRACTICE QUESTIONS

Data: The table lists the share of students (in per cent) per total population who had at least one smartphone available at home and the share of students (in per cent) who bought a new smartphone after 2020 in the ASER survey of

the year 2020 and 2021, categorised according to the level of education of their parents. Given that the total population of India is 1.5 billion.



PARENTS' EDUCATION	TOTAL FAMILIES	ASER 2020		ASER 2021	
		At least one smartphone at home	Bought a new smartphone for children's education since March 2020	At least one smartphone at home	Bought a new smartphone for children's education since March 2020
Low	50 billion	45	5	50	25
Medium	30 billion	60	10	65	25
High	0.5 billion	80	10	80	30
Nil	20 billion	60	10	65	25

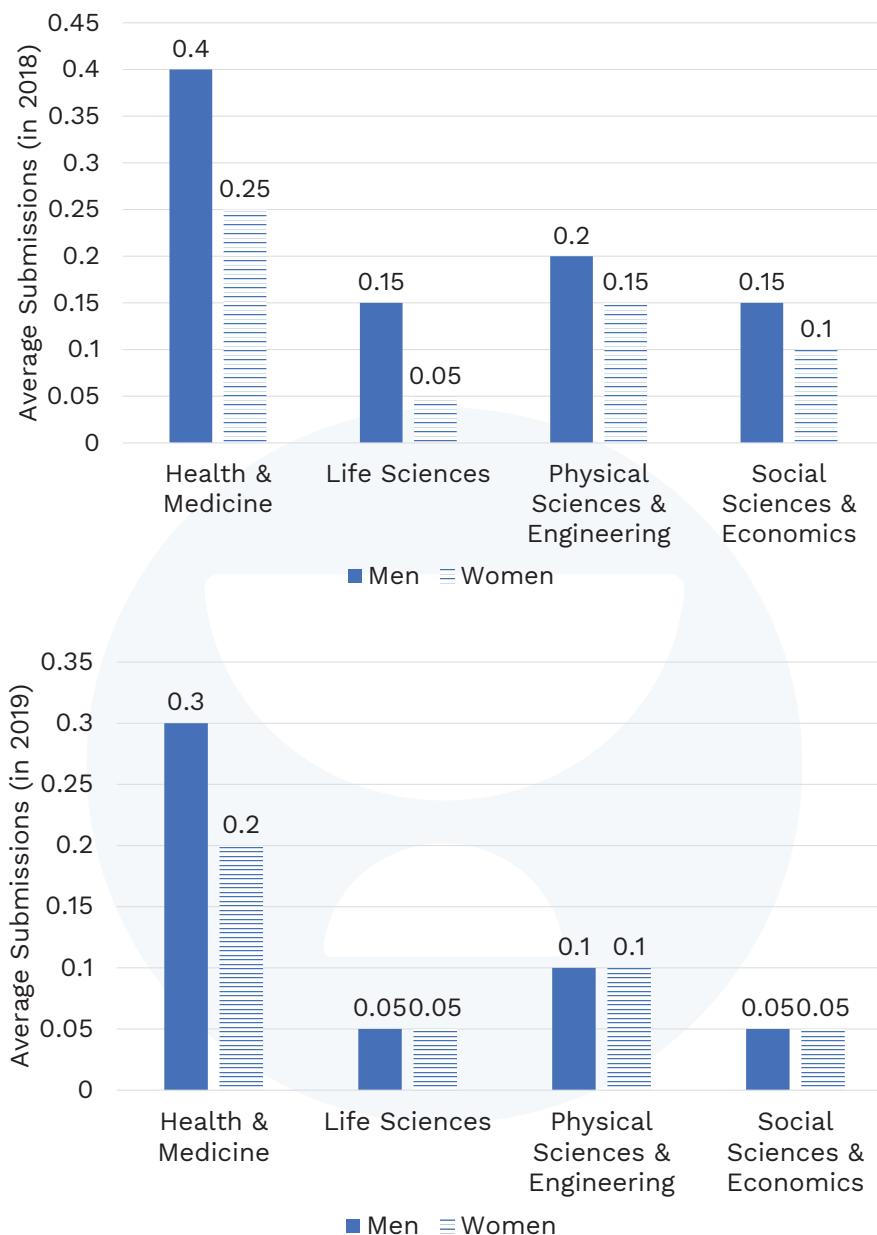
Source: The Hindu Data Point

1. The total of people who had at least one smartphone at home in the 2020 survey is what per cent of total people who bought a new phone after march 2020 in the 2021 survey? (use approximate values)
 - A. 190%
 - B. 210%
 - C. 250%
 - D. 175%
2. The number of nil parental education people who bought a new phone after march 2020 in the 2021 survey are what per cent less than the same in 2020 survey?
 - A. 60%
 - B. 50%
 - C. 80%
 - D. 30%
3. What is the respective ratio of people who had at least one smartphone at home whose parents had low income in 2020 and people who didn't have even one smartphone at home in a high parental education home in the same year survey?
 - A. 1 : 225
 - B. 150 : 4
 - C. 225 : 1
 - D. 175 : 2
4. What is the difference between the total number of people who had at least one smartphone in 2021 and people who bought a new phone after march 2020 in a medium parental education home in the 2020 survey?
 - A. 56.7 billion
 - B. 48.5 billion
 - C. 52 billion
 - D. 54.9 billion
5. What is the difference between the total number of people who didn't even have at least one smartphone in 2020 and that in 2021 in a nil education family?
 - A. 1 billion
 - B. 1.5 billion
 - C. 3.5 billion
 - D. 2 billion



Data: The chart shows the average change in scholarly article submissions of men compared to the average number of submissions of women in 2018 and 2019 across various research areas.

Source: The Hindu Data Point



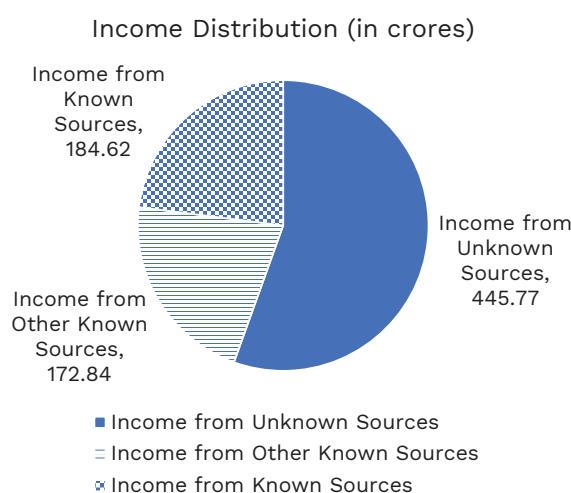
6. What is the ratio of the number of times when average submissions of women in any research area are higher/equal than men in the same to the number of times when average submissions of women in any research area are lower than men in both years?
 - A. 2 : 7
 - B. 3 : 4
 - C. 3 : 5
 - D. 4 : 3
7. What is the difference between the total average submissions of men and women in all the research areas in 2018?
 - A. 0.45
 - B. 0.35
 - C. 0.50
 - D. 1.25



- 8.** How many times did women have at least 50% of the average submissions of men in respective research areas?
- 4 times
 - 3 times
 - 7 times
 - 9 times
- 9.** Sum Average submission of men in life science area in both the years is approximately what per cent of sum Average submission of men in social science and economics area in both years?
- 100%
 - 50%
 - 75%
 - 25%
- 10.** The area in which the percentage decrease of total average submission of men and women in 2019 was maximum from its preceding year?
- health and medicine
 - life science
 - physical sciences and engineering
 - social sciences and economics

Data: The pie chart shows the sources of income of select regional parties in FY20. The total income for the selected regional parties in FY20 amounted to ₹803.24 crores, of which 55% was from unknown sources. The table depicts the total income of select regional parties in FY20 against the share from unknown sources.

Source: The Hindu Data Point



PARTY	UNKNOWN INCOME (IN %)	TOTAL INCOME (IN CRORE)
ABC	25	10
BCD	45	15
CDE	55	20
DEF	90	20
EFG	20	50
FGH	70	60
GHI	55	90
HIJ	10	90
IJK	80	90
JKL	90	90
KLM	40	110
LMN	70	130

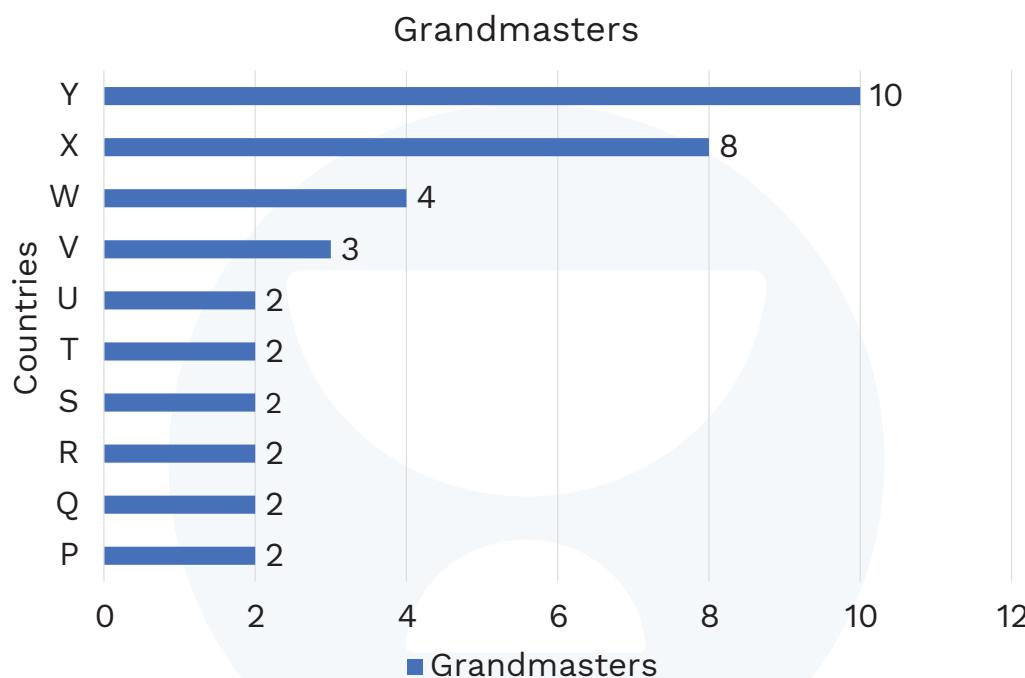
- 11.** The unknown income of ABC is less than the unknown income of GHU by (approximately)
- 80%
 - 95%
 - 85%
 - 70%
- 12.** What is the difference between the unknown income and the rest income of IJK?
- 54 Crores
 - 56 Crores
 - 52 Crores
 - 50 Crores
- 13.** The total income from other known sources is what per cent of total income from known sources? (approximately)
- 85%
 - 69%
 - 93%
 - 75%



14. What is the total income from unknown sources of KLM, ABC, and GHU together?
- A. 89
 - B. 98
 - C. 96
 - D. 81
15. What is the difference between the income which is not unknown of LMN and that of JKL?
- A. 10 Crores
 - B. 20 Crores
 - C. 30 Crores
 - D. 40 Crores

Data: The Bar chart shows the number of Chess Grandmasters from different countries born after 2000.

Source: The Hindu Data Point



16. How many times are the number of grandmasters from Y than the number of grandmasters from P?
- A. 2
 - B. 3
 - C. 4
 - D. 5
17. The total grandmasters from T and V are what per cent of total grandmasters from the X and Q?
- A. 75%
 - B. 25%
 - C. 70%
 - D. 50%
18. What is the per cent difference in Grandmasters from W than that from S?
19. What is the difference in the average grandmasters from P, Q, R, S and T taken together to the average of U, V, W, X, and Y?
- A. 35.5
 - B. 25.5
 - C. 20.5
 - D. 18.5
20. If there are 23 grandmasters from P who were born before 2000, then what is the percentage difference in the number of grandmasters born after 2000 than the number of grandmasters born before 2000 in P? (approximately)
- A. 97%
 - B. 80%
 - C. 91%
 - D. 89%



Data: 1.7 million migrants tried to illegally enter the US between FY 21. Of which, 0.15 million were unaccompanied minors. And 2500 of the 1.7 million were Indians. The line graph represents the share of individuals who were apprehended more than once by the Border Patrol out of total immigrants.

Source: The Hindu Data Point

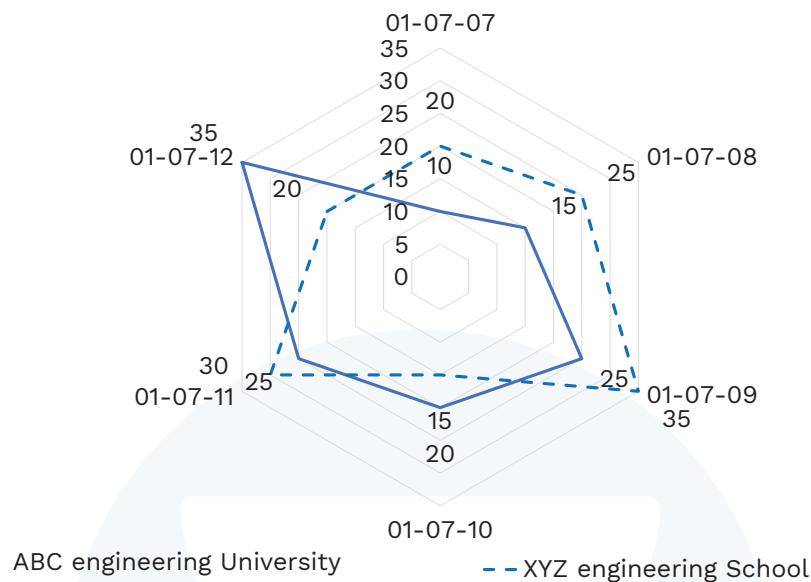


21. Which year has the highest per cent difference in repeated apprehenders as compared to the preceding year?
- FY 16
 - FY 21
 - FY 20
 - FY 17
22. What is the difference in people who crossed the US border in FY 19 and FY 21, if total immigrants in FY 19 were 90% of total immigrants in FY 21?
- 0.17 million
 - 0.15 million
 - 0.19 million
 - 0.13 million
23. Unaccompanied minors are what per cent of illegal immigrants in FY 21?
- 10%
 - 8.8%
24. If the number of repeated apprehenders in 2017 who were Indian were 2% of the total immigrants of the same year and were 100 less than in 2021, then what is the total number of illegal immigrants in FY 17.
- 0.13 million
 - 0.14 million
 - 0.12 million
 - 0.11 million
25. For how many years, repeated apprehender immigrants per cent were more than the average per cent overall years?
- 2 years
 - 3 years
 - 4 years
 - 5 years



Data: The radar graph below represents the number of students(in hundred) in ABC engineering university and XYZ engineering school in the starting of six different academic years.

Source: The Hindu Data Point



- 26.** What was the difference between the number of students in ABC engineering school in the year 2010 and the number of students in XYZ engineering school in the year 2012?
- A. 500
 - B. 1200
 - C. 700
 - D. 1000
- 27.** What is the sum of the number of students in ABC engineering university in the year 2007 and the number of students in XYZ engineering school in the year 2011 together?
- A. 2500
 - B. 2800
 - C. 3500
 - D. 3000
- 28.** If 20% of the students in XYZ in the year 2010 were females, what was the number of male students in XYZ in the same year?
- 29.** What was the per cent increase in the number of students in ABC engineering university in the year 2011 as compared to the previous year?
- A. 33.3%
 - B. 14.28%
 - C. 75%
 - D. 66.6%
- 30.** In which year was the difference between the number of students in ABC engineering university and XYZ highest?
- A. 2008
 - B. 2007
 - C. 2010
 - D. 2012



Data: The table shows the average spending on drugs per patient for both forms of treatment in the 10 high TB-burden countries in 2020.

Source: The Hindu Data Point

COUNTRY	FIRST-LINE TREATMENT (IN DOLLAR)	DRUG- RESISTANT TB TREATMENT (IN DOLLAR)
A	93	20287
B	46	2188
C	24	1723
D	45	1600
E	47	1352
F	67	1339
G	45	913
H	26	864
I	63	713
J	33	700

- 31.** Among the given countries, which country spent the total most approximately to the average total spending of all countries?

- A. H
- B. C
- C. I
- D. B

- 32.** What is the difference between the total spending of the top five countries and that of the bottom five countries?
- A. 23423 dollars
 - B. 22656 dollars
 - C. 34567 dollars
 - D. 43513 dollars
- 33.** How much will J spend next year if its total spending next year increases by 13%?
- A. 834.2 dollars
 - B. 849.6 dollars
 - C. 857.5 dollars
 - D. 828.2 dollars
- 34.** How much is the total spending of H more than that of C?
- A. 678 dollars
 - B. 756 dollars
 - C. 857 dollars
 - D. 934 dollars
- 35.** Which of the following countries has the lowest difference in respective spending over both types of treatment?
- A. J
 - B. I
 - C. H
 - D. B

SOLUTIONS

- 1. (B)** People who had at least one smartphone in 2020

$$= \frac{45}{100} \times 50 + \frac{60}{100} \times 30 + \frac{80}{100} \times 0.5 + \frac{60}{100} + 20 \\ = 22.5 + 18 + 0.4 + 12 = 53 \text{ billion (approx.)}$$

People who bought a new smartphone in 2021

$$= \frac{25}{100} \times 50 + \frac{25}{100} \times 30 + \frac{30}{100} \times 0.5 + \frac{25}{100} \times 20 \\ = 12.5 + 7.5 + 0.15 + 5 = 25 \text{ billion (approx.)}$$



$$\text{Required percentage} = \frac{53}{25} \times 100 = 210\%$$

2. (A) People who bought a new phone after march 2020 in 2021 = $\frac{10}{100} \times 20 = 2 \text{ billion}$

$$\text{People who bought a new phone after march 2020 in 2020} = \frac{25}{100} \times 20 = 5 \text{ billion}$$

$$\text{Required percentage} = \frac{5 - 2}{5} \times 100 = 60\%$$

3. (C) People who had at least one smartphone at home whose parents had low income in 2020 = $\frac{45}{100} \times 50 = \frac{45}{2} \text{ billion}$

$$\text{People who didn't have even one smartphone at home in high parental education home in 2020} = \frac{20}{100} \times 0.5 = \frac{1}{10} \text{ billion}$$

$$\text{Ratio} = \frac{45}{2} : \frac{1}{10} = \frac{225}{1} = 225:1$$

4. (D) Total number of people who had at least one smartphone in 2021
 $= \frac{50}{100} \times 50 + \frac{65}{100} \times 30 + \frac{80}{100} \times 0.5 + \frac{65}{100} \times 20$
 $= 25+19.5+0.4+13 = 57.9 \text{ billion}$

$$\text{People who bought a new phone after march 2020 in medium parental education home} = \frac{10}{100} \times 30 = 3 \text{ billion}$$

$$\text{Required difference} = 57.9 - 3 = 54.9 \text{ billion}$$

5. (A) Total number of people who didn't even had at least one smartphone in 2020 = $\frac{40}{100} \times 20 = 8 \text{ billion}$

$$\text{Total number of people who didn't even had at least one smartphone in 2021} = \frac{35}{100} \times 20 = 7 \text{ billion}$$

$$\text{Required difference} = 8 - 7 = 1 \text{ billion}$$

6. (C) Number of times when average submissions of women in any research area are higher than men = 3

Number of times when average submissions of women in any research area are lower than men = 5

$$\text{Required ratio} = 3:5$$

7. (B) Total average submissions of men = 0.4
 $+ 0.15 + 0.2 + 0.15 = 0.9$
 Total average submissions of women = 0.25
 $+ 0.05 + 0.15 + 0.1 = 0.55$
 Required difference = $0.9 - 0.55 = 0.35$

8. (C) Self-explanatory. 7 times

9. (A) Sum average submission of men in life science area in both years = $0.15 + 0.05 = 2$
 Sum average submission of men in social science and economics area in both years = $0.15 + 0.05 = 2$

$$\text{Required per cent} = \frac{2}{2} \times 100 = 100\%$$

10. (D) Percentage decrease of total average submission of men and women than its preceding year in one area

$$\text{Total submission in 2019} -$$

$$= \frac{\text{Total submission in 2018}}{\text{Total submission in 2018}} \times 100$$

$$\text{Percentage change in health and medicine area} = \frac{0.5 - 0.65}{0.65} \times 100 = 23\% \text{ decrease}$$

$$\text{Percentage change in life science area} = \frac{0.1 - 0.2}{0.2} \times 100 = 50\% \text{ decrease}$$

$$\text{Percentage change in physical sciences and engineering area} = \frac{0.2 - 0.35}{0.35} \times 100 = 42\% \text{ decrease}$$

$$\text{Percentage change in social sciences and economics area} = \frac{0.1 - 0.25}{0.25} \times 100 = 60\% \text{ decrease}$$

Thus, the maximum percentage decrease is in social sciences and economics.

11. (B) Unknown income of ABC = $\frac{25}{100} \times 10 = 2.5 \text{ crores approx.}$

$$\text{Unknown income of GHU} = \frac{55}{100} \times 90 = 50 \text{ crores approx.}$$



$$\text{Required per cent} = \frac{47.5}{50} \times 100 = 95\%$$

12. (A) Unknown income of IJK = $\frac{80}{100} \times 90 = 72$ crores

$$\text{Rest income of IJK} = \frac{20}{100} \times 90 = 18 \text{ crores}$$

$$\text{Required difference} = 72 - 18 = 54 \text{ crores}$$

13. (C) Income from other known sources = 172 crores

$$\text{Income from known sources} = 184 \text{ crores}$$

$$\text{Required per cent} = \frac{172}{184} \times 100 = 93\%$$

approximately

14. (C) Income from unknown sources of KLM

$$= \frac{40}{100} \times 110 = 44 \text{ crores}$$

$$\text{Income from unknown sources of ABC} = \frac{25}{100} \times 10 = 2.5 \text{ crores}$$

$$\text{Income from unknown sources of GHU} = \frac{55}{100} \times 90 = 49.5 \text{ crores}$$

$$\text{Total} = 44 + 2.5 + 49.5 = 96$$

15. (C) Income that is not unknown of LMN =

$$\frac{30}{100} \times 130 = 39 \text{ crores}$$

$$\text{Income which is not unknown of JKL} =$$

$$\frac{10}{100} \times 90 = 9 \text{ crores}$$

$$\text{Required difference} = 30 \text{ crores}$$

16. (D) Grandmasters from Y = 10

Grandmasters from P = 2

Self-explanatory 5 times.

17. (D) Total grand masters from T and V = 2 + 3 = 5

$$\text{Total grand masters from X and Q} = 8 + 2 = 10$$

$$\text{Required per cent} = \frac{5}{10} \times 100 = 50\%$$

18. (B) Grand masters from W = 4

Grand masters from S = 2

$$\text{Required per cent difference} = \frac{4 - 2}{2} \times 100$$

$$= 100\%$$

19. (B) Average grandmasters from P, Q, R, S, and T = 2

$$\text{Average of U, V, W, X, and Y} = 27.5$$

$$\text{Required difference} = 25.5$$

20. (B) Grandmasters from P who were born before 2000 = 23

Number of grandmasters born after 2000 in P = 2

$$\text{Required difference per cent} = \frac{21}{13} \times 100 =$$

$$91\%$$

21. (C) Per cent difference in repeated apprehenders as compared to the preceding year:

$$\text{FY 16} = \frac{12 - 14}{14} \times 100 = 0.14\%$$

$$\text{FY 17} = \frac{10 - 12}{12} \times 100 = 16.6\%$$

$$\text{FY 18} = \frac{11 - 10}{10} \times 100 = 10\%$$

$$\text{FY 19} = \frac{6 - 11}{11} \times 100 = 45.4\%$$

$$\text{FY 20} = \frac{26 - 6}{6} \times 100 = 333.3\%$$

$$\text{FY 21} = \frac{27 - 26}{26} \times 100 = 3.8\%$$

Thus, highest in FY 20

22. (A) People who crossed US border in the

$$\text{FY 19} = \frac{90}{100} \times 1.7 = 1.53 \text{ million}$$

People who crossed US border in the FY 21

$$= 1.7 \text{ million}$$

$$\text{Required difference} = 0.17 \text{ million}$$

23. (B) Required per cent = $\frac{0.15}{1.7} \times 100 = 8.8\%$.

24. (C) Number of Indian repeated apprehenders in 2017 = 2500 - 100

$$2400 = \text{total immigrants of the same year} \times \frac{1}{2/100}$$

$$\text{Total immigrants of the same year} = 0.12 \text{ million}$$



- 25. (A)** Average per cent overall years = $(14 + 12 + 10 + 11 + 6 + 26 + 27)/7 = \frac{106}{7} = 15\%$ approximately
Thus, repeated apprehender immigrants per cent were more than the average per cent overall years for 2 FY.
- 26. (A)** The number of students in ABC engineering school in the year 2010 = 2000
Number of students in XYZ engineering school in the year 2012 = 1500
Difference = 500
- 27. (C)** Self-explanatory
- 28. (C)** Female students in XYZ in the year 2010 = $\frac{20}{100} \times 1500 = 300$
Male students = $1500 - 300 = 1200$
- 29. (D)** Number of students in ABC engineering university in the year 2011 = 25
Number of students in ABC engineering university in the year 2010 = 15
Per cent increase = $\frac{25 - 15}{15 \times 100} \times 100 = 66.6\%$
- 30. (D)** Year 2007 difference = 10
Year 2008 difference = 10
Year 2009 difference = 10
Year 2010 difference = 5
Year 2011 difference = 5
Year 2012 difference = 15
Thus, highest in 2012.
- 31. (D)** Average spending of all countries = $(20380 + 2234 + 1747 + 1645 + 1399 + 1406 + 958 + 890 + 958 + 890 + 776 + 733)/10 = 3216.8$ dollars
Thus, B spent total most approximate to the average total spending of all countries (i.e., 2234 dollars)
- 32. (B)** Total spending of top five countries = $20380 + 2234 + 1747 + 1645 + 1399 = 27412$ dollars
Total spending of bottom five countries = $1406 + 958 + 890 + 958 + 890 + 776 + 733 = 4756$ dollars
Required difference = 22656 dollars
- 33. (D)** J's spending next year = $\frac{113}{100} \times 733 = 828.2$ dollars
- 34. (C)** Total spending of H = 890 dollars
Total spending of C = 1747 dollars
Required difference = 857 dollars
- 35. (A)** Difference in both spendings:
A = 20194
B = 2142
C = 1699
D = 1555
E = 1305
F = 1292
G = 868
H = 868
I = 838
J = 650
Thus, lowest in J.



SYNOPSIS

- Introduction
- Types of numbers
- Factors
- HCF
- LCM
- Simplification
- Fractions
- Types of fractions
- Surds and indices
- Divisibility rule

INTRODUCTION

Number system basically comprises ten digits, i.e., **0, 1, 2, 3, 4, 5, 6, 7, 8, and 9**. When these digits are combined together, they form a particular number. And when we form a number using these digits, then each digit takes a particular place, and that particular place of the digits we call the *place value* of that particular digit in that particular number. But the value of these

particular digits remains the same wherever they are placed in a number and called as *face value* of that particular number.

Example:

Let us form a number using some of the digits given above, 23789 is an example of a number that we formed using digits 2, 3, 7, 8, and 9.

Now, let us see the place value of these digits in this number, so here, the place value of each number is:

PLACE VALUE	TEN THOUSAND	THOUSANDS	HUNDREDS	TENS	ONES/UNITS
Digit	2	3	7	8	9

We use a number system in our day-to-day lives, for example, while playing cricket we count runs scored by a team, or how many wickets are taken by a team or by a particular bowler and so on.

After understanding the basic concept of the number system now comes the types of numbers in the number system.

TYPE OF NUMBERS

Natural numbers

Natural numbers are all positive counting numbers that begin with 1 and can be counted

indefinitely. Because natural numbers can never be negative, these numbers are also known as positive integers.

Example:

1, 2, 3, 4,... 267,...4999,... ∞ (infinity)

Whole numbers

If we add the number zero to the collection of natural numbers, we get the collection of whole numbers. Whole numbers are counting numbers that can be counted starting from digit 0 to infinity. Whole numbers include natural numbers that begin from 1 onwards.



Whole numbers include positive integers along with 0.

Example:

0, 1, 2, 3, 4, 5,...556, 721,...∞ (infinity)

Even numbers

Even numbers are basically integers, positive integers as well as negative integers, which can be divided exactly or evenly by digit 2. If an integer is exactly divided by 2, it implies that the number in question has a remainder of 0 upon the division of 2.

Example:

2, 4, 6, 8,... 498, 8888,...

Odd numbers

Odd numbers are integers, positive integers as well as negative integers, which cannot be divided exactly or evenly by digit 2. If an integer is not exactly divisible by 2, it implies that the number in question has a remainder of 1 upon the division of 2.

Example:

1, 3, 5, 7, ...379, 689,...

Prime numbers

Prime numbers are those positive integers that have only two factors, 1 and the number itself. They are not divisible by any other numbers.

Example:

2, 3, 5, 11,...71,...1381,...

Note

- There are only 25 prime numbers up to 100, i.e., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.
- 2 is the only even prime number.

Co-prime numbers

Co-prime numbers is basically a set of two numbers that have HCF as 1, i.e., there are

no common factors between these two numbers.

Example:

5 and 24, 6 and 37, etc.

Composite numbers

Composite numbers are those numbers that have more than 2 factors excluding the number itself. Also, a composite number is a positive integer that can be formed by multiplying two smaller positive integers. Equivalently, it is a positive integer that has at least one divisor other than 1 and itself.

Example:

4, 6, 8, 9, 10,... 656, 7864,...

Integers

From negative infinity to positive infinity, integers are all positive and negative counting numbers. Integers are a type of number that consists of both positive and negative numbers. Integers, like whole numbers, do not include the fractional portion. Integers, on the other hand, are numbers that can be positive, negative or zero but not a fraction.

Zero is also an integer.

Example:

...-23, -498,... 890, 4567,...

Positive integers

Positive integers are all whole numbers that are larger than zero and do not include fractions or decimals. The positive integers lie on the right side of 0 on a number line.

Example:

1, 2, 3, 4,... 666, 879,...

Negative integers

Negative integers are numbers that have a value less than zero. They do not include fractions or decimals. The negative integers lie on the left side of 0 on a number line.

Example:

...-37, -44, -990, -8769...



Note

- Zero is defined as neither negative nor positive. The ordering of integers is compatible with the algebraic operations in the following way: if $a < b$ and $c < d$, then $a + c < b + d$. If $a < b$ and $0 < c$, then $ac < bc$.

Rational numbers

Rational numbers are those numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$. Also, rational numbers include all integers, zero, or fractions.

Example:

$\frac{5}{2}, \frac{11}{13}, 0.7777, \frac{-1}{2}$, etc.

Irrational numbers

Irrational numbers are those numbers that cannot be expressed in the form of $\frac{p}{q}$.

Example:

$\pi, \sqrt{7}, 0.12122122212222\dots$

Real numbers

The collection of all rational numbers and irrational numbers together makes up what we call the collection of real numbers. All the concepts that we have discussed above, about different types of numbers, fall under the category of real numbers. So, all of the above-given examples are examples of real numbers also.

Complex numbers

Complex numbers are those numbers that can be written in the form $a + bi$ where a and b are real numbers, and i is the square root of -1 .

Example:

$\dots, -7 + 2i, 0, 1 + 3i\dots$

FACTORS

Factors of a number are the exact divisors of the provided numbers because they divide the

given number exactly with 0. For example, the number 15 is divisible by 1, 3, 5, and 15.

HCF

The Highest Common Factor is the full form of HCF. HCF is also known as Greatest Common Factor (GCF) or Greatest Common Divisor (GCD). The largest positive integer that divides all the provided integers with zero remainders is the greatest common factor (GCF, GCD, or HCF) for a subset of whole numbers.

Example: Consider two numbers 15 and 18.

Factors of 15 = 1, 3, 5, and 15

Factors of 18 = 1, 2, 3, 6, 9, and 18

The common factors of both 15 and 18 are 1 and 3. Here, 3 is the highest common factor of both numbers. Hence, 3 is the HCF of 15 and 18.

LCM

LCM stands for Lowest or Least Common Multiple in its full form. The smallest positive integer divisible by all the provided numbers is the LCM of two or more numbers.

Example:

Consider two numbers: 7 and 21.

The multiples of 7 are:

$$\begin{aligned}7 \times 1 &= 7, \\7 \times 2 &= 14, \\7 \times 3 &= 21, \\7 \times 4 &= 28, \text{ and so on...}\end{aligned}$$

The multiples of 21 are:

$$\begin{aligned}21 \times 1 &= 21, \\21 \times 2 &= 42, \\21 \times 3 &= 63, \\21 \times 4 &= 84, \text{ and so on...}\end{aligned}$$

21 is the lowest common multiple from all the above multiples of 7 and 21. So, 21 is the LCM of 7 and 21.

SOME OTHER IMPORTANT CONCEPTS

Simplification

As we know that in GATE and other engineering entrance examinations, fast and quick calculations are very important, and the simplification concept is one of the concepts



of quantitative aptitude, which helps you in solving questions fast and quick. Every question in the math section of your exam can be solved quickly with the help of this concept, and this makes this concept a very important one.

Finding an answer to a difficult calculation including division, multiplication, square roots, and cube roots, plus, and minus is known as simplification. When a calculation is given, it is sometimes observed that one of the numbers is missing. We must either approximate the given values or do basic operations to determine the missing number. We have to simplify the calculation when all the numbers are given with some operations between them.

Now, let's discuss some of the basic rules related to simplification:

1. Rule 1: Always replace 'of' by multiplication and '/' by division.

Example:

Find $\frac{1}{8}$ of 72.

Solution:

$$\frac{1}{8} \times 72 \text{ or } \frac{72}{8}$$

Therefore, 9 is the answer.

2. Rule 2: Always apply the rule of BODMAS in every number operation.

B	Bracket
O	Open, of
D	Division
M	Multiplication
A	Addition
S	Subtraction

So, whenever in any question when there is more than one type of calculation then we have to apply the rule of BODMAS while solving the question.

Example:

$$2 + 6 - (1 \times 4) \div 4 \times 5$$

Solution:

By applying the rule of BODMAS here, First, we will solve what is inside the bracket, i.e. $1 \times 4 = 4$

So, now the operation is $2 + 6 - 4 \div 4 \times 5$

Now, we will divide the number, i.e., $-4 \div 4 = -1$

Then, $2 + 6 - 1 \times 5$

Now, we will multiply the numbers, i.e. $-1 \times 5 = -5$

So, $2 + 6 - 5$

The next step is to add the number, i.e. $2 + 6 = 8$

So, $8 - 5$

Therefore, the answer is 3.

Fraction

Fractions are portions of a whole or a collection that are equal in size. Each part of a whole divided into equal pieces is a fraction of the total. Fractions are numbers that represent a piece of a larger whole. When an object or a set of objects is divided into equal pieces, each individual part becomes a fraction. Like most cases, a fraction is written as $\frac{1}{21}$, $\frac{50}{17}$, $\frac{7}{19}$, and

so on. The numerator represents the total number of equal parts into which the whole is divided, and the denominator represents the total number of equal parts into which the whole is divided. The total number of equal pieces removed is the numerator. For example, in the fraction $\frac{37}{4}$, the numerator is 37, and the denominator is 4.

We can also convert decimal numbers into fractions by putting 1 and the number of zeroes equal to the number of digits after the decimal, in the denominator.

Examples

If you divide a pizza into two equal portions, each half equals half of the total pizza,

$$0.2 = \frac{2}{10}, 1.879 = \frac{1879}{1000}, \text{etc.}$$



Example:

What is the greatest fraction among:

$$\frac{11}{16}, \frac{7}{8}, \frac{13}{20}, \text{ and } \frac{31}{40}$$

Solution:

If the denominator of some fractions is the same then the fraction with the largest numerator will be the largest fraction. So, to make the denominator equal to each fraction take LCM of 16, 8, 20, and 40 which is 80.

$$\text{Now, } \frac{80}{16} = 5 = \frac{11}{16} \times \frac{5}{5} = \frac{55}{80}$$

$$\frac{80}{8} = 10 = \frac{7}{8} \times \frac{10}{10} = \frac{70}{80}$$

$$\frac{80}{20} = 4 = \frac{13}{20} \times \frac{4}{4} = \frac{52}{80}$$

$$\frac{80}{40} = 2 = \frac{31}{40} \times \frac{2}{2} = \frac{62}{80}$$

$$70 > 62 > 55 > 52 \Rightarrow \frac{7}{8} > \frac{31}{40} > \frac{11}{16} > \frac{13}{20}$$

Therefore, the greatest fraction is $\frac{7}{8}$.

Types of fraction

1. Unit fraction:

In a fraction, the numerator with 1 is called a unit fraction. For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{99}$, etc.

2. Proper fraction:

If a numerator's value is less than the denominator's value, it is called a proper fraction. For example, $\frac{2}{7}, \frac{8}{11}, \frac{73}{79}$, etc.

3. Improper fraction:

If a numerator's value is greater than the denominator's value, then it is called an improper fraction. For example, $\frac{6}{5}, \frac{3}{2a}, \frac{97}{95}$, etc.

4. Mixed fraction:

If a fraction consists of a whole number with a proper fraction, it is called a mixed fraction. For example, $5\frac{3}{4}, 10\frac{1}{2}, 22\frac{1}{3}$, etc.

5. Like fractions:

The fractions with the same denominator are called like fractions. For example, $\frac{3}{2}, \frac{5}{2}, \frac{99}{2}$, etc.

6. Unlike fractions

The fractions with different denominators are called unlike fractions. For example, $\frac{3}{2}, \frac{4}{7}, \frac{67}{3}$, etc.

7. Equivalent fractions:

If two fractions result in the same value, after simplification, then they are equivalent to each other. For example, $\frac{2}{3}, \frac{4}{6}$ (when we

simplify it, it will become $\frac{2}{3}, \frac{6}{9}$ (when we

simplify it, it will become $\frac{2}{3}$), etc.

Surds and indices

Surds are the square roots ($\sqrt{}$) of numbers that cannot be reduced to a single whole or rational number. It is impossible to portray it accurately in a fraction. Surds are also the root values that cannot be expressed in whole numbers. Indices are a value's power or exponent.

Examples:

$\sqrt{2} \approx 1.414$, and therefore, we leave it as a surd $\sqrt{2}$.

There are some rules for surds and indices, which are:

1. Rules for indices:

- a. $X^0 = 1$
- b. $X^a \times X^b = X^{a+b}$
- c. $\frac{X^a}{X^b} = X^{a-b}$
- d. $(X^a)^b = X^{ab}$
- e. $(X^y)^a = X^a Y^a$
- $$\left(\frac{X}{Y}\right)^a = \frac{X^a}{Y^a}$$

2. Rules for surds:

- a. $\sqrt[x]{a} = a^{\frac{1}{x}}$
- b. $\sqrt[x]{ab} = \sqrt[x]{a} \times \sqrt[x]{b}$
- c. $\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$
- d. $(\sqrt[x]{a})^x = a$



$$\begin{aligned} \text{e. } & \sqrt[3]{\sqrt{x}} = \sqrt[3]{x} \\ \text{f. } & (\sqrt[3]{x})^y = \sqrt[3]{x^y} \end{aligned}$$

Example:

Find the value of $(\sqrt{64})^{\frac{1}{3}}$.

Solution:

Removing the root from the base value and putting it in the form of power, we get,

$$\begin{aligned} \Rightarrow & \left(64^{\frac{1}{2}}\right)^{\frac{1}{3}} = (64)^{\frac{1}{6}} \\ & (4^3)^{\frac{1}{6}} = 4^{\frac{1}{2}} = \sqrt{4} = 2 \end{aligned}$$

Therefore, the answer is 2.

Divisibility rule

Divisibility rule is when a number is divided by another number or digit without leaving a remainder, it is said to be divisible. To make division simple we use this rule, and here are some of the rules of few numbers:

1. **For 2:** If the unit or ones digit of any number is 0, 2, 4, 6, and 8, then that number is divisible by 2. For example, 2378, 666, and 3675344. As we see here the last digits of the numbers are 8, 6, and 4, respectively, so these numbers are divisible by 2.
2. **For 3:** A number is only divisible by 3 when the sum of all the digits of the number is divisible by 3. For example, 333 ($3 + 3 + 3 = 9$), 33455 ($3 + 3 + 4 + 5 + 5 = 20$). As we see here the sum of the digits of the first number is 9, which is divisible by 3, so 333 is also divisible by 3, but the sum of the digits of the second number is 20, which is not divisible by 3, so the number 33455 will also not be divisible by 3.
3. **For 4:** If the last digit of a number is 0, or the last two digits are divisible by 4, then that number is divisible by 4. For example, 2508, and 2506. As we see here, the last digits of the number 2508 are 08. Since 08 is divisible by 4, then the number 2508 is also divisible by 4. But 2506 is not divisible by 4 because the last two digits, 06, are not divisible by 4.
4. **For 5:** If a number has 0 or 5 at its unit's place then it is divisible by 5. For example, 2700, 23675, and 34583805. As we see here the last digit of the numbers are 0, 5, and 5 respectively, so these numbers are divisible by 5.
5. **For 6:** If a number is divisible by 2 and 3 both are also divisible by 6. For example, 36, 216, and 1296. As we see here, all the three numbers are divisible by 2 as well as 3. Hence, the numbers are divisible by 6 also.
6. **For 7:** The difference between twice the unit digit of the given number and the remaining part of the given number should be a multiple of 7, or it should be equal to 0. Let us learn this using an example: 161. The first step would be to double the number at units place, i.e., $1 \times 2 = 2$. Now subtract this from the remaining number $16 - 2 = 14$. Check if the reduced number is divisible by 7 or not. If yes then number 161 is also divisible by 7.
7. **For 8:** A number is only divisible by 8 when its last 3 digits, i.e., digits at hundred's, ten's, and unit's place, are divisible by 8, or if the last 3 digits are zero. For example, 4608, 36864, 294912. As we see here the last three digits of all the three numbers are divisible by 8. So, all three numbers are completely divisible by 8.
8. **For 9:** A number is only divisible by 9 if the sum of all its digits is divisible by 9. For example, 999 ($9 + 9 + 9 = 27$), 89919 ($8 + 9 + 9 + 1 + 9 = 36$). As we see here the sum of the digits of the first number is 27, which is divisible by 9, so 999 is also divisible by 9 likewise the sum of the digits of the second number is 36, which is also divisible by 9, so the number 89919 will also be divisible by 9.
9. **For 10:** If the last digit of any number is 0, then the number is divisible by 10. For example,



356821730, 2546270, and 9876540, etc. All these numbers are completely divisible by 10.

- 10. For 11:** If the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or multiple of 11, then the number is divisible by 11. For example, 1331 ($1 + 3 = 4$; $3 + 1 = 4$, difference

between the sum is 0), 161051 ($1 + 1 + 5 = 7$; $6 + 0 + 1 = 7$, difference between the sum is 0). As we see here, the difference between the sum of digits at odd places and the sum of digits at even places of the numbers is 0; hence, these numbers are divisible by 11.

PRACTICE QUESTIONS

1. $93 + 3 - 7 \div 43 \times 29 = ?$
A. 90.44
B. 91.27
C. 100
D. 98.77
2. $213 + 711 \times 322 + 23 - 9 = ?$
A. 229769
B. 229169
C. 229669
D. 229969
3. $1.8 \times 7 + 5 - 14 \div 15 + 3 = ?$
A. 11.99
B. 12
C. 11.27
D. 12.50
4. $121 + 517 + 47 - 17 \div 17 \times 41 = ?$
A. 600
B. 650
C. 666
D. 644
5. $2 - 7 + 3 + 4 - 2 \times 19 \div 12 = ?$
A. 1.166
B. 2.679
C. -2.679
D. -1.166
6. $\frac{1}{3 + \frac{8}{4 + \frac{3}{5 + \frac{1}{4}}}} = ?$
A. $\frac{4}{33}$
B. $\frac{4}{67}$
7. If $4x - 1 \times 6 \times \frac{82x}{2x} \times 3x - 1 = 12$, then the value of x is:
A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$
8. If $Y = \sqrt{2} + 1$, then the value of $Y + \frac{1}{Y}$ is:
A. $2\sqrt{2}$
B. $\sqrt{2}$
C. $2\sqrt[3]{2}$
D. 2
9. Which among the following is the greatest fraction? $\frac{2}{5}, \frac{3}{5}, \frac{1}{5}, \frac{7}{15}, \frac{4}{5}$
A. $\frac{2}{5}$
B. $\frac{4}{5}$
C. $\frac{3}{5}$
D. $\frac{7}{15}$



- 10.** The sum of the first squares of the first ten natural numbers is:
- A. 333
 - B. 388
 - C. 385
 - D. 365
- 11.** A pile of coconuts is divided into groups of 2, 3 and 5, with one coconut left out each time. What is the smallest number of coconuts in the heap?
- A. 62
 - B. 30
 - C. 35
 - D. 31
- 12.** The remainder is 3 when n is divided by 4. When $2n$ is divided by 4, what is the remainder?
- A. 2
 - B. 3
 - C. 4
 - D. 1
- 13.** What is the unit digit in the number $(211)^{111}$?
- A. 1
 - B. 2
 - C. 3
 - D. 4
- 14.** What is the sum of the first 45 natural numbers?
- A. 1000
 - B. 1022
 - C. 1035
 - D. 1099
- 15.** What is the smallest value that * must have in order for the number $451*603$ to be divisible by 9 exactly?
- A. 9
 - B. 6
 - C. 7
 - D. 8
- 16.** What is the smallest value that must be provided to * in order for $63576*2$ to be divisible by 8?
- A. 1
 - B. 3
 - C. 5
 - D. 2
- 17.** Which of the following is divisible by 99 exactly?
- A. 114345
 - B. 114344
 - C. 114343
 - D. 114342
- 18.** What is the greatest five-digit number that can be divided by 99?
- A. 99990
 - B. 90909
 - C. 99099
 - D. 90999
- 19.** When a number is divided by 338, the remainder is 68; nevertheless, when the same amount is divided by 13, the remainder is?
- A. 3
 - B. 5
 - C. 1
 - D. 7
- 20.** Between 200 and 600, how many numbers are divisible by 4, 5 and 6?
- A. 6
 - B. 8
 - C. 10
 - D. 12
- 21.** What is the closest number to 99547 that is divisible by 687 exactly?
- A. 99617
 - B. 99615
 - C. 99619
 - D. 99614



- 22.** Six bells began to toll in sync, at 3, 6, 9, 12, 15 and 18 seconds intervals, respectively. How many times did they toll collectively in 30 minutes?
- A. 9
B. 10
C. 11
D. 12
- 23.** What is the smallest five-digit number that can be divided by 11?
- A. 10000
B. 10001
C. 10009
D. 10010
- 24.** The sum of a two-digit number's digits is 9. If the digits are flipped, the number falls by 45, thus what is the original number?
- A. 72
B. 63
C. 90
D. 54
- 25.** Find the smallest integer that leaves a remainder of 2, 3 or 4 when divided by 3, 4 or 5.
- A. 56
B. 57
C. 58
D. 59
- 26.** When a number is divided by 943, the remainder is 76. When you divide the same number by 23, what is the remainder?
- A. 4
B. 6
C. 9
D. None of the above
- 27.** 2093 is the sum of two numbers. The quotient is 2 when the larger number is divided by the difference in the numbers, and the remainder is 118. What is the smaller of the two numbers?
- A. 734
B. 735
- 28.** Between 500 and 600, how many numbers include the number 9 only once?
- A. 18
B. 15
C. 8
D. 9
- 29.** When $(1923^{1924^{1925}})$ is divided by 1924, what is the remainder?
- A. 0
B. 2
C. 1
D. 11
- 30.** A chain smoker had used up all of his cash. He couldn't afford to buy cigarettes because he didn't have any money left. As a result, he joined the stubs and smoked them together. To make a single cigarette, he needed four stubs. How many cigarettes could he smoke in total if he received a pack of ten cigarettes as a gift?
- A. 12
B. 13
C. 11
D. 14
- 31.** How many of the numbers given below are divisible by 132?
- 264, 396, 462, 792, 968, 2178, 5184, 6336
- A. 7
B. 6
C. 5
D. 4
- 32.** Which of the following numbers is divisible by 11 exactly?
- A. 415624
B. 415625
C. 415626
D. 415627
- 33.** In total, how many three-digit numbers are divisible by six?



- A. 144
- B. 150
- C. 160
- D. 164

34. What is the smallest prime number?

- A. 0
- B. 1
- C. 2
- D. 3

35. How many prime numbers are less than 50?

- A. 12
- B. 13
- C. 14
- D. 15

36. What is the smallest three-digit prime number?

- A. 333
- B. 497
- C. 459
- D. None of the above

37. Which of the following is even if a and b are odd numbers?

- A. $a + b + 1$
- B. $a + 3b$
- C. $a + b$
- D. $a + \frac{b}{3}$

38. We obtain 269 as the quotient and 0 as the remainder when we divide an integer by 68. What will be the remainder if you divide the same number by 67?

- A. 3
- B. 2
- C. 1
- D. 0

39. When we divide a number by 56, the remainder is 29. What is the remainder when you divide the same number by 8?

- A. 7
- B. 6
- C. 5
- D. 4

40. When we divide a number by 357, the remainder is 39. What is the remainder when you divide the same number by 17?

- A. 3
- B. 4
- C. 5
- D. 6

41. We receive 3 as a remainder when we divide a number by 5. When the square of this number is divided by 5, what is the remainder?

- A. 4
- B. 5
- C. 6
- D. 7

42. 1365 is the difference between the two numbers. We obtain 6 as the quotient and 15 as the remainder when we divide the larger number by the smaller. What is the smaller of the two numbers?

- A. 278
- B. 270
- C. 275
- D. 289

43. The remainder in a division sum is 0. When a pupil multiplied the divisor by 12 instead of 21, the quotient was 35. What is the right quotient to use?

- A. 12
- B. 21
- C. 20
- D. 35

44. The product of two numbers is 35, while the sum of two numbers is 12. What is the sum of these numbers' reciprocals?

- A. $\frac{35}{12}$



- B. $\frac{1}{12}$
C. $\frac{1}{35}$
D. $\frac{12}{35}$

45. A positive proper fraction and its reciprocal have a difference of $\frac{9}{20}$. What is the fraction?

- A. $\frac{5}{4}$
B. $\frac{4}{5}$
C. $\frac{20}{9}$
D. $\frac{1}{5}$

46. We receive the same remainder when we divide 2272 by 3 digit number X as we do when we divide 875 by 3 digit number X . What is the sum of X 's digits?

- A. 9
B. 10
C. 11
D. 13

47. On multiplying a number by 7 the product is a number each of whose digits is 3. The smallest such number is:

- A. 47617
B. 47619
C. 47666
D. 47615

48. Which of the following integers is divisible by the difference of the squares of two successive even integers?

- A. 3
B. 4
C. 5
D. 6

49. Which of the following integers is divisible by the difference of the squares of two successive odd integers?

- A. 8
B. 9
C. 13
D. 17

50. If n is a natural number, then $(6n^2+6n)$ is always divisible by:

- A. 6
B. 12
C. Both 6 and 12
D. None of the above

SOLUTIONS

1. **(B)** As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and lastly the subtraction part.

$$\text{So, } 93 + 3 - \frac{7}{43} \times 29$$

$$\begin{aligned} &= 93 + 3 - 0.16 \times 29 \\ &= 93 + 3 - 4.72 \\ &= 96 - 4.72 \\ &= 91.279069767442. \end{aligned}$$

Therefore, the correct option is B.

2. **(B)** As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 213 + 711 \times 322 + 23 - 9 \\ &= 213 + 228942 + 23 - 9 \\ &= 229155 + 23 - 9 \\ &= 229178 - 9 = 229169 \end{aligned}$$

Therefore, the correct option is B.

3. **(C)** As there is no bracket in this question, first we will solve the division part of the



sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 1.8 \times 7 + 5 - \frac{14}{1.5} + 3 \\ &= 12.6 + 5 - \frac{14}{1.5} + 3 \\ &= 12.6 + 5 - 9.3 + 3 \\ &= 17.6 - 9.3 + 3 \\ &= 8.27 + 3 \\ &= 11.27 \end{aligned}$$

Therefore, the correct option is C.

4. (D) As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 121 + 517 + 47 - \frac{17}{17} \times 41 \\ &= 121 + 517 + 47 - 1 \times 41 \\ &= 121 + 517 + 47 - 41 \\ &= 638 + 47 - 41 \\ &= 685 - 41 = 644 \end{aligned}$$

Therefore, the correct option is D.

5. (D) As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned} &= 2 - 7 + 3 + 4 - 2 \times \frac{19}{12} \\ &= 2 - 7 + 3 + 4 - \frac{38}{12} \\ &= 2 - 7 + 3 + 4 - 3.16 \\ &= -5 + 3 + 4 - 3.16 \\ &= -2 + 4 - 3.16 \\ &= 2 - 3.166666666667 \\ &= -1.166 \end{aligned}$$

Therefore, the correct option is D.

6. (C) Solving the sum,

$$\begin{aligned} &= \frac{1}{3} + \frac{168}{96} \\ &= \frac{4}{19} \end{aligned}$$

Therefore, the correct option is C.

7. (B) $2^{2x-2} \times 2^x \times 3^x \times \frac{2^{6x}}{2^x} \times 3^x - 1$

$$\begin{aligned} &= 2^2 \times 3^1 \\ &2^{2x-2+x+x+6x-x} \times 3^{x-x+1} = 2^2 \times 3 \\ &x = \frac{1}{2} \end{aligned}$$

Therefore, the correct option is B

8. (A) $Y + \frac{1}{Y} = \frac{Y^2 + 1}{Y}$

Now put the value of $Y = \sqrt{2} + 1$

$$\begin{aligned} &\left[(\sqrt{2} + 1)^2 + 1 \right] \\ &\quad \sqrt{2} + 1 \\ &= 2\sqrt{2} \end{aligned}$$

Therefore, the correct option is A.

9. (B) If the denominator of some fractions is the same then the fraction with the largest numerator will be the largest fraction. So, to make the denominator equal to each fraction take LCM of 5, 5, 5, 15 and 5 which is 15.

$$\text{Now, } \frac{15}{5} = 3 \Rightarrow \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$$

$$\frac{15}{15} = 1 \Rightarrow \frac{7}{15} \times \frac{1}{1} = \frac{7}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$$

$$12 > 9 > 7 > 6 > 3 \frac{4}{5} > \frac{3}{5} > \frac{7}{15} > \frac{2}{5} > \frac{1}{5}$$

Hence, the greatest fraction is $\frac{4}{5}$.

Therefore, the correct option is B.

10. (C) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

$$\begin{aligned} &= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 \\ &= 385 \end{aligned}$$

Therefore, the correct option is C.

11. (D) Take the LCM of 2, 3, 5 which is 30



Therefore, the correct option is D.

- 27. (D)** Let the numbers be X and Y
So, $X + Y = 2093$...Equation (1)
According to the question

$$\Rightarrow \frac{X}{X} - Y = 2 + \frac{118}{X} - Y$$

$$\Rightarrow 2Y - X = 11$$
...Equation (2)
- Solving equations (1) and (2) we get $Y = 737$.
Therefore, the correct option is D.
- 28. (D)** All numbers between 500 and 600 in which 9 occurs only once are: 509, 519, 529, 539, 549, 559, 569, 579, 589. Hence there are 9 such numbers.
Therefore, the correct option is D.
- 29. (C)** The remainder is 1.
Therefore, the correct option is C.
- 30. (B)** 10 cigarettes give 10 stubs. From 10 stubs 3 more cigarettes can be made. 13 is the answer.
Therefore, the correct option is B.
- 31. (D)** For a number to be divisible by 132 it should have to be divisible by 3, 4 and 11 also. So, applying the divisibility rules of these three numbers on each of the given numbers, we find that there are only 4 such numbers (264, 396, 792 and 6336).
Therefore, the correct option is D.
- 32. (A)** Apply the divisibility rule of 11 on each option.
Therefore, the correct option is A.
- 33. (B)** There are 150 such numbers.
Therefore, the correct option is B.
- 34. (C)** 2 is the smallest prime number.
Therefore, the correct option is C.
- 35. (D)** There are 15 prime numbers less than 50 (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47).
Therefore, the correct option is D.
- 36. (D)** None of the given numbers is the smallest prime number.

Therefore, the correct option is D.

- 37. (C)** The sum of two odd numbers is always even.
Therefore, the correct option is C.
- 38. (C)** Dividend = $68 \times 269 + 0 = 18292$
Dividing 18292 by 67 we get 1 as remainder.
Therefore, the correct option is C.
- 39. (C)** Let's assume the number is $56 + 29 = 85$
 $85 = 56 \times 1 + 29$
When it is divided by 8,

$$\Rightarrow \frac{85}{8} = 8 \times 10 + 5$$
 or $29 = 3 \times 8 + 5$
Hence, the remainder is 5.
Therefore, the correct option is C.
- 40. (C)** Let x be the number and y be the quotient.
Then, $x = 357y + 39 =$

$$\Rightarrow (17 \times 21 \times y) + (17 \times 2) + 5 =$$

$$\Rightarrow 17 \times (21y + 2) + 5$$

Required number = 5.
Therefore, the correct option is C.
- 41. (A)** 4 will be the remainder in this case due to the square condition applied here.
Therefore, the correct option is A.
- 42. (B)** Let the smaller number be x . Then the larger number = $(x + 1365)$.

$$\Rightarrow x + 1365 = 6x + 15$$

$$\Rightarrow 5x = 1350$$

$$x = 270$$

Smaller number = 270.
Therefore, the correct option is B.
- 43. (C)** Number = (12×35)
Correct quotient = $420 \div 21 = 20$
Therefore, the correct option is C.
- 44. (D)** $\frac{12}{35}$ is reciprocal.
Therefore, the correct option is D.
- 45. (B)** Let the required fraction be X .
Then, $X - \frac{1}{X} = \frac{9}{20}$



$$\text{So, } 1 - \frac{x^2}{x} = \frac{9}{20}$$

$$\Rightarrow x = \frac{4}{5}$$

Therefore, the correct option is B.

- 46. (B)** Clearly, $(2272 - 875) = 1397$, is exactly divisible by x .

$$\text{Now, } 1397 = 11 \times 127$$

The required 3-digit number is 127, the sum of whose digits is 10.

Therefore, the correct option is B.

- 47. (B)** 47619 is such a number.

Therefore, the correct option is B.

- 48. (B)** Let the two consecutive even integers be $2n$ and $(2n+2)$.

Then,

$$(2n+2)^2 - (2n)^2 = (2n+2+2n)(2n+2-2n)$$

$$\Rightarrow 2(4n+2)$$

$\Rightarrow 4(2n+1)$, which is divisible by 4.

Therefore, the correct option is B.

- 49. (A)** Let the two consecutive odd integers be $(2n+1)$ and $(2n+3)$.

$$\text{Then, } (2n+3)^2 - (2n+1)^2$$

$$\Rightarrow (2n+3+2n+1)(2n+3-2n-1)$$

$$\Rightarrow (4n+4) \times 2$$

$$8(n+1), \text{ which is divisible by 8.}$$

Therefore, the correct option is A.

- 50. (C)** $(6n^2 + 6n) = 6n(n+1)$, which is always

divisible by 6 and 12 both, since $n(n+1)$ is always even.

Therefore, the correct option is C.

30 Percentages



SYNOPSIS

- Introduction
- Percentage chart
- Percentage difference formula
- Important percentage formulas
- Summary

INTRODUCTION

The percentage concept is used to calculate the proportion of a value in relation to its original value. Percentages are often used in business to calculate a company's profit or loss percentage. It is also used in schools and universities to indicate the grades that students have obtained. A percentage formula can be used to assess the students' grades. In general, a percentage is defined as a ratio of any value to the total value multiplied then by 100. The percentage symbol % is used to denote the percentage.

Percentage is a relative figure that represents one-hundredth of a quantity. One per cent (1%) equals one hundredth; two per cent (2%) equals two hundredths, and so on.

There are no dimensions to the %. It means they are integers with no dimensions. When we say 80% of a number, we mean 80 percentage of the whole. It can also be stated as a decimal or a fraction, for example, 0.76% or 0.17%. The proportion of marks earned by students in any subject is calculated during the examination by using the concept of percentage.

Example:

A student received a 78% on his exam. So, this percentage is derived by dividing a student's total marks into all subjects by the total maximum marks and then multiplying the result by 100.

Some other understanding-based examples of percentages are:

10% is equal to $\frac{1}{10}$ fraction.

20% is equal to $\frac{1}{5}$ fraction.

25% is equal to $\frac{1}{4}$ fraction.

50% is equal to $\frac{1}{2}$ fraction.

75% is equal to $\frac{3}{4}$ fraction.

90% is equal to $\frac{9}{10}$ fraction.

Example:

Find the 40% of 300.

Solution:

$$\frac{40}{100} \times 300 = 40 \times 3 = 120$$

Hence, 120 is 40% of 300.

Example:

In a class of 72 students, 87.5% of the students cleared the GATE examination. How many did not clear it?

Solution:

The number of students who did not clear the

$$\text{GATE exam} = 12.5 \times \frac{72}{100} = 9.$$

**Example:**

The population of a town increases at the rate of 3.7% each year. It is 31,110 now. What was it like last year?

Solution:

Let's assume last year it was x

$$\text{So, } 31,110 = x \left(1 + \frac{3.7}{100}\right)$$

Hence, $x = 30,000$.

Example:

Kyler needed Rs. 800 to cover her fees. Her brother provided 20% of the funding, and her mother provided 30% of the remaining funds. She had Rs. 200 in her bank account. How much extra (in Rs.) does she require?

Solution:

Required amount = 800

$$\text{From her brother she got} = 800 \times \frac{20}{100} = 160$$

$$\text{From her mother she got} = (800 - 160) \times \frac{30}{100} =$$

$$640 \times \frac{30}{100} = 192$$

From the bank she got Rs. 200

$$\text{Now, she needs } 800 - (160 + 192 + 200) = 800 - 552 = \text{Rs. 248 more}$$

Example:

Allen, the barber, shaved 40% of his customers and gave a haircut to 80% of his customers. He charged Rs. 7 for a shave and Rs. 5 for a haircut. If 20% of customers who opted for a shave also had a haircut, what were Allen's earnings if he had 75 customers (in Rs.)?

Solution:

Total customers = 75

$$\text{Numbers of customers shaved} = 75 \times \frac{40}{100} = 30$$

Number of customers who got hair cut

$$= 75 \times \frac{80}{100} = 60$$

$$\text{Hence, his total income} = (30 \times 7) + (60 \times 5) = 210 + 300 = \text{Rs. 510.}$$

Percentage chart

The percentage chart is given here for fractions converted into percentages.

FRACTIONS	PERCENTAGE
$\frac{1}{2}$	50%
$\frac{1}{3}$	33.33%
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	16.66%
$\frac{1}{7}$	14.28%
$\frac{1}{8}$	12.50%
$\frac{1}{9}$	11.11%
$\frac{1}{10}$	10%
$\frac{1}{11}$	9.09%
$\frac{1}{12}$	8.33%
$\frac{1}{13}$	7.69%
$\frac{1}{14}$	7.14%
$\frac{1}{15}$	6.66%
$\frac{1}{16}$	6.25%
$\frac{1}{17}$	5.88%
$\frac{1}{18}$	5.55%
$\frac{1}{19}$	5.26%
$\frac{1}{20}$	5%



Percentage difference formula

If we are given two numbers suppose X and Y , and the question asked us to find the percentage difference between the two numbers, then we will apply the below-given formula to find the percentage difference quickly:

$$\text{Percentage difference} = \frac{|X - Y|}{\left(\frac{X + Y}{2}\right)} \times 100$$

Example:

If two numbers are given suppose 20 and 30, and then the percentage difference between these two numbers is:

Solution:

$$\text{Percentage difference} = \frac{|20 - 30|}{\left(\frac{20 + 30}{2}\right)} \times 100$$

On solving the above equation, $\frac{10}{25} \times 100 = 40$

Therefore, the percentage difference between 20 and 30 is 40%.

Some other important percentage formulas

1. Percentage formula in case of fraction:

$$\frac{\text{Numerator}}{\text{Denominator}} \times 100$$

Example:

Convert $\frac{3}{16}$ into a percentage.

Chapter Summary

$$\frac{\text{Numerator}}{\text{Denominator}} \times 100$$

Solution:

Using above given formula,

$$\frac{3}{16} \times 100 = 3 \times \frac{25}{4}$$

$$\Rightarrow \frac{75}{4}\% \text{ or } 18.75\%$$

2. Percentage change formula:

$$\frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100$$

Example:

Annie works in a supermarket for \$10.00 per hour. If her pay is increased to \$12.00, then what is her percentage increase in pay?

Solution:

Here, original value = 10, and new value = 12

$$\frac{12 - 10}{10} \times 100$$

$$\frac{2}{10} \times 100 = 20$$

- In general, a percentage is defined as a ratio of any value to the total value multiplied then by 100.
- The percentage symbol % is used to denote the percentage.
- Percentage difference = $\frac{|X - Y|}{\left(\frac{X + Y}{2}\right)} \times 100$
- Percentage formula in case of fraction:



PRACTICE QUESTIONS

- Aaron receives 35% of the maximum marks in an exam, while John receives 50% of the maximum marks. Aaron would have received the same percentage as John if he had received 45 more marks.

What is the exam's maximum score?

- 300
- 400
- 350
- 450



2. Peter received 20% of the maximum marks in an exam, but he failed by 30 marks. Peter received 40% of the same and was passed by ten marks. What were the minimum passing marks for the exam?
- A. 70
B. 80
C. 120
D. 60
3. Tom's pay is 12% more than Jerry's. Jerry's pay is lesser than Tom's by what percentage?
- A. 10.71%
B. 10%
C. 9.71%
D. 11.71%
4. Benett is 25% older than Cyril, his younger brother. Cyril is younger than Benett by what percentage?
- A. 20%
B. 25%
C. 30%
D. 16.66%
5. Patrick's pay is reduced by 16% initially and increased by 16% in the next month. Patrick's current income is approximately how much less than his starting salary?
- A. 2% less
B. 2.16% less
C. 2.56% less
D. 2.009% less
6. James's salary is boosted by 20% for the first month and then reduced by 20% for the second month. What is James's current salary in relation to his starting salary?
- A. 2% more
B. 3% less
C. 4% less
D. 5% more
7. A commission of 5% on the first Rs. 5000 and 2.5% on the rest of the selling price is charged by an agent. What was his commission if the selling price was Rs. 30,600?
- A. 850
B. 1015
C. 890
D. 1020
8. In a two-candidate election, 6% of voters did not cast their ballots. Won by 3000 votes, the winning candidate received 48% of the total votes cast. How many people cast ballots in the election?
- A. 12000
B. 10000
C. 15000
D. 20000
9. To be eligible for a scholarship, you must get at least 80% in an examination. Tony received 1005 marks but was just 13% of the way to receiving the scholarship. What was the highest possible score?
- A. 2000
B. 2500
C. 3000
D. 2800
10. Marcos has a particular quantity of mangoes, with 15% of them being rotten. He offers his friend 60% of the remaining mangoes, leaving him with 102 mangoes. How many mangoes did he have when he first started?
- A. 200
B. 300
C. 400
D. 500
11. In a two-candidate election, one candidate received 55% of the total valid votes, while 20% of the total votes cast were invalid. What was the number of legitimate votes



- received by the other candidate if the total votes cast were 7500?
- A. 2500
B. 2700
C. 3000
D. 3200
- 12.** A person's daily income has been boosted by 75%, and he now receives Rs. 75. How much did he earn on a daily basis before the raise?
- A. 41
B. 42.85
C. 45.85
D. 45
- 13.** What is 66% of 66.67?
- A. 22
B. 11
C. 33
D. 44
- 14.** When Albert's income is 25% more than Pinto's, how much less does Pinto's income have to be?
- A. 10%
B. 15%
C. 18%
D. 20%
- 15.** In an exam, a candidate receives 20% of the maximum score and fails by ten marks. Another candidate receives 42% of the maximum possible marks, which are 12 marks more than the passing grade. What is the exam's maximum score?
- A. 80
B. 90
C. 110
D. 100
- 16.** In a poll between A, B, and C, a total of 600 votes were cast. A received 30% of the vote, B received 360 votes, with the remaining votes going to C. In relation to his nearest rival, what percentage of the overall votes did the winner receive?
- A. 100%
B. 500%
C. 300%
D. 200%
- 17.** When two candidates, A and B, are running for office election. A received 60% of the total valid votes, 15% of the total 5,00,000 ballots cast was considered illegitimate. What is the total number of votes that B has received?
- A. 100000
B. 150000
C. 170000
D. 190000
- 18.** An accountant charges a fee of 14% of the transaction amount. How much did Rory pay the accountant if he completed a transaction of Rs. 14,00,000 for him?
- A. 200000
B. 196000
C. 190000
D. 193000
- 19.** A carpenter manufactured 50 chairs, 14 of which are defective. What is the percentage of chairs that are in good condition?
- A. 50%
B. 75%
C. 80%
D. 72%
- 20.** In an election between A and B, A received 65% of the vote and won the election by a margin of 2748 votes. What is the total number of votes cast if no vote is declared invalid?
- A. 9160
B. 9000
C. 9169
D. 9167
- 21.** In a contest between Kurian and Pablo, Kurian received 25% of the vote but was defeated. Pablo received 14000 votes more



- than Kurian in the election. What is the total number of votes cast?
- A. 25000
B. 27000
C. 26000
D. 28000
- 22.** Nathan is 22% more productive than Lyon. In comparison to Nathan, how inefficient is Lyon?
- A. 17.03%
B. 18.03%
C. 19.03%
D. 20.03%
- 23.** Simon's total salary was reduced by 35%. He spent 45% of the money left over, leaving him with Rs. 2500. What was his overall salary?
- A. 6991
B. 6992
C. 6993
D. 6994
- 24.** If A's salary is 25% more than B's salary, what percentage of B's salary is less than A's salary?
- A. 10%
B. 20%
C. 15%
D. 18%
- 25.** Only 75% of the total number of eligible voters voted in a presidential election. The victorious candidate received 58% of all votes cast and won by a margin of 12000 votes. What was the total number of voters who were eligible to vote?
- A. 125000
B. 100000
C. 150000
D. 175000
- 26.** Stark's pay is set at 80% of that of his boss. If he spends 40% of his salary on rent and 20% on food and is left with Rs. 2400, what is Stark's boss's salary?
- A. 6200
B. 6250
C. 6225
D. 6275
- 27.** In the market, the price of rice increases by 14%. A family's rice intake must be reduced by what percentage in order to avoid an increase in rice expenditure?
- A. 12.28%
B. 12.50%
C. 12.47%
D. 12.98%
- 28.** Walter's pay has been boosted by 10%. At this point, his pay is 10% less than his boss's. What percentage of Walter's pay is smaller than his boss's salary if his salary is now lowered by 10%?
- A. 17%
B. 18%
C. 19%
D. 20%
- 29.** Jessi's salary is reduced by 40%, then boosted by 20%, and lastly increased by 20%. What percentage of his beginning pay does he earn now?
- A. 12.6% less
B. 11.6% less
C. 14.6% less
D. 13.6% less
- 30.** Robert, Sandy and Phil ran for office election in the same election. Robert received 40% of the vote and so won the election. Sandy and Phil received 35% and 25% of the overall vote, respectively. What is the amount of votes Robert received in the election if Sandy received 2000 more than Phil?
- A. 8000
B. 8100
C. 8200
D. 8500



- 31.** A total of 45,000 individuals took part in the exam, with 40% of them being female. What was the percentage of passed students if 70% of the boys and 75% of the girls passed the exam?
- A. 70%
B. 71%
C. 72%
D. 73%
- 32.** A mine's lead ore provides 60% metal, with 0.75% of that being silver. How much silver can be extracted from 8000 kg of ore?
- A. 34 grams
B. 35 grams
C. 36 grams
D. 37 grams
- 33.** What percentages will the volume of a cone rise if the height and base radius of the cone are both increased by 50%?
- A. 237%
B. 237.5%
C. 238%
D. 238.5%
- 34.** When 80% of a number is added to 80, the outcome is the same number. Which of the following numbers is the correct answer?
- A. 400
B. 500
C. 600
D. 700
- 35.** What percentage does the volume of a cone rise if the radius and height of the cone are increased by 200% and 100%, respectively?
- A. 1700%
B. 1800%
C. 2000%
D. 2500%
- 36.** What percentage of 800 is $\frac{2}{3}$ less than 0.2%?
- A. 0.50
B. 0.73
C. 0.93
D. 1
- 37.** Rehan's pay is equal to half of Mariyam's. Rehan's pay is equivalent to 80% of Denver. What is Mariyam's monthly income if the total compensation of the three for a month is Rs. 22000?
- A. 2000
B. 3000
C. 4000
D. 5000
- 38.** By how much is $\frac{2}{3}$ less than 0.2% of 800?
- A. 72%
B. 73%
C. 74%
D. 75%
- 39.** What is the increase in the area of a rectangle if the length and breadth of the rectangle are raised by 20% and decreased by 10%, respectively?
- A. 5%
B. 6%
C. 7%
D. 8%
- 40.** What is the number if 41% of a number is less than 76% of the same number by 105?
- A. 200
B. 300
C. 400
D. 500
- 41.** What is the increase in the area of a square if the sides of a square are increased by 20%?
- A. 33%
B. 44%
C. 55%
D. 66%



- 42.** A total of 4000 people applied for an engineering entrance exam. One-fourth of these were girls. What is the number of successful candidates if 80% of the girls and 95% of the boys did not make it through?
- A. 350
B. 450
C. 550
D. 650
- 43.** From a 6-litre sugar solution containing 4% sugar, 1 litre of water is evaporated. What is the sugar content of the remaining solution?
- A. 4%
B. 4.8%
C. 5%
D. 5.8%
- 44.** A fruit dealer has some bad oranges, about 2% of which are rotten. He sells 95% of the remaining oranges, leaving him with only 49. How many oranges did he start with?
- A. 100
B. 10000
C. 1000
D. 100000
- 45.** There are two papers in an examination. A student receives a 30% grade on the first paper out of 180. How much should he score in the second paper if he has to earn 50% overall and the second paper is worth 150 marks?
- A. 222
B. 333
C. 444
D. 111
- 46.** A fabric dealer claims to sell cloth at cost price, but his measurements are inaccurate.
- His scale reads 1 metre for 95 centimetres. What is his approximate % gain or loss?
- A. 5.25% gain
B. 5% gain
C. 5.50% gain
D. 5.75% gain
- 47.** A 20% increase in bus fares resulted in a 10% decrease in passenger numbers. Despite this, the bus depot's daily collection climbed by Rs. 160. Before the rise, how much money did they gather on a daily basis?
- A. 1000
B. 2000
C. 3000
D. 4000
- 48.** A 25% reduction in the price of rice allows a buyer to get 3 kg more for Rs. 189. What was the original per-kilogram price of rice?
- A. 19 per kg
B. 20 per kg
C. 21 per kg
D. 22 per kg
- 49.** At EPS, 35 boys and 25 girls took part in the annual sports day. 60% of the students obtained certificates. What is the percentage of girls who earned certificates if the total number of guys who received certificates was 12?
- A. 50%
B. 33.33%
C. 25%
D. 66.67%
- 50.** The diameter of a circle is expanded by 15%. What is the area's percentage increase?
- A. 32.25%
B. 32%
C. 32.50%
D. 32.75%



SOLUTIONS

1. (A) Let maximum marks be X

$$\text{Difference} = 15\%$$

$$\text{So, } 15\% \text{ of } X = 45$$

$$\frac{15}{100} \times X = 45$$

$$X = 300$$

Therefore, option A is correct.

2. (A) Let total marks be X

$$\text{So, } \frac{20X}{100} + 30 = \frac{40X}{100} - 10$$

$$\frac{20X}{100} = 40$$

$$X = 200$$

$$\text{Hence, passing marks} = 70$$

Therefore, option A is correct.

3. (A) Let Jerry's salary be Rs. 100

$$\text{So, Tom's salary be Rs. 112}$$

$$112 - \frac{100}{112} \times 100$$

$$= 10.71\%$$

Therefore, option A is correct.

4. (A) $\frac{25}{125} \times 100$

$$= 20\%$$

Therefore, option A is correct.

5. (C) Let Patrick's pay be Rs. 100

$$\text{So, after } 16\% \text{ reduction} = 84$$

$$\text{And after } 16\% \text{ increment} = 84 \times 1.16 = \text{Rs.}$$

$$97.44$$

$$\text{Hence, the difference} = 100 - 97.44$$

$$= \text{Rs. } 2.56$$

Therefore, option C is correct.

6. (C) Let James's starting salary be 100

$$\text{So, after } 20\% \text{ increment it becomes } 120$$

$$\text{and after } 20\% \text{ reduction} = 120 \times 0.80$$

$$= \text{Rs. } 96$$

$$\text{Hence, difference } 100 - 96 = 4$$

Therefore, option C is correct.

7. (C) Commission due = $\frac{5}{100} \times 5000 + \frac{2.5}{100}$

$$\times (30600 - 5000)$$

$$= \text{Rs. } 890$$

Therefore, option C is correct.

8. (C) Voters who voted = $100 - 6 = 94\%$

$$\text{So the losing candidate got} = 94 - 48 = 46\%$$

$$\text{Difference} = 2\%$$

$$\text{Let the total voters be } X$$

$$2\% \text{ of } X = 3000$$

On solving

$$X = 150,000$$

Therefore, option C is correct.

9. (B) Difference = $80 - 13 = 67\%$

$$\text{So, let's assume max marks be } X$$

$$67\% \text{ of } X = 1005$$

$$X = 1500$$

Therefore, option B is correct.

10. (B) Let's assume Marcos have 100 mangoes at the beginning

$$\text{So, the number of rotten mangoes} = 15$$

$$\text{And number of mangoes given to friend} = 85 \times 0.60 = 51$$

$$\text{Remaining mangoes} = 85 - 51 = 34$$

According to question,

$$34X = 102$$

$$\text{So, } 100X = \frac{102}{34} \times 100$$

$$= 300$$

Therefore, option B is correct.

11. (B) Valid votes = $7500 - \frac{20}{100} \times 7500$

$$= 6000$$

$$\text{So, } 55\% \text{ of } 6000 = 3300$$

$$\text{Other candidate got} = 6000 - 3300$$

$$= 2700$$

Therefore, option B is correct.

12. (B) Let his daily wage before increase be X

$$x + \frac{75x}{100} = 75$$

$$x = 42.85$$

Therefore, option B is correct.

13. (D) $\frac{200}{3} \times 66\%$

$$= 44$$

Therefore, option D is correct.



14. (D) Let Pinto's income be 100
So, Albert's income is 125
Hence, it has to be 20% less
Therefore, option D is correct.
15. (D) Let the max marks be X
 $\frac{20X}{100} + 10 = \frac{42X}{100} - 12$
 $X = 100$
Therefore, option D is correct.
16. (D) $A = \frac{30}{100} \times 600 = 180$ votes
 $B = 360$ votes
So, $C = 60$ votes.
Hence, winner B received 200% votes as compared to his rival A
Therefore, option D is correct.
17. (C) Invalid votes = $\frac{15}{100} \times 500000 = 75000$
So, valid votes = 425000
 $A = \frac{60}{100} \times 425000$
= 255000
So, $B = 170000$
Therefore, option C is correct.
18. (B) Fee paid = $\frac{14}{100} \times 1400000$
= 196000
Therefore, option B is correct.
19. (D) Good chairs = $50 - 14 = 36$
Percentage = $\frac{36}{50} \times 100$
= 72%
Therefore, option D is correct.
20. (A) A votes = 65%
B votes = $100 - 65 = 35\%$
Difference = $65 - 35 = 30\%$
So, 30% = 2748
So, 100% = 9160
Therefore, option A is correct.
21. (D) Let total votes be X
Pablo votes = 75%
Kurian votes = 25%
So, 50% of $X = 14000$
 $X = 28000$
Therefore, option D is correct.
22. (B) $\frac{22}{122} \times 100$
= 18.03%
Therefore, option B is correct.
23. (C) Remaining salary after reduction = 65%
45% of the remaining = $65 \times 0.45 = 29.25\%$
Remaining salary in hand = 35.75% which is equal to 2500
So, total salary = 6993
Therefore, option C is correct.
24. (A) Let B's salary be 100
So, A's salary be 125
 $125 - \frac{100}{125} \times 100$
= 20%
Therefore, option A is correct.
25. (B) Difference between votes = $58 - 42 = 16\%$ which is equal to 12000
So, 100% = 75000, i.e., voters voted, i.e., 75%
Hence, total eligible voters = 100000
Therefore, option B is correct.
26. (B) Let's assume Stark's salary be 100
So, rent = 40
Food = 12
Remaining salary = 48 which is equal to 2400
So, Stark's total salary = 5000
Hence, his boss's salary = 6250
Therefore, option B is correct.
27. (A) $\frac{14}{114} \times 100$
= 12.28%
Therefore, option A is correct.
28. (C) Let's take Walter's salary to be 100.
So, his new salary = 110
Percentage smaller as compared with boss
= 19%
Therefore, option C is correct.
29. (D) Let's take salary be 100
After reduction = 60
After increment = 72
After another increment = 86.4
Difference = $100 - 86.4 = 13.6$
Therefore, option D is correct.



- 30. (A)** Sandy got 2000 votes more than Phil
So $35 - 25\% = 2000$
Let total votes be X
So, 10% of $X = 2000$
 $X = 20000$
So, votes received by Robert = 8000
Therefore, option A is correct.

- 31. (C)** Total students = 45000
So, female = 18,000
Then, males = $45000 - 18000 = 27000$
70% of the boys = 18900
75% of the girls = 13500
So, total passed students = $18900 + 13500$
= 32400
And, 32400 is 72% of 45000.
Therefore, option C is correct.

- 32. (C)** Silver obtained = $\frac{0.75}{100} \times \frac{60}{100} \times 8000$
= 36 grams
Therefore, option C is correct.

- 33. (B)** New volume = $\frac{1}{3}\pi \left(\frac{3}{2}R\right)^2 \times \frac{3}{2}H$
= $\frac{9}{8}\pi R^2 H$
Difference = $(\frac{9}{8} - \frac{1}{3})\pi R^2 H$
= $\frac{19}{24}\pi R^2 H$
So, increase in volume = 237.5%
Therefore, option B is correct.

- 34. (A)** Let the number be X
 $\frac{80X}{100} + 80 = X$
 $X = 400$
Therefore, option A is correct.

- 35. (A)** The new radius and height are $3r$ and $2h$ respectively.
So, the new volume will become = $6\pi r^2 h$
Now, increase in volume
 $= 6\pi r^2 h - \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi r^2 h} \times 100$
 $17 \times 100 = 1700\%$
Therefore, option A is correct.

36. (C) 0.2% of 800 = $\frac{8}{5}$
Difference = $\frac{8}{5} - \frac{2}{3}$
= 0.93
Therefore, option C is correct.

- 37. (C)** Let Denver's salary be 100
Rehab's salary = 80
Mariyam's salary = 40
Total = $100 + 80 + 40 = 220 = 22000$
Hence, Mariyam's salary = 4000
Therefore, option C is correct.

38. (D) 0.2% of 800 = $\frac{8}{5}$
Now, difference = $\frac{8}{5} - \frac{2}{3} = \frac{14}{15}$
And, this difference is 75%
Therefore, option D is correct.

39. (D) New area = $1.2L \times 0.9B$
= 1.08LB
Change in area = $\frac{1.08LB}{LB} \times 100$
= 8%
Therefore, option D is correct.

- 40. (B)** Difference = $76 - 41 = 35$
35% of $X = 105$
 $X = 300$
Therefore, option B is correct.

41. (B) Let the original side be X , then area = X^2
Now, new area = $(1.2X)^2 = 1.44X^2$
Percentage change = $1.44 \left(\frac{X}{X}\right)^2 = 1.44$
OR 44%
Therefore, option B is correct.

- 42. (A)** Number of girls who gave the exam = 1000 ($\frac{1}{4}$ of 4000)
Number of boys = $4000 - 1000 = 3000$
Now, as given 80% girls failed then 20% succeeded
Similarly, 95% boys failed then 5% succeeded
So, girls who got through = 20% of 1000 = 200
Boys who got through = 5% of 3000 = 150



Hence, total successful candidates = 200 + 150 = 350
Therefore, option A is correct.

43. (B) Sugar = $\frac{4}{100} \times 6$
= 0.24L
% of sugar in remaining = $\frac{0.24}{5} \times 100$
= 4.8%
Therefore, option B is correct.

44. (C) Let he had X oranges

Now, according to the question, $\frac{2}{100} X + \frac{95}{100} (X - \frac{2}{100} X) + 49 = X$
 $\frac{1}{50} X + \frac{19}{20} X - \frac{19}{1000} X - X = -49$
 $20X + 950X - 19X - 1000X = -49 \times 1000$
 $X = 1000$
Therefore, option C is correct.

45. (D) Total marks = 180 + 150 = 330
50% of 330 = 165
30% of 180 = 54
He should score = 165 - 54 = 111
Therefore, option D is correct.

46. (A) On every 95 cm, the trader gains 5 cm
So, on a trade of Rs. 95 his gain is Rs. 5
So, gain % = $\frac{5}{95} \times 100 = 5.25\%$
Therefore, option A is correct.

47. (B) Let the bus fare be X and number of passenger be Y
So, total collection = XY
Now, after the hike, total collection,
 $\left(X + \frac{20}{100} X\right)\left(Y - \frac{10}{100} Y\right) = 1.08 XY$
Increase in collection = $1.08XY - XY = 0.08XY$
According to the question, $0.08XY = 160$
So, $XY = 2000$
Therefore, option B is correct.

48. (C) New price = $\frac{3}{4}$ Initial price and total amount is constant that is Rs. 189
New quantity = $\frac{4}{3}$ Initial quantity
New quantity is 3 kg more than original quantity. Hence, original quantity was 9 kg
Then, original price = $\frac{189}{9} = \text{Rs. } 21 \text{ per kg.}$
Therefore, option C is correct.

49. (D) Total students participating = 35 + 25 = 60
Total students receiving certificates = $60 \times 0.6 = 36$
Total girls receiving certificates = $36 - 12 = 24$
Percentage of girls receiving certificates = $\frac{24}{36} \times 100 = 66.66\%$
Therefore, option D is correct.

50. (A) $\pi \times r^2 = \pi \times (1.15r)^2 = \pi \times (1.3225)$
Hence, percentage increase = 32.25%
Therefore, option A is correct.

PRACTICE QUESTIONS

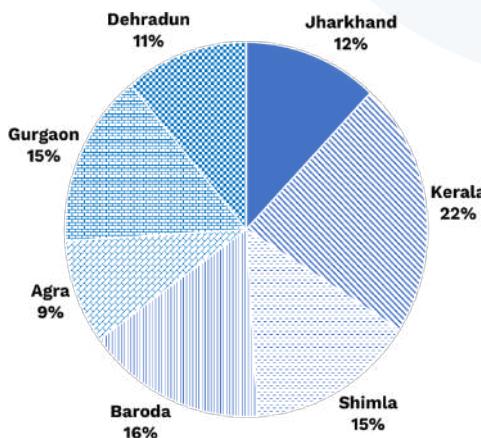
Questions 1–5: Vishesh is living in Delhi and works in a reputed company. The details of the monthly expenditure of Vishesh are given here. He spends 19% of the total monthly expenditure on Rent and 5% of the total monthly expenditure on Transport. He spends 21% of the total monthly expenditure on Food and 15% of the total monthly expenditure on Education. He spends 30% of the total monthly expenditure on other expenses and he saves 10% of his total expenditure.

- If Vishesh is earning Rs. 50,000 per month, then what's his total expenditure on rent, transport and education?
 - 20000
 - 19000
 - 20500
 - 19500
- If Vishesh is earning Rs. 20,000 per month, then how much more does he spend on food than on education and transportation put together?
 - 300
 - 200
 - 400
 - 250



3. In 'others,' Vishesh spends 40% on the accessories of his car which amounts to Rs. 3000. What's his total expenditure on transportation?
- 1050
 - 1200
 - 1200
 - 1250
4. If a pie chart is drawn by taking the savings as one part and total expenses as the second part, then what is the angle made by total expenses?
- 330°
 - 280°
 - 324°
 - 320°
5. Due to an increase in the price of fuel, Vishesh has to spend Rs. 300 more on transportation. This is 20% more than usual. If other expenses remain constant, find his new savings.
- 2700
 - 2600
 - 2500
 - 2400

Questions 6–10: The following pie gives the data of the total number of passengers travelling from Delhi to different districts = 12000



6. The number of passengers travelling from Delhi to Baroda is what percentage more than the number of passengers travelling to Jharkhand?
- 20%
 - 33%
 - $33 \frac{1}{3}\%$
 - 30%
7. The number of passengers travelling from Delhi to Gurgaon is what percentage of the total number of passengers travelling from Delhi to Dehradun and Agra together?
- 70%
 - 75%
 - 72%
 - 68%
8. What is the average number of passengers travelling from Delhi to Shimla, Kerala and Baroda together?
- 2120
 - 2020
 - 2000
 - 2100
9. What will be the central angle for passengers travelling from Delhi to Baroda if it is put in the form of a pie-chart?
- 45%
 - 57%
 - 57.6%
 - 50%
10. Among the passengers who are travelling from Delhi to Kerala, 42% are women and 38% are men. What is the number of children?
- 428
 - 528
 - 280
 - 410

SOLUTIONS

1. **(D)** Total percentage of salary spent on rent, transport and education = $\{19 + 5 + 15\} = 39\%$

Amount spent on rent, transport and education = 39% of 50,000 = Rs. 19500
Therefore, option D is correct.
Therefore, option B is correct.



2. (B) The total percentage of salary spent on education and transportation together is $(15 + 5) = 20\%$
His expenditure is 21% on food.
Vishesh spends $21 - 20 = 1\%$ more on food.
Amount = 1% of 20,000 = Rs. 200
3. (D) Expenditure of Vishesh on car = 40% of 30% = 12%
Given 12% = 3000
(Transportation) 5% = x
 $x = (5 \times \frac{3000}{12}) = \text{Rs. } 1250$
Therefore, option D is correct.
4. (C) Vishesh spends 90% of his income
Angle made by this division = $\frac{90 \times 360}{100} = 324^\circ$
Therefore, option C is correct.
5. (A) Vishesh spends 5% on transportation
His new expenditure on transportation = 120% of 5% = 6%
Increase in expenditure = 1%
His new savings = $10\% - 1\% = 9\%$
= 1% = 300
9% = x
 $x = 9(300) = \text{Rs. } 2700$
Therefore, option A is correct.
6. (C) The number of passengers from Delhi to Baroda = 16%
Number of passengers from Delhi to Jharkhand = 12%

$$\begin{aligned}\text{Required \%} &= \frac{16 - 12}{12} \times 100\% = \frac{4 \times 100}{12} \\ &= 33 \frac{1}{3}\%\end{aligned}$$

Therefore, option C is correct.

7. (B) Number of passengers travelling from Delhi to Gurgaon = 15%
Number of passengers travelling from Delhi to Dehradun and Agra = $11\% + 9\% = 20\%$
Required % = $\frac{15}{20} \times 100 = 75\%$
Therefore, option B is correct.
8. (A) Total percentage of Shimla, Kerala and Baroda = $(15\% + 22\% + 16\%) = 53\%$
Therefore, 53% of 12000
= $(50\% \text{ of } 12000 + 3\% \text{ of } 12000) = 6000 + 360 = 6360$
Required average = $\frac{6360}{3} = 2120$
Therefore, option A is correct.
9. (C) Central angle = $\frac{16}{100} \times 360^\circ = 57.6^\circ$
Therefore, option C is correct.
10. (B) Number of children travelling from Delhi to Kerala = $100\% - (42 + 38)\% = 100\% - 80\% = 20\%$
Total number of children travelling from Delhi to Kerala = $20\% \text{ of } 22\% \text{ of } 12000$
= $\frac{20}{100} \times \frac{22}{100} \times 12000 = 528$
Therefore, option B is correct.



SYNOPSIS

- Introduction
- Average
- Properties of average
- Weighted average or grouped average
- Common types of questions asked
- Summary

INTRODUCTION

The word average is quite often used in our day-to-day life. The average salary in a company, the average score of a team, the average marks of a class, and many other similar phrases are some examples of average that we encounter in our daily routine.

Questions based on the concepts of average are being frequently asked in GATE, JEE Main, and other engineering entrance examinations. This topic is easy to understand, which makes it easy for the test takers to score more in these exams.

Average

The average is a mean or middle value of a set of data, which can be calculated by dividing the total sum of all values by the total number of values in the given data.

The formula for calculating average:

$$\text{Average} = \frac{\text{Total sum of values}}{\text{Number of values}}$$

Example:

If A, B, C, and D are the partners in a firm, the firm gains a profit of Rs. 1 lakh, which is to be divided equally among the partners. Then the average profit gained by a partner can be calculated as:

Solution:

The average profit of the partner is given by

$$\text{Average} = \frac{\text{Total sum of values}}{\text{Number of values}}$$

$$\text{Average profit} = \frac{1,00,000}{4} = \text{Rs. } 25,000$$

Therefore, each partner would get an average profit of Rs. 25,000.

Properties of average

1. In a given set of data, the average always lies between its maximum and minimum value.
2. If a constant is added or subtracted to each value in the data, then the average also increases or decreases respectively by the same constant.
3. If each value in the given set of data is multiplied or divided by a constant, then the average also gets multiplied or divided by the same constant.

Weighted average or grouped average

When the average of two or more groups with the same or the different number of values is given, and we have to find the average of the values of all the groups taken together, then the combined average of the groups is known as the weighted average or grouped average. If there are two groups with 'A1' and 'A2' as the number of elements or values in them, let 'P1' and 'P2' be their averages, respectively. Then the combined average of these two groups taken together can be calculated by: weighted

$$\text{average} = \frac{A1P1 + A2P2}{A1 + A2}$$



Let us assume if there are 'n' groups with 'A₁, A₂, A₃, A₄, A₅, ... A_n' as the number of elements or values in these groups and 'P₁, P₂, P₃, P₄, P₅, ... P_n' be the averages of these groups, respectively. Then the combined average of all these groups taken together can be calculated by:

weighted average

$$= \frac{A_1P_1+A_2P_2+A_3P_3+A_4P_4+A_5P_5\dots+A_nP_n}{P_1+P_2+P_3+P_4+P_5\dots+P_n}$$

Example:

The XYZ International School has two divisions in standard A, P and Q. The number of students in divisions P and Q is 42 and 35, respectively. The average weight of students in division P is 63 and that of students in division Q is 52. What is the average weight of standard A of XYZ International School?

Solution:

The average weight of the two divisions taken together is given by

$$\text{Weighted average} = \frac{A_1P_1+A_2P_2}{A_1+A_2}$$

$$\text{Weighted average} = \frac{42 \times 63 + 36 \times 52}{35 + 42}$$

Therefore, the average weight of standard A is 58.

Common types of questions asked

Case I

When a person is replaced by another person in a group, for example in cases of age, weight, height, etc. related problems.

1. If the average is increased:

The following formula will apply:

New person's weight = weight of person who left + (increase in the average × total number of people)

Example:

The average weight of 8 persons is increased by 2.5 kg when one of them whose weight is 56 is replaced by a new man, the weight of the new man is:

Solution:

The new person's weight, if the average is increased, is given by

New person's weight = weight of person who left + (increase in the average × total number of people)

$$\text{New person's weight} = 56 + (2.5 \times 8) = 76 \text{ kg}$$

2. If the average is decreased:

The following formula will apply:

New person's weight = weight of person who left + (decrease in the average × total number of people)

Example:

The average weight of 10 persons is decreased by 2 kg when one of them whose weight is 50 is replaced by a new man; the weight of the new man is:

Solution:

The new person's weight if the average is decreased is given by

New person's weight = weight of person who left + (decrease in the average × total number of people)

$$\text{New man's weight} = 50 + (-2 \times 10) = 30 \text{ kg}$$

Case II

When a person joins a group then there may be two cases:

1. If there is an increase in average, the following formula will apply:

New person's weight = earlier average + (increase in the average × total number of people)

Example:

The average age of 10 students is 12 years. By the admission of a new student, the average age is increased to 13 years. The age of the new student is

Solution:

The average of new person's weight if there is an increase in average

New person's weight = earlier average + (increase in the average × total number of people)



Age of a new student is = $12 + (1 \times 11) = 23$ years

- 2.** If there is an increase in average, the following formula will apply:

New person's weight = earlier average - (decrease in the average × total number of people)

Example:

The average weight of 19 students is 15 kg. By the admission of a new student, the average weight is reduced to 14.8 kg. The weight of the new student is

Solution:

The average of new person's weight if there is a decrease in average

New person's weight = earlier average - (decrease in the average × total number of people)

$$\text{New student weight} = 15 - (0.2 \times 20) = 11 \text{ kg}$$

Case III

For calculating average speed in speed, time, and distance, the following formula will apply for calculating average speed:

$$\text{Average speed} = \frac{2xy}{x+y} \quad (\text{wherein, } x \text{ and } y \text{ denote}$$

for the different average speeds)

Example:

A covers a journey from Delhi to Jaipur by car at an average speed of 40 km/h and returns by scooter with an average speed of 24 km/h. What is his average speed during the whole journey?

Solution:

The average speed is given by

$$\text{Average speed} = \frac{2xy}{x+y}$$

$$\text{Average speed} = \frac{2(40 \times 24)}{40+24} = \frac{1920}{64} = 30 \text{ km/h}$$

Case IV

Average when arithmetic progression (AP) series is odd:

The average of the odd AP series will be the term that is in the middle of the series.

Example:

The average for series 3, 6, 9, 12, 15, 18, 21, 24, and 27 is:

Solution:

The middle term of the series is 15. Therefore, its average will be 15.

Case V

Average when AP series is even:

If the AP series is even, then the average can be calculated from the average of two middle terms.

Example:

The average for series 2, 4, 6, 8, 10, and 12 is:

Solution:

The two middle terms of the series are 6 and 8.

By calculating the average of both terms,

$$\text{i.e., } \frac{6+8}{2} = 7$$

Therefore, the average of the given series is 7.

Chapter Summary



- Average is an arithmetic mean of a given set of data.
- Average also changes accordingly if each value in the given set of data multiplies or divides by a constant.
- The average of two or more groups can be calculated by the weighted average.



PRACTICE QUESTIONS

1. The population of 5 cities is 35560, 27500, 30000, 25000, and 25600 people. What is the average population of a city?
 - A. 28742
 - B. 28562
 - C. 28732
 - D. 28730
2. The average weight of 48 employees is 70 kg. If the weight of two new employees is included, the average increases by 800 grams. What is the average weight of new employees?
 - A. 55 kg
 - B. 50 kg
 - C. 45 kg
 - D. 40 kg
3. The average of the first nine multiples of 5 is:
 - A. 23
 - B. 24
 - C. 25
 - D. 26
4. The average weight of 8 workers in a truck is increased by 1 kg when one of the workers, whose weight is 60 kg, is replaced by a new worker. What is the weight of the new worker?
 - A. 58 kg
 - B. 68 kg
 - C. 70 kg
 - D. 64 kg
5. Raj travels from Delhi to Gurgaon by train at an average speed of 80 km/h and returns by car with an average speed of 48 km/h. What is his average speed during the whole journey?
 - A. 60 km/h
 - B. 58 km/h
 - C. 54 km/h
 - D. 62 km/h
6. The average of 0.60204, 0.60203, 0.60202, and 0.60201 is:
 - A. 0.60203
 - B. 0.60204
 - C. 0.602025
 - D. 0.60201
7. The average mark of 40 students in a class is 60, and the average mark of 60 students in a different class is 40. What is the average of all the tests?
 - A. 48
 - B. 50
 - C. 46
 - D. 40
8. The average weight of 30 boys in a class of 40 is 50, and the average of remaining boys is 46. What is the average weight of the whole class?
 - A. 48
 - B. 50
 - C. 49
 - D. 40
9. The average of 40 numbers is 25. If two numbers 24 and 26 are removed, then the average of the remaining numbers will be:
 - A. 27
 - B. 24
 - C. 20
 - D. 25
10. The average number of persons crossing a toll plaza on the first four days of a week is 1163 and that on the last three days is 1660. What is the daily average for the whole week?
 - A. 1370
 - B. 1350
 - C. 1376
 - D. 1440
11. The average rainfall of the first three days of a week is 1 inch, and that of the next



- four days is 0.3 inch. What is the average rainfall for the whole week?
- A. 0.7 inch
 - B. 0.6 inch
 - C. 0.4 inch
 - D. 0.5 inch
- 12.** The average collection of a shop in the first three days is Rs. 2700, and that of the next three is Rs. 2900. If the average collection of the whole week is Rs. 2850, then what is the collection on the last day of the week?
- A. Rs. 3150
 - B. Rs. 3050
 - C. Rs. 2950
 - D. Rs. 3000
- 13.** The average of 3 numbers is 63. The first is twice the second and four times the third. Find the second number.
- A. 48
 - B. 54
 - C. 56
 - D. 50
- 14.** The average weight of 19 bags is 30 kg, and by adding one more bag, the average weight is reduced to 29.6 kg. What is the weight of the new bag?
- A. 28 kg
 - B. 20 kg
 - C. 26 kg
 - D. 22 kg
- 15.** The average weight of 8 students in a class is increased by 2.5 kg when one of them whose weight is 28 kg is replaced by the new student, what is the weight of the new student?
- A. 48
 - B. 50
 - C. 46
 - D. 40
- 16.** The average weight of 40 members in a joint family is 40 kg. If the weight of grandfather is included, the average weight increases by 500 grams. What is the weight of the grandfather?
- A. 58 kg
 - B. 60.5 kg
 - C. 60 kg
 - D. 55.5 kg
- 17.** The average mark of four siblings is 12. If the gap between their marks obtained is 4, then what are the least marks amongst all?
- A. 8
 - B. 5
 - C. 6
 - D. 4
- 18.** The average daily wage of 20 workers in a factory is Rs. 190 per day. If the daily wage of a new worker is added, the average daily wage becomes Rs. 200 per day. What is the daily wage of a newly added worker?
- A. 480
 - B. 500
 - C. 460
 - D. 400
- 19.** The average age of P, Q, R, and S ten years ago was 40 years. By including A, the present average age of all is 55 years. Find the present age of A.
- A. 75 years
 - B. 70 years
 - C. 76 years
 - D. 74 years
- 20.** A street vendor earned Rs. 1008 in 12 days. His average earnings for the first four days were Rs. 80 a day. What is his average earning for the remaining days?
- A. Rs. 88
 - B. Rs. 80
 - C. Rs. 86
 - D. Rs. 74



- 21.** The average height of 24 pillars in a class is 10 feet. If the height of the new pillar is included, the average height increases by 1 foot. The height of the new pillar is:
- A. 38
 - B. 35
 - C. 36
 - D. 40
- 22.** The average income of 4 employees in the office is Rs. 7350. One of the employees left the office, and therefore the average income was reduced to Rs. 6500. What is the income of the person who left the job?
- A. 9900
 - B. 9000
 - C. 9600
 - D. 9400
- 23.** Find the average of the given series: 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40.
- A. 24
 - B. 25
 - C. 26
 - D. 22
- 24.** The average weight of 12 students in a class is 20 kg. If the teacher's weight is included, the average weight increases by 2 kg. What is the weight of the teacher?
- A. 48 kg
 - B. 44 kg
 - C. 46 kg
 - D. 40 kg
- 25.** The average height of 5 persons is 1.2 m. If the height of a new person is included, the average height increases by 100 cm. What is the height of the newly added person?
- A. 1.8 m
 - B. 1.4 m
 - C. 1.6 m
 - D. 1.4 m
- 26.** Find the average of the first six multiples of 9.
- A. 28
 - B. 31.5
 - C. 29
 - D. 30
- 27.** Arpit travelled from Kolkata to Jaipur by car at an average speed of 60 km/h. He returned to Kolkata by motorcycle at an average speed of 40 km/h. What is the average speed of the whole journey?
- A. 48
 - B. 50
 - C. 46
 - D. 40
- 28.** The average of 40 values is 20, and the average of 20 different values is 40. What is the average of all the values?
- A. 22
 - B. 24
 - C. 26
 - D. 20
- 29.** The average score of the first six batsmen is 15, and that of the next three batsmen is 18. Find the average score of nine batsmen.
- A. 14
 - B. 15
 - C. 16
 - D. 14
- 30.** The average daily expenditure on four vehicles is Rs. 100. If one of the vehicles gets sold, the average daily expenditure becomes Rs. 90. What is the daily expenditure on the vehicle which gets sold?
- A. 130
 - B. 160
 - C. 150
 - D. 140



SOLUTIONS

1. (D) Average

$$= \frac{35560 + 27500 + 30000 + 25000 + 25600}{5}$$

$$= \frac{143660}{5}$$

$$= 28732$$

Therefore, the correct option is D.

2. (A) Total weight of 48 employees

$$= 48 \times 70 = 3430 \text{ kg}$$

Total weight of all the employees including 2 new employees = $50 \times 70.8 = 3540 \text{ kg}$

Average weight of new employees

$$= (3540 - 3430) \text{ kg} = 110$$

$$= \frac{110}{2} = 55 \text{ kg}$$

The average weight of new employees is 55 kg.

Therefore, option A is correct.

3. (C) The first nine multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, and 45

The given numbers or multiples of 5 form an AP series and the middle term of the series is 25, therefore the average of the first nine multiples of 5 is 25.

Therefore, option C is correct.

4. (B) Weight of new crew member = Weight of person who left + (Increase in the average \times Total number of people)

$$\text{Weight of new worker} = 60 + (1 \times 8) = 68 \text{ kg}$$

The weight of the new worker is 68 kg.

Therefore, option B is correct.

5. (A) The average speed of the whole journey

$$= \frac{2xy}{x+y}$$

$$= \frac{2(80 \times 48)}{80+48}$$

$$= \frac{7680}{128} = 60 \text{ km/h}$$

The average speed of the whole journey is 60 km/h.
Therefore, option A is correct.

6. (C) Average

$$= \frac{0.60204 + 0.60203 + 0.60202 + 0.60201}{4}$$

$$= \frac{2.4081}{4} = 0.602025$$

The average of given four values is 0.60205.
Therefore, option C is correct.

7. (A) Average marks of all the students can

$$\text{be calculated by} = \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(40 \times 60) + (60 \times 40)}{60 + 40}$$

$$= \frac{4800}{100} = 48$$

The average marks of all the students are 48.

Therefore, option A is correct.

8. (C) Average weight of whole class =

$$\frac{A1P1 + A2P2}{A1 + A2} = \frac{(30 \times 50) + (46 \times 10)}{30 + 10}$$

$$= \frac{1960}{40} = 49 \text{ kg}$$

The average weight of the whole class is 49 kg.

Therefore, option C is correct.

9. (D) Total sum of all numbers = $40 \times 25 = 1000$

Total sum of remaining numbers
 $= 1000 - (24 + 26) = 950$

$$\text{Average of remaining numbers} = \frac{950}{38} = 25$$

The average of remaining numbers will be 25.



Therefore, option D is correct.

- 10. (C)** Daily average for whole week

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(1163 \times 4) + (1660 \times 3)}{4 + 3}$$

$$= \frac{(4652 + 4980)}{7} = \frac{9632}{7}$$

$$= 1376$$

 The daily average for the whole week is 1376.
 Therefore, option C is correct.
- 11. (B)** Average rainfall of whole week

$$= \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(1 \times 3) + (0.3 \times 4)}{4 + 3}$$

$$= \frac{4.2}{7} = 0.6 \text{ inch}$$

 The average rainfall of the whole week is 0.6 inch.
 Therefore, option B is correct.
- 12. (A)** Total collection of first three days

$$= 2700 \times 3 = 8100$$

 Total collection of next three days

$$= 2900 \times 3 = 8700$$

 Total collection of first six days

$$= 8100 + 8700 = 16800$$

 Collection of the last day = Total collection in a week – Total collection in first six days
 Collection of the last day = $(2850 \times 7) - 16800$

$$= \text{Rs. } 3150.$$

 The collection of the last day of a shop is Rs. 3150.
 Therefore, option A is correct.
- 13. (B)** If the third number is x , the first number is $4x$ and the second is $2x$.
 The total sum of the numbers is $= 63 \times 3 = 189$

$$= x + 2x + 4x = 7x = 189$$

$$x = \frac{189}{7} = 27$$

 The numbers are

$$= (27 \times 4), (27 \times 2), 27 = 108, 54, 27$$

 Therefore, option B is correct.

- 14. (D)** The weight of new bag = Earlier average – (Decrease in average × Total number of bags)
 Weight of new bag = $30 - (0.4 \times 20)$

$$= 30 - 8 = 22 \text{ kg}$$

 The weight of the new bag is 22 kg.
 Therefore, option D is correct.
- 15. (A)** Weight of new student = Weight of student who left + (Increase in average × Total number of students)
 Weight of new student = $28 + (2.5 \times 8)$

$$= 48 \text{ kg}$$

 The weight of the new student is 48 kg.
 Therefore, option A is correct.
- 16. (B)** Weight of grandfather = Earlier average + (Increase in average × Total number of people)
 The average weight of joint family = 40 kg
 Weight of grandfather = $40 + (0.5 \times 41)$

$$= 60.5 \text{ kg}$$

 The weight of grandfather is 60.5 kg.
 Therefore, option B is correct.
- 17. (C)** Let the minimum marks obtained be x
 Total marks = $12 \times 4 = 48$
 Then from the question,

$$x + (x + 4) + (x + 4 + 4) + (x + 4 + 4 + 4) = 48$$

 So, $4x + 24 = 48, x = 6$
 The least mark obtained is 6.
 Therefore, option C is correct.
- 18. (D)** The daily wage of newly added worker = Earlier average + (Increase in average × Total number of people)

$$= 190 + (10 \times 21)$$

$$= \text{Rs. } 400$$

 The daily wage of a newly added worker is Rs. 400 per day.
 Therefore, option D is correct.
- 19. (A)** The total sum of ages of P, Q, R, and S ten years ago = $40 \times 4 = 160$
 Total sum of ages of P, Q, R, and S at present = $160 + 40 = 200$
 Total sum of ages of P, Q, R, S, and A = $55 \times 5 = 275$



Present age of A = $275 - 200 = 75$ years
 The present age of A is 75 years.
 Therefore, option A is correct.

- 20. (C)** Total earnings = 1008
 Total earnings in first four days = $80 \times 4 = 320$
 Total earnings for remaining days
 $= 1008 - 320 = 688$
 Average for remaining days = $\frac{688}{8} = \text{Rs. } 86$
 The average earning of the remaining days is Rs. 86.
 Therefore, option C is correct.

- 21. (B)** Height of new pillar = Earlier average + (Increase in average \times Total number of pillars)
 $= 10 + (1 \times 25)$
 $= 35$ feet
 The height of the new pillar is 35 feet.
 Therefore, option B is correct.

- 22. (A)** Total income of 4 employees
 $= 7350 \times 4 = 29400$
 The total income of 3 employees
 $= 6500 \times 3 = 19500$
 Income of the person who left
 $= 29400 - 19500 = \text{Rs. } 9900$
 The income of the person who left is Rs. 9900.
 Therefore, option A is correct.

- 23. (D)** The two middle terms of the given series are 20 and 24
 Average of two middle terms
 $= \frac{20 + 24}{2} = 22$
 The average of the given series is 22.
 Therefore, option D is correct.

- 24. (B)** Weight of teacher = earlier average + (increase in average \times total number of people)
 $= 20 + (2 \times 12)$
 $= 44$ kg
 The weight of the teacher is 44 kg.
 Therefore, option B is correct.

25. (A) Height of new person = earlier average + (increase in average \times total number of people)
 $= 1.2 + (0.1 \times 6)$

$= 1.8$ m
 The height of the newly added person is 1.8 m.
 Therefore, option A is correct.

- 26. (B)** The first six multiples of 9 are: 9, 18, 27, 36, 45, and 54.
 The middle terms of the formed series are 27 and 36.
 The average of the series is $= \frac{27 + 36}{2}$
 $= 31.5$
 The average of the first six multiples of nine is 31.5.
 Therefore, option B is correct.

- 27. (A)** The average speed of the whole journey
 $= \frac{2xy}{x + y}$
 $= \frac{2(60 \times 40)}{60 + 40} = \frac{4800}{100}$
 $= 48$ km/h
 The average speed of the whole journey is 48 km/h.
 Therefore, option A is correct.

- 28. (D)** Grouped average of all values
 $= \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$
 $= \frac{(40 \times 20) + (20 \times 40)}{20 + 40}$
 $= 20$
 The average of all the values is 20.
 Therefore, option D is correct.

- 29. (C)** Average score of nine batsman
 $= \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$
 $= \frac{(15 \times 6) + (18 \times 3)}{6 + 3} = \frac{144}{9}$
 $= 16$ runs
 The average score of nine batsmen is 16 runs.
 Therefore, option C is correct.

- 30. (A)** Total expenditure on four vehicles = $100 \times 4 = \text{Rs. } 400$



Total daily expenditure after one of the vehicles gets sold = $90 \times 3 = \text{Rs. } 270$

Daily expenditure of vehicle which gets sold = $400 - 270 = \text{Rs. } 130$

The daily expenditure of a vehicle that gets sold is Rs. 130.

Therefore, option A is correct.

PRACTICE QUESTIONS

Questions 1–5: The following table shows the data of sales of cars of different companies from 2015 to 2020.

COMPANY	2015	2016	2017	2018	2019	2020
A	440	480	470	500	520	510
B	400	410	415	415	420	430
C	380	390	390	400	420	495
D	360	380	400	415	440	500
E	480	440	440	420	425	435

1. What is the average sale of cars in company C from the year 2016–2020?
 A. 420
 B. 419
 C. 425
 D. 415
 2. In which year the average sale of cars is the highest?
 A. 2020
 B. 2019
 C. 2015
 D. 2018
 3. If the average sale of cars of company D in 2015–2017 is 320 and that of 2018–2020 is 340. Find the average sale of cars of company D from 2015 to 2021.
 A. 320
 B. 319
 C. 335
 D. 330
 4. What is the average sale of cars in company B from 2016 to 2019?
 A. 419
 B. 420
 C. 425
 D. 415
 5. What is the average sale of cars in company A from 2015 to 2019?
 A. 470
 B. 480
 C. 482
 D. 450
- Questions 6–10:** There are five sections in the 12th standard which are A, B, C, and D. The number of students in section A is 40 and the average weight of the class is 45 kg. The total sum of weights of 30 students of section C is 1200 kg. The total number of students in section B is 20. The average weight of 35 students of section D is 30 kg. The average weight of section B is 30 kg.
6. If the weight of the teacher is added, the average weight is 46 kg. Find the weight of the teacher.



- A. 86 kg
B. 79 kg
C. 75 kg
D. 85 kg
7. Find the average weight of students in section C.
A. 42 kg
B. 41 kg
C. 40 kg
D. 38 kg
8. If a student weighing 25 kg is replaced by a new student, the average weight of section D increases by 500 grams. What is the weight of the new student?
A. 42.5 kg
B. 42 kg
- C. 41 kg
D. 40.5 kg
9. What is the average weight of the 12th standard?
A. 37
B. 37.2
C. 37.5
D. 35
10. What will be the average weight of section B, if a student weighs 30 kg removed from the class?
A. 30
B. 31
C. 32
D. 31.5

Questions 11–15: The following table represents the data of marks obtained by students in different subjects.

NAME OF STUDENTS	ENGLISH	HINDI	MATHS	SCIENCE
P	30	32	38	40
Q	36	32	30	36
R	42	36	34	30
S	40	40	42	26

11. What are the average marks obtained by P?
A. 37
B. 36
C. 37
D. 35
12. What are the average marks obtained by students in science subjects?
A. 32
B. 31
C. 33
D. 34
13. What is the combined average of marks obtained by students in Hindi and Maths?
A. R
B. S
C. Q
D. P
14. What are the average marks obtained by students in English and Science?
A. 33
B. 35
C. 34
D. 33
15. Which student has scored the highest average marks?
A. R
B. S
C. Q
D. P



SOLUTIONS

1. (B) Average sale of C from 2016 to 2020

$$= \frac{390 + 390 + 400 + 420 + 495}{5}$$
$$= \frac{2095}{5} = 419$$

The average sale of company C from 2016 to 2020 is 419.

Therefore, option B is correct.

2. (A) Average sales for year 2015

$$= \frac{440 + 400 + 380 + 360 + 480}{5}$$
$$= \frac{2060}{5} = 412$$

Average sales for year 2016

$$= \frac{480 + 410 + 390 + 380 + 440}{5}$$
$$= \frac{2100}{5} = 420$$

Average sales for year 2017

$$= \frac{470 + 415 + 390 + 400 + 440}{5}$$
$$= \frac{2115}{5} = 423$$

Average sale for year 2018

$$= \frac{500 + 415 + 400 + 415 + 420}{5}$$
$$= \frac{2150}{5} = 430$$

Average sale for year 2019

$$= \frac{520 + 420 + 420 + 440 + 425}{5}$$
$$= \frac{2225}{5} = 445$$

Average sale for year 2020

$$= \frac{510 + 430 + 495 + 500 + 435}{5}$$
$$= \frac{2370}{5} = 474$$

The highest average sale of cars was in 2020.

Therefore, option A is correct.

3. (D) Average sale of company D from 2015 to 2020

$$= \frac{A1P1 + A2P2}{A1 + A2}$$
$$= \frac{(320 \times 3) + (340 \times 3)}{3 + 3} = \frac{1980}{6}$$

= 330

The average sale of company D from 2015 to 2020 is 330.

Therefore, option D is correct.

4. (D) The average sale of cars in B from 2016

$$to 2019 = \frac{410 + 415 + 415 + 420}{4}$$
$$= \frac{1660}{4} = 415$$

The average sale of cars in company B from 2016 to 2019 is 415.

Therefore, option D is correct.

5. (C) Average sale of cars in A

$$= \frac{440 + 480 + 470 + 500 + 520}{5}$$
$$= \frac{2410}{5} = 482$$

The average sale of cars in company A from 2015 to 2019 is 482.

Therefore, option C is correct.

6. (A) Weight of teacher = earlier average + (increase in average × total number of persons)

$$= 45 + (1 \times 41)$$

$$= 86 \text{ kg}$$

The weight of the teacher is 86 kg.

Therefore, option A is correct.

7. (C) The average weight of students in section C

$$= \frac{1200}{30}$$

$$= 40 \text{ kg}$$

The average weight of students in section C is 40 kg.

Therefore, option C is correct.



8. (A) Weight of new student = weight of person who left + (increase in average × total number of people)
 $= 25 + (0.5 \times 35)$
 $= 25 + 17.5 = 42.5 \text{ kg}$

The weight of the new student is 42.5 kg.
 Therefore, option A is correct.

9. (B) Average weight of 12th standard
 $= \frac{A_1P_1 + A_2P_2 + A_3P_3 + A_4P_4 + A_5P_5 + \dots + A_nP_n}{P_1 + P_2 + P_3 + P_4 + P_5 + \dots + P_n}$
 $= \frac{(40 \times 45) + 1200 + (35 \times 30) + (20 \times 30)}{40 + 30 + 35 + 20}$
 $= \frac{4650}{125} = 37.2 \text{ kg}$

The total average of the 12th standard is 37.2 kg.
 Therefore, option B is correct.

10. (A) Total sum of weights of section B
 $= 20 \times 30 = 600 \text{ kg}$

Total weight after removal of student
 $= 600 - 30 = 570 \text{ kg}$

$$\text{New average weight} = \frac{570}{19} = 30 \text{ kg}$$

The average of section B will be the same,
 i.e., 30 kg.
 Therefore, option A is correct.

11. (D) Average marks of P = $\frac{30 + 32 + 38 + 40}{4}$
 $= \frac{140}{4} = 35$

The average marks obtained by P are 35.
 Therefore, option D is correct.

12. (C) Average marks in science

$$= \frac{40 + 36 + 30 + 26}{4}$$

 $= \frac{132}{4} = 33$

The average marks of students in science subjects are 33.
 Therefore, option C is correct.

13. (D) Average marks in Hindi

$$= \frac{32 + 32 + 36 + 40}{4}$$

$$\text{Average marks in Hindi} = \frac{140}{4} = 35$$

$$\text{Average marks in Maths} = \frac{38 + 30 + 34 + 42}{4}$$

$$= \frac{144}{4} = 36$$

$$\text{Combined average} = \frac{35 + 36}{2} = 35.5$$

The combined average is 35.5.
 Therefore, option D is correct.

14. (B) Average marks in English

$$= \frac{30 + 36 + 42 + 40}{4}$$

$$= \frac{148}{4} = 37$$

$$\text{Average marks in Science} = \frac{40 + 36 + 30 + 26}{4}$$

$$= \frac{132}{4} = 33$$

$$\text{Combined average} = \frac{37 + 33}{2} = 35$$

Average marks obtained by students in English and Science is 35, therefore, option B is correct.

15. (B) Average marks of P = $\frac{30 + 32 + 38 + 40}{4}$

$$= \frac{140}{4} = 35$$

$$\text{Average marks of Q} = \frac{36 + 32 + 30 + 36}{4}$$

$$= \frac{144}{4} = 36$$

$$\text{Average marks of R} = \frac{42 + 36 + 34 + 30}{4}$$

$$= \frac{142}{4} = 35.5$$

$$\text{Average marks of S} = \frac{40 + 40 + 42 + 26}{4}$$

$$= \frac{148}{4} = 37$$

S has scored the highest average marks.
 Therefore, option B is correct.

32 Ratio and Proportion



SYNOPSIS

- Introduction
- Ratio
- Calculating ratio
- Important concepts relating to ratio
- Properties of ratio
- Tips and tricks
- Proportion
- Continued proportion
- Third proportion
- Fourth proportion
- Direct proportion
- Indirect proportion
- Points to remember
- Summary

INTRODUCTION

This topic covers concepts related to ratio and proportion, and it is considered as a very important topic in Mathematics. In daily life also, the concept of ratio and proportion is frequently used. Questions related to ratio and proportion are often asked in GATE, JEE Main, and other engineering entrance examinations; therefore, test takers must be cleared with the concepts related to this topic.

Ratio

Basically, a ratio is a comparison of two quantities or numbers. It refers to the relationship between the two quantities and represents how many times one quantity is equal to the other quantity. Ratios are represented by the symbol of colon, i.e. (:). The ratio of a to b is written as,

$$a : b = \frac{a}{b} = a \div b$$

In the ratio $a : b$, a and b are called the terms of the ratio, wherein ' a ' is the first term or known

as an antecedent, and ' b ' is the second term or known as consequent.

Example:

In the ratio $3 : 4$, 3 is the first term or antecedent, and 4 is the second term or consequent.

Calculating ratio

For calculating the ratio of two quantities of the same units, we can use the steps quoted hereunder.

Example:

If 30 kg of soil and 40 kg of cement are used to make a wall, let us calculate the ratio of soil and cement used to make a wall.

Solution:

Step 1: First find the quantities for which we have to determine the ratio, and in this case, it is 30 and 40 .

Step 2: Write both the quantities in the fraction form, such as $\frac{30}{40}$.

Step 3: Thereafter, simplify the fraction, if possible. The simplified fraction will give the final ratio. Here, $\frac{30}{40}$ can be simplified into $\frac{3}{4}$.



Step 4: Therefore, the ratio of soil to cement is $\frac{3}{4}$.

Important concepts relating to ratio

- To find the ratio of any two quantities, the quantities must be expressed in the same units.

Example:

4 m : 200 cm, therefore, for finding its ratio, both the units must be converted into the same. By converting any of the two units, we can find its ratio. $4 \text{ m} = 400 \text{ cm}$; therefore the ratio will be $400 \text{ cm} : 200 \text{ cm} = 2 : 1$

- The ratio would remain same if the antecedent and the consequent in a ratio are multiplied or divided by a constant.

Example:

$$a : b = \frac{a}{b} = \frac{a \times y}{b \times y} = \frac{\underline{a}}{\underline{y}}, \text{ wherein } y = \text{constant}$$

- If two different ratios are expressed in different units, then to find out the combined ratio of the two ratios, we have to compound the ratios. This is known as compounding.

The compound of $a : b$ and $p : q$ is $\frac{a \times p}{b \times q}$

Example:

To complete a task, 1 man works for 8 hours a day for 10 days, and 1 boy works for 12 hours a day for 20 days. How many boys can complete the same task that 1 man can do?

Solution:

To know how many boys are required to complete the same task that 1 man can do, we need to combine two different units, i.e., hours and days.

The ratio of days is $10 : 20$, the ratio of hours is $8 : 12$.

The compound ratio of days can be calculated as, $\frac{a \times p}{b \times q}$

$$= \frac{10 \times 8}{20 \times 12} = \frac{80}{240} = \frac{1}{3} = 1 : 3$$

This ratio signifies the ratio of men to boys. Therefore, 3 boys are required to complete the same task that one man can do in a day.

- There should be significant order of terms in the ratio.
- If there are two quantities in the ratio $x : y$, then the first quantity is $\frac{x}{x+y}$ times the total and the second quantity is $\frac{y}{x+y}$ times the total of both the quantities.

Properties of ratio

To solve the questions of ratio and proportion, you need to remember the different sets of rules. Particularly while solving questions on proportion, you are required to remember the following set of rules. They are invertendo, alternando, componendo, dividendo, and componendo-dividendo.

- This is the property of the ratio that if $a : b = c : d$, then the overall ratio of the series will be the sum of antecedent to the sum of all the consequents.

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \dots,$$

$$\begin{aligned} \text{Ratio} &= \frac{\text{Sum of numerators}}{\text{Sum of denominators}} \\ &= \frac{a+c+e+g+\dots}{b+d+f+h+\dots} \end{aligned}$$

- In invertendo, if $a : b = c : d$, then $b : a = d : c$. In this law, you need to remember that if the ratio is in proportion then the inverse of that ratio is also the proportion to each other.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times b = c \times d$$

$$\frac{a \times d}{a \times c} = \frac{c \times b}{a \times c} \text{ (dividing both sides by } a \times c)$$

$$\frac{d}{c} = \frac{b}{a}$$

$$\text{If, } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}$$



The inverse ratios of two equal ratios are equal. This property of ratio is called **invertendo**.

3. In this law, if $a : b :: c : d$, then $a : c :: b : d$. In this law, the property of continued proportion is used. Suppose you are given two ratios, and the values in both the ratios are proportional to each other. In this law, when you replace the denominator of the first ratio with the numerator of the second ratio, the two ratios remain proportional to each other.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times d = b \times c$$

$$\frac{a \times d}{c \times d} = \frac{b \times c}{c \times d} \text{ (dividing both sides by } c \times d)$$

$$\frac{a}{c} = \frac{b}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}$$

The ratios of the antecedents and the consequent of two equal ratios are equal. This property of ratio is called **alternando**.

4. In componendo, the basic rule that you need to remember is if $a : b :: c : d$, then $(a + b) : b :: (c + d) : d$. Thus, in componendo as you can see, you need to add the denominator to the numerator given in the ratios and then equate them. If you are using the rule on the left-hand side, then you also need to use the rule on the right-hand side.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} + 1 = \frac{c}{d} + 1 \text{ (adding 1 to both sides)}$$

$$\frac{a+b}{b} = \frac{c+d}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{b} = \frac{c+d}{d}$$

Therefore, this property of ratio is known as **componendo**.

5. In the law of dividendo, if $a : b :: c : d$, then $(a - b) : b :: (c - d) : d$. In dividendo, instead of addition, you are required to subtract the

denominator from the numerator in both ratios. Rest everything in this law is similar to the componendo.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} - 1 = \frac{c}{d} - 1 \text{ (subtracting 1 from both sides)}$$

$$\frac{a-b}{b} = \frac{c-d}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a-b}{b} = \frac{c-d}{d}$$

Therefore, this property of ratio is called **dividendo**.

6. If the ratio of any two numbers is equal to the ratio of another two numbers, then the ratios of the sum of numerator and denominator to the difference of numerator and denominator of both rational numbers are equal. This property of ratio is called componendo-dividendo.

$$\frac{a}{b} = \frac{c}{d}$$

$$\text{From componendo, } \frac{a+b}{b} = \frac{c+d}{d} \quad \dots (1)$$

$$\text{From dividendo, } \frac{a-b}{b} = \frac{c-d}{d} \quad \dots (2)$$

By dividing (1) by (2)

$$\text{We get, } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

Therefore, this property of ratio is called **componendo-dividendo**.

Example:

$$\text{If } \frac{3a^3 + 5b^3}{3a^3 - 5b^3} = \frac{109}{19}, \text{ find the value of } \frac{5a + 2b}{5a - 2b}.$$

Solution:

$$\frac{3a^3 + 5b^3}{3a^3 - 5b^3} = \frac{109}{19}$$

By applying componendo-dividendo,

$$\begin{aligned} &= \frac{\left[(3a^3 + 5b^3) + (3a^3 - 5b^3) \right]}{\left[(3a^3 + 5b^3) - (3a^3 - 5b^3) \right]} = \frac{109 + 19}{109 - 19} \\ &= \frac{6a^3}{10b^3} = \frac{128}{90} \end{aligned}$$



$$\begin{aligned}
 &= \frac{a^3}{b^3} = \frac{64}{27} \\
 &= \frac{a}{b} = \frac{4}{3} \text{ (by taking cube root)} \\
 &= \frac{5a}{2b} = \frac{20}{6} = \frac{10}{3}
 \end{aligned}$$

By applying componendo-dividendo,

$$\frac{5a+2b}{5a-2b} = \frac{13}{7}$$

Tips and Tricks

- In the ratio of $a : b$, if a and b are equal, then $a : b = 1$.
- In ratio $a : b$, if $a > b$, then $a : b > 1$.
- In ratio $a : b$, if $a < b$, then $a : b < 1$.
- To determine the ratio, quantities must be of similar units.

Proportion

When the ratio of two terms is equal to the ratio of two other terms, then these four terms are said to be in proportion. In simple words, the ratios are said to be in proportion when the two ratios are equal. If the two sets of given numbers are increasing or decreasing proportionally, then the ratios are said to be in proportion with each other.

To exemplify, if $\frac{a}{b} = \frac{c}{d}$ or $a : b = c : d$, then a, b, c , and d are in proportion.

Here, a and d are known as extremes, while b and c are known as means.

Example:

If $\frac{2}{3} = \frac{7}{14}$, then $2 : 3 = 7 : 14$.

1. Continued proportion:

The three quantities a, b , and c are said to be in continued proportion if the ratio between a and b is equal to the ratio between b and c .

Therefore, if $\frac{a}{b} = \frac{b}{c}$, then $b^2 = ac$, or we can say $b = \sqrt{ac}$

When quantities such as a, b , and c are in continued proportion, b is known as the geometric mean or mean proportion between a and c .

Example:

If $\frac{3}{x} = \frac{4}{14}$ are in continued proportion, then find the value of x .

Solution:

In continued proportion – $\frac{a}{b} = \frac{b}{c}$

Therefore, $x = 4$

2. Third proportion:

If a, b , and c are three quantities or numbers such that $a : b = b : c$, then c is called the third proportion to a and b .

Illustration: The third proportion of 6 and 15 is ' x ' such that $\frac{6}{15} = \frac{15}{x}$

So, $6x = 225$; therefore, $x = 37.5$.

3. Fourth Proportion:

If a, b, c , and d are four quantities or numbers such that $a : b = c : d$, then d is called the fourth proportion of a, b , and c .

Example:

The fourth proportion of 10, 18, and 30 is a number ' y ' such that $\frac{10}{18} = \frac{30}{y}$

So, $10y = 540$, therefore, $y = 54$.

4. Direct proportion:

If two quantities are related to each other in such a manner that an increase or decrease in one quantity will create a proportionate increase or decrease in the other quantity, then the two quantities are said to be in direct proportion. In simple words, if one quantity increases, the other quantity will increase proportionately, and if one quantity decreases, then the other quantity will also decrease proportionately.

Example:

The area of a circle increases with its increase in radius and decreases with its decrease in radius; therefore, the radius of a circle is directly proportional to its area.



If a and b are directly proportional to each other, then they will be denoted as $a \propto b$.

$a = kb$, where ' k ' is a non-zero constant, which is known as the constant of proportionality.

If a and b are directly proportional to each other, then $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

Example:

The cost of salt varies directly as its weight. 12 kg of salt costs Rs. 138. Find the cost of 5 kg of salt.

Solution:

Let us take the cost of salt to be ' a ', and its weight is ' b '.

Therefore, we have $a \propto b$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{138}{12} = \frac{a_2}{5}$$

$$a_2 = \frac{138 \times 5}{12}$$

$$a_2 = \text{Rs. } 57.5$$

The cost of 5 kg of salt is Rs. 57.5

5. Indirect proportion:

If two quantities are related to each other in such a manner that increases in one quantity will create a proportionate decrease in another quantity, and a decrease in one quantity will create a proportionate decrease in another quantity, then the two quantities are said to be in indirect proportion. In simple words, if one quantity increases, the other quantity will decrease proportionately, and if one quantity decreases, then the other quantity will increase proportionately.

If a and b are inversely proportional to each other, then they will be denoted as $a \propto \frac{1}{b}$

$a = \frac{k}{b}$, where k is a non-zero constant,

which is known as the constant of proportionality.

If a and b are inversely proportional to each other, then $a_1 b_1 = a_2 b_2$

Example:

The number of days required to finish a work is inversely proportional to the number of workers employed. 15 men finish the work in 8 days. Find the number of men to be employed if the same job is to be completed in 6 days.

Solution:

Let us take the number of days to finish the work as ' x ' and the number of men employed

as ' y ', then $x \propto \frac{1}{y}$.

$$x_1 y_1 = x_2 y_2$$

$$8 \times 15 = 6 \times y_2$$

$$y_2 = \frac{8 \times 15}{6} = 20$$

20 men should be employed to finish the work in 6 days.

Some brief points to remember

1. If $A \propto B$ and $B \propto C$, then $A \propto C$.
2. If $A \propto C$ and $B \propto C$, then $A + B \propto C$ or $A - B \propto C$.
3. If $A \propto BC$, then $B \propto \frac{A}{C}$ and $C \propto \frac{A}{B}$.
4. If $A \propto B$ and $C \propto D$, then $AC \propto BD$.
5. If $A \propto B$, then $A^n \propto B^n$.
6. If $A \propto B$ and $A \propto C$, then $A \propto (B-C)$ and $A \propto (B+C)$.
7. If $A \propto B$, then $AX \propto BX$ where X is any constant, variable or quantity.



Chapter Summary



- The ratio is used to compare two quantities of the same kind, and when two or more ratios are equal, then they are said to be in proportion.
- Comparison can only be done between the quantities of the same units.
- Two ratios are said to be in proportion if they are equal.
- The ratio will remain the same after multiplication and dividing each term of the ratio by a constant.
- We can denote ratio and proportion in two ways, either by $\frac{a}{b} = \frac{c}{d}$ or by $a : b = c : d$.

PRACTICE QUESTIONS

- What is the third proportion of 32 and 48?
 - 32
 - 48
 - 72
 - None of the above
- What must be subtracted from the terms of the ratio 3 : 7 to make it 2 : 6?
 - 3
 - 5
 - 6
 - Data insufficient
- The sum of the present ages of Abhay, Bhanu, and Chetan are 96 years. Eight years ago, their ages were in the ratio 1 : 2 : 3, what is the present age of Chetan?
 - 44
 - 45
 - 48
 - 40
- What is the fourth proportion of 12, 28, and 40?
 - 28
 - 56
 - 12
 - 40
- A person distributes chocolates among four children A, B, C, and D in the ratio $\frac{1}{6} :$
- $\frac{1}{8} : \frac{1}{10} : \frac{1}{12}$. What is the minimum number of chocolates before the distribution?
 - 52
 - 56
 - 57
 - 60
- How many factors are there of 120?
 - 16
 - 12
 - 10
 - 18
- If $6x = 8y$ and $5y = 9z$, then find the value of $\frac{z}{x}$.
 - $\frac{20}{50}$
 - $\frac{20}{38}$
 - $\frac{20}{40}$
 - $\frac{20}{48}$
- If $a : b = \frac{3}{5}$, find $\frac{2a + 3b}{4a - b}$.
 - 3 : 1
 - 1 : 2
 - 3 : 2
 - 4 : 3



9. The ratio between the three angles in the triangle is $3 : 4 : 5$. What is the difference between the smallest and the largest angle?
- A. 25°
 - B. 32°
 - C. 30°
 - D. 28°
10. If A has 10% less money than that of B who has 20% less than that of C. If C has Rs. 900, then what is the total amount of money A, B, and C have together?
- A. 2896
 - B. 2268
 - C. 3000
 - D. 2800
11. Rs. 9600 is distributed among Suresh, Ramesh, and Mahesh in the ratio of $6 : 5 : 4$. What is the difference in the shares of Suresh and Mahesh?
- A. 1280
 - B. 1250
 - C. 1200
 - D. 1300
12. If two friends Vineet and Karan earn in the ratio of $5 : 6$. If both of them earn more Rs. 100, then their earnings will be in the ratio $7 : 8$. What were Vineet's initial earnings?
- A. 260
 - B. 240
 - C. 250
 - D. 248
13. A person distributed papers among Kartik, Ajay, Vijay, and Deepak in the ratio of $5 : 2 : 4 : 3$. If Vijay got 100 papers more than Deepak, then how many papers did Kartik and Ajay together get?
- A. 760
 - B. 750
 - C. 650
 - D. 700
14. A handbag contains coins of 25 paise, 50 paise, and Re. 1 in the ratio of $5 : 6 : 4$. If Rs. 33 is in the handbag, then how many 1 rupee coins are there in the bag?
- A. 16
 - B. 12
 - C. 10
 - D. 18
15. A person distributed Rs. 770 among Ravi, Deepak, and Amit. If Ravi has received two-ninth of what Deepak and Amit together receive, then how much did Ravi receive?
- A. 160
 - B. 120
 - C. 140
 - D. 180
16. Chocolates are distributed among children A, B, and C. For every 10 chocolates that A gets, B gets 6 chocolates and for every 4 chocolates that A gets, C gets 3 chocolates. If a total number of 141 chocolates are distributed, then how many chocolates does B get?
- A. 36
 - B. 32
 - C. 30
 - D. 38
17. An office consists of 70 employees. The ratio of women to men in the office is $2 : 3$. How many more men should join the office so that the number of women is half the number of men in the office?
- A. 16
 - B. 12
 - C. 10
 - D. 14
18. A sum of Rs. 1248 was divided among A, B, and C in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, respectively. What is the total share of B?
- A. 360
 - B. 372
 - C. 380
 - D. 384



- 19.** A person has distributed a sum of money to Arpit, Bhim, and Chetan in such a manner that for each rupee Arpit gets, Bhim gets 65 paise and Chetan gets 35 paise. If Chetan's share is Rs. 560, then what is the total sum?
- A. 3600
B. 3200
C. 3100
D. 3800
- 20.** The ratio of the number of male employees and female employees in a company of 720 employees is 7 : 5. How many more female employees should join to make the ratio 1 : 1?
- A. 160
B. 120
C. 130
D. 180
- 21.** The prices of a laptop and its accessories are in the ratio 9 : 5. If the laptop costs Rs. 6800 more than its accessories, then what is the price of the laptop?
- A. 16000
B. 15200
C. 15300
D. 15800
- 22.** After the addition of 7 in both the numerator and denominator, a fraction becomes $\frac{3}{4}$. What is the original fraction?
- A. $\frac{5}{12}$
B. $\frac{7}{9}$
C. $\frac{2}{5}$
D. $\frac{8}{9}$
- 23.** If the salaries of three friends A, B, and C are in the ratio of 5 : 7 : 8. If there is an increase in the salary of all of them by 40%, 50%, and 75%, respectively, what will be the ratio of increased salaries?
- A. 1 : 2 : 3
B. 2 : 1 : 3
C. 3 : 2 : 1
D. 2 : 3 : 4
- 24.** The ratio of pens and pencils in a box is 7 : 8. If the percentage increases in pens and pencils by 20% and 10% respectively, then what will be the new ratio?
- A. 21 : 22
B. 12 : 13
C. 20 : 21
D. 22 : 21
- 25.** In the ratio of 5 : 8, the antecedent is 40, then the consequent is-
- A. 86
B. 62
C. 64
D. 60
- 26.** The population of two cities A and B are in the ratio 2 : 3. If the population is increased by 2000, then the new ratio of the population would become 40 : 57. What is B's population?
- A. 16000
B. 12000
C. 19000
D. 14000
- 27.** If the weights of A, B, and C are in the ratio 2 : 3 : 5. If there is an increase in weight by 15%, 10%, and 20%, then what will be the ratio of their weights?
- A. 23 : 33 : 60
B. 12 : 13 : 14
C. 22 : 21 : 40
D. 33 : 23 : 60
- 28.** Ramesh and Vijay invested in a joint venture in the ratio of 3 : 2. If 5% of the total gains or profits are donated to the trust, and Ramesh's share is Rs. 1710, what is the total gain?
- A. 2500
B. 3000
C. 2000
D. 2800



SOLUTIONS

1. **(C)** If A : B and B : C, then C is the third proportion.
As per the given options, 32 is A, hence cannot be the third proportion.
48 is B, hence cannot be the third proportion.
Therefore, option C, i.e., 72 can be the third proportion.
2. **(D)** The data given in this question are not sufficient to answer the same.
Therefore, option D is correct.
3. **(A)** Total sum of their ages before 8 years = $96 - (3 \times 8) = 72$
Let the age in ratio be x , therefore, eight years ago, ages of Abhay, Bhanu and Chetan be x , $2x$ and $3x$
So, $x + 2x + 3x = 72$ years
 $6x = 72$, $x = 12$ years
Age of Chetan eight years ago = $3x = 3(12) = 36$ years
Present age of Chetan = $36 + 8 = 44$ years
The present age of Chetan is 44 years.
Therefore, option A is correct.
4. **(B)** If $A : B = C : D$, then D is the fourth proportion.
Options A, C and D are the quantities already present in the ratio proportion; thus, they cannot be the fourth proportion.
Therefore, only option B being the different quantity, can be the fourth proportion.
5. **(C)** The ratio among the children A, B, C, and D = $\frac{1}{6} : \frac{1}{8} : \frac{1}{10} : \frac{1}{12}$
On rearranging the ratio = $\frac{120}{6}, \frac{120}{8}, \frac{120}{10}, \frac{120}{12} = 20 : 15 : 12 : 10$

Hence, the minimum number of chocolates can be when the common ratio is 1.

$$\text{Thus, minimum number of chocolates} = 20 + 15 + 12 + 10 = 57.$$

Therefore, option C is correct.

6. **(A)** The factors of 120 are the integers that divide 120 without leaving any remainder.
Therefore, such factors of 120 are, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, and 120.
Hence, there are a total 16 factors of 120.
Therefore, option A is correct.

7. **(D)** $6x = 8y \quad \dots\dots(1)$
 $9z = 5y \quad \dots\dots(2)$

Dividing (2) by (1), we get

$$\frac{9z}{6x} = \frac{5y}{8y}$$

$$\frac{z}{x} = \frac{5}{12} = \frac{20}{48}$$

Therefore, option D is correct.

8. **(A)** Let $a = 3x$ and $b = 5x$

$$\frac{2a + 3b}{4a - b} = \frac{2(3x) + 3(5x)}{4(3x) - 5x}$$

$$= \frac{21}{7} = 3 : 1$$

Therefore, option A is correct.

9. **(C)** Let the angles in the triangle be $3x$, $4x$, and $5x$
Sum of angles in a triangle = 180
Smallest angle = $\frac{3}{12} \times 180 = 45^\circ$
Largest angle = $\frac{5}{12} \times 180 = 75^\circ$
Difference between smallest and largest = $75 - 45 = 30^\circ$.
Therefore, option C is correct.

10. **(B)** Given that C has Rs. 900



$$B = 900 \times \frac{80}{100} = \text{Rs. } 720$$

$$C = 720 \times \frac{90}{100} = \text{Rs. } 648$$

Total amount A, B, and C have together
 $= 900 + 720 + 648 = \text{Rs. } 2268$
 Therefore, option B is correct.

11. (A) Let the shares be $6x$, $5x$ and $4x$
So, $6x + 5x + 4x = 9600$
 $15x = 9600$, $x = 640$
Share of Suresh = $6x = 6(640) = 3840$
Share of Mahesh = $4x = 4(640) = 2560$
Difference = $3840 - 2560 = \text{Rs. } 1280$
Therefore, option A is correct.

12. (C) Let Vineet and Karan's earnings be $5x$ and $6x$ then

$$= \frac{5x + 100}{6x + 100} = \frac{7}{8}$$
$$= 40x + 800 = 42x + 700$$
$$x = 50$$

Vineet's earnings = $5(50)$ = Rs. 250.
Therefore, option C is correct.

13. (D) Let the shares of papers among Kartik, Ajay, Vijay, and Deepak be $5x$, $2x$, $4x$, $3x$ respectively

Therefore, $4x - 3x = 100$

$$x = 100$$

Kartik and Ajay share = $5(100) + 2(100) = 700$

Therefore, option D is correct.

- 14. (A)** Let the number of coins of 25 paise, 50 paise and Re. 1 be $5x$, $6x$, and $4x$, respectively.

Then the net value = $0.25(5x) + 0.50(6x) + 1(4x) = 33$

$$1.25x + 3x + 4x = 33$$
$$8.25x = 33, x = 4$$

Number of Re. 1 coins = $4 \times 4 = 16$.

Therefore, option A is correct.

- 15. (C)** Let the amount received by Ravi, Deepak and Amit be x , y , and z .

$$x + y + z = 770$$

$$\frac{2(z + y)}{9} + y + z = 770$$

$$11x + 11y = 6930$$

$x + y = 630$, amount received by Deepak and Amit = Rs. 630

Amount received by Ravi = $770 - 630$ = Rs. 140

Therefore, option C is correct.

16. (A) A : B = 10 : 6 = 5 : 3(1)
 A : C = 4 : 3(2)

Multiplying (1) by 4 and (2) by 5 to get a common value of A

A : B = 20 : 12; A : C = 20 : 15
 A : B : C = 20 : 12 : 15

Share of B = $\frac{12}{47} \times 141 = 36.$

Therefore, option A is correct.

- 17. (D)** Number of women = $\frac{2}{5} \times 70 = 28$

Number of men = $\frac{3}{5} \times 70 = 42$

Let x be the more men should join so that

$$\frac{28}{42 + x} = \frac{1}{2}$$

$$56 = 42 + x, x = 14.$$

Therefore, option D is correct.

- 18. (D)** Share of A : B : C = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$
 $= \frac{6}{12} : \frac{4}{12} : \frac{3}{12} = 6 : 4 : 3$
Share of B = $\frac{4}{13} \times 1248 = \text{Rs. } 384.$
Therefore, option D is correct.

- 19. (B)** Share of Arpit = $\frac{560}{35} \times 100$ = Rs. 1600
 (Rs. 1 = 100 paisa)
 Share of Bhim = $\frac{540}{35} \times 65$ = Rs. 1040
 Total sum = 1600 + 1040 + 560 = Rs. 3200.
 Therefore, option B is correct.

- 20. (B)** Let the number of male employees and female employees be $7x$ and $5x$, respectively.

$$7x + 5x = 720$$
$$12x = 720, x = 60$$

Number of male employees = 420, number of female employees = 300



To make the ratio $1 : 1$, $420 - 300 = 120$ more female employees should join the company.

Therefore, option B is correct.

- 21. (C)** Let the price of the laptop and its accessories be $9x$ and $5x$, respectively.

Given that $9x - 5x = \text{Rs. } 6800$

$$4x = \text{Rs. } 6800, x = 1700$$

$$\text{Price of laptop} = 9(1700) = \text{Rs. } 15300.$$

Therefore, option C is correct.

$$\text{22. (C)} \text{ Option A } \frac{5+7}{12+7} = \frac{12}{19}$$

$$\text{Option B } \frac{7+7}{9+7} = \frac{14}{16} = \frac{7}{8}$$

$$\text{Option C } \frac{2+7}{5+7} = \frac{9}{12} = \frac{3}{4}$$

Therefore, option C is correct.

- 23. (D)** Let the salaries of A, B and C be $5x$, $7x$ and $8x$ respectively

Increment in salary (140% of $5x$), (150% of $7x$), (175% of $8x$)

$$\left(\frac{140}{100} \times 5x\right), \left(\frac{150}{100} \times 7x\right), \left(\frac{175}{100} \times 8x\right)$$

$$7x, \frac{21x}{2} \text{ and } 14x$$

Thus, the required ratio will be $14x : 21x : 14x = 2 : 3 : 4$.

Therefore, option D is correct.

- 24. (A)** Let the pens and pencils in the box be $7x$ and $8x$ respectively.

Increased number is (120% of $7x$) and (110% of $8x$)

$$\left(\frac{120}{100} \times 7x\right) \text{ and } \left(\frac{110}{100} \times 8x\right)$$

$$\text{So, } \frac{42x}{5} \text{ and } \frac{44x}{5}$$

Thus, the required ratio would be $42 : 44 = 21 : 22$.

Therefore, option A is correct.

- 25. (C)** Let the antecedent and consequent be $5x$ and $8x$, respectively.

$$5x = 40, x = 8$$

$$\text{Consequent} = 8(8) = 64$$

Therefore, option C is correct.

- 26. (C)** Let $2x$ and $3x$ be the original population of A and B, respectively.

$$\text{Then, } \frac{2x + 2000}{3x + 2000} = \frac{40}{57}$$

$$57(2x + 2000) = 40(3x + 2000)$$

$$6x = 34000$$

$$3x = 17000$$

$$\text{B's present population} = 17000 + 2000 = 19000$$

Therefore, option C is correct.

- 27. (A)** Let the weights of A, B and C be $2x$, $3x$ and $5x$ respectively.

$$\text{A's weight} = \frac{115}{100} \times 2x = \frac{23x}{10}$$

$$\text{B's weight} = \frac{110}{100} \times 3x = \frac{33x}{10}$$

$$\text{C's weight} = \frac{120}{100} \times 5x = \frac{6x}{10}$$

$$\text{New required ratio} = 23 : 33 : 60$$

Therefore, option A is correct.

- 28. (B)** Let the profit be Rs. 100

After donating to the trust, Ramesh's share is $\text{Rs. } (95 \times \frac{3}{5}) = \text{Rs. } 57$

If Ramesh's share is Rs. 57, total profit = Rs. 100

If Ramesh's share is Rs. 1710, the total profit = $\text{Rs. } (\frac{1710 \times 100}{57}) = \text{Rs. } 3000$

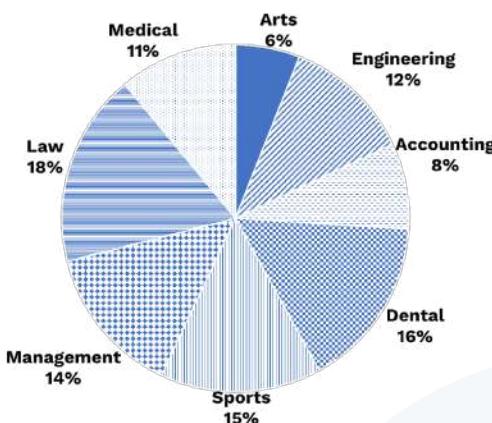
Therefore, option B is correct.



PRACTICE QUESTIONS

Questions 1–5: Analyse the given data and answer the following questions:

Total number of students = 2500



The ratio of boys to girls:

DEPARTMENT	BOYS	GIRLS
Engineering	7	5
Accounting	2	3
Dental	5	3
Sports	7	8
Management	3	4
Law	5	4
Medical	6	5
Arts	2	1

- What is the ratio between boys and girls students in engineering and arts respectively?
 - 7 : 4
 - 4 : 7
 - 3 : 4
 - 2 : 3
- What is the difference between the total number of students in management and law?
 - 80
 - 150

- C. 75
D. 100
- What is the ratio between the total number of boys in Dental and Sports and the total number of girls in these two departments, respectively?
 - 13 : 17
 - 17 : 14
 - 13 : 14
 - 14 : 13
- How many girls are pursuing accounts?
 - 130
 - 120
 - 125
 - 110
- What is the difference between boys and girls in the medical department?
 - 25
 - 75
 - 100
 - 40

Questions 6–10: Analyse the given data and answer the following questions:

Total citizens of Konoha are 8000 and there are 3 regions, i.e., P, Q and R in it. Ratio of citizens of these 3 regions (P : Q : R) is 8 : 5 : 3. Number of males in region P is 1900 more than that of females in region R. The ratio of number of females in region P to that of males in region R is 5 : 3. The number of females in region Q is 30% of the number of males in region P.

- What is the average of the number of males in region P and Q together is how much more than the number of females in region P?
 - 650
 - 625
 - 675
 - 610
- 60% and 88% of females of region P and Q, respectively, are working, then find non-working females in both regions.
 - 1000
 - 1200
 - 1400
 - 1600



working females of region P is how much per cent less than that working females of region Q?

A. $\frac{58}{11}\%$

B. $\frac{67}{11}\%$

C. $\frac{100}{11}\%$

D. $\frac{89}{11}\%$

8. Total female citizens of region P and R together is what per cent of total citizens of these two regions together?

A. 28.18%

B. 30.18%

C. 32.18%

D. 38.18%

9. What is the ratio of the total number of females in all regions to that of the total number of males in all regions?

A. $\frac{42}{89}$

B. $\frac{47}{139}$

C. $\frac{39}{55}$

D. $\frac{57}{103}$

10. The average of male citizens of region Q and R is how much more than the average of female citizens of these two regions?

A. 650

B. 600

C. 550

D. 500

SOLUTIONS

1. (A) Boys in engineering = $\frac{7}{12}$ of 12% of total students

$$\text{Boys in arts} = \frac{2}{3} \text{ of } 6\% \text{ of total students}$$

$$\text{Ratio} = \frac{\frac{7}{12} \text{ of } 12\% \times \text{total students}}{\frac{2}{3} \times 6\% \times \text{total students}}$$

$$= \frac{14}{12} \times \frac{3}{2} = 7 : 4$$

Therefore, option A is correct.

2. (D) Required difference = $2500 \times (18 - 14)\%$
 $= \frac{2500 \times 4}{100} = 100$

Therefore, D is correct.

3. (B) Total students in dental = 16% of total students
 $= \frac{16}{100} \times 2500 = 400$ students in dental
 Girls in dental = $400 - 250 = 150$

$$\text{Total students in sports} = \frac{7}{15} \times 375 = 175$$

$$\text{Girls in sports} = 375 - 175 = 200$$

$$\text{Required ration} = 425 : 350 = 17 : 14$$

Therefore, option B is correct.

4. (B) Total girls = $\frac{3}{5}$ of 8% of total students
 $= \frac{3}{5} \times \frac{8}{100} \times 2500 = 120$

Therefore, option B is correct.

5. (A) Difference = $\frac{6}{11} \times \frac{11}{100} \times 2500 - \frac{5}{11} \times \frac{11}{100} \times 2500$
 $= 150 - 125 = 25$
 Therefore, option A is correct.

Solutions 6–10:

Let the number of males in region R be $3x$, the number of females in region P will be $5x$. Ratio of the citizens in three regions P : Q : R = 8 : 5 : 3



$$\text{Citizens in P} = 8 \times \frac{8000}{16} = 4000$$

$$\text{Citizens in Q} = 5 \times \frac{8000}{16} = 2500$$

$$\text{Citizens in R} = 3 \times \frac{8000}{16} = 1500$$

Number of males in region P is 1900 more than that of females in region R.

The given information can be tabulated as:

P(4000)		Q(2500)		R(1500)	
MALES	FEMALES	MALES	FEMALES	MALES	FEMALES
1500 - 3x + 1900	5x		30 $\frac{\{1500 - 3x + 1900\}}{100}$	3x	1500 - 3x

The total citizens in region P = 4000

$$(1500 - 3x + 1900) + 5x = 4000$$

$$3400 + 2x = 4000$$

$$x = 300$$

Now, the table will be

P(4000)		Q(2500)		R(1500)	
MALES	FEMALES	MALES	FEMALES	MALES	FEMALES
2500	1500	1750	750	900	600

6. (B) Males in P = 2500, males in Q = 1750

$$\text{Average} = \frac{2500 + 1750}{2} = 2125$$

$$\text{Females in P} = 1500$$

$$\text{Required difference} = 2125 - 1500 = 625$$

Therefore, option B is correct.

7. (C) Females in P = 1500, working females in P = 60% of 1500, non-working females in P = 40% of 1500 = 600

$$\text{Females in Q} = 750, \text{working females in Q} = 88\% \text{ of } 750 = 660$$

$$\text{Required \%} = (660 - 600) \times \frac{100}{660} = \frac{100}{11} \%$$

Therefore, option C is correct.

8. (D) Females in P and R = 1500 + 600 = 2100

$$\text{Total citizens of P and R} = 4000 + 1500 = 5500$$

$$\text{Required \%} = 2100 \times \frac{100}{5500} = 38.18\%$$

Therefore, option D is correct.

9. (D) Total number of females in all regions =

$$1500 + 750 + 600 = 2850$$

$$\text{Total number of males in all regions} = 2500 + 1750 + 900 = 5150$$

$$\text{Required ratio} = \frac{2850}{5150} = \frac{57}{103}$$

Therefore, option D is correct.

10. (A) Males in Q = 1750

$$\text{Males in R} = 900$$

$$\text{Average} = \frac{1750 + 900}{2} = 1325$$

$$\text{Females in Q} = 750$$

$$\text{Females in R} = 600$$

$$\text{Average} = \frac{750 + 600}{2} = 675$$

$$\text{Required difference} = 1325 - 675 = 650$$

Therefore, option A is correct.



SYNOPSIS

- Mixtures
- Mean price
- Alligation
- Rule of alligation
- Criss-cross method
- Common types of questions asked
- Summary

MIXTURE

A mixture is the combination of two or more quantities.

Mean price

The cost price of the unit quantity of a mixture is called the mean price.

When a person buys x kg of the first kind of an item and y kg of the second kind of an item, the total amount he spends is $\text{Rs.}(px + qy)$ and the total amount of mixture is $(x + y)$ kg. But the ratio in which these two ingredients at the given price must be mixed to produce a mixture of a different desired price is calculated using the rule of alligation.

ALLIGATION

Rule of alligation is used to find the ratio in which two or more ingredients must be mixed at a given price to produce the mixture at a desired price.

Rule of alligation

If two items are mixed, then

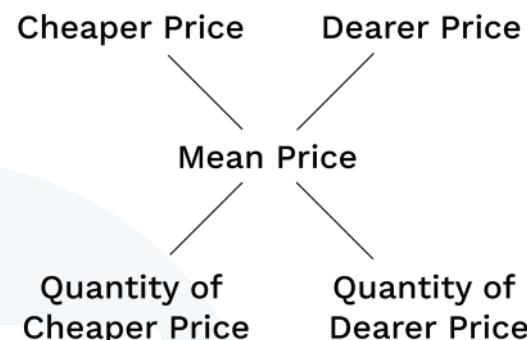
$$\left(\frac{\text{quantity of cheaper item}}{\text{quantity of dearer item}} \right) = \frac{(\text{CP of dearer}) - (\text{mean price})}{(\text{mean price}) - (\text{CP of cheaper})}$$

Or,

$$\text{cheaper quantity : dearer quantity} = (d - m) : (m - c)$$

Where, d , m , and c are dear price, mean price, and cheap price respectively.

Criss-cross method



The difference between mean price and cheaper price in the place of 'quantity of dearer price' and the difference between mean price and dearer price in the place of 'quantity of cheaper price'.

Example:

In what ratio were two different kinds of cereals costing Rs. 45 per kg and 60 per kg, mixed to get a mixture costing Rs. 50 per kg.

Solution:

Applying the Criss-Cross method:

$$\begin{aligned} \text{Ratio} &= \frac{(60 - 40)}{50 - 45} \\ &= \frac{10}{5} \end{aligned}$$

Therefore, the ratio of the two types of cereals = 2 : 1.

COMMON TYPES OF QUESTIONS ASKED

1. The ratio of items required

In this type of questions, the price and quantity of the mixture along with the price of items are provided, and the students are required to calculate the ratio in which the items are



to be mixed to get the required amount of mixture, using the given price and quantity of the mixture along with the price of items.

Example:

In what proportion must a shopkeeper mix rice type 1 at Rs. 4.08 per kg and rice type 2 at Rs. 5.76 per kg to make a mixture worth Rs. 5.04 per kg?

Solution:

Cheap price = Rs. 4.08

Mean price = Rs. 5.04

Dear price = Rs. 5.76

Thus, according to the alligation formula,
cheap quantity : dear quantity = $(d - m) : (m - c)$

$$\text{Required proportion} = (5.76 - 5.04) : (5.04 - 4.08) \\ = 0.72 :: 0.96 = 3 : 4$$

2. The final quantity of the original item

In these types of questions, the original quantity of a liquid is provided along with the amount of liquid that is being taken out and replaced by some other liquid two or three times, and the students have to calculate the final quantity of the original liquid left.

Example:

A container has 20 L milk. 4 L of milk is replaced with an equal quantity of water. What will be the final quantity of milk left in the container if this process is repeated once again?

Solution:

Original quantity of milk = 20 L

After taking out 4 L and replacing it with water, milk left = 16 L

When repeating the process, after taking out 4 L of this new mixture

$$\text{Quantity of milk taken out} = 4 \times \frac{16}{20} = 3.2 \text{ L}$$

Thus, the required quantity of milk left
 $= 32 - 3.2 \text{ L} = 28.8 \text{ L}$

3. The ratio of two liquids in the third container

In these types of questions, the ratio of two liquids in two different containers is given, and the students are asked to calculate the new ratio of these two liquids if both the containers are poured out in a third container together.

Example:

Two jars have oil and milk in the ratio 4 : 1 and 5 : 1, respectively. If both jars are emptied into a third jar, then what will be the ratio of oil and water in this third jar?

Solution:

Ratio of oil and milk in 1st jar = 4 : 1

$$\text{Quantity of oil in this 1st jar} = \frac{4}{5}$$

$$\text{Quantity of milk in this 1st jar} = \frac{1}{5}$$

The ratio of oil and milk in 2nd jar = 5 : 1

$$\text{Quantity of oil in this 2nd jar} = \frac{5}{6}$$

$$\text{Quantity of milk in this 2nd jar} = \frac{1}{6}$$

$$\text{In the third jar, the quantity of milk} = \frac{4}{5} + \frac{5}{6} = \frac{49}{30}$$

$$\text{Quantity of water} = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$$

$$\text{Therefore, required ratio} = \frac{49}{30} : \frac{11}{30} = 49 : 11$$

Note

- If not given, assume the cost of water to be Rs. 0.



Chapter Summary



- Rule of alligation: if two items are mixed, then

$$\left(\frac{\text{quantity of cheaper item}}{\text{quantity of dearer item}} \right) = \frac{(CP \text{ of dearer}) - (mean \text{ price})}{(mean \text{ price}) - (CP \text{ of cheaper})}$$

Or,

$$\text{cheaper quantity : dearer quantity} = (d - m) : (m - c)$$

- Where, d , m , and c are dearer price, mean price, and cheap price, respectively.

PRACTICE QUESTIONS

- If 50 L of oil and water mixture comprises of 50% oil, then how much more oil should be mixed to make it 70% in the mixture?
 - 33.33 L
 - 28.14 L
 - 16.66 L
 - 11.11 L
- A mixture of x mL milk and 8 mL water costs Rs. 90 per mL. If the cost price of milk is Rs. 108 mL, then how much milk is there in the mixture?
 - 40 mL
 - 32 mL
 - 36 mL
 - 44 mL
- In a container, liquid A and liquid B are in their ratio of 7 : 5. If 45 L more of liquid B is mixed in the mixture, the ratio of liquid B and A becomes 8 : 7. Find the amount of liquid B in the old mixture.
 - 65 L
 - 70 L
 - 75 L
 - 80 L
- A mixture of Rooh Afza and 16 L of water costs rupees 95 per litre. If pure Rooh Afza costs Rs.114 per litre, then what is the amount of Rooh Afza in the original mixture?
 - 7 mg
 - 10 mg
 - 15 mg
 - 12 mg
- A shopkeeper sells a mixture of 45 L milk and 9 L water at Rs. 66 per kg, what is the cost price of pure milk?
 - 86.4 L
 - 76.5 L
 - 79.2 L
 - 84.7 L
- In a mixture of 15 mL, ratio of two liquids is 4 : 2. What amount of liquid 2 should be mixed in this to make the ratio of the two liquids 1 : 2.
 - 5 mL
 - 8 mL
 - 15 mL
 - 18 mL
- 150 mg of sugar solution has 20% sugar in it. What quantity of sugar should be added in the sugar water solution to make sugar 25% in the solution?
 - 7 mg
 - 10 mg
 - 15 mg
 - 12 mg
- 5 kg of alloy X is mixed with 25 kg of alloy Y. If the amount of copper and zinc in alloy



X is in the ratio 1 : 2, and the amount of copper and zinc in alloy Y is in the ratio of 2 : 3, then what is the ratio of copper to tin in the resulting mixture?

- A. 1 : 3
- B. 3 : 5
- C. 5 : 7
- D. 7 : 11

9. A deodorant contains butane and benzyl alcohol in the ratio 5 : 3. If 8 L of the mixture is replaced with the same quantity of benzyl alcohol, the ratio becomes 3 : 5. What is the total quantity of the container?
- A. 5 L
 - B. 10 L
 - C. 15 L
 - D. 20 L

10. 27 litres of milk and water mixture in the ratio of 4 : 5 is added to a bottle containing x litre pure milk. If the new mixture formed contains 70% milk, what was the original quantity of pure milk in the bottle?
- A. 21 L
 - B. 23 L
 - C. 25 L
 - D. 27 L

11. A mixture of 10 g of wheat and maida contains 20% maida. How much maida should be added to this mixture of wheat and maida to get 33.33% maida?
- A. 6 g
 - B. 2 g
 - C. 4 g
 - D. 5 g

12. A jewellery shop has two qualities of gold, 24 carats, and 22 carats purity gold. In what proportion should he mix both qualities of gold to make an ornament having 20 carats purity?
- A. 1 : 2
 - B. 5 : 3
 - C. 3 : 4
 - D. 4 : 5

13. If selling price of milk is Rs. 100. In what ratio should a salesman mix water in that milk, to sell the water-milk mixture at Rs. 80, and obtain a profit of 50%?

- A. 7 : 8
- B. 8 : 9
- C. 9 : 11
- D. 11 : 13

14. How much amount of butter worth 25 per kg should be blended with 30 kg butter that costs 30 per kg to gain 10% and sell the mixture at 30 per kg?
- A. 36 kg
 - B. 35 kg
 - C. 40 kg
 - D. 38 kg

15. There are two different types of plastic available in market, one having the ratio of components A and B as 2 : 11 and other having same, 5 : 21. In what proportion should a manufacturer combine both the components to get A and B in 7 : 32 ratio?
- A. 1 : 2
 - B. 3 : 4
 - C. 2 : 3
 - D. 5 : 6

16. When 70% of the juice and 30% of the water were removed from a full vessel, it gets emptied by 55%, leaving only 160 L of mixture. What is the remaining amount of juice and water in the mixture?
- A. Water = 60 L, juice = 100 L
 - B. Water = 90 L, juice = 70 L
 - C. Water = 80 L, juice = 80 L
 - D. Water = 100 L, juice = 60 L

17. The ratio of acid to ethanol in a 6 mL liquid is 2 : 1. If the acid-to-ethanol ratio has to be 1 : 2, the amount of ethanol to be added to the mixture is
- A. 5.5 mL
 - B. 6 mL



- C. 6.5 mL
D. 5 mL
- 18.** 4 mL of liquid from an oil bottle is replaced with water. And then this action is repeated three times more, until the ratio of oil to water in the container equals 16 : 65. How much oil was in the container at the start?
A. 16 mL
B. 12 mL
C. 15 mL
D. 13 mL
- 19.** A container is filled with pulses, 6 parts of which are type 1 and 10 part type 2. How much of the mixture must be replaced with type 2 so that the mixture contains half type 1 and half type 2?
A. $\frac{2}{3}$
B. $\frac{3}{7}$
C. $\frac{1}{5}$
D. $\frac{1}{8}$
- 20.** A mixture of 48 mL contains H_2SO_4 and HCl in the ratio of 3 : 5. How much HCl must be added to this mixture to reverse the ratio?
A. 10 mL
B. 14 mL
C. 8 mL
D. 16 mL
- 21.** How many litres of refined oil costing Rs. 9 per L must be mixed with 27 L of refined oil costing Rs. 7 per L to get a gain of 10% by selling the mixture at Rs. 9.24 L.
A. 61 L
B. 58 L
C. 65 L
D. 63 L
- 22.** Two jars B and A have water and buttermilk mixed in the ratio 2 : 5 and 6 : 7. If a new mixture in vessel C must have water and buttermilk in the ratio 5 : 8, calculate the ratio in which mixture from these two jars must be mixed.
A. 3 : 5
B. 3 : 4
C. 1 : 5
D. 7 : 9
- 23.** A jar full of soda water contains 40% soda. A part of this soda water is replaced by another mixture containing 19% soda, and now the jar contains 26% soda. The quantity of soda water replaced is:
A. $\frac{2}{5}$
B. $\frac{1}{5}$
C. $\frac{2}{3}$
D. $\frac{3}{5}$
- 24.** A juice shop has two big jars of juice. First one contains 25% water and 75% fruit pulp. Second one contains 50% water and 50% fruit pulp. How much juice should the shop mix from each of the containers to get 12 litres of fruit pulp, and ratio of water and pulp being 3 : 5?
A. 2 L
B. 4 L
C. 6 L
D. 8 L
- 25.** The cost of onion A is Rs. 15 per kg and onion B is Rs. 20 per kg. If both types A and B are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of onion is:
A. 13 per kg
B. 18 kg
C. 16 kg
D. 12 kg
- 26.** In what proportion water must be added to ethanol to gain 20% by selling resulting mixture at the cost price?



- A. 4 : 3
B. 3 : 4
C. 5 : 6
D. 1 : 5
- 27.** In a pot, a mixture of curd and water is in the ratio of 4 : 5. When it is added an additional 8 g of curd, the pot gets full, and the ratio of curd and water becomes 6 : 5. What is the capacity of the pot?
A. 11 g
B. 22 g
C. 33 g
D. 44 g
- 28.** The diluted hydrochloric acid contains 8 litres of HCl and rest water. How many litres of the mixture should be replaced with pure hydrochloric acid to get mixture of 30% concentration if there were initially 32 litres of water in the mixture?
A. 2 L
B. 3 L
C. 4 L
D. 5 L
- 29.** In a mixture of sand and water, the proportion of sand by weight was 75%. If in 60 grams of mixture, 15 gram sand was added, what would be the percentage of sand?
A. 83%
- B. 70%
C. 75%
D. 62%
- 30.** The ratio of ethanol and isopropanol in two different sanitisers is 2 : 3 and 4 : 5. In what ratio are we required to mix the mixtures of two sanitizers in order to get the new mixture in which the ratio of ethanol and isopropanol is 7 : 5?
A. 11 : 7
B. 7 : 5
C. 5 : 3
D. 3 : 1
- 31.** A mixture contains diesel and oil in a ratio of 2 : 3. How much oil should be added to the 60 L mixture to make the ratio of diesel and mixture as 1 : 3?
A. 16
B. 12
C. 10
D. 18
- 32.** The ratio of water and dye is 5 : 2 in the mixture of 28 litres. If 2 litres of dye are added to the mixture, then what will be the ratio of water and dye in the new solution?
A. 1 : 3
B. 1 : 2
C. 3 : 1
D. 2 : 5

SOLUTIONS

- 1. (A)** Initial quantity of oil = 25 L
Oil to be added = x L
Since water remains same,
30% of $(50 + x) = 25$ L
 $150 + 3x = 250$ L
 $X = 33.33$ L
Therefore, option A is correct.
- 2. (A)** Water : Milk = $(108 - 90) : (90 - 0)$
= $18 : 90 = 1 : 5$

Thus, if the water in mixture is 8 mL, milk quantity will be $8 \times 5 = 40$ mL
Therefore, option A is correct.

- 3. (C)** Liquid A: liquid B = 7 : 5
Liquid B initially = $5x$
Liquid B after addition = $5x + 45$
New ratio = 7 : 8 = $7x : 5x + 45$
 $56x = 35x + 315$
 $21x = 315$



$$x = 15$$

Thus, original liquid B = 75 L

Therefore, option C is correct.

4. (D) Water: Rooh Afza = $(114 - 95) : (95 - 0)$
 $= 19 : 95 = 1 : 5$

Thus, if water in mixture is 16 L, Rooh Afza quantity will be $16 \times 5 = 80$ L

Therefore, option D is correct.

5. (C) $\frac{\text{Quantity of water}}{\text{quantity of milk}} = \frac{x - 66}{66 - 0}$
 $\Rightarrow \frac{45}{9} = \frac{66}{x - 66}$
 $\Rightarrow 5x - 330 = 66$
 $\Rightarrow x = \frac{396}{5} = 79.2$

Therefore, option C is correct.

6. (C) The initial quantity of liquid 1 = 10 mL
The initial quantity of liquid 2 = 5 mL
Since liquid 1 remains same,

$$\frac{1}{3} \times (15 + x) = 10 \text{ mL}$$

$$X = 15 \text{ mL}$$

To get the mixture in ratio 1 : 2, (i.e., liquid 2 double the amount of liquid 1)

Liquid 2 to be added = 15 mL

Therefore, option C is correct.

7. (B) Initial quantity of sugar = 30 g

Since water remains same,

$$75\% \text{ of } (150 + x) \text{ g} = 120 \text{ g}$$

$$x = 10 \text{ g}$$

Sugar to be added = 10 g

Therefore, option B is correct.

8. (C) Amount of copper in alloy $x = \left(\frac{1}{3}\right) \times 5 = \frac{5}{3}$

$$\text{Amount of zinc in alloy } x = \left(\frac{2}{3}\right) \times 5 = \frac{10}{3}$$

$$\text{Amount of copper in alloy } x = \left(\frac{2}{5}\right) \times 25 = 10 \text{ kg}$$

$$\text{Amount of zinc in alloy } y = \left(\frac{3}{5}\right) \times 25 = 15 \text{ kg}$$

When alloys x and y are mixed, then

$$\text{Amount of copper} = \frac{5}{3} + 10 = \frac{35}{3}$$

$$\text{Amount of zinc} = \frac{10}{3} + 15 = \frac{55}{3}$$

The ratio of copper and zinc in new mixture = 7 : 11

Therefore, option C is correct.

9. (D) Quantity of butane removed
 $= \frac{5}{5+3} \times 8 = 5 \text{ L}$

Quantity of benzyl alcohol removed
 $= \frac{3}{5+3} \times 8 = 3 \text{ L}$

New ratio,
 $3 : 5 = \frac{5x - 5}{3x - 3 + 8}$

$$9x + 15 = 25x - 25$$

$$x = 2.5$$

$$\text{Total quantity of container} = 20 \text{ L}$$

Therefore, option D is correct.

10. (B) Quantity of milk in 27 L mixture
 $= \frac{4}{4+5} \times 27 = 12 \text{ L}$

$$\text{Quantity of water} = 15 \text{ L}$$

In new mixture,

$$\Rightarrow \frac{7}{3} = \frac{x + 12}{15}$$

$$3x + 36 = 15 \times 7 = 105$$

$$x = 23 \text{ L}$$

Therefore, option C is correct.

11. (B) In the mixture of 10 litres, maida is 20%, i.e., 2 kg.

Let x kg of maida be added.

According to the question,

$$\Rightarrow \frac{10 + x}{2 + x} = \frac{100}{33.33} = \frac{3}{1}$$

$$x = 2$$

Therefore, option B is correct.

12. (A) Proportion = $\frac{22 - 20}{20 - 24} = 1 : 2$

Therefore, option A is correct.

13. (A) The total price of milk is now $80 \times (1 + x)$

But, he gets a profit of 50%.

$$\text{So, } 80(1 + x) = 150$$



$$1 + x = \frac{15}{8}$$

$$x = \frac{7}{8}$$

Therefore, option A is correct.

14. (A) $(25x + 30 \times 30) \times \frac{110}{100} = 30(30 + x)$

$$(275x + 9900) = (9000 + 300x)$$

$$\Rightarrow 300x - 275x = 900 \Rightarrow x = \frac{900}{25} = 36$$

Therefore, option A is correct.

15. (A) Quantity of component A in the first kind of plastic = 213

Quantity of component B in second kind plastic = 526

Required ratio = 178 : 139 = 1 : 2

Therefore, option A is correct.

16. (A) Juice : water = 5 : 3

Quantity of juice in the remaining mixture

$$= \frac{5}{8} \times 160 = 100 \text{ L}$$

Similarly, quantity of water

$$= \frac{3}{8} \times 160 = 60 \text{ L}$$

Therefore, option A is correct.

17. (B) Quantity of acid in the mixture = $\frac{2}{3} \times 6$

$$= 4 \text{ mL}$$

Quantity of ethanol in the mixture = $\frac{1}{3} \times 6$

$$= 2 \text{ mL}$$

Let the required quantity of ethanol be $x \text{ L}$.

$$\Rightarrow \frac{4}{2} + x = \frac{1}{3}$$

$$8 = 2 + x$$

$$x = 6 \text{ mL}$$

Therefore, option B is correct.

18. (B) Quantity of oil in container = $x \text{ mL}$.

Quantity of water left in cask = $[x(1 - 4x)4] \text{ mL}$

$$\text{Thus, } \left[x \left(1 - \frac{4}{x} \right) 4x \right] = 1681$$

$$\Rightarrow (1 - 4x)4 = (23)4$$

$$\Rightarrow (x - 4x) = \frac{23}{3x} - 12$$

$$x = 12$$

Therefore, option B is correct.

19. (C) Let the container initially contain 16 kg of pulses.

Let $a \text{ kg}$ of pulses be compressing type 2 pulse.

Quantity of type 1 pulse in the new mixture

$$= \left(6 + \frac{6a}{16} + a \right)$$

Quantity of type 2 pulse in the new mixture

$$= \left(10 - \frac{10a}{16} \right)$$

$$\Rightarrow 6 - \frac{6a}{16} + a = 10 - \frac{10a}{16}$$

$$\Rightarrow 96 - 6a + 16a = 160 - 10a$$

$$\Rightarrow 96 + 10a = 160 - 10a$$

$$\Rightarrow 20a = 64$$

$$\Rightarrow a = \frac{64}{20} = \frac{16}{5}$$

$$\text{Part of mixture replaced} = \left(\frac{1}{16} \right) \times \left(\frac{16}{5} \right) = \frac{1}{5}$$

Therefore, option C is correct.

20. (C) Initial quantity of HCl = $\left(\frac{3}{8} \right) \times 48 \text{ litres}$

$$= 18 \text{ mL}$$

To get the ratio 5 : 3, new quantity of HCl

$$= \frac{5}{8} = \frac{18 + x}{48 + x}$$

$$= 120 + 5x = 144 + 8x$$

$$3x = 24$$

$$x = 8 \text{ mL}$$

Therefore, option C is correct.

21. (D) S = Rs. 9.24

Gain of 10%

$$C = \text{Rs.} \left(\frac{110}{100} \times 9.24 \right) = \text{Rs.} 8.40$$

According to alligation,

The ratio of 1st and 2nd kinds = 14 : 6 = 7 : 3

Let $x \text{ kg}$ of butter of 1st be mixed with 27 L of 2nd kind

$$7 : 3 = x : 27$$



$$\Rightarrow x = 7 \times \frac{27}{3} = 63$$

Therefore, option D is correct.

- 22. (D)** Buttermilk in 1-litre mix of A = $\frac{5}{7}$ litre

$$\text{Buttermilk in 1-litre mix of B} = \frac{7}{13} \text{ litre}$$

$$\text{Buttermilk in 1-litre mix of C} = \frac{8}{13} \text{ litre}$$

By rule of the alligation,

$$\text{Required ratio} = \frac{13}{1} : \frac{9}{19}$$

$$= 7 : 9$$

Therefore, option D is correct.

- 23. (C)** Ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2

$$\text{Quantity of soda water replaced} = \frac{2}{3}$$

Therefore, option C is correct.

- 24. (C)** Juice from the first container = x
Juice from the second container = $(12 - x)$
Juice pulp in x litres of the first container = $0.75x$
Water in x litres of the first container = $0.25x$
Juice pulp in $(12-x)$ litres of the second container = $0.5(12-x)$
Water in $(12-x)$ litres of the second container = $0.5(12-x)$
Ratio of water to juice pulp = $[0.25x + 0.5(12 - x)] : [0.75x + 0.5(12 - x)]$

$$= 3 : 5$$

$$\Rightarrow \frac{(0.25x + 6 - 0.5x)}{(0.75x + 6 - 0.5x)} = \frac{3}{5}$$

$$\Rightarrow \frac{6 - 0.25x}{6 + 0.25x} = \frac{3}{5} (6 - 0.25x)(0.25x + 6) = 35$$

$$\Rightarrow 30 - 1.25x = 0.75x \cdot 18$$

$$\Rightarrow 2x = 12$$

$$\therefore x = 6$$

Therefore, option C is correct.

- 25. (B)** Price of the mixed variety = x per kg
According to alligation,

$$\therefore \frac{x - 15}{20 - x} = \frac{3}{2}$$

$$\Rightarrow 60 - 3x = 2x - 30$$

$$\Rightarrow 5x = 9$$

$$\Rightarrow x = 18$$

So, the price of the mixture is Rs. 18 per kg.

Therefore, option B is correct.

$$\text{26. (D)} \frac{\text{Quantity of water}}{\text{Quantity of ethanol}} = \frac{\frac{5}{3}}{\frac{25}{3}} = \frac{1}{5}$$

The ratio of water and ethanol = 1 : 5

Therefore, option D is correct.

- 27. (D)** Let the capacity of the pot be x g.
Quantity of curd in the mixture before adding more curd = $\left(\frac{4}{9}\right) \times (x - 8)$

After adding curd, the quantity of curd in the mixture = $\frac{6}{11}x$.

$$\Rightarrow \frac{6x}{11} - 8 = \left(\frac{4}{9}\right) \times (x - 8)$$

$$\Rightarrow 10x = 792 - 352 = x = 44.$$

Therefore, option D is correct.

- 28. (D)** Original quantity of water = 32 L
Original quantity of hydrochloric acid = 8 L
Initial hydrochloric acid to water ratio = 1 : 4
Required hydrochloric acid to water ratio = 3 : 7
Concentration ratio, $\frac{8+x}{40+x} = \frac{3}{10}$
 $80 + 8x = 120$
 $x = 5$ L
Therefore, option D is correct.

- 29. (A)** Sand in 60 gram mixture = $60 \times \frac{75}{100} = 45$ gram and water = 15 gram.

After adding 15 gram of sand in mixture, total sand = $45 + 15 = 60$ gram and weight of a mixture = $60 + 15 = 75$ gram.

$$\text{So \% of sand} = 100 \times \frac{60}{75} = 80\%$$

Therefore, option A is correct.

- 30. (C)** Let x be the quantity taken from the first container

$$\text{In } x \text{ litres, the volume of ethanol} = \frac{2x}{5}$$



In x litres, the volume of isopropanol = $\frac{3x}{5}$

Let y be the quantity taken from the second container

In y litres, the volume of ethanol = $\frac{4y}{9}$

In y litres, the volume of isopropanol = $\frac{5y}{9}$

When x litres from the first container and y litre from the second container is mixed, quantity of ethanol in the new mixture

$$= \frac{2x}{5} + \frac{4y}{9}$$

Quantity of isopropanol in the new mixture

$$= \frac{3x}{5} + \frac{5y}{9}$$

According to the question, the ratio of isopropanol to ethanol in the new mixture should be $7 : 5$

$$\Rightarrow \frac{3x}{5} + \frac{5y}{9} : \frac{2x}{5} + \frac{4y}{9} = 7 : 5$$

$$\Rightarrow 5\left(\frac{3x}{5} + \frac{5y}{9}\right) = 7\left(\frac{2x}{5} + \frac{4y}{9}\right)$$

$$\Rightarrow \frac{5(27x + 25y)}{45} = \frac{7(18x + 20y)}{45}$$

$$\Rightarrow 135x + 135y = 126x + 140y$$

$$\Rightarrow 9x = 15y$$

$$\Rightarrow \frac{x}{y} = \frac{15}{9}$$

$$\Rightarrow \frac{x}{y} = \frac{5}{3}$$

$$\Rightarrow x : y = 5 : 3$$

Therefore, option C is correct.

- 31. (B)** Let the ratio in the mixture be $2x, 3x$

$$2x + 3x = 60,$$

$$5x = 60, x = 12$$

Diesel in mixture = 24 L

To make the ratio $1 : 3$, let the mixture be $3x$

$$\text{Thus, } 3 \times 24 = 72$$

Oil to be added in the mixture = $72 - 60 = 12$ L

Therefore, option B is correct.

- 32. (C)** Let the water and dye be $5x$ and $2x$, respectively

$$\text{So, } 5x + 2x = 28$$

$$7x = 28, x = 4$$

Dye in the new solution = $2(4) + 2 = 10$ litres

Then the new ratio will be $30 : 10 = 3 : 1$

Therefore, option C is correct.

PRACTICE QUESTIONS

Questions 1–5: A dairy house has 5000 L of pure milk. It sells 40% of the total milk to six different milk sellers and utilises remaining 60% of total milk in the dairy itself. Also, each seller, A, B, C, D, E, and F mixes water in their milk. The following table shows the sales of milk to six different sellers by dairy as a percentage of total sales and also the concentration of water in each seller's milk.

PERSON	MILK SOLD BY EACH PERSON	WATER ADDED BY EACH
A	24%	24%
B	10%	10%
C	12%	18%
D	7%	15%
E	28%	25%
F	19%	12%



1. Find the difference between total quantity of water added by seller A and C.
 - A. 105 litres
 - B. 91 litres
 - C. 99 litres
 - D. 95 litres
 2. Calculate the respective ratio of quantity of water added by seller C and E.
 - A. 81 : 287
 - B. 3 : 11
 - C. 11 : 3
 - D. 287 : 81
 3. Which seller added the least quantity of water (in litres)?
 - A. A
 - B. C
 - C. B
 - D. D
 4. If sellers A, B, and C add their diluted milk in one can then what will be the concentration of pure milk in that can?
 - A. 70.75%
 - B. 80.25%
 - C. 90.50%
 - D. 60.25%
 5. How many litres of water were added by all the sellers in total? (approximately)
 - A. 500 L
 - B. 450 L
 - C. 440 L
 - D. 490 L
- Questions 6–10:** For his photographic studies, a scientist buys a 50-litre mixture of two chemicals, H_2SO_4 and HCl, on a weekly basis at the rates shown in the line graph and the percentage composition shown in the pie chart. After thoroughly combining them, he sells them to a manufacturing company. Solve the questions below.
6. The average per litre cost of the mixture for the 1st three weeks is:
 - A. $\frac{2863}{75}$
 - B. $\frac{2863}{50}$
 - C. $\frac{2863}{100}$
 - D. $\frac{2863}{25}$
 7. What is the scientist's profit or loss percentage in the fourth week if he sells that week's mixture to the company for 3297?
 - A. 75% loss
 - B. 75% profit
 - C. 50% profit
 - D. 50% loss
 8. During the third week, 20% of the mixture from container P was replaced in container Q. Calculate the ratio of the sum of H_2SO_4 in P and HCl in Q and the sum of H_2SO_4 in P and HCl in Q.
 - A. $\frac{29}{52}$
 - B. $\frac{73}{52}$
 - C. $\frac{113}{52}$
 - D. $\frac{155}{52}$
 9. If the scientist had sold both H_2SO_4 and HCl separately for Rs. 35 and Rs. X per litre in the second week, he would have lost 7%, but if he had sold them at Rs. 28 and Rs. 35 per litre, he would have lost Y per cent. Then what is the product of X and Y?
 - A. $\frac{4536}{17}$
 - B. $\frac{4536}{13}$
 - C. $\frac{4536}{15}$
 - D. $\frac{4536}{11}$
 10. If the scientist secretly converted some percentage of HCl to H_2SO_4 and then mixed them, the cost price difference of the mixture for the same volume increased by



Rs. $(\frac{248}{50})$ in the first week. What was the percentage of HCl that was transformed to H_2SO_4 ?

A. $\frac{557}{14}$

- B. $\frac{575}{14}$
C. $\frac{577}{14}$
D. $\frac{775}{14}$

SOLUTIONS

1. (C) The total quantity of solution = $480 + a$ litre, if seller A adds 'a' litre of water

water = 24%

Thus, 24% of $(480 + a) = a$

$$100a = 24 \times 480 + 24a$$

$$76a = 24 \times 480$$

$$A = \frac{2880}{19} \text{ litres} = \text{approximately } 151.58$$

litres = Quantity of water added by Seller A
Similarly, let the seller C adds c litres of water then

$$8\% \text{ of } (240 + c) = c$$

$$82c = 240 \times 18$$

$$c = \frac{2160}{41} \text{ litres} = 52.68 \text{ litres approximately}$$

= quantity of water added by seller C
required difference = $151.58 - 52.68 = 99$ litres approximately

Therefore, option C is correct.

2. (A) $\frac{2160}{41}$ litres = quantity of water added

by C

Let the seller E adds e litres of water

Then, water = 25% of $(560 + e)$

$$75e = 560 \times 25$$

$$E = \frac{560}{3}$$

$$\text{Required ratio} = \frac{2160}{41} : \frac{560}{3} = 81 : 287$$

Therefore, option A is correct.

3. (C) $\frac{2380}{19}$ litres = quantity of water added

by seller A

$\frac{2160}{41}$ litres = quantity of water added by seller C

$\frac{560}{3}$ litres = quantity of water added by seller E

Let the seller B adds b litres of water then,
10% of $(200 + b) = b$

$$\frac{200}{9} \text{ litres} = \text{approximately } 22.22 \text{ litres}$$

Let the seller D adds d litres of water

$$15\% \text{ of } (140 + d) = d$$

$$D = \frac{140 \times 15}{85} = 24.70 \text{ litres}$$

= quantity of water added by seller D

Let the seller F add f litres of water then,
12% of $(380 + f) = f$

$$88f = 380 \times 12$$

$$f = 51.82 \text{ litres}$$

= quantity of water added by seller F

B added the least quantity (in litres) of water

Therefore, option C is correct.

4. (B) The total quantity of new solution = $920 + 226.36 = 1146.36$ litres

Quantity of milk in the new can = $480 + 200 + 240 = 920$ litres

The required concentration of milk in the new solution

$$= \frac{920 \times 100}{1146.36} = 80.25\% \text{ approximately}$$

Therefore, option B is correct.

5. (D) Total quantity of water added by all the sellers together = $151.58 + 22.22 + 52.68$



$+ 24.70 + 186.67 + 51.82 = \text{approximately}$
 $489.67 \text{ litres} = \text{approximately } 490 \text{ litres}$
 Therefore, option D is correct.

6. (A) Litres of HCl = 64% of 50 litres

$$= \left(\frac{64}{100} \right) \times 50 = \frac{64}{2}$$

Litres of HCl = 32 litres

Litres of H_2SO_4 = $50 - 32 = 18$ litres

$$\text{Average CP of } \text{H}_2\text{SO}_4 = \frac{49 + 28 + 42}{3} = \frac{\text{Rs. } 119}{3}$$

$$\text{Average CP of HCl} = \frac{35 + 42 + 35}{3} = \text{Rs. } \frac{112}{3}$$

Let CP of per litre mixture be Rs. x .

$$\text{Then, } \left(\frac{119}{3} - x \right) 18 \text{ litres} = \left(x - \frac{112}{3} \right) 32 \text{ litres}$$

$$\Rightarrow \frac{119 \times 9}{3} + \frac{112 \times 16}{3} = 16x + 9x = 25$$

$$\Rightarrow \frac{1071 + 1792}{3} = 25x$$

$$x = \text{Rs. } \frac{2863}{75}$$

Therefore, option A is correct.

7. (C) CP for 4th week = Rs. x .

Then, $(49 - x) 32$ litres = $(x - 35) 18$ litres

$$x = \text{Rs. } \frac{2198}{50}$$

CP for 50 litres = Rs. 2198

Profit = Rs. $3297 - 2198 = \text{Rs. } 1099$

$$\text{Profit \%} = \left(\frac{1099}{2198} \right) \times 100 = 50\% \text{ profit}$$

Therefore, option C is correct.

8. (B) 20% of mixture = $\left(\frac{20}{100} \right) \times 50$ litres

= 10 litres (moved from P to Q)

H_2SO_4 in container P = 36% of 10 litres = 3.6 litres

HCl in container P = 64% of 10 litres = 6.4 litres

H_2SO_4 in container Q = 36% of 40 litres = 14.4 litres

HCl in container Q = 64% of 40 litres = 25.6 litres

$$\text{Ratio} = \frac{\text{H}_2\text{SO}_4 \text{ in P} + \text{HCl in Q}}{\text{H}_2\text{SO}_4 \text{ in P} + \text{HCl in Q}}$$

$$= \frac{3.6 + 25.6}{6.4 + 14.4}$$

$$= \frac{29.2}{20.8} = \frac{73}{52}$$

Therefore, option B is correct.

9. (D) In 2nd week, CP of H_2SO_4 = Rs. 28 and

SP = Rs. 35

CP of HCl = Rs. 42 and SP = Rs. X

Total CP = Rs. $(28 \times 18 + 42 \times 32) = \text{Rs. } (504 + 1344) = \text{Rs. } 1848 \dots (1)$

Total SP = Rs. $(35 \times 18 + X \times 32) = \text{Rs. } (630 + 32X)$

Loss = Rs. $[1848 - (630 + 32X)] = \text{Rs. } (1218 - 32X)$

$$\text{Loss \%} = \left[\frac{1218 - 32X}{1848} \right] \times 100 = 7$$

$$(1218 - 32X) = \frac{7 \times 1848}{100}$$

$$32X = \frac{121800}{100} - \frac{12936}{100}$$

$$X = \frac{3402}{100}$$

Also,

Total SP = Rs. $(28 \times 18 + 35 \times 32) = \text{Rs. } (504 + 1120) = \text{Rs. } 1624$

Loss = Rs. $[1848 - 1624] = \text{Rs. } 224$

$$\text{Loss \%} = Y = \left(\frac{224}{1848} \right) \times 100 = \frac{400}{33}$$

$$\text{Then, } X \times Y = \left(\frac{3402}{100} \right) \times \left(\frac{400}{33} \right) = \frac{3402 \times 4}{33}$$

$$= \frac{4536}{11}$$

Therefore, option D is correct.

10. (D) New volume of HCl = $32 - s$, if s litres of HCl be converted.

New volume of H_2SO_4 = $18 + s$

CP of mixture per litre for the 1st week = Rs. x .

Then, $[49 - x] (18 \text{ litres}) = [x - 35] (32 \text{ litres})$

$$882 - 18x = 32x - 1120$$

$$50x = 1120 + 882 = 2002$$

$$x = \text{Rs. } \frac{2002}{50}$$



Then cost price of mixture will be

$$\frac{2002}{50} + \frac{248}{50} = \frac{2250}{50} = 45$$

$$(18 + s)(49 - 45) = (45 - 35)(32 - s)$$

$$4(18 + s) = 10(32 - s)$$

$$72 + 4s = 320 - 10s$$

$$14s = 320 - 72 = 248$$

$$S = \frac{248}{14} = \frac{124}{7}$$

$$\% \text{ of HCl} = \left[\frac{\frac{124}{7}}{32} \right] \times 100 = \frac{775}{14}$$

Therefore, option D is correct.

34 Blood Relations



The questions from this section are generally easy to address, but they are framed in a challenging manner. The test takers are expected to address these questions attentively and draw a flowchart to arrive at the ultimate result when answering the questions in this part.

A blood relationship between two persons is described as a relationship formed by their birth or marriage or any other circumstance. Blood Relation questions require analysing data that demonstrate the blood relation between family members. A chain of relationships is provided to the candidates in the form of information, and on the basis of this information, the candidates are required to determine the relationship between two members of the chain.

Students should be familiar with various family hierarchical structures, which are normally three generations above and below the current generation. So, in a nutshell, Blood Relation involves questions aimed to identify a link between family members based on the facts provided in the question. For example, any relation through birth will be mother, father, son, daughter, and so on. Any marriage relationship will have a father-in-law, mother-in-law, sister-in-law, and so on.

Before proceeding with this chapter, here is a list of the common relationship terms you must know:

List of male relations [Males can be denoted by a (+) or a square]:

TERMS	RELATION
Mother's or father's son	Myself/Brother
Mother's or father's brother	Uncle

Mother's or father's brother	Uncle
Mother's or father's father	Grandfather
Daughter's husband	Son-in-law
Husband's wife's brother	Brother-in-law
Brother's son	Nephew
Uncle or aunt's son	Cousin
Sister's husband	Brother-in-law
Brother of wife	Brother-in-law
Brother of husband	Brother-in-law

List of female relations [Females can be denoted by a (-) or a circle]:

TERMS	RELATION
Mother's or father's daughter	Myself/Sister
Mother's or father's sister	Aunt
Mother's or father's mother	Grandmother
Son's wife	Daughter-in-law
Husband's wife's sister	Sister-in-law



Brother's daughter	Niece	Grandfather's only daughter-in-law	Mother
Uncle or aunt's daughter	Cousin	Grandmother's only daughter-in-law	Mother
Brother's wife	Sister-in-law	The only daughter of your father	Yourself
Sister of wife	Sister-in-law	Wife of your father	Mother
Sister of husband	Sister-in-law		

Hierarchy of blood relationships:

Stage 1	Grandparents	Grandfather, grandmother
Stage 2	Parents and in-laws	Father, mother, father-in-law, mother-in-law, uncle, aunt
Stage 3	Siblings, spouse, and in-laws	Brother, sister, wife, husband, brother-in-law, sister-in-law
Stage 4	Children and in-laws	Son, daughter, niece, nephew, son-in-law, daughter-in-law
Stage 5	Grandchildren	Grandson, granddaughter

Maternal: Relations on the mother's side are called maternal relations or relatives. For example, mother's brother is called 'maternal uncle.'

Paternal: Relations on the father's side are called paternal relations or relatives. For example, father's father is called 'paternal grandfather.'

There are three kinds of questions that can be asked under blood relations:

1. Dialogue/Conversation Based: In these types of questions, one person describes his/her relationship with another person, which may or

may not be related to the person with whom the conversation is being made.

Example:

Karan introduces Imran saying, "He is the husband of the granddaughter of the father of my father." How is Imran related to Karan?

- A. brother-in-law
- B. son-in-law
- C. father
- D. brother

Answer: Father's father—grandfather; grandfather's granddaughter—sister; sister's husband—brother-in-law. Therefore, Imran is Karan's brother-in-law. The right option is A.



2. Based on Puzzles: Blood relations based on puzzles contain a piece of brief information about multiple people being interrelated and sub-questions based on the same can be asked. It is easier to solve these questions by drawing a family tree. A family tree is a pictorial representation of genealogical data. Here are some points to note while drawing family trees for blood relations questions:

- Usually, the names of female members of the family are written inside a circle and the names of male members are written inside a square for differentiation.
- The family members are ranked in a hierarchical order, i.e., the older generations are at the top of the family tree and the latest generation members are at the bottom of the family tree. Refer to the earlier table ‘hierarchy of blood relationships’ on the first page.
- While most relationships in the family tree are connected by a normal arrow, you can differentiate relationships (for example, to denote a couple, by using an arrow, like this \rightleftharpoons or an equals to (=) sign).

SYMBOL	MEANING
+ or □	Male
- or ○	Female
\rightarrow or \leftarrow	Siblings (Brother or sister)
= or \rightleftharpoons	Husband-wife
\uparrow or \downarrow	Generation gap

Example:

Six friends, namely P, Q, R, S, T, and U, are shopping in a shopping mall. P and T are brothers. U is the sister of T. R is the only son of P’s uncle. Q and S are the daughters of the brother of R’s father. R’s father has only one brother.

1. What is the relation of R with U?

- A. Brother B. Sister
C. Cousin D. Uncle

Answer: U is T’s and hence P’s sister. R is also the son of U’s uncle and is, therefore, U’s cousin. Hence, the answer is option C.

2. How many boys are in this group of friends?

- A. 1 B. 2
C. 3 D. 4

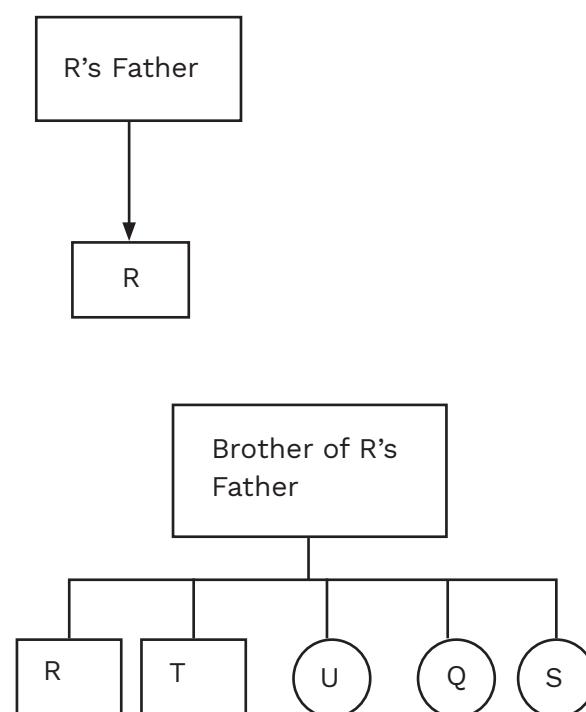
Answer: P and T are brothers, so they both are males. U is the sister of T, hence female. R is the son, hence male. Q and S are daughters, so they both are female. There are a total of three boys, so the answer is option C.

3. What is the relation of S with P?

- A. Uncle B. Cousin
C. Sister D. None of these

Answer: S’s father is the brother of R’s father and R’s father is P’s uncle. So, S’s father is also P’s father. Thus, S is P’s sister, and the right option is C.

Given below is the family tree for this example:





- 3. Coding-Decoding:** Under this type, the relationships are represented by certain codes or symbols like +, *, and =, which are to be decoded.

Example:

If P + Q means P is Q's mother; P # Q means P is the Q's brother; P @ Q means P is the Q's son and P * Q means P is the Q's daughter, which of the following means X is the niece of Y?

- A. Y * X B. Y @ Z * X
C. X * Z # Y D. Z + Y # X

Answer: X is the niece of Y means X is the daughter of the brother (say Z) of Y, i.e., X * Z # Y. Hence, option C is the correct answer.

Important things to keep in mind while solving blood relations questions:

1. The gender of a person should never be inferred from their given name. For example,

generally, Ram is the name of a boy but you are not expected to assume this on your own. In these types of questions, Ram can be the name of a girl as well.

2. If a particular statement says A is the son of B, the gender of B cannot be ascertained unless it is indicated in the question. B can be A's mother or father.
3. Make family trees using the information provided in the questions, since this will help you in answering the questions more quickly.
4. Visualising the scene and putting yourself in the shoes of one of the characters may help you in quickly resolving questions concerning blood relations.
5. The term 'only son' or 'only daughter' does not necessarily imply "only child." If the question states that A is B's only son, A could have one son and one daughter, or simply one son.

PRACTICE QUESTIONS

1. A lady, while referring to a person in an album, said, "His sister's father is the only son of my grandmother." How is the lady related to the person in the album?
A. sister-in law B. mother
C. daughter D. sister
2. Pointing out an officer, a lady said, "His wife is the only daughter of my father." How was that officer related to the lady?
A. husband B. father
C. brother D. none of the above
3. Waving to a woman, Kabir said, "She is the daughter of the woman who is the mother of the husband of my mother." Who is the woman to Kabir?
A. sister B. mother
C. aunt D. daughter
4. X is the son of Y while Y and W are sisters. L is the mother of W. If K is the son of L, find the correct statement?

- A. L is the brother of Y
B. K is the cousin of X
C. Y and K are sisters
D. K is the maternal uncle of X

(Questions 5 to 9) Read the following information carefully and answer the questions.

There are six family members, namely A, B, C, D, E, and F, living at Sharma's residence. There is one teacher, one singer, one dancer, one painter, and two professors in the family. There are two couples. B is a teacher and E's father. F is C's grandfather and is a singer. D is the grandmother of E and is a painter. Both the grandchildren are professors.

5. Who is the husband of A?
A. C B. F
C. B D. D
6. Who is the sister of E?
A. C B. E
C. B D. Information insufficient



7. Which of the following are two couples?
A. FD, BE B. FD, BA
C. ED, CF D. FD, CA
8. Which of the following groups is definitely of male members?
A. BF B. BFE
C. BFA D. FE
9. What is the profession of A?
A. teacher B. dancer
C. teacher or dancer D. professor

(Questions 10 to 13) Read the following information carefully and answer the questions.

There are 6 family members, namely P, Q, R, S, T, and U, in a family who are planning to go on a trip to Uttarakhand in October. T is R's brother. S is P's daughter. P and R are married to each other. U is Q's brother. Q is R's son, but R is not Q's mother.

10. How many of the members of the family are male?
A. 1 B. 3
C. 2 D. 4
11. Who is the mother of Q?
A. S B. U
C. T D. P
12. What is the number of children that P has
A. 1 B. 2
C. 3 D. 4
13. Who is the wife of T?
A. P B. U
C. Q D. Can't be determined

(Questions 14 to 20) Answer the questions based on the information given below.

X + Y means X is Y's father.

X - Y means X is Y's wife.

X × Y means X is Y's brother.

X ÷ Y means X is Y's daughter.

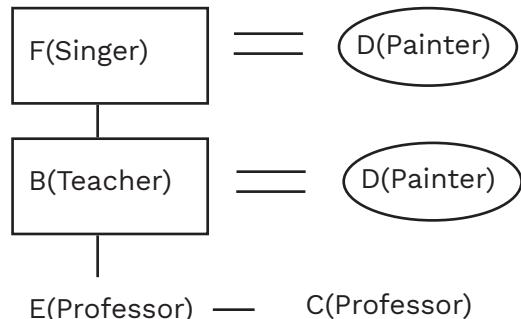
14. If A ÷ C + D + B, which of the following is

- correct?
A. A is B's mother
B. A is B's daughter
C. A is B's aunt
D. B is A's aunt
15. If A - C + B, which of the following statements is correct?
A. A is B's mother
B. B is A's daughter
C. A is B's aunt
D. A is B's sister
16. If A × C ÷ B, which of the following statements is correct?
A. A is B's uncle
B. A is B's father
C. A is B's brother
D. A is B's son
17. If A × C - B, which of the following is correct?
A. A is the brother-in-law of B
B. A is the brother of B
C. A is the uncle of B
D. A is the father of B
18. If A + C ÷ B, which of the following is correct?
A. A is B's brother
B. A is B's son
C. A is B's husband
D. A is B's father
19. If A ÷ C + B, which of the following is correct?
A. A is the father of B
B. A is the brother of B
C. A is the mother of B
D. A is the sister of B
20. If A - C × B, which of the following is correct?
A. A is the sister of B
B. B is the husband of A
C. A is the sister-in-law of B
D. B is the son of A



SOLUTIONS

- (D)** Only son of the lady's grandmother—lady's father. Person's sister's father—person's father. So, the lady is that person's sister.
- (A)** The only daughter of my father—myself. Therefore, the officer is the lady's husband.
- (C)** Mother's husband—father. Father's mother—grandmother. Grandmother's daughter—father's sister. Father's sister—aunt. Therefore, the woman is Kabir's aunt.
- (D)** Y and W are sisters. So, L is the mother of W means L is the mother of both Y and W. K is the son of L means L is the brother of Y. Thus, X is the son of Y means K is the maternal uncle of X.



(Solutions 5-9):

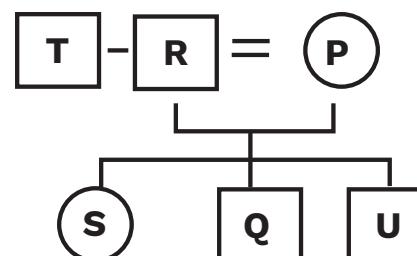
The teacher B is the father of E. The painter D is the grandmother of E and hence the mother of B. Since there are only two couples, one being that of B, C's grandfather (i.e., F) must be married to D. Thus, C and E are both children of B and are the professors. So, A who is remaining is the wife of B and she alone can be the dancer. Thus, F must be the singer.

- (C)** B is the husband of A.
- (D)** C and E are children of the same parents, but their gender is not given. Therefore, the information is insufficient to conclude the relation between C and E.
- (B)** The two couples are B, A and F, D.
- (A)** The father B and the grandfather F are definitely male.
- (B)** A is the dancer.

(Solutions 10-13):

Q is R's son, but R is not the mother of Q means that R is the father of Q. P is married to R so P must be Q's mother. U is the brother of Q, so U is the son of P and R. S is the daughter of P means she is the daughter of P and R.

- (D)** P—mother, Q—son, R—husband, S—daughter, T—brother, U—son. Hence, there are 4 males.



- (D)** P is the mother of Q.
- (C)** P has 3 children—son Q, son U and daughter S.
- (D)** The wife of T cannot be determined from the data available.
- (C)** $A \div C + D + B$ means A is the daughter of C, who is the father of D, who is the father of B, i.e., A is the sister of D, who is the father of B. Therefore, A is the aunt of B.
- (A)** $A - C + B$ means A is the wife of C, who is B's father. Hence, A is the mother of B.
- (D)** $A \times C \div B$ means A is the brother of C, who is the daughter of B. Hence, A is the son of B.
- (A)** $A \times C - B$ means A is the brother of C who is the wife of B. Hence, A is the brother-in-law of B.
- (C)** $A + C \div B$ means A is the father of C, who is the daughter of B. So, A and B are the father and mother of C, respectively. Hence, A is the husband of B.
- (D)** $A \div C + B$ means A is the daughter of C who is the father of B. Hence, A is the sister of B.
- (C)** $A - C \times B$ means A is the wife of C who is the brother of B. Hence, A is the sister-in-law of B.



Coding-decoding is a process of encrypting and decrypting any word, letter, set of patterns, sentences, or codes based on certain rules. In these types of questions, certain code values are assigned to a set of words or phrases and you have to find the original words and phrases.

When any letter/word/phrase is written in a way that hides the actual meaning of the particular word/phrase/sentence, then it is called coding. The process of finding the actual sentence from the hidden one is called decoding. Some useful points to solve such questions are:

1. Forward order of letters

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

2. Reverse order of numbering

A	B	C	D	E	F	G	H	I	J	K	L	M
26	25	24	23	22	21	20	19	18	17	16	15	14
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	12	11	10	9	8	7	6	5	4	3	2	1

3. Opposite letters

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
Z	Y	X	W	V	U	T	S	R	Q	P	O	N
26	25	24	23	22	21	20	19	18	17	16	15	14

To arrive quickly to the nearest possible position, use the concept of

E	J	O	T	Y
5	10	15	20	25



TYPES OF CODING-DECODING

1. Letter-to-letter coding

Here, the alphabets are coded using other letters in their place and are mostly opposites of each other. It can be from Table 2 like writing N in place of A or vice versa or it can be from Table 3 like Z in place of A or vice versa.

Illustration

In a certain language, BUNNY is written as YFMMB. What will be SHORT written in the same language?

- | | |
|----------|----------|
| A. HSLIG | B. HGSIL |
| C. HLGIS | D. HFLIS |

Answer: A

It can be seen that in the sequence:

A B C D E F G H I J K L M
Z Y X W V U T S R Q P O N.

The word stated in the question and its code are opposite pairs like B is Y, U is F, and so on.

2. Letter-to-number coding

Here, numerical code values are assigned to a word or alphabetical code letters are assigned to the number.

Illustration

If BIRD is written as 2518923. What will BASKET be written as?

- A. 2119115201
- B. 2526816227
- C. 2534516727
- D. 4567782317

Answer: B

It can be seen that the number allotted is as per reverse order of placement, which means, Z is 1 and A is 26. It can be seen that in BIRD B is 25, I is 18, and so on as per their reverse order placement in alphabets.

3. Substitution coding

In this type of coding and decoding, words of different statements are coded with letters, symbols, and numbers using different operations. We need to find logic in the operations.

Illustration

If in a certain language, pen is paper, paper is plastic, plastic is black, black is bird. On what do we write according to this code language?

- | | |
|------------|----------|
| A. Pen | B. Paper |
| C. Plastic | D. Bird |

Answer: C

We write on paper and in this language, paper is plastic. So, we write on plastic.

4. Conditional coding

Here, a few conditions will be provided and candidates need to answer based on the logic derived from the conditions.

Illustration

CODE	B	A	T
DIGIT	31	58	90

Conditions

- If the word starts with a consonant, then its digit is reversed in the code.

What will TAB be written as in this code?

- A. 905831
- B. 315890
- C. 135890
- D. 095831

Answer: D

Since it starts with T, a consonant, so the digit of T is reversed; others are written as per their digits.

5. Word coding

Here, a few statements consisting of the same words but in a different order will be coded as words, symbols, or letters. The same words are to be decoded with the common word as present in the code.

Illustration

If in a certain language 'ki ou ti' means 'I like singing', 'ti ji ka' means 'singing is good', 'ka ki ta' means 'I good person'. Then what is the code for 'good singing'?

- A. ti ka
- B. ti ji
- C. ki ti
- D. ou ti

Answer: A

From 1st and 2nd statements, singing = ti

From 2nd and 3rd statements, good = ka



PRACTICE QUESTIONS

1. In a certain language, ENGLISH is written as FPJPJNYO. What will SCIENCE be, if written in this language?
A. TEKGRFL B. TELGSFJ
C. TELISIL D. TELISIJ
2. In a certain language, TRIANGLE is written as SSHBMHKF. What will CREATIVE be written in the same language?
A. DQDBUHWF B. BSDBSKUG
C. BSDBSJUG D. BSDBSJUF
3. In a certain language, BEAUTY is written as YVZFGB. What will CONSIDER be written in the same language?
A. XMLHPWVS
B. XMLGOWVS
C. XLMHRWVI
D. XLMHSWVI
6. In a certain language, DRAGON is written as QENTBA. What will SQUARE be written in the same language?
A. HJFZIV B. FDHNER
C. HJFNER D. FCHNER
7. In a certain language, CAPITAL is written as MBUQBD. What is IMPORT written in the same language?
A. USPQNJ B. USPRNK
C. URPQNJ D. URPRNK
8. In a certain language, A is coded as 1, B is coded as 2, and so on. How will AROUND be written in this language?
A. 1171520144 B. 1181521134
C. 1181521144 D. 1181520144
9. If in a certain code, FUTURA is coded as 204091 and BUTTERY is coded as 5044798. What is BEAUTY, in the same language?
A. 571048 B. 571281
C. 581048 D. 581281
10. In a certain code, 1 is coded as Z, 4 is coded as S, 8 is coded as Q, 9 is coded as E, 5 is coded as V. What is 854891 according to the code?
A. QVSQZE B. QSVQZE
C. QVSQEZ D. QVQSEZ
11. In a certain code, CUTE is coded as 49. What will BOAT be coded as?
A. 37 B. 38
C. 39 D. 40
12. In a certain code, E is coded as 3, G is coded as 8, Y is coded as 7, N is coded as 5, R is coded as 1, A is coded as 6. What is ENERGY in the same code?
A. 363178 B. 358317
C. 353187 D. 353168
13. In a certain code, orange is called blue, blue is called green, green is called white, white is called black, black is called grey. What color is the grass?
A. Orange B. Blue
C. Green D. White
14. In a certain code, bus is called car, car is called taxi, taxi is called train, train is called airplane, and airplane is called brake. What is actually written in this language, if it is written as ‘car brake’?
A. Bus airplane B. Taxi bus
C. Taxi airplane D. Bus taxi
15. In a certain code language, ‘sim op tim’ means ‘apple is green’, ‘op ho tap’ means ‘green and black’, and ‘ho tim ko’ means ‘shirt is black’. Which of the following represents apple in that language?
A. sim B. op
C. tim D. tap
16. In a certain code ‘nae po tam’ means ‘sky is blue’, ‘me tam sam’ means ‘she is cute’, and ‘ism po me’ means ‘she stares sky’. What is ‘cute blue sky’ in this language?
A. me po tam B. sam nae po
C. nae me tam D. tam op sam



- 17.** In a certain code, '289' means 'read from book', '276' means 'tea from field' and '85' means 'wall book'. What is the code for 'field'?
- 2
 - 7
 - 6
 - Cannot be determined

- 18.** If in a certain code, '456' means 'she is pretty', '563' means 'she is cute', '689' means 'it is cold'. What will be the code for 'cute is pretty'?
- 653
 - 564
 - 543
 - 643

- 19.** If in the word EXISTENCE, each vowel is replaced by the 2nd next letter and each consonant is replaced by the previous letter and all the letters are arranged in

alphabetical order, then which letter is 4th from the right?

- K
- O
- R
- M

20.

CODE	O	R	D	E	T
DIGITS	5	2	6	3	7

Conditions:

If the first and last digits are odd, both are written as X

If the first and last digits are even, both are written as Y

With the following conditions and code, what will 253672 be written as?

- ROEDRR
- XOETRY
- YOEDTR
- YOEDTY

SOLUTIONS

1. (C)

E	N	G	L	I	S	H
+1 F	+2 P	+3 J	+4 P	+5 N	+6 Y	+7 O
S	C	I	E	N	C	E
+1 T	+2 E	+3 L	+4 I	+5 S	+6 I	+7 L

- 2. (D)** Subtract 1 from the first, third, fifth letters and so on and add 1 to the second, fourth, sixth, and so on.

- 3. (C)** Write the alphabets in the order as given in the table of reversing alphabets

A B C D E F G H I J K L M

Z Y X W V U T S R Q P O N.

We see that the letters are opposite to each other. For B there is Y, for E there is V, and so on.

- 6. (B)** Write the letters in the normal sequence

A B C D E F G H I J K L M

N O P Q R S T U V W X Y Z

We see the answer forms opposite pairs. For D it is Q, for R it is E, and so on.

- 7. (A)** Reverse the word and then add 1 to each letter of the reversed word.

- 8. (C)** As given in question A is 1, B is 2, and so each letter is coded as the number on which it falls in the alphabets.

- 9. (A)** As given in the question

F U T U R A B E Y
2 0 4 0 9 1 5 7 8

So, Beauty can be written in the same code where B is 5, E is 7, and so on.

- 10. (C)** 854891 is coded as QVSQEZ.

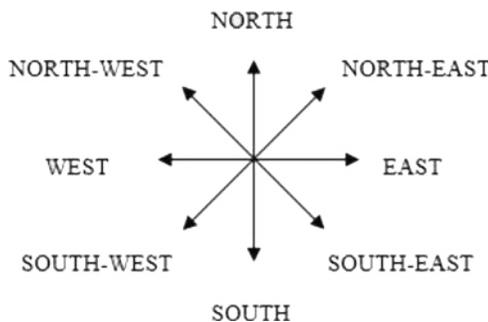
- 11. (B)** C (3) + U (21) + T (20) + E (5) = 49. In the same way, B (2) + O (15) + A (1) + T (20) = 38.

- 12. (C)** ENERGY can be written as 353187.



13. **(D)** We know grasses are green. As given, green is called white, so white is the color of grass.
14. **(A)** In this language, car means bus in the actual language and brake means airplane in the actual language.
15. **(A)** From the 1st and 3rd statements, is = tim
From the 1st and 2nd statements, green = op
From 1st statement and above, apple = sim
16. **(B)** From the 1st and 3rd statements, sky = po
From the 2nd statement, cute = sam
From the 1st statement blue = nae.
17. **(D)** We cannot find 'field' as we have no idea what 'tea' is and there is no other code with 'field'.
18. **(D)** From the 1st statement, pretty = 4
From the 1st, 2nd, and 3rd statements, is = 6
From the 2nd statement, cute = 3
19. **(D)** EXISTENCE, replacing all the letters we get GWKRSGMBG. Arranging the letters, we get BGGGKMRSW. Therefore, M is the 4th letter from the right.
20. **(D)** Since both first and last digits are even, it is written as Y and others are compared with the table.

36 Directions



Directions help us to trace a path from one place to another. Distance is a measurement of the position of one thing with respect to another thing or a reference point. The shortest distance between two different points is known as displacement. There are mainly 4 directions: north, south, east, and west. There are some other directions which fall between the 4 main directions, namely north-east, north-west, south-east and south-west.

The problems based on direction and distance will be regarding the movement of a person or an object from a starting point to an endpoint. The questions will provide the directions and magnitude on the basis of which the questions need to be solved.

The questions will be asked on the following basis:

1. Turns and rotations

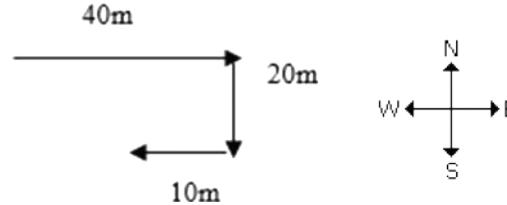
In this type of question, you will be provided with rotations like clockwise or anticlockwise or left or right and you need to find the final direction you will be facing. A right turn is clockwise and a left turn is anticlockwise.

Example

Ram walks 40 m towards east and then takes a right turn and walks 20 m and then turns right again and walks 10 m. Which direction is he facing now?

- A. North
- B. South
- C. East
- D. West

Answer: D



Distance and displacement

Here, you need to find the distance between the start point and endpoint. The shortest distance between start and endpoints is known as displacement. The shortest distance from a particular point after travelling distance x metres in the horizontal direction and y metres in the vertical direction is $(x^2 + y^2)^{1/2}$.

Illustration

Atul starts cycling from home towards his school. On the way to his school, he takes three left turns. He takes each one of the left turns after cycling 40 m straight, 50 m straight, and 60 m, respectively, and then 80 m straight. What is the minimum distance from his house to school?

- A. $10(13)^{1/2}$ m
- B. $5(13)^{1/2}$ m
- C. 10 m
- D. 50 m

Answer: A

His path can be seen as $80 - 50 = 30$ m, $60 - 40 = 20$ m.

Minimum distance = $[(30)^2 + (20)^2]^{1/2} = 10(13)^{1/2}$ m.

2. Shadow based

Shadows are formed with respect to the position of the Sun. In this type of question, you will be asked either the direction of the shadow or the direction of the Sun. Some important points to be remembered are:

- At the time of sunrise if a man stands facing the east, his shadow is in the west, i.e., behind him. At the time of sunset, the shadow of an object is always towards the east.



- If a man is facing north, at the time of sunrise his shadow will be towards his left and at the time of sunset, it will be towards the right.
- At noon, there will be no shadow.

Illustration

In the evening, Seeta is standing in her garden and facing west, in which direction will her shadow fall?

- A. North B. South
C. East D. West

Answer: C

During sunset, shadows are always formed in the east no matter what direction you face.

3. Direction puzzle

Here, the persons are seated in rows and then scattered or shuffled in different directions.

For a person facing north, right is east and left is west. The opposite is true for the south.

Illustration

Ram, Shyam, and Seeta are standing in a line one after another facing north. Ram takes two steps forward, Seeta turns right and walks two steps. Shyam turns 180 degrees. What are the directions the three are facing respectively?

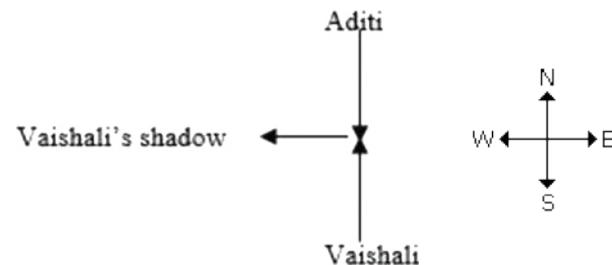
- A. North, South, East
B. South, North, East
C. South, North, West
D. North, South, West

Answer: A

Ram was facing north and just moved two steps forward so he is still facing north. Shyam turned 180 degrees, and so from the north, he turned to south and Seeta turned right from the north which is east.

PRACTICE QUESTIONS

1. Mr. X starts walking from his house to his office. He walks 5 km north, then takes a right turn and walks 10 km, he takes a left turn and walks 2 km to reach the office. Which direction is he currently facing with respect to the start point?
A. North B. North-east
C. East D. South-east
2. In the evening a man is walking from a park to his house. He walks 5 m towards the west then takes a turn, then walks 3 m towards the south and then takes a left turn, and walks 5 m. Which direction is he facing?
A. North B. South
C. East D. West
3. The door of Arun's house faces east. He walked 200 m from the door, turned right and walked 100 m, then turned left and walked 100 m to reach his office. In which direction is he with respect to the door?
4. Chintu walks southwards and then turns 45 degrees right and then takes a left turn. In which direction is he walking now?
A. South-west B. North-west
C. North-east D. South-east
5. One morning, Aditi and Vaishali were talking to each other face to face at a crossing. If Vaishali's shadow was exactly to the right of Aditi, which direction was Aditi facing?
A. East B. West
C. North D. South





6. Adil is in the east of Shyam, who is in the north of Zio. If Pankaj is in the south of Zio, then in which direction of Adil is Pankaj?
A. North B. South
C. South-east D. South-west
7. If north-east becomes north-west, north becomes west and so on, what will south become?
A. South-west B. South-east
C. East D. North-east
8. If south-east becomes north, north-east becomes west and so on, which direction will be the new south?
A. North B. North-west
C. West D. East
9. Ram put his timepiece on the table in such a way that at 6 A.M., the hour hand points to north. In which direction will the minute hand point at 10:15 A.M.?
A. South-east B. South
C. North D. West
10. One evening before sunset, Rina and Hetal were talking to each other face to face. If Hetal's shadow was exactly to the right of Hetal, which direction was Rina facing?
A. North B. South
C. East D. Data inadequate
11. A person rode southward, then turned right and rode 1 km and again turned right and rode 2 km. He found himself 1 km west of his starting point. How far did he ride southward initially?
A. 1 km B. 2 km
C. 3 km D. 4 km
12. Karan is 30 m south-west of Lina. If Mala is 30 m south-east of Lina, then Mala is in which direction of Karan?
A. East B. West
C. North-east D. South
13. Shina goes 4 km east, then turns right and goes 5 km. Then turns left and goes 4 km and then turns left and goes 5 km. At what distance is Shina from the starting point?
A. 5 km B. 6 km
C. 7 km D. 8 km
14. The length and breadth of a room are 3 m and 4 m, respectively. A cat runs along all four walls and finally along a diagonal order to catch a rat. How much total distance is covered by the cat?
A. 16 m B. 18 m
C. 19 m D. 17 m
15. One morning, Gita started to walk towards the sun. After covering some distance, she turned to the right, then again to the right, and after covering some distance she again turns to the right. Now in which direction is her shadow facing?
A. To her right
B. To her left
C. Behind her
D. In front of her
16. P started from his house towards west. After walking a distance of 20 m, he turned to the right and walked 5 m. He then again turned to the right and walked 15 m. After this, he is to turn right at 135° and to cover 30 m. In which direction is he going?
A. West B. South
C. South-west D. South-east
17. If $A \times B$ means A is to the south of B; $A + B$ means A is to the north of B; $A \% B$ means A is to the east of B; $A - B$ means A is to the west of B; then in $P \% Q + R - S$, S is in which direction with respect to P?
A. North-east B. North
C. South D. South-west
18. One morning after sunrise, Sanika was standing facing a pole. The shadow of the pole fell exactly to her right. To which direction was she facing?
A. East B. West
C. North D. South



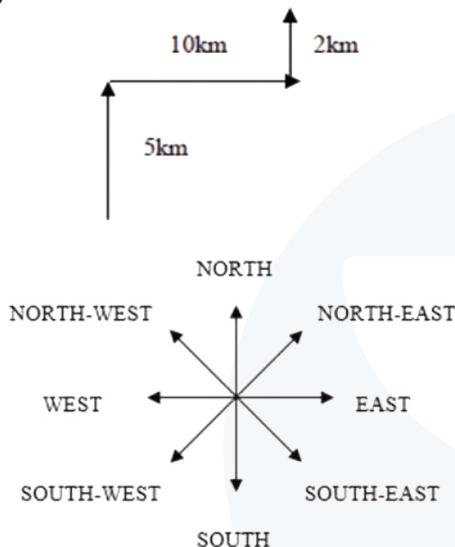
19. Kiyu started from his house, walked 10 km north, then 4 km west, and then 13 km south. How far is he from his house?
- A. 5 km B. 6 km
C. 10 km D. 4 km
20. Anita faces north and covers 24 km, turns west and covers 12 km, then turns south

and covers 6 km, and turns west again and covers 12 km. How far is she from the starting point and in which direction?

- A. 20 km north-east
B. 20 km north-west
C. 30 km north-east
D. 30 km north-west

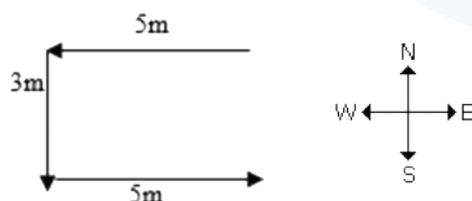
SOLUTIONS

1. (B)



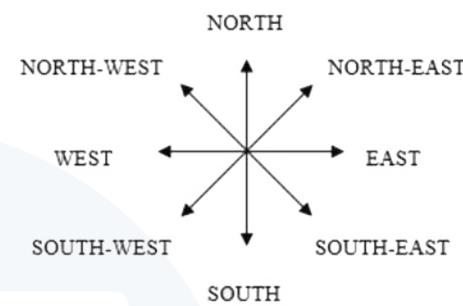
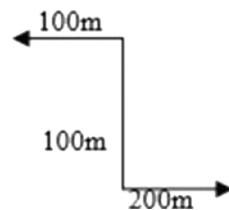
Option B is correct. Mr. X is in north-east direction with respect to the start point.

2. (C)



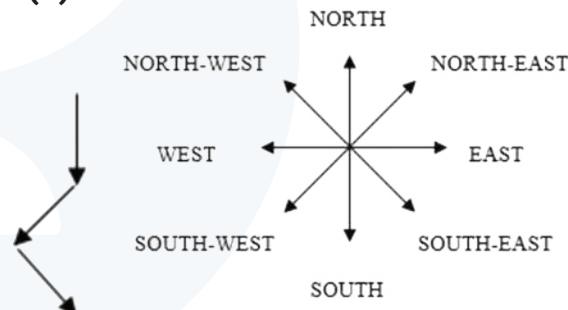
Option C is correct. The man is facing east.

3. (B)



Option B is correct. Arun is in north-west direction with respect to the door.

4. (D)

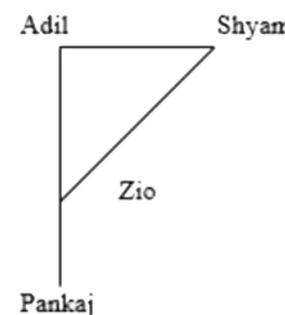


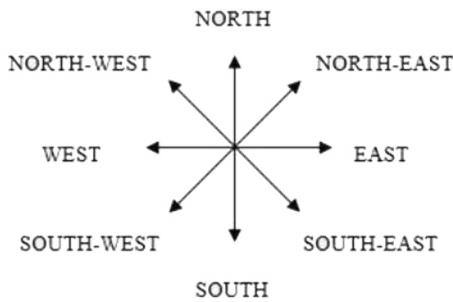
Hence, the direction is south-east. The correct answer is option D.

5. (D)

Aditi is facing south. Therefore, the correct answer is option D.

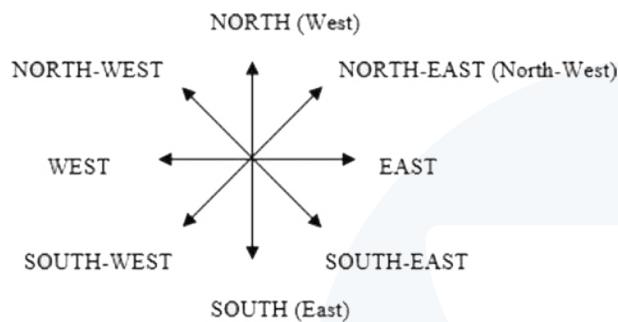
6. (D)





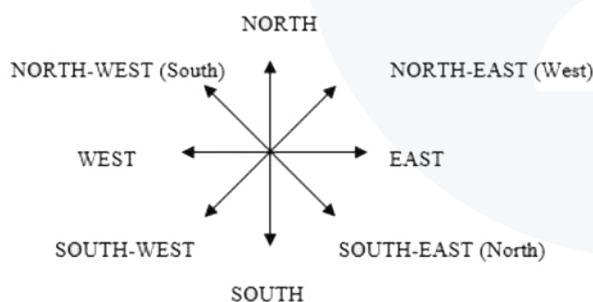
Pankaj is in the south-west of Shyam. Therefore, the correct answer is option D.

7. (C)



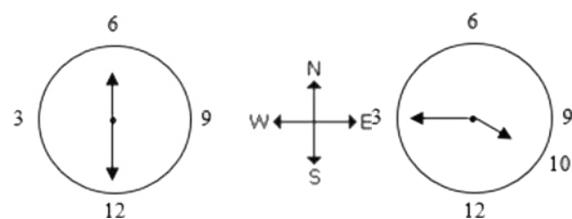
South will become east as per new details. Therefore, the correct answer is option C.

8. (B)



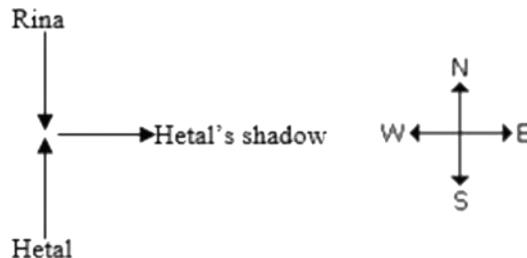
North-west will be the new south as per the diagram above. Therefore, the correct answer is option B.

9. (D)



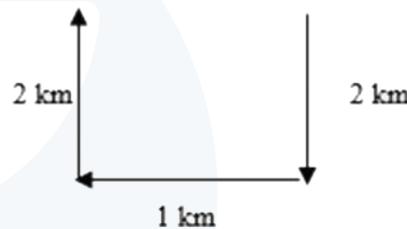
The minute hand will point in West direction at 10.15 A.M. Therefore, the correct answer is option D.

10. (B)



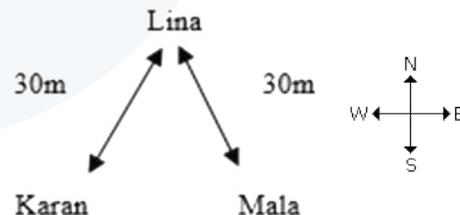
In the evening, sun sets in west. Hence then any shadow falls in the east. Since Hetal's shadow was to the right of Hetal, Rina was facing towards South. Therefore, the correct answer is option B.

11. (B)



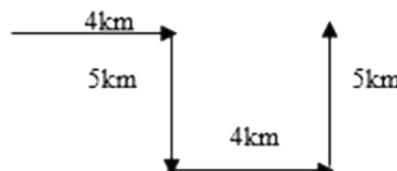
He rode 2 km southward. Therefore, the correct answer is option B.

12. (A)



Mala is in the east of Karan. Therefore, the correct answer is option A.

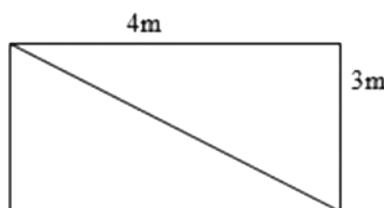
13. (D)



The required distance is $4 \text{ km} + 4 \text{ km} = 8 \text{ km}$. Therefore, the correct answer is option D.



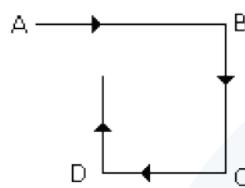
14. (C)



Using Pythagoras theorem, the diagonal length will be = 5 m. Total distance: $3 + 4 + 5 = 12$ m

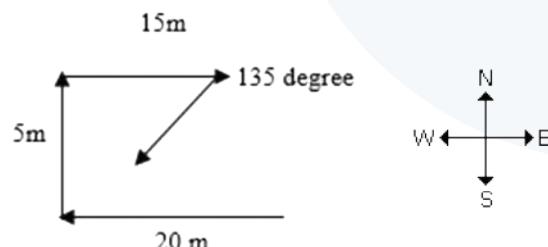
Therefore, the correct answer is option C.

15. (B)



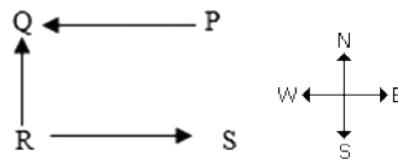
At the end, Gita is facing north. At the time of sunrise, the sun is in the east direction, and hence the shadow points towards the west at that time. As she is facing the north, then west is situated to her left. Therefore, her shadow is located to her left. Therefore, the correct answer is option B.

16. (C)



P is going in the south-west direction. Therefore, the correct answer is option C.

17. (C)

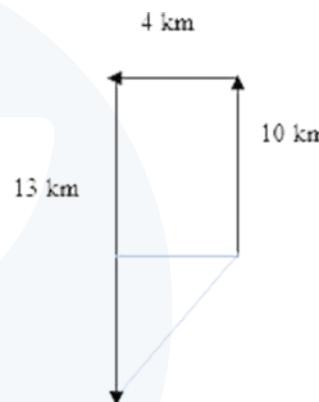


S is to the south of P. Therefore, the correct answer is option C.

18. (D)

Sun rises in the east in the morning. Since the shadow of Suresh falls to his right he is facing south. Therefore, the correct answer is option D.

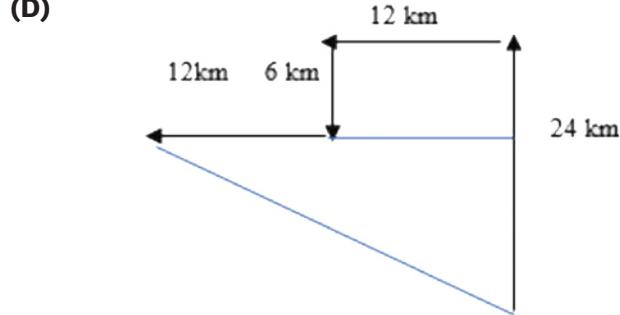
19. (A)



$13 \text{ km} - 10 \text{ km} = 3 \text{ km}$ is the breadth.
4 km is the length.
So, distance will be 5 km after applying Pythagoras theorem
 $(3^2 + 4^2 = 5^2)$
Therefore, the correct answer is option A.

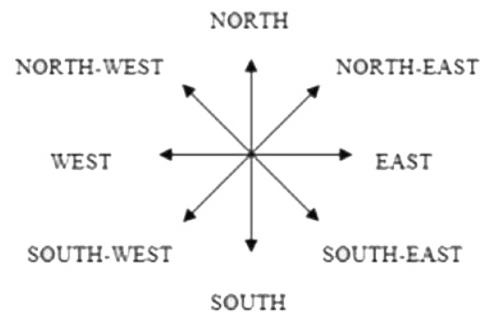


20. (D)



$$[(24)^2 + (18)^2]^{1/2} = 30 \text{ km}$$

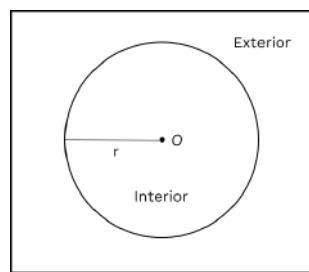
And it is north-west. Therefore, the correct answer is option D.



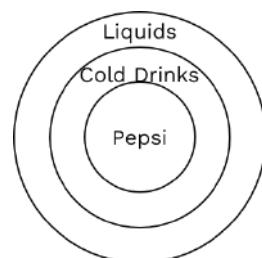


Venn diagram deals with questions that aim at analysing a candidate's ability to relate a certain group of given things or items and represent it diagrammatically. Generally, it uses circles to show the relations among things or finite groups of things. It helps to visually represent the similarities and differences between two concepts. The overlapping circles have something in common while circles that do not overlap do not have anything in common. For example, tomatoes and potatoes are two different things. So their circles won't overlap. However, if we consider tomatoes, potatoes, and vegetables, the circles of potatoes, and tomatoes will come inside a bigger circle of vegetables because potatoes and tomatoes both are vegetables. Given below are a few types of Venn diagrams with their implication made clear.

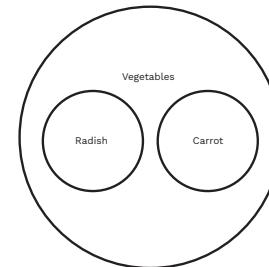
- Suppose you are given a group of three items and those items evidently belong to three different categories, they will be represented as illustrated below. For example, fruits, vegetables, and spices.



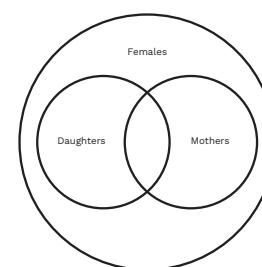
- If one item belongs to the class of second and the second item belongs to the class of third, then the representation is in the form of three concentric circles, as represented below. For example, Pepsi, Cold Drinks, and Liquids.



- If two separate items belong to the part of the third, for example, radish and carrot belong to the vegetable category, this will be represented as



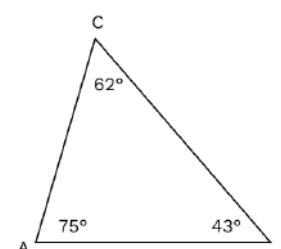
- If the given two items belong to the class of the third in such a way that some items of each of these two groups are common in some particular trait, then such a relationship will be illustrated by a big circle inclusive of two intersecting smaller circles. For example, females, daughters, and mothers.



- If three items are partly related to each other, they are represented as shown below.

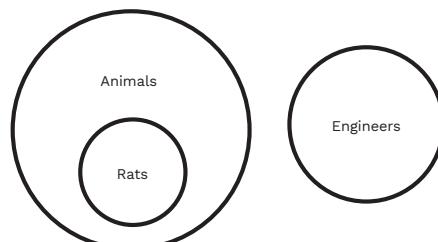
For example, males{A}, engineers{B}, and postgraduate{C}.

Some males may be engineers and some may be postgraduates. Similarly, some engineers may be males and some may be postgraduates. Also, some postgraduates can be engineers and some may be males.

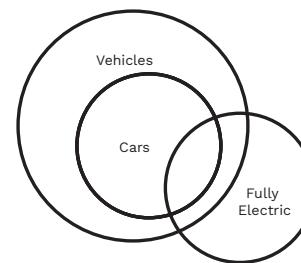




6. If one item belongs to the second, and the third is entirely different from the two, then they can be represented by the below diagram. For example, animals, rats, and engineers.

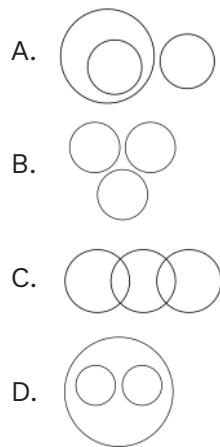


7. The first item is completely related to the second and the third item is partially related to the first and second items. For example, car, vehicles, fully-electric. All the cars belong to vehicles, but some cars are fully electric but not all.

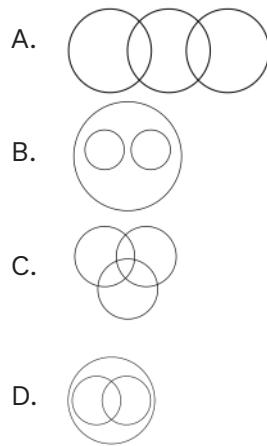


PRACTICE QUESTIONS

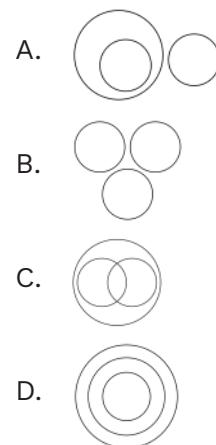
1. Which of the following diagrams indicates the best relation between passengers, car, and bus?



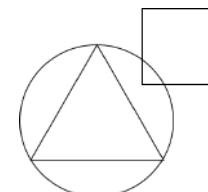
2. Which of the following diagrams indicates the best relation between amount, principal, and Interest?



3. Which of the following diagrams indicates the best relation between teacher, lawyer, and dancer?



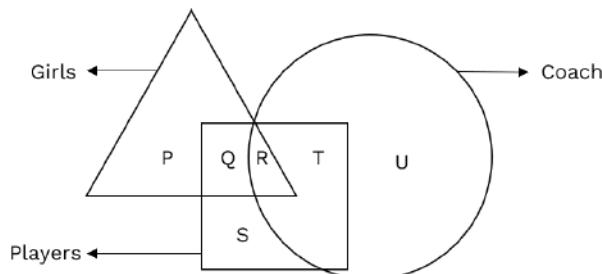
4. In an organisation of the environment board, professors are represented by a circle, legal experts by a square, and environmentalists by a triangle. Who is most represented in the board as shown in the following figure?



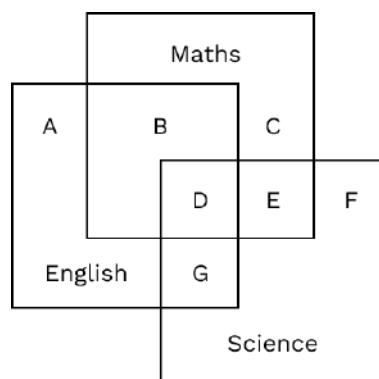
- A. Environmentalists
B. Legal experts
C. Professors with legal background
D. Professors who are also environmentalists



5. In the following figure, triangle represents girls {from Carmel school}, rectangle represents players {from Football team} and circle-coach {coach of indoor stadium}. Which part of the diagram represents the girls who are players but not coaches?

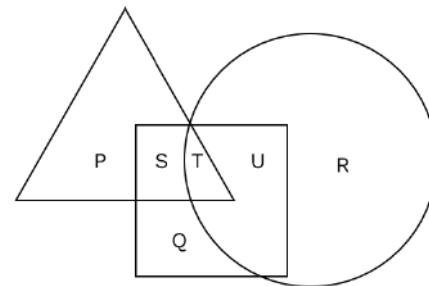


- A. P
B. Q
C. R
D. S
6. The diagram given below represents those students who like Maths, English, and Science.

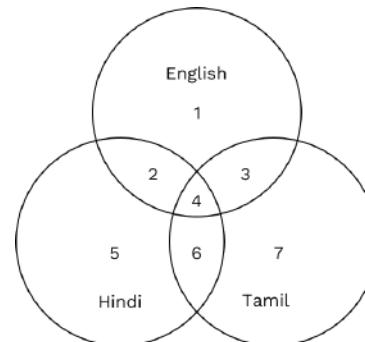


Study the diagram and identify the students who like all three subjects.

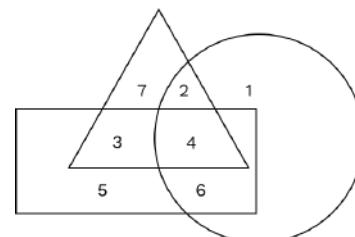
- A. A + B + C
B. E + G
C. D + E + G
D. D
7. In the figure given next, the square represents kites, the triangle represents blue, and the circle represents pentagon. By which letter are the kites both of which are Blue in colour and Pentagon in shape represented?



- A. U
B. T
C. S
D. P
8. From the following diagram, which number represents the people who can speak only one language.



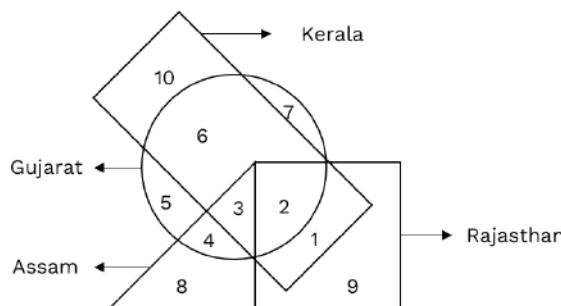
- A. $2 + 3 + 4$
B. $1 + 5 + 7$
C. 4
D. $1 + 4 + 5 + 7$
9. In the given figure, if triangle represents snacks, rectangle represents crunchy and circle represents sweet, then what is the number of those snacks which are sweet but not crunchy?



- A. 3
B. 4
C. 6
D. 2



10. In the following diagram, various figures represent the states which people have visited. Which number indicates the people who visited Kerala, Rajasthan, and Gujarat but not Assam?



- A. 1
- B. 2
- C. 10
- D. 6

(Directions for questions from 11 to 20) Each of the following questions contains three items. Using the relationship between these items, match each question with suitable diagrams.

- A.
- B.
- C.
- D.
- E.

11. Ostrich, deer, whale
12. Lion, deer, mammal

13. Human beings, teachers, graduates

14. Plums, tomatoes, fruits

15. Flowers, clothes, whites

16. Uncle, parents, friends

17. Jaipur, Rajasthan, Assam

18. Engineer, doctor, people

19. Thieves, lawyers, criminals

20. Sea, island, mountain

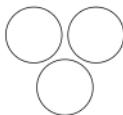
21. Find the diagram that shows the right relation between bike, furniture, and mouse?

- A.
- B.
- C.
- D.

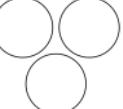
22. Which Venn diagram correctly illustrates the relationship between the following?
I. Train
II. Entertainment park
III. Satellite

- A.
- B.



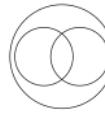
- C. 
- D. 

23. Identify the figure that best represents the relationship among the potato, vegetable, and pencil.

- A. 
- B. 
- C. 
- D. 

24. Which of the following diagrams best represents the relationship among brother, husband, and men?

- A. 

- B. 
- C. 
- D. 

25. Which of the four figures will best represent the relationship among peacocks, birds, and mice?

- A. 
- B. 
- C. 
- D. 



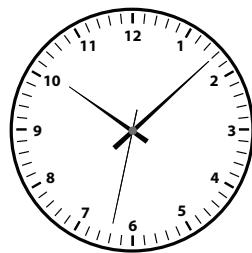
SOLUTIONS

1. **(C)** Bus and car are different from each other but some passengers travel by bus and some travel by train.
2. **(B)** Principal and interest are different from each other. But both are parts of amount.
3. **(B)** All three are different professions.
4. **(D)** Professors who are also environmentalists are most represented in the board as an environmentalist is illustrated by the triangle and professors by the circle.
5. **(B)** Q part of the figure represents those girls who are players but not coaches.
6. **(D)** D indicates those students who like all three subjects.
7. **(B)** Because T represents the kites which are blue and pentagon.
8. **(B)** The regions represented by the numbers 1, 5, and 7 denote such people who can speak only one language.
9. **(D)** When compared with the above-given statement, we will get the answer as 2 as it represents snacks that are sweet but not crunchy.
10. **(B)** The required number is 2 as it includes Kerala, Rajasthan, and Gujarat but not Assam.
11. **(A)** The three have nothing in common.
12. **(B)** Lions and deer come in the mammal's category.
13. **(C)** Teachers and graduates come under human beings and some graduates can be teachers.
14. **(D)** Plums come under fruits, whereas tomatoes come under vegetables.
15. **(E)** Some flowers are white and some clothes are also white.
16. **(A)** Uncle, parents, and friends are entirely different from each other.
17. **(D)** Jaipur is part of Rajasthan and Assam is a separate state.
18. **(B)** Both engineers and doctors are people. But both of them are different from each other.
19. **(D)** All thieves are criminals, but lawyers are entirely different.
20. **(D)** Island is part of the sea, but the mountain is entirely different.
21. **(B)** The three are different from each other.
22. **(C)** The given three things are totally different from each other.
23. **(A)** Potato falls under the vegetable category and pencil neither is related to any of the given species nor is part of those.
24. **(B)** Husband and brother both fall under the men category, and there can be some husbands who are brothers and vice versa.
25. **(B)** Peacocks fall under the category of birds, and mice are not related to any of the two.

38 Clocks



Clock test-related questions are one form of a question that requires a lot of practice. Although these questions appear less frequently in exams, they are still important for GATE, JEE Main, and other engineering entrance examinations.



We'll try to cover some of the clock test problems and discuss the tactics and procedures for answering them in depth. Following that, there will be some practice questions for you to answer and a clock test to see how strong you are. You must be familiar with the concept of angle difference to comprehend the clock.

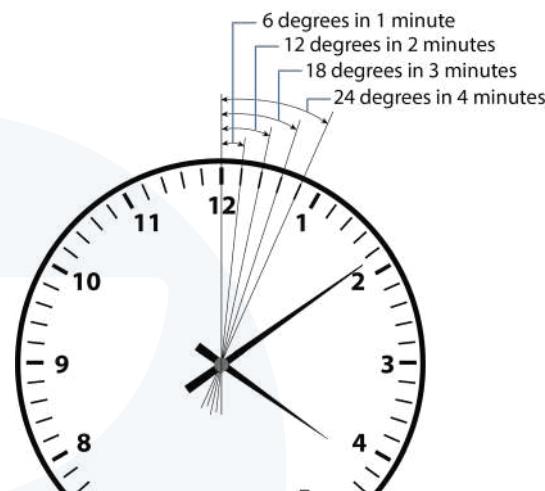
INTRODUCTION

Clocks are time-telling devices that measure and display the passage of time. Humans have been measuring time in various ways for millennia, including using sundials to measure the sun's movements, water clocks, candle clocks, and hourglasses.

The face of a dial watch or clock that we usually see is a circle whose circumference is divided into 60 equal parts, called minutes space. A clock generally has two hands. The smaller hand is called the hour hand or short hand, while the larger one is called the minute hand or long hand. The third hand is called the second hand. We don't take into consideration this second hand as the time-lapse or, say, the calculation of exact time up to seconds is a tedious exercise. But let's differentiate between these minute and second hands. First, consider the second and minute hands. In one hour, the minute hand makes one revolution

and the second-hand goes around 60 times. This means that, in one minute, the second-hand passes over the minute hand $60 - 1 = 59$ times and the two are also in line (but with 180 degrees between them) 59 times.

IMPORTANT POINTS



- One minute division in a watch is $= 6^\circ$ apart, i.e., in one minute, the 60-min hand moves 6° .
- One-hour division $= 6^\circ \times 5 = 30^\circ$ apart, i.e., in one hour, the hour hand moves 30° apart.
- Also, in one minute, the hour hand moves apart:
 - 15-min gap apart indicates a right angle or perpendicular.
 - 22 times in 12 h or 44 times in 24 h = right angle or perpendicular (1 day).
 - 30-minute interval apart at a straight angle or in a straight line (180°).
 - 11 times in 12 h or 22 times in 24 h for a straight angle (1 day).
 - In 12 h, the angle indicated by the hour hand equals 360° .
 - In 60 min, the angle drawn by the minute hand equals 360° .
 - Speed of hour hand $= 0.5$ DPM (degree per minute).



- Speed of minute hand = 6 DPM
- The angle of the hour hand from vertical at N o'clock = $30N$

After going through the basics of the clock, we will be further looking at the types of questions that usually come in examinations regarding this topic such as:

1. Basic overlapping and coinciding of hands of the clock type questions
2. Angle-finding theory
3. Relative speeds of the hour and minute hands

Basic overlapping and coinciding of hands of the clock-type questions

First, these are some of the questions that are based on angles but don't need to be formulated. They are simply logic-based questions. These

are called the basic overlapping and coinciding of hands of the clock-type questions.

The reason why we have these coinciding or overlapping situations between hands of the clocks is that the hands of a clock move at varying speeds, colliding and forming different angle forms among themselves at different times of the day.

These questions require thorough observation of a clock by a person and can be solved without applying any sort of formula or calculations. Look at the examples to understand these observational occurrences in a better way.

Point to remember: The hour hand and minute hand overlap each other 22 times in a day.

SOLVED EXAMPLES

1. In a day, how many times do the minute hand and hour hand coincide (or overlap or coincide together)?
- A. 12 B. 22
C. 24 D. 48

Explanation: A day starts at 12 a.m. and ends at 11:59 p.m.; from 12 a.m. again a new day starts. If the hour hand and minute hand coincide or overlap each other, that means the angle between them is 0° . The first time they coincide in a day is at 12 a.m., then anywhere between 1 a.m. and 2 a.m., and it carries on in this way. The last time they will coincide is anywhere between 11 p.m. and 12 a.m.; then another day starts. So, if we calculate them in a span of 12 hours, they are coinciding for 11 times, then for a day, they will be coinciding for $11 \times 2 = 22$ times. Therefore, option B is the right answer.

2. In a day how many times the minute hand and hour hand are opposite to each other?

- A. 44 B. 22
C. 42 D. 24

Explanation: Again we will be resorting to our earlier discussion. A day starts at midnight and ends at 11:59 p.m.; from midnight again a new day starts. If the hour hand and minute hand are opposite to each other, that means the angle between them is 180° . The first time they are opposite to each other in a day is somewhere between 12 and 1 a.m. (for now at this early stage, we would be solving this question by approximation method, but subsequently with the introduction of angle theory, we can do it precisely and accurately), then between 1 and 2 a.m., then between 2 and 3 a.m., and it carries on in this way. The last time they will coincide is between 11 p.m. and 12 a.m. So, if we calculate them in a span of 12 hours, they are opposite to each other for 11 times, then for a day they will be opposite to each other for $11 \times 2 = 22$ times. Therefore, option B is the right answer.



ANGLE-FINDING THEORY

We can find the angle between hour hand and minute hand of the clock by using this very important formula:

$$\text{Angle between hands} = \left[\frac{11}{2} \times \text{Minute} - (30) \times \text{Hour} \right] \text{ when } \frac{11}{2} \text{ Minute} > (30) \text{ Hour,}$$

$$\text{Angle between hands} = \left[(30) \times \text{Hour} - \frac{11}{2} \times \text{Minute} \right] \text{ when } (30) \text{ Hour} > \frac{11}{2} \text{ Minute}$$

SOLVED EXAMPLES

3. Find the angle between the hour hand and the minute hand at 2:20 o'clock?
- A. 30 B. 40
C. 50 D. 52

Explanation: By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

$$\text{Angle between hands} =$$

$$\left[\left(\frac{11}{2} \right) \text{Minute} - (30) \text{Hour} \right]$$

$$\begin{aligned}\text{Angle} &= \left[\left(\frac{11}{2} \right) 20 - 30 \times 2 \right] \\ &= [11 \times 10 - 60] \\ &= [110 - 60] \\ &= 50^\circ\end{aligned}$$

4. Find the time between 3 and 4 o'clock when the hour hand and minute hand are at right angle to each other?

- A. $30 \frac{8}{11}$ min past 3
B. $31 \frac{4}{11}$ min past 3
C. $32 \frac{8}{11}$ min past 3
D. $33 \frac{4}{11}$ min past 3

Explanation: When hour hand and minute hand are at right angles, then the angle between them is 90° . In such questions, where we are to find the angle between 1 hour span, say between 3 and 4, we will take the first unit, i.e., 3 because if we take

4 then the time will cross 4, then it will not be in between 3 and 4.

By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

$$\text{Angle between hands} =$$

$$\left[\left(\frac{11}{2} \right) \text{Minute} - (30) \text{Hour} \right]$$

$$90 = \left[\left(\frac{11}{2} \right) \text{minute} - 30 \times 3 \right]$$

$$180 = \left(\frac{11}{2} \right) \text{minute}$$

$$180 \times \left(\frac{2}{11} \right) = \text{minute}$$

$$360/11 = 32 \text{ minute } \left(\frac{8}{11} \right) \text{s}$$

we can write it as 32 min 43 s
 $\left(\frac{8}{11} \times 60 = 8 \times 5.4 \approx 43 \right)$

5. What is the possible time after 1, when a clock shows a 20° angle between the two hands of the clock between 1 p.m. and 2 p.m.?

- A. 9 min 5 s past 1
B. 9 min 25 s past 1
C. 9 min 18 s past 1
D. 9 min 10 s past 1

Explanation: In this question, angle is given and here we have to find the time between 1 and 2 o'clock when the angle between minute and hour hands is 20° .



By simply using, the angle-finding theory, we can find the angle between the two hands of the clock:

Angle between hands =

$$[(\frac{11}{2}) \times \text{Minute} - (30) \times \text{Hour}]$$

$$20^\circ = [(\frac{11}{2}) \text{ minute} - 30 \times 1]$$

$$50 = (\frac{11}{2}) \text{ minute}$$

$$50 * (\frac{2}{11}) = \text{minute}$$

$$\frac{100}{11} = 9 \text{ minute } (\frac{1}{11}) \text{ s}$$

we can write it as $9 \text{ min } 5 \text{ s } (\frac{1}{11} \times 60 = 1 \times 5.4 \approx 5)$

RELATIVE POSITION OF THE HANDS

- The position of the minute hand relative to the hour hand is said to be the same, whenever the minute hand is separated from the hour hand by the same number of minute divisions and is on the same side (clockwise or anticlockwise) of the hour hand.
- Relative speed of the angle between minute hand and hour hand is 5.5° .

Proof

We see, a clock or a watch is a complete circle, so it has 360 degrees. Further, it is divided into 12 equal parts; i.e., each part is $360/12 = 30^\circ$.

As the minute hand takes a complete round in one hour, it covers 360° in 60 minutes.

Therefore, in 1 minute, it covers $360/60 = 6^\circ$.

Also, as the hour hand covers just one part out of the given 12 parts in one hour. This implies it covers 30° in 60 minutes, i.e., $\frac{1}{2}^\circ$ per minute.

This implies that the relative speed of the minute hand is $6 - \frac{1}{2} = 5 \frac{1}{2} \text{ degrees per minute}$.

(Relative speed in the same direction for two objects M₁ and M₂, where M₁ > M₂ is = M₁ - M₂)

- We will use the concept of relative speed and relative distance while solving problems on clocks.

- Any relative position of the hands of a clock is repeated 11 times in every 12 h.
- In every hour, the two hands are at right angles 2 times.
- In a day, the two hands are at right angles 44 times.
- If both the hands coincide, then they will again coincide after 65 min, i.e., in correct clock, both hands coincide at an interval of 65 min approx.

We would be practically understanding this concept. Just imagine hour hand and minute hand starting at 12 o'clock. After 1 h, the minute hand will be at 12, whereas the hour hand will be at 1. So, the difference between the time covered by both the hands is 55 'Minute Space.' So, it takes 1 hour to cover 55 minute spaces; for overlapping they should cover a total 60 minute space.

Thus, here is the calculation:

$$1 \text{ h} \rightarrow 55 \text{ min space.}$$

$$? \text{ h} \rightarrow 60 \text{ min space.}$$

$$\frac{1 \times 60}{55} \text{ h} = \frac{60}{55} * 60 \text{ min (converting hour to minutes)}$$

$$= 65 \frac{5}{11} \text{ min.}$$

⇒ If the two hands coincide in time less than 65 min, then the clock is too fast and if the two hands coincide in time more than 65 min, then the clock is too slow.



SOLVED EXAMPLES

- 6.** A clock is started at afternoon, by 25 min past 8, the find the angle that the hour hand has turned through:
 A. 255.25° B. 252.5°
 C. 250° D. 241.5°

Explanation: In a clock, 12 h form 360° . So, one-hour forms 30° . Therefore, 8 h form 240° ($30^\circ \times 8$) degrees. In a minute, the hour hand moves 0.5° , so in 25 min it moves $0.5^\circ \times 25 = 12.5^\circ$. Thus, by 25 min past 8, the hour hand has turned through $240^\circ + 12.5^\circ = 252.5^\circ$.

- 7.** At 8 a.m., a clock is set to show the right time. Every day, the clock loses 10 min. When the clock strikes 5 p.m. the next day, what time will it be?
 A. 5:25 p.m. B. 4:45 p.m.
 C. 5:42 p.m. D. 4:46 p.m.

Explanation: Time from 8:00 a.m. a day to 5:00 p.m. the next day = 33 hours
 23 h 50 min of this clock = 24 h of the correct clock.

$$(23) \frac{5}{6} \text{ of this clock} = 24 \text{ h of the correct clock}$$

$$33 \text{ h of this clock} = \frac{33 \times 143}{6 \times 24}$$

= 32 h 46 min (approx.)

The correct time is 32 h 46 min after 08:00 a.m. = 46 min past 4 p.m.

- 8.** A watch that gains consistently is 2 min slower at 1 p.m. on Sunday and 5 min faster at 8 p.m. the following Sunday. When was it correct?
 A. 3 p.m. on Tuesday
 B. 8 a.m. on Wednesday
 C. 2 a.m. on Wednesday
 D. 4 a.m. on Tuesday

Explanation: Time from 1 p.m. on Sunday to 8 p.m. on following Sunday = 7 days and 7 h = $7 \times 24 + 7 = 175$ h

The watch gains $(2 + 5)$ min or 7 min in 175 h.
 Now 7 min are gained in 175 h.

$$2 \text{ min are gained in } (175 \times \frac{1}{7} \times 2) \text{ h} = 50 \text{ h} =$$

2 days and 2 h

Watch is correct 2 days 2 h after 1 p.m. on Sunday.

It will be correct at 3 p.m. on Tuesday.

- 9.** The minute hand of a clock overtakes the hour hand at an interval of 60 min of the correct time. How much a day does the clock gain or lose?

- A. $120\frac{5}{11}$ min gain in a day
 B. $135\frac{3}{11}$ min loss in a day
 C. $130\frac{10}{11}$ min gain in a day
 C. $126\frac{2}{11}$ min loss in a day

Explanation: In 1 h both the hands cover 55 min space.

$$\Rightarrow \frac{60}{55} = \frac{12}{11} \text{ min space covered in 1 min of}$$

the actual time.

For the hands to coincide, they have to cover 60 min space

$$\Rightarrow \frac{12}{11} \times 60 = \frac{720}{11} = 65\frac{5}{11} \text{ min in actual clock.}$$

But it is given in the question that the clock coincides every 60 min.

$$\text{Gain in 60 min} = 65\frac{5}{11} - 60 = 5\frac{5}{11} = \frac{60}{11} \text{ min}$$

in 60 min.

$$\text{Gain in 1 min} = \frac{60}{11} \times \frac{1}{60} = \frac{1}{11} \text{ min in 1 min.}$$

$$\text{Loss in 24 h} = \frac{60}{11} \times (24 \times \frac{60}{60}) = \frac{1440}{11} =$$

$$130\frac{10}{11} \text{ min is gained in a day.}$$



PRACTICE QUESTIONS

1. For how many times does the hands of a clock form a right angle in a day?
A. 22 B. 24
C. 44 D. 48
2. If the two hands in a clock are 4 min 48 s divisions apart, then the angle between them is
A. 23.6° B. 28°
C. 24.4° D. 28.8°
3. What will be the acute angle between the hands of a clock at 1:20?
A. 105° B. 90°
C. 80° D. 95°
4. In the same time that the hour hand moves by 21° , how many degrees does the minute hand move?
A. 248° B. 252°
C. 256° D. 260°
5. What is the possible time after 5, when a clock shows 45° angle between the two hands between 5 p.m. and 6 p.m.?
A. 32 min 5 s past 5 p.m.
B. 33 min 25 s past 5 p.m.
C. 34 min 18 s past 5 p.m.
D. 35 min 27 s past 5 p.m.
6. What is the total angle traced by both minute and hour hands at 4:24?
A. 131° B. 132°
C. 133° D. 134°
7. What will be the acute angle between hands of a clock at 5:40?
A. 105° B. 75°
C. 95° D. 70°
8. A clock is started in the morning, by 44 min past 6, the hour hand has turned through:
A. 215.25° B. 190°
C. 202° D. 241.5°
9. The reflex angle between the hands of a clock at 8 h 20 min is
A. 280° B. 192°
C. 230° D. 210°
10. Find the mirror image of the clock when the time is 06:12.
A. 5:20 B. 5:22
C. 5:48 D. 5:42
11. Approximately at what time between 12 p.m. and 1 a.m., will the minute hand and hour hand of the clock be exactly opposite to each other?
A. 01:36 B. 01:35
C. 01:43 D. 01:32
12. When will the minute and hour hands of the clock come together in a straight line between 4 and 5 o'clock?
A. 04:50 B. 04:52
C. 04:54 D. 04:56
13. What is the possible time when a clock shows 33° angle between the two hands of the clock between 1 p.m. and 2 p.m.?
A. 1:09 min B. 1:08 min
C. 1:07 min D. 1:06 min
14. A clock shows 7 o'clock in the morning. When the clock strikes 6 o'clock the next morning, how much will the hour's hand rotate?
A. 333° B. 300°
C. 330° D. 303°
15. At what time between 1:30 and 2 will the hands of a clock be at right angles?
A. 1:50 B. 1:52
C. 1:54 D. 1:56
16. A clock is set to show the correct time at 5 a.m. The clock uniformly loses 18 min in a day. What will be the actual time when the clock shows 5 p.m. the next day?



- A. 4:33 p.m. B. 4:35 p.m.
C. 5:22 p.m. D. 4:42 p.m.
- 17.** The minute hand of a clock overtakes the hour hand at an interval of 63 min of the correct time. How much a day does the clock gain or lose approximately?
A. 56 min gain in a day
B. 52 min loss in a day
C. 50 min loss in a day
D. 59 min gain in a day
- 18.** At 4 p.m., a clock is set right. What is the correct time when the clock reads 9 p.m. on the same day if it gains 1 min in an hour?
A. $55 \frac{5}{61}$ min past 8.
B. $54 \frac{5}{61}$ min past 8.
C. $53 \frac{6}{61}$ min past 8.
D. $55 \frac{6}{61}$ min past 8.
- 19.** The time was right at 1 a.m. on a watch. In a 24-h period, the clock loses 55 min. When the clock reads 1 p.m. on the fifth day, what is the true time?
A. 10 min past 1 p.m.
B. 14 min past 4 p.m.
C. 17 min past 2 p.m.
D. 20 min past 3 p.m.
- 20.** At 4 a.m., a clock is set right. In a 24 h period, the clock advances (gains) 10 min. When the clock reads 9 p.m. the next day, what is the true time?
A. 24 min past 8 p.m.
B. 32 min past 8 p.m.
C. 42 min past 8 p.m.
D. 48 min past 8 p.m.
- 21.** Mark the correct statements.
I. The angle between minute hand and hour hand at 3:20 o'clock is 30° .
- II. In half a day, the minute hands and hour hand coincide (or overlap or coincide together) for 22 times.
A. Only I B. Only II
C. Both I and II D. Neither I and II
- 22.** A clock shows the correct time at 10 a.m. The clock gains 10 min in 24 h. What will be the correct time when the clock shows 2 p.m. on the next day?
A. 24 min past 1 p.m.
B. 30 min past 1 p.m.
C. 40 min past 1 p.m.
D. 48 min past 1 p.m.
- 23.** A watch, which gains uniformly, is 4 min slow at 12 noon on Sunday, and is 4 min 48 s fast at 2 p.m. on the following Sunday. When was it correct?
A. 3:36 p.m. on Wednesday
B. 4:18 p.m. on Thursday
C. 2:52 p.m. on Friday
D. 5:16 p.m. on Wednesday
- 24.** The minute hand of a clock overtakes the hour hand at intervals of 62 min of the correct time. How much a day does the clock gain or lose approximately?
A. 86 min B. 89.2 min
C. 85.25 min D. 80.20 min
- 25.** A clock is set right at 5 p.m. If it gains one minute in an hour, then what is the true time when the clock indicates 12 a.m. on the same day?
A. $51 \frac{7}{61}$ min past 11.
B. $56 \frac{5}{61}$ min past 11.
C. $53 \frac{7}{61}$ min past 11.
D. $55 \frac{5}{61}$ min past 11.



SOLUTIONS

- 1. (C)** Since these types of questions can be easily solved here we will give an apt justification as to why they must be 44 times.

In 60 min, the minute hand rotates 360 degrees. This means that $6t$, where t is the number of minutes past midnight, equals the angle of the minute hand.

In 60 min, the hour hand rotates 30 degrees. This means that $0.5t$ determines the tilt of the hour hand.

The hands start together at midnight. The first time they make a 90° angle is when the minute hand has moved 90 degrees further than the hour hand, so this is given by the equation:

$$6t = 0.5t + 90$$

$$5.5t = 90$$

$$t = 16 \frac{4}{11} \text{ (16 min and } 4/11 \text{ s)}$$

To put it another way, it's around 16 min past midnight.

When the minute hand has gained another 180 degrees on the hour hand and is 90° degrees behind it, the next time is:

$$6t = 0.5t + 270$$

$$5.5t = 270$$

$$t = 49 \frac{1}{11} \text{ (49 min and } 1/11 \text{ s)}$$

At about 11 minutes to 1 o'clock.

For every 180 degrees that the minute hand gains on the hour hand, there will be one 90° angle, so every $49 \frac{1}{11} - 16 \frac{4}{11} = 32 \frac{8}{11}$ minutes

$$24 \text{ h is } 1440 \text{ min. } \frac{1440}{32 \frac{8}{11}} = 44$$

- 2. (D)** In a clock, each minute makes 6°

$\therefore 4 \text{ min } 48 \text{ s } (\frac{48}{60} = \frac{4}{5}) \text{ minutes will make }$

$$6 \times 4 \frac{4}{5} = 6 \times \frac{24}{5} = 28.8^\circ$$

- 3. (C)** We can simply do this question by the angle-finding theorem.

Angle between hands =

$$[(\frac{11}{2}) \text{ Minute} - (30) \text{ Hour}]$$

when $\frac{11}{2}$ Minute > (30) Hour,

$$\text{Angle} = [\frac{11}{2} 20 - (30) 1] = 110 - 30 = 80^\circ$$

Alternatively: At 1 o'clock, minute hand will be $5 \times 6 = 30^\circ$ behind the hour hand.

In 20 minutes, the minute hand will gain

$$\frac{11}{2} \times 20 = 110$$

\therefore Angle between hour hand and minute hand = $110 - 30 = 80^\circ$

- 4. (B)** The hour hand moves 30° when it completes 1 h or 60 min and in the same time the minute hand moves 360° . So 'x' number of minutes will be covered in 21°:

$$\Rightarrow \frac{21}{30} \times 60 = 42 \text{ min}$$

\Rightarrow In the meanwhile, the minute hands will trace $42 \times 6 = 252^\circ$ [1 min = 6°]

- 5. (D)** In this question, the angle is given and we have to find the time between 5 and 6 o'clock when the angle between minute and hour hands is 45° .

By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

Angle between hands =

$$[(\frac{11}{2}) \text{ Minute} - (30) \text{ Hour}]$$

$$45^\circ = [(\frac{11}{2}) \text{ minute} - 30 \times 5]$$

$$195 = (\frac{11}{2}) \text{ minute}$$

$$195 \times (\frac{2}{11}) = \text{minute}$$

$$\frac{390}{11} = 35 \text{ minute } \frac{5}{11}$$

we can write it as $35 \text{ min } 27 \text{ s } (\frac{5}{11} \times 60 \approx 27)$



6. (B) In a clock, 12 h form 360° . So, 1 h forms 30° . Therefore, 4 h form 120° ($30^\circ \times 4$) degrees. In a minute, the hour hand moves 0.5° , so in 24 min it moves $0.5^\circ \times 24 = 12^\circ$. Thus, by 24 min past 4, the hour hand has turned through $120^\circ + 12^\circ = 132^\circ$.

7. (D) We can simply do this question by angle-finding theorem.

Angle between hands =

$$\left[\frac{11}{2} \text{ Minute} - (30) \text{ Hour} \right]$$

when $\frac{11}{2}$ Minute > (30) Hour,

$$\text{Angle} = \left[\frac{11}{2} \cdot 40 - (30)5 \right] = 220 - 150 = 70^\circ$$

Alternatively: At 5 'oclock, the minute hand will be $5 \times 30 = 150^\circ$ behind the hour hand.

In 40 min, the minute hand will gain $\frac{11}{2} \times 40 = 220$

\therefore Angle between hour hand and the minute hand = $220 - 150 = 70^\circ$.

8. (C) In a clock, 12 h form 360° . So, 1 h forms 30° . Therefore, 6 h forms 180° ($30^\circ \times 6$) degrees. In a minute, hour hand moves 0.5° , so in 44 min it moves $0.5^\circ \times 44 = 22^\circ$. Thus, by 25 min past 8, the hour hand has turned through $180^\circ + 22^\circ = 202^\circ$.

9. (C) To find the reflex angle of a given angle, we need to subtract the given measure from 360° . So, the reflex angle of $60^\circ = 360^\circ - 60^\circ = 300^\circ$. Therefore, the reflex angle of 60° is equal to 300° .

The angle between hand =

$$\left[(30) \text{ Hour} - \frac{11}{2} \text{ Minute} \right]$$

when (30) Hour > $\frac{11}{2}$ Minute

$$\text{Angle} = 30 \times 8 - \frac{11}{2} \times 20$$

$$\text{Angle} = 240 - 110 = 130^\circ$$

Now the reflex angle is = $360^\circ - 130^\circ = 230^\circ$

10. (C) We need to subtract from 12:00 or 11:60 to get mirror image time. Mirror image of 06:12; 11:60 – 06:12 = 5:48

11. (D) When the hour hand and the minute hand are opposite to each other, then the angle between them is 180° . In such questions where we are to find an angle between 1 h span, say, between 3 and 4, we will take the first unit, i.e., 3 because if we take 4, then the time will cross 4, then it will not be in between 3 and 4.

By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

$$\text{Angle between hands} = \left[(30) \text{ Hour} - \frac{11}{2} \text{ Minute} \right]$$

when (30) Hour > $\frac{11}{2}$ Minute

$$180 = 30 \times 12 - \frac{11}{2} \times \text{min}$$

$$\frac{11}{2} \times \text{min} = 360 - 180 = 180$$

$$\text{Minute} = 180 \times \frac{2}{11} = \frac{360}{11} = 32 \text{ min } 43 \text{ min}$$

$$\left(\frac{8}{11} \times 60 \approx 43 \right)$$

12. (C) When the hour hand and minute hand of the clock come in a straight line then the angle between them is 180° . In such questions where we are to find the angle between 1 h span, say, between 3 and 4, we will take the first unit, i.e., 3 because if we take 4, then the time will cross 4, then it will not be in between 3 and 4.

By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

Angle between hands =

$$\left[\frac{11}{2} \text{ Minute} - (30) \text{ Hour} \right]$$

when (30) Hour < $\frac{11}{2}$ Minute

$$180 = \frac{11}{2} \times \text{min} - 30 \times 4$$

$$\frac{11}{2} \times \text{min} = 180 + 120 = 300^\circ$$

$$\text{Minute} = 300 \times \frac{2}{11} = \frac{600}{11} = 54 \text{ min } 32 \text{ s}$$

$$\left(\frac{6}{11} \times 60 \approx 32 \right)$$



- 13. (D)** In this question, the angle is given, and we have to find the time between 1 and 2 o'clock when the angle between the minute and hour hands is 33° .

By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

$$\text{Angle between hands} =$$

$$[(\frac{11}{2}) \text{ Minute} - (30) \text{ Hour}]$$

$$\text{when } (30) \text{ Hour} < \frac{11}{2} \text{ Minute}$$

$$33 = [(\frac{11}{2}) \text{ minute} - 30 \times 1]$$

$$33 = (\frac{11}{2}) \text{ minute}$$

$$33 \times (\frac{2}{11}) = 6 \text{ minutes}$$

- 14. (C)** In 12 hours, the hand turns 360° .

Here, the difference between time = 11 hours
Then, required angle = $11 \times 30 = 330^\circ$

- 15. (C)** In this question, the angle is given, and we have to find the time between 1:30 and 2 o'clock when the angle between the minute and hour hands is 90° .

By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

$$\text{Angle between hands} =$$

$$[(\frac{11}{2}) \text{ Minute} - (30) \text{ Hour}]$$

$$90 = [(\frac{11}{2}) \text{ minute} - 30 \times 1.5]$$

$$135, (90 + 45 = 135) = (\frac{11}{2}) \text{ minute}$$

$$135 \times \frac{2}{11} = 24 \text{ min } 22 \text{ s } (\frac{4}{11} \times 60 \approx 22)$$

So, the time between 1:30 and 2 at which the angle between hands of the clock is at 90° would be 24 min 22 s past 1:30, that's approximately at 1:54

- 16. (A)** Time from 5:00 a.m. a day to 5:00 p.m. the next day = 36 h

23 h 42 min of this clock = 24 h of the correct clock.

$$(23) \frac{7}{10} \text{ of this clock} = 24 \text{ h of the correct clock}$$

$$36 \text{ h of this clock} = \frac{36 \times 23 \left(\frac{7}{10} \right)}{24}$$

$$= 35 \text{ h } 33 \text{ min (Approx.)}$$

The correct time is 35 h 33 min after 05:00 a.m.

$$= 4 \text{ h } 33 \text{ min.}$$

Alternatively: We know that 18 min is lost in a day. Total no of hours from 5 a.m. to 5 p.m. is 36 h, that's 1.5 day.

If 18 min is lost in 1 day, then by unitary method, we can calculate the minutes lost in 1.5 day as well. It will be 27 min ($18 \times 1.5 = 27$).

Now we need to find the actual time when the clock shows 5 p.m. on the next day; It will be 27 min before 5 p.m., that's 4:33 p.m. ($5:00 - 0:27 = 4:33$).

- 17. (A)** In 1 hour both the hands cover 55 min space.

$$\Rightarrow 60/55 = 12/11 \text{ min space covered in 1 min of the actual time.}$$

for the hand to coincide, hands have to cover 60 min space

$$\Rightarrow 12/11 \times 60 = 720/11 = 65 \frac{5}{11} \text{ min in actual}$$

clock.

But the clock coincides every 63 min.

$$\text{Gain in 60 min} = 65 \frac{5}{11} - 63 = 2 \frac{5}{11} = \frac{27}{11} \text{ min in 60 min.}$$

$$\text{Gain in 1 min} = \frac{27}{11} \times \frac{1}{60} = \frac{27}{660} \text{ s in 1 min.}$$

$$\text{Gain in 24 h} \Rightarrow \frac{27}{11} \times (24 \times \frac{60}{63}) = \frac{38880}{693} \approx$$

56 min is gained in a day

- 18. (A)** Time interval indicated by incorrect clock = 9 p.m. - 4 p.m. = 5 h.

Time gained by incorrect clock in one hour

$$= +1 \text{ min} = +\frac{1}{60} \text{ h.}$$

Using the formula \rightarrow

$$\frac{\text{True time interval}}{\text{Time interval in incorrect clock}}$$



$$\begin{aligned}
 &= \frac{1}{1 + \text{hour gained in } 1 \text{ h by incorrect clock}} \\
 \Rightarrow \frac{\text{True time interval}}{5} &= \frac{1}{1 + \frac{1}{60}} \\
 \Rightarrow \text{True time interval} &= 5 \times \frac{60}{61} = 4 \frac{57}{61} \\
 \therefore \text{True time} &= 4 \text{ p.m.} + 4 \frac{57}{61} \text{ h} = 8 \text{ p.m.} + \\
 \frac{57}{61} \text{ h} &= 8 \text{ p.m.} + \frac{57}{61} \times 60 \text{ min.} \\
 &= 55 \frac{5}{61} \text{ min past 8.}
 \end{aligned}$$

- 19. (B)** Time from 1 a.m. on a day to 1 p.m. on the 5th day = 132 h.

23 h 05 min of this clock = 24 h of the correct clock.

$$(23 \frac{1}{12} \text{ h}) \frac{277}{12} \text{ h of this clock} = 24 \text{ h of the correct clock.}$$

$$132 \text{ h of this clock} = [24 \times (\frac{12}{277}) \times 132] \text{ h of the correct clock}$$

$$\Rightarrow \frac{38,016}{277} = 137 \text{ h } 14 \text{ min, } (\frac{67}{277} \times 60)$$

Therefore, the correct time is 137 h 14 min after 1 a.m.

This is 14 min past 4 p.m.

- 20. (D)** Time from 4 a.m. on a day to 9 p.m. on the following day = 29 h.

24 h 10 min of this clock = 24 h of the correct clock.

$$(24 \frac{1}{6} \text{ h}) \frac{145}{6} \text{ h of this clock} = 24 \text{ h of the correct clock.}$$

$$29 \text{ h of this clock} = [24 \times (\frac{6}{145}) \times 29] \text{ h of the correct clock}$$

= 28 h 48 min of the correct clock.

Therefore, the correct time is 28 h 48 min after 4 a.m.

This is 48 min past 8 p.m.

- 21. (D)** Statement I is incorrect. By simply using the angle-finding theory, we can find the angle between the two hands of the clock:

$$\text{Angle between hands} = [(\frac{11}{2}) \text{ Minute} - (30 \text{ Hour})]$$

$$\begin{aligned}
 \text{Angle} &= [(\frac{11}{2}) 20 - 30 \times 3] \\
 &= [11 \times 10 - 90] \\
 &= [110 - 90] \\
 &= 20^\circ
 \end{aligned}$$

Statement II is again incorrect. A day starts at 12 a.m. and ends at 11:59 p.m.; from 12 a.m. again a new day starts. If the hour hand and the minute hand coincide or overlap each other, that means the angle between them is 0. The first time they coincide in a day is at 12 a.m., then anywhere between 1 a.m. and 2 a.m., and it carries on in this way. The last time they coincide is anywhere between 11 p.m. and 12 a.m.; then another day starts. So, if we calculate them in a span of 12 h, they are coinciding for 11 times, then for a day they will be coinciding for $11 \times 2 = 22$ times. Here we have been asked to find for half a day, which will be 11 times.

- 22. (D)** Time from 10 a.m. on a day to 2 p.m. on the following day = 28 h.

24 h 10 min. of this clock = 24 h of the correct clock.

$$\frac{145}{6} \text{ h of this clock} = 24 \text{ h of the correct clock.}$$

$$28 \text{ h of this clock} = [24 \times \frac{6}{145} \times 28] \text{ h of the correct clock}$$

$\approx 27 \text{ h } 48 \text{ min of the correct clock.}$

Therefore, the correct time is 27 h 48 min. after 10 a.m., that's 48 min past 1 p.m.

- 23. (D)** From Sunday 12 noon to the following Sunday at 2 p.m. = 7 days 2 h = 170 h.

$$\text{The watch gains } 4 + 4 \frac{48}{60} = 8 \frac{4}{5} \text{ min in } 170 \text{ h.}$$

$$\therefore \text{The watch gains } 4 \text{ min in } \frac{4}{8 \frac{4}{5}} \times 170 = \frac{5}{11}$$

$$\times 170 = 77 \text{ h } 16 \text{ min } (\frac{3}{11} \times 60 \approx 16).$$



Now, $77 \text{ h } 16 \text{ min} = 3 \text{ days } 5 \text{ h } 16 \text{ min}$
 3 days 5 h 16 min from Sunday noon = 5:16 p.m. on Wednesday.

- 24. (D)** In 60 min, the minute hand gains 55 min over the hour hand in an accurate clock.

The minute hand must gain 60 min over the hour hand to re-join the hour hand.

$$55 \text{ min. are gained in } (\frac{60}{55} \times 60) \text{ min} = 65 \frac{5}{11} \text{ min.}$$

But, they are together after 62 min.

$$\therefore \text{Gain in } 62 \text{ min} = 65 \frac{5}{11} - 62 = 3 \frac{5}{11} \text{ min.}$$

$$\text{Gain in } 24 \text{ h} = \frac{38}{11} \times 60 \times \frac{24}{62} \text{ min} =$$

$$\frac{27,360}{341} \text{ min} \approx 80.2 \text{ min}$$

\therefore The clock gains 80.2 min approximately in 24 h.

- 25. (C)** Time interval indicated by incorrect clock = 5 p.m. – 12 a.m. = 7 h.

Time gained by incorrect clock in one hour

$$= +1 \text{ min} = +\frac{1}{60} \text{ h.}$$

Using the formula →

$$\frac{\text{True time interval}}{\text{Time interval in incorrect clock}}$$

$$= \frac{1}{1 + \text{hour gained in } 1 \text{ h by incorrect clock}}$$

$$\Rightarrow \frac{\text{True time interval}}{7} = \frac{1}{1 + \frac{1}{60}}$$

$$\Rightarrow \text{True time interval} = 7 \times \frac{60}{61} = 6 \frac{54}{61}$$

$$\therefore \text{True time} = 5 \text{ p.m.} + 6 \frac{57}{61} \text{ h} = 11 \text{ p.m.} +$$

$$\frac{54}{61} \text{ h} = 11 \text{ p.m.} + \frac{54}{61} \times 60 \text{ min.}$$

$$\Rightarrow 53 \frac{7}{61} \text{ min past 11.}$$



Calendar test-related questions require a lot of practice. Although such questions appear less frequently in tests, these are important with respect to GATE, JEE Main, and other engineering entrance examinations. If you understand the fundamentals of these problems, solving them will become second nature to you. It's possible that you'll be asked if the year in question is a leap year or not! You will be given a date and a day, and you will be asked to predict which day falls on that date the next year, as well as many others. In such questions, one has to find the days of the week on a particularly given date. The process of finding it is tied with obtaining the number of odd days.

We'll try to cover some of the calendar test problems and discuss the tactics and procedures for answering them in depth. Then some practice questions are given for you to answer and a calendar test to see how much you have mastered the concept. You must be familiar with the concept of odd days in order to comprehend the calendar.

INTRODUCTION TO CALENDAR

A calendar is a mechanism for keeping track of days. This is accomplished by naming time intervals, which are commonly days, weeks, months, and years. Within such a system, a date is the designation of a single, unique day. A calendar is a physical record of such a system (typically made of paper). The calendar we use today is called 'The Gregorian calendar'. The Gregorian calendar is the most widely used calendar on the planet. Pope Gregory XIII introduced it in October 1582 as a revision and replacement for the Julian calendar. The main adjustment was to space leap years differently, resulting in an average calendar year of 365.2425 days, which is closer to the 365.2422-day 'tropical'

or solar' year determined by the earth's movement around the sun. The time in which the earth travels around the sun is called as a solar year, and it is equal to 365 days 5 h 48 min and $47\frac{1}{2}$ s.

1. A year contains 365.2422 days approximately.
2. The common year or a non-leap year consists of 365 days.
3. The difference between a common year and a solar year is therefore 0.2422 of a day and we consider it by adding a whole day to every fourth year.
4. This is the reason why we have 366 days in every 4th year.
5. The years which have the extra day are called leap years. The day is inserted at the end of February, the difference between 4 common years and 4 solar years is 0.969 of a day.

If, therefore, we add a whole day to every 4th year, we add too much by 0.0312 of a day. To take account of this, we omit the extra day three times every 400 years.

Calendars: Important terms

1. **Day:** It is the basic unit of the calendar. A day has 24 h and 7 days makes up a week.
2. **Week:** A week consists of 7 days, which are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. Approximately 52 weeks makes up a year.
3. **Month:** A month has 28/29/30/31 days. A month is the 12th part of a year, or a year consists of 12 months.
4. **Year:** Year is the time taken by earth to make one revolution around the sun. A year is the 100th part of a century.
5. **Date:** A date is a term given to each day in general. The 28th/29th/30th/31st part of a month is the date. The 365th/366th part of a year (Lunar/Leap Year) is also known as the date.



6. **Century:** A block of 100 years is called a century.
7. **Ordinary Year:** The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.
8. **Leap Year:** There are 366 days in a leap year. If the year is not a century, any year divisible by four is a leap year. A leap year occurs once every fourth century and once every other century. If the year is divisible by both 4 and 400, it is a leap year.
For example: 1700 is not a leap year because it is not divisible by both 4 and 400.
9. **What is meant by odd days?**
On a given date, we must determine the day of the week. We use the concept of 'odd days' to do this. Odd days occur when the number of days in a certain time exceeds the total number of weeks.
We have clearly understood the relevance of finding odd days in relation to the concept of finding days that fall on a specific day up to this point. Now we'll look into how to figure out how to calculate these odd days for a particular date.

Counting of Odd Days: As we all know that, 1 ordinary year contains 365 days which is = 52 weeks + 1 day

Therefore, 1 ordinary year has 1 odd day.

So, we can say that 1 leap year = 366 days = (52 weeks + 2 days)

Therefore, 1 leap year has 2 odd days.
Now we will be calculating number of odd days in a century.

1 century or 100 years = 76 ordinary years + 24 leap years.

The number of odd days = $(76 \times 1 + 24 \times 2)$ odd days = 124 odd days.

Now, 124 days = (17 weeks + days) = 5 odd days.
So, the number of odd days in a century or 100 years = 5

Then the number of odd days in 200 years = (5×2) = 3 odd days.

Number of odd days in 300 years = (5×3) = 1 odd day.

Number of odd days in 400 years = $(5 \times 4 + 1)$ = 0 odd day.

Similarly, each one of 800 years, 1,200 years, 1,600 years, 2,000 years etc. has 0 odd days.

Day of the week related to odd days (assuming that 1 AD January 1st is a Monday):

NO. OF DAYS:	0	1	2	3	4	5	6
DAY:	Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.

PRACTICE QUESTIONS

1. How many leap years are there between 1900 and 2000?
 - A. 20
 - B. 25
 - C. 24
 - D. 26
2. Today is Wednesday. After 57 days, what day will it be?
 - A. Monday
 - B. Tuesday
 - C. Sunday
 - D. Thursday
3. What will be the day of the week on 17th August 2011?
 - A. Saturday
 - B. Sunday
 - C. Wednesday
 - D. Monday
4. If 7th April 2005 is Thursday, what was the day of the week on 7th April 2004?
 - A. Sunday
 - B. Tuesday
 - C. Saturday
 - D. Wednesday



5. On what date of March 2005 did the first Monday fall?
A. 4th March
B. 5th March
C. 7th March
D. 8th March
6. From the options given below, find the ones that are leap years
I. 1988 II. 1800
III. 1830 IV. 1920
A. Only IV and I
B. Only IV, I, and II
C. Only IV and III
D. Only IV and II
7. If 25th April 2020 was Saturday, what will be the day on 25th April 2021?
A. Monday B. Tuesday
C. Wednesday D. Sunday
8. If today is Monday, then what will be the day after 149 days?
A. Monday B. Tuesday
C. Wednesday D. Thursday
9. If 20th February 2012 was Monday, what was the day on 20th February 2013?
A. Saturday B. Sunday
C. Tuesday D. Wednesday
10. If today is Thursday, then what will be the day after 1,757 days?
A. Monday B. Tuesday
C. Wednesday D. Thursday
11. Which of the following is a leap year?
A. 1500 B. 1600
C. 1700 D. 1800
12. If 29th March 2006 was on Wednesday, what was the day on 1st April 2007?
A. Saturday B. Sunday
C. Tuesday D. Wednesday
13. How many days are there in y weeks y days?
A. $y^2 + y$ B. $2y + y$
C. $7y^2$ D. $8y$
14. How many non-leap years are there from 1701 to 1800?
A. 24 B. 26
C. 75 D. 76
15. For how many times does the 29th day of a month occur from 1601 to 2000?
A. 4097 B. 4703
C. 4497 D. 3333
16. If 21st February 1988 was on Sunday, what was the day on 20th April 1989?
A. Saturday B. Friday
C. Tuesday D. Wednesday
17. The date is February 21st, and the day is Sunday. It is a leap year. After 5 years, determine the day of the week on this date.
A. Thursday B. Sunday
C. Saturday D. Friday
18. It was Wednesday on 1st January, 1997. What was the day of the week 1st January, 2001?
A. Monday B. Tuesday
C. Wednesday D. Sunday
19. If every second Saturday and all Sundays are holidays for months of July and August 2020, then find the total number of working days for these months.
A. 47 B. 48
C. 49 D. 50
20. On what date of January 2005 did the first Saturday fall?
A. 1st January B. 2nd January
C. 3rd January D. 5th January
21. What was the day of the week on 23rd July 1776?
A. Monday B. Tuesday
C. Saturday D. Sunday
22. Which of the years next to 2004 will have the same calendar as that of the year 2004?
A. 2028 B. 2030
C. 2032 D. 2034



- 23.** On which date does the 2nd Friday of March 2023 fall?
A. 9th March B. 10th March
C. 11th March D. 12th March
- 24.** Mark the correct statements.
I. 1700 was a leap year.
II. The calendar of 2001 will be repeated in 2006.
III. 15th August 1947 was on Friday.
A. Only I and II B. Only II and III
C. Only III D. Only II
- 25.** What was the day of the week on 19th June 1998?
A. Monday B. Wednesday
C. Friday D. Sunday
- 26.** Fourth Saturday and every Sunday are a holiday in a 28 days' month. Read the following sentences and mark the correct inferences.
I. Total working days will be 23 in the given month
- II. 4th Saturday will fall on the 27th day.
A. Only I B. Only II
C. Both I and II D. Neither of them
- 27.** On which date does the 3rd Wednesday of November 2030 fall?
A. 18th November B. 19th November
C. 20th November D. 21st November
- 28.** The year next to 1997 will have the same calendar as that of the year 1997?
A. 1997 B. 2000
C. 2001 D. 2003
- 29.** What will be the day of the week on 23rd August 2015?
A. Sunday B. Saturday
C. Wednesday D. Monday
- 30.** On what dates of February 2002 did Friday fall?
A. 1st, 8th, 14th, and 22nd.
B. 1st, 8th, 15th, and 21st.
C. 1st, 8th, 15th, and 22nd.
D. 1st, 7th, 15th, and 22nd.

SOLUTIONS

- 1.** **(C)** We have been asked to calculate the years which are multiples of 4 between 1900 and 2000, so we must exclude 1900 and 2000. Though they are multiples of 4, the question demands years between them, so we must exclude them.
So, the leap year would be 1904, 1908, 1912..... 1996
Here, we will be solving this question by using Arithmetic Progression: -
$$N_t = [(L_2 - L_1)/d] + 1$$
 [where, N_t = no of terms, L_1 = first term, L_2 = last term, d = common difference.]
$$N_t = 1996 - 1904 = 92/4 = 23 + 1 = 24$$

So, the number of years which are divisible by 4 are 24. So, there are 24 leap years between 1900 and 2000.
- 2.** **(D)** Each day of the week is repeated after 7 days. If today is Wednesday, that means after 7 days, it will again be Wednesday. Dividing $(57/7 = 1$ odd day). So, after 57 days, it will be Thursday.
- 3.** **(C)** 17th August 2011 = (10 years from 2000 + period from 1/1/2011 to 17/8/2011)
The no. of odd days in 1600 years = 0
The no. of odd days in 400 years = 0
10 years = (2 leap years + 8 ordinary years) = $(2 \times 2 + 8 \times 1) = 12 = 5$ odd days.
No. of days from 1st January to 17th August = $(31 + 28 + 31 + 30 + 31 + 30 + 31 + 17) = 229$ days = (32 weeks + 5 days) = 5 odd days.
Total number of odd days = $(0 + 0 + 5 + 5) = 10 = 3$ odd days.



So, the day on 17th August 2011 was Wednesday.

4. (D) The year 2004 is a leap year. So, it has 2 odd days.

But February 2004 is not included because we are calculating from April 2004 to April 2005.

So, it has 1 odd day only.

The day on 7th April 2005 will be 1 day beyond the day on 7th April 2004.

Given that, 7th April 2005 is Thursday.

7th April 2004 is Wednesday (1 day before 7th April 2005).

5. (C) First we have to find the day on 1st March 2005

$1/3/2005 = (2004 \text{ years} + \text{period from } 1/1/2005 \text{ to } 1/3/2005)$

Odd days in 1600 years = 0

Odd days in 400 years = 0

4 years = (1 leap year + 3 ordinary years) = $(1 \times 2 + 3 \times 1) = 5$ odd days.

Jan., Feb., Mar. ($31 + 28 + 1 = 60$ days) = $(8 \text{ weeks} + 4 \text{ days}) = 4$ odd days.

Total numbers of odd days = $(0 + 5 + 4 = 9) = 2$ days

That 1st March was Tuesday. So, the first Monday falls on 7th March.

6. (A) A leap year is the one that is divisible by 4. So, 1988 and 1920 are leap years but not 1830. 1800 is not a leap year because it is a century year, and to be a leap year it must be divisible by both 4 and 400. Therefore, only statements I and IV are correct.

7. (D) Here we will use the concept of odd days. Since 2020 is a leap year so it has 2 odd days. But here we would not add 2 days to get the day on the same day next year because February 2020 which has an extra 1 day in leap year does not include in the span of 25th April 2020 to 25th April 2021. So, we will add only 1 odd day to get the day on the same date the next day. Therefore, it was Sunday on 25th April 2021.

8. (C) In questions where we have been asked to find the day after 'n' number of days, we have to simply divide that number

of days (n) from 7 because days repeat themselves after 7 days. If it is completely divisible by 7, then that day will be the same as the given day. In this question = 21 weeks + 2 odd days, so this is not completely divisible by 7. Therefore, we need to add this odd day to the present day to get the actual day after 149 days. So, it will be Wednesday.

9. (D) Now this question is on the same line as the previous question. Here we will use the concept of odd days. Since 2012 is a leap year, it has 2 odd days. But here is a twist as this would add 2 days to get the day on the same day next year because February 2012 which has an extra 1 day in a leap year is included in the span of 20th February 2012 to 20th February 2013. So, we will add 2 odd days to get the day on the same date next year. Therefore, it was Wednesday on 20th February 2013.

10. (D) In questions where we have been asked to find the day after 'n' number of days, we have to simply divide that number of days (n) from 7 because days repeat themselves after 7 days. If it is completely divisible by 7, then that day will be the same as the given day. In this question = 251, so this is completely divisible. Therefore, after 1757 days, it will again be Thursday.

11. (B) Leap year is the one which is divisible by both 4 and 400, but the twist is that for century years it must be divisible by 400 as well to be called a leap year. So out of all options, only 1600 is divisible by both 4 and 400. Therefore, it is only a century year.

12. (B) Now this question is on the same line as the previous questions. Here we will use the concept of odd days. Since 2006 is a non-leap year, it has 1 odd day. So, we will add a 1-day odd day to get the day on the same date the next day. Therefore, it was Thursday on 29th March 2007. Now, adding 3 (29th to 1st) more days to this outcome



date we will get the required day. So, it will be Sunday on 1st April 2007.

- 13. (D)** Let us understand this question through examples. Suppose $y = 2$, then the total number of days will be $7 \times 2 + 2 = 16$ or $y = 5$ then the number of days will be $7 \times 5 + 5 = 40$. So, by these examples that days are multiple of 8, so we can say that $7y + y = 8y$.
- 14. (D)** We have been asked to calculate the years which are not multiples of 4 from 1701 to 1800.

So, the leap years would be 1704, 1708, 1712..... 1796 {Note: 1800 is not a leap year} Here, we will solve this question by using Arithmetic Progression:

$N_t = [(L_2 - L_1)/d] + 1$ [where, N_t =no of terms, L_1 = first term, L_2 = last term, d = common difference.]

$$N_t = 1796 - 1704 = 92/4 = 23 + 1 = 24$$

So, the number of years which are divisible by 4 are 24. So, there are 24 leap years from 1701 to 1800. But here we have to find non-leap years so we will simply subtract 24 from number of years from 1701 to 1800, which is 100. There a total number of non-leap years are 76.

- 15. (C)** Total number of years from 1601 to 2000 is 400. So, in 400 years, we have 97 leap years. But February has only 28 days in a non-leap year and 29 days in leap year; therefore, we would not calculate non-leap years containing February months since it doesn't have 29 days.

The total number of months are $\Rightarrow 400 \times 11 = 4400$ (excluding all Februarys)

Now add these 97 Februarys to the total number of months to get the actual number of months that has 29th days. $(4400 + 97 = 4497)$

Therefore, we have 4497 months which have 29th days from 1601 to 2000.

- 16. (B)** Now this question is on the same line as the previous question. Here we will use the concept of odd days. Since 1988 was a leap year, it has 2 odd days. But here is a twist as this would add 2 days to get the day

on the same day next year because 29th February 1988, which has an extra 1 day in a leap year, is included in the span of 21st February 1988 to 20th April 1989. So, we will add 2 odd days to get the day on the same day the next day. Therefore, it was Tuesday on 21st February 1989. Now we will add 59 more days (22nd February 1988 to 20th April 1989 = $8 + 31 + 20 = 59$) to get the exact day. So, we have 3 odd days and adding this to Tuesday, we get Friday on 20th April 1989.

- 17. (C)** It has been given in the question that this is a leap year. As a result, each of the next three years will have one odd day (the idea of odd days in an ordinary year). Then there are two odd days in leap year, and one odd day the next year. Therefore $(3 + 2 + 1) = 6$ odd days will be there. Hence the day of the week will be 6 odd days that's 5 days beyond Sunday. So, it will be Saturday on the date after 5 years.
- 18. (A)** On 31st December 1996 it was Tuesday. Number of odd days from 1996 to 2000 = $(2 + 1 + 1 + 1 + 2) = 7$ days.
On 31st December 2000, it was Sunday.
Thus, on 1st January 2001, it is Monday.

- 19. (C)** For solving this question first we are supposed to find the day on 1st July 2020.
Counting of odd days:
1600 years have 0 odd days.
400 years have 0 odd days.
 $19 \text{ years} = (4 \text{ leap years} + 15 \text{ ordinary years}) = [(4 \times 2) + (15 \times 1)] = 23$ (3 weeks + 2 days) = 2 odd days
No. of days from 1st of January to 30th June = $(31 + 29 + 31 + 30 + 31 + 30) = 182$ days = 29 weeks = 0 odd days
Total number of odd days = $(0 + 2) = 2$.
Therefore, 1st July was on Wednesday.
Now the total number of days in July and August is 62. So we have these following days on which Sundays will fall: 5th July, 12th July, 19th July, 26th July, 2nd Aug, 9th Aug, 16th Aug, 23rd Aug, 30th Aug
2nd Saturdays will fall on: 11th July, 25th July, 8th Aug, 22nd Aug.



So, the total number of holidays will be 13 therefore total numbers of working days are $(62 - 13) = 49$

- 20. (A)** First we have to find the day on 1st January 2005

$1/1/2005 = (2004 \text{ years} + \text{period from } 1/1/2005 \text{ to } 1/3/2005)$

Odd days in 1600 years = 0

Odd days in 400 years = 0

4 years = $(1 \text{ leap year} + 3 \text{ ordinary years}) = (1 \times 2 + 3 \times 1) = 5$ odd days.

So, it was Friday on 31st December 2004; therefore, 1st January 2005 was Saturday.

- 21. (A)** $23\text{rd July } 1776 = (1775 \text{ years} + \text{Period from } 1\text{st Jan } 1776 \text{ to } 23\text{rd July } 1776)$

Counting of odd days:

for the 1600 years have 0 odd days.

for the 100 years have 5 odd days.

for the upcoming 75 years we have = $(18 \text{ leap years} + 57 \text{ ordinary years}) = [(18 \times 2) + (57 \times 1)] = 93$ (13 weeks + 2 days) = 2 odd days

1775 years have $(0 + 5 + 2)$ odd days = 7 odd days = 0 odd days.

No. of days from 1st of January to 23rd of July = $(31 + 29 + 31 + 30 + 31 + 30 + 23) = 205$ days = (29 weeks + 2 days)

Total number of odd days = $(0 + 2) = 2$.

The required day was 'Tuesday'.

- 22. (C)** We have a general rule for repetition of a calendar.

Repetition of leap year \Rightarrow Add + 28 to the given year.

The given year is 2004 which is a leap year. So simply adding 28 to the given year we will get the year for which the calendar for 2004 will be repeated.

Therefore, the calendar for the year 2004 will be the same for the year 2032.

- 23. (D)** To solve this question first, we are supposed to find the day on 1st March 2023.

Counting of odd days:

1600 years have 0 odd days.

400 years have 0 odd days.

$22 \text{ years} = (5 \text{ leap years} + 17 \text{ ordinary years}) = [(5 \times 2) + (17 \times 1)] = 27$ (3 weeks + 6 days) = 6 odd days

January and February $\rightarrow 31 + 28 = 59$ days = 8 weeks 3 odd days = 3 odd days

Total number of odd days = $(6 + 3) = 2$.

Therefore, 1st March 2023 will be on Wednesday

So, the 1st Friday will fall on 3rd of March and subsequently, the 2nd Friday will fall on the 10th of March.

- 24. (A)** Statement I is incorrect. For a century year, it can be called a leap year only when it is divisible by both 4 and 400. Since 1700 is not divisible by 400, it is not a leap year but an ordinary year.

Statement II is again incorrect because calendars repeat themselves in the given order:

leap year = after 28 years

1 year after leap year = 6 years from this year.

2 years after leap year = 6 years from this year.

3 years after leap year = 11 years from this year.

Now, the given year is 2001, which is 1 year from a leap year so it will repeat after 6 years from this year that is in 2007.

Statement III is correct. Odd days in 1600 years = 0

Odd days in 300 years = 1

$46 \text{ years} = (35 \text{ ordinary years} + 11 \text{ leap years}) = (35 \times 1 + 11 \times 2) = 57$ (8 weeks + 1 day) = 1 odd day

Number of days from 1st January to 15th of August $(31 + 28 + 31 + 30 + 31 + 30 + 31 + 15) = 227$ days = (32 weeks + 3 days) = 3 odd days.

Total number of odd days = $(0 + 1 + 1 + 3) = 5$ odd days.

Hence, as the number of odd days = 5, the given day is Friday.

- 25. (C)** $17\text{th June } 1998 = (1997 \text{ years} + \text{Period from } 1.1.1998 \text{ to } 17.6.1998)$

The number of odd days in 1600 years = 0

The number of odd days in 300 years = 1

For the upcoming 97 years we have = 24 leap years + 73 ordinary years.



Number of odd days in 97 years ($24 \times 2 + 73$) = 121 = 2 odd days.

Number of days from 1st January to 19th June $\Rightarrow (31 + 28 + 31 + 30 + 31 + 19) = 170$ days
 $= 24$ weeks 2 days = 2 odd days.

Total number of odd days = $(0 + 1 + 2 + 2) = 5$.

The given day is Friday.

- 26.** As given in the question, the month starts on Sunday so every 7th day from 1st date of this month would be a Sunday. Therefore, Sundays fall on 1st, 8th, 15th, 22nd.

Now the 1st Saturday will fall on the 7th then subsequently on the 14th, 21st, 28th.

Total no. of working days will be $(28 - 5 = 23)$ 23 days.

4th Saturday will fall on the 28th day

So, only statement I is true.

- 27.** To solve this question, first we need to find the day on 1st November 2030.

Counting of odd days:

1600 years have 0 odd days.

400 years have 0 odd days.

29 years = $(7$ leap years $+ 22$ ordinary years)
 $= [(7 \times 2) + (22 \times 1)] = 36$ (5 weeks + 1 day) = 1 odd days

From 1st January to 31st October $\rightarrow (31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31) = 304$ days = 43 weeks 3 odd days = 3 odd days

Total number of odd days = $(1 + 3) = 4$. So, on 31st October it will be Thursday.

Therefore, 1st November 2030 will be on Friday

So, 1st Wednesday will fall on 6th of November and subsequently 2nd Wednesday will fall on 13th and 3rd Wednesday will fall on 20th November 2030.

- 28.** We have a general rule for repetition of a calendar.

For the repetition of a leap year \Rightarrow Add + 28 to the given year.

For the repetition of a non-leap year, follow these steps:

Step 1: Add + 11 to the given year. If result is a leap year, go to step 2.

Step 2: Add + 6 to the given year.

Given year is 1997, which is a non-leap year.

Step 1: Add + 11 to the given year (i.e., $1997 + 11$) = 2008, which is a leap year.

Step 2: Add + 6 to the given year (i.e., $1997 + 6$) = 2003

Therefore, the calendar for the year 1997 will be the same for the year 2003.

- 29.** 23th August 2015 = (14 years from 2000 + period 1/1/2015 to 23/8/2015)

Odd days in 1600 years = 0

Odd days in 400 years = 0

14 years = $(3$ leap years $+ 11$ ordinary years)
 $= (2 \times 3 + 11 \times 1) = 17 = 3$ odd days.

No. of days from 1st of January to 23rd of August = $(31 + 28 + 31 + 30 + 31 + 30 + 31 + 23) = 235$ days = $(33$ weeks $+ 4$ days) = 4 odd days.

Total number of odd days = $(0 + 0 + 3 + 4) = 7 = 0$ odd days.

So, the day on 23rd August 2015 was Sunday.

- 30.** We shall find the day on 1st January 2002.

1st February 2002 = (2001 years + period from 1.1.2002 to 1.2.2002)

The no. of odd days in 1600 years = 0

The no. of odd days in 400 years = 0

Since 2001 is an ordinary year, it has 1 odd day.

January has 31 days = 3 odd days.

Total number of odd days = $(0 + 1 + 3) = 4$, so 31st January was Thursday.

On 1st February, 2002 it was Friday.

In February 2002, Friday fell on 1st, 8th, 15th, and 22nd.

40 Series



A series is a collection of letters, numbers, or both arranged in such a way that each term in the collection follows a set of rules. These rules can be based on mathematical operations, the place of letters in alphabetical order, and so on. In these questions, one is required to figure out the logic used to form the sequence or series of numbers/alphabets. By understanding the logic, one is required to deduce either a continuation, the immediate next term, a missing term, or a wrong term within the series.

The various types of series are discussed below:

1. Number series

A number series is a set of numbers that are arranged in a specific order and follow a predetermined pattern.

In this section, we'll look at questions in which you're given a sequence of numbers (which are referred to as the terms of the series). Throughout the series, these numbers/terms follow a consistent pattern. Candidates are prompted to either locate a missing word or the incorrect series term.

Prominently, the following patterns of reasoning have been observed in the analytical questions.

Addition or subtraction basic

This series only has questions that include addition or subtraction operations. They could be even or odd numbers or even a sub-series that follows a set pattern within itself.

This can be understood with the help of the following examples:

- With normal even number series:

53, 43, 35, 29, 25, ____.

Here, the pattern followed is:

$$\begin{aligned}(53 - 43) &= 10, \\(43 - 35) &= 8, \\(35 - 29) &= 6, \\(29 - 25) &= 4.\end{aligned}$$

Thus, the answer should be $(25 - 2 = 23)$.

- With prime numbers

13, 15, 18, 23, 30, ____.

Here, the pattern followed is –

$$\begin{aligned}(13 + 2) &= 15, \\(15 + 3) &= 18, \\(18 + 5) &= 23, \\(23 + 7) &= 30,\end{aligned}$$

Thus, the answer should be a sum of the next prime number and the previous number in the series, that is, $(30 + 11 = 41)$.

- Combination sub-series

16, (4,3) 23, (6,5) 34, (8,7) 49, ____.

Here, the series has a sub-series of even numbers (starting from 4) and prime numbers (starting from 3).

$$\begin{aligned}16 + 4 + 3 &= 23, \\23 + 6 + 5 &= 34, \\34 + 8 + 7 &= 49.\end{aligned}$$

Thus, the answer here should be the sum of the previous number in the series + the next number in the prime series + the next number in the even series, that is, $(49 + 10 + 11 = 70)$.

- Decimal addition or subtraction

1.5, 2.3, 3.1, 3.9, ____.

Here, the next figure is a sum of 0.8 and the previous figure. Thus,

$$\begin{aligned}(1.5 + 0.8), \\(2.3 + 0.8), \\(3.1 + 0.8).\end{aligned}$$

Hence, the answer should be $(3.9 + 0.8 = 4.7)$.

Multiplication or division of numbers

This can easily be understood through the following examples:

- 9, 81, 729, 6,561, ____.

Here, 9 is multiplied to get the next number.

$$\begin{aligned}(9 \times 9) &= 81, \\(9 \times 81) &= 729, \\(729 \times 9) &= 6,561,\end{aligned}$$

Thus, the answer is $(6561 \times 9 = 59,049)$.



- 54, 9, 36, 6, ___, 3.

Here, the bigger number once divided by 6 leaves the next smaller number as the quotient. Thus, $(54/6 = 9)$, $(36/6 = 6)$; therefore, the missing number should be $(6 \times 3 = 18)$.

- $1/4, 1/8, 1/16, 1/24, 1/32, \text{___}$.

Here, $1/4$ being the starting point, the next element is multiplied by $1/n$ where n is an even number starting from 2. Thus, the series goes like this:

$$\begin{aligned} &(1/4 \times 1/2), \\ &(1/4 \times 1/4), \\ &(1/4 \times 1/6), \\ &(1/4 \times 1/8). \end{aligned}$$

This leads us to conclude that $(1/4 \times 1/10 = 1/40)$ is the answer.

Squares or cubes and like powers

Needless to say, this form of number series focuses on patterns with squares and cubes of numbers.

- 626, 126, 26.

Here, 5 has been raised to consecutive powers and each result is thus increased by 1. Then it has been arranged in descending order.

Thus, $(5^4 + 1 = 626)$

$$(5^3 + 1 = 126),$$

$$(5^2 + 1 = 26).$$

- 1, 4, 27, 16, 125, 36, 343, ___.

Here, the series starts with 1. All the squares are of even numbers and cubes of odd numbers. Since 343 is a cube of 7, the next element will be the square of 8, that is, 64.

Combination

Sometimes in the given series, one has to make use of multiple logic to identify the pattern. Let us understand it better with the following examples:

- 49, 56, 64, 72, 81, ___.

Here, the series follows the following pattern:

$$\text{Square of } 7 = 7 \times 7 = 49.$$

Then, the number whose square it was, is multiplied by the next natural number.

$$7 \times 8 = 56.$$

Then again, the new number is square; thus,

$$8 \times 9 = 64.$$

Similarly,

$$8 \times 9 = 72.$$

$$9 \times 9 = 81.$$

Thus, the missing element should be $(9 \times 10 = 90)$.

- 8, 6, 9, 23, 87, ___.

In the given series, there are multiplication and subtraction operations performed with a particular logic:

$$8 \times 1 - 2 = 6$$

$$6 \times 2 - 3 = 9$$

$$9 \times 3 - 4 = 23$$

$$23 \times 4 - 5 = 87$$

Thus, the answer should be,

$$87 \times 5 - 6 = 429.$$

- 64, 45, 28, ___.

In the given series, there are multiplication and subtraction operations performed in descending order.

$$16 \times 4 = 64.$$

$$15 \times 3 = 45.$$

$$14 \times 2 = 28.$$

Thus, the answer should be,

$$13 \times 1 = 13.$$

- 0.2, 0.3/2, 0.9/8, ___.

$$0.2 \times \frac{3}{4} = 0.3/2.$$

$$0.3 \times \frac{3}{4} = 0.9/8.$$

Thus, the answer should be,

$$0.9/8 \times \frac{3}{4} = 2.7/32.$$

2. Letter series

It is a logical arrangement of the letters of the English alphabet in a specific way. They are arranged in a way that each term in the series is identified by following a set of rules. Some of these rules can be based on the positions of different letters in alphabetical order.

The numerical positions of the letters in the alphabets are given below:

There are mainly two types of letter series:



ALPHABET POSITIONS														
Letter	A	B	C	D	E	F	G	H	I	J	K	L	M	
Position	1	2	3	4	5	6	7	8	9	10	11	12	13	
Reverse Order	26	25	24	23	22	21	20	19	18	17	16	15	14	
Letter	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	
Position	26	25	24	23	22	21	20	19	18	17	16	15	14	
Reverse Order	1	2	3	4	5	6	7	8	9	10	11	12	13	

a. Alphabet series

The letters of the English alphabet are arranged in a particular pattern in this type of series. For example, reverse order of letters, the position of letters in alphabetical order, etc.

b. Continuous Pattern Series

Understand with example



Example 1: Which term will come next in the series? A, C, E, G, ?

Now, we can see that as per the positions of these letters in the alphabet, they are at 1, 3, 5 and 7. The gap between the two consecutive terms is two. So, the next term would be of position 9, i.e., I.

Example 2: Which term will come next?
ACE, CEF, EGG, GIH, ?

Now, for the first letters of each term, the pattern followed is +2. So the first letter in next term would be I. For the second letter of each term, the pattern followed is +2. So, the second letter in the next term would be K. Lastly, for the last letters in each term, the pattern followed is +1. So, the last letter in next term would be I. So, the next term would be IKI.

In this type, a series of small/capital letters given follow a specific pattern. However, some letters are missing from the series which have to be found out.

3. Alphanumeric series

Understand with example



Example 1: Find the missing letters:
abca_c_bc_b_a_c

Now, the pattern followed here is abc/_abc/_abc/_abc/_abc. So the missing letters in that order are baacb.

Example 2: Find the missing letters: ab_db_dcc_ad_bca

Now, the pattern followed here is abcd/_badc/_cbad/_dbca. So the missing letters in that order are cabd.

The term alphanumeric means a combination of numbers and alphabets. So, alphanumeric series questions are a combination of numbers, alphabets and symbol-based series which candidates need to answer. The alphanumeric series generally covers 2-3 questions in the reasoning ability section of various entrance examinations.



The questions might include upper and lower-case alphabets, symbols, and punctuation marks. These questions are easy to solve and carry a considerable weightage in the reasoning section.

Tips to solve alphanumeric series questions

Here are some tips to help you prepare for the alphanumeric series topic:

- Analyse the alphanumeric pattern and try to crack that pattern for the pattern-

based series. These questions usually revolve around changes in the pattern's ascending or descending order.

- There is no pattern in the short series, which mainly comprises alphabets, numbers, and symbols. In these, check the questions and straightaway answer them.
- In the missing number series, check the entire series to find the answer and recheck the series to ensure there is no error.

PRACTICE QUESTIONS

[Q. 1-15] Find the missing element.

1. 31, 2, 41, 3, ___, 4, 61, 5, 71, 6
A. 24 B. 52 C. 51 D. 55
2. 19, 37, 55, 73, ___.
A. 91 B. 83 C. 109 D. 82
3. 90, 110, 132, ___, 182, 210.
A. 146 B. 138 C. 164 D. 156
4. 1, 2, 6, 15, 31, 56, ___.
A. 90 B. 92 C. 89 D. 87
5. 13, 32, 59, ___.
A. 78 B. 97 C. 94 D. 65
6. 11, 17, 39, 85, ___.
A. 134 B. 157 C. 163 D. 129
7. 2, 1, 1/3, 1/15, ___.
A. 1/105. B. 1/115. C. 1/125. D. 1/95
8. 0.7, 1.2, 1.4, 1.4, 2.1, 1.6, ___.
A. 2.7 B. 3.1 C. 2.8 D. 5.6
9. 2, 3, 10, 39, ___, 885.
A. 687 B. 456 C. 234 D. 172

10. 3, 9, 21, ___, 93.
A. 21 B. 42 C. 63 D. 45
11. 110, 1331, 120, 1728, ___.
A. 121 B. 143 C. 2197 D. 130
12. 10, 6, 8, 15, ___.
A. 34 B. 17 C. 4 D. 18
13. 132, 156, ___, 210, 240, 272.
A. 135 B. 169 C. 182 D. 178
14. 2, 4, 16, 256, ___.
A. 65,536 B. 8,192 C. 16,384
D. 32,768
15. 53, 53, 40, 40, 27, 27, ___.
A. 12 B. 14 C. 16 D. 18
16. In the below statement, how many vowels are preceded by a symbol?
Statement: U ! 2 4 O T & I @ # E 1 *
A. One B. Two C. Three D. Four
17. In the below series, which is the second element to the left of the sixth element from the right?



- B * 6 8 J L % U @ # V 3 &
 A. @ B. & C. L D. U

- 18.** Which number becomes the highest when its digits are arranged in ascending order?
 A. 895 B. 768 C. 321 D. 742
- 19.** How many consonants are immediately followed by symbols and preceded by a number in the below series?
 @ 9 6 B F J 7 M & % A I
 A. One B. Two C. Three
 D. None
- 20.** In the given series, when numbers are multiplied with each other, whose product would be lowest?
 289 123 111 823 308
 A. 289 B. 123 C. 111 D. 308

Directions (21–25): Find the wrong number in the series given below.

- 21.** 27, 125, 343, 625, 1000
 A. 27 B. 625 C. 343 D. 1000
- 22.** 8, 24, 48, 80, 120, 166
 A. 24 B. 48 C. 120 D. 166
- 23.** 2, 4, 12, 60, 240, 1440
 A. 4 B. 12 C. 60 D. 240
- 24.** 4, 11, 25, 55, 109, 221
 A. 55 B. 109 C. 25 D. 11
- 25.** 3, 12, 4, 20, 5, 25, 6, 42
 A. 12 B. 20 C. 25 D. 42

SOLUTIONS

- 1. (C)** The sequence is a combination of two given series:
 i. 31, 41, 52, 61, 71 and
 ii. 2, 3, 4, 5, 6
 Thus, the answer is $(41 + 10) = 51$.
- 2. (A)** The pattern followed here is
 $2 \times 9 + 1 = 19$
 $4 \times 9 + 1 = 37$
 $6 \times 9 + 1 = 55$
 $8 \times 9 + 1 = 73$
 $10 \times 9 + 1 = 91$.
- 3. (D)** The pattern followed here is
 $9 \times 9 + 9 = 90$.
 $10 \times 10 + 10 = 110$.
 $11 \times 11 + 11 = 132$.
 Thus, the answer should be: $12 \times 12 + 12 = 156$.
- 4. (B)** Here, the pattern followed is:
 Difference between each element – 1, 4, 9, 16, 25 = squares of (1, 2, 3, 4, 5).
 Thus, the answer should be, $56 + 36 = 92$.
- 5. (C)** The following pattern is followed here:

$$\begin{aligned}3 \times 4 + 1 &= 13. \\5 \times 6 + 2 &= 32. \\7 \times 8 + 3 &= 59.\end{aligned}$$

Thus, the answer should be, $9 \times 10 + 4 = 94$.

- 6. (C)** The following pattern is followed here:
 11
 $11 + (3^2 - 3) = 17$
 $17 + (5^2 - 3) = 39$
 $39 + (7^2 - 3) = 85$
 $85 + (9^2 - 3) = 163$
 Hence, the next number in the given sequence is 163.
- 7. (A)** This problem is based on prime numbers and division. The series starts with 2 and thereafter each element is divided by each prime number. Thus, the answer should be
 $1/(3 \times 5 \times 7) = 1/105$
- 8. (C)** The series is divided into two sub-series:
 First sub-series: 0.7, 1.4, 2.1, _____. Addition of 0.7 after every element.



Second sub-series: 1.2, 1.4, 1.6. Addition of 0.2 after every element.

Thus, the answer should be $(2.1 + 0.7 = 2.8)$.

- 9. (D)** The series follows the following pattern:

$$2 \times 1 + (1^2) = 3$$

$$3 \times 2 + (2^2) = 10$$

$$10 \times 3 + (3^2) = 39$$

$$39 \times 4 + (4^2) = 172$$

$$172 \times 5 + (5^2) = 885$$

- 10. (D)** The first difference is 6 and the second is 12.

So, if we have to take the difference as 6, 12, 24 and 48, the answer is $21 + 24 = 45$.

- 11. (D)** The series follows the following pattern:

$$11 \times 10 = 110$$

$$11^3 = 1331$$

$$12 \times 10 = 120$$

$$12^3 = 1728$$

Thus, the answer should be $(13 \times 10 = 130)$.

- 12. (A)** The series follows the following pattern:

$$10$$

$$10 \times 0.5 + 1 = 6$$

$$6 \times 1 + 2 = 8$$

$$8 \times 1.5 + 3 = 15$$

Thus, the answer should be $15 \times 2 + 4 = 34$.

- 13. (C)** The series follows the following logic:

$$11 \times 12 = 132, 12 \times 13 = 156, 13 \times 14 = 182, 14 \times$$

$$15 = 210, 15 \times 16 = 272,$$

Thus, the answer is $13 \times 14 = 182$.

- 14. (A)** This is a simple series in which the following number is a square of the previous number. Thus, the answer is the square of 256, that is, 65,536.

- 15. (B)** In this series, each number is repeated, then 13 is subtracted to arrive at the next number. Thus, the answer here should be $(27 - 13 = 14)$.

- 16. (B)** Two vowels: 'E' is preceded by '#' and 'I' is preceded by '&'.

- 17. (C)** The sixth element from the left is U, and the second to its left is L.

- 18. (B)** The ascending order series becomes

$$895 — 589$$

$$768 — 678$$

$$321 — 123$$

$$742 — 247$$

- 19. (A)** 'M' is followed by '&' and preceded by 7.

- 20. (D)** Any number multiplied by 0 would be 0, which is the lowest.

- 21. (B)** All numbers except 625 are cubes.

- 22. (D)** The wrong term is 166.

- 23. (C)** $2 \times 2 = 4, 4 \times 3 = 12, 12 \times 4 = 48, 48 \times 5 = 240, 240 \times 6 = 1440$. The wrong term is 60.

- 24. (A)** $4 \times 2 + 3 = 11, 11 \times 2 + 3 = 25, 25 \times 2 + 3 = 53, 53 \times 2 + 3 = 109, 109 \times 2 + 3 = 221$. The wrong term is 55.

- 25. (C)** $3 \times 4 = 12, 4 \times 5 = 20, 5 \times 6 = 30, 6 \times 7 = 42$. The wrong term is 25.



The concept of the ‘odd one out’ engages a person’s thinking abilities around a certain area or subject that is presented in a confusing manner with similar objects or writings.

To identify the odd one out accurately, one must improve their logical thinking skills by distinguishing those objects (or set of alternatives) using well-established ideas.

You must differentiate the objects and select one alternative from the list that is unique or out of the ordinary, i.e., one that is unrelated to the rest.

For example, when we compare elements such as lion, cat, tiger, cheetah, bear, and fox, we can see that they form a group of animals. How do we categorize them? If we want to segregate one animal from the rest, it will almost certainly be the cat, because it is the only domestic animal in the herd. The remaining animals (Lion, Tiger, Cheetah, Bear, and Fox) are wild animals.

HOW TO FIND THE ODD ONE OUT

The aim is to solve it using common sense, but we can also pick the different one or odd one out by focusing on the following categories:

- Even or odd
- Category of the object
- Gender of the object
- The object’s use and application

Make a concerted effort to distinguish between the options provided. Except for one, all of the options can be linked to one another and will fall into the same category.

Examine all of the alternatives and try to connect them using their common characteristics/purposes.

There are two kinds of questions asked on this topic:

1. Single words
2. Pair of words

These can also be subdivided into three subtypes:

1. Numbers
2. Words
3. Letters

Let’s look at the following examples to understand the concept better.

SOLVED EXAMPLES

Example 1. Choose the odd one out from the following options.

- | | |
|------------|-------------|
| A. Scanner | B. Joystick |
| C. Stylus | D. Printer |

Answer: D

Explanation: Option D is the correct answer because except for the printer which is an output device, the remaining given options are input devices.

Example 2. Choose the odd one out from the following options.

- | | |
|-------------|-----------|
| A. 121, 163 | B. 49, 81 |
| C. 225, 289 | D. 1, 9 |

Answer: A

Explanation: Option A is the correct answer because the two given numbers in all the other options are squares of consecutive natural odd numbers.

Example 3. Choose the odd one out from the following options.

- | | |
|---------------|----------|
| A. Strawberry | B. Apple |
| C. Papaya | D. Guava |

**Answer: A**

Explanation: Option A is the correct answer because all the other fruits given in the options have seeds on the inside whereas strawberries have seeds on the outside.

Example 4. Choose the odd one out from the following options.

- | | |
|------------|-----------|
| A. Jar | C. Saucer |
| B. Pitcher | D. Mug |

Answer: B

Explanation: Option B is the correct answer because all the given options are a type of container that contain liquid whereas

saucer is a type of a small plate or shallow bowl that supports a cup used to serve tea or coffee.

Example 5. Choose the odd one out from the following options.

- | | |
|------------|-----------|
| A. Diamond | B. Iron |
| C. Gold | D. Silver |

Answer: A

Explanation: Option A is the correct answer because except for diamond, rest all the options given, i.e., Iron, Silver and Gold, are metals.

PRACTICE QUESTIONS

Directions: Each of the following questions contains four options, three of which are similar in a way and one of which is different. Choose the odd one out.

1. Choose the odd one out.
A. Freedom of speech
B. Right against exploitation
C. Freedom of religion
D. Right to make contract

2. Choose the odd one out.
A. Seismology B. Epicentre
C. Crater D. Richter Scale

3. Choose the odd one out.
A. Retinol B. Amylase
C. Calciferol D. Phylloquinone

4. Choose the odd one out.
A. Silicon B. Potassium
C. Arsenide D. Germanium

5. Choose the odd one out.
A. Alligator B. Humpback Whale
C. Coyote D. Leopard

6. Choose the odd one out.
A. Brother : Sister
B. Wife : Husband
C. King : Queen
D. Horse : Mare

7. Choose the odd one out.
A. HNUTRDE B. OLCDU
C. INRA D. OMSEU

8. Choose the odd one out.
A. V. V. Giri
B. Zail Singh
C. P. V. Narsimha Rao
D. K. R. Narayanan

9. Choose the odd one out.
A. Orange B. Watermelon
C. Cucumber D. Bitter Gourd

10. Choose the odd one out.
A. Kidney B. Eye
C. Leg D. Liver



11. Choose the odd one out.

- A. Dollar B. Yen
C. Won D. Ounce

12. Choose the odd one out.

- A.  B. 
C.  D. 

13. Choose the odd one out.

- A. Mercury B. Saturn
C. Venus D. Mars

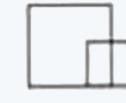
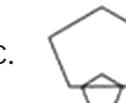
14. Choose the odd one out.

- A. Jute B. Wool
C. Paper D. Glass

15. Choose the odd one out.

- A. Garlic B. Lemon
C. Potato D. Ginger

16. Choose the odd one out.

- A.  B. 
C.  D. 

17. Choose the odd one out.

- A. 4721 B. 5682
C. 1891 D. 9154

18. Choose the odd one out.

- A. MNPR B. BCLQ
C. ACLT D. VYRS

19. Choose the odd one out.

- A. Nathu la B. Saltoro Kangri
C. Kongka la D. Khambatki Ghat

20. Choose the odd one out.

- A. Tel Aviv B. Czechia
C. Angola D. Lebanon

21. Choose the odd one out.

- A. 64 B. 343
C. 400 D. 512

22. Choose the odd one out.

- A. D B. O
C. P D. G

23. Choose the odd one out.

- A. 393 B. 482
C. 221 D. 510

24. Choose the odd one out.

- A. GHJLO B. LMORV
C. PQSVZ D. ABDGK

25. Choose the odd one out.

- A. 57 B. 17
C. 26 D. 37

SOLUTIONS

- (D) is the correct option because except the right to make contract, all the other options mention fundamental rights of an Indian citizen that are provided to him by the Indian Constitution.
- (C) is the correct answer because all the other options, except for craters which are circular depressions in the ground

generated by volcanic activity, are linked to earthquakes in one or the other way.

- (B) is the correct option because except for Amylase, which is an enzyme or special protein that helps us in digesting our food, all the other options are chemical names for vitamins like vitamin A, vitamin D, and vitamin K.



4. **(B)** is the correct answer because except potassium which is a chemical element, the rest, i.e., silicon, arsenide, and germanium, are semiconductors.
5. **(A)** is the correct answer because alligators are reptiles, whereas the remaining animals given in the options are classified as mammals.
6. **(B)** is the correct answer because in all the other pairs, the first one is masculine while the second one is feminine, but in option (b), the first one is feminine while the second one is masculine.
7. **(D)** is the correct answer because we get THUNDER, CLOUD, and RAIN by arranging the alphabets in HNUTRDE, OLCDU, and INRA, which are all connected to each other except OMSEU, which stands for MOUSE.
8. **(C)** is the correct answer because P.V. Narsimha Rao was the 9th Prime Minister of India, whereas all the other names mentioned have been the Presidents at various points.
9. **(A)** is the correct answer because oranges grow on trees, but the vegetables or fruits that are given in the rest of the options grow on creeper plants.
10. **(D)** is the correct answer because all the other organs mentioned like kidneys, eyes, and legs, are present in the human body in pairs, but there is only one liver in the human body.
11. **(D)** is the correct answer because an ounce is a unit of weight, whereas the rest of the given options are currencies of various nations like the USA, Japan, and South Korea.
12. **(C)** is the correct answer because only the circle in option C has an acute angle, whereas all the other circles given in the options have an obtuse angle.
13. **(B)** is correct because no other planet in the given options be it Mercury, Venus, or Mars has rings around it, except Saturn.
14. **(D)** is the correct answer because only glass is a non-biodegradable material among the given options, whereas jute, wool, and paper are all biodegradable materials and do not harm the environment.
15. **(B)** is the correct answer because except for the lemon plant which belongs to the family of shrubs, all other options like garlic, potato, and ginger are examples of modified stems.
16. **(D)** is the correct answer. With the exception of (D), all of the others have the same smaller pattern linked to them. The triangle, on the other hand, is linked to a rhombus in option (D).
17. **(A)** is the correct answer because in all the other options, the sum of the second and fourth digits of the number is equal to the third digit of the number.
 $1891 \rightarrow 8 + 1 = 9$ (third digit from right)
 $5682 \rightarrow 6 + 2 = 8$ (third digit from right)
 $4721 \rightarrow 7 + 1 = 8$ (not the third digit from right)
 $9154 \rightarrow 1 + 4 = 5$ (third digit from right)
18. **(C)** is the correct answer because none of the other options have a vowel in them, but the group of alphabets in option C consists of an 'A' too.
19. **(B)** is the correct answer because all the other options such as Nathu la, Kongka la, and Khambatki Ghat are various mountain passes, whereas Saltoro Kangri is a peak in Asia.
20. **(A)** is the correct answer because all the other options are the names of various countries, whereas Tel Aviv is the capital city of Israel.
21. **(C)** is the correct answer because all the other options are the perfect cube of a number such as:
 $8^3 = 512$ $7^3 = 343$ $4^3 = 64$
But 400 is not a perfect cube.



22. (B) is the correct answer because out of all the options given, only O is the alphabet that has the same mirror image.
23. (D) is the correct answer because in all the other options given, the product of the first and last digits, is equal to the digit in the middle but option D does not follow that pattern.
24. (A) is the correct answer because in all the other options that are given, between every two consecutive alphabets 0, 1, 2, and 3 alphabets are missing respectively. But option (A) does not follow that pattern.
25. (A) is the correct answer because the pattern that is followed here is as follows:
- $$(4 \times 4) + 1 = 17$$
- $$(5 \times 5) + 1 = 26$$
- $$(6 \times 6) + 1 = 37$$
- But option A does not follow this pattern.

42 Analogy

A comparison of two objects or systems of objects that emphasises the similarities between them is referred to as analogy. The word ‘analogy’ literally means ‘resemblance,’ as in possessing similar characteristics. Analogy questions assess an aspirant’s ability to comprehend the connection between two specified objects and apply that understanding to get the answer to the question. It is crucial to remember that while an aspirant’s intellectual ability is important in analysing the similarities between two or more options, a broad understanding of diverse word usages enhances a person’s efficiency.

In analogy, questions can be asked on any kind of relationship, a few types of which are mentioned below:

TYPES	EXAMPLES
Quantity and Unit	Length : Metre Weight : Kilogram Volume : Litres Force : Newton Power : Watt Pressure : Pascal
Worker and Tool	Carpenter : Hammer Farmer : Plough Gardener : Harrow Surgeon : Scalpel Sculptor : Chisel
Masculine and Feminine	Waiter : Waitress Monk : Nun Earl : Countess Fox : Vixen Bachelor : Spinster
Part to Whole	Fin : Fish Blade : Fan Chapter : Book Arc : Circle Steering : Car

TYPES	EXAMPLES
Word and Synonym	Inept : Incompetent Dearth : Scarcity Encumber : Burden Audacity : Impudence Dissipate : Squander
Word and Antonym	Contradict : Agree Varied : Monotonous Expand : Condense Arid : Fertile Plentiful : Sparse
Country and Currency	Bangladesh : Taka Italy : Lira Argentina : Peso Iran : Rial Burma : Kyat
Field of Study and Subject	Ornithology : Birds Onomatopoeia : Names Palaeontology : Fossils Oology : Eggs Pedology : Soil
Country and Capital	Canada : Ottawa UK : London France : Paris Nepal : Kathmandu Kenya : Nairobi Ukraine : Kyiv
Universal Pair	Cushion : Couch Cheese : Mouse Pepper : Salt Table : Chair Fork : Knife
Animal and Baby	Lion : Cub Kangaroo : Joey Cat : Kitten Deer : Fawn Swan : Cygnet



TYPES	EXAMPLES
Animal and Sound	Cow : Moo Owl : Hoot Goat : Bleat Donkey : Bray Camel : Grunt
Instrument and Measurement	Thermometer : Temperature Anemometer : Wind Seismograph : Earthquakes Ammeter : Current Odometer : Distance
Object and Function	Spoon : Eat Keyboard : Type Helmet : Protect Shovel : Dig Scissors : Cut
Cause and Effect	Cold : Shivering Itch : Scratch Spin : Dizzy Rain : Flood Study : Pass
Games and Playing Areas	Golf : Course Boxing : Ring Cricket : Pitch Skating : Rink Wrestling : Arena

In this topic, there are several kinds of questions that might be asked in the examinations. Let us discuss a few of such types.

- **'Complete the Analogous Pair' Type Questions:** Two words are given here. In some sense, these words are associated with each other. There is also a third word. The respondent must identify the relation between the first two words and select the word from the options that has the same relationship to the third word as the first two.

Example: Mouth : Speak :: Ears : ?

- A. See B. Hear
C. Breathe D. Touch

Solution: In this example, we know that mouth is used to speak. Therefore, we need to look for the same relation between the third word and the correct alternative as well. Now, we know that ears are used to hear sound; hence option B is the right answer.

- **Alphabet Type Questions:** Two pairs of alphabets are given in this type of questions, and they are related in the same way. The respondent must identify the relation between the two and select a group of alphabets that is related to the third group given in the question.

Example: ABC : DEF :: PQR : ?

- A. TUV B. XYZ
C. STU D. RQP

Solution: Here, we can see that DEF comes immediately after ABC in terms of the alphabetic order. Therefore, option C is the right answer as STU comes immediately after PQR.

- **Number-Type Questions:** In this, a pair of numbers is given in the question. The candidate needs to determine how the two numbers are related to each other, and select the alternative that is similarly related to the third number given in the question.

Example: 25 : 5 :: 81 : ?

- A. 9 B. 3
C. 6 D. 12

Solution: We know that 25 is the square of 5. So, we need to check if any of the alternatives has 81 as its square. Since 9 is the root of 81, option A is the right answer.

To understand the concept in a better way, let us look at the following examples.



SOLVED EXAMPLES

Example 1. Right to Information Act : 2005

:: Right to Education Act : ?

- A. 2006 B. 2009
C. 2012 D. 2007

Answer: B

Explanation: Option B is the correct answer because the Right to Information Act was enacted in the year 2005. Similarly, the Right to Education Act was enacted in the year 2009.

Example 2. Anaemia : Iron :: Scurvy : ?

- A. Vitamin C B. Iodine
C. Zinc D. Vitamin A

Answer: A

Explanation: Option A is the correct answer because anaemia is a disease which is caused by iron deficiency, whereas scurvy is a disease which is caused by vitamin C deficiency.

Example 3. Horse : Stable :: Pig : ?

- A. Burrow B. Coop
C. Fold D. Sty

Answer: D

Explanation: Option D is the correct answer because the place where a horse lives is called a stable, whereas the place where a pig lives is called a sty.

Example 4. Hand : Gloves :: Eyes : ?

- A. Pupil B. Sunglasses
C. Read D. Eyesight

Answer: B

Explanation: Option B is the correct answer because the second is worn on the first as gloves are worn in hands whereas sunglasses are worn on the eyes.

Example 5. Death : Life :: Fantasy : ?

- A. Fiction B. Reality
C. True D. Story

Answer: B

Explanation: Option B is the correct answer because death is the opposite of life. Similarly, reality is the opposite of fantasy.

PRACTICE QUESTIONS

Directions: On both sides of the ‘::’ in each of the questions given below, there is a specific link between two provided pairings. On the other side of ‘::’, one word is given, and another word must be picked from the alternatives, having the same relationship with this word as the words in the given pair. Identify the appropriate option from the given alternatives.

1. Carmine : Red :: Aureolin : ?

- A. Yellow B. Pink
C. White D. Green

2. Monday : Saturday :: January : ?

- A. March B. February

- C. November D. December

3. Patricide : Father :: Regicide : ?

- A. Parent B. Brother
C. King D. Old man

4. Mudhif : Iraq :: Chalets : ?

- A. Russia B. Switzerland
C. Africa D. Iceland

5. 2 : 6 :: 3 : ?

- A. 12 B. 14
C. 8 D. 15



6. Lal Bahadur Shastri: Uttar Pradesh:: Mahatma Gandhi: ?
A. Maharashtra B. Delhi
C. Madhya Pradesh D. Gujarat
7. CAT : 60 :: LOT : ?
A. 120 B. 3600
C. 1440 D. 828
8. Mt. Vesuvius : Italy :: Mauna Loa : ?
A. Martinique B. Africa
C. Turkey D. Hawaii
9. Gir National Park : Gujarat :: Nagarhole National Park : ?
A. Karnataka B. Kerala
C. Andhra Pradesh D. Nagaland
10. Kuchipudi : Andhra Pradesh :: Sattriya : ?
A. Bihar
B. Assam
C. Chhattisgarh
D. Himachal Pradesh
11. Quesadillas : Mexico :: Churros : ?
A. Greece B. France
C. Turkey D. Spain
12. Colombia : Bogota :: Ecuador : ?
A. Cairo B. Beirut
C. Baku D. Quito
13. World Tourism Day : 27th September :: World Population Day : ?
A. 1st July B. 12th October
C. 11th July D. 19th November
14. Estrange : Alienate :: Fastidious : ?
A. Lax B. Frivolous
C. Finicky D. Obdurate
15. rJA : Ajr :: mBA : ?
A. aBm B. Abm
C. ABm D. ABM
16. Cynophobia : Dogs :: Chionophobia : ?
A. Snow B. Touch
C. Travel D. Cats
17. Brain : Dura mater :: Heart : ?
A. Medulla
B. Epicardium
C. Pleural membrane
D. Aorta
18. Lion : Roar :: Dolphin : ?
A. Bray B. Click
C. Chortle D. Caw
19. Hunger : Satiate :: Thirst : ?
A. Water B. Drink
C. Liquid D. Quench
20. Notaphilist : Banknotes :: Arctophile : ?
A. Flies B. Postcards
C. Teddy bears D. Umbrellas
21. Absolve : Accuse :: Boisterous : ?
A. Placid
B. Desolate
C. Clamorous
D. Repugnant
22. 46 : 82 :: 37 : ?
A. 68 B. 55
C. 85 D. 99
23. Earth : Planet :: Sirius : ?
A. Asteroid B. Galaxy
C. Star D. Satellite
24. Flour : Cake :: Lumber : ?
A. Wood B. House
C. Tree D. Forest
25. Champagne : Grapes :: Rum : ?
A. Apple B. Rice
C. Flour D. Sugarcane



SOLUTIONS

1. (A) is the correct answer because Carmine is a shade of Red and in the same way, Aureolin is a shade of Yellow.
2. (C) is the correct answer because Monday and Saturday are alternate days of the week, in the same order, in which January and November are alternate months of the year.
3. (C) is the correct answer because in the term given in the question, ‘cide’ is the root word which means ‘to kill’. Therefore, patricide means killing one’s father, whereas regicide means killing a king.
4. (B) is the correct answer because in this question, the first is the type of houses traditionally famous in the country denoted by the second.
5. (A) is the correct answer because the square of 2 plus 2 is equal to 6. Similarly, the square of 3 plus 3 is equal to 12.
 $(2 \times 2) + 2 = 6$
 $(3 \times 3) + 3 = 12$
6. (D) is the correct answer because Lal Bahadur Shastri was born in the state of Uttar Pradesh in India and Mahatma Gandhi was born in the state of Gujarat in India.
7. (B) is the correct answer because if we take the place value of each letter in the word and multiply it, we get:
CAT = $3 \times 1 \times 20 = 60$
LOT = $12 \times 15 \times 20 = 3600$
8. (D) is the correct answer because Mt. Vesuvius is an active volcano located in Italy. Similarly, Mauna Loa is an active volcano located in Hawaii.
9. (A) is the correct answer because Gir National Park is located in the state of Gujarat, whereas Nagarhole National Park (also known as Nagarhole Tiger Reserve) is located in the state of Karnataka.
10. (B) is the correct answer because Kuchipudi is a famous folk dance, originated in the 17th century, in the state of Andhra Pradesh, whereas Sattriya is a famous folk dance, originated in the 15th century, in the state of Assam.
11. (D) is the correct answer because Quesadilla is a famous cuisine from Mexico, whereas Churro is a famous cuisine from Spain.
12. (D) is the correct answer because Bogota is the capital city of Colombia. Similarly, Quito is the capital city of Ecuador.
13. (C) is the correct answer because the World Tourism Day is observed on 27th September and, similarly, the World Population Day is observed on 11th of July.
14. (C) is the correct answer because alienate is a synonym of estrange which means to make someone feel isolated. Similarly, finicky is synonymous to fastidious which means the quality of being very attentive to and very concerned about the details and accuracy.
15. (B) is the correct answer because to get the 2nd group, reverse the order of the letters in the first group and substitute the middle capital letter with a small letter. So, mBA : Abm if we follow the same pattern.
16. (A) is the correct answer because in the term given in the question, the root word is ‘phobia’ which means an extreme or irrational fear of something. Therefore, fear of dogs is called cynophobia whereas fear of snow is called chionophobia.
17. (B) is the correct answer because dura mater is the outermost layer of the human brain, whereas epicardium is the outermost layer of the human heart.
18. (B) is the correct answer because the sound that a lion makes is called a roar. Similarly, the sound that a dolphin makes is called click.
19. (D) is the correct answer because the act of eradicating the first is referred to as the second. So, a person ‘satiates’ his



hunger by eating and ‘quenches’ his thirst by drinking.

20. (C) is the correct answer because in the term given in the question, the root word is ‘phil’, which means to love. A notaphilist collects banknotes, whereas an arctophile collects teddy bears.
21. (A) is the correct answer because placid is an antonym of boisterous as placid means being calm and boisterous means being noisy and energetic. In the same way, accusing is the opposite of absolving as accusing means claiming that someone has done something wrong and absolving means to declare someone free from guilt.
22. (B) is the correct answer because the following is the pattern followed in the question:

$$46 : 82 = (4 + 6) : (8 + 2)$$

$$10 : 10$$

Similarly,

$$37 : 55 = (3 + 7) : (5 + 5)$$

$$10 : 10$$

23. (C) is the correct answer because Earth is a planet in the solar system, whereas Sirius is the brightest star.

24. (B) is the correct answer because the first refers to the material used to make the second as flour is used to make a cake and lumber is used to make a house.

25. (D) is the correct answer because Champagne is made with grapes, whereas Rum is made of sugarcane. The relationship between the two terms is product and the raw material used to make it.



This section of the GATE exam looks into the seating arrangement problems that students have to practice with during their preparation for GATE and other engineering entrance exams. Seating arrangement problems test your ability to reach conclusions after decoding a particular scenario that is presented to you in the question paper. These questions could be highly misleading and confusing to solve if tackled with the wrong approach; however, if you have your basics correct, even the lengthiest of the questions take less than 5 minutes to solve.

In order to solve this section, one has to be aware of his ability to comprehend data. While some people prefer not making diagrams while trying to solve problems, it is highly encouraged if you can make them. Diagrams help you in keeping the track of events that are going on throughout the problem.

Basic tips to solve the seating arrangement section:

- Read the problem in its entirety before making any statements or diagrams.
- Jot down all the information in one place. It can be a diagram or a short note. This information does not have to be elaborate. This information is there so that you do not have to go back to the question for referencing again and again.
- Use your vivid imagination and be a part of a problem. Imagining yourself in the problem helps you understand the scenario better

Looking at the last line, we find out that A and D are at extremes; hence, the only possible combination can be ABCD, which is the answer.

and hence make decisions better. It will help you in keeping a track of where things or people are more accurately

- Always draw diagrams where the information is a lot. Once you are able to figure out the diagram, the rest of the problem becomes very easy for you.
- While making a diagram pay attention to the things which are definitely known in the problem first. Like:
FACT ONE: A is sitting at the right of B
FACT TWO: B is sitting at the immediate right of B.

Here the second fact must be noted first as it is an absolute fact. More about immediate and simple left/right is explained later.

Here is a sample problem to try on your own: A, B, C, and D are four people sitting in a row. If A is sitting on the immediate left of B and C is sitting on the immediate left of D. D and A are at the extremes.

What is the correct order of seating?

- | | |
|---------|---------|
| A. BCDA | B. ACDA |
| C. ABCD | D. DCAB |

Explanation:

From the first sentence we can figure out that A is sitting on the immediate left of B; hence, one pattern is fixed: **AB**

We can also infer that C is at the immediate left of D; hence, another pattern can be found: **CD**

Now the possible combinations we can have are **ABCD** and **CDAB**.

IMMEDIATE LEFT/RIGHT	LEFT/RIGHT
In this case, the person or the thing will always be exactly one position right/left from the other person or thing.	In this case, the person or the thing can be anywhere in any position left or right of the other person or thing
E.g.: If B is sitting at the immediate right of A, the sequence would be AB.	E.g.: If B is sitting at the right of A, the sequence can be: A_ _ B or AB or A_ _ _ _ B, etc.



Type of seating arrangements

Single-row arrangement problems: These are the easiest of the seating arrangement problems that you will face. The previous problems we solved were one of the same kinds. Let us have a look at a different and slightly more complex problem to have a better understanding.

Example:

Six students P, Q, R, S, T, and U are sitting in a row. T and U are sitting in the centre. P and Q are sitting at the ends. R is sitting to the left of P.

Who is sitting immediate right of Q?

Answer: S

Explanation:

If we go by the problem line by line, we get to this pattern: Q S T U R P, which makes it clear.

Double-row arrangement problems:

These double-row problems are a bit more complex than the single-row problems; however, if we move step by step, it gets easier

to solve. Remember not to be confused and go one step at a time.

Example:

Two volleyball teams of 6 players each assembled in two rows in front of each other facing north and south, respectively. Team Titans had Vyom, Raj, Yash, Manik, Suresh, and Deepak as players and they were facing north. Team Spikers had Andrew, Joe, Kevin, Paul, Jimmy, and Stuart as players and they were facing south.

Only one person stood between Vyom and Yash, who was standing at the extreme left end of the row. Joe, who was standing at the extreme left end, stood second to the left of Andrew. Kevin was facing the one who was an immediate neighbour of Vyom. Raj was facing Paul. Jimmy, who was standing at one of the extreme ends of the row, stood third to the right of the one who faced Deepak. Manik was facing the one who stood second to the left of Andrew.

Who among the following stood third to the right of the one, who was facing the one, who was second to the left of Jimmy?

Answer:

Manik was sitting third to the right of one, who is facing the one, who is second to the left of Jimmy.

Explanation:

Team Spikers Facing South	Jimmy	Kevin	Stuart	Andrew	Paul	Joe
---------------------------------	-------	-------	--------	--------	------	-----

← Right End

Left end →

Team Titans Facing North	Yash	Suresh	Vyom	Deepak	Raj	Manik
--------------------------------	------	--------	------	--------	-----	-------

← Left End

Right End →

Rectangular arrangement

This section is also moderately easy to solve. One must make a diagram in order to get a proper understanding of the question and hence be able to solve it.

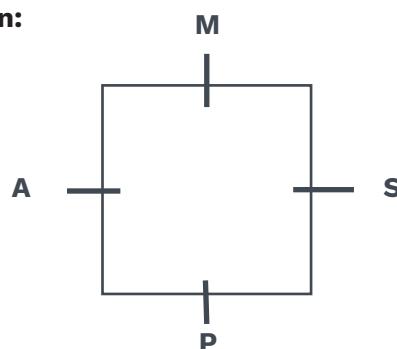
Example:

S, M, A, and P are playing UNO. A is to the right of M, who is to the right of S. Who is to the right of A?

Answer: P



Explanation:



Circular Arrangement:

This category deals with fairly tough questions revolving around the topic. In this area, the amount of details is a lot so a high degree of caution is to be maintained while jotting down the required info.

There are two kinds of questions one will face in this area:

- Inward facing: The right, in this case, is in the anticlockwise direction and the left is in the clockwise direction. Here the people are facing **towards the centre**.

- Outward-facing: The right, in this case, is in the clockwise direction and the left is in the anticlockwise direction. Here the people are facing **away from the centre**.

Example:

Eight friends P, Q, S, T, U, V, W, and X are sitting in a circle, but not necessarily in the same order. Four of them are facing outside and four of them are facing the centre.

U faces outside, both the immediate neighbours of U face the centre. X sits second to the right of U. Q sits third to the left of U. T faces the centre. Both the immediate neighbours of T face outside. W sits second to the left of P. Q sits third to the right of X. V is an immediate neighbour of T. S is an immediate neighbour of W. T is not an immediate neighbour of Q. Who amongst the following sits third to the right of P?

Answer: V

PRACTICE QUESTIONS

- Jashn, Prakrit, Rekha, Xeo, Surya, and Zara are sitting in a row facing north. Surya and Zara are in the centre. Jashn and Prakrit are at both ends. Rekha is sitting to the left of Jashn.
Who sits immediate right of Prakrit?
A. Jashn B. Xeo
C. Surya D. Zara
- Seven persons, M, N, O, P, Q, R, and S, are sitting in a circular manner all facing inside. M is third to the left of O, who is second to the right of the person, who sits immediately next to R. P sits second to the left of M. N sits third to right of Q.
Which of the following statement(s) is (are) true?
I. S sits immediately next to M.
II. Q is second to the left of R.
III. O is third to the right of S.
- A. Only I B. Only III
C. Both II and III D. Both I and II
- Arpit, Bhitam, Chanakya, Dhruv, and Emma are sitting on a bench facing north. Arpit is sitting next to Bhitam, Chanakya is sitting next to Dhruv, Dhruv is not sitting with Emma who is on the left end of the bench. Chanakya is in the second position from the right. Arpit is to the right of Bhitam and Emma. Arpit and Chanakya are sitting together.
In which position Arpit is sitting?
A. Between Bhitam and Dhruv
B. Between Bhitam and Chanakya
C. Between Emma and Dhruv
D. Between Chanakya and Emma
- Anamika, Bhoomika, Chandni, Deepika, Elisa, Falguni, and Gauri are sitting in a row facing north:
(a) Falguni is to the immediate right of Elisa.



- (b) Elisa is 4th to the right of Gauri.
- (c) Chandni is the neighbour of Bhoomika and Deepika.
- (d) The person who is third to the left of Deepika is at one of the ends.

What is the position of Anamika?

- A. Extreme Right
- B. 2nd from the left
- C. Centre of the row
- D. 3rd from the right

- 5.** In a bike race, seven bikes of different companies—Honda, Suzuki, Bajaj, TVS, Hero, Piaggio, and Yamaha—are standing facing to the east in the following order:
- (a) Honda is next to the right of Yamaha.
 - (b) Yamaha is fourth to the right of Bajaj.
 - (c) TVS is between Suzuki and Piaggio.
 - (d) Bajaj which is third to the left of Suzuki is at one end.

Which one of the following is the correct position of Hero?

- A. Next to the left of Honda
- B. Next to the left of Piaggio
- C. Between Piaggio and Yamaha
- D. Fourth to the right of TVS

- 6.** Five friends are sitting in a row in a park facing north. Sukesh is to the left of Rakesh and to the right of Bablesh. Mikhilesh is to the right of Reshma. Reshma is between Rakesh and Mikhilesh.

Who is second from the left in the row?

- A. Reshma
- B. Mikhilesh
- C. Bablesh
- D. Sukesh

- 7.** Eight friends (A, B, C, D, E, F, G, and H) are sitting in two opposite rows, facing each other. Each row has 4 persons. A is between F and G and is facing north. H is opposite to E who is to the immediate left of B. C is between D and B. H is to the immediate right of G.

Which of the following pairs of persons has a second person sitting to the immediate left of the first person?

- A. FA
- B. GA
- C. EB
- D. CD

(Questions 8 to 12) Study the following information to answer the questions given below.

Eight women (Aria, Betty, Cathy, Damaris, Emily, Fiona, Gina, and Hannah) are sitting around a circular table. A few of them are facing the centre of the table and others are facing the opposite direction.

Gina sits second to the left of Aria. Damaris and Hannah are sitting right next to each other and facing the same direction. Aria sits third to the left of Fiona. The immediate neighbours of Cathy face the same direction. Damaris sits second to the right of both Aria and Betty. The one who sits immediately to the right of Aria faces the centre of the table and her immediate neighbours face opposite directions. Fiona sits third to the right of Gina and both of them face opposite directions. Emily sits second to the right of both Cathy and Hannah.

- 8.** How many women are seated between Emily and Fiona?

- A. Two
- B. Three
- C. One
- D. No one

- 9.** What is Hannah's position with respect to Emily?

- A. Second to the right
- B. Immediate left
- C. Second to the left
- D. Immediate right

- 10.** How many women face outside?

- A. Three
- B. Four
- C. Five
- D. Six

- 11.** Four of the following women are alike in some way according to their seating arrangement. Which of the following women does not belong to the group?



- A. Fiona
- B. Aria
- C. Cathy
- D. Betty

- 12.** What is the position of Gina with respect to Hannah?
- A. Immediate left
 - B. Immediate right
 - C. Third to the left
 - D. Third to the right

(Questions 13 to 17) Study the following information to answer the questions given below.

Eight people—Ethan, Isaac, Kevin, Lokesh, Madhan, Otis, Shree, and Ravi—are sitting in a straight line, not necessarily in the same order facing North.

Otis sits third to the left of Madhan. Ravi sits at one of the extreme ends of the line, and only two people sit between Madhan and Kevin. Ethan is not an immediate neighbour of Lokesh. Only one person sits between Shree and Ravi. Kevin is not an immediate neighbour of Ravi. Only three people sit between Shree and Lokesh.

- 13.** Which is the correct seating arrangement?
- A. Ravi, Otis, Shree, Madhan, Isaac, Ethan, Lokesh, Kevin
 - B. Ravi, Otis, Ethan, Shree, Madhan, Isaac, Lokesh, Kevin
 - C. Ravi, Otis, Shree, Ethan, Madhan, Lokesh, Isaac, Kevin
 - D. Ravi, Otis, Shree, Ethan, Madhan, Isaac, Lokesh, Kevin
- 14.** Where is Ravi with respect to Ethan?
- A. Second to the right
 - B. Third to the left
 - C. Second to the left
 - D. Third to the right
- 15.** How many people sit between Otis and Shree?
- A. One
 - B. Two
 - C. Three
 - D. No one

- 16.** Which of the following is correct?
- A. Otis and Ravi are at the extreme ends of line
 - B. Lokesh sits second to the left of Madhan
 - C. Only one person sits between Madhan and Ethan
 - D. Shree and Ethan are immediate neighbours

- 17.** Who are the immediate neighbours of Madhan?
- A. Ethan, Kevin
 - B. Shree, Lokesh
 - C. Isaac, Ethan
 - D. Kevin, Shree

(Question 18 to 22) Study the following information to answer the questions given below.

In a conference hall, 10 persons are present, 5 men and 5 women are sitting in two parallel lines, facing each other. Five men, namely, Ajinkya, Bhoomij, Chirayu, Dharmendra, and Ehsan are facing the south and the five women, Mrignayani, Navya, Oshiyana, Priya, and Arunima are facing towards the north.

- Bhoomij, who is just next to the left of Dharmendra, is opposite Arunima.
- Chirayu and Navya are diagonally opposite to each other.
- Ehsan is opposite Oshiyana who is just next to Mrignayani.
- Priya, who is just to the left of Arunima, is opposite Dharmendra.
- Mrignayani is at the left corner of her line.

- 18.** Which two people are sitting at the two extreme ends of the line?
- A. Ehsan, Bhoomij
 - B. Ajinkya, Chirayu
 - C. Chirayu, Dharmendra
 - D. Dharmendra, Bhoomij
- 19.** Who is sitting right in front of Dharmendra?
- A. Navya
 - B. Mrignayani
 - C. Oshiyana
 - D. Priya



20. Who sits exactly in between Mrignayani and Navya?

- A. Priya
- B. Oshiyana
- C. Arunima
- D. Cannot be determined

21. Who is sitting opposite Oshiyana?

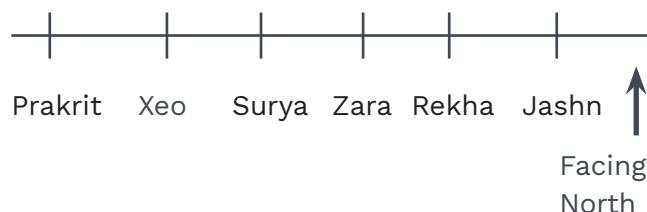
- A. Chirayu
- B. Ehsan
- C. Dharmendra
- D. Bhoomij

22. Who is sitting at the right corner of the women's line?

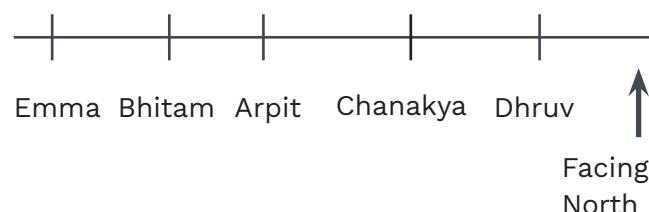
- A. Mrignayani
- B. Oshiyana
- C. Arunima
- D. Navya

SOLUTIONS

1. The seating arrangement would be as follows: Prakrit, Xeo, Surya, Zara, Rekha, Jashn



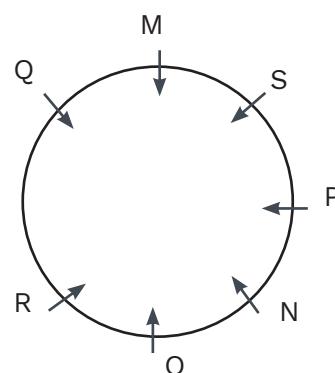
3. The following order will be there:



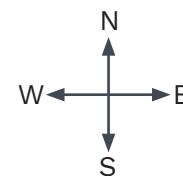
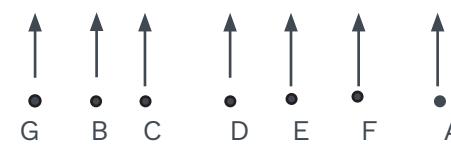
2. M is third to the left of O, who is second to the right of the person, who sits immediately next to R.

We get,
P sits second
to the left of M.

N sits third to
right of Q.
Hence, we get



4. The following order will be there:

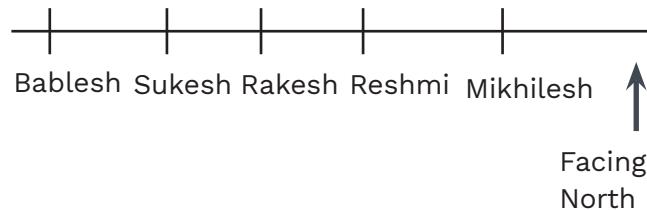


5. The order would be: Bajaj, Piaggio, TVS, Suzuki, Yamaha, Honda, and Hero.

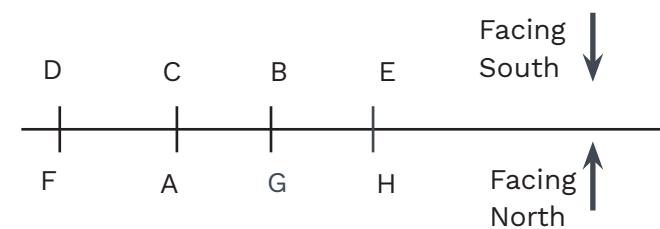
Facing East ↑	Bajaj	Piaggio	TVS	Yamaha	Suzuki	Honda	Hero
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6. The order would go like: Bablesh Sukesh
Rakesh Reshma Mikhilesh

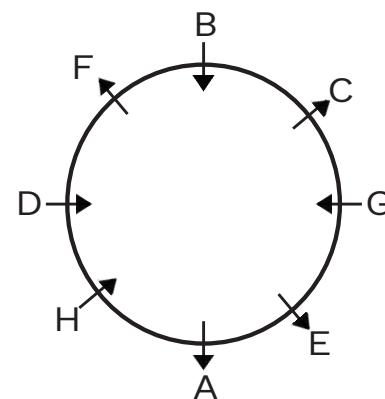


7. Given all eight friends are sitting in two opposite rows, facing each other. Each row has 4 persons. A is between F and G and is facing north. H is to the immediate right of G. H is opposite to E who is to the immediate left of B. C is between D and B. Among FA, GA, EB, CD, only GA is true because in this, second person A is sitting immediate left of the first person G. In all other second person is sitting immediate right to the first person.



(Solutions 8-12):

From the given observations, it can be concluded that this is the seating arrangement:



8. Three women (Betty, Cathy, and Gina) sit between Emily and Fiona
 9. Hannah is second to the right of Emily
 10. Four women
 11. Betty, Fiano, Aria, and Cathy all face outwards whereas Betty does not
 12. Gina is third to the right of Hannah

(Solutions 13-17):

From the given observation, it can be inferred that this is the seating arrangement:



- 13.** Ravi, Otis, Shree, Ethan, Madhan, Isaac, Lokesh, Kevin

14. Ravi is third to the left of Ethan

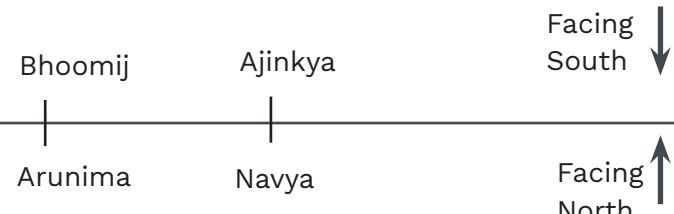
- 15. No one—Otis and Shree—are immediate neighbours
 - 16. Shree and Ethan are immediate neighbours
 - 17. Ethan and Isaac are the immediate neighbours of Madhan

(Solutions 18-22):

The arrangement would be as follows:



- 18.** Ajinkya and Chirayu sit at the extreme ends.
 - 19.** Priya is sitting right in front of Dharmendra.
 - 20.** Priya is sitting exactly between Mrignayani and Navya.



- 21.** Ehsan is sitting opposite Oshiyana.
22. Navya is sitting at the right corner of the women's line.



Puzzles are raw data for a sequence or order of events that must be arranged systematically for the sequence or order of events to be accurately depicted. Candidates are given information in a jumbled, haphazard format in puzzles. It tests the candidate's ability to decipher the sequence and analyse given information into a meaningful and judgmental form, to arrive at a final decision or conclusion by following a systematic pattern of linking and interlinking one or more pieces of information with one another.

There are various types of puzzles some of which are given below:

1. Floor Puzzle: In this type of puzzle, the information is given regarding the people living on different floors of the same or different buildings. They have to figure out the sequence which matches each person with the floor number or name.

2. **Scheduling Puzzle:** In this type of puzzle, the information is given regarding the different parameters based on the dates, months or years.
3. **Multiple-Variable Puzzle:** In this type of puzzle, different kinds of information of different subjects are given. It can be different things belonging to different people in different colours.
4. **Square/Circular Arrangement Puzzle:** This type of puzzle includes the seating arrangement of different people around a square or circular table. These are usually covered under the seating arrangement topic.
5. **Box Puzzle:** In this type of puzzle, information regarding different boxes is given and the boxes need to be arranged one above another.

PRACTICE QUESTIONS

Questions 1-5: Seven boxes P, Q, R, S, T, U, and V are placed one above the other, but not necessarily in the same order. Each of them has a different weight, i.e., 75 kg, 80 kg, 70 kg, 60 kg, 55 kg, 85 kg, and 45 kg, but not necessarily in the same order. S does not have a box weighing 45 kg. There are only two boxes below the box which weigh 85 kg. There are only three boxes between box V and the box which weigh 85 kg. V does not have a box either weighing 60 kg or weighing 55 kg. The box weighing 55 kg is immediately above box R. The box weighing 45 kg is immediately above the box which weighs 75 kg and immediately below the box which weighs 60 kg. R has a box weighing 80 kg. The box weighing 70 kg is immediately above box Q. There are only two boxes between the box whose weight is 60 kg and box U. Neither P nor S has a box weighing 55 kg.

1. Which of the following weights does box T have?

A. 85 kg	B. 70 kg
C. 55 kg	D. 45 kg
2. Which of the following weights does box P have?

A. 45 kg	B. 75 kg
C. 85 kg	D. 55 kg
3. Which of the following weights does box S have?

A. 45 kg	B. 75 kg
C. 85 kg	D. 55 kg
4. Who has the box weighing 85 kg?

A. Q	B. P
C. U	D. R



5. Who has the box weighing 60 kg?
- A. Q B. P
C. U D. R

Questions 6-10: Eight persons P, Q, R, S, T, U, V, and W are issued debit cards on the 13th and 17th of each month from March to June. No one is issued a debit card on the same day. U, V, and W got their cards on the 13th date of consecutive months in the same order. W gets his debit card earlier than P and Q, who got their cards in the month of June. Two persons got their debit card between W and P. T got his debit card earlier than S but later than R.

6. Who got their debit cards in May?
- A. W, S B. P, W
C. U, S D. R, V
7. Who got their debit cards in June?
- A. W, S B. Q, P
C. U, S D. R, V
8. Who got their debit cards in April?
- A. W, S
B. Q, P
C. V, S
D. V, T
9. Who got their debit cards in March?
- A. W, S
B. U, P
C. U, R
D. R, V
10. Who got their debit cards on 17th dates of all the 4 months?
- A. R, T, S, P
B. V, T, S, P
C. R, W, S, P
D. R, T, Q, P

Questions 11-15: Seven people A, B, C, D, E, F, and G live on 7 different floors in a building. The ground floor is numbered 1, the first floor is numbered 2, and so on. A lives on the floor immediately below B and immediately above C who lives on an even-numbered floor. There is a gap of two people between B and F. Also,

there is a gap of two people between A and E. E does not live below C and F lives below B. D lives somewhere above G.

11. Who lives on the 5th floor of the building?
- A. G B. D
C. A D. Either G or D
12. Who lives on the 7th floor of the building?
- A. G
B. D
C. A
D. Either G or D
13. Who lives on the 1st floor of the building?
- A. G B. B
C. F D. E
14. Who lives on the 4th floor of the building?
- A. G B. D
C. A D. B
15. Who lives on the 3rd floor of the building?
- A. G B. D
C. A D. B

Questions 16-20: There are seven members in a family: M, N, O, P, Q, R, S. Each of them owns a different car and their incomes are also different. There are four males among them. No female member owns a Hyundai or BMW car. The person with a BMW earns the most. P owns Audi and earns less than R who owns Mercedes. O who owns Skoda earns more than M and less than P. S's wife has the least income. Q is an unmarried lady who owns Honda. She earns less than M and more than only N. The person who owns Hyundai does not have the least income.

16. Who among the following has the least income?
- A. M B. N
C. M or N D. O
17. Which car is owned by M?
- A. Hyundai
B. Maruti
C. Mercedes
D. Cannot be determined



- 18.** How many of the family members earn more than the person who owns Mercedes?
- A. 4 B. 3
C. 2 D. 1
- 19.** Which of the following is the combination of three female members in the family?
- A. OQN
B. PQN
- 20.** Which of the following is the correct combination?
- A. S—Audi—M
B. O—Skoda—F
C. Q—Honda—F
D. N—Honda—F

SOLUTIONS

1. **(C)** Out of the seven boxes, there are only two boxes below the box that weighs 85 kg. There are only three boxes between box V and the box which weighs 85 kg.

So,

BOX	WEIGHT
V	
	85 kg

The box weighing 45 kg is immediately above the box which weighs 75 kg and immediately below the box which weighs 60 kg. V does not have a box either weighing 60 kg or weighing 55 kg.

BOX	WEIGHT
V	
	60 kg

BOX	WEIGHT
	45 kg
	75 kg
	85 kg

The box weighing 70 kg is immediately above the box Q. V does not have a box either weighing 60 kg or weighing 55 kg.

BOX	WEIGHT
V	70 kg
Q	60 kg
	45 kg
	75 kg
	85 kg

R has a box weighing 80 kg. The box weighing 55 kg is immediately above the box R.



BOX	WEIGHT
V	70 kg
Q	60 kg
	45 kg
	75 kg
	85 kg
	55 kg
R	80 kg

There are only two boxes between the box whose weight is 60 kg and box U.

BOX	WEIGHT
V	70 kg
Q	60 kg
	45 kg
	75 kg
U	85 kg
	55 kg
R	80 kg

Neither P nor S has a box weighing 55 kg. S does not have a box weighing 45 kg.

BOX	WEIGHT
V	70 kg
Q	60 kg
P	45 kg
S	75 kg

BOX	WEIGHT
U	85 kg
	55 kg
R	80 kg

One box T is left and only one place is remaining. So, he will sit there.

BOX	WEIGHT
V	70 kg
Q	60 kg
P	45 kg
S	75 kg
U	85 kg
T	55 kg
R	80 kg

Hence, T has a box weighing 55 kg.

2. (A)

BOX	WEIGHT
V	70 kg
Q	60 kg
P	45 kg
S	75 kg
U	85 kg
T	55 kg
R	80 kg

Hence, P has a box weighing 45 kg.



3. (B)

BOX	WEIGHT
V	70 kg
Q	60 kg
P	45 kg
S	75 kg
U	85 kg
T	55 kg
R	80 kg

Hence, S has a box weighing 75 kg.

4. (C)

BOX	WEIGHT
V	70 kg
Q	60 kg
P	45 kg
S	75 kg
U	85 kg
T	55 kg
R	80 kg

Hence, U has a box weighing 85 kg.

5. (A)

BOX	WEIGHT
V	70 kg
Q	60 kg
P	45 kg
S	75 kg

BOX	WEIGHT
U	85 kg
T	55 kg
R	80 kg

Hence, Q has a box weighing 85 kg.

- 6. (A)** U, V, and W got their debit cards on the 13th of consecutive months in the same order.

MONTH	CASE I		CASE II	
	13	17	13	17
March			U	
April	U		V	
May	V		W	
June	W			

W gets its card earlier than P and Q who got their cards in June. Thus, case (I) gets eliminated.

MONTH	CASE II	
	13	17
March	U	
April	V	
May	W	
June	Q/P	P/Q

Two people got their cards between W and P.



MONTH	CASE II	
	13	17
March	U	
April	V	
May	W	
June	Q	P

T got its card earlier than S but later than R.

MONTH	CASE II	
	13	17
March	U	R
April	V	T
May	W	S
June	Q	P

Thus, W and S got their debit cards in May.

7. (B)

MONTH	CASE II	
	13	17
March	U	R
April	V	T
May	W	S
June	Q	P

Thus, Q and P got their debit cards in June.

8. (D)

MONTH	CASE II	
	13	17
March	U	R
April	V	T
May	W	S
June	Q	P

Thus, V and T got their debit cards in April.

9. (C)

MONTH	CASE II	
	13	17
March	U	R
April	V	T
May	W	S
June	Q	P

Thus, U and R got their debit cards in March.

10. (A)

MONTH	CASE II	
	13	17
March	U	R
April	V	T
May	W	S
June	Q	P

Thus, R, T, S, P got their debit cards on 17th of all the 4 months.



- 11. (A)** A lives on the floor immediately below B and immediately above C, who lives on an even-numbered floor.

7		
6		B
5		A
4	B	C
3	A	
2	C	
1		

(Case 1) (Case 2)

There is a gap of two persons between B and F. Also, there is a gap of two persons between A and E.

E does not live below C. F lives below B.

7		
6	E	B
5		A
4	B	C
3	A	F
2	C	E
1	F	

(Case 1) (Case 2)

Since, E does not live below C, case 2 gets eliminated.

D lives somewhere above G.

7	D
6	E
5	G
4	B
3	A
2	C
1	F

Thus, G lives on the 5th floor.

- 12. (B)**

7	D
6	E
5	G
4	B
3	A
2	C
1	F

Thus, D lives on the 7th floor.

- 13. (C)**

7	D
6	E
5	G
4	B
3	A
2	C
1	F

Thus, F lives on the 1st floor.

- 14. (D)**

7	D
6	E
5	G
4	B
3	A
2	C
1	F

Thus, B lives on the 4th floor.



15. (C)

7	D
6	E
5	G
4	B
3	A
2	C
1	F

Thus, A lives on the 3rd floor.

16. (B) According to the given information, the arrangement is as follows in order of their income (from highest to lowest):

INCOME	PERSON	CAR	SEX
Highest	S	BMW	M
	R	Mercedes	M/F
	P	Audi	M/F
	O	Skoda	M/F
	M	Hyundai	M
	Q	Honda	F
Lowest	N	Maruti	F

Therefore, N has the least income. Option B is correct.

17. (A) According to the given information, the arrangement is as follows in order of their income (from highest to lowest):

INCOME	PERSON	CAR	SEX
Highest	S	BMW	M
	R	Mercedes	M/F
	P	Audi	M/F
	O	Skoda	M/F
	M	Hyundai	M
	Q	Honda	F
Lowest	N	Maruti	F

Therefore, M owns a Hyundai car. Option A is correct.

18. (D) According to the given information, the arrangement is as follows in order of their income (from highest to lowest):

INCOME	PERSON	CAR	SEX
Highest	S	BMW	M
	R	Mercedes	M/F
	P	Audi	M/F
	O	Skoda	M/F
	M	Hyundai	M
	Q	Honda	F
Lowest	N	Maruti	F

Therefore, there is only one member S who earns more than R who owns a Mercedes car. Option D is correct.

19. (D) According to the given information, the arrangement is as follows in order of their income (from highest to lowest):



INCOME	PERSON	CAR	SEX
Highest	S	BMW	M
	R	Mercedes	M/F
	P	Audi	M/F
	O	Skoda	M/F
	M	Hyundai	M
	Q	Honda	F
Lowest	N	Maruti	F

Therefore, Q and N are definitely the female members but the third female is among R, P, and O. So, it cannot be determined. Option D is correct.

20. (C) According to the given information, the arrangement is as follows in order of their income (from highest to lowest):

INCOME	PERSON	CAR	SEX
	S	BMW	M
	R	Mercedes	M/F
	P	Audi	M/F
	O	Skoda	M/F
	M	Hyundai	M
	Q	Honda	F
Lowest	N	Maruti	F

Therefore, Q—Honda—F is the correct combination. Option C is correct.

45 Data Sufficiency



INTRODUCTION

Data sufficiency is an important topic of the reasoning section. Data sufficiency questions contain a problem followed by two or three statements containing certain data or information, which may be required for solving the given problem. You have to determine which all statements are required to answer the problem. Thus, questions based on data sufficiency are meant to judge the candidate's ability to determine the information necessary to solve a given question (rather than the actual solution of the same). You should be able to determine the minimum information required for solving the problem so that you may arrive at the most appropriate answer choice.

Questions on data sufficiency may be asked from topics such as ranking, arrangements, coding-decoding, mathematical problems, blood relationships, directions, and so on. (a) You need to remember the steps involved in solving a particular data sufficiency question and follow them in this particular order: Check the first statement, then check the second statement, and lastly, if required, combine the two statements to get the answer. The answer may be in the form of YES or NO; in both cases, the given statement will be sufficient to give the answer. When you are unable to find a definite answer then the given statement is not sufficient. Note: Do not make any assumptions while solving data sufficiency questions.

EXEMPLAR QUESTIONS

1. In a certain code, 'jil jua jia' means 'please take care'. Which code word means 'please'?
 - I. 'tak sil jia' means 'we take him'.
 - II. 'est pros jil' means 'I care for'.
 - A. If the data in only statement I are sufficient to answer the question while the data in statement II are alone not sufficient to answer the question.
 - B. If the data in only statement II are sufficient to answer the question while the data in statement I are alone not sufficient to answer the question.
 - C. If data even in both the statements I and II together are not sufficient to answer the question.
 - D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From the statement I → 'jia' means 'take'.

From statement II → 'jil' means 'care' so, by resolving all the statements, we can say that 'jua' means 'please' Therefore, option D is correct.

2. How is Tia related to Aliya?
 - I. Tia's husband is the only son of Aliya's mother.
 - II. Tia's brother and Aliya's husband are cousins.
 - A. If the data in only statement I are sufficient to answer the question while the data in statement II are alone not sufficient to answer the question.
 - B. If the data in only statement II are sufficient to answer the question while the data in statement I are alone not sufficient to answer the question.
 - C. If the data either in statement I or in statement II alone are sufficient to answer the question



- D. If data in both statements I and II together are necessary to answer the question.

Answer: B

Explanation: From the statement I \rightarrow Tia's husband is Aliya's brother (only son of her mother) \Rightarrow Tia is Aliya's sister-in-law and Aliya is a male, then Tia should be the wife of Aliya. Hence, the relation is not defined. Therefore, option B is correct.

From statement II \rightarrow Tia's brother = cousin of Aliya's husband. Therefore, Tia will be the cousin of Aliya's husband.

Then, Tia is Aliya's cousin-in-law, whose gender need not be defined. So, the data in statement II are sufficient to answer the question while the data in statement I are alone not sufficient to answer the question.

3. Towards which direction is Charu from Riva?
- Anij is towards the west of Bilal and northeast of Riva.
 - Charu is towards the south of Anij.
- A. If the data in only statement I are sufficient to answer the question,

while the data in statement II alone are not sufficient to answer the question.

- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If data even in both the statements I and II together are not sufficient to answer the question.
- D. If data in both statements I and II together are necessary to answer the question.

Answer: C

Explanation: From the statement I \rightarrow we will get to know that Anij and Bilal are in straight lines and Anij is to the west of Bilal. Secondly, Anij is to the northeast of Riva.

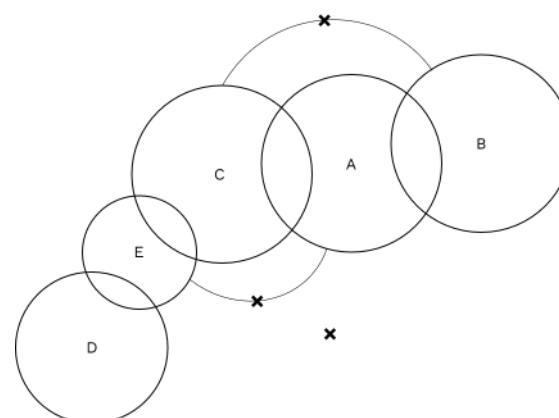
From statement II, we get to know that Charu is to the south of Anij, but the twist here is that we don't know the directions of Charu and Riva with respect to each other's whether they are in straight lines or not. Therefore, we can't answer this question. So, the data in both statements I and II together are not necessary to answer the question. Therefore, option C is correct.

PRACTICE QUESTIONS

1. Which statements show that $\rightarrow A$ can be either C or D but not both?
- some A is C and B but no C is B
 - some C is E but not A is E
 - no B is E and some E is D
- A. If data in only statements I and II will be suffice to answer
- B. If data in only statements I and III will be suffice to answer
- C. If data in all the statements I, II, and III will be suffice to answer
- D. If data in even all the statements will not be suffice to answer

Answer: D

Explanation: By resolving all the statements, we get this result





So, we can say that data in even all the statements will not be suffice to answer. Therefore, option D is correct.

2. In a certain code 'dua lopa jima' means 'we are best'. Which code word means 'we'?
 - I. 'al sim jima' means 'best of luck'.
 - II. 'est pros dua' means 'are you there'.
 - A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
 - B. If the data in only statement II are sufficient to answer the question, while the data in statement I are alone not sufficient to answer the question.
 - C. If data even in both the statements I and II together are not sufficient to answer the question.
 - D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From the statement I → 'jima' means 'best'.

From the statement, II → 'dua' means 'are'.

So, by resolving all the statements, we can say that 'lopa' means 'we'

Therefore, option D is correct.

3. In a certain code 'ar br cr dr' means 'let it be now'. And 'ar fr dr hr' means 'now just be yourself'. Which code word means 'it'?
 - I. 'fr br dr jr' means 'just it's be it'.
 - II. 'vr zr br ar' means 'now this is it'.
 - A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
 - B. If the data in only statement II are sufficient to answer the question while the data in statement I are alone not sufficient to answer the question.
 - C. If the data either in statement I or in statement II alone are sufficient to answer the question

- D. If data even in both statements I and II together are not sufficient to answer the question.

Answer: C

Explanation: From question → 'ar' and 'dr' can either be 'now' or 'be'.

From statement I and question statement → 'dr' means 'be' and 'br' means 'it'.

From statement II and question statement → 'ar' means 'now' and 'br' means 'it'.

So, by resolving both statements I and II individually, we can say that 'br' means 'it'; therefore, the data either in statement I or in statement II alone are sufficient to answer the question. Therefore, option C is correct.

4. Who is the granddaughter of T?

- I. I is the sister-in-law of G and G is the husband of K. A is the grandson of T who is the father-in-law of K.
- II. B has only two sons and one of them is F. K is sister-in-law of F. C is the sister of A who is the son of I.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question.
- D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From statement I → I and K are females; they are sister-in-law because I is G's sister-in-law, that is, she is G's brother's wife.

From statement II → 'T has only two sons and one of them is F and also given K is F's sister-in-law. As resolved from statement I, K is G's



wife. Therefore, F and G are brothers and sons of T.

From both the statements, we get to know that T and B are husband and wife who have 2 children, F and G whose wives are I and K respectively. From statement II, we get to know that A is the son of I and C is the sister of A. From statement I, A is the grandson of T; therefore, C is the granddaughter of T.

So, the data in both statements I and II together are necessary to answer the question. Therefore, option D is correct.

5. In a certain code 'jo so ro to' means 'I am a boy'. And 'ao do wo so' means 'where should I go'. Which code word means 'am'?
 - I. 'yo uo jo io' means 'there is a girl'.
 - II. 'no mo ro co' means 'boy you can leave'.
 - A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
 - B. If the data in only statement II are sufficient to answer the question while the data in statement I are alone not sufficient to answer the question.
 - C. If the data either in statement I or in statement II alone are sufficient to answer the question
 - D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From the statement, I → 'jo' means 'a'.

From statement II → 'ro' means 'boy'.

From the question → 'so' means 'I'.

So, by resolving all the statements, we can say that 'to' means 'am'.

Therefore, option D is correct.

6. Who is the aunt of X?

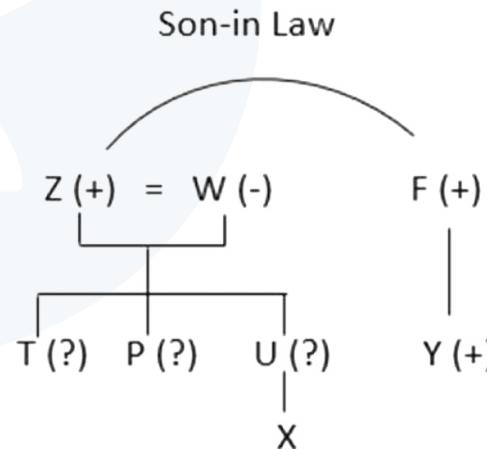
- I. F is the son-in-law of Z, who is the father of T and P. Y is the son of F.

II. W is the grandmother of X, who is the daughter of U. M is the daughter-in-law of W, who has 3 children T, P, and U.

- A. If the data in only statement I are sufficient to answer the question while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question while the data in statement I alone are not sufficient to answer the question.
- C. If data even in both statements I and II together are not sufficient to answer the question.
- D. If data in both statements I and II together are necessary to answer the question.

Answer: C

Explanation: From both the statements we would get the relations between these peoples as shown in the diagram.



Still, we can't determine the relationship between them because the genders of T, P, and U are not defined. Moreover, M's relationship with X could not be established till we get to know the gender of U. If U was a male, then probably M might be his wife and then she will be X's mother. In the second case, even if the gender of U is male, then M might be the wife of either P or T, whose gender is still unknown. So, the data even in both statements I and



II together are not sufficient to answer the question. Therefore, option C is correct.

7. Zen is in which direction with respect to Xavier?
- I. Yasir is to the south of Xavier and Zen is to the east of Prateek, which is to the north of Yasir.
 - II. Prateek is to the south of Xavier.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If data even in both statements I and II together are not sufficient to answer the question.
- D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From statement I → we will get two directions. (1) Xavier is to the north of Yasir. (2) Prateek is to the north of Yasir.

From statement II, we get to know that Prateek will lie somewhere between Xavier and Yasir and, therefore, Zen will be in the south-east with respect to Xavier.

So, the data in both statements I and II together are necessary to answer the question. Therefore, option D is correct.

8. Which direction is Abhay facing?
- I. Ajay is to the right of Abhay.
 - II. Anil is sitting opposite Ajay, who is facing north.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question,

- while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question
- D. If data even in both the statements I and II together are not sufficient to answer the question.

Answer: D

Explanation: From the statement I → we will get to know that Ajay and Abhay are sitting in straight lines, but their direction of the face is unknown.

From statement II, we get to know that Anil is south of Ajay who is facing north.

From both the statements, the face of the direction of Abhay is still unknown; therefore, the data in both statements I and II together are not necessary to answer the question. Therefore, option D is correct.

9. Towards which direction is Parul from Rohit?
- I. Sam is towards the west of Mohit and northeast of Rohit.
 - II. Parul is towards the south of Sam.
- A. If the data in only statement I are sufficient to answer the question while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question
- D. If data even in both statements I and II together are not sufficient to answer the question.

Answer: D

Explanation: From the statement I → we will get to know that Sam and Mohit are in straight lines and Sam is to the west of Mohit. Secondly, Sam is to the northeast of Rohit.



From statement II, we get to know that Parul is to the South of Sam, but the twist here is that we don't know the directions of Parul and Rohit with respect to each others, whether, they are in straight lines or not. Therefore, we can't answer this question. So, the data in both statements I and II together are not sufficient to answer the question. Therefore, option D is correct.

- 10.** In a certain code, 'sel ta rei jai' means 'love to be here'. And 'rei hoe mi sel' means 'you should be love'. Which code word means 'love'?

- I. 'mi del si ta' means 'you are to sign'.
- II. 'jai del sel bu' means 'love sign is here'.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question
- D. If data in both statements I and II together are necessary to answer the question.

Answer: B

Explanation: From question → 'rei' and 'sel' can either be 'love' or 'be'.

From the statement I → 'ta' means 'to'.

From statement II and question statement → 'sel' means 'love' and 'jai' means 'here'.

So, by resolving only statement II, we can say that 'sel' means 'love'.

Therefore, option B is correct.

- 11.** Who is the sister of D?

- I. G is the grandson of C who is the father-in-law of H. D is the uncle of L.
- II. A, who has only two children: one son and one daughter K. G and L are brothers, who are the children of H.

- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question
- D. If data even in both the statements I and II together are not sufficient to answer the question.

Answer: D

Explanation: By resolving both the sentences, we would get an unrecognised relationship between these people. There is a major missing relation between G and A as to whether they are husband and wife or not. If it would have been given, then D would be the brother of K, a lady. L and G would be the sons of K and H. And K and D would have been the children of G and A. So, the data even in both the statements I and II together are not sufficient to answer the question. Therefore, option D is correct.

- 12.** Towards which direction is Ranji from Rita?

- I. Salmond is towards the west of Moron and northeast of Rita. Calon is to the east of Rita.
- II. Ranji is towards the south of Salmond and Calon.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question



- D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From the statement I \rightarrow we will get to know that Salmon and Moron are in a straight line, and the former is the west of the latter. Calon and Rita are in straight lines and the former is to the east of the latter.

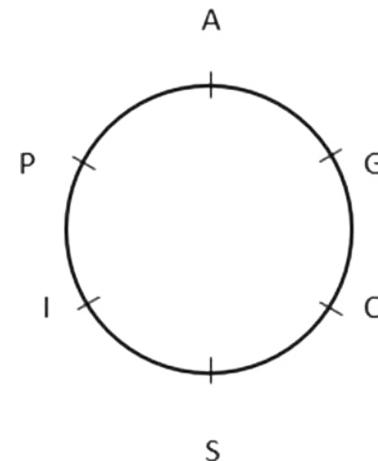
From statement II, we get to know Ranji is to the south of both Rita and Calon; therefore, she is to the southwest of Rita. So, the data in both statements I and II together are necessary to answer the question. Therefore, option D is correct.

- 13.** There are 6 people, G, I, S, A, P, Q, sitting around the circular table randomly. Find the position of A with respect to I?

- I. G is second left of P and I is third left of G.
- II. S is not an immediate neighbour of G and A sits third right of S.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question.
- D. If data in both statements I and II together are necessary to answer the question.

Answer: D

Explanation: From the statements I and II \rightarrow we will get to know that G is the second left of P and I is the third left of G. If S is not the immediate neighbour of G then only one place is left for it, i.e., second right of G. So, A and Q will be at immediate left and right of G, respectively. So in this way, A is to the second to the left of I.



So, the data in both statements I and II together are necessary to answer the question. Therefore, option D is correct.

- 14.** What is the possible time after 1, when a clock shows 20° angle between the two hands of the clock.

- I. The hour hand must be between 1 and 2.
- II. The minute hand must be after 2 o'clock in the watch.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question
- D. If data even in both the statements I and II together are not sufficient to answer the question.

Answer: A

Explanation: In this question, the angle is given, and here we have to find the time between when the angle between minute and hour hands is 20° . But the prerequisite for clock questions is that the position of the hour hand must be between 2 consecutive



hours like between 1 and 2 or between 3 and 4, but there are no such conditions for the minute hand. So, the data in only statements I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question. Therefore, option A is correct.

By simply using the angle-finding theory, we can find the angle between the two hands of the clock: Angle between hands =

$$[(11 \div 2) \text{ Minute} - (30 \times 1)] \times \frac{360}{60}$$

$20^\circ = [(\quad) \text{ Minute} - 30 \times 1] \rightarrow 50 = (\quad) \text{ Minute}$, we

can write it as: $9 \text{ min } 5 \text{ s} (\quad \times 60 = 1 \times 5.4 \approx 5)$

15. Among the seven students P, Q, R, S, T, U, and V, who scored 2nd highest in the English test?

- I. P scored more than only R. Two students scored more than V, who scored more than P and Q.
- II. T scored less than S, who scored more than Q.
- III. R scored less than S, who does not score the highest.
- A. If the data in only statements I and II are sufficient to answer the question, while the data in statements II and III alone are not sufficient to answer the question.
- B. If the data in only statements II and III are sufficient to answer the question, while the data in statements I and II alone are not sufficient to answer the question.
- C. If the data in all the statements are necessary to answer the question.
- D. If the data in only statements I and III are sufficient to answer the question, while the data in statements II and III alone are not sufficient to answer the question.

Answer: C

Explanation: From statements I and II:

P scored more than only R. Two students scored more than V, who scored more than P and Q $\rightarrow ? > ? > V > ? > ? > P > R$

T scored less than S, who scored more than Q. So, no proper information is given.

From statements II and III:

T scored less than S, who scored more than Q. R scored less than S, who did not score the highest. So, no proper information is given.

From both I, and III:

P scored more than only R. Two students scored more than V, who scored more than P and Q. R scored less than S, who did not score the highest.

$\rightarrow ? > ? > V > ? > ? > P > R$

So, no proper information is given.

From I, II, and III

$\rightarrow U > S > V > Q/T > Q/T > P > R$

So, from I, II and III, it is clear that S scored the 2nd highest. Therefore, option C is correct.

16. In which month of the year 1999 did Amit go to Delhi for vacation?

- I. Amit's son remembers that he went after 20th July 1999.
- II. Amit remembers that he went before 10th August 1999.
- III. Vinayak, a friend of Amit, remembers that he went to Delhi either in the 7th or 8th month of the year 1999.
- A. If the data in only statements I and II are sufficient to answer the question, while the data in statements II and III alone are not sufficient to answer the question.
- B. If the data in only statements II and III are sufficient to answer the question, while the data in statements I and II alone are not sufficient to answer the question.
- C. If the data in all the statements are necessary to answer the question.
- D. If data even in statements I, II, and III are not sufficient to answer the question.

Answer: D

Explanation: From statements I and II:



Amit's son remembers that he went after 20th July. Amit remembers that he went before 10th August 1999. So, from I and II, Amit went in either July or August.

From statements II and III:

Vinayak, a friend of Amit, remembers that he went to Delhi in the 7th or 8th month of the year 1999. Amit remembers that he went before 10th August 1999. So, from II, and III, Amit went in either July or August.

From I and III, Amit's son remembers that he went after 20th July 1999. Vinayak, a friend of Amit, remembers that he went to Delhi either in the 7th or 8th month of the year 1999. So, from I and III, Amit went either in July or August.

Combining all statements:

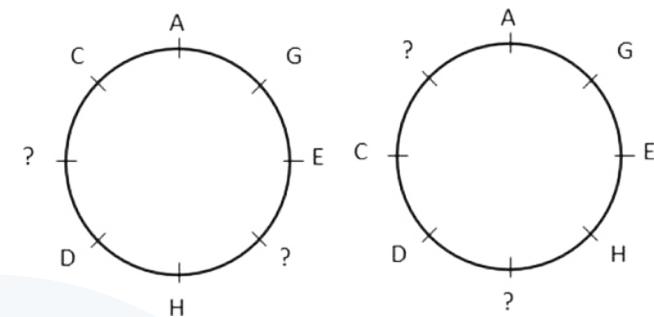
Amit can go in either July or August; hence, question cannot be answered even with the information in all three statements. Therefore, option D is correct.

17. There are 8 people, A, B, C, D, E, F, G, H, sitting around the circular table randomly facing outside. Find the position of B with respect to C?

- I. A is to the third right of D, and G is not an immediate neighbour of C.
- II. C is the third right of H, and D is the fourth right of G. E is the second to the right of A.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question.
- D. If data even in both the statements I and II together are not sufficient to answer the question.

Answer: D

Explanation: From statements I and II → we will have two settings on which H is the immediate neighbour of D and, in others, it is not as shown in the diagram. But in both the figures, we can't determine the position of B since nothing is talked about.



So, the data in both statements I and II together are not sufficient to find the position of B with respect to C. Therefore, option D is correct.

18. What was the price of the house P in 2010?

- I. The ratio of the price of P to Q in 2010 was 7:8.
- II. The price of house Q in 2011 was Rs 650,000 and it has increased by 30% from 2010.
- III. The ratio of the price of houses P and Q in 2011 was 6:7.
- A. If the data in only statements I and II are sufficient to answer the question, while the data in statements II and III alone are not sufficient to answer the question.
- B. If the data in only statements II and III are sufficient to answer the question, while the data in statements I and II alone are not sufficient to answer the question.
- C. If the data in all the statements are necessary to answer the question.
- D. If the data in only statements I and III are sufficient to answer the question, while the data in statements II and III alone are not sufficient to answer the question.

**Answer: A****Explanation:** From statements I and II:

$$\begin{aligned}\text{Price of house Q in 2010} &= 650000 \times \frac{A_1 P_1 + A_2 P_2}{A_1 + A_2} \\ &= \text{Rs } 500000\end{aligned}$$

$$\begin{aligned}\text{Price of house P in 2010} &= 500000 \times \frac{A_1 P_1 + A_2 P_2}{A_1 + A_2} \\ &= \text{Rs } 437500\end{aligned}$$

Therefore, the data in only statements I and II are sufficient to answer the question, while the data in statements II and III alone are not sufficient to answer the question. Therefore, option A is correct.

- 19.** How many boys are taller than Alana in his class?

- I. When students of Alan's class are ranked in descending order of their heights, Alan's rank is 19th from the top among all the students and 13th among girls.
- II. Alana's rank from the bottom on the basis of height among girls is 19th and among all students is 27th.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question.
- D. If data even in both the statements I and II together are not sufficient to answer the question.

Answer: A**Explanation:** From the statement I, we will get to know that $(19 - 13) = 6$ boy students are taller than Alana.

From statement II, we get to know that $(27 - 19) = 8$ boy students are shorter than Alana. But

from II alone it is not known how many girls are there in the class. So, data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question. Therefore, option A is correct.

- 20.** What is Kailash's rank in the class of 50 students?

- I. Reita, who is 9th from the top in the class, is above Radhe by 17 ranks, who is below Kailash by 6 ranks.
- II. Namita, who is between Kailash and Kaveri, is 25th from the bottom.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question.
- D. If data in both statements I and II together are necessary to answer the question.

Answer: A**Explanation:** From the statement I, we will get to know that

Reita's rank = 9th from the top, Radhe is 26th from the top and Kailash is 20th from the top. So, data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

From statement II, we get to know that Namita's rank is $(50 - 25 + 1) = 26$ th from the top.

Therefore, option A is correct.

- 21.** Who among Malan, Troy, Richard, Ken, and Peter, is the tallest?

- I. Troy is taller than Richard, Malan and Peter but shorter than Ken.



- II. Richard, Troy, and Malan are shorter than Ken but taller than Peter.
- A. If the data in only statement I are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- B. If the data in only statement II are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- C. If the data either in statement I or in statement II alone are sufficient to answer the question
- D. If data even in both the statements I and II together are not sufficient to answer the question.

Answer: C

Explanation: From the statement I, we will get to know that

Ken > Troy > Richard, Malan, and Peter. Hence, Ken is the tallest.

From statement II, we get to know that Ken > Richard, Troy, Malan > Peter. Hence Ken is the tallest.

Therefore, the data either in statement I or in statement II alone are sufficient to answer the question. Therefore, option C is correct.