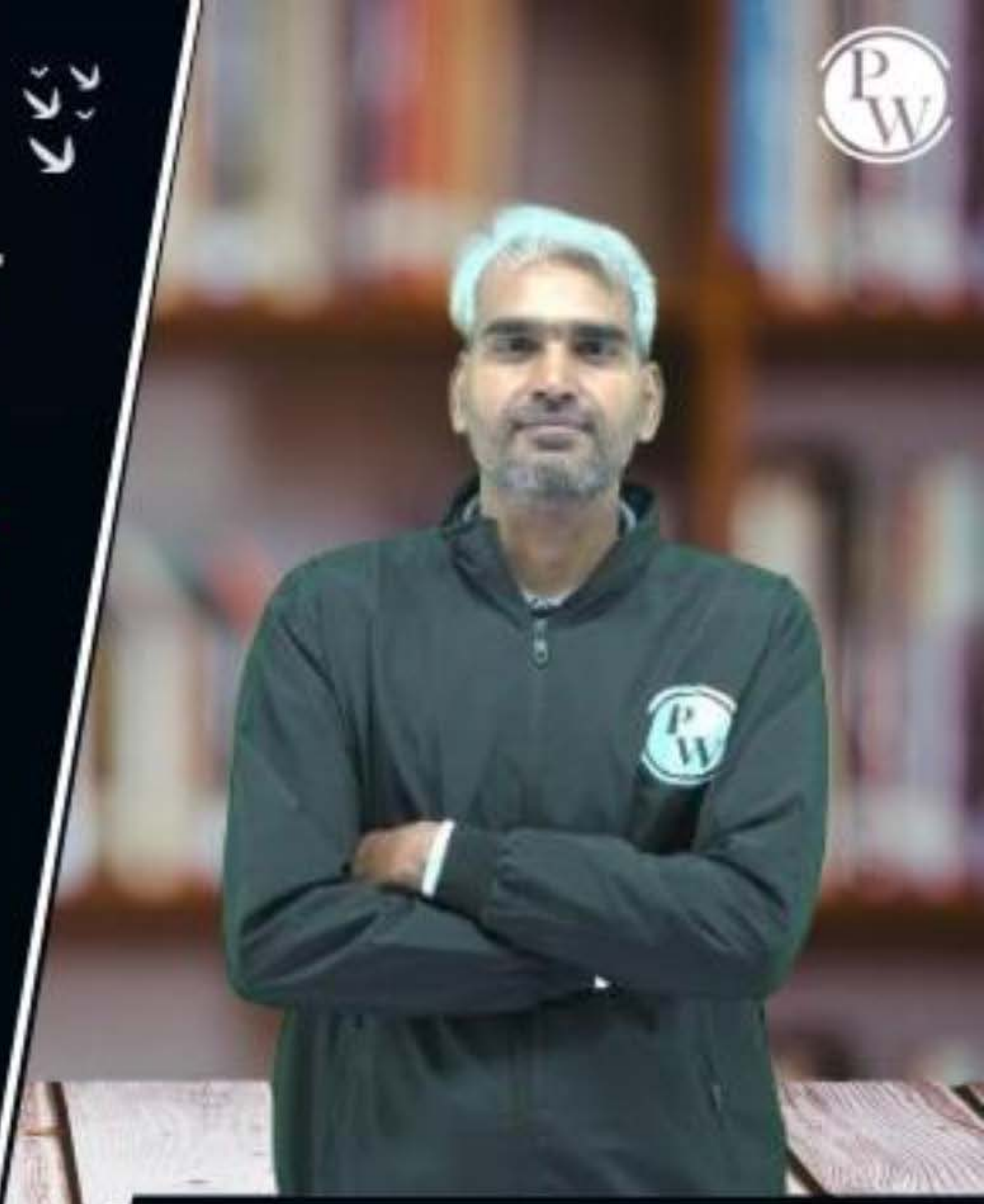
Computer Science & IT

Discrete Mathematics

Graph Theory

Lecture No. 04



By- Vishal Sir

Recap of Previous Lecture

- **Topic** Different types of graphs
  - Bipartite graph
  - Complete Bipartite graph
- **Topic** Sum of degree theorem
- **Topic** Degree Sequence

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Topics to be Covered

- Topic: Degree Sequence
- Topic: Havel Hakimi's algorithm
- Topic: Complement of a graph
- Topic: Graph isomorphism

Slide

Topic : Degree sequence

In a graph G if degrees of all the vertices are arranged in non-increasing or non-decreasing order, then it is called degree sequence of graph G.

H.W.

#Q. Which of the following degree sequences represent a simple non-directed graph?

$\boxed{2+3+3+4+4+5 = \text{odd}}$

No. of vertices with odd degree are 'odd'
∴ it can not represent any graph
{Simple (or) Not simple}

1. ✗ {2, 3, 3, 4, 4, 5}
2. {2, 3, 4, 4, 5}
3. {1, 3, 3, 3}
4. {0, 1, 2, 3,............,n-1}
5. {1, 3, 3, 4, 5, 6, 6}
6. {3, 3, 3, 3, 2}

H.W.

#Q. Which of the following degree sequences represent a simple non-directed graph?

In a simple graph with n-vertices degree of each vertex $\leq (n-1)$

1. {2, 3, 3, 4, 4, 5}
2. ✗ {2, 3, 4, 4, 5}

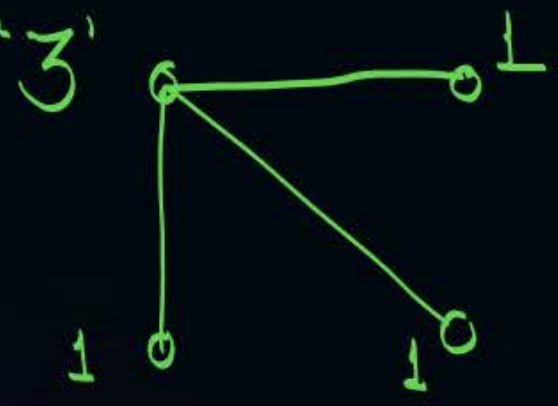
∴ In a simple graph with '5' vertices degree of each vertex $\leq (5-1) = 4$

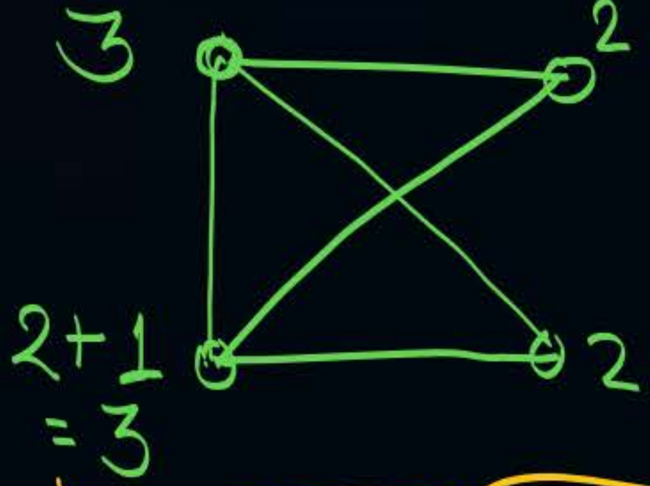
∴ Vertex of degree '5' is not possible

3. {1, 3, 3, 3}
4. {0, 1, 2, 3,............,n-1}
5. {1, 3, 3, 4, 5, 6, 6}
6. {3, 3, 3, 3, 2}

H.W.

#Q. Which of the following degree sequences represent a simple non-directed graph?





1. × {2, 3, 3, 4, 4, 5}
2. × {2, 3, 4, 4, 5}
3. × $\{1, \underbrace{3, 3, 3}\}$ Total vertices = 4; we have three vertices of degree = (4-1) = 3 ∴ Vertex of deg = 1 is not possible
4. {0, 1, 2, 3, ..., n-1}
5. {1, 3, 3, 4, 5, 6, 6}
6. {3, 3, 3, 3, 2}

n=4, if degree of any vertex in a simple graph is (n-1) = 4-1 = 3 then degree of each vertex is at least one

If there are two vertices with degree = n-1 = (4-1) = 3, (in a simple graph) then degree of each vertex will be at least '2'

$$\boxed{\begin{array}{l}\text{• In a simple graph } G \text{ with } n\text{-vertices.} \\ \text{if there are 'K' vertices of degree} = (n-1). \\ \text{then degree of each vertex in that graph is at least 'K'}\end{array}}$$

H.W.

#Q. Which of the following degree sequences represent a simple non-directed graph?

1. ✗ {2, 3, 3, 4, 4, 5}
2. ✗ {2, 3, 4, 4, 5}
3. ✗ {1, 3, 3, 3}
4. ✗ {0, 1, 2, 3,............,n-1}
5. {1, 3, 3, 4, 5, 6, 6}
6. {3, 3, 3, 3, 2}

Note:- In a simple graph there must be at least two vertices with same degree {degrees of all the vertices can not be distinct}

Total no. of vertices = n

one vertex with degree = (n-1)

∴ Vertex of deg = 0 is not possible in a simple graph

Note: If graph is not a simple graph, then all vertices may have distinct degrees.

eg.

H.W.

#Q. Which of the following degree sequences represent a simple non-directed graph?

1. ✗ {2, 3, 3, 4, 4, 5}
2. ✗ {2, 3, 4, 4, 5}
3. ✗ {1, 3, 3, 3}
4. ✗ {0, 1, 2, 3,............,n-1}
5. ✗ {1, 3, 3, 4, 5, 6, 6}
6. {3, 3, 3, 3, 2}

Total no. of vertices = 7

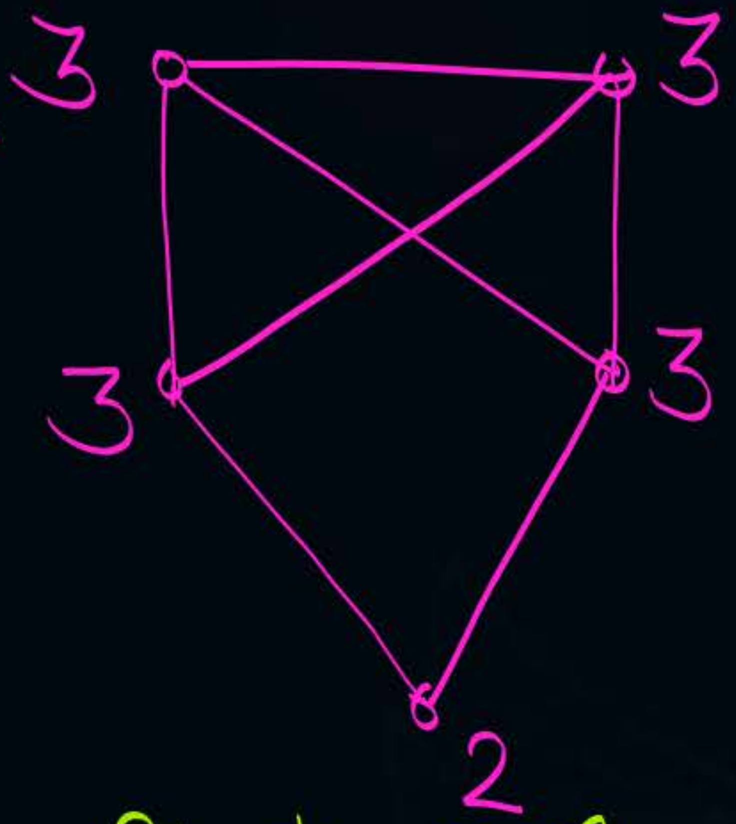
Two vertices of degree = $(n-1) = (7-1) = 6$

$\therefore$ vertex of deg = 1 is not possible in a simple graph

H.W.

#Q. Which of the following degree sequences represent a simple non-directed graph?

1. {2, 3, 3, 4, 4, 5}
2. {2, 3, 4, 4, 5}
3. {1, 3, 3, 3}
4. {0, 1, 2, 3,…………,n-1}
5. {1, 3, 3, 4, 5, 6, 6}
6. {3, 3, 3, 3, 2}



simple graph with degree sequence {3,3,3,3,2} is possible

Simple graph

deg. seq = {3,3,3,3,2}

Topic : Havel Hakimi's Result

Consider the following degree sequence in non-increasing order.

Seq 1: $\{S, t_1, t_2, t_3, \ldots, t_S, d_1, d_2, \ldots d_k\}$ {S, t_1, t_2, t_3, ... etc. are the degrees of different vertices}

To reduce the size of the problem, delete the first degree from the degree sequence {i.e. delete 'S'} and subtract '1' from the next 'S' degrees in the sequence

After deletion of 'S'

Seq 2: $\{(t_1-1), (t_2-1), (t_3-1), \ldots, (t_S-1), d_1, d_2, d_3, \ldots d_k\}$

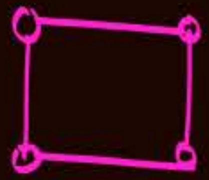
"Seq 1" can represent a simple non-directed graph if and only if "Seq 2" Can represent a simple non-directed graph.

If not able to visualize the graph w.r.t. "Seq 2" as well, then re-arrange "seq 2" in non-increasing order and repeat the process on "seq 2" to obtain "seq 3" {keep repeating until problem size is reduced sufficiently that you can visualize the graph}

Q: Check whether deg sequence $\{3, 3, 3, 3, 2\}$ can represent a simple non-directed graph or not

Sol^n

Seq 1 : $\{\cancel{3}, \underbrace{3, 3, 3}_{-1}, 2\} \equiv$  $= \{3, 3, 3, 3, 2\}$

Seq 2' : $= \{\cancel{2}, \underbrace{2, 2}_{-1}, 2\} \equiv$  $= \{2, 2, 2, 2\}$

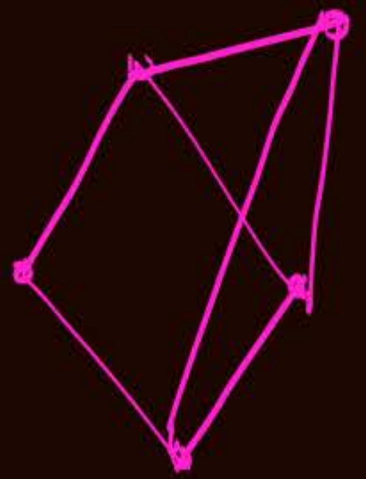
{Already in non-increasing order}

"Seq 3" $= \{1, 1, 2\} \equiv$  $= \{2, 1, 1\}$

rearranged "Seq 3" $= \{\cancel{2}, \underbrace{1, 1}_{-1}\}$

"Seq 4" $= \{0, 0\} \equiv$  $= \{0, 0\}$

two isolated vertices



#Q. Which of the following degree sequences represent a simple non directed graph?

S1 = {6, 6, 6, 6, 4,3,3,0}

S2 = {6, 5, 5, 4, 3, 3, 2, 2,2}

A) Only S1

B) Only S2

C) Both S1 and S2

D) Neither S1 nor S2

$\text{Seq } 1 = \{\cancel{6}, \underbrace{6, 6, 6, 4, 3, 3}, 0\}$

$\text{Seq } 2 = \{\cancel{5}, \underbrace{5, 5, 3, 2, 2}, 0\}$

$\text{Seq } 3 = \{\cancel{4}, \underbrace{4, 2, 1, 1}, 0\}$

$\text{Seq } 4 = \{3, 1, 0, 0, 0\}$

Not simple

{∴ "S_1" does not represent any simple non-directed graph}

#Q. Which of the following degree sequences represent a simple non directed graph?

S1 = {6, 6, 6, 6, 4,3,3,0}

S2 = {6, 5, 5, 4, 3, 3, 2, 2,2}

A Only S1

B Only S2

C Both S1 and S2

D Neither S1 nor S2

$\text{Seq } 1 = \{\cancel{6}, \underbrace{5, 5, 4, 3, 3, 2}, 2, 2\}$

$\text{Seq } 2 = \{4, 4, 3, 2, 2, 1, 2, 2\}$

Rearranges

$\text{Seq } 2 = \{\cancel{4}, \underbrace{4, 3, 2, 2}, 2, 2, 1\}$

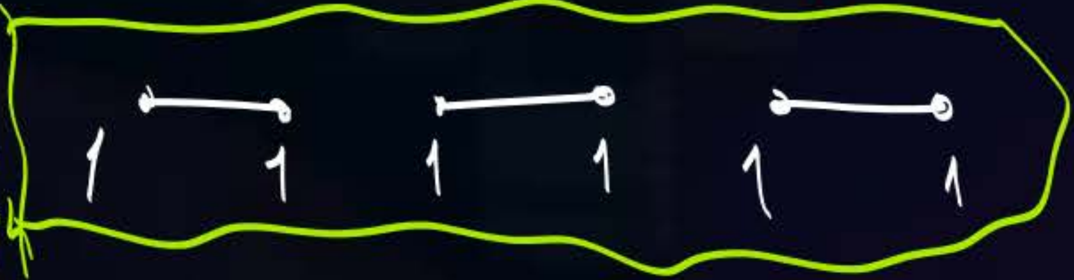
$\text{Seq } 3 = \{3, 2, 1, 1, 2, 2, 1\}$

Re-arranged

$\text{Seq } 3 = \{\cancel{3}, \underbrace{2, 2, 2}, 1, 1, 1\}$

$\text{Seq } 4 = \{1, 1, 1, 1, 1, 1\}$

final sequence represent a simple non-directed graph



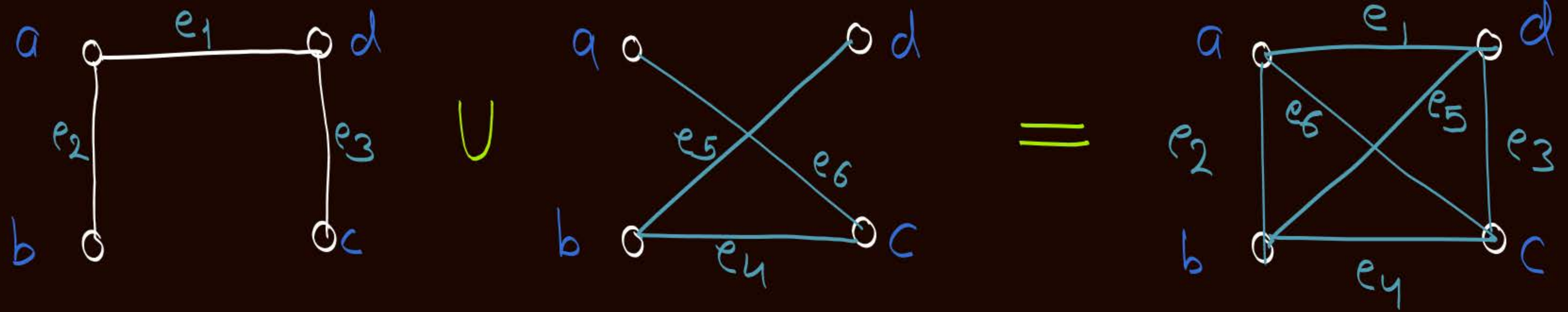
∴ S_2 represent a simple non-directed graph

Topic : Complement of a graph

{ Complement is defined Only for simple graphs }

Let G be a simple graph with n-vertices, then complement of graph G is a simple graph with same n-vertices as of G, but an edge is present in complement of graph G if and only if that edge is not present in G.

Complement of graph G is denoted by $\overline{G}$.



$G \cup \bar{G}$

$G \cup \bar{G} = \left\{ \begin{array}{c} \text{it is a} \\ \text{Complete} \\ \text{graph} \end{array} \right\}$

$G = (V, E_1)$

$V = \{a, b, c, d\}$

$E_1 = \{e_1, e_2, e_3\}$

$\bar{G} = (V, E_2)$

$V = \{a, b, c, d\}$

$E_2 = \{e_4, e_5, e_6\}$

$G \cup \bar{G} = (V \cup V, E_1 \cup E_2)$

$V \cup V = \{a, b, c, d\}$

$E_1 \cup E_2 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

Topic : Complement of a graph

- Let G be a simple graph with n- vertices, and $\bar{G}$ is Complement of graph G, then

$$\boxed{G \cup \bar{G} = K_n}$$

$$\boxed{E(G) \cap E(\bar{G}) = \phi}$$

$$\boxed{V(G) = V(\bar{G})}$$

$$|E(G)| + |E(\bar{G})| = |E(K_n)|$$

$$|E(G)| + |E(\bar{G})| = {}^{n}C_2 = \frac{n(n-1)}{2}$$

Q1: Let G be a simple graph with n-vertices & 21 edges, if there are 24 edges in the Complement of graph G. Find the number of vertices in graph G.?

$$|E(G)| + |E(\bar{G})| = {}^{n}C_2 = \frac{n(n-1)}{2}$$

$$21 + 24 = \frac{n(n-1)}{2}$$

$$90 = n(n-1)$$

$$\boxed{n = 10}$$

Topic : Graph Isomorphism

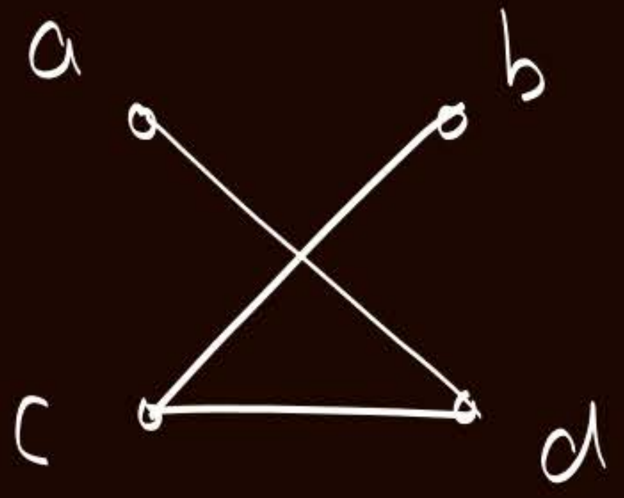
{ Two graphs are said to be isomorphic if they have exactly same properties }
{ Name of vertices & edges may be different }

Two graphs G and G' are said to be isomorphic iff there exists a function f: V(G)→V(G') such that

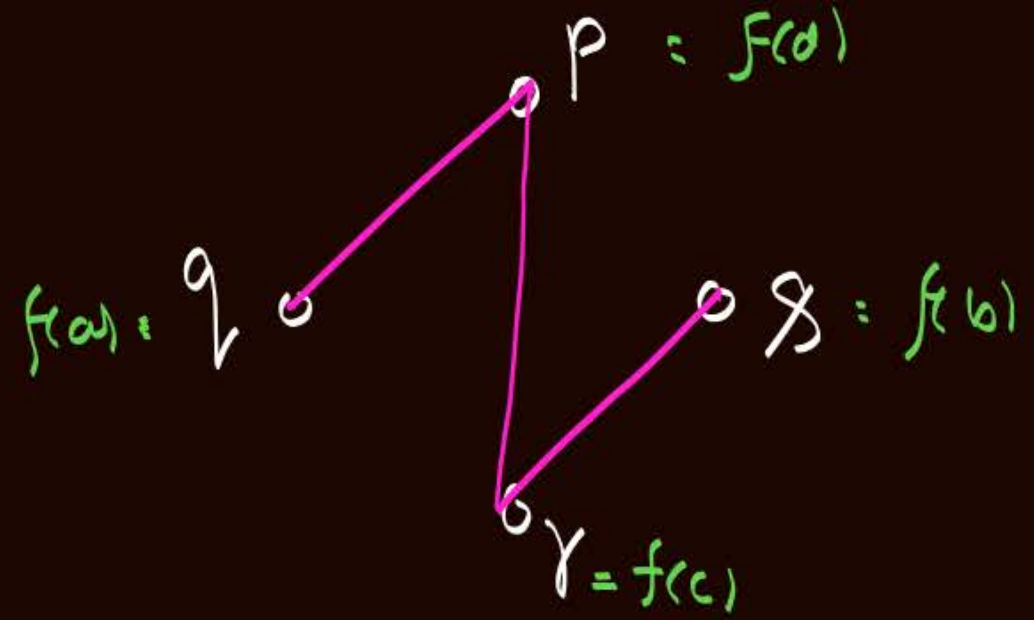
I. f is bijective { one-one + onto } $\Rightarrow |V(G)| = |V(G')|$

II. f preserves adjacency of vertices

If two vertices $a, b \in V(G)$ are adjacent to each other in graph G then their images in G', { i.e. $f(a), f(b) \in V(G')$ } should also be adjacent to each other in G'. $\Rightarrow |E(G)| = |E(G')|$



G



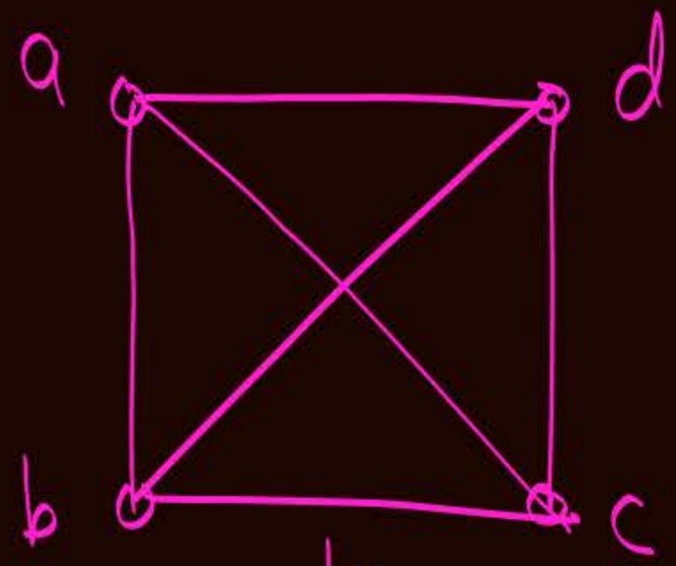
G'

let G & G'
are isomorphic

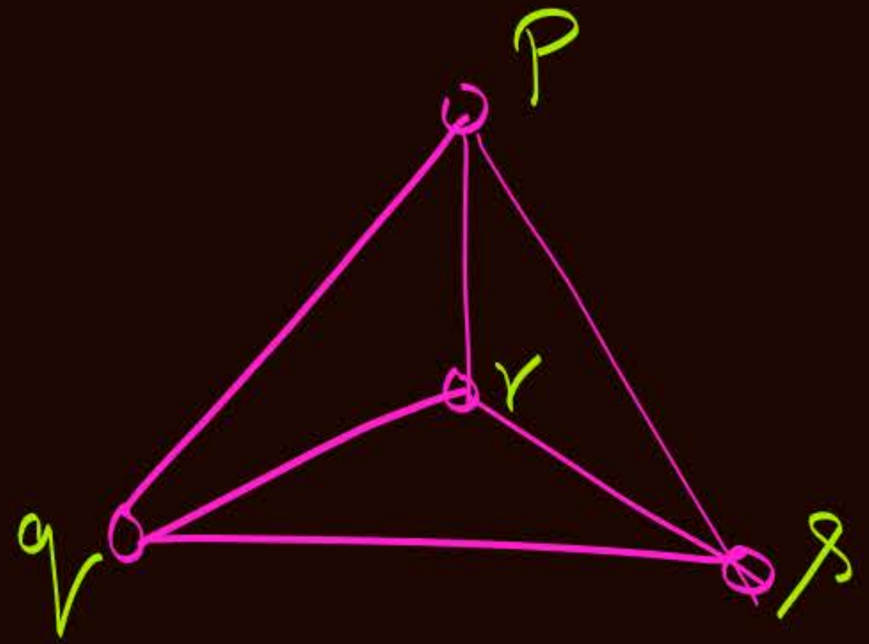
$\therefore |V(G)| = |V(G')|$

$\exists\ f : V(G) \to V(G')$
s.t. f is bijective.

Let $f(a) = q$
$f(b) = s$
$f(c) = r$
$f(d) = p$



K_4



W_4

K_4 & W_4 are isomorphic to each other

* If Graphs G & G' are isomorphic to each other then it is denoted by,

$$\boxed{G \cong G'}$$

2 mins Summary

Topic Degree Sequence

Topic Havel Hakimi’s algorithm

Topic Complement of a graph

Topic Graph isomorphism

THANK - YOU

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