

CS & IT ENGINEERING



THEORY OF COMPUTATION

✓ Pushdown Automata

Lecture – 02



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Recap of Previous Lecture



Topic

→ Pushdown Automata

?????

→ Designing PDA for non regular languages.



Topics to be Covered



Topic

Push down automat

Construction

Topic

?? NPDA Construction

Topic

?? DCFL

Topic

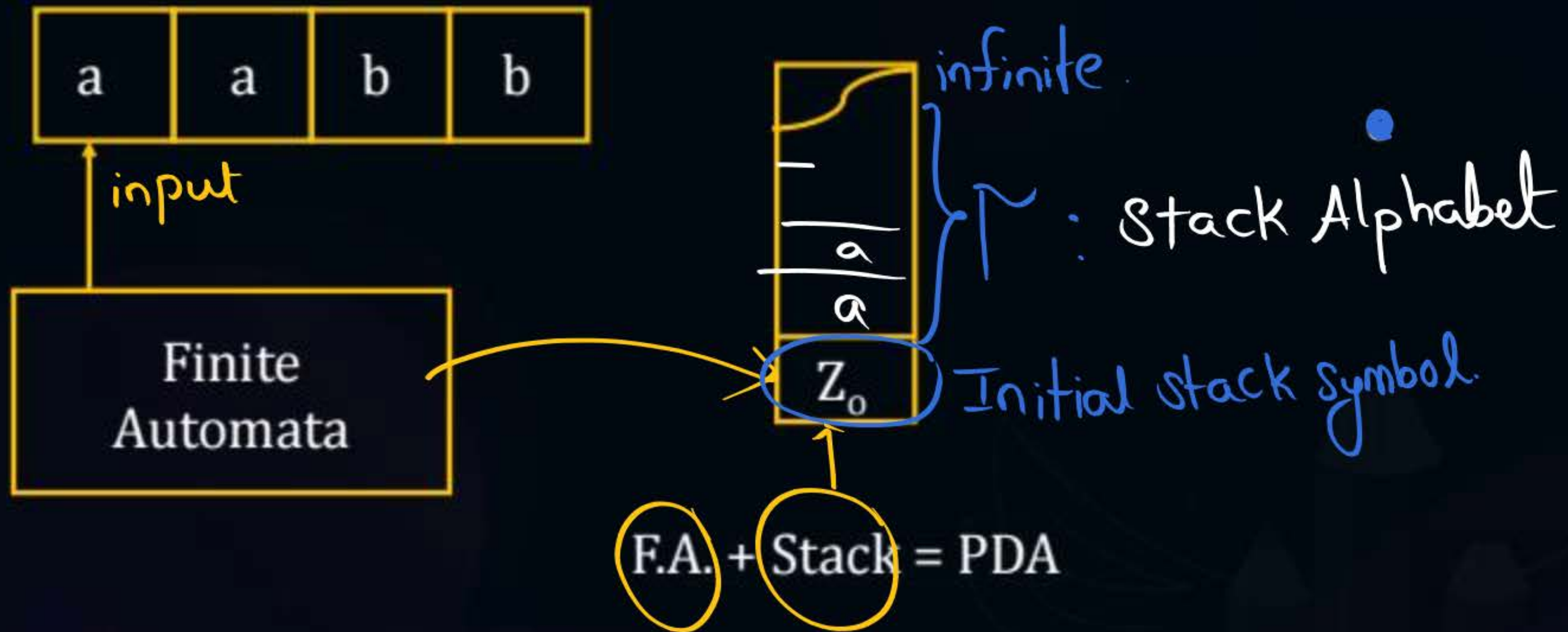
?? NCFL



Topic : Pushdown Automata



Pushdown Automata





Topic : PDA



- Finite Automata having additional power form of stack known as Push down automata.
- Size of stack in Push Down automata is infinite
- There exist only one type of push down automata i.e. “language recognisor”
- Push down automata can accept language in deterministic way or non-deterministic way

PDA $(Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$

Q :- Finite number of states

Σ :- Input alphabet

q_0 :- initial state

F :-set of final states

Z_0 :-initial stack symbol

Γ :-stack alphabet

δ :- transition function

$$Q \times \Sigma \cup \{\epsilon\} \times \Gamma \rightarrow Q \times \Gamma^*$$

PDA $(Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$

Q :- Finite number of states

Σ :- Input alphabet

δ :- Initial State

q_0 :- Set of final states

F :- Initial stack elements

Z_0 :- Stack alphabet

Γ :- transition function

$$Q \times \Sigma \cup \{\epsilon\} \times \Gamma \rightarrow Q \times \Gamma$$



Topic : Note:

Note:- The following operation possible with PDA stack.

Push operation:- Moving i/p symbol from i/p buffer stack.

POP operation:- removing element from stack.

By pass operation:- don't push & don't pop (just reading symbol only)

SKIP



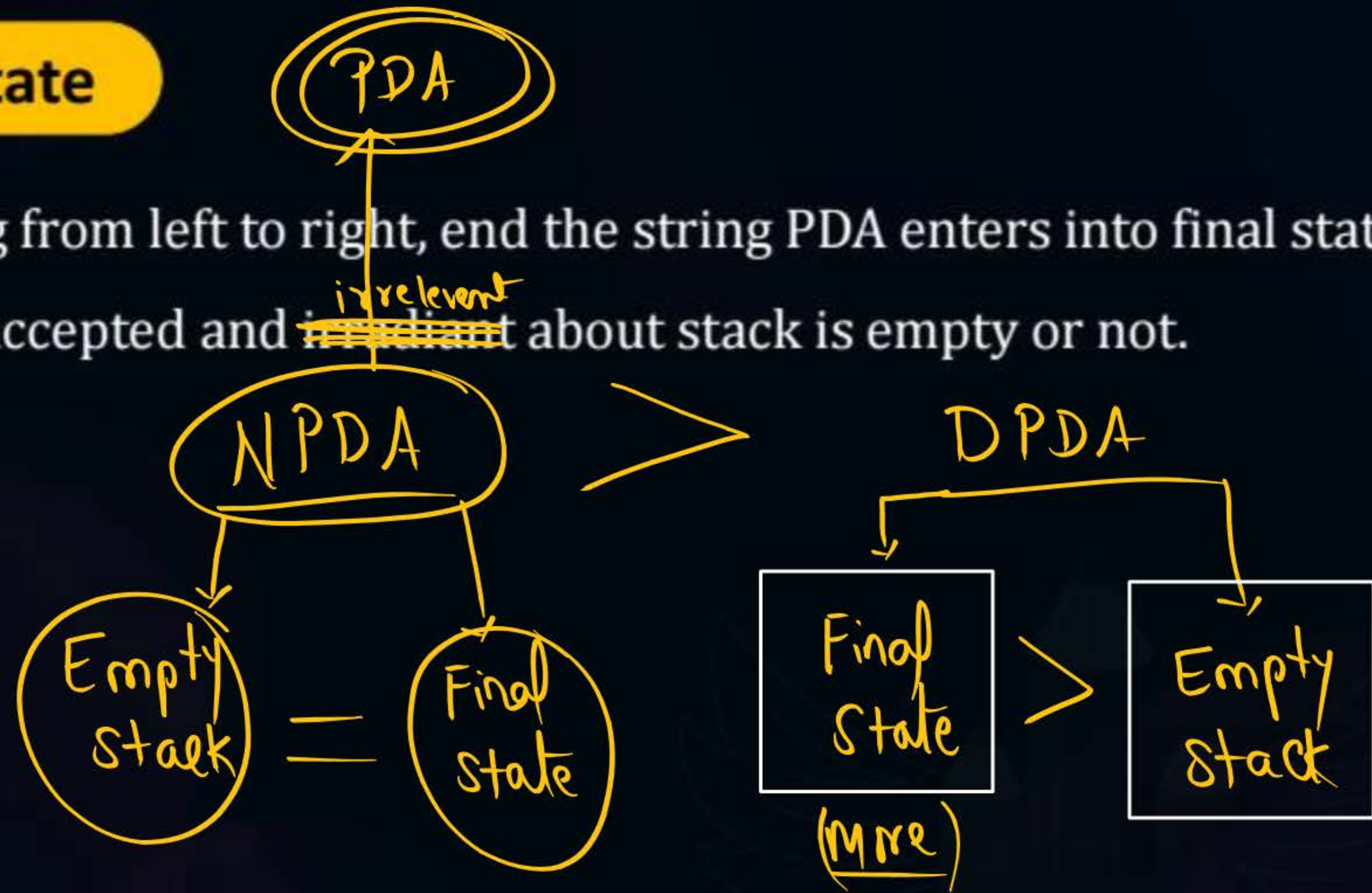
Topic : Empty Stack

By reading the string from left to Right by end of the string, if stack of the PDA is empty, then given string is accepted and ~~irrelevant~~ ^{irrelevant} of No of final state.



Topic : Final State

By reading the string from left to right, end the string PDA enters into final state then given string is accepted and ~~irrelevant~~ ^{irrelevant} about stack is empty or not.





Topic : Note

Note:- Number of language accepted by empty stack method and final state method is same in PDA.

The language L is accepted by empty stack if and only if L should be final state.



Topic PDA



- The expressive power of NPDA is more than DPDA.
- By Default PDA means NPDA.
- PDA practically used in compilers as parser.
- There are two types of acceptance method in PDA they are acceptance by empty stack and acceptance by final stack.

Notations:-



PDA (Acceptor)



[MCQ]

#Q. Let N_1 is number of language accepted by using empty stack method N_2 is number of lang accepted by using final state then which of the following is true.

A $n_1 = n_2$

C $n_1 < n_2$

B $n_2 > n_2$

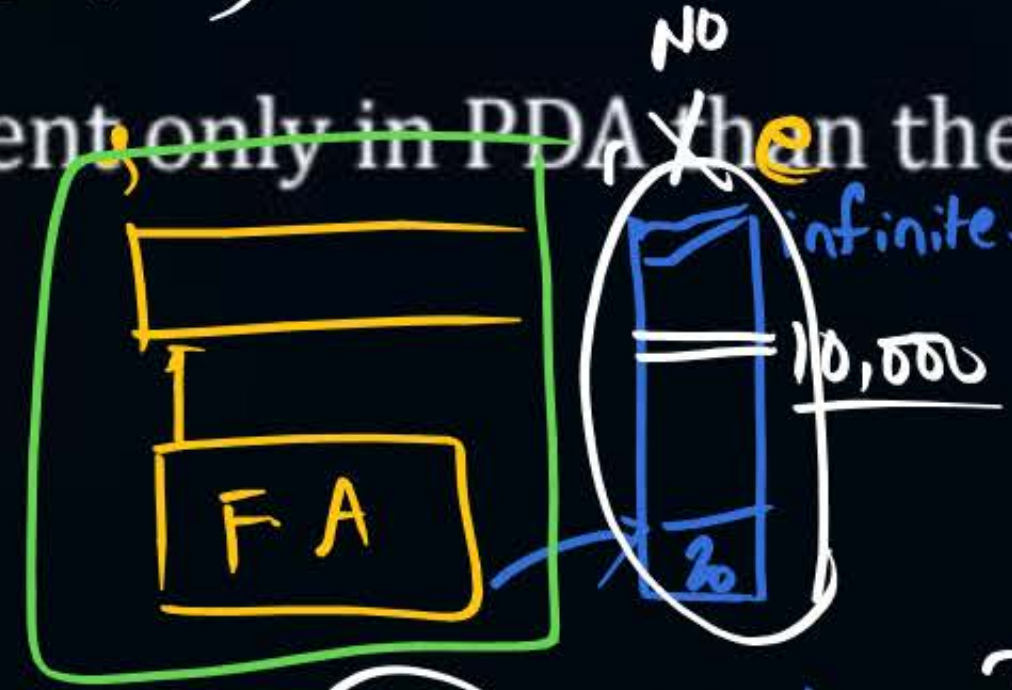
D We can't say

[MCQ]

a^*b^*

$\{a^n b^n \mid n \geq 1\}$

#Q. Size of the stack is restricted to 10000 element only in PDA, then the lang accepted by that type of PDA is-



$FA + \text{Stack} = PDA$

A Regular Lang

C Finite lang

B CFL but Not Reg.

D Reg. but not ~~reg.~~ finite

Note:-

- Lang accepted by push down automata known as CFL.
- The expressive power of PDA is more than finite automata because PDA can accept regular language as well as CFL.
-



Topic : Drawback of PDA

PDA fails to accept language which requires more than one stack.

Ex:- $L = \{a^n b^n c^n \mid n \geq 1\}$

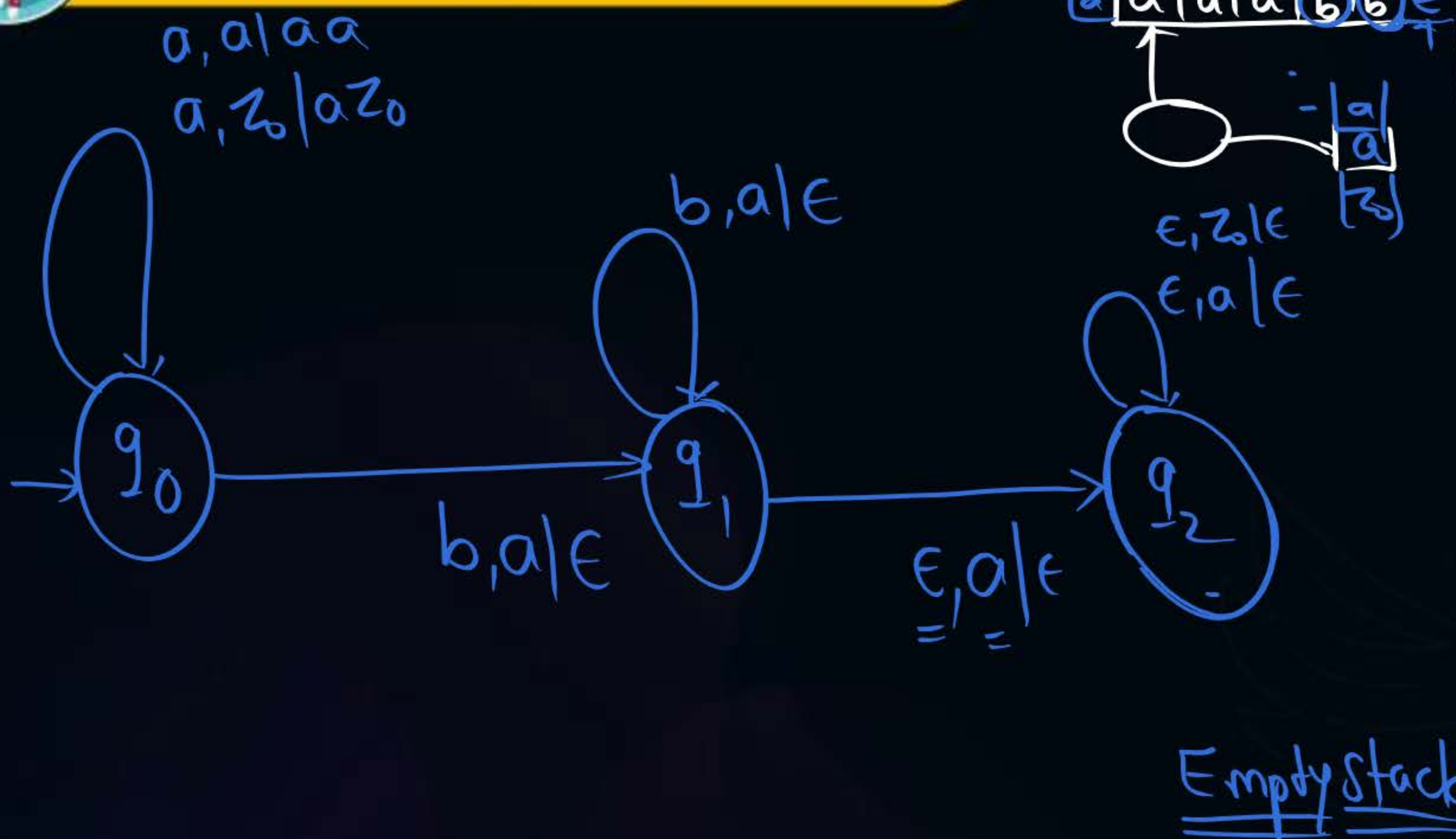
The language for which PDA Not possible known as non-cfL.



Topic : Pushdown Automata

①

$$L = \{a^m b^n \mid m > n, m, n \geq 1\}$$



Logic:- $a^n b^n$

- ① All a 's push
- ② for 1 $b \rightarrow 1$ a pop
- ③ $(q_2, \epsilon, a) \rightarrow \text{accept}$



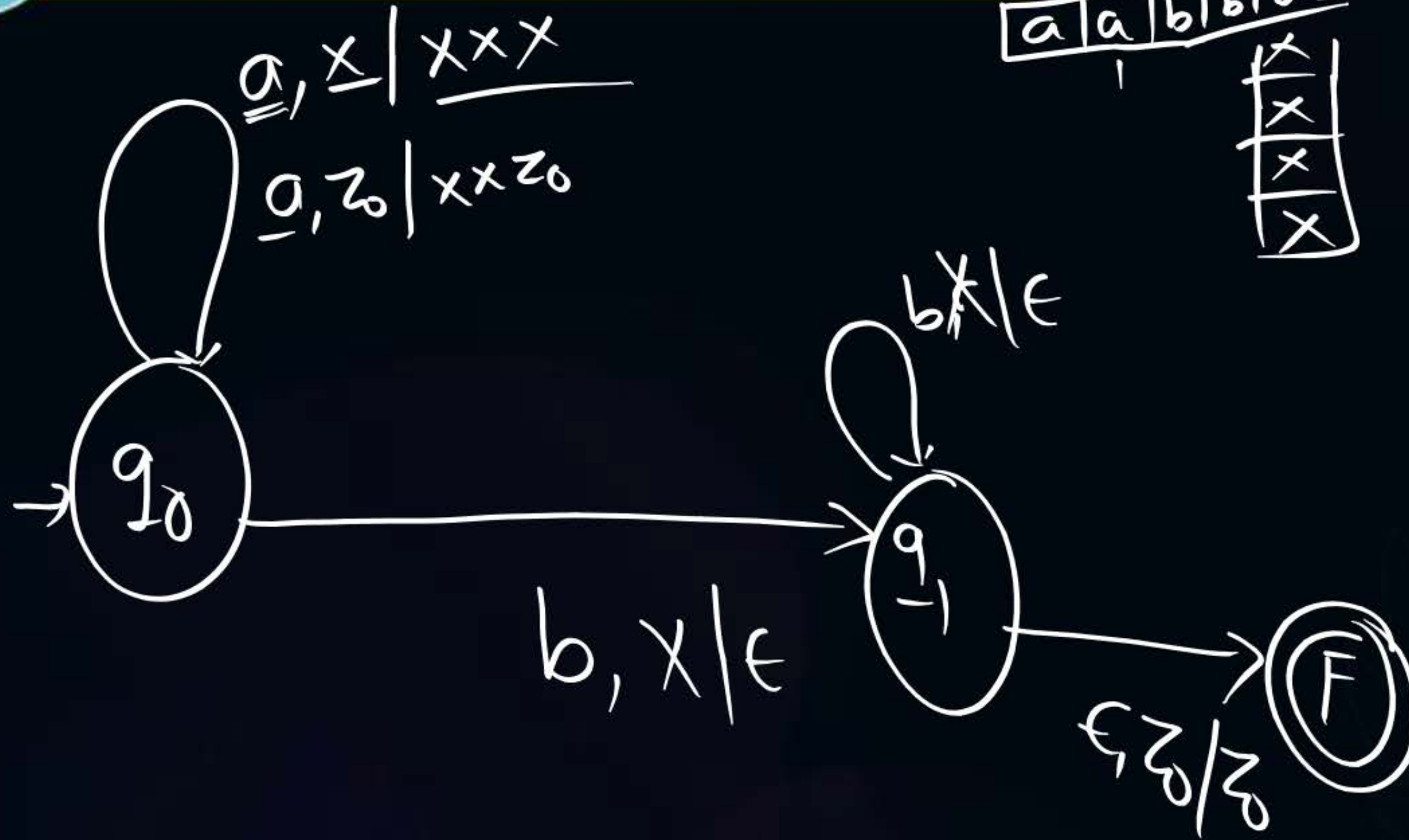
②

$$L = \{a^n b^{2n} \mid n \geq 1\}$$

$$a \cdot \epsilon = 2 \times 1$$



Topic : Pushdown Automata

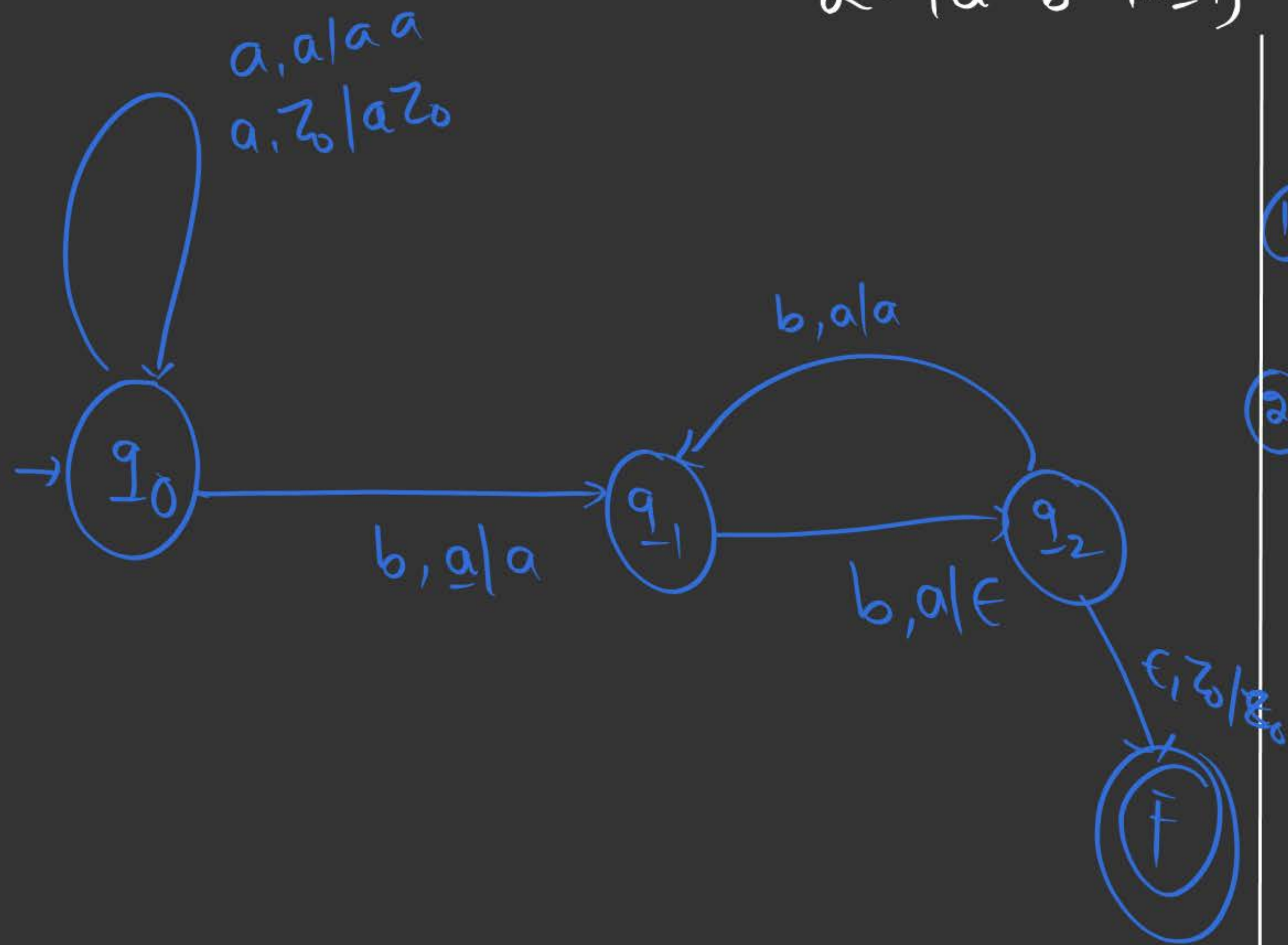


Logic ①

① for 1 $a \rightarrow$ push 2 x into stack

② for every $b \rightarrow$ pop x from stack.

$$L = \{a^n b^{2n} \mid n \geq 1\}$$



Logic:- ②

① All a's push

② 1 b → SKIP

2 b → pop

3 b → SKIP

4 b → pop

⋮



Topic : Pushdown Automata



$$L = \{a^n b^{2^n} \mid n \geq 1\}$$

$$L = \{a^n b^{2^n} \mid n \geq 1\} \} \underline{\underline{\text{Non CFL}}}$$

PDA not possible

Q : finite no. of states

F.A + stack

(Q) Which of the following language is CFL?

~~(a)~~ $L = \{a^n b^n \mid n \geq 1\} = \{a^1 b^1, a^2 b^2, a^3 b^3, a^4 b^4, \dots\}$

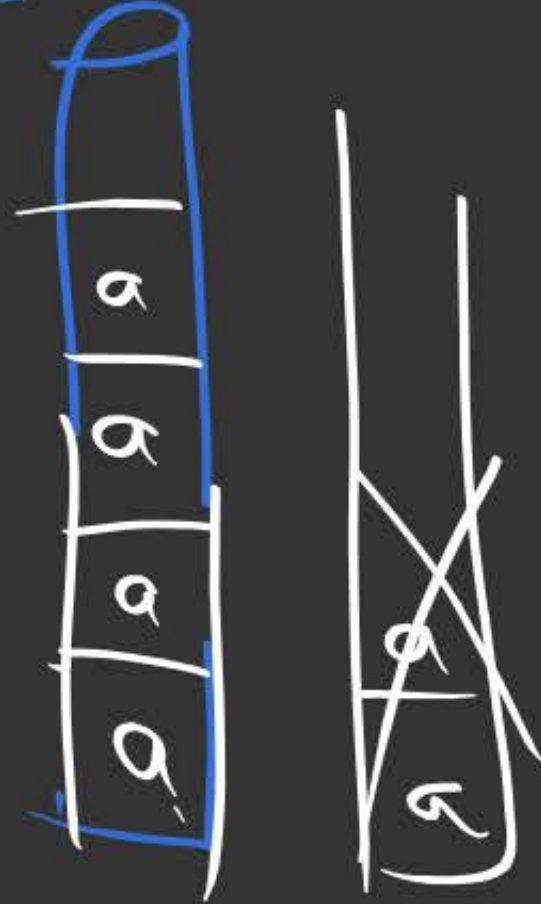
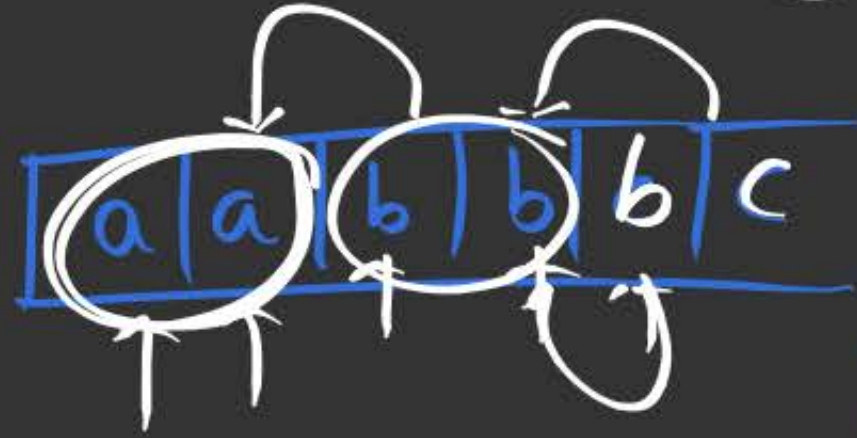
(b) $L = \{a^n b^{n^2} \mid n \geq 1\}$ non CFL

(c) $L = \{a^{n^2} b^{n^3} \mid n \geq 1\}$ non CFL

~~(d)~~ none

$L = \{a^n b^n c^n \mid n \geq 1\}$ Construct PDA for this Lang?

PDA not possible non CFL



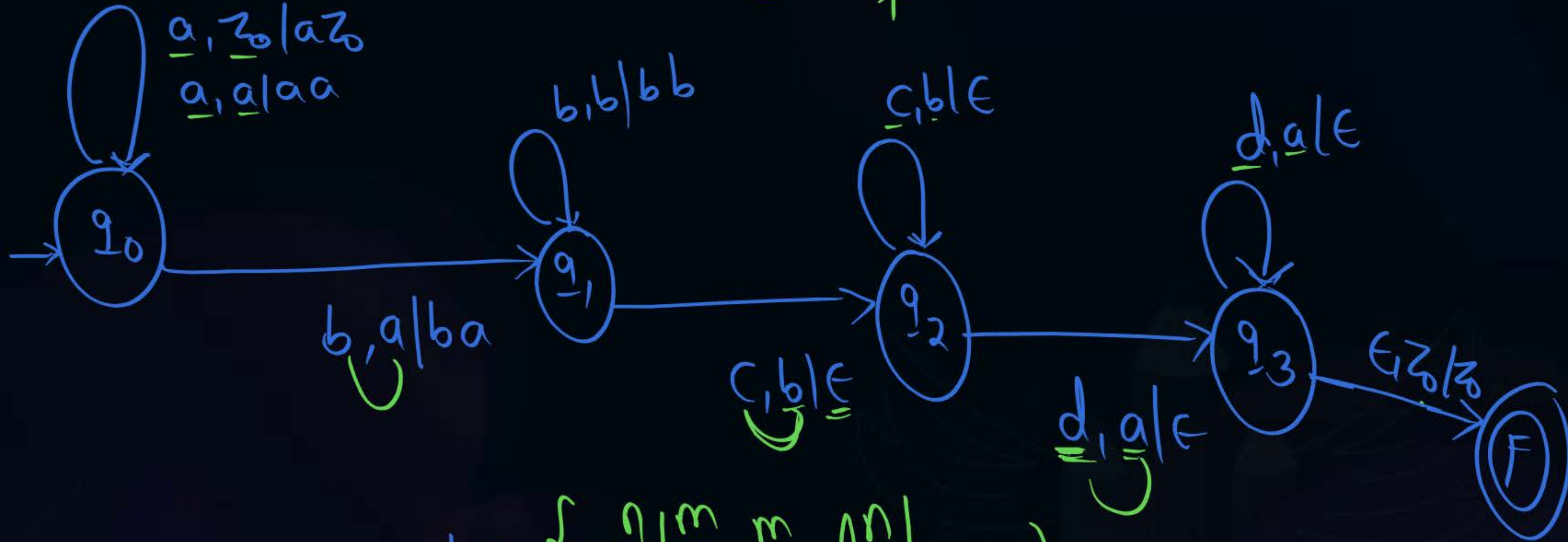


Topic : Pushdown Automata

I identify language of following PDA?



ccccdd



$$L = \{ a^n b^m c^m d^n \mid n, m \geq 1 \}$$



Topic : Pushdown Automata

Construct PDA for the Language

$$\mathcal{L} = \{a^n b^m c^n d^m \mid n, m \geq 1\}$$

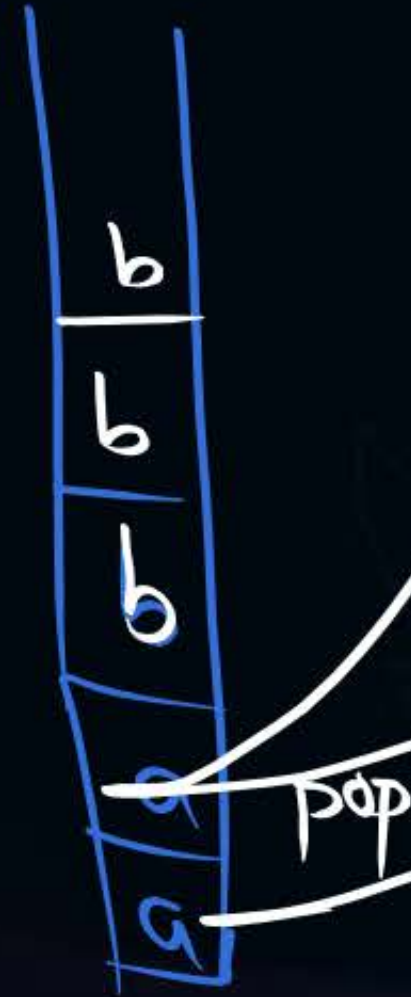
PDA not possible

aabbccddd

Logic:-

a's \rightarrow push

b's \rightarrow





Topic : Pushdown Automata



$$L_1 = \{a^n b^m c^m d^n \mid n, m \geq 1\} \rightarrow \text{CFL}$$

$$L_2 = \{a^n b^m c^n d^m \mid n, m \geq 1\} \rightarrow \underline{\underline{\text{Non CFL}}}$$



Topic : Pushdown Automata

Construct PDA for $L = \{wCw^R \mid w \in (a+b)^*\}$

CFL

ababcbaba → SKIP



- ① Total w push into stack.
- ② SKIP C
- ③ reading symbol and top of stack are same then pop



Construct PDA for $L = \{w w^R \mid w \in (a+b)^*\}$



Topic : Pushdown Automata

$$\delta(q_0, \epsilon, z_0) = (q_f, z_0)$$

$$\delta(q_0, a, z_0) = (q_0, a z_0)$$

$$\delta(q_0, b, z_0) = (q_0, b z_0)$$

$$\delta(q_0, a, b) = (q_0, ab)$$

$$\delta(q_0, b, a) = (q_0, ba)$$

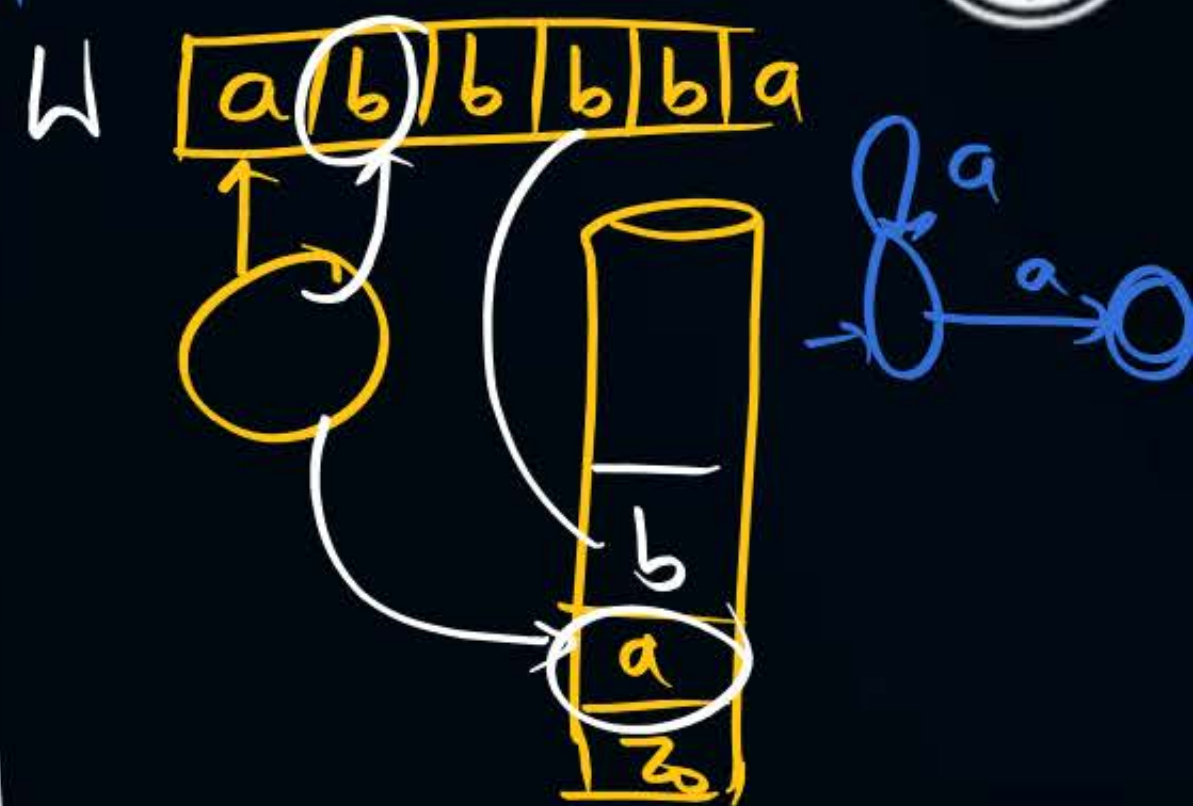
$$\delta(q_0, a, a) = (q_0, aa) \text{ (or) } (q_1, \epsilon)$$

$$\delta(q_0, b, b) = (q_0, bb) \text{ (or) } (q_1, \epsilon)$$

$$\delta(q_1, a, a) = (q_1, \epsilon)$$

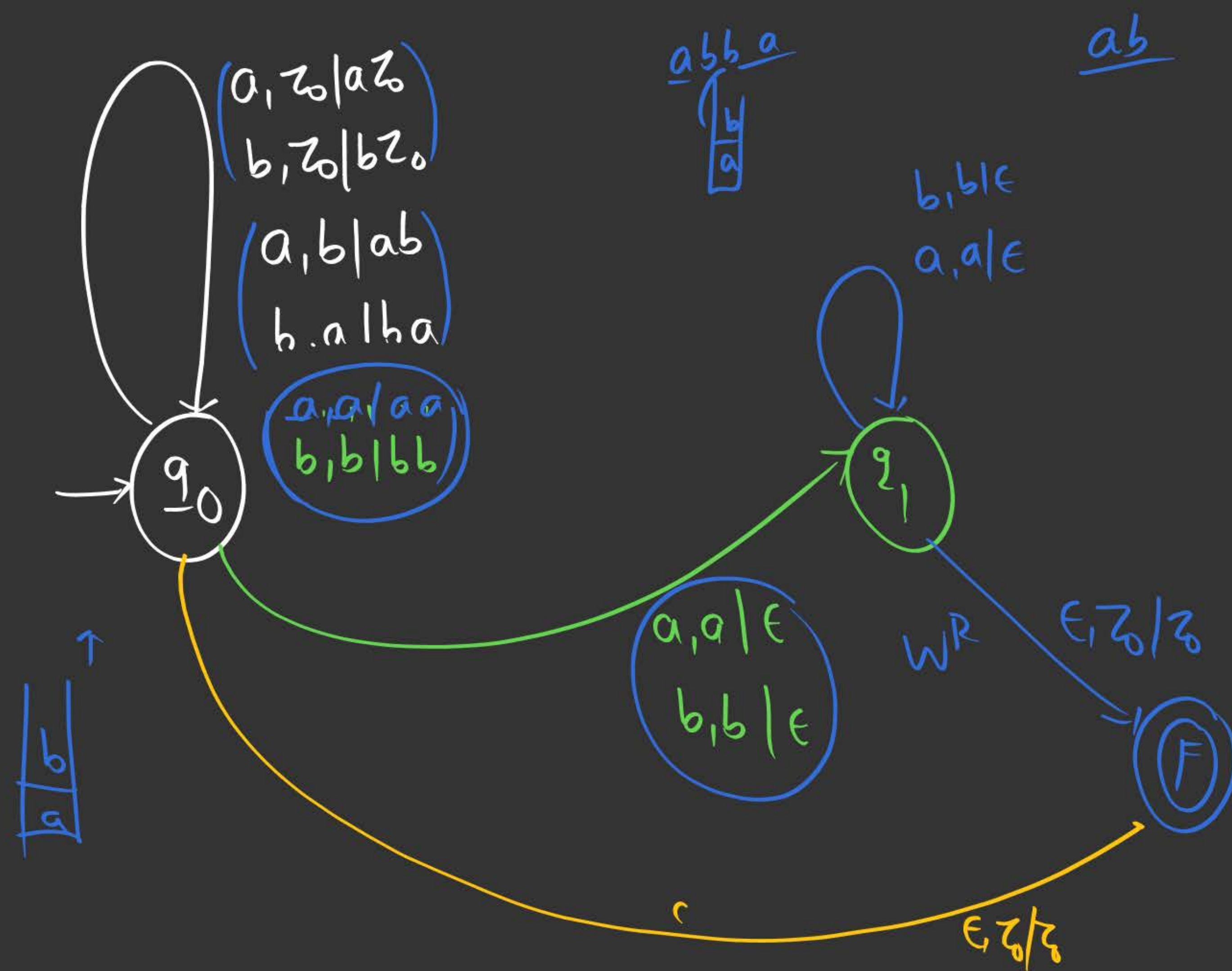
$$\delta(q_1, b, b) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_f, z_0)$$



$$\delta(q_1, b, b) = \text{push (or) pop}$$

$$\delta(q_1, a, a) = \text{push (or) pop}$$



Logic:-

- ① Initially $a(a) b$ push
- ② input symbol & stack symbol is different then also push (w possibility)
- ③ input & stack symbol are same then multiple possibility ($w(a)w^R$)

$$L = \{a^n b^{2n}\} \cup \{a^n b^{3n}\} = \{a a a \dots\}$$

$$\{a^n b^{2n}\}$$

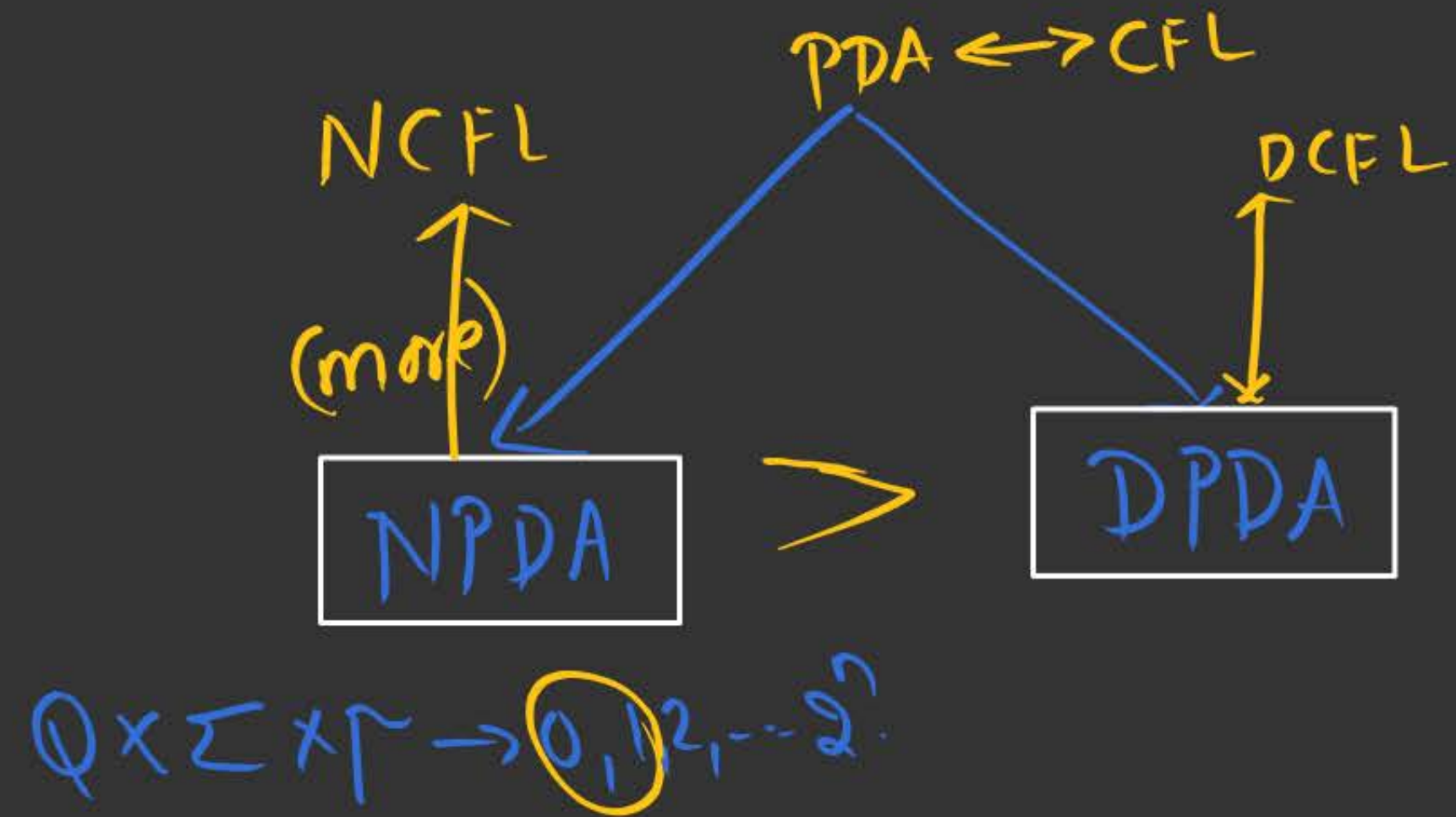
$$1a \rightarrow 2a$$

$$\{a^n b^{3n}\}$$

$$1a \rightarrow 3a$$

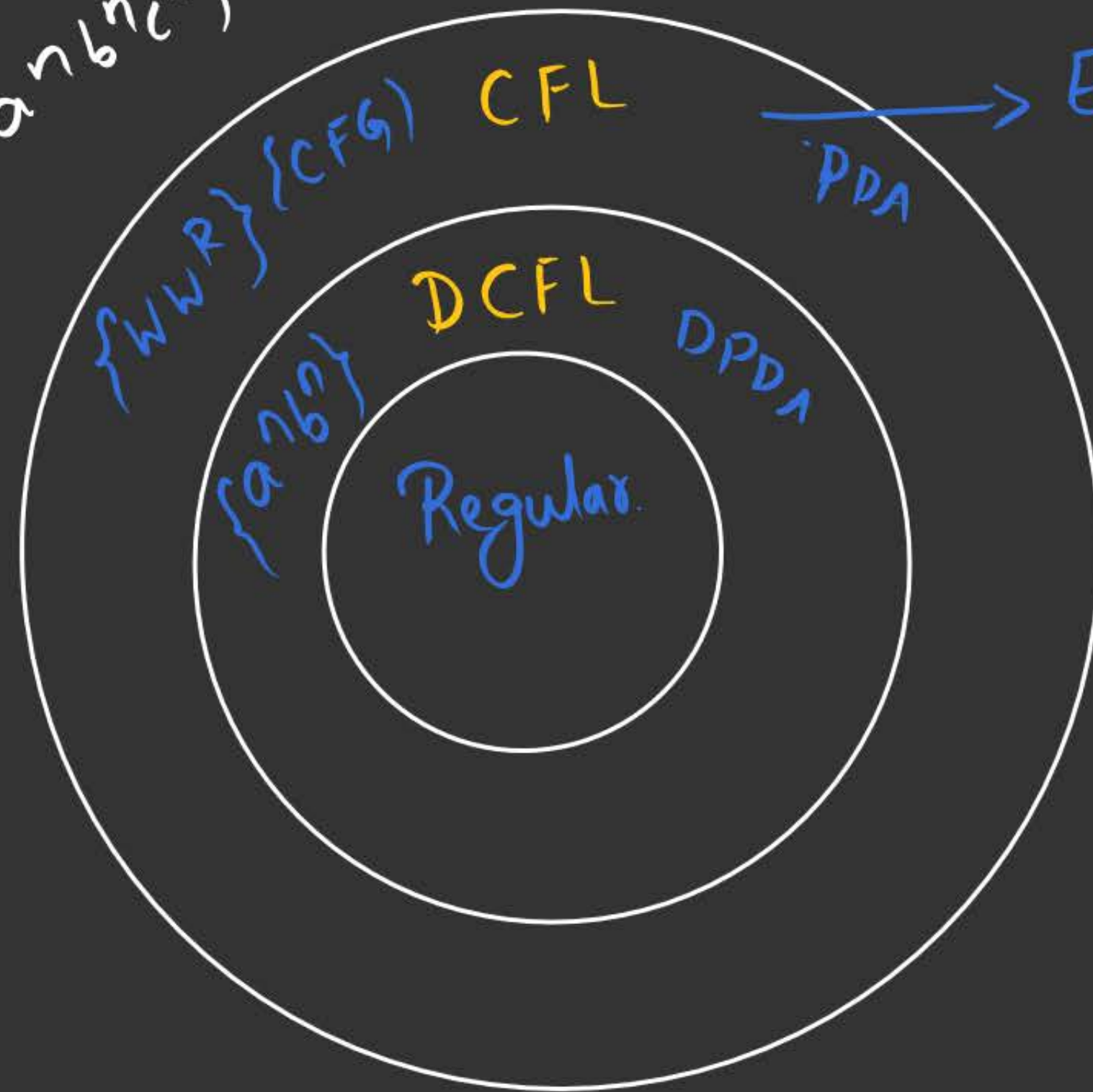
DPDA not possible

NPDA



Expressive power NPDA is more than DPDA
 Hence Every NPDA can't be converted to DPDA

$\{a^n b^n c^n\}$ non CFL



→ Every CFL need not be DCFL.

(Q) Which of the following is DCFL?
 \hookrightarrow DPDA

(a) $L = \{ww^R \mid w \in (a+b)^*\} \rightarrow$ CFL but not DCFL

(b) $L = \{a^n b^{2n}\} \cup \{a^n b^{3n}\} \rightarrow$ CFL but not DCFL

(c) $L = \{a^n b^n c^n \mid n \geq 1\} \rightarrow$ not CFL

~~(d) none~~

Every DPDA also called as NPDA

But Every NPDA need not be DPDA

$$L = \{ \underline{a}^n b^m c^m \} \cup \{ a^n \underline{b}^n c^m \}$$

$a \rightarrow \text{SKIP}$

$b \rightarrow \text{push}$

$c \rightarrow \text{pop } b's$

$a \cdot S \rightarrow \text{push}$

$b \cdot S \rightarrow \text{pop}$

$c \cdot S \rightarrow \text{SKIP}$

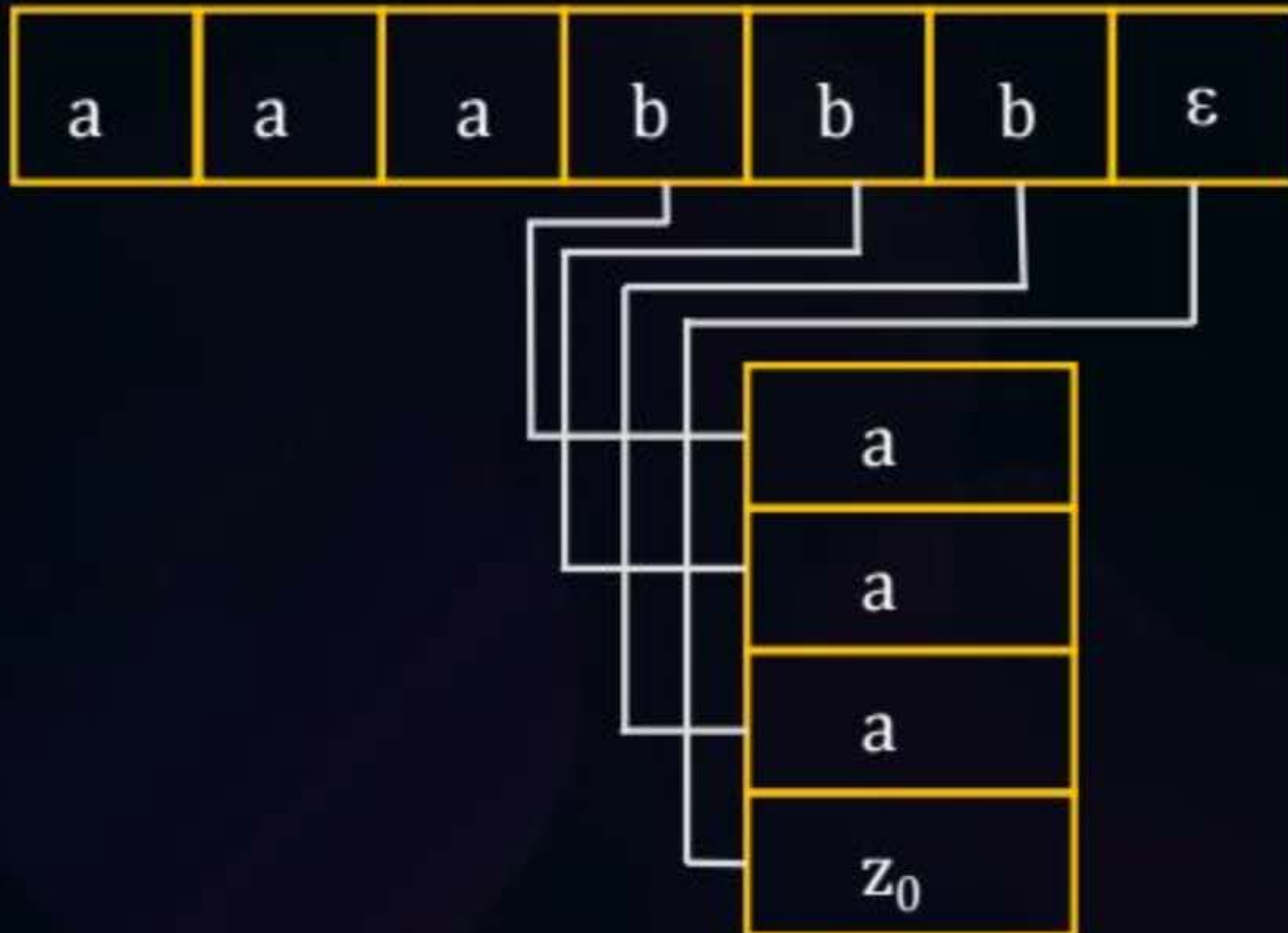
aaaa - - - -

↑
DPDA not possible

NPDA for this

Construct PDA for the language $L = \{a^n b^n / n \geq 1\}$

$$\delta: Q \times \Sigma \cup B \times \Gamma \rightarrow Q \times \Gamma$$

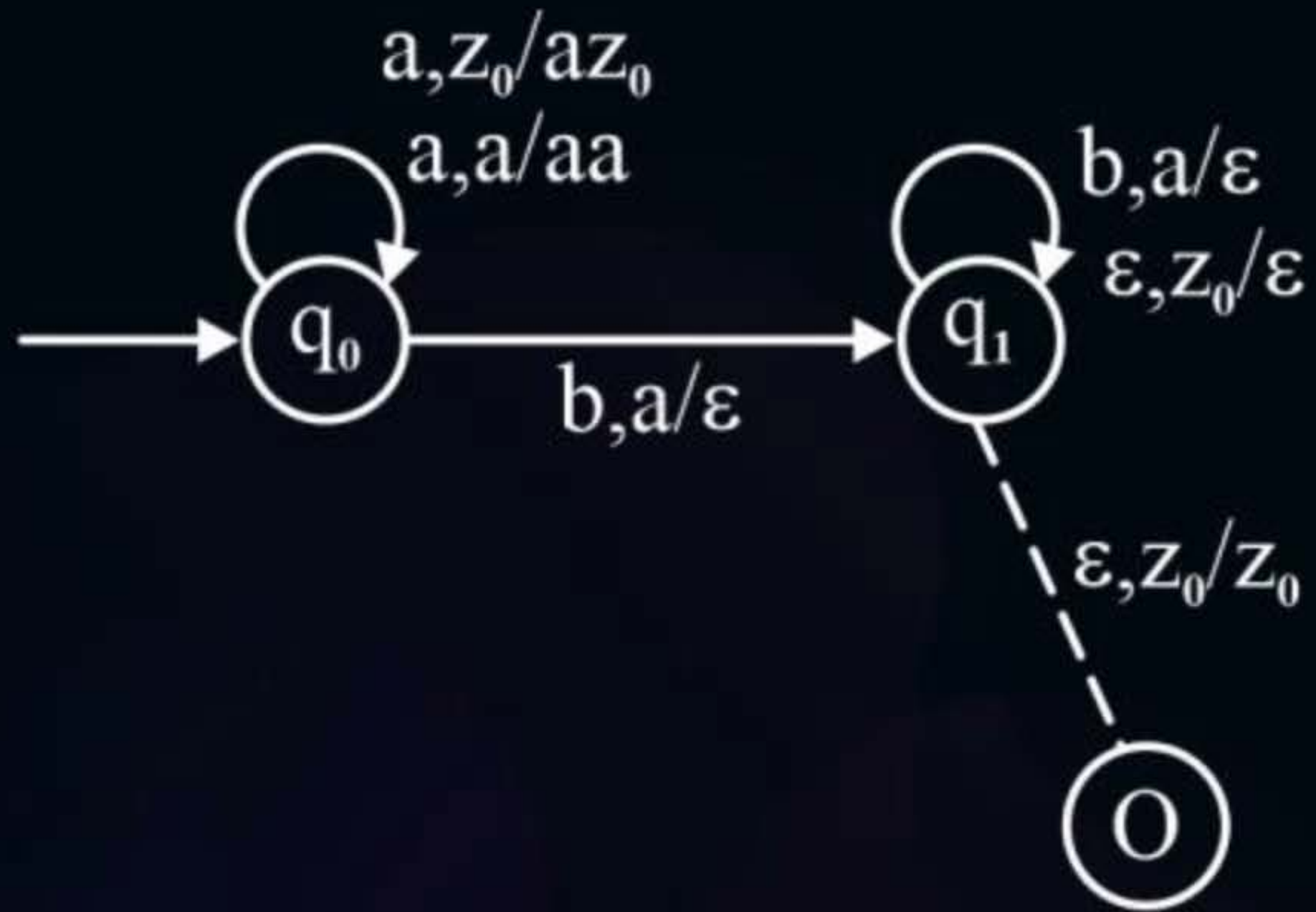


a's Push $\left[\begin{array}{l} S(q_0, a, z_0) = (q_0, az_0) \\ S(q_0, a, a) = (q_0, aa) \end{array} \right.$

b's PED $\left[\begin{array}{l} S(q_0, b, a) = (q_1, \epsilon) \\ S(q_1, b, a) = (q_1, \epsilon) \end{array} \right.$

accepted $\left[S(q_0, \epsilon, z_0) = (q_1, \epsilon) \right.$

Empty stack





Topic : Note

Note:- By reading the input string by the end of the string stack is non empty or starting is not ended is-

Whenever m/c is halted then that i/p is rejected.

- the input is valid only string is ended end 2 not be in there stack.
- In final state mechanism i/p is valid only when automata enters into final state whenever m/c is halted.



Topic : Context Free Language

10. $\{a^n b^n / n \geq 13\}$

11. $L = \{a^n b^m / n \leq m^2\}$

12. $\{a^n b^m c^{n+m} / n, m \geq 1\}$

13. $L = \{L = a^n b^{n+m} / n, m = 1\}$

$$14. \quad L = \{a^{m^2} b^{n^3} c^{k^5} / n, n, k > 1\}$$

$$15. \quad L = \{a^{3^n} b^{5^k} c^{2 \leq \ell} / n, k, \ell \geq 1\}$$

$$16. \quad L = \{a^i b^j c^k / j = i + k\}$$

$$17. \quad L = \{a^i b^j c^k / i > j \text{ or } j < k\}$$



Topic : Context Free Language

27. $L = \{a^i b^j / (i + j) \bmod 5 = 0\}$

28. $\{a^{2^n} / n \geq 1\}$

29. $\{a^{n^2} / n \geq 1\}$

30. $L = \{1^{2n+1} / n \geq 1\}$

31. $L = \{a^p / p \text{ is prime number}\}$

32. $L = \{a^k / k \text{ is odd number}\}$



Topic : Context Free Language

33. $L = \{wxw / w \in \{a, b\}^*\}$

34. $L = \{wxw / w, x \in \{a, b\}^*\}$

35. $ww^R x / \{w, x, \in \{a, b\}^+\}$

36. $\{\Sigma^* - \{ww / w \in \{a, b\}^+\}$

37. $\{ww^R w / w \in \{a, b\}^+\}$

38. $L = \{ww^R ww^R / w \in \{a, b\}^+\}$



2 mins Summary



Topic

One

Topic

Two

Topic

Three

Topic

Four

Topic

Five



THANK - YOU