GATE WALLAH

TOPICWISE

GATE PREVIOUS YEAR QUESTIONS

2008 – 2023

COMPUTER SCIENCE & IT

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PREFACE

A highly skilled professional team of GATE Wallah works arduously to ensure that the students receive the best content for their **GATE** exam.

A plethora of GATE Study Material is available in the market but GATE Wallah professionals at PW are continuously working to provide supreme quality study material for the GATE students.

From the beginning, the content team comprising Subject Matter Experts, Content Creators, Reviewers, DTP operators, Proofreaders, and others is involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

GATE Wallah Faculties have adopted a novel style of presenting the content in easy-to-understand language and have provided the content team with expert guidance and supervision throughout the creation and curation of this book.

PW's GATE Wallah strongly believes in conceptual and fun-based learning. GATE Wallah provides highly exam-oriented content to bring quality and clarity to the students.

This book adopts a multi-faceted approach to mastering and understanding the concepts by having a rich diversity of questions asked in the examination and equipping the students with the knowledge for this highly competitive exam.

The main objective of this book is to provide an edge to your preparation with high-quality content & video solutions.

BOOK FEATURES

This book, especially designed for GATE aspirants, contains

- a Topic-wise set of questions from the past 16 years (2008-2023)
- Embedded QR codes for Video Solutions (by expert faculties)
- Complete explanations to help you ace your exam
- Detailed Chapter Wise Analysis of PYQs

GATE Computer Science & IT Complete Syllabus

SECTION 1 : ENGINEERING MATHEMATICS

- **Discrete Mathematics:** Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.
- **Linear Algebra:** Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU
- decomposition.
- **Calculus:** Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.
- **Probability and Statistics:** Random variables. Uniform, normal, exponential, Poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem

SECTION 2 : DIGITAL LOGIC

- Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and
- computer arithmetic (fixed and floating point).

SECTION 3 : COMPUTER ORGANIZATION AND ARCHITECTURE

- Machine instructions and addressing modes. ALU, data ‑ path and control unit. Instruction pipelining, pipeline hazards. Memory hierarchy: cache, main memory and secondary storage; I/O interface (interrupt and DMA mode).

SECTION 4 : PROGRAMMING AND DATA STRUCTURES

- Programming in C. Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.

SECTION 5 : ALGORITHMS

- Searching, sorting, hashing. Asymptotic worst-case time and space complexity. Algorithm design techniques: greedy, dynamic programming and divide ‑ and ‑ conquer. Graph traversals, minimum spanning trees, shortest paths

SECTION 6: THEORY OF COMPUTATION

- Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and context-free languages, pumping lemma. Turing machines and undecidability.

SECTION 7: COMPILER DESIGN

- Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation. Local optimisation, Data flow analyses: constant propagation, liveness analysis, common subexpression elimination.

SECTION 8 : OPERATING SYSTEM

- System calls, processes, threads, inter ‐ process communication, concurrency and synchronization. Deadlock. CPU and I/O scheduling. Memory management and virtual memory. File systems.

SECTION 9 : DATABASES

- ER ‐ model. Relational model: relational algebra, tuple calculus, SQL. Integrity constraints, normal forms. File
- organization, indexing (e.g., B and B+ trees). Transactions and concurrency control.

SECTION 10 : COMPUTER NETWORKS

- Concept of layering: OSI and TCP/IP Protocol Stacks; Basics of packet, circuit and virtual circuit switching; Data link layer: framing, error detection, Medium Access Control, Ethernet bridging; Routing protocols: shortest path, flooding, distance vector and link state routing; Fragmentation and IP addressing, IPv4, CIDR notation, Basics of IP support protocols (ARP, DHCP, ICMP), Network Address Translation (NAT); Transport layer: flow control and congestion control, UDP, TCP, sockets; Application layer protocols: DNS, SMTP, HTTP, FTP, Email.

SPECIAL ABOUT THIS BOOK

We take pride in providing high-quality study material for the **GATE** exam. Our team at **GATE WALLAH** has meticulously prepared each solution to ensure they are error-free and easy to understand.

The **GATE** exam pattern has shifted in recent years, with an increased emphasis on topics that previously received fewer questions. At **GATE WALLAH**, we understand the importance of identifying these crucial topics, and that's why we've provided a chapter-wise analysis for the last 16 years. Our analysis will help you understand the significance of each chapter and focus your preparation accordingly. We believe that this will be a valuable resource in your journey to success in the **GATE** exam.

To ensure that you focus on the most relevant questions, we have arranged the questions in this book topic-wise and year-wise in descending order. This means that the questions from the most recent years are emphasized, as we believe they are the most relevant for your **GATE** exam preparation. By studying these questions, you'll gain a deeper understanding of the exam pattern and the type of questions asked in recent years, which will help you perform better in the upcoming **GATE** exam.

We've designed the text solutions in this book to be in sync with our video solutions recorded by expert faculties, making it easier for you to understand the concepts. We understand that every student has a unique learning style, and that's why we've included both text and video solutions to cater to your needs. By using our comprehensive guide and video solutions together, you'll gain a better understanding of the concepts and be better prepared to tackle the **GATE** exam.

Steps to Open Video solutions through mobile:

(1) Scan the given embedded QR Code for a particular solution.

(2) Visit the link generated & you'll be redirect to the video solution.

GATE

COMPUTER SCIENCE & IT

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Database Management System

Database Management System

Syllabus

ER - model. Relational model: relational algebra, tuple calculus, SQL. Integrity constraints, normal forms. File organization, indexing (e.g., B and B+ trees). Transactions and concurrency control.

Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5
2008	4	0	0	5	5
2009	0	2	0	6	2
2010	0	2	3	1	1
2011	0	1	0	4	0
2012	1	1	1	7	0
2013	2	2	0	2	1
2014(P1)	0	3	3	3	0
2014(P2)	0	2	2	4	0
2014(P3)	0	3	2	3	0
2015(P1)	2	0	0	4	2
2015(P2)	0	0	4	2	2
2015(P3)	0	1	0	4	2
2016(P1)	0	2	3	0	0
2016(P2)	0	0	3	2	1
2017(P1)	0	1	2	5	0
2017(P2)	1	1	2	2	2
2018	1	2	0	3	0
2019	0	2	2	2	0
2020	1	2	2	1	2
2021(P1)	0	2	2	3	0
2021(P2)	0	1	2	2	1
2022	0	3	2	4	0
2023	1	0	0	2	2

CHAPTER

1

DATABASE DESIGN AND ER MODEL

Introduction to RDBMS

1. **[MCQ]** **[GATE-2023: 1M]**

Which one of the options given below refers to the degree (or arity) of a relation in relational database systems?

(a) Number of attributes of its relation schema.
(b) Number of tuples stored in the relation.
(c) Number of entries in the relation.
(d) Number of distinct domains of its relation schema.

Key Concepts

2. **[NAT]** **[GATE-2014 : 1M]**

Given an instance of the STUDENTS relation as shown below:

Student ID	Student Name	Student Email	Student Age	CPI
2345	Shankar	shankar@math	X	9.4
1287	Swati	swati@ee	19	9.5
7853	Shankar	shankar@cse	19	9.4
9876	Swati	swati@mech	18	9.3
8765	Ganesh	ganesh@civil	19	8.7

For (Student Name, Student Age) to be a key for this instance, the value X should NOT be equal to ______.

3. **[MCQ]** **[GATE-2014 : 1M]**

A prime attribute of a relation scheme R is an attribute that appears

(a) in all candidate keys of R.
(b) in some candidate key of R.
(c) in a foreign key of R.
(d) only in the primary key of R.

4. **[MCQ]** **[GATE-2011 : 1M]**

Consider a relational table with a single record for each registered student with the following attributes.

1. Registration_Num: Unique registration number of each registered student

2. UID: Unique identity number, unique at the national level for each citizen.

3. Bank Account_Num: Unique account number at the bank. A student can have multiple accounts or joint accounts. This attribute stores the primary account number.

4. Name: Name of the student

5. Hostel_Room: Room number of the hostel

Which of the following option is INCORRECT?

(a) BankAccount_Num is a candidate key.
(b) Registration_Num can be a primary key.
(c) UID is a candidate key if all students are from the same country.
(d) If S is a super key such that S $\cap$ UID is NULL then S $\cup$ UID is also a super key.

Foreign Key Concepts

5. **[MCQ]** **[GATE-2021 : 1M]**

Consider the following statements S1 and S2 about the relational data model:

S1: A relation scheme can have at most one foreign key.

S2: A foreign key in a relation scheme R cannot be used to refer to tuples of R.

Which one of the following choices is correct?

(a) Both S1 and S2 are true
(b) S1 is true and S2 is false
(c) S1 is false and S2 is true
(d) Both S1 and S2 are false

6. [NAT] [GATE-2017 : 2M]

Consider the following tables T_1 and T_2.

In table T_1, P is the primary key and Q is the foreign key referencing R in table T_2 with on delete cascade and on-update cascade. In table T_2, R is the primary key and S is the foreign key referencing P in table T_1 with on-delete set NULL and on-update cascade. In order to delete record ⟨3, 8⟩ from table T_1, the number of additional records that need to be deleted from table T_1 is

T_1

P	Q
2	2
3	8
7	3
5	8
6	9
8	5
9	8

T_2

R	S
2	2
8	3
3	2
9	7
5	7
7	2

7. [MCQ] [GATE-2014 : 1M]

Given the following statements:

S1: A foreign key declaration can always be replaced by an equivalent check assertion in SQL.

S2: Given the table R(a, b, c) where a and b together form the primary key, the following is a valid table definition.

CREATE TABLE S

a INTEGER,

d INTEGER,

e INTEGER,

PRIMARY KEY (d),

FOREIGN KEY (a) references (R)

Which one of the following statements is CORRECT?

(a) S1 is TRUE and S2 is FALSE
(b) Both S1 and S2 are TRUE
(c) S1 is FALSE and S2 is TRUE
(d) Both S1 and S2 are FALSE

ER Model Concept

8. [MCQ] [GATE-2020 : 1M]

Which one of the following is used to represent the supporting many-one relationships of a weak entity set in an entity-relationship diagram?

(a) Diamonds with double/bold border.
(b) Ovals with double/bold border
(c) Ovals that contain underlined identifiers
(d) Rectangles with double/bold border

9. [MCQ] [GATE-2018 : 1M]

In an Entity-Relationship (ER) model, suppose R is a many-to-one relationship from entity set E_1 to entity set E_2. Assume that E_1 and E_2 participate totally in R and that the cardinality of E_1 is greater than the cardinality of E_2.

Which one of the following is true about R?

(a) Every entity in E_1 is associated with exactly one entity in E_2.
(b) Some entity in E_1 is associated with more than one entity in E_2.
(c) Every entity in E_2 is associated with exactly one entity in E_1.
(d) Every entity in E_2 is associated with at most one entity in E_1.

ER Model Concept and Conversion of ER Model to RDBMS

10. [MCQ] [GATE-2017 : 1M]

An ER model of a database consists of entity types A and B. These are connected by a relationship R which does not have its own attribute. Under which one of the following conditions, can the relational table for R be merged with that of A?

(a) Relationship R is one-to-many and the participation of A in R is total.
(b) Relationship R is one-to-many and the participation of A in R is partial.
(c) Relationship R is many-to-one and the participation of A in R is total.
(d) Relationship R is many-to-one and the participation of A in R is partial.

11. [NAT] [GATE-2015 : 2M]

Consider an Entity-Relationship (ER) model in which entity sets E_1 and E_2 are connected by an m: n relationship R_{12} . E_1 and E_3 are connected by a 1: n (1 on the side of E_1 and n on the side of E_3) relationship R_{13}.

E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute. E_2 has two single valued attributes a_{21} and a_{22} of which a_{21} is the key attribute. E_3 has two single valued attributes a_{31} and a_{32} of which a_{31} is the key attribute. The relationships do not have any attributes.

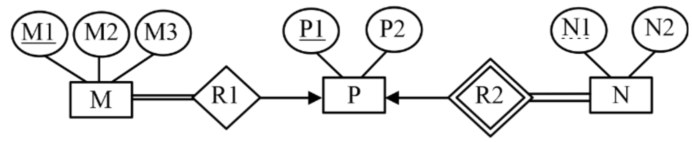
If a relational model is derived from the above ER model, then the minimum number of relations that would be generated if all the relations are in 3 NF is__________.

12. [MCQ] [GATE-2012 : 1M]

Given the basic ER and relational models, which of the following is INCORRECT?
(a) An attribute of an entity can have more than one value.
(b) An attribute of an entity can be composite.
(c) In a row of a relational table, an attribute can have more than one value.
(d) In a row of a relational table, an attribute can have exactly one value or a NULL value.

13. [MCQ] [GATE-2008 : 2M]

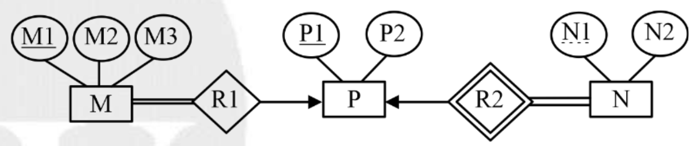
Consider the following ER diagram



The minimum number of tables needed to represent M, N, P R1, R2 is
(a) 2 (b) 3
(c) 4 (d) 5

14. [MCQ] [GATE-2008 : 2M]

Consider the following ER diagram



Which of the following is a correct attribute set for one of the tables for the minimum number of tables needed to represent M, N, P, R1 and R2
(a) {M1, M2, M3, P1}
(b) {M1, P1, N1, N2}
(c) {M1, P1, N1}
(d) {M1, P1}

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ANSWER KEY

1.	(a)	**2.**	(19 to 19)	**3.**	(b)	**4.**	(a)
5.	(d)	**6.**	(0 to 0)	**7.**	(d)	**8.**	(a)
9.	(a)	**10.**	(c)	**11.**	(4 to 4)	**12.**	(c)
13.	(b)	**14.**	(a)				

SOLUTIONS

1. (a)

Arity (Degree): Number of Attributes/fields

Cardinality: Number of Tuples/Records

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2. (19 to 19)

In order for Student Name, Student Age to be the key, it must be unique, therefore the value of X $\neq$ 19 satisfies the criteria of Student Name, Student Age being the key.

Hence, 19 is the answer.

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3. (b)

Prime attribute is a attribute that is present in some candidate key of a relation.

Example: Consider for relation R(ABCDE) if the candidate keys are AC, and BC, then the set of prime attributes are {A, B, C}.

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4. (a)

(a) BankAccount_Num is a candidate key: Incorrect.

Because a student can have multiple account and **joint account**.

Example

Assume 2 person Ramesh and Suresh have joint account they both have some account number in a primary account number. So, Bank account number can not uniquely determine and Bank account cannot be candidate key.

(b) Registration number can be primary key: True. Because registration number is a unique for each registered student

So, it can be uniquely determining each tuple of relation.

(c) UID is a candidate if all student is from **same country**: correct.

Because UID is unique at **National level**

Example

Indian Aadhar card is unique for each person in India.

(d) If S is a super key, then S $\cup$ UID is also super key: correct

Because any super set of super keys is also super key.

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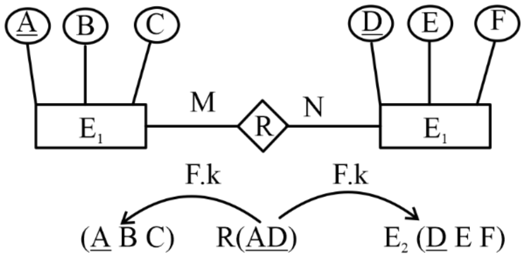
5. (d)



2 Foreign key

Employee

F.k

eid	ename	supervisor ID
E_1	A	E_2
E_2	B	E_3
E_3	C	E_4
E_4	D	E_2

- A relation scheme can have zero or more foreign key.
- A foreign key in a relation scheme R can be used to refer to tuples of R.

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6. (0 to 0)

<u>On delete cascade:</u> Whenever primary key is deleted from referenced (parent) table then that related value (Tuple/Row) (that primary key value) from referencing (CHILD table) has to be deleted.

<u>On delete set NULL:</u> Whenever primary key is deleted from reference (parent) table then that related value (Tuple/Row) in referencing [CHILD table (which contain foreign key)] is set to NULL.

By default foreign key references to the primary key of referenced relation.

On Delete set NULL
On update cascade

F.k

T_1

P	Q
2	2
3	**8**
7	3
5	8
6	9
8	5
9	8

T_2

R	S
2	2
8	~~3~~
3	2
9	7
5	7
7	2

For foreign key S, Referenced relation is T_1.

NULL

T_1: On delete cascade & On update cascade

T_2: On delete set NULL & On update cascade

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7. (d)

- Statement S1 is false as a foreign key declaration cannot be replaced by check assertion in SQL, because foreign key may have duplicate and Null values and having ON DELETE cascade operation then if any primary key deleted from referential relation then the referential integrity constraint may not be maintained.
- Statement S2: R(a b c)

  Here ab is primary key

  S(a d e)

  Primary key : d, foreign key a references R, foreign key reference to the primary key of referenced relation R. But in referenced relation ab is primary key, only a is not a primary key so here foreign key 'a' not references key (ab) of referenced relation R. therefore S2 is false.

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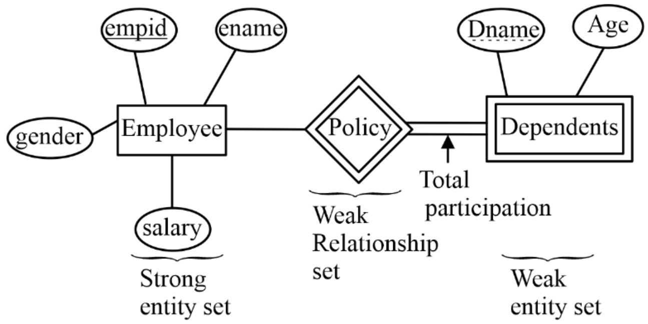
8. (a)



The relationship between strong entity set and weak entity set is represented by double/bold diamond.

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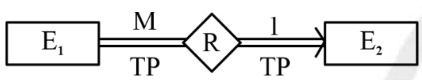
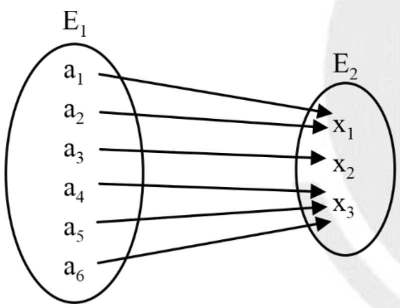
9. (a)



Cardinality of E_1 > Cardinality of E_2



(a) **Correct.** Every entity of E_1 is associated with exactly one entity in E_2.
($\because$ its total participation and many to one from E_1 to E_2).

(b) **Incorrect.** Because mention some entity in E_1 but here total participation and mention more than one entity in E_2 but here many to one (of E_2 side).

(c) **Incorrect.** Because there is a many to one from E_1 to E_2, not from E_2 to E_1.

(d) **Incorrect.** Because there is a many to one from E_1 to E_2.

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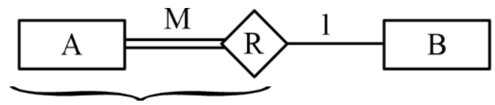
10. (c)



Relationship R is many to one and participation of A in R is total.

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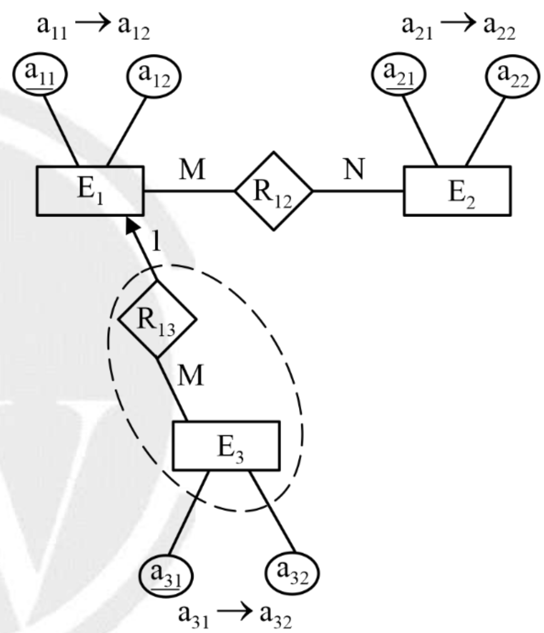
11. (4 to 4)



R_{12} is many to many relationships between E_1 and E_2. So, separate table is created

E_1 ($\underline{a_{11}}$ a_{12}) E_2 ($\underline{a_{21}}$ a_{22}) R_{12} ($\underline{a_{11}\ a_{21}}$)

$a_{11} \rightarrow a_{12}$ $a_{21} \rightarrow a_{22}$

E_3 R_{13}($\underline{a_{31}}$ a_{32} a_{11}) $a_{31} \rightarrow a_{32}a_{11}$

3NF (Third normal form): Let R be the relational schema and $X \rightarrow Y$ is non trivial FD is in 3NF

If X: super key

or

Y: prime attribute

E_1 ($\underline{a_{11}}$ a_{12}) $a_{11} \rightarrow a_{12}$

E_2 ($\underline{a_{21}}$ a_{22}) $a_{21} \rightarrow a_{22}$

R_{12} $(\underline{a_{11}a_{21}})$

E_3R_{13} $(\underline{a_{31}}\ a_{32}\ a_{11})a_{31} \rightarrow a_{32}a_{11}$

X is a super key

So, R is in 3NF

Minimum 4 table required

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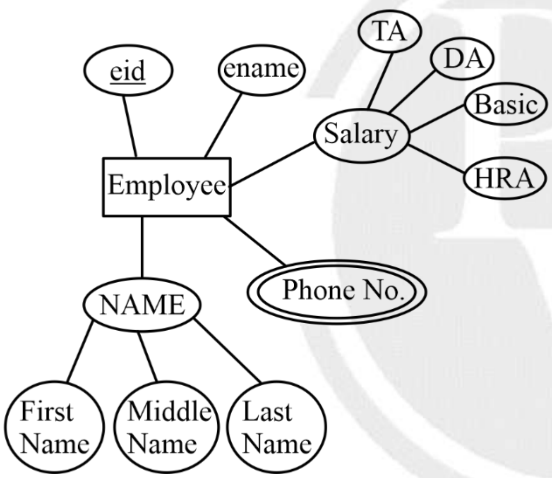
12. (c)

(a) Statement given is correct with respect to ER diagram.

(b) Statement given is correct with respect to ER diagram.



(c) In ER diagram multivalued attribute and composite attribute both are allowed, but in RDBMS Multivalued attribute and composite attribute are not allowed.

Each Tuple in RDBMS can have only one value or NULL for each field (Attribute exactly)

So, option (c) is incorrect.

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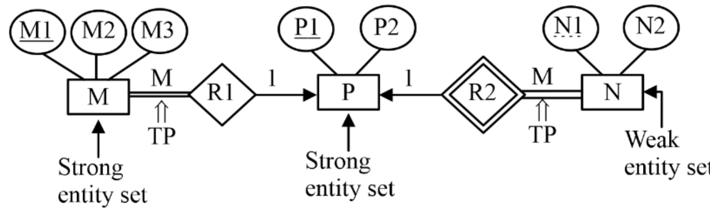
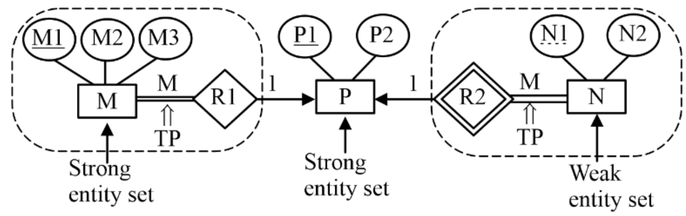
13. (b)





MR1P $\Rightarrow$ many to one and total participation at many side

MR1($\underline{M1}$ M2 M3 $\underline{P1}$) , here P1 is a foreign key

P($\underline{P1}$ P2)

NR2($\underline{N1}$ N2 $\underline{P1}$) , here P1 is a foreign key

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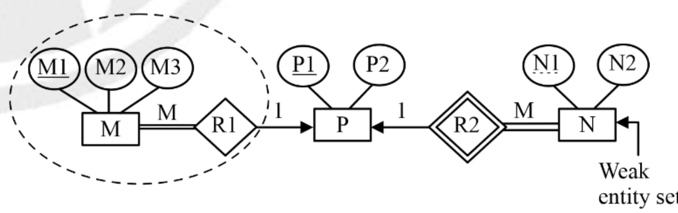
14. (a)



MR1($\underline{M1}$ M2 M3 $\underline{P1}$), here P1 is a foreign key

P($\underline{P1}$ P2), P1 is primary key

NR2($\underline{N1}$N2$\underline{P1}$), here P1 is a foreign key

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CHAPTER

2

FUNCTIONAL DEPENDENCY AND NORMALIZATION

Finding Number of Candidate Key

1. [NAT] [GATE-2022 : 1M]

Consider a relation R(A, B, C, D, E) with the following three functional dependencies.

$AB \rightarrow C$; $BC \rightarrow D$; $C \rightarrow E$;

The number of super keys in the relation R is _________.

2. [MCQ] [GATE-2016 : 1M]

Which of the following is NOT a super key in a relational schema with attributes V, W, X, Y, Z and primary key VY ?

(a) V X Y Z
(b) V W X Z
(c) V W X Y
(d) V W X Y Z

3. [NAT] [GATE-2014 : 1M]

The maximum number of superkeys for the relation schema R (E, F, G, H) with E as the key is ______________.

4. [MCQ] [GATE-2014 : 1M]

Consider the relation scheme R = (E, F, G, H, I, J, K, L, M, N) and the set of functional dependencies

$\{\{E, F\}\} \rightarrow \{G\}, \{F\} \rightarrow \{I, J\}, \{E, H\} \rightarrow \{K, L\}, \{K\} \rightarrow \{M\}, \{L\} \rightarrow \{N\}\}$ on R.

What is the key for R?

(a) {E, F}
(b) {E, F, H}
(c) {E, F, H, K, L}
(d) {E}

5. [MCQ] [GATE-2013 : 2M]

Relation R has eight attributes ABCDEFGH.

Fields of R contain only atomic values.

$F = \{CH \rightarrow G, A \rightarrow BC, B \rightarrow CFH, E \rightarrow A, F \rightarrow EG\}$ is a set of functional

dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R.

How many candidate keys does the relation R have?

(a) 3
(b) 4
(c) 5
(d) 6

Membership Set and Equality between FD Sets

6. [MSQ] [GATE-2021 : 2M]

Suppose the following functional dependencies hold on a relation U with attributes P, Q, R, S, and T:

- $P \rightarrow QR$
- $RS \rightarrow T$

Which of the following functional dependencies can be inferred from the above functional dependencies?

(a) $PS \rightarrow T$
(b) $R \rightarrow T$
(c) $P \rightarrow R$
(d) $PS \rightarrow Q$

7. [MCQ] [GATE-2015 : 1M]

Consider the relation X(P, Q, R, S, T, U) with the following set of functional dependencies

$F = \{ \{P, R\} \rightarrow \{S, T\}; \{P, S, U\} \rightarrow \{Q, R\};\}$

Which of the following is the trivial functional dependency in F^+ is closure of F ?

(a) $\{P, R\} \rightarrow \{S, T\}$
(b) $\{P, R\} \rightarrow \{R, T\}$
(c) $\{P, S\} \rightarrow \{S\}$
(d) $\{P, S, U\} \rightarrow \{Q\}$

Minimal Cover

8. [MCQ] [GATE-2017 : 2M]

The following functional dependencies hold true for the relational schema R {V, W, X, Y, Z}:

[$V \rightarrow W$; $VW \rightarrow X$; $Y \rightarrow VX$; $Y \rightarrow Z$]

Which of the following is irreducible equivalent for this set of functional dependencies?

(a) $V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow Z$

(b) $V \rightarrow W$, $W \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow Z$

(c) $V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

(d) $V \rightarrow W$, $W \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

Properties of Decomposition

9. [MCQ] [GATE-2021 : 2M]

Consider the relation R(P, Q, S, T, X, Y, Z, W) with the following functional dependencies

$PQ \rightarrow X$; $P \rightarrow YX$; $Q \rightarrow Y$; $Y \rightarrow ZW$

Consider the decomposition of the relation R into the constituent relations according to the following two decomposition schemes

D_1: R = [(P, Q, S, T); (P, T, X); (Q, Y); (Y, Z, W)]

D_2: R = [(P, Q, S); (T, X); (Q, Y); (Y, Z, W)]

Which one of the following options is correct?

(a) D_1 is a lossless decomposition, but D_2 is a lossy decomposition.

(b) D_1 is a lossy decomposition, but D_2 is a lossless decomposition.

(c) Both D_1 and D_2 are lossless decompositions.

(d) Both D_1 and D_2 are lossy decompositions.

10. [MCQ] [GATE-2019 : 2M]

Let the set of functional dependencies F = {$QR \rightarrow S$, $R \rightarrow P$, $S \rightarrow Q$} hold on a relation schema X = (PQRS). X is not in BCNF. Suppose X is decomposed into two schemas Y and Z, where Y = (P R) and Z = (Q R S).

Consider the two statements given below.

I. Both Y and Z are in BCNF

II. Decomposition of X into Y and Z is dependency preserving and lossless

Which of the above statements is/are correct?

(a) Both I and II
(b) I only
(c) II only
(d) Neither I nor II

11. [MCQ] [GATE-2008 : 2M]

Let R(A, B, C, D) be a relational schema with the following functional dependencies:

$A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$ and $D \rightarrow B$

The decomposition of R into (A, B),(B, C), (B, D)

(a) Gives a lossless join, and is dependency preserving

(b) Gives a lossless join, but is not dependency preserving

(c) Does not give a lossless join, but is dependency preserving

(d) Does not give a lossless join and is not dependency preserving.

Normal Form

12. [MCQ] [GATE-2022 : 1M]

In a relational data model, which one of the following statements is TRUE?

(a) A relation with only two attributes is always in BCNF.

(b) If all attributes of a relation are prime attributes, then the relation is in BCNF.

(c) Every relation has at least one non-prime attribute.

(d) BCNF decompositions preserve functional dependencies.

13. [MCQ] [GATE-2020 : 2M]

Consider a relational table R that is in 3 NF, but not in BCNF. Which one of the following statements is TRUE?

(a) R has a non-trivial functional dependency $X \rightarrow A$, where X is not a superkey and A is a prime attribute.
(b) R has a non-trivial functional dependency $X \rightarrow A$, where X is not a superkey and A is a non-prime attribute and X is not a proper subset of any key.
(c) R has a non-trivial functional dependency $X \rightarrow A$, where X is not a superkey and A is a non-prime attribute and X is a proper subset of some key.
(d) A cell in R holds a set instead of an atomic value.

14. [MCQ] [GATE-2018 : 2M]

Consider the following four relational schemas. For each schema, all non-trivial functional dependencies are listed. The underlined attributes are the respective primary keys.

Schema I:

Registration (<u>rollno</u>, courses)

Field 'courses' is a set-valued attribute containing the set of courses a student has registered for.

Non-trivial functional dependency:

rollno $\rightarrow$ courses

Schema II:

Registration (<u>rollno</u>, <u>courseid</u>, email)

Non-trivial functional dependencies:

rollno, courseid $\rightarrow$ email

email $\rightarrow$ rollno

Schema III:

Registration (<u>rollno</u>, <u>courseid</u>, marks, grade)

Non-trivial functional dependencies:

rollno, courseid $\rightarrow$ marks, grade

marks $\rightarrow$ grade

Schema IV:

Registration (<u>rollno, courseid</u>, credit)

Non-trivial functional dependencies:

rollno, courseid $\rightarrow$ credit

courseid $\rightarrow$ credit

Which one of the relational schemas above is in 3 NF but not in BCNF?

(a) Schema I (b) Schema II
(c) Schema III (d) Schema IV

15. [MCQ] [GATE-2014 : 2M]

Given the following two statements:

S1: Every table with two single-valued attributes is in 1 NF, 2 NF, 3 NF and BCNF.

S2: $AB \rightarrow C$, $D \rightarrow E$, $E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C$, $D \rightarrow E$, $AB \rightarrow E$, $E \rightarrow C$.

Which one of the following is CORRECT?

(a) S1 is TRUE and S2 is FALSE.
(b) Both S1 and S2 are TRUE.
(c) S1 is FALSE and S2 TRUE.
(d) Both S1 and S2 are FALSE.

16. [MCQ] [GATE-2013 : 2M]

Relation R has eight attributes ABCDEFGH.

Fields of R contain only atomic values.

F = {$CH \rightarrow G$, $A \rightarrow BC$, $B \rightarrow CFH$, $E \rightarrow A$, $F \rightarrow EG$} is a set of functional dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R.

The relation R is

(a) in 1 NF, but not in 2 NF.
(b) in 2 NF, but not in 3 NF.
(c) in 3 NF, but not in BCNF.
(d) in BCNF.

17. [MCQ] [GATE-2012 : 1M]

Which of the following is TRUE?

(a) Every relation in 3 NF is also in BCNF
(b) A relation R is in 3 NF if every non-prime attribute of R is fully functionally dependent on every key of R
(c) Every relation in BCNF is also in 3 NF
(d) No relation can be in both BCNF and 3 NF

18. [MCQ] [GATE-2009 : 2M]

Consider the following relational schema:

Suppliers(sid: integer, sname: string, city: string, street: string)

Parts(pid: integer, pname:string, color:string)

Catalog (sid:integer, pid:integer, cost:real)

Assume that, in the suppliers relation above, each supplier and each street within a city has a unique name, and (sname, city) forms a candidate key. No other functional dependencies are implied other than those implied by primary and candidate keys. Which one of the following is TRUE about the above schema?

(a) The schema is in BCNF
(b) The schema is in 3NF but not in BCNF
(c) The schema is in 2NF but not in 3NF
(d) The schema is not in 2NF

19. [MCQ] [GATE-2008 : 2M]

Consider the following relational schemes for a library database:

Book (Title, Author, Catalog_no, Publisher, Year, price)

Collection (Title, Author, Catalog_no)

With the following functional dependencies:

I. TitleAuthor → Catalog_no

II. Catalog_no → Title Author Publisher Year

III. Publisher Title Year → Price

Assume { Author, Title} is the key for both schemes.

Which of the following statements is true?

(a) Both Book and Collection are in BCNF
(b) Both Book and Collection are in 3 NF only
(c) Book is in 2 NF and Collection is in 3 NF
(d) Both Book and Collection are in 2 NF only

20. [MCQ] [GATE-2008 : 2M]

Let R(A, B, C, D, E, P, G) be a relational schema in which the following functional dependencies are known to hold:

AB→CD, DE→P, C→E, P→C and B→G.

The relational schema R is

(a) in BCNF
(b) in 3NF, but not in BCNF
(c) in 2NF, but not in 3 NF
(d) not in 2NF

Decomposition into Higher Normal Form

21. [MCQ] [GATE-2016 : 1M]

A database of research articles in a journal uses the following schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE

(VOLUME, NUMBER) → YEAR

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE)

(VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(a) 1 NF (b) 2 NF
(c) 3 NF (d) BCNF

❑❑❑

ANSWER KEY

1.	(8 to 8)	**2.**	(b)	**3.**	(8 to 8)	**4.**	(b)
5.	(b)	**6.**	(a, c, d)	**7.**	(c)	**8.**	(a)
9.	(a)	**10.**	(c)	**11.**	(a)	**12.**	(a)
13.	(a)	**14.**	(b)	**15**	(a)	**16.**	(a)
17.	(c)	**18.**	(a)	**19.**	(c)	**20.**	(d)
21.	(b)						

SOLUTIONS

1. (8 to 8)

Given Relation R(ABCDE) having following FD sets: {AB $\rightarrow$ C, BC$\rightarrow$ D, C$\rightarrow$ E}

$[AB]^+ = \{A, B, C, D, E\}$

$[A]^+ = \{A\}$

$[B]^+ = \{B\}$

The proper subset of "AB" is not determining all attributes. Thus, AB is minimal set of attributes which determines all attributes. So, AB is candidate key.

Now, R (A, B, C, D, E)

Candidate key = AB

Any combination of non-prime attribute with the candidate key will yield super key:

AB combined with {Φ, C, D, E, CD, DE, CE, CDE}

Total Number of super keys with combinations with the remaining (n – 2) attributes (where n is the number of Attributes)

$= 2^{5-2} \Rightarrow 2^3$

$\Rightarrow$ 8 Super keys for the relation R that is{AB, ABC, ABD, ABE, ABCD, ABDE, ABCE, ABCDE}

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2. (b)

Method 1:

Super key: Any super set of candidate key (primary key) is super key.

Given that "VY" is primary key so, any super set of "VY" will be a super key:

(a) vxyz

(b) vwxz (y is not present) $\therefore$ not super key

(c) vwxy

(d) vwxyz

So, vwxz is not a super key.

Method 2:

Attributes = V, W, X, Y, Z

Primary key = VY

Total super key = $2^3 = 8$

= {VY, VYX, VYW, VYZ, VYXW, VYXZ, VYWZ, VYWXZ}

VWXZ is not a super key

Hence, option b is correct.

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3. **(8 to 8)**

Super key:

Any super set of candidate key is a super key.

R(EFGH)

Candidate Key = [E]

Number of super keys = 2^{n-1} (n is number of attributes)

$= 2^{4-1} = 2^3$

= 8 super keys

OR

R(EFGH)

$2^3 = 8$ super keys.

The super keys of R(EFGH)-

E, EF, EG, EH, EFG, EGH, EFH, EFGH}

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4. **(b)**

Relation R(EFGHIJKLMN)

{EF → G, F → IJ, EH → KL, K → M, L → N}

(a) $[EF]^+ = \{EFGIJ\}$

(b) $[E]^+ = \{E\}$

(c) $[EFH]^+ = \{EFGHIJKLMN\}$;

Since, $(EFH)^+$ contains all the attributes of R, EFH is the key for R.

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5. **(b)**

Given FD set:

{CH → G, A → B, A → C, B → C, B → F,

B → H, E → A, F → E, F → G}.

$[A]^+ = [ABCEFGH]$

D is not present in FD set, whenever any attribute not present in FD then add that attribute in a candidate key.

$[AD]^+ = \{ABCDEFGH\}$

$[ED]^+ = \{ABCDEFGH\}$

$[FD]^+ = \{ABCDEFGH\}$

$[BD]^+ = \{ABCDEFGH\}$

Candidate keys = [AD, ED, FD, BD]

Hence, 4 Candidate keys for the relation R.

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6. **(a, c, d)**

Given relation: U(P, Q, R, S, T)

Given FD set: {P → QR, RS → T}

(a) $(PS)^+ = \{PQRS\mathbf{T}\}$; PS → T can be inferred.

(b) $(R)^+ = \{R\}$; R → T cannot be inferred.

(c) $(P)^+ = \{PQ\mathbf{R}\}$;P → R can be inferred.

(d) $(PS)^+ = \{P\mathbf{Q}RST\}$; PS → Q can be inferred.

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7. **(c)**

X → Y is trivial functional dependency if and only if, $X \supseteq Y$

PS → S

PS $\supseteq$ S; So, it is a trivial FD.

Option b is a semi-trivial FD. Hence, it is incorrect.

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8. (a)

Given FDs are as follows:

$V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow VX$, $Y \rightarrow Z$

Step 1: Split the FD such that R.H.S contain single attribute.

$V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

Step 2: Find the redundant (extraneous) attribute from L.H.S of FD and delete them.

$VW \rightarrow X$, $[W]^+ = \{W\}$; V is not extraneous.

NOTE: The attribute V will be extraneous if $[W]^+$ contain V. So, $[V]^+ = \{VW\ldots\}$; W is extraneous attribute. Thus, W is extraneous as $[V]^+$ contain W

$V \rightarrow W$, $V \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$

Step 3: Find the redundant FD and delete them from FD set.

NOTE: A functional dependency in the set is redundant if it can be derived from the other functional dependencies in the set.

- $Y \rightarrow X$ is redundant as it can be derived from $Y \rightarrow V$ and $V \rightarrow X$.

Hence, the minimal cover is:

$V \rightarrow W$

$V \rightarrow X$

$Y \rightarrow V$

$Y \rightarrow Z$

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9. (a)

F: $[PQ \rightarrow X, P \rightarrow XY, Q \rightarrow Y, Y \rightarrow ZW]$

D_1: R_1(PQST) R_2 (PTX) R_3 (QY) R4 (YZW)

R_1: (PQST) $\cap$ R_2 (PTX) = PT

$[PT]^+$ = [PTXY…] super key of R_2

R_{12} (PQSTX) $\cap$ R_3 (QY) = Q

$[Q]^+$ = [QY…..] Super key of R_3

R_{123} (PQSTXY) $\cap$ R_4 (YZW) = Y

$[Y]^+$ = [YZW] Super key of R_4

R_{1234} (PQSTXYZW)

D_1 is lossless decomposition

R_1(PQST) R_2(PTX) R_3(QY) R_4(YZW)

R_{12}(PQSTX) R_3(QY) R_4(YZW)

R_{123}(PQSTXY) R_4(YZW)

R_{1234}(PQSTXYZW)

D_1: Lossless Decomposition

R (PQSTXYZW)

F: $[PQ \rightarrow X, P \rightarrow YX, Q \rightarrow Y, Y \rightarrow ZW]$

R_1(PQS) R_2(TX) R_3(QY) R_4(YZW)

R_3(QY) $\cap$ R_4(YZW) $\Rightarrow$Y

R_{34}(QYZW)

$[Y]^+$ = [YZW] Super key of R_4

R_1(PQS) $\cap$ R_{34}(QYZW) $\Rightarrow$Q

$[Q]^+$ = [QYZW…..] Super key of R_{34}

But in R_{134} (PQSYZW) $\cap$ R_2 (TX)-

No Common Attribute is present.

D_2 is a lossy decomposition.

R_1(PQS) R_2(TX) R_3(QY) R_4(YZW)

R_2(TX) R_1(PQS) R_{34}(QYZW)

R_2(TX) R_{134}(PQSYZW)

No Common Attribute

D_2: Lossy Decomposition

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10. (c)

I: INCORRECT

X (PQRS) [$QR \rightarrow S$, $R \rightarrow P$, $S \rightarrow Q$]

$[QR]^+ = [QRSP]$

QR is Candidate key

$S \rightarrow Q$

$[SR]^+ = [SRQP]$

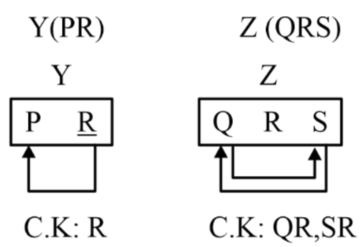
SR is Candidate key

Candidate key = [QR, SR]

$X \rightarrow Y$

$\left.\begin{matrix} S \rightarrow Q \\ R \rightarrow P \end{matrix}\right\}$ X: is Not Superkey

So, relation is not in BCNF



So, relation Y is in BCNF | $S \rightarrow Q$
$\downarrow$
Not a super key
Z is not in BCNF

II: CORRECT

The union of the attribute set of Y and Z is equivalent to R.

Y (PR) $\cap$ Z (QRS) = R

$[R]^+ = [RP]$; Super key of Y.

$R \rightarrow P$ preserved in Y and $QR \rightarrow RS$, $S \rightarrow Q$ preserved in Z.

Lossless and dependency preserving decomposition.

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11. (a)

Consider the following relation R (ABCD) with FD set as {$A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$, $D \rightarrow B$}

Decompose relation: (A, B) (B, C) (B, D)

(1) $R_1(AB) \cup R_2(BC) \cup R_3(BD) \Rightarrow R[ABCD]$

$R_1(AB) \cap R_2(BC) = B$

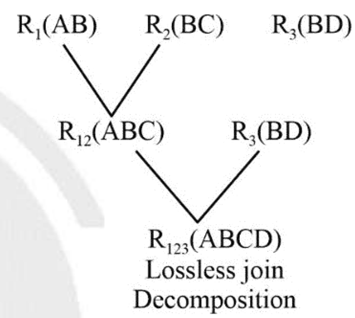
$[B]^+ = [BCD]$ Super key of R_2

$R_{12}(ABC) \cap R_3(BD) = B$

$[B^+] = [BCD]$ super key of R_3

R_{123} (ABCD)

Lossless join



$R_1(AB)$ $R_2(BC)$ $R_3(BD)$

Dependency preserving:

$R_1(AB)$	$R_2(BC)$	$R_3(BD)$
$A \rightarrow B$	$B \rightarrow C$	$D \rightarrow B$
	$C \rightarrow B$	$B \rightarrow D$

$[A]^+ = [ABCD]$

$[B]^+ = [BCD]$

$[C]^+ = [CDB]$

$[D]^+ = [DBC]$

$A \rightarrow B$, $B \rightarrow C$, $D \rightarrow B$, $C \rightarrow B$, $B \rightarrow D$

$A \rightarrow B$, $B \rightarrow C$, $D \rightarrow B$, $C \rightarrow D$

If $F_1 \cup F_2 \cup F_3 \ldots\ldots \cup F_n \equiv F$ (Dependency preserving)

OR

$R_1(AB)$	$R_2(BC)$	$R_3(BD)$
$R_1(AB)$	$R_2(BC)$	$R_3(BD)$
$A \rightarrow B$	$B \rightarrow C$	$D \rightarrow B$

$C \to D$

$C \to B$ $\quad$ $B \to D$

$[C]^+ = [CDB]$ $\quad$ $[B]^+ = [CDB]$

$C \to D$ Indirectly preserved

$C \to B$ in R_2, and $B \to D$ in R_3

$\therefore$ Dependency is preserved.

Hence, option (a) is correct

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12. (a)

(a) A relation with two attributes always in BCNF: **True**

BCNF: Every non-trivial FD $x \to y$ is in BCNF if and only if x is super key.

Let us consider the following instances with two attributes:

R (AB) [$A \to B$]

Candidate key = [A]

A is super key

R (AB) [$B \to A$]

Candidate key = [B]

B is super key

R (AB) [$A \to B$, $B \to A$]

Candidate key = A, B

A & B are super keys. .

(b) If all attributes of relation are prime attributes, then relation is in BCNF: **False**

There may exist a possibility that one prime attribute may determine another prime attribute. Such a FD is allowed in 3NF but not in BCNF design.

(c) **False**

R(ABCD) [$A \to B$, $B \to C$, $C \to A$]

Candidate key = [AD, BD, CD]

Key/Prime Attribute = [A, B, C, D]

Here all attributes are prime/key attribute and R is in 3NF. So, it is not mandatory that there must exist at least one non-prime attribute for a 3NF design. However, the correct statement is "Every relation has at-least one prime attribute".

(d) BCNF Preserve functional Dependency: **False**

R (ABCD) [$AB \to CD$, $D \to A$]

Candidate key = [AB, DB]

Not in BCNF ($\because D \to A$; D is not a super key)

BCNF Decomposition

R_1	R_2
B C D	D A

The above decomposition satisfies BCNF design. It is lossless but not dependency preserving.

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13. (a)

3NF: R is in 3NF

If every $X \to Y$ (Non-Trivial FD)

satisfies the following conditions

X: Super key or

Y: Prime key Attribute

BCNF

R is in BCNF if every non-trivial

$X \to Y$ FD must satisfy the following **condition:**

X: Superkey

$X \to A$ is in 3NF But Not in BCNF

X : is Not Superkey (Not in BCNF)

Then

For 3NF design, A must be a prime Attribute.

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14. (b)

SCHEMA–I:

Registration (rollno. course)

rollno $\rightarrow$ course is in BCNF because rollno is a super key. So, schema I is in BCNF.

SCHEMA–II:

Registration (rollno. courseid. email)

I. rollno courseid $\rightarrow$ email

II. email $\rightarrow$ rollno.

Candidate keys = [rollno courseid, email courseid]

FD – I: rollno course id $\rightarrow$ email $\rightarrow$ BCNF

FD–II: email $\rightarrow$ roll no; email is Not superkey, but rollno is key/Prime Attribute.

So, SCHEMA-II is in 3NF But Not is BCNF.

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15. (a)

Consider the following examples:

R (AB) [A→B] Candidate key = [A] A is super key So, R is in BCNF	R(AB) [A→B, B → A] Candidate key = (A, B) A and B are super keys So, R is in BCNF	R(AB) [B → A] Candidate key = [B] B is super key So, R is in BCNF

A relation with two attributes is always in BCNF implies it is in 1NF, 2NF, 3NF and BCNF. S1 is correct.

OR

A set A is called the minimal cover of set B iff

(a) A covers B and
(b) B covers A and
(c) A is irreducible

Given FD set (B): {AB $\rightarrow$ C, D $\rightarrow$ E, AB $\rightarrow$ E, E $\rightarrow$ C}

Given minimal cover(A): {AB $\rightarrow$ C, D $\rightarrow$ E, E $\rightarrow$ C}

(1) Check if A covers B

A cover B means all FD's of B will be reducible using FD's of A or Every FD of B is A member of A

$(AB)^+ = \{A\ B\ C\}$ $(D)^+ = \{D\ E\}$ $(E)^+ = \{E\ C\}$

(AB) + is not determining E

$\Downarrow$

AB $\rightarrow$ E is not covered by the given minimal cover.

$\therefore$ Statement S2 is FALSE.

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16. (a)

Given FD set –

F = {CH $\rightarrow$ G, A $\rightarrow$ BC, B $\rightarrow$ CFH, E $\rightarrow$A, F $\rightarrow$ EG}

Candidate keys – {AD, ED, FD, BD}

Non-key/non-prime attribute = {C, G, H}

Testing of normal forms

	CH→G	A→BC	B→CFH	E→A	F→EG
BCNF	×	×	×	×	×
3NF	×	–	–	–	–

A $\rightarrow$ C, B $\rightarrow$ C, B $\rightarrow$ H, F $\rightarrow$ G violates 2NF

2NF – proper subset of CK should not determine non-prime subset of CK –

$\{A\}^+ = \{A\ B\ \underline{C}\ F\ \underline{H\ G}\}$

$\{B\}^+$ non-prime

$\{D\}^+$

$\{E\}^+$

$\{F\}^+$

$\therefore$ The relation is not in 2NF

(a) It is in 1NF but not in 2NF.

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17. (c)

3NF definition

Every non trivial FD X → Y is in 3 NF iff

X: super key or

Y: key/Prime attribute

Every non trivial FD X → Y is in BCNF iff , X : Super key

Option (a): False

Let us assume-

R (ABCD) with FD set as [AB → CD, D → A]

Candidate key = [AB, DB]

	AB → CD	D→A
3NF	YES (as AB is a candidate key)	YES (as A is a prime attribute)
BCNF	YES (as AB is a candidate key)	NO (D is not a candidate key)

(c) True.

Let us assume, R (AB) with FD set [A → B]

Candidate key = [A]

Since, A is the key, R is in BCNF and also in 3NF.

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18. (a)

Supplier (<u>Sid</u>, Sname, City, Street)

Each supplier, each street within a city has a unique name

Sid Street City → Sname

Sname City form a candidate key

Sname City → Sid Street

Sid is primary key.

Sid → Sname City Street

Keys (Sid, Sname City)

Check 2NF, as there is no partial dependency so R is in 2NF

Check 3NF in X → Y, either X is a super key or Y is the prime attribute

I. Sid Street City → Sname

II. Sname City → Sid Street

III. Sid → Sname City Street

∴ R is 3NF

Check for BCNF

X is super key, therefore R is in BCNF.

Scan for Video solution

19. (c)

Book (Title, Author, Catalog_no, Publisher, Year, price)

Collection (Title, Author, Catalog_no)

FD I. TitleAuthor ⟶ Catalog_no

FD II. Catalog_no ⟶ Title Author Publisher Year

FD III. Publisher Title Year ⟶ Price

Key = [Author Title]

Collection (Title, Author, Catalog_no)

FD I. TitleAuthor ⟶ Catalog_no

The relation Collection is in BCNF because Title Author is a super key

Collection is in BCNF so its in also in 3NF, 2NF and 1NF.

Key = [Author Title]

Book (Title, Author, Catalog_no, Publisher, Year, price)

FD I. Title Author → Catalog_no

FD II. Catalog_no → Title Author Publisher Year

FD III. Publisher Title Year → Price

In FD II Catalog_no → **Title Author**

Publisher Year

Catalog no is a key

Candidate key = [Title author, catalog]

In book schema

FD I. Satisfy BCNF

FD II. Satisfy BCNF

FD III. Publisher Title Year → Price

Satisfy 2NF but not 3NF

Because neither publisher title year is super key

Nor price is key/prime attribute

∴ Book is in 2NF only.

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20. (d)

Consider relation R(ABCDEPG) with FD set as [AB → CD, DE → P, C → E, P → C, B → G]

$[AB]^+ = [ABCDEPG]$

$[A]^+ = [A]$

$[B]^+ = [BG]$

AB is candidate key ….(1)

There aren't multiple candidate keys.

Prime/key attributes = [A, B]

Non-prime/ non key attribute= [C, D, E, P, G]

A relational design is not in 2NF if there exists B → G (partial dependency).

B (Proper subset of candidate key) → G (non key attribute) Violation of 2NF.

∴ R is not in 2 NF

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21. (b)

Journal (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

FD I: (VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE

FD II: (VOLUME, NUMBER) → YEAR

FD III: (VOLUME, NUMBER, STARTPAGE, ENDPAGE) → PRICE

Keys: (VOLUME, NUMBER, STARTPAGE, ENDPAGE)

FD I: is in BCNF or satisfy BCNF definition

(Because in x → y, x is super key)

FD III: is in BCNF or satisfy BCNF definition

But **FD II (**VOLUME, NUMBER) → YEAR not satisfy 2NF definition because the non-key attribute is not fully dependent on key or proper subset of candidate key → non-key attribute, violation of 2NF exists.

FD II:

Journal Not in 2 NF

Redesign

Schema

SC I: Volume, Number, Start page, end page, Title price

SC II: Volume Number year

Schema I: Volume number start page end page Title price

FD I: Volume Number start page end page → Title price.

FD II: Volume Number start page end page → Price

∴ Satisfy BCNF definition is schema I

Schema II: Volume number year

Volume Number → year

∴ Satisfy BCNF definition

New design is in BCNF but old design

Only 1 NF but not satisfy 2NF

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CHAPTER

3

TRANSACTION AND CONCURRENCY CONTROL

Transaction Concept

1. **[MCQ]** **[GATE-2021 : 1M]**

Suppose a database system crashes again while recovering from a previous crash. Assume checkpointing is not done by the database either during the transactions or during recovery. Which of the following statement is/are correct?

(a) The same undo and redo list will be used while recovering again
(b) The system cannot recover any further
(c) All the transactions that are already undone and redone will not be recovered again
(d) The database will become inconsistent

2. **[MCQ]** **[GATE-2016 : 2M]**

Suppose a database schedule S involves transaction T_1……, T_n. Construct the precedence graph of S with vertices representing the transactions and edges representing the conflicts. If S is serializable, which one of the following orderings of the vertices of the precedence graph is guaranteed to yield a serial schedule?

(a) Topological order
(b) Depth-first order
(c) Breadth-first order
(d) Ascending order of transaction indices

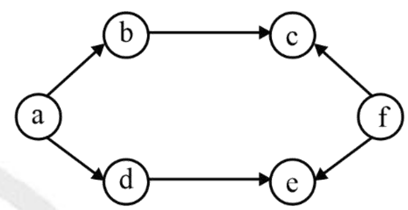
3. **[MCQ]** **[GATE-2016 : 1M]**

Which one of the following is NOT a part of the ACID properties of database transactions?

(a) Atomicity (b) Consistency
(c) Isolation (d) Deadlock-freedom

4. **[NAT]** **[GATE-2016 : 1M]**

Consider the following directed graph:



The number of different topological ordering of the vertices of the graph is

5. **[MCQ]** **[GATE-2015 : 2M]**

Consider a simple checkpointing protocol and the following set of operations in the log.

(start, T4); (write, T4, y, 2, 3); (start, T1);
(commit, T4); (write, T1, z, 5, 7);
(checkpoint);
(start, T2); (write, T2, x, 1, 9); (commit, T2);
(start, T3); (write, T3, z, 7, 2);

If a crash happens now and the system tries to recover using both undo and redo operations, what are the contents of the undo list and the redo list?

(a) Undo: T3, T1; Redo: T2
(b) Undo: T3, T1; Redo: T2, T4
(c) Undo: none; Redo: T2, T4, T3, T1
(d) Undo: T3, T1, T4; Redo: T2

6. **[MCQ]** **[GATE-2015 : 1M]**

Consider the following transaction involving two bank accounts x and y

[read (x) ; x: = x-50 ; write(x) ; read(y) ; y: = y + 50; write(y);]

The constraint that the sum of the accounts x and y should remain constant is that of

(a) Atomicity (b) Consistency
(c) Isolation (d) Durability

Serializable Schedule(Conflict and View)

7. [MCQ] [GATE-2022: 2M]

Let R_i,(z) and W_i (z) denote read and write operations on a data element z by a transaction T_i, respectively. Consider the schedule S with four transactions.

S: $R_4(x)R_2(x)\ R_3(x)\ R_1(y)\ W_1(y)\ W_2(x)\ W_3(y)\ R_4(y)$

Which one of the following serial schedules is conflict equivalent to S ?

(a) T1 → T3 → T4 → T2
(b) T1 → T4 → T3 → T2
(c) T4 → T1 → T3 → T2
(d) T3 → T1 → T4 → T2

8. [MCQ] [GATE-2021 : 2M]

Let S be the following schedule of operations of three transactions T_1, T_2 and T_3 in a relational database system:

R_2 (Y), $R_1(X)$, $R_3(Z)$, $R_1(Y)$, $W_1(X)$, $R_2(Z)$, $W_2(Y)$, $R_3(X)$, $W_3(Z)$

Consider the statements P and Q below:

P: S is conflict-serializable.

Q: If T3 commits before T1 finishes, then S is recoverable.

Which one of the following choices is correct?

(a) Both P and Q are true.
(b) P is true and Q is false.
(c) P is false and Q is true.
(d) Both P and Q are false.

9. [MCQ] [GATE-2021 : 2M]

Let $r_1(z)$ and $w_1(z)$ denote read and write operations respectively on a data item z by a transaction Ti. Consider the following two schedules:

S_1: $r_1(x)\ r_1(y)\ r_2(x)\ r_2(y)\ w_2(y)\ w_1(x)$

S_2: $r_1(x)\ r_2(x)\ r_2(y)\ w_2(y)\ r_1(y)\ w_1(x)$

Which one of the following options is correct?

(a) S_1 is conflict serializable, and S_2 is not conflict serializable
(b) S_1 is not conflict serializable, and S_2 is conflict serializable
(c) Both S_1 and S_2 are conflict serializable
(d) Neither S_1 nor S_2 is conflict serializable

10. [MCQ] [GATE-2020: 2M]

Consider a schedule of transaction T_1 and T_2:

T_1	RA			RC		WD		WB	Commit	
T_2		RB	WB		RD		WC			Commit

Here, RX stand for "Read (X) and WX stands for "Write(X)". Which one of the following schedules is conflict equivalent to the above schedule?

(a)

T_1				RA	RC	WD	WB		Commit	
T_2		RB	WB	RD				WC		Commit

(b)

T_1	RA	RC	WD	WB					Commit	
T_2					RB	WB	RD	WC		Commit

(c)

T_1	RA	RC	WD				WB		Commit	
T_2				RB	WB	RD		WC		Commit

(d)

T_1					RA	RC	WD	WB	Commit	
T_2	RB	WB	RD	WC						Commit

11. [MCQ] [GATE-2014 : 2M]

Consider the transactions T1, T2 and T3 and the schedules S1 and S2 given below.

T1: r1(X); r1(Z); w1(X); w1(Z)

T2: r2(Y) ; r2(Z) ; w2(Z)

T3: r3(Y); r3(X); w3(Y)

S1: r1(X); r3(Y); r3(X); r2(Y); r2(Z); w3(Y); w2(Z); r1(Z); w1(X); w1(Z)

S2: r1(X); r3(Y); r2(Y); r3(X); r1(Z); r2(Z); w3(Y); w1(X); w2(Z); w1(Z)

Which one of the following statements about the schedules is TRUE?

(a) Only S1 is conflict-serializable.
(b) Only S2 is conflict-serializable.
(c) Both S1 and S2 are conflict-serializable.
(d) Neither S1 nor S2 is conflict-serializable.

12. [MCQ] [GATE-2014 : 2M]

Consider the following schedule S of transactions T1, T2, T3 and T4;

T1	T2	T3	T4
	Reads(X)		
		Writes (X) Commit	
Writes (x) Commit			
	Writes(Y); Reads (Z); Commit		
			Reads (X); Reads (Y); Commit

Which one of the following statements is CORRECT?

(a) S is conflict-serializable but not recoverable
(b) S is not conflict-serializable but is recoverable
(c) S is both conflict-serializable and recoverable
(d) S is neither conflict-serializable nor is it recoverable

13. [MCQ] [GATE-2014 : 2M]

Consider the following four schedules due to three transactions (indicated by the subscript using read and write on a data item x, denoted by r(x) and w(x) respectively. Which one of them is conflict serializable?

(a) $r_1(x)$; $r_2(x)$; $w_1(x)$; $r_3(x)$; $w_2(x)$
(b) $r_2(x)$; $r_1(x)$; $w_2(x)$; $r_3(x)$; $w_1(x)$
(c) $r_3(x)$; $r_2(x)$; $r_1(x)$; $w_2(x)$; $w_1(x)$
(d) $r_2(x)$; $w_2(x)$; $r_3(x)$; $r_1(x)$; $w_1(x)$

14. [MCQ] [GATE-2012: 1M]

Consider the following transactions with data items P and Q initialized to zero:

T1: read (P);
read (Q);
if P = 0 then Q := Q + 1 ;
write (Q).

T2: read (Q) ;
read (P);
if Q = 0 then P := P + 1 ;
write (P).

Any non-serial interleaving of T_1 and T_2 for concurrent execution leads to

(a) a serializable schedule
(b) a schedule that is not conflict serializable
(c) a conflict serializable schedule
(d) a schedule for which a precedence graph cannot be drawn

15. [MCQ] [GATE-2010 : 2M]

Consider the following schedule for transactions T1, T2 and T3:

T1	T2	T3
Read (X)		
	Read (Y)	
		Read (Y)
	Write (Y)	
Write (X)		
		Write (X)
	Read (X)	
	Write (X)	

Which one of the schedules below is the correct serialization of the above?

(a) T1 → T3 →T2
(b) T2 →T1 →T3
(c) T2 → T3 →T1
(d) T3 →T 1 →T 2

16. [MCQ] [GATE-2009 : 2M]

Consider two transactions T_1 and T_2, and four schedules S_1, S_2, S_3, S_4 of T_1 and T_2 as given below:

T_1: R_1 [x] W_1 [x] W_1 [y];

T_2: R_2 [x] R_2 [y] W_2 [y];

S_1: R_1 [x] R_2 [x] R_2 [y] W_1 [x] W_1 [y] W_2 [y];

S_2: R_1 [x] R_2 [x] R_2 [y] W_1 [x] W_2 [y] W_1 [y];

S_3: R_1 [x] W_1 [x] R_2 [x] W_1 [y] R_2 [y] W_2 [y];

S_4: R_2 [x] R_2 [y] R_1 [x] W_1 [x] W_1 [y] W_2 [y];

Which of the above schedules are conflict serializable?

(a) S_1 and S_2 (b) S_2 and S_3
(c) S_3 only (d) S_4 only

17. [MCQ] [GATE-2008 : 2M]

Consider the following three schedules of transactions T1, T2 and T3. [Notation: In the following NYO represents the action Y (R for read, W for write) performed by transaction N on object O.]

S1: 2RA 2WA 3RC 2WB 3WA 3WC 1RA 1RB 1WA 1WB

S2: 3RC 2RA 2WA 2WB 3WA 1RA 1RB 1WA 1WB 3WC

S3: 2RA 3RC 3WA 2WA 2WB 3WC 1RA 1RB 1WA 1WB

Which of the following statements is TRUE?

(a) S1, S2 and S3 are all conflict equivalent to each other
(b) No two of S1, S2 and S3 are conflict equivalent to each other
(c) S2 is conflict equivalent to S 3, but not to S1
(d) S1 is conflict equivalent to S2, but not to S3

Finding Number of Serializable Schedule

18. [NAT] [GATE-2017 : 2M]

Two transaction T_1 and T_2 are given as

T_1: $r_1(X)\ w_1(X)\ r_1(Y)\ w_1(Y)$

T_2: $r_2(Y)\ w_2(Y)\ r_2(Z)\ w_2(Z)$

Where r_i (V) denotes a read operation by transaction T_i on a variable V and w_i (V) denotes a write operation by transaction T_i on a variable V. The total number of conflict serializable schedules than can be formed by T_1 and T_2 is _______

Recoverable Schedule

19. [MCQ] [GATE-2016 : 2M]

Consider the following database schedule with two transactions, T_1 and T_2.

$S = r_2(X)\ ;\ r_1(X)\ ;\ r_2(Y)\ ;\ w_1(X)\ ;\ r_1(Y)\ ;\ w_2(X)\ ;\ a_1\ ;\ a_2$

Where r_i (Z) denotes a read operation by transaction T_i on a variable Z, w_i(Z) denotes a write operation by T_i on a variable Z and ai denotes an abort by transation T_i.

Which one of the following statements about the above schedule is TRUE?

(a) S is non-recoverable
(b) S is recoverable, but has a cascading abort
(c) S does not have a cascading abort
(d) S is strict

20. [MCQ] [GATE-2015 : 2M]

Consider the following partial Schedule S involving two transactions T1 and T2. Only the read and the write operations have been shown. The read operation on data item P is denoted by read (P) and the write operation on data item P is denoted by write (P).

Time	Transaction-id	
	T1	T2
1.	read(A)	
2.	write(A)	
3.		read(C)
4.		write(C)
5.		read(B)
6.		write(B)
7.		read(A)
8.		commit
9.		

Suppose that the transaction T1 fails immediately after time instance 9. Which one of the following statements is correct?

(a) T2 must be aborted and then both T1 and T2 must be restarted to ensure transaction atomicity.
(b) Schedule S is non recoverable and cannot ensure transaction atomicity.
(c) Only T2 must be aborted and then restarted to ensure transaction atomicity.
(d) Schedule S is recoverable and can ensure atomicity and nothing else needs to be done.

Lock Based/TME Stamp Protocol

21. [MCQ] [GATE-2019 : 1M]

Consider the following two statements about database transaction schedules:

I. Strict two-phase locking protocol generates conflict serializable schedules that are also recoverable.

II. Timestamp-ordering concurrency control protocol with Thomas Write Rule can generate view serializable schedules that are not conflict serializable.

Which of the above statements is/are True?

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

22. [MCQ] [GATE-2017 : 2M]

In a database system, unique timestamps are assigned to each transaction using Lamport's logical clock. Let TS(T1) and TS(T2) be the timestamps of transactions T1 and T2 respectively. Besides, T1 holds a lock on the resource R and T2 has requested a conflicting lock on the same resource R. The following algorithm is used to prevent deadlocks in the database system assuming that a killed transaction is restarted with the same timestamp.

if TS(T2)<TS(T1) then

T1 is killed

else T2 waits

Assume any transaction that is not killed terminates eventually. Which of the following is TRUE about the database system that uses the above algorithm to prevent deadlocks?

(a) The database system is both deadlock-free and starvation-free.
(b) The database system is deadlock-free, but not starvation-free
(c) The database system is starvation-free, but not deadlock-free.
(d) The database system is neither deadlock-free nor starvation-free.

23. [MCQ] [GATE-2016 : 2M]

Consider the following two phase locking protocol. Suppose a transaction T accesses (for read or write operations), a certain set of objects $\{O_1, \ldots\ldots O_k\}$. This is done in the following manner:

Step 1. T acquires exclusive locks to $O_1, \ldots O_K$, in increasing order of their addresses.

Step 2. The required operations are performed.

Step 3. All locks are released.

This protocol will

(a) guarantee serializability and deadlock-freedom
(b) guarantee neither serializability nor deadlock-freedom
(c) guarantee serializability but not deadlock-freedom
(d) guarantee deadlock-freedom but not serializability

24. [MCQ] [GATE-2010 : 1M]

Which of the following concurrency control protocols ensure both conflict serializability and freedom from deadlock?

I. 2-phase locking **II.** Time-stamp ordering

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

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ANSWER KEY

1.	(a)	**2.**	(a)	**3.**	(d)	**4.**	(6 to 6)
5.	(a)	**6.**	(b)	**7.**	(a)	**8.**	(b)
9.	(b)	**10.**	(a)	**11.**	(a)	**12.**	(c)
13	(d)	**14.**	(b)	**15.**	(a)	**16.**	(b)
17.	(d)	**18.**	(54 to 54)	**19.**	(c)	**20.**	(b)
21.	(c)	**22.**	(a)	**23.**	(a)	**24.**	(b)

SOLUTIONS

1. (a)

Recovery system maintains a log of all the current and previous transactions.

Those transaction that commit perform **Redo** operation while the transactions that do not commit perform **undo** operation.

Old value: $x = \cancel{9}\ \cancel{7}\ 5$

$\boxed{x = 5}$ **Undo**

	Data Item	Old value	New value
Time t = 1	x	5	7
Time t = 2	x	7	9

$\boxed{x = 9}$ **Redo**

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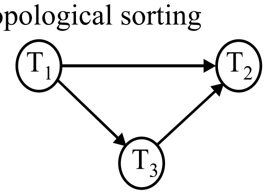
2. (a)

S is serializable, as the graph is Acyclic

Topological sorting



Serializable order (T_1 T_3 T_2)

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3. (d)

In A C I D:

A stands for Atomicity

C stands for Consistency

I stands for Isolation

D stands for Durability

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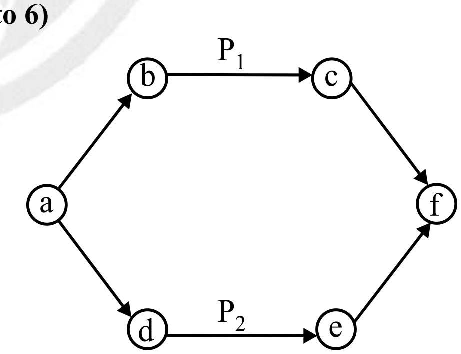
4. (6 to 6)



a	_	_	_	_	f
a	b	c	d	e	f
a	d	e	b	c	f
a	b	d	c	e	f
a	b	d	e	c	f
a	d	b	e	c	f
a	d	b	c	e	f

There exists 6 topological orders for the given graph.

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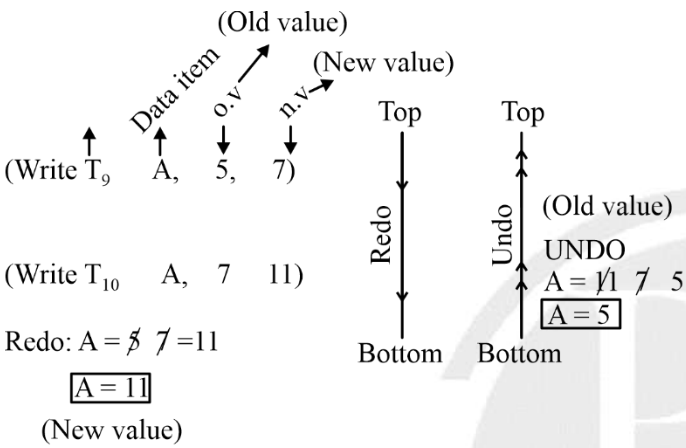
5. **(a)**

Redo: T_2

Undo: T_1, T_3



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6. **(b)**

Consistency ensures integrity of the data before and after the transaction operations have been performed that is the initial value before the start of the operations should match with the value that is obtained after the operations have completed.

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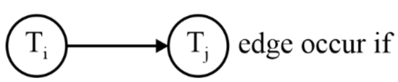
7. **(a)**

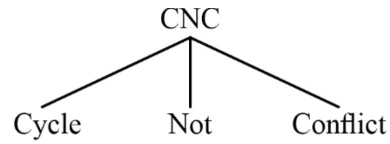


T_i: R(Q) Before T_j = W(Q)

T_i: W(Q) Before T_j = R(Q)

T_i: W(Q) Before T_j = R(Q)

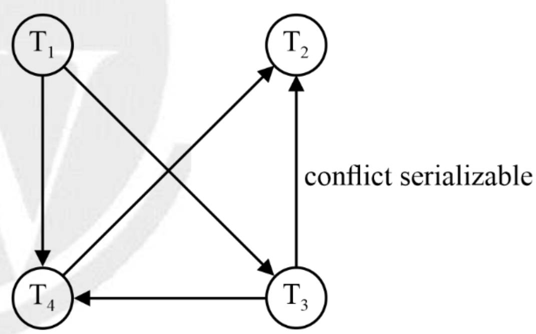
Same Data item (Q)



$$\left.\begin{array}{l} R(Q) \quad - W(Q) \\ W(Q) \quad - R(Q) \\ W(Q) \quad - W(Q) \end{array}\right\} \text{conflict operation}$$

S: $R_4(x) R_2(x)\ R_3(x)\ R_1(y)\ W_1(y)\ W_2(x)\ W_3(y)\ R_4(y)$

T_1	T_2	T_3	T_4
			R(x)
	R(x)		
		R(x)	
R(y)			
W(y)			
	W(x)		
		W(y)	
			R(y)



Seralizablility order as $T_1 \to T_3 \to T_4 \to T_2$

Alternate Method

S: $R_4(x) R_2(x)\ R_3(x)\ R_1(y)\ W_1(y)\ W_2(x)\ W_3(y)\ R_4(y)$

For data item x:

$T_4 \to T_2$ ($R_4(x) \to W_2(x)$)

$T_3 \to T_2$ ($R_3(x) \to W_2(x)$)

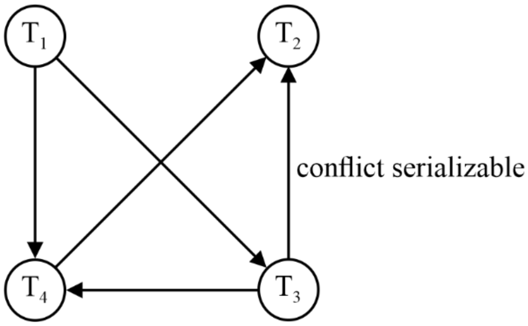
For data item y:

$T_1 \to T_3$ ($R_1(y) \to W_3(y)$)

$T_1 \to T_3$ ($W_1(y) \to W_3(y)$)

$T_1 \to T_4$ ($W_1(y) \to R_4(y)$)

$T_3 \to T_4$ ($W_3(y) \to R_4(y)$)



Seralizablility order as $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$

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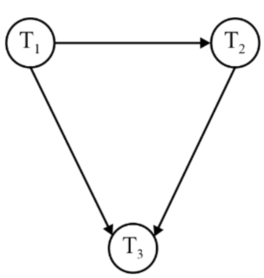
8. (b)

R_2 (Y), R_1(X), R_3(Z), R_1(Y), W_1(X), R_2(Z), W_2(Y), R_3(X), W_3(Z)

T_1	T_2	T_3
	R(y)	
R(x)		
		R(z)
R(y)		
W(x)		
	R(z)	
	W(y)	
		R(x)
		W(z)
		Commit



Conflict Serializable

S_1: True < T_1 T_2 T_3>

Irrecoverable or Non-Recoverable schedule

(T_i)	(T_j)
T_1	T_2
W(Q)	
	R(Q)
C/R	
	$\rightarrow$ Commit

Recoverable schedule

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9. (b)

<u>Conflict operation</u>

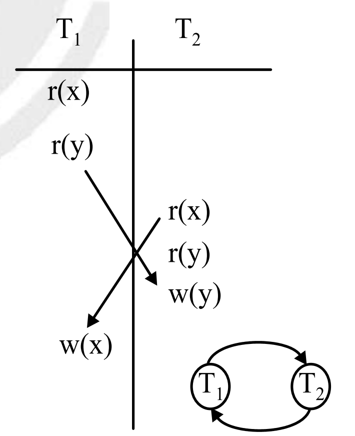
R(A) — W(A)

W(A) — R(A)

W(A) — W(A)

S_1: $r_1(x)$ $r_1(y)$ $r_2(x)$ $r_2(y)$ $w_2(y)$ $w_1(x)$

If cycle exists in the precedence graph, not conflict serializable.

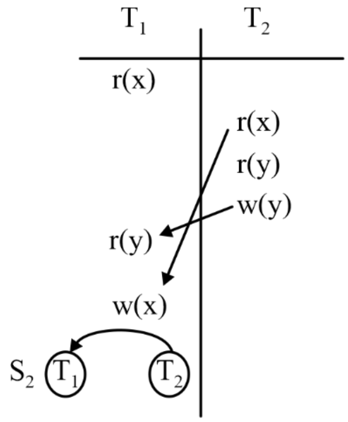


S_1 Cycle Not conflict

S_2: $r_1(x)$ $r_2(x)$ $r_2(y)$ $w_2(y)$ $r_1(y)$ $w_1(x)$

Conflict serializable as the precedence graph is acyclic.

$T_2 \rightarrow T_1$



Conflict Serializable (T_2T_1)

Scan for Video solution

10. (a)

In the given question,

Conflict operations

$R_2(B) - W_1(B)$

$W_2(B) - W_1(B)$

$R_1(C) - W_2(C)$

$R_2(D) - W_1(D)$

OR

$R(B) - W(B) : T_2 \to T_1$

$W(B) - W(B) : T_2 \to T_1$

$R(C) - W(C) : T_1 \to T_2$

$R(D) - W(D) : T_2 \to T_1$

In option (a), the conflicts are as follows:

$R_2(B) - W_1(B)$

$W_2(B) - W_1(B)$

$R_1(C) - W_2(C)$

$R_2(D) - W_1(D)$

OR

$R(B) - W(B) : T_2 \to T_1$

$W(B) - W(B) : T_2 \to T_1$

$R(C) - W(C) : T_1 \to T_2$

$R(D) - W(D) : T_2 \to T_1$

In option (b), the conflicts are as follows:

$W_1(B) - R_2(B)$ $\times$

$W_1(B) - W_2(B)$ $\times$

$R_1(C) - W_2(C)$ $\checkmark$

$W_1(D) - R_2(D)$ $\times$

OR

$W(B) - R(B) : T_1 \to T_2$ $\times$

$W(B) - W(B) : T_1 \to T_2$ $\times$

$R(C) - W(C) : T_1 \to T_2$ $\checkmark$

$W(D) - R(D) : T_1 \to T_2$ $\times$

In option (c), the conflicts are as follows:

$R_2(B) - W_1(B)$

$W_2(B) - W_1(B)$

$R_1(C) - W_2(C)$

$W_1(D) - R_2(D)$ $\times$

OR

$R(B) - W(B) : T_2 \to T_1$

$W(B) - W(B) : T_2 \to T_1$

$R(C) - W(C) : T_1 \to T_2$

$W(D) - R(D) : T_1 \to T_2$ $\times$

In option (d), the conflicts are as follows:

$R_2(B) - W_1(B)$

$W_2(B) - W_1(B)$

$W_2(C) - R_1(C)$ $\times$

$R_2(D) - W_1(D)$

OR

$R(B) - W(B) : T_2 \to T_1$

$W(B) - W(B) : T_2 \to T_1$

$W(C) - R(C) : T_2 \to T_1$ $\times$

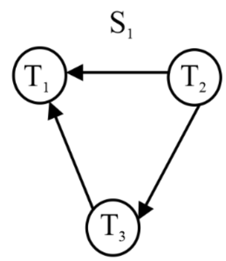
$R(D) - W(D) : T_2 \to T_1$

Scan for Video solution

11. (a)

Schedule S_1 :

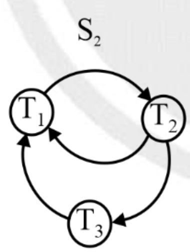
T_1	T_2	T_3
r(x)		
		r(y)
		r(x)
	r(y)	
	r(z)	
		w(y)
	w(z)	
r(z)		
w(x)		
w(z)		



No cycle
conflict serializable
(T_2, T_3, T_1)

Schedule S_2: below schedule wrong including precedence graph:

T_1	T_2	T_3
r(x)		
		r(y)
	r(y)	
		r(x)
r(z)		
	r(z)	
		w(y)
w(x)		
	w(z)	
w(z)		



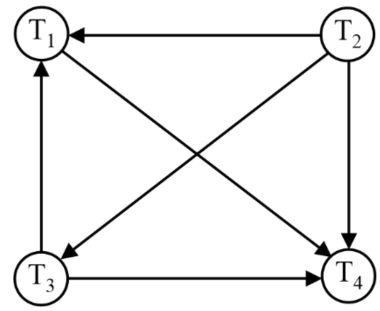
Cycle
Not conflict
Serializable

Therefore, only S_1 is conflict serializable schedule but not schedule S_2.

Scan for Video solution

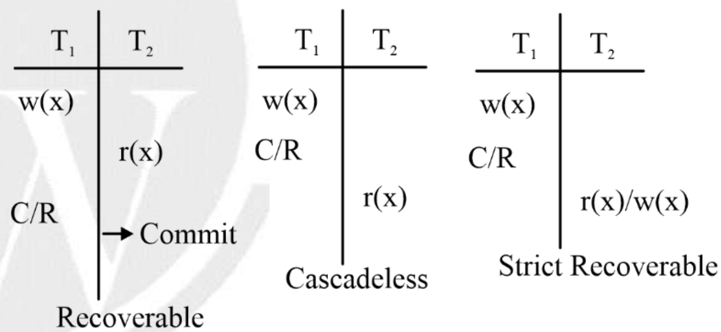
12. (c)

To check for conflict serializability we check the precedence graph, which is as below:



Since there exists no cycle in the precedence graph therefore it is conflict serializable schedule.

For Recoverable schedule we check for write-read uncommitted / dirty Read. In the given Schedule there exists no uncommitted Read, so it is Recoverable and Cascade-less also, no write-write conflict so it is strict recoverable.



Here C/R represents Commit or Rollback operation.

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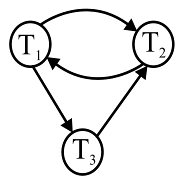
13. (d)

For conflict serializability we check the precedence graph:

1. If cycle present then not conflict serializable schedule.
2. If no cycle, then schedule is conflict serializable schedule.

(a)

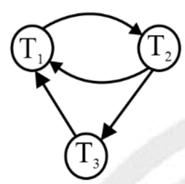
T_1	T_2	T_3
r(x)		
	r(x)	
w(x)		
		r(x)
	w(x)	



cycle, Not conflict Serializable

(b)

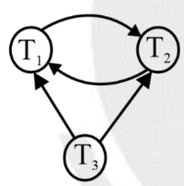
T_1	T_2	T_3
	r(x)	
r(x)		
	w(x)	
		r(x)
w(x)		



cycle, Not conflict Serializable

(c)

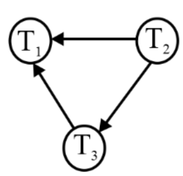
T_1	T_2	T_3
		r(x)
	r(x)	
r(x)		
	w(x)	
w(x)		



cycle, Not conflict Serializable

(d)

T_1	T_2	T_3
	r(x)	
	w(x)	
		r(x)
r(x)		
w(x)		



Not cycle,
conflict serializable
equivalent serial serializable
(T_2, T_3, T_1)

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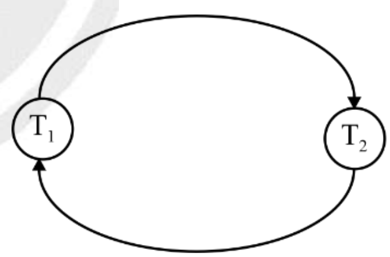
14. (b)

T_1: read (P)
read (Q)
if P = 0 then Q = Q +1
write (Q)

T_2: read (Q)
read (P)
if Q = 0 then P = P +1
write (P)

T_1	T_2
read(P)	
	read (Q) read (P) If Q = 0 then P: P + 1 write (P)
read (Q) If P = 0 then Q = Q + 1 write (Q)	



- Cycle in the precedence graph therefore not conflict serializable but it may be view serializable.
- View serializable (view equivalent)

For each data item

(1) Initial Read same order

(2) Final -write same order

(3) Write-Read (updated-read) same order

Check for view serializable:

T_1	T_2
read(P)	
read (Q)	
if P = 0 then Q; = Q + 1	
write (Q)	
	read (Q)
	read (P)
	if Q = 0 then P: P + 1
	write (P)

<T_1, T_2>

Updated read on data item Q is $T_1 \rightarrow T_2$ exists therefore it fails.

T_1	T_2
	read (Q)
	read (P)
	If Q = 0 then P: P +1
	write (P)
read (P)	
read (Q)	
If Q = 0 then Q: Q + 1	
write (Q)	

<T_2, T_1>

Updated read on data item Q is $T_2 \rightarrow T_1$ exists therefore it fails.

The given schedule is not view serializable as well.

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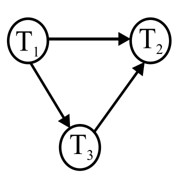
15. (a)



No cycle in the precedence graph, therefore conflict serializable schedule and equivalent to the serial schedule: (T_1 T_3 T_2).

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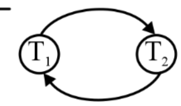
16. (b)

Schedule S_1

T_1	T_2
r(x)	
	r(x)
	r(y)
w(x)	
w(y)	
	w(y)



S_1: Cycle, not conflict

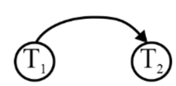
Schedule S_2

T_1	T_2
r(x)	
	r(x)
	r(y)
w(x)	
	w(y)
w(y)	

No cycle
conflict serializable
(T_2, T_1)

Schedule S_3

T_1	T_2
r(x)	
w(x)	
	r(x)
w(y)	
	r(y)
	w(y)

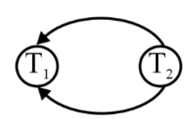


No cycle
conflict serializable
(T_1, T_2)

Schedule S_4

S_4

T_1	T_2
	r(x)
	r(y)
r(x)	
w(x)	
w(y)	
	w(y)



Cycle, Not conflict

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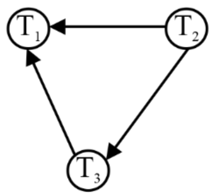
17. (d)

Schedule S_1

T_1	T_2	T_3
	R(A)	
	W(A)	
		R(C)
	W(B)	
		W(A)
		W(C)
R(A)		
R(B)		
W(A)		
W(B)		

Precedence graph



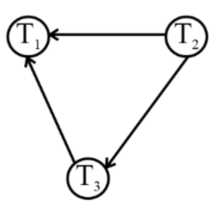
No cycle.
Conflict serializable
(T_2, T_3, T_1)

Conflict pairs are as follows:

1. $R_2(A) – W_3(A)$
2. $R_2(A) – W_1(A)$
3. $W_2(A) – W_3(A)$
4. $W_2(A) – R_1(A)$
5. $W_2(A) – W_1(A)$
6. $W_2(B) – R_1(B)$
7. $W_2(B) – W_1(B)$
8. $W_3(A) – R_1(A)$
9. $W_3(A) – W_1(A)$

Schedule S_2

T_1	T_2	T_3
		R(C)
	R(A)	
	W(A)	
	W(B)	
		W(A)
R(A)		
R(B)		
W(A)		
W(B)		
		W(C)



No cycle.
Conflict serializable
$(T_2\ T_3\ T_1)$

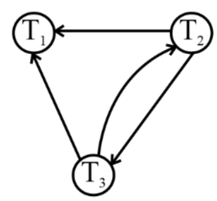
Conflict pairs are as follows:

1. $R_2(A) – W_3(A)$
2. $R_2(A) – W_1(A)$
3. $W_2(A) – W_3(A)$
4. $W_2(A) – R_1(A)$
5. $W_2(A) – W_1(A)$
6. $W_2(B) – R_1(B)$
7. $W_2(B) – W_1(B)$
8. $W_3(A) – R_1(A)$
9. $W_3(A) – W_1(A)$

All conflict operation pair in S_1 and S_2 are in same order.

Schedule S_3

T_1	T_2	T_3
	R(A)	
		R(C)
		W(A)
	W(A)	
	W(B)	
		W(C)
R(A)		
R(B)		
W(A)		
W(B)		



cycle exist
$\therefore S_3$ is Not
Conflict serializable

So, here S_1 is conflict equivalent to S_2, but not to S_3.

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18. (54 to 54)

$T_1 \rightarrow T_2$

$T_2 \rightarrow T_1$

$T_1 : r_1(x) w_1(x) r_1(y) w_1(y)$

$T_2 : r_2(y) w_2(y) r_2(z) w_2(z)$

How to find number of conflict serializable schedules.

$T_1 \rightarrow (T_2)$ & $T_2 \rightarrow (T_1)$

$T_1 \rightarrow T_2$

$r_1(x)\ w_1(x)\ r_1(y)\ w_1(y)\ r_2(y)\ w_2(y)\ r_2(z)\ w_2(z)$

$T_1 \rightarrow (T_2) : 1$

$T_1 : r_1(x)\ w_1(x)\ r_1(y)\ w_1(y)$

$T_2 : r_2(y)\ w_2(y)\ r_2(z)\ w_2(z)$

$T_2 \rightarrow (T_1)$

$r_2(z)\ w_2(z)$ can be placed any where but only after $w_2(y)$

✓ $w_2(y)$ ✓ $w_2(y)$ ✓ $w_2(y)$ × $w_2(y)$

↑ $r_1(x)$ ↑ $w_1(x)$ ↑ $r_1(y)$ ↑ $w_1(y)$ ↑

$w_2(y)$ $\boxed{\text{Case III}}$ $\boxed{\text{Case II}}$ $\boxed{\text{Case I}}$

Case I	$r_1(x)$	$w_1(x)$	$\underline{w_2(y)}$	$r_1(y)$	$w_1(y)$
Case II	$r_1(x)$	$\underline{w_2(y)}$	$w_1(x)$	$r_1(y)$	$w_1(y)$
Case III	$\underline{w_2(y)}$	$r_1(x)$	$w_1(x)$	$r_1(y)$	$w_1(y)$

Now check for $r_2(y)$.

Case I

✓ $r_2(y)$ ✓ $r_2(y)$ ✓ $r_2(y)$ $\quad R_2(z)W_2(z)$

↑ $r_1(x)$ ↑ $w_1(x)$ ↑ $\boxed{w_2(y)}$ ① $r_1(y)$ ② $w_1(y)$ ③

for $r_2(z)\ w_2(z)$

out of three places either we can place together $[^3C_1]$ OR place them separately $[^3C_2]$

$3 \times [^3C_1 + {^3C_2}]$

$3 \times [3 + 3]$

Case I – 18 ways

Case II

$\underline{r_2(y)}$ $r_2(y)$ $\quad R_2(z)\ W_2(z)$

↑ $r_2(x)$ ↑ $\boxed{\underline{w_2(y)}}$ ① $w_2(x)$ ② $r_1(y)$ ③ $w_1(y)$ ④

Sit together (or) separately

$^4C_1 + {^4C_2}$

$r_2(y) \rightarrow 2$

$2 \times [^4C_1 + {^4C_2}]$

$2 \times [4 + 6]$

Case II = 20

Case III

$r_2(y)$

↑ $w_2(y)$ ① $r_1(x)$ ② $w_2(x)$ ③ $r_1(y)$ ④ $w_1(y)$ ⑤

1 way

$1 \times [^5C_1 + {^5C_2}]$

Sit together $^5C_1$ OR sit Separately $^5C_2$

$1 \times [^5C_1 + {^5C_2}]$

$5 + 10 = 15$

Case III : 15 ways

Therefore, $T_2 \rightarrow T_1$: Case I + Case II + Case III

= 18 + 20 + 15 =53

$T_2 \rightarrow (T_1)$ = 53 ways

$T_1 \rightarrow (T_2)$: 1 way

Total conflict serializable 54 schedules.

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19. (c)

T_1	T_2
	r(x)
r(x)	
	r(y)
w(x)	
r(y)	
	w(x)
a_1	
	a_2

Recoverable, Cascadeless (no cascadeding roll back) because there exists no uncommitted read but not strict schedule.

T_1	T_2
w(A)	
C/R	
	r(A)/w(A)

Strict Recoverable

T_1	T_2
w(A)	
	r(A)
C/R	
	Commit

Recoverable

T_1	T_2
w(A)	
C/R	
	r(A)

Cascadeless

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20. (b)

Time	Transaction-id	
	T1	T2
1.	read(A)	
2.	write(A)	
3.		read(C)
4.		write(C)
5.		read(B)
6.		write(B)
7.		read(A)
8.		commit
9.	read(B)	

* Fail

Irrecoverable or Non-recoverable

(a) Both T_1 & T_2 Restarted, ensure atomicity: Incorrect.

(b) Non Recoverable & cannot ensure Atomicity: Correct

→ Cannot perform undo of Transaction T_2 because T_2 commits.

(c) Incorrect, Because T_2 performs commit so only Redo operation can be performed.

(d) Recoverable: Incorrect.

(T_i) T_1	(T_j) T_2
w(a)	
	r(a)
C/R	
	Commit

Recoverable

T_1	T_2
w(a)	
	r(a)
	Commit
C/R	

Irrecoverable

or

Non Recoverable Schedule

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21. (c)

Strict 2PL: 2PL+ All Exclusive [x Lock] held by the transaction until Commit (or) Rollback

(OR)

2PL + All Exclusive Lock (x Lock) Release by the transaction after commit or rollback.

T_1	T_2
x(A)	
C/R	
Unlock x(A)	
	S(A) / X(A)
	R(A) / W(A)

Strict Recoverable

T_1	T_2
W(A)	
C/R	
	R(A)/W(A)

<u>**Thomas write Rule:**</u> Obsolete write ensures view serializable but not conflict serializable.

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22. (a)

Deadlock prevention algorithm

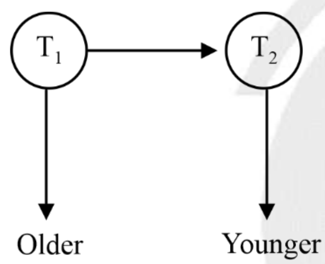
(1) Wait – Die: Transaction T_1 Request a Data item currently held by T_j, T_i is allowed to wait only Time stamp of T_i [TS (T_i)] is smaller than that of T_j.

(2) Wound -wait: Transaction T_i Request a data item currently held T_j then T_i is allowed to wait only if time stamp of T_i [TS(T_i)] is larger than that of T_j.

Wound wait: Older Transaction kills the younger Transaction. But restarted with same time stamp.

TS(T_1): 10

TS(T_2): 20



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23. (a)

STEP-1: T acquire x lock (Exclusive lock) O_1 …. O_k in increasing order of the address i.e. other transaction cannot take any lock on that data item.

		T_j	
	Same Data Item	S	X
T_i	S	Yes	No
	X	No	No

STEP-2: Required operation are performed.

STEP-3: All lock are released.

2PL (2 Phase locking Protocol) → Growing Phase, Shrinking Phase

→ Ensure conflict serializable Schedule (Serializability)

& Serializability order is determined by lock point.

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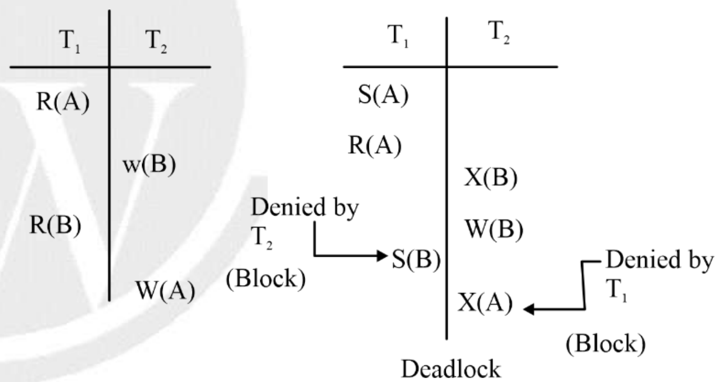
24. (b)

2PL ensures conflict serializability but may not be free from deadlock.



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❑❑❑

CHAPTER

4

RELATIONAL ALGEBRA, SQL QUERIES, TRC

Relational Algebra

1. [MSQ] **[GATE-2022 : 1M]**

Consider the following three relations in a relational database.

Employee (eId, Name), Brand(bId, bName), Own(eId, bId)

Which of the following relational algebra expressions return the set of eIds who own all the brands?

(a) Π_{eId} ($\Pi_{eId,\ bId}$ (Own) / Π_{bId} (Brand))

(b) Π_{eId} (Own) – Π_{eId} ((Π_{eId} (Own) × Π_{bId} (Brand)) – $\Pi_{eId,\ bId}$ (Own))

(c) Π_{eId} ($\Pi_{eId,\ bId}$ (Own)/Π_{bId} (Own))

(d) Π_eld ((Π_{eId} (Own)×Π_{bId} (Own))/Π_bld (Brand))

2. [NAT] **[GATE-2021 : 1M]**

A relation r(A, B) in a relational database has 1200 tuples. The attribute A has integer values ranging from 6 to 20 , and the attribute B has integer values ranging from 1 to 20 . Assume that the attributes A and B are independently distributed.

The estimated number of tuples in the output of $\sigma_{(A>10)}$ V (B = 18) (r) is

3. [MCQ] **[GATE-2020 : 1M]**

The following relation records the age of 500 employees of a company, where empNo {indicating the employee number} is the key:

empAge(empNo, age)

Consider the following relational algebra expression:

$\prod_{empNo.}$(empAge $\bowtie_{(age > age1)}$ $\rho_{\ empNo1}$, age1) (empAge))

What does the above expression generate?

(a) Employee numbers of only those employees whose age is the maximum

(b) Employee numbers of only those employees whose age is more than the age of exactly one other employee

(c) Employee numbers of all employees whose age is not the minimum

(d) Employee numbers of all employees whose age is the minimum

4. [NAT] **[GATE-2019 : 2M]**

Consider the following relations P(X, Y, Z), Q(X, Y, T) and R(Y, V)

P			Q			R	
X	Y	Z	X	Y	T	Y	V
X1	Y1	Z1	X2	Y1	2	Y1	V1
X1	Y1	Z2	X1	Y2	5	Y3	V2
X2	Y2	Z2	X1	Y1	6	Y2	V3
X2	Y4	Z4	X3	Y3	1	Y2	V2

How many tuples will be returned by the following relational algebra query?

[$\Pi_{X\ (\sigma\ (P.Y=R.Y\ \wedge\ R.V=V2)}$ (P × R)) – $\Pi_{X\ (\sigma\ (Q.Y=R.Y \wedge\ Q.T\ >\ 2)}$ (Q × R))];

5. [MCQ] **[GATE-2018 : 2M]**

Consider the relations r(A, B) and s(B, C), where s.B is a primary key and r.B is a foreign key referencing s.B. Consider the query.

Q: r ⋈ ($\sigma_{B<5}$ (S))

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following queries is NOT equivalent to Q ?

(a) $\sigma_{B<5}$ (r⋈s)
(b) $\sigma_{B<5}$ (r LOJ s)
(c) r LOJ ($\sigma_{B<5}$ (s))
(d) $\sigma_{B<5}$ (r) LOJ s

6. [NAT] [GATE-2017 : 2M]

Consider a database that has the relation schema CR(StudentName, CourseName). An instance of the schema CR is as given below:

The following query is made on the database.

T1 ← $\pi_{\text{course Name}}$ ($\sigma_{\text{Studtn Name} = \text{'SA'}}$ (CR))

T2 ← CR÷T1;

The number of rows in T2 is _____________.

CR	
Student Name	**Course Name**
SA	CA
SA	CB
SA	CC
SB	CB
SB	CC
SC	CA
SC	CB
SC	CC
SD	CA
SD	CB

Student Name	Course Name
SD	CC
SD	CD
SE	CD
SE	CA
SE	CB
SF	CA
SF	CB
SF	CC

7. [MCQ] [GATE-2015 : 2M]

Consider two relations R_1(A,B) with the tuples (1,5), (3,7) and R_2(A,C) = (1,7), (4,9). Assume that R(A.B,C) is the full natural outer join of R_1 and R_2. Consider the following tuples of the form (A,B,C): a = (1,5,null), b = (1,null,7), c = (3, null, 9), d = (4,7,null), e = (1,5,7), f = (3,7,null), g = (4,null, 9). Which one of the following statements is correct?

(a) R contains a, b, e, f, g but not c, d.
(b) R contains all of a, b, c, d, e, f, g.
(c) R contains e,f, g but not a, b.
(d) R contains e but not f , g.

8. [MCQ] [GATE-2014 : 1M]

What is the optimized version of the relation algebra expression π_{A1} (π_{A2} (σ_{F1} (σ_{F2} (r)))), where A1, A2 are sets of attributes in r with A1 ⊂ A2 and F1, F2 are Boolean expression based on the attributes in r ?

(a) π_{A1} ($\sigma_{F1 \wedge F2}$) (r))
(b) π_{A1} ($\sigma_{F1 \vee F2}$) (r))
(c) π_{A2} ($\sigma_{F1 \wedge F2}$) (r))
(d) π_{A2} ($\sigma_{F1 \vee F2}$) (r))

9. [MCQ] [GATE-2014 : 2M]

Consider the relational schema give below, where eId of the relation dependent is a foreign key referring to empId of the relation employee. Assume that every employee has at least one associated dependent in the dependent relation.

employee (<u>empId</u>, empName, empAge)

dependent (<u>depId, eId</u>, depName, depAge)

Consider the following relational algebra query:

Π_{empId} (employee)

– Π_{empId} (employee $\bowtie_{\text{empId=eId}} \wedge$ (empAge ≤ dep Age)) (dependent)

The above query evaluates to the set of empIds of employees whose age is greater than that of

(a) some dependent.
(b) all dependents.
(c) some of his/her dependents.
(d) all of his/her dependents.

10. [MCQ] [GATE-2014 : 2M]

Consider a join (relation algebra) between relations r(R) and s(S) using the nested loop method. There are 3 buffers each of size equal to disk block size, out of which one buffer is reserved for intermediate results.

Assuming size (r(R))<size(s(S)), the join will have fewer number of disk block accesses if

(a) relation r(R) is in the outer loop.
(b) relation s(S) is in the outer loop.
(c) join selection factor between r(R) and s(S) is more than 0.5.
(d) join selection factor between r(R) and s(S) is less than 0.5.

11. [MCQ] [GATE-2013 : 2M]

Suppose R_1 (A, B) and R_2 (C, D) are two relation schemes. Let r_1 and r_2 be the corresponding relation instances. B is a foreign key that refers to C in R_2. If data in r_1 and r_2 satisfy referential integrity constraints, which of the following is ALWAYS TRUE?

(a) $\Pi_B(r_1) - \Pi_C(r_2) = \phi$ (b) $\Pi_C(r_2) - \Pi_B(r_1) = \phi$
(c) $\Pi_B(r_1) = \Pi_C(r_2)$ (d) $\Pi_B(r_1) - \Pi_C(r_2) \neq \phi$

12. [MCQ] [GATE-2012 : 2M]

Consider the following relation A, B and C:

A		
ID	Name	Age
12	Arun	60
15	Shreya	24
99	Rohit	11

B		
ID	Name	Age
15	Shreya	24
25	Hari	40
98	Rohit	20
99	Rohit	11

C		
ID	Phone	Area
10	2200	02
99	2100	01

How many tuples does the result of the following relational algebra expression contain? Assume that the schema of $A \cup B$ is the same as that of A.

$(A \cup B) \bowtie_{A.Id > 40 \vee C.Id < 15} C$

(a) 7 (b) 4
(c) 5 (d) 9

13. [MCQ] [GATE-2011: 2M]

Consider a relational table r with sufficient number of records, having attributes $A_1, A_2, \ldots, A_n$ and let $1 \le p \le n$. Two queries Q_1 and Q_2 are given below.

Q_1: $\pi_{A1, \ldots, AP}(\sigma_{AP = C}(r))$ where c is a constant

Q_2: $\pi_{A1, \ldots, AP}(\sigma_{C_1 \le A_P \le C_2}(r))$ where C_1 and C_2 are constants

The database can be configured to do ordered indexing on A_p or hashing on A_p. Which of the following statements is TRUE?

(a) Ordered indexing will always outperform hashing for both queries.
(b) Hashing will always outperform ordered indexing for both queries.
(c) Hashing will outperform ordered indexing on Q_1, but not on Q_2.
(d) Hashing will outperform ordered indexing on Q_2, but not on Q_1.

14. [MCQ] [GATE-2010 : 2M]

The following functional dependencies hold for relations R(A, B, C) and S(B, D, E) FD for both the tables.

$B \to A$ $A \to C$

The relation R contains 200 tuples and the relation S contains 100 tuples. What is the maximum number of tuples possible in the natural join $R \bowtie S$?

(a) 100 (b) 200
(c) 300 (c) 2000

15. [MCQ] [GATE-2008 : 2M]

Let R and S be two relations with the following schema

[R(P, Q, R1, R2, R3) S(P, Q, S1, S2)]

Where {P, Q} is the key for both schemas. Which of the following queries are equivalent?

I. $\Pi_P(R \bowtie S)$

II. $\Pi_P(R) \bowtie \Pi_P(S)$

III. $\Pi_P(\Pi_{P,Q}(R) \cap \Pi_{P,Q}(S))$

IV. $\Pi_P(\Pi_{P,Q}(R) - (\Pi_{P,Q}(R) - \Pi_{P,Q}(S)))$

(a) Only I and II (b) Only I and III
(c) Only I, II and III (d) Only I, III and IV

Structured Query Language(SQL)

16. [NAT] [GATE-2023: 2M]

Consider the following table named Student in a relational database. The primary key of this table is rollNum.

Student

Roll Num	Name	Gender	Marks
1	Naman	M	62
2	Aliya	F	70
3	Aliya	F	80
4	James	M	82
5	Swati	F	65

The SQL query below is executed on this database.

```
SELECT *
FROM Student
WHERE gender = 'F' AND marks > 65;
```

The number of rows returned by the query is

17. [NAT] [GATE-2022 : 2M]

Consider the relational database with the following four schemas and their respective instances.

Student (sNo, sName, dNo) Dept (dNo, dName)
Course (cNo, cName, dNo) Register (sNo, cNo)

Student

sNo.	s Name	dNo
S01	James	D01
S02	Rocky	D01
S03	Jackson	D02
S04	Jane	D01
S05	Milli	D02

Register

sNo	cNo
S01	C 11
S01	C 12
S02	C 11
S03	C 21
S03	C 22
S03	C 23
S04	C 11
S04	C 12
S05	C 11
S 05	C 21

Course

cNo	cName	dNo
C11	DS	D01
C12	SO	D01
C21	DE	D02
C22	PT	D02
C23	CV	D03

Dept

DNo	DName
D01	CSE
D02	EEE

SQL Query:

```
SELECT * From Student AS S WHERE NOT EXIST
(SELECT cNo FROM Course WHERE dNo = "D01"
EXCEPT
SELECT cNo FROM Register WHERE sNo=S.sNo)
```

The number of rows returned by the above SQL query is ______________ .

18. [MCQ] [GATE-2021 : 2M]

The relation scheme given below is used to store information about the employees of a company, where empId is the key and dept Id indicates the department to which the employee is assigned. Each employee is assigned to exactly one department.

emp(empId, name, gender, salary, deptId)

Consider the following SQL query:

```
select deptId, count(*)
from emp
where gender = "female" and salary > (select avg(salary) from emp) group by deptId;
```

The above query gives, for each department in the company, the number of female employees whose salary is greater than the average salary of

(a) employees in the department.
(b) employees in the company.
(c) female employees in the department.
(d) female employees in the company.

19. [MCQ] [GATE-2020 : 1M]

Consider a relational database containing the following schemas.

Catalogue		
sno	pno	cost
S1	P1	150
S1	P2	50
S1	P3	100
S2	P4	200
S2	P5	250
S3	P1	250
S3	P2	150
S3	P5	300
S3	P4	250

Suppliers		
sno	sname	location
S1	M/s Royal furniture	Delhi
S2	M/s Balaji furniture	Bangalore
S3	M/s Premium furniture	Chennai

Parts		
pno	pname	part_spec
P1	Table	Wood
P2	Chair	Wood
P3	Table	Steel
P4	Almirah	Steel
P5	Almirah	Wood

The primary key of each table is indicated by underlining the constituent fields.

SELECT s.sno, s.name FROM Suppliers s, Catalogue c WHERE s.no = c.sno AND cost > (SELECT AVG(cost) FROM Catalogue WHERE pno = 'P4' GROUP BY pno);

The number of rows returned by the above SQL query is

(a) 4 (b) 5

(c) 2 (d) 0

20. [NAT] GATE-2019 : 2M]

A relational database contains two tables Student and Performance as shown below:

Student

Roll_no.	Student_name
1	Amit
2	Priya
3	Vinit
4	Rohan
5	Smita

Performance

Roll_no.	Subject_code	Marks
1	A	86
1	B	95
1	C	90
2	A	89
2	C	92
3	C	80

The primary key of the Student table is Roll_no. For the Performance table, the columns Roll_no. and Subject_code together form the primary key. Consider the SQL query given below:

SELECT S.Student_name, sum(P.Marks)

FROM Student S, Performace P

WHERE P.Marks >84

GROUP BY S.Student_name;

The number of rows returned by the above SQL query is ____________.

21. [MCQ] [GATE-2018 : 1M]

Consider the following two tables and four queries in SQL.

Book (isbn, bname), Stock (isbn, copies)

Query 1:

SELECT B.isbn, S.copies

FROM Book B INNER JOIN Stock S

ON B.isbn= =S.isbn;

Query 2:

SELECT B.isbn, S.copies

FROM Book B LEFT OUTER JOIN Stock S

ON B.isbn = S.isbn;

Query3:
SELECT B.isbn, S.copies
FROM Book B RIGHT OUTER JOIN Stock S
ON B.isbn =S.isbn;

Query4:
SELECT B.isbn, S.copies
FROM Book B FULL OUTER JOIN Stock S
ON B. isbn=S.isbn;

Which one of the queries above is certain to have an output that is superset of the outputs of the other three queries?

(a) Query 1 (b) Query 2
(c) Query 3 (d) Query 4

22. [NAT] [GATE-2017 : 2M]

Consider the following database table named top_scorer.

Consider the following SQL query:
SELECT ta.player FROM top_scorer AS ta
WHERE ta.goals > ALL (SELECT tb.goals FROM top_scorer AS tb
WHERE tb.country='Spain')
AND
ta.goals > ANY (SELECT tc.goals FROM top_scorer AS tc
WHERE tc.country ='Germany')

The number of tuples returned by the above SQL query is ________

Player	Country	Goals
Klose	Germany	16
Ronaldo	Brazil	15
G Muller	Germany	14
Fontaine	France	13
Pele	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11
Batistuta	Argentina	10
Cubillas	Peru	10
Lato	Poland	10
Lineker	England	10
T Muller	Germany	10
Rahn	Germany	10

23. [NAT] [GATE-2017 : 1M]

Consider a database that has the relation schema EMP(EmpId, EmpName, DeptName). An instance of the schema EMP and a SQL query on it are given below:

EMP		
EmpId	**EmpName**	**DeptName**
1	XYA	AA
2	XYB	AA
3	XYC	AA
4	XYD	AA
5	XYE	AB
6	XYF	AB
7	XYG	AB
8	XYH	AC
9	XYI	AC
10	XYJ	AC
11	XYK	AD
12	XYL	AD
13	XYM	AE

SELECT AVG(EC.Num)
FROM EC
WHERE (DeptName, Num) IN
(SELECT DeptName, COUNT(EmpId) AS EC(DeptName, Num)
FROM EMP
GROUP BY DeptName)

The output of executing the SQL query is ________.

24. [NAT] [GATE-2016 : 2M]

Consider the following database table named water_schemes:

water_schemes		
scheme_no	**district_name**	**Capacity**
1	Ajmer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	10
2	Churu	20
1	Dungargarh	10

The number of tuples returned by the following SQL query is

with total(name, capacity) as

select district_name, sum (capacity)

from water_schemes

group by district_name

with total_avg(capacity) as

select avg (capacity)

from total

select name

from total, total_avg

where total.capacity > total_avg.capacity

25. [MCQ] [GATE-2015 : 2M]

Consider the following relation

Cinema (theater, address, capacity)

Which of the following options will be needed at the end of the SQL query

SELECT P1.address

FROM Cinema P1

such that it will always finds the addresses of theaters with maximum capacity?

(a) WHERE P1.capacity ≥ All (select P2.capacity from Cinema P2)
(b) WHERE P1.capacity ≥ Any (select P2. capacity from Cinema P2)
(c) WHERE P1.capacity > All (select max (P2.capacity) from Cinema P2)
(d) WHERE P1.capacity > Any(select max (P2.capacity) from Cinema P2)

26. [MCQ] [GATE-2015 : 1M]

SELECT operation in SQL is equivalent to

(a) the selection operation in relational algebra
(b) the selection operation in relational algebra except that SELECT in SQL retains duplicates
(c) the projection operation in relational algebra
(d) the projection operation in relational algebra, except that SELECT in SQL retains duplicates

27. [NAT] GATE-2015 : 2M]

Consider the following relations:

Student	
Roll_No	**Student_Name**
1	Raj
2	Rohit
3	Raj

Performance		
Roll_No	**Course**	**Marks**
1	Math	80
1	English	70
2	Math	75
3	English	80
2	Physics	65
3	Math	80

Consider the following SQL query.

SELECT S.Student_Name, sum(P.Marks)

FROM Student S, Performance P

WHERE S.Roll_No=P.Roll_No

GROUP BY S.Student_Name

The number of rows that will be returned by the SQL query is ___________

28. [MCQ] [GATE-2014 : 2M]

Consider the following relational schema:

employee (empId, empName, empDept)

customer(custId, custName, salesRepId, rating)

salesRepId is a foreign key referring to empId of the employee relation. Assume that each employee makes a sale to at least one customer.

What does the following query return?

SELECT empName

FROM employee E

WHERE NOT EXISTS

(SELECT custId FROM customer C

WHERE C.salesRepId = E.empId

AND C.rating < > 'GOOD');

(a) Name of all the employees with at least one of their customers having a 'GOOD' rating.
(b) Names of all the employees with at most one of their customers having a 'GOOD' rating.

(c) Names of all the employees with none of their customers having a 'GOOD' rating.
(d) Names of all the employees with all their customers having a 'GOOD' rating.

29. [MCQ] [GATE-2014 : 2M]

SQL allows duplicate tuples in relations, and correspondingly defines the multiplicity of tuples in result of joins. Which one of the following queries always gives the same answer as the nested query shown below:

Select * from R where a in (select S.a from S)

(a) select R.* from R, S where R.a=S.a
(b) select distinct R . * from R, S where R.a = S.a
(c) select R.* from R, (select distinct a from S) as S1 where R.a=S1.a
(d) select R.* from R, S where R.a=S.a and is unique R

30. [MCQ] [GATE-2014 : 2M]

Given the following schema:

employees(emp-id, first-name, last-name, hire-date, dept-id, salary)

departments (dept-id, dept-name, manager-id, location-id)

You want to display the last names and hire dates of all latest hires in their respective departments in the location ID 1700. You issue the following query:

```
SQL>SELECT last-name, hire-date
FROM employees
WHERE (dept-id, hire-date) IN
(SELECT dept-id, MAX(hire-date)
FROM employees JOIN departments
USING(dept-id)
WHERE location-id =1700
GROUP BY dept-id);
```

What is the outcome?

(a) It executes but does not give the correct result.
(b) It executes and gives the correct result.
(c) It generates an error because of pairwise comparison.
(d) It generates an error because the GROUP BY clause cannot be used with table joins in a subquery.

31. [MCQ] [GATE-2012: 1M]

Which of the following statements are TRUE about an SQL query?

P: An SQL query can contain a HAVING clause even if it does not have a GROUP BY clause.
Q: An SQL query can contain a HAVING clause only if it has a GROUP BY clause
R: All attributes used in the GROUP BY clause must appear in the SELECT clause
S: Not all attributes used in the GROUP BY clause need to appear in the SELECT clause

(a) P and R (b) P and S
(c) Q and R (d) Q and S

32. [MCQ] [GATE-2012 : 2M]

Consider the following relations A, B and C :

A		
Id	**Name**	**Age**
12	Arun	60
15	Shreya	24
99	Rohit	11

B		
Id	**Name**	**Age**
15	Shreya	24
25	Hari	40
98	Rohit	20
99	Rohit	11

C		
Id	**Phone**	**Area**
10	2200	02
99	2100	01

How many tuples does the result of the following SQL query contain?

```
SELECT A.Id
FROM A
WHERE A.Age > All (SELECT B.Age
FROM B
WHERE B. Name='Arun')
```

(a) 4 (b) 3
(c) 0 (d) 1

33. [MCQ] [GATE-2011 : 2M]

Database table by name Loan_Records is given below:

Borrower	Bank_Manager	Loan_Amount
Ramesh	Sunderajan	10000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00

What is the output of the following SQL query?

SELECT Count (*) FROM
((SELECT Borrower, Bank_Manager FROM Loan_Records) AS S NATURAL JOIN (SELECT Bank-Manager, Loan_Amount FROM Loan_Records) AS T);

(a) 3 (b) 9
(c) 5 (d) 6

34. [MCQ] [GATE-2011 : 2M]

Consider a database table T containing two columns X and Y each of type integer. After the creation of the table, one record (X=1, Y=1) is inserted in the table.

Let MX and MY denote the respective maximum values of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values being MX+1, 2*MY+1 respectively. It may be noted that each time after the insertion, values of MX and MY change.

What will be the output of the following SQL query after the steps mentioned above are carried out?

SELECT Y FROM T WHERE X=7;

(a) 127 (b) 255
(c) 129 (d) 257

35. [MCQ] [GATE-2010: 1M]

A relation schema for a train reservation database is given below:

Passenger (pid, pname, age)

Reservation (pid, class, tid)

Table; Passenger		
Pid	**Pname**	**Age**
0	'Sachin'	65
1	'Rahul'	66
2	'Sourav'	67
3	'Anil'	69

Table; Reservation		
Pid	**Class**	**Tid**
0	'AC'	8200
1	'AC'	8201
2	'SC'	8201
5	'AC'	8203
1	'SC'	8204
3	'AC'	8202

What pids are returned by the following SQL query for the above instance for the tables?

SELECT pid
FROM Reservation
WHERE class = 'AC' AND
EXITST (SELECT *
FROM Passenger
WHERE age > 65 AND
Passenger pid = Reservation pid)

(a) 1, 0 (b) 1, 2
(c) 1, 3 (d) 1, 5

36. [MCQ] GATE-2009 : 2M]

Consider the following relational schema:

Suppliers(sid: integer, sname: string, city: string, street: string)

Parts(pid: integer, pname:string, color:string)

Catalog (sid:integer, pid:integer, cost:real)

Consider the following relational query on the above database:

SELECT S.sname
FROM Suppliers S
WHERE S.sid NOT IN (SELECT C.sid
FROM Catalog C

WHERE C.pid NOT IN (SELECT P.pid FROM Parts P

WHERE P.color <> 'blue'))

Assume that relations corresponding to the above schema are not empty. Which one of the following is the correct interpretation of the above query?

(a) Find the names of all suppliers who have supplied a non-blue part

(b) Find the names of all suppliers who have not supplied a non-blue part

(c) Find the names of all suppliers who have supplied only blue parts

(d) Find the names of all suppliers who have not supplied only blue parts

37. [MCQ] [GATE-2008 : 2M]

Consider the following relational schema:

Student (school-id, sch-roll-no, sname, saddress)

School (school-id, sch-name, sch-addres, sch-phone)

Enrolment(school-id, sch-rollno, erollno, examname)

ExamResult (erollno, examname, marks)

What does the following SQL query output?

SELECT sch-name, COUNT (*)

FROM School C, Enrolment E,

ExamResult R

WHERE E.school-id=C.school-id

AND E.examname = R.examname

AND E.erollno=R.erollno

AND R.marks=100 AND S.school-id

IN

(SELECT school-id FROM student

GROUP BY school-id

HAVING COUNT (*)>200)

GROUP By school-id;

(a) for each school with more than 200 students appearing in exams, the name of the school and the number of 100s scored by its students.

(b) for each school with more than 200 students in it, the name of the school and the number of 100s scored by its students.

(c) for each school with more than 200 students in it, the name of the school and the number of its students scoring 100 in at least one exam.

(d) nothing; the query has a syntax error.

TRC

38. [MCQ] [GATE-2017 : 2M]

Consider a database that has the relation schemas EMP(EmpId. EmpName. DeptId). and DEPT (DeptName. Deptld). Note that the DeptId can be permitted to be NULL in the relation EMP. Consider the following queries on the database expressed in tuple relational calculus.

(i) $\{t \mid \exists u \in EMP(t[EmpName] = u[EmpName] \wedge \forall v \in DEPT(t[DeptId] \neq [Deptld]))\}$

(ii) $\{t \mid \exists u \in EMP(t[EmpName] = u[EmpName] \wedge \exists v \in DEPT(t[DeptId] \neq v[Deptld]))\}$

(iii) $[t \mid \exists u \in EMP(t[EmpName] = u[EmpName] \wedge \exists v \in DEPT(t[DeptId] = v[Deptld]))\}$

Which of the above queries are safe?

(a) (i) and (ii) only

(b) (i) and (iii) only

(c) (ii) and (iii) only

(d) (i), (ii) and (iii)

39. [MCQ] [GATE-2013 : 2M]

Consider the following relational schema.

Student (rollno: integer, sname: string)

Courses (courseno:integer, cname:string)

Registration (rollno: integer, courseno: integer, percent: real)

Which of the following queries are equivalent to this query in English?

Find the distinct names of all students who score more than 90 % in the course numbered 107

I. SELECT DISTINCT S.sname
FROM Students as S, Registration as R
WHERE R.rollno=S.roll.no AND
R.courseno =107 and R.percent >90

II. $\Pi_{sname}(\sigma_{courseno=107} \wedge \text{percent} > 90)$ (Registration ⋈ Student))

III. {T|∃S∈Students, ∃R∈Registration
(S. rollno = R. rollno ∧ R. courseno =107 ∧ R. percent > 90 ∧
T. sname = S. sname) }

IV. $\{<S_N> | \exists S_R \exists S_P (<S_R, S_N> \in \text{Students} \wedge <S_R, 107, R_P> \in \text{Registration} \wedge R_P > 90)\}$

(a) I, II, III and IV
(b) I, II and III only
(c) I, II and IV only
(d) I, III and IV only

40. [MCQ **[GATE-2009 : 2M]**

Let R and S be relation schemes such that R = {a, b, c} and S = {c}. Now consider the following queries on the database:

I. $\pi_{R-S}(r) - \pi_{R-S}(\pi_{R-S}(r) \times s - \pi_{R-S,S}(r))$

II. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall u \in s\ (\exists v \in r(u = v[s] \wedge t = v[R-S]))\}$

III. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall v \in r\ (\exists u \in s(u = v[s] \wedge t = v[R-S]))\}$

IV. Select R.a, R.b
from R,S
where R.c = S.c

Which of the above queries are equivalent?

(a) I and II
(b) I and III
(c) II and IV
(d) III and IV

41. [MCQ] **[GATE-2008 : 2M]**

Consider the following relational schema:

Student (**school-**<u>id, sch-roll-no</u>, sname, saddress)

School (<u>school-id</u>, sch-name, sch-addres, schphone)

Enrolment(<u>school-id, sch-roll-no</u>, erollno, examname)

ExamResult (<u>erollno, examname</u>, marks)

Consider the following tuple relational calculus query:

{t| ∃ E ∈ Enrolment t = E· school-id ∧ |{x|x ∈ ExamResult B.school-id=t ∧
(∃ B ∈ Exam Result B.erollno = x. erollno ∧ B. examname = x. examname ∧ B. marks > 35)}|÷|{x|x ∈ Enrolment ∧ x .school-id=t}| * 100 > 35}

If a student needs to score more than 35 marks to pass an exam, what does the query return?

(a) The empty set
(b) schools with more than 35% of its students enrolled in some exam or the other
(c) schools with a pass percentage above 35% over all exams taken together
(d) schools with a pass percentage above 35% over each exam

❑❑❑

ANSWER KEY

1.	(a, b)	**2.**	(819 to 820 or 205 to 205)	**3.**	(c)	**4.**	(1 to 1)
5.	(c)	**6.**	(4 to 4)	**7.**	(c)	**8.**	(a)
9.	(d)	**10.**	(a)	**11.**	(a)	**12.**	(a)
13.	(c)	**14.**	(a)	**15.**	(d)	**16.**	(2 to 2)
17.	(2 to 2)	**18.**	(b)	**19.**	(a)	**20.**	(5 to 5)
21.	(d)	**22.**	(7 to 7)	**23.**	(2.6 to 2.6)	**24.**	(2 to 2)
25.	(a)	**26.**	(d)	**27.**	(2 to 2)	**28.**	(b)
29.	(c)	**30.**	(b)	**31.**	(c)	**32.**	(b)
33.	(c)	**34.**	(a)	**35.**	(c)	**36.**	(d)
37.	(d)	**38.**	(d)	**39.**	(a)	**40.**	(a)
41.	(c)						

SOLUTIONS

1. (a, b)

$$\boxed{\frac{\pi_{AB}(R)}{\pi_B(S)} = \pi_A(R) - \pi_A[\pi_A(R) \times \pi_B(S) - \pi_{AB}(R)]}$$

For example, consider below relations:

Brand

Bid	B_name
b_1	AT
b_2	WC

Own

eid	Bid
e_1	b_1
e_1	b_2
e_2	b_1
e_3	b_1
e_3	b_2

π_{eid} (own) – π_{eid} [π_{eid} (Own) × π_{bid} (Brand)
– $\pi_{eid,bid}$ (Own)]

π_{eid} (own)
$\downarrow$

$$\begin{array}{|c|}\hline \text{eid} \\ e_1 \\ e_2 \\ e_3 \\ \hline\end{array} \times \begin{array}{|c|}\hline \text{bid} \\ b_1 \\ b_2 \\ \hline\end{array} \Rightarrow \begin{array}{|cc|}\hline \text{eid} & \text{bid} \\ e_1 & b_1 \\ e_1 & b_2 \\ e_2 & b_1 \\ e_2 & b_2 \\ e_3 & b_1 \\ e_3 & b_2 \\ \hline\end{array} - \text{Own} \quad \begin{array}{|cc|}\hline \text{eid} & \text{bid} \\ e_1 & b_1 \\ e_1 & b_2 \\ e_2 & b_1 \\ e_3 & b_1 \\ e_2 & b_2 \\ \hline\end{array}$$

$$\pi_{eid} \xrightarrow{e_2 \text{ Not own } b_2} \begin{array}{|cc|}\hline \text{eid} & \text{bid} \\ e_1 & b_1 \\ \hline\end{array}$$

π_{eid} – π_{eid}[π_{eid}(Own) × π_{bid}(Brand)
(Own) – $\pi_{eid,\,bid}$ (Own)]
$\downarrow$ $\downarrow$

$$\begin{array}{|c|}\hline \text{eid} \\ e_1 \\ e_2 \\ e_3 \\ \hline\end{array} - \begin{array}{|c|}\hline \text{eid} \\ e_2 \\ \hline\end{array} \Rightarrow \begin{array}{|c|}\hline \text{eid} \\ e_1 \\ e_3 \\ \hline\end{array}$$

Eid's who own all brands.

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2. (819 to 820 or 205 to 205)

Ist approach

r(AB) = 1200 Tuples

A range is = 6 to 20 ⇒ 15 distinct values

B range is = 1 to 20 ⇒ 20 distinct values

$\sigma_{(A>10)} V_{(B=18)}(r)$

A has = 15 distinct values

B has = 20 distinct values

$$\underset{\substack{\Downarrow \\ 11,12,13....20}}{P(A>10)} = \frac{10}{15} = \frac{2}{3}$$

$$P(B=18) = \frac{1}{20}$$

$P(A > 10 \wedge B = 18)$

$$= \frac{2}{3} \times \frac{1}{20} = \frac{1}{30}$$

$\sigma_{(A>10)} V_{(B=18)}(r) = P(A > 10) + P(B = 18) - P(A > 10 \wedge B = 18)$

$$\Rightarrow \frac{2}{3} + \frac{1}{20} - \frac{1}{30}$$

$$\Rightarrow \frac{40+3-2}{60} = \frac{41}{60}$$

Estimated number of tuples

$$= \frac{41}{\cancel{60}} \times {}^{20}\cancel{1200} = 820$$

IInd approach

$\sigma_{(A>10)} V_{(B=18)}(r)$

A range 6 to 20 = 15 distinct values

B range 1 to 20 = 20 distinct values

(1) A > 10 ⇒ For (11, 12, 13, ….. 20), for 10 A value there are 20 distinct value of B (11, 12, 13 ….. 20)

So total for A = 10 × 20 = 200

For B = 18, one tuple of B value there are total 15 distinct value of A.

So total for B = 1 × 15 = 15

But there are some value A > 10, B = 18 appearing more than once, so there are 10 tuples that are appearing more than once = 10

Estimated number of tuples is = 200 + 15 – 10 = 205

Or

By Relational Algebra:

In A distinct value = 15 (6 to 20)

B distinct value = 20

Estimated number of tuples

$$\sigma_{(A>10 \vee B=18)} = \frac{41}{\cancel{60}_3} \times \left[\cancel{20} \times \cancel{15}^5\right] = 205$$

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3. (c)

$\Pi_{empNo.}$ **(empAge ⋈ $_{(age > age1)}$ $\rho_{empNo1,\ age1}$ (empAge))**

⇓

Employee number whose age is greater than any employee age.

empAge	
empNo	**age**
e_1	30
e_2	40
e_3	50

\>

empAge1	
empNo1	**age1**
e_1	30
e_2	40
e_3	50

e_1 30 > 30 false

e_2 40 > 30 true

e_3 50 > 30 true

e_3 50 > 40 true

Output

empNo.
e_2
e_3

→ emp No. whose age is not Minimum.

Therefore, option C is the correct expression generated for given RA query.

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4. **(1 to 1)**

$x_2 - x_1 = x_2 \Rightarrow$ 1 tuple

P	R
4 tuples	4 tuples
3 attributes	2 attributes

$P \times R = 4 \times 4 = 16$ tuples

$3 + 2 = 5$ attributes

P.X	P.Y	P.Z	Ry	R.V
x_2	y_2	z_2	y_2	v_2

$\Pi_x(-) \Rightarrow$

x
x_2

output of part I

Q	R
4 tuples	4 tuples
3 attributes	2 attributes

$Q \times R = 4 \times 4 = 16$ tuples

$3 + 2 = 5$ attributes

$Q.T > 2$

Q.X	Q.Y	Q.T	Ry	R.V
x_1	y_2	5	y_1	v_3
x_1	y_2	5	y_2	v_2
x_1	y_1	6	y_1	v_1

$\Pi_x \Rightarrow$

x
x_1

output of part II

Part I – Part II

$x_2 - x_1 \Rightarrow x_2$, only 1 tuple in output.

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5. **(c)**

Q: $r \bowtie (\sigma_{B<5}(S))$, Outputs:

A	B	C
a_{11}	3	C_{21}
a_{12}	3	C_{21}
a_{13}	4	C_{22}

Now, checking for given options

R

A	B
a_{11}	3
a_{12}	3
a_{13}	4
a_{14}	7
a_{15}	9
a_{16}	9

$\xrightarrow{\text{FK}}$

S

B	C
3	C_{21}
4	C_{22}
7	C_{23}
9	C_{24}

R ⋈ S

A	B	C
a_{11}	3	C_{21}
a_{12}	3	C_{21}
a_{13}	4	C_{22}
a_{14}	7	C_{23}
a_{15}	9	C_{24}
a_{16}	9	C_{24}

In option (a) the output is same as given query

A	B	C
a_{11}	3	C_{21}
a_{12}	3	C_{21}
a_{13}	4	C_{22}

$\therefore$ Equivalent

In option (b) the output is same as given query therefore equivalent

A	B	C
a_{11}	3	C_{21}
a_{12}	3	C_{21}
a_{13}	4	C_{22}

In option (c) the output is not same as given query

A	**B**	**C**
a_{11}	3	C_{21}
a_{12}	3	C_{21}
a_{13}	4	C_{22}
a_{14}	7	NULL
a_{15}	9	NULL
a_{16}	9	NULL

In option (d) the output is same as given query

A	**B**	**C**
a_{11}	3	C_{21}
a_{12}	3	C_{21}
a_{13}	4	C_{22}

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6. (4 to 4)

$\pi_{\text{course Name}}\ (\sigma_{\text{student Name = 'SA'}}\ (\text{CR}))$

T_1 :

Course Name
CA
&CB
CC

T_2 : CR ÷ T_1

CR ÷

CA
CB
CC

- When CR ÷ CA, the output is as below:

SA
SC
SD
SE
SF

- When CR ÷ [CA, CB] the output is as below:

SA
SC
SD
SE
SF

- When CR÷ [CA, CB, CC] the output is as below:

SA
SC
SD
SF

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7. (c)

R_1(AB)	
1	5
3	7

R_2 (AC)	
1	7
4	9

Natural join between R_1 and R_2:

$R_1 \bowtie R_2$ =

A	B	C
1	5	7

Full natural outer join between R_1 and R_2:

	A	**B**	**C**
a	1	5	Null
b	1	Null	7
c	3	Null	9
d	4	7	Null
e	**1**	**5**	**7**
f	**3**	**7**	**Null**
g	**4**	**Null**	**9**

R_1 ⟗ R_2 =

A	B	C	
1	5	7	e
3	7	Null	f
4	Null	9	g

R_1 ⟗ R_2 = contain e, f and g.

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8. (a)

$\pi_{A1}(\pi_{A2}(\sigma_{F1}(\sigma_{F2}(r))))$

A_1 & A_1 one set of Attributes such that $\boxed{A_1 \subset A_2}$

I. $\pi_{A1}(\pi_{A2}(r)) \Rightarrow \pi_{A1}(r)\ [\because\ A_1 \subset A_2]$

II. $(\sigma_{F1}(\sigma_{F2}(r) \Rightarrow \sigma_{F_1 \wedge F_2}(r)$

$\pi_{A1}(\pi_{A2}(\sigma_{F1}(\sigma_{F2}(r)))) \Rightarrow \pi_{A1}\left(\sigma_{F_1 \wedge F_2}(r)\right)$

F1 & F2 are Boolean expression based on attribute in Relation r.

Example:

I. Assume $\boxed{A_1 \subset A_2}$

A_2 : Sid Sname

A_1 : Sid

$$\underset{(A1)}{\pi_{Sid}}\left[\underset{(A2)}{\pi_{Sid,Sname}}(r)\right]$$
$$\Downarrow$$
$$\pi_{Sid} \equiv \pi_{A1}$$

II. F_1 CGPA > 8

F_2 Branch = 'CS'

$$\sigma_{CGPA>8}\left[\sigma_{Branch:='CS'}(r)\right] \Rightarrow \sigma_{CGPA>8\ \wedge\ Branch='CS'}(r)$$

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9. (d)

The inner query chooses workers whose ages are lower than or equal to those of at least one of his dependents. Employees whose ages exceed those of all of his dependents are obtained by removing those from the collection of employees.

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10. (a)

Two factors are involved when A is in the outer loop of a nested loop to join A and B.

The number of blocks that include all the rows in A should be fetched.

Number of blocks comprising all rows of B divided by the number of rows of A.

(Worst case scenario) All rows of B match all rows of A.

In the above question |R| < |S|

(i) Will decrease when the outer table's row count decreases since fewer rows require fewer blocks.

(ii) If we continue R in the outer loop, there are fewer rows and more blocks.

If we continue S in the outer loop, there are more rows and fewer blocks in R.

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11. (a)

$R_1(AB)$

A	**B**
1	7
2	7
3	8
4	9

$\xrightarrow{FK}$

C	D
6	1
7	2
8	5
9	6

(a) $\pi_B(r_1) - \pi_c(r_2) = \phi$

$[7, 8, 9] - [6, 7, 8, 9] = \phi$

(b) $\pi_C(r_2) - \pi_B(r_1) = \phi$

$[6, 7, 8, 9] - [7, 8, 9] = 6 \neq \phi$

(c) $\pi_B(r_1) = \pi_c(r_2)$

$[7, 8, 9] \neq [6, 7, 8, 9]$

(d) $\pi_B(r_1) - \pi_c(r_2) \neq \phi$

$[7, 8, 9] - [6, 7, 8, 9] = \phi$

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12. (a)

$A \cup B \equiv \underline{A}$		
ID	**Name**	**Age**
12	Arun	60
15	Shreya	24
99	Rohit	11
25	Hari	40
98	Rohit	20

C		
ID	**Phone**	**Area**
10	2200	02
99	2100	01

A	**C**
5 tuple	2 tuple
3 attribute	3 attribute

$A \times C = 5 \times 2 = 10$ Tuples

$= 3 + 3 = 6$ Attribute

$(A \cup B) \bowtie_{A.I.d > 40 V C.I.d < 15} (C)$

A.Id	A.Name	A.Age	C.Id	C.Phone	C. Area
12	**Arun**	**60**	**10**	**2200**	**02**
12	Arun	60	99	2100	01
15	**Shreya**	**24**	**10**	**2200**	**02**
15	Shreya	24	99	2100	01
99	**Rohit**	**11**	**10**	**2200**	**02**
99	**Rohit**	**11**	**99**	**2100**	**01**
25	**Hari**	**40**	**99**	**2200**	**02**
25	Hari	40	99	2100	01
98	**Rohit**	**20**	**10**	**2200**	**02**
98	**Rohit**	**20**	**99**	**2100**	**01**

The output relation will consist of 7 tuples marked in bold in the above relation.

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13. (c)

Query I: Condition applied on a particular value.

Query II: Condition applied on a Range C_1 to $C_2 \Rightarrow$ range queries.

Hashing will perform better for a particular value Q_1.

Ordered index perform better on range queries Q_2 (B^+ Tree).

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14. (a)

R(A$\underline{B}$C) S(BDE) R(A$\underline{B}$C) S(BDE)

$\downarrow$ $\downarrow$ $R \bowtie S = \pi_{ABCDE}[\sigma_{R.B = S.B}(R \times S)]$

200 Tuples 100 Tuples

FD'S : $[B \rightarrow A, A \rightarrow C]$

$[B]^+ = [ABC]$ getting all attribute of relation R

$\therefore$ B is the candidate key for relation R.

Relation R having 200 unique values (entries)

$R \bowtie S = 100$ Tuples

Because maximum 100 entries (value) of relation S match with relation R.

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15. (d)

R ($\underline{PQ}$, R_1 R_2, R_3) S ($\underline{P.Q}$, S_1S_2)

I. $\pi_p (R \bowtie S)$

$$\pi_P \left[\sigma_{\substack{R.P = S.P \\ R.Q = S.Q}} \wedge (R \times S) \right]$$

II. $\pi_p (R) \bowtie \pi_p (S)$

Here we get the value of P in which Q not equal.

R ($\underline{PQ}$ R_1 R_2 R_3)	
P	Q
1	2
3	5
4	7

S ($\underline{PQ}$ S_1 S_2)	
P	Q
1	2
3	6
4	7

I. $\pi_p(R \bowtie S) = \begin{array}{|cc|}\hline P & Q \\ 1 & 2 \\ 4 & 7 \\ \hline\end{array} \xrightarrow{\pi_p} \begin{array}{|c|}\hline P \\ 1 \\ 4 \\ \hline\end{array}$

II. $\pi_P(R) \times \pi_P(S)$

$$\begin{array}{|c|}\hline P \\ 1 \\ 3 \\ 4 \\ \hline\end{array} \bowtie \begin{array}{|c|}\hline P \\ 1 \\ 3 \\ 4 \\ \hline\end{array} \xrightarrow{\text{Output}} \begin{array}{|c|}\hline P \\ 1 \\ 3 \\ 4 \\ \hline\end{array}$$

(III) $\pi_P[\pi_{PQ}(R) \cap \pi_{PQ}(S)]$

$$\pi_P \leftarrow \begin{array}{|cc|}\hline P & Q \\ 1 & 2 \\ 4 & 7 \\ \hline\end{array} \; \begin{array}{|c|}\hline P \\ 1 \\ 4 \\ \hline\end{array}$$

(IV) $\boxed{R \cap S = R - (R - S)}$

Therefore, I, III & IV are equivalent.

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16. (2 to 2)

Roll Num	Name	Gender	Marks
1	Naman	M	62
2	**Aliya**	**F**	**70**
3	**Aliya**	**F**	**80**
4	James	M	82
5	Swati	F	65

The given query on above relation will yield two tuples in the output relation

"SELECT *

FROM Student

WHERE gender = 'F' AND marks > 65;"

Number of tuples (Records) = 2

Output

Roll No.	Name	Gender	Marks
2	Aliya	F	70
3	Aliya	F	80

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17. (2 to 2)

Not exist: Return true if inner Query Result empty.

Working of EXCEPT/minus:

a_{11} a_{12} Except $a_{11} \Rightarrow a_{12}$

a_{11} a_{12} Except $a_{12} \Rightarrow a_{11}$

a_{11} a_{12} Except a_{12} $a_{11} \Rightarrow$ Empty

a_{11} a_{12} Except a_{22} $a_{23} \Rightarrow a_{11}$ a_{12}

S01: C_{11} C_{12} Except C_{11} $C_{12} \Rightarrow$ Empty, therefore S01 returns true

S02: C_{11} C_{12} Except C_{11} results C_{12}, non-empty relation (false)

S03: C_{11} C_{12} Except C_{21} C_{22} C_{23} results C_{11} C_{12} in non-empty relation (false)

S04 C_{11} C_{12} Except C_{11} C_{12} results in empty relation (true)

S05: C_{11} C_{12} Except C_{11} C_{21} results C_{12} non-empty relation (false)

Therefore, the output:

SNo
S01
S04

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18. (b)

It's a nested query but not Co-related query.

Evaluate the innermost query first:

$$\text{Inner Query} \left(\begin{array}{l} \text{Select average (salary)} \\ \text{from emp.} \\ \text{Average salary} \end{array} \right)$$

will yield the average salary of all the employees and the outer query will generate the department id of employees whose gender is female and whose salary is greater than the salaries generated in the inner query.

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19. (a)

Running the inner query, the average cost from catalogue where part number is P4

$$\text{Average cost} = \frac{200+250}{2}$$

Average cost = 225

Now the outer query executes and we select sno., sname from the join of Suppliers and Catalogue relation where s.no = c.sno AND cost > 225, the output relation is as below:

sno.	sname
S_2	Balaji furniture
S_3	Premium furniture
S_3	Premium furniture
S_3	Premium furniture

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20. (5 to 5)

5 Tuples in Student [S] and 5 Tuples in Performance [P] so total 25 tuples in the CROSS Product. On applying the condition in the given query **WHERE P.Marks > 84** we get only 5 tuples for each Student_name after that is:

Roll_no.	Student_name	Roll_no	Subject_code	Marks
1	Amit	1	A	86
1	Amit	1	B	95
1	Amit	1	C	90
1	Amit	2	A	89
1	Amit	2	C	92
1	Amit	3	C	80
⋮	⋮	⋮	⋮	⋮
5	Smita	3	C	80

If we group by Student_name that have marks greater than 84,finally get the below relation:

Name	Sum (marks)
Amit	452
Priya	452
Vinit	452
Rohan	452
Smita	452

The output will consist of 5 tuples.

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21. (d)

Book (B)

Isbn	Bname
1	A
2	B
3	C
4	D
5	E
6	F

Stock (S)

Isbn	Bname
2	10
4	20
5	30
6	40
7	50

Query I: Inner join

Isbn	Copies
2	10
4	20
5	30
6	40

Query II: B left outer join S

Isbn	Copies
2	10
4	20
5	30
6	40
1	NULL
3	NULL

Query III B right outer join S

Isbn	Copies
2	10
4	20
5	30
6	40
7	50

Query IV B full outer join S

Isbn	Copies
2	10
4	20
5	30
6	40
1	Null
3	Null
7	50

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22. (7 to 7)

Lets say the query structure is Q_1 AND Q_2 where Q_1 is:

SELECT ta.player FROM top_scorer AS ta
WHERE ta.goals > ALL (SELECT tb.goals FROM top_scorer AS tb
WHERE tb.country='Spain'), and Q_2 is:
ta.goals > ANY (SELECT tc.goals FROM top_scorer AS tc
WHERE tc.country ='Germany')

Note:

ALL returns true when the condition is false.

ANY returns true when the condition is true for atleast one value.

In Q_1 , ALL (empty set) returns true as there is no entry for the country 'Spain'

Q_2 returns the following tuples:

$$Q_2\begin{cases}\text{Germany}:(16,14,11,10,10)\\ \text{ta.goal>Any}(16,14,11,10,10)\end{cases}$$

ta.goal will yield 7 Tuples, So Q_1 AND Q_2 returns 7 tuples. The output relation will be:

Player	Country	Goals
Klose	Germany	16
Ronaldo	Brazil	15
G Muller	Germany	14
Fontaine	France	13
Pele	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11

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23. (2.6 to 2.6)

The given query is a related nested query. Output of inner Query:

"(SELECT DeptName, COUNT(EmpId) AS EC(DeptName, Num)
FROM EMP
GROUP BY DeptName)" is as follows:

EC:	
DeptName	**Num**
AA	4
AB	3
AC	3
AD	2
AE	1

After the inner query execution, the outer query will find the average as follows:

$$\text{Average} = \frac{4+3+3+2+1}{5} = \frac{13}{5} = 2.6$$

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24. (2 to 2)

Total	
Name	**Capacity**
Ajmer	20
Bikaner	40
Churu	30
Dungargarh	10

≥ 25

$$\text{Total Average} = \frac{20+40+30+10}{4} = \frac{100}{4} = 25$$

Total Average Capacity
25

Total Capacity $\geq$ Total Average Capacity

$\boxed{\text{Total Capacity} \geq 25}$

Name	Capacity
Bikaner	40
Churu	30

2 Tuples in the output relation.

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25. **(a)**
Consider the given relation Cinema along with the data:
Cinema

Theater	Address	Capacity
T_1	A	100
T2	B	200
T3	C	300
T4	D	400

When we execute the query with the condition mentioned in options on the above relation we get:

(a) WHERE P_1. Capacity ≥ ALL (100, 200, 300, 400)
(x ≥ 100) AND (x ≥ 200) AND (x ≥ 300) AND (x ≥ 400)
400 is in output i.e theater with maximum Capacity.

(b) WHERE P_1 Capacity ≥ ANY (100, 200, 300, 400)
(x ≥ 100) OR (x ≥ 200) OR (x ≥ 300) (OR) (x ≥ 400)
100, 200, 300, 400 is in output, i.e not getting theater with maximum Capacity

(c) WHERE P_1. Capacity > ALL (400) returns 0 tuples.

(d) WHERE P_1. Capacity > ANY (400) returns 0 tuples.

Therefore, condition in option(a) will always find the addresses of theaters with maximum capacity.

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26. **(d)**
SELECT operation in SQL is equivalent to PROJECTION operation in relational algebra except the fact that PROJECTION contains only distinct values and eliminates duplicates while SELECT in SQL retains the duplicate values.
For example consider SELECT and PROJECTION operation outputs on the given relation R with A, B, and C fields:
R (ABC)

A	B	C
2	5	7
3	6	9
2	5	8

SELECT AB
FROM R $\xrightarrow{\text{O/P}}$

A	B
2	5
3	6
2	5

$\pi_{AB}(R) \longrightarrow$

A	B
2	5
3	6

= SELECT [DISTINCT]AB FROM R;

SQL:

$$\left\{\begin{array}{l}\text{Select[Distinct] } A_1\, A_2\, A_3 \ldots\ldots A_n = \text{Projection}[\pi] \\ \text{From } R_1\, R_2\, R_3 \ldots R_n = \text{Cross product}[x] \\ \text{Where condition } [P] = \text{Selection}[\sigma]\end{array}\right.$$

Relation Algebra query:
$\pi A_1 A_2 \ldots\ldots A_n\, [\sigma_{\text{condition}}\, (R_1 \times R_2 \times R_3 \times \ldots. \times R_m)]$

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27. **(2 to 2)**
The cross product of Student S, Performance P will yield the following tuples:

S. Roll_No.	S. Student_Name	P. Roll no.	P. course	P. marks
1	Raj	1	Math	80
1	Raj	1	English	70
2	Rohit	2	Math	75
2	Rohit	2	Physics	65
3	Raj	3	English	80
3	Raj	3	Math	80

After performing the group by operation on the the above output relation based on Student_Name we get the following output:

S_Name	P. marks
Raj	310
Rohit	140

The output will consists of 2 tuples.

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28. (b)
The query is a co-related nested query. The inner query yields the customer Id whose rating is NOT GOOD. The Operator **NOT EXISTS** returns true if inner query result is empty. It works like a complement operation, so the outer query returns the names of all the employees with all their customers having a 'GOOD' rating.
All A.Id will be in output

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29. (c)
R:

A	B	C
1	4	8
1	4	8
5	6	10
5	6	10

S:

A	D	E
1	4	8
4	5	11
5	8	9
5	8	9

The **output of the given query:** Select * from R where a in (select S.a from S) is as follows:

A	B	C
1	4	8
1	4	8
5	6	10
5	6	10

In option (a):
$4 \times 4 = 16$ tuple
$3 + 3 = 6$ attribute

A	B	C
1	4	8
1	4	8
5	6	10
5	6	10
5	6	10
5	6	10

In option (b):

A	B	C
1	4	8
5	6	10

In option (c):

A	B	C
1	4	8
1	4	8
5	6	10
5	6	10

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30. (b)
Inner query gives dept id and max(Hire data) in each department where location id is 1700.
Complete query gives last name and hire date
Of maximum (latest hiring) in their respective dept at which location id = 1700

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31. (c)

- Having even does not have group by clause.
- Having clause applies condition on each group.
- All Attribute used in the group by clause must be (Present) in select clause.

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32. (b)

ALL returns **True** if inner query returns no tuples or ϕ. So, Number of tuples returned will be number of tuples in relation A that is 3.

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33. (c)

S:	
Borrower	**Bank_ Manager**
Ramesh	Sunderajan [SJ]
Suresh	Ramgopal [RG]
Mahesh	Sunderajan [SJ]

⋈

T:	
Bank_ Manager	**Loan_ Amount**
Sunderajan [SJ]	10000
Ram Gopal [RG]	5000
Sunderajan [SJ]	7000

3×3 = 9 Tuple

2+2 = 4 Attribute

S. Borrower	**S. Bank_ Manager**	**T. Bank_ Manager**	**T.Loan_ Amount**
Ramesh	**[SJ]**	**[SJ]**	**10,000**
Ramesh	[SJ]	RG	5000
Ramesh	**[SJ]**	**[SJ]**	**7000**
Suresh	RG	[SJ]	10,000
Suresh	**RG**	**RG**	**5000**
Suresh	RG	[SJ]	7000
Mahesh	**[SJ]**	**[SJ]**	**10000**
Mahesh	[SJ]	RG	5000
Mahesh	**[SJ]**	**[SJ]**	**7000**

5 Tuples in the output.

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34. (a)

X	Y
1	1

[MX + 1]	X	Y	2 * MY + 1
	1	1	
X = 1 + 1 = 2	2	3	$2 \times 1 + 1 = 3$
2 + 1 = 3	3	7	$2 \times 3 + 1 = 7$
3 + 1 = 4	4	15	$2 \times 7 + 1 = 15$
4 + 1 = 5	5	31	$2 \times 15 + 1 = 31$
5 + 1 = 6	6	63	$2 \times 31 + 1 = 63$
6 + 1 = 7	7	127	$2 \times 63 + 1 = 127$
7 + 1 = 8	8	255	$2 \times 127 + 1 = 255$
8 + 1 = 9	9	511	$2 \times 255 + 1 = 511$

Therefore, the query: SELECT Y FROM T WHERE X = 7; will yield 127.

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35. (c)

EXIST returns true if inner query result is non-empty. It is a corelated nested query & inner query condition is Age > 65 & passenger Pid = reservation Pid.

Corelated nested query: Top → Bottom → Top

Outer → Inner → Outer query

In outer query ⇒ condition class = 'AC'

will return Pid 0, **1**, 5, **3** and inner query Age > 65.

returns Pid **1**, 2, **3**, final the output relation is as follows:

Pid
1
3

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36. (d)

Supplier			
Sid	Sname	City	Street
1	A	JP	X
2	B	UDP	Y
3	C	AJMER	Z

Catalouge		
Sid	Pid	Cost
1	11	1cr
1	21	2cr
2	11	1cr
3	21	2cr

Parts		
Pid	Pname	Color
11	Px	Red
21	Py	Blue

Hence, option (d) generates the output in the given query.

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37. (d)

If the select clause includes columns with and without aggregate data. The Select clause's non-aggregate columns must all exist in the Group By clause. Nevertheless, this query's Group by clause uses school-id rather than school-name. Hence, the query generates the syntax error.

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38. (d)

All three expressions are identical before $\wedge$ the operation, i.e.return true if for each tuple t we have finite number of tuple u in employee table for which they have same employee_name.

(I) But in part two, for each tuple v in department there may exist infinite number of tuple t for which they may not be equal. i.e. true for finite number of tuples $\wedge$ true for infinite number of tuples, over all true for finite tuple.

(II) There may exist infinite number of tuple for which at least one tuple v belongs to department table for which they may not be equal. i.e. true for finite number of tuples $\wedge$ true for infinite number of tuples, over all true for finite tuple.

(III) This statement is true for a finite number of tuples because only a finite number of tuples that are identical to at least one tuple in department. Due to the limited number of tuples in department tables, tuples that are identical may not exceed all tuples in department table in the event of equity. Specifically, true for a finite tuple, true for a finite tuple overall.

Therefore, all TRC queries will yield finite tuples, indicating that all are safe.

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39. (a)

(a) This is a SQL query expression. It first perform a cross product of Students and Registration, then WHERE clause only keeps those rows in the cross product set where the student is registered for course no 107, and percentage is > 90. Then select distinct statement gives the distinct names of those students as the result set.

(b) This is a relational algebra expression. It first perform a NATURAL JOIN of Students and Registration (NATURAL JOIN implicitly joins on the basis of common attribute, which here is rollno), then the select operation(sigma) keeps only those rows where the student is registered for courseno 107, and percentage is > 90. And then the projection operation (pi) projects only distinct student names from the set.

Note: Projection operation (pi) always gives the distinct result.

(c) This is a Tuple Relational Calculus (TRC) language expression, It is not a procedural language (i.e. it only tells "what to do", not "how to do"). It just represents a declarative mathematical expression.

Here T is a Tuple variable.

From left to right, it can be read like this, "It is a set of tuples T, where, there exists a tuple S in Relation Students, and there exist a tuple R in relation Registration, such that S.rollno = R.rollno AND R.couseno = 107 AND R.percent > 90 AND T.sname = S.sname". And the schema of this result is (sname), i.e. each tuple T will contain only student name, because only T.sname has been defined in the expression.

As TRC is a mathematical expression, hence it is expected to give only distinct result set.

(d) This is a Domain Relational Calculus (DRC) language expression. This is also not procedural. Here SN is a Domain Variable. It can be read from left to right like this "The set of domain variable SN, where, there exist a domain variable SR, and a domain variable Rp, such that, SN and SR domain variables is in relation Students and SR, 107, RP is a domain variables set in relation Registration, AND RP > 90 "Above, SN represents sname domain attribute in Students relation, SR represents rollno domain attribute in Students relation, and RP represents percentage domain attribute in Registration relation. The schema for the result set is (SN), i.e. only student name.

As DRC is a mathematical expression, hence it is expected to give only distinct result set.

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40. (a)

$$\frac{\pi_{AB}(R)}{\pi_B(S)} = \pi_A(R) - \pi_A\left[\pi_A(R) \times \pi_B(S) - \pi_{AB}(R)\right]$$

$$\frac{\pi_{(R-S)S}(R)}{S}$$

$$\pi_{R-S}(r) - \pi_{R-S}\left\lfloor \pi_{(R-S)}(r) \times S - \pi_{(R-S)S}(r) \right\rfloor$$

Same as Division expansion.

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41. (c)

t | ∃E Enrolment t = E. school-id

Returns school-ids from Enrolment table SUCH THAT

- |{x; | x ∈ Enrolment ∧ x. school-id = t ∧ (∃B ∈ ExamResult B. erollno x. erollno ∧ B.examname = x. examname ∧ B.marks > 35)}÷
- the number of student enrolments from the school for exams with marks > 35 divides
    - |{x | x ∈ Enrolment ∧ x.school-id = t}|
    - total number of student enrolments from the school
      *100 > 35
    - percentage of student enrolments with mark > 35 is > 35

Selecting the school-ids where the pass percentage of students across all the examinations taken combined is > 35 is necessary because passing an exam requires a mark of > 35.

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CHAPTER 5

FILE ORGANIZATION AND INDEXING

File Organization and Indexing

1. **[MCQ]** **[GATE-2015 : 1M]**

A file is organized so that the ordering of data records is the same as or close to the ordering of data entries in some index. Then that index is called.

(a) Dense (b) Sparse
(c) Clustered (d) Unclustered

2. **[MCQ]** **[GATE-2013 : 1M]**

An index is clustered, if

(a) it is on a set of fields that form a candidate key.
(b) it is on a set of fields that include the primary key.
(c) the data record of the file are organized in the same order as the date entries of the index
(d) the data records of the file are organized not in the same order as the data entries of the index.

3. **[MCQ]** **[GATE-2008 : 1M]**

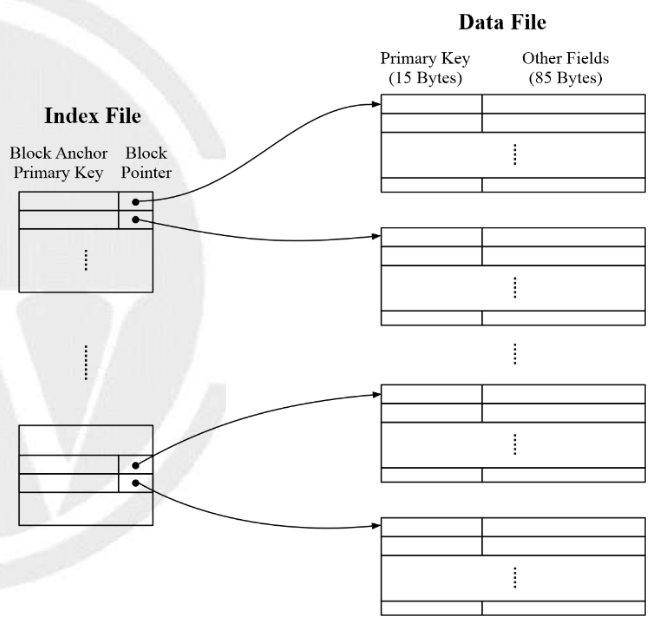
A clustering index is defined on the fields which are of type

(a) non-key and ordering
(b) non-key and non-ordering
(c) key and ordering
(d) key and non-ordering

Multi Level Indexing

4. **[NAT]** **[GATE-2023: 2M]**

Consider a database of fixed-length records, stored as an ordered file. The database has 25,000 records, with each record being 100 bytes, of which the primary key occupies 15 bytes. The data file is block-aligned in that each data record is fully contained within a block. The database is indexed by a primary index file, which is also stored as a block-aligned ordered file. The figure below depicts this indexing scheme.



Suppose the block size of the file system is 1024 bytes, and a pointer to a block occupies 5 bytes. The system uses binary search on the index file to search for a record with a given key. You may assume that a binary search on an index file of b blocks takes [log2 b] block accesses in the worst case.

Given a key, the number of block accesses required to identify the block in the data file that may contain a record with the key. in the worst case, is_____.

5. **[NAT]** **[GATE-2021 : 1M]**
A data file consisting of 1,50,000 student-records is stored on a hard disk with block size of 4096 bytes. The data file is sorted on the primary key RollNo. The size of a record pointer for this disk is 7 bytes. Each student-record has a candidate key attribute called ANum of size 12 bytes. Suppose an index file with records consisting of two fields, ANum value and the record pointer to the corresponding student record, is built and stored on the same disk. Assume that the records of data file and index file are not split across disk blocks. The number of blocks in the index file is ________________.

6. **[MCQ]** **[GATE-2008 : 2M]**
Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is built on the key field of the file, and a multilevel index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multilevel index are respectively-
(a) 8 and 0 (b) 128 and 6
(c) 256 and 4 (d) 512 and 5

B Tree and B^+ Tree

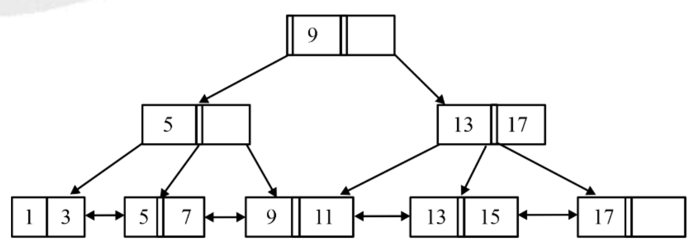
7. **[NAT]** **[GATE-2020 : 2M]**
Consider a database implemented using B+ tree for file indexing and installed on a disk drive with block size of 4 KB. The size of search key is 12 bytes and the size of tree/disk pointer is 8 bytes. Assume that the database has one million records. Also assume that no node of the B^+ tree and no records are present initially in main memory. Consider that each record fits into one disk block. The minimum number of disk accesses required to retrieve any record in the database is_______.

8. **[NAT]** **[GATE-2017 : 2M]**
In a B^+ tree, if the search-key value is 8 bytes long, the block size is 512 bytes and the block pointer size is 2 bytes, then the maximum order of the B^+ tree is _____________.

9. **[MCQ]** **[GATE-2016 : 1M]**
B^+ Tree are considered BALANCED because
(a) the length of the paths from the root to all leaf nodes are all equal.
(b) the lengths of the paths from the root to all leaf nodes differ from each other by at most 1
(c) the number of children of any two nonleaf sibling nodes differ by at most 1 .
(d) the number of records in any two leaf nodes differ by at most 1

10. **[NAT]** **[GATE-2015 : 2M]**
Consider a B^+ tree in which the search key is 12 bytes long, block size is 1024 bytes, record pointer is 10 bytes long and block pointer is 8 bytes long. The maximum number of keys that can be accommodated in each non-leaf node of the tree is _____________

11. **[NAT]** **[GATE-2015 : 2M]**
With reference to the B^+ tree index of order 1 shown below, the minimum number of nodes (including the Root node) that must be fetched in order to satisfy the following query: "Get all records with a search key greater than or equal to 7 and less than 15" is?



12. **[MCQ]** **[GATE-2010 : 1M]**
Consider a B^+ tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?
(a) 1 (b) 2
(c) 3 (d) 4

13. [MCQ] [GATE-2009: 2M]

The following key values are inserted into a B^+ -tree in which order of the internal nodes is 3, and that of the leaf nodes is 2, in the sequence given below. The order of internal nodes is the maximum number of tree pointers in each node, and the order of leaf nodes is the maximum number of data items that can be stored in it. The B^+ -tree is initially empty.

10, 3, 6, 8, 4, 2, 1

The maximum number of times leaf nodes would get split up as a result of these insertions is

(a) 2
(b) 3
(c) 4
(d) 5

14. [MCQ] [GATE-2008: 2M]

A B-tree of order 4 is built from scratch by 10 successive insertions. What is the maximum number of node splitting operations that may take place?

(a) 3 (b) 4
(c) 5 (d) 6

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ANSWER KEY

1.	(c)	**2.**	(c)	**3.**	(a)	**4.**	(6 to 6)
5.	(698 to 698)	**6.**	(c)	**7.**	(4 to 4)	**8.**	(52 to 52)
9.	(a)	**10.**	(50 to 50)	**11.**	(5 to 5)	**12.**	(b)
13	(c)	**14.**	(c)				

SOLUTIONS

1. (c)

A file is organized so that the ordering of data records is the same as or close to the ordering of data entries in some index then the arrangement is culstered.

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2. (c)

The data record of the file is organized in the same order as the date entries of the index. A clustered index consists of non-key + ordered file.

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3. (a)

Clustering Index: Non key + ordered field

Primary Index: Key + ordered field

Secondary Index:

(a) Non key + unordered field [Secondary index over non key attribute].

(b) Key + unordered field [Secondary index over key attribute].

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4. (6 to 6)

Total number of Records = 25,000

Block size = 1024 Byte, Records size = 100 Byte

Key = 15 byte, B_P = 5 byte, Fixed length, Unspanned organization, primary index

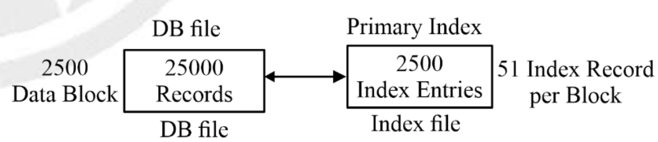
Block factor of DB file $[B_{FDB}] = \left\lfloor \frac{\text{Block size}}{\text{Record size}} \right\rfloor \Rightarrow$

$\left\lfloor \frac{1024\text{ B}}{100\text{ B}} \right\rfloor$ = 10 Records per Block

Total number of Records = 25,000

Total number of DB block $= \left\lceil \frac{25000}{10} \right\rceil$

= 2500 Data Blocks



One Index record size = Size of key + size of block pointer

15 + 5 = 20 Byte

Block factor of Index file $= \left\lfloor \frac{1024\text{ B}}{20\text{ B}} \right\rfloor = \lfloor 51.2 \rfloor = 51$

51 index record per Block (entries).

Primary Index:

SPARSE $\Rightarrow$ Total number of Index Entries = 2500 (Number of DB Block)

Total number Index entries = 2500

Block factor of Index file = 51 Index entries per Block.

$\lfloor$B: Index Block$\rfloor$

Total number of Index Block [b] = $\left\lceil \frac{2500}{51} \right\rceil$

= $\lceil 49.01 \rceil$ = 50 Index Block

In worst case number of Block Access Required = $\lceil \log_2 b \rceil \Rightarrow \lceil \log_2 50 \rceil$

= $\boxed{\text{6 Block Access}}$

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5. (698 to 698)

Total number of Record = 1,50,000, Block size = 4096 Byte, Record Pointer = 7 Byte, key (ANum) size =12 Byte.

Data file is sorted based on primary key (Roll No), but index is build on the field ANum. [candidate key] secondary Index.

$\therefore$ so its dense index i.e number of index entries = number DB records = 1,50,000 and Record of Data file & Index file are not split across Disk Block, i.e un-spanned organization.

One index record size = size of key (ANum) + size of (Record Pointer) R_p = 12 + 7

One index record size = 19 Byte

Block Factor of Index File (Unspanned) = $\left\lfloor \frac{\text{Blocksize}}{\text{Record size}} \right\rfloor \left\lfloor \frac{4096}{19} \right\rfloor = 215$ Index Record Per Block.

Total Number of Index Block = $\left\lceil \frac{1,50,000}{215} \right\rceil = 698$ Index Blocks.

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6. (c)

Total number of records = 16384 (2^{14})

Record size = 32 Byte, Block size = 1024 Byte, Key size = 6 Byte, Block Pointer = 10 Byte

The index is unspanned, secondary (dense) index.

One index record size = Size of search key + size of block pointer $\Rightarrow$ 6 + 10 = 16 Byte

Block factor of index file (BF_i) (Unspanned organization) $= \left\lfloor \frac{\text{Block size}}{\text{Record size}} \right\rfloor = \left\lfloor \frac{1024B}{16B} \right\rfloor$

$\Rightarrow \frac{2^{10}}{2^4} = 2^6 = 64$. Index entries per Block

Secondary Index: Total number of Index entries = Total number of records = 16,384.

I^st level- Total number of Index Block = $\left\lceil \frac{\text{Total Index Entries}}{\text{Number of Entries per Block}} \right\rceil$

$= \frac{16384}{64} = \frac{2^{14}}{2^6} = 2^8 = 256$ Index Block.

II^nd Level-

Total number of Index Entries = 256 (Number of I^st level Block) & BFi = 64 Index Entries per Block

Total number of Index Block = $\left\lceil \frac{\text{Total Index Entries}}{\text{Number of Entries per Block}} \right\rceil$

$= \frac{256}{64} = \frac{2^8}{2^6} = 2^2 = 4$ Index Blocks.

So, first Level and second level blocks are: 256 & 4, respectively.

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7. (4 to 4)

Block size = 4KB,

Search key = 12 Byte

B_P = 8B

Total number of records = 10^6

ORDER of B^+ Tree

$P \times B_P + (P - 1)$ key $\leq$ Block size

$P \times 8 + (P - 1)\, 12 \leq 4 \times 1024$

$8P + 12P - 12 \leq 4096$

$20P - 12 \leq 4096$

$20P \leq 4108$

$$P = \left\lfloor \frac{4108}{20} \right\rfloor = \lfloor 205.4 \rfloor$$

$\boxed{P = 205}$

Minimum number of levels means that we get maximum number of keys at a block.

(B_P) ORDER (P) = 205

(P – 1) maximum key = 204

Or (R_P)

1st level:

Total number of Record = 10^6 (Entries)

$$\text{Total number of Block} = \left\lceil \frac{10^6}{204} \right\rceil = 4902 \text{ Blocks}$$

2nd level

Total number of entries = 4902 (No. of 1st level block)

$$\text{Total number of blocks} = \left\lceil \frac{4902}{205} \right\rceil = 24 \text{ blocks}$$

3rd level

Total number of entries = 24 (No. of 2nd level block)

$$\text{Total number of Block} = \left\lceil \frac{24}{205} \right\rceil = 1 \text{ block}$$

Here, 3 Block access for 3 level + 1 block access for accessing the records = 3 + 1 = 4.

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8. (52 to 52)

Key = 8 Byte, Block pointer(Bp) = 2 Byte, Block size = 512 Byte

Maximum order of B^+ tree: Internal node

Internal Node: ORDER P: $P \times Bp + (P - 1)$ key $\leq$ Block size

$P \times 2 + (P - 1)\, 8 \leq 512$

$2P + 8P - 8 \leq 512$

$10P \leq 520$

$$P = \left\lfloor \frac{520}{10} \right\rfloor = 52$$

P = 52

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9. (a)

In a B^+ tree the length of the paths from the root to all leaf nodes are all equal.

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10. (50 to 50)

Key = 12 Byte, Block Pointer(BP) = 8 Byte, Block size = 1024Byte, Record Pointer (RP) = 10 Byte.

B^+ tree: Non leaf Node:

ORDER(P): $P \times BP + (P-1)$ key $\leq$ Block size

$P \times 8 + (P - 1)\, 12 \leq 1024$

$8P + 12P - 12 \leq 1024$

$20P \leq 1036$ Maximum Block Pointer

$$P = \left\lfloor \frac{1036}{20} \right\rfloor = \lfloor 51.8 \rfloor = \boxed{P = 51}$$

Maximum Number of keys = P – 1

= 51 – 1

= 50.

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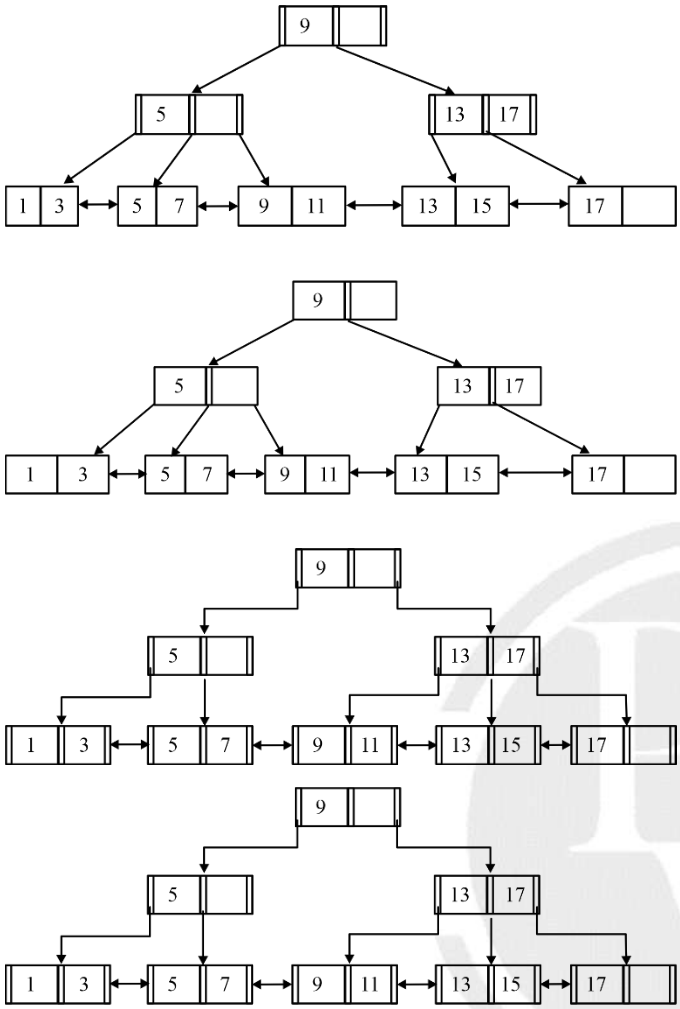
11. (5 to 5)



The minimum number of nodes including the root node that must be fetched in order to satisfy the given query is 5.

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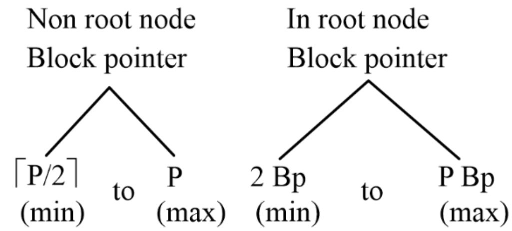
12. (b)

Maximum Number of keys = 5

ORDER P : Maximum Number of key = P – 1

$\Rightarrow$ P – 1 = 5 P = 6



Order of B^+ tree = 6

Minimum Number of Block Pointer (in non-root (internal) node = $\lceil P/2 \rceil$

Minimum number of key in non-root node $= \lceil P/2 \rceil - 1$

$\lceil 6/2 \rceil - 1 \Rightarrow 3 - 1 = 2.$

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13. (c)

B^+ tree, keys to be inserted are: 10, 3, 6, 8, 4, 2, 1

Order = 3

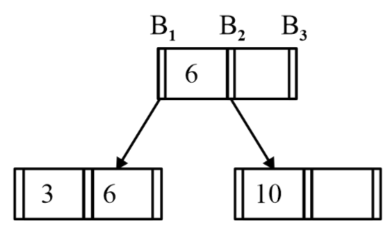
Maximum key = 2

Insert 10: | 10 | |

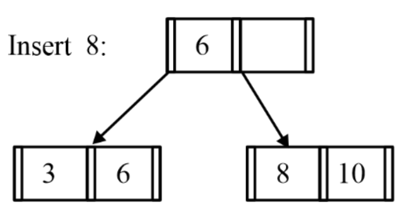
Insert 3: | 3 | 10 |

Insert 6: | 3 | 6 | 10 |

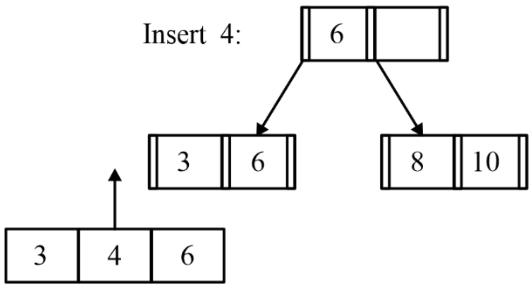
Violates definition so split.
Split number (1) when '6' inserted.



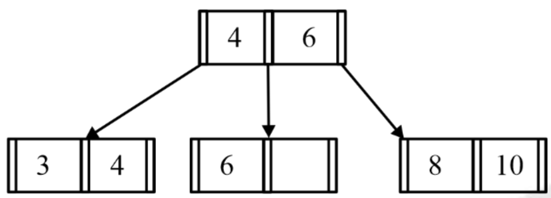
Insert 8:



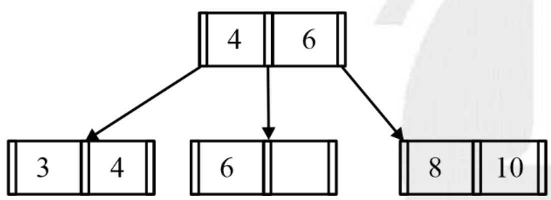
Insert 4:



Split number (2) when '4' inserted.



10, 3, 6, 8, 4, 2, 1



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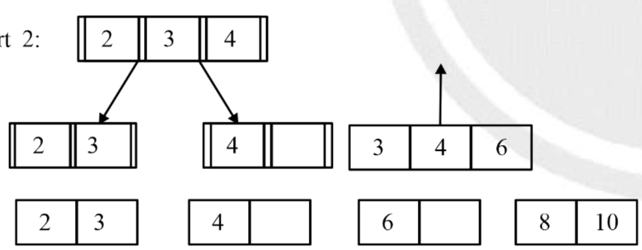

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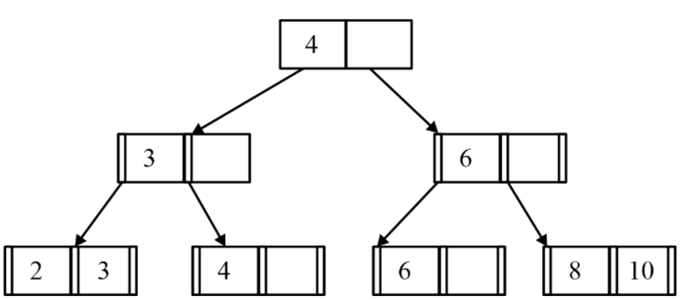
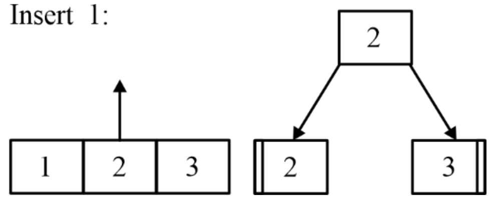
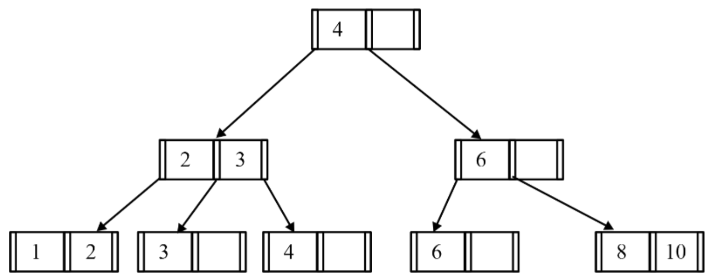


Insert 1:





Split number (4) when '1' inserted.

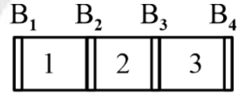
So, total number of splits at leaf node = 4.

Scan for Video solution

14. (c)

B Tree Order: 4

Max key =3

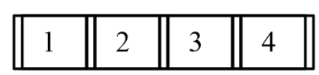


Insert – 1

Insert – 2

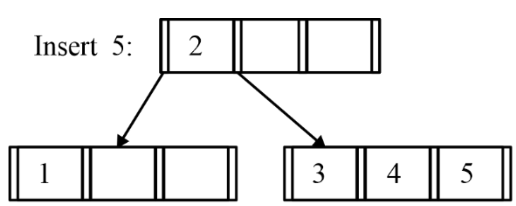
Insert – 3

Insert – 4



Violate definition of B tree, do splitting

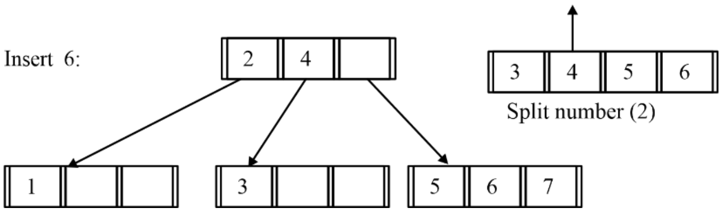
Insert 5:



Split number (1) at leaf node

Let assume: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Insert 6:



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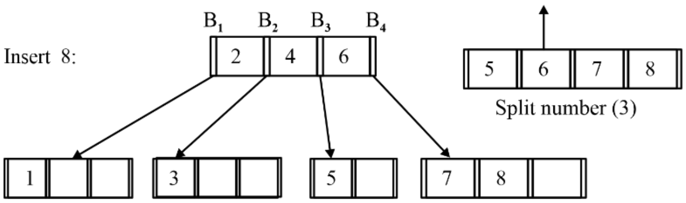

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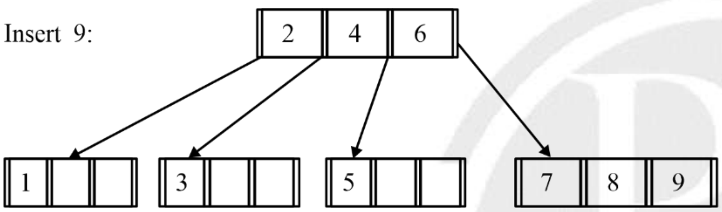

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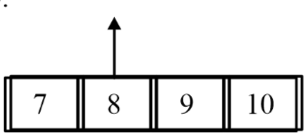

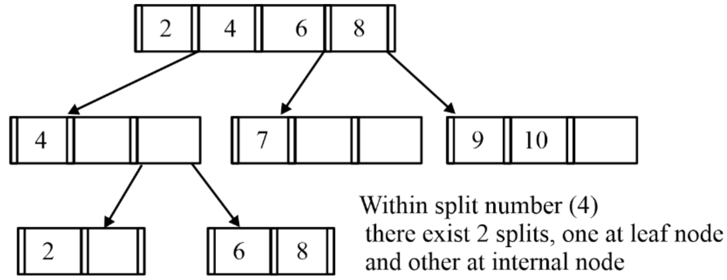

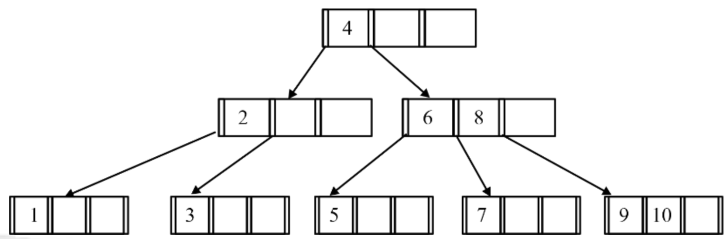

Total number of splits = 4 + 1 = 5

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NOTE :

INDEX

Operating System

Operating System

Syllabus

System calls, processes, threads, inter-process communication, concurrency and synchronization. Deadlock. CPU and I/O scheduling. Memory management and virtual memory. File systems.

Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5
2008	4	4	4	8	1
2009	0	2	2	4	2
2010	2	3	2	1	0
2011	5	0	0	1	0
2012	3	4	0	2	2
2013	1	5	0	2	0
2014 (P1)	3	0	2	2	1
2014 (P2)	2	2	0	2	1
2014 (P3)	2	0	2	3	0
2015 (P1)	2	2	0	3	3
2015 (P2)	0	0	1	5	2
2015 (P3)	3	2	2	0	0
2016 (P1)	1	2	0	4	2
2016 (P2)	2	4	0	1	0
2017 (P1)	2	2	0	2	0
2017 (P2)	3	0	2	0	1
2018	0	2	3	1	2
2019	3	1	2	2	2
2020	3	2	0	3	2
2021 (P1)	2	2	0	1	0
2021 (P2)	1	4	0	2	0
2022	2	1	1	4	2
2023	3	0	0	4	0

CHAPTER 1

PROCESS MANAGEMENT & CPU SCHEDULING

Introduction & Background

1. [MCQ] [GATE-2011 : 1M]

A computer handles several interrupt sources of which of the following are relevant for this question.

- Interrupt from CPU temperature sensor (raises interrupt if CPU temperature is too high)
- Interrupt from Mouse (raises interrupt if the mouse is moved or a button is pressed)
- Interrupt from Keyboard (raises interrupt when a key is pressed or released)
- Interrupt from Hard Disk (raises interrupt when a disk read is completed)

Which one of these will be handled at the HIGHEST priority?

(a) Interrupt from Hard Disk
(b) Interrupt from Mouse
(c) Interrupt from Keyboard
(d) Interrupt from CPU temperature sensor

Process concept

2. [MSQ] [GATE-2023 : 1M]

Which one or more of the following need to be saved on a context switch from one thread (T1) of a process to another thread (T2) of the same process?

(a) Page table base register
(b) Stack pointer
(c) Program counter
(d) General purpose registers

3. [MCQ] [GATE-2020 : 1M]

Consider the following statements about process state transitions for a system using preemptive scheduling.

I. A running process can move to ready state.
II. A ready process can move to running state.
III. A blocked process can move to running state.
IV. A blocked process can move to ready state.

Which of the above statements arc TRUE?

(a) I, II, III and IV
(b) II and III only
(c) I, II and IV only
(d) I, II and III only

4. [MCQ] [GATE-2015 : 1M]

The maximum number of processes that can be in Ready state for a computer system with n CPUs is

(a) n
(b) n^2
(c) 2^n
(d) Independent of n

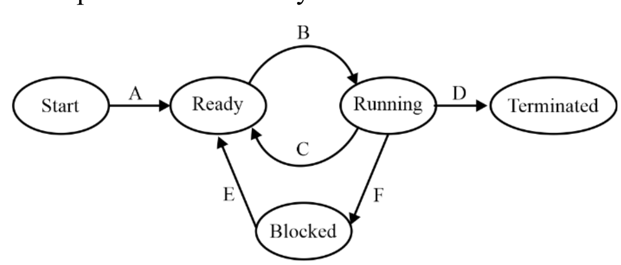
5. [MCQ] [GATE-2011 : 1M]

Let the time taken to switch between user and kernel modes of execution be T_1 while the time taken to switch between two user processes be T_2. Which of the following is TRUE?

(a) $T_1 > T_2$
(b) $T_1 = T_2$
(c) $T_1 < T_2$
(d) Nothing can be said about the relation between T_1 and T_2

6. [MCQ] [GATE-2009 : 2M]

In the following process state transition diagram for a uniprocessor system, assume that there are always some process in the ready state:



Now consider the following statements:

I. If a process makes a transition D, it would result in another process making transition A immediately
II. A process P_2 in blocked state can make transition E while another process P_1 is in running state
III. The OS uses preemptive scheduling
IV. The OS uses non-preemptive scheduling

Which of the above statements are TRUE?

(a) I and II (b) I and III
(c) II and III (d) II and IV

CPU Scheduling Algorithms

7. [MSQ] [GATE-2023 : 1M]

Which one or more of the following CPU scheduling algorithms can potentially cause starvation?

(a) First-in First-Out
(b) Round Robin
(c) Priority Scheduling
(d) Shortest Job First

8. [MCQ] [GATE-2022 : 2M]

Consider four processes P, Q, R, and S scheduled on a CPU as per round robin algorithm with a time quantum of 4 units. The processes arrive in the order P, Q, R, S, all at time t = 0. There is exactly one context switch from S to Q, exactly one context switch from R to Q, and exactly two context switches from Q to R. There is no context switch from S to P. Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible as CPU burst time (in time units) of these processes?

(a) P = 4, Q = 10, R = 6, S = 2
(b) P = 2, Q = 9, R = 5, S = 1
(c) P = 4, Q = 12, R = 5, S = 4
(d) P = 3, Q = 7, R = 7, S = 3

9. [MSQ] [GATE-2021 : 1M]

Which of the following statement(s) is/are correct in the context of CPU scheduling?

(a) Turnaround time includes waiting time.
(b) The goal is to only maximize CPU utilization and minimize throughput.
(c) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori.
(d) Implementing preemptive scheduling needs hardware support.

10. [NAT] [GATE-2021 : 1M]

Three processes arrive at time zero with CPU bursts of 16, 20 and 10 milliseconds. If the scheduler has prior knowledge about the length of the CPU bursts, the minimum achievable average waiting time for these three processes in a non-preemptive scheduler (rounded to nearest integer) is _______ milliseconds.

11. [NAT] [GATE-2020 : 2M]

Consider the following set of processes, assumed to have arrived at time 0. Consider the CPU scheduling algorithms Shortest Job First (SJF) and Round Robin (RR). For RR, assume that the processes are scheduled in the order P_1, P_2, P_3, P_4.

Processes	P_1	P_2	P_3	P_4
Burst time (in ms)	8	7	2	4

If the time quantum for RR is 4 ms, then the absolute value of the difference between the average turnaround times (in ms) of SJF and RR (round off to 2 decimal places) is _______.

12. [NAT] [GATE-2019 : 2M]

Consider the following four processes with arrival times (in milliseconds) and their length of CPU bursts (in milliseconds) as shown below:

Process	P_1	P_2	P_3	P_4
Arrival time	0	1	3	4
CPU burst time	3	1	3	Z

These processes are run on a single processor using preemptive Shortest Remaining Time First scheduling algorithm. If the average waiting time of the processes is 1 millisecond, then the value of Z is __________.

13. [NAT] [GATE-2017 : 2M]

Consider the set of processes with arrival time (in milliseconds), CPU burst time (in milliseconds), and priority (0 is the highest priority) shown below. None of the process have I/O burst time.

Process	Arrival Time	Burst Time	Priority
P_1	0	11	2
P_2	5	28	0
P_3	12	2	3
P_4	2	10	1
P_5	9	16	4

The average waiting time (in milliseconds) of all the processes using preemptive priority scheduling algorithm is ___.

14. [NAT] [GATE-2017 : 1M]

Consider the following CPU processes with arrival times (in milliseconds) and length of CPU bursts (in milliseconds) as given below:

Process	Arrival Time	Burst Time
P_1	0	7
P_2	3	3
P_3	5	5
P_4	6	2

If the pre-emptive shortest remaining time first scheduling algorithm is used to schedule the processes, then the average waiting time across all processes is ___ milliseconds.

15. [NAT] [GATE-2016 : 2M]

Consider the following processes, with the arrival time and the length of the CPU burst given in milliseconds. The scheduling algorithm used is preemptive shortest remaining-time first.

Process	Arrival Time	Burst Time
P_1	0	10
P_2	3	6
P_3	7	1
P_4	8	3

The average turn around time of these processes is___ milliseconds.

16. [MCQ] [GATE-2016 : 1M]

Consider an arbitrary set of CPU-bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithms would minimize the average waiting time in the ready queue?

(a) Shortest remaining time first
(b) Round – robin with time quantum less then the shortest CPU burst
(c) Uniform random
(d) Highest priority first with priority proportional to CPU burst length

17. [MCQ] [GATE-2015 : 2M]

For the processes listed in the following table, which of the following scheduling schemes will give the lowest average turnaround time?

Process	Arrival Time	Processing Time
A	0	3
B	1	6
C	4	4
D	6	2

(a) First come First serve.
(b) Non-preemptive shortest job first.
(c) Shortest Remaining time.
(d) Round Robin with Quantum value two.

18. [NAT] [GATE-2015 : 2M]

Consider a uniprocessor system executing three tasks T_1, T_2 and T_3, each of which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period, and the available tasks are scheduled in order of priority, which is the highest priority task scheduled first. Each instance of T_1, T_2 and T_3, requires an execution time of 1, 2 and 4 milliseconds, respectively. Given that all task initially arrive at the beginning of the 1st millisecond and task preemptions are allowed, the first instance of T_3 completes its execution at the end of ___ milliseconds.

19. [NAT] [GATE-2014 : 2M]

An operating system uses shortest remaining time first scheduling algorithm for pre-emptive scheduling of processes. Consider the following set of processes with their arrival times and CPU burst times (in milliseconds):

Process	Arrival Time	Burst Time
P_1	0	12
P_2	2	4
P_3	3	6
P_4	8	5

The average waiting time (in milliseconds) of the processes is ______.

20. [NAT] [GATE-2014 : 2M]

Three processes A, B and C each execute a loop of 100 iterations. In each iteration of the loop, a process performs a single computation that requires t_c CPU milliseconds and then initates a single I/O operation that lasts for t_{io} milliseconds. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also, the scheduling overhead of the OS is negligible. The processes have the following characteristics:

Process id	t_c	t_{io}
A	100 ms	500 ms
B	350 ms	500 ms
C	200 ms	500 ms

The process A, B and C are started at times 0, 5 and 10 milliseconds respectively, in a pure time-sharing system (round robin scheduling) that uses a time slice of 50 milliseconds. The time in milliseconds at which process C would complete its first I/O operation is ___.

21. [NAT] [GATE-2014 : 2M]

Consider the following set of processes that need to be scheduled on a single CPU. All the times are given in milliseconds.

Process Name	Arrival Time	Execution Time
A	0	6
B	3	2
C	5	4
D	7	6
E	10	3

Using the shortest remaining time first scheduling algorithm, the average process turnaround time (in msec) is __

22. [MCQ] [GATE-2013 : 1M]

A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts with priority zero (the lowest priority). The scheduler re-evaluates the process priorities every T time units and decides the next process to schedule. Which one of the following is TRUE if the processes have no I/O operations and all arrive at time zero?

(a) This algorithm is equivalent to the first-come first-serve algorithm

(b) This algorithm is equivalent to the round-robin algorithm

(c) This algorithm is equivalent to the shortest-job-first algorithm

(d) This algorithm is equivalent to the shortest-remaining-time-first algorithm

23. [MCQ] [GATE-2012 : 2M]

Consider the 3 processes, P_1, P_2 and P_3 shown in table.

Process	Arrival Time	Time Units Required
P_1	0	5
P_2	1	7
P_3	3	4

The completion order of the 3 processes under the policies FCFS and RR2 (round robin scheduling with CPU quantum of 2-time units) are:

(a) FCFS: P_1, P_2, P_3, RR2: P_1, P_2, P_3

(b) FCFS: P_1, P_3, P_2, RR2: P_1, P_3, P_2

(c) FCFS: P_1, P_2, P_3, RR2: P_1, P_3, P_2

(d) FCFS: P_1, P_3, P_2, RR2: P_1, P_2, P_3

24. [MCQ] [GATE-2011 : 2M]

Consider the following table of arrival time and burst time for three processes P_0, P_1 and P_2.

Process	Arrival Time	Burst Time
P_0	0 ms	9 ms
P_1	1 ms	4 ms
P_2	2 ms	9 ms

The pre-emptive shortest job first scheduling algorithm is used. Scheduling is carried out only at arrival or completion of processes. What is the average waiting time for the three processes?

(a) 5. 0 ms (b) 4.33 ms

(c) 6.33 ms (d) 7. 33 ms

25. [MCQ] [GATE-2010 : 1M]

Which of the following statements are true?

I. Shortest remaining time first scheduling may cause starvation.

II. Preemptive scheduling may cause starvation.

III. Round robin is better than FCFS in terms of response time.

(a) I only (b) I and III only

(c) II and III only (d) I, II and III

26. [MCQ] [GATE-2008 : 2M]

If the time-slice used in the round-robin scheduling policy is more than the maximum time required to execute any process, then the policy will

(a) degenerate to shortest job first

(b) degenerate to priority scheduling

(c) degenerate to first come first serve

(d) None of the above

System Call

27. [MSQ] [GATE-2023 : 1M]

Which one or more of the following options guarantee that a computer system will transition from user mode to kernel mode?

(a) Function Call (b) Malloc Call

(c) Page Fault (d) System Call

28. [MSQ] [GATE-2021: 1M]

Which of the following standard C library functions will always invoke a system call when executed from a single-threaded process in a UNIX/Linux operating system?

(a) exit (b) malloc

(c) sleep (d) strlen

29. [NAT] [GATE-2019 : 1M]

The following C program is executed on a Unix/Linux system:

```
#include<unistd.h>
 int main( )
 {
    int i;
    for (i=0;i<10;i++)
     if (i%2==0)
             fork( );
   return 0;
  }
```

The total number of child processes created is ___________.

30. [MCQ] [GATE-2012 : 1M]

A process executes the code

```
fork ();
fork ();
fork ();
```

The total number of child processes created is

(a) 3 (b) 4
(c) 7 (d) 8

31. [MCQ] [GATE-2008 : 2M]

A process executes the following code

```
for (i = 0; i < n; i++) fork ();
```

The total number of child processes created is

(a) n (b) $2^n - 1$
(c) 2^n (d) $2^{n+1} - 1$

Threads

32. [MCQ] [GATE-2017 : 1M]

Which of the following is/are shared by all the threads in a process?

I. Program counter II. Stack
III. Address space IV. Registers

(a) I and II only (b) III only
(c) IV only (d) III and IV only

33. [MCQ] [GATE-2017 : 1M]

Threads of a process share

(a) global variables but not heap
(b) heap but not global variables
(c) neither global variables nor heap
(d) both heap and global variables

34. [MCQ] [GATE-2014 : 1M]

Which of the following is FALSE?

(a) User level thread are not scheduled by the kernel.
(b) When a user level thread is blocked, all other threads of its process are blocked.
(c) Context switching between user level threads is faster than context switching between kernel level threads.
(d) Kernel level threads cannot share the code segment.

35. [MCQ] [GATE-2011 : 1M]

A thread is usually defined as a "light weight process" because an Operating System (OS) maintains smaller data structures for a thread than for a process. In relation to this, which of the following is TRUE?

(a) On per-thread basis, the OS maintains only CPU register state.
(b) The OS does not maintain a separate stack for each thread.
(c) On per-thread basis, the OS does not maintain virtual memory state.
(d) On per-thread basis, the OS maintains only scheduling and accounting information.

❑❑❑

ANSWER KEY

1.	(d)	**2.**	(b, c, d)	**3.**	(c)	**4.**	(d)
5.	(c)	**6.**	(c)	**7.**	(a, c, d or c, d)	**8.**	(d)
9.	(a, c, d)	**10.**	(12 to 12)	**11.**	(5.25 to 5.25)	**12.**	(2 to 2)
13.	(29 to 29)	**14.**	(3 to 3)	**15.**	(8.2 to 8.3)	**16.**	(a)
17.	(c)	**18.**	(12 to 12)	**19.**	(5.5 to 5.5)	**20.**	(1000 to 1000)
21.	(7.2 to 7.2)	**22.**	(b)	**23.**	(c)	**24.**	(a)
25.	(d)	**26.**	(c)	**27.**	(c, d)	**28.**	(a, c)
29.	(31 to 31)	**30.**	(c)	**31.**	(b)	**32.**	(b)
33.	(d)	**34.**	(d)	**35.**	(c)		

SOLUTIONS

1. (d)

Interrupt from CPU temperature sensor should be given highest priority. Delaying a CPU temprature sensor could result in major consequences, overheated CPU can damage internal circuits, and hence hampering entire computer system.

The priority of above events is given as:

CPU temprature sensor > Hard disk > Mouse > Keyboard.

Therefore, option d is the correct answer.

Scan for Video solution

2. (b, c, d)

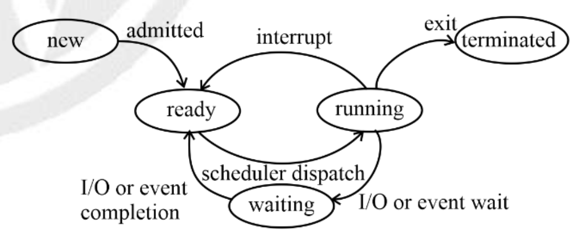
Page table base register is used to access the virtual address space, as all threads share the same address space, so there is no need to save it. Therfore, option A is incorrect.

Every thread has its own stack pointer, program counter, and general purpose registers, so we need to save them. Therefore, Option B, C, and D are correct.

Scan for Video solution

3. (c)

A process state diagram with pre-emptive scheduling is as follows:



So, from the given diagram we can say

I. A running process can move to ready state.

II. A ready process can move to running state.

IV. A blocked process can move to ready state.

Therefore, option C is correct.

Scan for Video solution

4. (d)

The maximum number of processes present in the ready state is independent of number of CPU's the system have. Number of processes in ready state depends upon the size of ready queue. Therefore, option D is the correct answer.

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5. (c)

Given,

User Mode → Kernel Mode = T_1

User process 1 → User process 2 = T_2 [Context Switching]

Context switching includes mode shifting, first preempt the process, shift the mode from user to kernel, OS will save and load the PCB, again shift the mode from kernel to user.

From this we can say context switching time is superset and mode shifting is a subset. Therefore, T_2 is greater than T_1.

Hence, option C is the correct answer.

Scan for Video solution

6. (c)

If a process is completed it is not necessary that you should load a new process into memory. Therefore, statement I is false.

A process P_2 in blocked state can make transition E while another process P_1 is in running state. TRUE. Process moving from block to ready state and another process is in running state, this situation is possible and hence statement II is true.

The OS uses preemptive scheduling. TRUE. The given diagram makes a transition from running to ready state and hence this indicates that processes can be preempted while running. So, statement III is also true and statement IV is false.

Hence, option C is the correct answer.

Scan for Video solution

7. (a, c, d, or c, d)

First-In-First-Out may suffer from starvation in some exceptional case. In FIFO if a process has arrived then it will definitely get CPU once all the processes arrived before that are serviced. But processes may starve when a process has very long burst time and wait in waiitng queue.

Round robin never suffers from starvation because every process gets time to execute on CPU in circular manner.

Priority Scheduling suffers from starvation as processes with lower priority may starve for processor if higher prioirty process keeps arriving.

Shortest Job First also suffers from starvation as processes with higher burst time may starve for processor when processes with lower burst time keeps on arriving.

Therefore, option C and D are correct

Scan for Video solution

8. (d)

Given,

1. Switching to ready processes after termination of currently executing process also a context switch.
2. There is no context switch from S to P. (Therefore P should be completed within 1 TQ)
3. There are exactly two context switches from Q to R (Therefore Q and R both should requires more than 1 TQ)

4. There is exactly one context switch from R to Q (Therefore Q require more time than R)
5. Exactly one context swich from R to S. (Therefore S should be complete with in 1 TQ otherwise another CS from R to S required.)
6. Exactly one conext switch from S to Q.

$S \rightarrow Q = 1$

$R \rightarrow Q = 1$

$Q \rightarrow R = 2$

$S \rightarrow P$ = No context switch

From the given points, we can conclude:

1. Burst Time (P) ≤ 4
2. Burst Time (Q) > 8
3. Burst Time (S) ≤ 4

→ Round Robin (Time Quantum = 4)

R.Q; | ~~P~~ ~~Q~~ ~~R~~ ~~S~~ ~~Q~~ ~~R~~ Q | |

CPU | P Q R S Q R Q | |

0

We have said that Burst Time (Q) > 8. Only option D does not satisfy this situation and hence it is NOT possible as CPU bursts.

Scan for Video solution

9. (a, c, d)

(a) Turnaround time includes waiting time. CORRECT. As, Turnaround time = waiting time + burst time.

(b) The goal is to only maximize CPU utilization and minimize throughput. INCORRECT. The goal of CPU scheduling is to increase the CPU utilization and throughput, and minimise the response time.

(c) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori. CORRECT. Round robin policy works irrespective of burst time of the process, it gives equal amount of time quantum to each process on processor.

(d) Implementing preemptive scheduling needs hardware support. CORRECT. To preempt a process, hardware support is required, a timer interrupt is generated whenever time quantum expires and the interrupted process is enqueued in ready queue and anohter process is scheduled to the CPU.

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10. (12)

It is given that we have to find out the minimum achievable average waiting time for these three processes in a non-preemptive scheduler. This is always possible with Shortest Job First CPU scheduling algorithm. Hence, we will be using SJF to solve this problem.

Gantt chart using SJF:

P_3	P_1	P_2

0 10 26 46

$$\text{AV.W.T} = \frac{10+26+0}{3}$$

$$= \frac{36}{3} = 12$$

Therefore, 12 is the correct answer.

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11. (5.25)

P.NO	Arrive Time	Burst Time
P_1	0	8
P_2	0	7
P_3	0	2
P_4	0	4

Gantt chart using SJF:

CPU

P_3	P_4	P_2	P_1

0 2 6 13 21

Time

TAT = Completion time- arrival time

TAT $P_1 = 21 - 0 = 21$

TAT $P_2 = 13 - 0 = 13$

TAT $P_3 = 2 - 0 = 2$

TAT $P_4 = 6 - 0 = 6$

Average TAT $= \frac{42}{4} = 10.5$

Round Robin

Ready Queue- $\boxed{P_1}$; $\boxed{P_2}$; $\boxed{P_3}$; $\boxed{P_4}$; $\boxed{P_1}$; $\boxed{P_2}$;

Gantt chart using round robin:

CPU

P_1	P_2	P_3	P_4	P_1	P_2

0 4 8 10 14 18 21

TAT

TAT $P_1 = 18$

TAT $P_2 = 21$

TAT $P_3 = 10$

TAT $P_4 = 14$

Total = 63

Average TAT $= \frac{63}{4} = 15.75$

Difference = [15.75 – 10.5] = 5.25

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12. (2 ms)

P.NO	Arrive Time	Burst Time
P_1	0	3
P_2	1	1
P_3	3	3
P_4	4	Z

Average Waiting Time = 1 ms

CASE I: If $Z < 3$

Gantt chart using SRTF:

P_1	P_2	P_1	P_4	P_3

0 1 2 4 4 + Z 7 + Z

P.No	Arrival Time	Burst Time	Priority	Completion Time	Turnaround Time	Waiting Time
P_1	0	11	2	49	49	38
P_2	5	28	0 (Highest)	33	28	0
P_3	12	2	3	51	39	37
P_4	2	10	1	40	38	28
P_5	9	16	4 (Lowest)	67	58	42

Average waiting Time $= \dfrac{1+0+(z+1)}{4}$

$= \dfrac{z+2}{4} + 1$ ms

$\Rightarrow \dfrac{z+2}{4} + 1 \Rightarrow Z + 2 = 4 \Rightarrow Z + 4 - 2 = 2$

$\Rightarrow Z = 2$ ms

CASE II: If $Z > 3$

Average Waiting Time > 1 ms

CPU

P_1	P_2	P_1	P_3	P_4	

0 1 2 4 7 7 + Z

Average waiting Time $= \dfrac{1+0+1+3}{4} = 1$ ms

$\Rightarrow 5/4 \neq 1$

$\Rightarrow 1.25 \neq 1$

So, the value of Z could not be greater than 3, and hence the correct answer is 2.

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13. (29)

P.No	Arrival Time	Burst Time	Priority
P_1	0	11	2
P_2	5	28	0 (Highest)
P_3	12	2	3
P_4	2	10	1
P_5	9	16	4 (Lowest)

Gantt chart using Pre-emptive priority scheduling algorithm:

P_1	P_4	P_2	P_4	P_1	P_3	P_5

0 2 5 33 40 49 51 67

$$\text{Average Waiting Time} = \frac{38+0+37+28+42}{5}$$

$$= \frac{145}{5} = 29$$

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14. (3.0)

P.NO	Arrive Time	Burst Time
P_1	0	7
P_2	3	3
P_3	5	5
P_4	6	2

Gantt chart using SRTF scheduling algorithm:

P_1	P_2	P_4	P_1	P_3

0 3 6 8 12 17

P.NO	Arrival Time	Burst Time	Completion time	Turnaround time	Waiting time
P_1	0	7	12	12	5
P_2	3	3	6	3	0
P_3	5	5	17	12	7
P_4	6	2	8	2	0

$$\text{AV.W.T} = \frac{5+0+7+0}{4} = \frac{12}{4}$$

Average Waiting Time = 3.0

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15. (8.25)

Gantt chart using SRTF:

P_1	P_2	P_3	P_2	P_4	P_1

0 3 7 8 10 13 20

Process	Arrival Time	Burst Time	Completion Time	Turnaround Time
P_1	0	10	20	20
P_2	3	6	10	7
P_3	7	1	8	1
P_4	8	3	13	5

$$\text{AV.TAT} = \frac{20+7+1+5}{4} = \frac{33}{4} = 8.25$$

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16. (a)

Shortest job first (SJF) minimizes the waiting time, SJF gives the minium average waiting time as comapred with other algorithms. Here it is given that arrival time of all processes is same, so SRTF acts same as SJF.

Therefore, option A is correct.

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17. (c)

Shortest remaining time first or Shortest job first will always generate minimum average turnaround time and average waiting time. Here also preemptive SJF or SRTF will always give minimum average waiting time as comapred to non-preemptive SJF. Therefore, option C is the correct answer.

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18. (12)

Prio	Period	P.NO	A.T	B.T	Insta
1/3	3	T1	1	1	1; 4; 7; 10; 13; 16…
1/7	7	T2	1	2	1; 8; 15; …
1/20	4	T3	1	4	1; 21; 41; …

Pre-Prio:

R.O $\boxed{T_{1_1}}$; $\boxed{T_{2_1}}$; $\boxed{T_{3_1}}$; $\boxed{T_{1_2}}$; $\boxed{T_{1_3}}$; $\boxed{T_{2_2}}$; $\boxed{T_{1_4}}$;

Gantt Chart:

	T_{1_1}	T_{2_1}	T_{1_2}	T_{3_1}	T_{1_3}	T_{2_2}	T_{1_4}	T_{3_1}	

0 1 2 4 5 7 8 10 11 13

So, at the end of 12 millisecond and at the beginning of 13 millisecond the first instance of T3 completed its execution.

Therefore, 12 is the correct answer.

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19. (5.5)

Gantt Chart using SRTF:

P_1	P_2	P_3	P_4	P_1

0 2 6 12 17 27

Process	Arrival Time	Burst Time	Completion Time	Turnaround Time	Waiting Time
P_1	0	12	27	27	15
P_2	2	4	6	4	0
P_3	3	6	12	9	3
P_4	8	5	17	9	4

$$\text{AV.W.T} = \frac{15+0+3+4}{4} = \frac{22}{4} = 5.5$$

So, 5.5 ms is the average waiting time.

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20. (1000)

Given,

Time Quantum= 50

P.No	Arrival Time	Burst Time	IOBT
A	0	100	500
B	5	350	500
C	10	200	500

Gantt Chart:

CPU	A	B	C	A	B	C	B	C	B	C	

0 50 100 150 200 250 300 350 400 450 500

I_0 A 700
500

I_0 C 1000
500

C has completed its IO at time 1000 ms.

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21. (7.2)

Gantt Chart using SRTF:

A	B	A	C	E	D

0 3 5 8 12 15 21

Process Name	Arrival Time	Burst Time	Completion Time	Turnaround Time
A	0	6	8	8
B	3	2	5	2
C	5	4	12	7
D	7	6	21	14
E	10	3	15	5

$$\text{AV.TAT} = \frac{8+2+7+14+5}{5}$$

$$\text{AV.TAT} = \frac{36}{5} = 7.2$$

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22. (b)

This algorithm will behave as preemptive FCFS and is equivalent to the round-robin algorithm. Therefore, option B is the correct answer.

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23. (c)

In FCFS all the process's gets executed as per their time of arrival, so using FCFS processes will be completed as P1, P2, P3

Now, checking the completion sequence using round-robin scheduling:

P.No	Arrival Time	Burst Time
1	0	5
2	1	7
3	3	4

Round robin Time Qunatum = 2

Ready Queue-

$\boxed{P_1}; \boxed{P_2}; \boxed{P_1}; \boxed{P_3}; \boxed{P_2;} \boxed{P_1}; \boxed{P_3}; P_2$

Gantt Chart:

P_1	P_2	P_1	P_3	P_2	P_1	P_3	P_2

0 2 4 6 8 10 11 13 16

Here, Process are completed as P1, P3, P2.

So, option C is the correct answer.

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24. (a)

Gantt Chart Preemptive SJF:

CPU

P_0	P_1	P_0	P_2

0 1 5 13 22

P. No	Arrival Time	Burst Time	Completion Time	Turnaround Time	Waiting Time
P_0	0	9	13	13	4
P_1	1	4	5	4	0
P_2	2	9	22	20	11

$$\text{AV.W.T} = \frac{4+0+11}{3} = \frac{15}{3} = 5$$

Hence, option A is the correct answer.

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25. (d)

I. Shortest remaining time first scheduling may cause starvation. TRUE. Processes with longer burst time may starve if processes with shorter burst time keeps on arriving.

II. Preemptive scheduling may cause starvation. TRUE. Preemptive scheduling may cause starvation for example SRTF is preemptive scheudling and it can cause starvation. So this statement is also correct.

III. Round robin is better than FCFS in terms of response time. TRUE. Round robin has defined time quantum for each process, therefore it has better response time than FCFS.

Hence, all statements are true and option D is correct.

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26. (c)

If the time-slice used in the round-robin scheduling policy is more than the maximum time required to execute any process, then the policy will degenerate to first come first serve.

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27. (c, d)

A computer system has to make a transition from user mode to kernel mode, whenver a page fault occurs, the page fault handler will run in kernel mode.

System call alaways makes a transition from user to kernel mode.

Therfore, option C and D are correct.

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28. (a, c)

(a) exit will always invoke a system call.

(b) malloc will not invoke a system call always, it may or may not invoke a system call.

(c) sleep will always invoke a system call.

(d) strlen will never invoke a system call.

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29. (31)

We know that, n-forks creates (2^n –1) child processes

n = 10; (n % 2 = 0) ⇒ 5 times

∴ No of child processes = 25 – 1 = 31

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30. (c)

For, n-fork system call ⇒ Total child processes = 2^n – 1

Here, n = 3

So, $2^3 - 1 = 7$

So, option C is the correct answer.

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31. (b)

For, n fork( ) system calls, total number of child processes created is $2^n - 1$.

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32. (b)

Thread is the light weight process and each thread comprises its own stack, program counter, register. Only the address space of the thread is shared among threads of a single process. So, option B is the correct answer.

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33. (d)

A thread of a process shares with other threads are: Data section, Code section, Address space, permissions, etc. So, option D is the correct answer. Threads of a process shares both heap and global variables.

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34. (d)

User level threads are scheduled by the thread library not by kernel. Kernel even don't know about the existence of user level threads. So, A is correct.
When a user level thread is blocked, all other thread of its process are blocked. Option B is also correct.
Context switching between user level threads is faster than kernel level thread because user level threads have to store nothing, whereas kernel level thread save and restore registers, program counter, etc. Therefore, option C is also correct.
Kernel level threads cannot share the code segment. Threads can share the code segment, threads have seprate address space but they can share same code segment.
So, option D is incorrect.

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35. (c)

On per-thread basis, the OS maintains only CPU register. **INCORRECT**. As it also maintains thread state, thread ID, thread priority, etc. are also maintained.
The OS maintains a seprate stack for each thread, so option B is also **INCORRECT**.
On per-thread basis, the OS does not maintain virtual memory state. **CORRECT**. Virtual memory state is actually address space, and it is not unique for each thread, address space is shared among multiple threads.
On per-thread basis, the OS maintains only scheduling and accounting information. **INCORRECT**. As thread mainatains other data also like thread priority, thread state, etc.

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❑❑❑

CHAPTER

2

PROCESS SYNCHRONIZATION

Critical Section Problem

1. **[MCQ]** **[GATE-2016 : 2M]**

Consider the following proposed solution for the critical section problem. There are n processes: $P_0 \ldots P_{n-1}$. In the code, function pmax returns an integer not smaller than any of its arguments.

For all i, t[i] is initialized to zero.

Code for Pi

```
do
{
c[i] = 1; t[i] = pmax(t[0],….,t[n-1])+1; c[i]=0;
for every  in {0,…, n-1}
{
while (c[j]);
while (t[j]!=0 && t[j] <= t[i]);
}
Critical Section; t[i] = 0;
Remainder Section;
} while (true);
```

which one of the following is TRUE about the above solution?

(a) At most one process can be in the critical section at any time.
(b) The bounded wait condition is satisfied.
(c) the progress condition is satisfied.
(d) It cannot cause a deadlock.

2. **[MCQ]** **[GATE-2016 : 2M]**

Consider the following two-process synchronization solution

```
Process 0
    Entry: loop while (turn == 1);
    (critical section)
    Exit: turn = 1;
Process 1
    Entry: loop while (turn == 0);
    (critical section)
    Exit: turn = 0;
```

The shared variable turns in initialized to zero.

Which one of the following is TRUE?

(a) This is a correct two-process sysnchronization solution.
(b) This solution violates mutual exclusion requirement.
(c) This solution violates progress requirement.
(d) This solution violates bounded wait requirement.

3. **[MCQ]** **[GATE-2015 : 2M]**

Two processes X and Y need to access a critical section. Consider the following synchronization construct used by both the processes.

```
Process X
/* other code for process X */
while (true)
{
varP = true;
while (varQ == true)
    { /* Critical Section */
        varP = false;
    }
}
/* other code for process X */
Process Y
/* other code for process Y */
while (true)
{
varQ = true;
while (varP == true)
```

```
        { /* Critical Section */
            varQ = false;
        }
    }
/* other code for the process Y */
```

Here, varP and varQ are shared variables and both are initialized to false. Which one of the following statements is true?

(a) The proposed solution prevents deadlock but fails to guarantee mutual exclusion.

(b) The proposed solution guarantees mutual exclusion but fails to prevent deadlock

(c) The proposed solution guarantees mutual exclusion and prevents deadlock

(d) The proposed solution fails to prevent deadlock and fails to guarantee mutual exclusion

Hardware Solution (TSL, etc)

4. [MCQ] [GATE-2012 : 2M]

Fetch_And_Add (X, i) is an atomic Read-Modify-Write instruction that reads the value of memory location X, increments it by the value i, and returns the old value of X, it is used in the presudocode shown below to implement a busy-wait lock. L is unsigned integer shared variable initialized to 0. The value of 0 corresponds to lock being available, while any non-zero value corresponds to the lock being not available.

```
AcquireLock (L) {
    while (Fetch_And_Add (L, 1))
    L = 1;
}
ReleaseLock (L) {
    L = 0;
}
```

This implementation

(a) fails as L can overflow.

(b) fails as L can take on a non-zero value when the lock is actually available.

(c) works correctly but may starve some processes

(d) works correctly without starvation.

5. [MCQ] [GATE-2009 : 2M]

The enter_CS () and leave_CS () functions to implement critical section of a process are realized using test-and-set instruction as follows:

```
void enter_CS(X)
{
    while (test-and-set(X));
}
    void leave_CS(X)
{
    X = 0;
}
```

In the above solution, X is a memory location associated with the CS and is initialized to 0.

Now consider the following statements

I. The above solution to CS problem is deadlock free.

II. The solution is starvation free.

III. The processes enter CS in FIFO order.

IV. More than one process can enter CS at the same time.

Which of the above statements are TRUE?

(a) I only (b) I and II

(c) II and III (d) IV only

Software Solution

6. [MSQ] [GATE-2021 : 2M]

Consider the following multi-threaded code segment (in a mix of C and pseudo-code), invoked by two processes P_1 and P_2, and each of the processes spawns two threads T_1 and T_2.

```
int x = 0;   // global
Lock L1;  // global
main() {
 create a thread to execute foo( );  // Thread T1
 create a thread to execute foo( );  // Thread T2
 wait for the two threads to finish execution;
print (x);}
foo( ) {
int y = 0;
Acquire L1;
x = x + 1;
y = y + 1;
Release L1;
```

print (y); }

Which of the following statements(s) is/are correct?

(a) Both P_1 and P_2 will print the value of x as 2.
(b) At least one of P_1 and P_2 will print the value of x as 4.
(c) At least one of the threads will print the value of y as 2.
(d) Both T_1 and T_2, in both the processes, will print the value of y as 1.

7. [MCQ] [GATE-2017 : 2M]

A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are non-reentrant, i.e., if a thread holds a lock l, then it cannot re-acquire lock l without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:

(a) x = 1, y = 2 (b) x = 2, y = 1
(c) x = 2, y = 2 (d) x = 1, y = 1

8. [MCQ] [GATE-2010 : 1M]

Consider the methods used by processes P1 and P2 for accessing their critical sections whenever needed, as given below. The initial values of shared boolean variables S1 and S2 are randomly assigned.

Method used by P1	Method used by P2
while (S1==S2); Critical Section S1=S2;	While(S1!=S2) Critical Section S2=not(S1);

Which one of the following statements describes the properties achieved?

(a) Mutual exclusion but not progress
(b) Progress but not mutual exclusion
(c) Neither mutual exclusion nor progress
(d) Both mutual exclusion and progress

9. [MSQ] [GATE-2021-CS: 2M]

Consider a computer system with multiple shared resource types, with one instance per resource type. Each instance can be owned by only one process at a time. Owning and freeing of resources are done by holding a global lock (L). The following scheme is used to own a resource instance:

```
function OWNRESOURCE(Resource R)
Acquire lock L // a global lock
if R is available then
    Acquire R
     Release lock L
 else
      if R is owned by another process P then
     Terminate P, after releasing all resources owned by P
     Acquire R
     Restart P
     Release lock L
     end if
 end if
end function
```

Which of the following choice(s) about the above scheme is/are correct?

(a) The scheme ensures that deadlocks will not occur.
(b) The scheme may lead to live-lock.
(c) The scheme may lead to starvation.
(d) The scheme violates the mutual exclusion property.

OS Based Solution

10. [MSQ] [GATE-2021 : 2M]

Consider the following pseudocode, where S is a semaphore initialized to 5 in line # 2 and counter is a shared variable initialized to 0 in line # 1. Assume that the increment operation in line # 7 is not atomic.

```
1. int counter = 0;
2. Semaphore S = init(5);
3. void parop (void)
4. {
5.     wait (S);
6.     wait (S);
7.     counter++;
```

8. signal (S);
9. signal (S);
10. }

If five threads execute the function parop concurrently, which of the following program behavior(s) is/are possible?

(a) The value of counter is 5 after all the threads successfully complete the execution of parop
(b) The value of counter is 1 after all the threads successfully complete the execution of parop
(c) The value of counter is 0 after all the threads successfully complete the execution of parop
(d) There is a deadlock involving all the threads

11. [MCQ] [GATE-2020 : 2M]

Each of a set of n processes executes the following code using two semaphores a and b initialized to 1 and 0, respectively. Assume that count is a shared variable initialized to 0 and not used in CODE SECTION P.

CODE SECTION P

```
wait (a); count = count + 1;
if (count == n) signal (b);
signal (a); wait (b); signal (b);
```

CODE SECTION Q

What does the code achieve?

(a) It ensures that no process executes CODE SECTION Q before every process has finished CODE SECTION P.
(b) It ensures that all processes execute CODE SECTION P mutually exclusively.
(c) It ensures that two processes are in CODE SECTION Q at any time.
(d) It ensures that at most n-1 process are in CODE SECTION P at any time.

12. [NAT] [GATE-2016 : 2M]

Consider a non-negative counting semaphore S. The operation P(S) decrements S, and V(S) increments S. During an execution, 20 P(S) operations and 12 V(S) operations are issued in some order. The largest initial value of S for which at least one P(S) operation will remain blocked is ____

13. [MCQ] [GATE-2014 : 1M]

Consider the procedure below for the Producer-Consumer problem which uses semaphores:

```
semaphore n = 0;
semaphore s = 1;
Void producer ()              Void consumer ()
{                             {
 while (true)                     while(true)
 {                             {
    produce ();                  semWait(s);
    semWait(s);                  semWait(n);
    addToBuffer ();              removeFromBuffer();
    semSignal(s);                semSignal(s);
    semSignal(n);                consume ();
  }                              }
 }                             }
```

Which one of the following is TRUE?

(a) The producer will be able to add an item to the buffer, but the consumer can never consume it.
(b) The consumer will remove no more than one item from the buffer.
(c) Deadlock occurs if the consumer succeeds in acquiring semaphore s when the buffer is empty.
(d) The starting value for the semaphore n must be 1 and not 0 for deadlock-free operation.

14. [MCQ] [GATE-2013 : 2M]

A certain computation generates two arrays a and b such that a[i] = f(i) for $0 \le i < n$ and b [i] = g(a[i]) for $0 \le i < n$. Suppose this computation is decomposed into two concurrent processes X and Y such that X computes the array a and Y computes the array b. The processes employ two binary semaphores R and S, both initialized to zero. The array a is shared by the two processes. The structure of the processes are shown below.

```
Process X                      Process Y
private i;                       private i;
for (i = 0; i < n; i++)        for(i = 0; i < n; i++)
    {                               {
```

```
a[i] = f(i)              EntryY(R, S);
ExitsX(R, S);            b[i] = g(a[i]);
}                        }
```

Which one of the following represents the CORRECT implementations of ExistsX and EntryY?

(a)

```
Exits X (R, S)
        {
  P(R);
  V(S)
}
Entry Y (R, S) {
  P(S);
  V(R);
}
```

(b)

```
Exits X (R, S)
        {
  V(R);
  V(S);
}
Entry Y (R, S) {
  P(R);
  P(S);
}
```

(c)

```
Exits X (R, S)
        {
  P(S);
  V(R);
}
Entry Y (R, S){
  V(S);
  P(R);
}
```

(d)

```
Exits X (R, S)
        {
  V(R);
  P(S);
}
Entry Y (R, S){
  V(S);
  P(R);
}
```

15. [MCQ] [GATE-2013 : 2M]

A shared variable x, initialized to zero, is operated on by four concurrent process W, X, Y, Z as follows. Each of the processes W and X reads x from memory, increments by one, stores it to memory, and then terminates. Each of the processes Y and Z reads x from memory, decrements by one, stores it to memory and then terminates. Each process before reading x invokes the P operations (i.e., wait) on a counting semaphore S and invokes the V operation (i.e., signal) on the semaphore S after storing x to memory. Semaphore S is initialized to two. What is the maximum possible value of x after all processes complete execution?

(a) –2 (b) –1
(c) 1 (d) 2

16. [MCQ] [GATE-2013 : 1M]

Three concurrent processes X, Y and Z execute three different code segments that access and update certain shared variables. Process X execute the P operation (i.e., wait) on semaphores a, b and c; process Y executes the P operation on semaphores b, c and d; process Z executes the P operation on semaphores c, d and a before entering the respective code segments. After completing the execution of its code segments each process invokes the V operation (i.e., signal) on its three semaphores. All semaphores are binary semaphores initialized to one. Which one of the following represents a deadlock-free order of invoking the P operations by the processes?

(a) X: P(a)P(b)P(c)
Y: P(b)P(c)P(d)
Z: P(c)P(d)P(a)

(b) X: P(b)P(a)P(c)
Y: P(b)P(c)P(d)
Z: P(a)P(c)P(d)

(c) X: P(b)P(a)P(c)
Y: P(c)P(b)P(d)
Z: P(a)P(c)P(d)

(d) X: P(a)P(b)P(c)
Y: P(c)P(b)P(d)
Z: P(a)P(c)P(d)

17. [MCQ] [GATE-2010 : 2M]

The following program consists of 3 concurrent processes and 3 binary semaphores. The semaphores are initialized as S0 = 1, S1 = 0, S2 = 0.

Process P0	Process P1	Process P2

while (true) {
wait (S0);
print '0'
release (S1);
release (S2);
}
	wait (S1);
	release(S0);
		wait (S2)
		release (S0);

How many times will process P0 print '0'?

(a) At least twice (b) Exactly twice
(c) Exactly thrice (d) Exactly once

18. [MCQ] [GATE-2008 : 2M]

The P and V operations on counting semaphores, where s is a counting semaphore, are defined as follows:

P(s): s = s – 1;
if s < 0 then wait;
V(s): s = s + 1;
if s ≤ 0 then wakeup a process waiting on s;

Assume that P_b, and V_b, the wait and signal operations on binary semaphores are provided. Two binary semaphores X_b and Y_b are used to implement the semaphore operations P(s) and V(s) as follows:

```
P(s):  Pb (Xb);
       s = s – 1;
       if (s < 0) {
       Vb (Xb);
       Pb (Yb);
       }
       else Vb(Xb);
V(s):  Pb (Xb);
       s = s + 1;
       if (s ≤ 0) Vb(Yb);
       Vb(Xb);
```

The initial values of X_b and Y_b are respectively

(a) 0 and 0 (b) 0 and 1
(c) 1 and 0 (d) 1 and 1

19. [MCQ] [GATE-2008 : 2

The following is a code with two threads, producer and consumer, that can run in parallel. Further S and Q are binary semaphores equipped with the standard P and V operations.

```
Semaphore S = 1, Q = 0;
integer x;
producer:              consumer:
while (true) do        while (true) do
  P(S);                  P(Q);
x = produce ();        consume (x);
  V(Q);                  V(S);
done                   done
```

Which of the following is TRUE about the program above?

(a) The process can deadlock
(b) One of the threads can starve
(c) Some of the items produced by the producer may be lost
(d) Values generated and stored in 'x' by the producer will always be consumed before the producer can generate a new value.

20. [MCQ] [GATE-2023-CS: 2M]

Consider the two functions incr and decr shown below.

There are 5 threads each invoking incr once, and 3 threads each invoking decr once, on the same shared variable X. The initial value of X is 10. Suppose there are two implementations of the semaphore s, as follows:

```
incr(){               decr(){
wait(s);                 wait(s);
X = X +1;                X = X -1;
Signal(s);               Signal(s);
}                     }
```

I-1: s is a binary semaphore initialized to 1.
I-2: s is a counting semaphore initialized to 2.

Let V1, V2 be the values of X at the end of executiong of all the threads with implementations I-1, I-2, respectively.

Which one of the following corresponds to the minimum possible values of V1, V2, respectively?

(a) 15, 7 (b) 7, 7
(c) 12, 7 (d) 12, 8

21. [MCQ] [GATE-2022-CS: 1M]

Consider the following threads, T_1, T_2, and T_3 executing on a single processor, synchronized using three binary semaphore variables, S1, S2, and S3, operated upon using standard wait() and signal(). The threads can be context switched in any order and at any time.

T_1	T_2	T_3
while (true) { wait (S_3); print("C"); signal (S_2); }	while (true) { wait (S_1); print("B"); signal (S_3); }	while (true) {wait (S_2); print("A"); signal (S_1); }

Which initialization of the semaphores would print the sequence BCABCABCA…..?

(a) $S_1 = 1; S_2 = 1; S_3 = 1$
(b) $S_1 = 1; S_2 = 1; S_3 = 0$
(c) $S_1 = 1; S_2 = 0; S_3 = 0$
(d) $S_1 = 0; S_2 = 1; S_3 = 1$

Classical IPC Problems

22. [MCQ] [GATE-2015 : 1M]

Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is N. Three semaphores empty, full and mutex are defined with respective initial value of 0, N and 1. Semaphore empty denotes the number of available slots in the buffer, for the consumer to read from. Semaphore full denotes the number of available slots in the buffer, for the producer to write to. The placeholder variables, denoted by P, Q, R and S, in the code below can be assigned either empty or full. The valid semaphore operations are: wait () and signal ().

Producer:	Consumer:

do {
wait (P);
wait (mutex);
//Add item to buffer
signal (mutex);
signal (Q);
}while (1);
	do {
	wait (R);
	wait (mutex);
	//consume item from buffer
	signal (mutex);
	signal (S);
	}while (1);

Which one of the following assignment to P, Q, R and S will yield the correct solution?

(a) P: full, Q: full, R: empty, S: empty
(b) P: empty, Q: empty, R: full, S: full
(c) P: full, Q: empty, R: empty, S: full
(d) P: empty, Q: full, R: full, S: empty

Concurrency Mechanism

23. [NAT] [GATE-2019 : 1M]

Consider three concurrent process P1, P2 and P3 as shown below, which access a shared variable D that has been initialized to 100.

P_1	P_2	P_3
:	:	:
:	:	:
D = D + 20	D = D – 50	D = D + 10
:	:	:
:	:	:

The processes are executed on a uniprocessor system running a time-shared operating system. If the minimum and maximum possible values of D after the three processes have completed execution are X and Y respectively, then the value of Y-X is ___________.

24. [NAT] [GATE-2015 : 1M]

The following two functions P1 and P2 that share a variable B with an initial value of 2 execute concurrently.

```
P1()                    P2()
{                       {
   C = B -1;                D = 2 * B;
   B = 2 * C                B = D – 1;
}                       }
```

The number of distinct values that B can possibly take after the execution is _______.

❑❑❑

ANSWER KEY

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(b)
5.	(a)	**6.**	(a, d)	**7.**	(d)	**8.**	(a)
9.	(a, b, c)	**10.**	(a, b, d)	**11.**	(a)	**12.**	(7 to 7)
13.	(c)	**14.**	(c)	**15.**	(d)	**16.**	(b)
17.	(a)	**18.**	(c)	**19.**	(d)	**20.**	(c)
21.	(c)	**22.**	(c)	**23.**	(80 to 80)	**24.**	(3 to 3)

SOLUTIONS

1. (a)

The given algorithm is an updated version of Bakery Algorithm, this algorithm is basically for multi process solution.

The condition while **(t[j]!=0 && t[j] <= t[i]);** ensures that no two processes can be in critical section at the same time.

The given algorithm satisfies mutual exclusion but does not satisfies bounded waiting, progress, and deadlock. So, at most one process can be in the critical section at any time.

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2. (c)

The given code is of strict alternation. Both processes are strictly changing the value of turn to 1 and 0. Strict alternation guarantees mutual exclusion but fails to guarantee process.

Therefore, This solution violates progress requirement. Hence, option (c) is correct.

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3. (a)

Suppose
Process Y starts execution
made varQ= true; and pre-empted.
Process X starts execution
made varP = true;
Now both varP and varQ are true and both process X and Y can enter into CS simultaneously. Hence, no mutual exclusion.
Here deadlock is not possible, for deadlock the processes must stuck, here no process is stucked both are executing hence no deadlock.
Therefore, the proposed solution prevents deadlock but fails to guarantee mutual exclusion.

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4. (b)

Fetch-And-Add(x, i), this will returns value of (x) and increment 'x' by 'i'.

Implementation :

```
int fetch-And-Add (x, i)
{
    int rv;
    rv = x;
    x = x + i;
```

```
    return(rv);
}
while (F-A(L, 1))        // Entry section
<cs>
L = 0                    // Exit section
```

A process acquires a lock only when L is equals to 0, when L=1, then the process repeates the while loop. There is no overflow because after every time l is incremented, L is again made equals to 1.

Acquiring lock is successful only when Fetch_And_Add gets executed with L = 0.

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5. (a)

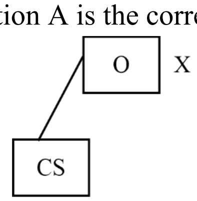
The given solution is deadlock free. So, statement I is true.

The given solution does not guarantee bounded waiting, this can lead to starvation. So, statement II is false.

There is no restriction on processes to enter the CS in a certain manner. Therefore, statement III is false.

The given solution guarantees mutual exclusion. So, statement IV is also incorrect.

Therefore, option A is the correct answer.



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6. (a, d)

Here value of x is not assigned, but in C programming all the global and static variables are initialised to 0. y is a local variable.

So,

1. Process P_1:

Two threads are created in main. Both threads execute foo() function and do not wait for each other and because of explicit locking mechanism [Acquire L1 and Release L1], mutual exclusion is there and no race condition will take place inside function foo().

y is local variable, both threads will print its value as 1.

Because of wait in the main (); print (x) will only execute when both threads are completed. So, the x value will become 2.

2. Process P_2:

P_2 will be executed same as P1. So, it will also print value of x as 2 and value of y as 1. Therefore, option A and D are true.

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7. (d)

The question is asking for the minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:

So, checking for the minimum value x = 1, y = 1.

Deadlock is possible in this scenario the condition is process should perform down/wait operation twice. If down operation on lock is performed twice, then deadlock is possible.

Therefore, option D is correct.

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8. (a)

The given code is purely Strict alternation method. Strict alternation always gurantee mutual exclusion but do not gurantee progress. So, option A is correct.

Scan for Video solution

9. (a, b, c)

When every process is in the waiting state, a system is said to be in deadlock. This is comparable to a stop-and-go traffic situation.

When processes carry out repeated tasks without advancing the system (still producing no valuable work), the system is said to be in livelock. This is comparable to a traffic gridlock when some cars drive forward, strike a block, then reverse and move forward again.

Now, deadlock and livelock are mutually exclusive; in a system, only one of them can occur at any given time. However, they both suggest that the system will not advance, starving the associated processes in the process.

Now, regarding the query, any process can terminate another process and then obtain the necessary resource, and this can result in

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10. (a, b, d)

The value of semaphore is initialised to 5.

If all the threads execute sequentially one by one, like first thread arrives and increases the value of counter by 1, then next thread increases it to 2 and so on. So, the final value will become 5. Therefore, option A is correct.

If the first thread arrives and performed counter++, increases the value of counter by 1, load the value in the register and gets pre-empted. Now subsequent thread arrives and completed the execution, increases the value of counter. At last, first thread will arrive (as it was pre-empted) to complete its execution it will store the value into register which is counter =1 (the value loaded onto register before pre-emption). So, option B is also correct.

The value of counter cannot be 0, because counter's value has to be updated at least one time. Therefore, it's maximum value will be 5 and minimum value will be 1. So, option C is incorrect.

Deadlock can be present in the system, if first thread arrives and performed step-5(wait(S);) and gets pre-empted, second thread arrives perform step-5 (wait(S);) and gets pre-empted and all other subsequent threads does the same. Now, the semaphore value will become 0 and if thread wants to perform step-6 (wait(S);), it will get blocked, and all the subsequent threads will be blocked after performing second wait(S); operation. None of the threads are able to execute further and hence a deadlock is possible involving all the threads. So, option D is also correct.

So, option A, B, and D are correct.

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11. (a)

Given,

There are two semaphores a and b initialized to 1 and 0 respectively. Count is a shared variable it is initialized to 0.

1. CODE SECTION P
2. wait (a);
3. count = count + 1;
4. if (count == n) signal (b);
5. signal (a);
6. wait (b);
7. signal (b);
8. CODE SECTION Q

Process will arrive,

Complete the CODE SECTION P //line 1.

and then only it can perform wait (a); //line 2

now, it will increase the value of count to 1; //line 3

check if (count==n), if yes then perform signal (b); else we will not execute signal(b); //line 4

It will perform signal (a); //line 5

wait (b) [b was initialized to 0, so it will be blocked]. //line 6

Now, processes will continuously block till count ==n. Once count becomes equals to n,

it will execute signal(b) // line 4

and this unblocks one process, that unblocked process will execute signal (b) // line 7

Now, subsequent processes will be unblocked and can enter CODE SECTION Q //line 8.

Means,

No processes can enter into CODE SECTION Q until N processes finishes CODE SECTION P and N processes has incremented the count variable.

OR

To execute CODE SECTION Q, N processes must completed their CODE SECTION P and has incremented the count variable.

So, It ensures that no process executes CODE SECTION Q before every process has finished CODE SECTION P. Therefore, option A is correct.

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12. (7)

Suppose, the initial value of S = x

Now, 20P means 20 down operations(-20) and 12 V means 12 up operations (+12).

It is saying that at least one operation should remain blocked.

So, this can be written as,

$x - 20 + 12 = -1$

$x - 8 = -1$

$x = 7$

So, the largest initial value of S for which at least one operation will remain blocked is 7.

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13. (c)

In the given code, if the producer will produce the item, then consumer will definitely consume it. So, the option A is incorrect.

If producer is producing two or more items then consumer can consume all of them, So, option B is incorrect.

If the buffer is empty, then

n = 0

S = $\cancel{1}$ 0 [wait operation on semWait(S)]

$\because$ n = 0; consumer gets blocked

S = 0; producer gets blocked

Both the processes are blocked, therfore there exists a deadlock. Hence, option C is correct.

In option D, with n =1, then also there will be a deadlock, So, option D is also incorrect.

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14. (c)

(a) With option A, deadlock happens.

(b) Process 'X' may compute all values (iterations) and process would be able to make only one iteration and then gets blocked. So, option B is also incorrect.

(c) Processes 'X and 'Y' alternates and signal each other so that all iterations of both 'X' and 'Y' completes successfully

(d) Some of the signals issued by 'X' cannot be captured by 'Y'

Therfore, process 'Y' will not be able to complete all iterations. So, option D is also incorrect.

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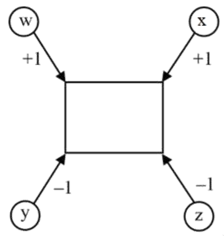
15. (d)

The code can be written as:

```
CSEM S = 2;
Pri
i = W, X, Y, Z
{
    P(S);
    <update x>
    V(S);
}
```



W: Completes, x =1
X: Load ; increment; Preempted, x =2
Y: Completes
Z: Completes
X: Store, x = 2 is stored as the final value

Therfore, 2 is the correct answer hence option D is true.

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16. (b)

In option A:

X: P(a) // a = 0
Y: P(b) // b= 0
Z: P(c) //c =0

X: P(a)P(b) // b= blocked
Y: P(b)P(c) // c= blocked
Z: P(c)P(d) P(a) // d = 0, a = blocked
Here we can have deadlock.

In option B:

X: P(b) // b = 0
Y: P(b) // b = blocked
Z: P(a) // a = 0
X: P(b)P(a) // a = blocked
Y: P(b)P(c) // already blocked by b
Z: P(a)P(c) // c = 0

X: P(b)P(a)P(c) // already blocked by a
Y: P(b)P(c)P(d) // already blocked by b
Z: P(a)P(c)P(d) // d= 0 [Success]
No deadlock.

If one process is successful other processes can also complete successfully. So, option B is the correct answer.

In option C:

X: P(b) // b=0
Y: P(c) // c= 0
Z: P(a) // a = 0
X: P(b)P(a)P(c) // a = blocked
Y: P(c)P(b)P(d) // b = blocked
Z: P(a)P(c)P(d) // c = blocked.
Here we can have deadlock.

In option D:

X: P(a) // a = 0
Y: P(c) // c = 0
Z: P(a) // a = blocked
X: P(a) P(b)P(c) // b= 0
Y: P(c)P(b)P(d) // b = blocked
Z: P(a)P(c)P(d)
Here also deadlock is possible.

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17. (a)

If the process run in following order

- P_0; P_1; P_2; P_0; P_0 → "0" will be printed 2 times
  If the process run in following order
- P_0; P_1; P_0; P_2; P_0; P_0 → "0" will be printed 3 times

So, "0" will be printed maximum 3 times, and minimum 2 times. Therefore, option A is correct.

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18. **(c)**

Initial

The value of X_b compulsarily should be 1, So that any one of $P_b(X_b)$ should be successful in the beginenning itself.

$Y_b = 0$

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19. **(d)**

Only after the producer has made the thing may the consumer consume it, and only after the consumer has consumed the item can the producer manufacture it again (except for the first time).

Let's go over how this code functions.

The parallel execution of Producer and Consumer is mentioned.

Producer:

S value is 1 in st1, P(S) on S reduces it to 0, and st2 produces the x item.

Q value for st3 is 0. V(Q) on Q results in 1.

Since this is an endless while loop, it ought to loop endlessly. S is already 0 in the subsequent while loop iteration; further P(S) on 0 sends P to S's blocked list. Producer is thus obstructed.

Consumer:

P(Q) on Q causes Q to equal 0 and then eats the thing.

Instead of setting S to 1 with V(S) on S, the consumer now reawakens the stalled process on Q's queue. Process P thus becomes active. Since P was blocked at statement 1, it continues from statement 2. P then creates the subsequent item. Therefore, a product is consumed by a consumer before a producer creates a new one.

The right answer is (d).

Option for this query:

Deadlock cannot occur because both the producer and the consumer are using distinct semaphores (there is no hold and wait). Likewise, there is no starvation since there is a change in the relationship between the producer and the consumer, which also results in bounded waiting.

(C) The producer might lose some of the products it produces, but

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20. **(c)**

Case I: Let's the minimum possible value of x for v1, when semaphore 's' is binary semaphore initialized to 1.

P4 …P8 $\Rightarrow$ Incr() Decr () $\Rightarrow$ P1 , P2, P3

1. Wait (s);	1. Wait(s);
2. x = x +1	2. x = x –1;
3. signal(s);	3. Signal(s)

Semaph~~ore~~ s = 1 0 1

P_1 = Decr ()

1. wait (s); s = 0

2. Read x; x = 10

Now, if you pre-empt P1, before performing decrement operation then other process would not be able to execute on 'x' as semaphore value is s = 0.

x – 1: 10 – 1: 9

x = 9; update x by 9

3. signal(s); s = 1

P_2 Decr()

1, 2, 3: Final value of x = 8

P_3: Decr ()

1, 2,3 : Final value of x = 7

P_4 P_5: Incr()

1,2,3 Increment x 5 times: x $\Rightarrow$ 12

Hence, minimum possible value is 12 for V_1.

Case II: Let's find the minimum possible value of x for V_2. when semaphore S is counting semaphore initialized to 2.

P_1: Decr ()

1. wait (s): $s \neq 2$ 1
2. read (x); x = 10

Preempt Process P_1.

Now. execute Incr()

P_4 ... P_5: Incr()

1, 2, 3: Final x value will be, x = 15

Now. revoke the process P_1 from pre-emption

2. Perform x – 1: 10–1 = 9

Update x: $x \equiv 9$

P_2 : Decr()

1,2,3 x = 8

P_3 Decr()

1,2,3 x = 7

Hence, find value of V_2 is 7 for counting semaphore

So, option C is the correct answer.

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21. (c)

We need the output to be BCABCABCA, i.e. it should start with B, means T_2 should be 1 and T_1, T_3 should be zero. This means S_1 should be 1; S_2 and S_3 would be 0.

So, option C is the correct answer.

Scan for Video solution

22. (c)

Given,

Empty is initialized with 0.

Full is initialized with N.

Mutex is initialized with 1.

Initially the buffer is empty therefore, empty is initialized to 0. And if buffer is full then Full will be 0 as there will be no slots available.

The given code is of classical IPC problem, producer consumer problem.

So, in Producer; wait (P), if P is full, then producer will be blocked. Means P should be full. So, option B and D are eliminated and are incorrect.

Now, if we are performing wait (P) // means wait on full

Then, signal(Q) operation should be on empty.

Hence, option A is also incorrect and option C is the correct answer.

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23. (80)

Given,

D = 100

X is minimum value

Y is maximum value

To calculate maximum value:

P_1: D = D + 20 $\quad \therefore$ D =120

P_3: D = D + 10 // executed load, executed increment and got pre-empted. [Means the value of register is 130 here but this value is not stored into register yet.]

P_2: D = D – 50 // value of D was 130 and now 130-50 = 80, we will store 80 into register.

Now, P_3 will complete its execution and perform store operation. When P3 preempted the value of register was 130, so it will store value as 130

Therefore, **Y = 130**

Similarly, To calculate maximum value:

P2: D = D-50 // Load the value; Decrement the value of register[100-50 = 50]; Pre-empt

Execute P1;

Execute P3;

Complete execution of P2, perform store operation.

The final value of register will be 50.

Therefore, **X = 50**

So, the difference between maximum and minimum value is Y – X = 130 – 50 = 80

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24. (3)

Initially B = 2

P1()

{

1. C = B -1;
2. B = 2 * C;

}

P2()

{

3. D = 2 * B;
4. B = D – 1;

}

CASE I:

1. C = B -1 ⇒ 2- 1 ⇒ 1
2. B = 2 * C ⇒ 2*1 ⇒ 2
3. D = 2* B ⇒ 2* 2 ⇒ 4
4. B = D -1 ⇒ 4-1 ⇒ 3

B = 3

CASE II:

3. D = 2 * B ⇒ 2 * 2 ⇒ 4
4. B = D - 1 ⇒ 4 - 1 ⇒ 3
1. C = B - 1 ⇒ 3 - 1 ⇒ 2
2. B = 2 * C ⇒ 2 * 2 ⇒ 4

B = 4

CASE III:

1. C = B - 1 ⇒ 2 – 1 ⇒ 1
3. D = 2 * B ⇒ 2 * 2 ⇒ 4
4. B = D – 1 ⇒ 4 -1 ⇒ 3
2. B = 2* C ⇒ 2 * 1 ⇒ 2.

B = 2

CASE IV:

3. D = 2 * B ⇒ 2 * 2 ⇒ 4
1. C = B - 1 ⇒ 2 – 1 ⇒ 1
4. B = D – 1 ⇒ 4 -1 ⇒ 3
2. B = 2* C ⇒ 2 * 1 ⇒ 2.

B = 2

The number of distinct values that B can possibly take after the execution is 3 [3, 4, 2].

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❑❑❑

CHAPTER

3 DEADLOCKS

Deadlock Handling

1. **[MCQ]** **[GATE-2019 : 2M]**

Consider the following snapshot of a system running n concurrent processes. Process i is holding X_i instances of a resources R, $1 \le i \le n$. Assume that all instances of R are currently in use. Further, for all i, process i can place a request for at most Y_i additional instances of R while holding the X_i instances it already has. Of the n processes, there are exactly two processes p and q such that $Y_p = Y_q = 0$. Which one of the following conditions guarantees that no other process apart form p and q can complete execution?

(a) $X_p + X_q < \text{Min}\ \{Y_k \mid 1 \le k \le n, k \ne p, k \ne q\}$

(b) $X_p + X_q < \text{Max}\ \{Y_k \mid 1 \le k \le n, k \ne p, k \ne q\}$

(c) $\text{Min}\ (X_p, X_q \ge \text{Min}\ \{Y_k \mid 1 \le k \le n, k \ne p, k \ne q\}$

(d) $\text{Min}\ (X_p, X_q \le \text{Max}\ \{Y_k \mid 1 \le k \le n, k \ne p, k \ne q\}$

2. **[MCQ]** **[GATE-2018 : 2M]**

In a system, there are three types of resources: E, F and G. Four processes P_0, P_1, P_2 and P_3 execute concurrently. At the outset, the processes have declared their maximum resource requirements using a matrix named Max as given below. For example, Max[P_2, F] is the maximum number of instances of F that P_2 would require. The number of instances of the resources allocated to the various processes at any given state is given by a matrix named Allocation.

Consider a state of the system with the Allocation matrix as shown below and in which 3 instances of E and 3 instances of F are the only resources available.

Form the perspective of deadlock avoidance, which one of the following is true?

Allocation			
	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max			
	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

(a) The system is in safe state.

(b) The system is not in safe state, but would be safe if one more instance of E were available.

(c) The system is not in safe state, but would be safe if one more instance of F were available.

(d) The system is not in safe state, but would be safe if one more instance of G were available.

3. **[MCQ]** **[GATE-2017 : 2M]**

A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are non-reentrant, i.e., if a thread holds a lock l, then it cannot re-acquire lock l without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:

(a) x = 1, y = 2 (b) x = 2, y = 1

(c) x = 2, y = 2 (d) x = 1, y = 1

4. **[MCQ]** **[GATE-2017 : 2M]**

A system shares 9 tape drives. The current allocation and maximum requirement of tape drives for that processes are shown below:

Process	Current Allocation	Maximum Requirement
P1	3	7
P2	1	6
P3	3	5

Which of the following best describes current state of the system?
(a) Safe, Deadlocked
(b) Safe, Not Deadlocked
(c) Not Safe, Deadlocked
(d) Not Safe, Not Deadlocked

5. [MCQ] [GATE-2015: 2M]
Consider the following policies for preventing deadlock in a system with mutually exclusive resources.
I. Processes should acquire all their resources at the beginning of execution. If any resource is not available, all resources acquired so far are released.
II. The resources are numbered uniquely, and processes are allowed to request for resources only in increasing resource numbers.
III. The resources are numbered uniquely, and processes are allowed to request for resources only in decreasing resource numbers.
IV. The resources are numbered uniquely. A process is allowed to request only for a resource with resource number larger than its currently held resources.
Which of the above policies can be used for preventing deadlock?
(a) Any one of I and III but not II or IV
(b) Any one of I, III and IV but not II
(c) Any one of II and III but not I or IV
(d) Any one of I, II, III and IV

6. [MCQ] [GATE-2014 : 2M]
An operating system uses the Banker's algorithm for deadlock avoidance when managing the allocation of three resource types X, Y and Z to three processes P_0, P_1, and P_2. The table given below represents the current system state. Here, the Allocation matrix shows the current number of resources of each type allocated to each process and the Max matrix shows the maximum number of resources of each type required by each process during its execution.

	Allocation			Max		
	X	Y	Z	X	Y	Z
P0	0	0	1	8	4	3
P1	3	2	0	6	2	0
P2	2	1	1	3	3	3

There are 3 units of type X, 2 units of type Y and 2 units of type Z still available. The system is currently in a safe state. Consider the following independent requests for additional resources in the current state:
REQ 1: P_0 requests 0 units of X, 0 units of Y and 2 units of Z
REQ 2: P_1 requests 2 units of X, 0 units of Y and 0 units of Z
Which one of the following is TRUE?
(a) Only REQ 1 can be permitted.
(b) Only REQ 2 can be permitted.
(c) Both REQ 1 and REQ 2 can be permitted
(d) Neither REQI nor REQ 2 can be permitted

7. [MCQ] [GATE-2008 : 2M]
Which of the following is NOT true of deadlock prevention and deadlock avoidance schemes?
(a) In deadlock prevention, the request for resources is always granted if the resulting state is safe
(b) In deadlock avoidance, the request for resources is always granted if the resulting state is safe
(c) Deadlock avoidance is less restrictive than deadlock prevention
(d) Deadlock avoidance requires knowledge of resource requirements a priori.

Resource Allocation Graph

8. [MSQ] [GATE-2022: 1M]
Which of the following statements is/are TRUE with respect to deadlocks?
(a) Circular wait is a necessary condition for the formation of deadlock.
(b) In a system where each resource has more than one instance, a cycle in its wait-for graph indicates the presence of a deadlock.b

(c) If the current allocation of resources to processes leads the system to unsafe state, then deadlock will necessarily occur.
(d) In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

9. [NAT] [GATE-2018 : 1M]
Consider a system with 3 processes that share 4 instances of the same resource type. Each process can request a maximum of K instances. Resource instances can be requested and released only one at a time. The largest value of K that will always avoid deadlock is ______.

10. [MCQ] [GATE-2015 : 2M]
A system has 6 identical resources and N processes competing for them. Each process can request at most 2 resources. Which one of the following values of N could lead to a deadlock?
(a) 1 (b) 2
(c) 3 (d) 4

11. [NAT] [GATE-2014: 2M]
A system contains three programs, and each requires three tape units for its operation. The minimum numbe of tape units which the system must have such that deadlocks never arise is______.

12. [MCQ] [GATE-2010 : 2M]
A system has n resources R_0, R_1, …, R_n – 1, and k processes P_0, P_1, …, Pk – 1 . The implementation of the resource request logic of each process Pi, is as follows:
```
if (i% 2 == 0) {
if (i < n) request Ri;
if (i + 2 < n) request Ri + 2;
}
else {
if (i < n) request Rn – i;
if (i + 2 < n) request Rn – i– 2 ;
}
```
In which one of the following situations is a deadlock possible?
(a) n = 40, k = 26 (b) n = 21, k = 12
(c) n = 20, k = 10 (d) n = 41, k = 19

13. [MCQ] [GATE-2009 : 2M]
Consider a system with 4 types of resources R1 (3 units), R2(2 units), R3(3 units), R4(2 units). A non-preemptive resource allocation policy is used. At any given instance, a request is not entertained if it cannot be completely satisfied. Three processes P1, P2, P3 request the resources as follows if executed independently.

Process Pl:	Process P2:	Process P3:
t = 0 : requests 2 units of R2	t = 0: requests 2 units of R3	t = 0: requests 1 unit of R4
t = 1: requests 1 unit of R3	t = 2 : requests 1 unit of R4	t = 2: requests 2 units of Rl
t = 3: requests 2 units of RI	t = 4 : requests 1 unit of Rl	t = 5: releases 2 units of Rl
t = 5: releases 1 unit of R2 and 1 unit of RI	t = 6 : releases 1 unit of R3	t = 7: requests 1 unit of R2
t = 7 : releases 1 unit of R3	t = 8 : Finishes	t = 8: requests 1 unit of R3
t = 8 : requests 2 units of R4		t = 9: Finishes
t = 10 : Finishes		

Which one of the following statements is TRUE if all three processes run concurrently starting at time t = 0?
(a) All processes will finish without any deadlock.
(b) Only P1 and P2 will be in deadlock.
(c) Only P1 and P3 will be in deadlock.
(d) All three processes will be in deadlock

14. [MCQ] [GATE-2008: 2M]
An operating system implements a policy that requires a process to release all resources before making a request for another resource.
Select the TRUE statement from the following:
(a) Both starvation and deadlock can occur.
(b) Starvation can occur but deadlock cannot occur.
(c) Starvation cannot occur but deadlock can occur.
(d) Neither starvation nor deadlock can occur.

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ANSWER KEY

1.	(a)	**2.**	(a)	**3.**	(d)	**4.**	(b)
5.	(d)	**6.**	(b)	**7.**	(a)	**8.**	(a, d)
9.	(2 to 2)	**10.**	(d)	**11.**	(7 to 7)	**12.**	(b)
13.	(a)	**14.**	(b)				

SOLUTIONS

1. (a)

p_i	alloc	request	Avail/R
p_1	x_1	y_1	0
p_2	x_2	y_2	
.	.	.	
.	.	.	
p_p	x_p	0	
P=	x_p	0	
.	.	.	
.	.	.	
p_n	x_n	y_n	

$<P_p, p_q$

't' : $(x_p + x_q)$: Avail

$(x_p + x_q) < \min (y_k)$

$k \neq (p, q)$

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2. (a)

Resource Allocation Matrix:

Pid	Max E F G	Allocation E F G	Need E F G	Available E F G
Po	4 3 1	1 0 1	3 3 0	3 0 0
P1	2 1 4	1 1 2	1 0 2	
P2	1 3 3	1 0 3	0 3 0	
P3	5 4 1	2 0 0	3 4 1	

P0 can be executed. And the available will be

Pid	Max E F G	Allocation E F G	Need E F G	Available E F G
Po	4 3 1	1 0 1	3 3 0	3 0 0
P1	2 1 4	1 1 2	1 0 2	4 3 1
P2	1 3 3	1 0 3	0 3 0	
P3	5 4 1	2 0 0	3 4 1	

Now, P2's can be satisfied, so P2 will be executed and

Pid	Max E F G	Allocation E F G	Need E F G	Available E F G
Po	4 3 1	1 0 1	3 3 0	3 0 0
P1	2 1 4	1 1 2	1 0 2	4 3 1
P2	1 3 3	1 0 3	0 3 0	5 3 4
P3	5 4 1	2 0 0	3 4 1	

Now, P1's need can be satisfied, so P1 will be executed and

Pid	Max E F G	Allocation E F G	Need E F G	Available E F G
Po	4 3 1	1 0 1	3 3 0	3 0 0
P1	2 1 4	1 1 2	1 0 2	4 3 1
P2	1 3 3	1 0 3	0 3 0	5 3 4
P3	5 4 1	2 0 0	3 4 1	6 4 6

Now, we can satisfy needs of P3. Therefore,

Pid	Max E F G	Allocation E F G	Need E F G	Available E F G
Po	4 3 1	1 0 1	3 3 0	3 0 0
P1	2 1 4	1 1 2	1 0 2	4 3 1
P2	1 3 3	1 0 3	0 3 0	5 3 4
P3	5 4 1	2 0 0	3 4 1	6 4 6
				8 4 6

Safe sequence will be: $\boxed{< P_o, P_2, P_1, P_3 >}$

System state is safe

So, option A is True.

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3. **(d)**

The question is asking for the minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:

So, checking for the minimum value x = 1, y = 1.

Deadlock is possible in this scenario the condition is process should perform down/wait operation twice. If down operation on lock is performed twice, then deadlock is possible.

Therefore, option D is correct.

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4. **(b)**

Resource Allocation Matrix:

Pid	Max R	Allocation R	Need R	Avail R
P1	7	3	4	2
P2	6	1	5	
P3	5	3	2	

P3's need can be satisfied and the available will become

Pid	Max R	Allocation R	Need R	Avail R
P1	7	3	4	2
P2	6	1	5	5
P3	5	3	2	

Now, P2's need can be fulfilled.

Pid	Max R	Allocation R	Need R	Avail R
P1	7	3	4	2
P2	6	1	5	5
P3	5	3	2	6

Now, P1's need can be fulfilled.

Pid	Max R	Allocation R	Need R	Avail R
P1	7	3	4	2
P2	6	1	5	5
P3	5	3	2	6
				9

R = 9

Safe sequence: P_3, P_2, P_1

∴ System is safe and not deadlocked

So, option B is correct.

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5. **(d)**

A deadlock will never take place if any one of the following condition is preempted.

1. Hold & Wait
2. Circular Wait

3. Mutual Exclusion
4. No-Preemption

Statement I violates hold & wait.

Statement II violates circular wait.

Statement III violates circular wait.

Statement IV also violates circualr wait by making dependency graph acyclic.

Therefore, option D is correct.

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6. (b)

Resource allocation table:

Pid	Max X Y Z	Alloc X Y Z	Need X Y Z	Avail X Y Z
P_0	8 4 3	0 0 1	8 4 2	3 2 2
P_1	6 2 0	3 2 0	3 0 0	
P_2	3 3 3	2 1 1	1 2 2	

Request 1 needs [0, 0, 2] resources for process P_1, then the system will become

Pid	Max X Y Z	Alloc X Y Z	Need X Y Z	Avail X Y Z
P_0	8 4 3	0 0 3	8 4 0	3 2 0
P_1	6 2 0	3 2 0	3 0 0	
P_2	3 3 3	2 1 1	1 2 2	

Now, we can satisfy need of process P_1, after allotting resources to P_1,

Pid	Max X Y Z	Alloc X Y Z	Need X Y Z	Avail X Y Z
P_0	8 4 3	0 0 3	8 4 0	3 2 0
P_1	6 2 0	3 2 0	3 0 0	6 4 0
P_2	3 3 3	2 1 1	1 2 2	

Now, with the given available resources, the need of neither P_2 nor P_0 can be satisfied, system is unsafe, Therefore, Req_1 cannot be granted.

Now, checking for Req_2. Request 2 needs [2, 0, 0]

Pid	Max X Y Z	Alloc X Y Z	Need X Y Z	Avail X Y Z
P_0	8 4 3	0 0 1	8 4 2	1 2 2
P_1	6 2 0	5 2 0	1 0 0	
P_2	3 3 3	2 1 1	1 2 2	

P_1's need can be satisfied and available will become

Pid	Max X Y Z	Alloc X Y Z	Need X Y Z	Avail X Y Z
P_0	8 4 3	0 0 1	8 4 2	1 2 2
P_1	6 2 0	5 2 0	1 0 0	6 4 2
P_2	3 3 3	2 1 1	1 2 2	

Now, P_2's request can be granted and after completing P_2 the new available will become.

Pid	Max X Y Z	Alloc X Y Z	Need X Y Z	Avail X Y Z
P_0	8 4 3	0 0 1	8 4 2	1 2 2
P_1	6 2 0	5 2 0	1 0 0	6 4 2
P_2	3 3 3	2 1 1	1 2 2	8 5 3

With 8, 5, 3. Need of P_0 can be fulfilled easily.

Hence Req_2 can be granted.

(a) System is unsafe
$\therefore$ Req1 is Not granted

(b) < P_1, P_2, P_0>

System is safe
Req2 is granted

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7. **(a)**

In deadlock prevention, we need to ensure one of the four conditions (hold & wait, circular wait, mutual exclusion and no-pre-emption) to avoid deadlock, but there may be a case where resulting state is safe but the resource request is rejected. So, option A is not always true.

In deadlock avoidance, if the resulting state is safe, then request is always granted and deadlock avoidance is less restrictive than deadlock prevention. So, option B, C both are true.

In deadlock avoidance, knowledge of resource requirements must be known apriori. True.

Therefore, option A is correct.

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8. **(a, d)**

(a) Circular wait is a necessary condition for the formation of deadlock. TRUE

(b) In a system where each resource has more than one instance, a cycle in its wait-for graph indicates the presence of a deadlock. FALSE. Every cycle is not a deadlock but every deadlock have a cycle.

(c) If the current allocation of resources to processes leads the system to unsafe state, then deadlock will necessarily occur. FALSE. Every unsafe state is not deadlock.

(d) In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state. TRUE. Only hold without wait doesn't cause deadlock.

Therefore, option A, D are correct.

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9. **(2 to 2)**

Given,

R = 4

To avoid deadlock, so each process can have maximum of k-1 resources.

Required	
P_1	K –1
P_2	K –1
P_3	K –1

R = 4

Condition for deadlock = (3K – 3) +1

$\Rightarrow$ 3K –2 = 4

$\Rightarrow$ 3K = 6

$\boxed{\therefore K=2}$

So, 2 is the correct answer.

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10. **(d)**

$P_i = 2$ (R)

If we allot 2 resources to 3 processes, then total resource required = 6. Hence, no deadlock.

The number of processes, so the system should be in deadlock will be 4. Each processes requires 2 resources at most. Total resource required= 8.

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11. **(7 to 7)**

Number of processes, N = 3

Min (R)

$P_i \rightarrow 3$ (R)

Minimum number of resource unit so the deadlock never arises is, 7, upto 6 resources each process can have 2 resources each and result into deadlock. But with 7 resources, at least one processes' need will be satisfied and hence other processes can also proceed.

$P_1 - 2$

$P_2 - 2 \quad +1 = \boxed{7}$

$P_3 - 2$

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12. (b)

- Based on Resource Request logic, Even Numbered processes are Requesting Even number resource.
- For Preventing/Avoiding Deadlock, odd numbered processes should request odd numbered resources.
- In the else part, when 'n' is odd, then process will request even numbered resources & may form cycle, which will lead to Deadlock.

$\therefore$ Only in option (B) & (D) there is a possibility of deadlock.

Option (B):

M = 2k; k = 12

$P_0 \rightarrow R_0, R_2$	$P_1 \rightarrow R_{20}, R_{18}$
$P_2 \rightarrow R_2, R_4$	$P_3 \rightarrow R_{18}, R_{16}$
$P_4 \rightarrow R_4, R_6$	$P_5 \rightarrow R_{16}, R_{14}$
$P_6 \rightarrow R_6, R_8$	$P_7 \rightarrow R_{14}, R_{12}$
$P_8 \rightarrow R_8, R_{10}$	$P_9 \rightarrow R_{12}, R_{10}$
$P_{10} \rightarrow R_{10}, R_{11}$	$P_{11} \rightarrow R_{10}, R_8$

$\boxed{\begin{matrix} P_8 \rightarrow R_8, R_{10} \\ P_{11} \rightarrow R_{10}, R_8 \end{matrix}}$ Deadlock

Similarly, for Option (d) there is no – overlap with resources.

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13. (a)

List of events at time T_i

T_0 : $P_1 \rightarrow 2(R_2)$; $P_2 \rightarrow 2(R_3)$; $P_3 \rightarrow 1(R_4)$
T_1 : $P_1 \rightarrow 1(R_3)$;
T_2 : $P_2 \rightarrow 1(R_4)$; $P_3 \rightarrow 2(R_1)$;
T_3 : $P_1 \rightarrow 2(R_1)$;
T_4 : $P_2 \rightarrow 1(R_1)$
T_5 : $P_1 \rightarrow$ releases 1 unit of R_2 ; 1 unit of R_1 ; P_3 is releasing 2 (R_1)
T_6 : $P_2 \rightarrow$ releases 1 unit of R_3
T_7 : event is satisfied
T_8 : event is satisfied
T_9 : event is satisfied
T_{10} : event is satisfied

$\boxed{\text{No Deadlock}}$

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14. (b)

In such a policy starvation can occur as every time a process requests for a resource it has to release all the acquired resources. If the process has not utilized acquired resources properly then it has to repeat this when the process requests for another resource and improper utilization of resources can lead to starvation.

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CHAPTER

4 MEMORY MANAGEMENT

Memory Management Techniques

1. **[MCQ]** **[GATE-2020: 2M]**

Consider allocation of memory to a new process. Assume that none of the existing holes in the memory will exactly fit the process's memory requirement. Hence, a new hole of smaller size will be created if allocation is made in any of the existing holes. Which one of the following statements is TRUE?

(a) The hole created by next fit is never larger than the hole created by best fit.
(b) The hole created by first fit is always larger than the hole created by next fit.
(c) The hole created by best fit is never larger than the hole created by first fit.
(d) The hole created by worst fit is always larger than the hole created by first fit.

2. **[MCQ]** **[GATE-2015 : 2M]**

Consider six memory partitions of size 200 KB, 400 KB, 600 KB, 500 KB, 300 KB, and 250 KB, where KB refers to kilobyte. These partitions need to be allotted to four processes of sizes 357 KB, 210 KB, 468 KB and 491 KB in that order. If the best fit algorithm is used, which partitions are NOT allotted to any process?

(a) 200 KB and 300 KB (b) 200 KB and 250 KB
(c) 250 KB and 300 KB (d) 300 KB and 400 KB

Paging

3. **[NAT]** **[GATE-2023 : 2M]**

Consider a computer system with 57-bit virtual addressing using multi-level tree-structured page tables with L levels for virtual to physical address translation. The page size is 4 KB (1 KB = 1024 B) and a page table entry at any of the levels occupies 8 bytes.

The value of L is______.

4. **[MCQ]** **[GATE-2022 : 2M]**

Which one of the following statements is FALSE?

(a) The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.
(b) If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in a cache miss, then the word will always be present in the main memory.
(c) The memory access time using a given inverted page table is always same for all incoming virtual addresses.
(d) In a system that uses hashed page tables, if two distinct virtual addresses V1 and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same

5. **[MSQ]** **[GATE-2021 : 1M]**

In the context of operating systems, which of the following statements is/are correct with respect to paging?

(a) Paging helps solve the issue of external fragmentation
(b) Page size has no impact on internal fragmentation
(c) Paging incurs memory overheads
(d) Multi-level paging is necessary to support pages of different sizes

6. **[NAT]** **[GATE-2021 : 2M]**
Consider a three-level page table to translate a 39-bit virtual address to a physical address as shown below. The page size is 4KB (1KB $=2^{10}$ bytes) and page table entry size at every level is 8 bytes. A process P is currently using 2GB (1GB $=2^{30}$ bytes) virtual memory which is mapped to 2GB of physical memory. The minimum amount of memory required for the page table of P across all levels is _______KB.

← 39-bit virtual address →

Level 1 offset	Level 2 offset	Level 3 offset	Page offset
9 bits	9 bits	9 bits	12 bits

7. **[NAT]** **GATE-2016 : 2M]**
Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process page table is ___ megabytes.

8. **[NAT]** **[GATE-2015 : 2M]**
A computer system implements 8 kilobyte pages and a 32-bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table of a process is 24 megabytes, the length of the virtual address supported by the system is _____ bits.

9. **[NAT]** **[GATE-2015 : 2M]**
A computer system implements a 40 bit virtual address, page size of 8 kilobytes, and a 128-entry translation look-aside buffer (TLB) organized into 32 sets each having four ways. Assume that the TLB tag does not store any process id. The minimum length of the TLB tag in bits is ___

10. **[NAT]** **[GATE-2015 : 2M]**
Consider a system with byte-addressable memory, 32-bit logical addresses, 4 kilobyte page size and page table entries of 4 bytes each. The size of the page table in the system in megabytes is _____

11. **[NAT]** **[GATE-2014 : 2M]**
Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 milliseconds to search the TLB and 80 milliseconds to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is ____

12. **[MCQ]** **[GATE-2013: 2M]**
A computer uses 46-bit virtual address, 32-bit physical address, and a three-level paged page table organization. The page table base register stores the base address of the first-level (T1), which occupies exactly one page. Each entry of T1 stores the base address of a page of the second-level table (T2). Each entry of T2 stores the base address of a page of the third-level table (T3). Each entry of T3 stores a page table entry (PTE). The PTE is 32 bit in size. The processor used in the computer has a 1 MB 16-way-set associative virtually indexed physically tagged cache the cache block size is 64 bytes.
What is the size of a page in KB in this computer?
(a) 2 (b) 4
(c) 8 (d) 16

13. **[MCQ]** **[GATE-2009 : 2M]**
The essential content(s) in each entry of a page table is/are
(a) Virtual page number
(b) Page frame number
(c) Both virtual page number and page frame number
(d) Access right information

14. **[MCQ]** **[GATE-2009: 2M]**
A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because.

(a) it reduces the memory access time to read or write a memory location.
(b) it helps to reduce the size of page table needed to implement the virtual address space of a process.
(c) it is required by the translation look a side buffer.
(d) it helps to reduce the number of page faults in page replacement algorithms.

15. [MCQ] [GATE-2008: 2M]
A processor uses 36-bit physical addresses and 32-bit virtual addresses, with a page frame size of 4 Kbytes. Each page table entry is of size 4 bytes. A three-level page table is used for virtual-to-physical address translation, where the virtual address is used as follows:

- bits 30-31 are used to index into the first level page table
- bits 21-29 are used to index into the second level page table
- bits 12-20 are used to index into the third level page table
- bits 0-11 are used as offset within the page.

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are respectively.
(a) 20, 20 and 20
(b) 24, 24, and 24
(c) 24, 24 and 20
(d) 25, 25 and 24

16. [MCQ] [GATE-2008 : 2M]
A paging scheme uses a Translation Look-aside Buffer (TLB). A TLB-access takes 10 ns and a main memory access takes 50 ns. What is the effective access time (in ns) if the TLB hit ratio is 90% and there is no page-fault?
(a) 54 (b) 60
(c) 65 (d) 75

Virtual Memory

17. [NAT] [GATE-2023: 2M]
Consider the following two-dimensional array D in the C programming language, which is stored in row-major order:
int D[128][128];
Demand paging is used for allocating memory and each physical page frame holds 512 elements of the array D. The Least Recently Used (LRU) page-replacement policy is used by the operating system. A total of 30 physical page frames are allocated to a process which executes the following code snippet:

```
for (int i = 0; i < 128; i++)
        for (int j = 0; j < 128; j++)
                D[j][i] *= 10;
```

The number of page faults generated during the execution of this code snippet is_______.

18. [NAT] [GATE-2022 : 2M]
Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string 7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7,1, 5, 6,1 the page fault rate, defined as the ratio of number of page faults to the number of memory accesses (rounded off to one decimal place) is_________.

19. [NAT] [GATE-2020 : 2M]
Consider a paging system that uses 1-level page table residing in main memory and a TLB for address translation. Each main memory access takes 100 ns and TLB lookup takes 20 ns. Each page transfer to/from the disk takes 5000 ns. Assume that the TLB hit ratio is 95%, page fault rate is 10%. Assume that for 20% of the total page faults, a dirty page has to be written back to disk before the required page is read from disk. TLB update time is negligible. The average memory access time in ns (round off to 1 decimal places) is _____

20. [MCQ] [GATE-2019 : 2M]
Assume that in a certain computer, the virtual addresses are 64 bits long and the physical addresses are 48 bits long. The memory is word addressable.

The page size is 8 kB and the word size is 4 bytes. The translation Look-aside Buffer(TLB) in the address translation path has 128 valid entries. At most how many distinct virtual addresses can be translated without any TLB miss?

(a) 16×2^{10} (b) 256×2^{10}
(c) 4×2^{20} (d) 8×2^{20}

21. [MCQ] [GATE-2018 : 2M]

Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is M units if the corresponding memory page is available in memory and D units if the memory access causes a page fault. It has been experimentally measured that the average time taken for a memory access in the process is X units.

Which one of the following is the correct expression for the page fault rate experienced by the process?

(a) (D – M) / (X – M)
(b) (X – M) / (D – M)
(c) (D – X) / (D – M)
(d) (X – M) / (D – X)

22. [NAT] [GATE-2016 : 2M]

Consider a computer system with ten physical page frames. The system is provided with an access sequence (a_1, a_2, ….. a_{20}, a_1, a_2, …., a_{20}), where each ai is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is _____

23. [MCQ] [GATE-2011 : 2M]

Let the page fault service time be 10 ms in a computer with average memory access time being 20 ns. If one-page fault is generated for every 10^6 memory accesses, what is the effective access time for the memory?

(a) 21 ns (b) 30 ns
(c) 23 ns (d) 35 ns

24. [MCQ] [GATE-2008 : 2M]

Match the following flag bits used in the context of virtual memory management on the List-I (Name of the bit) with the different purposes on the List-II (Purpose) of the table below.

List-I	List-II
I. Dirty	a. Page initialization
II. R/W	b. Write-back policy
III. Reference	c. Page protection
IV. Valid	d. Page replacement policy

Codes:

(a) I-d, II-a, III-b, IV-c (b) I-b, II-c, III-a, IV-d
(c) I-c, II-d, III-a, IV-b (d) I-b, II-c, III-d, IV-a

Page Replacement Techniques

25. [MCQ] [GATE-2009 : 1M]

In which one of the following page replacement policies, Belady's anomaly may occur?

(a) FIFO (b) Optimal
(c) LRU (d) MRU

26. [MCQ] [GATE-2017 : 2M]

Recall that Belady's anomaly is that the page-fault rate may increase as the number of allocated frames increases. Now, consider the following statements:

S_1: Random page replacement algorithm (where a page chosen at random is replaced) suffers from Belady's anomaly

S_2: LRU page replacement algorithm suffers from Belady's anomaly

Which of the following is CORRECT?

(a) S_1 is true, S_2 is true (b) S_1 is true, S_2 is false
(c) S_1 is false, S_2 is true (d) S_1 is false, S_2 is false

27. [MCQ] [GATE-2016 : 2M]

In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases?

(a) LRU (Least Recently Used)
(b) OPT (Optimal Page Replacement)
(c) MRU (Most Recently Used)
(d) FIFO (First In First Out)

28. **[MCQ]** **[GATE-2015 : 2M]**
Consider a main memory with five-page frames and the following sequence of page references:
3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3
Which one of the following is true with respect to page replacement policies First-In-First Out (FIFO) and Least Recently Used (LRU)?
(a) Both incur the same number of page faults
(b) FIFO incurs 2 more-page faults than LRU
(c) LRU incurs 2 more-page faults than FIFO
(d) FIFO incurs 1 more page faults than LRU

29. **[NAT]** **[GATE-2014 : 2M]**
A system uses 3 pages frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below?
4, 7, 6, 1, 7, 6, 1, 2, 7, 2

30. **[MCQ]** **[GATE-2014 : 2M]**
A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2, …, 100 in that order, and repeats the access sequence THRICE.
Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?
(a) Least-recently-used
(b) First-in-first-out
(c) Last-in-first-out
(d) Most-recently-used

31. **[NAT]** **[GATE-2014 : 2M]**
Assume that there are 3-page frames which are initially empty. If the page reference string is 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6, the number of page faults using the optimal replacement policy is _____

32. **[MCQ]** **[GATE-2012 : 2M]**
Consider the virtual page reference string
1, 2, 3, 2, 4, 1, 3, 2, 4, 1
On a demand paged virtual memory system running on computer system that has main memory size of 3 page frames which are initially empty. Let LRU, FIFO and OPTIMAL denote the number of page faults under the corresponding page replacement policy. Then
(a) OPTICAL < LRU < FIFO
(b) OPTIMAL < FIFO < LRU
(c) OPTIMAL = LRU
(d) OPTIMAL = FIFO

33. **[MCQ]** **[GATE-2010 : 2M]**
A system uses FIFO policy for page replacement. It has 4-page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur?
(a) 196 (b) 192
(c) 197 (d) 195

34. **[MCQ]** **[GATE-2008 : 2M]**
Assume that a main memory with only 4 pages, each of 16 bytes, is initially empty. The CPU generates the following sequence of virtual addresses and uses the Least Recently Used (LRU) page replacement policy.
0, 4, 8, 20, 24, 36, 44, 12, 68, 72, 80, 84, 28, 32, 88, 92
How many page faults does this sequence cause? What are the page numbers of the page present in the main memory at the end of the sequence?
(a) 6 and 1, 2, 3, 4
(b) 7 and 1, 2, 4, 5
(c) 8 and 1, 2, 4, 5
(d) 9 and 1, 2, 3, 5

❑❑❑

ANSWER KEY

1.	(c)	**2.**	(a)	**3.**	(5 to 5)	**4.**	(c)
5.	(a, c)	**6.**	(4108 to 4108)	**7.**	(384 to 384)	**8.**	(36 to 36)
9.	(22 to 22)	**10.**	(4 to 4)	**11.**	(112 to 112)	**12.**	(c)
13.	(b)	**14.**	(b)	**15.**	(d)	**16.**	(c)
17.	(4096 to 4096)	**18.**	(0.6 to 0.6)	**19.**	(154.5 to 155.5)	**20.**	(b)
21.	(b)	**22.**	(1 to 1)	**23.**	(b)	**24.**	(d)
25.	(a)	**26.**	(b)	**27.**	(d)	**28.**	(a)
29.	(6 to 6)	**30.**	(d)	**31.**	(7 to 7)	**32.**	(b)
33.	(a)	**34.**	(b)				

SOLUTIONS

1. (c)

Best fit always search for the smallest hole that can accommodate a request. So, hole created by best fit will always be less than or equal to the hole created by any other policy.

Worst fit always search for the largest hole to accommodate a request. If the first hole is the largest one, then worst fit and first fit both will accommodate request in the same hole.

So, we can clearly say option c is the correct option.

Scan for Video solution

2. (a)

Here we have 6 memory partitions of sizes 200 KB, 400 KB, 600 KB, 500 KB, 300 KB, and 250 KB, and the partition allotted to four processes using best fit algorithm is as follows:

200	400	600	500	300	250
	357	491	468		210
	P1	P4	P3		P2

So, we can clearly see that partition 200KB and 300KB are empty and not allotted to any process. Hence, option A is correct.

Scan for Video solution

3. (5 to 5)

L : number of paging.

Virtual Address space = 57 bits

Page Size = 4 KB

Page table entry = 8 B

Page number = (57-12) = 45 bits

Number of entries in P.T = $\frac{4KB}{8B} = 512 = 2^9$. Means, we need 9 bits to index the page table.

level 1	level 2	level 3	level 4	level 5	offset
9	9	9	9	9	12

57 bits

So, number of levels = 9 × L = 45

$$\boxed{L = \frac{45}{9} = 5}$$

Scan for Video solution

4. (c)

(a) The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address. TRUE.

(b) If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in a cache miss, then the word will always be present in the main memory. TRUE. If the content is not in the physical address space, then it will present in main memory.

(c) The memory access time using a given inverted page table is always same for all incoming virtual addresses. FALSE. Inverted paging has issue of time, every virtual address may present at different locations. In worst case, we may have to search entire page table.

(d) In a system that uses hashed page tables, if two distinct virtual addresses V1 and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same. TRUE. Because we have to search in linked list addresses may present in different locations.

Therefore, option C is the correct answer.

Scan for Video solution

5. (a, c)

Paging helps solve the issue of external fragmentation. Correct.

Page size has no impact on internal fragmentation. Incorrect, paging has impact on internal fragmentation

Paging incurs memory overheads. Correct. Page tables are stored in memory and are overhead to memory.

Multi-level paging in necessary to support pages of different sizes. Incorrect.

Scan for Video solution

6. (4108 to 4108)

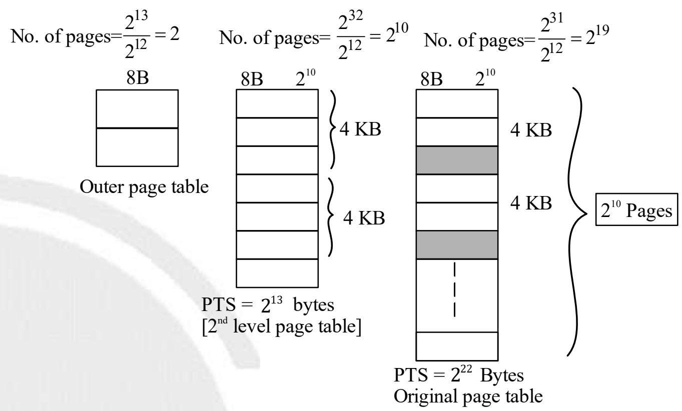
39 bits

V.A	Level 1	Level 2	Level 3	Offset d
	9	9	9	12

Page Size = 4 KB

Page Table Entry size = 8B

$$\frac{P}{\text{V.M.S} = 2\text{ GB}}$$



Total number of pages of PT'S needed.

= 1 k of 1 – level paging

= 2 k of 2 – level paging

= 1 k of Outer Page Table

= (1+2+1024) * 4 KB = $\boxed{\begin{array}{l}1027 \times 4\text{KB} \\ 4108\text{KB}\end{array}}$

Scan for Video solution

7. (384 to 384)

Virtual Address = 40 bits

Page Size = 16 KB

Page Table Entry = 48 bits = 6B

Page Table Size = N*e

$N = \frac{2^{40}}{2^{14}} = 2^{26}$

e = 6B

$\therefore$ P.T.S $= 2^{26} \times 6\text{B} = 64 \times 6$ MB

$\boxed{\text{P.T.S.} = 384 \text{ MB}}$

Scan for Video solution

8. (36 to 36)

Given,

Page Size = 8 KB

Physical Address = 32 bits

Page table size = 24 MB

19	1	1	3
Frame No.	Valid/Invalid	Dirty	Permission

Number of frames, $M = \frac{2^{32}}{2^{13}} = 2^{19}$

Page Table Entry = 24 bits = 3 B

P.T.S. = N *e

24 MB = N * 3B

$N = \frac{24\text{MB}}{3B} = 8\text{M}$

$N = 8\text{M} \Rightarrow P = 23$ bits

$\therefore$ V.A.

P	d
23	13

36 bits

So, the length of the virtual address is 36 bits.

Scan for Video solution

9. (22 to 22)

Given,

Virtual Address = 40 bits

Page Size = 8 KB

TLB lines = 128

Sets = 32 sets

TLB Tag bits =?

Page offset (d) = 13 bits

Set offset = 5 bit

TLB = Tag bit = 40 – (13 + 5) = 40 – 18 = 22 bits

So, the minimum length of the TLB tag is 22 bits.

Scan for Video solution

10. (4 to 4)

Logical Address space = 32 bits

Page Size = 4 KB

Page Table Entry = 4B

Page Table Size = N * e

$$N = \frac{2^{32}}{2^{12}} = 2^{20}$$

$\boxed{\text{P.T.S.} = 1\text{M} * 4\text{B} = 4\text{MB}}$

Scan for Video solution

11. (112 to 112)

TLB access time (c)= 10 ms

Main Memory access time (m) = 80 ms

TLB Hit ratio (x) = 0.6

EMAT $= x\,(c + m) + (1 - x)\,(c + 2m)$

$= 0.6\,(10 + 80) + 0.4\,(10 + 160)$

$= 0.6 \times 90 + 0.4 \times 170$

$\boxed{\text{EMAT} = 54 + 64 = 122ms}$

Scan for Video solution

12. (c)

Page size =?

Given,

Outer Page Table fit in recently one page

Virtual Address = 46 bits

Page Table Base Register = 4 B

Let P.S. = 2^x bytes

46 bits

P_1	P_2	P_3	d
y	y	y	x

$3y + x = 46$….(i)

OPT size = $(2^y) * 4$ B = 2 bytes

$\therefore 2^{y+2} = 2^x \Rightarrow x = y + 2$

$3y + y + 2 = 46$

$4y = 44$

$\therefore y = 11$

$\boxed{x = 13}$

Therefore,

$$\boxed{\begin{aligned}\therefore \text{Page size} &= 2^{13}\\ &= 8\text{KB}\end{aligned}}$$

Scan for Video solution

13. (b)

The essential content(s) in each entry of a page table is page frame number. Virtual page number os nt stored in page table, so option A and C are clearly eliminated. Access right information is stored in page table but it is not an essential content. Therefore, option B is the correct answer.

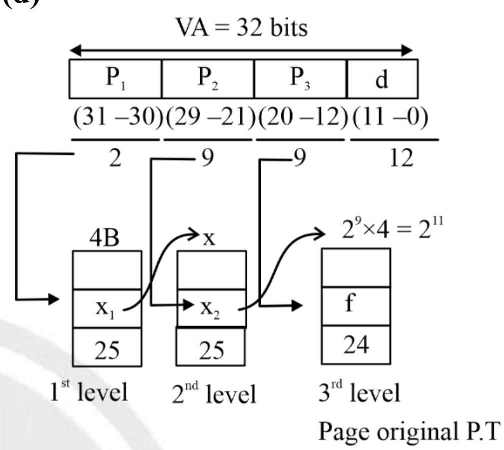
Scan for Video solution

14. (b)

A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because it helps to reduce the size of page table needed to implement the virtual address space of a process. The main purpose of multilevel paging is to reduce the space overhead page table size overhead.

Scan for Video solution

15. (d)



P.A 36 bits

$$M = \frac{2^{36}}{2^{12}} = 2^{24}$$

F = 24

$$\frac{2^{36}}{2^{12}} = 2^{25}$$

Scan for Video solution

16. (c)

Given,

Hit ratio (x) = 90%

TLB access time (c) = 10

Main memory access time (m)= 50

So,

EMAT $= x\,(c + m) + (1–x)\,(c + 2m)$

$= 0.9\,(60) + 0.1\,(10+100)$

$= 54 + 11 = 65$

Scan for Video solution

17. (4096 to 4096)

Each value i causes 32 Page Faults

→ matrix is stored in R.M.O

→ matrix size int D[128] [128]

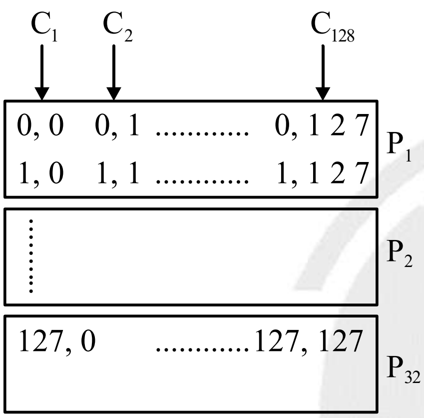
→ Page Size = 512

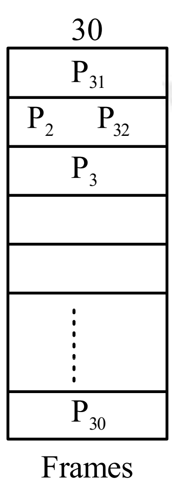
→ Number of frames = 30

→ Page replacement technique is LRU.

Number of pages needed to store the matrix

$= \frac{128 \times 128}{512} = \frac{2^{14}}{2^{9}} = 2^{5} = 32$ pages





→ each value of ‘i’ causes 32 Page Faults

→ All value of i(128) causes a total of 128 × 32 = 2^{12} = 4096 Page Faults

Scan for Video solution

18. (0.6 to 0.6)

5	~~7~~	~~4~~
	~~2~~	6
	~~3~~	1
	~~5~~	7

4 = frames

Number of Page faults= 9

Total references = 15

Page fault ratio = $\frac{9}{15} = 0.6$

Scan for Video solution

19. (154.5 to 155.5)

Given,

MMAT = 100ns;

TLBAT = 20 ns;

Disk R/W = 5000ns;

TLB Hit ratio = 95%;

Page fault rate = 10%;

Dirty page 20%

Average MAT= 0.95 (20 + 100) ns + 0.05[20ns + 0.9 (100ns + 100ns) + 0.1 (100ns + 0.2 (5000 + 500)) ns + 0.8 (500) ns]

$\boxed{\text{Average MAT} = 154.5\text{ns}}$

Scan for Video solution

20. (b)

Given,

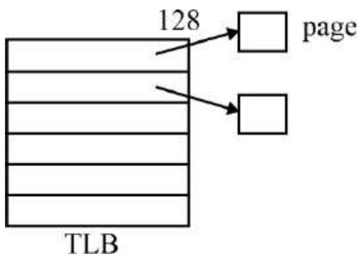
Page Size = 8KB = 2^{11}

Word Size = 4 KB

Virtual Memory Size = 2^{64} words

Number of pages possible = 2^{53}

Page offset= 64-53 = 11 bits



TLB contains 128 or 2^7 distinct page numbers.
One-page hit implies 2^{11} distinct virtual address hits.

$\boxed{\text{So, Number of V.A} \rightarrow \text{P.A}=128 \times 2k = 256 \times 2^{10}}$

Therefore, option B is the correct answer.

Scan for Video solution

21. (b)

Main Memory Access Time = M

Page Fault Service Time = D

Effective Memory Access Time = X

Page Fault rate = 'P' = ?

EMAT = (1– P) MMAT + P * PFST

$X = (1- P)\, M + P * D$

$= M - M.P + D.P$

$X - M = P\,(D - M)$

$\boxed{\therefore P = \dfrac{X-M}{D-M}}$

Scan for Video solution

22. (1 to 1)

10 frames;

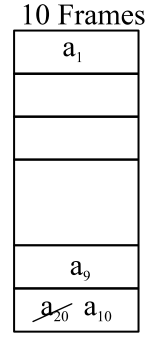
Reference string <a_1, a_2, a_3 … a_{20}, a_1, a_2, a_3, a_4 ..a_9, a_{10} a_{11}, a_{12}… a_{20} >

I. L.I.F.O

10 Frames

(a) $a_1 \ldots a_{10} \rightarrow 10$ PF's
(b) $a_{11} \ldots a_{20} \rightarrow 10$ PF's
(c) $a_1 \ldots a_9 \rightarrow X$
(d) $a_{10} \ldots a_{20} \rightarrow 11$

P.F. 31

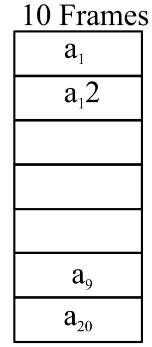


II. optimal

10 Frames

(a) $a_1 \ldots a_{10} \rightarrow 10$ PF's
(b) $a_{11} \ldots a_{20} \rightarrow 10$ PF's
(c) $a_1 \ldots a_9 \rightarrow X$
(d) $a_{10} \ldots a_{19} \rightarrow 10$ PF's
(e) $a_{20} \ldots$ No. P.F.

Total 30



Using LIFO, first 20 pages will result in page faults and next 9 pages will be page hit and then next 11 pages will be page faults.
Using optimal page replacement algorithm, first 20 pages will result in page faults and next 9 pages will be page hit and then next 10-page faults followed by page hit for last page.

Scan for Video solution

23. (b)

Memory access time (x) = 20ns
Page fault rate (p) = $1/10^6$
Page fault service time (y) = 10ms = 10×10^{-3} ns
Effective Memory Access Time = (1-p)*x + p*y

$$\text{E.M.A.T} = \left(1 - \frac{1}{10^6}\right) \times 20\text{ns} + \frac{1}{10^6} \times 10 \times 10^{-3}$$
$$= 1 \times 20\text{ns} + 10 \times 10^{-9}$$
$$= 20 + 10 = 30\text{ns}$$

So, option B is the correct answer,

Scan for Video solution

24. (d)

Dirty bit is used in write-back policy.

Read/write bit is used for page protection.

Reference bit is used for page replacement policies.

Valid bit is used for page initialization.

So, option D is the correct answer.

Scan for Video solution

25. (a)

FIFO (First in First out) page replacement algorithm suffers from Belady's Anomaly. So, option A is correct.

Scan for Video solution

26. (b)

S1: Random page replacement may suffer from belady's anomaly when it starts behaving like FIFO page replacement algorithm.

S2: LRU does not suffers from belady's anamoly. LRU is a stack algorithm and stack algorithm does not suffers from belady's anamoly.

So, option B is correct.

Scan for Video solution

27. (d)

FIFO page replacement algorithm can result in increasing page fault rate even when the number of allocated frames are decreasing.

Scan for Video solution

28. (a)

Reference string:

3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3

Page fault using FIFO and LRU page replacement algorithm:

FIFO

9 PF's

$\cancel{3}$ 6
$\cancel{8}$ 3
$\cancel{2}$ 8
$\cancel{9}$ 2
1

5 – Frames

LRU

9 PF's

$\cancel{3}$
$\cancel{8}$ 6
$\cancel{2}$ $\cancel{8}$ 1
$\cancel{9}$
$\cancel{1}$ 2

5 – Frames

Scan for Video solution

29. (6 to 6)

Reference string: 4, 7, 6, 1, 7, 6, 1, 2, 7, 2

4	7	6	1	7	6	1	2	7	2
4	4	4	1	1	1	1	1	1	1
	7	7	7	7	7	7	2	2	2
		6	6	6	6	6	6	7	7
PF	PF	PF	PF	PH	PH	PH	PF	PF	PH

PF is page fault and PH is page hit, so total 6 number of page faults.

Scan for Video solution

30. (d)

In optimal page replacement policy, the page which will next occur farthest in the future will be swapped out from the memory.

It is given that, computer has 20-page frames and initially the pages are numbered from 1 to 20. And the program accesses the pages numbered 1, 2, …., 100 and repeats the sequence THRICE.

The first 20 page access (i.e. 1 to 20) will cause page fault. Now, when page 21 will arrive it will swap out page 20 as 20 is going to be used farthest in future, similarly when 22 will arrive so 21 will be swapped out and so on.

So, the given algorithm is working as most-recently used. Hence, option D is the correct answer.

1	2	3	4	2	1	5	3	2	4	6
1	1	1	1	1	1	5	3	3	3	6
	2	2	2	2	2	2	2	2	2	2
		3	4	4	4	4	4	4	4	4
PF	PF	PF	PF	PH	PH	PF	PF	PH	PH	PF

Scan for Video solution

31. (7 to 7)

Reference string: 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6

1	2	3	4	2	1	5	3	2	4	6
1	1	1	1	1	1	5	3	3	3	6
	2	2	2	2	2	2	2	2	2	2
		3	4	4	4	4	4	4	4	4
PF	PF	PF	PF	PH	PH	PF	PF	PH	PH	PF

Scan for Video solution

32. (b)

The page faults using FIFO, optimal and LRU are as follows:

1 FIFO

6 PF'S

~~1~~ 4
~~2~~ 1
~~2~~ 2

3 Frames

2 Optimal

5 PF'S

1
~~2~~ 4
~~3~~ 2

3 Frames

3 L.R.U

9 PF'S

~~1~~ ~~4~~ 2
~~2~~ ~~3~~ 1
~~3~~ ~~1~~ 4

3 Frames

$\boxed{\text{Optimal} < \text{FIFO} < \text{LRU}}$

So, option B is correct.

Scan for Video solution

33. (a)

Given that the system has 4 page frames.

Initially for page number 1 to 100 it will cause page faults, then in reverse order for page number 100, 99, 98, and 97 it will be a page hit and then again page faults for page 96 to page 1.

[1, 2, …. 100,100….2, 1]

So, $2n - 4$, we are accessing n pages for 2 times and only 4 page hits are there. Here, N equals to 100.

200 – 4 = 196.

Therefore, option A is correct.

Scan for Video solution

34. (b)

Page Size = 16 bytes

Number of frames, M =4;

Virtual Addresses:

0, 4, 8, 20, 24, 36, 44, 12, 68, 72, 80, 84, 28, 32, 88, 92

Reference String corresponding to given virtual addresses:

<0, 0, 0, 1, 1, 2, 2, 0, 4, 4, 5, 5, 1, 2, 5, 5 >

$\langle \underline{0, 1, 2, 0, 4, 5, 1, 2, 5} \rangle$

LR U :

~~0~~ 2
~~1~~ 5
~~2~~ 1
4

Number of P.F's : 7

The page numbers of the page present in the main memory at the end of the sequence is 1, 2, 4, 5.

So, option B is the correct answer.

Scan for Video solution

❑❑❑

CHAPTER

5

FILE SYSTEM & DISK MANAGEMENT

File System

1. [NAT] [GATE-2022 : 2M]

Consider two files systems *A* and *B* , that use contiguous allocation and linked allocation, respectively. A file of size 100 blocks is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50^{th} and 51^{st} block), whose data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in *A* and *B* are n_A and n_B, respectively, then the value of $n_A + n_B$ is_________.

2. [MSQ] [GATE-2021 : 1M]

Consider a linear list-based directory implementation in a file system. Each directory is a list of nodes, where each node contains the file name along with the file metadata, such as the list of pointers to the data blocks. Consider a given directory foo.

Which of the following operations will necessarily require a full scan of foo for successful completion?

(a) Creation of a new file in foo

(b) Deletion of an existing file from foo

(c) Renaming of an existing file in foo

(d) Opening of an existing file in foo

Disk Scheduling Algorithm

3. [MCQ] [GATE-2020 : 2M]

Consider the following five disk access requests of the form (request id, cylinder number that are present in the disk scheduler queue at a given time.

(P, 155), (Q, 85), (R, 110), (S, 30), (T, 115)

Assume the head is positioned at cylinder 100. The scheduler follows Shortest Seek Time First scheduling to service the requests,

Which one of the following statements is FALSE?

(a) R is serviced before P

(b) Q is serviced after S but before T.

(c) T is serviced before P.

(d) The head reverses its direction of movement between servicing of Q and P

4. [NAT] [GATE-2018 : 2M]

Consider a storage disk with 4 platters (numbered as 0, 1, 2 and 3), 200 cylinders (numbered as 0, 1, …., 199) and 256 sectors per track (numbered as 0, 1, …., 255). The following 6 disk requests of the form [sector number, cylinder number, platter number] are received by the disk controller at the same time:

[120, 72, 2], [180, 134, 1], [60, 20, 0],

[212, 86, 3], [56, 116, 2], [118, 16, 1]

Currently the head is positioned at sector number 100 of cylinder 80 and is moving towards higher cylinder numbers. The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15 milliwatts. Power dissipation associated with rotational latency and switching of head between different platters is negligible.

The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First disk scheduling algorithm is

5. [NAT] [GATE-2016 : 2M]

Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is ____

6. [MCQ] [GATE-2015 : 2M]

Suppose the following disk request sequence (track numbers) for a disk with 100 tracks is given: 45, 20, 90, 10, 50, 60, 80, 25, 70. Assume that the initial position of the R/W head is on track 50. The additional distance that will be traversed by the R/W head when the Shortest Seek Time First (SSTF) algorithm is used compared to the SCAN (Elevator) algorithm (assuming that SCAN algorithm moves towards 100 when it starts execution) is ______ tracks.

(a) 8 (b) 9
(c) 10 (d) 11

7. [NAT] [GATE-2014 : 2M]

Suppose a disk has 201 cylinders, numbered from 0 to 200. At some time the disk arm is at cylinder 100, and there is a queue of disk access requests for cylinders 30, 85, 90, 100, 105, 110, 135 and 145. If Shortest-Seek Time First (SSTF) is being used for scheduling the disk access, the request for cylinder 90 is serviced after servicing ____ number of requests.

8. [MCQ] [GATE-2009 : 2M]

A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because.

(a) it reduces the memory access time to read or write a memory location.
(b) it helps to reduce the size of page table needed to implement the virtual address space of a process.
(c) it is required by the translation look a side buffer.
(d) it helps to reduce the number of page faults in page replacement algorithms.

9. [MCQ] [GATE-2009 : 2M]

Consider a disk system with 100 cylinders. The requests to access the cylinders occur in following sequence:

4, 34, 10, 7, 19, 73, 2, 15, 6, 20

Assuming that the head is currently at cylinder 50, what is the time taken to satisfy all requests if it takes 1 ms to move from one cylinder to adjacent one and shortest seek time first policy is used?

(a) 95 ms (b) 119 ms
(c) 233 ms (d) 276 ms

File Allocation Methods

10. [NAT] [GATE-2019 : 2M]

The index node (inode) of a Unix-like file system has 12 direct, one single-indirect and one double-indirect pointers. The disk block size is 4 kB, and the disk block address is 32-bits long. The maximum possible file size is (rounded off to 1 decimal place) ___________ GB.

11. [MCQ] [GATE-2017 : 1M]

In a file allocation system, which of the following allocation scheme(s) can be used if no external fragmentation is allowed?

I. Contiguous II. Linked
III. Indexed

(a) I and III only (b) II only
(c) III only (d) II and III only

12. [NAT] [GATE-2014 : 1M]

A FAT (file allocation table) based file system is being used and the total overhead of each entry in the FAT is 4 bytes in size. Given a 100×10^6 bytes disk on which the file system is stored and data block size is 10^3 bytes, the maximum size of a file that can be stored on this disk in units of 10^6 bytes is _____

13. [MCQ] [GATE-2012 : 2M]

A file system with 300 GByte disk uses a file descriptor with 8 direct block addresses, 1 indirect block address and 1 doubly indirect block address. The size of each disk block is 128 Bytes and the size of each disk block address is 8 Bytes. The maximum possible file size in this file system is

(a) 3 KBytes
(b) 35 KBytes
(c) 280 Kbytes
(d) dependent on the size of the disk.

14. [MCQ] [GATE-2008: 2M]

The data block of a very large file in the Unix file system are allocated using.

(a) Contiguous allocation
(b) Linked allocation
(c) Indexed allocation
(d) An extension of indexed allocation

❑❑❑

ANSWER KEY

1.	(153 to 153)	**2.**	(a, c)	**3.**	(b)	**4.**	(85 to 85)
5.	(346 to 346)	**6.**	(c)	**7.**	(3 to 3)	**8.**	(b)
9.	(b)	**10.**	(4 to 4)	**11.**	(d)	**12.**	(99.55 to 99.65)
13.	(b)	**14.**	(d)				

SOLUTIONS

1. (153 to 153)

F_A : Contiguous allocation : 100 blocks (Array)
F_B : Linked allocation: 100 blocks

We have oved total data if 100

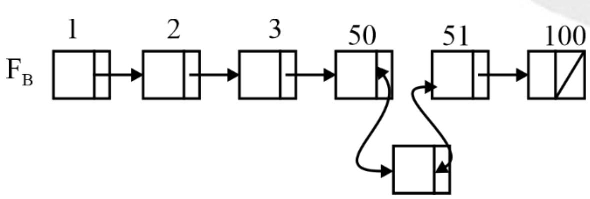
I) FA : 50 × 2 = 100 + 1 = 101 (Total disk accesses)

II.

Contiguous Allocation can directly traverse to the 50th element as it is already given the file control blocks is already present.

Now, we need 50 operations to read the rest of the 50 elements and another 50 operations to write those 50 blocks and 1 operation is needed to write the new block.

Therefore, total operations in this case = 50 + 50 + 1.



Linked Allocation needs to traverse to the 50th element.

Now, simply change the pointers just like the Linked list where Adding element in the middle of it.

So, here it takes 50 operations to read the first 50 elements and 2 operations to change the pointer of the 50th block and a new block. So, 52 operations in this case.

$$\boxed{F_B = 50 + 1 + 1 = 52\ n_B}$$

$$\therefore \boxed{n_A + n_B = 101 + 52 = 153}$$

Scan for Video solution

2. (a, c)

Creating a new file in foo will definitely require a full scan, so that we can check that the new file is not conflicting with any other already existing file.

Also, renaming of a exisiting file will require a full scan, to ensure that the updated name is not conflicting with nay other exisiting file name.

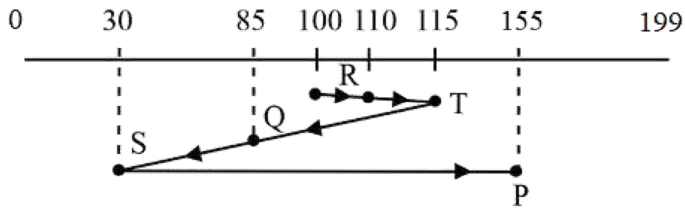
Deleting or opening an exsiitng file, do not need a full scan.

Scan for Video solution

3. (b)

Request: ⟨(P, 155), (Q, 85), (R, 110), (S, 30), (T, 115)⟩

Initially head at cylinder 100, scheduling using SSTF:



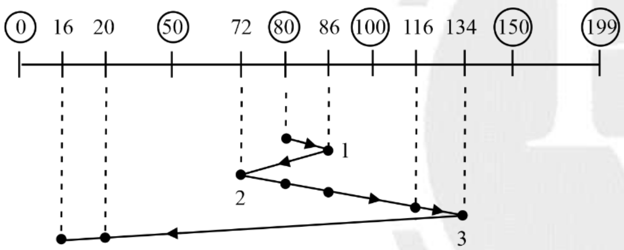
From the above diagram we can see that: Q is serviced after S but before T is wrong. Q is serviced before S and after T. So, option B is the answer.

Scan for Video solution

4. (85 to 85)

Request:⟨72, 134, 20, 86, 116, 16⟩

Initially the head is positioned at 100, and scheduling using **S.S.T.F:**



Total seeks: 200

Power consumption for seek movements: 2 × 20 = 40 mw

No. of direction changes 3

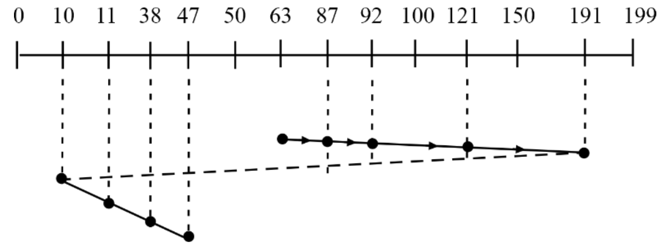
Power consumption for R/W head direction changing: 3 × 15 = 45 mw

Total power consumption : 40 + 45 = 85 mw

Scan for Video solution

5. (346 to 346)

Initially the head is at 63, and moving toward larger cylinder number, scheduling using C-LOOK:



The total head movement incurred:

(191 – 63) + (191 – 10) + (47 – 10)

= 128 + 181 + 37

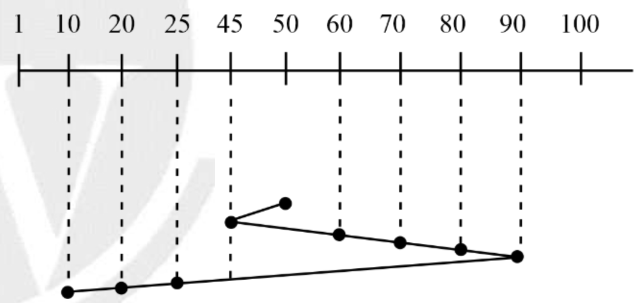
= 346

Scan for Video solution

6. (c)

Requests: ⟨45, 20, 90, 10, 50, 60, 80, 25, 70⟩

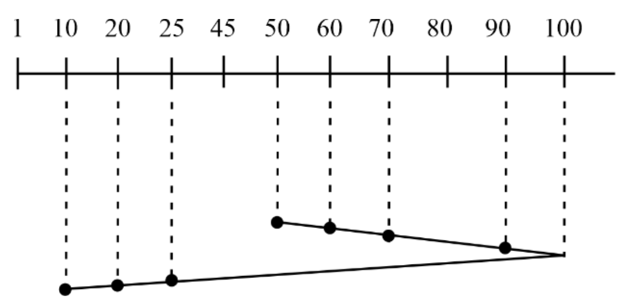
Scheduling using **S.S.T.F:**



Number of seeks: (90 – 45) + (50 – 45) + (90 – 10)

= 45 + 5 + 80 = 130

Scheduling using SCAN:



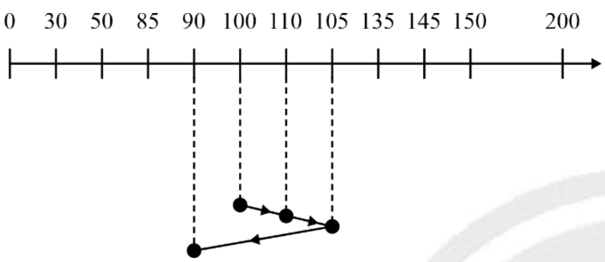
No of seeks: (100 – 50) + (100 – 10)

= 50 + 90 = 140

Additional seeks = 140-130 = 10
So, option C is the correct asnwer.

Scan for Video solution

7. **(3 to 3)**
Scheduling using SSTF:



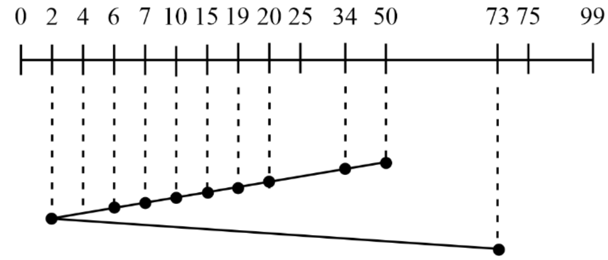
Before servicing 90, we have serviced, 100, 105, 110. So, 3 is the correct answer..

Scan for Video solution

8. **(b)**
A multilevel page table is preferred in comparison to a single level page table for translating virtual address to physical address because it helps to reduce the size of page table needed to implement the virtual address space of a process. The main purpose of multilevel paging is to reduce the space overhead page table size overhead.

Scan for Video solution

9. **(b)**
Request: ⟨4, 34, 10, 7, 19, 73, 2, 15, 6, 20⟩
Initially head is at 50, Scheduling using **SSTF:**

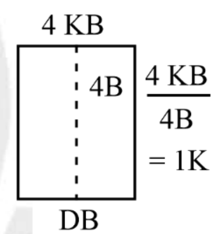


Total seeks: (73-2) + (50-2) = 71 + 48 = 119.

Time to go frame one cylinder to ment adjust cylinder = 1ms
Total seek time = 119 × 1 ms = 119 ms
So, option B is the correct asnwer.

Scan for Video solution

10. **(4 to 4)**
DBS = 4KB; DBA = 32 bits (4B)



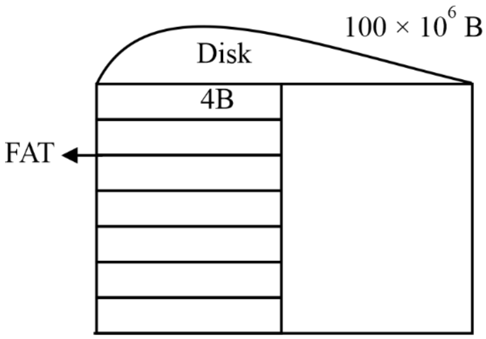
1K disk block addresses can be stored.
Maximum file size:
(12 * 4KB + 1 K * 4KB + 1 K * 1K * 4 KB)
= 48KB + 4MB + 4GB = 0.004 GB + 4GB = 4.0 GB

Scan for Video solution

11. **(d)**
Linked and indexed allocation can be used when no external fragmentation is allowed.

Scan for Video solution

12. (99.55 to 99.65)



No. of blocks on disk = $\frac{100 \times 10^6}{10^3} = 100 \times 10^3$

FAT size = $100 \times 10^3 \times 4B$

$= 4 \times 100 \times 10^3$

$= 4 \times 10^5$

$= 0.4 \times 10^6$

Maximum file size = Disk size – FAT size

$= 100 \times 10^6 - 0.4 \times 10^6$

= 100 MB – 0.4 MB

= 99.6 MB

Scan for Video solution

13. (b)

Disk Block Size = 128B

Disk Block Address = 8B

In one block we can store:

$\frac{128}{8} = \frac{2^7}{2^3} = 2^4 =$ 16 disk block pointer.

Maximum file size = $8 \times 128B + 16 \times 128B + 16 \times 16 \times 128B$

= 1KB + 2KB + 32KB = 35KB

So, option B is the correct answer.

Scan for Video solution

14. (d)

The data block of a very large file in the Unix file system are allocated using an extension of indexed allocation

Scan for Video solution

❑❑❑

INDEX

Computer Networks

Computer Networks

Syllabus

Concept of layering: OSI and TCP/IP Protocol Stacks; Basics of packet, circuit and virtual circuit switching; Data link layer: framing, error detection, Medium Access Control, Ethernet bridging; Routing protocols: shortest path, flooding, distance vector and link state routing; Fragmentation and IP addressing, IPv4, CIDR notation, Basics of IP support protocols (ARP, DHCP, ICMP), Network Address Translation (NAT); Transport layer: flow control and congestion control, UDP, TCP, sockets; Application layer protocols: DNS, SMTP, HTTP, FTP, Email.

Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6
2008	2		5		2	
2009		4			2	
2010	2			1	2	1
2011					4	2
2012	3	2			2	2
2013	1	2			3	
2014 (P1)	5	1			3	
2014 (P2)		2	1	1		5
2014 (P3)	2		2	1	2	1
2015 (P1)	1	2	2		1	1
2015 (P2)	2				2	2
2015 (P3)	3	4			1	
2016 (P1)		4		3		1
2016 (P2)		3	4			1
2017 (P1)	1	3			1	
2017 (P2)		4		2		1
2018		1	2		4	
2019	2	2				1
2020	2			2	1	1
2021 (P1)		4		1	2	2
2021 (P2)	2			2	1	
2022	2	1		2	2	2
2023	2	1		1	2	2

CHAPTER 1

BASIC CONCEPTS AND IPv4 ADDRESSING

OSI Model

1. [MCQ] [GATE-2014 : 1M]

In the following pairs of OSI protocol layer/sub-layer and its functionality, the INCORRECT pair is

(a) Network layer and Routing
(b) Data Link Layer and Bit synchronization
(c) Transport layer and End-to-end process communication
(d) Medium Access Control sub-layer and Channel sharing.

Classful Addressing

2. [MCQ] [GATE-2012: 1M]

In the IPv4 addressing format, the number of networks allowed under Class C addresses is:

(a) 2^{14} (b) 2^{7}
(c) 2^{21} (d) 2^{24}

Subnetting

3. [NAT] [GATE-2023 : 2M]

The forwarding table of a router is shown below.

Subnet Number	Subnet Mask	Interface ID
200.150.0.0	255.255.0.0	1
200.150.64.0	255.255.224.0	2
200.150.68.0	255.255.255.0	3
200.150.68.64	255.255.255. 224	4
default		0

A packet addressed to a destination address 200.150.68.118 arrives at the router. It will be forwarded to the interface with ID_____ .

4. [MCQ] [GATE-2019 : 2M]

Consider three machines M, N and P with IP addresses 100.10.5.2, 100.10.5.5, and 100.10.5.6 respectively. The subnet mask is set to 255.255.255.252 for all the three machines. Which one of the following is true?

(a) M, N, and P all belong to the same subnet
(b) Only M and N belong to the same subnet
(c) Only N and P belong to the same subnet
(d) Only N and P belong to the same subnet

5. [MCQ] [GATE-2015: 2M]

Consider the following routing table at an IP router:

Network No.	Net Mask	Next Hop
128.96.170.0	255.255.254.0	Interface 0
128.96.168.0	255.255.254.0	Interface 1
128.96.166.0	255.255.254.0	R2
128.96.164.0	255.255.252.0	R3
0.0.0.0	Default	R4

For each IP address in Group-I identify the correct choice of the next hop from Group-II using the entries from the routing table above.

List-I		List-II	
A.	128.96.171.92	1.	Interface 0
B.	128.96.167.151	2.	Interface 1
C.	128.96.163.151	3.	R2
D.	128.96.165.121	4.	R3
		5.	R4

Codes:

	a	b	c	d
(a)	1	3	5	4
(b)	1	4	2	5
(c)	2	3	4	5
(d)	2	3	5	4

6. [MCQ] [GATE-2010: 2M]

Suppose computers A and B have IP addresses 10.105.1.113 and 10.105.1.91 respectively and they both use the same netmask N. Which of the values of N given below should not be used if A and B should belong to the same network?

(a) 255.255.255.0
(b) 255.255.255.128
(c) 255.255.255.192
(d) 255.255.255.224

Common Data for Next Two Questions

Host X has IP address 192.168.1.197 and is connected through two routers R1 and R2 to another host Y with IP address 192. 168. 1. 80. Router R1 has IP addresses 192.168.1.135 and 192.168.1.110. R2 has IP addresses 192.168.1.67 and 192. 168.1.155. The netmask used in the network is .

7. [MCQ] [GATE-2008: 2M]

Given the information above, how many distinct subnets are guaranteed to already exist in the network?

(a) 1 (b) 2
(c) 3 (d) 6

8. [MCQ] [GATE-2008: 2M]

Which IP address should X configure its gateway as?

(a) 192.168.1.67 (b) 192.168.1.110
(c) 92.168.1.135 (d) 192.168.1.155

9. [MCQ] [GATE-2008: 2M]

If a class B network on the Internet has a subnet mask of 255.255.248.0, what is the maximum number of hosts per subnet?

(a) 1022 (b) 1023
(c) 2046 (d) 2047

Classless Addressing

10. [MCQ] [GATE-2020: 2M]

An organization requires a range of IP addresses to assign one to each of its 1500 computers. The organization has approached an Internet Service Provider (ISP) for this task. The ISP uses CIDR and serves the requests from the available IP address space 202.61.0.0/17. The ISP wants to assign an address space to the organization, which will minimize the number of routing entries in the ISP's router using route aggregation. Which of the following address spaces are potential candidates from which the ISP can allot any one to the organization?

I. 202.61.84.0/21 II. 202.61.104.0/21
III. 202.61.64.0/21 IV. 202.61.144.0/21

(a) I and II only (b) II and III only
(c) III and IV only (d) I and IV only

11. [NAT] [GATE-2015 : 2M]

In the network 200.10.11.144/27, the fourth octet (in decimal) of the last IP address of the network which can be assigned to a host is _________

12. [NAT] [GATE-2014 : 2M]

An IP router implementing Classless Inter-domain Routing (CIDR) receives a packet with address 131.23.151.76. The router's routing table has the following entries:

Prefix	Output Interface Identifier
131.16.0.0/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

The identifier of the output interface on which this packet will be forwarded is _____

13. [MCQ] [GATE-2012: 2M]

An Internet Service Provider (ISP) has the following chunk of CIDR-based IP addresses available with it: 245.248.128.0/20. The ISP wants to give half of this chunk of addresses to Organization A, and a quarter of Organization B, while retaining the remaining with itself. Which of the following is a valid allocation of addresses to A and B?

(a) 245.248.136.0/21 and 245.248.128.0/22

(b) 245.248.128.0/21 and 245.248.128.0/22

(c) 245.248.132.0/22 and 245.248.132.0/21

(d) 245.248.136.0/24 and 245.248.132.0/21

Supernetting

14. [MSQ] [GATE-2022: 2M]

Consider routing table of an organization's router shown below:

Subnet Number	Subnet Mask	Next Hop
12.20.164.0	255.255.252.0	R1
12.20.170.0	255.255.254.0	R2
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
default		R3

Which of the following prefixes in CIDR notation can be collectively used to correctly aggregate all of the subnets in the routing table?

(a) 12.20.164.0/20

(b) 12.20.164.0/22

(c) 12.20.164.0/21

(d) 12.20.168.0/22

❑❑❑

ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(3 to 3)	**4.**	(c)
5.	(a)	**6.**	(d)	**7.**	(c)	**8.**	(b)
9.	(c)	**10.**	(b)	**11.**	(158 to 158)	**12.**	(1 to 1)
13.	(a)	**14.**	(b, d)				

SOLUTIONS

1. (b)

Option b is incorrect because bit synchronization is the functionality of physical layer.

Scan for Video solution

2. (c)

NID	**HID**
24	8

$\boxed{110 \mid 21bit}$

↓

The number of networks in class C = 2^{21}

So, option C is the correct answer.

Scan for Video solution

3. (3)

First start with the longest subnet mask

I 200.150.68.01110110

AND

255.255.255.11100000

NID = 200.150.68.96

Not matched with interface (4)

II 200.150.68.118

AND

255.255.255.0

NID = 200.150.68.0

Matched with interface (3)

No need to check further if it matches with more than one interfaces then we choose longest subnet mask.

Scan for Video solution

4. (c)

IP address

AND

SubnetMask

Subnetid

M: 100.10.5.00000010

AND

255.255.255.11111100

SID = 100.10.5.0

N : 100.10.5.00000101

AND

255.255.255.11111100

SID = 100.10.5.00000100

SID = 100.10.5.4

P : 100.10.5.00000110

AND

255.255.255.11111100

SID = 100.10.5.00000100

100.10.5.4

N & P belong to same subnet. So, option c is the answer.

Scan for Video solution

5. **(a)**

Note: First start with longest mask

(a)

128.96.171.92

AND

255.255.254.0

NID = 128.96.170.0

Matched with interface (0)

(b)

128.96.167.151

AND

255.255.254.0

NID = 128.96.166.0

Matched with R2.

(c)

128.96.163.151

AND

255.255.254.0

NID = 128.96.162.0

Not matched

128.96.163.151

AND

255.255.252.0

NID = 128.96.160.0

Not matched

Note:- Does not match with any given interface then 128.96.163.151 will forward to default interface R_4.

(d)

128.96.165.121

AND

255.255.254.0

128.96.164.0

128.96.165.121

AND

255.255.252.0

128.96.164.0

Matched with R3.

So, option A is correct answer.

Scan for Video solution

6. **(d)**

(a) 10.105.1.113

AND

255.255.255.0

NID = 10.105.1.0

10.105.1.91

AND

255.255.255.0

NID = 10.105.1.0

So, option a is incorrect.

(b) 10.105.1.113

AND

255.255.255.128

NID = 10.105.1.0

10.105.1.91

AND

255.255.255.128

NID = 10.105.1.0

So, option b is incorrect

(c) 10.105.1.113

AND

<u>255.255.255.192</u>

NID = 10.105.1.64

10.105.1.91

AND

<u>255.255.255.192</u>

NID = 10.105.1.64

So, option c is incorrect

(d) 10.105.1.113

AND

<u>255.255.255.224</u>

NID = 10.105.1.96

10.105.1.91

AND

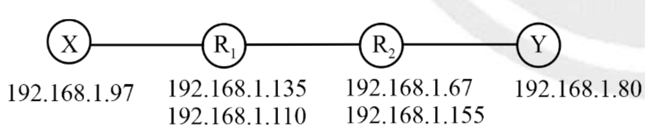
<u>255.255.255.224</u>

NID = 10.105.1.64

So, option d is the correct answer.

Scan for Video solution

7. (c)



Netmask: 255.255.255.224

I 192.168.1.97

AND

<u>255.255.255.224</u>

SID = 192.168.1.96

II 192.168.1.135

AND

<u>255.255.255.224</u>

SID = 192.168.1.128

III 192.168.1.110

AND

<u>255.255.255.224</u>

SID = 192.168.1.96

IV 192.168.1.67

AND

<u>255.255.255.224</u>

SID = 192.168.1.64

V 192.168.1.155

AND

<u>255.255.255.224</u>

SID = 192.168.1.128

VI 192.168.1.80

AND

<u>255.255.255.224</u>

SID = 192.168.1.64

Different subnet id's

$$\left.\begin{array}{l}192.168.1.96\\192.168.1.128\\192.168.1.64\end{array}\right\}\text{3 different subnet id's}$$

So, option C is the correct answer.

Shortcut

<u>255.255.255.</u><u>111</u> <u>00000</u>

NID SID HID

97 : 011
135 : 100
110 : 011
67 : 010
155 : 100
80 : 010

<u>3 different subnet id's</u>

011 → 96
100 → 128
010 → 64

Scan for Video solution

8. **(b)**

X : 192.168.1.97

AND

255.255.255.224

192.168.1.96

Note: Gateway must also have the same subnet id

(a) 192.168.1.67

AND

255.255.255.224

SID = 192.168.1.64

So, option a is incorrect.

(b) 192.168.1.110

AND

255.255.255.224

SID = 192.168.1.96

Therefore, option B is the correct answer.

Scan for Video solution

9. **(c)**

Given,

Subnet Mask: 255.255.248.0

SM : 11111111.11111111.11111<u>000.00000000</u>

HID = 11 bit

Maximum number of Hosts/Subnet $= 2^{11} - 2$

$= 2048 - 2$

$= 2046$

So, option C is the correct answer.

Scan for Video solution

10. **(b)**

For 1500 computers HID must be minimum 11 bits

202.61.0.0/17

NID = 17bit, HID = 32 – 17 = 15bit

202.61.0 <u>- - - - - - - - . - - - - - - - -</u>

<u>8 + 8 + 1</u> HID

NID

<u>202.61.0</u>0000000 00000000 → 202.61.0.0

⋮ ⋮ ⋮

202.61.01111111.11111111 → 202.61.127.255

I 202.61.84.0/21

NID = 21, HID = 11 bit

First IP address of the block must be divisible by size of the block means all host id bits should be zero.

202.61.01010<u>100.00000000</u>/2^{11}

HID

It is incorrect

II 202.61.01101<u>000.00000000</u>/2^{11}

HID

It is correct

III 202.61.01000<u>000.00000000</u>/2^{11}

HID

It is correct

So, option b is the correct answer.

Scan for Video solution

11. **(158)**

200.10.11.144/27

NID = 27 bit, HID = 5 bit

200.10.11.100<u>10000</u>

<u>8 + 8 + 8 + 3</u> **HID**

NID

200.10.11.100 <u>- - - - -</u>

HID

200.10.11.<u>100</u>11110

↓

200.10.11.<u>158</u>

Scan for Video solution

12. (1)

Prefix	**Subnet mask**	**Output Interface Identifier**
131.16.0.0/12	255.240.0.0	3
131.28.0.0/14	255.252.0.0	5
131.19.0.0/16	255.255.0.0	2
131.22.0.0/15	255.254.0.0	1

First start with the longest subnet mask

I. DIP = 131.23.151.76

AND AND

<u>SM</u> <u>255.255.0.0</u>

NID = 131.23.0.0

Not matched with interface (2)

II. DIP = 131.23.151.76

AND AND

<u>SM</u> <u>255.254.0.0</u>

NID = 131.22.0.0

Matched with interface (1)

23 : 00010111

AND AND

254 <u>11111110</u>

(22) 00010110

Scan for Video solution

13. (a)

Given,

245.248.128.0/20

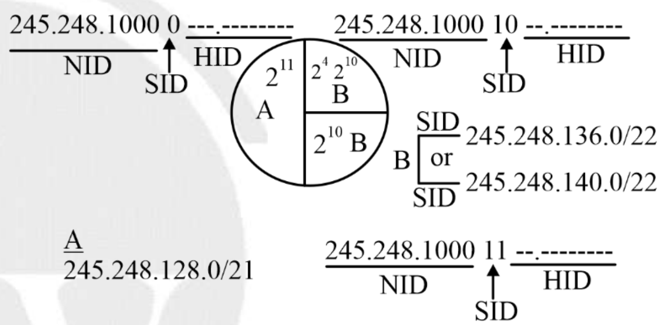
NID = 20 bit, HID = 32 – 20 = 12 bit

$\underbrace{245.248.1000}_{\substack{8+8+4 \\ \text{NID}}}\underbrace{0000.00000000}_{\text{HID}}$

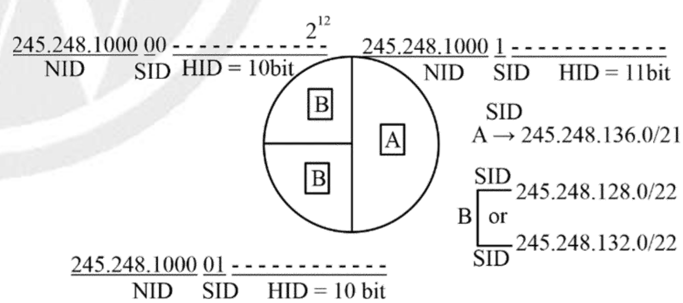
<u>245.248.1000</u> <u>. . . . .</u>

NID HID

Number of IP Addresses = 2^{12}



Or



So, option a is the correct answer.

Scan for Video solution

14. (b, d)

12.20.164.0/23

12.20.170.0/23

12.20.168.0/23

12.20.166.0/23

I.

$\underbrace{12.\ 20.\ 101001}_{\substack{8+8+6 \\ \text{NID}}} \mid \underbrace{00.\ 00000000}_{\text{HID}}$

$\underbrace{12.20.\ 101001}_{} \ 00 \ \ 00000000 \rightarrow 12.20.164.0$

⋮

$\underbrace{12.20.\ 101001}_{\text{NID}} \ 11 \ \ 11111111 \rightarrow 12.20.167.255$

II. $\underbrace{12.20.\ 1010101}_{8+8\ +\ 7}\underbrace{0.00000000}_{\text{HID}} \rightarrow 12.20.164.0$

12.20. 10101010.00000000→ 12.20.170.0

⋮

12.20. 10101011.11111111→ 12.20.171.255

III. $\underbrace{12.20.\ 1010100}_{\substack{8+8\ +\ 7 \\ \text{NID}}}\underbrace{0.00000000}_{\text{HID}}$

12.20. 1010100 0.00000000 →12.20.168.0

⋮

12.20. 1010100 1.11111111 →12.20.169.255

IV. $\underbrace{12.20.\ 1010011}_{\substack{8+8\ +\ 7 \\ \text{NID}}}\underbrace{0.00000000}_{\text{HID}}$

12.20. 1010011 0.00000000 →12.20.166.0

⋮

12.20. 1010100 1.11111111 →12.20.167.255

Network IV is the part of Network I so we can just ignore Network IV

I. 12. 20. 164. 0/22

II. 12.20.170.0/23

III. 12.20.168.0/23

12. 20. 164. 0 /22
12. 20. 168. 0 /23
12. 20. 170. 0 /23

supernet id = 12.20.168.0

→12. 20. 168. 0 /22

→12. 20. 164. 0 /22

1. contiguous (True)
2. Same size $= 2^9$ & No. of n/w's = $2 = 2^1$
3. total size of supernet = $2^9 + 2^9 = 2^{10}$

$12.20.101010\underbrace{\boxed{00.00000000}}_{\text{HID}}/2^{10}$ (True)

12. 20. 164. 0 /22
12. 20. 168. 0 /22

1. contiguous (True)
2. Same size $= 2^{10}$ & No. of n/w's = $2 = 2^1$
3. Total size of supernet = $2^{10} + 2^{10} = 2^{11}$

$12.20.10100\underbrace{\boxed{100.00000000}}_{\text{HID}}/2^{11}$ (False)

Can't combine further so answer is option b and d.

Scan for Video solution

❑❑❑

CHAPTER

2 DATA LINK LAYER

Flow Control Policies of Data Link Layer

1. [MCQ] [GATE-2023: 1M]

Suppose two hosts are connected by a point-to-point link and they are configured to use Stop-and-Wait protocol for reliable data transfer. Identify in which one of the following scenarios, the utilization of the link is the lowest.

(a) Longer link length and lower transmission rate
(b) Longer link length and higher transmission rate
(c) Shorter link length and lower transmission rate
(d) Shorter link length and higher transmission rate

2. [NAT] [GATE-2022: 2M]

Consider a 100Mbps link between an earth station (sender) and a satellite(receiver) at an altitude of 2100Km. The signal propagates at a speed of 3×10^8 m/s. The time taken (in milliseconds, rounded off to two decimal places) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is __________.

3. [NAT] [GATE-2021: 2M]

Consider the sliding window flow-control protocol operating between a sender and a receiver over a full-duplex error-free link. Assume the following:

- The time taken for processing the data frame by the receiver is negligible.
- The time taken for processing the acknowledgement frame by the sender is negligible.
- The sender has infinite number of frames available for transmission.
- The size of the data frame is 2000 bits and the size of the acknowledgement frame is 10 bits.
- The link data rate in each direction is 1mbps (= 10^6 bits per second).
- One way propagation delay of the link is 100 milliseconds.

The minimum value of the sender's window size in terms of the number of frames, (rounded to the nearest integer) needed to achieve a link utilization of 50% is _____.

4. [NAT] [GATE-2017: 2M]

The values of parameters for the Stop-and-Wait ARQ protocol are as given below:

- Bit rate of the transmission channel = 1 Mbps
- Propagation delay from sender to receiver = 0.75 ms.
- Time to process a frame = 0.25 ms.
- Number of bytes in the information frame = 1980.
- Number of bytes in the acknowledgement frame = 20.
- Number of overhead bytes in the information frame = 20.

Assume that there are no transmission errors. Then, the transmission efficiency (expressed in percentage) of the Stop-and-Wait ARQ protocol for the above parameters is _______ (correct to 2 decimal places).

5. [MCQ] [GATE-2017: 2M]

Consider two hosts X and Y connected by a single direct link of rate 10^6 bits/sec. The distance between the two hosts is 10,000 km and the propagation speed along the link is 2×10^8 m/sec. Hosts X sends a file of 50,000 bytes as one large message to host Y continuously. Let the transmission and propagation delays be p milliseconds and q milliseconds, respectively. Then the values of p and q are.

(a) p = 50 and q = 100 (b) p = 50 and q = 400
(c) p = 100 and q = 50 (d) p = 400 and q = 50

6. **[NAT]** **[GATE-2016: 2M]**

A sender used the Stop-and-wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps (1 Kbps = 1000 bits/second). Size of an acknowledgement is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds. Assuming no frame is lost, the sender throughput is _______ bytes/second.

7. **[NAT]** **[GATE-2015: 2M]**

Consider a 128×10^3 bits/second satellite communication link with one way propagation delay of 150 milliseconds. Selective retransmission (repeat) protocol is used on this link to send data with a frame size of 1 kilobyte. Neglect the transmission time of acknowledgement. The minimum number of bits required for the sequence number field to achieve 100% utilization is ________.

8. **[NAT]** **[GATE-2015: 2M]**

Consider a network connecting two systems located 8000 kilometers apart. The bandwidth of the network is 500×10^6 bits per second. The propagation speed of the media is 4×10^6 meters per second. It is needed to design a Go-Back-N sliding window protocol for this network. The average packet size is 10^7 bits. The network is to be used to its full capacity.

Assume that processing delays at nodes are negligible. Then, the minimum size in bits of the sequence number field has to be_____.

9. **[NAT]** **[GATE-2015: 1M]**

A link has a transmission speed of 10^6 bits/sec. It uses data packets of size 1000 bytes each. Assume that the acknowledgement has negligible transmission delay, and that its propagation delay is the same as the data propagation delay. Also assume that the processing delays at nodes are negligible. The efficiency of the stop-and-wait protocol in this setup is exactly 25%. The value of the one-way propagation delay (in milliseconds) is _______.

10. **[NAT]** **[GATE-2015: 2M]**

Suppose that the stop-and -wait protocol is used on a link with a bit rate of 64 kilobits per second and 20 milliseconds propagation delay. Assume that the transmission time for the acknowledgement and the processing time at nodes are negligible. Then the minimum frame size in bytes to achieve a link utilization of at least 50% is _______.

11. **[NAT]** **[GATE-2014: 2M]**

Consider a selective repeat sliding window protocol that uses a frame size of 1 KB to send data on a 1.5 Mbps link with a one-way latency of 50 msec. To achieve a link utilization of 60%, the minimum number of bits required to represent the sequence number field is _____________.

Common Data for Next Two Questions

12. **[MCQ]** **[GATE-2009: 2M]**

Frames of 1000 bits are sent over a 10^6 bps duplex link between two hosts. The propagation time is 25ms. Frames are to be transmitted into this link to maximally pack them in transit (within the link).

What is the minimum number of bits (I) that will be required to represent the sequence numbers distinctly? Assume that no time gap needs to be given between transmission of two frames.

(a) I = 2 (b) I = 3
(c) I = 4 (d) I = 5

13. **[MCQ]** **[GATE-2009: 2M]**

Let I be the minimum number of bits (I) that will be required to represent the sequence numbers distinctly assuming that no time gap needs to be given between transmission of two frames.

Suppose that the sliding window protocol is used with the sender window size of 2^I, where I is the numbers of bits as mentioned earlier and acknowledgements are always piggy backed. After sending 2^I frames, what is the minimum time the sender will have to wait before starting transmission of the next frame? (Identify the closest choice ignoring the frame processing time)

(a) 16 ms (b) 18 ms

(c) 20 ms (d) 22 ms

14. [MCQ] [GATE-2008: 2M]

A satellite link connects two ground stations. The altitude of the satellite is and speed of the signal is 3×10^8 m/s. What should be the packet size for a channel utilization of 25% for a satellite link using go-back- 127 sliding window protocol? Assume that the acknowledgment packets are negligible in size and that there are no errors during communication.

(a) 120 bytes (b) 60 bytes

(c) 240 bytes (d) 90 bytes

Error Control Policies

15. [MCQ] [GATE-2021: 2M]

Assume that a 12-bit Hamming codeword consisting of 8-bit data and 4 check bits is d_8 d_7 d_6 d_5 c_8 d_4 d_4 d_3 d_2 c_4 d_1 c_2 c_1, where the data bits and the check bits are given in the following tables:

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_8	c_4	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of x and y?

(a) x is 0 and y is 0 (b) x is 0 and y is 1

(c) x is 1 and y is 0 (d) x is 1 and y is 1

16. [MCQ] [GATE-2021: 2M]

Consider the cyclic redundancy check (CRC) based error detecting scheme having the generator polynomial $X^3 + X + 1$. Suppose the message $m_4m_3m_2m_1m_0$ =11000 is to be transmitted. Check bits $c_2c_1c_0$ are appended at the end of the message by the transmitter using the above CRC scheme, The transmitted bit string is denoted by $m_4m_3m_2m_1m_0c_2c_1c_0$. The value of the check bit sequence $c_2c_1c_0$ is

(a) 101 (b) 110

(c) 100 (d) 111

17. [MCQ] [GATE-2017: 2M]

Consider a binary code that consists of only four valid codewords as given below: 00000, 01011, 10101, 11110.

Let the minimum Hamming distance of the code be P and the maximum number of erroneous bits that can be corrected by the code be q. Then the values of p and q are.

(a) p = 3 and q = 1 (b) p = 3 and q = 2

(c) p = 4 and q = 1 (d) p = 4 and q = 2

18. [MCQ] [GATE-2017: 2M]

A computer network uses polynomials over GF (2) for error checking with 8 bits as information bits and uses $x^3 + x + 1$ as the generator polynomial to generate the check bits. In this network, the message 01011011 is transmitted as

(a) 01011011010

(b) 01011011011

(c) 01011011101

(d) 01011011100

19. [MCQ] [GATE-2009: 2M]

Let G(x) be the generator polynomial used for CRC checking. What is the condition that should be satisfied by G(x) to detect odd number of bits in error?

(a) G(x) contains more than two terms

(b) G(x) does not divide $1 + x^k$, for any k not exceeding the frame length.

(c) 1 + x is a factor of G(x)

(d) G(x) has an odd number of terms.

20. [MCQ] [GATE-2008: 2M]

Data transmitted on a link use the following 2D parity scheme for error detection:

Each sequence of 28 bits is arranged in a 4 × 7 matric (rows r_0 through r_3, and columns d_7 through d_1) and is padded with a column d_0 and row r_4 of parity bits computed using the Even parity scheme. Each bit of column d_0 (respectively, row r_4) gives the parity of the corresponding row (respectively, column). These 40 bits are transmitted over the data link.

	D_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0
r_0	0	1	0	1	0	0	1	1
r_1	1	1	0	0	1	1	1	0
r_2	0	0	0	1	0	1	0	0
r_3	0	1	1	0	1	0	1	0
r_4	1	1	0	0	0	1	1	0

The table shows data received by a receiver and has n corrupted bits, What is the minimum possible value of n?

(a) 1 (b) 2

(c) 3 (d) 4

21. [MCQ] [GATE-2008: 1M]

How many bytes of data can be sent in 15 seconds over a serial link with baud rate of 9600 in asynchronous mode with odd parity and two stop bits in the frame?

(a) 10,000 bytes (b) 12,000 bytes

(c) 15,000 bytes (d) 27,000 bytes

Framing

22. [MCQ] [GATE-2014: 2M]

A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is:

(a) 0111110100

(b) 0111110101

(c) 0111111101

(d) 0111111111

❑❑❑

ANSWER KEY

1.	(b)	**2.**	(7.07 to 7.09)	**3.**	(50 to 52)	**4.**	(86.5 to 89.5)
5.	(d)	**6.**	(2500 to 2500)	**7.**	(4 to 4)	**8.**	(8 to 8)
9.	(12 to 12)	**10.**	(320 to 320)	**11.**	(5 to 5)	**12.**	(d)
13.	(c)	**14.**	(a)	**15.**	(a)	**16.**	(c)
17.	(a)	**18.**	(c)	**19.**	(c)	**20.**	(c)
21.	(b)	**22.**	(b)				

SOLUTIONS

1. (b)

Efficiency $(\eta) = \dfrac{1}{1+2(a)}$

$$\boxed{a = \frac{T_p}{T_t}}$$

Transmission Time $(T_t) = \dfrac{\text{Frame Size}}{\text{Bandwidth}}$

Propagation time $(T_P) = \dfrac{\text{Length}}{\text{Velocity}}$

$$\eta = \frac{1}{1+2\left(\frac{T_p}{T_t}\right)}$$

- T_P increases then overall efficiency will decrease (as denominator bigger than the numerator)
- Also, if bandwidth (Transmission Rate) is higher than T_t value will be small and hence T_p / T_t will be a large value.
- Therefore, to get overall η low, higher link length and higher transmission rate is required.

Scan for Video solution

2. (7.08)

Given,

d = 2100 km, v = 3 ×10^8 m /sec.

$P_d = \dfrac{d}{u}$

$= \dfrac{2100 \text{ km}}{3\times10^5 \text{ km/sec}}$

$= 7 \times 10^{-3}$ sec. = 7 msec

Packet size = 1000 byte

= 8000 bits

B = 100 mbps

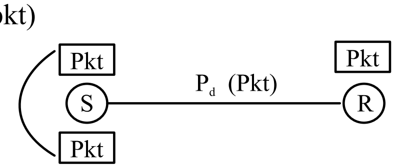
$= 100 \times 10^6$ bits/sec.

$T_d = \dfrac{\text{Pkt size}}{\text{Bandwidth}}$

$= 0.08 \times 10^{-3}$ sec.

= 0.08 m sec.

T_d (pkt)



Total time taken to send a packet form source to destination = T_d (pkt) + P_d (pkt)

= 0.08 + 7 = 7.08 m sec.

Scan for Video solution

3. (51)

Frame size $= 2000$ bits

Bandwidth $= 10^6$ bit/sec.

$P_d = 100$ m /sec.

$\eta = 50\% \left(\frac{1}{2}\right)$

$$T_d \text{ (frame)} = \frac{\text{Frame size}}{\text{Bandwidth}}$$

$$= \frac{200 \text{ bits}}{10^6 \text{ bit/sec.}}$$

$= 2\times10^{-3}$ sec.

$= 2$ msec.

Ack size $= 10$ bits

$$T_d \text{ (Ack)} = \frac{\text{Ack size}}{\text{Bandwidth}}$$

$$= \frac{10 \text{ bits}}{10^6 \text{ bits/sec}}$$

$= 10^{-5} \times 10^3$ m sec.

$= 10^{-2}$ msec.

$= \frac{1}{100}$ m sec .

$= 0.01$ m sec.

$$\text{Link utilization } (\eta) = \frac{\text{Useful time}}{\text{Total time}}$$

$$\frac{1}{2} = \frac{N\times T_d(\text{frame})}{T_d(\text{frame}) + 2\times P_d + Q_d + P_{rd} + T_d(\text{Ack})}$$

$$\frac{1}{2} = \frac{N\times 2}{2+2\times100+0.01}$$

$4N = 202.01$

$$N = \frac{202.01}{4}$$

$N = \lceil 50.51 \rceil$

$N = 51$

Scan for Video solution

4. (89.33)

$B = 10^6$ bit /sec.

$P_d = 0.75$ m sec/

Packet size = data + Header

Frame size $= 1980 + 20$

$= 20{,}00$ byte

$= 16{,}000$ bits

$$T_d \text{ (frame)} = \frac{\text{Frame size}}{\text{Bandwidth}}$$

$$= \frac{16{,}000 \text{ bits}}{10^6 \text{ bits / sec.}}$$

$= 16$ msec.

Ack size $= 20$ byte $= 160$ bits

$$T_d \text{ (ack)} = \frac{\text{Ack size}}{\text{Bandwidth}}$$

$$= \frac{160 \text{ bits}}{10^6 \text{ bit/sec}}$$

$= 160 \times 10^{-6}$ sec.

$= .16 \times 10^{-6}$ sec.

$= 0.16$ m sec.

$$\eta = \frac{\text{useful time}}{\text{total time}}$$

$$\eta = \frac{T_d(\text{frame})}{T_d(\text{frame}) + 2\times P_d + Q_d + P_{rd} + T_d(\text{Ack})}$$

$$= \frac{16}{16+2\times0.75+0+0.25+0.16}$$

$$\eta = \frac{16}{17.91}$$

$\eta = 0.8933$

$\eta = 89.33\%$

Scan for Video solution

5. (d)

$B = 10^6$ bit /sec.

Packet size $= 50{,}000$ byte

$= 8 \times 50{,}000$ bits

$= 4{,}00000$ bits

T_d (P) $= \dfrac{\text{Packet size}}{\text{Bandwidth}}$

$= \dfrac{4{,}00000 \text{ bits}}{10^6 \text{ bits/sec}}$

$= 400\times10^{-3}$ sec.

P = 400 m/sec.

d = 10,000 km

$v = 2 \times 10^8$ m/sec. $= 2\times10^5$ km/sec.

P_d (q) $= \dfrac{\text{Distance}}{\text{Velocity}}$

$= \dfrac{10{,}000 \text{ km}}{2\times10^5 \text{ km/sec}}$

$= 50 \times10^{-3}$ sec.

q = 50 msec.

Scan for Video solution

6. **(2500)**

P_d = 100 m sec.

$= 100 \times 10^{-3}$ sec.

$= 10^{-1}$ sec. $= \frac{1}{10}$sec.

Frame size = 1000 Byte = 8000 bits

B (Sender) = 80 kbps

$= 80 \times 10^3$ bits/sec.

T_d (frame) $= \dfrac{\text{Frame size}}{\text{Bandwidth}} = \dfrac{8000 \text{ bits}}{80\times10^8 \text{ bits/sec}}$

$= \dfrac{1}{10}$ sec.

Ack size = 100 byte

= 800 bits

B (Receiver) $= 8 \times10^3$ bit/sec.

T_d (Ack) $= \dfrac{\text{Ack size}}{\text{Bandwidth}}$

$= \dfrac{800}{8 \times 10^3 \text{ bit/sec}} = \dfrac{1}{10}$ sec

Throughput $= \dfrac{\text{Frame size}}{\text{Bandwidth}}$

$= \dfrac{800\text{bits}}{T_d(\text{frame}) + 2 \times P_d + Q_d + P_{rd} + T_d(\text{Ack})}$

$= \dfrac{800 \text{ bits}}{\frac{1}{10}\text{sec} + 2 \times \frac{1}{10}\text{sec}, + \frac{1}{10}\text{sec},}$

$= \dfrac{800 \text{ bits}}{0.4 \text{ sec}}$

= 20,000 bits/sec.

$= \dfrac{20{,}000}{8}$ byte/sec

Throughput = 2500 byte /sec.

Scan for Video solution

7. **(4)**

$B = 128 \times 10^3$ bits/sec.

P_d = 150 m sec.

Frame size = 1 KB

= 1024 byte

= 8 × 1024 bits

T_d(frame) $= \dfrac{\text{Frame size}}{\text{Bandwidth}}$

$= \dfrac{8\times1024\text{bit}}{128\times10^8 \text{bits/sec}}$

$= 64 \times 10^{-3}$ sec.

= 64 m sec.

$\eta = \dfrac{\text{useful}}{\text{total time}}$

$1 = \dfrac{W_s T_d(\text{frame})}{T_d(\text{frame}) + 2 \times P_d + Q_d + P_{rd} + T_d(\text{Ack})}$

$\dfrac{1}{1} = \dfrac{W_s \times 64}{64 + 2\times150}$

$W_s = \dfrac{364}{64}$

$W_s = 5.68$

$W_s \cong 6$

In selective repeat minimum sequence number required = 6 + 6 = 12

$2^k = 12$

$2^k = 2^4$

K = 4 bit

Scan for Video solution

8. **(8)**

Given,

d = 8000 Km

$B = 500\times10^6$ bits/sec.

$u = 4 \times 10^6$ m /sec.

Packet size/Frame size = 10^7 bits

$u = 4 \times 10^3$ km/sec.

$$P_d = \frac{d}{u}$$

$$= \frac{8000\text{km}}{4\times10^3\,\text{km/sec}} = 2\,\text{sec}$$

$$T_d\,(\text{frame}) = \frac{\text{Frame size}}{\text{Bandwidth}}$$

$$= \frac{10^7\,\text{bits}}{500\times10^6\,\text{bits/sec}} = 0.02\,\text{sec}\,.$$

$$\eta = \frac{\text{useful time}}{\text{total time}}$$

$$1 = \frac{N \times T_d\,(\text{frame})}{T_d\,(\text{frame}) + 2 \times P_d + Q_d + P_{rd} + T_d\,(\text{Ack})}$$

$$\frac{1}{I} = \frac{N \times 0.02}{0.02 + 2 \times 2}$$

$$N = \frac{4.02}{0.02}$$

N = 201

Minimum sequence number required in GB–N = 201 +1 = 202

$2^k = 202$

$2^k = 2^8$

K = 8 bit

Scan for Video solution

9. **(12)**

Given,

Frame size = 1000 byte

= 8000 byte

$$\eta = \frac{1}{4}$$

$B = 10^6$ bits/sec.

$$T_d\,(\text{frame}) = \frac{\text{Frame size}}{\text{Bandwidth}}$$

$$= \frac{8000\text{ bits}}{10^6\text{ bits/sec}}$$

= 8 × 10–3 sec.

= 8 m sec.

$$\eta = \frac{\text{useful time}}{\text{total time}}$$

$$\frac{1}{4} = \frac{T_d\,(\text{frame})}{T_d\,(\text{frame}) + 2 \times P_d + Q_d + P_{rd} + T_d\,(\text{Ack})}$$

$$\frac{1}{4} = \frac{8}{8 + 2 \times p_D}$$

$8 + 2\times P_d = 32$

$2\times P_d = 32 - 8$

$2\times P_d = 24$

P = 12 m sec.

Scan for Video solution

10. **(320)**

Given,

$B = 64\times10^3$ bit/sec, $P_d = 20$ m sec.

$\eta \geq 0.5$

$$\eta \geq \frac{1}{2}$$

$$\frac{\text{use ful ltime}}{\text{total time}} {}_3 \frac{1}{2}$$

$$\Rightarrow \frac{T_d\,(\text{frame})}{T_d\,(\text{frame}) + 2 \times P_d + Q_d + P_{rd} + T_d\,(\text{Ack})} \geq \frac{1}{2}$$

$$\Rightarrow \frac{T_d\,(\text{frame})}{T_d\,(\text{frame}) + 2 \times P_d} \geq \frac{1}{2}$$

$\Rightarrow 2 \times T_d\,(\text{Frame}) \geq T_d\,(\text{frame}) + 2 \times P_d$

$\Rightarrow T_d\,(\text{frame}) \geq 2 \times P_d$

$\Rightarrow \dfrac{\text{Frame size}}{\text{Bandwidth}} \geq 2 \times P_d$

$\Rightarrow \frac{L}{B} \geq 2 \times P_d$

$\Rightarrow L \geq 2 \times P_d \times B$

$\Rightarrow L \geq 2 \times 20 \times 10^{-3}$ sec $\times 64 \times 10^{-3}$ bits/sec.

$\Rightarrow L \geq 2 \times 20 \times 64$ bits

$\Rightarrow L \geq \dfrac{2 \times 20 \times 64}{8}$ byte

$\Rightarrow L \geq 320$ byte

Scan for Video solution

11. (5)

Given,

Frame size = 1KB = 1024 Byte = 8 × 1024 bits
= 8192 bits

B = 1.5 mbps
= 1.5×10^6 bits/sec.

P_d = 50 msec.

$$T_d(\text{frame}) = \frac{\text{Frame size}}{\text{Bandwidth}}$$
$$= \frac{8192 \text{ bits}}{1.5 \times 10^6 \text{ bits/sec.}}$$
$$= 5461.33 \times 10^{-6} \text{ sec.}$$
$$= 5.461 \times 10^{-3} \text{ sec.}$$
$$= 5.461 \text{ m sec.}$$

$$\eta = \frac{\text{useful time}}{\text{total time}} = 60\% = 0.6$$

$$0.6 = \frac{W_s \times T_d(\text{Frame})}{T_d(\text{Frame}) + 2 \times P_d + Q_d + P_{rd} + T_d(\text{Ack})}$$

$$\frac{0.6}{1} = \frac{W_s \times 5.461}{5.461 + 2 \times 50}$$

$$W_s = \frac{105.461 \times 0.6}{5.461}$$

$$W_s = 11.58 \cong 12$$

Minimum sequence Number required in SR
= 12+12 = 24

$2^k = 24$

$2^k = 2^5$

K = 5 bit

Scan for Video solution

12. (d)

Frame size = 1000 bits

B = 10^6 bps = 10^6 bits/sec, P_d = 25 msec.

$$T_d(\text{frame}) = \frac{\text{Framesize}}{\text{Bandwidth}}$$
$$= \frac{1000 \text{ bits}}{10^6 \text{ bits / sec.}} = 10^{-3} \text{ sec.} = 1 \text{ msec.}$$

Maximally packed means capacity of link

Capacity of link $= B \times P_d$
$= 10^6$ bit/sec. $\times 25 \times 10^{-3}$ sec.
$= 25 \times 10^3$ bits
= 25,000 bits

$$\text{Number of frame} = \frac{25{,}000 \text{ bits}}{1000 \text{ bits}} = 25$$

$2^l = 25$

$2^l = 2^5$

$\boxed{l = 5 \text{ bit}}$

Scan for Video solution

13. (c)

Sender window size = $2^l = 2^5 = 32$

Minimum time the sender will have to wait before transmitting the next frame = total time – useful time

= Total time – Ws × T_d

= 52 m sec. – 32 × 1 msec.

= 20 msec.

Total time – $T_d(\text{frame}) + 2 \times P_d + \cancel{Q_d} + \cancel{P_{rd}} + T_d(\text{Ack})$

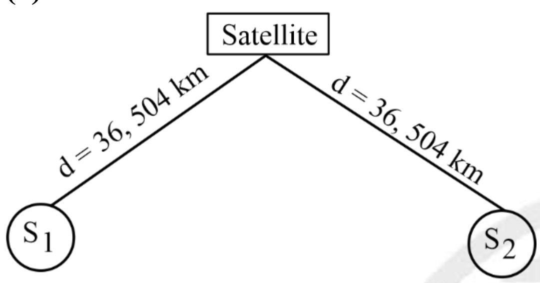
= 1 msec. + 2× 25msec. + 1 msec.

(ACK is piggy backed we are assuming frame size for piggybacking is also 1000 bits.)

= 52 msec.

Scan for Video solution

14. (a)



$v = 3 \times 10^8$ m/sec.

$\eta = \frac{1}{4}$, GB – 127

N = 127

d = 36,504 ×2 = 73008 km

$$P_d = \frac{d}{v} = \frac{73{,}008 \text{ km}}{3\times 10^5 \text{ km}}$$

$$= 24336 \times 10^{-5} \text{ sec.} = 0.24336 \text{ sec.}$$

$$\text{Efficiency} = \frac{\text{Useful time}}{\text{Total time}}$$

$$\frac{1}{4} = \frac{N\times T_d(\text{PKt})}{T_d(\text{PKt}) + 2\times P_d + \underbrace{\cancel{Q_d}}_{0} + \underbrace{\cancel{P_{rd}}}_{0} + \underbrace{\cancel{T_d(\text{ACk})}}_{0}}$$

$$\frac{1}{4} = \frac{127\times T_d(\text{Pkt})}{T_d(\text{Pkt})+2\times P_d}$$

$127 \times 4 \times T_d(\text{Pkt}) = T_d(\text{Pkt}) + 2 \times P_d$

$508\ T_d = T_d + 2\times P_d$

$507\ T_d = 2\times P_d$

$$T_d = \frac{2\times P_d}{507}$$

$$T_d = \frac{2\times 0.24336\text{sec.}}{507}$$

$$\frac{L}{B} = \frac{2\times 0.24336\text{sec.}}{507}$$

$$L = \frac{2\times 0.24336\,\cancel{\text{sec}}.\times 10^6 \text{bits}/\cancel{\text{sec.}}}{507}$$

L = 960 bits

$$L = \frac{960}{8}\text{Byte}$$

L = 120 Byte.

Scan for Video solution

15. (a)

C_1	C_2	3	C_4	5	6	7	C_8	9	10	11	12
0	1	1	0	0	1	0	y	x	0	1	1

C_1

1	3	5	7	9	11
0	1	0	0	X	1

X = 0 (for even parity)

C_2

2	3	6	7	10	11
1	1	1	0	0	1

Number of 1's→ even (even parity)

C_4

4	5	6	7	12
0	0	1	0	1

Number of 1's → even (even parity)

C_8

8	9	10	11	12
y	x	0	1	1
0	0	0	1	1

→ even parity (y = 0)

Scan for Video solution

16. (c)

Generator = $X^3 + X + 1$

$= 1.X^3 + 0.X^2 + 1.X^1 + 1.X^0$

$= (1011)_2$

```
1011)11000000
     1011
     01110000
      1011
     0101000
       1011
       0001 0 0
           c2c1c0
```

Scan for Video solution

17. (a)

0 0 0 0 0 (a)

0 1 0 1 1 (b)

1 0 1 0 1 (c)

1 1 1 1 0 (d)

Hamming distance (a, b) = 3

Hamming distance (a, c) = 3

Hamming distance (a, d) = 4

Hamming distance (b, c) = 4

Hamming distance (b, d) = 3

Minimum Hamming distance = 3

Min Hamming distance required to correct 'd' bit error = 2d + 1

2d + 1 = 3

2d = 2

$\boxed{d = 1}$

Scan for Video solution

18. (c)

Given, $x^3 + x + 1$

$= 1 - x^3 + 0.x^2 + 1.x^1 + 1x^o$

$= 1011$

```
1011)01011011000
      1011
     0000011000
           1011
           01110
            1011
           0[101]
             CRC
```

CRC = 101

Transmitted data: 01011011101

Scan for Video solution

19. (c)

If G(x) has a factor of x+1 then it can detect odd number of bits error.

Scan for Video solution

20. (c)

2–D parity scheme and column do and row r_4 of parity bits computed using even parity scheme.

	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0
r_0	0	1	0	1	0	0	1	1
r_1	1	1	$\cancel{0}$ 1	0	1	1	1	0 × (error)
r_2	0	0	0	1	0	1	0	0
r_3	0	1	1	0	1	0	1	0
r_4	1	1	0	0	0	$\cancel{1}$ 0	1	$\cancel{0}$ 1
	✓	✓	× (error)	✓	✓	× (error)	✓	× (error)

Bits got corrupted are:

(r_1, d_5), (r_4, d_2), (r_4, d_0)

Total three bits got corrupted.

Scan for Video solution

21. (b)

Given

Baud rate = 9600 Bps

Mode: Asynchronous mode with odd parity and stop bits

Note: In asynchronous mode data is transmitted in the form of byte or character.

Star bit = 1

Odd parity bit =1

Stop bit = 2

Number of bits sent in 1 byte = 8 + 1 + 1 + 2 = 12 bits

Baud rate = $\frac{9600}{12}$ bytes /sec = 800 bytes/sec.

In 1 sec. ________800 bytes

Then In 15 sec. ________15 × 800 = 12000 bytes

Scan for Video solution

22. (b)

8-bit delimiter pattern (flag) = 01111110

Output bit string = 01111100101

In bit stuffing a bit in stuffed in the input data after every continuous 1's, one less than in the flag or delimiter pattern.

Output data: 01111100101

↑

Bit is stuffed in the input data

So, the input data will be: 0111110101

Scan for Video solution

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CHAPTER

3

MAC SUBLAYER

Pure Aloha

1. **[NAT]** **[GATE-2021: 2M]**

Consider a network using the pure ALOHA medium access control protocol, where each frame is of length 1000 bits. The channel transmission rate is 1 Mbps (= 10^6 bits per second). The aggregate number of transmissions across all the nodes (including new frame transmissions and retransmitted frames due to collisions) is modelled as a Poisson process with a rate of 1000 frames per second. Throughput is defined as the average number of frames successfully transmitted per second. The throughput of the network (rounded to the nearest integer) is ______________

CSMA, CSMA/CD

2. **[NAT]** **[GATE-2018 : 2M]**

Consider a simple communication system where multiple nodes are connected by a shared broadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for 5 units of time. If the node does not detect any other transmission in this duration, it starts transmitting its packet in the next time unit. If the node detects another transmission, it waits until this other transmission finishes, and then begins to carrier-sense for 5-time units again. Once they start to transmit, nodes do not perform any collision detection and continue transmission even if a collision occurs. All transmissions last for 20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium.

Assume that the system has two nodes P and Q, located at a distance d meters from each other. P starts transmitting a packet at time t = 0 after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time t = 0 and begins to carrier-sense the medium.

The maximum distance d (in meters, rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is ________.

3. **[NAT]** **[GATE-2019: 2M]**

Consider that 15 machines need to be connected in a LAN using 8-port Ethernet switches. Assume that these switches do not have any separate uplink ports. The minimum number of switches needed is___________.

4. **[MCQ]** **[GATE-2016: 2M]**

In an Ethernet local area network, which one of the following statement is TRUE?

(a) A station stops to sense the channel once it starts transmitting a frame.
(b) The purpose of the jamming signal is to pad the frames that are smaller than the minimum frame size.
(c) A station continues to transmit the packet even after the collision is detected.
(d) The exponential backoff mechanism reduces the probability of collision on retransmission.

5. **[NAT]** **[GATE-2016: 2M]**

A network has a data transmission bandwidth of 20×10^6 bits per second. It uses CSMA/CD in MAC Layer. The maximum signal propagation time from one node to another node is 40 microseconds. The minimum size of a frame in the network is _____ bytes.

6. [MCQ] [GATE-2015: 1M]

Consider a CSMA/CD network that transmits data at a rate of 100 Mbps (10^8 bits per second) over a 1 km (kilometer) cable with no repeaters. If the minimum frame size required for this network is 1250 bytes. What is the signal speed (km/sec) in the cable?

(a) 8000 (b) 10000
(c) 16000 (d) 20000

7. [NAT] [GATE-2015: 2M]

Consider a LAN with four nodes S_1, S_2, S_3 and S_4. Time is divided into fixed-size slots, and a node can begin its transmission only at the beginning of a slot. A collision is said to have occurred if more than one node transmits in the same slot. The probabilities of generation of a frame in a time slot by S_1, S_2, S_3 and S_4 are 0.1, 0.2, 0.3 and 0.4, respectively. The probability of sending a frame in the first slot without any Collison by any of these four stations is ______.

8. [MCQ] [GATE-2013: 2M]

Determine the maximum length of the cable (in km) for transmitting data at a rate of 500 Mbps in an Ethernet LAN with frames of size 10,000 bits. Assume the signal speed in the cable to be 2,00,000 km/s.

(a) 1 (b) 2
(c) 2.5 (d) 5

9. [MCQ] [GATE-2008: 2M]

The minimum frame size required for a CSMA/CD based computer network running at 1 Gbps on a 200 m cable with a link speed of 2×10^8 m/s is:

(a) 125 bytes
(b) 250 bytes
(c) 500 bytes
(d) None of these

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ANSWER KEY

1.	(130 to 140)	**2.**	(50 to 50)	**3.**	(3 to 3)	**4.**	(d)
5.	(200 to 200)	**6.**	(d)	**7.**	(0.40 to 0.46)	**8.**	(b)
9.	(b)						

SOLUTIONS

1. (135)

Frame size = 1000 bits

$B = 10^6$ bits/sec

$$\text{Transmission time} = \frac{\text{Frame size}}{\text{Bandwidth}} = \frac{1000 \text{ bits}}{10^6 \text{ bits / sec}}$$

$$= 10^{-3} \text{ sec.} = 1 \text{ msec.}$$

1 sec. ____________1000 frames

1 m sec. = 10^{-3} sec. ____________1000×10^3 Frame

= 1 frame

So, G = 1 [Number of frames generated in one frame transmission time]

Throughput. S $= G \times e^{-2G}$

$= 1 \times e^{-2}$

$$= \frac{1}{e^2} = 0.135$$

Throughput = 13.5 %

Average number of frames successfully

Transmitted per second = 1000×0.135

= 135

Scan for Video solution

2. (50)

Carrier sense time t = 5 unit

Transmission time = 20 unit

Speed = 10 meter/unit

$t \le 5$ unit

Distance = speed × time

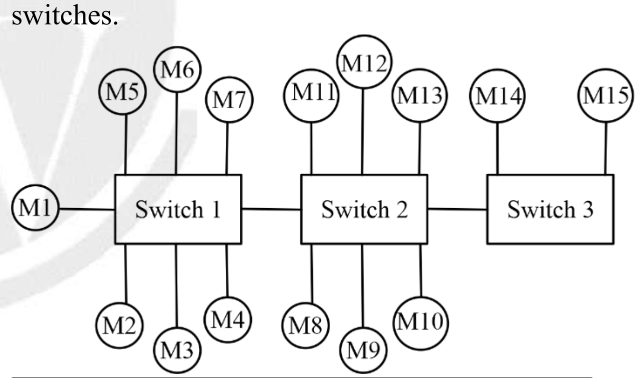
Distance $\le$ 10 meter/unit × 5 unit

Distance $\le$ 50 meter.

Scan for Video solution

3. (3)

Using 8-port ethernet switch with no separate unlink port 7 machines or computers can be connected. So, for 15 machines we required 3 such kind of switches.



Scan for Video solution

4. (d)

The exponential back off algorithm is a collision resolution mechanism. This mechanism reduces the probability of collision on retransmission.

Scan for Video solution

5. **(200)**

B = 20 * 106 bits/sec, Pd = 40 sec= 40 * 10^{-6} sec

$T_{d(frame)} \geq 2 * P_d$

$\frac{L}{B} \geq 2 * P_d$

$L \geq 2 * P_d * B$

$L \geq 2 * 40 * 10^{-6}$ sec * 20 * 10^6 bits/sec

= 1600 bits

$= \frac{1600}{8} Byte$ = 200 Byte.

Scan for Video solution

6. **(d)**

B = 10^8 bits/sec.

d = 1 km

Minimum Frame size (l) = 1250 bytes

= 8 × 1250 bits

= 10,000 bits

v = ?

$$\boxed{T_d(\text{frame}) \geq 2\times P_d + \underbrace{\cancel{T_{d(\text{JAM signal})}}}_{0}}$$

$$\frac{L}{B} \geq 2 \times \frac{d}{v}$$

$$\frac{10{,}000 \text{ bits}}{10^8 \text{ bits/sec}} \geq \frac{2 \times 1 \text{ km}}{v}$$

$$\frac{1}{10^4} \geq \frac{2}{v}$$

v = 2 ×10^4 km/sec

v = 20,000 km/sec

Scan for Video solution

7. **(0.4404)**

Probability = $S_1 (1 – S_2) (1 – S_3) (1 – S_4) + (1 – S_1) S_2 (1 – S_3) (1 – S_4) + (1 – S_1) (1 – S_2) S_3 (1 – S_4) +(1 – S_1) (1 – S_2) (1 – S_3) S_4$

= (0.1 × 0.8 × 0.7 × 0.6) + (0.9 × 0.2 × 0.7 × 0.6) + (0.9 × 0.8 × 0.3 × 0.6) + (0.9 × 0.8× 0.7 × 0.4) = 0.4404

Scan for Video solution

8. **(b)**

$T_{d(frame)} \geq 2 * P_d$

$$\frac{L}{B} \geq \frac{2*d}{v}$$

$$\frac{10000 \text{ bits}}{500*10^6 \text{ bits/sec}} \geq \frac{2*d}{200000\, km/\text{sec}}$$

$\boxed{d \leq 2\text{KM}}$

Scan for Video solution

9. **(b)**

B = 10^9 bits/sec

d = 200 mtr

v = 2 ∗ 10^8 m/sec

$$P_d = \frac{d}{v} = \frac{200}{2*10^8\, m/\text{sec}}$$

= 100 * 10^{-8} sec.

Minimum frame size in CSMA/CD

$$T_{d\,(frame)} \geq 2 * P_d + T_{d(JAM\ signal)}$$

$$\frac{L}{B} \geq 2 * P_d$$

$L \geq 2 * P_d * B$

$L \geq 2 * 100 * 10^{-8}$ sec * 10^9 bits/sec

$L \geq 2 * 100 * 10$ bits

$L \geq 2000$ bits

$L \geq \frac{2000}{8}$ Byte = 250 Byte.

Scan for Video solution

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CHAPTER

4 NETWORK LAYER

IP Header

1. [MSQ] [GATE-2021: 1M]

Consider two hosts P and Q connected through a router R. The maximum transfer unit (MTU) value of the link between P and R is 1500 bytes, and between R and Q is 820 bytes. A TCP segment of size 1400 bytes was transferred from P to Q through R, with IP identification value as 0x1234. Assume that the IP header size is 20 bytes. Further, the packet is allowed to be fragmented, i.e, Don't Fragment (DF) flag in the IP header is not set by P.

Which of the following statements is/ are correct?

(a) Two fragments are created at R and the IP datagram size carrying the second fragment is 620 bytes.

(b) If the second fragment is lost, R will resend the fragment with IP identification value 0x1234

(c) If the second fragment is lost, P is required to resend the whole TCP segment.

(d) TCP destination port can be determined by analyzing only the second fragment.

2. [MCQ] [GATE-2020: 1M]

Consider the following statements about the functionality of an IP based router.

I. A router does not modify the IP packets during forwarding.

II. It is not necessary for a router to implement any routing protocol.

III. A router should reassemble IP fragments if the MTU of the outgoing link is larger than the size of the incoming IP packet.

Which of the above statements is/are TRUE?

(a) I only (b) I and II only

(c) II and III only (d) II only

3. [NAT] [GATE-2018: 2M]

Consider an IP packet with a length of4500 bytes that includes a 20-byte IPv4 header and a 40-byte TCP header. The packet is forwarded to an IPv4 router that supports a Maximum Transmission Unit MTU) of 600 bytes. Assume that the length of the IP header in all the outgoing fragments of this packet is 20 bytes. Assume that the fragmentation offset value stored in the first fragment is 0.

The fragmentation offset value stored in the third fragment is_____.

4. [NAT] [GATE-2017: 1M]

The maximum number of IPv4 router addresses that can be listed in the record route (RR) option field of an IPv4 header is _______.

5. [NAT] [GATE-2016: 2M]

An IP datagram of size 1000 bytes arrives at a router. The router has to forward this packet on a link whose MTU (Maximum transmission Unit) is 100 bytes. Assume that the size of the IP header is 20 bytes. The number of fragments that the IP datagram will be divided into for transmission is __________.

6. [MCQ] [GATE-2015: 1M]

Which one of the following fields of an IP header is NOT modified by a typical IP router?

(a) Checksum (b) Source address

(c) Time to Live (TTL) (d) Length

7. [MCQ] [GATE-2015: 2M]

Host A sends a UDP datagram containing 8880 bytes of user data to host B over an Ethernet LAN. Ethernet frames may carry data up to 1500 bytes (i.e. MTU = 1500 bytes). Size of UDP header is 8 bytes and size of IP header is 20 bytes. There is no option field in IP header. How may total number of IP fragments will be transmitted and what will be contents of offset field in the last fragment?

(a) 6 and 925
(b) 6 and 7400
(c) 7 and 1110
(d) 7 and 8880

8. [NAT] [GATE-2014: 2M]

Every host in an IPv4 network has a 1-second resolution real-time clock with battery backup. Each host needs to generate up to 1000 unique identifiers per second. Assume that each host has a globally unique IPv4 address. Design a 50-bit globally unique ID for this purpose. After what period (in seconds) will the identifiers generated by a host wrap around?

9. [MCQ] [GATE-2014: 1M]

Host A (on TCP/IP v4 network A) sends an IP datagram D to host B (also on TCP/IP v4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field(s) may be different from that of the original datagram D?

(i) TTL
(ii) Checksum
(iii) Fragment Offset

(a) (i) only
(b) (i) and (ii) only
(c) (ii) and (iii) only
(d) (i), (ii) and (iii)

10. [MCQ] [GATE-2014: 2M]

An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are

(a) MF bit:0, Datagram Length:1444;Offset:370
(b) MF bit:1, Datagram Length:1424;Offset:185
(c) MF bit:1, Datagram Length:1500;Offset:370
(d) MF bit:0, Datagram Length:1424;Offset:2960

11. [MCQ] [GATE-2013: 2M]

In an IPv4 datagram, the M bit is 0, the value of HLEN is 10, the value of total length is 400 and the fragment offset value is 300. The position of the datagram, the sequence numbers of the first and the last bytes of the payload, respectively are

(a) Last fragment, 2400 and 2789
(b) First fragment, 2400 and 2759
(c) Last fragment, 2400 and 2759
(d) Middle fragment, 300 and 689

12. [MCQ] [GATE-2010: 1M]

One of the header fields in an IP datagram is the Time-to-Live (TTL) field. Which of the following statements best explains the need for this field?

(a) It can be used to prioritize packets
(b) It can be used to reduce delays
(c) It can be used to optimize throughput
(d) It can be used to prevent packet looping

IPv4 Protocol

13. [MCQ] [GATE-2021 : 1M]

Consider the following two statements

S_1: Destination MAC address of an ARP reply is a broadcast address.

S_2: Destination MAC address of an ARP request is a broadcast address.

Which one of the following choices is correct?

(a) Both S_1 and S_2 are true
(b) S_1 is true and S is false
(c) S_1 is false and S_2 is true
(d) Both S_1 and S_2 are false

14. [MCQ] [GATE-2019 : 2M]

Suppose that in an IP-over-Ethernet network, a machine X wishes to find the MAC address of another machine Y in its subnet. Which one of the following techniques can be used for this?

(a) X sends an ARP request packet to the local gateway's MAC address which then finds the MAC address of Y and sends to X.

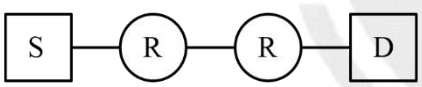
(b) X sends an ARP request packet with broadcast MAC address in its local subnet.

(c) X sends an ARP request packet with broadcast IP address in its local subnet.

(d) X sends an ARP request packet to the local gateway's IP address which then finds the MAC address of Y and sends to X.

15. [MCQ] [GATE-2013 : 1M]

Assume that source S and destination D are connected through two intermediate routers labels R. Determine how many times each packet has to visit the network layer and the data link layer during a transmission form S to D.



(a) Network layer – 4 times and Data link layer – 4 times

(b) Network layer – 4 times and Data link layer – 3 times

(c) Network layer – 4 times and Data link layer – 6 times

(d) Network layer – 2 times and Data link layer – 6 times

Routing Algorithm

16. [MCQ] [GATE-2023: 1M]

Which of the following statements is/are INCORRECT about the OSPF (Open Shortest Path First) routing protocol used in the Internet?

(a) OSPF implements Bellman-Ford algorithm to find shortest paths.

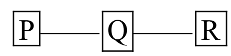
(b) OSPF uses Dijkstra's shortest path algorithm to implement least-cost path routing.

(c) OSPF is used as an inter-domain routing protocol.

(d) OSPF implements hierarchical routing.

17. [NAT] [GATE-2022: 2M]

Consider a network with three routers P,Q,R shown in the figure below. All the links have cost of unity.



The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q-R fails. Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation (rounded off to one decimal place) between P and Q, leading to count-to-infinity problem, is ______.

18. [MCQ] [GATE-2021: 2M]

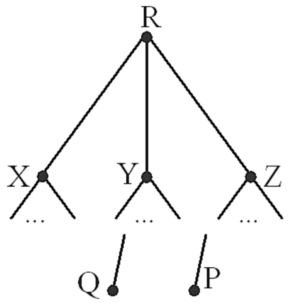
Consider a computer network using the distance vector routing algorithm in its network layer. The partial topology of the network is as shown below.

The objective is to find the shortest-cost path from the router R to routers

P and Q. Assume that R does not initially know the shortest routes to P and Q.

Assume that R has three neighboring routers denoted as X, Y and Z.

During one iteration, R measures its distance to its neighbors X, Y and Z as 3,2 and 5, respectively. Router R gets routing vectors from its neighbors that indicate that the distance to router P from routers X, Y and Z are 7, 6 and 5, respectively. The routing vector also indicates that the distance to router Q from routers X, Y and Z are 4,6 and 8, respectively. Which of the following statement(s) is/are correct with respect to the new routing table of R, after updation during this iteration?



(a) The distance from R to Q will be stored as 7.

(b) The distance from R to P will be stored as 10.

(c) The next hop router for a packet from R to Q is Z.

(d) The next hop router for a packet from R to P is Y.

19. [MCQ] [GATE-2017: 1M]

Consider the following statements about the routing protocols, Routing Information Protocol (RIP) and Open Shortest Path first (OSPF) in an IPv4 network.

I: RIP uses distance vector routing

II: RIP packets are sent using UDP

III: OSPF packets are sent using TCP.

IV: OSPF operation is based on link-state routing.

Which of the statements above are CORRECT?

(a) I and IV only (b) I, II and III only

(c) I, II and IV only (d) II, III and IV only

20. [MCQ] [GATE-2014: 1M]

Which one of the following is TRUE about the interior gateway routing protocols - Routing Information Protocol (RIP) and Open Shortest Path First (OSPF)

(a) RIP uses distance vector routing and OSPF uses link state routing.

(b) OSPF uses distance vector routing and RIP uses link state routing

(c) Both RIP and OSPF use link state routing

(d) Both RIP and OSPF use distance vector routing

21. [MCQ] [GATE-2014: 1M]

Consider the following three statements about link state and distance vector routing protocols, for a large network with 500 network nodes and 4000 links.

S1: The computational overhead in link state protocols is higher than in distance vector protocols.

S2: A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.

S3: After a topology change, a link state protocol will converge faster than a distance vector protocol.

Which one of the following is correct about S_1, S_2, and S_3?

(a) S_1, S_2, and S_3 are all true.

(b) S_1, S_2, and S_3 are all false.

(c) S_1 and S_2 are true, but S_3 is false.

(d) S_1 and S_3 are true, but S_2 is false.

Combined data for Next Two Questions

Consider a network with five nodes, N1 to N5, as shown below.

The network uses a Distance Vector Routing protocol. Once the routes have stabilized, the distance vectors at different nodes are as following.

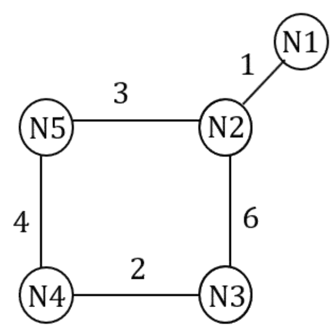
N1 : (0, 1,7, 8, 4)

N2 : (1, 0, 6, 7, 3)

N3 : (7, 6, 0, 2, 6)

N4 : (8, 7, 2, 0, 4)

N5 : (4, 3, 6, 4, 0)



Each distance vector is the distance of the best known path at that instance to nodes, N1 to N5, where the distance to itself is 0. Also, all links are symmetric and the cost is identical in both directions. In each round, all nodes exchange their distance vectors with their respective neighbors. Then all nodes update their distance vectors. In between two rounds, any change in cost of a link will cause the two incident node's to change only that entry in their distance vectors.

22. [MCQ] [GATE-2011: 1M]

The cost of link N2-N3 reduces to 2 (in both directions). After the next round of updates, what will be the new distance vector at node, N3?

(a) (3, 2, 0, 2, 5) (b) (3, 2, 0, 2, 6)
(c) (7, 2, 0, 2, 5) (d) (7, 2, 0, 2, 6)

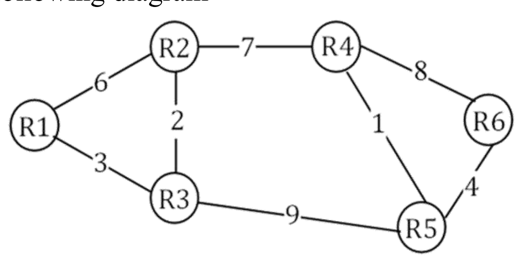
23. [MCQ] [GATE-2011: 1M]

After the update in the previous question, the link N1-N2 goes down. N2 will reflect this change immediately in its distance vector as cost, ∞. After the NEXT ROUND of update, what will be the cost to N1 in the distance vector of N3?

(a) 3 (b) 9
(c) 10 (d) ∞

Combined Data for Next Two Questions

Consider a network with 6 routers R1 to R6 connected with links having weights as shown in the following diagram



24. [MCQ] [GATE-2010: 1M]

All the routers use the distance vector based routing algorithm to update their routing tables. Each router starts with its routing table initialized to contain an entry for each neighbur with the weight of the respective connecting link. After all the routing tables stabilize, how many links in the network will never be used for carrying any data?

(a) 4 (b) 3
(c) 2 (d) 1

25. [MCQ] [GATE-2010: 1M]

Suppose the weights of all unused links in the previous question are changed to 2 and the distance vector algorithm is used again until all routing tables stabilize. How many links will now remain unused?

(a) 0 (b) 1
(c) 2 (d) 3

26. [MCQ] [GATE-2008: 1M]

Two popular routing algorithms are Distance Vector(DV) and Link State (LS) routing. Which of the following are true?

(S1): Count to infinity is a problem only with DV and not LS routing

(S2): In LS, the shortest path algorithm is run only at one node

(S3): In DV, the shortest path algorithm is run only at one node

(S4): DV requires lesser number of network messages than LS

(a) S1, S2 and S4 only
(b) S1, S3 and S4 only
(c) S2 and S3 only
(d) S1 and S4 only

Switch, Router and Gateway

27. [NAT] [GATE-2015: 2M]

Two hosts are connected via a packet switch with 10^7 bits per second links. Each link has a propagation delay of 20 microseconds. The switch begins forwarding a packet 35 microseconds after it receives the same. If 10000 bits of data are to be transmitted between the two hosts using a packet size of 5000 bits, the time elapsed between the transmission of the first bit of data and the reception of the last bit of the data in microsecond is _________.

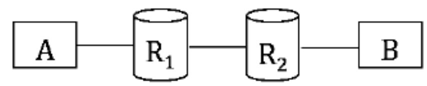
28. [MCQ] [GATE-2012: 2M]

Consider a source computer (S) transmitting a file of size 10^6 bits to a destination computer (D) over a network of two routers (R_1 and R_2) and three link (L_1, L_2 and L_3). L_1 connects S to R_1; L_2 connects R_1 to R_2; and L_3 connects R_2 to D. Let each link be of length 100 km. Assume signals travel over each link at a speed of 10^8 meters per second. Assume that the link bandwidth on each link is 1Mbps. Let the file be broken down into 1000 packets each of size 1000 bits. Find the total sum of transmission and propagation delays in transmitting the file from S to D?

(a) 1005 ms (b) 1010 ms
(c) 3000 ms (d) 3003 ms

29. [MCQ] [GATE-2014: 2M]

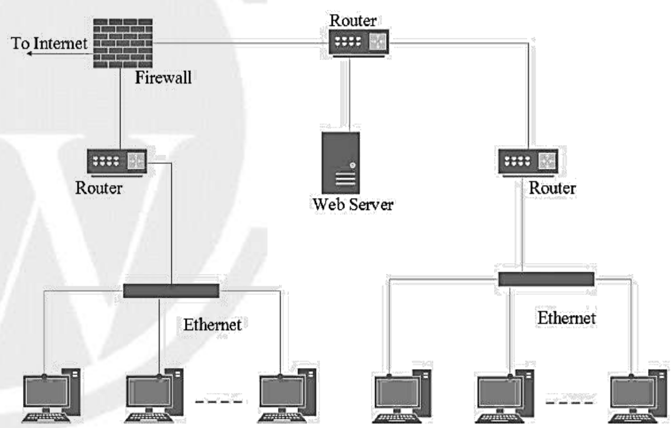
Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes/sec. A user on host A sends a file of size 10^3 bytes to host B through routers R1 and R2 in three different ways. In the first case a single packet containing the complete file is transmitted from A to B. In the second case, the file is split into 10 equal parts, and these packets are transmitted from A to B. In the third case, the file is split into 20 equal parts and these packets are sent from A to B. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no error during transmission. Let T1, T2 and T3 be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT?



(a) T1 < T2 < T3 (b) T1 > T2 > T3
(c) T2 = T3, T3 < T1 (d) T1 = T3, T3 > T2

30. [MCQ] [GATE-2022: 1M]

Consider an enterprise network with two Ethernet segments, a web server and a firewall, connected via three routers as shown below.



What is the number of subnets inside the enterprise network?

(a) 3 (b) 12
(c) 6 (d) 8

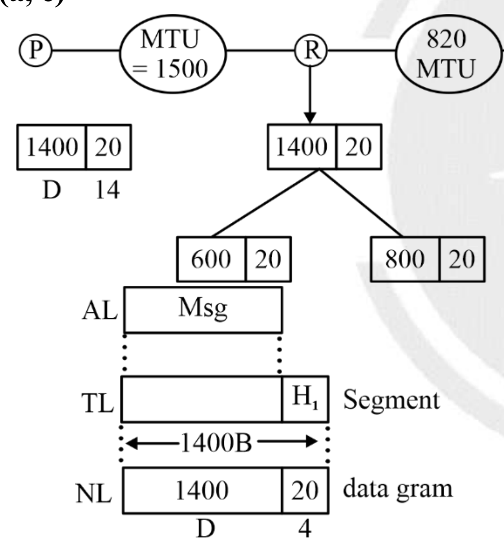
❑❑❑

ANSWER KEY

1.	(a, c)	**2.**	(d)	**3.**	(144 to 144)	**4.**	(9 to 9)
5.	(13 to 13)	**6.**	(b)	**7.**	(c)	**8.**	(262 to 263)
9.	(d)	**10.**	(a)	**11.**	(c)	**12.**	(d)
13.	(c)	**14.**	(b)	**15.**	(c)	**16.**	(a, c)
17.	(0.5 to 0.5)	**18.**	(a, d)	**19.**	(c)	**20.**	(a)
21.	(d)	**22.**	(a)	**23.**	(c)	**24.**	(c)
25.	(d)	**26.**	(d)	**27.**	(1575 to 1575)	**28.**	(a)
29.	(d)	**30.**	(c)				

SOLUTIONS

1. (a, c)



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2. (d)

Due to the TTL value changing during forwarding, a router alters the IP packets. Since it can simply forward packets in all directions without performing any routing, a router is not required to implement any routing protocols. The packets are not assembled by the router. At the destination system, assembly is completed.

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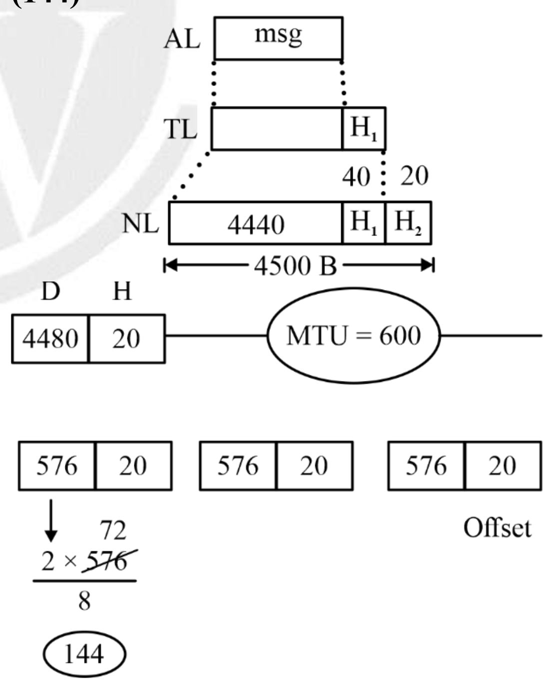
3. (144)



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4. **(9)**

Option = 40 Byte

2 Bytes are resered for option type & length

40 – 2 = 38 byte

In 38 bytes he can store 9 IPv4 address as each IPv4 address is of 4 Bytes.

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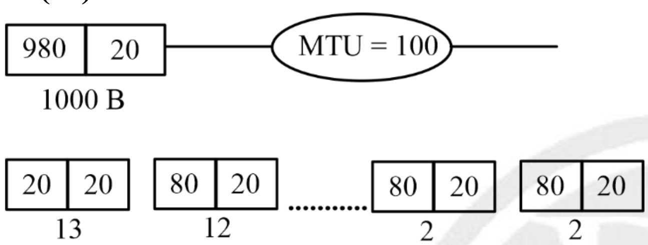
5. **(13)**



12 × 80 = 960

980 × 960 = 20B (last fragment)

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6. **(b)**

Not Changed	May be Changed	Definitely Changed
1. VER 2. Header Length 3. Services 4. Identification number 5. DF (Don't fragment 6. Protocol 7. SIP, DIP	1. Total length 2. MF (more fragment) 3. Fragment offset	1. TTL 2. Checksum

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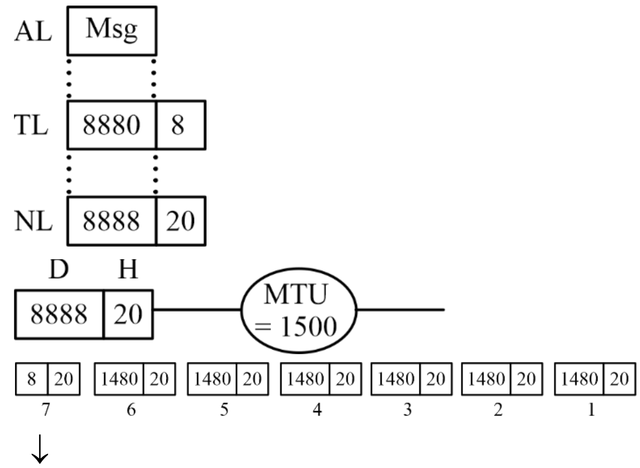
7. **(c)**



$$\text{Offset} = \frac{6\times1480}{8} = \frac{8880}{8} = 1110$$

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8. **(262.14)**

Number of bits in IPV4 = 32

Total number of hosts = 2^{32}

Total number of unique identifier (ID) generated per second = $2^{32} \times 1000$

Total number of $\text{ID}_\text{S} = 2^{50}$

$$\text{Wrap around time} = \frac{\text{Total IDs}}{\text{IDs generated/sec}}$$

$$= \frac{2^{50}}{2^{32} \times 1000} = 262.14 \text{ sec.}$$

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9. **(d)**

All the given filed TTL, Checksum, and Fragment Offset value may have change in their values.

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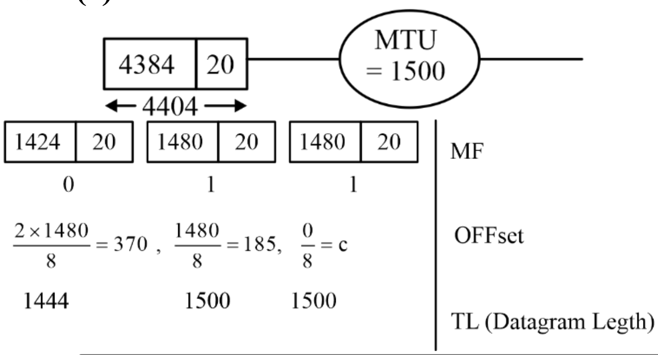
10. (a)



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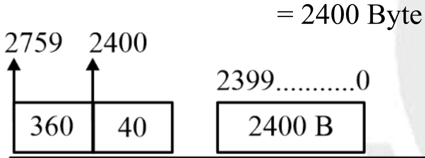
11. (c)

HLEN = 10
Header size = 10 × 4 = 10 Byte
Total Length = 400B
Total Length = Data + Header
Data (Payload) = 400 – 40 = 360B
Offset = 300 Byte
Number of byte alned = 3 × 300
= 2400 Byte



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12. (d)

TTL is used to Avoid Infinite looping

Scan for Video solution

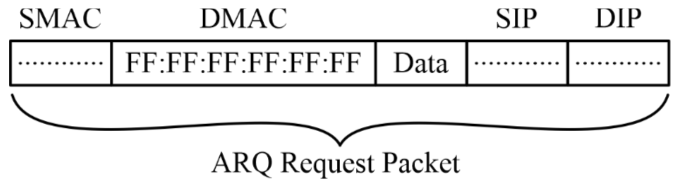
13. (c)

ARP request is broadcasting
ARP reply is unicasting

IP Address	MAC Address
–	?

Scan for Video solution

14. (b)



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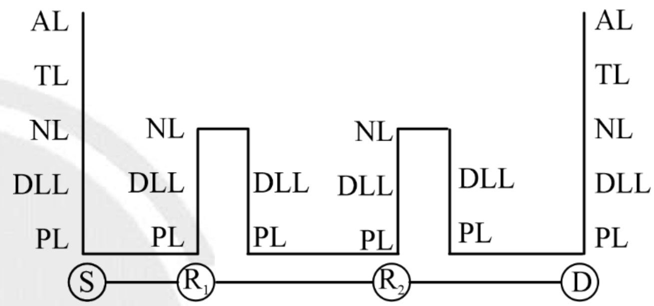
15. (c)



AL→2times	N→Intermediate Node or Router
TL→2times	AL→2 times
NL→4times	TL→2 times
DLL→6times	NL→ N + 2 times
PL→6times	DLL→2N + 2 times
	PL→2N + 2 times

Scan for Video solution

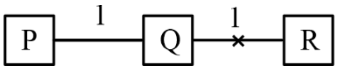
16. (a,c)

OSPF uses Dijkstra's algorithm not bellman ford algorithm. OSPF is an intra-domain routing protocol not inter domain. OSPF supports hierarchical network design (implemented using two layers).

Scan for Video solution

17. (0.5)

In distance vector routing every router will know the complete information of the network only with the help of neighbors.



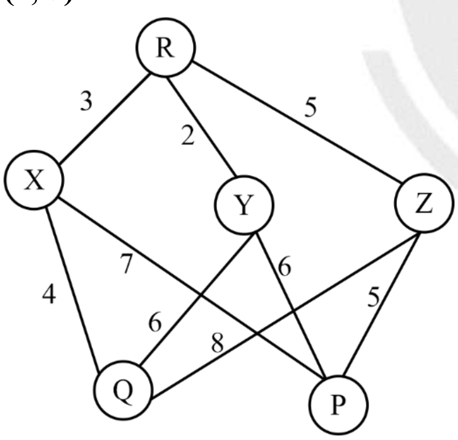
If link Q–R is broken. Then there are two possibility that among P and Q which will send information first to each other.

- If P will send the information first to Q then Q will update then its entry to reach R as 3 and network will enter in count to infinity problem.
- If Q sends first then P will come to know that link Q–R is broken and will update its entry to reach R as ∞. No count to infinity problem.

So, the probability that network will enter in count to infinity problem will be: $\frac{1}{2} = 0.5$

Scan for Video solution

18. (a, d)



R to Q

R to Q = min {R-X-Q, R-Y-Q, R-Z-Q}

3 + 4   2 + 6   5 + 8

7   8   13

Min = 7 (through x)

R to P

R to P = min {R-X-P, R-Y-P, R-Z-P}

3 + 7   2 + 6   5 +5

R to p = 8 (Through y)

Scan for Video solution

19. (c)

RIP is based on distance vector routing. RIP uses UDP as transport layer protocol. And OSPF is based on link state routing. OSPF does not uses either TCP or UDP.

Scan for Video solution

20. (a)

Open Shortest Path First (OSPF) and Routing Information Protocol (RIP) are both Interior Gateway Protocols, i.e., they are both employed within an autonomous system. RIP is based on distance vector routing. And OSPF is based on link state routing.

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21. (d)

- In link state protocols due to LSD (link state database) the computational overhead is higher than in distance vector protocols.
- Distance vector protocols with split horizon avoids persistent loop, but link state protocol there is no loop. It avoids persistent routing loops.
- The states of the links can be known to the router with the help of trigger update and this values will be known immediately. Whenever there is a change in topology trigger updates are used.

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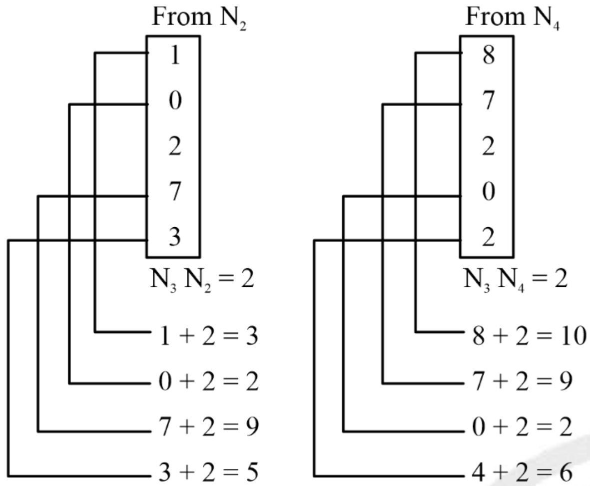
22. (a)

At N_3

N3 Receive the distance vector from N_2, N_4



New Routing table at N_3

Destination	Distance	NH
N_1	3	N_2
N_2	2	N_2
N_3	0	-
N_4	2	N_4
N_5	5	N_2

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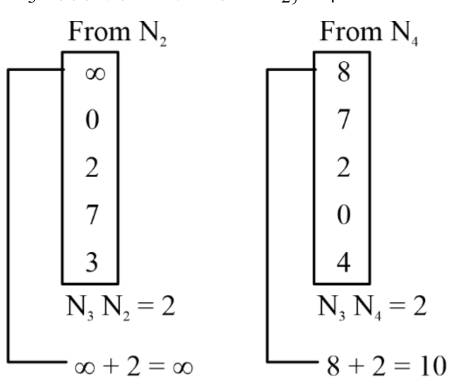
23. (c)

At N_3

N_3 receiver DV from N_2, N_4



Routing table at N_3

Destination	Distance	NH
N_1	10	N_4
N_2		
N_3	0	-
N_4		
N_5		

Scan for Video solution

24. (c)

Shortest distance from R_1 to R_2 = 5
R_1 to R_3 = 3
R_1 to R_4 = 12
R_1 to R_5 = 12
R_1 to R_6 = 6

Shortest distance from R_2 to R_3 = 2
R_2 to R_4 = 7
R_2 to R_5 = 8
R_2 to R_6 = 12

Shortest distance from R_4 to R_5 = 1
R_3 to R_5 = 9
R_3 to R_6 = 13

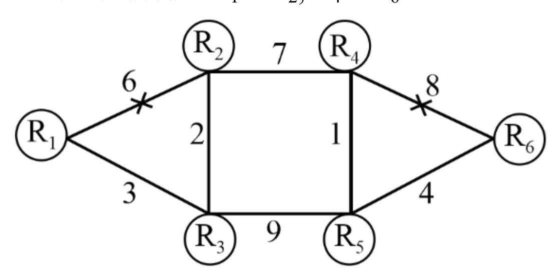
Shortest distance from R_4 to R_5 = 1
R_4 to R_6 = 5

Shortest distance from R_5 to R_6 = 4

The links used for the shortest path. From every Routery to every other router are:

$R_1 - R_3$, $R_2 - R_3$, $R_2 - R_4$, $R_4 - R_5$, $R_3 - R_5$, $R_5 - R_6$

Links Not used = $R_1 - R_2$, $R_4 - R_6$

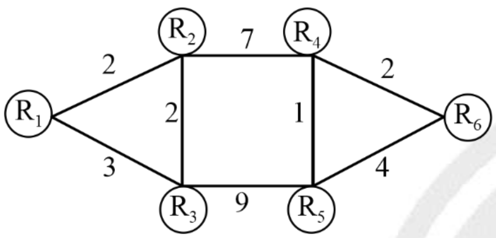


$\begin{pmatrix} R_1 \text{ to } R_2 \text{ not used} \\ R_4 \text{ to } R_6 \text{ not used} \end{pmatrix}$

Scan for Video solution

25. (b)

If we change the unused link value to 2 i.e. $R_1 \xrightarrow{2} R_2$, $R_4 \xrightarrow{2} R_6$. Then $R_5 - R_6$ will remains unused.



($R_5 - R_6$) Not used

Scan for Video solution

26. (d)

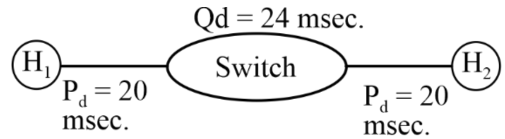
Count to infinity is a problem in distance vector routing. But there is no count to infinity problem in link state routing.

In link state and distance vector routing the shortest path algorithm is run at every node.

In link state LSDP (link state database packets) generated with latest into of the network. So, in link state a greater number of network message are required.

Scan for Video solution

27. (1575)



$B = 10^7$ bits sec.

Pkt size = 5000 byte

Data size = 10,000 byte

$$\text{Number of Pkt} = \frac{10000\text{B}}{5000\text{B}} = 2$$

$$T_d = \frac{\text{Pkt size}}{B} = \frac{50{,}000 \text{ Bits}}{10^7 \text{ Bits/sec.}}$$

$$= 500 \times 10^{-6} \text{ sec.} = 500 \text{ msec.}$$

X $\rightarrow$HOP $\Rightarrow$ N – Pkts

TT = X [T_d + P_d] + X–1(Q_d + Prd] + N–1 (T_d)

= 2 [500 + 20] + 1 (35) + 1 × 500

= 1000 + 40 + 35 + 500

= 1575 msec.

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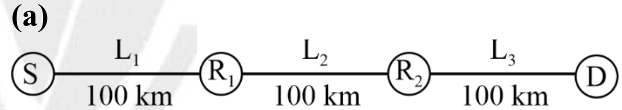
28. (a)



$v = 10^8$ m/sec. $\quad = 10^5$ km/sec

B = 1 Mbps

$$T_d \frac{\text{Pkt size}}{B} = \frac{1000 \text{ bits}}{10^6 \text{ bits/sec.}} = 1\text{m sec.}$$

X $\rightarrow$HOP $\Rightarrow$ N Pkts

$$\boxed{TT = X\left[T_d + P_d\right] + X–1\left(Q_d + P_{rd}\right) + N–1\left(T_d\right)}$$

[Queuing delay & processing delay are not given so assume 0]

= 3 [1+1] + 999 ×1

= 6 + 999 = 1005 msec.

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29. **(d)**

Header size = 100B

$B = 10^6$ Byte/sec

File size = 10^3 Byte

= 1000 Byte

1st Case:

PKT size = 1000 + 100 = 1100 Byte

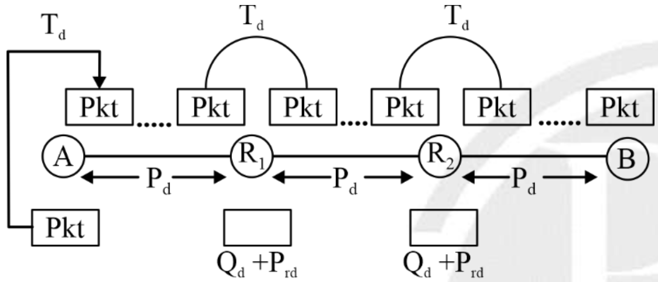
$B = 10^6$ Byte/sec.

$$T_d = \frac{\text{Pkt size}}{\text{Bandwidth}} = \frac{1100 \text{ Byte}}{10^6 \text{ Byte/sec.}}$$

$= 11 \times 10^{-4}$ sec.

$= 1.1 \times 10^{-3}$ sec.

$= 1.1 \times 10^{-3}$ sec.



X→ Number of Hops/ Links

Total time = $x(T_d) + X(P_d) + X - 1\ (Q_d + P_{rd})$

= 3× 1.1 msec. + 3 × 0 + 2 × 0 (Processing dealy, Queuing dealy & propagation delay are given negligiable)

= 3.3 msec. (T_1)

2nd Case:

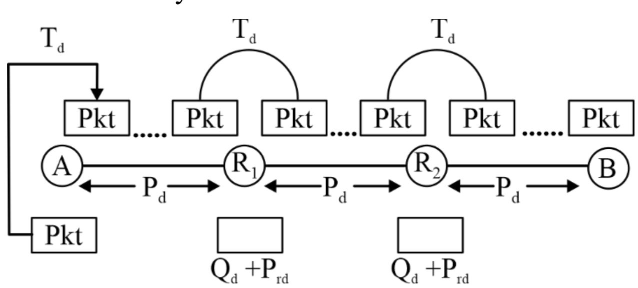
10 Parts

$$\text{Data size in each Pkt} = \frac{1000 \text{ Byte}}{10} = 100 \text{ Byte}$$

Pkt size = 100 + 100 = 200 Byte

$$T_d = \frac{\text{Pkt Byte}}{\text{Bandwidth}}$$

$$= \frac{200 \text{ Byte}}{10^6 \text{ Byte/sec.}} = 0.2 \text{ msec.}$$



For X →HOP ⇒ N Pkts

Total time = Time for 1st Pkt + Time for Remainin (N–1) Pkt.

$$\boxed{\text{Total time } = X[T_d + P_d] + X{-}1(Q_d + P_{rd}) + N{-}1(T_d)}$$

$= 3\times 0.2 + 9\times 0.2 = 0.6 + 1.8 = 2.4\ (T_2)$

3rd Case:

20 Part $\boxed{N = 20}$

$$\text{Data size in each Pkt} = \frac{1000\text{B}}{20} = 50 \text{ Byte}$$

Pkt size = 50 + 100 = 150 Byte

$$T_d = \frac{\text{Pkt size}}{\text{Bandwith}} = \frac{150\text{B}}{10^6 \text{ B/sec.}} = 0.15 \text{ msec.}$$

X → HoP & N– Pkt

TT $= X\ [T_d + P_d] + X{-}1\ (Q_d + P_{rd}] + N{-}1\ (T_d)$

= 3× 0.15 + 19 × 0.15

= 0.45 + 2.28

= 3.3 (T_3)

$T1 = T3 > T_2$

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30. **(c)**

Number of subnets are calculated are number of interfaces for a router.

Hence in the question one router is having 3 interfaces (firewall web server and router) rest both the routers are having 2 interfaces each.

So, total number of interfaces (subnets)

= 3 + 2 ×2 = 7

But among these three routers 2 routers are having a common interface.

So, total number of subnets = 7 –1 = 6

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❑❑❑

CHAPTER

5

TRANSPORT LAYER

TCP Header

1. [MCQ] [GATE-2023 : 1M]

Suppose you are asked to design a new reliable byte-stream transport protocol like TCP. This protocol, named myTCP, runs over a 100 Mbps network with Round Trip Time of 150 milliseconds and the maximum segment lifetime of 2 minutes. Which of the following is/are valid lengths of the Sequence Number field in the myTCP header?

(a) 30 bits (b) 32 bits

(c) 34 bits (d) 36 bits

2. [MCQ] [GATE-2015 : 1M]

Consider the following statements:

I: TCP connections are full duplex.

II: TCP has no option for selective acknowledgement.

III: TCP connections are message streams.

(a) Only I is correct

(b) Only I and III are correct

(c) Only II and III are correct

(d) All of I, II and III are correct

3. [MCQ] [GATE-2015 : 2M]

Assume that the bandwidth for a TCP connection is 1048560 bits/sec. Let α be the value of RTT in milliseconds (rounded off to the nearest integer) after which the TCP window scale option is needed. Let β be the maximum possible window size with window scale option. Then the values of α and β are

(a) 63 milliseconds 65535×2^{14}

(b) 63 milliseconds 65535×2^{16}

(c) 500 milliseconds 65535×2^{14}

(d) 500 milliseconds 65535×2^{16}

4. [MCQ] [GATE-2013: 1M]

The transport layer protocols used for real time multimedia, file transfer, DNS and email respectively are

(a) TCP, UDP, UDP and TCP

(b) UDP, TCP, TCP and UDP

(c) UDP, TCP, UDP and TCP

(d) TCP, UDP, TCP and UDP

UDP Header

5. [MCQ] [GATE-2017 : 1M]

Consider socket API on a Linux machine that supports connected UDP sockets. A connected UDP socket is a UDP socket on which connect function has already been called. Which of the following statements is/are CORRECT?

I. A connected UDP socket can be used to communicate with multiple peers simultaneously.

II. A process can successfully call connect function again for an already connected UDP socket.

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

6. [MCQ] [GATE-2018: 1M]

Match the following:

	Field		Length in bits
P.	UDP Header's Port Number	I.	48
Q.	Ethernet MAC Address	II.	8
R.	IPv6 Next Header	III.	32
S.	TCP Header's Sequence Number	IV.	16

(a) P-III, Q-IV, R-II, S-I

(b) P-II, Q-I, R-IV, S-III

(c) P-IV, Q-I, R-II, S-III

(d) P-IV, Q-I, R-III, S-II

State Transitions of TCP

7. [MSQ] [GATE-2021 : 2M]

A TCP server application is programmed to listen on port number P on host S. A TCP client is connected to the TCP server over the network.

Consider that while the TCP connection was active, the server machine S crashed and rebooted. Assume that the client does not use the TCP keepalive timer Which of the following behaviors is/are possible?

(a) If the client was waiting to receive a packet, it may wait indefinitely
(b) The TCP server application on S can listen on P after reboot
(c) If the client sends a packet after the server reboot, it will receive a RST segment
(d) If the client sends a packet after the server reboot, it will receive a FIN segment

8. [MCQ] [GATE-2017 : 1M]

Consider a TCP client and a TCP server running on two different machines. After completing data transfer, the TCP client calls close to terminate the connection and a FIN segment is sent to the TCP server. Server-side TCP responds by sending an ACK. Which is received by the client-side TCP. As per the TCP connection state diagram (RFC 793). In which state does the client-side TCP connection wait for the FIN from the server-side TCP?

(a) LAST-ACK (b) TIME-WAIT
(c) FIN-WAIT-1 (d) FIN-WAIT-2

9. [MCQ] [GATE-2015 : 1M]

Identify the correct order in which a server process must invoke the function calls accept, bind, listen, and recv according to UNIX socket API.

(a) listen, accept, bind, recv
(b) bind, listen, accept, recv
(c) bind, accept, listen, recv
(d) accept, listen, bind, recv

10. [MCQ] [GATE-2014 : 1M]

Which one of the following socket API functions converts an unconnected active TCP socket into a passive socket?

(a) connect (b) bind
(c) listen (d) accept

11. [MCQ] [GATE-2008: 2M]

A client process P needs to make a TCP connection to a server process S. Consider the following situation: the server process S executes a socket(), a bind() and a listen () system call in that order, following which it is preempted. Subsequently, the client

process P executes a socket () system call followed by connect() system call to connect to the server process S. The server process has not executed any accept () system call. Which one of the following events could take place?

(a) connect() system call returns successfully
(b) connect() system call blocks
(c) connect() system call returns an error
(d) connect() system call results in a core dump

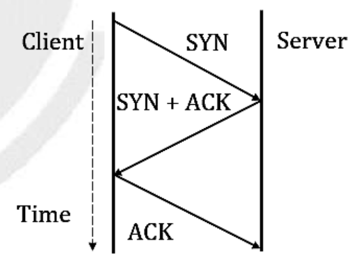
12. [MCQ] [GATE-2008: 2M]

Which of the following system calls results in the sending SYN packets?

(a) socket (b) bind
(c) listen (d) connect

13. [MCQ] [GATE-2008: 2M]

Which of the following statements are TRUE?



S1 : Loss of SYN + ACK from the server will not establish a connection

S2 : Loss of ACK from the client cannot establish the connection

S3 : The server moves LISTEN → SYN_RCVD → SYN_SENT → ESTABLISHED in the state machine on no packet loss

S4 : The server moves LISTEN → SYN_RCVD → ESTABLISHED in the state machine on no packet loss.

(a) S_2 and S_3 only (b) S_1 and S_4 only
(c) S_1 and S_3 only (d) S_2 and S_4 only

Flow Control Policies

14. [NAT] [GATE-2022 : 2M]

Consider the data transfer using TCP over a 1 Gbps link. Assuming that the maximum segment lifetime (MSL) is set to 60 seconds, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is ______.

15. [MCQ] [GATE-2021 : 1M]

Consider the three-way handshake mechanism followed during TCP connection establishment between hosts P and Q. Let X and Y be two random 32-bit starting sequence numbers chosen by P and Q respectively. Suppose P sends a TCP connection request message to Q with a TCP segment having SYN bit = 1, SEQ number = X, and ACK bit = 0. Suppose Q accepts the connection request. Which one of the following choices represents the information present in the TCP segment header that is sent by Q to P?

(a) SYN bit = 1, SEQ number = Y, ACK bit = 1, ACK number = X + 1, FIN bit = 0

(b) SYN bit = 1, SEQ number = Y, ACK bit = 1, ACK number = X , FIN bit = 0

(c) SYN bit = 0, SEQ number = X + 1, ACK bit = 0, ACK number = Y, FIN bit = 1

(d) SYN bit = 1, SEQ number = X + 1, ACK bit = 0, ACK number = Y, FIN bit = 0

16. [MCQ] [GATE-2020 : 2M]

Consider a TCP connection between a client and a server with the following specifications: the round trip time is 6 ms, the size of the receiver advertised window is 50 KB, slow-start threshold at the client is 32 KB, and the maximum segment size is 2 KB. The connection is established at time t = 0. Assume that there are no timeouts and errors during transmission. Then the size of the congestion window (in KB) at time t+60 ms after all acknowledgments are processed is ______.

17. [NAT] [GATE-2018 : 1M]

Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps (= 109 bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds, rounder to the closest integer) before this sequence number can be used again is _____

18. [MCQ] [GATE-2018 : 1M]

Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note the cwnd stands for the TCP congestion window and MSS denotes the Maximum Segment Size:

(i) The cwnd increases by 2 MSS on every successful acknowledgement.

(ii) The cwnd approximately doubles on every successful acknowledgement.

(iii) The cwnd increases by 1 MSS every round trip time.

(iv) The cwnd approximately doubles every round trip time.

Which one of the following is correct?

(a) Only (ii) and (iii) are true

(b) Only (i) and (iii) are true

(c) Only (iv) is true

(d) Only (i) and (iv) are true

19 [NAT] [GATE-2016 : 2M]

For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Tokens arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data.

The minimum time required to transmit the data is _____ seconds.

20. [MCQ] [GATE-2015 : 1M]

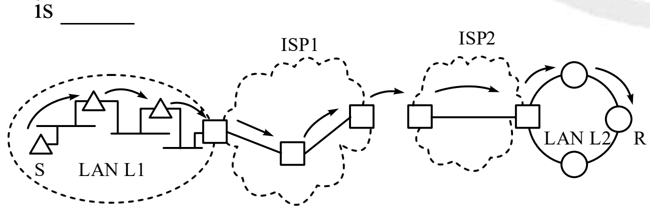
Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are FALSE with respect to the TCP connection?

1. If the sequence number of a segment is m, then the sequence number of the subsequent segment is always m + 1.
2. If the estimated round-trip time at any given point of time is t sec, the value of the retransmission timeout is always set to greater than or equal to t sec.
3. The size of the advertised window never changes during the course of the TCP connection.
4. The number of unacknowledged bytes at the sender is always less than or equal to the advertised window.

(a) 3 only (b) 1 and 3 only
(c) 1 and 4 only (d) 2 and 4 only

21. [NAT] [GATE-2014 : 1M]

In the diagram shown below, L1 is an Ethernet LAN and L2 is a Token-Ring LAN. An IP packet originates from sender S and traverses to R, as shown. The links within each ISP and across the two ISPs, are all point-to-point optical links. The initial value of the TTL field is 32. The maximum possible value of the TTL field when R receives the datagram is _____



22. [NAT] [GATE-2014: 2M]

Let the size of congestion window of a TCP connection be 32 KB when a timeout occurs. The round trip time of the connection is 100 msec and the maximum segment size used is 2 KB. The time taken (in msec) by the TCP connection to get back to 32 KB congestion window is_________.

23. [MCQ] [GATE-2012: 2M]

Consider an instance of TCP's Additive Increase Multiplicative Decrease (AIMD) algorithm where the windows size at the start of the slow start phase is 2 MSS and the threshold at the start of the first transmission is 8 MSS. Assume that a time-out occurs during the fifth transmission. Find the congestion window size at the end of the tenth transmission.

(a) 8 MSS (b) 14 MSS
(c) 7 MSS (d) 12 MSS

24. [MCQ] [GATE-2008: 1M]

A computer on a 10 Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 2 Mbps. It is initially filled to capacity with 16Megabits. What is the maximum duration for which the computer can transmit at the full 10 Mbps?

(a) 1.6 seconds (b) 2 seconds
(c) 5 seconds (d) 8 seconds

25. [MCQ] [GATE-2008: 2M]

Which of the following statements are TRUE?

S1: TCP handles both congestion and flow control.

S2: UDP handles congestion but not flow control.

S3: Fast retransmit deals with congestion but not flow control.

S4: Slow start mechanism deals with both congestion and flow control.

(a) S1, S2 and S3 only
(b) S1 and S3 only
(c) S3 and S4 only
(d) S1, S3, and S4 only

26. [MCQ] [GATE-2008: 2M]

In the slow-start phase of the TCP congestion algorithm, the size of the congestion window:

(a) does not increase
(b) increase linearly
(c) increases quadratically
(d) increases exponentially

❑❑❑

ANSWER KEY

1.	(b, c, d)	**2.**	(a)	**3.**	(c)	**4.**	(c)
5.	(b)	**6.**	(c)	**7.**	(a, b, c)	**8.**	(d)
9.	(b)	**10.**	(c)	**11.**	(c)	**12.**	(d)
13.	(b)	**14.**	(33 to 33)	**15.**	(a)	**16.**	(44 to 44)
17.	(34 to 35)	**18.**	(c)	**19.**	(1.1 to 1.1)	**20.**	(b)
21.	(26 to 26)	**22.**	(1100 to 1300)	**23.**	(c)	**24.**	(b)
25.	(b)	**26.**	(d)				

SOLUTIONS

1. (b, c, d)

LT = 2 min

LT = 120 sec.

B = 100×10^6 Bit/sec.

$\frac{100}{8} \times 10^6$ Byte/sec.

12.5×10^6 Byte/sec.

1 sec.______________ 12.5×10^6 Byte

1 sec.______________ 12.5×10^6 Seq. Number

120 sec.____________ $120 \times 12.5 \times 10^6$ Seq. Number

$= 1500 \times 10^6$ Seq. Number

$= 15 \times 10^8$ Seq. Number

Minimum sequence number required in the Life time $= 15 \times 10^8$ Seq. No

$= \lceil 10g_2\ 15 \times 10^8 \rceil = \lceil 30.32 \rceil = 31$ bits.

So, option (b, c, d) are correct

Scan for Video solution

2. (a)

1. TCP connection is full duplex.
2. TCP can use both selective and cumulative acknowledgement.
3. TCP is byte stream not message stream.

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3. (c)

In one RTT, the amount of data that can be sent in the maximum window size.

Here in the question 2 ⇒ RTT

$\alpha \times 1048560$ bps ≥ 65535 bytes

$$\alpha \geq \frac{65535 \times 8 \text{ bits}}{1048560 \text{ bits/sec.}}$$

$\alpha \geq 0.5$ sec.

$\alpha \geq 500$ m sec.

$\boxed{\alpha_{min} = 500 \text{ m sec.}}$

β = maximum possible window size

$\boxed{\beta = 65535 \times 2^{14}}$

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4. (c)

TCP	**VDP**
SMTP	DNS
FTP	SNMP
POP	DHCP
IMAP	TFTP
Telnet	All real time and multimedia protocol

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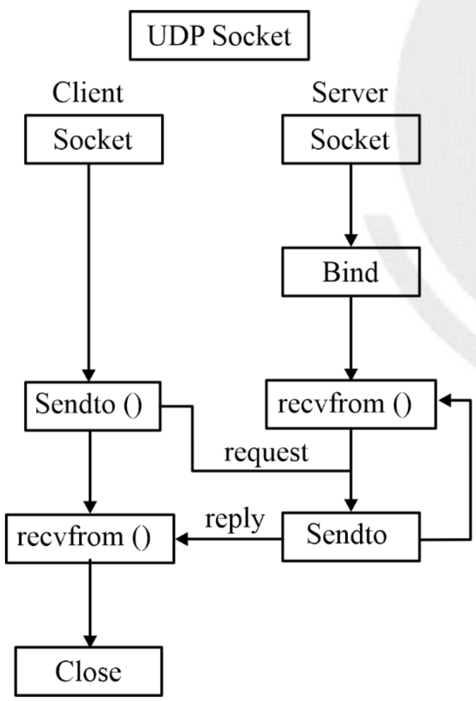
5. (b)



A process can successfully call connect function again for an already connected UDP.

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6. (c)

UDP Header's Port Number – 16 bits
Ethernet MAC Address – 48 bits
IPv6 Next Header – 8 bits
TCP Header's Sequence number – 32 bits

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7. (a, b, c)

If a client is not using the keepalive timer then it may wait indefinitely for a packet to receive. The TCP server application on S can listen on P after reboot because session still present. If the client sends a packet after the server reboot, it will receive a RST segment.

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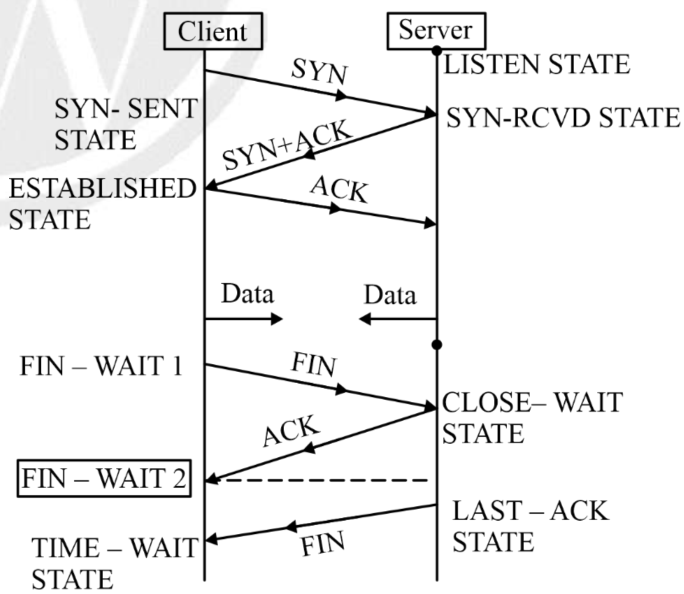
8. (d)

Consider the state transition diagram of TCP.



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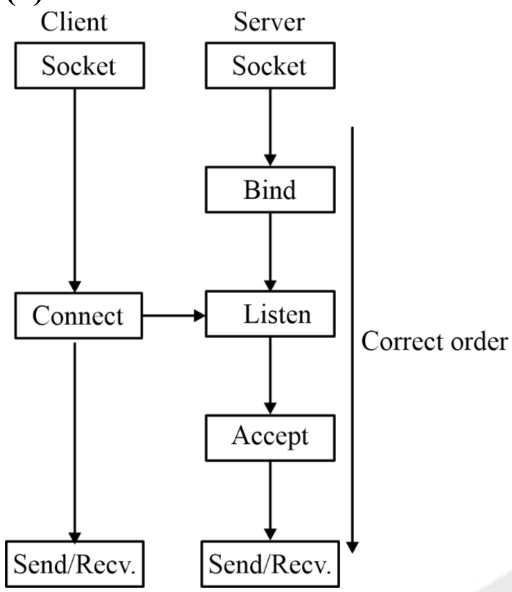
9. (b)



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10. (c)

Listen system call (socket API function) keeps the socket in a passive mode, where it waits for the client to approach the server to make a connection.

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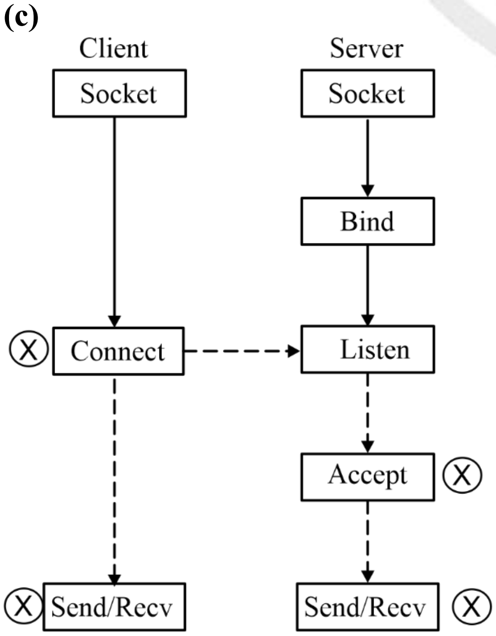
11. (c)



If the server Process has not executed any accept () system call that means client's process connect () system call returns an error.

There will not be any connection established between client and server.

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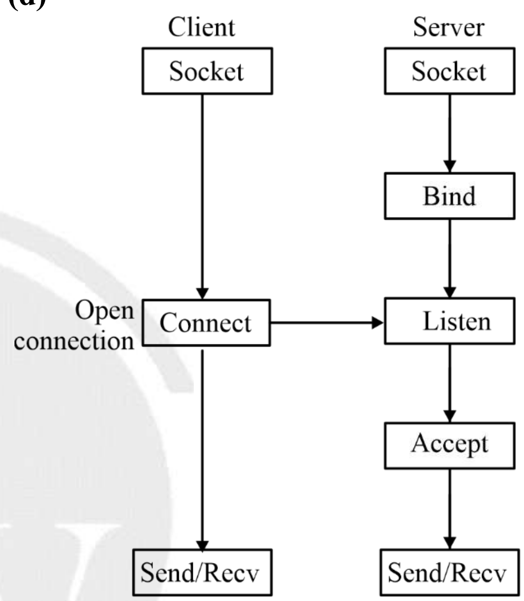
12. (d)



Connect system call used to send the SYN packets.

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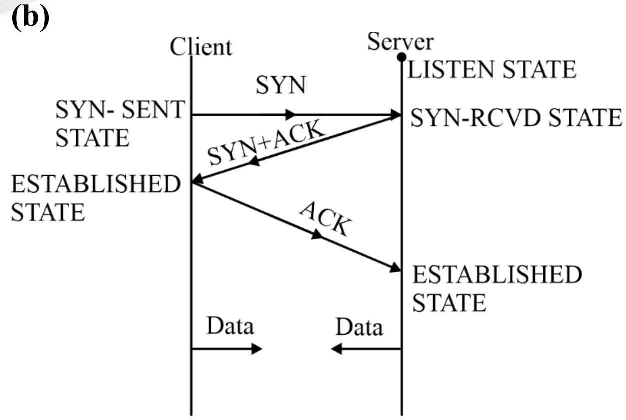
13. (b)



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14. (33)

$B = 109 \text{ bits/sec} = \frac{10^9}{8} \text{byte/sec.}$

LT = 60 sec.

Minimum sequence number required to Avoid with in the life time = $\lceil 10g_2 \, B \times LT \rceil$

$= \left\lceil 10g_2 \frac{10^9}{8} \times 60 \right\rceil = \lceil 10g_2 \, 7.5 \times 10^9 \rceil = \lceil 32.8 \rceil = 33$

bits

OR

1 sec.______________ $\frac{10^9}{8}$ Byte

1 sec.______________ $\frac{10^9}{8}$ seq No

60sec.______________ $\frac{60 \times 10^9}{8}$ seq No

$= 7.5 \times 10^9$ seq. No

Minimum sequence number required to Avoid wrap Around = 7.5×10^9

$= \lceil 10g_2 7.5 \times 10^9 \rceil = \lceil 32.8 \rceil = 33$ bit

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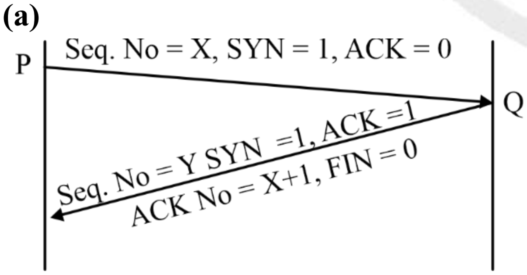
15. (a)



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16. (44)

RTT = 6msec.

W_R = 50 km

Slow start threshold = 32 KB

MSS = 2 KB

t = 0

(t + 60) m sec. , Wc = ?

$$\left.\begin{array}{l} \text{At } t = 0 : 2 \text{ KB} \\ \text{At } t + 6 : 4 \text{ KB} \\ \text{At } t + 12 : 8 \text{ KB} \\ \text{At } t + 18 : 16 \text{ KB} \\ \text{At } t + 24 : 32 \text{ KB} \end{array}\right\} \text{Slow start}$$

$$\left.\begin{array}{l} \text{At } t + 30 : 34 \text{ KB} \\ \text{At } t + 36 : 36 \text{ KB} \\ \text{At } t + 42 : 38 \text{ KB} \\ \text{At } t + 48 : 40 \text{ KB} \\ \text{At } t + 54 : 42 \text{ KB} \\ \text{At } t + 60 : 44 \text{ KB} \end{array}\right\} \text{Congestion Avoidance}$$

OR

44 KB

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17. (34)

$B = 10^9$ bits/sec.

$B = \frac{10^9}{8}$ bits/sec.

$\frac{10^9}{8}$ Bits______________1 sec.

$\frac{10^9}{8}$ Seq No.___________1 sec.

1 Seq. No_____________ $\frac{1}{\frac{10^9}{8}} \text{sec.} = \frac{8}{10^9} \text{sec.}$

2^{32} Seq. No__________ $= \frac{2^{32} \times 8 \text{sec.}}{10^9} = 34.35 = 34\text{sec.}$

OR

$$\text{WAT} = \frac{\text{Total Sec. No}}{(\text{Bandwidth})\text{Byte/sec.}}$$

$$= \frac{2^{32}}{8} = \frac{8\times 2^{32}}{10^9} = 34.35 \qquad = 34.\ \text{Sec.}$$

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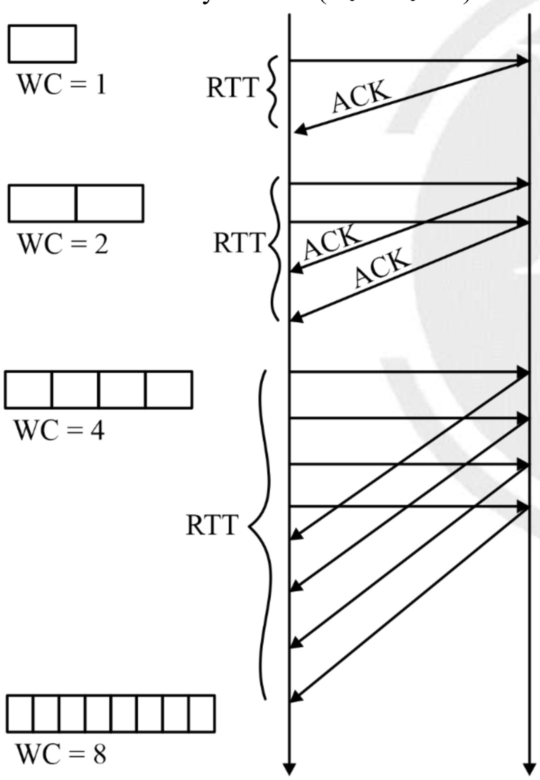
18. (c)

Slow Start Phase:

- After one RT congestion window will be double in slow start phase.
- If an Ack arrives then congestion window increased by one i.e. ($w_c = w_c + 1$)



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19. (1.1)

C = 1 mb, m = 20mb/sec, r =10mb/sec

$$t = \frac{c}{m-r} \quad \frac{1\text{ mb}}{20\text{ mb/sec.} - 10\text{ mb/sec.}}$$

$$= \frac{1\text{mb}}{10\text{ mb/sec}} = \frac{1}{10}\text{sec.} = 0.1\text{ sec.}$$

Time taken to empty the bucket = $\frac{1}{10}$sec.

= 0.1 sec.

In 0.1 sec. we can transfer = 0.1 × 20MB byte

= 2 MB byte

Total size = 12 MB

Remaining data = 12 MB – 2 MB = 10 MB

So, for transferring 10 MB we need only 1 Sec.

Total time to send 12 MB = 0.1 + 1 = 1.1 sec.

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20. (b)

Statement 1 and 3 are the false statements.

- If a sequence number of a segment in m, then the sequence number of the subsequent segment need not to be m +1 always because TCP can accept out of order segment but always sends in order acknowledgement.
- Advertised window size can be change during the course of TCP connection.

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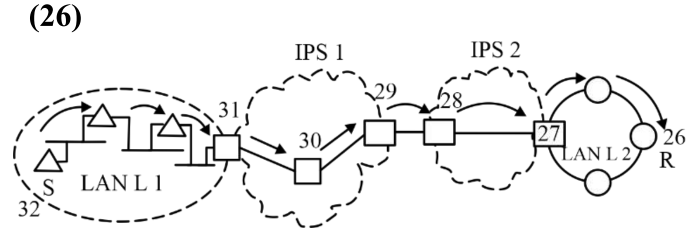
21. (26)



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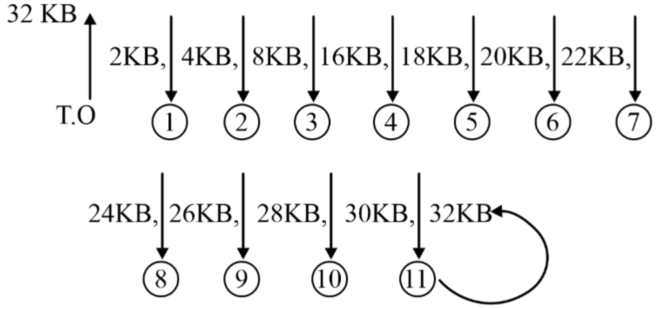
22. (1100-1300)

N + H = 16KB

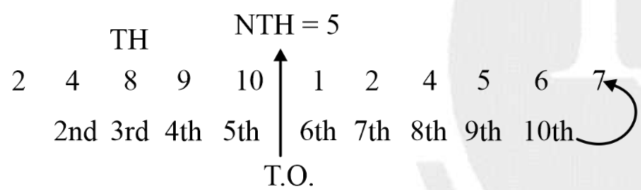


RTT= 100 × 11= 1100 msec.

Scan for Video solution

23. (c)

For this gate question marks given to all students some of students has picked the answer as 8 MSS & some of students has picked the answer as 7 MSS.



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24. (b)

Capacity of token bucket = 16m bits

Token added into the bucket at the rate of '*r*' token per sec. *r* = 2m bits/sec.

Maximum transmission rate 'm' = 10m bits/sec.

Maximum burst time 't' = ?

$$\boxed{t = \frac{C}{m - r}}$$

$$t = \frac{10 \text{ Mbits}}{10 \text{ Mbps} - 2\text{Mpbs}},\ t = \frac{16\text{M bits}}{8 \text{ M bits/sec}} = t = 2\text{sec.}$$

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25. (b)

- TCP uses the congestion policies to deal with congestion. And with the help of window scaling factor it deals with the flow control.
- Fast retransmit deals with congestion.

So, statements S1 and S3 are the only correct statement.

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26. (d)

In slow start-phase of the TCP congestion algorithm, the size of the congestion window increases exponentially.

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CHAPTER

6

APPLICATION LAYER

Hypertext Transfer Protocol

1. [MSQ] [GATE-2023 : 1M]

Suppose in a web browser, you click on the www.gate-2023.in URL. The browser cache is empty. The IP address for this URL is not cached in your local host, so a DNS lookup is triggered (by the local DNS server deployed on your local host) over the 3-tier DNS hierarchy in an iterative mode. No resource records are cached anywhere across all DNS servers.

Let RTT denote the round trip time between your local host and DNS servers in the DNS hierarchy. The round trip time between the local host and the web server hosting www.gate-2023.in is also equal to RTT. The HTML file associated with the URL is small enough to have negligible transmission time and negligible rendering time by your web browser, which references 10 equally small objects on the same web server.

Which of the following statements is/are CORRECT about the minimum elapsed time between clicking on the URL and your browser fully rendering it?

(a) 7 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

(b) 5 RTTs, in case of persistent HTTP with pipelining.

(c) 9 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

(d) 6 RTTs, in case of persistent HTTP with pipelining.

2. [NAT] [GATE-2020 : 1M]

Assume that you have made a request for a web page through your web browser to a web server. Initially the browser cache is empty. Further, the browser is configured to send HTTP requests in non-persistent mode. The web page contains text and five very small images. The minimum number of TCP connections required to display the web page completely in your browser is_____.

3. [MCQ] [GATE-2012 : 1M]

The protocol Data Unit (PDU) for the application layer in the Internet stack is

(a) Segment (b) Datagram

(c) Message (d) Frame

4. [MCQ] [GATE-2010 : 1M]

Which one of the following is not a client server application?

(a) Internet chat (b) Web browsing

(c) E-mail (d) Ping

5. [MCQ] [GATE-2008 : 1M]

What is the maximum size of data that the application layer can pass on to the TCP layer below?

(a) Any Size

(b) 2^{16} bytes- size of TCP header

(c) 2^{16} bytes

(d) 1500 bytes

File Transfer Protocol

6. [MCQ] [GATE-2015 : 1M]

In one of the pairs of protocols given below, both the protocols can use multiple TCP connections between the same client and the server. Which one is that?

(a) HTTP, FTP (b) HTTP, TELNET

(c) FTP, SMTP (d) HTTP, SMTP

7. [MCQ] [GATE-2019 : 1M]

Which of the following protocol pairs can be used to send and retrieve e-mails (in that order)?

(a) IMAP, SMTP (b) SMTP, MIME

(c) IMAP, POP3 (d) SMTP, POP3

8. [MCQ] [GATE-2016(set-1) : 1M]

Which of the following is/are example(s) of stateful application layer protocols?

(i) HTTP (ii) FTP

(iii) TCP (iv) POP3

(a) (i) and (ii) only

(b) (ii) and (iii) only

(c) (ii) and (iv) only

(d) (iv) only

9. [MCQ] [GATE-2012 : 1M]

Which of the following transport layer protocols is used to support electronic mail?

(a) SMTP (b) IP

(c) TCP (d) UDP

10. [MCQ] [GATE-2011 : 1M]

Consider different activities related to email.

m_1: Send an email from a mail client to a mail server

m_2: Download an email from mailbox server to a mail client

m_3: Checking email in a web browser

Which is the application level protocol used in each activity?

(a) m1 : HTTP m2 : SMTP m3 : POP

(b) m1 : SMTP m2 : FTP m3 : HTTP

(c) m1 : SMTP m2 : POP m3 : HTTP

(d) m1 : POP m2 : SMTP m3 : IMAP

Domain Name Space Protocol

11. [MCQ] [GATE-2016 : 1M]

Which one of the following protocols is NOT used to resolve one form of address to another one?

(a) DNS (b) ARP

(c) DHCP (d) RARP

12. [NAT] [GATE-2022 : 1M]

Consider the resolution of the domain name www.gate.org.in by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that iterative query mechanism is used in the resolution. The number of DNS query-response pairs involved in completely resolving the domain name is.

13. [MCQ] [GATE-2016 : 1M]

Identify the correct sequence in which the following packets are transmitted on the network by a host when a browser requests a webpage from a remote server, assuming that the host has just been restarted.

(a) HTTP GET request, DNS query, TCP SYN

(b) DNS query, HTTP GET request, TCP SYN

(c) DNS query, TCP SYN, HTTP GET request

(d) TCP SYN, DNS query, HTTP GET request

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ANSWER KEY

1.	(c, d)	**2.**	(6)	**3.**	(c)	**4.**	(d)
5.	(a)	**6.**	(a)	**7.**	(d)	**8.**	(c)
9.	(c)	**10.**	(c)	**11.**	(c)	**12.**	(4 to 4)
13.	(c)						

SOLUTIONS

1. (c, d)

9 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

6 RTTs, in case of persistent HTTP with pipelining.

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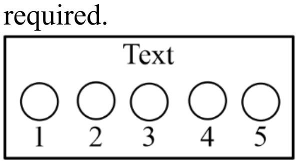
2. (6)

In Non-persistent mode one TCP connection is required for each request/response web page contain text and 5 same images so TCP connection are required.



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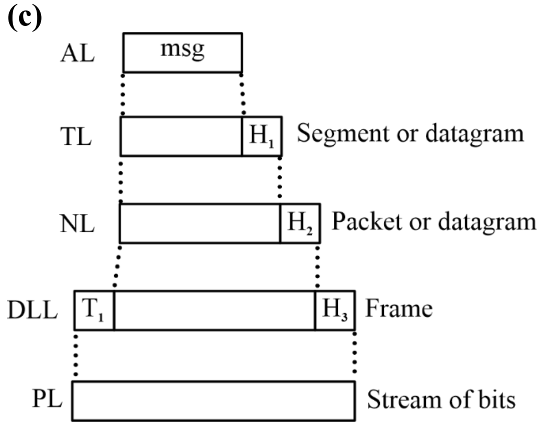
3. (c)



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4. (d)

Ping is not a client server application.

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5. (a)

Application Layer can pass any size of data to the transport layer.

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6. (a)

FTP uses two types of connection

(i) Control connection

(ii) Data Connection

In Non-persistent mode HTTP can use different TCP connections for different objects of webpage.

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7. (d)

Protocol used for sending email – SMTP

Protocols used to retrieve emails – POP3, IMAP4

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8. (c)

Steless	Stefull
DNS	POP
SMTP	IMAP
HTTP	FTP

Scan for Video solution

9. (c)

SMTP uses TCP as transport layer protocol. In transport layer TCP is used to support electronic mail.

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10. (c)

M1- SMTP (Push the mail from client to server)

M2- POP (is usded for downloading the mail from mail server)

M3 – HTTP (Messaging from receiving server to web browser is done through HTTP)

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11. (c)

DNS→ Domain Name to IP Add

ARP→ IP to MAC to Address

RARP→ MAC add to IP Address

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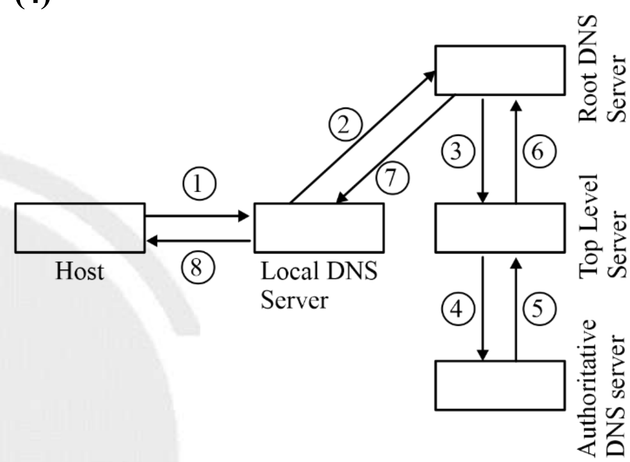
12. (4)



Number of query response pairs = 4

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13. (c)

Initially we send DNS request to find IP address then we send SYN segment for connection establishment and then we get the result through HTTP.

The correct sequence is:

DNS query, TCP SYN, HTTP GET request

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INDEX

Computer Organization and Architecture

Computer Organization and Architecture

Syllabus

Machine instructions and addressing modes. ALU, data - path and control unit. Instruction pipelining, pipeline hazards. Memory hierarchy: cache, main memory and secondary storage; I/O interface (interrupt and DMA mode).

Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5
2008	8	2	10	0	0
2009	1	0	2	0	0
2010	2	1	2	0	0
2011	3	2	2	0	0
2012	1	3	1	0	0
2013	0	2	2	0	0
2014 (P1)	1	0	2	2	0
2014 (P2)	1	0	2	2	0
2014 (P3)	1	0	2	2	0
2015 (P1)	1	0	2	0	0
2015 (P2)	2	0	2	0	0
2015 (P3)	0	0	2	1	0
2016 (P1)	0	1	2	0	2
2016 (P2)	3	0	2	4	0
2017 (P1)	3	0	2	6	0
2017 (P2)	2	0	4	0	0
2018	3	1	2	3	0
2019	1	0	0	3	0
2020	2	2	2	3	1
2021 (P1)	2	0	2	1	0
2021 (P2)	0	0	2	3	1
2022	0	0	2	4	1
2023	4	0	1	4	1

CHAPTER

1 MACHINE INSTRUCTION AND ADDRESSING MODES

Basics of COA

1. [MCQ] [GATE-2023 : 2M]

Consider the IEEE-754 single precision floating point numbers

P = 0xC1800000 and Q = 0x3F5C2EF4.

Which one of the following corresponds to the product of these numbers

(i.e., P × Q), represented in the IEEE-754 single precision format?

(a) 0x404C2EF4 (b) 0x405C2EF4

(c) 0xC15C2EF4 (d) 0xC14C2EF4

2. [MCQ] [GATE-2014 : 2M]

The value of a float type variable is represented using the single-precision 32-bit floating point format of IEEE-754 standard that uses 1 bit for sign, 8 bits for biased exponent and 23 bits for mantissa. A float type variable X is assigned the decimal value of −14.25. The representation of X in hexadecimal notation is

(a) C1640000H (b) 416C0000H

(c) 41640000H (d) C16C0000H

3. [MCQ] [GATE-2012 : 1M]

The amount of ROM needed to implement at 4 bit multiplier is

(a) 64 bits (b) 128 bits

(c) 1Kbits (d) 2 Kbits

Registers and its Types

4. [MCQ] [GATE-2010 : 2M]

The program below uses six temporary variables

a, b, c, d, e, f.

a = 1
b = 10
c = 20
d = a + b
e = c + d
f = c + e
b = c + e
e = b + f
d= 5 + e
return d + f

Assuming that all operations take their operands from registers, what is the minimum number of registers needed to execute this program without spilling?

(a) 2 (b) 3

(c) 4 (d) 6

5. [MCQ] [GATE-2008 : 1M]

A processor that has carry, overflow and sign flag bits as part of its program status word (PSW) performs addition of the following two 2's complement numbers 01001101 and 11101001. After the execution of this addition operation, the status of the carry, overflow and sign flags, respectively will be:

(a) 1, 1, 0 (b) 1, 0, 0

(c) 0, 1, 0 (d) 1, 0, 1

6. [MCQ] [GATE-2008 : 2M]

The use of multiple register windows with overlap causes a reduction in the number of memory accesses for:

1. function locals and parameters
2. register saves and restores
3. instruction fetches

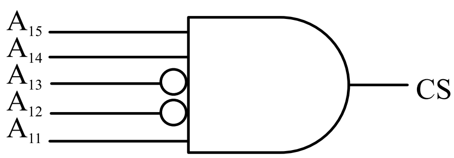
(a) 1 only (b) 2 only

(c) 3 only (d) 1, 2 and 3

Types of Buses

7. [MCQ] [GATE-2019 : 1M]

The chip select logic for a certain DRAM chip in a memory system design is shown below. Assume that the memory system has 16-address lines denoted by A15 to A0. What is the range of addresses (in hexadecimal) of the memory system that can get enabled by the chip select (CS) signal?



(a) C800 to CFFF (b) CA00 to CAFF
(c) C800 to C8FF (d) DA00 to DFFF

Instruction Set Architecture

8. [MCQ [GATE-2023 : 2M]

Consider the given C-code and its corresponding assembly code, with a few operands U1–U4 being unknown. Some useful information as well as the semantics of each unique assembly instruction is annotated as inline comments in the code. The memory is byte-addressable.

```
//C-code      ;assembly-code (; indicates comments)
              ;r1-r5 are 32-bit integer registers
              ;initialize r1=0, r2=10
              ;initialize r3, r4 with base address of a, b

int a[10],    L01: jeq r1, r2, end      ;if(r1==r2) goto end
b[10], i;     L02: lw r5, 0(r4)         ;r5 <- Memory[r4+0]
// int is     L03: shl r5, r5, U1       ;r5 <- r5 << U1
32-bit        L04: sw r5, 0(r3)         ;Memory[r3+0] <- r5
for (i=0;     L05: add r3, r3, U2       ;r3 <- r3+U2
i<10;i++)     L06: add r4, r4, U3
a[i] = b[i]   L07: add r1, r1, 1
* 8;          L08: jmp U4               ;goto U4
              L09: end
```

Whicho of the following options is a correct replacement for operands in the position (U1, U2, U3, U4) in the above assembly code?

(a) (8, 4, 1, L02) (b) (3, 4, 4, L01)
(c) (8, 1, 1, L02) (d) (3, 1, 1, L01)

9. [NAT] [GATE-2021 : 2M]

Consider the following instruction sequence where registers R1, R2 and R3 are general purpose and MEMORY [X] denotes the content at the memory location X.

Instruction	Semantics	Instruction Size (bytes)
MOV R1, (5000)	R1 ← MEMORY [5000]	4
MOV R2, (R3)	R2 ← MEMORY [R3]	4
ADD R2, R1	R2 ← R1 + R2	2
MOV (R3), R2	MEMORY [R3] ← R2	4
INC R3	R3 ← R3 + 1	2
DEC R1	R1 ← R1 – 1	2
BNZ 1004	Branch if not zero to the given absolute address	2
HALT	Stop	1

Assume that the content of the memory location 5000 is 10, and the content of the register R3 is 3000. The content of each of the memory locations from 3000 to 3010 is 50. The instruction sequence starts from the memory location 1000. All the numbers are in decimal format. Assume that the memory is byte addressable.

After the execution of the program, the content of memory location 3010 is _____.

10. [NAT] [GATE-2020 : 2M]

A processor has 64 registers and uses 16-bit instruction format. It has two types of instructions: I-type and R-type. Each I-type instruction contains an opcode, a register name, and a 4-bit immediate value. Each R-type instruction contains an opcode and two register names. If there are 8 distinct I-type opcodes, then the maximum number of distinct R-type opcodes is ______.

11. [NAT] [GATE-2018 : 2M]

A processor has 16 integer registers (R0, R1, …., R15) and 64 floating point registers (F0, F1, …., F63). It uses a 2-byte instruction format. There are four categories of instructions: Type-1, Type-2, Type-3 and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands (3Rs). Type-2 category consists of eight instructions, each with 2 floating point register operands (2Fs). Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand (1R+1F). Type-4 category consists of N instructions, each with a floating-point register operand (1F).

The maximum value of N is ________.

12. [MCQ] [GATE-2018 : 1M]

The following are some events that occur after a device controller issues an interrupt while process L is under execution.

(P) The Processor pushes the process status of L onto the control stack.

(Q) The processor finishes the execution of the current instruction.

(R) The processor executes the interrupt service routine.

(S) The processor pops the process status of L from the control stack.

(T) The processor loads the new PC value based on the interrupt.

Which one of the following is the correct order in which the events above occur?

(a) QPTRS (b) PTRSQ

(c) TRPQS (d) QTPRS

13. [NAT] [GATE-2016 : 2M]

Consider a Processor with 64 registers and an instruction set of size twelve. Each instruction has five distinct fields, namely, opcode, two source register identifiers, one destination register identifier, and a twelve-bit immediate value. Each instruction must be stored in memory in a byte-aligned fashion. If a program has 100 instructions, the amount of memory (in bytes) consumed by the program text is _____.

14. [NAT] [GATE-2016 : 1M]

A Processor has 40 distinct instructions and 24 general purpose registers. A 32-bit instruction word has an opcode, two register operands and an immediate operand. The number of bits available for the immediate operand field is _____.

15. [MCQ] [GATE-2015 : 1M]

For computers based on three-address instruction formats, each address field can be used to specify which of the following:

S_1: A memory operand

S_2: A processor register

S_3: An implied accumulator register

(a) Either S_1 or S_2 (b) Either S_2 or S_3

(c) Only S_2 and S_3 (d) All of S_1, S_2 and S_3

16. [MCQ] [GATE-2015 : 2M]

Consider a processor with byte-addressable memory. Assume that all registers, including Program Counter (PC) and Program Status Word (PSW), are of size 2 bytes. A stack in the main memory is implemented from memory location $(0100)_{16}$ and it grows upward. The stack pointer (SP) points to the top element of the stack. The current value of SP is $(016E)_{16}$. The CALL instruction is of two words, the first word is the op-code and the second word is the starting address of the subroutine (one word = 2 bytes). The CALL instruction is implemented as follows:

- Store the current value of PC in the stack.
- Store the value of PSW register in the stack.
- Load the starting address of the subroutine in PC.

The content of PC just before the fetch of a CALL instruction is $(5FA0)_{16}$. After execution of the CALL instruction, the value of the stack pointer is

(a) $(016A)_{16}$ (b) $(016C)_{16}$

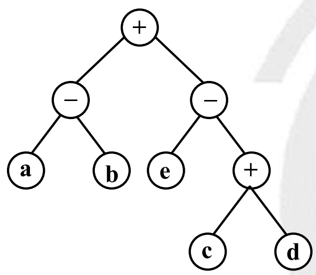
(c) $(0170)_{16}$ (d) $(0172)_{16}$

17. [NAT] [GATE-2014: 2M]

A machine has a 32-bit architecture, with 1-word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have an immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is ________.

18. [MCQ] [GATE-2011 : 2M]

Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables a, b, c, d and e are initially stored in memory. The binary operators used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register. If no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(a) 2 (b) 9

(c) 5 (d) 3

19. [MCQ] [GATE-2009 : 1M]

A CPU generally handles an interrupt by executing an interrupt service routine

(a) As soon as an interrupt is raised.

(b) By checking the interrupt register at the end of fetch cycle.

(c) By checking the interrupt register after finishing the execution of the current instruction.

(d) By checking the interrupt register at fixed time intervals.

20. [MCQ] [GATE-2008 : 2M]

Which of the following must be true for the RFE (Return from Exception) instruction on a general-purpose processor.

1. It must be a trap instruction
2. It must be a privileged instruction
3. An exception cannot be allowed to occur during execution of an RFE instruction.

(a) 1 only

(b) 2 only

(c) 1 and 2 only

(d) 1, 2 and 3 only

Addressing Modes

21. [MCQ] [GATE-2017 : 1M]

Consider the C struct defined below:

```
struct data {
int marks [100];
char grade;
int cnumber;
};
struct data student;
```

The base address of student is available in register R1. The field student, grade can be accessed efficiently using

(a) Post-increment addressing mode, (R1)+

(b) Pre-decrement addressing mode, - (R1)

(c) Register direct addressing mode, R1

(d) Index addressing mode, X(R1). Where X is an offset represented in 2’s complement 16-bit representation.

22. [MCQ] [GATE-2011 : 1M]

Consider a hypothetical processor with a instruction of type LW R1, 20(R2), which during execution reads a 32-bit word from memory and stores it in a 32-bit register R1. The effective address of the memory location is obtained by the addition of a constant 20 and the contents of register R2. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory?

(a) Immediate Addressing

(b) Register Addressing

(c) Register Indirect Scaled Addressing

(d) Base Indexed Addressing

23. [MCQ] [GATE-2008 : 2M]

Which of the following is/are true of the auto-increment addressing mode?

1. It is useful in creating self-relocating code.
2. If it is included in an instruction Set Architecture, then an additional ALU is required for effective address calculation
3. The amount of increment depends on the size of the data item accessed.

(a) 1 only (b) 2 only
(c) 3 only (d) 2 and 3 only

24. [MCQ [GATE-2008 : 1M]

Assume that EA = (X)+ is the effective address equal to the contents of location X, with X Incremented by one word length after the effective address is calculated; EA =-(X) is the effective address equal to the contents of location X, with X decremented by one word length before the effective address is calculated; EA = (X)- is the effective address equal to the contents of location X, with X decremented by one word length after the effective address is calculated. The format of the instruction is (opcode, source, destination), which means (destination ← source op destination). Using X as a stack pointer, which of the following instructions can pop the top two elements from the stack, perform the addition operation and push the result back to the stack.

(a) ADD (X)–,(X)
(b) ADD (X), (X)–
(c) ADD –(X), (X)+
(d) ADD –(X), (X)

❑❑❑

ANSWER KEY

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(b)
5.	(b)	**6.**	(a)	**7.**	(a)	**8.**	(b)
9.	(50 to 50)	**10.**	(14 to 14)	**11.**	(32 to 32)	**12.**	(a)
13.	(500 to 500)	**14.**	(16 to 16)	**15.**	(a)	**16.**	(d)
17.	(16383 to 16383)	**18.**	(d)	**19.**	(c)	**20.**	(d)
21.	(d)	**22.**	(d)	**23.**	(c)	**24.**	(a)

SOLUTIONS

1. (c)

P = O_x C1800000 Q = O_x 3F5C2EF4

1bit	8bit	23bit
S	E	M

Bias = $2^{8-1} - 1$ bias = 127

P = O_x C1800000

Sign 1bit	E(8bit)	Mantissa (23bit)
1	100 0001 1	000 0000 0000 0000 0000 0000

S = 1(–ve)

E = 10000011 = 131

BE or E = 131

M = 00000000

$(-1)^S$ 1.M × 2^e

$(-1)^1$ 1.00000000 × $2^{131-127}$

P = –(1.00000000) × 2^{+4}

BE = AE + bias

Or

E = e + bias

e = E – bias

E = 131

Bias = 127

Q = 3F5C2EF4

Sign 1bit	E(8bit)	Mantissa (23bit)
0	011 1111 0	101 1100 0010 1110 1111 0100

Sign = 0 (+ve) E = 126 bias = 127

E = 01111110 ⇒ E = 126 e = E – bias

BE or E = 126 126 – 127

M = 101 1100 0010 1110 1111 0100.

Bias = 127

E = e + bias

e = E – bias

(-1)s 1.M × 2^e

$(-1)^0$ 1.101 1100 0010 1110 1111 0100 ×$2^{126-127}$

Q = (1.101 1100 0010 1110 1111 0100) × 2^{-1}

Sign = –ve.

P × Q = exponent = (+4) + (–1) = +3.

Mantissa

= (1.0000) *(1.101 1100 0010 1110 1111 0100)

– (1.101 1100 0010 1110 1111 0100) × 2^{+3}

Sign = 1(–ve)

e = +3

bias = 127

E = e + bias

= 3 + 127

E = 130

E = 10000010

s(bit) E(8bit) Mantissa (23 bit)

s(bit)	E(8bit)	Mantissa (23 bit)
1	100 0001 0	101 1100 0010 1110 1111 0100

↓ C 1 5 C 2 E F 4

(C15C2EF4 H)

2nd Approach.

Alternate Approach. P = C1800000

1bit	8bit	23bit
S	E	M

s(bit)	E(8bit)	Mantissa (23 bit)
1	100 0001 1	000 0000 0000 0000 0000 0000

S = 1(–ve) $(-1)^S$ 1.M × 2^e

E = 10000011 $(-1)^1$ 1.000000 × 2

BE or E = 131 – 1.000000 × 2^{+4}

M = 00000000 – 10000.00

Bias = 2^{8-1}–1 P = – 16

Bias = 127

E = e + bias

e = E – bias

Q = 3F5C2EF4

Sign	E(8bit)	Mantissa (23 bit)
0	011 1111 0	101 1100 0010 1110 1111 0100

1bit

S = 0 (+ve)

E = 01111100

BE or E = 126

M = 10111000010 … Q = 1.101 1100 × 2^{-1}

Bias = 129 = 0.1101 1100

E = e + bias Q = 0.8593

e = E – bias

$(-1)^S$ 1.M × 2^e

$(-1)^0$ 1.101 1100 0010 1110 × $2^{126-127}$

P * Q = – 16 × .8593

= – (13.75)

P * Q = – 13.75

– 1101.11

⇒ – 1.10111 × 2^{+3}

$\boxed{e = +3}$ $\boxed{\text{bias} = 127}$

E or BE = e + bias ⇒ 3 + 127

= 130

E = 130

M = 10111 → 10000010

$\boxed{S = 1}$ –ve

S(1bit)	E(8bit)	Mantissa (23 bit)
1	100 0001 0	101 11 - - - - - - - - - - - - - -

12 C 1 5 C 12

↓

(C15C2EF4)

Scan for Video solution

2. **(a)**

– 14.25

⇒ 1110.01

⇒ –1110.01

⇒ – 1.11001×2^{+3}

sign = 1 [–ve]

Mantissa [m] = 11001

[AE]e = +3

[BE] $\boxed{E = e + \text{bias}}$

$\boxed{BE = AE + \text{bias}}$

E = 3+bias = 3 +127

$\boxed{E = 130}$

$\boxed{BE = 130}$

IEE E 754

Single Precision

← 32bit →

S	E	Mantissa
1bit	8bit	23bit

bias = 2^{8-1} –1 = 127

$\boxed{\text{bias} = 127}$

S = 1

E = 130 ⇒ 10000010

M : 11001

S(1 bit)	E(8 bit)	Mantissa(23 bit)
1	10000010	11001000000000000000000

↓ C ↓ 1 ↓ 6 ↓ 4 ↓ 0 ↓ 0 ↓ 0 ↓ 0

[C1640000H]

Scan for Video solution

3. (d)

When we multiply two 4 bit number then each result is 8 bit

4 bit 4 bit

↓ ↓

Combination 2^4 2^4

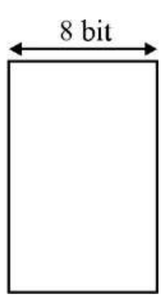
Total ROM size = $2^4 \times 2^4 \times 8$ bit

$\Rightarrow 2^8 \times 8$ bit

$\Rightarrow 2^8 \times 2^3$ bit $\Rightarrow 2^{11}$ bit

$\Rightarrow 2 \times 2^{10}$ bit

$= \boxed{\text{2k bits}}$



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4. (b)

$R_1 = 1(a)$
$R_2 = 10(b)$
$R_3 = 20(c)$

I $R_1 \leftarrow R_1 + R_2$	$[R_1[d] = a + b]$
II $R_1 \leftarrow R_3 + R_1$	$[R_1[e] = c + d]$
III $R_2 \leftarrow R_3 + R_1$	$[R_2[f] = c + e]$
IV $R_2 \leftarrow R_3 + R_1$	$[R_2[b] = c + e]$
V $R_1 \leftarrow R_2 + R_2$	$[R_1[e] = b + f]$
VI $R_3 \leftarrow R_1 + 5$	$R_3[d] = 5 + e$
VII return $R_2 + R_3$	

Minimum 3 register required

OR

a = 1, b = 10, c = 20

$R_1 \leftarrow R_1 + R_2$	I. d = a + b
$R_1 \leftarrow R_3 + R_1$	II. e = c + d
$R_2 \leftarrow R_3 + R_1$	III. f = c + e
$R_3 \leftarrow R_3 + R_1$	IV b = c + e
$R_1 \leftarrow R_2 + R_3$	V e = b + f
$R_3 \leftarrow R_1 + 5$	VI d = 5 + e
return $R_2 + R_3$	VIII return d + f

Scan for Video solution

5. (b)

C_{in}

C_{out} C_{in}

$$\begin{array}{r} 0\,1\,0\,0\,1\,1\,0\,1 \\ +\,1\,1\,1\,0\,1\,0\,0\,1 \\ \hline 0\,0\,1\,1\,0\,1\,1\,0 \end{array}$$

Overflow

C in + cout = 1

1 + 1 = 0

Carry = 1, Sign = 0

Overflow flag = 0

Scan for Video solution

6. (a)

1. Function, Local parameter, memory Access ↓

Scan for Video solution

7. (a)

A_{15}, A_{14}, A_{11} is enabled (1), A_{13} and A_{12} = 0

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_7	A_8	A_6	A_5	A_4	A_3	A_2	A_1	A_0
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
C				8				0				0			
1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1
C				F				F				F			

C800 to CFFF

Scan for Video solution

8. (b)

$U_1 = 3$ multiply by 8.

$U_2 = 4$ 32 bit given

$U_3 = 4$ memory is byte (8bit) Addressable

$U_4 = L01$

$r_1 = 0$ $r_2 = 10$

$r_3 \in [a]$

$r_4 \in [b]$

$r_5 \leftarrow m\,[b_0 + 0]$

$U_1 = 3$

$M\,[a_0 + 0]$

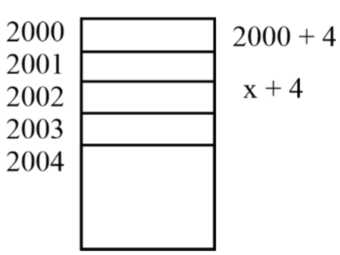
$U_2 = 4$

$U_3 = 4$

L03 : Multiple by 8 then left shift by 3 bit

U_3 memory is Byte addressable but size is 32 bit

32 bit ≈ 4 Byte



$U_2 = 4$

$\underline{U_3 = 4}$

L08 Jmp $U_4 \Rightarrow U_4$ = L01

Because at L01 condition checking.

Scan for Video solution

9. (50 to 50)

	Instruction	Semantics	Instruction Size (bytes)	
1000 –1003	MOV R1, (5000) I_1	R1 ← MEMORY [5000]	4	R1 = 10
1004 –1007	MOV R2, (R3) I_2	R2 ← MEMORY [R3] R2 ←m[3000]	4	R2 = 50
1008 –1009	ADD R2, R1 I_3	R2 ← R1 + R2, R2 ←10 + 50	2	R2 = 10 + 50 = R2 = 60
1010 –1013	MOV (R3), R2 I_4	MEMORY [R3] ← R2 m[3000] ← R2	4	M [3000] = 60
1014 –1015	INC R3 I_5	R3 ← R3 + 1, R3 = 3000+1	2	R3 = 3000 + 1 ⇒ R3 = 3001
1016 –1017	DEC R1 I_6	R1 ← R1 – 1, R1 = 10–1	2	R1 = 10 – 1 ⇒ R1 = 9
1018 –1019	BNZ 1004 I_7	Branch if not zero to the given absolute address	2	2
1020	HALT I_8	Stop	1	Go to 1004 (I_2)

M [5000] = 10
R3 = 3000

3000	~~50~~ 60
3001	~~50~~ 59
3002	~~50~~ 58
3003	~~50~~ 57
3004	~~50~~ 56
3005	~~50~~ 55
3006	~~50~~ 54
3007	~~50~~ 53
3008	~~50~~ 52
3009	~~50~~ 51
3010	50

$R_2 \leftarrow m\,[3001] = \boxed{R_2 = 50}$

$R_2 \leftarrow 9 + 50 = \boxed{R_2 = 59}$

$M\,[R_3] \leftarrow R_2 \rightarrow m\,[3001] = 59$

$R_3 \leftarrow 3001 + 1 \Rightarrow \boxed{R3 = 3002}$

$R_1 \leftarrow 9 - 1 \Rightarrow \boxed{R_1 = 8}$

2nd Part

$R_2 \leftarrow m\,[3003]\ R_2 = 50$

$R_2 \leftarrow R1 + R_2 \Rightarrow 7 + 50 \Rightarrow \boxed{R_2 = 57}$

M [3003] = 57

$R_3 = 3003 + 1 \Rightarrow \boxed{R_3 = 3004}$

Simiarly execute

$R_1 = 7 \Rightarrow \boxed{R_1 = 6}$

In the last $\boxed{R_1 = 0}$ so, m [3010] will not change [update]

Scan for Video solution

10. (14 to 14)

I type — 16 bit

OPCODE	Register	Immediate
6 bit	6 bit	4 bit

64 register $\Rightarrow$ $\boxed{\text{Register AF=6 bit}}$

R type — 16 bit

OPCODE	Register	Register
4 bit	6 bit	6 bit

Total number of operation in R type = $2^4 = 16$ operatin

Assume R type instruciton = x

Number of free opcode after allocating R type = $(16 - x)$

R type — 16 bit

OPCODE	Register 1	Register 2
4 bit	6 bit	6 bit

I type — 16 bit

OPCODE	Register	Immediate
6 bit	6 bit	4 bit

Total number of operations in I type = free opcode $\times\ 2^{\text{increment bit in opcode}}$

$= (16 - x) \times 2^{6-4} \Rightarrow (16 - x) \times 2^2$

I type (Given) = 8

$8 = (16 - x) \times 4$

$2 = 16 - x$

$x = 16 - 2$

$\boxed{x = 14}$

Scan for Video solution

11. (32 to 32)

16 Integer Register (R_0, R_1 R_2….. R_{15})

$\boxed{IR = 4\text{ bit}}$

64 floating point Register (F_0, F_1 ….. F_{63})

$\boxed{FR = 6\text{ bit}}$

TYPE-1: $\boxed{\text{OP CODE}}\boxed{IR}\boxed{IR}\boxed{IR}$

4 Instruction

Given:

TYPE-2: | OP CODE | FR | FR |

8 Instruction

64 floating point Register (F_0, F_1, F_2 ……F_{64})

$\boxed{\text{FR}= 6 \text{ bit}}$

Instruction size = 16 bit (2 byte)

TYPE: 3 | OP CODE | IR | FR |

14 Instructions

TYPE: 4 | OP CODE | FR |

N Instructions

TYPE:- 1

← 16 bit →

OP Code	IR	IR	IR
4 bit	4 bit	4 bit	4 bit

N bit can perform 2^n operation

Total number of operation in type 1 = 2^4 = 16 operation.

Given = 4 Instruction

Total number of free after allocating type

= 16 – 4 = 12

TYPE -2

← 16 bit →

OP Code	FR	FR
4 bit	6 bit	6 bit

Total number of operation in type 2: Free opcode $\times 2^{\text{Increment bit in opcode}}$

Total number of operations = 12 × 2^{4-4} =12×2^0 = 12 operation

Type 2 (given) = 8 Instruction (operation)

Total number of free opcode after Allocating type 2 = 12 – 8 = 4 free code

TYPE -3

← 16 bit →

OP Code	IR	FR
6 bit	4 bit	4 bit

Total number of operations in type 3 = free opcode × 2 Increment bit in opcode

$\Rightarrow 4 \times 2^{6-4} = 4 \times 2^2$

= 16 operation/ Instruction

Type 3 (given) = 14 Instruction

Total number of free opcodes after allocating = 16 – 14

Type 3 instruction = 2 Free opcodes

TYPE -4

← 16 bit →

OP Code	FR
10 bit	6 bit

Total number of operations in type 4 = Free opcode × $2^{\text{Increment bit in opcode.}}$

(N) $\Rightarrow 2 \times 2^{10-6}$

$\Rightarrow 2 \times 2^4$

$\Rightarrow 32$

$\boxed{N= 32}$

Scan for Video solution

12. (a)

When Interrupt occur, after completion of current Instruction.

Interrupt will be serviced, It push the program current [PC] value into stack & control transfer to ISR.

Q: Process finish the current instruction execution

P: PUSH the PC value into stack

T: Interrupt → PC

R: Service the Interrupt

S: Pop the PC value

Scan for Video solution

13. (500 to 500)

64 Register, Instruction set size = 12

5 Fields = Opcode, Source Reg1, S Reg2, D Reg2, 12 bit Immediate field

64 Register ⇒ $\boxed{\text{Reg. AF} = 6\text{bit}}$

Inst. Set = 12 ⇒ Op code = 4 bit

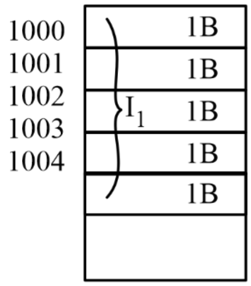
Opcode	Reg1	Reg2	Reg3	Immediate Field
9 bits	6 bits	6 bits	6 bits	12 bits

Instruction size = 4 + 6 + 6 + 6 +12

= 34 bits ≈ 5 Byte

Program having = 100 Inst.

Program size = 100 × 5B = 500 byte.



Byte aligned memory

Scan for Video solution

14. (16 to 16)

40 Distinct Instruction/operation ⇒ OP code = $\lceil \log_2 40 \rceil$ = 6 bit

$\boxed{\text{OP code} = 6 \text{ bit}}$

24 Register ⇒ $\boxed{\text{Reg.AF} = 5\text{bit}}$

32 bits Instruction

← 32 bit →

OP Code	Reg.1	Reg.2	Immediate field
6 bit	5 bit	5 bit	5 bit

Immediate field = 32 – (5 + 5 + 6)

= 32 – 16

$\boxed{\text{Immediate field} = 16\text{bit}}$

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15. (a)

OP Code	AF1	AF2	AF3

Memory operand

Or

Register operand

But Accumulator is a special purpose Register

Scan for Video solution

16. (d)

Given

Current SP value is = $(016E)_{16}$

PC = 2 Byte, PSW = 2 byte

(1) Store the current value of PC into the stack.

Here PC is 2 byte so stack pointer to increased by 2 bytes

$(016E)_2 + 2 = [0170]_{16}$

01	0110	1110	016E + 1
01	0110	1111	016F
01	0111	0000	016F +1
			0170_{16}

(2) Store the value of PSW in the stack

Here PSW is also 2 byte long. So SP increased by 2 byte

$(0170)_{16} + 2 = (0172)_{16}$

$SP = (0172)_{16}$

(3) Load the starting address of the subroutine in the PC.

PC ← ISR

$\boxed{SP = (0172)_{16}}$

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17. (16383 to 16383)

Instruction size = 1 word = 32 bit

45 Operation $\Rightarrow$ $\boxed{\text{OP code = 6 bit}}$

64 Register $\Rightarrow$ $\boxed{\text{Register = 6 bit}}$

$\longleftarrow$ 32 bit $\longrightarrow$

OP Code	Reg.1	Reg.2	Immediate field
6 bit	6 bit	6 bit	

Immediate field $= 32 - (6 + 6 + 6)$

$= 32 - 18 = 14$ bit

n bit unsigned Range $= 0$ to $2^n - 1$

Immediate field Range $= 0$ to $2^{14} - 1 = 16,383$

Scan for Video solution

18. (d)

LOAD R_1 c;

LOAD R_2 d;

ADD R_1 R_1 R_2; $R_1 \leftarrow R_1 + R_2 \Rightarrow R_1 = c + d$

LOAD R_2 e;

SUB R_2, R_2, R_1; $R_2 \leftarrow R_2 - R_1 \Rightarrow R_2 = e - (c + d)$

LOAD R_1 a;

LOAD R_3 b;

SUB R_1, R_1, R_3; $R_1 \leftarrow R_1 - R_3$; $R_1 = (a - b)$

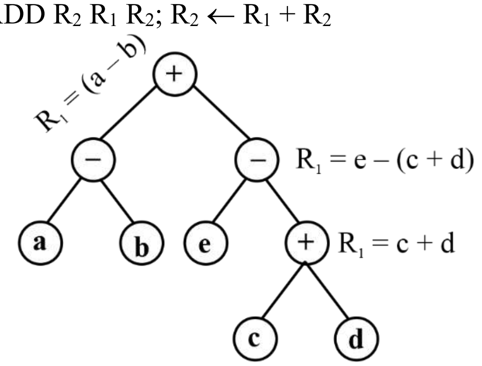
ADD R_3, R_1, R_2; $R_3 = (a - b) + [e - (c + d)]$

OR

ADD R_1 R_1 R_2; $R_1 \leftarrow R_1 + R_2$

OR

ADD R_2 R_1 R_2; $R_2 \leftarrow R_1 + R_2$



Minimum 3 Registers required

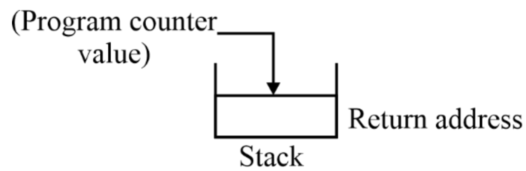
Scan for Video solution

19. (c)

CPU will check the interrupt after finishing of current instant execution.

If interupts is present

Push PC value



Scan for Video solution

20. (d)

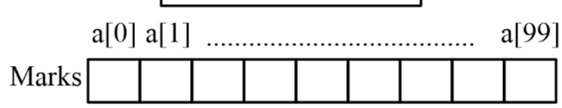
The RFE (Return from Exception) instruction must be a trap instruction, a privilege instruction and an exception cannot be allowed to occur during execution of an RFE instruction.

Scan for Video solution

21. (d)

	Struct Data
2×100 ←	int marks [100]
1 ←	Char Grade
2 ←	int Number
203	Short date Student

Base address is available in Register R1. Want to access grade student field



$X(R_1) \Rightarrow M[X + R_1]$

Scan for Video solution

22. (d)

LW R_1, $20(R_2)$

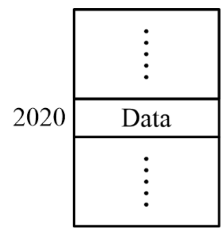
$R_1 \leftarrow M[20 + R_2]$

Read 32 bit word from memory and store into 32 bit Register R.

$EA = M[20 + R_2]$

20: Index

$R_2 = 2000$
$EA = M[20 + 2000]$
$= M[2020]$



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23. (c)

In auto-increment addressing mode, the amount of increment depends on the size of the data item accessed.

Scan for Video solution

24. (a)

OPCODE source destination.

$$\text{Destination} \leftarrow \underset{\text{Source}}{(S_1)} \ \text{OPERATION} \ \underset{\text{destination}}{(S_2)}$$

1. (X)+: <u>Post increment</u>

   First Fetch the operand (Content) from the location X then <u>Increment</u>.

2. (X)– <u>Post Decrement</u>

   First Fetch the operand (Content) from the location X. then Decrement in X.

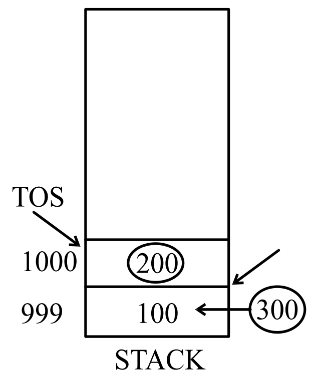
3. +(X) <u>Pre Increment</u>

   First Increment in the X, then Updated Location [After Increment X content (Operand) is Fetch

4. –(X): <u>Pre decrement</u>

   First Decrement in the X, then Updated Location (After Decrement) X content (operand) is Fetch.

Assume start from 1000



(a) ADD (X)–, X

(X) – : Fetch then Decrement operand

X = 999 M[1000] then X–

X = 999 ⇒ M [999] = 100

200 + 100 = 300.

ADD

POP (Top Of Stack)

POP (Top Of Stack)

ALU operation (ADD) push into POP.

200 + 100

STACK: LIFO

(b) ADD X, (X)–

M [1000] = 200

M [1000] =200

(c) ADD – (X), (X) +

X= First Decrement X = 999

M [999] = 100

X + Post Increment M [999] = 100

(d) ADD –(X), X

–X: M [999] = 100

M [999] = 100

Scan for Video solution

❑❑❑

CHAPTER

2 ALU AND CONTROL UNIT

Micro-Operation and Micro Program

1. [MCQ] [GATE-2013 : 2M]

Consider the following sequence of micro - operations.

MBR $\leftarrow$ PC

MAR $\leftarrow$ X

PC $\leftarrow$ Y

Memory $\leftarrow$ MBR

Which one of the following is a possible operation performed by this sequence

(a) Instruction fetch
(b) Operand fetch
(c) Conditional branch
(d) Initiation of interrupt service

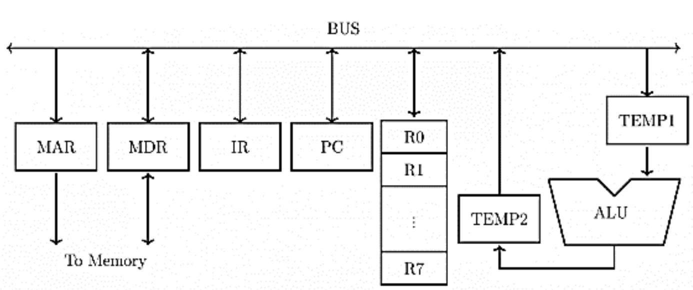
ALU Data Path

2. [NAT] [GATE-2020 : 1M]

A multiplexer is placed between a group of 32 registers and an accumulator to regulate data movement such that at any given point in time the content of only one register will move to the accumulator, the minimum number of select lines needed for the multiplexer is _____

3. [MCQ] [GATE-2020 : 1M]

Consider the following data path diagram



Consider an instruction: R0 $\leftarrow$ R1 + R2. The following steps are used to execute it over the given data path. Assume that PC is incremented appropriately. The subscripts r and w indicate read and write operations, respectively.

1. $R2_r$ $TEMP1_R$, ALU_{add}, $TEMP2_W$
2. $R1_r$ $TEMP1_W$,
3. PC_r, MAR_W, MEM_r
4. $TEMP2_R$,$R0_W$
5. MDR_r, IR_W

Which one of the following is the correct order of execution of the above steps?

(a) 3, 5, 1, 2, 4
(b) 2, 1, 4, 5, 3
(c) 3, 5, 2, 1, 4
(d) 1, 2, 4, 3, 5

Microprogrammed Control

4. [MCQ] [GATE-2008 : 2M]

Consider a CPU where all the instructions require 7 clock cycles to complete execution. There are 140 instructions in the instruction set. It is found that 125 control signals are needed to be generated by the control unit. While designing the horizontal micro-programmed control unit, single address field format is used for branch control logic. What is the minimum size of the control word and control address register?

(a) 125, 7
(b) 125, 10
(c) 135, 9
(d) 135, 10

RISC and CISC

5. [MCQ] [GATE-2018 : 1M]

Consider the following processor design characteristics:

I. Register-to-register arithmetic operations only.

II. Fixed-length instruction format.

III. Hardwired control unit.

Which of the characteristics above are used in the design of a RISC processor?

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

❑❑❑

ANSWER KEY

1.	(d)	**2.**	(5 to 5)	**3.**	(c)	**4.**	(d)
5.	(d)						

SOLUTIONS

1. **(d)**

MBR ← PC

MAR ← X

PC ← Y

Memory ← MBR

(a) Instruction fetch

PC → MAR → Memory → MBR → IR

(Mem to CPU (IR))

PC value stored in Memory then PC is updated

(b) Operand fetch

IR (AF) → MAR → Memory → MBR → ALU

(c) Conditional branch

Not store the PC value in the memory

(d) Initiation of Interrupt service

Scan for Video solution

2. **(5 to 5)**

Multiplexer

3 select line then 2^3 Input line (8 input line)

$\left[\log_2 2^3\right]$ or $\log_2 8$

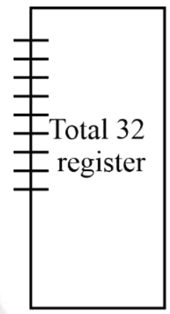
$[2^3]$ input line then 3 select line required

M select line ⇒ 2^m line

If 2^m line then $[\log 2^m]$ select line

For 32 input line

select line = $\log_2(32) = 5$



32
register

Scan for Video solution

3. **(c)**

$R_0 \leftarrow R_1 + R_2$

Fetch cyle (Mem to CPU (IR))

PC → M [MAR] → MBR → IR

3. PCR MAR_w MBR_r

5. MBR_r, IR_w ; MBR → IR

2. $R1_R$ $Temp_w$ ⇒ R1 → Temp1

1. $R2_R$ $temp1_R$ ALU $temp_w$; temp2 ← R_2 + temp1

4. $Temp2_R$ $R0_w$; R0 ← Temp2

$R_0 \Leftarrow R_1 + R_2$.

Scan for Video solution

4. (d)

Total number of instructions = 140

Each intruction requires = 7 cycle

Total number of micro operations = 140×7

$= 980\ \mu$ inst^n |μ operations|cw

$\boxed{\text{Control memory} = 980\text{ CW}}$

NIA|A.F|CAR = $[\log_{10} 980]$

$\boxed{\text{CAR} = 10\text{ bit}}$

Horizontal μ program

125 CS = 125 bits required

CW=135bit

C.F	CAR (AF)
125 bit	10 bit

$125 + 10 = 135$

CW = 135

CAR = 10 bit

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5. (d)

RISC

- Register to Register
- More number of Register
- Fixed length Instruction
- CPI = 1 [cycle Pre Instruction]
- Fortest [Using Hardwired CU]

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CHAPTER

3 INSTRUCTION PIPELINING

Basics of Pipelining

1. [NAT] [GATE-2014 : 2M]

Consider two processors P_1 and P_2 executing the same instructions set. Assume that under identical conditions, for the same input, a program running on P_2 takes 25% less time but incurs 20% more CPI (clock cycles per instruction) as compared to the program running on P_1. If the clock frequency of P_1 is $1GH_Z$, then the clock frequency of P_2 (in GH_Z) is ________.

2. [MCQ] [GATE-2008 : 2M]

In an instruction execution pipeline, the earliest that the data TLB (Translation Look a side Buffer) can be accessed is

(a) Before effective address calculation has started
(b) During effective address calculation
(c) After effective address calculation has completed
(d) After data cache lookup has completed

Performance Evaluation of Pipeline

3. [NAT] [GATE-2023 : 1M]

Consider a 3-stage pipelined processor having a delay of 10 ns (nanoseconds), 20 ns, and 14 ns, for the first, second, and the third stages, respectively. Assume that there is no other delay and the processor does not suffer from any pipeline hazards. Also assume that one instruction is fetched every cycle.

The total execution time for executing 100 instructions on this processor is __________ ns.

4. [NAT] [GATE-2020 : 2M]

Consider a non-pipelined processor operating at 2.5 GHz. It takes 5 clock cycles to complete an instruction. You are going to make a 5-stage pipeline out of this processor. Overheads associated with pipelining force you to operate the pipelined processor at 2 GHz. In a given program, assume that 30% are memory instructions, 60% are ALU instructions and the rest are branch instructions. 5% of the memory instructions cause stalls of 50 clock cycles each due to cache misses and 50% of the branch instructions cause stalls of 2 cycles each. Assume that there are no stalls associated with the execution of ALU instructions. For this program, the speedup achieved by the pipelined processor over the non-pipelined processor (round off to 2 decimal places) is _________.

5. [NAT] [GATE-2018 : 2M]

The instruction pipeline of a RISC processor has the following stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Writeback (WB) The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions, In the PO stage, 40 instructions take 3 clock cycles each, 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control hazards.

The number of clock cycles required for completion of execution of the sequence of instructions is ______.

6. [NAT] [GATE-2017 : 2M]

Instruction execution in a processor is divided into 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Execute (EX) and Write Back (WB). These stages take 5, 4, 20, 10 and 3 nanoseconds (ns) respectively. A pipelined implementation of the processor requires buffering between each pair of consecutive stages with a delay of 2 ns. Two pipelined implementations of the processor are contemplated:

(I) a naive pipeline implementation (NP) with 5 stages and (II) an efficient pipeline (EP) where the OF stage is divided into stages OF1 and OF2 with execution times of 12 ns and 8 ns respectively.

The speedup (correct to two decimal places) achieved by EP over NP in executing 20 independent instructions with no hazards is _____.

7. [MCQ] [GATE-2017 : 1M]

Consider the following processors (ns stands for nanoseconds). Assume that the pipeline registers have zero latency.

P1: Four – stage pipeline with stage latencies 1ns, 2 ns, 2 ns, 1 ns.

P2: Four-stage pipeline with stage latencies 1ns, 1.5 ns, 1.5 ns, 1.5 ns.

P3: Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns. 1 ns.

P4: Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Which processor has the highest peak clock frequency?

(a) P1 (b) P2

(c) P3 (d) P4

8. [NAT] [GATE-2016 : 2M]

Consider a 3 GH_z (gigahertz) processor with a three-stage pipeline and stage latenciesτ_1, τ_2, and τ_3 such that $\tau_1 = 3\tau_{2/4} = 2\tau_3$. If the longest pipeline stage is split into two pipeline stages of equal latency, the new frequency is _____ GH_Z, ignoring delays in the pipeline registers.

9. [NAT] [GATE-2016 : 2M]

The stage delays in a 4-stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is ____ percent.

10. [NAT] [GATE-2015 : 2M]

Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of four. The same processor is upgraded to a pipelined processor with five stages; but due to the internal pipeline delay, the clock speed is reduced to 2 gigahertz. Assume that there are no stalls in the pipeline. The speed up achieved in this pipelined processor is______.

11. [NAT] [GATE-2014 : 2M]

An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register write back (WB) with stage latencies 1 ns, 2.2 ns, 2ns ,1 ns, and 0.75 ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are P and Q nanoseconds, respectively. The value of P/Q is ______.

12. [NAT] [GATE-2014 : 2M]

Consider a 6-stage instruction pipeline, where all stages are perfectly balanced. Assume that there is no cycle-time overhead of pipelining. When an application is executing on this 6-stage pipeline, the speedup achieved with respect to non-pipelined execution if 25% of the instructions incur 2 pipeline stall cycles is _______.

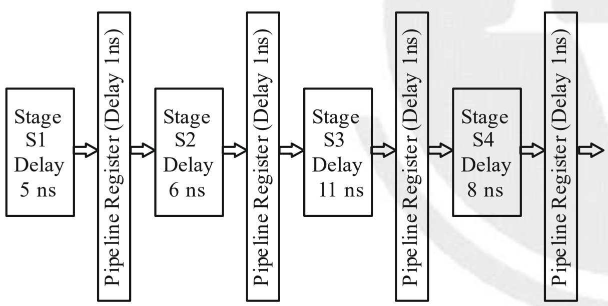
13. [MCQ] [GATE-2013 : 2M]

Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7 ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instruction I_1, I_2, I_3, ….I_{12} is executed in this pipelined processor. Instruction I_4 is the only branch instruction and its branch target is I_9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is

(a) 132 (b) 165
(c) 176 (d) 328

14. [MCQ] [GATE-2011 : 2M]

Consider an instruction pipeline with four stages (S1, S2, S3 and S4) each with combinational circuit only. The pipeline registers are required between each stage and at the end of the last stage. Delays for the stage and for the pipeline registers are as given in the



What is the approximate speed up of the pipeline in steady state under ideal conditions when compared to the corresponding non-pipeline implementation?

(a) 4.0 (b) 2.5
(c) 1.1 (d) 3.0

15. [MCQ] [GATE-2009 : 2M]

Consider a 4-stage pipeline processor. The number of cycles needed by the four instructions I1, I2, I3, I4 in stages S1, S2, S3, S4 is shown below:

	S1	S2	S3	S4
I1	2	1	1	1
I2	1	3	2	2
I3	2	1	1	3
I4	1	2	2	2

What is the number of cycles needed to execute the following loop? For (i = 1 to 2) {I1; I2; I3; I4;}

(a) 16 (b) 23
(c) 28 (d) 30

16. [MCQ] [GATE-2008 : 2M]

A non-pipelined single cycle processor operating at 100 MHZ is converted into a synchronous pipelined processor with five stages requiring 2.5 nsec, 1.5 nsec, 2 nsec, 1.5 nsec and 2.5 nsec, respectively. The delay of the latches is 0.5 nsec. The speedup of the pipeline processor for a large number of instructions is:

(a) 4.5 (b) 4.0
(c) 3.33 (d) 3.0

Pipelining Dependencies

17. [NAT] [GATE-2022: 2M]

A processor X_1 operating at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program P that has 30% branch instructions, control hazards incur 2 cycles stall for every branch. A new version of the processor X_2 operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminates stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for X_1 and X_2. If the BPU has a prediction accuracy of 80%, the speed up (*rounded off to two decimal places*) obtained by X_2 over X_1 in executing P is____________.

18. [NAT] [GATE-2021 : 2M]

A five-stage pipeline has stage delays of 150,120,150,160 and 140 nanoseconds. The registers that are used between the pipeline stages have a delay of 5 nanoseconds each.

The total time to execute 100 independent instructions on this pipeline, assuming there are no pipeline stalls, is ______ nanoseconds.

19. [NAT] [GATE-2021 : 1M]

Consider a pipelines processor with 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Execute (EX), Memory Access (MEM), and Write Back (WB). Each stage of the pipeline, except the EX stage, takes one cycle. Assume that the ID stage merely decodes the instruction and the register read is performed in the EX stage. The EX stage takes one cycle for ADD instruction and two cycles for MUL instructions. Ignore pipeline register latencies.

Consider the following sequence of 8 instructions:

ADD, MUL, ADD, MUL, ADD, MUL, ADD, MUL

Assume that every MUL instruction is data-dependent on the ADD instruction just before it and every ADD instruction (except the first ADD) is data-dependent on the MUL instruction just before it. The Speedup is defined as follows:

$$\text{Speedup} = \frac{\text{Execution time without operand forwarding}}{\text{Execution time with operand forwarding}}$$

The Speedup achieved in executing the given instruction sequence on the pipelined processor (rounded to 2 decimal places) is _____.

20. [NAT] [GATE-2017 : 2M]

Consider a RISC machine where each instruction is exactly 4 bytes long. Conditional and unconditional branch instructions use PC-relative addressing mode with Offset specified in bytes to the target location of the branch instruction. Further the Offset is always with respect to the address of the next instruction in the program sequence. Consider the following instruction sequence

Instr. No	Instruction
i	add R2, R3, R4
i + 1	sub R5, R6, R7
i + 2	cmp R1, R9, R10
i + 3	beq R1, Offset

If the target of the branch instruction is i, then the decimal value of the Offset is ______.

21. [MCQ] [GATE-2015 : 2M]

Consider the following code sequence having five instructions I_1 to I_5. Each of these instructions has the following format.

OP Ri, Rj, Rk

Where operation OP is performed on contents of registers Rj and Rk and the result is stored in register Ri.

I_1: ADD R1, R2, R3 **I_2**: MUL R7, R1, R3

I_3 : SUB R4, R1, R5 **I_4** : ADD R3, R2, R4

I_5 : MUL R7, R8, R9

Consider the following three statements:

S1: There is an anti-dependence between instructions I_2 and I_5.

S2: There is an anti-dependence between instructions I_2 and I_4.

S3: Within an instruction pipeline an anti-dependence always creates one or more stalls.

Which one of above statements is/are correct?

(a) Only S_1 is true

(b) Only S_2 is true

(c) Only S_1 and S_3 are true

(d) Only S_2 and S_3 are true

22. [NAT] [GATE-2015 : 2M]

Consider the sequence of machine instructions given below:

MUL R5, R0, R1

DIV R6, R2, R3

ADD R7, R5, R6

SUB R8, R7, R4

In the above sequence, R0 to R8 are general purpose registers. In the instructions shown, the first register stores the result of the operation performed on the second and the third registers. This sequence of instructions is to executed in a pipelined instruction processor with the following 4 stages: (1) Instruction Fetch and Decode (1F), (2) Operand Fetch (OF), (3) Perform Operation (PO) and (4) Write back the Result (WB). The IF, OF and WB stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD or SUB instruction, 3 clock cycles for MUL instruction and 5 clock cycles for DIV instruction. The pipelined processor uses operand forwarding from the PO stage to the OF stage. The number of clock cycles taken for the execution of the above sequence of instructions is ____.

23. [MCQ] [GATE-2012 : 1M]

Register renaming is done in pipelined processors

(a) As an alternative to register allocation at compile time

(b) For efficient access to function parameters and local variables

(c) To handle certain kinds of hazards

(d) As part of address translation

24. [MCQ] [GATE-2010 : 2M]

A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write Operand (WO) stage. The IF, ID, OF and WO stage take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and SUB instructions ,3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?

Instruction	**Meaning of Instruction**
I_0: MUL R_2, R_0, R_1	$R_2 \leftarrow R_0 * R_1$
I_1: DIV R_5, R_3, R_4	$R_5 \leftarrow R_3/R_4$
I_2: ADD R_2, R_5, R_2	$R_2 \leftarrow R_5 + R2$
I_3: SUB R_5, R_2, R_6	$R_5 \leftarrow R_2 - R_6$

(a) 13 (b) 15

(c) 17 (d) 19

Common Data for next two questions:

Delayed branching can help in the handling of control hazards

25. [MCQ] [GATE-2008 : 2M]

The following code is to run on a pipelined processor with one branch delay slot:

I1: ADD R2 ← R7 + R8

I2: SUB R4 ← R5 - R6x

I3: ADD R1 ← R2 + R3

I4: STORE Memory [R4] ← R1

BRANCH to Label if /R1 = = 0

Which of the instruction I1, I2, I3 or I4 can legitimately occupy the delay slot without any other program modification?

(a) I1 (b) I2

(c) I3 (d) I4

26. [MCQ] [GATE-2008 : 2M]

Delayed branching can help in the handling of control hazards.

For all delayed conditional branch instructions, irrespective of whether the condition evaluates to true or false,

(a) The instruction following the conditional branch instruction in memory is executed

(b) The first instruction in the fall through path is executed

(c) The first instruction in the taken path is executed

(d) The branch takes longer to execute than any other instruction

27. [MCQ] [GATE-2008 : 2M]

Which of the following are NOT true in a pipelined processor?

1. Bypassing can handle all RAW hazards.
2. Register renaming can eliminate all register carried WAR hazards.
3. Control hazard penalties can be eliminated by dynamic branch prediction.

(a) 1 and 2 only

(b) 1 and 3 only

(c) 2 and 3 only

(d) 1, 2 and 3

❑❑❑

ANSWER KEY

1.	(1.6 to 1.6)	**2.**	(c)	**3.**	(2040 to 2040)	**4.**	(2.15 to 2.18)
5.	(219 to 219)	**6.**	(1.49 to 1.52)	**7.**	(c)	**8.**	(3.9 to 4.1)
9.	(33 to 34)	**10.**	(3.2 to 3.2)	**11.**	(1.54 to 1.54)	**12.**	(4 to 4)
13.	(b)	**14.**	(b)	**15.**	(b)	**16.**	(c)
17.	(1.42 to 1.42)	**18.**	(17160 to 17160)	**19.**	(1.87 to 1.88)	**20.**	(–16 to –16)
21.	(b)	**22.**	(13 to 13)	**23.**	(c)	**24.**	(b)
25.	(d)	**26.**	(a)	**27.**	(b)		

SOLUTIONS

1. (1.6 to 1.6)

Total execution time = Number of Instruction × CPI × cycle time

But in the Question number of Instruction are same for processor P_1 & P_2

$$\boxed{\begin{aligned} ET_{P_1} &= CPI \times \text{cycle time}_1 \\ &= CPI \times 1 \text{ nsec.}\end{aligned}}$$

Clock frequency P_1 = 1GHZ

Cycle time $P_1 = \frac{1}{1\text{ G}}$ sec. $\Rightarrow 10^{-9}$ sec. = 1 nsec.

$\boxed{ET_{P2} = 1.2\text{ CPI} \times \text{cycle time}_2}$ [20% more CPI]

$\boxed{\text{Cycle time}_{P1} = 1\text{nsec.}}$

$\boxed{ET_{P2} = 0.75\ ET_{P1}}$ (25% less time that of P_1)

$0.75 \times ET_{P1} = 1.2$ CPI × cycle time_2.

$0.75 \times$ CPI × 1nsec. = 1.2CPI × cycle time_2

0.75 CPI = 1.2 CPI × cycle time_2

Cycle $\text{time}_2 = \frac{0.75}{1.2} = 0.625$ nsec.

Clock frequency $_{P2} = \frac{1}{\text{cycle time }_{P2}}$

$$= \frac{1}{0.625 \times 10^{-9}} = 1.6\text{GHZ.}$$

Scan for Video solution

2. (c)

Effective adress (EA) →: Actual address of the object

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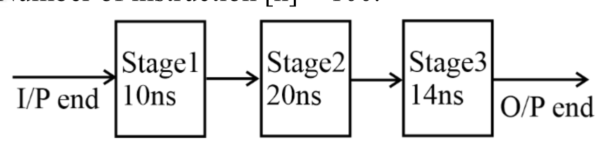
3. (2040 to 2040)

Number of stage (k) = 3.

Number of instruction [n] = 100.



T_p = max (stage delay) = max (10, 20, 14)

T_p = 20 nsec

ET_{pipe} = [k + (n – 1)] tp

= [3 + (100 –1)]20 = 102 × 20

ET_{pipe} = 2040 nsec

Scan for Video solution

4. (2.15 to 2.18)

Non pipeline processor:

Frequency = 2.5 GHZ

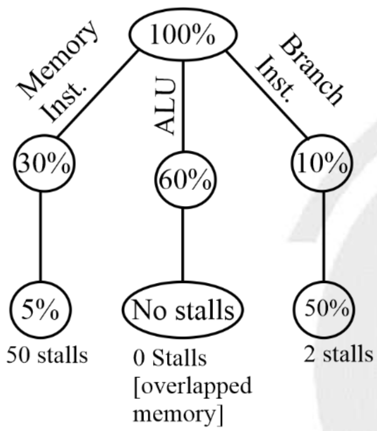
Cycle time = $\dfrac{1}{\text{frequency}} \Rightarrow \dfrac{1}{2.5\text{ G}}$ sec.

$\Rightarrow \dfrac{1}{2.5} 10^{-9} \Rightarrow 0.4$ n sec

$\boxed{\text{Cycle time} = 0.4 \text{ nsec}}$

$ET_{\text{NON PIPE}}$ = CPI × Cycle time = 5 × 0.4 n sec.

$\boxed{ET_{\text{PIPE}} = 2 \text{ nsec.}}$



Number of Stalls / Inst.= .30 × 0.05 × 50 +.60 × 0 + .10× .50 × 2 = .75 + .1

$\boxed{\text{Number of stalls/Inst}^{n} = 0.85}$

ET_{PIPE} = (1 + Number of stalls/Inst.) × cycle time pipe.

Cycle time pipeline = $\dfrac{1}{2\text{ G}}$ sec. $\Rightarrow \dfrac{1}{2} \times 10^{-9}$

0.5 nsec

$ET_{\text{PIPE}} = (1 + 0.85) \times 0.5$ n sec. = 1.85 ×0.5

$\boxed{ET_{\text{PIPE}} \Rightarrow 0.925 \text{ ns}}$

$$S = \frac{ET_{\text{NONPIPE}}}{ET_{\text{PIPE}}} = \frac{2}{0.925} = 2.16$$

Scan for Video solution

5. (219 to 219)

IF, ID, OF, PO, WB

Number of stages = 5

(Number of Inst.) n = 100

IF ID OF WB

ET_{PIPE} = [k + (n–1)] cycle

$ET_{\text{PIPE} \frac{\text{without stalls}}{\text{without forwards}}}$ = [5 + (100 –1)] cycle

= 104 cycle

40 Inst. Takes 3 clocks cycle

35 Inst. Takes 2 clocks cycle

25 Inst. Takes 1 clocks cycle

Number of stalls = 40 × 2 + 35 ×1 + 25 × 0

= 80 + 35 = 115 cycle

Total number of clock cycle take = 104 + 115

= 219 cycle

Alternate Approch.

WB				I_1
PO				I_1
OF			I_1	
ID		I_1		
IF	I_1			

IF ID OF PO WB

For 1^{st} Inst.$\Rightarrow$ $\underbrace{1+1+1+1+}_{4}$ Po stages

Total times = 1 + 1 + 1 + 1 + (40 × 3 + 35 × 2 + 25 × 1) = 4 + 120 + 70 + 25 = 219

Scan for Video solution

6. (1.49 to 1.52)

Naïve pipeline [NP]

5 stages: IF, ID, OF, EX, WB.

$\boxed{K = 5}$ 5ns, 4ns, 20ns, 10ns, 30ns

Buffer Delay = 2ns

Number of Instruction = 20 $\boxed{n = 20}$

Max (stage Delay + Buffer Delay)

NP

tp = max (5, 4, 20, 10, 3) + Buffer Delay = 20 + 2

$\boxed{tp_{NP} = 22 \text{ ns}}$

$ET_{NP} = [k + n - 1]\, tp_{NP}$

$\Rightarrow [5 + (20 - 1)] \times 22 \text{ ns} \Rightarrow 24 \times 22 \text{ ns}$

$\boxed{ET_{NP} = 528 \text{ nsec}}$

Efficient pipe line [EP]

6 Stages IF, ID, OF_1 OF_2 EX WB

$\boxed{k = 6}$ (5, 4, 12, 8, 10, 3) n sec.

Buffer Delay = 2 nsec.

EP

tp = max (stage Delay + Buffer Delay)

$\Rightarrow$ max (5, 4, 12, 8, 10, 3) + 2ns = 12 + 2

$\boxed{tp_{EP} = 14 \text{ nsec.}}$

n = 20

$$EP_{EP} = [k + (n-1)]\, tp_{EP}$$

$$\Rightarrow [6 + (20 - 1)] \times 14 = 25 \times 14$$

$\boxed{ET_{NP} = 350 \text{ nsec}}$

$$\text{Speed up factor} = \frac{\text{Perfomance of EP}}{\text{Perfomance of NP}} = \frac{1/ET_{EP}}{1/ET_{NP}}$$

$$= \frac{ET_{EP}}{ET_{NP}} = \frac{528}{350}$$

$\boxed{\text{Speed up foctor} = 1.508}$

Scan for Video solution

7. **(c)**

P_1 = tp = max (1ns, 2ns, 2ns, 1ns) = 2 nsec.

P_2 = tp = max (1ns, 1.5ns, 1.5ns, 1.5ns) = 1.5 nsec.

P_3 = tp = max (0.5ns, 1ns, 1ns, 0.6ns, 1ns) = 1 nsec.

P_4 = tp = max (0.5ns, 0.5ns, 1ns, 1ns, 1.1ns) =1.1 nsec.

$$\text{Clock Frequency} \propto \frac{1}{\text{cycle time}}$$

P_3 having lowest time

$\therefore$ P_3 having highest clock frequency

Scan for Video solution

8. **(3.9 to 4.1)**

$$\tau_1 = \frac{3\tau_2}{4} \approx 2T_3$$

$$\tau_1 = \frac{3\tau_2}{4} \quad \& \quad \frac{3\tau_2}{4} = 2\tau_3$$

$$\frac{\tau_1}{\tau_2} = \frac{3}{4} \text{ or } \frac{6}{8} \qquad \frac{T_2}{T_3} = \frac{8}{3}$$

$\boxed{\tau_1 : \tau_2 : \tau_3 \Rightarrow 6 : 8 : 3}$

Let x is time

$\tau_1 = 6x$, $\tau_2 = 8x$, $\tau_3 = 3_x$

tp = max (6x, 8x, 3x)

$\boxed{tp = 8x}$

$$\text{Frequency} = \frac{1}{tp} = \text{Frequency} \Rightarrow \frac{1}{8x}$$

$$3\text{GHz} = \frac{1}{8x}$$

$$\Rightarrow \boxed{\frac{1}{x} = 24 \text{ GHZ}}$$

New Design

$$tp = \max(6x, \overbrace{4x, \ 4x}^{8x}, 3x)$$

$\boxed{tp_{New} = 6x}$

$$\text{Frequency}_{New} = \frac{1}{tp_{new}}$$

$$\text{Frequency}_{New} = \frac{1}{6x} \Rightarrow \frac{1}{6} \times \frac{1}{x}$$

$$\Rightarrow \frac{1}{6} \times 24 \text{ GHZ}$$

Frequency_{New} = 4 GHZ.

Scan for Video solution

9. **(33 to 34)**

OLD Design:

4 Stage, (800, 500, 400, 300)

tp_{OLD} = max (800PS, 500PS, 400PS, 300 PS)

$\boxed{tp_{OLD\ Desgin} = 800\ PS}$

Instruction takes = 800PS

In 1 sec. how many # of Inst.

$\boxed{tp_{OLD} = \frac{1}{800}}$

New Design:-

5 stages Delay = (600, 350, 500, 400, 300) PS

tp_{New} = max. (600, 350, 500, 400, 300)

$\boxed{tp_{New} = 600PS}$

Instruction takes = 600 PS

Ins.1 sec. how many number of instruction

$\boxed{trp_{new} = \frac{1}{600}}$

% of throughput increase in pipeline = $\frac{\text{New} - \text{OLD}}{\text{OLD}}$

$$\Rightarrow \frac{\frac{1}{600} - \frac{1}{800}}{\frac{1}{800}} \Rightarrow \frac{\frac{1}{6} - \frac{1}{8}}{\frac{1}{8}} \Rightarrow \frac{\frac{8-6}{48}}{\frac{1}{8}} = \frac{2 \times 8}{48} = \frac{2}{6}$$

$\Rightarrow \frac{1}{3} = 33.33\%$

Scan for Video solution

10. **(3.2 to 3.2)**

Non pipelined processer

I Instruction takes 4 cycle

Cycle time = $\frac{1}{2.5\text{ G}}$ sec. $\Rightarrow \frac{1}{2.5} \times 10^{-9}$ sec.

Cycle time = 0.4 n sec.

In non-pipelined

I Instruction execution time = 4 × 0.4 sec. = 1.6 n sec.

Pipelined Processer:

5 stage

1 Inst^n takes = 1 cycle

Cycle time = $\frac{1}{2}$ sec. $\Rightarrow \frac{1}{2} \times 10^{-9}$ sec.

1 Inst. ET in pipeline = 0.5 nsec.

Speed up factor =

$$\frac{\text{Performance of pipeline}}{\text{Performance of Non-pipeline}} = \frac{1/\text{ETpipeline}}{1/\text{ETnon-pipeline}}$$

$$= \frac{\text{ET in non-pipeline}}{\text{ET in pipeline}} = \frac{tn}{tp} = \frac{1.6}{0.5}$$

= Speed up factor = 3.2

Scan for Video solution

11. **(1.54 to 1.54)**

Old Design

5 Stage	1 IF	2 ID/RF	3 Ex	4 MEM	5 WB
	1ns	2.2ns.	2ns	1ns	0.75 nsec

$tp_{OLD} = \max(1, 2.2, 2, 1, 0.75)$

$\boxed{tp_{OLD} = 2.2\text{n sec.}}$ & OLD Design 'Ex' Stage

Branch Penalty = 3–1

$\boxed{\text{B.P} = 2}$

$\boxed{\text{Branch Frequency} = 20\%}$

Number of stalls / Inst. = .20×2 = 0.4

Average Inst. ET_{OLD} = (1+Number of stalls/Ins.) tp_{OLD}

$\Rightarrow$ (1 + 0.4) × 2.2

$\Rightarrow$1.4 ×2.2

$\boxed{ET_{OLD[p]} = 3.08\text{ n sec.}}$

New Design

8 Stage	1 IF	2 ID	3 RF1	4 RF2	5 EX1	6 EX2	7 MEM	8 WB
	1ns.	$\frac{2.2}{3}$	$\frac{2.2}{3}$	$\frac{2.2}{3}$	1ns	1ns	1ns	0.75 nsec.

$$tp_{new} = \max\left(1, \frac{2.2}{3}, \frac{2.2}{3}, \frac{2.2}{3}, 1, 1, 1, 0.75\right) ns$$

$\boxed{tp_{new} = 1\text{ n sec.}}$

Branch Penalty = 6 – 1 = 5

Branch frequency = 20%

Branch required = .20 × 5 = 1

$ET_{\text{new Design}} = (1 + \text{number of stalls/ins.}) \times tp_{\text{new}}$

$\Rightarrow (1 + 1) \times 1$ nsec.

$\boxed{Q = 2n\,\text{sec.}}$

$\frac{P}{Q} = \frac{3.08}{2} = 1.54$

Scan for Video solution

12. (4 to 4)

Number of stalls/Inst. = Branch frequency × Branch Penalty

= .25× 2 = 0.5

Speed up factor = $\frac{ET_{NP}}{ET_{pipe}} \left(\text{Preformance} \propto \frac{1}{ET}\right)$

$= \frac{\text{Pipeline Depath (number stage)}}{(1 + \text{number stalls/Inst.})}$

∵ Perfactly balanced

$= \frac{6}{1+0.5} = \frac{6}{1.5} = 4$

Scan for Video solution

13. (b)

K = 5 Stage

FI	DI	FO	EI	WO
5ns	7ns	10ns	8ns	6 ns

Buffer Delay = 1 nsec.

tp = max (stage Delay + Buffer Delay) = 10 + 1

$\boxed{tp\ [\text{cycle time}] = 11\text{nsec.}}$

Without stalls

$ET_{PIPE} = [K + (n + 1)]$ cycle.

= [5 + 8 –1)] cycle. = 12 cycle

Total Inst^n = 12

$I_1, I_2, I_3, I_4, \boxed{I_5, I_6, I_7, I_8}, I_9, I_{10}, I_{11}, I_{12}$

Number of Inst^n[n] = 8

(Out of 12 only 8 Inst^n executing)

Number of stall(extra cycles) = 4 –1 = 3

Number of (extra cycle) = 3

Total cycle = 12 + 3 cycle = 15 cycles

ET_{PIPE} = 15 cycle

Cycle time (tp) = 11 nsec. = 15 × 11 = 165 nsec.

Alternate approach : By timing Diagram

tp (cycle time) =11 nsec.

5 stages

I_1	I_2	I_3	I_4				I_9	I_{10}	I_{11}	I_{12}
5	6	7	8	9	10	11	12	13	14	15

Total = 15 cycle

$\Rightarrow$ 15 × 11 nsec.

$\Rightarrow$ 165 nsec.

Scan for Video solution

14. (b)

S_1 = 5 nsec, S_2 = 6 nsec, S_3 = 11 nsec, S_4 = 8 nsec

P.R/IR or Buffer Delay = 1 nsec.

Speed up factor = ?

Execution time (ET) in non-pipeline = 5 + 6 + 11 + 8

$ET_{\text{Non-pipeline}}$ = 30 nsec.

Execution time in pipeline tp = max (stage Delay + Buffer Delay)

Max (5 + 1, 6 + 1, 11+1, 8+1)

Max (6, 7, 12, 9)

ET_{pipe} = 12 nsec.

Speed up factor = $\frac{ET_{\text{non-pipe}}}{ET_{pipe}} \Rightarrow \frac{30}{12}$

$\boxed{\text{Speed up factor} = 2.5}$

Scan for Video solution

15. (b)

S_4					I_1				I_2	I_2
S_3				I_1			I_2	I_2	I_3	I_4
S_2			I_1	I_2	I_2	I_2	I_3	I_4	I_4	I_1
S_1	I_1	I_1	I_2	I_3	I_3	I_4	I_1	I_1	I_2	I_3
	1	2	3	4	5	6	7	8	9	10

S_4	I_3	I_3	I_3	I_4	I_4	I_1	I_2	I_2	I_3	I_3
S_3	I_4	I_1		I_2	I_2	I_3	I_4	I_4		
S_2	I_2	I_2	I_2	I_3	I_4	I_4				
S_1	I_3	I_4								
	11	12	13	14	15	16	17	18	19	20

S_4	I_3	I_4	I_4							
S_3										
S_2										
S_1										
	21	22	23	24	25					

23 cycles.

Scan for Video solution

16. (c)

Non pipe line processor

Frequency = 100 MHZ

Cycle time non-pipe = $\dfrac{1}{100\times10^{6}} = 10^{-8}$ sec.

$= \dfrac{10}{10}\times10^{-8} = 10\times10^{-9}$ sec.

$\boxed{ET_{\text{non-pipe}} = 10 \text{ nsec.}}$

$\boxed{\text{Cycle time} \Rightarrow 10 \text{ nsec.}}$

PIPELINE 5 stages (k = 5)

Tp = 2.5, 1.5, 2, 1.5, 2 nsec.

Buffer/ Latch Delay = 0.5 nsec.

tp = max (stage Delay + Buffer Delay)

= 2.5 + 0.5

Tp = 3 nsec.

When very large Number of Instruction are executed

$S = \dfrac{tn}{tp} = \dfrac{10 \text{ nsec.}}{3 \text{ nsec.}}$

S = 3.33

Scan for Video solution

17. (1.42 to 1.42)

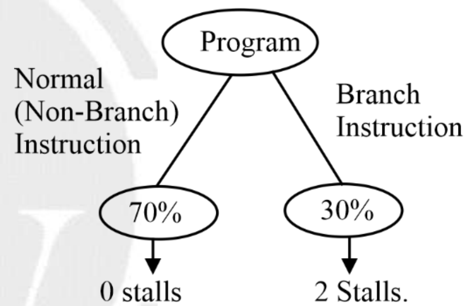
X_1 : 5 stage RISC pipeline, 30% branch instruction 2GHz clock frequency

Cycle Time = $\dfrac{1}{2G}$ sec Cycle Time = 0.5ns.

Number stalls/instruction = Branch frequency × Branch penalty.

= .70 × 0 + .30 × 2

The number stalls/Instruction = 0.6



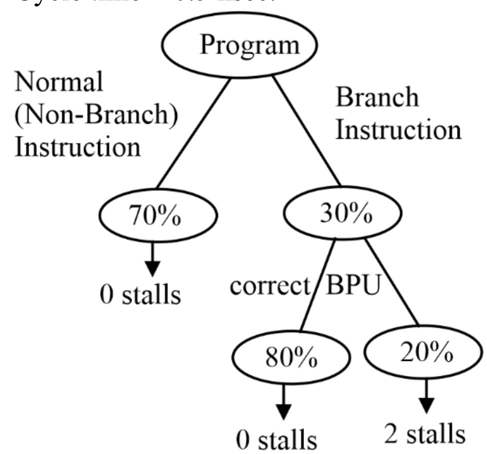
X_1: Avg Inst^n ET = (1 + number of stells/inst^n) × Cycle time

$\Rightarrow$ (1 + 0.6) × 0.5ns $\Rightarrow$ 1.6 × 0.5 ns

$\boxed{ET_{X_1} = 0.8\text{ns}}$

<u>New version X2 with Branch Prediction.</u>

Cycle time = 0.5 nsec.



The number of stalls/instructions = .70 × 0 + .30 × .20 × 2 + .30 × .80 × 0

The number of stalls/Instruction = 0.12

X_2: Avg instruction ET = (1 + number of stalls/instruction) × cycle time

= (1 + 0.12) × 0.5 ⇒ 1.12 × 0.5

$\boxed{ET_{X_2} = 0.56}$

$$\text{SPEED UP} = \frac{\text{performance of } X_2}{\text{performance of } X_1} \Rightarrow \frac{1/ET_{X_2}}{1/ET_{X1}}$$

$$\Rightarrow \frac{ET_{X_1}}{ET_{X2}} \Rightarrow \frac{0.8}{0.56}$$

S = 1.42

Scan for Video solution

18. (17160 to 17160)

Stage Delays = [150, 120, 150, 160, 140 ns]

Buffer delay = 5ns

n = 100

k = 5 stage

$\boxed{ET_{PIPE} = (K + (n-1))tp}$

tp = max. (Stage Delay + Buffer Delay (5ns))

max. [155, 125, 155, 165, 145] ns.

ET_{PIPE} = [5 + (100 –1)] × 165 nsec.

⇒ 104 × 165 ×10^9 sec.

ET_{PIPE} = 17160 ×10^{-9} sec.

$\boxed{ET_{PIPE} = 17160 \text{ nsec.}}$

Scan for Video solution

19. (1.87 to 1.88)

Number of stages = 5

IF, ID, EX, MEM, WB

With operand Forwarding	ADD	MUL	ADD	MUL	ADD	MUL	ADD	MUL
	1	2	1	2	1	2	1	2

Number of Inst^n = 8

Operand forwarding: No additional stalls (Extra cycle) are required to fetch the operand from the previous Inst^n output.

$R_1 \leftarrow R_1 + R_3$

$R_4 \leftarrow R_1 \times R_5$

Example for data dependency

Or

No extra cycle (stall) Due to Data Dependency

Each inst^n takes total 2 cycles 4 MUL Inst^n takes 1extra cycle

Stall = 4 × 1 = 4 cycles

ET_{PIPE} = [k + (n –1)] tp + stalls ⇒ [5 + (8 –1)] × 1+ 4 = 16 cycles

With operand forwards ET_{PIPE} = 16 cycle

Without opened forwarding

Extra cycle (stalls) due to data Dependency

Or

Additional cycle (stall) Required to fetch the opened from the previous Instruction.

$$\underbrace{\text{ADD MUL ADD MUL ADD MUL ADD MUL}}_{\text{7 Instruction data Dependency occur}}$$

'2 stalls per Instruction'

Total additional stall Due to data dependency

= 7 × 2 = 14 cycle

Without operand forwarding:

ET pipeline = 16 + 14 = 30 cycle

$S = \frac{30}{16} = 1.875$

Alternate approach:

Can do by. Time diagram $S = \frac{30}{16} = 1.875$

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20. **(–16 to –16)**

Each inst[n] is 4 bytes long

Assume start at location 4000

	Instr. No	Instruction
4000 – 4003	i	add R2, R3, R4
4004 – 4007	i +1	sub R5, R6, R7
4008 – 4011	i + 2	cmp R1, R9, R10
4012 – 4015	i + 3	beq R1, Offset
4016 –		

PC = 4016

Target address = i

PC ⇒ denotes the starting address of the next Inst[n]

In PC Relative Addressing (AM) ⇒ Target address = Current PC value + OFFSET

4000 = 4016 + OFFSET

$\boxed{\text{OFFSET} = -16}$

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21. **(b)**

OP R_i R_j R_k

$Ri \leftarrow Rj$ any operation R_k

S_1: Anti Dependency between I_2 & I_5 :

I_2 : MUL R_7 R_1 R_3 : $R_7 \leftarrow R_1 \times R_3$

I_5 : MUL R_7 R_8 R_9 : $R_7 \leftarrow R_8 \times R_9$

False because its output Dependency.

S_2: Anti Dependency between I_2 & I_4 :

I_2 : MUL R_7 R_2 R_3 : $R_7 \leftarrow R_1 \times R_3$

I_4 : ADD R_3 R_2 R_4 : $R_3 \leftarrow R_2 \times R_4$

True

Anti Dependency & output Dependency

Sol. Register Remaining

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22. **(13 to 13)**

PO

3	I_1	MUL	R_5	R_0	R_1	$R_5 \leftarrow R_0 \times R_1$
5	I_2	DIV	R_6	R_2	R_3	$R_6 \leftarrow R_2 / R_3$
1	I_3	ADD	R_7	R_5	R_6	$R_7 \leftarrow R_5 + R_6$
1	I_4	SUB	R_8	R_7	R_4	$R_8 \leftarrow R_7 - R_4$

IF

OF

PO

WB

WB						I_1				
PO			I_1	I_1	I_1	I_2	I_2	I_2	I_2	I_2
OF		I_1	I_2							I_3
IF	I_1	I_2	I_3	I_4						
	1	2	3	4	5	6	7	8	9	10

WB	I_2	I_3	I_4						
PO	I_3	I_4							
OF	I_4								
IF									
	11	12	13	14	15	16	17	18	19

13 clock cycle.

Alternate approach:

Stall (extra cycles)

I_1	MUL	3	→ 2
I_2	DIV	5	→ 4
I_3	ADD	1	→ 6 extra cycle stalls
I_4	SUB	1	

n = 4, k = 4

$ET = [K + (n - 1)]\ tp + t\ stalls$

$= [4 + (4 - 1)] \times 1 + 6 = 7 + 6 = 13$ cycle

Scan for Video solution

23. (c)

Register Renaming: Handles Hazards

Scan for Video solution

24. (b)

WB							I_0				
PO				I_0	I_0	I_0	I_1	I_1	I_1	I_1	I_1
OF			I_0	I_1							
ID		I_0	I_1	I_2	I_3						
IF	I_0	I_1	I_2	I_3							
	1	2	3	4	5	6	7	8	9	10	11

WB		I_1	I_2	I_3						
PO	I_1	I_2	I_3							
OF	I_2	I_3								
ID										
IF										
	12	13	14	15	16	17	18	19	20	21

15 clock cycle

	PO
I_0	3
I_1	6
I_2	1
I_3	1

OR

Alternate apporach

		PO stage		Extra cycle
I_0	MUL	3	$\rightarrow$	2
I_1	DIV	6	$\rightarrow$	5
				7 extra cycle (stalls)
I_2	ADD	1		
I_3	SUB	1		

$n = 4$, number of stage $= 5$

ET = [k + (n −1)]+P + (extra cycle +stalls due to hazards)

$\Rightarrow$ $[5 + (4 - 1)] \times 1 + 7 = 8 + 7 = 15$ cycle

Scan for Video solution

25. (d)

I_1 ADD R_2 R_7 R_8 ; $R_2 \leftarrow R_7 + R_8$

I_2 SUB R_4 R_5 R_6 ; $R_4 \leftarrow R_5 - R_6$

I_3 ADD R_1 R_2 R_3 ; $R_1 \leftarrow R_2 + R_3$

I_4 STORE Memory [R] R_1 ; $M[R_4] \leftarrow R_1$

Branch to level if $R_1 == 0$

(a) $I_1 : R_2 \leftarrow R_7 + R_8$

Used in I_3 result of I_1 (R_2) used as operand.

(b) I_2 : ?

$R_4 \leftarrow R_5 - R_6$

I_4 is used as memory [R_4] for memory store purpose

(c) **I_3 :** $R_1 \leftarrow R_2 + R_3$

Used in Branch to total if $R_1 = 0$

Not proper working

(d) **I_4:** memory [R_4] store

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26. (a)

Delayed Branch concept:

→ NOP operation
or
Execute Independent Inst^n

1000	I_1	
1004	I_2	
1008	I_3	JNZ Any Address 2000
1012	I_4	
1016	I_5	
⋮		
2000	I_{101}	
2004	I_{102}	

(a) False

(b) False: (Only branch)

(When condition is True)

WB						
EX						
ID		I_1				
IF	I_1	I_2				

I_2: Next sequential Instruction
Condition Branch Inst. JNZ.

$I_1 - I_{101} - I_{102}$

$\rightarrow\rightarrow$

Taken Path.

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27. (b)

1. By passing (operand forwarding): Handle All Raw Hazards: (False)

If I_1 Load Inst^n & I_2 Next Inst^n using the results of I_1 as a operand

2. WAR (ANTI Dep.) $\Rightarrow$ Register Remaining $\Rightarrow$ True

3. Control Hazards: eliminate by Branch Predication: (False)

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❑❑❑

CHAPTER

4 MEMORY HIERARCHY

Cache Memory and Cache Organisation

1. [NAT] [GATE-2017 : 1M]

Consider a two-level cache hierarchy with L_1 and L_2 caches. An application incurs 1.4 memory accesses per instruction on average. For this application, the miss rate of L_1 cache is 0.1; the L_2 cache experiences on average, 7 misses per 1000 instructions. The miss rate of L_2 expressed correct to two decimal places is ______.

2. [MCQ [GATE-2017 : 2M]

In a two-level cache system, the access times of L_1 and L_2 caches are 1 and 8 clock cycles, respectively. The miss penalty from the L_2 cache to main memory is 18 clock cycles. The miss rate of L_1 cache is twice that of L_2. The average memory access time (AMAT) of this cache system is 2 cycles. The miss rates of L_1 and L_2 respectively are:

(a) 0.111 and 0.056
(b) 0.056 and 0.111
(c) 0.0892 and 0.1784
(d) 0.1784 and 0.0892

3. [MCQ] [GATE-2014 : 2M]

In designing a computer's cache system, the cache block (or cache line) size is an important parameter. Which one of the following statements is correct in this context?

(a) A Smaller block size implies better spatial locality
(b) A Smaller block size implies a smaller cache tag and hence lower cache tag overhead
(c) A Smaller block size implies a larger cache tag and hence lower cache hit time
(d) A Smaller block size incurs a lower cache miss penalty

Mapping Techniques

4. [NAT] [GATE-2023 : 2M]

An 8-way set associative cache of size 64 KB (1 KB = 1024 bytes) is used in a system with 32-bit address. The address is sub-divided into TAG, INDEX, and BLOCK OFFSET. The number of bits in the TAG is ______________.

5. [NAT] [GATE-2021 : 1M]

Consider a computer system with a byte-addressable primary memory of size 2^{32} bytes. Assume the computer system has a direct-mapped cache of size 32 KB (1 KB = 2^{10} bytes), and each cache block is of size 64 bytes.

The size of the tag field is _______bits.

6. [NAT] [GATE-2021 : 1M]

Consider a set-associative cache of size 2 KB (1 KB = 2^{10} bytes) with cache block size of 64 bytes. Assume that the cache is byte - addressable and a 32-bit address is used for accessing the cache. If the width of the tag field is 22 bits, the associativity of the ache is _____.

7. [MCQ] [GATE-2020 : 2M]

A computer system with a word length of 32 bits has a 16 MB byte-addressable main memory and a 64 KB, 4-way set associative cache memory with a block size of 256 bytes. Consider the following four physical addresses represented in hexadecimal notation.

A1 = 0 × 42C8A4, A2 = 0 × 546888, A3 = 0 × 6A289C, A4 = 0 × 5E4880

Which one of the following is TRUE?

(a) A1 and A3 are mapped to the same cache set.
(b) A2 and A3 are mapped to the same cache set.
(c) A3 and A4 are mapped to the same cache set.
(d) A1 and A4 are mapped to different cache sets.

8. [MCQ] [GATE-2019 : 1M]

A certain processor uses a fully associative cache of size 16 kB. The cache block size is 16 bytes. Assume that the main memory is byte addressable and uses a 32-bit address. How many bits are required for the Tag and the Index fields respectively in the addresses generated by the processor?

(a) 24-bits and 0-bits (b) 28-bits and 4-bits
(c) 24-bits and 4-bits (d) 28-bits and 0-bits

9. [MCQ] [GATE-2018 : 2M]

The size of the physical address space of a processor is 2^P bytes. The word length is 2^Wbytes. The capacity of cache memory is 2^N bytes. The size of each cache block is 2^M words. For a K-way set-associative cache memory, the length (in number of bits) of the tag field is

(a) $P - N - \log_2 K$
(b) $P - N + \log_2 K$
(c) $P - N - M - W - \log_2 K$
(d) $P - N - M - W + \log_2 K$

10. [MCQ] [GATE-2017 : 2M]

Consider a machine with a byte addressable main memory of 2^{32} bytes divided into blocks of size 32 bytes. Assume that a direct mapped cache having 512 cache lines is used with this machine. The size of the tag field in bits is _____.

(a) 12 (b) 16
(c) 18 (d) 24

11. [NAT] [GATE-2017 : 2M]

A cache memory unit with capacity of N words and block size of B Words is to be designed. If it is designed as a direct mapped cache, the length of the TAG field is 10 bits. If the cache unit is now designed as a 16-way set-associative cache, the length of the TAG field is _____ bits.

12. [MCQ] [GATE-2016 : 2M]

The width of the physical address on a machine is 40 bits. The width of the tag field in a 512 KB 8-way set associative cache is ______bits.

(a) 24 (b) 20
(c) 30 (d) 40

13. [MCQ] [GATE-2015 : 1M]

Consider a machine with a byte addressable main memory of 2^{20} bytes, block size of 16 bytes and a direct mapped cache having 2^{12} cache lines. Let the addresses of two consecutive bytes in main memory be $(E201F)_{16}$ and $(E2020)_{16}$. What are the tag and cache line address (in hex) for main memory address $(E201F)_{16}$?

(a) E, 201 (b) F, 201
(c) E, E20 (d) 2, 01F

14. [NAT] [GATE-2014 : 1M]

A 4-way set-associative cache memory unit with a capacity of 16 KB is built using a block size of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. The number of bits for the TAG field is _______.

15. [MCQ] [GATE-2014 : 2M]

If the associativity of a processor cache is doubled while keeping the capacity and block size unchanged, which one of the following is guaranteed to be NOT affected?

(a) Width of tag comparator
(b) Width of set index decoder
(c) Width of way selection multiplexer
(d) Width of processor to main memory data bus

16. [MCQ] [GATE-2013 : 1M]

In a k-way set associative cache, the cache is divided into v sets, each of which consists of k lines. The lines of a set are placed in sequence one after another. The lines in set s are sequenced before the lines in set (s + 1). The main memory blocks are numbered 0 onwards. The main memory block numbered 'j' must be mapped to any one of the cache lines from

(a) (j mod v) * k to (j mod v) * k + (k-1)
(b) (j mod v) * (j mod v) + (k-1)
(c) (j mod k) to (j mod k) + (v-1)
(d) (j mod k) * v (j mod k) * v+ (v-1)

Common Data for next two questions:
Consider a computer with a 4-ways set-associative mapped cache of the following characteristics: a total of 1 MB of main memory, a word size of 1 byte, a block size of 128 words and a cache size of 8 KB.

17. [MCQ] [GATE-2008 : 2M]
While accessing the memory location 0C795H by the CPU, the contents of the TAG field of the corresponding cache line is
(a) 000011000 (b) 110001111
(c) 00011000 (d) 110010101

18. [MCQ] [GATE-2008 : 2M]
The number of bits in the TAG, SET and WORD fields, respectively are:
(a) 7, 6, 7 (b) 8, 5, 7
(c) 8, 6, 6 (d) 9, 4, 7

Common Data for next tree questions:
Consider a machine with a 2-way set associative data cache of size 64Kbytes and block size 16 bytes. The cache is managed using 32 bit virtual addresses and the page size is 4 Kbytes. A program to be run on this machine begins as follows:

```
double ARR [1024] [1024]
Int i, j;
/ * Initalize array ARR to 0.0 */
for (i = 0; i < 1024; i ++)
for (j = 0; j < 1024; j++)
ARR [i] [j] = 0.0;
```

The size of double 8 bytes. Array ARR is in memory starting at the beginning of virtual page 0×FF000 and stored in row major order. The cache is initially empty and no pre-fetching is done. The only data memory references made by the program are those to array ARR.

19. [MCQ] [GATE-2008 : 2M]
The cache hit ratio for this initialization loop is
(a) 0% (b) 25%
(c) 50% (d) 5%

20. [MCQ] [GATE-2008 : 2M]
Which of the following array elements has the same cache index as APR [0][0]?
(a) ARR [0][4] (b) ARR [4][0]
(c) ARR [0][5] (d) ARR [5][0]

21. [MCQ] [GATE-2008 : 2M]
The total size of the tags in the cache directory is
(a) 32 kbits (b) 34 kbits
(c) 64 kbits (d) 68 kbits

Cache Replacement Techniques

22. [MSQ] [GATE-2022 : 2M]
Consider a system with 2 KB direct mapped data cache with a block size of 64 bytes. The system has a physical address space of 64 KB and a word length of 16 bits. During the execution of a program, four data words P, Q, R, and S are accessed in that order 10 times (*i.e.*, PQRSPQRS…). Hence, there are 40 accesses to data cache altogether. Assume that the data cache is initially empty and no other data words are accessed by the program. The addresses of the first bytes of P, Q, R, and S are 0xA248, 0xC28A, 0xCA8A, and 0xA262, respectively. For the execution of the above program, which of the following statements is/are TRUE with respect to the data cache?
(a) Every access to S is a hit.
(b) Once P is brought to the cache it is never evicted.
(c) At the end of the execution only R and S reside in the cache.
(d) Every access to R evicts Q from the cache.

23. [NAT] [GATE-2017 : 2M]
Consider a 2-way set associative cache with 256 blocks and uses LRU replacement. Initially the cache is empty. Conflict misses are those misses which occur due to contention of multiple blocks for the same cache set. Compulsory misses occur due to first time access to the block. The following sequence of accesses to memory blocks (0, 128, 256, 128, 0, 128, 256, 128, 1, 129, 257, 129, 1, 129, 257, 129) is repeated 10 times. The number of conflict misses experienced by the cache is ______.

24. [MCQ] [GATE-2014 : 2M]

An access sequence of cache block address of length N and contains n unique block addresses. The number of unique block addresses between two consecutive accesses to the same block address is bounded above by k-. What is the miss ratio if the access sequence is passed through a cache of associativity $A \geq k$ exercising least-recently used replacement policy?

(a) $\frac{n}{N}$ (b) $\frac{1}{N}$

(c) $\frac{1}{A}$ (d) $\frac{k}{n}$

25. [MCQ] [GATE-2009 : 2M]

Consider a 4-way set associative cache (initially empty) with total 16 cache blocks. The main memory consists of 256 blocks and the request for memory blocks is in the following order:

0, 255, 1, 4, 3, 8, 133, 159, 216, 129, 63, 8, 48, 32, 73, 92, 155

Which one of the following memory block will NOT be in cache if LRU replacement policy is used?

(a) 3 (b) 8

(c) 129 (d) 216

Cache Updation Policy

26. [MSQ] [GATE-2022: 1M]

Let WB and WT be two set associative cache organizations that use LRU algorithm for cache block replacement. WB is a write back cache and WT is a write through cache. Which of the following statements is/are FALSE?

(a) Each cache block in WB and WT has a dirty bit.

(b) Every write hit in WB leads to a data transfer from cache to main memory.

(c) Eviction of a block from WT will not lead to data transfer from cache to main memory.

(d) A read miss in WB will never lead to eviction of a dirty block from WB.

27. [NAT] [GATE-2022 : 1M]

A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate.

However, the optimization results in an increase of cache access latency to 15 ns, whereas the miss penalty is not affected. The minimum hit rate (rounded off to two decimal places) needed after the optimization such that it should not increase the average memory access time is _____________.

28. [MCQ] [GATE-2021 : 2M]

Assume a two-level inclusive cache hierarchy. L1 and L2, where L2 is the larger of the two. Consider the following statements.

S_1: Read misses in a write through L1 cache do not result in write backs of dirty lines to the L2.

S_2: Write allocate policy must be used in conjunction with write through caches and no-write allocate policy is used with write back caches.

Which of the following statements is correct?

(a) S_1 is true and S_2 is true.

(b) S_1 is true and S_2 is false.

(c) S_1 is false and S_2 is true.

(d) S_1 is false and S_2 is false.

29. [NAT] [GATE-2020 : 1M]

A direct mapped cache memory of 1 MB has a block size of 256 bytes. The cache has an access time of 3 ns and a hit rate of 94%. During a cache miss, it takes 20 ns to bring the first word of a block from the main memory, while each subsequent word takes 5 ns. The word size is 64 bits. The average memory access time in ns (round off to 1 decimal place) is _____.

30. [NAT] [GATE-2019 : 2M]

A certain processor deploys a single-level cache. The cache block size is 8 words and the word size is 4 bytes. The memory system uses a 60-MHz clock. To service a cache miss, the memory controller first takes 1 cycle to accept the starting address of the block, it then takes 3 cycles to fetch all the eight words of the block, and finally transmits the words of the requested block at the rate of 1 word per cycle. The maximum bandwidth for the memory requested block at the rate of 1 word per cycle. The maximum bandwidth for the memory system when the program running on the processor issues a series of read operations is _______$\times 10^6$ bytes/sec.

31. [NAT] [GATE-2017 : 2M]

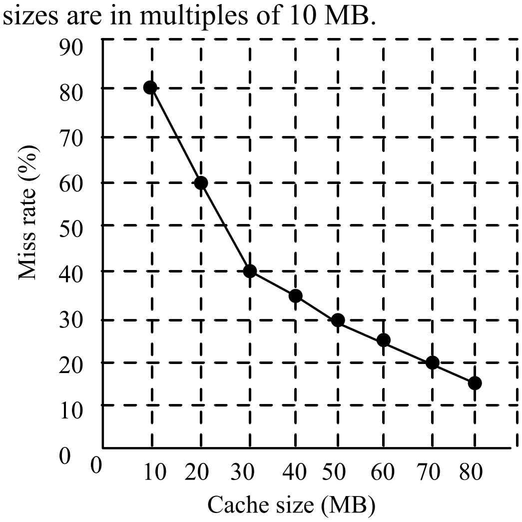
The read access times and the hit ratios for different caches in a memory hierarchy are as given below:

cache	Read access time (in nano seconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred word-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program, 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. The average read access time in nanoseconds (up to 2 decimal places) is ________.

32. [NAT] [GATE-2016 : 2M]

A file system uses an in-memory cache to cache disk blocks. The miss rate of the cache is shown in the figure. The latency to read a block from the cache is 1 ms and to read a block from the disk is 10 ms. Assume that the cost of checking whether a block exists in the cache is negligible. Available cache sizes are in multiples of 10 MB.



The smallest cache size required to ensure an average read latency of less than 6 ms is ______MB.

33. [NAT] [GATE-2015 : 1M]

Assume that for a certain processor, a read request takes 50 nanoseconds on a cache miss and 5 nanoseconds on a cache hit. Suppose while running a program, it was observed that 80% of the processor's read requests result in a cache hit. The average read access time in nanoseconds is _______.

34. [NAT] [GATE-2014 : 2M]

The memory access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations, 60 memory operand read operations and 40 memory operand write operations. The cache hit-ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is _____.

Common Data for next two questions:

A Computer has a 256 Kbyte, 4-way set associative, write back data cache with block size of 32 Bytes. The processor sends 32-bit addresses to the cache controller. Each cache tag directory entry contains, in addition to address tag, 2 valid bits, 1 modified bit and 1 replacement bit.

35. [MCQ] [GATE-2012 : 2M]

The number of bits in the tag field of an address is

(a) 11 (b) 14

(c) 16 (d) 27

36. [MCQ] [GATE-2012 : 2M]

The size of the cache tag directory is

(a) 160 Kbits

(b) 136 Kbits

(c) 40 Kbits

(d) 32 Kbits

37. [MCQ] [GATE-2011 : 2M]

An 8KB direct-mapped write back cache is organized as multiple blocks, each of size 32 bytes. The processor generates 32-bit addresses. The cache controller maintains the tag information for each cache block comprising of the following.

1Valid bit

1Modified bit

As many bits as the minimum needed to identify the memory block mapped in the cache.

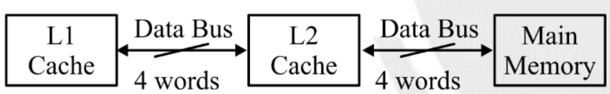
What is the total size of memory needed at the cache controller to store meta-data (tags) for the cache?

(a) 4864 bits (b) 6144bits

(c) 6656 bits (d) 5376 bits

Common Data for next two questions:

A Computer system has an L1 and L2 cache, an L2 cache, and a main memory unit connected as shown below. The block size in L1 cache is 4 words. The block size in L2 cache is 16 words. The memory access times are 2 nanoseconds, 20 nanoseconds and 200 nanoseconds for L1 cache, L2 cache and main memory unit respectively.



38. [MCQ] [GATE-2010 : 2M]

When there is a miss in L1 cache and a hit in L2 cache, a block is transferred from L2 cache to L1 cache. What is the time taken for this transfer?

(a) 2 nanoseconds (b) 20 nanoseconds

(c) 22 nanosecond (d) 88 nanoseconds

39. [MCQ] [GATE-2010 : 2M]

When there is a miss in both L1 cache and L2 cache, first a block is transferred from main memory to L2 cache, and then a block is transferred from L2 cache to L1 cache. What is the total time taken for these transfers?

(a) 222 nanoseconds (b) 888 nanoseconds

(c) 902 nanoseconds (d) 968 nanoseconds

40. [MCQ] [GATE-2008 : 2M]

For inclusion to hold between two cache level L1 and L2 in a multilevel cache hierarchy, which of the following are necessary?

1. L1 must be a write-through cache
2. L2 must be write-through cache
3. The associativity of L2 must be greater that of L1
4. The L2 cache must be at least as large as the L1 cache

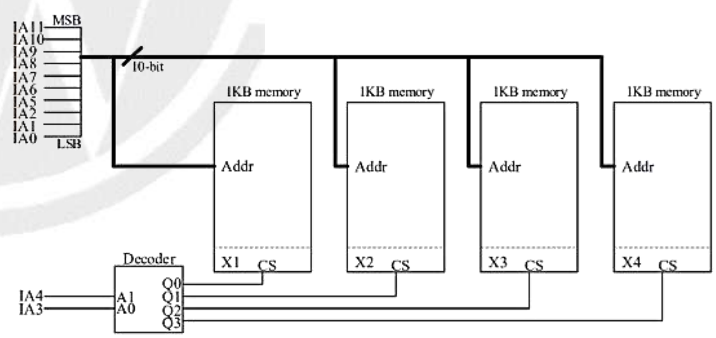
(a) 4 only (b) 1 and 4 only

(c) 1,2 and 4 only (d) 1,2,3 and 4

Main Memory

41. [MCQ] [GATE-2023 : 2M]

A 4 kilobyte (KB) byte-addressable memory is realized using four 1 KB memory blocks. Two input address lines (IA4 and IA3) are connected to the chip select (CS) port of these memory blocks through a decoder as shown in the figure. The remaining ten input address lines from IA11–IA0 are connected to the address port of these blocks. The chip select (CS) is active high



The input memory addresses (IA11–IA0), in decimal, for the starting locations

(Addr=0) of each block (indicated as X1, X2, X3, X4 in the figure) are among the

options given below. Which one of the following options is CORRECT?

(a) (0, 1, 2, 3)

(b) (0, 1024, 2048, 3072)

(c) (0, 8, 16, 24)

(d) (0, 0, 0, 0)

42. **[NAT]** **[GATE-2018 : 1M]**
A 32-bit wide main memory unit with a capacity of 1 GB is built using 256 M ×4-bit DRAM chips. The number of rows of memory cells in the DRAM chip is 2^{14}. The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds. The percentage (Rounded to the closest integer) of the time available for performing the memory read/write operations in the main memory unit is________.

43. **[NAT]** **[GATE-2016 : 1M]**
A processor can support a maximum memory of 4 GB, where the memory is word-addressable (a word consists of two bytes). The size of the address bus of the processor is at least ______bits.

44. **[NAT]** **[GATE-2014 : 2M]**
Consider a main memory system that consists of 8 memory modules attached to the system bus, which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address, and control signals. During the same 100 ns, and for 500 ns thereafter, the addressed memory module executes one cycle accepting and storing the data. The (internal) Operation of different memory modules may overlap in time, but only one request can be on the bus at any time. The maximum number of stores (of one word each) that can be initiated in 1 millisecond is _____.

45. **[MCQ]** **[GATE-2013 : 2M]**
A RAM chip has capacity of 1024 words of 8 bits each (1K × 8). The number of 2 × 4 decoders with enable line needed to construct a 16 K×16 RAM from 1K × 8 RAM is
(a) 4
(b) 5
(c) 6
(d) 7

46. **[MCQ]** **[GATE-2010 : 2M]**
A main memory unit with a capacity of 4 megabytes is built using 1M × 1-bit DRAM chips. Each DRAM chip has 1K rows of cells with 1K cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the cells in the memory unit is
(a) 100 nanoseconds
(b) $100*2^{10}$ nanoseconds
(c) $100*2^{20}$ nanoseconds
(d) $3200*2^{20}$ nanoseconds

47. **[MCQ]** **[GATE-2009 : 1M]**
How many 32K × 1 RAM chips are needed to provide a memory capacity of 256 K-bytes?
(a) 8
(b) 32
(c) 64
(d) 128

❑❑❑

ANSWER KEY

1.	(0.05 to 0.05)	**2.**	(a)	**3.**	(d)	**4.**	(19 to 19)
5.	(17 to 17)	**6.**	(2 to 2)	**7.**	(b)	**8.**	(d)
9.	(b)	**10.**	(c)	**11.**	(14 to 14)	**12.**	(a)
13.	(a)	**14.**	(20 to 20)	**15.**	(d)	**16.**	(a)
17.	(a)	**18.**	(d)	**19.**	(c)	**20.**	(b)

21.	(d)	**22.**	(a, b, d)	**23.**	(76 to 76)	**24.**	(a)
25.	(d)	**26.**	(a, b, d)	**27.**	(0.85 to 0.85)	**28.**	(d)
29.	(13.3 to 13.5)	**30.**	(160 to 160)	**31.**	(4.72 to 4.72)	**32.**	(30 to 30)
33.	(14 to 14)	**34.**	(1.68 to 1.68)	**35.**	(c)	**36.**	(a)
37.	(d)	**38.**	(c)	**39.**	(c)	**40.**	(a)
41.	(c)	**42.**	(59 to 60)	**43.**	(31 to 31)	**44.**	(10000 to 10000)
45.	(b)	**46.**	(b)	**47.**	(c)		

SOLUTIONS

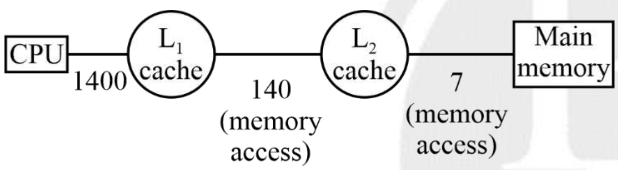
1. (0.05 to 0.05)

1 instruction takes = 1.4 memory access **(reference)**

1000 instruction takes = 1.4 × 1000 = 1400 memory reference (access)

Total memory reference (access) = 1400

L_1 miss rate = 0.1



L_2 Miss rate = $\frac{7}{140} = \frac{1}{20} = 0.05$

Alternate approach:

Number of misses occurred in

$L_2 = \frac{7}{1400} = \frac{1}{200} = 0.005$

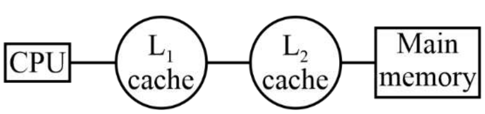
Miss rate of L_2 = $\frac{\text{Total misses occured in } L_2}{\text{Miss rate of } L_1}$

$= \frac{0.005}{0.1} = 0.05$

Scan for Video solution

2. (a)

L_1 miss rate twice that of L_2



L_1 cache access time = 1 clock cycle

L_2 cache access time = 8 clock cycle

Mm access = 18 clock cycle

Tavg = 2 clock cycle

Assume,

L_2 miss rate = x

L_1 = 2x

T_{avg} = Hit time L_1 + (miss rate L_1)

[(Hit time L_2 + miss rate L_2) (mm access time)]

2 = 1 + 2x [8 + x (18)]

1 = 1 + 2x[8 + 18 x]

1 = 1 + 16 x+ 36x^2

$\boxed{36x^2 + 16x - 1 = 0}$

a = 36

b = 16

c = –1

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \frac{-16 \pm \sqrt{256 - 4 \times 36 \times (-1)}}{2 \times 36}$

$\Rightarrow \frac{-16 \pm \sqrt{256 + 144}}{72}$

$\Rightarrow \frac{-16 \pm \sqrt{400}}{72}$ [miss rate cannot –v]

$\Rightarrow \frac{-16 + \sqrt{400}}{72}$

$\Rightarrow \frac{-16 + 20}{72} = \frac{4}{72} = \frac{1}{18} = 0.0556$

$\boxed{x = 0.0556}$

L_2 miss rate (x) = 0.0556 ≈ 0.056

L_1 = miss rate = 2 × 0.056 = 0.1112 ≈ 0.111

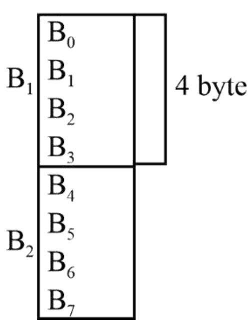
Scan for Video solution

3. (d)

(a) Incorrect because:

Spatial locality: Adjacent word (Data)

If block size 4B

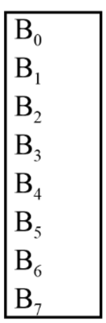


$B_0\ B_1\ B_2\ B_3 \mid B_4\ B_5\ B_6\ B_7$

M H H H ⁝ M H H H

Mm to cm complete block transferred

If block size 8 B



$B_0\ B_1\ B_2\ B_3\ B_4\ B_5\ B_6\ B_7$

M ———— Hit

(b) Incorrect because

PA = 10 bits, cm = 32 bytes, Block size = 2 bytes

Case I:

Direct mapping

P.A=10 bit

TAG	LINE OFFSET	WORD OFFSET
5 bit	4 bit	1 bit

Number of lines = $\frac{32}{2} = 16$

Case II: block size = 4Byte

P.A=10 bit

TAG	LINE OFFSET	WORD OFFSET
5 bit	3 bit	2 bit

Number of lines = $\frac{32}{4} = 8$

Case III: Block size = 16 byte

P.A=10 bit

TAG	LINE OFFSET	WORD OFFSET
5 bit	1 bit	4 bit

Number of lines = $\frac{32}{16} = 2$

But in fully associative :

Case I : Block size = 2bytes

P.A=10 bit

TAG	WORD OFFSET
9 bit	1 bit

Case II: Block size = 4 byte

P.A=10 bit

TAG	WORD OFFSET
8 bit	2 bit

Case III: Block size = 16 byte

P.A=10 bit

TAG	WORD OFFSET
6 bit	4 bit

(c) incorrect

(d) correct because

Smaller block size ⇒ then lower miss penalty

If B.S = 2 words or 2 bytes ⇒ only 2 W or 2 bytes bring from mm to cm

If B.S = 16 words or 16 byte ⇒ 16 W or 16 byte mm to cm ie miss peanalty increase.

Scan for Video solution

4. **(19 to 19)**

8 way set Associative.

Cache size = 64kB,

Physical address = 32 bit

Cache size = 64 KB = 2^{16} Byte.

Direct Mapping:

32bit

TAG	LINE OFFSET	WORD (Block) OFFSET
16 bit	16bit	

But In Question 8 way set Associative.

$\text{Log}_2(8) = 3\text{bit}$

8 Way set Associative

32bit

TAG	LINE OFFSET	WORD (Block) OFFSET
	13bit	

TAG = 32–13 = 19bit

Alternate approach

8 Way set Associative

Cache size = 64 KB

Physical address = 32bit

Assume Block size = 1 Byte

Number of lines = $\frac{\text{CM size}}{\text{Block size}}$

$$= \frac{64\text{KB}}{1\text{B}} = 64\text{K}\left(2^{16}\right)\text{Line}$$

Line off set = $\log_2 2^{16} = 16$ bit

32bit

TAG	1.0	W.D
16bit		

8 Way set associative

$$\text{Number of set} = \frac{\#\text{Lines}}{\text{N} - \text{ways}} = \frac{2^{16}}{2^3} = 2^{13}$$

Set offset = 13 bit

P.A = 32bit

TAG	LINE OFFSET	WORD OFFSET
	13bit	

Tag = 32–13 = 19bit.

Scan for Video solution

5. **(17 to 17)**

Main memory size = 2^{32} Byte

Cache size = 32 KB

Block size = 64 byte

Direct mapped cache:

Mm size 2^{32} byte = physical address (P.A) = 32 bit

Cache size = 32 kB = 2^{15} Byte

Block size = 64 Byte ⇒ word offset = $\log_2(64)$ = 6 bits

$$\text{Number of Lines:} = \frac{\text{CM size}}{\text{Block size}} = \frac{2^{15}\text{B}}{2^6\text{B}} = 2^9$$

Line offset = $\log_2(2^9)$

$\boxed{\text{9 bits}}$

Direct mapped: -

32 bit

TAG	LINE OFFSET	WORD OFFSET
	9 bit	6 bit

TAG = 32 – (9 + 6) = 32 – 15 = 17 bits

Scan for Video solution

6. **(2 to 2)**

Physical address (P.A) = 32 bit

Cache size = 2 KB = 2^{11} Bytes

Block size = 64 byte = 2^6 byte

Tag field = 22 bit

Number of sets = 2^4 = 16 set

Set offset (SO) = 4

Word offset = [$\log_2$ block size] ⇒ [$\log_2$ 64]

$\boxed{\text{Word offset} = 6 \text{ bit}}$

Set associative cache.

32 bit

TAG	SET OFFSET	WORD OFFSET
22 bit	4 bit	6 bit

$$\# \text{ LINES [\#CM blocks]} = \frac{\text{cm Size}}{\text{Block size}}$$

$$\Rightarrow \frac{2^{11}\text{B}}{2^{6}\text{B}} = 2^5 = 32 \text{ lines}$$

$$\# \text{ Set} = \frac{\#\text{Lines}}{\text{N} - \text{way}}$$

$$16 = \frac{32}{\text{N} - \text{way}}$$

$$\text{N-way} = \frac{32}{16} = 2$$

Scan for Video solution

7. **(b)**

Main memory = 16 MB = 2^{24} byte

Cache size = 64 MB = 2^{16} Byte

Block size = 256 B = 2^8 Byte

4 way set associative

A_1 : Ox 42C 8A4

A_2 Ox 546 888

A_3: Ox 6A2 89C

A_4: Ox 5E4 880

Physical address = 24 bit

Word offset = $\log_2$ (block size)

= [$\log_2$ 256] ⇒ 8 bit

$\boxed{\text{Word Offset} = 8 \text{ bit}}$

$$\text{Number of line} = \frac{\text{CM size}}{\text{Block size}} = \frac{2^{16}\text{B}}{2^8\text{B}} = 2^8$$

$$\text{Number of Set} = \frac{\text{Number of LINES}}{\text{No} - \text{way}} = \frac{256}{4} = 64$$

Set offset (S.O) = 6 bit

24 bit

TAG	SET OFFSET	WORD OFFSET
10 bit	6 bit	8 bit

A1: Ox 42C8A4	4211	001000 Set	A4	$\boxed{8 = A_1}$
A2: Ox 54 6888	5401	101000 Set	88	$\boxed{40 = A_2}$
A3: ox 6A289C	6A00	101000 Set	9C	$\boxed{40 = A_3}$
A4: Ox 5E 4880	5E01	001000 Set	80	$\boxed{8 = A_4}$

W.O

A_1 and A_4 mapped to the same cache set

A_2 and A_3 are mapped to the same cache set

Scan for Video solution

8. **(d)**

Physical address = 32 bit

Cache size = 16 kB

Block size = 16 byte

Word offset = [$\log_2$ block size] ⇒[$\log_2$16]

$\boxed{\text{Word offset} = 4\text{bit}}$

Fully associative cache

P.A = 32 bit

TAG	WORD OFFSET
28 bit	4 bit

TAG: 28 bit word offset = 4 bit

Index: 0 bit

Scan for Video solution

9. **(b)**

P.A.S = 2^p byte , word length = 2^w byte

Cache = 2^n byte

Block size = 2^m words

K-way set associative

Block size = 2^m word (each word size = 2^w byte)

$\Rightarrow 2^m \times 2^w$ Byte

$\boxed{\text{Block size} = 2^{m+w}\ \text{Byte}}$

$\boxed{\text{Word offset} = m+w\ \text{Bits}}$

$$\text{Number of LINEs} = \frac{\text{CM size}}{\text{Block size}} = \frac{2^n}{2^{m+w}}$$

$$\text{Number of Set} = \frac{\text{Number of Lines}}{\text{N} - \text{way}} = \frac{2^{N-M-W}}{K}$$

$$\text{Set offset} \Rightarrow \log_2 (\#\ \text{SET}) \Rightarrow \log_2\left(\frac{2^{N-M-W}}{K}\right)$$

$\Rightarrow \text{Log}_2(2^{(N-M-W)}) - \log_2 k$

$\Rightarrow \boxed{N - M - W - \log_2 k}$

P.A = P bit

TAG	SET OFFSET	WORD OFFSET
	N–M–W–$\log_2 k$	(m+w)

TAG : = P.A – (S.O + W.O)

$$\Rightarrow P - \left(N - \cancel{M} - \cancel{W} - \log_2 k + \cancel{M} + \cancel{W}\right)$$

$\boxed{\text{TAG} \Rightarrow P - N + \log_2 k}$

Scan for Video solution

10. (c)

MM size of 2^{32} byte

Physical address (P.A) = 32 bit

Block size = 32 bytes

Word offset = $[\log_2 32]$

$\boxed{\text{Word offset} = 5\ \text{bit}}$

Number of lines = 512; direct mapped cache

Line offset (L.O) = $[\log_2 512]$

$\boxed{\text{Line Offset} = 9\ \text{bit}}$

P.A = 32 bit

TAG	LINE OFFSET	WORD OFFSET
	9 bit	5 bit

TAG = 32 – (9 + 5)

TAG = 32 – 14

$\boxed{\text{TAG} = 18\ \text{bit}}$

Scan for Video solution

11. (14 to 14)

Direct mapped cache

P.A

TAG	LINE OFFSET	WORD OFFSET
10 bit	$\log_2 \frac{N}{B}$	$\log_2 B$

16 ways set associative (assume log = n bit)

$$\text{Number of set} = \frac{\text{Number of Line}}{\text{N ways}} = \left(\frac{\frac{N}{B}}{16}\right)$$

$$\text{Set offset} = \log_2\left(\frac{\frac{N}{B}}{16}\right)$$

$$\Rightarrow \log_2\left(\frac{N}{B}\right) - \log_2 16 \qquad \Rightarrow \log_2\left(\frac{N}{B}\right) - 4$$

$$10 + \cancel{\log_2}\frac{N}{B} + \cancel{\log_2}B = n + \cancel{\log_2}\left(\frac{N}{B}\right) - 4 + \cancel{\log_2}B$$

$10 = n - 4 \Rightarrow n = 14$ bit

$\boxed{\text{TAG} = 14\ \text{bit}}$

Alternate Approach

Direct mapped cache:

TAG	Line offset	Word offset
10 bits		

16 way set associate cache:

TAG (10 bit)	Set offset	Word offset

16-way set associate 1 set we can store 16 way (16 mm block), so in index now we require 4 less bits compare to direct mapped

Now, TAG = 10 + 4

TAG = 14 bits

Proof.

MM = 1 MB cache size – 16 kB, block size = 128 byte

PA = 20 bit

Direct method cache

Word offset = $[\log_2 128]$ = 7 bit

$$\text{Number of LINES} = \frac{\text{CM size}}{\text{block size}} = \frac{16\text{ k}\cancel{B}}{128\cancel{B}} = \frac{2^{14}}{2^{7}}$$

$$= 2^7 \text{ lines}$$

L.O = 7 bit

P.A=20 bit

TAG	LINE OFFSET	WORD OFFSET
6 bits	7 bits	7 bits

16-way set associate cache

$$\text{Number of set} = \frac{\text{Number of line}}{\text{N} - \text{ways}}$$

$$= \frac{2^7}{2^4(16\text{ways})} = 2^3 \text{set} = 8 \text{ set}$$

$$\boxed{\text{set offset} = 3\text{bit}}$$

P.A=20 bit

TAG	LINE OFFSET	WORD OFFSET
10 bits	3 bits	7 bits

cache size = N word, block size = B word

Tag = 10 bits

Word offset = $\log_2 B$

$$\text{Number of line} = \frac{\text{CM size}}{\text{Block size}} = \frac{N}{B}$$

$$\text{L.O} = \log_2\left(\frac{N}{B}\right)$$

Scan for Video solution

12. (a)

Physical address (P.A) = 40 bits

Cache size = 512 KB = 2^{19}

8-way set associative

P.A = TAG + set offset + word offset

$$\boxed{\text{TAG} + \text{set offset} + \text{word offset} = 40\text{bits}} \quad \text{.....(1)}$$

Cache = Number of sets × block per set × block size

512 KB = Number of Sets × 8 × block size

[2^{16}] KB = Number of sets × block size

$$\boxed{\text{set offset} + \text{word offset} = 16 \text{ bit}} \quad \text{......(2)}$$

P.A = 40 bit

TAG	SET OFFSET	WORD OFFSET

TAG + set offset + word offset = 40 bit

TAG + 16 bits = 40 bits

TAG = 40 – 16

$$\boxed{\text{TAG} = 24 \text{ bits}}$$

Scan for Video solution

13. (a)

Main memory = 2^{20} byte

Block size = 16 bytes

Number of lines = 2^{12}

Mm size = 2^{20} Byte

P.A = 20 bit

Word offset = $[\log_2 16]$

$$\boxed{\text{Word offset} = 4\text{bit}}$$

lines = 2^{12}

$$\boxed{\text{Line offset} = 12\text{bit}}$$

Direct mapped cache:

P.A = 20 bits

TAG	LINE OFFSET	WORD OFFSET
4 bit	12 bit	14 bit

E201F ⇒ 1110 | 0010 0000 0001 | 1111

TAG | LINE | W.O

E, 201 → E 2 0 1

Note: A : 10

B : 11

C : 12

D : 13

E : 14

F : 15

Main memory address = $(E2021F)_{16}$

E	201	F
4 bits	12 bits	4 bits

Tag = E

Cache line number = 201

Scan for Video solution

14. (20 to 20)

Main (physical) memory = 4GB = 2^{32} Byte

Cache size = 16 kB

Word size = 32 bits

Block size = 8 words

4-way set associative

Physical address = 32 bits

1 word size 32 bit = 4 byte

Block size = 8 words

= 8 × 4 bytes

= 32 bytes

Word offset = $[\log_2 32]$ = 5 bit

Number of lines = $\dfrac{\text{CM Size}}{\text{Block size}}$

$$= \frac{16\text{kB}}{32\text{B}} = \frac{2^{14}\text{B}}{2^5\text{B}} = 2^9 \text{ lines}$$

Number of set = $\dfrac{\text{Number of LINE}}{\text{N} - \text{way}}$

$$= \frac{2^9}{2^2} = 2^7 = 128 \text{ set}$$

Set offset = 7bit

P.A = 32 bit

TAG	SET OFFSET	WORD OFFSET
20 bit	7 bit	5 bit

TAG = 32 – (7 + 5)

$\boxed{\text{TAG} = 20 \text{ bit}}$

Scan for Video solution

15. (d)

PA = 20 bit (mm = 1MB), cach size = 16 MB, Block size = 64 byte

Direct mapping:

P.A=20 bit

TAG	LINE OFFSET	WORD OFFSET
6 bit	8 bit	6 bit

Number of line = $\dfrac{\text{CM size}}{\text{Block size}} = \dfrac{16\text{kB}}{64\text{B}} = \dfrac{2^{14}}{26} = 2^8$

2-way set associative;

P.A=20 bit

TAG	SET OFFSET	WORD OFFSET
7 bits	7 bits	6 bits

Number of set = $\dfrac{\text{Lines}}{\text{N} - \text{way}} = \dfrac{2^8}{2^1} = 2^7$

P.A=20 bit

TAG	LINE OFFSET	WORD OFFSET
8 bit	6 bit	6 bit

4 way set associative

Number of set = $\dfrac{\text{Lines}}{\text{N – way}} = \dfrac{2^8}{2^2} = 2^6$

Scan for Video solution

16. (a)

Number of sets = each set consit 'k' lines

Set associative mapping

K mod S or mm request (mod) # set

(J mod V)

Find first Line of any set

Each set contiant 4 lines (k = 4)

Set 0 ⇒ 0 × 4 = 0

Set 1 = 1 × 4 = 4

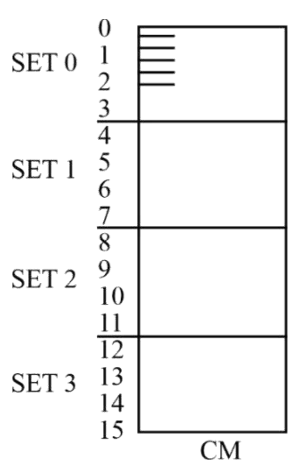
Set 2 = 2 × 4 = 8

Set 3 = 3 × 4 = 12

For first Line, set number × k

0 × 4 to 0 × 4 + (4 – 1) = 0 to 3

1 × 4 to 1 × 4 + (4 – 1) = 4 to 7x × k to x × k + (k – 1)



Each set containt 8 lines

K = 8

First line of any set

Set 0 = 0 × 8 = 0

Set 1 = 1 × 8 = 8

Set 2 = 2 × 8 = 16

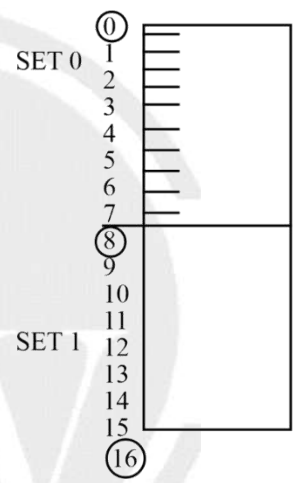
Set number × k

$\boxed{\text{0 to 7}}$

0 × 8 to 0 × 8 + (8 – 1)

1 × 8 to 1 × 8 + (8 – 1)

$\boxed{\text{8 to 15}}$



(j mod v) × k to
(j mod v) × k + (k – 1)

Scan for Video solution

17. (a)

P.A=20 bit

TAG	LINE OFFSET	WORD OFFSET
9 bits	4 bits	7 bits

$$\underbrace{\overset{0}{0000}\ \overset{C}{1100}}_{\text{TAG}}\ \underbrace{\overset{7}{0111}}_{\text{Set}}\ \underbrace{\overset{9}{1001}\ \overset{5}{0101}}_{\text{Word offset}}$$

Scan for Video solution

18. (d)

Main memory size = 1 MB (2^{20} B)

Cache size = 8 KB

1 word size = 1 byte

Block size = 128 words

Block size = 128 ×1 B = 128 bytes

Word offset = $\log_2 128$ = 7 bits

Number of lines $= \frac{\text{cm size}}{\text{Block size}} = \frac{8\text{ KB}}{128\text{ B}} = 2^6$

Number of sets $= \frac{2^6}{2^2} = 2^4 = 16$ sets

Set offset = $\log_2 16 = 4$

P.A=20 bit

TAG	SET offset	WORD OFFSET
9 bits	4 bits	7 bits

Tag = 20 – (4 + 7) = 9 bits

Scan for Video solution

19. (c)

Block size = 16 byte

Double = 8 byte

i.e each block cache [cache block] contain 2 element of the Array

Given

Row major = Accessing in serial order APR [0, 0] [0, 1] [0, 2] [0, 3]

For i = 0 ⇒ j = 0 to 1023 (1024 times)

For i = 1 ⇒ j = 0 to 1023 (1024 times)

Frist element of array

APR [0, 0] = miss

APR [0, 1] = HIT

APR [0, 2] = Miss

APR [0, 3] = HIT

APR [0, 4] = miss

APR [0, 5] = HIT

APR [0, 0]	APR [0, 1]
APR [0, 2]	APR [0, 3]
APR [0, 4]	APR [0, 5]
.....	.....

Cache Hit ratio = 50%

Scan for Video solution

20. (b)

Page size = 4 KB

Page offset(d) = [$\log_2$ page size] = [$\log_2$ 4KB]

d = 12 bit

Start address = $(FF000)_{16}$

V.A = 32 bit

TAG	SET OFFSET	WORD OFFSET
17 bit	11 bit	4 bit

$\underbrace{14\ 13\ 12\ 11\ 10\ 9\ 8\ 7\ 6\ 5\ 4}_{11\text{ bit}}$ $\underline{3\ 2\ 1\ 0}$

Paging V.A = 32 bit

Page Number	Page Offset (d)
20 bit	12 bit

APR[0][0] FF000

↓

I[st] element of the Array ⇒ Start from $(FF000)_{16}$[Page Number] and Page offset $(000)_F$

(FF000 <u>000</u>)

APR[0][0] =

1111 1111 0000 0000 0 | 000 0000 0000 | 0000

Set Block offset

Row Major

APR[0][4] = 5^{th} element of the Ist row, i.e [0] [0], [0] [1], [0, 2] [0, 3]

I^{st} row 5^{th} element (i.e already 4 element passed)

$4 \times 8 = 32$

V.A = $(F\ F000020)_{16}$

1111 1111 0000 0000 0|000 0000 0010| 0000

Set Index

Row Major

APR[4][0] = I^{st} element of the 5^{th} row [Row No 0, 1, 2, 3]

And each row 1024 element

$4 \times 1024 \times 8$

$2^2 \times 2^{10} \times 2^3 = 2^{15}$

OR

$4 \times 1024 \times 8$

$2^2 \times 2^{10} \times 8$

$21^2 \times 8 = 4k \times 8$

…. $2^{17}\ 2^{16}\ 2^{15}\ 2^{14}\ 2^{13}\ 2^{12}\ 2^{11}\ 2^{10}\ 2^9\ 2^8\ 2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

F F 0 0 $\frac{1|000}{|8}$ $\frac{0\,0\,0\,0}{0}$ $\frac{0\,0\,0\,0|}{0}$ $\frac{0000}{0}$

11 bit (Set Index)

Row Major

APR[0][5] = First row, 6^{th} element

$\underbrace{[[0][0], 0.1, 0.2, 0.3, 0.4]}_{5\text{ element}}$

$5 \times 8 = 40\ (\ )_{16}$

F F $\frac{0}{0000}$ $\frac{0}{0000}$ $\frac{|0}{0|000}$ $\frac{0}{0000}$ $\frac{2}{0010}$ | $\frac{8}{1000}$

32 8

	(in Hex virtual address)	Set Index
APR [0] [0]	FF 000 <u>000</u>	000 0000 0000
APR [0] [4]	FF 000 020	000 0000 0010
APR [4] [0]	FF 008 000	000 0000 0000
APR [0] [5]	FF 000 028	000 0000 0010

Match APR [4] [0]

Scan for Video solution

21. (d)

2 way set associative, block size = 16 B, VA = 32 bit

Cache size = 64 KB

Block size = 16 Byte

Block offset = [$\log_2$ block size] = $\log_2 16$ = 4 bit

Number of lines = $\frac{\text{cm size}}{\text{block size}} = \frac{64\text{ KB}}{16\text{B}}$

$= \frac{2^{16}}{2^4} = 2^{12}$ lines

Number of set = $\frac{\text{Number of lines}}{\text{N- way}} = \frac{2^{12}}{2} = 2^{11}$ set

Set offset = 11 bit

VIVT

Virtual Index and virtually Tag

← 32 bit →

TAG	Set offset	Block 1 word offset
17 bit	11 bit	4 bit

2 way set associative

TAG = 32 – (11 + 4)

Tag memory size = number of lines × Tag bits

$= 2^{12} \times 17$ bit $= 2^2 \times 17 \times 2^{10}$ bit

= 68 k bits

OR

Tag memory size = Number of set × Block per set × Tag bit

$= 2^{11} \times 2 \times 17$ bit $= 2 \times 2 \times 17 \times 2^{10}$ bit

Tag memory size = 68 k bits

Scan for Video solution

22. (a, b, d)

Cache size = 2KB $\Rightarrow 2^{11}$ B

Block size = 64 Byte = 2^6 B

Word offset = $\log_2$(block size) $\Rightarrow \log_2(2^6)$

Word offset (W.O) ⇒ 6 bits

Physical memory (MM) = 64KB = 2^{16}Byte.

Physical address (P.A) = 16 bits

Number of lines = $\dfrac{\text{Cache size}}{\text{Block size}} = \dfrac{2^{11}\text{B}}{2^{6}\text{B}} = 2^5$ Lines

Line offset (L.O) = 5 bits.

Direct Mapping:

⟵ 16bit ⟶

TAG	LINE(L.O) OFFSET	WORDOFFSET (W.O)
5bit	5bit	6bit

P:OxA248

TAG(5bit)	Line offset	Word offset (6bit)
10100	01001	001000

Line No : 9

Q:OxC28A

TAG(5bit)	Line Offset (5bit)	Word offset (6bit)
11000	01010	001010

Line No : 10

R:OxCA8A

TAG(5bit)	Line Offset (5bit)	Word offset (6bit)
11001	01010	001010

Line No : 10

S:OxA262

TAG(5bit)	Line Offset (5bit)	Word offset (6bit)
10100	01001	100010

Line No : 9

Line Number

p : 9

Q : 10

R : 10

S : 9

P : TAG(5bit) 10100 | Line offset 01001 — Line No : 9

S : TAG(5bit) 10100 | Line offset 01001 — Line No : 9

Here P and S same main memory block and mapped to the same cache line number.

Q : TAG(5bit) 11000 | Line offset (5bit) 01010 — Line No : 10

R : TAG(5bit) 11001 | Line offset (5bit) 01010 — Line No : 10

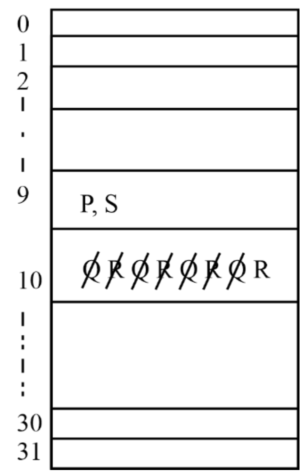
Here Q and R are the different main memory block but mapped to the same cache line number.

1st Interation
P : Miss
Q : Miss
R : Miss
S : Hit

2nd Interations
P : Hit
Q : Miss
R : Miss
S : Hit

3rd Interation
P : Hit
Q : Miss
R : Miss
S : Hit

4th Interation
P : Hit
Q : Miss
R : Miss
S : Hit



Every access to S is a hit.**: True.**

Once P is brought to the cache it is never evicted.

: True.

At the end of the execution only R and S reside in the cache.**: False**

Every access to R evicts Q from the cache. **: True.**

Scan for Video solution

23. (76 to 76)

Number of CM blocks = 256

(Number of Lines)

2-way set associative

Number of set $= \frac{256}{2} = 128$

$\boxed{\text{SET}[S] = 128}$

K MODS = i

$\boxed{\text{K MOD128} = i}$

K : mm block number

S : Number of cm set

i = cache set number

1st time

K MOD 128 = i

MM block	CM set number
0	MOD 128 = 0 → A
↑ 128	= 0 A
256	= 0 A
↑ 128	= 0 → HIT
0	= 0 → B → 1
↑ 128	= 0 → HIT
256	= 0 → B → 1
128	= 0 → HIT
↑ 1	= 1 → A
129	= 1 → A
257	= 1 → A
129	= 1 → HIT
↑ 1	= 1 → B → 3
129	= 1 → HIT
↑ 257	= 1 → B → 4
129	= 1 → HIT

compulsory miss (A)
conflict miss (B)
2 way set associative with LRU

1st time

Set		
0	~~0~~ ~~256~~ ~~0~~ 256	128
1	~~1~~ ~~257~~ ~~1~~ 257	129
2		
⋮		
127		

CM

2nd time

1st time = 4 conflict miss
2nd time access ⇒ conflict miss = 8
3rd = 8 conflict miss
...
10th time access = 8 conflict miss
Total conflict = 4 + 8 × 9 times
= 4 + 72
= 76 miss

K MOD 128 = i

MM block	CM set number
0	MOD 128 = 0 → B → 1
↑ 128	= 0 → HIT
256	= 0 → B → 2
↑ 128	= 0 → HIT
0	= 0 → B → 3
↑ 128	= 0 → HIT
256	= 0 → B → 4
128	= 0 → HIT
1	= 1 → B → 5
↑ 129	= 1 → HIT
257	= 1 → B → 6
↑ 129	= 1 → HIT
1	= 1 → B → 7
↑ 129	= 1 → HIT
257	= 1 → B → 8
129	= 1 → HIT

compulsory miss (A)
conflict miss (B)
2 way set associative with LRU

1st time

Set	Lines	
SET 0	~~256~~ ~~0~~ ~~256~~ ~~0~~	256
	128	
SET 1	~~257~~ ~~1~~ ~~257~~ ~~1~~	257
	129	
	⋮	
SET 127		

CM

Scan for Video solution

24. (a)

$A \geq k$

1 2 3 1 2 5 1 4 6 ⇒ $\boxed{N = 9}$

Unique block address (1, 2, 3, 4, 5, 6) ⇒ $\boxed{n = 6}$

Min miss $\frac{6}{9}$ [cold/compulsory miss.]

OR

1 2 4 1 5 6 1 8 9 1 7 8 2 5 3 ⇒ $\boxed{N = 15}$

n = 9 Unique block address $\boxed{N = 9}$

min miss $= \frac{9}{15}$ (compulsory)

minimum number of misses = n. [compulsory miss]

for maximum misses

k concept

Worst case: 7 $\boxed{1\ 3\ 6}$ 7

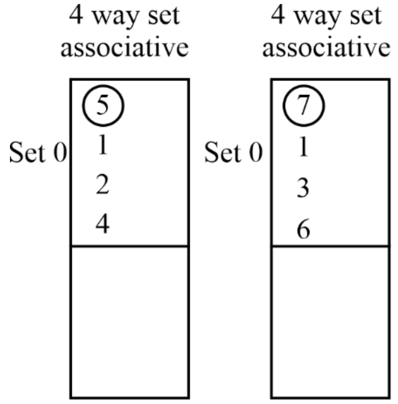
5 $\boxed{1\ 2\ 4}$ 5

At most k [k =3]

$\boxed{\text{Associativity } A \geq k}$

Case I:

Maximum allocation in same cache set (Any)



Case II: If that unique address repeated

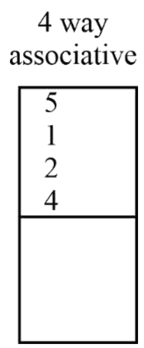
5 1 2 4 1 4 2 4 1 2 4 2 1 2 4 1 4 5

k = 3

At most k [unique = 3 address]

1, 2, 4

No other extra miss



Total number of misses = n

$\boxed{\text{Miss ratio} = \frac{n}{N}}$

Alternate approach:

Total access = N so divide by 'N' so option (c) and (d) wrong

Unique block address = n

So minimum not $\frac{1}{N}$ option (b) also wrong

$\boxed{\frac{n}{N}}$

Scan for Video solution

25. (d)

Mm = 256 block

Cache = 16 block

number of line = 16

4 way set Associative

number of set = $\frac{\text{Number of LINES}}{\text{N} - \text{ways}} = \frac{16}{4}$

$\boxed{\text{Number of SETS} = 4}$

K MOD S = i

Or

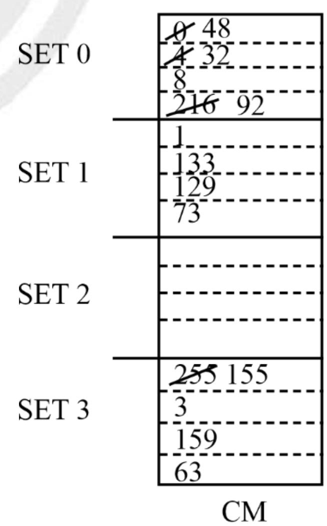
Cache set address = mm block mod number of cm set

$\boxed{\text{k MOD } 4 = \text{i}}$

0 = 0
255 = 3
1 = 1
4 = 0
3 = 3
8 = 0
133 = 1
159 = 3
216 = 0
129 = 1
63 = 3
8 = 0 — Hit
48 = 0
32 = 0
73 = 1

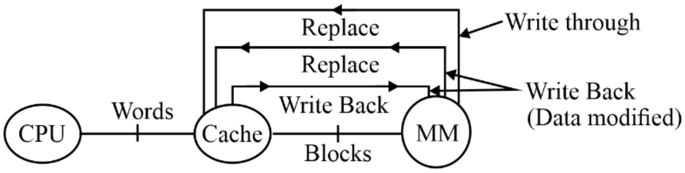
LRU ↑

92 MODE 4 = '0'
155 MOD 4 = 3

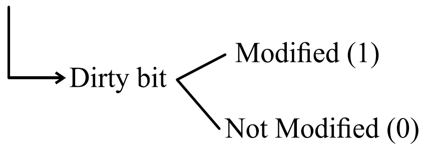


Scan for Video solution

26. (a, b, d)



Write through (Both cm & mm update at a same time) (simultaneously) so no need of dirty bit
Write Back: (first cache is update & mm update later).



False

(a) Incorrect, WB has dirty bit.

(b) Incorrect/False:

(c) Corect because in WT both cache memory & main memory update at the same time.

(d) False/Incorrect, depends on mapping technique & replacement algorithm.

Scan for Video solution

27. (0.85 to 0.85)

Range (0.85 to 0.85)

Cache Hit Rate [h] = 0.8

Cache Access Time [t_c] = 10 ns.

Miss penalty (M.P) = 100 ns.

$T_{avg} = h \times t_c + (1 - h)$ (M.P + t_c)

$= 0.8 \times 10 + (1 - 0.8)(100 + 10)$

$= 8 + 0.2(110)$

$T_{avg} = 30$ ns.

Memory with optimization (new)

Wants to reduce the miss rate ie increase the Hit Rate.

Miss penalty = 100 ns (not affected in new)

$T_{avg} = 30$ ns.

Cache Access Time($t_{c_{new}}$) = 15 ns

Cache Hit Rate = h_{new}.

$T_{avg} = h_{new} \times t_{c_{new}} + (1 - h_{new})(\text{M.P} + t_{c_{new}})$

$30 = 15 \times h_{new} + (1 - h_{new})(100 + 15)$

$30 = 15\, h_{new} + 115 - 115\, h_{new}$

85 = 100 hnew

$h_{new} = 85/100$

$h_{new} = 0.85$

Method 2

Cache Hit Rate [h] = 0.8

Cache Access Time [t_c] = 10 ns.

Miss Penalty (M.P) = 100 ns.

So, its Hierarchical Access.

T_{avg} = cache time + (1 – h) miss penalty

$= 10 + (1 - 0.8)\,100$

$T_{avg} = 30$ ns

<u>Memory with optimization (new)</u>

Wants to reduce the miss rate ie increase the Hit Rate.

New miss penalty = 100 ns (not affected in new)

$T_{avg_{new}} = 30$ ns. (not affected in new)

Cache Access Time ($t_{c_{new}}$) = 15 ns.

Cache Hit Rate = h_{new}

$T_{avg_{new}}$ = Cache time + (1 – h_{new}) miss penalty

$30 = 15 + (1 - h_{new})\,100$

$30 = 15 + 100 - 100\, h_{new}$

$100\, h_{new} = 85$

$h_{new} = 85/100$

$h_{new} = 0.85$

Scan for Video solution

28. (d)

S_1: Write through → simultaneous update in L_1 and NO role of dirty bit because here, its simultaneous update.

If miss occur directly replace the block

S_2: Write through: No write allocate.

Write back: Write allocation

Incorrect

29. (13.3 to 13.5)

Block size =256 bytes

Word size = 64 bit = 8 bytes

Cache bit ratio = 94%

Cache access time $[t_c]$ = 3 nesec.

First word takes 20 and each subsequent word take 5 nsec.

Number of word = $\frac{256 \text{ B}}{8 \text{ B}} = 32$

$T_{avg} = 0.94 \times 3 + (1 - 0.94)(3 + 20 + 31 \times 5)$

$\Rightarrow 2.82 + 0.06\,[178]$

$\Rightarrow 2.82 + 10.68$

$\Rightarrow$ 13.5 sec

30. (160 to 160)

Clock frequency = 60 MHz

Cycle time = $\frac{1}{\text{clock frequency}} = \frac{1}{60 \times 10^6}$ sec

Cache block size = 8 Words

1 word size = 4 Bytes

Block size = 4 Bytes

Block size = 8 × 4 byte = 32 Byte

Total time taken to

Transfer cache block =

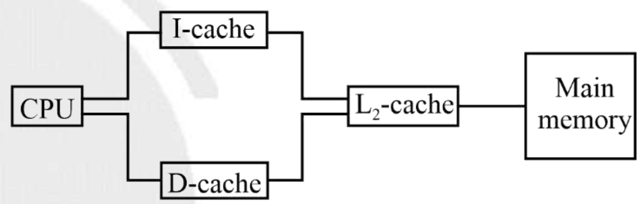
$$\overset{1}{(\text{Accept})} + \overset{3}{(\text{Fetch})} + \overset{8}{\begin{pmatrix}\text{Transmit 1 word}\\ \text{per cycle}\end{pmatrix}} = 12 \text{ cycle}$$

$$\text{Band width} = \frac{\text{Total data}}{12 \text{ cycle}} = \frac{32\text{Byte}}{12 \times \left(\frac{1}{60 \times 10^6}\right)}$$

$$\Rightarrow \frac{32\text{Byte}}{\cancel{12}} \times \overset{5}{\cancel{60}} \times 10^6 \text{ sec}$$

$= 160 \times 10^6$ bytes/sec

31. (4.72 to 4.72)



$T_{avg\ read\ inst^n\ fetch} = H_1T_1 + (1 - H_1)H_2(T_2 + T_1)$
$+ (1 - H_1)(1 - H_2)H_3(T_m + T_2 + T_1)$

Inst^n fetch $= 0.8 \times 2 + (1 - 0.8)\,0.9\,(8 + 2) + (1 - 0.8)$
$(1 - 0.9)\,[90 + 8 + 2]$

$\Rightarrow 1.6 + (0.2)(0.9)10 + (0.2)(0.1)(100)$

$T_{avgread\ inst^n\ fetch}$ = 5.4 nsec

$T_{avgread\ operand\ fetch} = H_1T + (1 - H_1)H_2(T_2 + T_1)$
$+ (1 - H_1)(1 - H_2)H_3(T_m + T_2 + T_1)$

$= 0.9 \times 2 + (1 - 0.9)\,0.9\,(8 + 2)\,(0.1)(0.1)$
$(90 + 8 + 2)$

$= 1.8 + 0.9 + 1 = 3.7$ nsec

$T_{avgread\ acceesstime}$ = frequency of inst^n fetch × $T_{avgread\ inst^n\ fetch}$ + frequency of operand fetch × $T_{avg\ operand\ fetch}$

$= 0.60 \times 5.4 + - 0.40 \times 3.7$

$$\boxed{T_{avg_{read}\ access\ time} = 4.72 \text{ nsec}}$$

Scan for Video solution

32. (30 to 30)

Cache access time [t_c] = 1msec

Disk access time [t_d] = 10 msec

Case I: When cache size is 10 MB that time miss rate 10.8.

$T_{avg} = h \times t_c + *(1 - h)(t_D + t_C)$

$\Rightarrow 0.2 \times 1 + (0.8)(10 + 1) \Rightarrow 0.2 + 0.8(11)$

T_{avg} = 9 msec at 10 MB cache size

Case II**:** When size is 20 MB that time miss rate = 0.6

$T_{avg} = 0.4 \times 1 + 0.6[10 + 1]$

$\Rightarrow 0.4 + 0.6[11] \Rightarrow 0.4 + 6.6$

$\boxed{T_{avg} = 7 \text{msec}}$ at 20 MB

Case III: When cache size is 30 MB, that time miss rate = 0.4

Hit rate (h) = 1– 0.4 = 0.6

$T_{avg} = h \times t_c + (1 - h)(t_D + t_c)$

$\Rightarrow 0.6 \times 1 + (0.4)(10 + 1) \Rightarrow 0.6 + 0.4(11)$

$\Rightarrow 0.6 + 4.4$

$\boxed{T_{avg} = 5 \text{msec}}$ at 30 MB cache size

Case IV: When size 40 MB, that time miss rate 0.35

$T_{avg} = h \times t_c + (1 - h)(t_D + t_C)$

$\Rightarrow 0.65(1) + (0.35)(10 + 1) \Rightarrow 0.65 + 0.35(11)$

$\Rightarrow 0.65 + 3.85$

$\boxed{T_{avg} = 4.5 \text{msec}}$ at cache size 40 MB

Case V: when cache size is 50 MB that time miss rate = 0.3

$T_{avg} = h \times t_C + (1 - h)(t_D + t_C)$

$\Rightarrow 0.7 \times 1 + (0.3)(10 + 1)$

$\boxed{T_{avg} = 4 \text{msec}}$

Hence, the smallest cache size required to ensure an average read latency of less than 6 ms will be 30 MB.

Scan for Video solution

33. (14 to 14)

Hit ratio = 80%

Cache read hit takes = 5 nsec

Read miss = 50 nsec

$T_{avg} = 0.80 \times 5 + (1 - 0.80) \times 50$

$\Rightarrow 4 + (0.20)\,50$

$\Rightarrow = 4 + 10$

$\boxed{T_{avg} = 14 \text{ nsec}}$𝔽

Scan for Video solution

34. (1.68 to 1.68)

100 instruciton fetch operation + 60 memmory operand read + 40 memory operand write

Total # Inst^n/operatin = 200

100 instruction fetch operation (memory read) and 60 memory read

Total memory read operation = 100 + 60 = 160

Read: t_C = 1 ns

Hit ratio = 0.9

$T_{readmiss}$ = 5 nsec

$T_{avgread} = 0.9 \times 1 + (0.1)(5)$

$\Rightarrow 0.9 + 0.5 = 1.4$ nsec

Total time required to perform read operation = 160 × 1.4 = 224 nsec

40 memory write operation

Write:

t_C = 2nsec

Hit ratio = 0.9

$T_{wirtemiss}$ = 10 ns

$T_{avgwrite} = 0.9 \times 2 + 0.1(10)$

$\Rightarrow 1.8 + 1$

$\boxed{T_{avgwrite} = 2.8}$

Total time taken to perform write operation = 40 × 2.8 = 112 nsec

Total time taken for 200 instructions = 224 + 112 = 336 nsec

Average memory access time = $\frac{336}{200} = 1.68 \text{ nsec}$

Scan for Video solution

35. (c)

$\boxed{\text{Tag} = 16 \text{ bit}}$

Physical address = 32 bit

Cache size = 256 k byte

Block size = 32 byte

4 way set associative $[2^2]$

Word offset = [$\log_2$ blog size] ⇒ [$\log_2$ 32]

Word offset = 5 bit

$$\text{Number lines} = \frac{\text{CM size}}{\text{Block size}} = \frac{256\text{kB}}{32\text{B}} = \frac{2^{18}\text{B}}{2^{T}\text{B}}$$

$$= 2^{13} \text{ Lines}$$

$$\text{Number set} = \frac{\text{Number Lines}}{\text{N.way}} = \frac{2^{13}}{2^2} = 2^{11}\text{set}$$

S. O = 11 bit

32 bit

TAG	LINE OFFSET	WORD OFFSET
16 bit	11 bit	5 bit

$\boxed{\text{TAG} = 16\text{bits}}$

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36. (a)

Tag directory size = Number of of lines × tag entry in bits

$= 2^{13} \times [16 + 2 + 1 + 1]$ bits

$= 2^3 \times 20 \times 2^{10}$ bits = 160 k bits

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37. (d)

Physical address = 32 bit   1 valid bit

Cache size = 8 kB   1 modified bit

Block size = 32 byte

Word offset = [$\log_2$ Block size] ⇒ [$\log_2 32$]

$\boxed{\text{Word offset} = 5 \text{ bit}}$

$$\# \text{ lines} = \frac{\text{CM size}}{\text{Block size}} = \frac{8\text{kB}}{32\text{byte}} = \frac{2^{13}}{2^5} = 2^8 \text{ lines}$$

$\boxed{\#\text{LINES} = 256}$

Line offset [L.0] =[$\log_2 256$]

$\boxed{\text{L.O} = 8 \text{ bit}}$

(direct mapped cache)

32 bit

TAG	LINE OFFSET	WORD OFFSET
	8 bit	5 bit

TAG ⇒ 32 – (8 + 5)

$\boxed{\text{TAG} = 19 \text{ bit}}$

Write back cache = Tag entry = 19 + 1 + 1 = 21 bits

Tag memory size = number of lines × Tag entry size

⇒ 256 × 21 bit

$\boxed{\text{TAG memory size} = 5376 \text{ bit}}$

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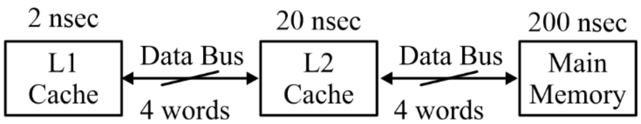
38. (c)



Time taken to transfer L_2 to L_1 = time to read word from L_2 + update or write/store in L_1

= 20 + 2 = 22 nsec

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39. (c)

Time taken to transfer data from mm to L2 = Time to read word from mm + time to store or update write in L2.

$4[200 + 20] \Rightarrow 4 \times 220 = 880$ nsec

Time taken to transfer data from L2 to L1 = time to read word form L2 + time to store or update / write word in L1

= 20 + 2 = 22 nsec.

Total time to transfer data

= 880 (mm to L2) + 22 (L2 to L1)

From mm to L2 and L2 to L1 = 902 nsec.

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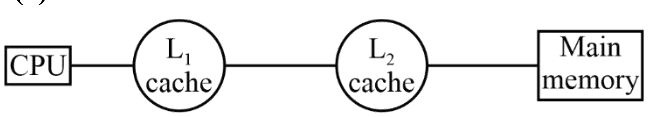
40. (a)



1. L_1 must be write through cache: not correct always (we can use write back cache)

2. L_2: not correct

3. Not correct

4. Correct : its necessary

$\boxed{L_2 \geq L_1}$

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41. (c)

IA_4 IA_3 Connect to decodes.

IA4	IA3		
0	0	:	x_1 enabled
0	1	:	x_2 enabled
1	0	:	x_3 enabled
1	1	:	x_4 enabled

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0

IA_6 IA_5 IA_4 IA_3 IA_2 IA_1 IA_0

0 $0 \rightarrow x_1$ enabled (0)

0 $1 \rightarrow x_2$ enabled (8)

1 $0 \rightarrow x_3$ enabled (16)

1 $1 \rightarrow x_4$ enabled (24)

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42. (59 to 60)

In chip total number of rows = 2^{14}

Time to per form one refresh operation = 50 nsecs

Total time taken to refresh operation in all rows

$= 2^{14} \times 50$ nsecs

$\Rightarrow 2^4 \times 2^{10} \times 50 = 819200$nsec = 0.8192 msec.

Refresh period = 2 msec

= 0.8192 msec

Percentage of time spend in refersh operation = $\frac{0.8192}{2} = 40.96\%$

Percentage of time available for read write operation

=100 – 40.96 = 59.04%

Alternate method

Total time taken for refresh operation = 0.8192 msec

Refresh period = 2 msec

Time available for read/write operationi = 2 – 0.8192

$\Rightarrow$ 1.1808 msec

%time available for read/write operation =

$\frac{1.1808}{2} = 59.04\%$

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43. (31 to 31)

Main memory size = 4 GB [2^{32} byte]

1 word size = 2 Byte

Memory is word addressable

$\boxed{1 \text{ word} = 2\text{Byte}}$

4 G Byte $\rightarrow \frac{4 \text{ G Byte}}{2 \text{ Byte}}$ Word $\rightarrow$ 2 G word

Word addressable

Mm size = 2G word

$\Rightarrow 2^1 2^{30}$ word = 2^{31} words

$\boxed{\text{Address bus size} = 31 \text{ Bit}}$

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44. (10000 to 10000)

Starting 0 to 100 ns $\Rightarrow$ data is available in bus for module 0

0 to 100 $\Rightarrow$ module 0 $\Rightarrow$ 100 ns + 500 = 600 nsec

100 to 200 $\Rightarrow$ module 1 $\Rightarrow$ 200 ns + 500 = 700 nsec

200 to 300 $\Rightarrow$ module 2 $\Rightarrow$ 300 ns + 500 = 800 nsec

300 to 400 $\Rightarrow$ module 3 $\Rightarrow$ 400 ns + 500 = 900 nsec

400 to 500 $\Rightarrow$ module 4 $\Rightarrow$ 500 ns + 500 = 1000 nsec

500 to 600 $\Rightarrow$ module 5 $\Rightarrow$ 600 ns + 500 = 1100 nsec

600 to 700 $\Rightarrow$ module 6 $\Rightarrow$ 700 ns + 500 = 1200 nsec

700 to 800 $\Rightarrow$ module 7 $\Rightarrow$ 800 ns + 500 = 1300 nsec

In every 100 ns, initiate 1 word data to the bus.

$\Rightarrow$ bus$\rightarrow$ initate one request (1 word request)

For 1 milli second number of words we can initate

$$\Rightarrow \frac{1 \text{ milli sec}}{100 \text{ n sec}} = \frac{10^{-3}}{100 \times 10^{-9}} = 10^4 = 10{,}000$$

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45. (b)

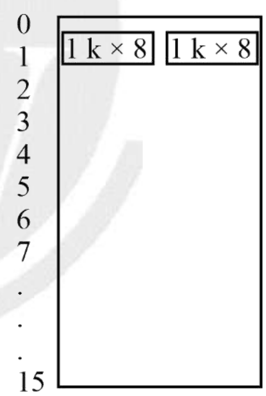
RAM CHIP size = 1 k × 8 (1024 word, 8 bit)

Wants to contruct RAM of size = 16 k × 16

Number of RAM chip requried = $\frac{\cancel{16} \text{ k} \times \cancel{16}}{\cancel{1}\text{k} \times \cancel{8}}$

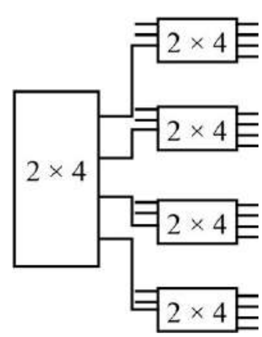
$\Rightarrow$ 16 × 2 RAM chip $\Rightarrow$ 16 lines

Decoder (4 × 16) required



Asking how many 2 × 4 decodes required

Total 2 × 4 decodes is = 4 + 1 = 5



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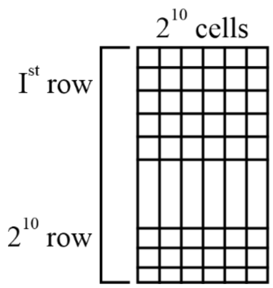
46. (b)

Mm capacity = 4m byte

1DRAM size = 1 m × 1 bit



$= 2^{10} \times 2^{10}$
$= 2^{20} \times 1$
$= 1\text{m} \times 1$ bit

Number of RAM chip = $\dfrac{4\text{ m} \times \text{Byte}}{1\text{m} \times 1\text{bit}} = \dfrac{4\text{ m} \times 8\text{bit}}{1\text{m} \times 1\text{bit}}$

= 32 RAM chip

Note: Dram chip one refresh ⇒ one row is refresh (one row cells)

Total number of rows = 2^{10}

In 1 chip, total number of refresh required = 2^{10} refresh operations

1 refresh operation takes = 100 nsec.

1 Chip refresh time = $2^{10} \times 100$ nsec

Note: all ram chip are refresh in parallel

Total refersh time = $\boxed{2^{10} \times 100\text{nsec}}$

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47. (c)

Memory capcity = 256 k Byte

Each ram chip size = 32 k × 1 bit

Numer of ram chip required = $\dfrac{\text{MM size}}{\text{RAM chip size}}$

$= \dfrac{256\text{ k Byte}}{32\text{k} \times 1\text{bit}} = \dfrac{{}^{8}\cancel{256\text{ k}} \times {}^{8}\cancel{8\text{ Bit}}}{\cancel{32\text{k}} \times \cancel{1\text{ bit}}} = 64$

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❑❑❑

CHAPTER

5

I/O INTERFACE

Modes of Data Transfer

1. [MCQ] [GATE-2022: 1M]

Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput?

(a) DMA based I/O transfer
(b) Interrupt driven I/O transfer
(c) Polling based I/O transfer
(d) Programmed I/O transfer

2. [NAT] [GATE-2021 : 1M]

Consider a computer system with DMA support. The DMA module is transferring one 8-bit character in one CPU cycle from a device to memory through cycle stealing at regular intervals. Consider a 2 MHZ processor. If 0.5% processor cycles are used for DMA, the data transfer rate of the device is _____ bit per second.

3. [MCQ] [GATE-2020 : 1M]

Consider the following statements:

I. Daisy chaining is used to assign priorities in attending interrupts.
II. When a device raises a vectored interrupt, the CPU does polling to identify the source of interrupt.
III. In polling, the CPU periodically checks the status bits to know if any device needs its attention.
IV. During DMA, both the CPU and DMA controller can be bus masters at the same time.

Which of the above statements is/are TRUE?

(a) I, II only (b) I and IV only
(c) I and III only (d) III only

4. [NAT] [GATE-2016 : 2M]

The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29, 154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA Controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is_____.

5. [MCQ] [GATE-2011 : 2M]

On a non-pipelined sequential processor, a program segment, which is a part of the interrupt service routine, is given to transfer 500 bytes from an I/O device to memory.

Initialize the address register

Initialize the count to 500

LOOP: Load a byte from device

Store in memory at address given by address register

Increment the address register

Decrement the court

If count! = 0 go to LOOP

Assume that each statement in this program is equivalent to a machine instruction which takes one clock cycle to execute if it is a non-load/store instruction. The load-store instructions take two clock cycles to execute.

The designer of the system also has an alternate approach of using the DMA controller to implement the same transfer. The DM controller requires 20 clock cycles for initialization and other overhead. Each DMA transfer cycle takes two clock cycles to transfer one byte of data from interrupt driven program-based input-output?

(a) 3.4 (b) 4.4
(c) 5.1 (d) 6.7

Secondary Memory

6. [NAT] [GATE-2023 : 1M]

A keyboard connected to a computer is used at a rate of 1 keystroke per second. The computer system polls the keyboard every 10 ms (milli seconds) to check for a keystroke and consumes 100 μs (micro seconds) for each poll. If it is determined after polling that a key has been pressed, the system consumes an additional 200 μs to process the keystroke. Let T_1 denote the fraction of a second spent in polling and processing a keystroke.

In an alternative implementation, the system uses interrupts instead of polling. An interrupt is raised for every keystroke. It takes a total of 1 ms for servicing an interrupt and processing a keystroke. Let T_2 denote the fraction of a second spent in servicing the interrupt and processing a keystroke.

The ratio $\frac{T_1}{T_2}$ is ___________. (Rounded off to one decimal place)

7. [NAT] [GATE-2015 : 2M]

Consider a typical disk that rotates at 15000 rotations per minute (RPM) and has a transfer rate of 50×10^6 bytes/sec. If the average seek time of the disk is twice the average rotational delay and the controller's transfer time is 10 times the disk transfer time, the average time (in milliseconds) to read or write a 512-byte sector of the disk is _____.

8. [NAT] [GATE-2015 : 2M]

Consider a disk pack with a seek time of 4 milliseconds and rotational speed of 10000 rotations per minute (RPM). It has 600 sectors per track and each sector can store 512 bytes of data. Consider a file stored in the disk. The file contains 2000 sectors. Assume that every sector access necessitates a seek, And the average rotational latency for accessing each sector is half of the time for one complete rotation. The total time (in milliseconds) needed to read the entire file is ____.

9. [MCQ] [GATE-2013 : 2M]

Consider a hard disk with 16 recording surfaces (0-15) having 16384 cylinders (0-16383) and each cylinder contains 64 sectors (0-63). Data storage capacity in each sector is 512 bytes. Data are organized cylinder-wise and addressing format is <cylinder no., surface no., sector no>. A file of size 42797 KB is stored in the disk and the starting disk location of the file is < 1200, 9, 40>. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner?

(a) 1281
(b) 1282
(c) 1283
(d) 1284

10. [MCQ] [GATE-2011 : 2M]

An application loads 100 libraries at start-up. Loading each library requires exactly one disk access. The seek time of the disk to a random location is given as 10 ms. Rotational speed of disk is 6000 rpm. If all 100 libraries are loaded from random locations on the disk, how long does it take to load all libraries? (The time to transfer data from the disk block once the head has been positioned at the start of the block may be neglected.)

(a) 0.50s
(b) 1.50s
(c) 1.25s
(d) 1.00s

Common Data for next two questions:

A hard disk has 63 sectors per track, 10 platters each with 2 recording surfaces and 1000 cylinders. The address of a sector is given as a triple ⟨c,h,s⟩, where c is the cylinder number, h is the surface number and s is the sector number. Thus, the 0th sector is addressed as⟨0,0,0⟩, the 1st sector as ⟨0,0,1⟩, and so on.

11. [MCQ] [GATE-2009 : 2M]

The address ⟨400,16,29⟩ corresponds to sector number:

(a) 505035 (b) 505036
(c) 505037 (d) 505038

12. [MCQ] **[GATE-2009 : 2M]**

The address of 1039th sector is

(a) ⟨0,15,31⟩
(b) ⟨0,16,30⟩
(c) ⟨0,16,31⟩
(d) ⟨0,17,31⟩

13. [MCQ] **[GATE-2008 : 2M]**

For a magnetic disk with concentric circular tracks, the latency is not linearly proportional to the seek distance due to

(a) Non-uniform distribution of requests
(b) Arm starting and stopping inertia
(c) Higher capacity of tracks on the periphery of the platter
(d) Use of unfair arm scheduling policies.

❑❑❑

ANSWER KEY

1.	(a)	**2.**	(80000 to 80000)	**3.**	(c)	**4.**	(456 to 456)
5.	(a)	**6.**	(10.2 to 10.2)	**7.**	(6.1 to 6.2)	**8.**	(14020 to 14020)
9.	(d)	**10.**	(b)	**11.**	(c)	**12.**	(c)
13.	(b)						

SOLUTIONS

1. (a)

In DMA bulk amount of data transferred from Hard disk (secondary Memory) to main memory with the highest through put.

In programmed I/O transfer CPU Time is depend on speed of I/O device, so utilization is very less & not used for bulk amount of data transfer.

Interrupt driven I/O transfer is used for small amount of data transfer with the involvement of the CPU.

But in DMA (Direct Memory Access) based I/O transfer bulk amount of data transfer without the involvement of the CPU. & DMA has the highest priority so through put is very high.

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2. (80000 to 80000)

Clock frequency = 2 MHz

Cycle time = $\frac{1}{2\text{MHz}}\text{sec} = 0.5\,\mu\text{sec}$

1 cycle time = $\frac{1}{2}\times 10^{-6}$ sec

In one second total number of CPU cycle = 2×10^6 cycle

0.5 % of the CPU cycle are used to data transfer by the DMA = $0.5\% \times 2 \times 10^6$ =

$$\frac{0.5}{100}\times 2\times 10^6 = 10000 \text{ cycle}$$

Total number of cycle taken by the DMA for data transfer = 10, 000 cycle

In 1 cycle – 8 bit data transfer

In 10,000 cycle = 10000 × 8 = 80000

Data transfer rate = 80000

OR

CPU cycles can be completed in ½ * 10^6 seconds.

Therefore, there will be 2 * 10^6 cycles in a second plus an additional 10,000 DMA cycles, or 0.5%, at which time 8 bits are transmitted.

Thus, 80,000 bits will be sent in total in one second.

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3. (c)

I Daisy Channing: true

II false

III true

IV false

At a time either CPU or DMA can be master on bus.

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4. **(456 to 456)**

Data count register = 16 bit

Count value : How many number of byte/word transferred by the DMA from input output (I/O) to memory in one cycle

Total count value = $2^{16} - 1$

In one time. Total # byte transferred

$= 2^{16} - 1 = 65{,}535$ Byte

Total # byte to be transferred = 29, 154 Byte

Number of time DMA needs the control on system bus

$$= \left\lceil \frac{29{,}154 \times 1024 \text{ Byte}}{65{,}535 \text{ Byte}} \right\rceil = 456$$

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5. **(a)**

Initialize the address register → 1 cycle

Initialize the count to 500 → 1 cycle

LOOP7: Load a byte from device → 2 cycle

Store in memory at address given by address register → 2 cycle

Increment the address register → 1 cycle

Decrement the court → 1 cycle

If count! = 0 go to LOOP → 1 cycle

In each one interation of loop takes

= 2+ 2 + 1 + 1 +1 = 7 Cycle

Loop execute = 500 times (Iteration)

Total time taken in 500 interatin = 500 × 7 = 3500 Cycle

Total time in ISR = 3500 + 2 = 3502 Cycle

500 Byte transfer using DMA

In DMA transfer Tim = 20 + 2 × 500 = 1020 Cycle

$$\text{Speed up} = \frac{3502}{1020} = 3.4$$

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6. **(10.2 to 10.2)**

1 key stroke per second,

Each polling takes = 100μsec.

After every 10 m sec polling is done.

If key is press then additional = 200 μsec

In every 10×10^{-3} sec →1 poll.

$$\text{In 1 sec} \rightarrow \frac{1}{10 \times 10^{-3}} \Rightarrow 10^{+2} = 100 \text{ poll in one second.}$$

Each poll takes = 100μsec

Total 100 poll takes = 100 × 100 = 10000μsec = 10 msec.

If key stroke is pressed then addition = 200μsec

Total time for polling and processing pressing key stroke = 10.000 + 200 = 10,200μsec = 10.2msec

T_1 in 1 sec — 10.2 msec.

$$T_1 = \frac{10.2}{1000}$$

T_2 Alternative approach: Interrupt

Total time taken to servicing interrupt = 1msec.

and processing key stroke.

$$T_2 = \frac{1}{1000}$$

(In 1 sec = 1000 msec)

$= 1000 \times 10^{-3} = 1$ sec

$$\frac{T_1}{T_2} = \frac{10.2}{1000} \times \frac{1000}{1} = 10.2$$

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7. (6.1 to 6.2)

Average disk access time or average read/write time

= Average S.T + Average R.T + D.T.T + overhead (if any given)

15000 RPM

15000 rotation in 60 sec

In 1 rotation = $\frac{^{2}\cancel{60}}{\cancel{15000}_{500}} = \frac{1}{\cancel{250}}\text{sec} \times \frac{\cancel{1000}}{1000}$

1 rotation time = 4 m sec

$\boxed{\text{Average rotational Latency} = \frac{1}{2} \times \text{RT} = 2\text{msec}}$

Average S.T = 2 × average R.T

⇒ 2 × 2 = 4msec

$\boxed{\text{Average S.T} = 4\text{msec}}$

Transfer rate = 50 × 10^6 Byte/second

50 × 10^6 Byte …..1second

1 byte ……… $\frac{1}{50 \times 10^6}$sec

512 Byte _____ $\frac{512}{50} \times 10^{-6}$

⇒ 10.24 × 10^{-6}

0.01024 msec

$\boxed{\text{D.T.T} = 0.0102 \text{ msec}}$

Control time = 10 × 0.01024 msec.

= 0.1024 m sec.

D.A.T. = 4 + 2 + 0.01024 + 0.1024

D.A.T = 6.11 msec

6.1 m sec

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8. (14020 to 14020)

Time required to read one sector = seek time + average R.T + Data transfer time

Seek time (S.T) = 4 msec

600 sector /track each sector capcity = 512 Byte

10000 rpm

10000 rotation in 60 second

1 rotation ….. $\frac{6\cancel{0}}{1000\cancel{0}}$

⇒ 6 × 10^{-3} sec

1 rotation takes = 6 msec

Average roational latency = $\frac{1}{2} \times 6$

Average Rotational latency = 3 msec

1 Track capacity = 600 × 512 Byte

In 1 Rotational 1 complete track

600 × 512 Byte takes _______6msec

1 Byte → $\frac{6}{600 \times 512}$msec

512 Byte (1 sector) → $\frac{512 \times 6}{600 \times 12} = \frac{1}{100}$msec

D.T.T = 0.01 msec

Total time required for 1 sector

= 4 + 3 + 0.01 = 7.01 msec

Total time requird for 2000 sector = 2000 × 7.01

= 14020 msec

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9. (d)

<c, h, s > <1200, 9, 40 >

16 Recording surface, 64 sector

Starting sector = <c, h, s >

Number of the file = <1200, 9, 40>

⇒ 1200 × 16 × 64 + 9 × 64 + 40 = 1229416

Starting (first) sector number of the file = 1229416

16384 cylinder

$<c, \quad h, \quad s>$

$<c, \quad h, \quad s>$

$<0, \quad 0, \quad 0>$
1
2
3
⋮
63>

$<0, \quad 1, \quad 0>$
⋮
63>

2, 0
⋮ ⋮
1 15

File size = 42,797 kB & each sector capacity = 512 byte.

Total number of sector needed (Required) to store the file $= \frac{42797 \text{ KB}}{512 \text{ Byte}} = 85594$ Sector

Range [o to 85593]

Starting (first) sector Address of the file = 1229416

Last sector number of the file = 1229416 + 85594 – 1 = 1315009

Last Sector number = 1315009

<c, h, s >

Cylinder number $= \frac{1315009}{64 \times 6} = 1284.188$

$\boxed{\text{1284 cylinder number}}$

Number of sectors covered = 1284 × 16 × 64

= 1314816 sectors.

Remaining sector = 1315009 – 1314816 = 193 sector

Surface Number $= \frac{193}{64} = 3$

64 sector per surface 16 Recording surface

$$\begin{array}{cccc} < c, & h, & s & > \\ & \Downarrow & \Downarrow & \\ & 16 & 64 & \end{array}$$

OR

Alternative Approach

Number of sector required = 85594

Need to cross = 85594 sector

Number of cylinders required to cross $= \frac{85594}{16 \times 64}$

= 83 cylinder.

In 83 cylinder the number of sector = 83×16 ×64

= 84992

Number of sector remaining = 85594 – 84992

= 602 sector

Surface number $= \frac{602}{64} = 9$ Surface.

(1 more cylinder required)

Number of cylinder = 83 + 1 = 84

Starting = 1200 + 84 = 1284

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10. **(b)**

Total 100 libraries

Seek time = 10 msec

D.T.T (neglected)

Rotational speed = 6000 rpm

Average disk access time = S.T + Average R.T + D.T.T

Average R.T $= \frac{1}{2} \times$ rotational time

6000 rotation in 60 second

$\Rightarrow 1 \text{ Rotation} = \frac{60}{6000} = 10 \text{ msec}$

Average R.T $= \frac{1}{2} \times 10$ msec = 5 msec

Total time 1 library access = S.T + R.L + D.T.T

$\Rightarrow$ 10 + 5 = 15 msec

Total tme taken for 100 libraries = 100 × 15 msec

$= 100 \times 5 \times 10^{-3}$ m sec = 1.5 second

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11. (c)

$$\langle c,\ h,\ s \rangle$$

c → cylinder no, h → surface, s → sector

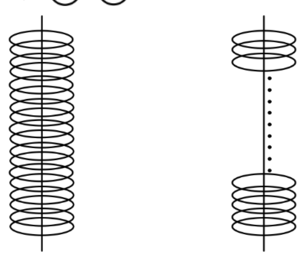
63 Sector per track

10 platter: 2 recording surface each

1000 cylinder

$\langle c,\ h,\ s \rangle$
$\langle 0,\ 0,\ 0 \rangle$
$\langle 0,\ 0,\ 1 \rangle$
$\langle 0,\ 0,\ 2 \rangle$
⋮
$\langle 0,\ 0,\ 62 \rangle$
1 0
⋮ ⋮
  63
⋮
20

$\langle c,\ h,\ s \rangle$



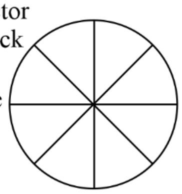
Cylinder 1 **Cylinder 1**

10 platter & each platter 2 (Number of surface) Recording surface Number of track / cylinder =20

$\langle c,\ h,\ s, \rangle$
1 cylinder: $10 \times 2 \times 63$

63 sector for track

1 surface



< c, h, s>		<400, 16, 29>
400 cylinder	=	400 [10 ×2 × 63]
16 surface	=	16 × 63
29 sector	=	29
Sector Number	=	400 [10 ×2 × 63] + 16 × 63 + 29
	=	400 × [1260] + 16 × 63 + 29
	=	504000 + 1008 + 29 = 505037

Alternate approach:

By formula: <c, h, s> <400, 16, 29>

St: number of sector per track = 63

tc: number of track per cylinder = 2×10 = 20

$$\boxed{\text{Sector Numebr} = s + st \times h + st \times tc \times c}$$

$\Rightarrow$ S + St (h + tc × c)

$\Rightarrow$ 29 + 63 [16 + 20 × 400]

$= 29 + 63 \times 16 + 63 \times 20 \times 400 = \boxed{505037}$

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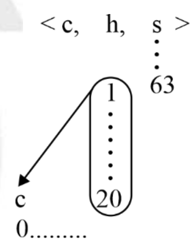
12. (c)

63 sector per track

1 cylinder = 10 × 2 × 63 = 1260



(a) $\langle 0, 15, 31\rangle \Rightarrow 15 \times 63 + 31 = 945 + 31 = 976$

(b) $\langle 0, 16, 30\rangle \Rightarrow 16 \times 63 + 30 = 1008 + 30 = 1038$

(c) $\langle 0, 16, 31\rangle \Rightarrow 16 \times 63 + 31 = 1008 + 31 = 1039$

(d) $\langle 0, 17, 31\rangle \Rightarrow 17 \times 63 + 31 = 1071 + 31 = 1102$

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13. (b)

Every time the head changes tracks, its speed and direction change, which is just a change in motion or the result of inertia.

Hence option (b) is correct.

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INDEX

Programming and Data Structures

Programming and Data Structures

Syllabus

Programming in C. Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.

Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6	Ch.7	Ch.8
2008	0	2	0	4	2	0	13	2
2009	0	0	1	0	0	0	6	2
2010	0	2	0	3	2	0	0	4
2011	0	0	4	1	0	0	3	0
2012	0	0	4	0	0	2	2	0
2013	0	0	0	0	0	2	2	0
2014 (P1)	0	2	0	0	1	0	2	2
2014 (P2)	1	2	2	0	0	0	3	0
2014 (P3)	0	0	1	0	2	2	1	2
2015 (P1)	0	2	2	0	2	0	1	2
2015 (P2)	0	0	2	2	0	2	3	0
2015 (P3)	0	2	2	1	1	1	2	1
2016 (P1)	0	2	2	3	0	2	4	0
2016 (P2)	0	2	0	4	0	1	4	0
2017 (P1)	2	2	2	4	1	0	2	0
2017 (P2)	0	2	2	5	0	1	1	0
2018	0	2	3	3	0	1	1	0
2019	0	2	5	3	0	0	0	0
2020	0	0	2	3	0	0	3	1
2021 (P1)	0	0	3	2	0	1	0	0
2021 (P2)	0	2	1	1	0	0	1	2
2022	2	0	0	3	1	2	1	1
2023	0	0	3	0	1	2	5	1

CHAPTER

1

DATA TYPES AND OPERATORS

Data Types

1. [MCQ] [GATE-2014 : 1M]

Suppose n and p are unsigned int variables in a C program. We wish to set p to n_{C_3}. If n is large, which one of the following statements is most likely to set p correctly?

(a) p = n * (n – 1)*(n – 2)/6;
(b) p = n * (n – 1)/2 * (n – 2)/3;
(c) p = n * (n – 1)/3 * (n – 2)/2;
(d) p = n * (n – 1)*(n – 2)/6.0;

Operators

2. [MCQ] [GATE-2022 : 2M]

What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[]){
char a = 'P';
char b = 'x';
char c = (a & b) + '*';
char d = (a | b) - '-';
char e = (a ^ b) + '+';
printf("%c %c %c\n", c, d, e);
return 0;
}
```

ASCII encoding for relevant characters is given below

A	B	C	…..	Z
65	66	67	…..	90

a	b	c	…..	z
97	98	99	…..	122

*	+	–
42	43	45

(a) z K S
(b) 122 75 83
(c) * – +
(d) P x +

3. [NAT] [GATE-2017 : 2M]

Consider the following C program

```
#include<stdio.h>
int main ( ) {
int m=10
int n, n1;
n=++m ;
n1=m++;
n– –;
– –n1 ;
n–=n1;
printf( ("%d", n)
return 0 ;
}
```

The output of the program is____________

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ANSWER KEY

1.	(b)	**2.**	(a)	**3.**	(0 to 0)

SOLUTIONS

1. (b)

We know that

$$n_{C_3} = \frac{n(n-1)(n-2)}{3!}$$

Let n = 50 int size = 2 byte (ranges from 0 to 65535)

Option (A):

$$50 \times 49 \times \frac{48}{6}$$

$\Rightarrow$ Out of range because of cyclic property. No error, but the results is incorrect.

Hence, Option (a) is wrong

Option (D):

Same as above reason option (d) is wrong.

Option (C):

$$\frac{n(n-1)}{3} \times \frac{(n-2)}{2}$$

n * (n – 1) is not always divisible by 3.

n = 20

n – 1= 19

$\frac{20 \times 19}{3}$ will produce incorrect result.

Hence, (c) is wrong.

Option (B):

$$\frac{n(n-1)}{2}$$

Product of any 2 successive number is always even (because out of these 2 numbers 1 must be even.)

Product of 3 consecutive number is always divisible by 3 (3!), Because out of these 3 numbers1 must be multiple of 3.

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2. (a)

*	+	–
42	43	45

$\therefore$ ASCII code of P is 80

ASCII code of x is 120

char c = (a & b) + '*'

( ) is having more priority

a & b

	a –	0 1 0 1 0 0 0 0	– 80
			&
	b –	0 1 1 1 1 0 0 0	– 120
a & b		0 1 0 1 0 0 0 0	= 80

char c = (a & b) + '*';

80 + '*'

80 + 42

char c = 1 2 2

z

On option elimination we get option (a) as correct option.

char = (a/b) – '–';

a 0 1 0 1 0 0 0 0

OR

b 0 1 1 1 1 0 0 0

0 1 1 1 1 0 0 0 = 120

120 – 45 = 75

char d = 75

K

char e = (a ∧ b) + ‘+’;

0 1 0 1 0 0 0 0

0 1 1 1 1 0 0 0

0 0 1 01 0 0 0 ⇒ 40

40 + 43 = 83

char e = 83

S

∴ z K S is printed Hence, (a) is correct option.

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3. **(0 to 0)**

Taking the snippet of code from program.

$$\text{n} = ++\text{m}; \rightarrow \left[\begin{array}{l}(\text{i})\ \ \text{m} = \text{m} + 1\ (\text{will make m as } 11)\\ (\text{ii})\ \text{n} = \text{m}\ (\text{This will assign } 11 \text{ to n})\end{array}\right.$$

$$\text{n1} = \text{m}{+}{+} \rightarrow \left[\begin{array}{l}\text{n1} = \text{m};\ \text{Assign } 11 \text{ to n1}\\ \text{m} = \text{m} + 1; \text{ increment the m by } 11\end{array}\right.$$

m $\boxed{\cancel{10}\ \cancel{11}\ 12}$

m $\boxed{\cancel{G}\ \cancel{11}\ \cancel{10}\ 0}$

m $\boxed{\cancel{G}\ \cancel{11}\ 10}$

n-- → n will decrease by 1 i.e. n becomes 10

n1→ n1 will decrease by 1 (10)

n – = n1 ⇒ n = n –n1 (this will make n as 0,0 is printed)

∴ 0 is printed

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CHAPTER

2

CONTROL FLOW STATEMENTS

Decision Control Statements

1. [NAT] [GATE-2019 : 2M]

Consider the following C program:

```
# include<stdio.h>
int main (){
float sum = 0.0, j = 1.0, i = 2.0;
while (i / j  >  0.0625){
    j = j + j;
    sum = sum + i/j;
    printf("%f \n", sum);
}
return 0;
}
```

The number of times the variable sum will be printed, when the above program is executed, is _________.

2. [MCQ] [GATE-2017 : 2M]

Consider the C program fragment below which is meant to divide x by y using repeated subtractions. The variables x, y, q and r are all unsigned int.

```
while (r >= y)
{
 r = r – y;
q = q + 1;
}
```

Which of the following conditions on the variables x y, q and r before the execution of the fragment will ensure that the loop terminates in a state satisfying the condition x == (y * q + r) ?

(a) (q==r) &&(r==0)

(b) (x > 0) &&(r==x) &&(y>0)

(c) (q==0) &&(r==x) &&(y>0)

(d) (q==0) &&(y>0)

3. [MCQ] [GATE-2016 : 2M]

The following function computes the maximum value contained in an integer array p[ ] of size n(n ≥ 1).

```
int max ( int * p, int n )
{ int a = 0, b = n – 1;
while (_______ )
{
    if(p[a] ≤ p[b]){a = a + 1;}
    else            { b = b – 1;}
}
return p[a];
}
```

The missing loop condition is

(a) a !=n (b) b != 0

(c) b>(a+1) (d) b !=a

4. [MCQ] [GATE-2016 : 2M]

The following program is to be tested for statement coverage:

```
begin
  if (a==b){S1 ; exit; }
  else if (c==d){S2 ;}
  else {S3 ; exit; }
  S4;
end
```

The test cases T1, T2, T3 and T4 given below are expressed in terms of the properties satisfied by the values of variables a, b, c and d. The exact values are not given.

T1: a, b, c and d are all equal

T2: a, b, c and d are all distinct

T3 : a=b and c !=d

T4 : a !=b and c=d

Which of the test suites given below ensures coverage of statements S1, S2, S3 and S4 ?

(a) T1, T2, T3

(b) T2, T4

(c) T3, T4

(d) T1, T2, T4

5. [MCQ] [GATE-2015 : 2M]

Consider the following pseudo code, where x and y are positive integers.

```
begin
q:=0
r:=x ;
while r≥y do
begin r:=r-y ; q:=q+1 ; end
end
```

The post condition that needs to be satisfied after the program terminates is

(a) $\{r=qx + y \wedge r < y\}$

(b) $\{x=qy + r \wedge r < y\}$

(c) $\{y=qx + r \wedge 0 < r < y\}$

(d) $\{q+1< r - y \wedge y > 0\}$

6. [NAT] [GATE-2014 : 2M]

Consider the following function

```
double f( double x){
if (abs(x*x-3)<0.01) return x;
else return f(x / 2+1.5 / x);
```

Given a value q (to 2 decimals) such that f(q) will return q : _________.

7. [MCQ] [GATE-2008 : 2M]

Choose the correct option to fill ?1 and ?2 so that the program below prints an input string in reverse order. Assume that the input string is terminated by a newline character

```
void reverse(void)  {
    int c;
    if (?1) reverse();
    ?2
}
main() {
printf ("Enter Text") ; printf ("\n");
reverse(); printf ("\n");
}
```

(a) ? 1 is (getchar () ! = '\n')
  ? 2 is getchar (c);

(b) ? 1 is (c = getchar ()) ! = '\n')
  ? 2 is getchar (c)

(c) ? 1 is (c ! = '\n')
  ? 2 is putchar (c);

(d) ? 1 is ((c = getchar ()) ! = '\n')
  ? 2 is putchar (c);

Switch Statements

8. [NAT] [GATE-2015 : 2M]

Consider the following C program:

```
#include<stdio.h>
int main()
{
int i, j, k = 0;
j=2 * 3 / 4 + 2.0 / 5 + 8 / 5;
k-=--j;
for (i=0; i<5; i++)
{
    switch(i+k)
    {
      case 1:
      case 2: printf("\n%d", i+k);
      case 3: printf("\n%d", i+k);
      default: printf("\n%d", i+k);
    }
  }
  return 0;
}
```

The number of times printf statement executed is ___.

Iterative Statements

9. [MCQ] [GATE-2021: 2M]

Consider the following ANSI C program

```
# include<stdio.h>
int main()
{
   int i, j, count;
   count =0;
   i=0;
   for(j=-3 ; j<=3 ; j++)
   {
     if((j>=0) &&(i++))
        count = count +j ;
   }
   count = count +i ;
   printf ("%d", count ) ;
   return 0 ;
```

Which one of the following options is correct?

(a) The program will not compile successfully

(b) The program will compile successfully and output 10 when executed

(c) The program will compile successfully and output 8 when executed.

(d) The program will compile successfully and output 13 when executed.

10. [MCQ] [GATE-2018 : 2M]

Consider the following C code. Assume that unsigned long int type length is 64 bits.

```
unsigned long int fun (unsigned long int n) {
        unsigned long int i, j=0, sum=0;
        for (i=n ; i>1 ; i=i / 2) j++;
        for ( ;  j>1 ; j = j / 2) sum++;
        return sum;
}
```

The value returned when we call fun with the input 2^{40} is:

(a) 4 (b) 5

(c) 6 (d) 40

11. [NAT] [GATE-2017 : 2M]

The output of executing the following C program is__________.

```
#include <studio.h>
int total (int v){
  static int count = 0;
      while (v){
      count += v&1;
      v >>= 1;
    }
     return count;
 }
void main()
{
    static int x=0;
    int i=5;
     for( ; i>0; i– –){
         x = x+ total (i);
        }
 printf (‘’%d\n‘’, x);
}
```

12. [MCQ] [GATE-2016 : 2M]

The following function computes X^Y for positive integers X and Y.

```
int exp (int X, int Y )  {
int res = 1, a = X, b = Y ;
while (b != 0){
    if (b % 2==0)
      { a=a* a ; b = b / 2 ;}
    else { res = res*a;  b = b –1 ;
}
return res;    }
```

Which one of the following conditions is TRUE before every iteration of the loop?

(a) $X^Y = a^b$

(b) $(res * a)^Y = (res * X)^b$

(c) $X^Y = res * a^b$

(d) $X^Y = (res * a)^b$

13. [MCQ] **[GATE-2014 : 2M]**

Consider the following pseudo code. What is the total number of multiplications to be performed?

```
D = 2
for i = 1 to n do
   for j = i to n do
      for k = j + 1  to n do
        D=D * 3
```

(a) Half of the product of the 3 consecutive integers.

(b) One-third of the product of the 3 consecutive integers.

(c) One-sixth of the product of the 3 consecutive integers.

(d) None of the above

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ANSWER KEY

1.	(5 to 5)	**2.**	(c)	**3.**	(d)	**4.**	(d)
5.	(b)	**6.**	(1.72 to 1.74)	**7.**	(d)	**8.**	(10 to 10)
9.	(b)	**10.**	(b)	**11.**	(23 to 23)	**12.**	(c)
13.	(c)						

SOLUTIONS

1. (5 to 5)

i = 20, j = 1.0

(1) j = 1.0

= i/j > 0.0625

= 2.0/1.0 > 0.0625(True,Printed)

(2) j = 2.0

2.0/2.0 > 0.0625 = 1.0 > 0.0625(True,Printed)

(3) j = 4.0

2.0/4.0 > 0.0625

0.5 > 0.0625(True,Printed)

(4) j = 8.0

2.0/8.0 > 0.0625

0.25 > 0.0625(True,Printed)

(5) j = 16.0

2.0/16.0 > 0.0625

0.125 > 0.0625(True,Printed)

(6) j = 32.0

2.0/32.0 > 0.0625

0.0625 > 0.0625(False,Not printed)

Hence 5 times printed.

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2. (c)

Let us first understand the concept before solving.

Assume x = 12, y = 5

Let's assume that we want to divide 12(x) by 5(y)

$$\begin{array}{r} 2 \rightarrow \text{Quotient} \\ 5\overline{)12} \\ \underline{10} \\ 2 \rightarrow \text{Remainder} \end{array}$$

Performing above operation by using repeated subtraction method then we subtract 5 two times from 12(x).

r = x = 12, y =5

Initially r =12(everything is remaining as we did not subtract 5 from 12 any number of times).

q = 0, because it denotes that how many times 5 has been subtracted from 12.

Now,

(1) r = 12, y = 5 can we subtract 5 from r? yes, because r is greater than y.
r = 12 – 5 = 7
q = 0 + 1 = 1(1 signifies that we have subtracted 5 one time from 12)

(2) r =7, y = 5 can we subtract 5 from r? yes, because r is greater than y.
r = 7 – 5 = 2
q = 1 + 1 = 2(2 signifies that we have subtracted 5 two time from 12)

(3) r = 2, y = 5 can we subtract 5 from r? no, because r is less than y so we need to stop further operation.
r = 2, q = 2

Now, solving question by option elimination method.

Initially q==0 and r = = x

Before execution of fragment the value of q is zero and value of r is x. which matches with option c.

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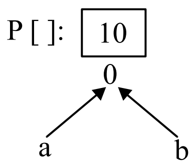
3. (d)

Let us take 1^{st} case

$n = 1 \Rightarrow a = 0, b = 0$



This is the maximum element (1), hence loop will not run, if there is single element then loop will not run.

Option (a)

a!=n

0!=1(true)

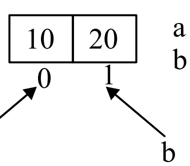
above condition is true for only one element too. Instead, it has to be returned by return statement.

Hence it is false option.

Option (b)

Let us take 2^{nd} case

n = 2



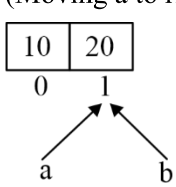
a = 0
b = 1

1^{st} iteration:

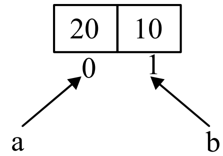
$p[a] \leq p[b] \rightarrow$ true

$a = a + 1$

(Moving a to right side)



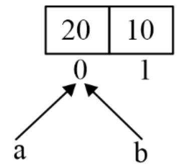
Let us take case 3



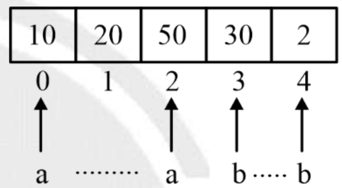
1^{st} iteration:

$p[a] \leq p[b] \Rightarrow$ False

$b = b - 1 \Rightarrow b = 0$



Case: 4



(i) $p[0] \leq p[4] \Rightarrow$ false

$b = b - 1 \Rightarrow 3$

Why?

Any Smaller element cannot be maximum if an element at a^{th} index is bigger than element at b^{th} index then p[b] cannot be maximum $\Rightarrow$ so we are moving to the left (skipping this element p[b]).

(ii) $p[a] \leq p[b] \Rightarrow$ true

$10 < 30$

10 cannot be maximum

Skip it by moving.

How $\Rightarrow$ by $a = a + 1$

(iii) $p[a] \leq p[b] \Rightarrow$ true

$20 \leq 30$

Skip 20 by $a = a + 1$

$p[a] \leq p[b]$

False

$50 \leq 30 \Rightarrow$ false

50 can be maximum, But 30 cannot be maximum.

b = b =1

b = 2

10	20	50	30	2
0	1	2	3	4

a b

*we are left with one element, this must be a maximum element. Now we need not iterate further.
*we need to stop whenever a=b. therefore loop condition will be b!=a or a!=b . i.e. option d is correct answer.

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4. **(d)**

T_1 : S1 T_1, T_2, T_4(covers all)
T_2 : S3 T_2, T_3, T_4(covers all)
T_3 : S1
T_4 : S2, S4

Hence option (d) is correct answer.

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5. **(b)**

Rewriting the code for better understanding
begin q: = 0// q is representing Quotient
r: = x // r is representing Remainder
while r $\geq$ y do
begin
r: = r – y
q: = q + 1
end
end

The abvoe code is representing, How to divide x by y using repeated subtraction.
Let x = 13, y = 5 and intially q = 0
r = 23 (everyting is remaining)

(i) Can we subtract y(5) from r ?
Yes, r = 13 – 5 = 18
q = q + 1 = 0 +1 =1
r = 8; q = 1 (one time 5 is subtrated from 13)

(ii) r = 8, q = 1 y = 5
can we subtract 5 from r ?
yes, r = 8 – 5 = 3
q = q + 1 = 1 + 1 = 2
r = 3; q = 2 (2 times 5 is subtracted from 13)

(iii) r = 3, q = 2
can we subtract 5 from r ?
No, because $\boxed{x < y}$ which is

Post condition
After code execution
r = 3, q = 2, x = 13 y = 5

$$5\overline{)\,13\,}(2$$
$$\underline{10}$$
$$3$$

we can write, $13 = \underset{2}{\overset{q}{\downarrow}} \times \underset{5}{\overset{y}{\downarrow}} + \underset{3}{\overset{r}{\downarrow}}$

$\boxed{x = qy + r}$ where, r < y

$\therefore$ on option verification, option (b) is correct.

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6. **(1.72 to 1.74)**

abs $(x^2 - 3) < 0.01$

absolute value is like an value of mod x
$$|x| = \begin{cases} -x & x < 0 \\ x & x >= 0 \end{cases}$$

$(x^2 - 3) < 0.01$ and $-(x^2 - 3) < 0.01$
$x^2 < 3.01$ and $x^2 > 2.99$
x < 1.735 and x > 1.729
x > 1.729 and x < 1.735
$\boxed{x = 1.73}$

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7. (d)

(1) option A and option B can be eliminated, because they are not printing anything (No putchar()).

(2) In option (c), there is No getchar() to read input Hence c is wrong.

(3) ∴ correct option is option (d).

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8. (10 to 10)

firstly, solving

j=2*3/4+2.0/5+8/5

we know that, *, / having same priority

*, / high priority
+ low priority

and also int, int →int

So, before loop started

$j = \underbrace{2*3/4}_{1} + 2.0/5 + 8/5$

$j = \underset{2}{6/4} + 2.0/5 + 8/5$

$j = 1 + \underset{3}{2.0/5} + 8/5$

2.0 is of double type

5 ⇒ 5.0 (double, promoted implicitly)

Results is also of type double type i.e. 0.4

$j = 1 + 0.4 + \underset{4}{8/5}$

$j = \underbrace{1 + 0.4}_{5\ \text{double}} + \underset{\text{int}}{1}$

j = 1.4 +1

j = 2.4

j is of int type.

j = 2, 2 is assigned to j

Now,

k = --j;

--j is performed

⇒ j becomes 1

k – = j;

k = 0 –1 = –1

So, before the loop started, $\boxed{k = -1 \text{ and } j = 1}$

Now evaluating loop

```
for (i = 0; i<5; i + +)
{
switch (i + k) {
case 1:
case 2: printf ("\n%d", i +k);
case 3: printf ("\n%d", i +k);
default: printf ("\n%d", i +k);
}
```

i	k	i +k	which case execute	No. of printf
0	–1	–1	only default	1
1	–1	0	only default	1
2	–1	1	case 1, case 2, case 3, default+ No break	3
3	–1	2	case 2,case 3, default	3
4	–1	3	case 3, default	2
loop ends 5				

Number of times printf executed = 1 + 1 + 3 + 3 + 2

= 10 times

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9. (b)

Initially variable count and i are initialized to zero.

for(j=-3;j<=3;j++) statement states that, for loop will run from -3 to +3(-3,-2-1,0,1,2,and 3).

Evaluating if condition inside for loop

j = –3

if((j>=0)&&(i++))

–3>=0&&(i++) →false

Here we are using concept of Short Circuiting

Short Circuiting: when the first operand of logical AND operator is 0 then 2nd operand will not be evaluated, if the 1st operand is 1 then 2nd operand will be evaluated.

$$0\&\&\underbrace{(i++)}_{\text{will never be evaluated}}$$

That's why

if(0)

count = count + j will not be executed

now, j= -2

if((j>=0)&&(i++))

$$\underbrace{\underbrace{-2>=0}_{\text{False}}\ \&\&\ \underbrace{(i++)}_{\text{will not be evaluated}}}_{(0)\text{false}}$$

0 && (i++)

count = count + j will not be executed because, if(0)(if condition fails) as same as above reason.

j = –1

if((j>=0)&&(i++))

(–1>=0 && (i++))

$$\underbrace{0\ \&\&\ \underbrace{(i++)}_{\text{will not be evaluated}}}_{\Downarrow\ 0(\text{false})}$$

again count = count +j will not be executed because, if(0)(if condition fails) as same as above reason above mentioned.

j = 0

$$\text{if}\left(\underbrace{0>=0}_{\text{true}}\ \&\&(i++)\right)$$

and becomes

1

$$\left(1\ \&\ \&\ \ \underbrace{i\ +\ +}_{\text{Post Increment}}\right)$$

Post increment: Post increment operator is used when it is required to increment the value of the variable after evaluating the expression.

Initially variable i has value 0.

$$\underbrace{1\&\&0}_{(0)\text{false}} \rightarrow \boxed{\text{after evaluation of this statement value of i becomes 1}}$$

Hence once again statement count= count + j will not be executed.

j = 1

if(1>=0 && i++)

$$\underbrace{1\&\&1}_{(1)\ \text{True}} \rightarrow \boxed{\text{after evaluation of this statement value of i becomes 2.}}$$

Hence if condition becomes true, so count = count +j will be executed

count = count + j

= 0 + 1

count = 1

j = 2

if(j>=0 &&(i++))

2>=0 && 2

1 && 2

(1) True $\boxed{\text{value of i will become 3}}$

Again, if condition becomes true and statement inside if condition will be executed

count = count + j

= 1 + 2

count = 3

j = 3

if (j>=0 && (i++))

3>=0 && 3

$\underbrace{1 \,\&\&\, 1}_{(1)\ \text{True}} \rightarrow \boxed{\text{Value of i will become 4}}$

Again, if condition becomes true and statement inside if condition will be executed.

count = count + j

3 + 3

count = 6

j = 4

condition in for loop gets failed and control of execution comes outside for loop

count=count+i

6 + 4(as value of count is 6 and i is 4)

Hence, count = 10 and lastly 10 is printed by printf statement in the end. Hence option b is correct.

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10. (b)

for (i = n; i > 1, i = 1/2)

{

j + 1;

}

Initially j = 0

Let us reduce the input size

$n = 2^6$

i	i>1
2^6	$2^6 > 1 \rightarrow$ True $\rightarrow$ j ++, j = 1
2^5	$2^5 > 1 \rightarrow$ True $\rightarrow$ j ++, j = 2
2^4	$2^4 > 1 \rightarrow$ True $\rightarrow$ j ++, j = 3
2^3	$2^3 > 1 \rightarrow$ True $\rightarrow$ j ++, j = 4
2^2	$2^2 > 1 \rightarrow$ True $\rightarrow$ j ++, j = 5
2^1	$2^1 > 1 \rightarrow$ True $\rightarrow$ j ++, j = 6

i	i>1
2^0	$2^0 > 1 \rightarrow$ Flase

for $n = 2^6$, the first loop

will make j as 6

for $n = 2^{40}$, the loop will make j as 40

Before second loop

j= 40, sum = 0

After second loop

Sum is 5

for (; j > 1; j = j/2)

{

sum ++;

}

j	j > 1
40	40 > 1 → True → sum ++
j = j/2 20	20 > 1 → True → sum ++
j = j /2 10	10 > 1 → True → sum ++
j = j/2 5	5 > 1 → True → sum ++
j = j/2 2	2 > 1 → True → sum ++
j = j/2 1	1 > 1 → True → loop ends

After second loop sum is 5.

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11. (23 to 23)

(i) **v & 1:** is checking if the rightmost bit of v is 1. (If it is 1, then count value is increment by 1).

(ii) **v>> =1:** Ensuring that every bit is checked.

Because count is a static variable, total(v) is increasing the value of count by the number of set bits in v and also returns the incremented value.

Let us assume that total (5) is called.

Before total(5)

count $\boxed{0}$

5 ⇒ 00000101 (only 2 set bits)

count $\boxed{\cancel{0}\ 2}$

count is increased by 2 and returns the same value.

Again, if total (4) is called

Before call count $\boxed{2}$

4 → 00000100

count is increased by 1 and same value is returned

count $\boxed{\cancel{2}\ 3}$

Now, understanding below code

for (; i > 0 ; i--){

x = x + total (1)

}

This loop will execute for i = 5, 4, 3, 2, 1 and

x = x + total (i) , x is initially 0

(i) x $\boxed{0}$

count $\boxed{0}$

i = 5

x = x + total (5)

5 ⇒ 00000101 (2 set bits)

total (5) ⇒ increases the value of count by 2.

count becomes 2 & same value is returned.

x $\boxed{\cancel{0}\ 2}$

count $\boxed{\cancel{0}\ 2}$

x = x + total

0 + 2

x becomes 2

(ii) x $\boxed{2}$

count $\boxed{\cancel{2}\ 3}$

i = 4

x = x + total (4)

Binary of 4 contains only one set bit, so it will increase value of count by 1 and same is returned

x $\boxed{\cancel{2}\ 5}$

count $\boxed{\cancel{2}\ 3}$

x = x + total (4)

2 + 3

$\boxed{x = 5}$

(iii) i = 3

x $\boxed{\cancel{5}\ 10}$

count $\boxed{\cancel{3}\ 5}$

x = x + total (3)

Binary of 3 contains 2 one's (00000011) that's why count is increased by 2 i.e. count becomes 5 and is reutned.

x = x + total (3)

x = 5 + 5

$\boxed{x = 10}$

(iv) i = 2

x $\boxed{\cancel{10}\ 16}$

count $\boxed{\cancel{5}\ 6}$

00000010

x = x + total (2)

Binary of two contains only one set value so count is increased by 1, count becomes 6 and same is returned.

x = x + total (2)

10 + 6

$\boxed{x = 16}$

(v) i = 1

x $\boxed{23}$

count $\boxed{\cancel{6}\ 7}$

→ 00000001

x = x + total (1)

Binary of 1 contains only one set value so count is increased by1, count becomes 7 and same is returned,

x = x+ total (1)

16 + 7

$\boxed{x = 23}$

Finally loop terminates and value of x gets printed i.e. 23 is printed.

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12. (c)

if (b % 2 = = 0)

{ a = a*a; b= b/2; }// when b is even

else

{

res = res *a;// executes when b is odd

b = b –1;}

Let X = 3, Y = 5

This code will compute = 3^5

Initially, res = 1, a = 3, b = 5

<u>1st iteration</u>

As b is odd,

res = 1*3 = 3

b = 5 – 1 = 4

After 1st iteration

a = 3, b = 4, res = 3, X = 3, Y = 5

Let us check all option after

1st iteration (before 2nd iteration)

(a) $X^Y = a^b$

$3^5 = 3^4$(false)

(b) $(res*a)^Y = (res*X)^b$

$(3*3)^5 = (3*3)^4$(false)

(c) $X^Y = res*a^b$

$3^5 = 3*3^4$(true)

(d) $X^Y = (res*a)^b$

$3^5 = (3*3)^4$(false)

Hence option C is correct option (this is option elimination method).

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13. (c)

for i = 1 to n do

for j = 1 to n do

for k = j + 1 to n do

$\boxed{D = D* 3}$

1 multiplication

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=j+1}^{n} 1$$

$$\sum_{i=1}^{n} \sum_{j=i}^{n} \left(\sum_{k=j+1}^{n} 1 \right)$$

$$\sum_{k=j+1}^{n} 1$$

= 1 + 1 + 1 + ……………+1 (How many times ?)

k = j + 1 ………..k = n

= last – first + 1 = n– (j + 1) + 1

= n – j –1 + 1 = n –j

$$\boxed{\sum_{k=j+1}^{n} = (\text{n}-\text{j})}$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{n}(\text{n}-\text{j})\right)$$

(n –i) + (n – (i +1)) + (n – (i + 2)) +… (n – (n – 1) + (n – n)

= (n – i) + (n – (i + 1)) + (n – (i + 2)) +….+ 1 + 0

= 0 + 1 + 2 +…. (n – i)

= 1 + 2 + …… (n – i)

$$= \frac{(\text{n}-\text{i})(\text{n}-\text{i}+1)}{2}$$

$$= \sum_{i=1}^{n}\frac{(\text{n}-\text{i})(\text{n}-\text{i}+1)}{2}$$

$$= \sum_{i=1}^{n}\left[\frac{n^2+i^2+2in+n-1}{2}\right]$$

$$= \frac{1}{2}\sum_{i=1}^{n}n^2+\frac{1}{2}\sum_{i=1}^{n}i^2-\sum_{i=1}^{n}i.n+\frac{1}{2}\sum_{i=1}^{n}n-\frac{1}{2}\sum_{i=1}^{n}i$$

$$= \frac{n^3}{2}+\frac{n(n+1)(2n+1)}{12}-\frac{n.n(n+1)}{2}+\frac{n^2}{2}-\frac{n(n+1)}{2}$$

$$= \cancel{\frac{n^3}{2}}+\frac{n(n+1)(2n+1)}{12}-\cancel{\frac{n^3}{2}}-\cancel{\frac{n^2}{2}}+\cancel{\frac{n^2}{2}}-\frac{n^2}{4}-\frac{n}{4}$$

$$= \frac{n(n+1)(2n+1)}{12}-\frac{n(n+1)}{4}$$

$$= \frac{n(n+1)}{12}[2n+1-3]$$

$$= \frac{n(n+1)}{12}(2n-2)$$

$$= \frac{2\cdot\ n(n+1)(n-1)}{12}$$

$$= \frac{(n-1)(n)(n+1)}{6}$$

Therefor option (c) is correct answer.

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❑❑❑

CHAPTER

3

FUNCTIONS AND STORAGE CLASSES

Functions

1. [NAT] [GATE-2019 : 1M]

Consider the following C program:

```
#include <stdio.h>
int jumble (int x, int y){
        x = 2 * x + y;
        return x;
}
int main () {
         int x = 2, y = 5;
         y = jumble(y, x);
         x = jumble (y, x);
         printf ("%d \n", x);
         return 0;
}
```

The value printed by the program is ___________.

2. [MCQ] [GATE-2019 : 2M]

Consider the following C program:

```
#include<stdio.h>
int r ( ){
    static int num = 7;
    return num – –;
}
int main( ){
    for (r ( ); r( ); r( ))
      printf ("%d", r( ));
    return 0;
}
```

Which one of the following values will be displayed on execution of the programs?

(a) 41 (b) 52

(c) 63 (d) 630

3. [NAT] [GATE-2018 : 2M]

Consider the following program written in pseudo-code. Assume that x and y are integers:

```
Count (x, y) {
if (y != 1) {
   if (x != 1) {
   print ("*");
          Count (x/2, y);
}
   else {
      y = y – 1;
          Count (1024, y);
      }
    }
}
```

The number of times that the print statement is executed by the call Count (1024, 1024) is _______.

Storage Classes

4. [NAT] [GATE-2015 : 1M]

The output of the following C program is________

```
void f1 (int a, int b) {
int c;
c = a; a = b; b  =c;
}
    void f2 (int *a, int *b) {
     int c;
     c = *a; *a = *b; *b = c;
  }
     int main()   {
     int a = 4;b = 5,c = 6;
     f1 (a,b);
     f2 (&b,&c);
     printf ("%d",c-a-b);
}
```

5. [MCQ] [GATE-2014 : 2M]

Consider the C function given below.

```
int f( int j){
static int i = 50 ;
int k ;
if (i==j){
printf ("something");
k=f(i) ;
return 0
} else return 0 ;
```

Which one of the following is TRUE?

(a) The function returns 0 for all values of j.
(b) The function prints the string something for all values of j.
(c) The function returns 0 when j = 50.
(d) The function will exhaust the runtime stack or run into an infinite loop when j = 50.

Recursion

6. [NAT] [GATE-2023 : 1M]

The integer value printed by the ANSI-C program given below is _______________ .

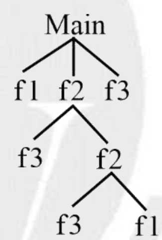
```
#include<stdio.h>
int funcp(){
static int x = 1;
x++;
return x;
}
int main(){
int x,y;
x = funcp();
y = funcp()+x;
printf("%d\n", (x+y));
return 0;
```

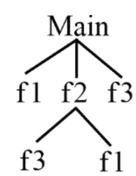
7. [MCQ] [GATE-2023 : 2M]

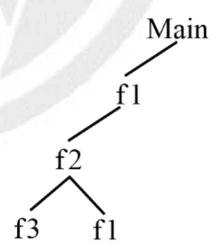
Consider the following program:

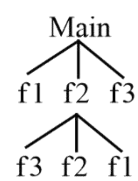
```
int main()
{
   f1();
   f2(2);
   f3();
      return(0);
}
int f1()
{
      return(1);
}
int f2(int X)
{
      f3();
    if (X==1)
        return f1();
   else
       return (X*f2(X-1));
}
int f3()
{
   return(5);
}
```

Which one of the following options represents the activation tree corresponding to the main function?

(a)


(b)


(c)


(d)


8. [NAT] [GATE-2021 : 1M]

Consider the following ANSI C function:

```
 int SomeFunction(int x, int y )
{
  if ((x==1) || (y==1)) return 1;
 if (x==y) return x;
 if (x >y) return SomeFunction (x-y, y);
 if (y >x) return SomeFunction (x, y-x);
}
```

The value returned by SomeFunction(15, 255) is____.

9. [NAT] [GATE-2021 : 2M]

Consider the following ANSI C program

```
#include < stdio.h >
int foo(int x, int y, int q) {
if ((x ≤ 0) &&(y ≤ 0))
return q;
if (x ≤ 0)
return foo (x, y–q, q);
if (y ≤ 0)
return foo (x–q, y, q);
return foo (x, y–q, q) + foo (x–q, y, q);
}
int main(){
   int r = foo (15, 15, 10);
   printf(" %d", r);
   return 0;
}
```

The output of the program upon execution is ______.

10. [MCQ] [GATE-2019 : 2M]

Consider the following C function.

```
void convert (int n){
   if (n < 0)
      printf("%d", n);
   else {
      convert(n/2);
      printf("%d", n % 2);
   }
}
```

Which one of the following will happen when the function convert is called with any positive integer n as argument?

(a) It will print the binary representation of n and terminate.

(b) It will print the binary representation of n in the reverse order and terminate

(c) It will print the binary representation of n but will not terminate

(d) It will not print anything and will not terminate.

11. [NAT] [GATE-2018 : 1M]

Consider the following C program:

```
#include<stdio.h>
int counter = 0;
int calc(int a, int b) {
    int c;
    counter++;
    if (b = = 3) return (a*a*a);
    else {
    c = calc(a, b/3);
    return(c*c*c);
    }
}
int main ( ) {
    calc(4, 81);
    printf ("%d", counter);
}
```

The output of this program is ______.

12. [MCQ] [GATE-2017 : 2M]

Consider the following two functions.

```
void fun1(int n) {
   if(n == 0) return;
   printf("%d", n);
   fun2(n – 2);
   printf("%d", n);
}
void fun2(int n) {
   if(n == 0) return;
   printf("%d", n);
   fun1(++n);
   printf("%d", n);
}
```

The output printed when fun1(5) is called is

(a) 53423122233445

(b) 53423120112233

(c) 53423122132435

(d) 53423120213243

13. [MCQ] [GATE-2017 : 2M]

Consider the C functions foo and bar given below:

```
int foo (int val)   {
int x=0;
while (val > 0)   {
        x = x + foo(val – –);
        }
  return val;
}
int bar (int val) {
int x = 0 ;
while (val >0){
     x = x + bar(val –1) ;
    }
return val;
}
```

Invocation of foo(3) and bar(3) will result in:

(a) Return of 6 and 6 respectively.

(b) Infinite loop and abnormal termination respectively

(c) Abnormal termination and infinite loop respectively

(d) Both terminating abnormally.

14. [MCQ] [GATE-2016 : 2M]

What will be the output of the following C program?

```
void count (int n ) {
   static int d=1;
   printf ("%d", n);
   printf ("%d", d);
   d++;
   if (n>1) count (n – 1);
   printf ("%d", d);
}
void main(){
   count (3);
}
```

(a) 312213444

(b) 312111222

(c) 3122134

(d) 3121112

15. [MCQ] [GATE-2016 : 2M]

Consider the recursive C function that takes two arguments unsigned int foo(unsigned int n, unsigned int r) {

```
if (n  > 0) return (n%r +  foo (n/r, r ));
else return 0;
}
```

What is the return value of the function foo when it is called as foo(513, 2)?

(a) 9
(b) 8
(c) 5
(d) 2

16. [MCQ] [GATE-2015 : 2M]

Consider the following recursive C function.

```
void get (int n) {
if (n < 1) return;
get (n – 1);
get (n – 3);
print f (“%d”, n);
}
```

If get (6) function is being called in main( ) then how many times will the get( ) function be invoked before returning to the main()?

(a) 15
(b) 25
(c) 35
(d) 45

17. [MCQ] [GATE-2015 : 1M]

Consider the following function written in the C programming language.

```
void foo ( char * a){
if (* a&& * a !=’  ’)   {
foo (a+1)
putchar (* a)
}
}
```

The output of the above function on input "ABCD EFGH" is

(a) ABCD EFGH

(b) ABCD

(c) HGFE DCBA

(d) DCBA

18. [NAT] [GATE-2015 : 1M]

Consider the following C function.

```
int fun (int n){
int x=1, k ;
if (n==1) return x ;
for (k=1 ; k<n ;++k)
x=x+ fun (k) *  fun (n-k) ;
return x ;
```

The return value of fun (5) is

19. [NAT] [GATE-2014 : 1M]

Consider the function func shown below:

```
int func (int num)
{   int count =0 ;
   while (num)
  { count ++;
    num >>=1 ;
  }
   return (count);
}
```

The value returned by function (435) is________.

20. [MCQ] [GATE-2012 : 2M]

Consider the following C code segment.

```
int a, b, c = 0;
void prtFun(void);
main()
{
    static int a = 1;      /* Line 1 */
    prtFun();
    a += 1;
    prtFun();
     printf(" \n %d %d ", a, b);
}
void prtFun(void)
{
    static int a = 2;      /* Line 2 */
     int b = 1;
     a += ++b;
     printf(" \n %d %d ", a, b);
}
```

What output will be generated by the given code segment?

(a) 3 1 4 1 4 2	(b) 4 2 6 1 6 1
(c) 4 2 6 2 2 0	(d) 4 2 4 2 2 0

21. [MCQ] [GATE-2012 : 2M]

Consider the following C code segment

```
int a, b, c = 0;
void prtFun(void);
main()
{
    static int a = 1;      /* Line 1 */
    prtFun();
    a += 1;
    prtFun();
     printf(" \n %d %d ", a, b);
}
void prtFun(void)
{
    static int a = 2;      /* Line 2 */
     int b = 1;
     a += ++b;
     printf(" \n %d %d ", a, b);
}
```

What output will be generated by the given code segment if:

Line 1 is replaced by auto int a = 1;

Line 2 is replaced by register int a = 2;

What output will be generated by the given code segment?

(a) 3 1 4 1 4 2	(b) 3 1 5 2 5 2
(c) 4 2 6 2 2 0	(d) 4 2 4 2 2 0

22. [MCQ] [GATE-2011 : 2M]

Consider the following recursive C function that takes two arguments. unsigned int foo (unsigned int n, unsigned int r)

```
 {
  if (n>0) return ((n%r)+ foo(n/r,r));
  else return 0;
 }
```

What is the return value of the function foo when it is called as foo (345,10) ?

(a) 345 (b) 12
(c) 5 (d) 3

23. [MCQ] [GATE-2009 : 1M]

Consider the program below:

```
#include<stdio.h>
int fun(int n, int*f_p) {
int t,f;
if(n<=1) {
 *f_p=1;
    return 1;
}
t=fun(n-1,f_p);
f=t+*f_p;
*f_p=t;
  return f;
}
int main( )
{
   int x=15;
   printf("%d\n",fun(5,&x));
   return 0;
}
```

The value printed is

(a) 6
(b) 8
(c) 14
(d) 15

Scoping

24. [NAT] [GATE-2020 : 2M]

Consider the following C functions:

```
int fun1(int n){
static int i = 0
if (n > 0) {
      ++i;
      fun1(n-1);
     }
return (i);
}
int fun2 (int n) {
static int i = 0;
if (n > 0) {
     i = i + fun1(n);
     fun2(n–1);
     }
return (i);
}
```

The return value of fun2 (5) is _________.

25. [NAT] [GATE-2015 : 2M]

Consider the following C program.

```
#include<stdio.in>
int  f1(void);
int  f2(void);
int  f3(void);
int  x=10;
int main()
{
    int x=1;
     x+=f1()+f2()+f3()+f2();
    print f("%d", x);
    return 0;
    }
    int f1(){int  x=25;x++;return x;}
    int f2(){static int=50;x++;return x;}
    int f3(){x*=10;return x;}
```

The output of the program is _____

❑❑❑

ANSWER KEY

1.	(26 to 26)	**2.**	(b)	**3.**	(10230 to 10230)	**4.**	(–5 to –5)
5.	(d)	**6.**	(7 to 7)	**7.**	(a)	**8.**	(15 to 15)
9.	(60 to 60)	**10.**	(d)	**11.**	(4 to 4)	**12.**	(a)
13.	(c)	**14.**	(a)	**15.**	(d)	**16.**	(b)
17.	(d)	**18.**	(51 to 51)	**19.**	(9 to 9)	**20.**	(c)
21.	(d)	**22.**	(b)	**23.**	(b)	**24.**	(55 to 55)
25.	(230 to 230)						

SOLUTIONS

1. (26 to 26)

$x_{main} = \cancel{2}\ \ 26$

$y_{main} = \cancel{5}\ \ 12$

1) y_{main} = jumble(5,2)

x=12, y=2

x=2*x+y

x=2*5+2

x=12

after 1st jumble function call

x_{main}=2, y_{main}=12

2) x_{main}=jumble(12,2)

x=26,y=2

x=2*x+y

x=2*12+2

x=26

after 2nd jumble function call

x_{main}=26, and y_{main}=12

lastly value of x is printed which is 26.

Scan for Video solution

2. (b)

r(): it will return the current value of num and returns num=num-1(post decrement).

```
                (5) (8)
      (1)      (2)    (4) (7)
for   (r ()) ;  r();   r( ))
             {
            (3) printf ("% d ", r ())
             (6)
             {
```

Post-decrement: A post-decrement operator is used to decrement the value of a variable after executing the expression in which the operator is used.

(1) r() will return 7 and then num is decreased by 1 that is num =7, value 7 is printed and then decremented.

(2) r() will return 6(which is non zero) that's why condition will become true and code inside the loop will be executed, but before reaching printf statement num is decresed by 1 ie…. num becomes 5.

(3) printf("%d"r());
r() inside printf will return 5 and same is printed and after printing num becomes 4.

(4) r() is called and 4 is returned and then decreses the num value by 1, hence num becomes 3.

(5) r() will return 3 which is non zero(true) so printf will be executed, before printf num value becomes 2 because of post decrement.

(6) printf("%d",r()), r() returns 2 and same value is printed. And then num becomes 1.

(7) r(), will return 1 and decrements the value to zero.

(8) r(), Here zero is returned and condition of for loop becomes false hence loop gets terminated.

Therefore, output printed is 52 which is option b.

Scan for Video solution

3. (10230 to 10230)

* If y =1, then the function will terminate without printing anything.

Count (x, 1) will not print anything

else statement will be executed when if condition fails.

Count (1024, 1024)

$\downarrow$ (1) print

Count (512, 1024)

$\downarrow$ (2) print

Count (256, 1024)

$\downarrow$ (3) print

Count (128, 1024)

$\downarrow$ (4) print

Count (64, 1024)

$\downarrow$ (5) print

Count (32, 1024)

$\downarrow$ (6) print

Count (16, 1024)

$\downarrow$ (7) print

Count (8, 1024)

$\downarrow$ (8) print

Count (4, 1024)

$\downarrow$ (9) print

Count (2, 1024)

$\downarrow$ (10) print

Count (1, 1024)

Note:- for each such y, 10 times printf will take place

Count (1024, 1024)

$\downarrow$After 10 times printf

Count (1, 1024)

Because x = 1 else part will be executed

Count (1024, y) $y \neq 1$

$\downarrow$10 times print

Count (1, y)

$\Rightarrow$ x = 1 else part will be executed

1. Count (1024, 1024)

   $\downarrow$ after 10 time printing

   Count (1, 1024)

   $y = y - 1 \Rightarrow y = 1023$

2. Count (1024, 1023) will execute

   Count (1024, 1023)

   $\downarrow$ 10 time

   Count (1, 1023)

   Again, else part will execute

   y = 1022

   count (1024, 1022) will execute

   count (1024, 1024) $\rightarrow$ 10 times

   count (1024, 1023) $\rightarrow$ 10 times

   count (1024, 1022) $\rightarrow$ 10 times

   .

   .

   .

   Count (1024, 2) $\rightarrow$ 10 times

   i.e. count (1024, y)where y = 2, 3, …1024

   i.e. for 1023 value of y count (1024, y) will print 10 times

   Total number of printf = $10 \times 1023 = 10230$

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4. **(–5 to –5)**

f2() function is swapping.

⇒ As f2 is called by passing address of b and address of c.

f2 () ⇒ will swap the contents of b, c.

a[4   b[~~5~~6   c[~~6~~5

printf will point the value of c – a – b

5 – 4 – 6

= 5 – 10

= – 5

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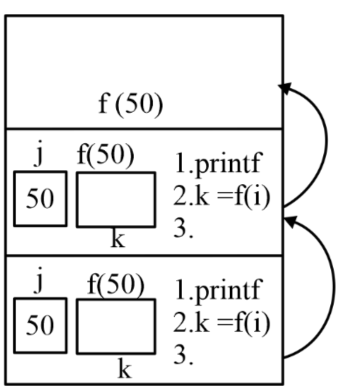
5. **(d)**

```
int f(int j){
static int i=50;
int k;
if(i= =j){
1.printf("something");
2.k=f(i);
3.return 0;
}
else
return 0
}
```



f(50) will call f(50) and keeps on calling which will lead to stack overflow.

for j = 50, function will never reach return statement.

Option (a): is wrong because for j=50, stack overflow occurs and function never reaches return statement.

Option (c): when j=50, function will never reach return statement.

Option(b): for j!=50, function will not print anything that's why (b) is also wrong.

Option(d): as explained through diagram which shows that function will exhaust the runtime stack or run into an infinite loop when j=50, therefore (d) is correct answer.

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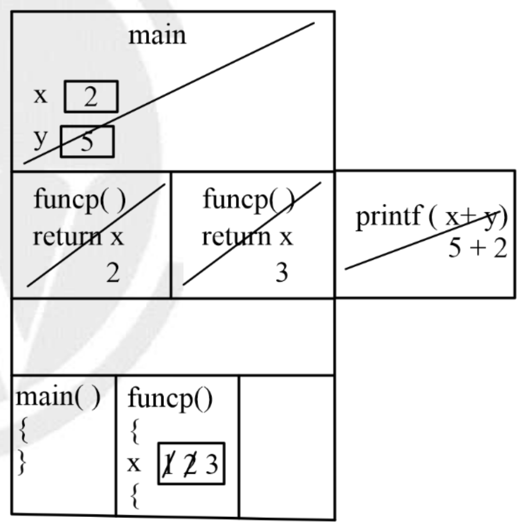
6. **(7 to 7)**



After execution of funcp 2 times, printf function prints the output (7) and gets deleted from activation record.

Similarly, control is returned to the main function, main function also gets deleted from activation record and control is returned to operating system.

Hence 7 is printed.

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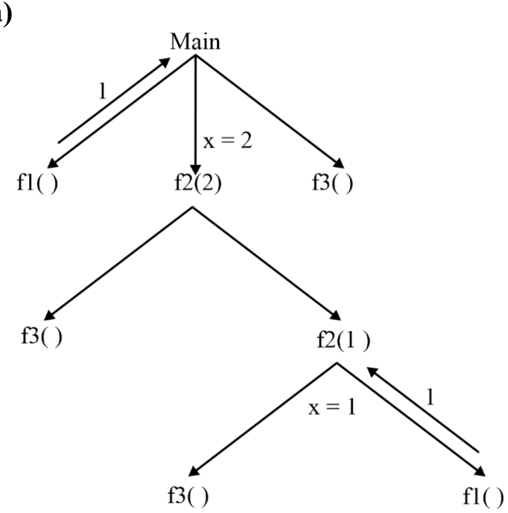
7. (a)



Comparing the above tree with the given trees in options. Above tree matches with the tree given in option A, hence a is correct answer.

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8. (15 to 15)

x y
SomeFunction (15, 255)
15 y > x

x y
SomeFunction (15, 240)
15 y > x

x y
SomeFunction (15, 225)
15 y > x

x y
SomeFunction (15, 210) → 210 is a multiple of 15 (15* 14 = 210)
15 y > x

SomeFunction (15, 30)
15 y > x

Note: The above code is finding the GCD of 2 number using repeated subtraction.

SomeFunction (15, 15)
15 x = = y

return 15

∴ Correct answer is 15

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9. (60 to 60)

Lets first understand the fragments of code.

1) if((x ≤ 0)&&(y ≤ 0))

The above case states that when both x and y are less than 0 (or) equal to zero then return 3rd argument(q).

2) if(x ≤ 0)

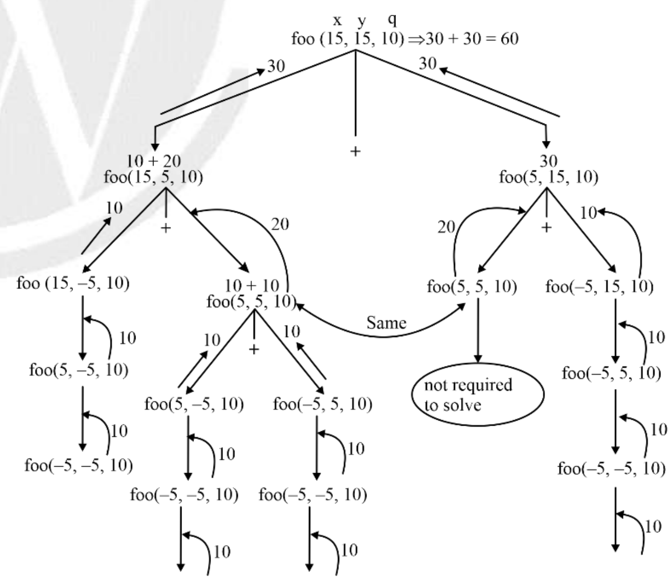
The above case states that x ≤ 0 and y > 0 then decrement 2nd argument using 3rd argument.

3) if(y ≤ 0)

The above case states that x >0, y ≤ 0 then decrement first argument using 3rd argument.

4) return foo (x, y – q, q) + foo (x – q, y, a)

The above case states that, when both x, y > 0 then from first function call, decrease 2nd argument using 3rd argument and from 2nd function call, decrease first argument using 3rd argument.



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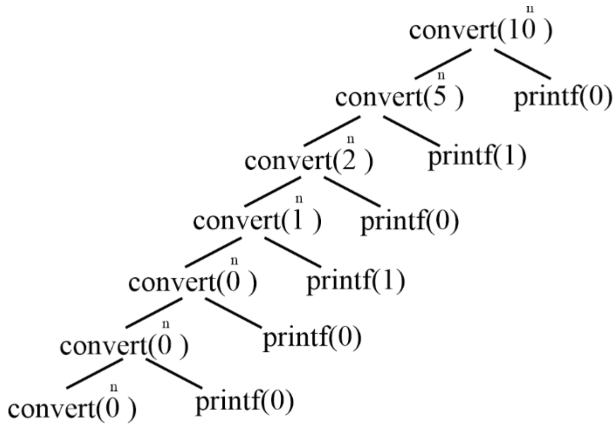
10. **(d)**

Let us try for n = 10



- Every printf is waiting, statement written after recursive call executes in opposite order of call.
- Nothing will be printed because convert(0) keeps on calling convert(0) and it will never terminate.

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11. **(4 to 4)**

In this program we need not focus on the return value, we must focus on the counter variable. We need to just look for the number of times calc function is being called.

a b

calc (4, 81)

↓ counter = 1, b = 81

calc (4, 27)

↓ counter = 2, b =27

calc (4, 9)

↓ counter = 3, b =9

calc (4, 3)

↓ counter = 4, b =3

No further call

$\therefore$ value of counter variable is printed is 4

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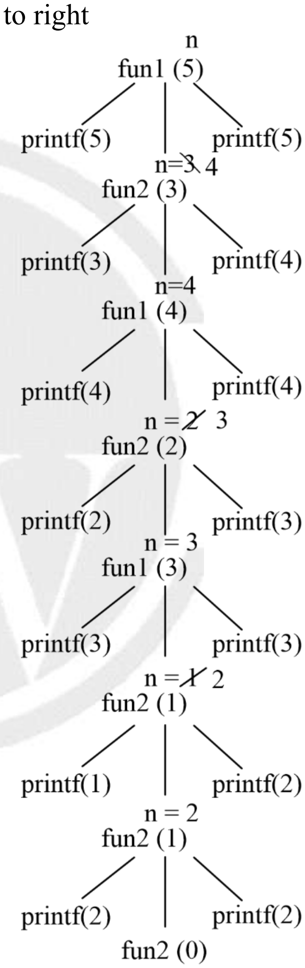
12. **(a)**

$$\text{fun1}(++n);\left[\begin{array}{l} n = n+1 \\ \text{fun1}(n) \end{array}\right.$$

printf ("% d", n) $\rightarrow$ will print incremented value of n

We need to trace this tree from top to bottom & left to right



We need to trace this tree from top to bottom & left to right.

Hence, 53423122233445 is printed, therefore option (a) is correct

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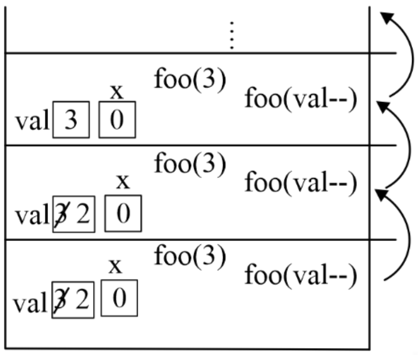
13. (c)

Understanding foo function

x=x+foo(val--) ;

val--performs post decrement.foo(val) will be called and then val=val-1 is performed later(after function call)

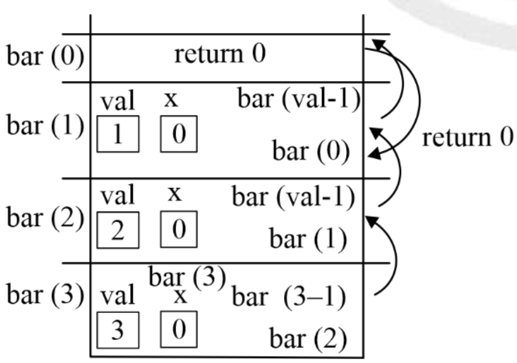


foo(3) will call foo(3) which will again call foo(3) and gets called every time.

Infinite recursive call keeps on increasing stack size, but stack size is limited. Hence at some point of time stack overflow occurs ie…. abnormal termination takes place.

Understanding bar function:

```
int bar(int val){
int x=0;
while(val>0){
x=x+bar(val-1);}
return val;
}
```



- We are still inside loop and val > 0 is true, bar (1) will again call bar(0) and again bar(0) will return 0,
- This process will continue .. loop count wont be getting terminated.
- But, stack size is not being exceeded at any moment of time.
- Leading to infinite loop Hence, correct option is (c).

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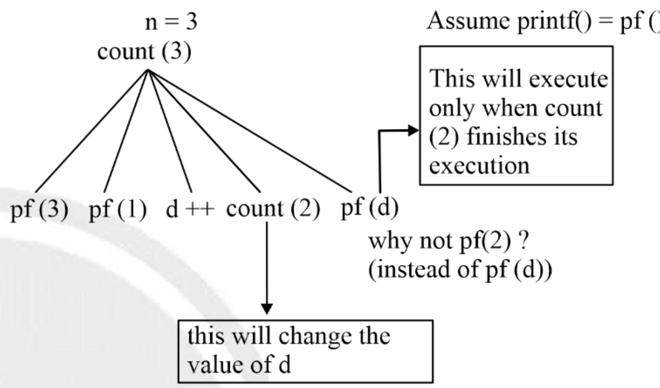
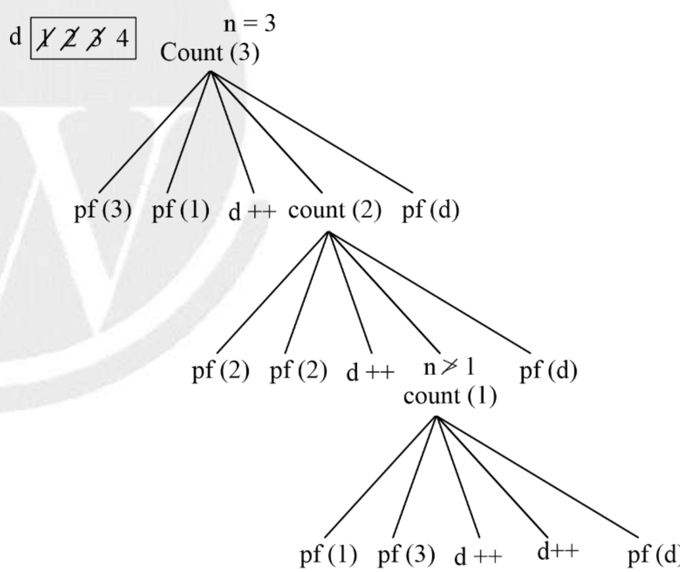
14. (a)





On tracing the tree from top to bottom and left to right we get

3,1,2,2,1,3,4,4,4

∴ Option a is correct answer.

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15. (d)

2
n r
foo (513, 2)
1
n r
1+ foo (256, 2)
1
n r
0+ foo (128, 2)
1
n r
0+ foo (64, 2)
1
n r
0+ foo (32, 2)
1
n r
0+ foo (16, 2)
1
n r
0+ foo (8, 2)
1
n r
0+ foo (4, 2)
1
n r
0+ foo (2, 2)
1
0+ foo (1, 2)
0
1+ foo (0, 2)
0

Answer is option (d)

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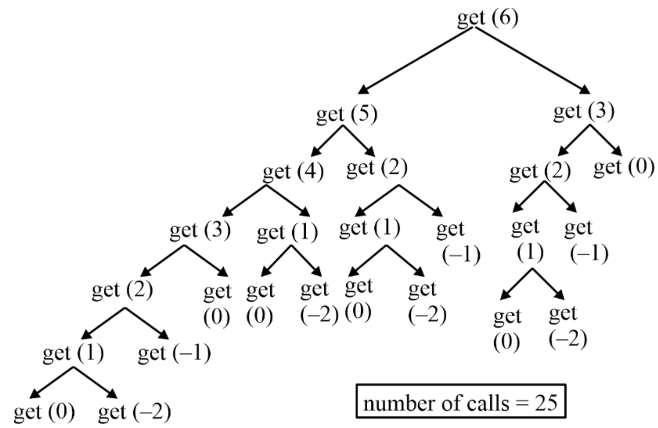
16. (b)

void get (int n) {
if (n < 1) return;
get (n – 1); ⇒ Recursion call with 1 less argument
get (n – 3); ⇒ Recursion call with 3 less argument
printf("%d", n);

Need not to focus on printf statement



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17. (d)

&&! =

Priority of && is less than !=

if ((*a)) && (*a! = ' '))

if will become false, Either

(1) *a = 0 (ASCI to or \0)

(2) *a become ' '

Now,

Recursion 'A'→'B'→'C'→'D'→ Recursion ends and then it starts printing.

Printing is done because putchar (*a); is written after recursive call.

Hence, DCBA is printed, i.e. option D is correct.

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18. (51 to 51)

x is a local variable so in every call x →1

fun(1) =1

fun(2) ⇒ = 1

x = x + fun(1)*fun (2 –1)

$= 1 + \text{fun}(1) * \text{fun}(1) = 1 + 1 \times 1 = 2$

$\text{fun}(3) \Rightarrow k = 1, 2$

$k = 1$

$x = x + \text{fun}(1) * \text{fun}(2) + \text{fun}(2) * \text{fun}(1)$

$= 1 + 1 \times 2 + 2 \times 1 = 1 + 2 + 2 = 5$

fun (4) $k = 1, 2, 3$ $n = 4$

$x = x + \text{fun}(1) \times \text{fun}(3) + \text{fun}(2) \times \text{fun}(2) + \text{fun}(3) \times \text{fun}(1)$

$= 1 + 1 \times 5 + 2 \times 2 + 5 \times 1 = 15$

$k = 1, 2, 3, 4$ $n = 5$

fun(5)

$= 1 + \text{fun}(1) \times \text{fun}(4) + \text{fun}(2) \times \text{fun}(3) + \text{fun}(3) \times \text{fun}(2) + \text{fun}(4) \times \text{fun}(1)$

$= 1 + 1 \times 15 + 2 \times 5 + 5 \times 2 + 15 \times 1$

$= 1 + 15 + 10 + 10 + 15$

$= 51$

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19. (9 to 9)

num = 435		count ++ (✓)	
num >> =1	num = 217	count ++ (✓)	
num >> =1	num = 108	count ++ (✓)	
num >> =1	num = 54	count ++ (✓)	
num >> =1	num = 27	count ++ (✓)	9 times
num >> =1	num = 13	count ++ (✓)	
num >> =1	num = 6	count ++ (✓)	
num >> =1	num = 3	count ++ (✓)	
num >> =1	num = 1	count ++ (✓)	

num>>1 (divided by 2) $\rightarrow$ num = 0

Hence, 9 is the correct answer.

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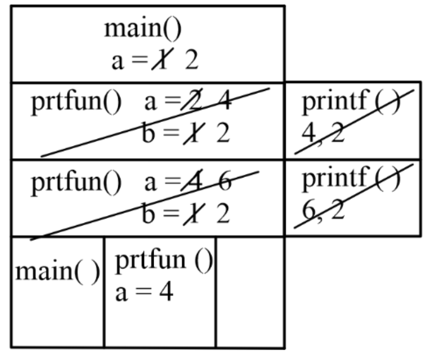
20. (c)

Part-A

a = 0, b = 0, c = 0



- 4, 2 is printed and it is deleted from activation record.
- Again prtfun () is called
- 6,2 is printed and it is deleted from activation Record, now control is retunred to main and 2,0 is printed.

$\left.\begin{matrix} 4\ 2 \\ 6\ 2 \\ 2\ 0 \end{matrix}\right\}$ is pointed, Hence (c) is correct option

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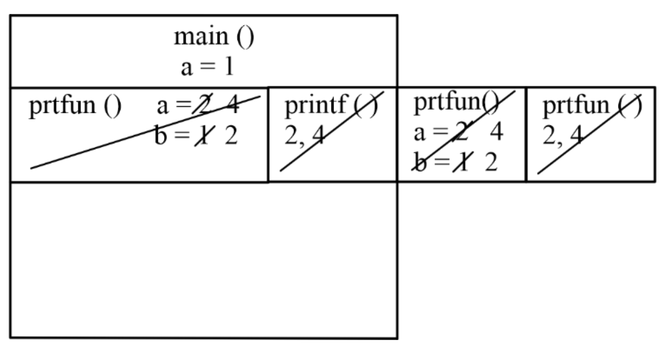
21. (d)

Part -B

a = 0,

b = 0

c = 0



Note that register vairable is as same as local vairable except the storage area

- After printing 4, 2 printf function and printf( ) gets deleted from activation record and control is returned to main, again prtfun is called.
- Again 4, 2 is printed and both the functions get deleted from activation record and control is returned to main and 2,0 is printed.

$$\therefore \left.\begin{matrix} 4\ 2 \\ 4\ 2 \\ 2\ 0 \end{matrix}\right\} \text{is pointed Hence, option } (d) \text{ is correct answer}$$

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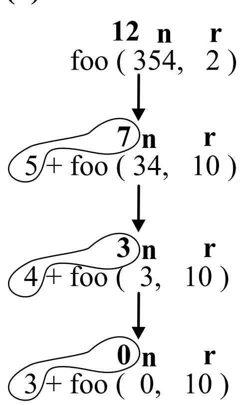
22. (b)



Hence (b) is correct option

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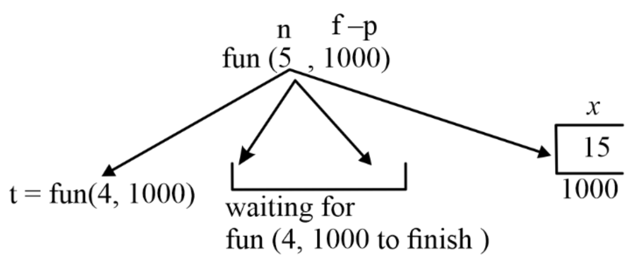
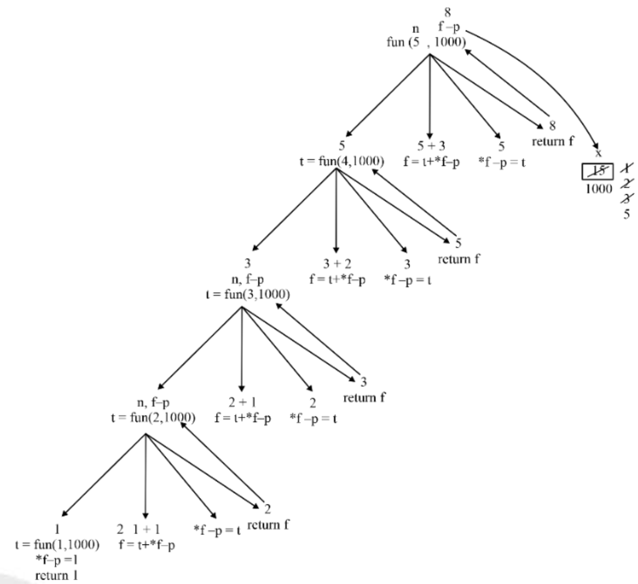
23. (b)

x = 15





Correct answer is 8 i.e. option (b).

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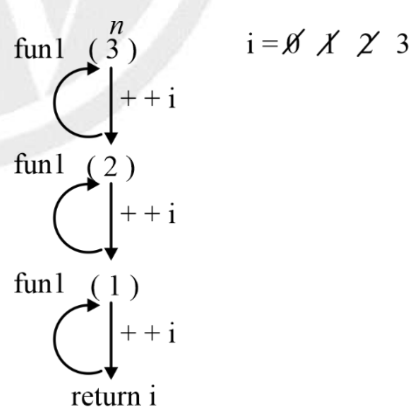
24. (55 to 55)

Let us try to understand the working of function fun1

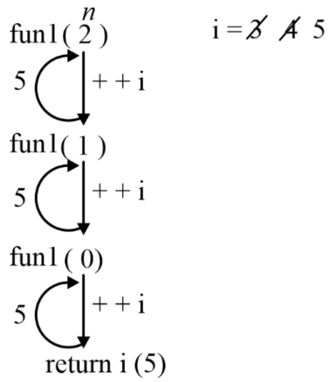
Let fun1(3) is called first

i is an static variable



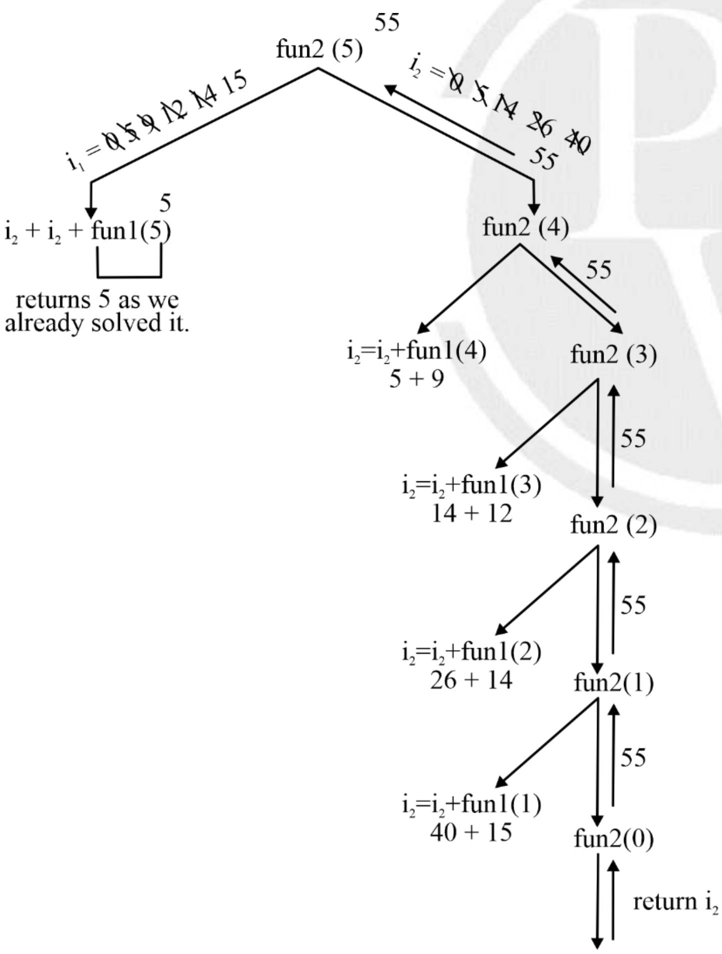
Before fun1(3) the value of i was 0 and after fun1(3) the value of i becomes 3.

Let us assume fun1() is called again i.e. for the 2nd time.



- Before fun1(2) value of static variable i was 3 and after fun1(2) value returned is 5. Which is the value of i variable
- fun 1(n) is increasing the value of static int i variable and returning the incremented value.

Now let us understand fun2(5).



$\therefore$ 55 is printed

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25. (230 to 230)

x_{main} $\boxed{1}$ x_{global} $\boxed{10}$

$x_{main} = x_{main} + f_1( ) + f_2() + f_3() + f_2();$

$f_1()$ is called:

x_1 $\boxed{\cancel{25}}$

26

26 is retuned by $f_1()$

$x_{main} = x_{main} + f_1( ) + f_2() + f_3() + f_2();$
$= 1 + 26$

$f_2()$ is called:

There is local static variable which resides in memory throughout the program.

x $\boxed{\cancel{50}}$

51

51 is returned but x being static, still remains in memory.

$x_{main} = x_{main} + f_1( ) + f_2() + f_3() + f_2();$
$= 1 + 26 + 51$

$f_3()$ is called:

x_{global} * = 10
$x_{global} = 10 * 10$
$x_{global} = 100$

100 is returned

$x_{main} = x_{main} + f_1( ) + f_2() + f_3() + f_2();$
$= 1 + 26 + 51 + 100$

f_2 () is called again

Variable x is already residing in memory already.

x $\boxed{\cancel{51}\ 52}$

52 is returned

$x_{main} = x_{main} + f_1( ) + f_2() + f_3() + f_2();$
$= 1 + 26 + 51 + 100 + 52= 230$

$\therefore$ 230 is printed

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❑❑❑

CHAPTER

4 POINTERS AND STRINGS

Arrays and Pointers

1. [MCQ] [GATE-2022 : 1M]

What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[])
{
int x = 1, z[2] = {10, 11};
int *p = NULL;
p = &x;
*p = 10;
p = &z[1];
*(&z[0] + 1) += 3;
printf("%d, %d, %d\n", x, z[0], z[1]);
return 0;
}
```

(a) 1, 10, 11 (b) 1, 10, 14

(c) 10, 14, 11 (d) 10, 10, 14

2. [MCQ] [GATE-2022 : 2M]

What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[])
{
int a[3][3][3] =
{{1, 2, 3, 4, 5, 6, 7, 8, 9},
 {10, 11, 12, 13, 14, 15, 16, 17, 18},
 {19, 20, 21, 22, 23, 24, 25, 26, 27}};
    int i = 0, j = 0, k = 0;
    for( i = 0; i < 3; i++ ){
        for(k = 0; k < 3; k++ )
     printf("%d ", a[i][j][k]);
        printf("\n");
    }
        return 0;
}
```

(a) 1 2 3
10 11 12
19 20 21

(b) 1 4 7
10 13 16
19 22 25

(c) 1 2 3
4 5 6
7 8 9

(d) 1 2 3
13 14 15

3. [NAT] [GATE-2021: 2M]

Consider the following ANSI C function:

```
int SimpleFunction (int Y[ ], int n, int x)
{
int total =Y[0], loopIndex;
for ( loopIndex =1 ; loopIndex <=n-l; loopIndex ++)
total = x * total +Y[ loopIndex ] ;
return total;
}
```

Let Z be an array of 10 elements with Z[i]=1, for all i such that $0 \le i \le 9$.

The value returned by SimpleFunction (Z, 10,2) is____

4. [MCQ] [GATE-2021 : 1M]

Consider the following ANSI C program.

```
#include <stdio.h >
int main( ){
int arr[4][5];
int i, j;
for (i=0 ; i<4 ; i++){
for (j=0 ; j<5 ; j++){
arr [i][j]=10 * i + j;
```

```
}
}
printf(" % d ", *(arr[1]+9));
return 0;
}
```

What is the output of the above program?

(a) 14 (b) 20

(c) 24 (d) 30

5. [NAT] [GATE-2020 : 1M]

Consider the following C program:

```
#include < stdio.h >
int main ( ) {
   int a [4][5] = {{1, 2, 3, 4, 5},
                   {6, 7, 8, 9, 10},
                   {11, 12, 13, 14, 15},
                   {16, 17, 18, 19, 20}};
   printf("%d", *(*(a+**a+2)+3));
   return (0);
}
```

The output of the program is________.

6. [NAT] [GATE-2020 : 2M]

Consider the following C functions:

```
int tob(int b, int* arr){
         int i;
    for (i = 0; b>0; i++) {
        if (b%2) arr [i] = 1;
        else     arr [i] = 0;
        b= b/2;
        }
  return (i);
}
int pp (int a, int b){
          int arr [20];
    int i, tot = 1, ex, len;
     ex = a;
    len = tob (b, arr);
    for (i = 0; i< len; i++){
            if (arr[i] ==1)
             tot = tot * ex;
            ex = ex * ex;
          }
          return (tot);
    }
```

The value returned by pp(3, 4) is______.

7. [NAT] [GATE-2019 : 1M]

Consider the following C program:

```
#include <stdio.h>
int main ()
{
 int arr [ ] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 5}, *ip = arr + 4;
 printf ("%d \n", ip[1]);
 return 0;
}
```

The number that will be displayed on execution of the program is _______.

8. [NAT] [GATE-2019 : 2M]

Consider the following C program:

```
#include<stdio.h>
int main( )
{
   int a[ ] = {2, 4, 6, 8, 10};
   int i, sum = 0, *b = a + 4;
   for (i = 0; i < 5; i++)
   sum = sum + (*b – i) –*(b – i);
   printf("%d\n", sum);
   return 0;
}
```

The output of the above C program is____________.

9. [MCQ] [GATE-2018 : 1M]

Consider the following C program:

```
# include<stdio.h>
struct Ournode {
    char x, y, z;
} ;
int main () {
    struct Ournode p = {'1' , '0' ,  'a' + 2};
    struct Ournode *q = &p;
```

```
    printf ("%c , %c",*((char*)q + 1), *((char*)
    q + 2));
    return 0;
}
```

The output of this program is:

(a) 0, c (b) 0, a+2

(c) ' 0 ', ' a+2 ' (d) ' 0 ', ' c '

10. [MCQ] [GATE-2017 : 1M]

Consider the following C code:

```
#include<stdio.h>
int *assignval (int  *x , int val)
  { *x = val;
  return x;
  }
  void main ( )
{
            int *x = malloc (sizeof(int));
if (NULL == x) return;
x = assignval (x, 0);
if ( x )
{   x = (int *) malloc (sizeof(int));
 if (NULL = = x) return;
 x = assignval (x, 10);
 }
    printf ("%d\n", *x);
    free(x);
}
```

The code suffers from which one of the following problems:

(a) compiler error as the return of malloc is not typecast appropriately.

(b) compiler error because the comparison should be made as x== NULL and not as shown.

(c) compiles successfully but execution may result in dangling pointer

(d) compiles successfully but execution may result in memory leak.

11. [NAT] [GATE-2017 : 2M]

Consider the following snippet of a C program. Assume that swap (&x, &y) exchanges the content of x and y.

```
int main ( ) {
int array [ ] = {3 , 5 , 1 , 4 , 6 , 2};
int done = 0;
int i;
while ( done ==0){
      done =1
      for (i=0 ; i<=4 ; i++){
            if (array [i] < array [i+1]){
                  swap(&array[i], &array[ i  + 1]);
                 done =0;
                }
         }
}
for (i=5 ; i>=1 ; i– –)
     {
        if (array [i] > array [i-1])
         {
          swap (&array[i], &array[i–1]);
         done =0;
          }
      }
   }
printf ("%d", array [3]);
}
```

The output of the program is__________

12. [NAT] [GATE-2017 : 2M]

Consider the following C program.

```
#include<stdio.h>
#include<string.h>
int main( )
{
    char* c= "GATECSIT2017";
    char* p=c ;
    printf("%d ", (int)strlen(c+2[p]–6[p]–1)) ;
    return 0 ;
}
```

The output of the program is______.

13. [MCQ] [GATE-2017 : 1M]

Consider the following function implemented in C :

```
void printxy(int  x , int  y )
{
int  *ptr;
x = 0;
ptr = &x;
y = * ptr;
*ptr = 1;
printf("%d, %d ", x, y) ;
}
```

The output of invoking printxy(1, 1) is:

(a) 0, 0
(b) 0, 1
(c) 1, 0
(d) 1, 1

14. [MCQ] [GATE-2017 : 1M]

Match the following:

List-I	List-II
P. static char var;	i. Sequence of memory locations to store addresses
Q. m=malloc(10) ; m= NULL;	ii. A variable located in data section of memory
R. char *ptr [10];	iii. Request to allocate a CPU register to store data
S. register int var1;	iv. A lost memory which cannot be freed

(a) P → (ii), Q → (iv), R → (i), S → (iii)
(b) P → (ii), Q → (i), R → (iv), S → (iii)
(c) P → (ii), Q → (iv), R → (iii), S → (i)
(d) P → (iii), Q → (iv), R → (i), S → (ii)

15. [NAT] [GATE-2016 : 2M]

Consider the following program:

```
int  f(int *p, int n){
   if (n <= 1) return 0;
   else return max(f(p+1, n – 1),p[0] – p[1]);
}
int main(){
   int a[ ] = {3 , 5 , 2 , 6 , 4};
   printf("%d", f(a, 5));
}
```

Note: max (x, y) returns the maximum of x and y:

The value printed by this program is____________.

16. [NAT] [GATE-2016 : 1M]

The value printed by the following program is____.

```
void f(int  * p,  int m) {
      m = m + 5;
     *p = *p + m;
      return;
}
void main( ) {
     int i = 5, j = 10;
     f( &i, j);
     printf("%d"  ,i + j);
}
```

17. [MCQ] [GATE-2016 : 2M]

What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```
a = 3;
void n(x) {
x = x * a; print (x);
}
void m(y) {
a = 1 ; a = y – a;  n(a);
print (a); }
void main () {
m(a);
}
```

(a) 6, 2
(b) 6, 6
(c) 4, 2
(d) 4, 4

18. [NAT] [GATE-2016 : 1M]

Consider the following C program.

```
#include<stdio.h>
void mystery(int*ptra,int*ptrb)
{     int*temp;
      temp=ptrb;
      ptrb=ptra;
      ptra=temp;
}
```

```
    int main()
{   int a=2016, b=0, c=4, d=42;
    mystery(&a, &b);
    if(a<c)
mystery(&c, &a);
mystery(&a, &d);
printf("%d\n",a);
}
```

The output of the program is____.

19. [MCQ] [GATE-2016 : 1M]

Consider the following C program.

```
void f (int, short);
void main (){
int i=100;
short s=12;
short  * p=& s ;
___________ ; / /  call to f()
}
```

Which one of the following expressions, when placed in the blank above, will NOT result in a type checking error?

(a) f(s, * s)
(b) i=f(i, s)
(c) f (i,* s)
(d) f(i, * p)

20. [NAT] [GATE-2015 : 2M]

Consider the following C program.

```
# include < stdio.h >
int main () {
    static int a[ ] = {10, 20, 30, 40, 50};
    static int *p[ ] = {a,a + 3, a+4, a + 1, a + 2);
    int **ptr = p;     ptr++;
    printf ("%d % d", prt-p, **ptr);
}
```

The output of the program is _______.

21. [MCQ] [GATE-2015 : 1M]

Consider the following C program segment.

```
# include <stdio.h >
int main (){
char s1 [7]=" 1234 ", * p;
p = s1 + 2;
    * p=' 0 ';
printf("% s" ,  s1);
}
```

What will be printed by the program?

(a) 12
(b) 120400
(c) 1204
(d) 1034

22. [MCQ] [GATE-2014 : 1M]

Consider the following program in C language:

```
#include <stdio.h>
main()
{
  int i;
  int*pi = &i;
  scanf("%d",pi);
  printf("%d\n", i+5);
}
```

Which one of the following statements is TRUE?

(a) Compilation fails.
(b) Execution results in a run-time error.
(c) On execution, the value printed is 5 more than the address of variable i.
(d) On execution, the value printed is 5 more than the integer value entered.

23. [MCQ] [GATE-2010 : 2M]

What is the value printed by the following C program?

```
#include < stdio.h>
int f(int *a, int n){
if (n <= 0) return 0;
else if (*a % 2 == 0)
return *a + f(a + 1, n-1);
else return *a - f(a + 1, n -1);
}
int main ()
{
int a[] = {12, 7,13.4.11,6);
printf(<,%d', f(a, 6));
return 0;
}
```

(a) –9
(b) 5
(c) 15
(d) 19

24. [MCQ] [GATE-2010 : 1M]

What does the following program print?

```
# include <studio.h>
void f( int * p, int * q){
p = q ;
* p = 2 ;
} int i = 0, j = 1 ;
int main( )
{
f(&i, &j) ;
printf(" % d% d\n ", i, j) ;
getchar(); return 0; }
```

(a) 22 (b) 21

(c) 01 (d) 02

Common Data for next two questions:

Consider the following C program that attempts to locate an element x in an array Y[] using binary search. The program is erroneous.

```
1.  f( int Y[10], int x){
2.  int i, j, k;
3.  i=0 ; j=9;
4.  do {
5.  k = (i + j) / 2;
6.  if (Y[K]<x) i = k; else j = k;
7.  } while ((Y[k]   !=x) &&(i<j));
8.  if (Y[k]==x)
    printf("x is in the array ") ;
9.  else
    printf ("x is not in the array");
10. }
```

25. [MCQ] [GATE-2008 : 2M]

On which of the following contents of Y and x does the program fail?

(a) Y is [1 2 3 4 5 6 7 8 9 10] and x < 10

(b) Y is [1 3 5 7 9 11 13 15 17 19] and x < 1

(c) Y is [2 2 2 2 2 2 2 2 2 2] and x > 2

(d) Y is [2 4 6 8 10 12 14 16 18 20] and 2 < x < 20 and x is even

26. [MCQ] [GATE-2008 : 2M]

The correction needed in the program to make it work properly is

(a) change line 6 to: if (Y[k] < x) i = k + 1; else j = k – 1;

(b) change line 6 to: if (Y[k] < x) i = k - 1; else j = k + 1;

(c) change line 6 to: if (Y[k] < x) i = k; else j = k;

(d) change line 7 to: while ((Y[k] = = x) && (i < j));

Strings

27. [MCQ] [GATE-2018 : 2M]

Consider the following C program:

```
#include<stdio.h>
void fun1 (char*  s1 , char* s2) {
   char* temp;
   temp = s1;
   s1 = s2;
   s2 = temp;
}
void fun2 (char** s1 , char** s2) {
   char* temp;
   temp = *s1;
  *s1 = *s2
  *s2 = temp ;
}
int main( ){
    char *str1 = “Hi”, *str2= “Bye”;
fun1(str1, str2);
printf(“%s %s”, str1, str2);
fun2(&str1, &str2);
printf(“%s, %s”, str1 , str2);
return 0;
}
```

The output of the program above is

(a) Hi Bye Bye Hi

(b) Hi Bye Hi Bye

(c) Bye Hi Hi Bye

(d) Bye Hi Bye Hi

28. [NAT] [GATE-2017 : 2M]

Consider the following C program.

```
#include<stdio.h>
#include<string.h>
void printlength(char *s, char *t) {
   unsigned int c=0;
   int len = ((strlen(s) – strlen(t)) > c) ? strlen(s) :
   strlen(t);
   printf("%d\n", len);
}
void main() {
   char *x = "abc";
   char *y = "defgh";
   printlength(x, y);
}
```

Recall that strlen is defined in string.h as returning a value of type size_t, which is an unsigned int. The output of the program is_____.

29. [MCQ] [GATE-2011 : 1M]

What does the following fragment of C program print?

```
char c[ ] = "GATE 2011";
char *p = c;
printf ("%s", p + p[3] – p[1]);
```

(a) GATE 2011 (b) E 2011

(c) 2011 (d) 011

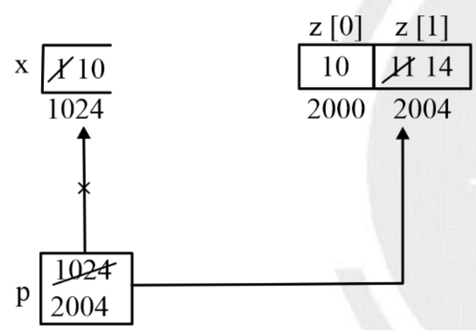
❑❑❑

ANSWER KEY

1.	(d)	**2.**	(a)	**3.**	(1023 to 1023)	**4.**	(c)
5.	(19 to 19)	**6.**	(81 to 81)	**7.**	(6 to 6)	**8.**	(10 to 10)
9.	(a)	**10.**	(d)	**11.**	(3 to 3)	**12.**	(2 to 2)
13.	(c)	**14.**	(a)	**15.**	(3 to 3)	**16.**	(30 to 30)
17.	(c)	**18.**	(2016 to 2016)	**19.**	(d)	**20.**	(140 to 140)
21.	(c)	**22.**	(d)	**23.**	(c)	**24.**	(d)
25.	(c)	**26.**	(a)	**27.**	(a)	**28.**	(3 to 3)
29.	(c)						

SOLUTIONS

1. **(d)**



→ *p=&x; stores the address of x into pointer p.

→ *p= 10; replaces value of x =1 by 10.

→ *p = &z[1]; stores the address of z[1] into pointer p.

Now,

&z [0] +1=&z [1]

(&z [0] +1) =*&z [1]

*(&z [0] + 1) = z [1]

*(&z [0] + 1) + 3 is same as z [1] + = 3

→ z [1] = z [1] + 3 = 11 + 3 =14

Because of statement *(&z[0] + 1) +=3, value at address 2004 is changed from 11 to 14.

Hence printf () prints 10, 10 and 14. Therefore correct option is (d).

Scan for Video solution

2. **(a)**

Given

$$\text{int a [3] [3] [3]} = \left\{\left\{\left\{\overbrace{\underbrace{1,2,3}_{a[0][0]},\underbrace{4,5,6}_{a[0][1]},\underbrace{7,8,9}_{a[0][2]}}^{a[0]}\right\}\right\}\right.,$$

$$\left\{\overbrace{\underbrace{10,11,12}_{a[1][0]},13,14,15,16,17,18}^{a[1]}\right\},$$

$$\left.\left\{\overbrace{19,20,21,22,23,24,25,26,27}^{a[2]}\right\}\right\}$$

i = 0, j = 0, k = 0,1,2

i	j	k	Printing
a[0]	[0]	[0]	1
a[0]	[0]	[1]	2
a[0]	[0]	[2]	3

i = 1, j = 0, k = 0,1,2

A → 65	Q → 81	a → 97
B → 66	R → 82	b → 98
C → 67	S → 83	c → 99
D → 68	T → 84	d → 100
E → 69	U → 85	.
F → 70	V → 86	.
G → 71	W → 87	.
H → 72	X → 88	.
I → 73	Y → 89	.
J → 74	Z → 90	.
M → 77		.
N → 78		x→ 120
O → 79		y→ 121
P → 80		z→ 122

i	j	k	Printing
a[0]	[0]	[0]	10
a[0]	[0]	[1]	11
a[0]	[0]	[2]	12

i = 2, j = 0, k = 0, 1, 2

i	j	k	Printing
a[0]	[0]	[0]	19
a[0]	[0]	[1]	20
a[0]	[0]	[2]	21

On verifying properly with options, option (a) is correct.

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3. (1023 to 1023)

Given that Z is an array which has index 0 to 9 and each index has value 1.

0	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1

Simple Function (Z, 10, 2) → calling function

↑

name of array

Array Z and array Y are same, because array is always call by reference.

As calling function is having Z, 10, 2 in parameters. n and x will take the values 10 and 2 respectively.

x = 2, n = 10

As initial value of Y[0] is 1, therefore initially value of variable total is 1.

Rewriting expression total = x × total + Y[i] for better understanding to

$total_i = x \times toal_{i-1} + Y[i]$, initially x = 2, each Y[i] = 1 and $total_0 = 1$

$\boxed{i=1}$ $total_1 = x \times total_0 + Y[1]$

$total_1 = 2 \times 1 = (2 + 1)$

$\boxed{i=2}$ $total_2 = x \times total_1 + Y[i]$

$total_2 = 2 \times (2 + 1) + 1 = 2^2 + 2^1 + 1$

$\boxed{i=3}$ $total_3 = 2 \times total_2 + Y[i]$

$= 2 \times (2^2 + 2^1 + 1) + 1$

$Total_3 = 2^3 + 2^2 + 2^1 + 1$

.

.

.

Similarly,

$\boxed{i=9}$ $total_9 = \underline{2^9 + 2^8 + 2^7 + ...2^1 + 1}$

Above series is in GP with 10 terms

We know that

$$\boxed{\text{Sum of first n terms of a GP} = \frac{a(r^n - 1)}{r - 1}}$$

Where,

a = first term

r = common ratio

n = number of terms

Writing $2^9 + 2^8 + 2^7 + 2^6 + \ldots\ldots 2^1 + 1$ in reverse order we get,

$1 + 2^1 + 2^2 + 2^3 + \ldots. 2^9$

a = 1, n = 10, common ratio (r) = 2

Substituting in above formula

$$S_{10} = \frac{1(2^{10} - 1)}{2 - 1}$$

= 1023 is the answer, which is the value of total_9.

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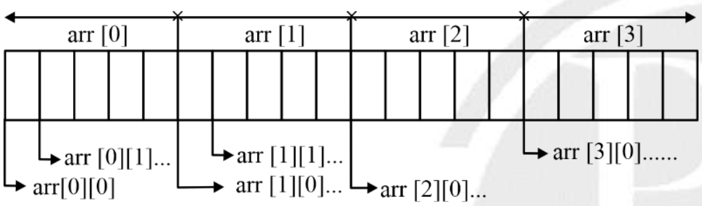
4. (c)

Given array int arr[4][5], it is read as arr is an array of 4 1-Dimensional array and each 1-Dimension array contains 5 elements

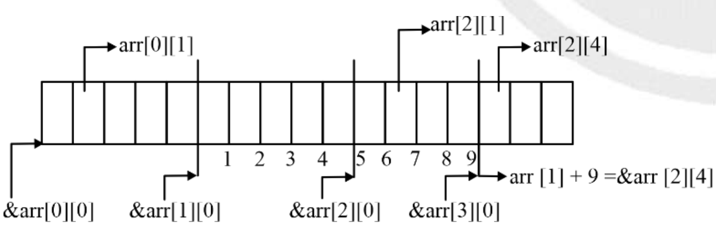


Note: array name represents the address of its first element.

Example: arr[1] = &arr[1][0]

arr[1] + 9 = &arr[1][0] + 9

(9 crepresenting to move 9 locations in forward direction from address of arr[1][0](&arr[1][0]))



arr[1] + 9 = &arr[2][4]

*(arr[1] + 9) = arr[2][4]

Therefore arr[2][4] = 10 * 2 + 4 = 24.

*(arr[1] + 9) = arr[2][4] = 24

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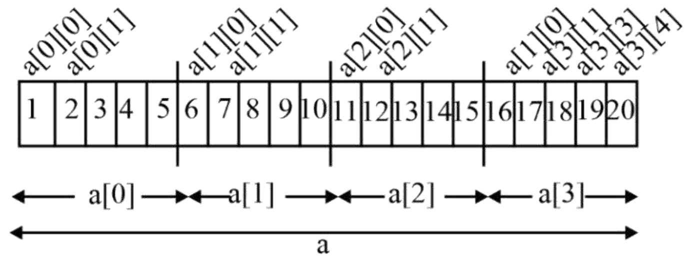
5. (19 to 19)



: array name (a is collection of 4 elements a[0],a[1], a[2],a[3])

Array name represent address of its first elemetns.

a = &a[0]

*a = *&a[0]

*a = a[0]\

<u>What is a [0]?</u>

a[0] is an array of 5 elements i.e a[0][0], a[0][1], a[0][2] & a[0][3],a[0][4]

a[0] ⇒ address of its first element

a[0] = &a[0][0]

*a = a[0]

*a = a[0] = &a[0][0]

*a = &a[0][0]

**a = a[0][0]

$\boxed{a[0][0] = 1}$

*(a + **a + 2) = *(a + 1 + 2) = * (a + 3) = a[3]

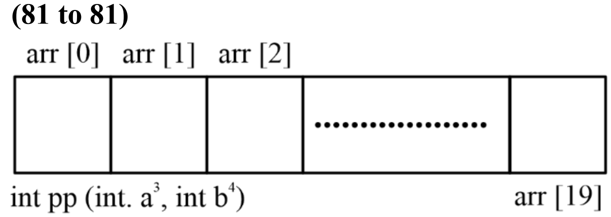
*(a + **a + 2) = a[3]

*(a+ **a+2) + 3 = a[3]+3

*(*a+ **a + 2) + 3) = a[3][3] = 19

Scan for Video solution

6. (81 to 81)



int pp (int. a^3, int b^4)

tot = 1 ex = 3

Note:- that arr[i] in tob is same as arr[i] in pp and also array contains garbage value initially.

Understanding tob function:

b = $\cancel{4}$ 2

Ø 0	Ø0	Ø1	G	G	G	G
arr [0]	arr [1]	arr [2]	arr [3]	....		arr [19]

$\left.\begin{array}{l}\text{if}\left(b\%2\right)\\ \text{arr}\left[i\right]=1;\end{array}\right|$ exeutes when b is odd

$\left.\begin{array}{l}\text{else}\\ \text{arr}\left[i\right]=0;\end{array}\right|$ exeutes when b is even

$\left.b=\frac{b}{2}\right|$ exeutes every time

(1) i = 0, b > 0 ⇒ 4 > 0 ⇒ true(b is even)
arr[i] = 0
arr[0] = 0
b = b/2 = 4/2 = 2

(2) i =1 b > 0 ⇒ 2 > 0 ⇒ true(b is even)
arr[i] = 0
arr[1] = 0
b = b/2 = 2/2 = 1

(3) i = 2, b > 0 ⇒ 1 > 0 ⇒ true (b is odd)
arr[i] = 1
arr[2] = 1
b = b/2 = ½ = 0

(4) i = 3, b > 0 ⇒ 0 > 0 ⇒ false

Here loop terminates.

Understanding pp function:

ex: = $\cancel{3}$ $\cancel{9}$ 81

tot = $\cancel{1}$ 81

len = 3

```
for(i = 0;i < len;i++)
{
if(arr[i]==1)
tot=tot*ex;
ex=ex*ex;
}
```

above code is executed from i=0 to 2(len =3)

(i) <u>i=0</u>
arr[i]= =1
0= =1(false)
ex=ex*ex
ex=3*3
ex=9

(ii) <u>i=1</u>
arr[i]==1
0==1(false)
ex=ex*ex
ex=9*9=>81
ex=81

(iii) <u>i=2</u>
arr[i]==1
1==1(true)
tot=tot*ex
tot=1*81
tot=81
ex=ex*ex
ex=81*81
81 is returned.

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7. (6 to 6)

1	2	3	4	5	6	7	8	9	0	1	2	5
arr &arr [0]	arr+1	arr+2	arr+3	arr+4								

arr is name of the given array, we know that array name represents the address of first element.

arr=&arr[0]

arr+4=&arr[0]+4(moving 4 locations ahead from &arr[0])

arr+4 = &arr[4]

$\boxed{\text{ip=\&arr}[4]}$

ip[1] = *(ip+1) = *(&arr[4]+1)

ip[1] = *(&arr[5])

ip[1] = arr[5]

ip[1] = 6

Hence 6 is the correct answer

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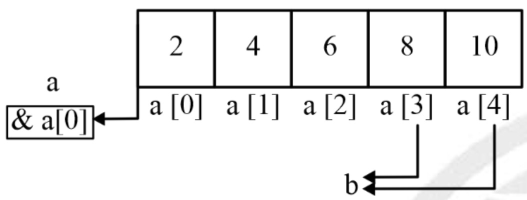
8. (10 to 10)

Constructing the array according to the given data.



Initially, value of sum = 0

b = a + 4

b = &a[0] + 4

b = &a[4]

*b = a[4]

b-0 = &a[4]

b-1 = &a[3]

b-2 = &a[2]

b-3 = &a[1]

b-4 = &a[0]

Note: Scope of for loop remains till the first semicolon (;).

i = 0 sum = sum + (*b – 0) – *(b – 0)

  sum=0 + (10 – 0) – 10 = 0

<u>i = 1</u> sum=0 + (10 – 1) – 8 = 1

<u>i = 2</u> sum=1 + (10 – 2) – 6 = 3

<u>i = 3</u> sum=3 + (10 – 3) – 4 = 6

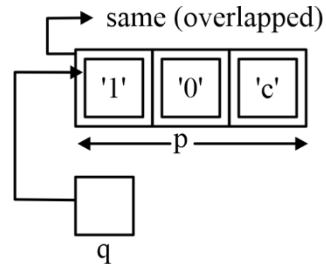
<u>i = 4</u> sum=6 + (10 – 4) – 2 = 10

Finally, value which is stored in sum variable is printed,

Therefore 10 is printed.

Scan for Video solution

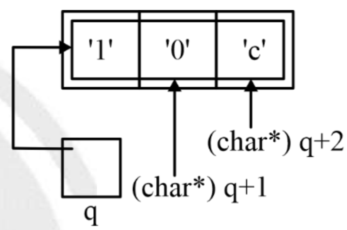
9. (a)



'a' + 2⇒ 'c'

Intitally q is pointing to address of whole structure variable p.

(char*)q : Here typecasting is being done,q is holding address of a character i.e q is pointing to addresses of '1' (First member)



(char*) q : Address of char '1'

(char*) q + 1 : Address of next character i.e. '0'

(char) q+1: '0'

(char*) q+2 : Address of 'c'

(char)q+2: 'c'

printf("%c",*((char*)q+1), *((char*)q+2)); will print 0,c without any quotes. Hence option A is correct.

Scan for Video solution

10. (d)

A (false): Compiler error as the return of malloc is not type casted appropriately.

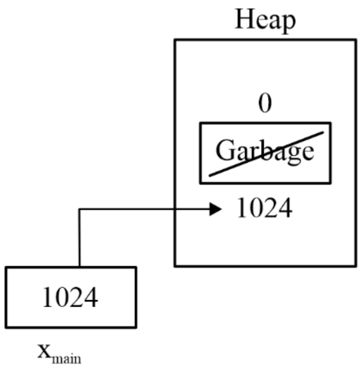
False because no typecasting is required. In C++, it is required but not in C language.

B(false): Compiler error because the comparison should be made as x==NULL and not as shown.

False, as we know that NULL==x or x==NULL are same, hence option B is also false.

C(false): Compiles successfully but execution may result in dangling pointer.

void main(){
int *x = malloc(sizeof(int));
if(NULL==x)
return; //if no memory is available then return.



assignval (1024, 0) is called

x_{local} = 1024

val= 0

x_{local} =0

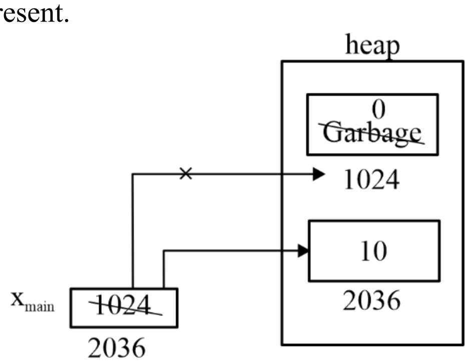
value at memory location pointed by x_{local} = 0

value at memory location 1024 = 0

return x, returns 1024 to x in main.

if(x){
x=(int*)malloc(sizeof(int))
if(NULL==x)
return;
x=assignval(x,10);
}
printf("%d",*x);
free(x);
}

Note that code written in solution are just used to represent.



we are again allocating memory without freeing old memory, x is now pointing to memory location 2036 and we cannot access to memory location 1024 which is lost (also known as memory leak problem), and also there is no pointing of memory location which is deleted, hence there is no dangling pointer problem.

Therefore, above code suffers from memory leak problem, hence option D is correct answer.

Scan for Video solution

11. (3 to 3)

Firstly, understanding the first loop in the given program.

Initial value of done variable and content of the array shown below.

while (done = = 0) {
done =1;
for(i=0;i<=4;i++){
if(array[i]<array[i+1]){
swap(&array[i], &array[i+1]);
done= 0;

done

~~0~~ ~~1~~ 0

3	5	1	4	6	2

(i) done = = 0 (true)

code inside while loop will execute

→ (i) done = = 1
→ (ii) for loop will execute from i = 0, 1, 2, 3, 4

<u>Executing for loop</u>

(1) <u>i = 0</u>

we are comparing element of i^{th} index and $i+1^{th}$ index

array[0]<array[1] →3<5 true, swapping will take place.

swap(&array[0],&array[1])

0	1	2	3	4	5
5	3	1	4	6	2

Again, done variable value becomes 0

(2) **i = 1**

array [1]< array [2] →3<1 false, no swapping will take place.

(3) **i = 2**

array [2]< array [3] →1<4 true, swapping will take place.

5	3	4	1	6	2
0	1	2	3	4	5

done = 0 (No effect because done is already 0)

(4) **i = 3**

array [3]< array [4]→1<6 true, swapping will take place.

5	3	4	6	1	2
0	1	2	3	4	5

done =0(no effect)

(5) **i = 4**

array[4]<array[5] true, swapping will take place

5	3	4	6	2	1
0	1	2	3	4	5

⇒ 1st loop terminate

Now understanding 2nd loop of the given program.

done = 0

5	3	4	6	2	1
0	1	2	3	4	5

```
for(i= 5;i> =1;i--){
if(array[i] >array [i– 1]){
swap(&array [i], &array [i –1]);
done =0;
  }
}
```

$\boxed{i = 5}$ array [5] > array [4] ⇒ 1 > 2 false, now swap

$\boxed{i = 4}$ array [4] > array [3] ⇒ 2 > 6 false, no swap

$\boxed{i = 3}$ array [3] > array [2] ⇒ 6 > 4 (true), swap

5	3	6	4	2	1
0	1	2	3	4	5

done = 0 (No effect)

$\boxed{i = 2}$ array [2] > array [1] ⇒ 6 > 3 (true), swap

5	6	3	4	2	1
0	1	2	3	4	5

done = 0 (No effect)

array [1] > array [0] ⇒6 > 5→ true, swap will take place.

6	5	3	4	2	1
0	1	2	3	4	5

done = 0 (No effect)

(5) $\boxed{i = 1}$

Both the loop ends

done

$\boxed{0}$

6	5	3	4	2	1
0	1	2	3	4	5

```
While(done==0)
{
 done = 1;
    1st loop
    2nd loop
    }
```

Another iteration of while loop takes place

Value of done becomes 1.

1st loop will execute for i = 0, 1, 2, 3, 4

6	5	3	4	2	1
0	1	2	3	4	5

$\boxed{i = 0}$ array [0] < array [1] ⇒ false, no swap

$\boxed{i = 1}$ array [1] < array [2] ⇒ false, no swap.

$\boxed{i = 2}$ array [2] < array [3] ⇒ true, swap

6	5	4	3		2	1
0	1	2	3		4	5

done = 0 is made

Array is sorted in decreasing order now i.e. arra y [1] > array [i +1], As this is true that means array [i] < array [i +1] is false for all i, 1st loop is terminated.

6	5	4	3	2	1

done = 0

2nd loop : because array is sorted in decreasing order.

$\Rightarrow$ array [i] < array [i –1]

i.e. array [i] > array [i –1] is false for all i.

Hence 2nd loop terminates

while (done = = 0)

0 = = 0 (true)

Another iteration of while loop is executed.

done

done						
~~0~~ 1	6	5	4	3	2	1
	0	1	2	3	4	5

1st loop, no change because array is already sorted.

2nd loop, no change in array because of same above reason.

While (done = = 0)

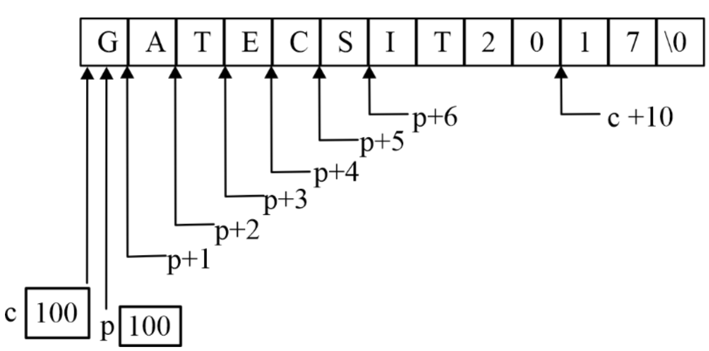
1 = = 0(False) while loop terminates.

Finally, array [3] is printed, therefore 3 is printed as output

Scan for Video solution

12. (2 to 2)

```
int  main ()
{
char *c = "GATECSIT2017";
char *p = c;
printf ('%d', (int) strlen(c +2 [p] – 6 [p] –1));
return 0;
}
```



p + 2 = moving 2 locations in forward direction from address of 'G'.

c is holding base address (address of G)

p + 2 = Address of 'T'

$\boxed{*(p+2) = 'T'}$

$\boxed{p[2] = 2[p] = *(p+2) = *(2+p) = 'T'}$

p + 6 = Address of 'I'

* (p + 6) = 'I'

$\boxed{p[6] = 6[p] = *(p+6) = *(6+p) = 'I'}$

Example:

c + 'B' – 'A': What are value for 'B', 'A' Here ASCII values are used.

c + 66 – 65

= c + 1; address of 'A'

c + 2 [p] – 6 [p] –1
= c + 'T' – 'I' –1
= $c + \cancel{x} + 11 - \cancel{x} - 1$
= c + 10

I $\rightarrow$ x
J $\rightarrow$ x + 1
K .
L .
M .
N .
O .
P .
Q .
R .
S .
T $\rightarrow$ x + 11

c + 10 $\Rightarrow$ Address of 'G' + 10
= Moving 10 location in forwards direction from address of 'G'

c + 10 $\Rightarrow$ address of char '1'.

c + 10 $\Rightarrow$ represents string "17"

strlen (c + 10) = 2

Note: strlen returns unsigned integer

Therefore 2 is returned.

Scan for Video solution

13. (c)

void printxy (int x int y)

{

int *ptr;

x =0;→ will make x as 0

ptr =&x; → assign 1026 to ptr

y = *ptr;→ y = value at (memory location 1026)

y = 0

*prt = 1;→ value at memory location 1026 = 1)

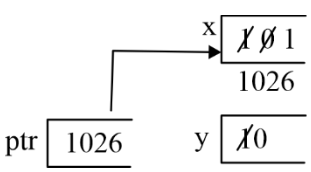
printf (“%d%d” x,y);

}

printxy (1 1)

↑ ↑

x y



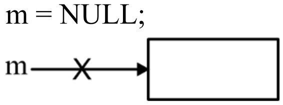
lastly value of x and y is printed as 1, 0; Hence option (c) is correct answer.

Scan for Video solution

14. (a)

(P) static char var: Stored in data segment
P – ii

(Q) m = malloc (10);
m = NULL;



No way to access memory location i.e.
Lost memory
Q → IV

(R) char *ptr [10]
ptr is an array of 10 pointers to chaaracter i.e. 10 pointer can hold 10 address.
R → (i)

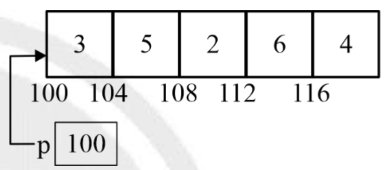
(S) register int var 1
Register variable is only a requests /recommendation
S → (iii)

P→ (ii), Q→ (iv), R → (i), S →(iii)

Hence, option A is correct.

Scan for Video solution

15. (3 to 3)



p + 1 = moving 1 location in forward direction from address holding pointer variable p.

p + 1 = memory location 104

⇒*(p + 1) = Then it would be value at memory location 104

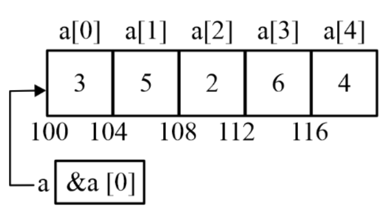
*(p + 1) = 5

p[1] = 5

NOTE: If p is pointing to address of integer, then

p[0]: is value at the address

p[1]: is value at next address



NOTE: array name always points to the address of first index of an array.

f(a,5) → f(100,5)

max(f(p+1),n-1,p[0]-p[1]) will be executed whenever n>1

p **n**
f (100, 5)
3
p[0] = 3
p [1] = 5
n > 1

p **3** **n** **–2**
max (f (104, 4) , 3 – 5)
p [0] = 5
p [1] = 2
n > 1

p **2** **n** **3**
max (f (108, 3) , 5 – 2)
n > 1
p [0] = 2
p [1] = 6

p **2** **n** **–4**
max (f 112, 2) , 2 – 6)
n > 1
p [0] = 6
p [1] = 4
0 **2**
max (f 116, 2) , 6 – 4)

f(116,1) $\Rightarrow$ n<=1 is true, returns 0.
Hence 3 is returned.

Scan for Video solution

16. (30 to 30)

f(&i, j) $\rightarrow$ f (1024, 10)
$\uparrow$ $\uparrow$
(Address, value)

p | 1024 |

i | ~~5~~ 20 |
1024

j | 10 |
2048

m | ~~10~~ |
15

*p = *p + m
(value at memory 1024 + 15)
*p = 5 + 15 = 20
Hence, 20 + 10 = 30 is printed

Scan for Video solution

17. (c)

a = 3 (declaring new variable)

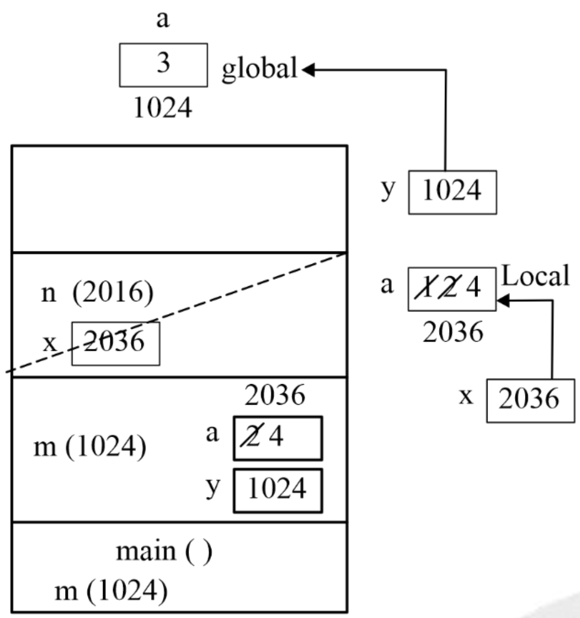
a = 1(declaring new variable)

a=3(Global scope, because it is not inside any function)

a=1(local scope, because declared inside m())

As it is mentioned in question that it is using dynamic scoping and pass by reference, rewriting the code for better understanding.

```
int a=3;
void n(int(*x)){
*x=*x*a;
print(*x);
}
void m(int *y){
int a=1;
a=*y-a;
n(&a);
print(a);
}
void main(){
m(&a);//there is no local a so, a is global variable
```



a = y –a (update local a)
a = value at (1024) –1
a = 3 – 1 = 2

a = y –a (update local a)

a = value at (1024) –1

a = 3 – 1 = 2

at n(&a), n(2036) is called

*x=*x*a;

= value at(2036)*a(here, previous call will be resolved)

i.e…..a of m()

=2 * 2 = 4 i.e….value at (2036) = 4

Lastly 4 is printed (value of x)

As function n gets deleted from activation record, updating has no meaning, m() will also print 4 and gets deleted from activation record and lastly main also gets deleted from activation record.

Therefore, C is correct option.

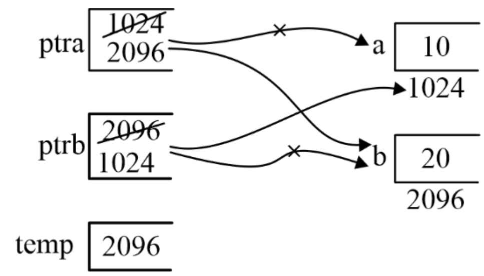
Scan for Video solution

18. (2016 to 2016)

Let mystery (&a, &b) is called

mystery $\left(\overset{\text{ptra}}{1024}, \overset{\text{ptrb}}{2096}\right)$

Note:-No dereferencing is taking palce

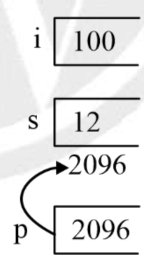


As soon as execution of this function ends, there is nothing available in activation record, no change in actual argument ie…. mystery function is not making any changes in actual argument. Hence actual value of a is printed ie…2016

Scan for Video solution

19. (d)

Function is expecting 1st argument as int type and 2nd argument as short type.



Option (A):

f(s, *s)

1st agreement is short type but function is expecting this argument to be type of int, but because of implicit conversion, short is converted into int. so 1st argument will not cause any error 2nd argument is pointer but function is expecting it to be short type which will generate an error.

∴ (A) is wrong

Option B

i = f(i,s)

Return type is not void, hence option (b) is also wrong.

Option C:

f(i, *S)

1^{st} argument is int but 2^{nd} argument is pointer i.e. There exists system error. So option c is also wrong. (s is not holding any address.)

Option D:

f(i, *p)

Both types are matching

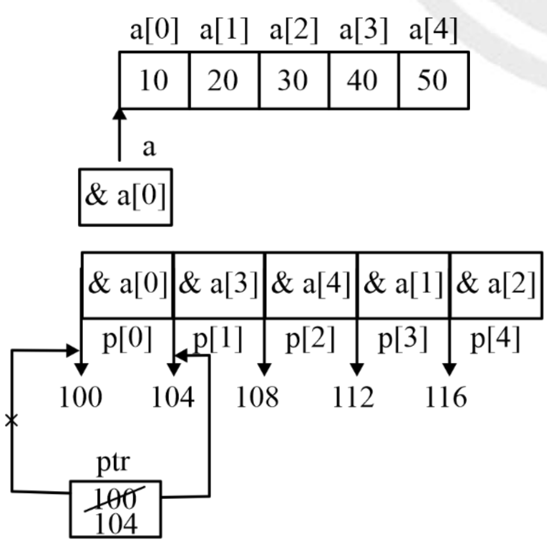
First argument is int and p is pointer to short type, Hence, (f) is correct.

s is not (a) pointer.

Scan for Video solution

20. (140 to 140)

static int * p[] is read as p is an array of pointer to integer each element of p[ ] holds an address of integer type variable.



(array name) a = &a[0]

a +1 = moving one location in forward direction from &a[0] = &a [1]

NOTE: Array name represents address of its first element.

a + 2 = & a [2]

a + 3 = & a [3]

a + 4 = & a [4]

int ** ptr = p;

Here p is array name, which represents address of its first elements.

ptr++=ptr will point to next address or location ie….104

(i) &p[1] – &p[0]

$= \frac{\text{Actual difference}}{4} = \frac{104 - 100}{4} = 1$

(ii) ** ptr

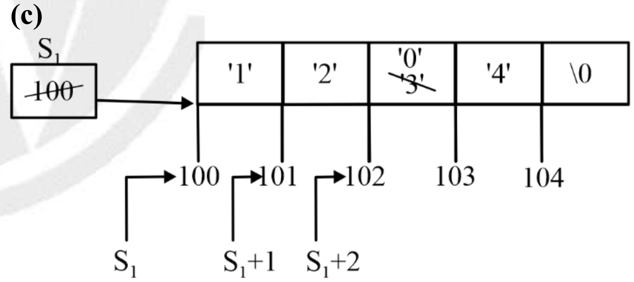
**(& p [1])

= ** &p[1] =*p[1] =*&a[3] = a[3] = 40

Hence, 140 is printed.

Scan for Video solution

21. (c)



p = s_1 + 2

⇒ Address + 2 (moving 2 locations in forward direction)

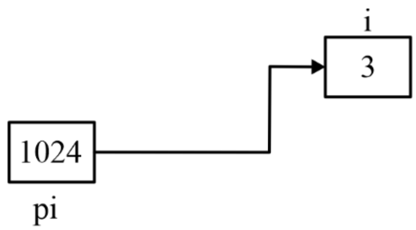
p = 102 (address of ‘3’)

* p = ‘0’ (Assign character ‘0’ (zero) at address 102)

Hence, 1204 is printed, option c is correct.

Scan for Video solution

22. (d)



scanf("%d", pi)

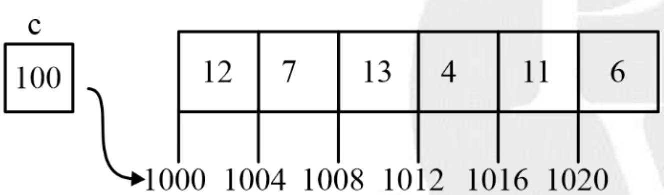
scanf takes address as input, whatever the input given will be stored at memory location 1024.

Let the input is 3

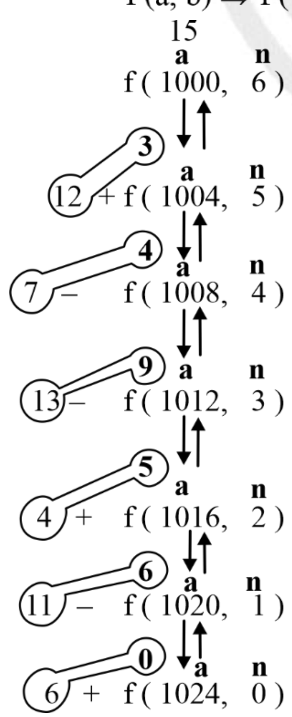
The printed value is 5 more than the input entered therefore option (d) is correct answer.

Scan for Video solution

23. (c)



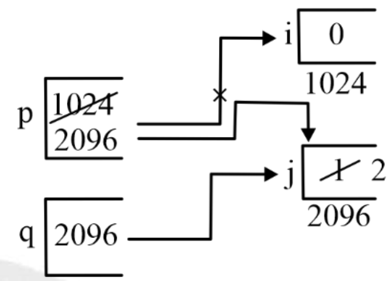
f (a, b) ⇒ f (1000, 6)



Hence 15 is printed and option (c) is correct answer

Scan for Video solution

24. (d)



Both p and q are pointing to j (2096)

* p = 2 that means insert value at address 2096

0, 2 is printed hence, correct option is(d).

Scan for Video solution

25. (c)

Firstly, let us understand how binary search works.

10	20	30	40	50	60	70	80	90	100
0	1	2	3	4	5	6	7	8	9
Begin									End

Case: 1

Let us suppose we are searching for x =90

$$\text{Mid} = \frac{\text{Begin} + \text{end}}{2} = \frac{0+9}{2} = 4$$

$$\underbrace{A[\text{mid}]}_{40} \underset{\text{No}}{==} \underbrace{x}_{90}$$

x is greater than elements at Mid position.

(1) x cannot be present from any location between Begin and mid new list must be from Mid +1 to end index.

Begin = Mid + 1

Begin = 5

Again, Mid = $\frac{(5+9)}{2} = 7$

Check A [Mid] = = x

A [7] = = 90 (No)

A [Mid] < x

Update ⇒ Begin = Mid + 1 = 8

Mid = $\frac{8+9}{2} = 8$

Check A [Mid] = x

yes

break and return.

Case: 2

Begin = 0, End = 9, x = 120

(i) Mid = $\frac{0+9}{2} = 4$

A [Mid] = = x No

(ii) Begin = Mid + 1 = 5

Mid = $\frac{5+9}{2} = 7$

A [7] = = x No

A [Mid] < x

(iii) Begin = Mid + 1 = 8

Mid = $\frac{8+9}{2} = 8$

A [Mid] = = x No

A [Mid] < x

(iv) Begin = Mid + 1 = 9

Mid = $\frac{9+9}{2} = 9$

A [Mid] = = x No

A [Mid] < x

Begin = Mid + 1 = 10

Begin becomes greater than End, unsuccessful search, element is not present.

Repeat till Begin < End && A [Mid]! = x

Case: 3

x = = 40

(i) Begin = 0, End = 9

Mid = $\frac{0+9}{2} = 4$

A [Mid] = = x No

x < A [Mid]

⇒ x cannot be present from Mid to End position.

New list must be from Begin to Mid –1

End = Mid –1

(i) We need to calculate mid and check every time if element is equal then search result is found.

(ii) If element is small then Mid search should be performed on left side.

(iii) If x is bigger than Mid then search should be performed on right side of list.

Now, focusing on the actual code.

(i) k = $\frac{i+j}{2}$ is performing mid operation

(ii) If (Y[k] < x) i = k; else j = k;

- In above code, if element to be searched is larger than Mid, then there should be i = k +1 (Begin = Mid +1) but above it is being assigned without incrementing.
- And if element is smaller then j = k – 1 (End = Mid–1) must have been performed but above it is being assigned without decrementing in else part.

The above given code will not work if

(i) x (Element to be searched) is greater than the maximum element (last element)

(ii) Or we are searching for the last element itself.

Case: 4

10	20	30	40	50	60	70	80	90	100
0	1	2	3	4	5	6	7	8	9
↓ i								↓ j	

(i) Let us assume we are searching x = 100

j = 9

k = $\frac{0+9}{2} = 4$

Y [4] < x (True)

i = k ⇒ i = 4 (i and k are printing same index)

(ii) $k = \frac{4+9}{2} = 6$

Y [6] < x (True)

i = k ⇒ i = 6

(iii) $k = \frac{6+9}{2} = 7$

Y [7] < x (True)

i = k ⇒ i = 7

(iv) $k = \frac{7+9}{2} = 8$

Y [8] < x (True)

i = k ⇒ i = 8

(v) $k = \frac{8+9}{2} = 8$

y [8] < x (True)

i = k ⇒ i = 8

i remains same (no update)

Program goes in to infinite loop, i will never become equal to j or greater then j.

∴ As above example is illustrated in (case 4), therefore correct option is option c, because program goes int. infinite loop.

Scan for Video solution

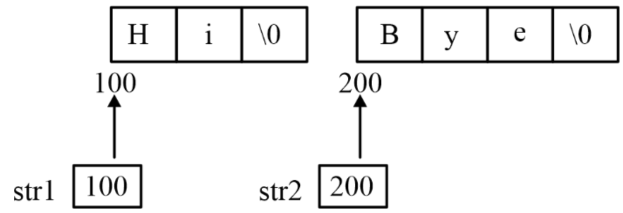
26. (a)

- i = k has to be replaced by i = k + 1 if elements is larger than Mid (k is Mid).
- j =k has to be replaced by j = k – 1 if elements is smaller than Mid.

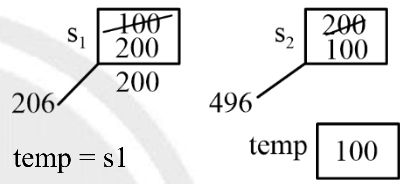
Hence, option A is correct answer.

Scan for Video solution

27. (a)



fun1 () is called, fun1 (100, 200)



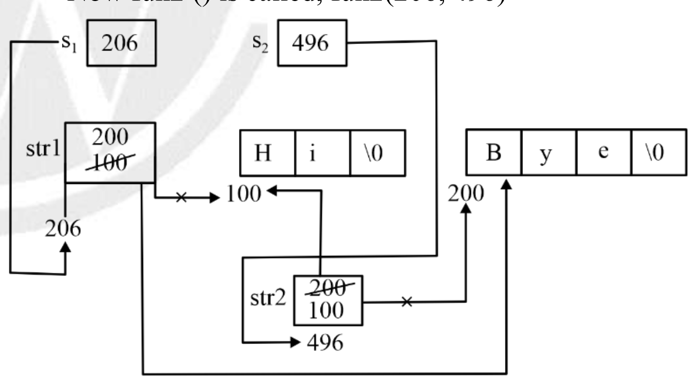
temp = s1

$s_1 = s_2$

s_2 = temp

No change in str1 and str2, so HiBye.. is printed from the first printf statement.

Now fun2 () is called, fun2(206, 496)



temp = *s1 (value of at memory location 206)

temp = 100

* s_1 = *s_2 (value at memory location 496)
* s_1 = 200
* s_2 = temp
* s_2 = 100

- contents of str1 and str2 are swapped
- Now str1 is pointing to 'Bye' and str2 is pointing to "Hi"

Therefore 2nd printf prints ByeHi.

Hence (a) is correct answer.

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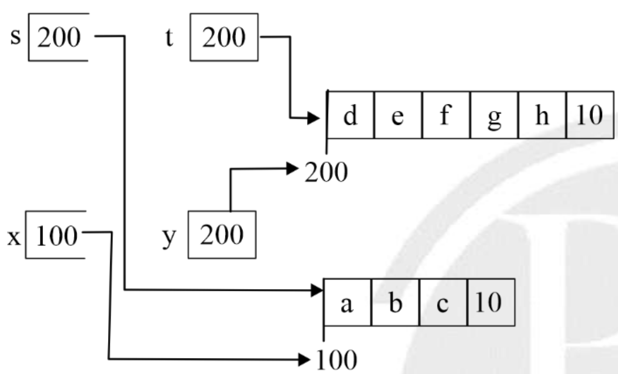
28. (3 to 3)

char*x = “abc”;→ Read only area

printlength (x, y); → printlength(100,200)



strlen(s) – strlen(t)

3 – 5 = –2

Unsigned int – unsigned int

Will also be an unsigned int

⇒ – 2 is treated as max integer value – 2

= large +ve value

$\overset{3}{\text{len}}$ = ($\underset{\text{true}}{\text{very large positive val >0}}$) ? $\overset{3}{\text{strlen(s)}}$: strlen(t)^{5}

Hence 3 is printed.

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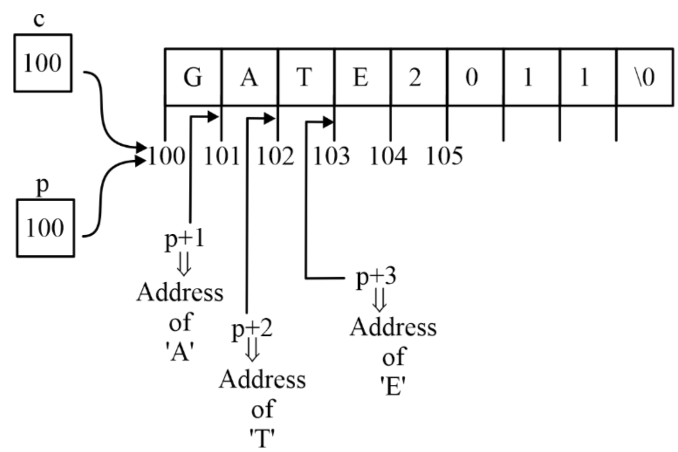
29. (c)

char c [ ] = “GATE 2011”;

char *p= c;

printf (“%s”, p + p [3] – p[1])



p+ 1 ⇒ Address of ‘A’

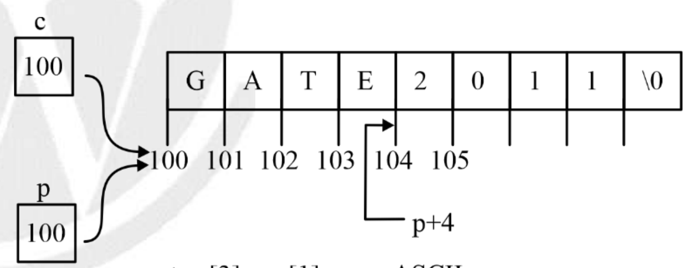
* (p +1) ⇒ ‘A’

$\boxed{p[1] = 'A'}$

p + 3 ⇒ Address of ‘E’

* (p +3) ⇒ ‘E’

$\boxed{p[3] = 'E'}$



$$\begin{aligned}&p + p[3] - p[1]\\&= p + 'E' - 'A'\\&= p + x + 4 - x\\&= p + 4\\&= \text{Address } 104\end{aligned}$$

ASCII

$A \to x$

$B \to x + 1$

$C \to x + 2$

$D \to x + 3$

$E \to x + 4$

2011 is printed till \0, therefore option c is correct.

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CHAPTER

5 ARRAYS AND LINKED LIST

Arrays

1. [NAT] [GATE-2015: 2M]

Suppose c = ⟨c[0],… , c[k-1] ⟩ is an array of length k, where all the entries are from the set {0,1}. For any positive integers a and n, consider the following pseudocode.

```
DOSOMETHING (c, a, n)
Z ← 1
for i ← 0 to k-1
do z ← z^2  mod n
if c[i] = 1
then z ← (z × a) mod n
return z
```

If k = 4, c = ⟨1,0,1,1⟩, a = 2 and n = 8, then the output of DOSOMETHING (c, a, n) is _____

2. [MCQ] [GATE-2014: 1M]

Let A be a square matrix of size n × n. Consider the following pseudo code. What is the expected output?

```
C = 100;
for i = 1 to n do
     for j = 1 to n do
   {
     Temp = A[i][j] + C;
     A[i][j] = A[j][i];
     A[j][i] = Temp – C;
     }
for  i = 1 to n do
     for j = 1 to n do
     Output  (A[i][j]);
```

(a) The matrix A itself
(b) Transpose of the matrix A
(c) Adding 100 to the upper diagonal elements and subtracting 100 from lower diagonal elements of A
(d) None of the above

3. [MCQ] [GATE-2014: 2M]

Consider the following C function in which size is the number of elements in the array E:

```
int MyX(int * E, unsigned int size) {
     int Y = 0;
     int Z;
     int i, j, k;
    for (i = 0; i < size; i++)
    Y = Y + E[i];
for (i= 0; i < size; i++)
  for (j = i; j< size; j++)   {
     Z = 0;
     for (k = i; k <= j; k++)
                        Z = Z + E[k];
     if (Z>Y)
                        Y = Z;
       }
return Y;
}
```

The value returned by the function MyX is the-

(a) maximum possible sum of elements in any sub-array of array E.
(b) maximum element in any sub-array of array E.
(c) sum of the maximum elements in all possible sub-arrays of array E
(d) the sum of all the elements in the array E.

Sorting Arrays

4. [MCQ] [GATE-2015: 2M]

Consider the following two C code segments. Y and X are one and two dimensional arrays of size n and n × n respectively, where $2 \le n \le 10$. Assume that in both code segments, elements of Y are initialized to 0 and each element X[i][j] of array X is initialized to i + j. Further assume that when stored in main memory all elements of X are in same main memory page frame.

Code segment 1:

```
// initialize elements of Y to 0
// initialize elements X [i][j] of X to i + j
    for (i = 0; i < n; i++)
    Y[i] += X[0][i];
```

Code segments 2:

```
// initialize elements of Y to 0
// initialize elements X[i][j] of X to i + j
    for (i = 0; i < n; i + +)
     Y[i] += X[i][0];
```

Which of the following statements is/are correct?

S1: Final contents of array Y will be same in both code segments.

S2: Elements of array X accessed inside the for loop shown in code segment 1 are contiguous in main memory.

S3: Elements of array X accessed inside the for loop shown in code segment 2 are contiguous in main memory.

(a) Only S2 is correct
(b) Only S3 is correct
(c) Only S1 and S2 are correct
(d) Only S1 and S3 are correct

Linked List

5. [MCQ] [GATE-2023 : 1M]

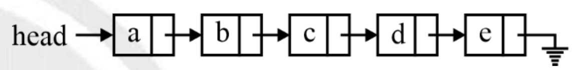
Let SLLdel be a function that deletes a node in a singly-linked list given a pointer to the node and a pointer to the head of the list. Similarly, let DLLdel be another function that deletes a node in a doubly-linked list given a pointer to the node and a pointer to the head of the list.

Let n denote the number of nodes in each of the linked lists. Which one of the following choices is TRUE about the worst-case time complexity of SLLdel and DLLdel?

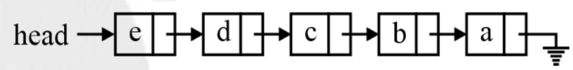
(a) SLLdel is O(1) and DLLdel is O(n)
(b) Both SLLdel and DLLdel are O(log(n))
(c) Both SLLdel and DLLdel are O(1)
(d) SLLdel is O(n) and DLLdel is O(1)

6. [MCQ] [GATE-2022: 1M]

Consider the problem of reversing a singly linked list. To take an example, given the linked list below,



the reversed linked list should look like



Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in O(1) space?

(a) The best algorithm for the problem takes $\theta(n)$ time in the worst case.
(b) The best algorithm for the problem takes $\theta(n \log n)$ time in the worst case.
(c) The best algorithm for the problem takes $\theta(n^2)$ time in the worst case.
(d) It is not possible to reverse a singly linked list in O(1) space.

7. [MCQ] [GATE-2021: 2M]

Consider the following ANSI C program:

```
#include <stdio.h>
#include <stdlib.h>
struct Node{
int value;
struct Node *next;};
int main( ){
```

```
struct Node *boxE, *head, *boxN; int index = 0;
boxE = head = (struct Node *) malloc(size of(struct
Node));
head → value = index;
for (index = 1; index <= 3; index++){
boxN = (struct Node *) malloc(size of(struct Node));
boxE → next = boxN;
boxN → value = index;
boxE = boxN; }
for (index = 0; index ≤ 3; index++) {
printf ("Value at index %d is %d\n", index, head →
value);
head = head → next;
printf ("Value at index %d is %d\n", index+1, head
→ value); } }
```

Which one of the statements below is correct about the program?

(a) Upon execution, the program creates a linked-list of five nodes.
(b) Upon execution, the program goes into an infinite loop.
(c) It has a missing return which will be reported as an error by the compiler.
(d) It dereferences an uninitialized pointer that may result in a run-time error.

8. [MCQ] [GATE-2017: 1M]

Consider the C code fragment given below.

```
typedef struct node
{ int data;
node* next;
} node;
void join (node *m, node *n){
node *p = n;
while (p → next ! = NULL){
p = p → next;
}
p → next = m;
}
```

Assuming that m and n point to valid NULL-terminated linked lists, invocation of join will

(a) append list m to the end of list n for all inputs.
(b) either cause a null pointer dereference or append list m to the end of list n.
(c) cause a null pointer dereference for all inputs
(d) append list n to the end of list m for all inputs

9. [MCQ] [GATE-2014: 2M]

What is the worst case time complexity of inserting n elements into an empty linked list, if the linked list needs to be maintained in sorted order?

(a) $\Theta(n)$ (b) $\Theta(n \log n)$
(c) $\Theta(n^2)$ (d) $\Theta(1)$

10. [MCQ] [GATE-2010: 1M]

The following C function takes a singly-linked list as input argument. It modified the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

```
typedef struct node{
int value;
struct node * next;
}Node;
Node * move_ to _ front (Node * head){
Node * p, *q;
if ((head = = NULL)|| (head→next = =NULL))
return head;
q = NULL;
p = head;
while (p →next ! = NULL){
q = p;
p = p → next;
}
________________________________
 return head;
}
```

Choose the correct alternative to replace the blank line.

(a) q = NULL ; p → next = head ; head = p;

(b) q→ next = NULL; head = p; p → next = head;

(c) head = p; p → next = q; q→ next = NULL;

(d) q→ next = NULL; p→ next = head; head = p;

11. [MCQ] [GATE-2008: 2M]

The following C function takes a singly-linked list of integers as a parameter and rearranges the elements of the list. The function is called with the list containing the integers 1,2,3,4,5,6,7 in the given order. What will be the contents of the list after the function completes execution?

```
struct node{
int value;
struct node * next;
};
void rearrange (struct node * list){
struct node * p, *q;
int temp;
if(!list||!list→next) return;
p=list; q=list->next;
while (q){
temp = p→value;
p→value = q→value;
q→ value = temp;
p = q→next;
q = p? p→next: 0:
}
}
```

(a) 1, 2, 3, 4, 5, 6, 7 (b) 2, 1, 4, 3, 6, 5, 7

(c) 1, 3, 2, 5, 4, 7, 6 (d) 2, 3, 4, 5, 6, 7, 1

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ANSWER KEY

1.	(0 to 0)	**2.**	(a)	**3.**	(a)	**4.**	(c)
5.	(d)	**6.**	(a)	**7.**	(d)	**8.**	(b)
9.	(c)	**10.**	(d)	**11.**	(b)		

SOLUTIONS

1. (0 to 0)

$Z \leftarrow 1$

for $i \leftarrow 0$ to $k-1$

do $z \leftarrow z^2 \bmod n$

If (c [i] = 1)

Then $z \leftarrow z \times a \bmod n$

return z

k = 4 c = [0,0,1,1], n = 8, a = 2;

(i) $i = 0, z = z^2 \bmod n \Rightarrow z = 1^2 \bmod 8 = 1$

if (c [i] =1) $\Rightarrow$ true because c [0] =1

$z = z \times a \bmod n$

$z = 1 \times 2 \bmod 8$

$z = 2$

(ii) $i = 1, z = z^2 \bmod n = 4$

if (c [i] = 1) $\Rightarrow$ false because c[1] =0

(iii) $i = z, z = 4^2 \bmod 8 = 0$

if (c [i] =1) $\Rightarrow$ true

No need to check anything further because z is 0 and all the operations involves multiplication of z.

$z = z^2 \bmod n$ & $z = 2 \times a \bmod n$. Because, z will never change from 0.

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2. (a)

$$A = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 3 & 4 \\ 7 & 9 \end{bmatrix} \end{array} n = 2$$

```
for (i = 1; i < 2; i ++)
{
for (j = 1; j ≤ 2;  j++)
{
    temp = A [i] [j] + C;
    A [i] [j] = A [j] [i];
    A [j][i] = temp – C;
}
}
```

(i) if i = 1 for j = 1

temp = A [1] [1] + 100 = 103

A [1] [1] = A [1] [1]

A [1] [1] = 103 – 100 = 3

No effect on A_{11}

for j = 2

temp = A [1] [2] + 100 = 104

A [1] [2] = 7

A [2] [1] = 104 – 100 = 4

swap

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 3 & 7 \\ 4 & 9 \end{bmatrix} \end{array}$$

New matrix: A_{12} swapped with A_{21}.

(ii) i = 2

for j = 1

Temp = A [2] [1] + 100 = 104

A [2] [1] = A [1] [2] = 7

A [1] [2] = 104 – 100 = 4

Temp = 9 + 100 = 109

Matrix = $\begin{bmatrix} 3 & 4 \\ 7 & 9 \end{bmatrix}$

Again, A_{21} swapped with A_{12}.

for j = 2

temp = 9 + 100 = 109

A [2] [2] = A [2] [2]

A [2] [2] = 109 – 100 = 9

No effect.

Finally, we get the same matrix

- A_{ij} is swapped with its mirror image A_{ji} (Lower triangular gular/upper triangular)
- And when the code executes for the mirror image again both elements are swapped

⇒ Same matrix

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3. (a)

for (i = 0; i < size; i++)

// find the sum of all elements of the array E and storing the results in Y.

for (i = 0; i < size; i++)

for (j = i; j < size; j++)

- i = 0

then, j = 0 to size

if i = 1

then j = 1 to size

for i = 0	for i = 1
K = 0 to 0	k = 1 to1
= 0 to 1	= 1 to 2
= 0 to 2	= 1 to 3
= 0 to 3	= 1 to size
= 0 to size	

Finding the sum of each subarray from i.e. starting from 0 to n–1 position.

Check whether the sum of elements of subarray is greater then, the maximum sum of any sub – array found till now⇒if its is then update current maximum.

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4. (c)

Code – 1

for (i = 0; i < n ; 1+ +)

Y [i]+ = X [0][i]

X [i] [i] = + I + j

X [0] [i]

for (i =0; I < n, i + +)

Y [i]+ = i

Code – 2

for (i = 0; i < n; i+j)

Y [i]+ = X [i] [0]

X [i] [j] = i + j

X [i] [0] = i

For (i = 0; i < n; i ++)

Y [i] + = i;

Result is same in both segment

For (i= 0; i < n ; 1+j)

Y [i] + = X [0][i];

Suppose, n = 3;

i = 0 X [0] [0]

i = 1 X [0] [1]

i = 2 X [0] [2]

i = 3 X [0] [3]

In C, by default row major order is followed

⇒ elements are stored contiguously.

In Code – 2

i = 0 X [0] [0]

i = 1 X [0] [0]

i = 2 X [2] [0]

i = 3 X [3] [0]

In C, Row major order is followed

⇒ elements are not stored contiguously.

Row major order:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0 & 1 & 2 & 3 \\ X_{00} & X_{01} & X_{02} & X_{03} \\ X_{10} & X_{11} & X_{12} & X_{13} \\ X_{20} & X_{21} & X_{22} & X_{23} \\ X_{30} & X_{31} & X_{32} & X_{33} \end{bmatrix}$$

Stored data row wise and access the data row wise.

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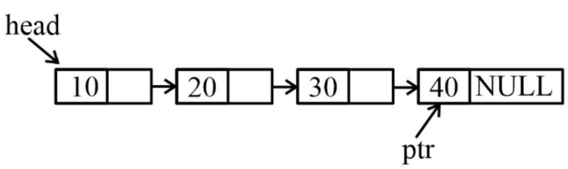
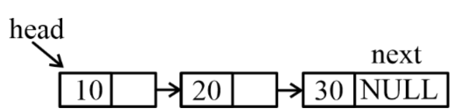
5. (d)

SLLdel:



Assume that we want to delete last node.
After Deletion we need to NULL in the second last node next field.

After deletion:

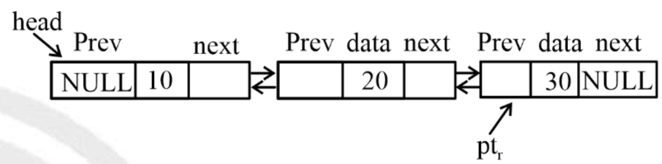


One way traversal is possible given ptr and head, how can we reach second last node ⇒ Traverse from start (head) till second last node ⇒ Traverse n – 1 nodes.

SLLDel Time complexity = O(n)

DLLdel:

- we can go to previous node as well as next node. No need to traverse from head to node to be deleted.
- In doubly linked list ptr → prev : Point to second last node



Delete operation:

ptr → Prev → next = NULL

free (ptr)

It will take constant time i.e. O(1).

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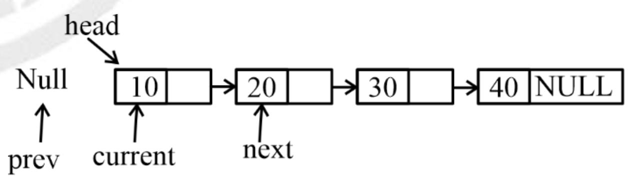
6. (a)

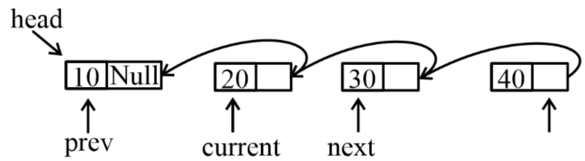
Suppose,



Program:

```
Struct node * prev, * current, * next
prev = Null;
current = head;
While (current! = NULL)
{
next = current → next;
current → next = prev;
Prev = current;
current = next;
}
```

head = prev;



TC = θ(n)

Space = θ(1)

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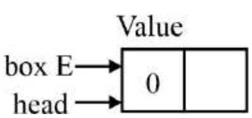
7. (d)

Index = 0

boxE = head = (struct node*) malloc (size of (struct node))

head → value = index



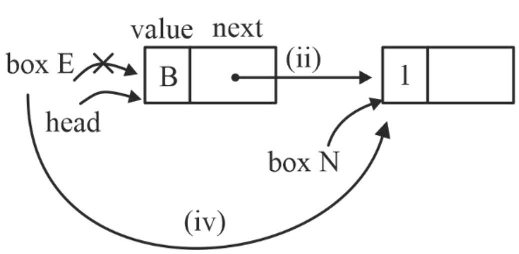
Index = 1

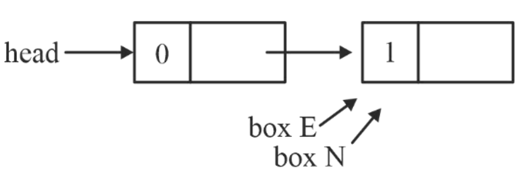
(i) box N = (struct Node*) malloc (size of (struct node))

(ii) box E → next = box N;

(iii) box N → value = index;

(iv) box E = box N





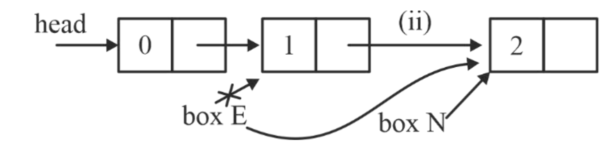
Index = 2

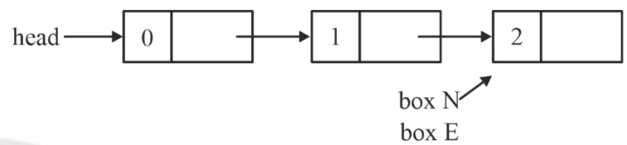
(i) box N = (struct Node*) malloc (size of (struct node))

(ii) box E → next = box N;

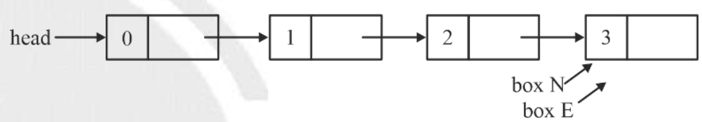
(iii) box N → value = index;

(iv) box E = box N





Same will happen for index = 3



(Last node next field contains garbage value)

Now 2nd loop starts

Index = 0

1st printf: value of index 0 is 0

head = head → next ⇒ head points to 2nd node.

2nd printf: value at Index 1 is 1

Index = 1

1st printf: value of index 1 is 1

head = head → next ⇒ head points to 3rd node.

2nd printf: value at Index 2 is 2.

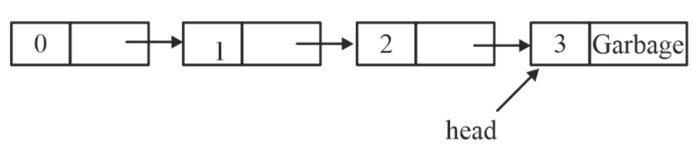
Index = 2

1st printf: value of index 2 is 2

Head = head → next ⇒ head points to last node.

2nd printf: value at Index 3 is 3.

Index = 3



1st printf: value of index 3 is 3

Head = head → next (Garbage value)

Head is pointing to some Garbage value or uniutilized memory.

2nd printf : while dereferencing head → value code may get a runtime error.

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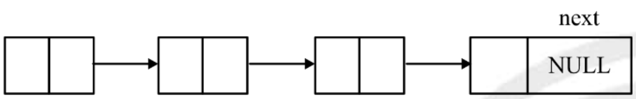
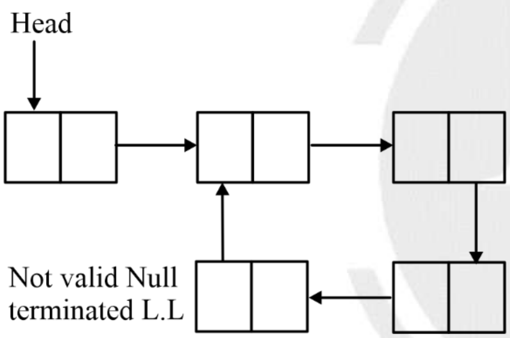
8. **(b)**

Valid NULL-terminated linked list means the linked-list with last node as NULL. It can be empty or it can be non-empty

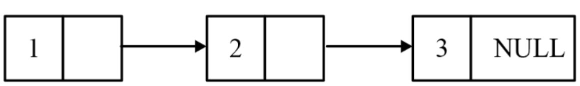


For example:

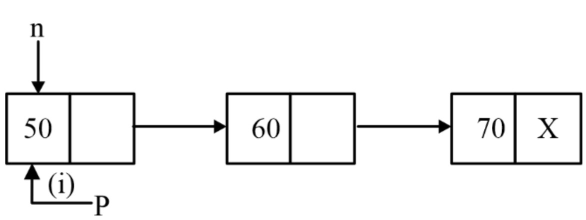


Case:-1: When both m and n are non empty linked list.

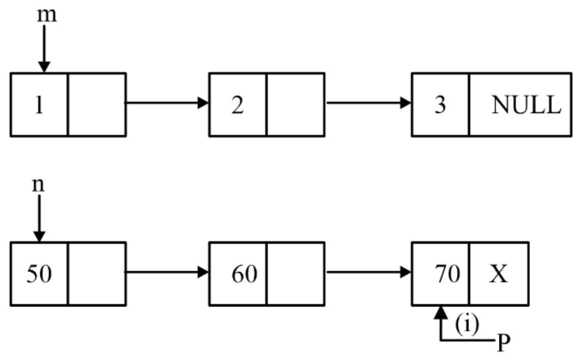
m:



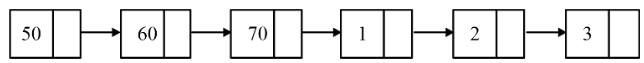
n:



Finding last node when we came out of loop, P is pointing to last node.



After while loop:



Appended m at the end of n.

Case 2: Both m and n are null

n = NULL

m = NULL

(i) If both m and n are null, then P = NULL

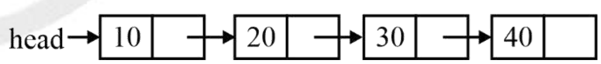
Null → Next ! = NULL

This will create an error because we are trying to dereference a null pointer. So, option (b) is the correct answer.

Scan for Video solution

9. **(c)**

Consdier the sorted list



If we want to insert 50 in this list

We need to traverse till end i.e, number of comparison = number of elements

i.e., to insert an element in a sorted linked list with n element n comparisons are needed, in worst case

For 1st element = 0 comparison

For 2nd element = 1 comparison

.

.

.

For nth element = (n – 1) comparison

Total comparisons needed to insert n elements

$= 0 + 1 + 2 + \ldots.(n - 1)$

$= \dfrac{(n-1)n}{2} = O(n^2)$

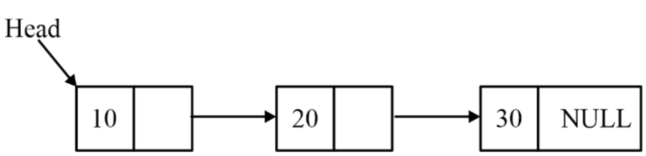
Option (c) is correct.

Scan for Video solution

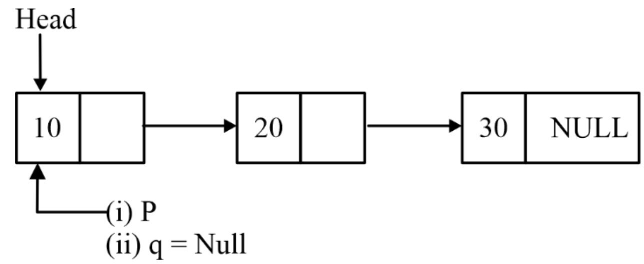
10. (d)

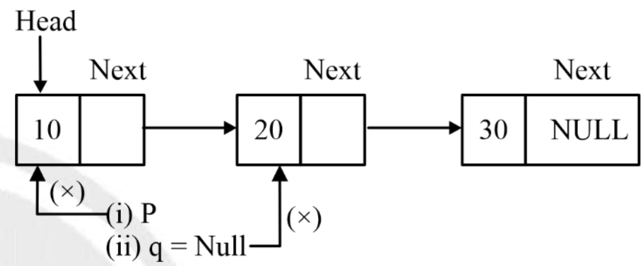
```
typedef struct node{
int value;
struct node * next;
}Node;          // Template for node
Node * move_ to _ front (Node * head){
Node * p, *q;
if ((head = = NULL)|| (head→next = =NULL))
return head;  // Either 0 or 1 node; last node is the first node
q = NULL;
p = head;
while (p →next ! = NULL){
q = p;
p = p → next;
}
______________________
 return head;

}
```

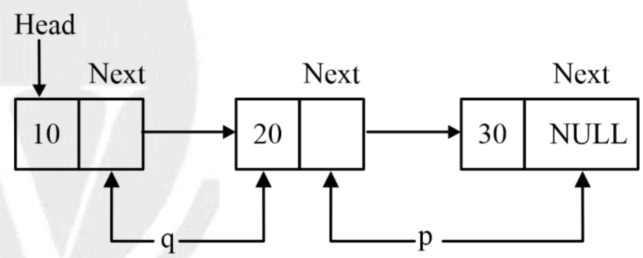


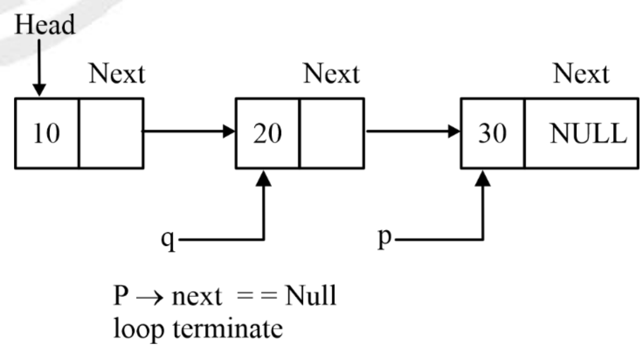
Ist iteration:

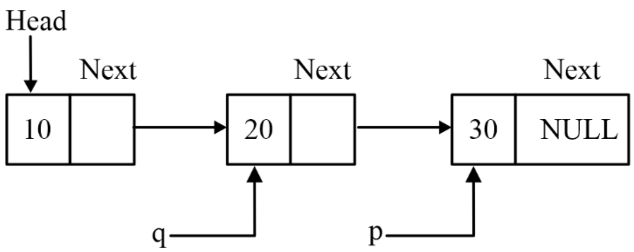




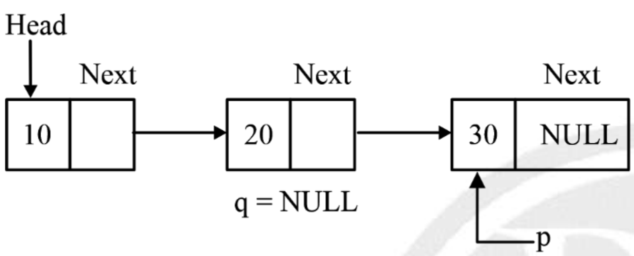
II iteration:

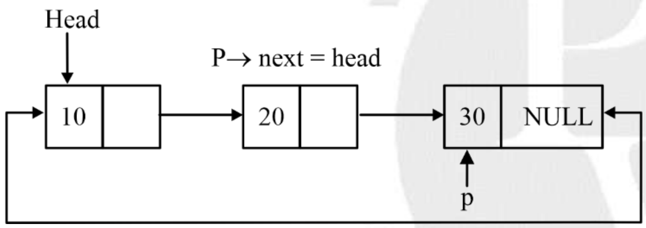


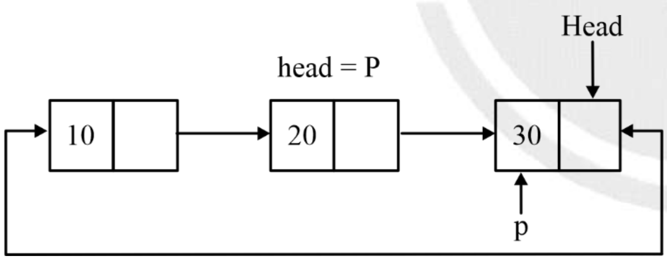




Option (a) q = NULL ; p → next = head ; head = p;

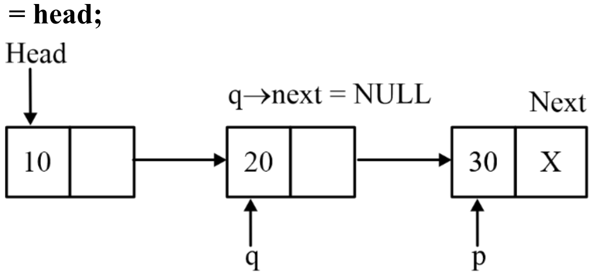


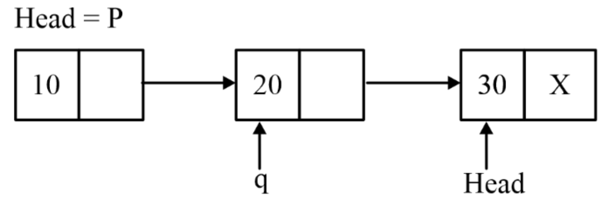


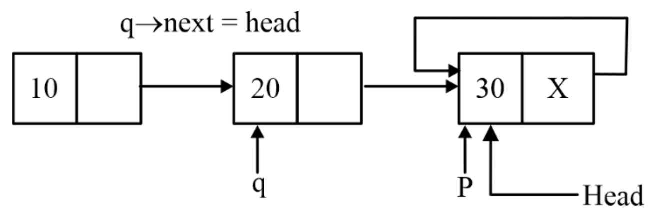


This is incorrect, this is not singly linked list. So, option (a) is wrong.

Option (b) q → next = NULL; head = p; p → next = head;

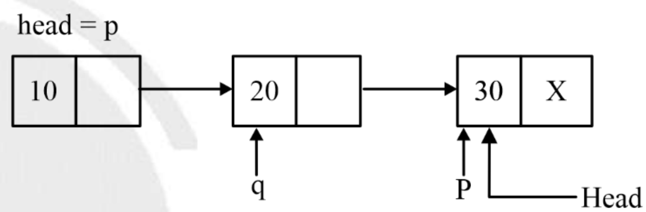




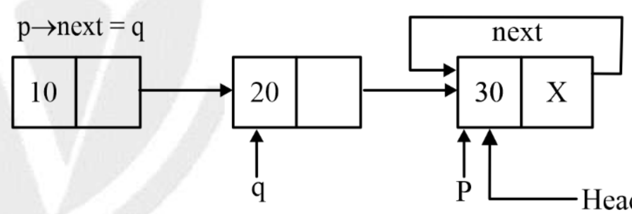


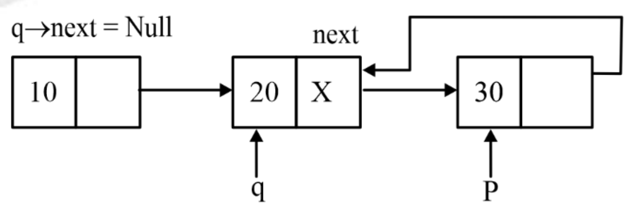
Option (b) is wrong.

Option (c) head = p; p → next = q; q → next = NULL;



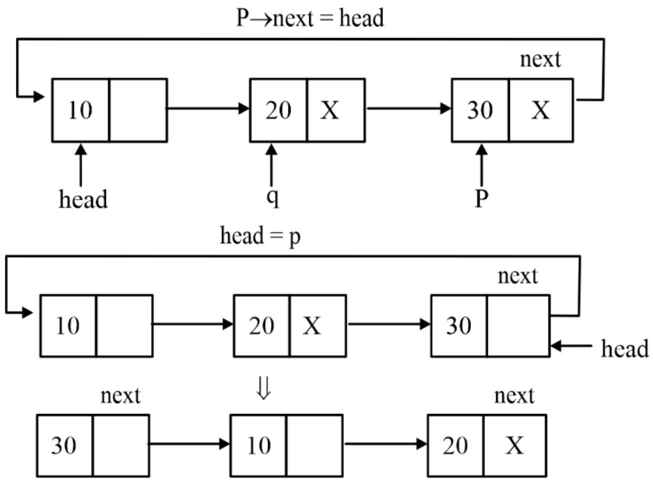
No way to reach first node





Option (c) is wrong.

Option (d) q → next = NULL; p → next = head; head = p;



Therefore, option (d) is the correct answer.

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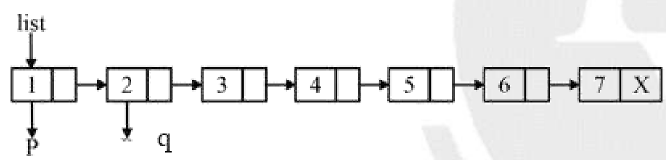
11. (b)

Given,

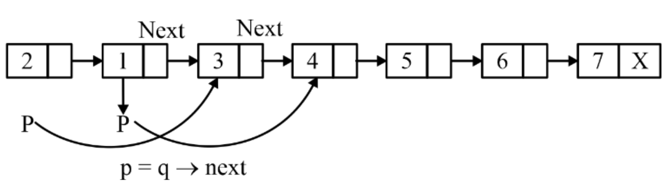


if(!list||!list→next) return; // 0 or 1 node

Swap data of node pointed by p,q

Q is valid address

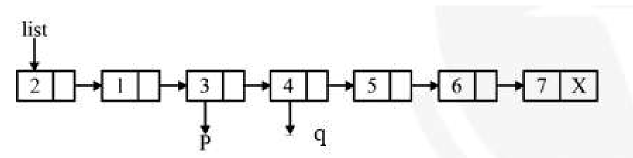
⇒ code inside loop ⇒ execute swap



p = q → next

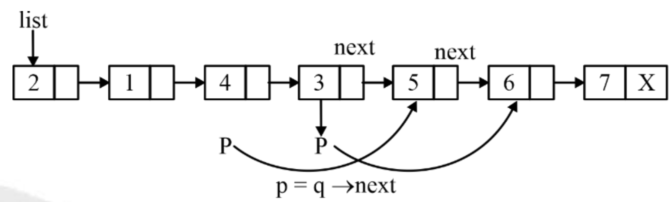
Q = p? P→ next: 0

After one iteration:



q is valid address (non-zero/true) code inside loop will execute

(i) Swap

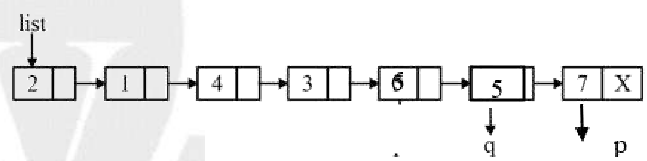


True non-zero

q = p ? P→ next : 0

⇒ p = q → next

After two iterations:



if(!list||!list→next) return;// 0 or 1 node

q is valid address code inside loop swap.

if(!list||!list→next) return;// 0 or 1 node

q = p ? p→ next: 0

q = p → next ⇒ NULL

while (q) Null ⇒ False

Loop terminates

2, 1, 4, 3, 6, 5, 7

Therefore, option (b) is the correct answer.

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❑❑❑

CHAPTER

6 STACKS AND QUEUES

Operation on Stack

1. [NAT] [GATE-2023: 2M]

Consider a sequence a of elements $a_0 = 1$, $a_1 = 5$, $a_2 = 7$, $a_3 = 8$, $a_4 = 9$, and $a_5 = 2$. The following operations are performed on a stack S and a queue Q, both of which are initially empty.

I: push the elements of a from a_0 to a_5 in that order into S.

II: enqueue the elements of a from a_0 to a_5 in that order into Q.

III: pop an element from S.

IV: dequeue an element from Q.

V: pop an element from S.

VI: dequeue an element from Q.

VII: dequeue an element from Q and push the same element into S.

VIII: Repeat operation VII three times.

IX: pop an element from S.

X: pop an element from S.

The top element of S after executing the above operations is _______.

2. [NAT] [GATE-2015: 2M]

Consider the C program below.

```
#include <stdio.h>
int * A, stkTop;
int stkFunc (int opcode, int val)
{
static int size = 0, stkTop = 0;
switch (opcode)
{
case -1: size = val; break;
case 0: if (stkTop < size)  A[stkTop++] = val; break;
default: if (stkTop) return A[– –stkTop];
}
return – 1;
}
int main ()
{
int B[20]; A = B; stkTop = – 1;
stkFunc (-1, 10);
stkFunc (0, 5);
stkFunc (0, 10);
printf("%d \ n", stkFunc(1, 0) + stkFunc (1, 0) );
}
```

The value printed by the above program is _____

Infix Postfix and Prefix Notations

3. [MCQ] [GATE-2015: 1M]

The result evaluating the postfix expression 10 5 + 60 6 / * 8 – is

(a) 284 (b) 213

(c) 142 (d) 71

Applications of Stack

4. [MCQ] [GATE-2014: 2M]

Suppose, a stack implementation supports an instruction REVERSE, which reverses the order of elements on the stack, in addition to the PUSH and POP instructions. Which one of the following statements is TRUE (with respect to this modified stack)?

(a) A queue cannot be implemented using this stack.

(b) A queue can be implemented where ENQUEUE takes a single instruction and DEQUEUE takes sequence of two instructions.

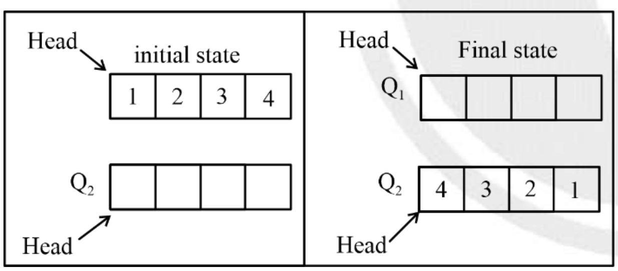
(c) A queue can be implemented where ENQUEUE takes a sequence of three instructions and DEQUEUE takes a single instruction

(d) A queue can be implemented where both ENQUEUE and DEQUEUE take a single instruction each.

Operations on Queues

5. [NAT] [GATE-2022: 2M]

Consider the queues Q_1 containing four elements and Q_2 containing none (shown as the Initial State in the figure). The only operations allowed on these two queues are Enqueue(Q, element) and Dequeue(Q). The minimum number of Enqueue operations on Q_1 required to place the elements of Q_1 in Q_2 in reverse order (shown as the Final State in the figure) without using any additional storage is___________.



6. [NAT] [GATE-2021: 1M]

Consider the following sequence of operations on an empty stack.

push(54); push(52); pop(); push(55); push(62); s = pop();

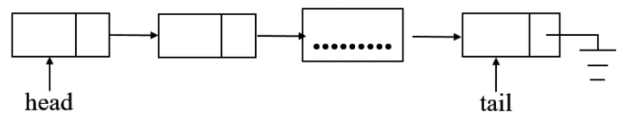
Consider the following sequence of operations on an empty queue.

enqueue(21); enqueue(24); dequeue(); enqueue(28); enqueue(32); q = dequeue();

The value of s + q is ________

7. [MCQ] [GATE-2018: 1M]

A queue is implemented using a non-circular singly linked list. The queue has a head pointer and tail pointer, as shown in the figure. Let n denote of number of nodes in the queue. Let 'enqueue' be implemented by inserting a new node at the head and 'dequeue' be implemented by deletion of a node from the tail.



Which one of the following is the time complexity of the most time-efficient implementation of enqueue and dequeue, respectively, for this data structure?

(a) $\theta(1),\theta(1)$ (b) $\theta(1),\theta(n)$

(c) $\theta(n),\theta(1)$ (d) $\theta(n),\theta(n)$

8. [MCQ] [GATE-2017: 1M]

A circular queue has been implemented using a singly linked list where each node consists of a value and a single pointer pointing to the next node. We maintain exactly two external pointers FRONT and REAR pointing to the front node and the rear node of the queue, respectively. Which of the following statements is/are CORRECT for such a circular queue, so that insertion and deletion operations can be performed in O(1) time?

I. Next pointer of front node points to the rear node.

II. Next pointer of rear node points to the front node

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

9. [MCQ] [GATE-2016: 1M]

A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is CORRECT (n refers to the number of items in the queue)?

(a) Both operations can be performed in O(1) time.
(b) At most one operation can be performed in O(1) time but the worst case time for the other operation will be $\Omega(n)$.
(c) The worst case time complexity for both operations will be $\Omega(n)$.
(d) Worst case time complexity for both operations will be $\Omega(\log n)$.

10. [MCQ] [GATE-2013: 2M]

Consider the following operation along with Enqueue and Dequeue operations on queues, where k is a global parameter.

```
MultiDequeue(Q){
m = k;
while (Q is not empty and m > 0) {
Dequeue (Q);
m = m – 1;
 }
}
```

What is the worst case time complexity of a sequence of n MultiDequeue () operations on an initially empty queue?

(a) $\Theta(n)$ (b) $\Theta(n + k)$
(c) $\Theta(nk)$ (d) $\Theta(n^2)$

11. [MCQ] [GATE-2012: 2M]

Suppose a circular queue of capacity (n-1) elements is implemented with an array of n elements. Assume that the insertion and deletion operations are carried out using REAR and FRONT as array index variables, respectively. Initially, REAR = FRONT = 0. The conditions to detect queue full and queue empty are

(a) Full: (REAR + 1) mod n = = FRONT
Empty: REAR = = FRONT
(b) Full: (REAR + 1) mod n = = FRONT
Empty: (FRONT + 1) mod n = = REAR
(c) Full: REAR = = FRONT
Empty:(REAR + 1) mod n = = FRONT
(d) Full:(FRONT + 1) mod n = = REAR
Empty: REAR = = FRONT

Applications of Queues

12. [NAT] [GATE-2016: 2M]

Let Q denote a queue containing sixteen numbers and S be an empty stack. Head(Q) returns the element at the head of the queue Q without removing it from Q. Similarly Top(S) returns the element at the top of S without removing it from S. Consider the algorithm given below.

```
While Q is not Empty do
If S is Empty OR Top (S) ≤ Head (Q) then
X: = Dequeue (Q);
Push (S, x):
else
X: = Pop (S);
Enqueue (Q, x);
end
end
```

The maximum possible number of iterations of the while loop in the algorithm is _____.

❑❑❑

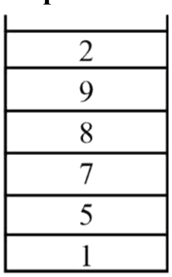
ANSWER KEY

1.	(8)	**2.**	(15)	**3.**	(c)	**4.**	(c)
5.	(0)	**6.**	(86)	**7.**	(b)	**8.**	(b)
9.	(a)	**10.**	(a)	**11.**	(a)	**12.**	(256)

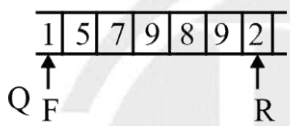
SOLUTIONS

1. (8)

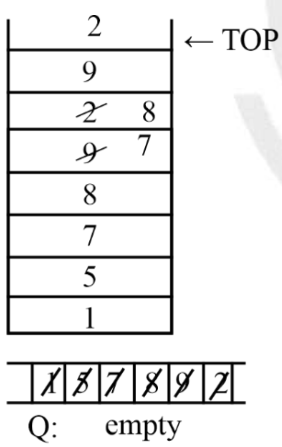
Step 1:



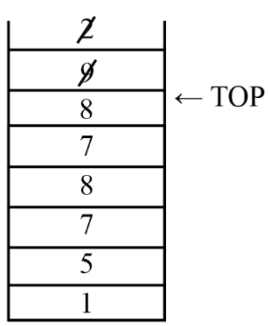
Step 2:



Step 3 to 8:



Step 9 & 10:



TOP element = 8

Scan for Video solution

2. (15)

stkFunc:

Implementation of stack with 3 choice/

Case 1:

Set the size of the stack

Case 0:

Push ( )

Whether stack is full or not if the stack is not full,

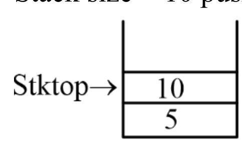
⇒ Push an element

⇒ Increment stk top

Default:

If stack is not empty then, Pop the top element & return it and decrease the top pointer.

Stack size = 10 push 5 and 10 on to stack.



stkFunc (1, 10);→ POP and return top most element 10 stkFunc (0, 5); → POP and return 5

Final output = 10 + 5 = 15.

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3. (c)

Two methods:

(i) without using stack

(ii) using stack

10, 5, + 60, 6, 1, *, 8, –

(i) Without Stack:

Scan from left to right & whenever an operator is encountered just put in between previous 2 operand and find result.

- 10, 5, +

10 + 5 = 15

15, 60, 6, 1, *, 8, –

(i) 15, 60, 6, /, *, 8 – put division in between 60 and 6

(ii) 15, 10, *, 8, – put * between 15 and 10

(iii) 15 * 10, 8, –

150, 8 –

(iv) 150, 8, – put – in between 150 and 8

150 – 8 = 142

Using stack:

- Scan ⇒ left to right

(i) Operand ⇒ Push it

(ii) Operator

(a) Pop top 2 elements from stack let first popped element is A & second popped elements is B.

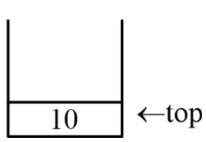
(b) Evaluate B ⊕ A (where ⊕ is the operator found)

(c) Push the result of previous step onto stack.

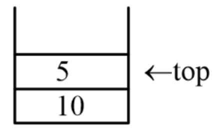
Keep repeating the above, once the input is over at the end, stack top element is the answer.

10, 5, +, 60, 6, /, *, 8, –

Push 10:



Push 5:

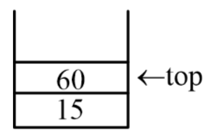


+ Operator is coming, pop the top two elements and calculating the result.

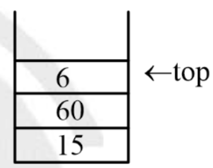
Result = 10 + 5 = 15

And push the result into stack.

Push 60:



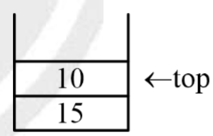
Push 6:



/ Operator is coming, pop the top two elements and calculating the result.

Result = 60 / 6 = 10. And push the result into stack.

Push 10 (result):

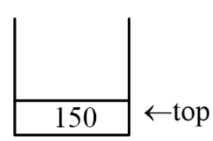


* Operator is coming, pop the top two elements and calculating the result.

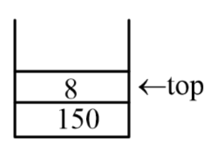
Result = 15 × 10 = 150

And push the result into stack.

Push 150 (result):



Push 8:



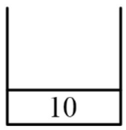
– Operator is coming, pop the top two elements and calculating the result.

Result = 150 – 8 = 142

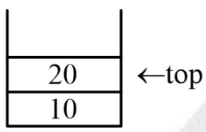
Scan for Video solution

4. (c)

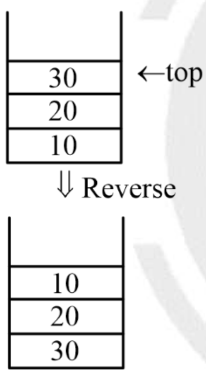
(i) Push (S, 10)

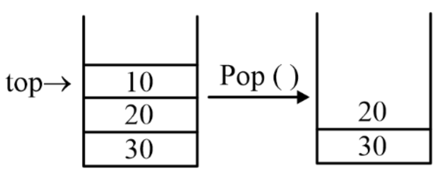


(ii) Push (S, 20)



(iii) Push (S, 30)

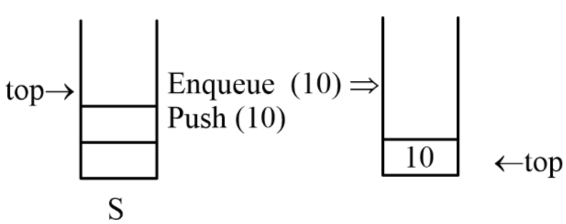




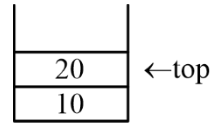
Given: Stack with 3 operation (Push/Pop/Reverse)

Queue: First in first out

(i) Let Enqueue is implemented using Push

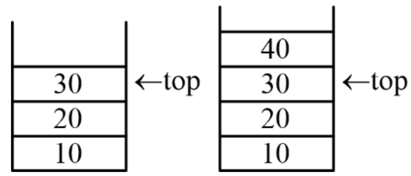


Enqueue (20) ⇒ Push (20)



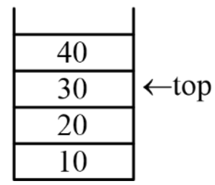
Enqueue (30) ⇒ Push (30)

Enqueue (40) ⇒ Push (40)



Dequeue?

(i) Enqueue 10
(ii) Enqueue 20
(iii) Enqueue 30
(iv) Enqueue 40



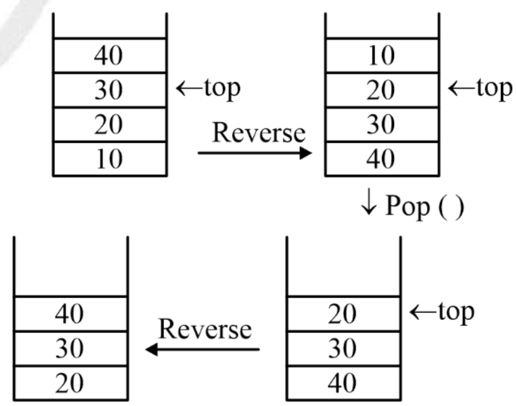
Order: $\underrightarrow{10, 20, 30, 40}$

So, as per Queue Policy, 0 is to be deleted first ⇒ but we cannot delete any other element except top element from stack.

We need 10 at the top of stack:- **REVERSE**

Dequeue?

(i) Enqueue 10
(ii) Enqueue 20
(iii) Enqueue 30
(iv) Enqueue 40



Order: $\underrightarrow{10, 20, 30, 40}$

(1) Dequeue → (i) Reverse, (ii) Pop, (iii) Reverse

(2) Enqueue ⟶ Push ( ) inst.

Another implement can be just reversed way

(1) Enqueue 3 ints. → (i) Reverse, (ii) Pop, (iii) Reverse — 3 ints.

(2) Dequeue 1 ints. → Push ( ) inst. — 1 ints.

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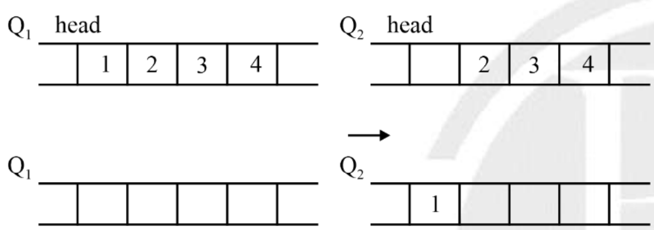
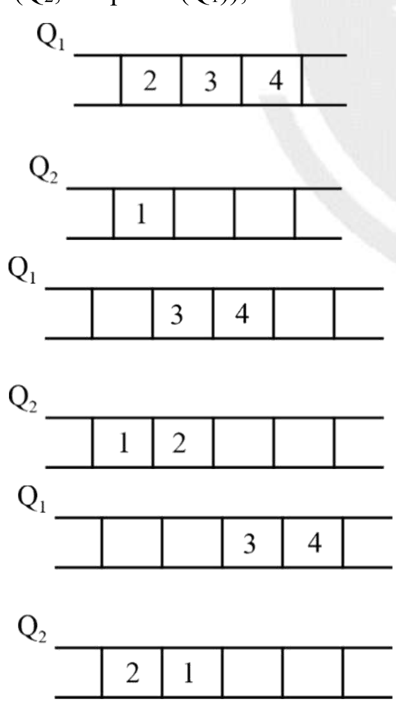
5. (0)

(i) Assume: size of $Q_2 > 4$

(ii) No addition storage

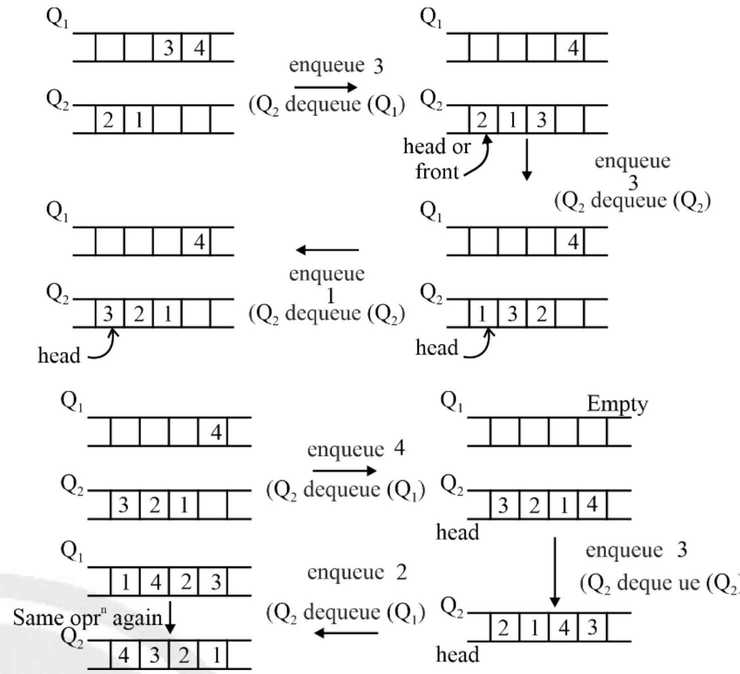


Enqueue (Q_2, Dequeue (Q_1)); No additional storage



Enque(Q_2 Deque (Q_1))

Enque(Q_2 Deque (Q_2))



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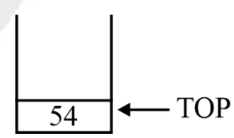
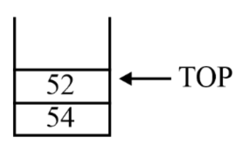
6. (86)

Stack operation:

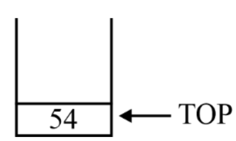
(1) Push (54):



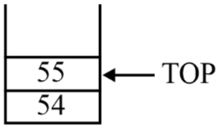
(2) Push (52):



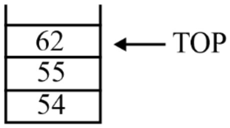
(3) POP ( ):



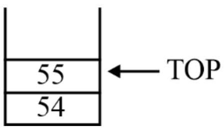
(4) Push (55):



(5) Push (62):



(6) s = POP ( ):

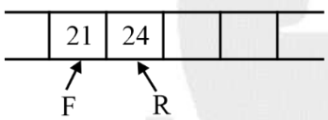


s = 62

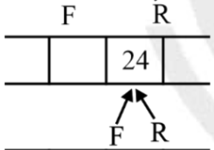
Queue Operation:

1. Enqueue(21)

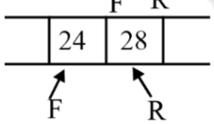
2. Enqueue(24)



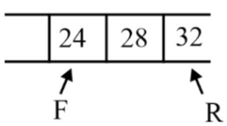
3. Dequeue



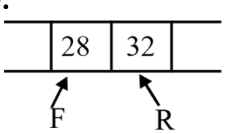
4. Enqueue(28)



5. Enqueue(28)



q = dequeue:



q = 24

s + q = 62 + 24 = 86

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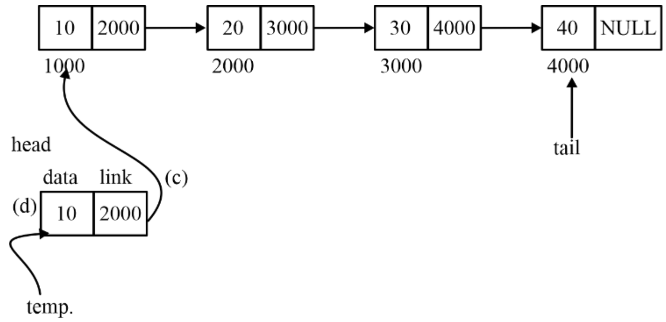
7. (b)

struct node temp = malloc (size of (struct node))



Enqueue:

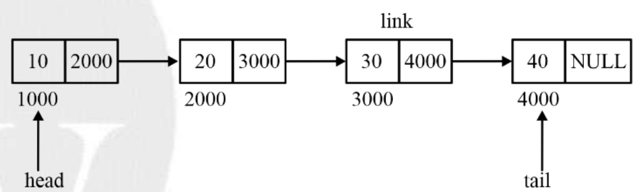
(a) struct node temp = malloc (size of (struct node))

(b) fill the data

(c) temp → link = head

(d) head = temp

fixed number of instructions ⇒ $\theta(1)$



Dequeue:

(1) After deletion of last node second last node will become the last node & that's why we must assign NULL in the link field of 2nd last node.

How to delete last node?

We need to reach second last node or to delete a node you need to travel from 1st node to last node.

Because we have singly limited list

⇒ Traversal till last node is depends on the number of nodes.

Time Complexity = $\Theta(n)$

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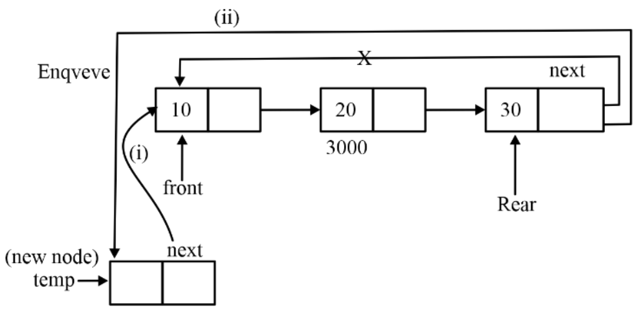
8. (b)

Enqueue → O (1)

Dequeue → O (1)

Rear → next points to front

Enqueue:

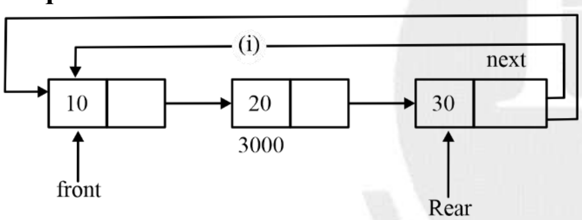


(i) temp → next = front or temp → next = Rear → next;

(ii) Rear → next = temp;

(iii) Rear = temp;

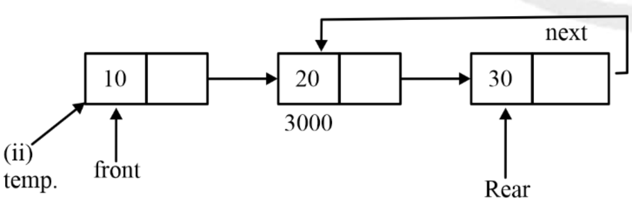
Dequeue:



Delete node 10.

(i) Rear → next (always point to front)

Rear → next = front → next



(ii) temp = front

(iii) front = Rear → next

(iv) free (temp)

Hence, option (b) is correct.

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9. (a)

Enqueue:

- If the array is full no need to do anything.
- Else Insert the new element at the end (Rear end) & also increment rear value.

Dequeue:

If the array is empty then stop else delete element from front end & also update the value of front.

Both can be perform in O(1) time.

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10. (a)

(1) **Enqueue:** Insert an element

(2) **Dequeue:** Delete an element

3 Operations:

(1) Dequeue

(2) Enqueue

(3) Multi Dequeue

Because the queue is initially empty

⇒ Number of dequeue $\not>$ Number of Enqueue total number of operation = $\theta(n)$

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11. (a)

Condition for Empty: Rear = Front = 0

(b) According to option b, Empty: (FRONT + 1) mod n = = REAR

n = 5 size

(Front +1) mode 5 = = 0

(0+1) mode 5 = = 0

1 = = 0

False, That's why option b is wrong.

Condition for FULL: (REAR +1) mod n == Front

Therefore, option (a) is the correct answer.

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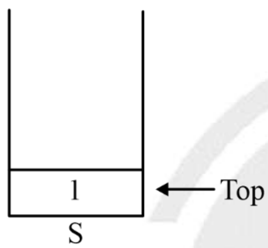
12. (256)

Let reduce the number of element from 16 to 3 to solve the questions in easy way.

(i) Q : $\cancel{1}$ 2 3

↑ Head

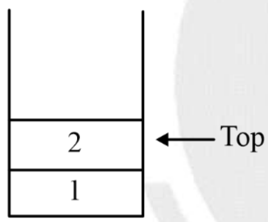
if execute



(ii) Q : $\cancel{2}$ 3

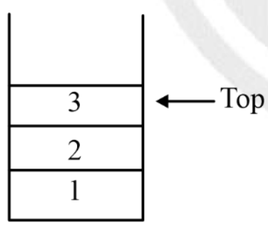
↑ Head

if execute



(iii) Q : $\cancel{3}$

if execute



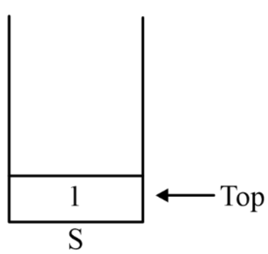
We can say

(i) Q: 1,2,3 $\rightarrow$ 3 iterations

Now let's take

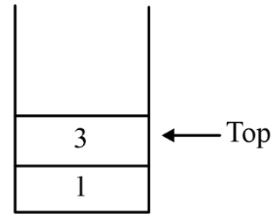
(i) Q : $\cancel{1}$ 3 2

if execute



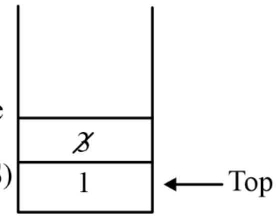
(ii) Q : $\cancel{3}$ 2

if execute



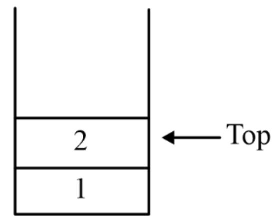
(iii) Q : 2

if will not execute
else will execute
Head (Q) < Top(S)



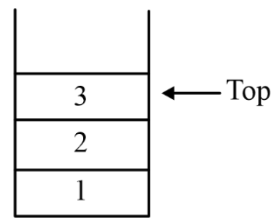
(iv) Q : $\cancel{2}$ 3

if execute



(v) Q : $\cancel{3}$

if execute



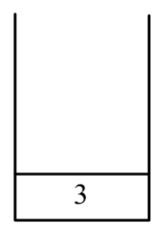
We can say for

Q: 1, 3, 2 $\rightarrow$ 5 iterations

Now let's take

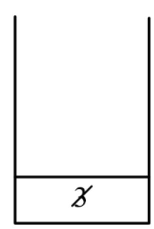
(i) Q : $\cancel{3}$ 2 1

if execute



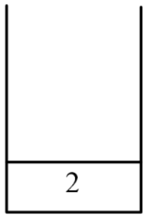
(ii) Q : 2 1 3

else execute



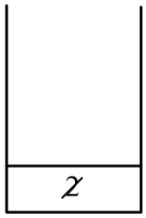
In these 2-operation 3 pushed to stack and then popped and go back to queue.

(iii) Q : $\cancel{2}$ 1 3



if execute

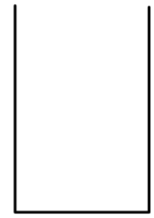
(iv) Q : 1 3 2



else execute

In these 2-iteration 2 pushed and then popped and go back to queue.

(v) Q : 1 3 2



Already know it will take 5 more iteration.

Total iteration = 4 + 5 = 9 $\Rightarrow$ (3×3)

So, for 16 element it will take

16 × 16 = 256 operation.

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❑❑❑

CHAPTER

7

TREES

Implementation of Tree

1. **[MCQ]** **[GATE-2023 2M]**

Let A be a priority queue for maintaining a set of elements. Suppose *A* is implemented using a max-heap data structure. The operation EXTRACT-MAX(A) extracts and deletes the maximum element from *A*. The operation INSERT(A, key) inserts a new element *key* in *A*. The properties of a max-heap are preserved at the end of each of these operations. When A contains n elements, which one of the following statements about the worst case running time of these two operations is TRUE?

(a) Both EXTRACT-MAX(A) and INSERT(A, key) run in $O(1)$.

(b) Both EXTRACT-MAX(A) and INSERT(*A*, key) run in $O(\log(n))$.

(c) EXTRACT-MAX(*A*) runs in $O(1)$ whereas INSERT(*A*, *key*) runs in $O(n)$.

(d) EXTRACT-MAX(*A*) runs in $O(1)$ whereas INSERT(A, key) runs in $O(\log(n))$.

2. **[MCQ]** **[GATE-2023: 1M]**

Which one of the following sequences when stored in an array at locations

A[1], . . . , A[10] forms a max-heap?

(a) 23, 17, 10, 6, 13, 14, 1, 5, 7, 12

(b) 23, 17, 14, 7, 13, 10, 1 ,5, 6, 12

(c) 23, 17, 14, 6, 13, 10, 1, 5, 7, 15

(d) 23, 14, 17, 1, 10, 13, 16, 12, 7, 5

3. **[NAT]** **[GATE-2020: 2M]**

Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is_____.

4. **[MCQ]** **[GATE-2019: 2M]**

Consider the following statements:

I. The smallest element in a max-heap is always at a leaf node

II. The second largest element in a max-heap is always a child of a root node

III. A max-heap can be constructed from a binary search tree in $\theta(n)$ time

IV. A binary search tree can be constructed from a max-heap in $\theta(n)$ time

Which of the above statements are TRUE?

(a) I, II and III
(b) I, II and IV
(c) I, III and IV
(d) II, III and IV

5. **[MCQ]** **[GATE-2021: 1M]**

Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst case time complexity of an optimal algorithm to find the maximum element in H?

(a) $\Theta(1)$
(b) $\Theta(\log n)$
(c) $\Theta(n)$
(d) $\Theta(n \log n)$

6. **[MCQ]** **[GATE-2017: 1M]**

Consider the following array of elements $\langle 89,19,50,17,12,15,2,5,7,11,6,9,100 \rangle$.

The minimum number of interchanges needed to convert it into a max-heap is

(a) 4
(b) 5
(c) 2
(d) 3

7. **[NAT] [GATE-2016: 2M]**
A complete binary min-heap is made by including each integer in [1,1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is _______.

8. **[MCQ] [GATE-2016: 2M]**
An operator delete(i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?
(a) O(1)
(b) O(d) but not O(1)
(c) O(2^d) but not O(d)
(d) O($d2^d$) but not O(2^d)

9. **[MCQ] [GATE-2014: 1M]**
Consider the C function given below. Assume that the array list A contains n (> 0) elements, sorted in ascending order.

```
int ProcessArray (int *list A, int x, int n){
int i, j, k;
i = 0;
j = n − 1;
do {
k = ( i + j ) / 2;
if ( x < = listA[k] )
j = k − 1;
If ( listA [k] < = x )
i = k + 1;
}
while ( i < = j );
if( listA[k] = = x ) return ( k );
else
return – 1;}
```

Which one of the following statements about the function Process Array is CORRECT?
(a) It will run into an infinite loop when x is not in list A.
(b) It is an implementation of binary search.
(c) It will always find the maximum element in list A
(d) It will return – 1 even when x is present in list A.

10. **[MCQ] [GATE-2014: 2M]**
Consider the pseudocode given below. The function **DoSomething ( )** takes as a argument a pointer to the root of an arbitrary tree represented by the **leftMostChild - rightSibling** representation. Each node of the tree is of type treeNode.

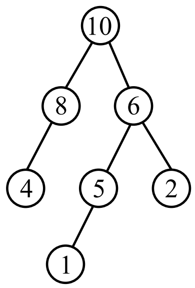
```
typedef struct treeNode* treeptr;
struct treeNode
{
treeptr leftMostChild, rightSibling;
};
int DoSomething (treeptr tree)
{
int value = 0;
if (tree ! = NULL) {
if (tree → left MostChild = = NULL) value = 1;
else
value = DoSomething(tree → leftMostChild);
value = value + DoSomething(tree → rightSibling);
}
return (value);
}
```

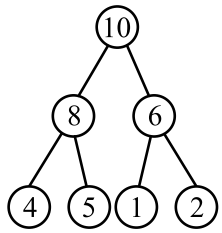
When the pointer to the root of a tree is passed as the argument to **DoSomething,** the value returned by thye function corresponds to the
(a) number of internal nodes in the tree.
(b) height of the tree.
(c) number of nodes without a right sibling in the tree.
(d) number of leaf nodes in the tree.

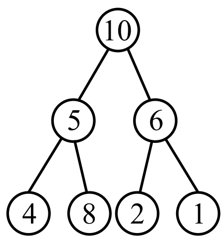
11. **[MCQ] [GATE-2014: 1M]**
A priority queue is implemented as Max-Heap. Initially, it has **5** elements. The level-order traversal of the heap is: **10, 8, 5, 3, 2**. Two new elements **1** and **7** are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:
(a) 10, 8, 7, 3, 2, 1, 5
(b) 10, 8, 7, 2, 3, 1, 5
(c) 10, 8, 7, 1, 2, 3, 5
(d) 10, 8, 7, 5, 3, 2, 1

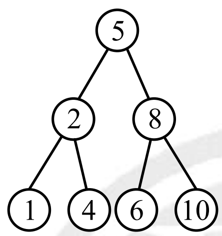
12. [MCQ] [GATE-2011: 1M]

A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?

(a) 

(b) 

(c) 

(d) 

Statements for linked answer for next two question(13 & 14)

13. [MCQ] [GATE-2009: 2M]

Which one of the following array represents a binary max-heap?

(a) {25, 12, 16, 13, 10, 8, 14}
(b) {25, 14, 13, 16, 10, 8, 12}
(c) {25, 14, 16, 13, 10, 8, 12}
(d) {25, 14, 12, 13, 10, 8, 16}

14. [MCQ] [GATE-2009: 2M]

Consider a binary max-heap implemented using an array.

What is the content of the array after two delete operations on the correct answer to the previous question?

(a) { 14, 13, 12, 10, 8 } (b) { 14, 12, 13, 8, 10 }
(c) { 14, 13, 8, 12, 10 } (d) { 14, 13, 12, 8, 10 }

15. [MCQ] [GATE-2008: 1M]

We have a binary heap on **n** elements and wish to insert **n** more elements (not necessarily one after another) into this heap. The total time required for this is:

(a) $\Theta(log\ n)$ (b) $\Theta(n)$
(c) $\Theta(n\ log\ n)$ (d) $\Theta(n^2)$

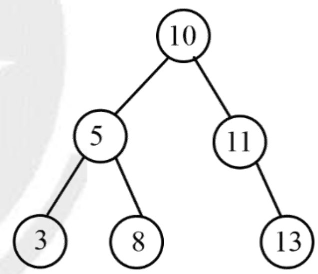
Tree Traversal

16. [MCQ] [GATE-2023: 2M]

Consider the C function foo and the binary tree shown.

```
typedef struct node {
    int val;
   struct node *left, *right;
} node;
int foo( node *p ) {
  int retval;
  if ( p = = NULL )
      return 0;
  else {
  retval = p → val + foo( p  → left ) + foo( p →right
         );
     printf( "%d ", retval );
     return retval;
  }
}
```

When foo is called with a pointer to the root node of the given binary tree, what will it print?



(a) 3 8 5 13 11 10 (b) 3 5 8 10 11 13
(c) 3 8 16 13 24 50 (d) 3 16 8 50 24 13

17. [MCQ] [GATE-2021: 1M]

Consider the following statements.

S_1: The sequence of procedure calls corresponds to a preorder traversal of the activation tree.

S_2: The sequence of procedure returns corresponds to a postorder traversal of the activation tree.

Which one of the following options is correct?

(a) S_1 is true and S_2 is false
(b) S_1 is false and S_2 is true
(c) S_1 is true and S_2 is true
(d) S_1 is false and S_2 is false

18. [MCQ] [GATE-2020: 1M]

The preorder traversal of a binary search tree is 15 10, 12, 11, 20, 18, 16, 19.

Which one of the following is the postorder traversal of the tree?

(a) 20, 19, 18, 16, 15, 12, 11, 10
(b) 11, 12, 10, 16, 19, 18, 20, 15
(c) 19, 16, 18, 20, 11, 12, 10, 15
(d) 10, 11, 12, 15, 16, 18, 19, 20

19. [NAT] [GATE-2018: 1M]

The postorder traversal of a binary tree is 8,9,6,7,4,5,2,3,1. The inorder traversal of the same tree is 8,6,9,4,7,2,5,1,3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is ______

20. [MCQ] [GATE-2017: 2M]

The pre-order traversal of a binary search tree is given by 12,8,6,2,7,9,10,16,15,19,17,20. Then the post-order traversal of this tree is:

(a) 2,6,7,8,9,10,12,15,16,17,19,20
(b) 2,7,6,10,9,8,15,17,20,19,16,12
(c) 7,2,6,8,9,10,20,17,19,15,16,12
(d) 7,6,2,10,9,8,15,16,17,20,19,12

21. [MCQ] [GATE-2015: 1M]

Which of the following is/are correct inorder traversal sequence(s) of binary search tree(s)?

1. 3, 5, 7, 8, 15, 19, 25
2. 5, 8, 9, 12, 10, 15, 25
3. 2, 7, 10, 8, 14, 16, 20
4. 4, 6, 7, 9, 18, 20, 25

(a) 1 and 4 only (b) 2 and 3 only
(c) 2 and 4 only (d) 2 only

22. [MCQ] [GATE-2013: 2M]

The preorder traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the postorder traversal sequence of the same tree?

(a) 10, 20, 15, 23, 25, 35, 42, 39, 30
(b) 15, 10, 25, 23, 20, 42, 35, 39, 30
(c) 15, 20, 10, 23, 25, 42, 35, 39, 30
(d) 15, 10, 23, 25, 20, 35, 42, 39, 30

23. [MCQ] [GATE-2008: 2M]

The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

I. MBCAFHPYK
II. **KAMCBYPFH**
III. **MABCKYFPH**

Pick the true statement from the following.

(a) I and II are preorder and inorder sequences, respectively
(b) I and III are preorder and postorder sequences, respectively
(c) II is the inorder sequence, but nothing more can be said about the other two sequences
(d) II and III are the preorder and inorder sequences, respectively

Binary Search Trees

24. [NAT] [GATE-2022: 1M]

Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index_____________.

25. [MCQ] [GATE-2021: 1M]

A binary search tree T contains n distinct elements. What is the time complexity of picking an element in T that is smaller than the maximum element in T?

(a) θ (n log n) (b) θ (n)
(c) θ (log n) (d) θ (1)

26. [MCQ] [GATE-2017: 1M]

Let T be a binary search tree with 15 nodes.

The minimum and maximum possible heights of T are:

Note: The height of a tree with a single node is 0.

(a) 4 and 15 respectively (b) 3 and 14 respectively
(c) 4 and 14 respectively (d) 3 and 15 respectively

27. [NAT] [GATE-2016: 2M]

The number of ways in which the numbers 1,2,3,4,5,6,7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is _______.

Note: The height of a tree with a single node is 0.

28. [MCQ] [GATE-2015: 1M]

While inserting the elements 71,65,84,69,67,83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

(a) 65 (b) 67

(c) 69 (d) 83

29. [NAT] [GATE-2015: 1M]

A binary tree T has 20 leaves. The number of nodes in T having two children is _______

30. [MCQ] [GATE-2015: 1M]

Consider a max heap, represented by the array: 40, 30, 20, 10, 15, 16, 17, 8, 4.

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

(a) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35

(b) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15

(c) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15

(d) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

31. [MCQ] [GATE-2015: 1M]

The height of a tree is the length of the longest root-to-leaf path in it.

The maximum and minimum number of nodes in a binary tree of height 5 are

(a) 63 and 6, respectively

(b) 64 and 5, respectively

(c) 32 and 6, respectively

(d) 31 and 5, respectively

32. [NAT] [GATE-2014: 1M]

Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $O(n^a \log^b n)$. Then the value of a + 10b is____.

33. [MCQ] [GATE-2012: 2M]

The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudo-code below is invoked as the height (root) to compute the height of a binary tree rooted at the tree pointer root.

```
int height (treeptr n)
 {
 if ( n = = NULL) return –1;
 if ( n → left = = null )
 if ( n → right = = NULL ) return 0;
    else return  B1 ;//Box1
 else {
    h1 = height (n → left);
    if (n → right = = NULL ) return (1 + h1);
   else {
     h2 = height ( n → right );
     return  B2 ;//Box2
    }
  }
}
```

The appropriate expression for the two boxes B1 and B2 are

(a) B1: (1 + height (n → right))
B2: (1 + max (h1, h2))

(b) B1: (height (n → right))
B2: (1 + max (h1, h2))

(c) B1: height (n → right))
B2: max (h1, h2)

(d) B1: (1 + height (n → right))
B2: max (h1, h2))

34. [MCQ] [GATE-2011: 2M]

We are given a set of **n** distinct elements and an unlabeled binary tree with **n** nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

(a) 0 (b) 1

(c) n! (d) $\frac{1}{(n+1)}\,{}^{2n}C_n$

35. [MCQ] [GATE-2008: 2M]

You are given the postorder traversal, **P**, of a binary search tree on the **n** elements 1, 2, …., n. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

(a) $\Theta(log\ n)$

(b) $\Theta(n)$

(c) $\Theta(nlog\ n)$

(d) none of the above, as the tree cannot be uniquely determined.

36. [MCQ] [GATE-2008: 2M]

A binary tree with n > 1 nodes has n_1, n_2 *and* n_3 nodes of degree one, two and three respectively. The degree of a node is a defined as the number of its neighbours.

Starting with the above tree, while there remains a node v of degree two in the tree, add an edge between the two neighbors of v and then remove v from the tree.

How many edges will remain at the end of the process?

(a) $2 * n_1 - 3$ (b) $n_2 + 2 * n_1 - 2$

(c) $n_3 - n_2$ (d) $n_2 + n_1 - 2$

37. [MCQ] [GATE-2008: 2M]

A binary tree with n > 1 nodes has n_1, n_2 *and* n_3 nodes of degree one, two and three respectively. The degree of a node is a defined as the number of its neighbours.

n_3 can be expressed as:

(a) $n_1 + n_2 - 1$ (b) $n_1 - 2$

(c) $[((n_1 + n_2)/2)]$ (d) $n_2 - 1$

38. [MCQ] [GATE-2008: 2M]

A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

I. 81, 537, 102, 439, 285, 376, 305

II. 52, 97, 121, 195, 242, 381, 472

III. 142, 248, 520, 386, 345, 270, 307

IV. 550, 149, 507, 395, 463, 402, 270

Suppose the BST has been unsuccessfully searched for key **273**. Which all of the above sequence list nodes in the order in which we could have encountered them in the search?

(a) II and III only (b) I and III only

(c) III and IV only (d) III only

AVL Trees

39. [MCQ] [GATE-2020: 1M]

What is the worst case time complexity of inserting n^2 elements into an AVL-tree with n elements initially?

(a) $\Theta(n^4)$ (b) $\Theta(n^2)$

(c) $\Theta(n^2 \log n)$ (d) $\Theta(n^3)$

40. [MCQ] [GATE-2009: 2M]

What is the maximum height of any AVL-tree with 7 nodes? Assume that the height of a tree with a single node is 0.

(a) 2 (b) 3

(c) 4 (d) 5

41. [MCQ] [GATE-2008: 1M]

Which of the following is TRUE?

(a) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is O(n)

(b) The cost of searching an AVL tree is $\Theta(\log n)$ but that of a complete binary tree is $\Theta(n \log n)$

(c) The cost of searching a binary search tree is O (log n) but that of an AVL tree is $\Theta(n)$

(d) The cost of searching an AVL tree is $\Theta(n \log n)$ but that of a binary search tree is O(n)

❑❑❑

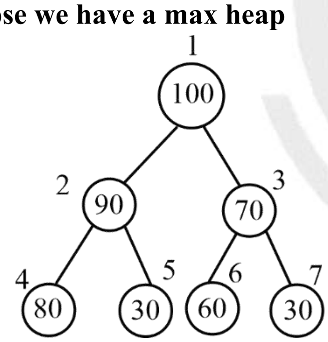
ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(511 to 511)	**4.**	(a)
5.	(c)	**6.**	(d)	**7.**	(8 to 8)	**8.**	(b)
9.	(b)	**10.**	(d)	**11.**	(a)	**12.**	(b)
13.	(c)	**14.**	(d)	**15.**	(b)	**16.**	(c)
17.	(c)	**18.**	(b)	**19.**	(4 to 4)	**20.**	(b)
21.	(a)	**22.**	(d)	**23.**	(d)	**24.**	(509 to 509)
25.	(d)	**26.**	(b)	**27.**	(64 to 64)	**28.**	(b)
29.	(19 to 19)	**30.**	(b)	**31.**	(a)	**32.**	(1 to 1)
33.	(a)	**34.**	(b)	**35.**	(b)	**36.**	(a)
37.	(b)	**38.**	(d)	**39.**	(c)	**40.**	(b)
41.	(a)						

SOLUTIONS

1. (b)

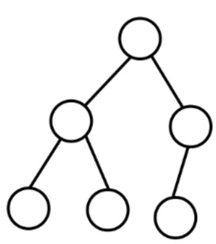
Suppopse we have a max heap



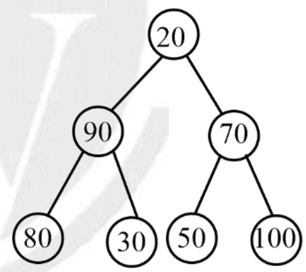
Array Representation:

1	2	3	4	5	6	7
100	90	70	80	30	50	20

Structure after delete an element:



Swap last with first element:



[Now Heapify]

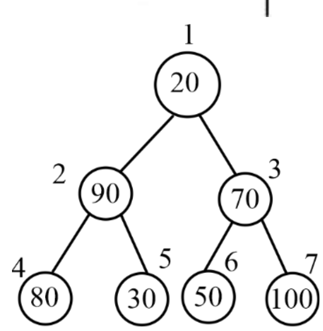
Operations:

A[1]: Max element

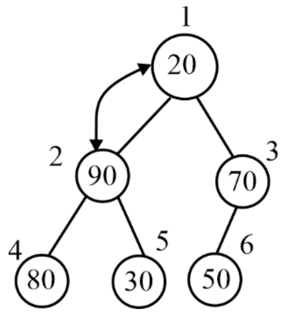
Swap A[1] → A[n]

n = n – 1

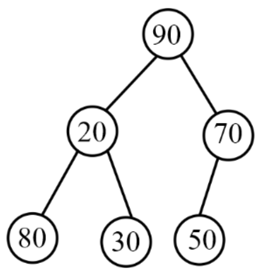
1	2	3	4	5	6	7
20	90	70	80	30	50	100



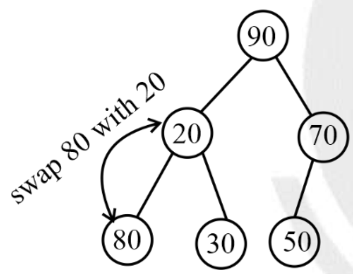
Delete element 100:



Swap 90 with 20:



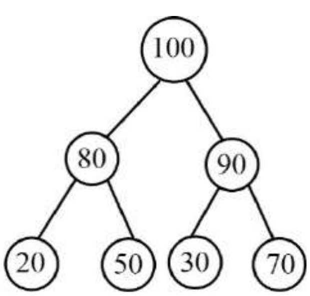
Apply heapify method:
Heapify method will take O(n) time.



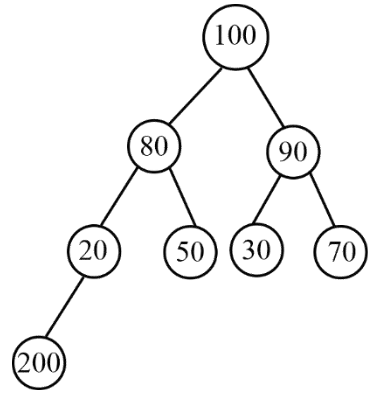
Height of heap = O(logn)

EXTRACT Max = O(logn)time

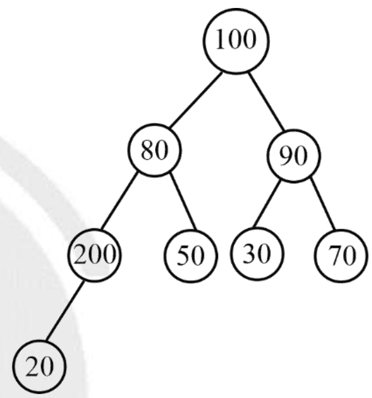
INSERT:
Suppose a max heap



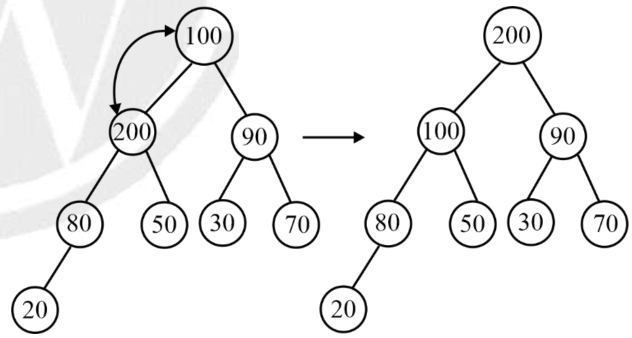
Insert 200 in given heap
Insert (A,key) ⇒ O(logn)



Parent is smallelr (swap)



200 is greater than 80. So, we need to swap



Now, a max-heap

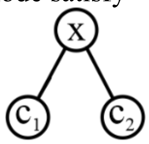
Total inertion time = O(logn)

So, operation will take O(n) time.

Scan for Video solution

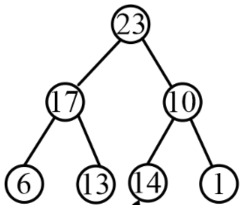
2. (b)

Max heap : every node satisfy



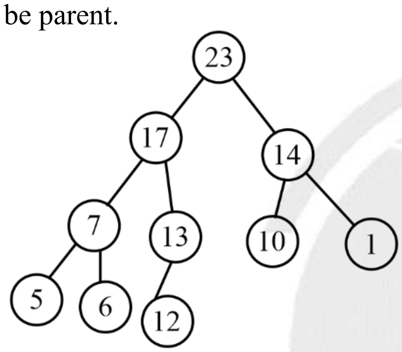
x >(c_1, c_2)

Node value > its child value



so, option (a) is wrong

14 must be parent.



It is satisfying all the property of Max- Heap.

Like that we will check option c and d.

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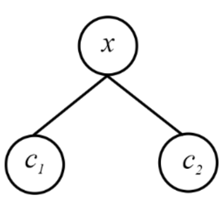
3. (511 to 511)

In a min-heap the maximum element can be present at a leaf node.

Property of min heap

For any node x

$\boxed{x < c_1, c_2}$



Largest among x, c_1, c_2 cannot be x.

Either of child can be maximum

$\Rightarrow$ Greatest element can be at a node which does not have any child $\Rightarrow$ leaf node.

Number of leaf node a heap with n element = $\left\lceil \frac{n}{2} \right\rceil$

In the question, n = 1023

Number of leaf nodes = $\left\lceil \frac{1023}{2} \right\rceil = 512$

To find max among 10 elements $\Rightarrow$ 9 comparisons are required

To find max among n element $\Rightarrow$ (n–1) comparisons are required

To find max among 512 element $\Rightarrow$ (512 –1) comparisons are required

= 511 comparisons.

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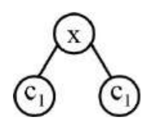
4. (a)

I. First statement is true as in a max-heap, the samllest element can not be a node having some child.



$x > c_1, c_2$

So, the smallest can be either c_1 and c_2

II. 2nd largest elements is a max-heap most be at level 1. So, this statement is also correct.

III. Third statement is true as we can use build-heap method to construct the heap in $\theta(N)$ time.

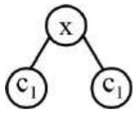
IV. is false because it will take O(n logn) time

So, (a) is correct.

Scan for Video solution

5. (c)

In a min-heap, every node satisfy the property that node-value is samller than child- value.



$x < c_1, c_2$

i,e the largest among node and children can not be node i.e only child can be larger.
So, a node with some child can not be largest i.e largest element must not have any child.
$\Rightarrow$ Only leaf nodes can have largest element

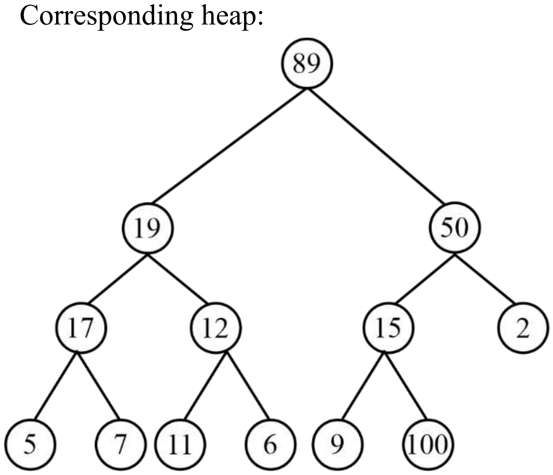
Almost $\left(\frac{n}{2}\right)$ number of leaf nodes are present in a heap to find largest among these $O\left(\frac{n}{2}\right) = O(n)$ leaves possible

Number of compareisons for n elemnts $= O(n) = \Theta(n)$

Scan for Video solution

6. (d)

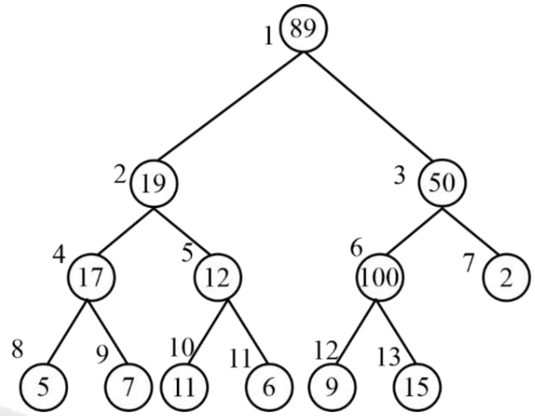
The given question is saying about build-heap algorithm.
Given,
89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100
Corresponding heap:

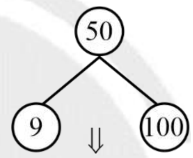


Swap

We need to work internal nodes 1,2,3,4,5,6

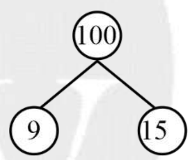
(1) with index 6

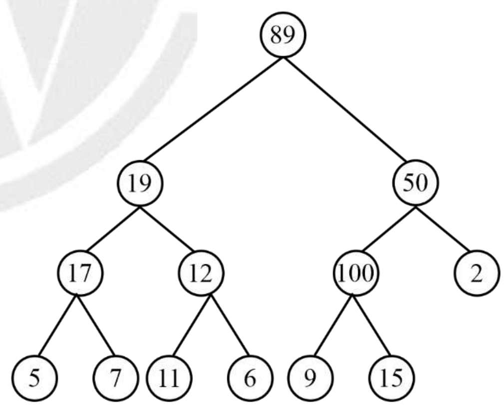


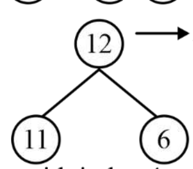


Largest among root, C_1, C_2 $\Rightarrow$ must be at root node

$\Downarrow$
1 swap

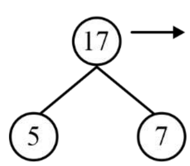






$\rightarrow$ Already heap property satisfied

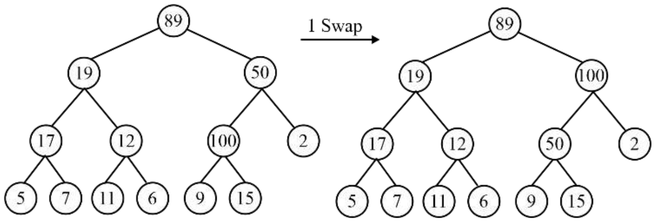
with index 4



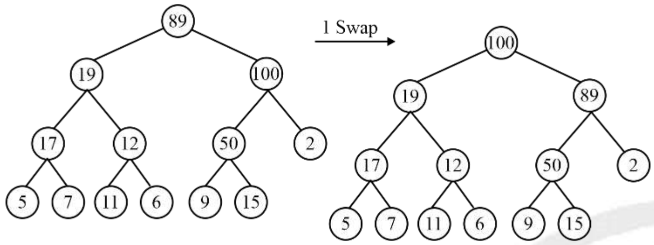
$\rightarrow$ Already heap

Build- Heap

With index 3



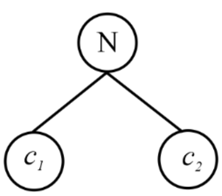
Index 2 $\Rightarrow$ heap prop. Already satisfied



Total 3 swaps required. So, option (d) is the correct answer.

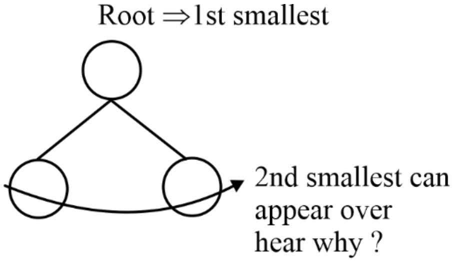
Scan for Video solution

7. **(8 to 8)**



N< c_1, c_2 . In min-heap, every node is smaller than its child.

Root $\Rightarrow$1st smallest



2^{nd} smaller cannot go beyond level 1.

k^{th} smallest cannot be beyond (k–1) level

Why $\Rightarrow$ there are only (k –1) smaller elements than k^{th} smallest and path from root to k^{th} smallest must go through these (k–1) elements.

Ensure: Enough nodes

Level 0 $\rightarrow 2^o = 1$

Level 1 $\rightarrow 2^1 = 2$

upto level 1 = $2^{1+1} - 1 = 3$

upto level 2 $\Rightarrow 2^{2+1} - = 7$

:

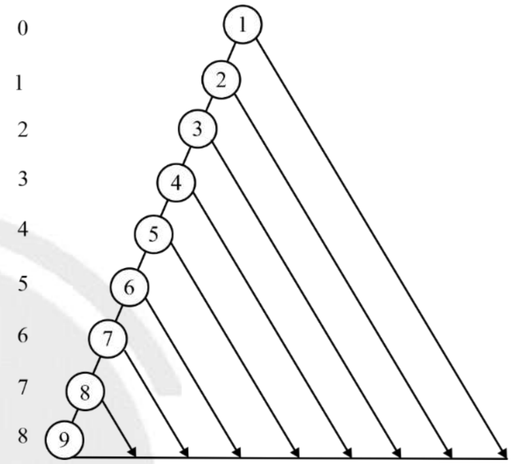
:

upto level 7 $\Rightarrow 2^{7+1} - 1 = 2^8 - 1 = 255$

upto level $8^{th} \Rightarrow 2^{8+1} = 511$

We are having more than 511.

$\Rightarrow$ Deepest level for 9 is 8



So, 8 is the correct answer.

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8. **(b)**

Delete (i) $\rightarrow$ delete ith element

O(1) time

But we need to maintain heap property after deletion

To delete i^{th} element $\Rightarrow$ replace it with last element

i.e. A[i]$\leftrightarrow$ A [n]

The, n = n –1

Apply Heapify algorithm to re- fix the heap

It takess $\Rightarrow$ O (logn) time in worst case

But here it is given that d is the depth

So, heapify will take O(d).

Therefore, option (b) is the correct answer.

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9. (b)

Given code is Implementation of binary search whenever we found the target element i.e. x both if conditions become true.

Code for first if:

j =k – 1 ⇒ update j to k – 1

Code of second if:

i = k + 1

$$\left.\begin{matrix} j = k - 1 \\ i = k + 1 \end{matrix}\right\}$$

⇒ i > j

⇒ Condition inside while loop become false and we come out of loop.

So, given code is an implementation of binary search.

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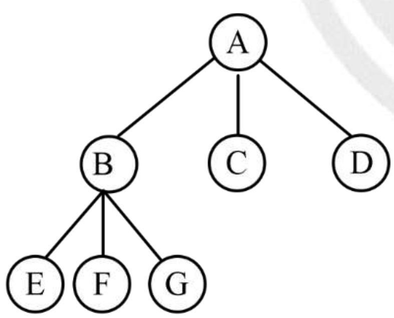
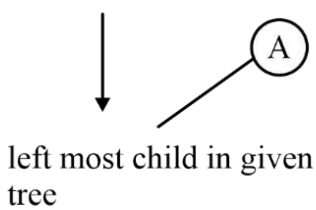
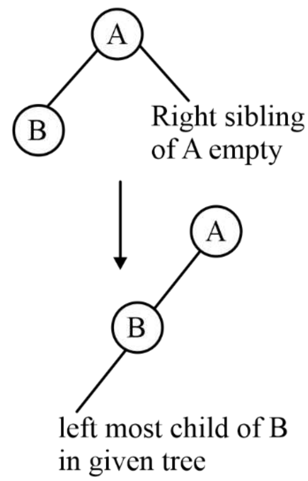
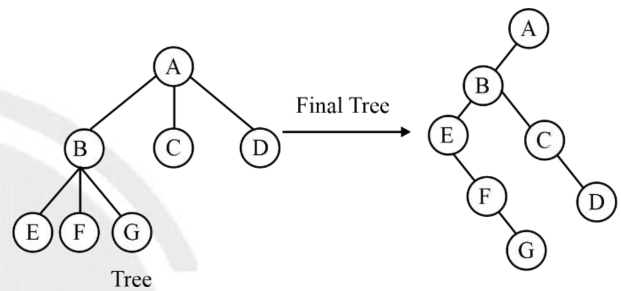
10. (d)



Tree







Current node is a leaf node

// if left most child pointer is NULL

⇒ there is no child of node under consideration.

If there is no left child we are counting the number of leaf nodes.

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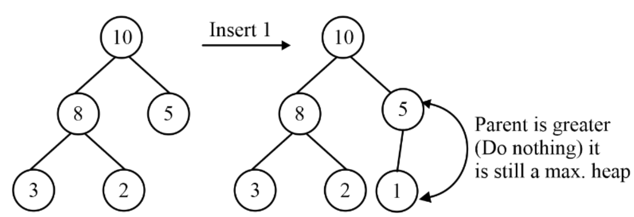
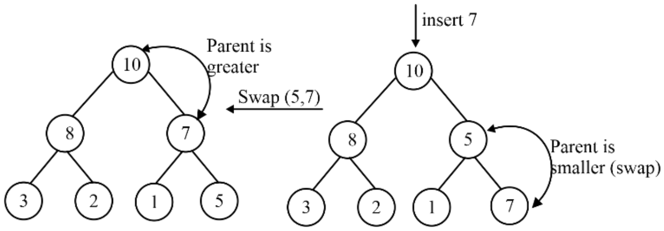
11. (a)

Initial:





Level order traversal of max heap

= 10, 8, 7, 3, 2, 1, 5

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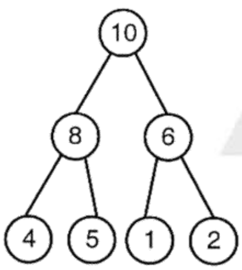
12. **(b)**



Max– heap

Satisfy max heap property only option (b)

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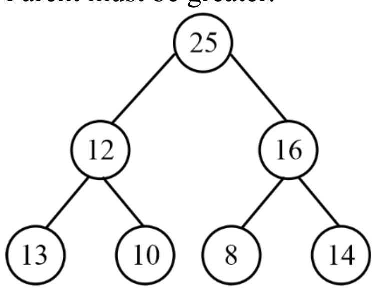
3. **(c)**

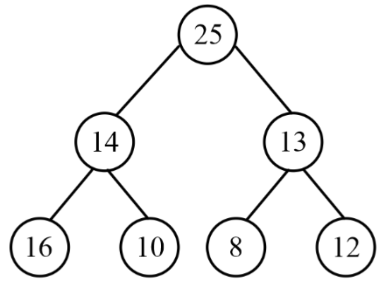
(a) Parent must be greater.



13 is greater than 12 (Parent)

Not a max-heap

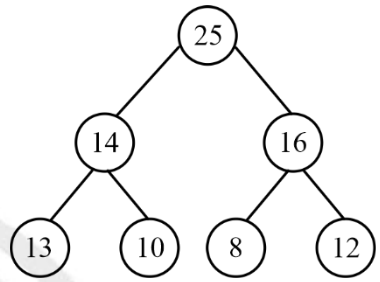
(b)



Not a max– heap

Not a max heap because 16 must be parent of 14 and 10.

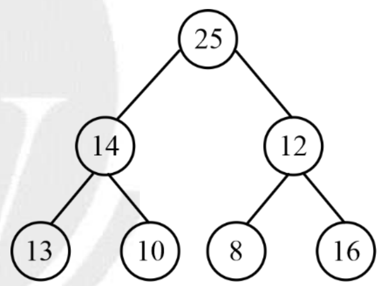
(c)



All nodes satisfy Property

Max- heap

(d)



12 is smaller than its right child 16

Not a max- heap

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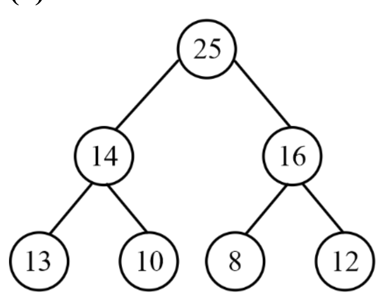
14. **(d)**



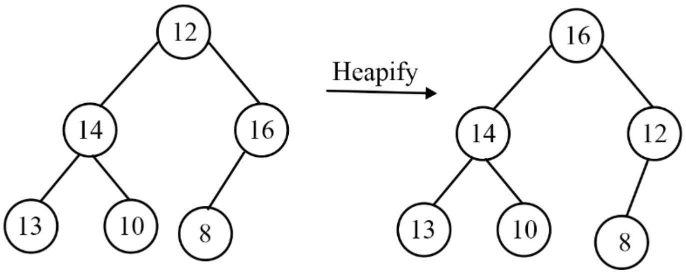
1	2	3	4	5	6	7
25	14	16	13	10	8	12

n = 7

A = [1] ↔ A [n] swap root element with last element.

n = n –1

heapify on A [1]

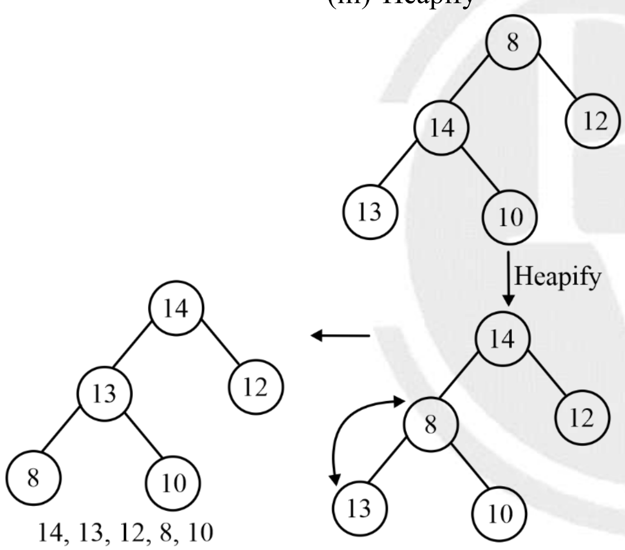


another delete

(i) swap 12, 16

(ii) n = n–1

(iii) Heapify



14, 13, 12, 8, 10

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15. (b)

1 insert ⇒O (logn)

(i) Inset n element without anything

(ii) Apply build heap algorithm on 2n element

Time Complexity = O(2n) = O(n)

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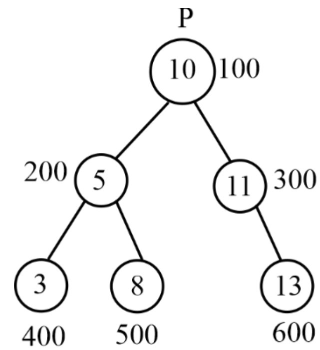
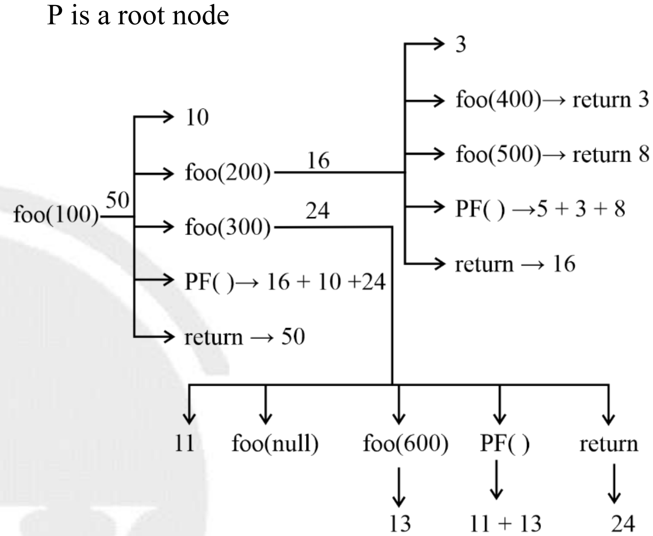
16. (c)



P is a root node



Output = 3, 8, 16, 13, 24, 50

Output matched with option (c)

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17. (c)

S1 : Ture

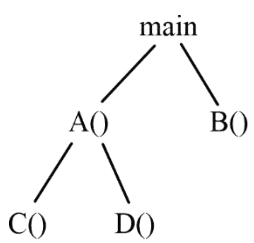
To perform func/procedure calls.

A functioni/procedure (parent)

must call child functions

```
Main A( )
{
A( );
B( );
}
A( )
{
C( );
```

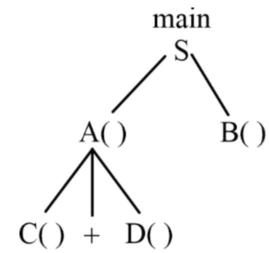
D( );
}
C( )
{
-
-
}
D( )
{
-
-
}



S$_2$: True because in order that parent func return its vaule, its child must provide (return) value to it.

```
Main ()
{
Int s;
S = A( ) + B( )
Pf ("%d, S")
}
Int A( )
{
Return (C) + D( );
}
Int B( )
{
Retrun 1;
}
int
C( );
{
return 2
}
int D( )
{
return 3;
}
```

Procedure return call:



Procedure return call = C ( ), D( ), A( ), B( ), S

It is nothing but post order.

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18. (b)

The in-order traversal of a BST is increasing order of keys.

In order:	10	11	12	$\boxed{15}$	16	18	19	20
Pr order:	15	10	12	11	20	18	16	19

we can construct the BST

We know, first element of preorder traversal is the root node of entire tree.

Mark this node in Inorder traversal

Now we will insert 10 (next node in preorder) in the BST.

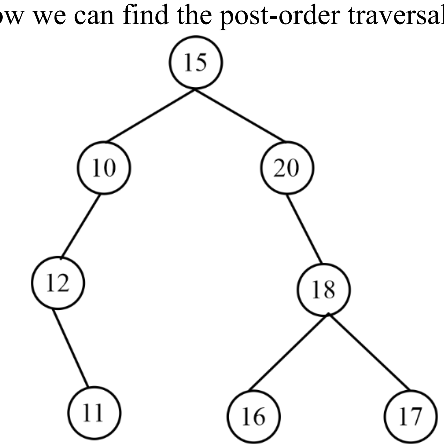
⇒ to find the position of 10

⇒ just check position of 10 in Inorder traversal w.r.t. the marked node.

{10 is the left of 15}

Similarly, finding position for each other element

Now we can find the post-order traversal



Post-order traversal: 11,12, 10, 16, 19, 18, 20, 15.

Therefore, option B is the correct answer.

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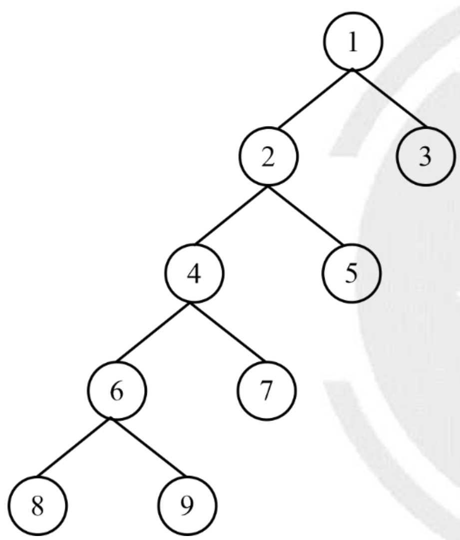
19. (4 to 4)

In order:	8	9	6	7	4	5	2	3	1
Pr order:	8	6	9	4	7	2	5	1	3

We will construct the binary tree using these 2 traversals & then find the height of binary tree.



Height = 4 of the tree.

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20. (b)

The Inorder traversal of a BST is increasing order of keys.

Inorder: Left subtree- Root Node- Right subtree

Pre-order: Root Node- Left subtree - Right subtree

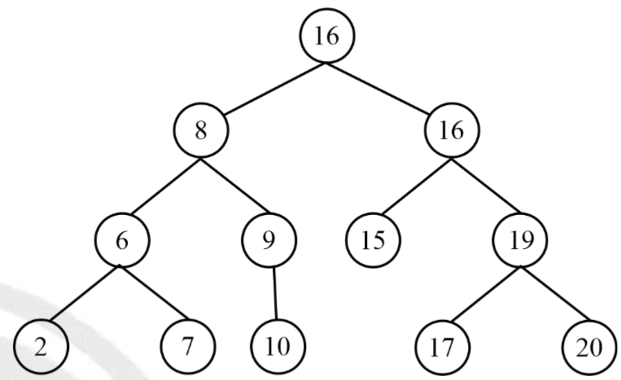
L_T ,Root, R_T $\boxed{2}$ $\boxed{6}$ $\boxed{7}$ $\boxed{8}$ $\boxed{9}$ $\boxed{10}$ $\boxed{12}$ $\boxed{15}$ $\boxed{16}$

Root, L_T, R_T 12 8 6 2 7 9 10 16 15

$\boxed{17}$ $\boxed{19}$ 20

19 17 20

insert nodes into initially empty BST in this order



Post-order traversal of this tree: 2, 7, 6, 10, 9, 8, 15, 17, 20 19, 12. So, option (b) is the correct answer.

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21. (a)

In-order traversal of a BST is increasing order of keys.

1. Increasing order of keys (✓)
2. Not increasing order of key cannot be in-order traversal of a BST (×)
3. Not increasing order of keys (×)
4. Increasing order of keys (✓)

Only 1 and 4 are Inorder traversals, therefore, option a is correct answer.

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22. (d)

BST: The in-order traversal is increasing order of keys.

In order: 10 15 20 23 25 $\boxed{30}$ 35 39 42

Pr order: 30 20 10 15 25 23 39 35 42

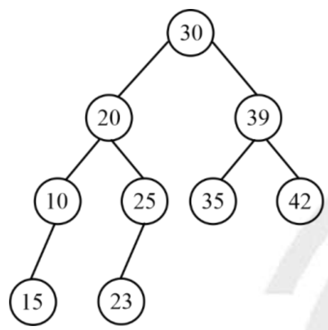
Insert nodes in the tree from left to right in pre-order.

Because pre-order gives first node as a root nods

In order: $\boxed{10}$ $\boxed{15}$ $\boxed{20}$ $\boxed{23}$ $\boxed{25}$ $\boxed{30}$ $\boxed{35}$ $\boxed{39}$ 42

Pr order: 30 20 10 15 25 23 39 35 42

Tree:



Post order = 15, 10, 23, 25, 20, 35, 42, 39, 30

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23. (d)

I. MBCAFHPYK

II. KAMCBYPFH

III. MABCKYFPH

Preorder: Root, L_T, R_T

Postorder: L_T, R_T, Root

Let I be preorder

Root $\Rightarrow$ M

Out of II and III one must be the postorder & therefore M must be the lost node in any then but it is not.

Hence, I is not preorder

Let II be preorder

$\Rightarrow$ K is root node I has k in last

$\Rightarrow$ I is postorder

& remaining trarversal i.e. III is Inorder.

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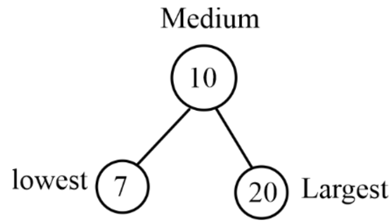
24. (509 to 509)

BST: keys are stored in some orderd way

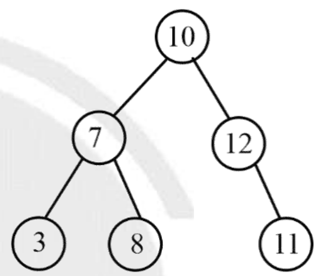
The in-order traversal is increasing order of keys.

1. Consider an BST



In - order : 7 10 20

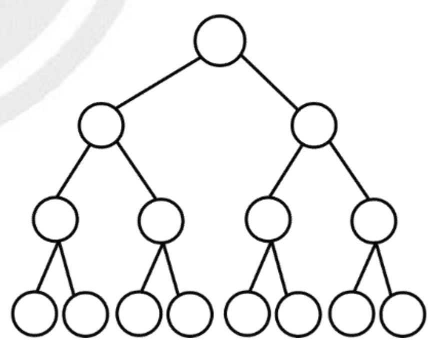
2.Take another BST



In – Order = 3 7 8 11 11 12

For a binary tree of height h, if all levels are full,then number of nodes = $2^{h+1} - 1$

If a height is 3 of CBT then number of nodes must be $2^4 - 1 = 15$



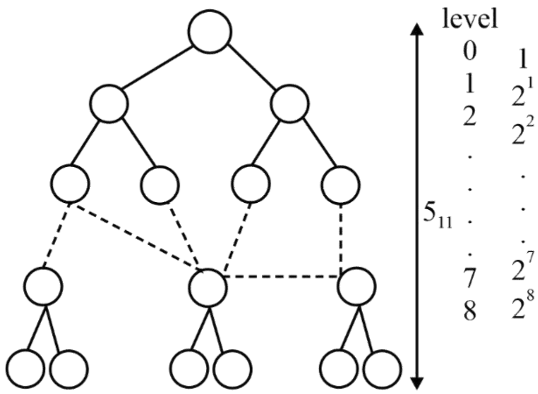
Height = 3

Level = 4

Number of nodes = 15

If in CBT having L level then number of nodes = $2^L - 1$

If N = 1000, all the levels are not full.



Till Height 8 or level 9 Number of nodes = $2^9 - 1$

Number of nodes = 511

For last level to be full number of nodes = $2^9 = 512$

Number of nodes remaining = 1000 – 511 = 489

In second last level (256 Nodes) 244 nodes having 2 child and 1 node having only 1 child and remaining are countion in leaf node.

3 largest element index value must be 509.

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25. **(d)**

We want any element other than maximum or we can say any element which is not maximum would be our element.

So, we need to pick any 2 elements starting from root node, then return the smaller from these 2 element.

⇒ Complexity would be Θ(1)

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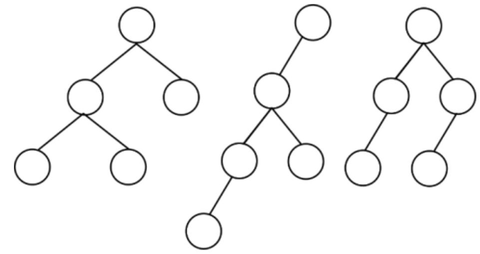
26. **(b)**

Let n = 5

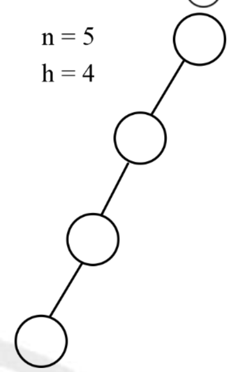
Maximum Height can be achieve when minimum nodes are present at every level.

i.e. only 1 node at each level.



n = 5

h = 4

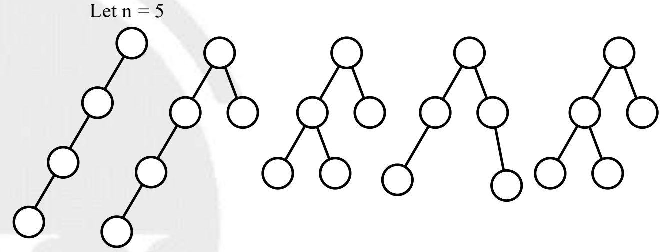


⇒ max. height of any binary tree or BST with n nodes = (n –1)

max. height of BST with 15 nodes = 15 – 1 = 14

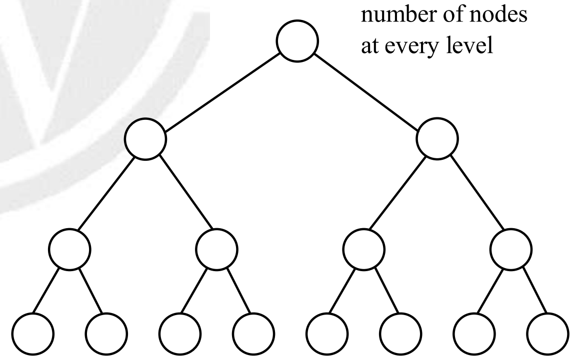
Min. height of a binary tree with 5 nodes

Let n = 5



n = 15

put max. number of nodes at every level



$\boxed{\text{height} = 3}$

Minimum height =3

Maximum height = 14

Therefore, option B is the correct answer.

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27. (64 to 64)

We cannot have any node with 2 child nodes.

At every level $\Rightarrow$ 1 node can be there (either to left or to right)

From 1st level till 2nd last level$\Rightarrow$ 2 choices are there

Last level $\Rightarrow$ only one choice

For six levels: $2\times2\times2\times2\times2\times1 = 64$

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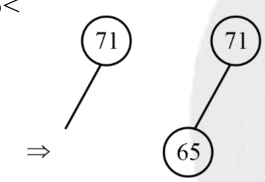
28. (b)

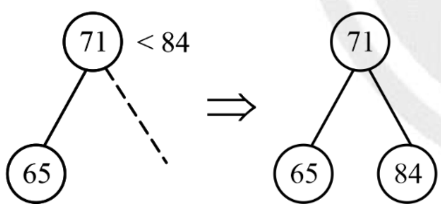
71, 65, 84, 69, 67, 83

(1) Insert 71

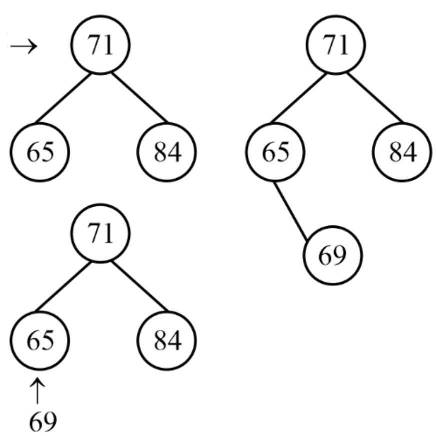
(2) Insert 65 65<



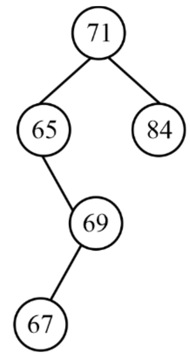
(3) Insert 84



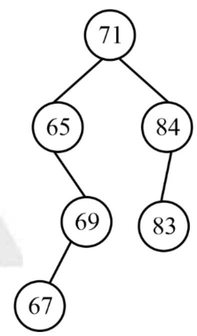
(4) Inset 69



(5) Insert 67



(6) Insert 83



$\Rightarrow$Lowest level element= 67. So, option (b) is the correct answer.

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29. (19 to 19)

Number of nodes with two child = Number of nodes with no child (leaf node) –1

$$\boxed{n_2 = n_0 - 1}$$

$$n_2 = 20 - 1$$
$$n_2 = 19$$

So, 19 is the correct answer

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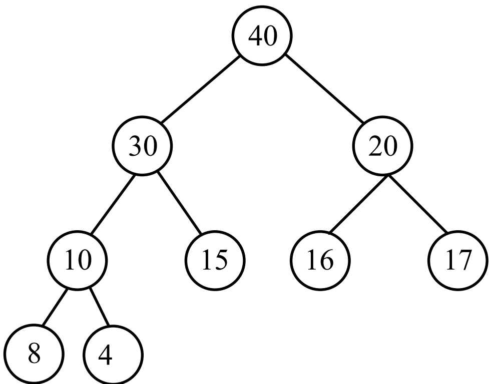
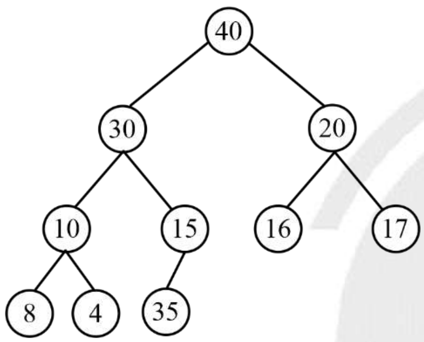
30. (b)

Given, 40, 30, 20, 10, 15, 16, 17, 8, 4

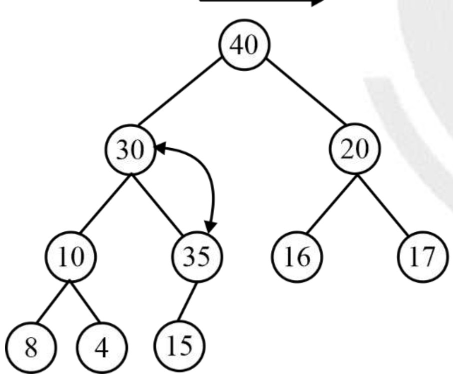
Corresponding heap:



Insert 35

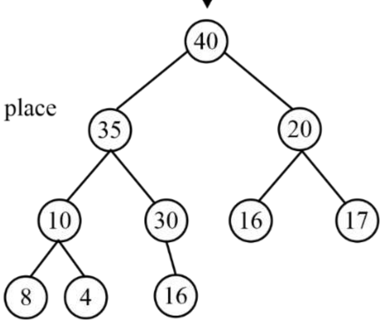


Swap



Swap

Right place



So, 40, 35, 20, 10, 30, 16, 17, 8, 4, 15. Option b is the correct answer.

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31. (a)

$\boxed{n_{min} = h + 1}$

$\boxed{n_{max} = 2^{h+1}}$

$r_{min} = 5 + 1 = 6$

$n_{max} = 2^{5+1} = 2^6 - 1$

$= 64 - 1$

$= 63$

So, option a is the correct answer.

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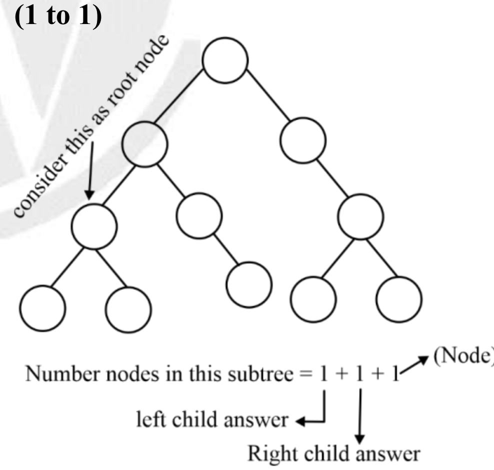
32. (1 to 1)



we need to use bottom up approach

```
Count = 0; // global
int f (struct node * Root)
{
if (Root = = NULL)
return 0 ;
int l = f (Root → left);
```

```
int r = f (Root → Right);
if (l + r +1 = = 4)
count + +;
return l + r + 1;
}
```

Time = O(n)

Given time = $O\left(n^a \log^b n\right)$

a = 1

b = 0

$O\left(n^a \log^b n\right) = a + 10 \times b = 1 + 10 \times 0 = 1$

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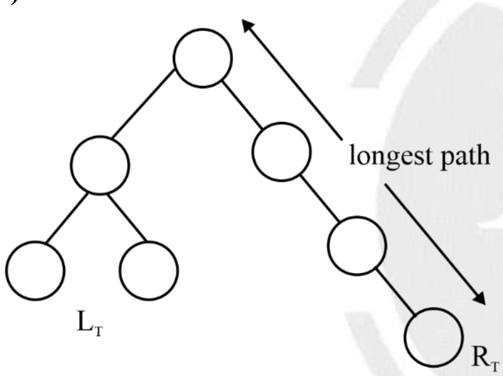
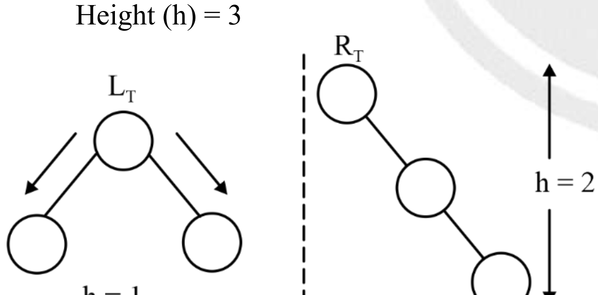
33. (a)



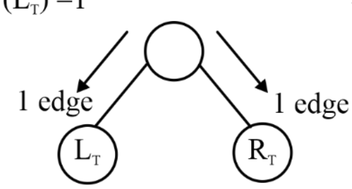
Height (h) = 3



h = 1

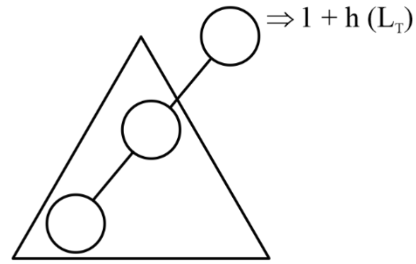
h (L_T) = 1

h (R_T) = 2



h = 1 + max (h (L_T), h (R_T))

- If L_T = empty and R_T = empty



h (L_T) = 1

If R_T = = NULL

h = 1 + h (L_T)

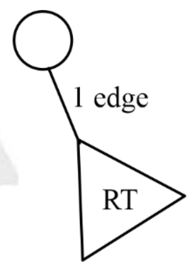
If L_T = = NULL

h = 1 + h(R_T)

- R_T, $L_T \Rightarrow$ non - empty

  1 + max (h (L_T), h (R_T))

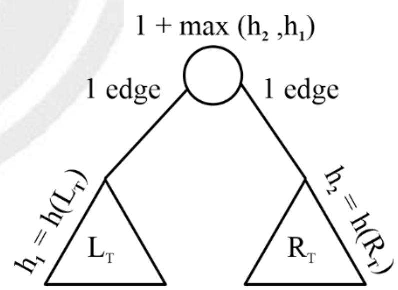
B1:



h = 1 + h(R_T)

B1: 1 + height (n → Right)

B2:



B2 = 1 + max (h_1, h_2)

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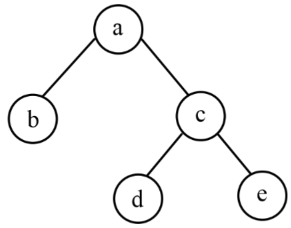
34. (b)

- Given a binary tree structure with n nodes and also given n distinct elements.
- Let n = 5

10, 13, 18, 19, 35

Because it is BST,

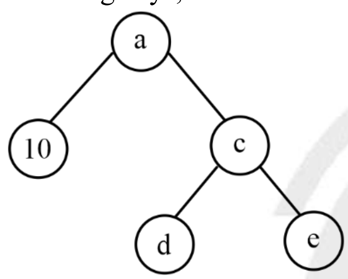
keys are ordered in some way.
Smallest element must be to the left of root node (1 choice)



b < a < (c, d, e)

$\Rightarrow \boxed{b=10}$

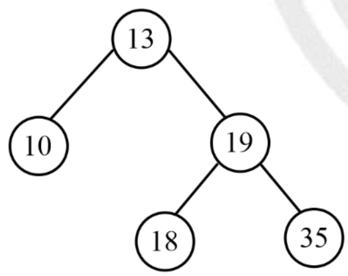
13, 18, 19, 35 are remaining
a < all keys in right subtree
among remaining keys, the smallest is the root node



$\boxed{a=13}$

Remaining: 18, 19, 35

$\begin{matrix} d < c < e \\ 18 < 19 < 35 \end{matrix}\{\text{only1 choice}\}$



$\Rightarrow$ only 1 way

Hence, option (b) is correct.

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35. (b)

BST: The in-order traversal is increasing order of keys.

In-order: 1, 2, 3, 4, ……..n

Post-order ----------------------Root

(i) from Post-order → lost node is the root node O(1) time.

Finding this root node in in-order traversal (sorted array)

BS→ O($\log_n$)

In-order: 1, 2, 3, 4………n

Post order: _____________Root

(1) From Post-order →last node is the root node O(1)

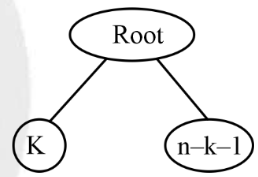
Finding this root node in inorder traversal (if sorted array then, BS→ O($\log_n$))

⇒ Split the postorder
- Smaller than pivot (left)
- Greater than pivot (Right)

If k element in left subtree then in right subtree element must be n – k – 1.

(n–k –1)

T(n) = O($\log_n$) + T(k) + T(n – k – 1)



Recursively follow.

T(n) = T(k) + T (n–k–1) + O ($\log_n$)

T(n) = O(n logn)

But

In-order [ ]

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10

$\underbrace{1, 2, 3, ... n}_{\text{let}}$ root

we can find any element in O(1) time

Left (1…. 5) 6 (7….10)

T(n) = O(1) + T(k) + T(n – k –1)

Hence, option (b) is correct.

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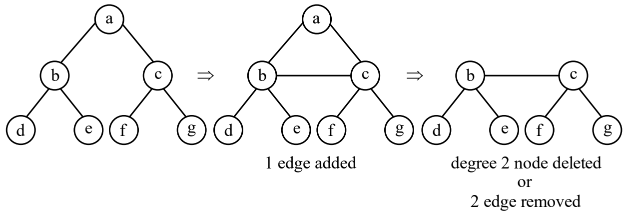
36. (a)

$\boxed{n_3 = n_1 - 2}$ Already know



In 1 such removal $\Rightarrow$ 1 edge remove

Repeat this process until all nodes of degree 2 removed

$\Rightarrow n_2$ times we need to repeat

$\Rightarrow n_2 \times 1$ edge will be removed in over all process

Number of edges initially = $n - 1 = n_1 + n_2 + n_3 - 1$

After complete process edges, remaining = $(n_1 + n_2 + n_3 - 1) - 1 \times n_2$

$= n_1 + n_3 - 1 = n_1 + n_1 - 2 - 1 = 2 * n_1 - 3$

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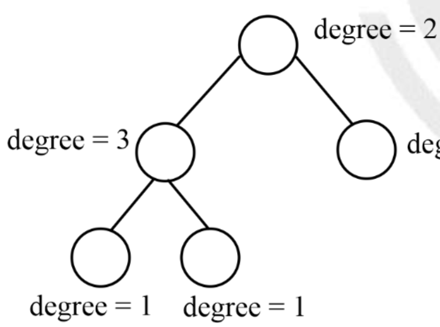
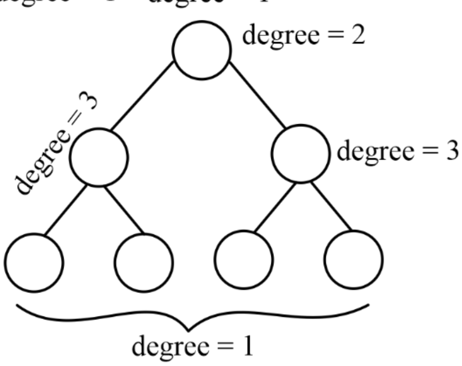
37. (b)

Basics:





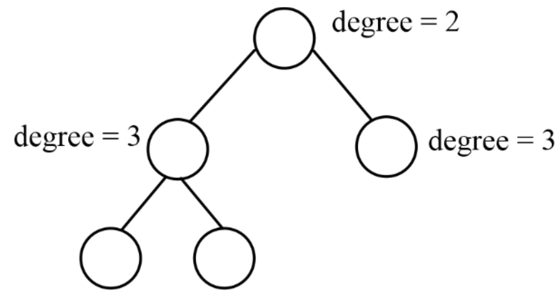
$n_1 = 4$ $n_2 = 2$ $n_3 = 2$

$4 + 1 - 1 = 4$ (False) n_3 most be 2

(b) $4 - 2 = 2$ (True)

(c) $[5/2] = [2.5]$ = Either 2 or 3

(d) 0



$n_1 = 3$, $n_2 = 1$, $n_3 = 1$,

(a) 1

(b) $(3 + 1)/2 = 2$ cannot be our answer.

Method 2:

Total number of nodes $n = n_1 + n_2 + n_3$

Sum of degree of all the nodes = $2|E|$

$= 2(n-1)$

$= 2(n_1 + n_2 + n_3 - 1)$

$= n_1 \times 1 + n_2 \times 2 + n_3 \times 3$

$n_1 + 2n_2 + 3n_3 = 2n_1 + 2n_2 + 2n_3 - 2$

$n_3 = 2n_1 - n_1 - 2$

$n_3 = n_1 - 2$

So, option (b) is correct.

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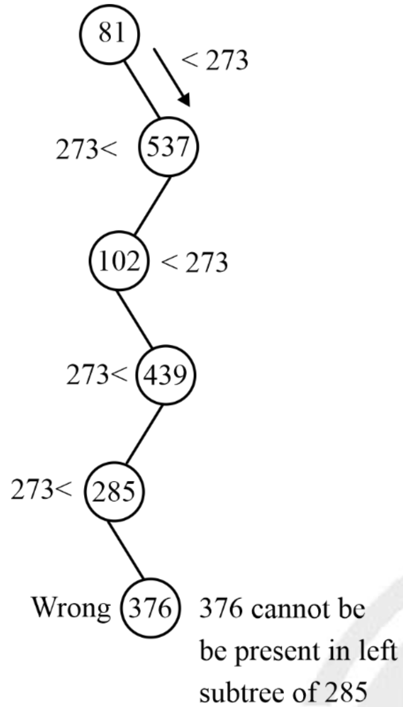
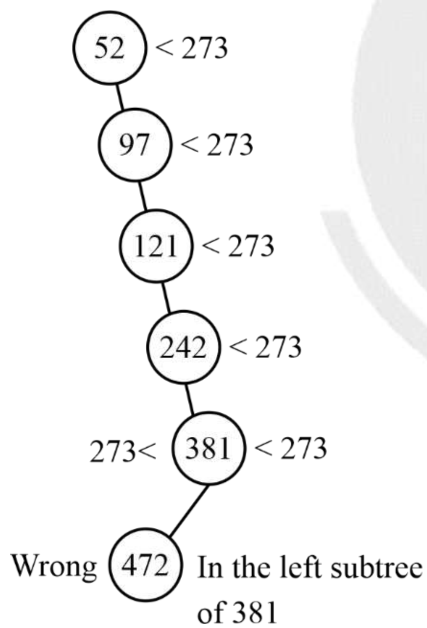
38. **(d)** **I.**

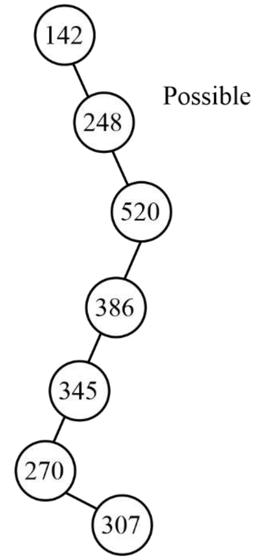


II.

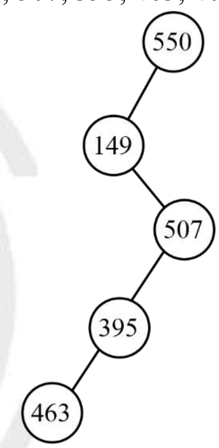


All keys must be smaller than 381.

III. 142, 248, 520, 386, 345, 270, 307



IV. 550, 149, 507, 395, 463, 402, 270



463 cannot be in left subtree of 395.

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39. **(c)**

AVL tree is height balanced tree and the height of an AVL tree with n element is $O(\log_2 n)$

1^{st} insertion time = O(logn)

2^{nd} insertion time = O(log(n + 1))

As AVL tree contain (n + 1) elements after 1^{st} insertion

3^{rd} insertion = O(log(n + 2))

.

.

n^2 insert = $O(\log(n + n^2 - 1))$

Total T.C = O(logn) + O(log (n + 1) + O(log (n + 2)) +……+ O(log(n + n^2 – 1))

= Olog (n * (n + 1)) * (n + 2) * … * (n + n^2 – 1)

$O\left(\log n^{n^2}\right)$

$O(n^2 \log n)$

(c) is correct

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40. (b)

n(h) = 1+ n (h–1) + n(h –2)

minimum nodes with height h.

n (0) =1

n (1) = 2

n (2) = 1+ n(1) +n(0) = 1+2 +1 = 4

n(3) = 1 + n(2) + n(1)

= 1 + 4 + 2 = 7

$\boxed{n(3) = 7}$

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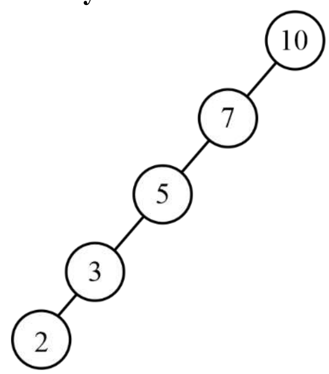
41. (a)

AVL tree is height balance tree (also a search tree)

Height of AVL tree = $O(\log_2 n)$

searching time = $O(\log_2 n)$

Binary search tree:



Searching time = O(n)

The height of complete binary tree = $O(\log_2 n)$ but it is not a search tree. Element order arranged randomly (not in some ordered way)

Searching time = O(n)

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CHAPTER

8

HASHING

Hash Functions

1. [MCQ] [GATE-2022 : 1M]

Suppose we are given n keys, m hash table slots, and two simple uniform hash functions h_1 and h_2 . Further suppose our hashing scheme uses h_1 for the odd keys and h_2 for the even keys. What is the expected number of keys in a slot?

(a) $\frac{m}{n}$ (b) $\frac{n}{m}$

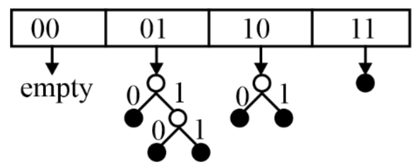
(c) $\frac{2n}{m}$ (d) $\frac{n}{2m}$

2. [MSQ] [GATE-2021 : 2M]

Consider a dynamic hashing approach for 4-bit integer keys:

1. There is a main hash table of size 4
2. The 2 least significant bits of a key is used to index into the main hash table.
3. Initially, the main hash table entries are empty.
4. Thereafter, when more keys are hashed into it, to resolve collisions, the set of all keys corresponding to a main hash table entry is organized as a binary tree that grows on demand.
5. First, the 3rd least significant bit is used to divide the keys into left and right subtrees.
6. To resolve more collisions, each node of the binary tree is further sub-divided into left and right subtrees based on the 4th least significant bit.
7. A split is done only if it is needed, i.e., only when there is a collision.

Consider the following state of the hash table.



Which of the following sequences of key insertions can cause the above state of the hash table (assume the keys are in decimal notation)?

(a) 5, 9, 4, 13, 10, 7

(b) 9, 5, 10, 6, 7, 1

(c) 10, 9, 6, 7, 5, 13

(d) 9, 5, 13, 6, 10, 14

3. [NAT] [GATE-2020 : 1M]

Consider a double hashing scheme in which the primary hash function is $h_1(k) = k \mod 23$ and the secondary hash function is $h_2(k) = 1 + (k \mod 19)$. Assume that the table size is 23. Then the address returned by probe 1 in the probe sequence (assume that the probe sequence begins at probe 0) for key value $k = 90$ is _______

4. [NAT] [GATE-2015 : 1M]

Given a hash table T with 25 slots that stores 2000 elements, the load factor α for T is ________.

5. [MCQ] [GATE-2015 : 2M]

Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2020?

(a) $h(i) = i^2 \mod 10$

(b) $h(i) = i^3 \mod 10$

(c) $h(i) = (11 * i^2) \mod 10$

(d) $h(i) = (12 * i) \mod 10$

Collision Resolution Techniques

6. [MCQ] [GATE-2023 : 1M]

An algorithm has to store several keys generated by an adversary in a hash table. The adversary is malicious who tries to maximize the number of collisions.

Let k be the number of keys, m be the number of slots in the hash table, and k > m. Which one of the following is the best hashing strategy to counteract the adversary?

(a) Division method, i.e., use the hash function h(k) = k mod m.
(b) Multiplication method, i.e., use the hash function $h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$, where A is a carefully chosen constant.
(c) Universal hashing method.
(d) If k is a prime number, use Division method. Otherwise, use Multiplication method.

7. [MCQ] [GATE-2014 : 2M]

Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?

(a) $(97 \times 97 \times 97)/100^3$
(b) $(99 \times 98 \times 97)/100^3$
(c) $(97 \times 96 \times 95)/100^3$
(d) $(97 \times 96 \times 95)/(3! \times 100^3)$

8. [MCQ] [GATE-2014 : 2M]

Consider a hash table with 9 slots. The hash function is h(k) = k mod 9. The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are

(a) 3, 0, and 1
(b) 3, 3 and 3
(c) 4, 0 and 1
(d) 3, 0, and 2

Common Data for next two questions (9-10):

A hash table of length 10 uses open addressing with hash function h(k) = k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

9. [MCQ] [GATE-2010 : 2M]

Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

(a) 46, 42, 34, 52, 23, 33
(b) 34, 42, 23, 52, 33, 46
(c) 46, 34, 42, 23, 52, 33
(d) 42, 46, 33, 23, 34, 52

10. [MCQ] [GATE-2010 : 2M]

How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(a) 10
(b) 20
(c) 30
(d) 40

11. [MCQ] [GATE-2009 : 2M]

The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(k) = k mod 10 and linear probing. What is the resultant hash table?

(a)

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	18
9	

(b)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(c)

0	
1	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(d)

0	
1	
2	12,2
3	13,3,23
4	
5	5,15
6	
7	
8	18
9	

12. [MCQ] [GATE-2008 : 2M]

Consider a hash table of size 11 that uses open addressing with linear probing. Let h(k) = k mod 11 be the hash function used. A sequence of records with keys 43 36 92 87 11 4 71 13 14 is inserted into an initially empty hash table, the bins of which are indexed from zero to ten. What is the index of the bin into which the last record is inserted?

(a) 2 (b) 4

(c) 6 (d) 7

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ANSWER KEY

1.	(b)	**2.**	(c)	**3.**	(13 to 13)	**4.**	(80 to 80)
5.	(b)	**6.**	(c)	**7.**	(a)	**8.**	(a)
9.	(c)	**10.**	(c)	**11.**	(c)	**12.**	(d)

SOLUTIONS

1. (b)

Either h_1 or h_2 for any key k every slot is equal likely.

Each bucket /slot is equal likely occupied.

For every slot the probability that key will be occupied $= \frac{1}{m}$

Expected number of keys in the slots $= n \times \frac{1}{m} = \frac{n}{m}$

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2. (c)

Given

00 = empty

01 = 3 keys

10 = 2 keys.

11 = 1key

(a) 5 = 0101   01 = keys (5, 9, 15)

9 = 1001   00 = 1key fail it must be empty.

4 = 0100   10 = 1 key (must be 2 key)

13 = 1101   11 = 1 key

10 = 1010

7 = 0111

(b) 9 = 1001   01 = 3 keys

5 = 0101   10 = 2 keys

10 = 1010   11 = 1 key

6 = 0110

7 = 0111

1 = 0001

According to 3^{rd} bit in 01, 1 entries in right and 2 in left but given hash table having 2 in right and 1 in left.

(c) 10 = 1010   01 = 3 keys

9 = 1001   10 = 2 keys

6 = 0110   11 = 1 key

7 = 0111

5 = 0101

13 = 1101

According to 3^{rd} bit in 01 entry 2 key in right and 1 in left.

If is matched with given hash table.

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3. (13 to 13)

m = 23

$h_1(k)$ = k mod 23 = Primary hashing

$h_2(k)$ = 1 + (k mod 19) = Secondary hashing

H(k, i) = $(h_1(k) + i.h_2(k))$ mod m

i = Collision number

k = 90

$h_1(k)$ = 90 mod 23 = 21

$h_2(k) = 1 + (90 \bmod 19)$
$= 1 + 14 = 15$
$H(k, i) = (h_1(k) + i.h_2(k)) \bmod m$
$H(90, i) = (h_1(90) + i.h_2(90) \bmod 23$
$= (21 + 15)$ mode 23
$= 13$

13 is the right answer

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4. (80 to 80)

Formula: $\boxed{\alpha = \frac{\text{Number of keys (n)}}{\text{Table size (m)}}}$

Number of keys = 2000

Table size (m) = 25

$\alpha = \frac{2000}{25} \Rightarrow$

$\boxed{\alpha = 80}$

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5. (b)

Number of buckets = 10

m = 10 [0 to 9]

(a) $1^2 = 1 \bmod 10 = 1$
$2^2 = 4 \bmod 10 = 4$
$3^3 = 9 \bmod 10 = 9$
$4^2 = 16 \bmod 10 = 6$
$5^2 = 25 \bmod 10 = 5$
$6^2 = 36 \bmod 10 = 6$
$7^2 = 49 \bmod 10 = 9$
$8^2 = 64 \bmod 10 = 4$
$9^2 = 81 \bmod 10 = 1$
$10^2 = 100 \bmod 10 = 0$

Bucket that is filled = 1,4,9,6,5,0

Some buckets remain empty. So, option (a) is wrong as well as option (c) also wrong.

(b) $1^3 = 1 \bmod 10 = 1$
$2^3 = 8 \bmod 10 = 8$
$3^3 = 27 \bmod 10 = 7$
$4^3 = 64 \bmod 10 = 4$
$5^3 = 125 \bmod 10 = 5$
$6^3 = 216 \bmod 10 = 6$
$7^3 = 343 \bmod 10 = 3$
$8^3 = 512 \bmod 10 = 2$
$9^3 = 729 \bmod 10 = 9$
$10^3 = 1000 \bmod 10 = 0$

All buckets will be filled uniformly.

(d) In option (d) all odd number of buckets will be empty

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6. (c)

Division method:-

- $h(k) = k \bmod m$
  $m = 2^P$
  $m = 2^4$
  $h(100) = 100 \bmod 2^4 = 100 \bmod 16 = 4$
  $h(128 + 4)$
  $= h(132) = 132 \bmod 16 = 4$
  132 = 1011100
  Last 4 bits is deciding the slot.
  A is not correct
- $h(k) = Lm\ (kA \cdot kA)$
  m = 100
  $A = \frac{1}{3}$
  $h(10) = \lfloor m(KA.\lfloor KA \rfloor) \rfloor$
  $h(11) = 66$
  $h(12) = 99$

B is wrong we will get only 3 values of h(k)

Optimal choice of a depends of keys themseleve

$$A = \frac{\sqrt{5-1}}{2}$$

- Malicious adversary can always choose keys. So, that all the keys are mapped to same slot $\Rightarrow$ O(n).

  Worst case retrieval time Set of uniform hash function Minimum collision Randomly hash function are picked.

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7. (a)

Number of slots = 100

m = 100

- First 3 slots must be empty.
- Number of insertions = 3

$$\text{Probability} = \frac{97}{100}\times\frac{97}{100}\times\frac{97}{100}$$

$$= \frac{97\times97\times97}{100\times100\times100}$$

Matched with option (a)

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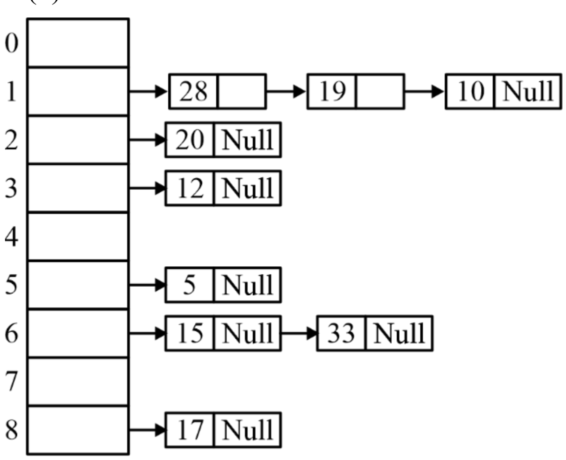
8. (a)

Number of slots = 9 [0 to 8]

m = 9

h (k) = k mod 9



h (5) = 5 mod 9 = 5

h (28) = 28 mod 9 = 1

h (19) = 19 mod 9 = 1

h (15) = 15 mod 9 = 6

h (20) = 20 mod 9 = 2

h (33) = 33 mod 9 = 6

h (12) = 12 mod 9 = 3

h (17) = 17 mod 9 = 8

h (10) = 5 mod 9 = 1

Maximum keys in a slot = 3

Minimum keys in a slot = 0

$$\text{Average keys in a slot} = \frac{9}{9} = 1$$

(3,0 and 1) is a right answer.

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9. (c)

Solve with the option

(a)

46 mod 10 = 6
42 mod 10 = 2
34 mod 10 = 4
52 mod 10 = $\cancel{2}$ = 3

0	
1	
2	42
3	52
4	34
5	
6	46
7	
8	
9	

Not matched with given Hash Table.

(b)

34 mod 10 = 4
42 mod 10 = 2
23 mod 10 = 3
52 mod 10 = $\cancel{2}\,\cancel{3}\,\cancel{4}$ = 5
38 mod 10 = $\cancel{3}\,\cancel{4}\,\cancel{5}$ = 6

0	
1	
2	42
3	23
4	34
5	52
6	33
7	
8	
9	

(c)

46 mod 10 = 6
34 mod 10 = 4
42 mod 10 = 2
23 mod 10 = 3
52 mod 10 = $\cancel{2}\,\cancel{3}\,\cancel{4}$ = 5
33 mod 10 = $\cancel{3}\,\cancel{4}\,\cancel{5}\,\cancel{6}$ = 7

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

Matched with given Hash Table

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10. (c)

- No collision for keys 46, 34, 42 and 23 to enter 52 in hash table total 6 possibilities. After insert 52 in table, to enter key 23 there are 5 possibilities.

Total number of possibilities = 6 × 5 = 30

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11. (c)

h (k) = k mod 10
h (12) = 12 mod 10
= 2
h (18) = 18 mod 10
= 8
h (13) = 13 mod 10
= 3
h (2) = 2 mod 10
= 2 $\cancel{3}$ = 4
h (3) = 3 mod 10
= $\cancel{3}\,\cancel{4}$ = 5
h (23) = 23 mod 10
= $\cancel{3}\,\cancel{4}\,\cancel{5}$ = 6
h (5) = 5 mod 10
= $\cancel{5}\,\cancel{6}$ = 7
h (15) = 15 mod 10
= $\cancel{5}\,\cancel{6}\,\cancel{7}\,\cancel{8}$ = 9

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

Match with option (c)

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12. (c)

h (k) = k mod 11
h (43) = 43 mod 11
= 10
h (36) = 36 mod 11
= 3
h (92) = 92 mod 11
= 4
h (87) = 87 mod 11
= $\cancel{10}$ = 0
h (11) = 11 mod 11
= $\cancel{0}$ = 1
h (4) = 4 mod 11
= $\cancel{4}$ = 5
h (71) = 71 mod 11
= $\cancel{5}$ = 6
h (13) = 13 mod 11
= 2
h (14) = 14 mod 11
= $\cancel{3}\,\cancel{4}\,\cancel{5}\,\cancel{6}$ = 7

0	87
1	11
2	13
3	36
4	92
5	4
6	71
7	
8	
9	43

Last record position = 7
Correct answer = (d)

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INDEX

Algorithms

Algorithms

Syllabus

Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques: greedy, dynamic programming and divide ‐ and ‐ conquer. Graph traversals, minimum spanning trees, shortest paths.

Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6
2008	6	4	2	3	8	3
2009	2	3	2	0	5	0
2010	1	2	0	4	0	0
2011	1	1	2	6	5	0
2012	1	2	0	2	1	1
2013	2	2	0	0	1	1
2014 (P1)	0	3	0	2	0	0
2014 (P2)	1	0	2	1	2	1
2014 (P3)	1	2	0	1	0	2
2015 (P1)	4	3	2	2	0	4
2015 (P2)	2	1	0	0	0	3
2015 (P3)	3	2	0	0	0	0
2016 (P1)	0	1	5	3	0	0
2016 (P2)	2	3	0	1	3	0
2017 (P1)	1	0	0	1	1	0
2017 (P2)	2	0	2	0	0	1
2018	0	2	2	2	4	0
2019	0	4	0	0	0	0
2020	1	2	2	1	0	5
2021 (P1)	4	3	1	2	0	0
2021 (P2)	0	3	2	1	0	3
2022	1	0	0	0	0	4
2023	4	0	0	2	0	0

CHAPTER

1 ASYMPTOTIC ANALYSIS

Asymptotic Notation

1. [MSQ] **[GATE-2023 : 2M]**

Let f and g be functions of natural numbers given by $f(n) = n$ and $g(n) = n^2$.

Which of the following statements is/are TRUE?

(a) $f \in O(g)$ (b) $f \in \Omega(g)$

(c) $f \in o(g)$ (d) $f \in \theta(g)$

2. [MSQ] **[GATE-2023 : 2M]**

Consider functions Function_1 and Function_2 expressed in pseudocode as follows:

```
Function_1
  while n > 1 do
     for i = 1 to n do
        x = x + 1;
     end for
     n = ⌊n/2⌋
  end while
```

```
Function_2
  for i = 1 to 100 ∗ n
do
        x = x + 1;
  end for
```

Let $f_1(n)$ and $f_2(n)$ denote the number of times the statement "x = x + 1" is executed in Function_1 and Function_2, respectively.

Which of the following statement is/are TRUE?

(a) $f_1(n) \in \Theta(f_2(n))$ (b) $f_1(n) \in o(f_2(n))$

(c) $f_1(n) \in \omega(f_2(n))$ (d) $f_1(n) \in O(n)$

3. [MCQ] **[GATE-2022 : 1M]**

Which one of the following statements is TRUE for all positive functions f(n)?

(a) $f(n^2) = \theta(f(n)2)$, when $f(n)$ is a polynomial.

(b) $f(n^2) = o(f(n)^2)$

(c) $f(n^2) = O(f(n)^2)$, when f(n) is an exponential function.

(d) $f(n^2) = \Omega(f(n)^2)$

4. [MCQ] **[GATE-2021 : 1M]**

Which of the given options provides the increasing order of asymptotic complexity of functions f_1, f_2, f_3 and f_4?

$F_1(n) = 2^n$ $F_2(n) = n^{3/2}$

$F_3(n) = n \log_2 n$ $F_4(n) = n^{\log 2\, n}$

(a) f_3, f_2, f_4, f_1 (b) f_3, f_2, f_1, f_4

(c) f_2, f_3, f_1, f_4 (d) f_2, f_3, f_4, f_1

5. [MCQ] **[GATE-2021 : 2M]**

Consider the following three functions:

$f_1 = 10^n$, $f_2 = n(^{\log n}$, $f_3 = n^{\sqrt{n}}$

Which one of the following options arranges the functions in the increasing order of asymptotic growth rate?

(a) f_3, f_2, f_1 (b) f_2, f_1, f_3

(c) f_1, f_2, f_3 (d) f_2, f_3, f_1

6. [MCQ] **[GATE-2017 : 1M]**

Consider the following functions from positive integers to real numbers:

$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$

The CORRECT arrangement of the above functions in increasing order of asymptotic complexity is:

(a) $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

(b) $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

(c) $10, \frac{100}{n}, \sqrt{n}, \log_2 n, n$

(d) $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

7. **[MCQ]** **[GATE-2015 : 2M]**
Let $f(n) = n$ and $g(n) = n^{(1 + \sin n)}$, where n is a positive integer. Which of the following statements is/are correct?
I: $f(n) = O(g(n))$ **II:** $f(n) = \Omega(g(n))$
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

8. **[MCQ]** **[GATE-2015 : 1M]**
Consider the equality $\sum_{i=0}^{n} i^3 = X$ and the following choices for X.
I: $\Theta(n^4)$ II: $\Theta(n^5)$
III: $O(n^5)$ IV: $\Omega(n^4)$
The equality above remains correct if X is replaced by
(a) Only I
(b) Only II
(c) I or III or IV but not II
(d) II or III or IV but not I

9. **[MCQ]** **[GATE-2015 : 2M]**
An algorithm performs $(\log N)^{1/2}$ find operations, N insert operation, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decrease-key operations on a set of data items with keys drawn from a linearly ordered set. For a delete operation, a pointer is provided to the record that must be deleted. For the decrease-key operation, a pointer is provided to the record that has its key decreased. Which one of the following data structures is the most suited for the algorithm to use, if the goal is to achieve the best total asymptotic complexity considering all the operations?
(a) Unsorted array
(b) Min – heap
(c) Sorted array
(d) Sorted doubly linked list

10. **[MCQ]** **[GATE-2012 : 1M]**
Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is ALWAYS TRUE?
(a) $A(n) = \Omega(W(n))$} (b) $A(n) = \Theta(W(n))$
(c) $A(n) = O(W(n))$ (d) $A(n) = o(W(n))$"

11. **[MCQ]** **[GATE-2008 : 2M]**
Consider the following functions:
- $f(n) = 2^n$
- f(n) n!
- $h(n) = n^{\log n}$

Which of the following statements about the asymptotic behaviour of f(n), g(n), and h(n) is true?
(a) $f(n) = O(g(n))$; $g(n) = O(h(n))$
(b) $f(n) = \Omega(g(n))$; $g(n) = O(h(n))$
(c) $g(n) = O(f(n))$; $h(n) = O(f(n))$
(d) $h(n) = O(f(n))$; $g(n) = \Omega(f(n))$

12. **[MCQ]** **[GATE-2008 : 2M]**
Arrange the followign functions in increasing asymptotic order:
A. $n^{1/3}$ **B.** e^n **C.** $n^{7/4}$
D. $n\log^9 n$ **E.** 1.0000001^n
(a) A, D, C, E, B (b) D, A, C, E, B
(c) A, C, D, E, B (d) A, C, D, B, E

Finding Time Complexity

13. **[MCQ]** **[GATE-2020 : 1M]**
What is the worst-case time complexity of inserting n elements into an empty linked list, if the linked list needs to be maintained in sorted order?
(a) $\Theta(n \log n)$ (b) $\Theta(n)$
(c) $\Theta(1)$ (d) $\Theta(n)^2$

14. **[MCQ]** **[GATE-2017 : 1M]**
Consider the following C function.
```
int fun (int n)
{   int i, j;
    for (i = 1; i < = n; i ++){
      for (j = 1; j < n; j + = i)
     {
      printf{"%d %d", i, j);
     }
    }
}
```

Time complexity of fun in terms of Θ notation is

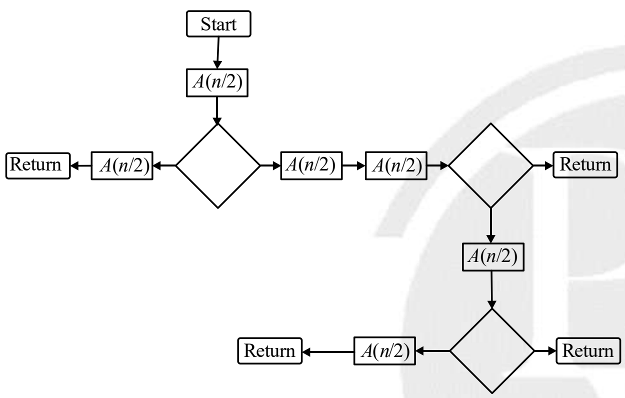
(a) $\Theta(n\sqrt{n})$ (b) $\Theta(n^2)$

(c) $\Theta(n \log n)$ (d) $\Theta(n^2 \log n)$

15. [NAT] [GATE-2016 : 2M]

The given diagram shows the flowchart for a recursive function A(n). Assume that all statements, except for the recursive calls, have O(1) time complexity. If the worst-case time complexity of this function is $O(n^\alpha)$, then the least possible value (accurate up to two decimal positions) of α is _____.

Flow chart for Recursive Function A(n)



16. [MCQ] [GATE-2015 : 2M]

Consider the following C function

```
int fun1 (int n)
{
    int i, j, k, p, q = 0;
    for (i = 1; i < n; ++i)
    {
      p = 0;
      for (j = n; j > 1; j = j /2)
          ++p;
      for (k =1; k < p; k = k*2)
          ++q;
    }
    return q;
}
```

Which one of the following most closely approximates the return value of the function fun1?

(a) n^3 (b) n(log n)2

(c) n log n (d) n log (log n)

17. [MCQ] [GATE-2013 : 2M]

Consider the following function:

```
int unknown (int n)
  {
    int i, j, k = 0;
    for (i = n/2; i < = n; i++)
        for(j = 2; j < = n; j = j*2)
            k = k + n/2;
    return(k);
  }
```

The return value of the function is ___.

(a) $\Theta(n^2)$ (b) $\Theta(n^2 \log n)$

(c) $\Theta(n)^3$ (d) $\Theta(n^3 \log n)$

18. [MCQ] [GATE-2010 : 1M]

Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001n^2$ time units and package B requires $10n\log_{10}n$ time units to process n records. what is the smallest value of k for which package B will be preferred over A?

(a) 12 (b) 10

(c) 6 (d) 5

19. [MCQ] [GATE-2009 : 2M]

The running time of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} T\left(\frac{n}{3}\right)^n & n \le 3 \\ +\,cm & \text{otherwise} \end{cases}$$

Which of the following represents the time complexity of the algorithm?

(a) $\Theta(n)$ (b) $\Theta(n \log n)$

(c) $\Theta(n)^2$ (d) $\Theta(n^2 \log n)$

Solving Recurrence Relation

20. [MCQ] [GATE-2021 : 2M]

Consider the following recurrence relation:

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right) + T\left(\frac{2n}{5}\right) + 7n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$$

Which one of the following options is correct?

(a) $T(n) = \theta(n \log n)$ (b) $T(n) = \theta(n^{5/2})$

(c) $T(n) = \theta(n)$ (d) $T(n) = \theta(\log n)^{5/2}$

21. [MCQ] [GATE-2020 : 1M]

For parameters a and b, both of which are $\omega(1)$, $T(n) = T(n^{1/a}) + 1$, and $T(b) = 1$. Then T(n) is

(a) $\Theta(\log_2 \log_2 n)$ (b) $\Theta(\log_a \log_b n)$

(c) $\Theta(\log_b \log_a n)$ (d) $\Theta(\log_{ab} n)$

22. [MCQ] [GATE-2017 : 1M]

Consider the recurrence function

$$T_{(n)} = \begin{cases} 2T(\sqrt{n})+1 & n > 2 \\ 2 & 0 < n \le 2 \end{cases}$$

Then T(n) in terms of Θ notation is?

(a) $\Theta(\log \log n)$ (b) $\Theta(\log n)$

(c) $\Theta(\sqrt{n})$ (d) $\Theta(n)$

23. [MCQ] [GATE-2014 : 1M]

Which one of the following correctly determines the solution of the recurrence relation with T(1) = 1?

$$T(n) = \left\{ 2T\left(\frac{n}{2}\right) + \log n \right.$$

(a) $\Theta(n)$ (b) $\Theta(n \log n)$

(c) $\Theta(n^2)$ (d) $\Theta(\log)$

24. [MCQ] [GATE-2008 : 2M]

When $n = 2^{2k}$ for some $k \ge 0$, the recurrence relation $T(n)\ \sqrt{2}T(n/2) + \sqrt{n}, T(1) = 1$ evaluated to:

(a) $\sqrt{n}(\log n + 1)$

(b) $\sqrt{n}\log n$

(c) $\sqrt{n}\log\sqrt{n}$

(d) $n\log\sqrt{n}$

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ANSWER KEY

1.	(a, c)	**2.**	(a, d)	**3.**	(a)	**4.**	(a)
5.	(d)	**6.**	(b)	**7.**	(d)	**8.**	(c)
9.	(a)	**10.**	(c)	**11.**	(d)	**12.**	(a)
13.	(d)	**14.**	(c)	**15.**	(2.2 to 2.4)	**16.**	(d)
17.	(b)	**18.**	(c)	**19.**	(a)	**20.**	(c)
21.	(b)	**22.**	(b)	**23.**	(a)	**24.**	(a)

SOLUTIONS

1. **(a, c)**

Given f is 'n' and g is "n^2" we can conclude that f(n) = O(g) that is f(n) can be smaller than or equal to O(g).

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2. **(a, d)**

```
Function_1                  | Function_2
  while n > 1 do            |   for i = 1 to 100 * n
    for i = 1 to n do       |   do
        x = x + 1;          |       x = x + 1;
    end for                 |   end for
  end while                 |
```

$f_2(n) = O(n)$

$$f_1(n) = n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \ldots \frac{n}{2^k}$$

$$= n\left[\sum_{i=0}^{k} \frac{1}{2}i\right] = n\left[1 - \frac{1}{2^k}\right]$$

$$= n - \frac{n}{2^k} = n - 1 = O(n)$$

$$\boxed{f_1(n) = O(n); f_2(n) = O(n)}$$

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3. **(a)**

$f(n^2) = \theta((f(n)^2)$

$f(n) = n$

$f(n^2) = n^2$

$(f(n))^2 = n^2$

$f(n) = 2^n$, $(f(n))^2 = (2^n)^2 = 2^{2n}$

$f(n^2) = f\left(n^2\right) = 2^{n^2}; 2^{2n}$

n^2; $2n.\log_2$

$n^2 > 2n.$

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4. **(a)**

$n. \log < n^{3/2} < n^{\log n} < 2^n$

$\boxed{f_3 < f_2 < f_4 < f_1}$

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5. **(d)**

$10^n; n^{\log n}; n^{\sqrt{n}}$

$n.\log_{10}; (\log n)^2; \sqrt{n}.\log n$

$f_2 < f_3 < f_1$

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6. **(b)**

$\sqrt{n}$ grows faster than log and slower than linear whereas n grows linearly and $\frac{100}{n}$ decreases with n.

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7. **(d)**

$f(n) = n; g(n) = n^{1+\sin n}$

sin value ranges from –1 to +1 (using trichotomy property)

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8. **(c)**

$$\sum i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$= O(n^4)$

$= \Omega(n^4)$

$= \Theta(n^4)$

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9. **(a)**

1. Find: $\sqrt{\log n}$;
2. Insert: N
3. Delete: $\sqrt{\log n}$
4. Decrement key: $\sqrt{\log n}$

Data Structure	FIND	INSERT	DELETE	DEC – KEY	Total time
Unsorted Array	$N.\sqrt{\log n}$	N	$\sqrt{\log n}$	$\sqrt{\log n}$	$N.\sqrt{\log n}$
Min-Heap	$N.\sqrt{\log n}$	N.log N	$(\log n)^{3/2}$	$(\log n)^{3/2}$	N.log N
Sorted Array	$(\log n)^{3/2}$	N^2	$N.\sqrt{\log n}$	$N.\sqrt{\log n}$	N^2
Sorted D.L.L	$N.\sqrt{\log n}$	N^2	$\sqrt{\log n}$	$N.\sqrt{\log n}$	N^2

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10. **(c)**

$B(n) \le A(n) \le W(n)$

$\therefore A(n) = O(W(n))$

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11. **(d)**

2^n	n^n	$n^{\log n}$
$n.\log_2 2$	$n\log n$	$(\log n)^2$
n	$n\log n$	$(\log n)^2$

Therefore: $\boxed{h < f < g}$

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12. (a)

For the given scenario we can observe that:

- $n^{1/3} < n^{7/4}$ and $n^{1/3} < n\log^9 n$
- $1.0000001^n < e^n$ and $1.0000001^n < n^{7/4}$
- $n\log^9 n < 1.0000001^n$
- $n^{7/4}$ is asymptotically larger than $n\log^9 n$. (If we take n common from both the functions then $n^{3/4}$ is larger than $\log^9 n$).
- Thus, $n^{1/3} < n\log^9 n < n^{7/4} < 1.0000001^n < e^n$.

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13. (d)

Total comparision's: $(1 + 2 + 3 + \ldots + n - 1)$

$$: \frac{n(n-1)}{2}$$
$$: O(n^2)$$
$$: \Omega(n^2)$$
$$: \Theta\,(n^2)$$

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14. (c)

i = 1, n, +1

j =1, n, +i

i = 1, i = 2, i = 3

$$j = 1,\ n,\ j = \frac{n}{2},\ j = \frac{n}{3}$$

$$n + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{n}$$

$$n\left[\sum_{x=1}^{n} \frac{1}{x}\right]$$

$T(n) = n.\log n = \Theta(n.\log n)$

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15. (2.2 to 2.4)

$T(n) = 5.T(n/2) + C$

First check C is it $O\left(n^{\log_2 5-\epsilon}\right)$, yes

$\therefore T(n)$ is $\Theta\left(n^{\log_2 5}\right) = \Theta\left(n^{2.32}\right)$

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16. (d)

$[\log \log n + \log \log n + \ldots +]^n$

return(q) = n.log $\log_n$

for (i = 1; i < n; ++i): n

for (j = n; j > 1; j = j /2): log n

for (k =1; k < p; k = k*2): log p(log log n)

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17. (b)

- for (i = n/2; i < = n; i++) $: \frac{n}{2}$
- for(j = 2; j < = n; j = j*2)

  k = k + n/2; $: \frac{n}{2}\log n$

Overall Complexity: $\frac{n}{2}\left[\frac{n}{2}.\log n\right]$.

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18. (c)

$10.n.\log_{10} n < 0.0001n^2$

$10.\cancel{10^k}.k < 10^{-4}.10^{\cancel{2}k}$

$$k < \frac{10^k}{10^5} \Rightarrow k = 6 \Rightarrow 6 < \frac{10^6}{10^5} < 10$$

$\therefore k = 6$

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19. (a)

$T(n) = T(n/3) + cn$

$a = 1;\ b = 3;\ f(n) = cn$

Master method:

I. cn is $O(n^{0-\epsilon})$, dissatified

II. cn is $\Theta(n^0 \cdot \log^k n)$, dissatified

III. cn is it $\Omega(n^{0+\epsilon})$ fn $\in = 1$

a. $f(n/b) \le \delta.f(n)$ satisfied

$1 \cdot \frac{cn}{3} \le \delta.cn;\ \delta = \frac{1}{3} < 1$

$\therefore$ T(n) is $\Theta(f(n))$

$\Theta(n)$

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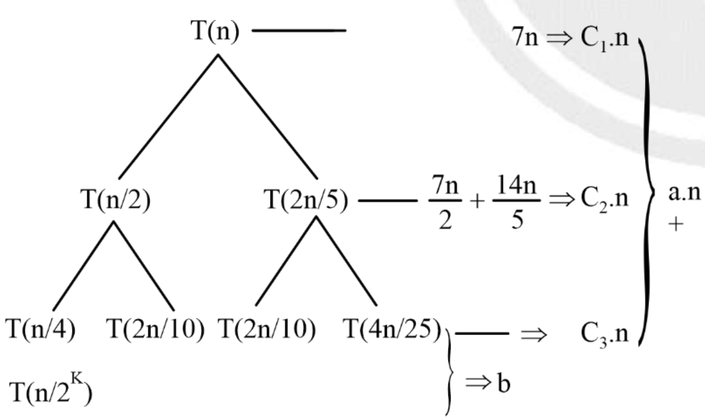
20. (c)

$T(n) = T(n/2) + T(2n/5) + 7n,\ n > 1$

$T(n/2) = T(n/4) + T(2n/10) + 7n/2$

$T(2n/5) = T(2n/10) + T(4n/25) + 7.2n/5$

Recursive Tree



Total time = an + b = O(n) = $\Omega(n) = \theta(n)$.

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21. (b)

$T(n) = T(n^{1/a}) + 1;\ T(b) = 1$

$T(n) = T(n^{1/a}) + 1$ …(1)

$T\left(n^{1/a}\right) = T\left(n^{1/a^2}\right) + 1$ …(2)

$T(n) = T\left(n^{1/a^2}\right) + 2$ …(3)

$= T\left(n^{1/a^3}\right) + 3$ …(4)

$= T\left(n^{1/a^K}\right) + K$ …(5)

$= T(b) + \log_a(\log_b n)$

$= 1 + \log_a . \log_b n$

$n^{1/a^K} = b$

$\frac{1}{a^K}.\log_2 n = \log_2 b$

$\frac{\log_2 n}{\log_2 b} = a^K \Rightarrow a^K = \log_b n$

$\therefore K = \log_a \log_b n$

$T(n) = \theta(\log_a . \log_b n)$

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22. (b)

$T(n) = 2.T\left(\sqrt{n}\right) + 1$

$T(n) = 2.T(n^{1/2}) + 1$ …(1)

$T(n^{1/2}) = 2.T(n^{1/4}) + 1$ …(2)

$T(n) = 2[2.T(n^{1/4}) + 1] + 1$

$= 4.T(n^{1/4}) + 3$ …(3)

$= 2^2.T\left(n^{1/2^2}\right) + \left(2^2 - 1\right)$ …(3)

$= 2^K.T\left(n^{1/2^K}\right) + \left(2^K - 1\right)$ …(4)

$n^{1/2^K} = 2$

$\frac{1}{2^K}.\log n = \log_2 2$

$2^K = \log n$

$T(n) = \log n.T(2) + \log n - 1$

$T(n) = 2.\log n + \log n - 1$

$T(n) = \Theta(\log n)$

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23. **(a)**

$a = 2;\ b = 2;\ f(n) = \log n$

$\log n$ is it $O(n^{1-\in})$, yes, here $\in = 0.5$

$\therefore$ T(n) is $\Theta(n)$

Scan for Video solution

24. **(a)**

$n = 2^{2k};\ k \geq 0$

$T(n) = \sqrt{2}\cdot\ T\left(\frac{n}{2}\right) + \sqrt{n}\ ;\ T(1) = 1$

$$T(n) = 2^{1/2}\cdot T\left(\frac{n}{2}\right) + n^{1/2} \quad \text{....(1)}$$

$$T\left(\frac{n}{2}\right) = 2^{1/2}\cdot\ T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^{1/2} \quad \text{....(2)}$$

$$T\left(\frac{n}{4}\right) = 2^{1/2}\cdot\ T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^{1/2}$$

$$T(n) = 2^{1/2}\left[2^{1/2}\cdot T\left(\frac{n}{4}\right) + \frac{n^{1/2}}{2^{1/2}}\right] + n^{1/2}$$

$$= \boxed{2^1.T\left(\frac{n}{4}\right) + 2\sqrt{n} + 2^1\sqrt{n}} \quad \text{.....(3)}$$

$$= 2\left[2^{1/2}\cdot T\left(\frac{n}{8}\right) + \frac{n^{1/2}}{2}\right] + 2\sqrt{n}$$

$$= 2^{3/2}\cdot T\left(\frac{n}{8}\right) + 3\sqrt{n} \quad \text{.....(4)}$$

$$T(n) = 2^2.T\left(\frac{n}{16}\right) + 4\sqrt{n} \quad \text{.....(5)}$$

$$= 2^2\cdot\ T\left(\frac{n}{2^{2.2}}\right) + 2.2\sqrt{n}$$

$$= 2^k\cdot\ T\left(\frac{n}{2^{2.k}}\right) + 2.k\sqrt{n} \quad \text{......(6)}$$

$$\frac{n}{2^{2k}} = 1$$

$$\Rightarrow n = 2^{2k}$$

$$2k = \log n$$

$$\Rightarrow k = \frac{\log n}{2}$$

$$= \left(2^{\log_2 n}\right)^{1/2}.T(1) + \cancel{2}.\frac{\log n}{\cancel{2}}.\sqrt{n}$$

$$\sqrt{n}\cdot 1 + \sqrt{n}\cdot \log n$$

$$\boxed{T(n) = \sqrt{n}\,(1 + \log n)}$$

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CHAPTER

2 DIVIDE AND CONQUER

Basics of Divide and Conquer

1. [MCQ] [GATE-2021 : 1M]

Let P be an array containing n integers. Let t be the lowest upper bound on the number of comparisons of the array elements, required to find the minimum and maximum values in an arbitrary array of n elements. Which one of the following choices is correct?

(a) $t > n$ and $t \le 3\left\lceil \frac{n}{2} \right\rceil$

(b) $t > n\left\lceil \frac{n}{2} \right\rceil$ and $t \le 2n-2$

(c) $t > 2n - 2$

(d) $t > \lceil \log_2(n) \rceil$ and $t \le n$

2. [MCQ] [GATE-2021 : 2M]

For Constants $a \ge 1$ and $b > 1$, consider the following recurrence defined on the non-negative integers:

$$T(n) = a.T\left(\frac{n}{b}\right) + f(n)$$

Which one of the following options is correct about the recurrence T(n)?

(a) If $f(n)$ is $\Theta(n^{\log_b(a)})$ then $T(n)$ is $\Theta\left(n^{\log_b(a)}\right)$

(b) If $f(n)$ is $O(n^{\log_b(a)-e})$ for some $e > 0$, then $T(n)$ is $\Theta\left(n^{\log_b(a)}\right)$

(c) If $f(n)$ is $\frac{n}{\log_2(n)}$, then $T(n)$ is $\Theta(n \log_2(n))$

(d) If f(n) is $n\log_2(n)$, then T(n) is $\Theta(n \log_2(n))$

3. [MCQ] [GATE-2019 : 2M]

There are n unsorted arrays: $A_1, A_2, \ldots, A_n$. Assume that n is odd. Each of $A_1, A_2, \ldots, A_n$ contains n distinct elements. There are no common elements between any two arrays. The worst-case time complexity of computing the median of the medians of $A_1, A_2, \ldots$, An is

(a) O(n log n)
(b) $O(n^2)$
(c) O(n)
(d) $\Omega(n^2 \log n)$

4. [MCQ] [GATE-2008 : 2M]

The minimum number of comparisons required to determine if an integer appears more than n/2 times in a sorted array of n integers is

(a) $\Theta(n)$
(b) $\Theta(\log n)$
(c) $\Theta(\log^* n)$
(d) $\Theta(1)$

Maximum Subarray Problem

5. [NAT] [GATE-2019 : 1M]

Consider a sequence of 14 elements: A = [–5, –10, 6, 3, –1, –2, 13, 4, –9, –1, 4, 12, –3, 0]. The subsequence sum $S(i, j) = \sum_{k=i}^{j} A[k]$. Determine the maximum of S(i, j), where $0 \le i \le j < 14$. (Divide and conquer approach may be used.)

6. [MCQ] [GATE-2015 : 1M]

An unordered list contains n distinct elements. The number of comparisons to find an element in this list that is neither maximum nor minimum is?

(a) $\Theta(n \log n)$
(b) $\Theta(n)$
(c) $\Theta(\log n)$
(d) $\Theta(1)$

7. **[NAT] [GATE-2014: 2M]**
The minimum number of comparisons required to find the minimum and the maximum of 100 numbers is _______.

8. **[MCQ] [GATE-2010 : 2M]**
The weight of a sequence $a_0, a_1, ..., a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + ... a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, ... a_{n-1}$ and Y the maximum possible weight of subsequence of $a_1, a_2, ..., a_{n-1}$. Then X is equal to
(a) $\max(Y, a_0 + Y)$
(b) $\max(Y, a_0 + Y/2)$
(c) $\max(Y, a_0 + 2Y)$
(d) $a_0 + Y/2$

Quick Sort

9. **[NAT] [GATE-2019 : 1M]**
An array of 25 distinct elements is to be sorted using quicksort. Assume that the pivot element is chosen uniformly at random. The probability that the pivot element gets placed in the worst possible location in the first round of partitioning (rounded off to 2 decimal places) is_____.

10. **[MCQ] [GATE-2015 : 1M]**
Which one of the following is the recurrence equation for the worst-case time complexity of the Quicksort algorithm for sorting n (≥ 2) numbers? In the recurrence equations given in the option below, c is a constant.
(a) $T(n) = 2T(n/2) + cn$
(b) $T(n) = T(n - 1) + T(1) + cn$
(c) $T(n) = 2T(n - 1) + cn$
(d) $T(n) = T(n/2) + cn$

11. **[MCQ] [GATE-2014 : 1M]**
You have an array of n elements. Suppose you implement quicksort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst-case performance is
(a) $O(n^2)$
(b) $O(n\log n)$
(c) $\Theta(n\log n)$
(d) $O(n^3)$

12. **[MCQ] [GATE-2014 : 1M]**
Let P be a quicksort program to sort numbers in ascending order using the first element as the pivot. Let t_1 and t_2 be the number of comparisons made by P for the inputs [1 2 3 4 5] and [4 1 5 3 2] respectively. Which one of the following holds?
(a) $t_1 = 5$
(b) $t_1 < t_2$
(c) $t_1 > t_2$
(d) $t_1 = t_2$

13. **[MCQ] [GATE-2009 : 2M]**
In quick sort, for sorting n elements, the $(n/4)^{th}$ smallest element is selected as pivot using an O(n) time algorithm. What is the worst case time complexity of the quick sort?
(a) $\theta(n)$
(b) $\theta(n \log n)$
(c) $\theta(n^2)$
(d) $\theta(n^2 \log n)$

14. **[MCQ] [GATE-2008 : 2M]**
Consider the Quicksort algorithm. Suppose there is a procedure for finding a pivot element which splits the list into two sub-lists each of which contains at least one-fifth of the elements. Let T(n) be the number of comparisons required to sort n elements. Then
(a) $T(n) \leq 2T(n / 5) + n$
(b) $T(n) \leq T(n/5) + T(4n/5) + n$
(c) $T(n) \leq 2T(4n/5) + n$
(d) $T(n) \leq 2T(n/2) + n$

Merge Sort

15. **[MCQ] [GATE-2015 : 1M]**
Assume that a merge sort algorithm in the worst case takes 30 seconds for an input of size 64. Which of the following most closely approximates the maximum input size of a problem that can be solved in 6 minutes?
(a) 256
(b) 512
(c) 1024
(d) 2048

16. [MCQ] [GATE-2012 : 2M]

A list of n strings, each of length n, is sorted into lexicographic order using the merge sort algorithm The worst-case running time of this computation is

(a) $O(n \log n)$ (b) $O(n^2 \log n)$

(c) $O(n^2 + \log n)$ (d) $O(n^2)$

Insertion Sort

17. [MCQ] [GATE-2021 : 1M]

Consider the following array:

23	32	45	69	72	73	89	97

Which algorithm out of the following options uses the least number of comparisons (among the array elements) to sort the above array in ascending order?

(a) Quicksort using the last element as pivot

(b) Selection Sort

(c) Mergesort

(d) Insertion sort

18. [MCQ] [GATE-2016 : 1M]

The worst-case running times of Insertion sort, Merge sort and Quick sort, respectively, are:

(a) $\Theta(n \log n)$, $\Theta(n \log n)$, and $\Theta(n^2)$

(b) $\Theta(n^2)$, $\Theta(n^2)$ and $\Theta(n \log n)$

(c) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n \log n)$

(d) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n^2)$

Bubble Sort

19. [MCQ] [GATE-2016 : 1M]

Assume that the algorithms considered here sorts the input sequences in ascending order. If the input is already in ascending order, which of the following are TRUE?

I: Quick sort runs in $\Theta(n^2)$ time

II: Bubble sort runs in $\Theta(n^2)$ time

III: Merge sort runs in $\Theta(n)$ time

IV: Insertion sort runs in $\Theta(n)$ time

(a) I and II only (b) I and III only

(c) II and IV only (d) I and IV only

Selection Sort

20. [MCQ] [GATE-2013 : 1M]

Which one of the following is the tightest upper bound that represents the number of swaps required to sort n numbers using selection sort?

(a) $O(\log n)$ (b) $O(n)$

(c) $O(n \log n)$ (d) $O(n^2)$

21. [MCQ] [GATE-2009 : 1M]

What is the number of swaps required to sort n elements using selection sort, in the worst case?

(a) $\Theta(n)$ (b) $\Theta(n \log n)$

(c) $\Theta(n)^2$ (d) $\Theta(n^2 \log n)$

Heap Sort

22. [MCQ] [GATE-2021 : 1M]

We have a binary heap on n elements and wish to insert n more elements (not necessarily one after another) into this heap. The total time require for this is

(a) $\Theta(\log n)$ (b) $\Theta(n)$

(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

23. [MCQ] [GATE-2021 : 1M]

Let H be a binary min-heap consistiong of n elements implemented as an array. What is the worst-case time complexity of an optimal algorthm to find the maximum elemets in H?

(a) $\Theta(1)$ (b) $\Theta(\log n)$

(c) $\Theta(n)$ (d) $\Theta(n \log n)$

24. [NAT] [GATE-2020 : 2M]

Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is _____.

25. [NAT] [GATE-2018 : 2M]

The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6 7} exactly once is ________.

26. [NAT] [GATE-2016 : 2M]

A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is ______.

27. [MCQ] [GATE-2015 : 1M]

Consider the following array of elements <89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100>. The minimum number of interchanges needed to convert it into a max-heap is

(a) 4 (b) 5

(c) 2 (d) 3

28. [MCQ] [GATE-2015 : 2M]

Consider a max heap, represented by the array:
40, 30, 20, 10, 15, 16, 17, 8, 4

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

(a) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35

(b) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15

(c) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15

(d) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

29. [MCQ] [GATE-2013: 1M]

The number of elements that can be sorted in $\Theta(\log n)$ time using heap sort is

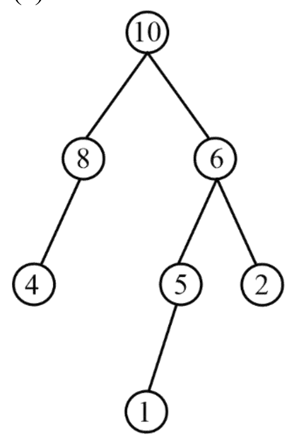
(a) $\Theta(1)$ (b) $\Theta\sqrt{\log n}$

(c) $\Theta\left(\frac{\log n}{\log \log n}\right)$ (d) $\Theta(\log n)$

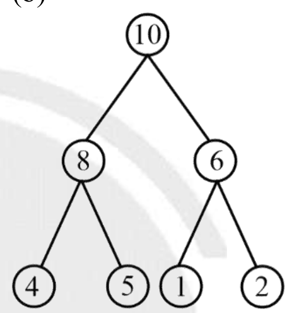
30. [MCQ] [GATE-2011 : 1M]

A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?

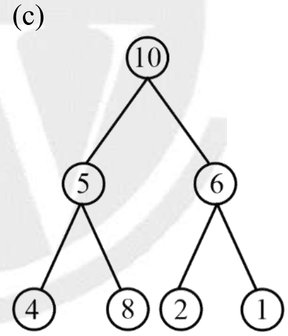
(a)



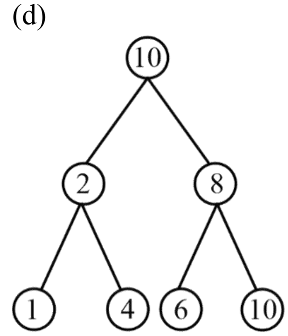
(b)



(c)



(d)



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ANSWER KEY

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(c)
5.	(29 to 29)	**6.**	(d)	**7.**	(148 to 148)	**8.**	(b)
9.	(0.08 to 0.08)	**10.**	(b)	**11.**	(a)	**12.**	(c)
13.	(b)	**14.**	(b)	**15.**	(b)	**16.**	(b)
17.	(d)	**18.**	(d)	**19.**	(d)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(c)	**24**	(511 to 511)
25.	(80 to 80)	**26.**	(8 to 8)	**27.**	(d)	**28.**	(b)
29.	(c)	**30.**	(b)				

SOLUTIONS

1. **(a)**

$$\boxed{\begin{aligned} T(n) &= 2T\left(\frac{n}{2}\right) + 2, \quad n > 2 \\ &= 1, \ n = 2 \end{aligned}}$$

$$\boxed{T(n) = \frac{3n}{2} - 2}$$

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2. **(b)**

According to standard case of master's theorem option (b) is correct.

Master Theorem

Let a ≥ 1 and b > 1 be constants, let f(n) be a function, and let T(n) be defined on the non-negative integers by the recurrence

$$T(n) = aT(n/b) + f(n),$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lfloor n/b \rfloor$. Then T(n) has the following asymptotic bounds:

1. If $f(n) = \left(n^{\log_b a - \in}\right)$ for some constant ε > 0, then $T(n) = \Theta\left(n^{\log_b a}\right)$.
2. If $f(n) = \Theta\left(n^{\log_b a}\right)$, then $T(n) = \Theta\left(n^{\log_b a} \log n\right)$.
3. If $f(n) = \Omega\left(n^{\log_b a + c}\right)$ for some constant ε > 0, and if f(n/b) ≤ cf(n) for some constant c < 1 and all sufficiently large n, then T(n) = Θ(f(n)).

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3. **(b)**

- To compute the median of an array (n) = O(n)
- To compute All medians of n – array(n)
  = n × O(n)
  = $O(n^2)$
- To compute median of all n-medians = O(n)

$$\boxed{\text{Total time } = O\left(n^2\right) + O(n) = O\left(n^2\right)}$$

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4. **(c)**

	1	2	3	4	5	6
A	1	2	2	2	2	5

Algorithm:

Majority element (A, n, x) $\rightarrow$ O(logn)

{

1. i = first occurrence of x in A.
   from left, using modified binary search
2. if $\left(A[i] = A\left[\frac{n}{2}+i\right]\right)$ them
   print(A[i] is majority);
   else
   print (No);

}

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5. **(29 to 29)**

[Maximum sum sub-array]

[By Kadane's algorithm]

S(2, 11) = (6 + 3 – 1 – 2 + 13 + 4 – 9 – 1 + 4 + 12)

= 29

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6. **(d)**

	1	2	3	4	5	6
A:	10	5	8	12	3	9

Consider first three elements: 10, 5, 8 from the array, the element that is neither maximum nor minimum is 8. The time required to find it requires only three comparisons that is constant time complexity.

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7. **(148 to 148)**

Maximum – Minimum(Using Divide and Conquer strategy) : $\left(\frac{3n}{2}-2\right)$

$T(n) = 2T\left(\frac{n}{2}\right) + 2, n > 2$

$= 1, \quad n = 2$

$= 0, \quad n = 1$

$\frac{3\times100}{2} - 2$

: 3 × 50 – 2 = 150 – 2 = 148

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8. **(b)**

$[a_0, a_1 \ldots.. a_{n-1}] \Rightarrow x = a_0 + \frac{a_1}{2} + \frac{a_2}{4} + \ldots\ldots + \frac{a_{n-1}}{2^{n-1}}$

$[a_1, a_2 \ldots.. a_{n-1}] \Rightarrow y = a_1 + \frac{a_2}{2} + \frac{a_3}{4} + \ldots\ldots + \frac{a_{n-1}}{2^{n-1}}$

$\boxed{X = \max\left(y,\ a_0 + \frac{y}{2}\right)}$

$a_0 + \frac{a_1}{2} + \frac{a_2}{4} + \ldots. + \frac{a_{n-1}}{2^{n-1}}$

$a_0 + \frac{a_1}{2} + \frac{a_2}{4} + \ldots. \left(a_0 + \frac{4}{2}\right)$

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9. **(0.08 to 0.08)**

In the worst case the pivot can get fixed up at either the first or last position;

$\therefore \boxed{\text{Probability } = \frac{1}{25} + \frac{1}{25} = \frac{2}{25} = 0.08}$

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10. **(b)**

For worst case scenario, the pivot should get fixed either at the first or at the last position.

$\boxed{T(n) = T(n-1) + O(n) + C}$

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11. (a)

The pivot can get fixed up at the first or last place in the list at all levels leading to the worst case behaviour = $O(n^2)$.

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12. (c)

$[1\ 2\ 3\ 4\ 5]$
$\downarrow$ $\quad$ t_1 is $O(n^2)$

$\frac{\text{Sorted list}}{t_1}$

$[4\ 1\ 5\ 3\ 2]$
$\downarrow$ $\quad$ t_2 is $O(n \log n)$

$\frac{\text{Unsorted list}}{t_2}$

$O(n^2) > O(n \log n)$

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13. (b)

$$\boxed{T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + O(n)}$$

$$\boxed{T(n) = O(n) + O(n) + T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right)}$$

By recursion tree method = $\Theta(n\log n)$

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14. (b)

In the given scenario the list is divided into two sublists with n/5 and 4n/5 elements and n is common so the answer is $T(n) \le T(n/5) + T(4n/5) + n$.

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15. (b)

30 sec $\rightarrow$ 64 elements

6 minutes $\rightarrow$ n = ?

Time for merge sort = n·logn

= 64·log64

64.6 units

30 sec – 64.6 units

? – 1 unit

$$\boxed{1 \text{ unit} = \frac{30}{64 \times 6} \text{s}}$$

$$1 \text{ unit} = \frac{30}{64 \times 6} \text{s}$$

(n) ? – 360 s

$$\frac{{}^{12}\cancel{360} \times 64 \times 6}{30} \text{unit} \_\_\_ \text{'n' elements}$$

$$\boxed{12 \times 6 \times 64 \_\_\_\_ n \cdot \log_2 n}$$

4608 = n·logn

$$\boxed{n = 512 = 512 \times 9 = 4608}$$

$\therefore$ n = 512

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16. **(b)**

Merg sort (n) = n log n

(n log n) n = n^2 log n

a	b	x	y
p	q	r	t
l	m	c	d
y	z	k	q

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17. **(d)**

Quicksort using the last element as pivot = n^2

Selection Sort = n^2

Mergesort = n log n

Insertion sort = n

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18. **(d)**

- For insertion sort, we need to run two loops both of order n, so worst case time complexity is $\Theta(n^2)$.
- Merge sort takes Θ(n log n) in all cases.
- Quick sort takes $\Theta(n^2)$ in worst case if the array is sorted or almost sorted.

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19. **(d)**

- If the input is already sorted then running quick sort will give worst case time complexity of $\Theta(n^2)$.
- Bubble sort runs in constant time complexity.
- Merge sort runs in Θ(n log n).
- Insertion sort will run in $\Theta(n)$ time only.

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20. **(b)**

In selection sort the maximum number of swaps can be of order of n that is selecting the smallest element among all other elements and placing it to the correct position.

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21. **(a)**

Considering the worst case, there will be one swap in each loop except the last one, that is n-1 swaps for 1 to n. Hence the number of swaps in the worst case is Θ(n).

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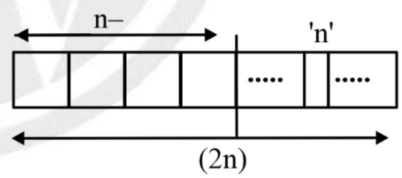
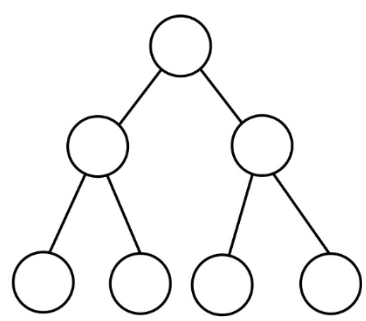
22. **(b)**

Heap with 'n' element





n more elements

Heapify method / Build heap

$\boxed{\text{Time: } \Theta(2n) = \Theta(n)}$

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23. **(c)**

- Maximum element in a heap lies at the leaf-level.
- Maximum of n/2 elements; $\boxed{O\left(\frac{n}{2}\right) = \Theta(n)}$

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24. **(511 to 511)**

Maximum will lie at the leaf.

Number of nodes at leaf $= \left\lceil \frac{1023}{2} \right\rceil = 511.5 = 512$

Number of comparisons needed = 512 – 1 = 511

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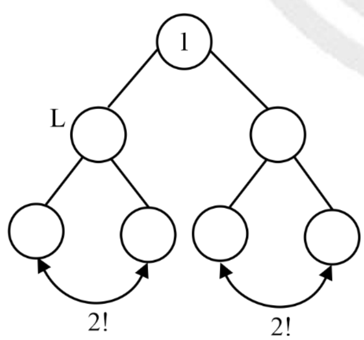
25. **(80 to 80)**

- Left subtree heap: ${}^6C_3$
- Left and Right Nodes



Can be arranged in 2! way

$\Rightarrow$ Total so far: $({}^6C_3 * 2!)$

Total number of possibilities = $(6C_3)$ * (2!) * (2!)

$= \frac{\cancel{6} \times 5 \times 4}{\cancel{3 \times 2}} \times 2 \times 2 = 20 * 2 * 2 = 80.$

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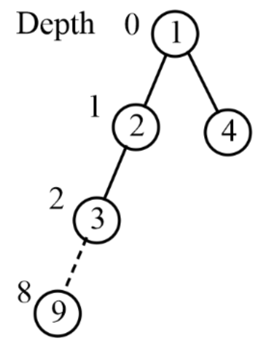
26. **(8 to 8)**



The maximum depth at which integer 9 can appear is 8 for the given condition.

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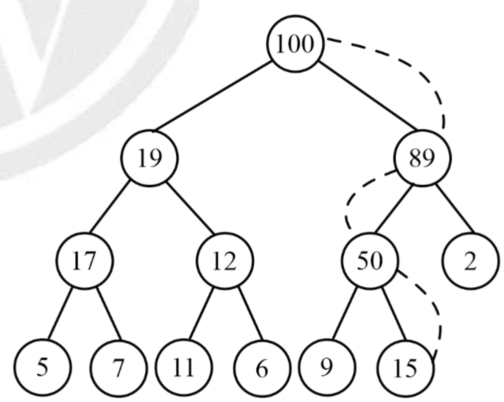
27. **(d)**



The minimum number of interchanges needed to convert it into a max heap = 1 + 1 + 1 = 3.

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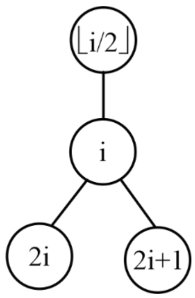
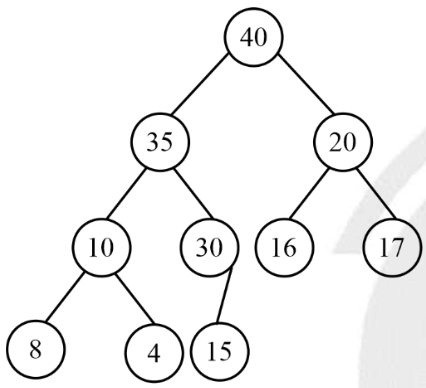
28. (b)

A:

40	30	20	10	15	16	17	8	4
1	2	3	4	5	6	7	8	9



A: 40, 35, 20, 10, 30, 16, 17, 8, 4, 15



Max-Heap

Scan for Video solution

29. (c)

Time of heap sort: O(n log n)

$$O\left(\sqrt{\log n}\,\log\log n\right)\times O\left(\log n\log\log n\right)$$

Time of heap sort = n.log n

$$\text{Time} = \frac{\log n}{\log\log n}\times\left(\log\left(\frac{\log n}{\log\log n}\right)\right)$$

$$= \frac{\log n}{\log n}\times\left(\log\log n - \log\log\log n\right)$$

$$= \boxed{\log n - \frac{\log n}{\log\log n}\times\log\log\log n = \Theta\left(\log n\right)}$$

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30. (b)

The max heap property is only satisfied by option (b), as in option (a) the heap is not a complete binary tree or an almost complete binary tree.

In option (c) max heap property is not satisfied as node value 8 is the leaf node and a child of a value that is smaller than itself, that is 5.

In option (d) max heap property is violated as node value 10 is in leaf node and a child of a value that is smaller than itself.

However, in option (b) the max heap property is satisfied.

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CHAPTER

3 GREEDY TECHNIQUES

Basics of Greedy

1. [NAT] [GATE-2014 : 2M]

Suppose P, Q, R, S, T are sorted sequences having lengths 20, 24, 30, 35, 50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is______.

Huffman Code Problem

2. [MSQ] [GATE-2021: 2M]

Consider the string abbccddeee. Each letter in the string must be assigned a binary code satisfying the following properties:

1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

Among the set of all binary code assignments which satisfy the above two properties, what is the minimum length of the encoded string?

(a) 23 (b) 25

(c) 21 (d) 30

3. [NAT] [GATE-2017 : 2M]

A message is made up entirely of characters from the set X = {P, Q, R, S, T}. The table of probabilities for each of the characters is shown below:

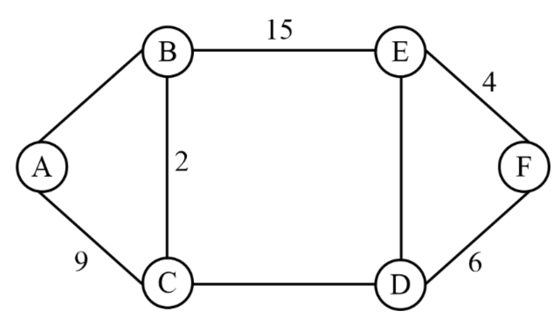
Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is _____.

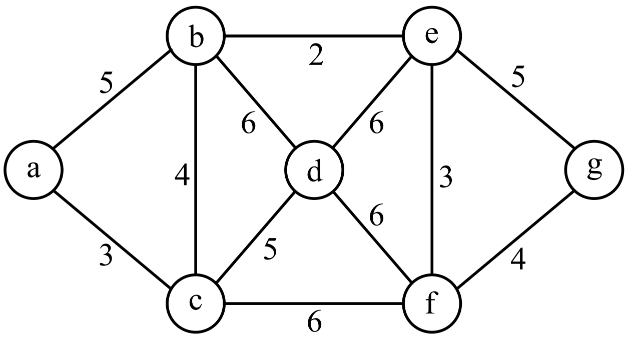
Kruskal's Algorithm

4. [NAT] [GATE-2015 : 2M]

The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: {(A, C), (B, C), (B, E), (E, F), (D, F)}. The edge weights of only those edges which are in the MST are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is ________.



5. **[MCQ]** **[GATE-2009 : 2M]**
Consider the following graph:



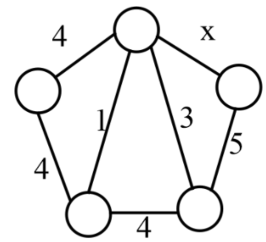
Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm?
(a) (b, e) (e, f) (a, c) (b, c) (f, g) (c, d)
(b) (b, e) (e, f) (a, c) (f, g) (b, c) (c, d)
(c) (b, e) (a, c) (e, f) (b, c) (f, g) (c, d)
(d) (b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

Prim's Algorithm

6. **[MCQ]** **[GATE-2021 : 1M]**
Let G = (V, E) be any connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:
I: Minimum spanning Tree of G is always unique.
II: Shortest path between any two vertices of G is always unique.
Which of the above statements is/are necessarily true?
(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

7. **[NAT]** **[GATE-2020 2M]**
Consider a graph G = (V, E), where V = $\{v_1, v_2 \ldots v_{100}\}$, E = $\{v_i, v_j\}$ $|1 \le i \le j \le 100\}$, and weight of the edge (v_i, v_j) is $|i - j|$. The weight of minimum spanning tree of G is ____.

8. **[NAT]** **[GATE-2018 : 2M]**
Consider the following undirected graph G:
Choose a value of x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is _______.



9. **[MCQ]** **[GATE-2016 : 1M]**
Let G be weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?
- **P:** Minimum spanning tree of G does not change.
- **Q:** Shortest path between any pair of vertices does not change.

(a) P only (b) Q only
(c) Neither P nor Q (d) Both P and Q

10. **[NAT]** **[GATE-2016 : 2M]**
Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5 and 6. The maximum possible weight that a minimum weight spanning tree of G can have is _____.

11. **[MCQ]** **[GATE-2016 : 2M]**
G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?
I: If e is the lightest edge of some cycle in G, then every MST of G includes e.
II: If e is the heaviest edge of some cycle in G, then every MST of G excludes e.
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

12. [MCQ] [GATE-2011 : 2M]

Let G = (V, E) be a weighted undirected graph and let T be a Minimum Spanning Tree (MST) of G maintained using adjacency lists. Suppose a new weighted edge $(u, v) \in V \times V$ is added to G. The worst-case time complexity of determining if T is still an MST of the resultant graph is

(a) $\Theta(|E| + |V|)$

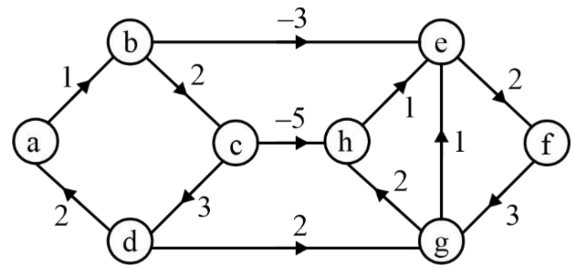
(b) $\Theta(|E|\ \ |V|)$

(c) $\Theta(|E| \log |V|)$

(d) $\Theta(|V|)$

13. [MCQ] [GATE-2008 : 2M]

Dijkstra's single source shortest path algorithm when run from vertex a in the above graph, computes the correct shortest path distance to



(a) Only vertex a

(b) Only vertices a, e, f, g, h

(c) Only vertices a, b, c, d

(d) All the vertices

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ANSWER KEY

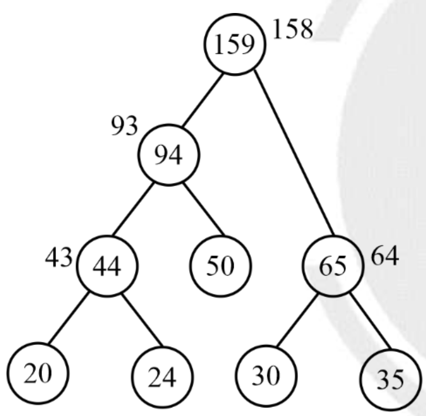
1.	(358 to 358)	**2.**	(a)	**3.**	(225 to 225)	**4.**	(69 to 69)
5.	(d)	**6.**	(a)	**7.**	(99 to 99)	**8.**	(4 to 4)
9.	(a)	**10.**	(7 to 7)	**11.**	(b)	**12.**	(d)
13.	(d)						

SOLUTIONS

1. (358 to 358)

L_1: ⟨n⟩ L_2: ⟨m⟩: The number of comparisons needed to merge them into a single sorted list = (n + m – 1)

⟨P, Q, R, S, T⟩ = ⟨20, 24, 30, 35, 50⟩



Total comparisons: 158 + 93 + 64 + 43 = 358.

Scan for Video solution

2. (a)

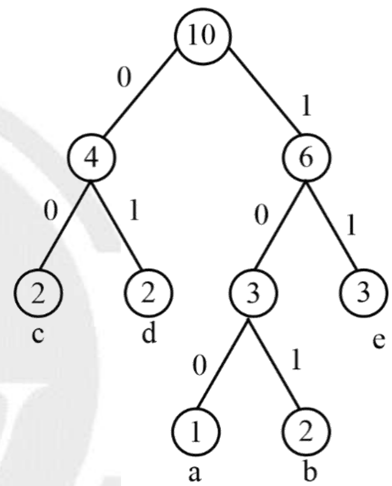
a → 1

b → 2

c → 2

d → 2

e → 3



<abbccddeee> : 3 + (3 + 3) + (2 + 2) + (2 + 2) + (2 + 2 + 2)

a = 100 (3)

b = 101 (3)

c = 00 (2)

d = 01 (2)

e = 11 (2)

<abbccddeee> : 1001011010000010111111 1.

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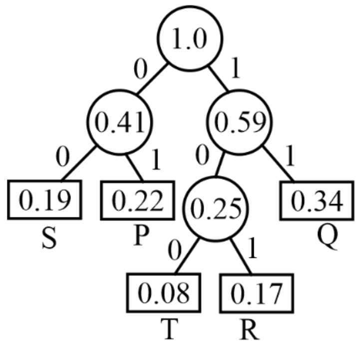
3. (225 to 225)

<P, Q, R, S, T> = <0.22, 0.34, 0.17, 0.19, 0.08



P → 01 (2 bits)

Q → 11 (2 bits)

R → 101 (3 bits)

S → 00 (2 bits)

T → 100 (3 bits)

$$\frac{\text{Average number of bits}}{\text{char}} = \sum_{i=1}^{n} di.qi$$

$= 2 * 0.22 + 2 * 0.34 + 3 * 0.17 + 2 * 0.19 + 3 * 0.08$

$= 0.44 + 0.68 + 0.51 + 0.38 + 0.24 = 2.25$

Number of bits for 100 char's = $100 \times 2.25 = 225$.

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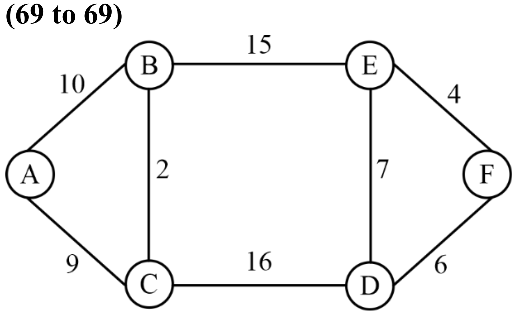
4. (69 to 69)



Sum = 36 + 10 + 16 + 7 = 69.

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5. (d)

In kruskals the edges are added in the non-increasing order. Only option (d) applies the algorithm correctly.

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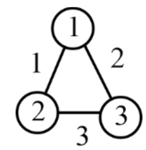
6. (a)



For the above graph statement, I follow but for the same graph statement II fails. The shortest path between node 2 and node 3 is of weight 3 but it is not unique. There are two paths:

- Node 2 to Node 3 via Node 1.
- Node 2 to Node 3.

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7. (99 to 99)

$$V_1 \xrightarrow{1} V_2 \xrightarrow{1} V_3 \xrightarrow{1} V_4 \xrightarrow{1} ....V_{99} \xrightarrow{1} V_{100}$$

Number of edges needed = $99 \times 1 = 99$.

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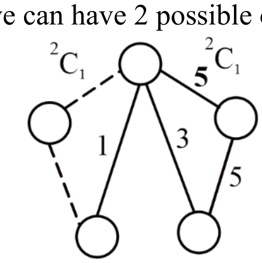
8. (4 to 4)

If x = 5, we can have 2 possible options



The number of spanning trees = $^2C_1 * {}^2C_1$

$= 2 * 2 = 4.$

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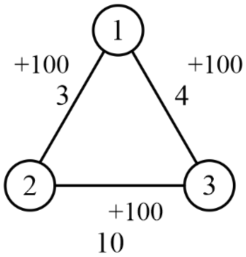
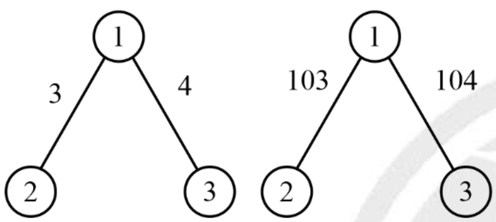
9. (a)



To the above graph we are adding a constant value 100. The M.S.T of the graph before adding the constant and after is shown below:

M.S.T



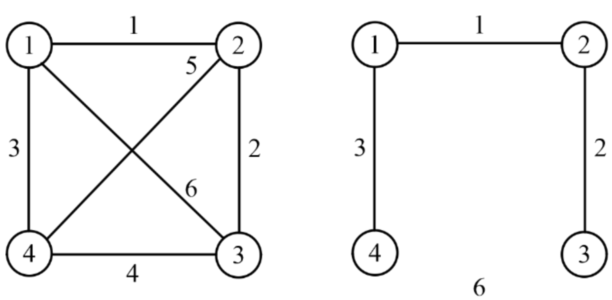
After adding a constant 100 the shortest path from node 2 to node 3 = 107.

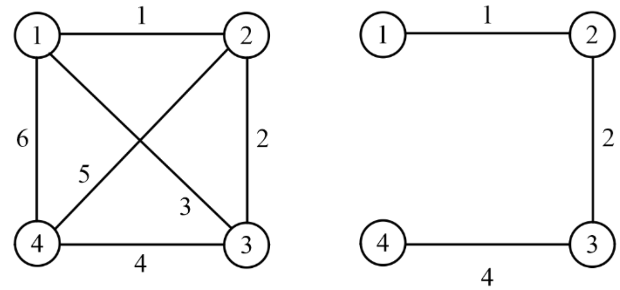
After adding a constant 100 the path from node 2 to node 3 = 110.

This proves that even after adding a constant value to the edge weights the minimum spanning tree of the graph does not change.

Scan for Video solution

10. (7 to 7)



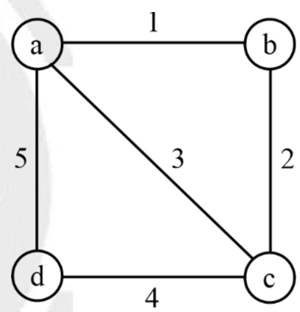


The maximum possible weight that a minimum weight spanning tree of G can have is 7.

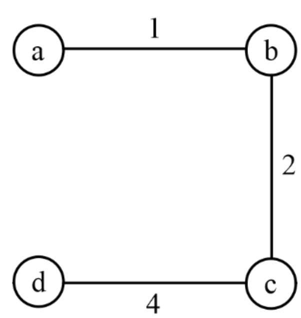
Scan for Video solution

11. (b)

Given that V = a, b, c, d and E = ab, bc, cd, da, ac, let G = (V, E) be a graph.

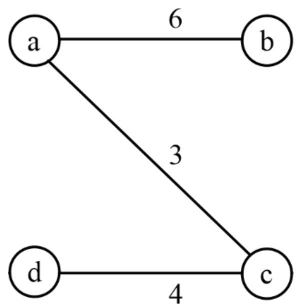


- Assign the following weights to the edges: ab = 1, bc = 2, cd = 4, da = 5, ac = 3. The MST abcd with cost 7 (= ab + bc + cd) does not include ac, which is obviously the cycle's lightest edge.



- Assign the following weights to the edges: ab = 6, bc = 7, cd = 4, da = 5, and ac = 3. The MST bacd with cost 13 (= ba + ac + cd) includes ac since it is the cycle cdac's lightest edge.

Therefore, the lightest edge may or may not be included in the MSTs of G.



- Let e be the heaviest edge. Consider the least spanning tree that includes e. A cycle will be produced if we increase the spanning tree's edge count by one. Suppose we add edge e' to the spanning tree which generated cycle C. We can reduce the cost of the minimum spanning tree if we choose an edge other than e from C for removal which implies that e must not be in minimum spanning tree and we get a contradiction.

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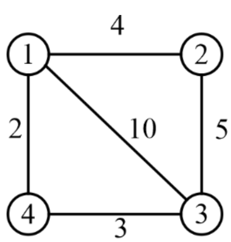
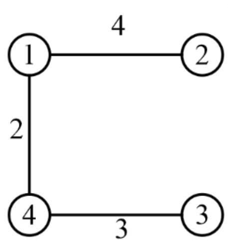
12. (d)

A Tree having 'n' vertices will always have (n – 1) edges.

$|V| = |E'| + 1$



G



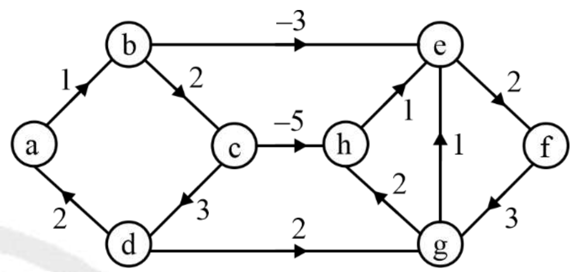
T

$E' : O(E') : O(v)$

A tree having n vertices will always have n – 1 edges.

Scan for Video solution

13. (d)



	Destination-d-values							
Vertex selected	a	b	c	d	e	f	g	h
{a}	–	1	∞	∞	∞	∞	∞	∞
{a, b}	–	1	3	∞	–2	∞	∞	∞
{a, b, e}	–	1	3	∞	–2	0	∞	∞
{a, b, e, f}	–	1	3	∞	–2	0	3	∞
{a, b, e, f, g}	–	1	3	∞	–2	0	3	5
{a, b, e, f, g, c}	–	1	3	6	–2	0	3	5
{a, b, e, f, g, c, h}	–	1	3	6	–2	0	3	5

Thus, the above graph computes the correct shortest path distance two all the vertices.

Scan for Video solution

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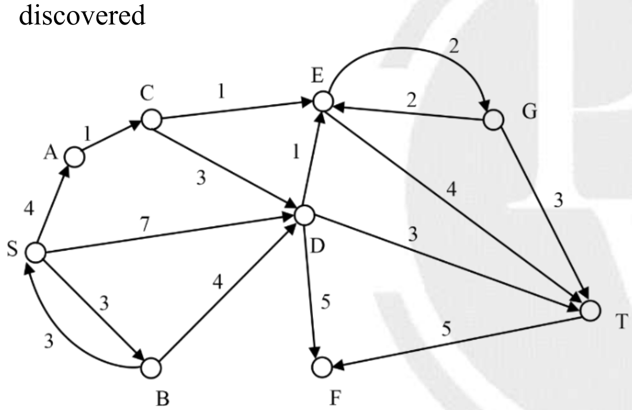
CHAPTER

4 GRAPH BASED ALGORITHM

Basics of Graph

1. [MCQ] [GATE-2012 : 2M]

Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered



(a) SDT (b) SBDT

(c) SACDT (d) SACET

Single Source Shortest Path

2. [MCQ] [GATE-2020 : 1M]

Let G = (V, E) be a directed, weighted graph with weight function w: E → R. For some function f: V → R, for each edge (u, v) ∈ E, define W′ (u, v) as W (u, v) + f(u) – f(v).

Which one of the options completes the following sentence so that it is TRUE?

"The shortest path in G under W are shortest paths under W′ too,. ______".

(a) for every f: V → R

(b) if and only if $\forall u \in V$, f(u) is positive

(c) if and only if $\forall u \in V$, f(u) is negative

(d) if and ony if f(u) is the distance from s to u in the graph obtained by adding a new vertex s to G and edges of zero weights from s to every vertex of G.

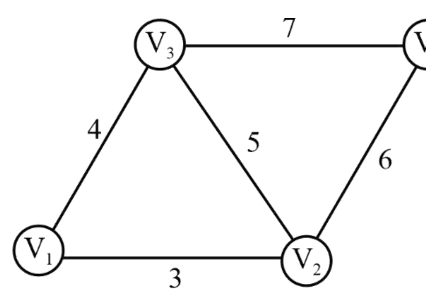
3. [NAT] [GATE-2016: 2M]

Consider the weighted undirected graph given with 4 vertices, where the weight of edge {i, j} is given by the entry W_{ij} in the $W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$

The largest possibel integer value of x, for which at least one shrotest path between some pair of vertices will contian the edge with weights x is ___.

Common data for next two Question

An undirected graph G(V, E) contains n(n > 2) nodes named $v_1, v_2, ..., v_n$. Two nodes v_i, v_j are connected if and only if $0 < |i - j| \leq 2$. Each edge (v_i, v_j) is assigned a weight i + j. A sample graph with n = 4 is shown below.



4. [MCQ] [GATE-2011 : 2M]

What will be the cost of the Minimum Spanning Tree (MST) of such a graph with n nodes?

(a) $\frac{1}{12}(11n^2 - 5n)$ (b) $n^2 - n + 1$

(c) $6n - 11$ (d) $2n + 1$

5. [MCQ] [GATE-2011 : 2M]

The length of the path from v_5 to v_6 in the MST of previous question with n = 10 is

(a) 11 (b) 25

(c) 31 (d) 41

6. [MSQ] [GATE-2011 : 2M]

Let G be a weighted graph with edge weights greater than one and G' be the graph constructed by squaring the weights of edge in G. Let T and T' be the minimum spanning trees of G and G' respectively, with total weights t and t'. Which of the following statements is TRUE?

(a) T' = T with total weight $t' = t^2$

(b) T' = T with total weight $t' < t^2$

(c) $T' \neq T$ but total weight $t' = t^2$

(d) None of the above

Common data for next two question

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

7. [MCQ] [GATE-2010 : 2M]

What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

(a) 7 (b) 8

(c) 9 (d) 10

8. [MCQ] [GATE-2010 : 2M]

What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

(a) 7 (b) 8

(c) 9 (d) 10

Graph Searching

9. [NAT] [GATE-2023 : 2M]

Let U = {1, 2, 3}. Let 2^U denote the powerset of U. Consider an undirected graph G whose vertex set is 2^U. For any A, B $\in 2^U$, (A, B) is an edge in G if and only if (i) $A \neq B$, and (ii) either $A \subsetneq B$ or $B \subsetneq A$. For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by B(A).

If $\varnothing$ denotes the empty set, then the cardinality of $B(\varnothing)$ is__________ .

10. [MSQ] [GATE-2021 : 2M]

An articulation point in connected graph is a vertex such that removing the vertex and its incident edges disconnects the graph into two or more connected components.

Let T be a DFS tree obtained by doing DFS in connected undirected graph G.

Which of the following options is/are correct?

(a) Root of T can never be an articulation point in G.

(b) Root of T is an articulation point in G if and only if it has 2 or more children.

(c) A leaf of T can be an articulation point in G.

(d) If u is an articulation point in G such that x is an ancestor of u in T and y is a descendent of u in T, then all paths from x to y in G must pass through u.

11. [NAT] [GATE-2021 : 1M]

Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing Breadth–first search (BFS) starting from the root. Let B denote the set of first 3 elements obtained by performing Depth – First Search (DFS) starting from the root.

The value of |A – B | is _________.

12. [MCQ] [GATE-2018 : 2M]

Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements:

I: No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D).

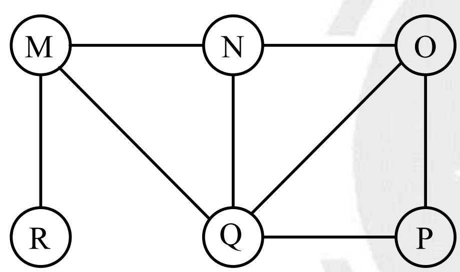
II: For every edge (u, υ) of G, if u is at depth i and υ is at depth j in T_B, then $|i - j| = 1$.

Which of the statements above must necessarily be true?

(a) I only (b) II only
(c) Both I and II (d) Neither I nor II

13. [MCQ] [GATE-2017 : 1M]

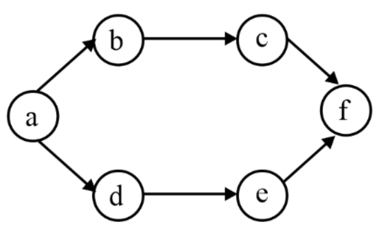
The Breadth First Search (BFS) algorithm has been implemented using the queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below?



(a) MNOPQR (b) NQMPOR
(c) QMNROP (d) POQNMR

14. [NAT] [GATE-2016 : 1M]

Consider the following directed graph:



The number of different topological orderings of the vertices of the graph is ______

15. [NAT] [GATE-2016 : 1M]

Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n^{th} vertex in his BFS traversal, then the maximum possible value of n is_______.

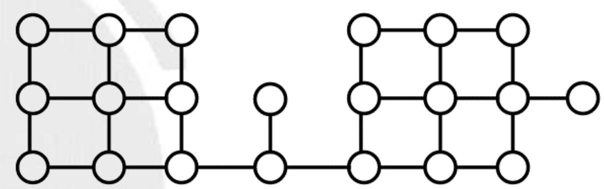
16. [MCQ] [GATE-2015 : 2M]

Let G = (V, E) be a simple undirected graph, and s be particular vertex in it called the source. For $x \in V$, let d(x) denote the shortest distance in G from s to x. A breadth first search (BFS) is performed starting at s. Let T be the resultant BFS tree. If (u, v) is an edge of G that is not in T, then which one of the following CANNOT be the value of d(u) – d(v)?

(a) –1 (b) 0
(c) 1 (d) 2

17. [NAT] [GATE-2014 : 1M]

Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is ______.



18. [MCQ] [GATE-2014 : 1M]

Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G. When G is represented as an adjacency matrix.

(a) $\Theta(n)$ (b) $\Theta(n + m)$
(c) $\Theta(n^2)$ (d) $\Theta(m^2)$

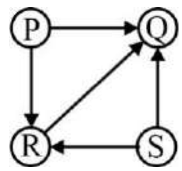
19. [MCQ] [GATE-2014 : 1M]

Consider the tree arcs of a BFS traversal from a source node W in an unweighted, connected, undirected graph. The tree T formed by the tree arcs is a data structure for computing

(a) The shortest path between every pair of vertices.
(b) The shortest path from W to every vertex in the graph.
(c) The shortest paths from W to only those nodes that are leaves of T.
(d) The longest path in the graph.

20. [MCQ] [GATE-2014 : 1M]

Consdier the directed graph given below:



Which of one the following is TRUE?

(a) The graph does not have any topological ordering.

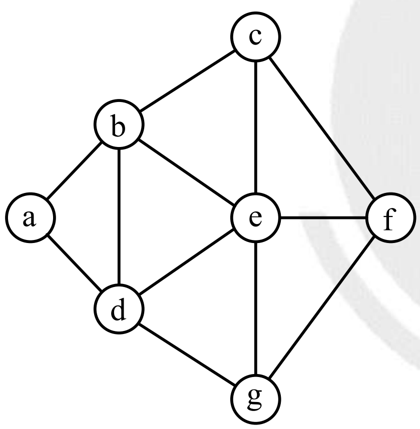
(b) Both PQRS and SRQP are topological orderings.

(c) Both PSRQ and SPRQ are topological orderings.

(d) PSRQ is the only topological ordering.

21. [MCQ] [GATE-2008 : 2M]

Consider the following sequence of nodes for the undirected graph given below:



1. a b e f d g c
2. a b e f c g d
3. a d g e b c f
4. a d b c g e f

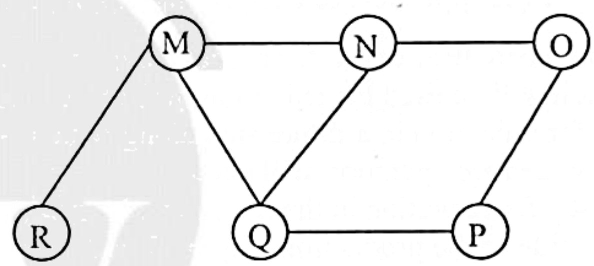
A Depth First Search (DFS) is started at node a. The nodes are listed in the order they are first visited. Which all of the above is (are) possible output(s)?

(a) 1 and 3 only (b) 2 and 3 only

(c) 2, 3 and 4 only (d) 1, 2 and 3

22. [MCQ] [GATE-2008 : 1M]

The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



(a) MNOPQR

(b) NQMPOR

(c) QMNPRO

(d) QMNPOR

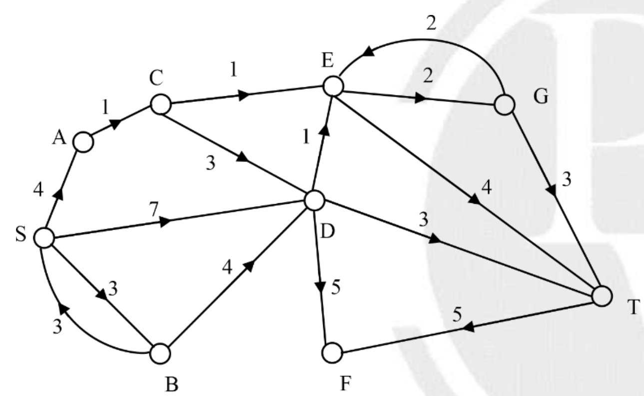
❑❑❑

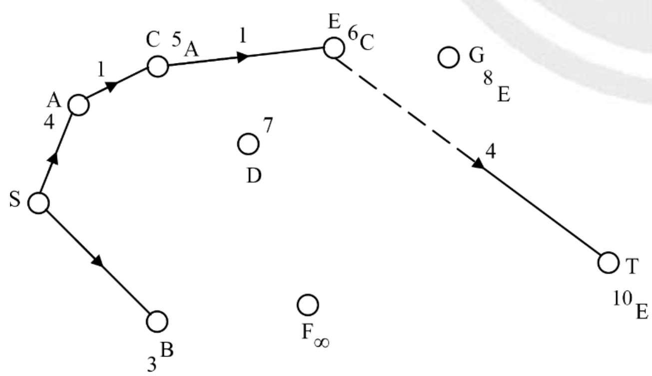
ANSWER KEY

1.	(d)	**2.**	(a)	**3.**	(12 to 12)	**4.**	(b)
5.	(c)	**6.**	(a, b)	**7.**	(d)	**8.**	(b)
9.	(5040 to 5040)	**10.**	(b)	**11.**	(1 to 1)	**12.**	(a)
13.	(d)	**14.**	(6 to 6)	**15.**	(31 to 31)	**16.**	(d)
17.	(19 to 19)	**18.**	(c)	**19.**	(b)	**20.**	(c)
21.	(b)	**22.**	(c)				

SOLUTIONS

1. (d)

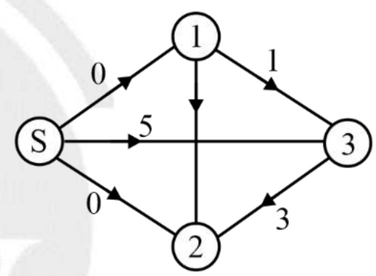




$\boxed{S \to A \to C \to E \to T}$

Scan for Video solution

2. (a)



W(1, 3) + W(3, 2) + f(1) – f(3) + f(3) – f(2)

W(1, 2) = 1 + 3 = 4

W′ (1, 2) = Transformation of W(u, v) = W(1, 2) + f(1) – f(2).

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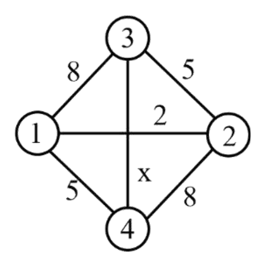
3. (12 to 12)



G = (V, E)

Shortest path between 3 and 4 has to be minimum {x, 12}.

There can exist multiple shortest paths with same cost.

<1, 2> = 2

<1, 3> = {8, 5 + x} x = 2 = 3

<1, 4> = 5

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

At least one shortest path must include edge cost 'x';

Let (4, r) = (3, 4)

1. 3 – 4 = x
2. 3 – 2 – 4 = 13 **(Incorrect)**
3. 3 – 1 – 4 = 13 **(Incorrect)**
4. 3 – 2 – 1 – 4 = 12

   x = 12

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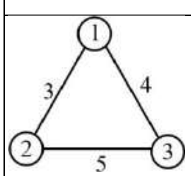
4. (b)

n = 4

n = 3	n = 4	n = 5	'n'
 3 + 4	:3 + 4 + 6 :3 + 2 (2 + 3)	:3 + 4 + 6 + 8 :3 + 2 (2 + 3 + 4)	:3 + 2 (2 + 3 + 4 +….+ n – 1)

$$:3+2\left[n\frac{(n-1)}{2}-1\right]=3+\cancel{2}\left[\frac{n^2-n-2}{\cancel{2}}\right]$$

The weight of minimum spanning tree is $n^2 - n + 1$.

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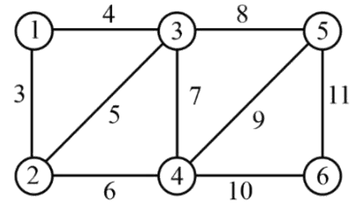
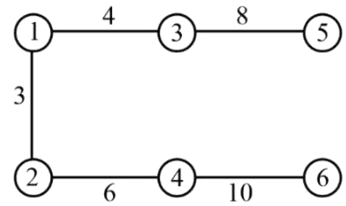
5. (c)





∴ The length of the path = (3 + 4 + 6 + 8 + 10)

From V_5 to V_6 in MST = 31.

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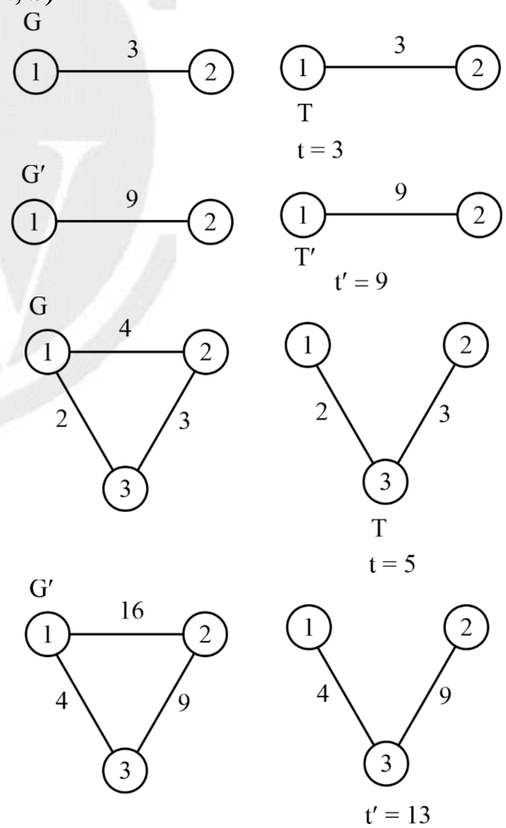
6. (a, b)



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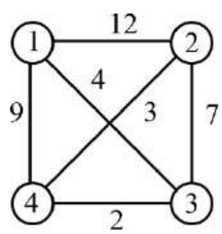
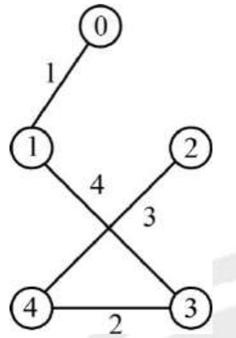
7. (d)

$$W = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{bmatrix} \end{array}$$





	1	2	3	4
1	0	12	4	9
2	12	0	7	3
3	4	7	0	2
4	9	3	2	0

$\boxed{\text{Minimum cost spanning tree} = 10}$

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8. (b)

Vertex set	0	1	2	3	4
{1}	1	–	12	4	9
{1, 0}	1	–	9	2	5
{1, 0, 3}	1	–	9	2	4
{1, 0, 3, 4}	1	–	8	2	4

Dijkstra's algorithm

$1 - 0 - 3 - 2 = 1 + 1 + 7 = 9$

$\boxed{1 - 0 - 4 - 2 = 1 + 4 + 3 = 8}$

Minimum possible weight of a path P from vertex 1 to vertex 2 is 8.

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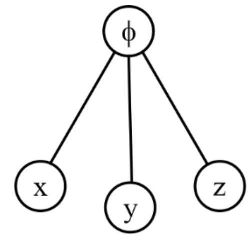
9. (5040 to 5040)



$$\left.\begin{array}{llll} \phi & (x, & y, & z) \\ \phi & y, & z, & x \\ \phi & z, & x, & y \\ \phi & x, & z, & y \\ \phi & y, & x, & z \\ \phi & z, & y, & x \end{array}\right\} 3!$$

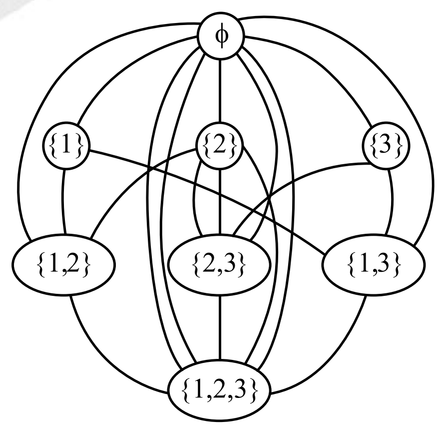
$U = \{1, 2, 3,\}$; $2^4 = V = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

$B(\phi) = 71$

$= 7 * 6 * 5 * 4 * 3 * 2 * 1$

$= 42 * 20 * 6 = 42 * 120$

$= \boxed{5040}$



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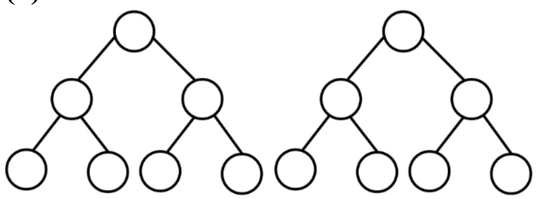
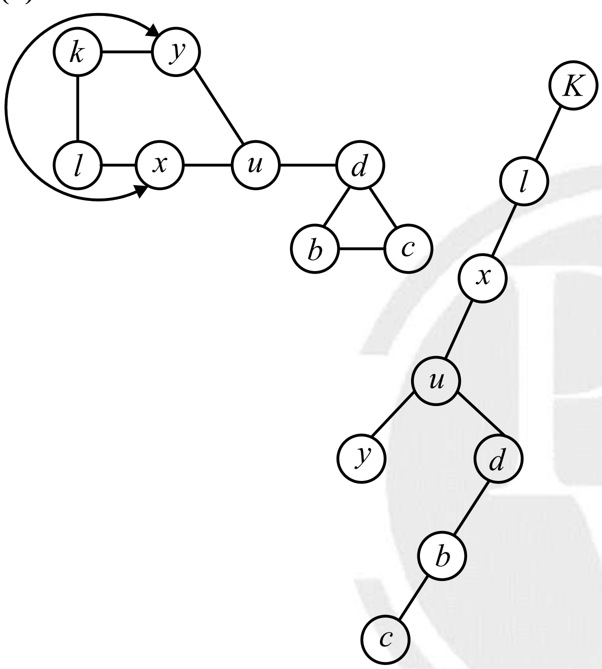
10. (b)



T G

If the Root has only one child it can never disconnect a tree on its removal. **Therefore, option (b) is correct.**



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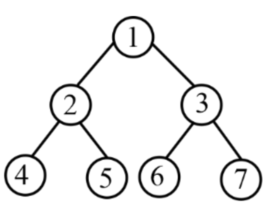
11. (1 to 1)



Breadth First Search = 1,2,3, 4,5, 6,7

Depth First Search = 1, 2, 4, 5, 3, 6, 7

A = {1, 2, 3}

B = {1, 2, 4}

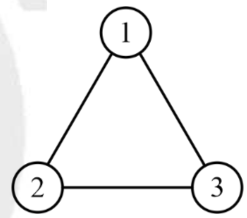
$$\boxed{\begin{array}{l} A - B = \{3\} \\ |A - B| = 1 \end{array}}$$

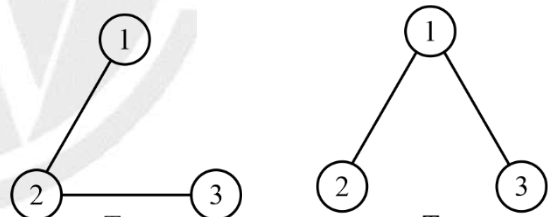
The value of |A – B | will be 1 after performing the said operation on a complete binary tree.

Scan for Video solution

12. (a)

- No edge of G is a cross edge with respect to T_D. (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D). **TRUE**
- For every edge (u, υ) of G, if u is at depth i and υ is at depth j in T_B, then |i –j| = 1. **FALSE**





T_D T_B

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13. (d)

Using breadth first search algorithm the order of visiting nodes will be: POQNMR.

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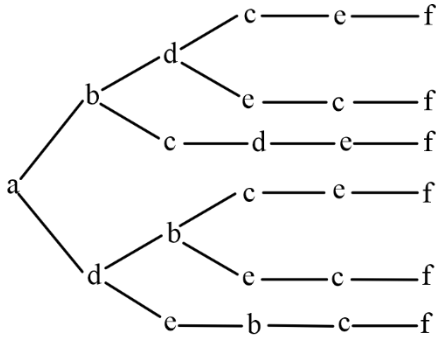
14. (6 to 6)

a_, _,_,_f

$$= \frac{4!}{(2!)(2!)} = \frac{4\times3\times2}{2\times2} = 6$$



Different number of topological sorting for the given graph will be 6.

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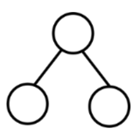
15. (31 to 31)



$: 2^{1+1} - 1 = 3$

$: 2^{2} - 1 = 3.$

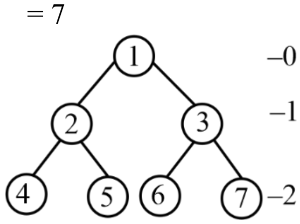
The maximum possible value of n for h=1 is 3.

The maximum possible value of n for h=2 is 7.

$: 2^{h+1} - 3$

$: 2^{3} - 1$

$= 7$

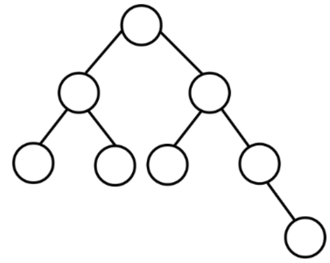


h = 1

For h = 4

$\boxed{h \equiv 4}$

Total Nodes $= 2^{h+1} - 1 = 2^{5} - 1 = 31$

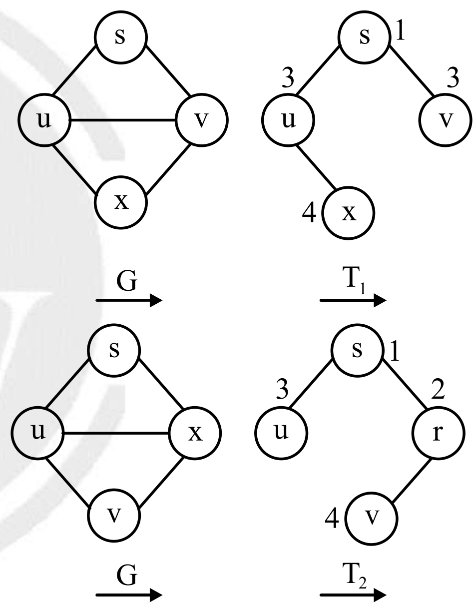


The maximum number of nodes will be 31.

Scan for Video solution

16. (d)

2 cannot be the value of d(u) – d(v).



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17. (19 to 19)

There exists total 21 nodes and only 2 nodes require back track therefore the maximum recursion depth is 21 – 2 = 19.

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18. (c)

$$A = \begin{bmatrix} & \end{bmatrix}_{n \times n}$$

$\Theta\left(n^2\right)$

For an adjacency matrix the DFS algorithm will take $\Theta(n^2)$.

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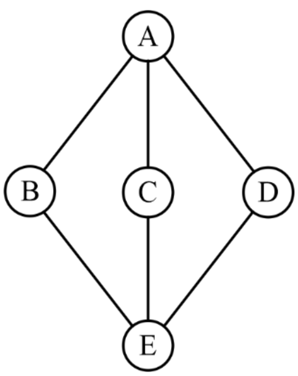
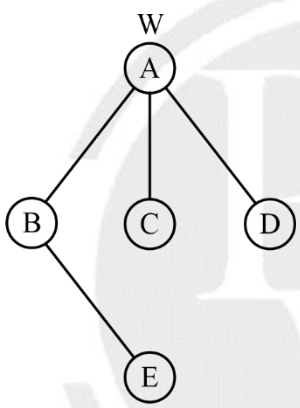
19. (b)

Shortest paths (unit distance) from 'w' to all other vertices in the graph





BFS spanning tree

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20. (c)

The topological orderings PSRQ and SPRQ are identical.

Select P or S as the starting vertices for DFS.

As soon as the vertex has finished, add it to the head of a linked list.

The needed topological ordering is the linked list.

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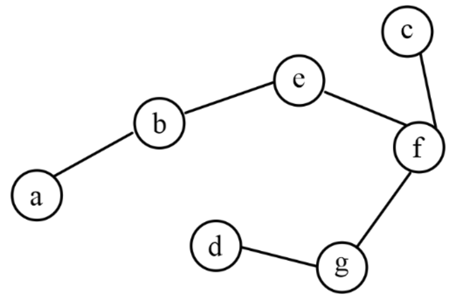
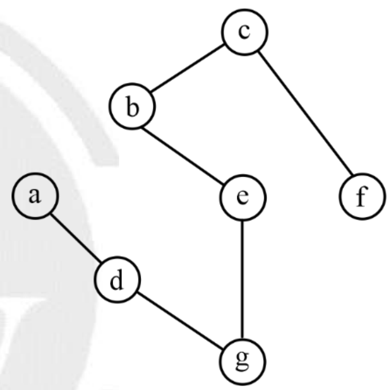
21. (b)



The possible outputs are: a b e f c g d



The possible outputs are: a d g e b c f

Scan for Video solution

22. (c)

The possible order of visiting the node is QMNPRO.

Scan for Video solution

❑❑❑

CHAPTER

5

DYNAMIC PROGRAMMING

Basics of Dynamic Programming

1. **[NAT]** **[GATE-2014: 2M]**
Consider two strings A = "qpqri"B = "pqprqrp". Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let be the number of such longest common subsequences between A and B.Then x + 10 y = _____.

2. **[MCQ]** **[GATE-2013 : 1M]**
What is the time complexity of Bellman-Ford single-source shortest path algorithm on a complete graph of n vertices?
(a) $\Theta(n^2)$ (b) $\Theta(n^2 \log n)$
(c) $\Theta(n^3)$ (d) $\Theta(n^3 \log n)$

3. **[MCQ]** **[GATE-2012 : 1M]**
The recurrence relation capturing the optimal execution time of the Tower of Hanoi problem with n discs is
(a) T(n) = 2T(n – 2) + 2 (b) T(n) = 2T(n – 1) + n
(c) T(n) = 2T(n/2) + 1 (d) T(n) = 2T(n – 1) + 1

4. **[MSQ]** **[GATE-2011 : 2M]**
Define R_n to be the maximum amount earned by cutting a rod of length *n* meters into one or more pieces of integer length and selling them. For i > 0, let p[i] denote the selling price of a rod whose length is i meters. Consider the array of prices:
p[1] = 1, p[2] = 5, p[3] = 8, p[4] = 9,
p[5] = 10, p[6] = 17, p[7] = 18
Which of the following statements is/are correct about R_7?
(a) $R_7 = 18$
(b) $R_7 = 19$
(c) R_7 is achieved by three different solutions.
(d) R_7 cannot be achieved by a solution consisting of three pieces.

5. **[MCQ]** **[GATE-2011 : 1M]**
An algorithm to find the length of the longest monotonically increasing sequence of number in an array A[0: n – 1] is given below.
Let Li denotes the length of the longest monotonicaly increasing sequence starting at index i in the array
Initialize $L_{n-1} = 1$
For all i such that $0 \le i \le n - 2$
$$L_i = \begin{cases} 1 + L_{i+1} & \text{If } A[i] < A[i+1] \\ 1 & \text{otherwise} \end{cases}$$
Finally the length of the longest monotonically increasing sequcnce is $\text{Max}(L_0, L_1, \ldots L_{n-1})$.
Which of the followign statements is TURE?
(a) The alogrithm uses dynamic programming paradigm.
(b) The alogrithm has a linear complexity and uses branch and bound paradigm
(c) The algorithm has non-linear polynomial complexity and uses brach and bound paradigm.
(d) The algorithm uses divide and conquer paradigm

6. **[MCQ]** **[GATE-2009 : 1M]**
Which of the following statement(s) is / are correct regarding Bellman-Ford shortest path algorithm?
P: Always finds a negative weighted cycle, if one exists.
Q: Finds whether any negative weighted cycle is reachable from the source.
(a) P only
(b) Q only
(c) both P and Q
(d) Neither P nor Q

Common data for next two questions:

A sub-sequence of a given sequence is just the given sequence with some elements(possibly none or all) left out. We are given two sequence X[m] and Y[n] of lengths m and n, respectively with indexes of X and Y starting from 0.

7. [MCQ] [GATE-2009 : 2M]

We wish to find the length of the longest common subsequence (LCS) of X[m] and Y[n] as l(m,n), where an incomplete recursive definition for the function l(i, j) to compute the length of the LCS of X[m] and Y[n] is given below:

$l(i, j) = 0$, if either $i = 0$ or $j = 0$

$= expr1$, if $i, j > 0$ and $x[i-1] = Y[j-1]$

$= expr2$, if $i, j > 0$ and $x[i-1] \neq Y[j-1]$

Which one of the following options is correct?

(a) $expr1 = l(i-1, j) + 1$

(b) $expr1 = l(i, j-1)$

(c) $expr2 = \max(l(i-1, j), l(i, j-1))$

(d) $expr2 = \max(l(i-1, j-1), l(i, j))$

8. [MCQ] [GATE-2009 : 2M]

The values of l(i, j) could be obtained by dynamic programming based on the correct recursive definition of l(i, j) of the form given above, using an array L[M, N], where M = m + 1 and N = n + 1, such that L[i, j] = l(i, j).

Which one of the following statements would be TRUE regarding the dynamic programming solution for the recursive definition of l(i, j)?

(a) All elements of L should be initialized to 0 for the values of l (i, j) to be properly computed.

(b) The values of l(i, j) may be computed in a row major order or column major order of L [M, N].

(c) The values of l(i, j) cannot be computed in either row major order or column major order of L [M, N].

(d) L [p, q] needs to be computed before L [r, s] if either p < r or q < s.

9. [MCQ] [GATE-2008: 2M]

Conider the following C funcitons:

```
intf1(int n)
{
    if(n = = 0||n = = 1) return n;
    else
        return (2* f1(n – 1) + 3* f1 (n – 2));
}
int f2 (int n)
{
    int i;
    int X[N], Y[N], Z[N];
    X[0] = Y[0] Z [0] = 0;
    X[1] = 1; Y[1] = 2; Z[1] = 3;
    for (i = 1; i ≤n; i ++)
    {
      X[i] = Y[i – 1] + Z [i – 2];
      Y[i] = 2* X[i];
      Z[i] = 3*X[i];
    }
    return X[n];
}
```

The running time of f1(n) and f2(n) are

(a) $\Theta(n)$ and $\Theta(n)$ (b) $\Theta(2^n)$ and $\Theta(n)$

(c) $\Theta(n)$ and $\Theta(2^n)$ (d) $\Theta(2^n)$ and $\Theta(2^n)$

Common data for next two questions:

The subset-sum problem is defined as follows. Given a set of n positive integers, $S = \{a_1, a_2, a_3, \ldots\ldots, a_n\}$ and a positive integer W, is there a subset S whose elements sum of W? A dynamic program for solving this problem uses a 2-dimensional Boolean array, X with n rows and W + 1 columns X [i, j], $1 \le i \le n$, $0 \le j \le W$, is TRUE if and only if there is a subset of $\{a_1, a_2 \ldots\ldots, a_i\}$ whose elements sum to j.

10. [MCQ] [GATE-2008 : 2M]

Which entry of the array X, if TRUE, implies that there is a subset whose elements sum of W?

(a) X [1, W] (b) X [n, 0]

(c) X [n, W] (d) X [n – 1, n]

11. [MCQ] [GATE-2008 : 2M]

Which of the following is valid for $2 \le i \le n$, and $a_i \le j \le W$?

(a) X [i, j] = X [i – 1, j] ∨ X[i, j – ai]

(b) X [i, j] = X [i – 1, j] ∨ X[i – 1, j – ai]

(c) X [i, j] = X [i – 1, j] ∧ X[i, j – ai]

(d) X [i, j] = X [i – 1, j] ∧ X[i – 1, j – ai]

12. [MCQ] [GATE-2008 : 2M]

The subset-sum problem is defined as follows: Given a set S of n positive integers and a positive integer W, determine whether there is a subset of S whose elements sum to W.

An algorithm Q solves this problem in O(nW) time. Which of the following statements is false?

(a) Q solves the subset−sum problem in polynomial time when the input is encoded in unary.

(b) Q solves the subset sum problem is polynomial time when the input is encoded in binary.

(c) The subset sum problem belongs to the class NP.

(d) The subset sum problem in NP−hard.

Floyd Warshall's Algorithm

13. [MCQ] [GATE-2017 : 1M]

Consider the followign table:

Algorithm	Design paradigms
(P) Kruskal	(i) Divide and conquer
(Q) Quicksort	(ii) Greedy
(R) Floyd-Warshall	(iii) Dynamic programming

(a) P-ii, Q-iii, R-i (b) P-iii, Q-i, R-ii

(c) P-ii, Q-i, R-iii (d) P-i, Q-ii, R-iii

14. [MCQ] [GATE-2016 : 1M]

The Floyd-Warshall alorithm for all-pair shortest paths computation is based on

(a) Greedy paradigm

(b) Divide-and-conquer paradigm

(c) Dynamic programming paradigm

(d) Neither Greedy nor Divide-and-conquer nor Dynamic Programming paradigm

0/1 Knapsack Problem

15. [NAT] [GATE-2018 : 2M]

Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item Number	Weight in (Kgs)	Value in (Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is not more than 11 kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}.

The value of $V_{opt} - V_{greedy}$ is ________.

Matrix Chain Multiplication

16. [MCQ] [GATE-2018 : 2M]

Assume that multiplying a matrix G_1 of dimension P × Q with another matrix G_2 of dimension q × r requires pqr scalar multiplications. Computing the product of n matrices $G_1 G_2 G_3 \ldots G_n$ can be done by parenthesizing in different ways. Define G_iG_i + l., as an explicitly computed pair for a given parenthesizing if they arc directly multiplied. For example, in the matrix multiplication chain $G_1G_2G_3G_4G_5G_6$ using parenthesization $(G_1(G_2G_3))(G_4(G_5G_6))$, G_2G_3 and G_5G_6 are the only explicitly computed pairs.

Consider a matrix multiplication chain $F_1F_2F_3F_4F_5$, where matrices F_1, F_2, F_3, F_4 and F_5 are of dimensions 2 × 25, 25 × 3, 3 × 16, 16 × 1 and 1 × 1000, respectively. In the parenthesization of $F_1F_2F_3F_4F_5$ that minimizes the total number of scalar multiplications, the explicitly computed pairs is/are

(a) F_1F_2 and F_3 F_4 only (b) F_2F_3 only
(c) F_3F_4 only (d) F_1F_2 and F_4F_5 only

17. [NAT] [GATE-2016 : 2M]
Let A_1, A_2, A_3, and A_4 be four matrices of dimensions 10×5, 5×20, 20×10, and 10×5, respectively. The minimum number of scaler multiplication required to find the product $A_1A_2A_3A_4$ using the basic matrix multiplication method is_________.

18. [MCQ] GATE-2011 : 2M]
Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$ and $s \times t$ respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplications is $pqr + prs + pst$. If $p = 10$, $q = 100$, $r = 20$, $s = 5$ and $t = 80$, then the minimum number of scalar multiplications needed is?
(a) 248000
(b) 44000
(c) 19000
(d) 25000

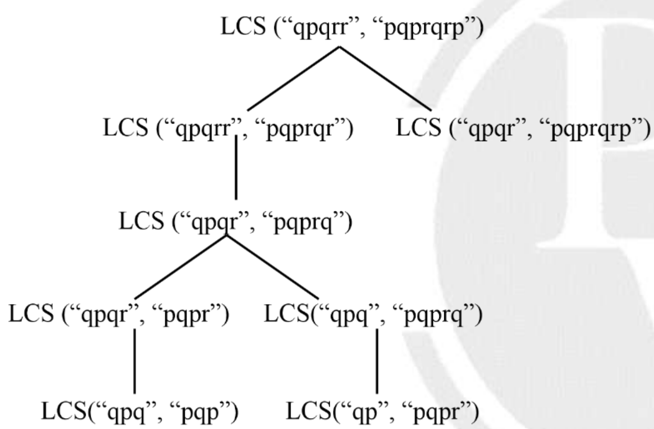
❑❑❑

ANSWER KEY

1.	(34 to 34)	**2.**	(c)	**3.**	(d)	**4.**	(a, c)
5.	(a)	**6.**	(b)	**7.**	(c)	**8.**	(b)
9.	(b)	**10.**	(c)	**11.**	(b)	**12.**	(b)
13.	(c)	**14.**	(c)	**15.**	(16 to 16)	**16.**	(c)
17.	(1500 to 1500)	**18.**	(c)				

SOLUTIONS

1. (34 to 34)



1. “qpqr”
2. “qprr”
3. “pqrr”

Length = 4 = x

Number of LCS = 3 = y

x + 10y = 4 + 10 × 3 = 34.

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2. (c)

$\boxed{\begin{aligned}\text{Bellman ford} &= \Theta(n.e) = \Theta(n.n^2)\\ &= \Theta(n^3)\end{aligned}}$ for complete graph $e = \Theta(n^2)$

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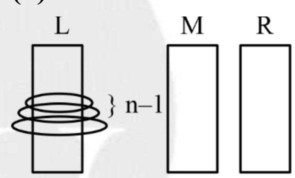
3. (d)



$\boxed{\begin{aligned}T(n) &= 2.T(n\text{–}1) + C\\ &= 2.T(n\text{–}1) + 1\end{aligned}}$

Algorithm for tower of hanoi problem:

```
TOH (n,L,M,R)
{
if (n = 0) return
else
{
TOH (n–1, L, R, M);
Move (1, L, R)
TOH (n–1, M, L, R);
}
}
```

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4. (a, c)

P[1] = 1; P[2] = 5; P[3] = 8; P[4] = 9; P[5] = 10; P[6] = 17; P[7] = 18;

No. of pieces	Combination	Selling Price	
1	1(7)	18	**Max (R_7)**
2	2(6, 1)	18	
3	3(2, 2, 3)	18	
4	4(2, 2, 2,1)	16	
5	5(2, 2, 1, 1, 1)	13	
6	6(2, 1, 1, 1, 1, 1)	10	
7	7(1, 1, 1, 1, 1, 1, 1)	7	

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5. (a)

The algorithm storts the optimal solutions to subproblems at every point, and then uses it to derive the optimal solution for a bigger problem and which is dynamic programming approach. The program runs in linear time complexity.

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6. (b)

The Bellman-Ford algorithm may not always find the negative weighted cycle. However, it finds any negative weighted cycle that is reachable from the given source.

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7. (c)

When the currently compared elements doesn't match, we have two possibilities for the LCS, one including Y[j] but not X[i] and including X[i] but not Y[j].

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8. (b)

The previously discovered LCS is saved using dynamic programming. Therefore, all smaller ones for any index should have been computed earlier.

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9. (b)

```
{
 if(n = = 0||n = = 1) return n;
 else
 return (2* f1(n – 1) + 3* f1 (n –
   2));
}
```

$T(n) = c, n = 0, 1$

$= T(n-1) + T(n-2) + a, n>1$

$= T(n) = O(2^n)$

```
{
   int i;
   int X[N], Y[N], Z[N];
   X[0] = Y[0] Z [0] = 0;
   X[1] = 1; Y[1] = 2; Z[1] = 3;
   for (i = 1; i ≤n; i ++)
   X[i] = Y[i – 1] + Z [i – 2];
   Y[i] = 2* X[i];
   Z[i] = 3*X[i];
}
```

$T(n) = O(n)$

The running time of f1(n) is $O(2^n)$ and f2(n) is O(n).

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10. (c)

X[0 …. n, …..]

X[1 …. n, …..]

Data set: n = 5; **S:** ⟨2, 8, 4, 11, 9⟩; W = 6

i \ j (X)	0	1	2	3	4	5	6
0	T	F	F	F	F	F	F
1	T	F	T	F	F	F	F
2	T	F	T	F	F	F	F
3	T	F	T	F	T	F	T
4	T						
5	T						T

X[n, W] = Entry stating whether subset exists/Not

X [i, j] = True/False, whether there exist a subset from first ‘i’ elements that sum to ‘j’.

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11. (b)

The right answer is B X[i, j] = X[i − 1, j] ∨ X[i − 1, j − a_i].

It is NP-complete (the subset-sum problem).

For instance, the technique checks to see if the sum of the elements in the set S' is zero such that S' ≠ S given the set S = {–7, –3, –2, 5, 7}.

For instance, say if S' = {–3, –2, 5}.

x is a two-dimensional boolean array with n rows and W + 1 columns in the proposed dynamic method, where W is a positive weight or sum.

x[i, j]1 ≤ i ≤ n, 0 ≤ j ≤ W

The element in the subset of S is determined by the i^{th} row, and the associated weight is determined by the j^{th} column.

When 2 ≤ i ≤ n and a ≤ j ≤ W

Let S' be a subset with weight W and a_1, a_2,...., a_i.

This means that X[i, j] = X[i – 1, j] ∨ X[i – 1, j – a_i].

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12. (b)

Subset of Sum is a NP Complete problem.

⇒ Subset sum problem ∈ NP

⇒ Subset sum problem is NP-hard problem.

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13. (c)

Algorithm	**Design paradigms**
(P) Kruskal	(ii) Greedy
(Q) Quicksort	(i) Divide and conquer
(R) Floyd-Warshall	(iii) Dynamic programming

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14. (c)

The Floyd-Warshall’s algorithm is based on dynamic programming. It is used to find all the pair shortest path for every pair (i, j), there exists two possibilities for every pair:

- If k is not an intermediate vertex in the shortest path from (i) to (j) then the value of dist[i][j] remains same.
- If k is an intermediate vertex in the shortest path from (i) to (j), then the value is updated as dist[i][j] + dist[k].

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15. (16 to 16)

M = 11

Using optimal algorithm, $V_{opt} = 60$

Item No.	Weight	Value	V/W	
1	10	60	6	×
2	7	28	4	×
3	4	20	5	✓
4	2	24	12	✓

Greedy approach:

Weight $\langle 2 + 4 \rangle$

$\therefore$ Value = 44

$V_{Greedy} = 44$

$V_{opt} - V_{Greedy} = 60 - 44 = 16.$

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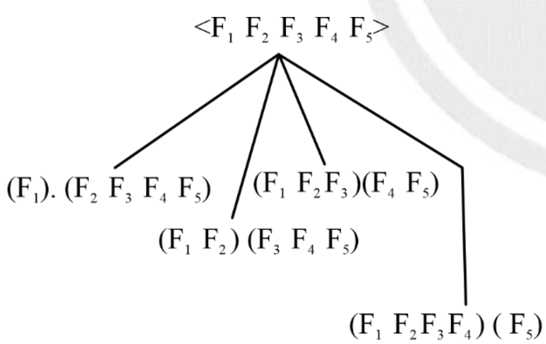
16. (c)

$\left(F_1\left(F_2\left(F_3F_4\right)\left(F_5\right)\right)\right)$



If we perform parenthesization such that F_3F_4 forms a pair then it will reduce the total number of scalar multiplications.

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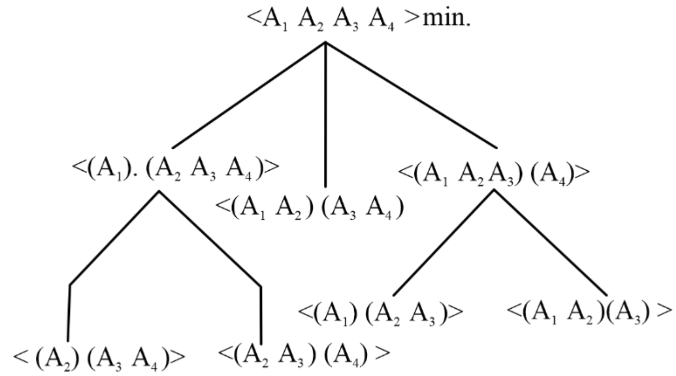
17. (1500 to 1500)



5 * 20 * 5 + 0 + 20 * 10 * 5 = 1500.

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18. (c)

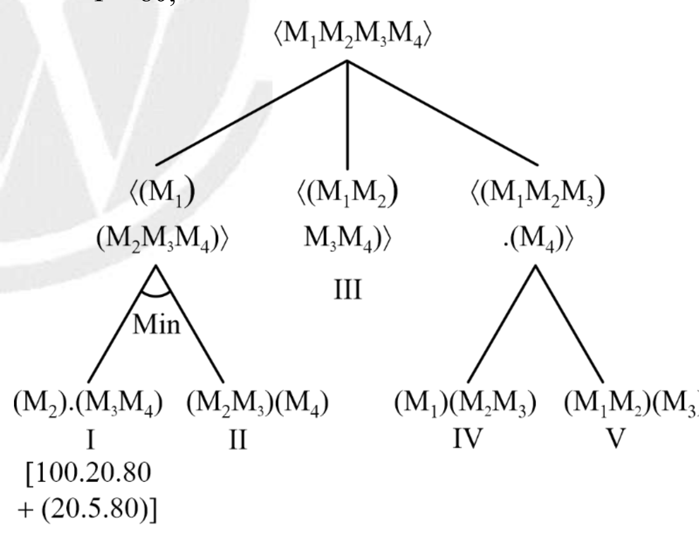
$M_1 = P \times Q$, $M_2 = Q \times R$, $M_3 = R \times S$

$M_4 = S \times T$, P = 10; Q = 100;

R = 20;

S = 5;

T = 80;



Number of scalar multiplications = minimum of (I, II, III, IV, V).

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❑❑❑

CHAPTER

6 MISCELLANEOUS TOPICS

Graph

1. [MSQ] [GATE-2022: 2M]

Consider a simple undirected weighted graph G, all of whose edge weights are distinct. Which of the following statements about the minimum spanning trees of G is/are TRUE?

(a) The edge with the second smallest weight is always part of any minimum spanning tree of G.

(b) One or both of the edges with the third smallest and the fourth smallest weights are part of any minimum spanning tree of G .

(c) Suppose $S \subseteq V$ be such that $S \neq \phi$ and $S \neq V$. Consider the edge with the minimum weight such that one of its vertices is in S and the other in V \ S. Such an edge will always be part of any minimum spanning tree of G .

(d) G can have multiple minimum spanning trees.

2. [NAT] [GATE-2022 : 2M]

Let G(V, E) be a directed graph, where V = {1, 2, 3, 4, 5} is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix

A. $A[i][j] = \begin{cases} 1, & 1 \le j \le i \le 5 \\ 0, & \text{otherwise} \end{cases}$

A[[i][j] = 1 indicates a directed edge from node i to node j. A directed spanning tree of G, rooted at r ϵ V, is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V. The number of such directed spanning trees rooted at vertex 5 is_____________.

3. [MCQ] [GATE-2021 : 1M]

Let G be a connected undirected weigthed graph. Consider the following two statements.

S_1: There exists a minimum weight edge in G which is present in every minimum spanning tree of G.

S_2: If every edge in G has distinct weight, then G has a unique minimum spanning treed.

Which one of the following options is correct?

(a) S_1 is false and S_2 is true

(b) Both S_1 and S_2 are flase

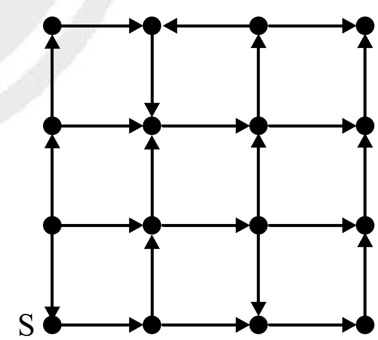
(c) S_1 is true and S_2 is false

(d) Both S_1 and S_2 are true.

4. [MSQ] [GATE-2021: 2M]

Consider the following directed graph:

Which of the following is/are correct about the graph?



(a) The graph does not have a topological order.

(b) A depth-frist traversal starting at vertex S classifies three directed edges as back edges.

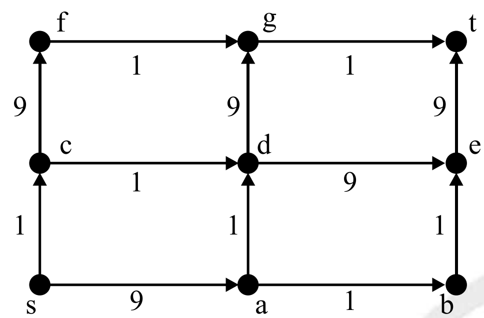
(c) For each pair of vertices u and v, there is a directed path from u to v.

(d) The graph does not have a strongly connected compnont.

5. **[NAT]** **[GATE-2021: 2M]**

In a directed acryclic graph with a source vertex s, the qauility-score of a directed path is defined to be the product of the weights of the edges on the path.

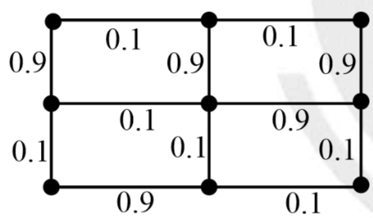
Further, for a vertex v other than s, the quality-score of v is defined to be the maximum among the quality-scores of all the paths from s to v. the quality score of s is assumed to be 1.



The sum of the quality -scores of all the vertieces in the graph shown above is___.

6. **[NAT]** **[GATE-2020 : 2M]**

Consider the following undirected graph with edge weights as shown:



The number of minimum-weight spanning trees of the graph is _____.

7. **[MCQ]** **[GATE-2015 : 1M]**

Match the following:

List-I	List -II
A: Prim's algrithm for minimum spanning tree	**1.** Backtraking
B: Floyd-Warshall algorithm for all pairs shortest paths	**2.** Greedy Method
C: Mergesort	**3.** Dynamic programming
D: Hamiltonian circuit	**4.** Divide and conquer

(a) A-3, B 2, C-4, D-1

(b) A-1, B-2, C-4, D-3

(c) A-2, B-3, C-4, D-1

(d) A-2, B-1, C-3, D-4

8. **[MCQ]** **[GATE-2015 : 2M]**

Given below are some algorithms, and some algorithm design paradigms.

List-I		List-II	
A	Dijkstra's Shortest path	**1.**	Divide and Conquer
B	Floyd-Warshall algorithm to compute all pairs shortest path.	**2.**	Dynamic programming
C	Binary search on a sorted array	**3.**	Greedy Design
D	Backtracking search on a graph	**4.**	Depth-first search
		5.	Breadth-first search

Math the above algorithm (List-I) to the corresponding design paradigam (List-II) they follow:

(a) A-1, B-3, C-1, D-5

(b) A-3, B-22, C-1, D-5

(c) A-3, B-2, C-1, D-4

(d) A-3, B-2, C-1, D- 5

9. **[MCQ]** **[GATE-2014 : 1M]**

In an adjacency list representation of an undirected simple graph G = (V, E), each edge (u, v) has two adjacency list entries: [v] in the adjacency list of u, and [u] in the adjacency list of v. These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If |E| = m and |V| = n, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?

(a) $\Theta(n^2)$ (b) $\Theta(n + m)$

(c) $\Theta(m^2)$ (d) $\Theta(n^4)$

10. [MCQ] [GATE-2008 : 1M]

The most efficient algorithm for finding the number of connected components in an undirected graph on n vertices and m edges has time complexity-

(a) $\Theta(n)$ (b) $\Theta(m)$

(c) $\Theta(m + n)$ (d) $\Theta(mn)$

Tree

11. [MCQ] [GATE-2020 : 1M]

What is the worst-case time complexity of inserting n^2 elements into an AVL-tree with n elements initially?

(a) $\theta(n^4)$ (b) $\theta(n^2 \log n)$

(c) $\theta(n^3)$ (d) $\theta(n^2)$

12. [MCQ] [GATE-2020 : 2M]

In a balanced binary search tree with n elements, what is the worst-case time complexity of reporting all elements in range [a, b]? Assume that the number of reported elements is K.

(a) $\Theta(\log n)$ (b) $\Theta(n \log k)$

(c) $\Theta(\log n + k)$ (d) $\Theta(k \log n)$

13. [MCQ] [GATE-2015 : 1M]

What are the worst-case complexities of insertion and deletion of a key in a binary search tree?

(a) $\Theta(\log n)$ for both insertion and deletion

(b) $\Theta(n)$ for both insertion and deletion

(c) $\Theta(n)$ for insertion and $\Theta(\log n)$ for deletion

(d) $\Theta(\log n)$ for insertion and $\Theta(n)$ for deletion

14. [NAT] [GATE-2015 : 2M]

Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes______.

15. [MCQ] [GATE-2015 : 1M]

Consider a complete binary tree where the left and the right subtrees of the root are max-heaps. The lower bound for the number of operations to convert the tree to a heap is?

(a) $\Omega(\log n)$ (b) $\Omega(n)$

(c) $\Omega(n \log n)$ (d) $\Omega(n^2)$

16. [NAT] [GATE-2014 : 2M]

Suppose we have a balanced binary search tree T holding n-numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. Suppose there are m such numbers in T.

If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is ____.

17. [MCQ] [GATE-2013 : 1M]

Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes?

(a) $O(1)$ (b) $O(\log n)$

(c) $O(n)$ (d) $O(n \log n)$

18. [MCQ] [GATE-2012 : 1M]

The worst-case running time to search for an element in a balanced binary search tree with $n2^n$ elements is?

(a) $\Theta(n \log n)$ (b) $\Theta(n2^n)$

(c) $\Theta(n)$ (d) $\Theta(\log n)$

Recurrence Relation

19. [MCQ] [GATE-2017 : 1M]

Match the algorithms with their time complexities:

List-I (Algorithm)

(P) Tower of Hanoi with n disks.

(Q) Binary search given n sorted numbers

(R) Heap sort given n numbers at the worst case.

(S) Addition to two $n \times n$ matrices

List-II (Time complexity)

(i) $\Theta(n^2)$ (ii) $\Theta(n \log n)$

(iii) $\Theta(2n)$ (iv) $\Theta(\log n)$

(a) P – (iii), Q – (iv), R – (i), S – (ii)

(b) P – (iv), Q – (iii), R – (i), S – (ii)

(c) P – (iii), Q – (iv), R – (ii), S – (i)

(d) P – (iv), Q – (iii), R – (ii), S – (i)

20. [MCQ] [GATE-2008 : 2M]

If we use Radix Sort to sort n integers in the range $(n^{k/12}, n^k)$, for some $k > 0$ which is independent of n, the time taken would be

(a) $\Theta(n)$ (b) $\Theta(kn)$

(c) $\Theta(n \log n)$ (d) $\Theta(n^2)$

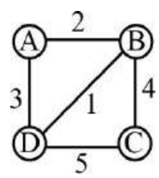
❑❑❑

ANSWER KEY

1.	(a, b, c)	**2.**	(24 to 24)	**3.**	(a)	**4.**	(a, b)
5.	(929 to 929)	**6.**	(3 to 3)	**7.**	(c)	**8.**	(c)
9.	(b)	**10.**	(c)	**11.**	(b)	**12.**	(c)
13.	(a)	**14.**	(995 to 995)	**15.**	(a)	**16.**	(110 to 110)
17.	(b)	**18.**	(c)	**19.**	(c)	**20.**	(c)

SOLUTIONS

1. (a, b, c)



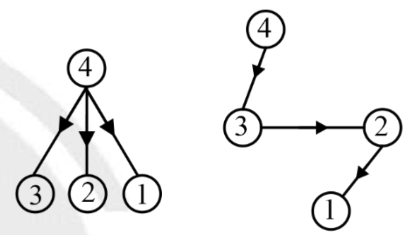
- Since G has Exactly One MST, as all the edge weights are unique so the statement in option (a) is correct.
- As already the given graph G has distinct as weights, the third and fourth smallest edge weights will always be a part of MST.
- This is a popular and basic property of Minimum Spanning Tree, known as "Cut Property." The cut property is the basis for the algorithms that we consider for the MST problem. "Cut Property" and "Cycle Property" are basic theorems.

Scan for Video solution

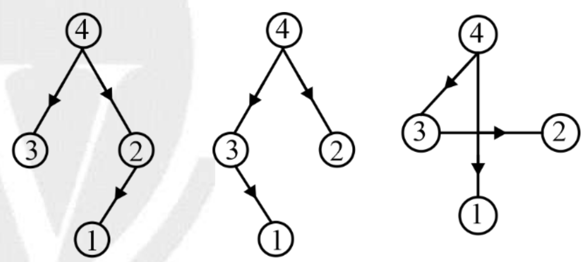
2. (24 to 24)

A	1	2	3	4	5
1	1	0	0	0	0
2	1	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	0
5	1	1	1	1	1

n = 4

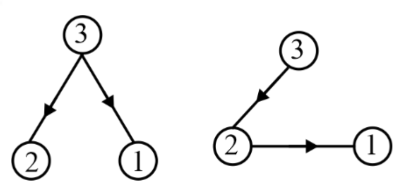


ST_1 ST_2



ST_3 ST_4 ST_5

n = 3



ST_1 ST_2

Total of 6 spannig tree

n = 3; r = 3; 2 ST's

n = 4; r = 4; 3! = 6 ST's

n = 5; r = 5; 4! = 24 ST's

Scan for Video solution

3. **(a)**

The Kruskals algorithm can be used to discover the Minimum Spanning Tree on a graph G.

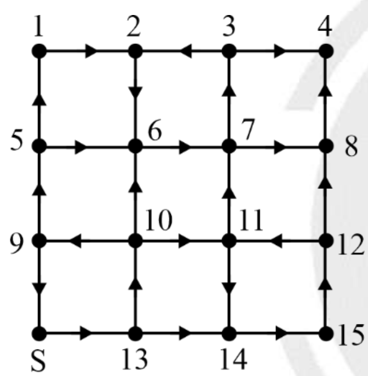
While doing that, we sort the edges according to their weight and begin choosing the smallest edge.

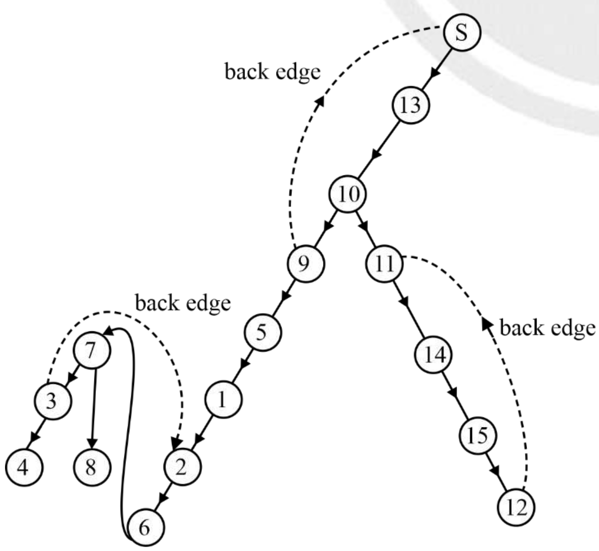
Problem with S_1: The Kruskals method may not always choose a particular weighted edge if there are numerous copies of it.

S_2 is True, the Kruskals method will always choose a distinct set of edges, resulting in a distinct minimal spanning tree, if the sorted order of the edges contains only distinct values.

Scan for Video solution

4. **(a, b)**





There are 3 back edges from node 3 to 2, node 9 to S, and node 12 to 11.

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5. **(929 to 929)**

Let q(s) denote the quality score of v, then:

q(s) = 1; q(d) = 9; q(b) = 9;

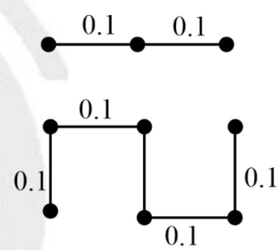
q(a) = 9; q(f) = 9; q(c) = 81;

q(c) = 1; q(g) = 81; q(t) = 729;

$$\boxed{\sum_{i=1}^{9} q(i) = 929}$$

Scan for Video solution

6. **(3 to 3)**



The number of minimum-weight spanning trees of the graph is ${}^3C_1 = 3$.

Scan for Video solution

7. **(c)**

List-I	List -II
A: Prim's algrithm for minimum spanning tree	**2.** Greedy Method
B: Floyd-Warshall algorithm for all pairs shortest paths	**3.** Dynamic programming
C: Mergesort	**4.** Divide and conquer
D: Hamiltonian circuit	**1.** Backtraking

Scan for Video solution

8. **(c)**

List-I		List-II	
A	Dijkstra's Shortest path	**3.**	Greedy Design
B	Floyd-Warshall algorithm to compute all pairs shortest path.	**2.**	Dynamic programming
C	Binary search on a sorted array	**1.**	Divide and Conquer
D	Backtracking search on a graph	**4.**	Depth-first search

Scan for Video solution

9. **(b)**

Using BFS, you can get all twin pts

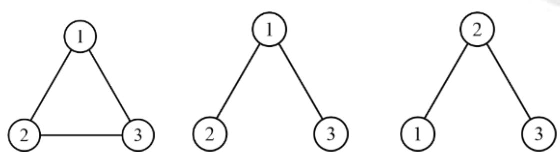
$\langle 1, 2\rangle$

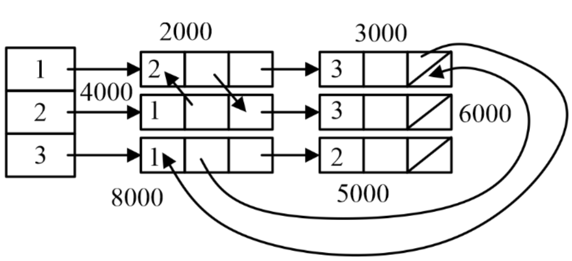
$\langle 1, 3\rangle$

$\langle 2, 1\rangle$

$\langle 2, 3\rangle$

H	1	2	3
1		2000	3000
2	4000		6000
3	8000	5000	





$\boxed{\text{size of Adj list} = O(n+m)}$

Scan for Video solution

10. **(c)**

The connected components of an undirected graph can be found by applying DFS in $\Theta(n + m)$.

Scan for Video solution

11. **(b)**

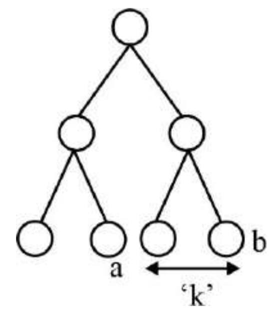
To, insert one element into an AVL – tree(n) = log n

Total time = $\theta(n^2.\log n)$ to insert n^2 elements.

Scan for Video solution

12. **(c)**

- Time of find 'a' and 'b' in binary search tree = O(logn)
- Time of sum up elements between a, b in binary search tree, using O(k) inorder sorting.



$\boxed{\text{Total time} = O(\log n + k)}$

Scan for Video solution

13. **(a)**

For both insertion and deletion the worst case time complexity in a binary search tree is $\Theta(\log n)$.

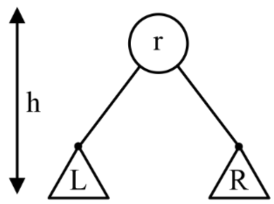
Scan for Video solution

14. **(995 to 995)**

Final output = [500 + 99 * 5] = 500 + 495 = 995.

Scan for Video solution

15. **(a)**



; Max-heap

Height of a heap with ‘n’ elements is logn.

Scan for Video solution

16. **(110 to 110)**

1. To locate ‘L” and ‘H’ in Binary Search Tree, time taken = O(log n)
2. Time to sum up all numbers between L and H, using

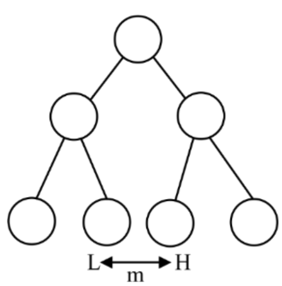
   In order sorting will be O(m)

   Total time: O(log n + m)

   a = 0; b = 1

   c = 1; d = 0

   (10 + 100 + 0) = 110



Scan for Video solution

17. **(b)**

Time to insert an element into a binary search tree (n) = logn.

Scan for Video solution

18. **(c)**

Time = $\log(n.2^n) = \log n + \log 2^n$

$T(n) = (\log n + n) = O(n) = \Omega(n) = \Theta(n)$.

Scan for Video solution

19. **(c)**

Tower of Hanoi with n disks takes Θ (2^n)

Binary search given n sorted numbers takes Θ (log n)

Heap sort given n numbers at the worst case takes Θ (n.log n)

Addition to two n × n matrices takes Θ (n^2)

Scan for Video solution

20. **(c)**

Time of radix sort: O(d(n +b))

d: Number of bits in the representation of highest number.

n: number of elements

b: base (radix)

$d = \log n^k = k.\log n$

$= k.\log n.n + k.\log n.b$

$= \Theta(n.\log n)$.

Scan for Video solution

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INDEX

Compiler Design

Compiler Design

Syllabus

Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation. Local optimization, Data flow analyses: constant propagation, liveness analysis, common subexpression elimination.

Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6	Ch.7
2008	0	0	3	0	0	1	2
2009	0	0	0	1	0	2	0
2010	1	0	2	0	0	0	1
2011	0	1	0	1	0	2	0
2012	0	0	4	0	0	0	2
2013	0	0	1	0	0	4	2
2014 (P1)	0	0	2	0	0	0	1
2014 (P2)	1	0	1	0	2	2	0
2014 (P3)	0	0	0	1	0	0	1
2015 (P1)	0	0	0	2	0	0	2
2015 (P2)	0	0	2	0	0	1	0
2015 (P3)	0	2	1	0	0	0	0
2016 (P1)	0	0	0	4	1	0	0
2016 (P2)	1	0	4	0	0	0	0
2017 (P1)	0	1	0	1	2	2	0
2017 (P2)	1	0	1	0	2	0	0
2018	0	2	0	2	0	0	1
2019	0	0	4	1	0	0	0
2020	0	0	0	2	0	0	1
2021 (P1)	0	0	1	4	2	0	0
2021 (P2)	1	0	0	1	2	3	0
2022	0	0	1	2	0	2	0
2023	1	0	0	2	0	2	0

CHAPTER

1

INTRODUCTION TO COMPILER

Phases of Compiler

1. [MCQ] **[GATE-2023 : 1M]**

Consider the following statements regarding the front-end and back-end of a compiler.

S1: The front-end includes phases that are independent of the target hardware.

S2: The back-end includes phases that are specific to the target hardware.

S3: The back-end includes phases that are specific to the programming language used in the source code.

Identify the CORRECT option.

(a) Only S1 is TRUE

(b) Only S1 and S2 are TRUE

(c) S1, S2, and S3 are all TRUE

(d) Only S1 and S3 are TRUE

2. [MCQ] **[GATE-2021 : 1M]**

Consider the following ANSI C program:

```
int main() {
    Integer x;
return ();
}
```

Which one of the following phases in a seven-phase C complier will throw an error?

(a) Lexical analyzer

(b) Syntax analyzer

(c) Semantic analyzer

(d) Machine dependent optimizer

3. [MCQ] **[GATE-2017 : 1M]**

Match the following according to input from List-I to the compiler phase in the List-II that processes it:

List-I	List-II
(P) Syntax tree	(i) Code generator
(Q) Character stream	(ii) Syntax analyser
(R) Intermediate representation	(iii) Semantic analyser
(S) Token stream	(iv) Lexical analyser

(a) P → (ii), Q → (iii), R → (iv), S → (i)

(b) P → (ii), Q → (i), R → (iii), S → (iv)

(c) P → (iii), Q → (iv), R → (i), S → (ii)

(d) P → (i), Q → (iv), R → (ii), S → (iii)

4. [MCQ] **[GATE-2016 : 1M]**

Match the following

List-I	List-II
P: Lexical analysis	(i) Leftmost derivation
Q: Top down parsing	(ii) Type checking
R: Semantic analysis	(iii) Regular expressions
S: Runtime environment	(iv) Activation records

(a) P → (i), Q → (ii), R → (iv), S → (iii)

(b) P → (iii), Q → (i), R → (ii), S → (iv)

(c) P → (ii), Q → (iii), R → (i), S → (iv)

(d) P → (iv), Q → (i), R → (ii), S → (iii)

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ANSWER KEY

1. (b)	**2.** (c)	**3.** (c)	**4.** (b)

SOLUTIONS

1. (b)

Phases of compiler:

LA, SyA, SeA ICG, CO,   CG, CO

M/C independent   M/C Dependent

(Frontend)   (Backend)

S1: The front-end includes phases that are independent of the target hardware.True

S2: The back-end includes phases that are specific to the target hardware. True

S3: The back-end includes phases that are specific to the programming language used in the source code. False

Therefore, option b is the correct asnwer.

Scan for Video solution

2. (c)

int main() {

Integer x;

return ();

}

Total 13 tokens

There is no lexical error.

In syntax analysis, the **Integer x;** is valid, and this will be checked in semantic analysis that whether **Integer** is a keyword or not. So, there is no syntax error.

In semantic analysis,

Integer x ;

id   id ;

Here, x has no type definition. **typedef int Integer;** is missing before **Integer x;** in given ANSI C program. Therfore, the given program is semantically incorrect.

Hence, option C is correct.

Scan for Video solution

3. (c)

List I representing the input to the phase and List-II has corresponding phases.

Syntax/Parse tree is an input to semantic analyser.

Character stream is source code and it is input to lexical analyser.

Intermediate representation is the intermediate code and it is input to code generator.

Token stream is output of lexical analyser and input to syntax analyser.

Hence, option c is correct.

Scan for Video solution

4. (b)

Lexical analyser can be designed/implemented with the help of regular expressions,

Top down parser follows leftmost derivation.

Semantic analysis does the type checking.

Runtime enviornment maintains the activation records.

Hence, option b is correct.

Scan for Video solution

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CHAPTER

2 LEXICAL ANALYSIS

Basics of Lexical Analysis

1. [MCQ] [GATE-2021 : 1M]

In a compiler, keywords of a language are recognized during

(a) parsing of the program

(b) the code generation

(c) the lexical analysis of the program

(d) dataflow analysis

2. [MCQ] [GATE-2021 : 1M]

Which data structure in a compiler is used for managing information about variables and their attributes?

(a) Abstract syntax tree

(b) Symbol table

(c) Semantic stack

(d) Parse table

3. [MCQ] [GATE-2011 : 1M]

The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

(a) Finite state automata

(b) Deterministic pushdown automata

(c) Non-deterministic pushdown automata

(d) Turing Machine

Finding the Tokens

4. [MCQ] [GATE-2018 : 2M]

A lexical analyzer uses the following patterns to recognize three tokens T_1, T_2 and T_3 over the alphabet {a, b, c}.

T_1: a ? (b | c)* a

T_2: b ? (a | c)* b

T_3: c ? (b | a)* c

Note that ′x? ′ means 0 or 1 occurrence of the symbol x. Note also that the analyzer outputs the token that matches the longest possible prefix.

If the string bbaacabc is processed by the analyzer, which one of the following is the sequence of tokens it outputs?

(a) T_1 T_2 T_3

(b) T_1 T_1 T_3

(c) T_2 T_1 T_3

(d) T_3 T_3

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ANSWER KEY

1.	(c)	**2.**	(b)	**3.**	(a)	**4.**	(d)

SOLUTIONS

1. (c)

In a compiler keywords, identifiers, constans, punctuations, and operators are recognized during the lexical analysis.

During the syntax analysis compiler does the verification of the syntax with the help of a parser.

During, the code generation compiler produes the assembly code.

Dataflow analysis can perform liveness analysis, available expression analysis, etc.

So, option c is the correct answer.

Scan for Video solution

2. (b)

Symbol table is used for managing information about variables and their attributes. Symbol table maintains all attributes information of each identifier recognized during the lexical analysis.

Hence, option c is the correct answer.

Scan for Video solution

3. (a)

Finite Automata(regular expression) is sufficient to represent all the tokens and identifiers used during lexical analysis.

Scan for Video solution

4. (d)

Given,

Here, x? means 0 or 1 occurrence of the symbol x.

We can represent x? as $\in + x$ using regular expression.

Given string: b b a a c a b c

T_1 can generate upto: bba

T_2 can generate upto bb

T_3 can generate upto **bbaac**

So, we can see T_3 can generate longest prefix of given string.

Again, check remaining portion of input for next longest prefix generator.

String left: a b c

T_1 cannot generate abc, it can generate abca but we require only abc

T_2 cannot generate abc, it can generate ab, but not abc

T_3 can generate **abc**.

So, $\mathbf{T_3T_3}$ is the correct sequence of tokens for the complete input. Hence. option d is correct answer.

Scan for Video solution

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CHAPTER

3 SYNTAX ANALYSIS

Context Free Grammar

1. [MCQ] [GATE-2017 : 2M]

Consider the following expression grammar G:

$E \to E - T \mid T$

$T \to T + F \mid F$

$F \to (E) \mid id$

Which of the following grammars is not left recursive, but is equivalent to G?

(a) E → E−T | T
T → T+F | F
F → (E) | id

(b) E → TE′
E → −TE′ | ε
T → T+F | F

(c) E → TX
X → −TX | ε
T → FY
Y → +FY | ε
F → (E) | id

(d) E → TX | (TX)
X → −TX | +TX | ε
T → id

Top Down Parser

2. [MCQ] [GATE-2021 : 2M]

Consider the following context-free grammar where the set of terminals is {a, b, c, d, f}

S → d a T | R f

T → a S | b a T | ε

R → c a T R | ε

The following is a partially-filled LL(1) parsing table

	a	b	c	d	f	$
S			①	S → da T	②	
T	T→aS	T→baT	③		T → ϵ	④
R			R → caTR		R → ϵ	

Which one of the following choices represents the correct combinations for the numbered cells in the parsing table (‘blank’ denotes that the corresponding cell is empty)?

(a) (1) S → R f
(2) S → R f
(3) T → ε
(4) T → ε

(b) (1) blank
(2) S → R f
(3) T → ε
(4) T → ε

(c) (1) S → R f
(2) blank
(3) blank
(4) T → ε

(d) (1) blank
(2) S → R f
(3) blank
(4) blank

3. [NAT] [GATE-2019 : 1M]

Consider the grammar given below:

S → Aa

A → BD

B → b | ε

D → d | ε

Let a, b, d, and $ be index as follows:

A	b	d	$
3	2	1	0

Compute the FOLLOW set of the non-terminal B and write the index values for the symbols in the FOLLOW set in the descending order. (For example, if the FOLLOW set is {a, b, d, $}, then the answer should be 3210)

4. [MCQ] [GATE-2017 : 1M]

Consider the following grammar:

P → xQRS

Q → yz | z

R → w | ε

S→y

What is FOLLOW(Q)?

(a) {R} (b) {w}

(c) {w,y} (d) {w,$}

(Common Data for next 2 questions)

5. [MCQ] [GATE-2012 : 2M]

For the grammar below, a partial LL(1) parsing table is also presented along with the grammar. Entries that need to be filled are indicated as E1, E2 and E3. ε is the empty string, $ indicates end of input, and | separates alternate right hand sides of productions.

$S \to aAbB \mid bAaB \mid \varepsilon$

$A \to S$

$B \to S$

	a	b	$
S	E1	E2	$S \to \varepsilon$
A	$A \to S$	$A \to S$	Error
B	$B \to S$	$B \to S$	E3

The FIRST and FOLLOW sets for the non-terminals A and B are

(a) FIRST (A)={a, b, ε}=FIRST (B)
FOLLOW (A)={a, b}
FOLLOW (B)={a, b, $}

(b) FIRST (A)={a, b, $}
FIRST (B)={a, b, ε}
FOLLOW (A)={a,b}
FOLLOW (B)={$}

(c) FIRST (A)={a, b, ε}=FIRST (B),
FOLLOW (A)={a,b}
FOLLOW (B)=$

(d) FIRST (A)={a,b}=FIRST (B)
FOLLOW (A)={a, b}
FOLLOW (B)={a, b}

6. [MCQ] [GATE-2012 : 2M]

The appropriate entries for E1, E2 and E3 are

(a) E1: $S \to aAbB$, $A \to S$
E2: $S \to bAaB$, $B \to S$
E3: $B \to S$

(b) E1: $S \to aAbB$, $S \to \varepsilon$
E2: $S \to bAaB$, $S \to \varepsilon$
E3: $S \to \varepsilon$

(c) E1: $S \to aAbB$, $S \to \varepsilon$
E2: $S \to bAaB$, $S \to \varepsilon$
E3: $B \to S$

(d) E1: $A \to S$, $S \to \varepsilon$
E2: $B \to S$, $S \to \varepsilon$
E3: $B \to S$

LR Parser

7. [MCQ] [GATE-2022 : 1M]

Which one of the following statements is TRUE?

(a) The LALR(1) parser for a grammar G cannot have reduce – reduce conflict if the LR(1) parser for G does not have reduce – reduce conflict.

(b) Symbol table is accessed only during the lexical analysis phase.

(c) Data flow analysis is necessary for run-time memory management.

(d) LR(1) parsing is sufficient for deterministic context – free languages.

8. [NAT] [GATE-2021 : 1M]

Consider the augmented grammar with {+, ∗ , (,), id} as the set of terminals.

$S' \to S$

$S \to S + R \mid R$

$R \to R * P \mid P$

$P \to (S) \mid id$

If I_0 is the set of two $LR(0)$ items {[S→S.], [S →S. + R]}, then $goto(closure(I_0), +)$ contains exactly ______ items.

9. [MCQ] [GATE-2021 : 1M]

Consider the following statements

S_1: Every SLR (1) grammar is unambiguous but there are certain unambiguous grammars that are not SLR(1).

S_2: For any context-free grammar, there is a parser that takes at most $O(n^3)$ time to parse a string of length n.

Which one of the following options is correct?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) S_1 is true and S_2 is true

(d) S_1 is false and S_2 is false

10. [NAT] [GATE-2021 : 2M]

Consider the following augmented grammar with {#, @,<, >,a, b, c} as the set of terminals

$S' \rightarrow S$

$S \rightarrow S \# cS$

$S \rightarrow SS$

$S \rightarrow S@$

$S \rightarrow <S>$

$S \rightarrow a$

$S \rightarrow b$

$S \rightarrow c$

Let I_0 = CLOSURE({S′ →"." S}). The number of items in the set GOTO(GOTO(I_0,<),<) is ________

11. [NAT] [GATE-2021 : 2M]

Consider the following grammar:

$S \rightarrow aSB \mid d$

$B \rightarrow b$

The number of reduction steps taken by a bottom-up parser while accepting the string aaadbbb is _______.

12. [MCQ] [GATE-2019 : 1M]

Which one of the following kinds of derivation is used by LR parsers?

(a) Leftmost

(b) Leftmost in reverse

(c) Rightmost

(d) Rightmost in reverse

13. [MCQ] [GATE-2017 : 1M]

Which of the following statements about parser is/are CORRECT?

I. Canonical LR is more powerful than SLR.

II. SLR is more powerful than LALR.

III. SLR is more powerful than Canonical LR.

(a) I only (b) II only

(c) III only (d) II and III only

14. [MCQ] [GATE-2015 : 2M]

Consider the following grammar G.

$S \rightarrow F \mid H$

$F \rightarrow p \mid c$

$H \rightarrow d \mid c$

Where S, F and H are non-terminal symbols, p, d and c are terminal symbols. Which of the following statements(s) is/are correct?

S_1: LL(1) can parse all strings that are generated using grammar G.

S_2: LR(1) can parse all strings that are generated using grammar G.

(a) Only S_1 (b) Only S_2

(c) Both S_1 and S_2 (d) Neither S_1 and S_2

15. [MCQ] [GATE-2015 : 1M]

Among simple LR (SLR), canonical LR, and look-ahead LR (LALR), which of the following pairs identify the method that is very easy to implement and the method that is the most powerful, in that order?

(a) SLR, LALR

(b) Canonical LR, LALR

(c) SLR, canonical LR

(d) LALR, canonical LR

16. [MCQ] [GATE-2015 : 1M]

Which one of the following is TRUE at any valid state in shift-reduce parsing?

(a) Viable prefixes appear only at the bottom of the stack and not inside

(b) Viable prefixes appear only at the top of the stack and not inside

(c) The stack contains only a set of viable prefixes

(d) The stack never contains viable prefixes

17. [MCQ] [GATE-2014 : 1M]

A canonical set of items is given below

$S \rightarrow L. > R$

$Q \rightarrow R.$

On input symbol > the set has

(a) a shift-reduce conflict and a reduce-reduce conflict.

(b) a shift-reduce conflict but not a reduce-reduce conflict.

(c) a reduce-reduce conflict but not a shift-reduce conflict.

(d) neither a shift-reduce nor a reduce-reduce conflict.

18. [MCQ] [GATE-2015 : 2M]

Consider the following two sets of LR(1) items of LR(1) grammar.

X → c.X, c/d	X → c.X, $
X → .cX, c/d	X → .cX, $
X → .d, c/d	X → .d, $

Which of the following statement related to merging of the two sets in the corresponding LALR parser is/are FALSE?

1. Cannot be merged since look aheads are different.
2. Can be merged but will result in S-R conflict.
3. Can be merged but will result in R-R conflict.
4. Cannot be merged since goto on c will lead to two different sets

(a) 1 only (b) 2 only

(c) 1 and 4 only (d) 1, 2, 3 and 4

19. [MCQ] [GATE-2013 : 1M]

What is the maximum number of reduces moves that can be taken by a bottom up parser for a grammar with no epsilon and unit productions (i.e. of type $A \rightarrow \varepsilon$ and $A \rightarrow a$) to parse a string with n tokens?

(a) n/2 (b) n – 1

(c) 2n – 1 (d) 2^n

20. [MCQ] [GATE-2010 : 2M]

The grammar $S \rightarrow aSa \mid bS \mid c$ is

(a) LL(1) but not LR(1)

(b) LR(1) but not LL(1)

(c) Both LL(1) and LR(1)

(d) Neither LL(1) nor LR(1)

21. [MCQ] [GATE-2008 : 2M]

An LALR(1) parser for a grammar G can have shift-reduce (S-R) conflicts if and only if

(a) The SLR(1) parser for G has S-R conflict

(b) The LR(1) parser for G has S-R conflicts

(c) The LR(0) parser for G has S-R conflicts

(d) The LALR(1) parser for G has reduce-reduce conflicts.

22. [MCQ] [GATE-2008 : 1M]

Which of the following describes a handle (as applicable to LR-parsing) appropriately?

(a) It is the position in a sentential form where the next shift or reduce operation will occur.

(b) It is non-terminal whose production will be used for reduction in the next step

(c) It is a production that may be used for reduction in a future step along with a position in the sentential form where the next shift or reduce operation will occur.

(d) It is the production p that will be used for reduction in the next step along with a position in the sentential form where the right-hand side of the production may be found.

23. [NAT] [GATE-2019 : 2M]

Consider the augmented grammar given below:

$S' \rightarrow S$

$S \rightarrow \langle L \rangle \mid id$

$L \rightarrow L, S \mid S$

Let I_0 = CLOSURE ({[S' → ·S]}). The number of items in the set GOTO (I_0, ⟨) is: __________.

Operator Precedence Parsing

24. [MCQ] [GATE-2014 : 1M]

Consider the grammar defined by the following production rules, with two operator * and +

S → T * P

T → U | T * U

P → Q + P | Q

Q → Id

U → Id

Which one of the following is TRUE?

(a) + is left associative, while * is right associativ

(b) + is right associative, while * is left associative

(c) Both + and * are right associative

(d) Both + and * are left associative

25. [NAT] [GATE-2016 : 2M]

The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
–	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression **2 – 5 + 1 – 7 * 3** in this language is ________.

❑❑❑

ANSWER KEY

1.	(c)	**2.**	(b)	**3.**	(31 to 31)	**4.**	(c)
5.	(a)	**6.**	(c)	**7.**	(d)	**8.**	(5 to 5)
9.	(c)	**10.**	(8 to 8)	**11.**	(7 to 7)	**12.**	(d)
13.	(a)	**14.**	(d)	**15.**	(c)	**16.**	(c)
17.	(d)	**18.**	(d)	**19.**	(b)	**20.**	(c)
21.	(b)	**22.**	(d)	**23.**	(5 to 5)	**24.**	(b)
25.	(9 to 9)						

SOLUTIONS

1. (c)

Eliminating left recursion from the given expression grammar:

I. E → E –T | T will be converted:

$$\boxed{\begin{array}{c} E \rightarrow TX \\ X \rightarrow -TX \mid \varepsilon \end{array}}$$

II. T → T + F | F will be converted:

$$\boxed{\begin{array}{c} T \rightarrow FY \\ Y \rightarrow +FY \mid \varepsilon \end{array}}$$

III. $\boxed{F \rightarrow (E) \mid id}$ is same as it is already free from left recursion

The resultant grammar after removing left-recursion is

E → TX

X → −TX | ε

T → FY

Y → +FY | ε

F → (E) | id

The given expression grammar is equivalent to grammar in option C.

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2. (b)

To find out correct combinations for the numbered cells in the parsing table, we need to first find out FIRST and FOLLOW of S, T, and R.

FIRST(S) = {d, c, f}
FOLLOW(S) = {c, f, $}
FIRST(T) = {a, b, e}
FOLLOW(T) = {$, c, f}
FIRST(R) = {c, ε }
FOLLOW(R) = {f}

So,

1. S → Rf [This production can produce c]
2. S → Rf [This production can produce f]
3. T → ε
4. T → ε

So, option A is the correct answer

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3. (31 to 31)

Sol- Follow(B) = {d, a}
Index for d is 3
Index for a is 1

So, 31 is the correct answer.

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4. **(c)**

Follow of Q: It depends on First of RS

$P \rightarrow \times \boxed{Q} R S$

$\quad\quad\quad\quad\quad w \; y$

Follow(Q) = {w, y}

Option c is the correct answer.

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5. **(a)**

First (S) = {a, b, e}

Follow(S) = {$, a, b}

First(A) = {a, b, ε}

First (B) = {a, b, ε}

Follow (A) = {a, b}

Follow (B) = {$, a, b}

So, option A is the correct answer.

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6. **(c)**

Given,

$S \rightarrow aAbB \mid bAaB \mid \varepsilon$

$A \rightarrow S$

$B \rightarrow S$

	a	b	$
S	E1	E2	$S \rightarrow \varepsilon$
A	$A \rightarrow S$	$A \rightarrow S$	Error
B	$B \rightarrow S$	$B \rightarrow S$	E3

First (S) = {a, b, ε }

Follow(S) = {$, a, b}

E_1 : {$S \rightarrow a\ AbB$, $S \rightarrow \varepsilon$ }

E_2 : {$S \rightarrow bAaB$, $S \rightarrow \varepsilon$ }

E_3: {$B \rightarrow S$}

Therefore, option C is the correct answer.

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7. **(d)**

(a) **FALSE.** The LALR(1) parser for a grammar G may have reduce – reduce conflict if the LR(1) parser for G does not have reduce – reduce conflict.

(b) **FALSE**. Symbol table can be accessed in all phases of compiler.

(c) **FALSE.** Data flow analysis is required to imporve the code but not necessary for run-time memory mangement.

(d) **TRUE.** Every LR(1) CFG going to generate DCFL.

Hence, option d is the correct answer.

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8. **(5 to 5)**

$$\boxed{\begin{array}{l} S' \rightarrow S. \\ S \rightarrow S.+R \end{array}} \xrightarrow{+} \boxed{\begin{array}{l} S \rightarrow S+.R \\ R \rightarrow .R*P \\ R \rightarrow .P \\ P \rightarrow .(S) \\ P \rightarrow .id \end{array}}$$

So, there are total 5 items in the goto(closure(I_0), +).

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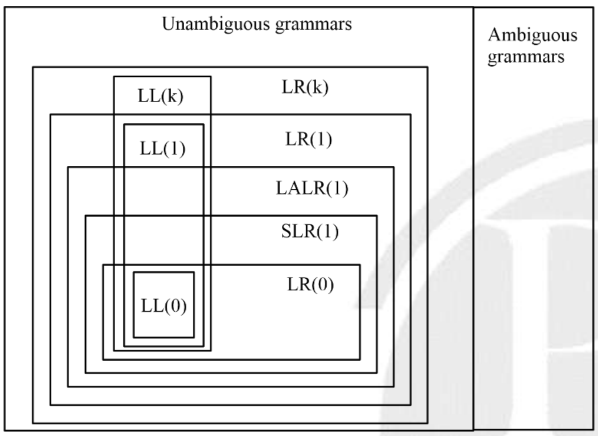
9. (c)

S_1: Every SLR (1) grammar is unambiguous but there are certain unambiguous grammars that are not SLR(1). **TRUE.** For example, LALR(1), CLR(1) are certain unambiguous grammars which are not SLR(1).



S_2: For any context-free grammar, there is a parser that takes at most $O(n^3)$ time to parse a string of length n. **TRUE.** For example, CYK algorithm takes $O(n^3)$ time to parse a string of length n.

So, option c is the correct answer.

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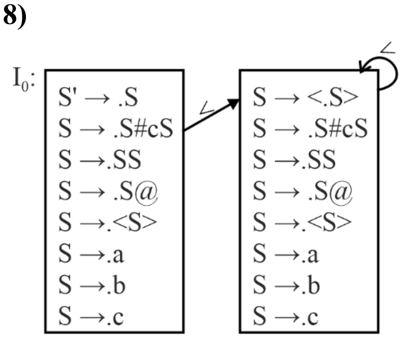
10. (8 to 8)



There are 8 items in the set $GOTO(GOTO(I_0,<),<)$.

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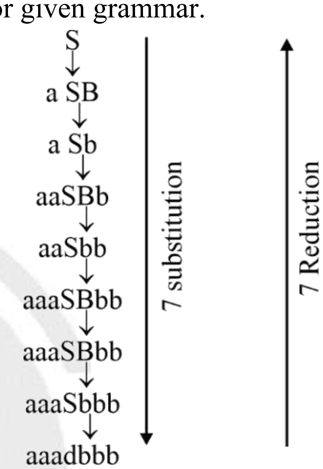
11. (7 to 7)

Bottom-up parser follows reverse of RMD (Right most derivation).

And, Number of reduction steps in reverse of RMD = Number of substitutions in RMD

RMD for given grammar.



So, total 7 number of reduction steps taken by bottom-up parser while verifying the string aaadbbb.

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12. (d)

LR parser also known as bottom-up parser and bottom-up parser uses rightmost derivation in reverse.

Hence, option d is the correct answer.

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13. (a)

SLR is less powerful than LALR and LALR is less powerful than CLR.

SLR < LALR < CLR.

I. Canonical LR is more powerful than SLR. **TRUE**

II. SLR is more powerful than LALR. **FALSE**

III. SLR is more powerful than Canonical LR. **FALSE**

Statement I is only correct, option a is the right answer.

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14. (d)

If we want to drive the string "c", it can be done in two ways S → F→c or S → H → c

So, we can say the given grammar is ambiguous. With the ambiguous grammar, parser cannot parse the string. So, in case of LL(1) or LR(1) parser is going to have a problem, that's why both the statements are incorrect. Hence, option D is the right answer.

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15. (c)

Easy to implement, Most powerful.

SLR is easy to implement and CLR is most powerful. So SLR, CLR, is the correct answer.

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16. (c)

(a) INCORRECT. Viable prefixes can be present anywhere in the stack.

(b) INCORRECT. Viable prefixes can be present anywhere in the stack.

(c) CORRECT. The stack contains only a set of viable prefixes.

(d) INCORRECT. The stack contains viable prefixes.

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17. (d)

$$\boxed{\begin{array}{l} S \rightarrow L.>R \\ Q \rightarrow R. \end{array}} \xrightarrow{>} \boxed{S \rightarrow L>.R}$$

This state does not have any conflict; therefore, option D is the correct answer.

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18. (d)

There is no conflicts in the given sets and there are no reduced items.

So,

1. "Cannot be merged since look aheads are different" is FALSE. Because they can be merged when lookaheads are different.
2. "Can be merged but will result in S-R conflict" is FALSE. Because no conflict present.
3. "Can be merged but will result in R-R conflict" is FALSE. Because no conflict when merged.

X → C**.**X, C/d/$

X → **.**CX, C/d/$

X → **.**d, C/d/$

4. "Cannot be merged since goto on c will lead to two different sets" FALSE. While merging we do not look at goto.

So, option D is the correct answer.

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19. (b)

n = 2 ⇒ 1 reduced move

n = 2 ⇒ 2 reduced moves

n = 4 ⇒ 3 reduced moves

w = ab	w = abc	w = abcd
S ↓ ab	S ↓ S → a × ax ↓ X → bc ↓ abc	S ↓ AB ↓ Acd ↓ abcd

For n length string we need n-1 moves. So, option B is the correct answer.

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20. **(c)**

Given,

S → aSa | bS | c

First check for LL(1) because Every LL(1) is LR(1)

The given grammar is LL(1). So, option C is the correct answer.

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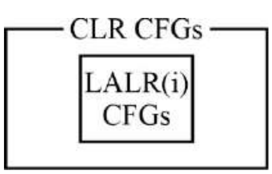
21. **(b)**

LALR parser uses the LR(1) items of CLR parser. LALR will contain SR conflicts if and only if LR parser has SR conflicts.

LALR can have RR conflicts even though LR parser do not have RR conflicts.



So, option B is correct answer

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22. **(d)**

In a production whole RHS part is known as handle. It can be defined as the production p that will be used for reduction in the next step along with a position in the sentential form where the right-hand side of the production may be found.

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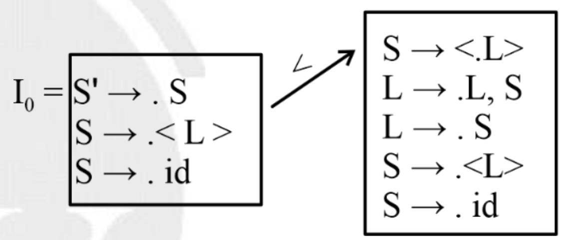
23. **(5 to 5)**



There are total 5 items in in the set GOTO (I_0, ⟨).

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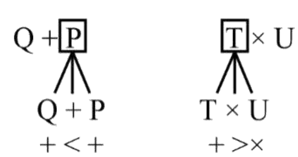
24. **(b)**

Operator < Operator	Operator > Operator
Right associate	Left associate



+ is right associative

* is left associative

So, option B is correct.

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25. (9 to 9)

Given,

+ has highest prcedence and left associative.

– has medium precedence and right associative.

* has lowest precedence and left associative.

Expression = 2 – 5 + 1 – 7 * 3

= 2- [5+1] – 7 * 3

= 2 – 6 – 7 * 3

= [2- [6 - 7]]*3

= [2-(-1)] * 3

= 3 * 3

= 9

So, 9 is the correct answer.

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CHAPTER

4

SYNTAX DIRECTED TRANSLATION

Basics of Syntax Directed Translation

1. [MCQ] [GATE-2021 : 2M]

Consider the following grammar (that admits a series of declarations, followed by expressions) and the associated syntax directed translation (SDT) actions, given as pseudo-code:

$P \rightarrow D^* E^*$

$D \rightarrow$ int ID {record that ID, lexeme is of type int}

$D \rightarrow$ bool ID {record that ID. lexeme is of type bool}

$E \rightarrow E_1 + E_2${check that E_1. Type = E_2. Type = int; set E. type := int}

$E \rightarrow\ !E_1${check that E_1. Type = bool; set E. type: = bool}

$E \rightarrow$ ID{set E type : = int}

With respect to the above grammar, which one of the following choices is correct?

(a) The actions can be used to correctly type-check any syntactically correct program

(b) The actions can be used to type-check syntactically correct integer variable declarations and integer expressions

(c) The actions can be used to type-check syntactically correct boolean variable declarations and boolean expressions

(d) The actions will lead to an infinite loop

2. [MCQ] [GATE-2016:2M]

A student wrote two context-free grammars G1 and G2 for generating a single C-like array declaration. The dimension of the array is at least one. For example,

int a[10][3];

The grammars use D as the start symbol, and use six terminal symbols int; id [ ] num.

Grammar G1	**Grammar G2**
D → int L;	D → int L;
L → id [E	L → id E
E → num]	E → E[num]
E → num][E	E → [num]

Which of the grammars correctly generate the declaration mentioned above?

(a) Both G1 and G2 (b) Only G1

(c) Only G2 (d) Neither G1 nor G2

Evaluation of Syntax Directed Translation

3. [NAT] [GATE-2023: 2M]

Consider the syntax directed translation given by the following grammar and semantic rules. Here N, I, F and B are non-terminals. N is the starting non-terminal, and #, 0 and 1 are lexical tokens corresponding to input letters "#", "0" and "1", respectively. X.val denotes the synthesized attribute (a numeric value) associated with a non-terminal X. I_1 and F_1 denote occurrences of *I* and *F* on the right hand side of a production, respectively. For the tokens 0 and 1, 0.val = 0 and 1.val = 1.

N	→	I#F	N. *val* = I.*val* + F.*val*
I	→	I_1 B	I. *val* = (2 I_1.val) + B. *val*
I	→	B	I. *val* = B.val
F	→	B F_1	F. *val* = ½ (B. *val* + F_1. *val*)
F	→	B	F. *val* = ½ B. *val*
B	→	0	B. *val* = 0. *val*
B	→	1	B. *val* = 1. *val*

The value computed by the translation scheme for the input string 10 # 011 is_________.(Rounded off to three decimal places)

4. [NAT] [GATE-2022 : 2M]

Consider the following grammar along with translation rules.

$S \to S_1 \# T \quad \{S_{.val} = S_{1 \cdot val} * T_{\cdot val}\}$

$S \to T \quad \{S_{.val} = T_{\cdot val}\}$

$T \to T_1 \% R \quad \{T_{.val} = T_{1 \cdot val} \div R_{\cdot val}\}$

$T \to R \quad \{T_{.val} = R_{\cdot val}\}$

$R \to id \quad \{R_{.val} = id_{\cdot val}\}$

Here # and % are operators and id is a token that represents an integer and $id_{.val}$ represents the corresponding integer value. The set of non-terminals is {S, T, R, P} and a subscripted non – terminal indicates an instance of the non – terminal. Using this translation scheme, the computed value of $S._{val}$ for root of the parse tree for the expression 20 #10%5#8%2%2 is _________.

5. [MCQ] [GATE-2020 : 2M]

Consider the productions $A \to PQ$ and $A \to XY$. Each of the five non-terminals A, P, Q, X and Y has two attributes: s is a synthesized attribute, and i is an inherited attribute. Consider the following rules.

Rule 1: P .i = A .i+2, Q .i = P .i + A .i and A .s = P .s + Q .s

Rule 2: X .i = A .i + Y .s and Y .i = X .s + A .i

Which one of the following is TRUE?

(a) Neither Rule 1 nor Rule 2 is L-attributed.

(b) Only Rule 1 is L-attributed.

(c) Both Rule 1 and Rule 2 are L-attributed.

(d) Only Rule 2 is L-attributed.

6. [MCQ] [GATE-2019 : 1M]

Consider the following grammar and the semantic actions to support the inherited type declaration attributes. Let X_1, X_2, X_3. X_4, X_5, and X_6 be the placeholders for the non-terminals D, T, L or L_1 in the following table:

Production rule	Semantic action
$D \to TL$	X_1.type = X_2.type
$T \to int$	T.type = int
$T \to float$	T.type = float
$L \to L_1$, id	X_3.type = X_4.type add Type (id.entry, X_5.type)
$L \to id$	add Type (id.entry, X_6.type)

Which one of the following are the appropriate choice for X_1, X_2, X_3 and X_4?

(a) $X_1 = L$, $X_2 = T$, $X_3 = L_1$, $X_4 = L$

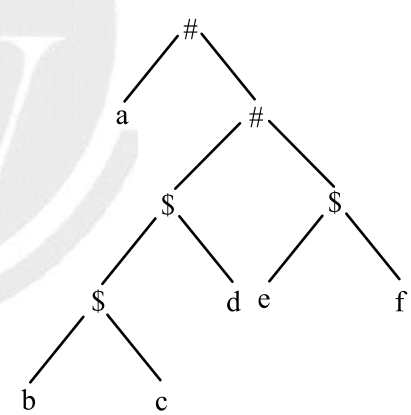
(b) $X_1 = T$, $X_2 = L$, $X_3 = L_1$, $X_4 = T$

(c) $X_1 = L$, $X_2 = L$, $X_3 = L_1$, $X_4 = T$

(d) $X_1 = T$, $X_2 = L$, $X_3 = T$, $X_4 = L_1$

7. [MCQ] [GATE-2018 : 2M]

Consider the following parse tree for the expression $a \# b \$ c \$ d \# e \# f$, involving two binary operators \$ and #.



Which one of the following is correct for the given parse tree?

(a) \$ has higher precedence and is left associative; # is right associative

(b) # has higher precedence and is left associative; \$ is right associative

(c) \$ has higher precedence and is left associative; # is left associative

(d) # has higher precedence and is right associative; \$ is left associative.

8. [MCQ] [GATE-2016 : 2M]

Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals {S, A} and terminals {a, b}.

$S \rightarrow aA$	{print 1}
$S \rightarrow a$	{print 2}
$A \rightarrow Sb$	{print 3}

Using the above SDTS, the output printed by a bottom-up parser, for the input aab is:

(a) 132 (b) 223
(c) 231 (d) Syntex error

9. [NAT] [GATE-2016 : 2M]

The attribute of three arithmetic operators in some programming language are given below.

OPERATOR	PRECEDENCE	ASSOCIATIVITY	ARITY
+	High	Left	Binary
-	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression 2 – 5 + 1 – 7 * 3 in this language is __________.

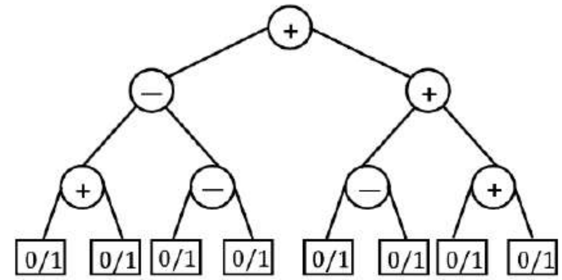
10. [MCQ] [GATE-2014 : 1M]

One of the purposes of using intermediate code in compilers is to

(a) make parsing and semantic analysis simpler.
(b) improve error recovery and error reporting.
(c) increase the chances of reusing the machine-independent code optimizer in other compilers
(d) improve the register allocation.

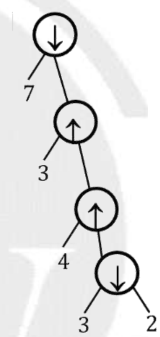
11. [NAT] [GATE-2014 : 2M]

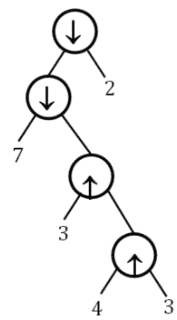
Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is ________.

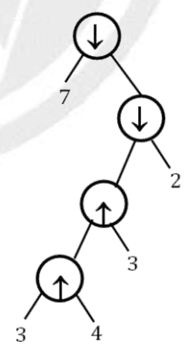


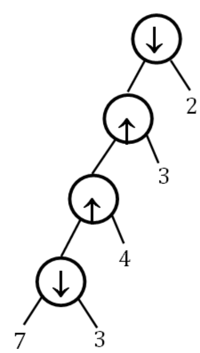
12. [MCQ] [GATE-2021 : 1M]

Consider two binary operators '$\uparrow$' and '$\downarrow$' with the precedence of operator $\downarrow$ being lower than that of the operator $\uparrow$. Operator $\uparrow$ is right associative while operator $\downarrow$ is left associative. Which one of the following represents the parse tree for expression ($7 \downarrow 3 \uparrow 4 \uparrow 3 \downarrow 2$)

(a)


(b)


(c)


(d)


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ANSWER KEY

1.	(b)	**2.**	(a)	**3.**	(2.375 to 2.375)	**4.**	(80 to 80)
5.	(b)	**6.**	(a)	**7.**	(a)	**8.**	(c)
9.	(9 to 9)	**10.**	(c)	**11.**	(6 to 6)	**12.**	(b)

SOLUTIONS

1. (b)

"The actions can be used to correctly type-check any syntactically correct program" is **FALSE.** Here we are not discussing the whole program, because program has lots of syntax.

"The actions can be used to type-check syntactically correct integer variable declarations and integer expressions" **TRUE**. With integer variable we can correctly represent the expressions. We can clearly see ID gets integer type whether keyword int or bool is used in the declaration of variable.

Therefore, option B is correct.

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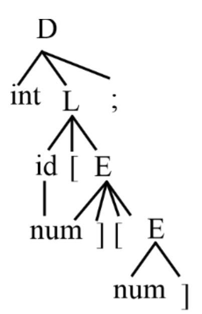
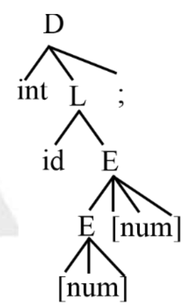
2. (a)

Given two Grammars G1 and G2 can generate all array declarations. We can see approach to derive any declaration using parse tree. Example: int a[10][3];

Using G1:



Using G2:



So, both G1 and G2 can generate the array declaration. So, option A is the correct answer.

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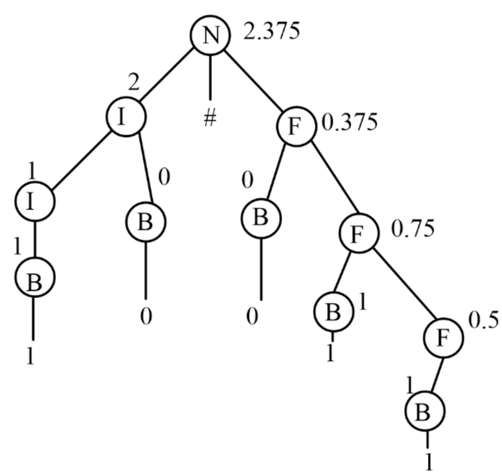
3. (2.375 to 2.375)

The parse tree for the given SDT is



The end of the production is 2.375.

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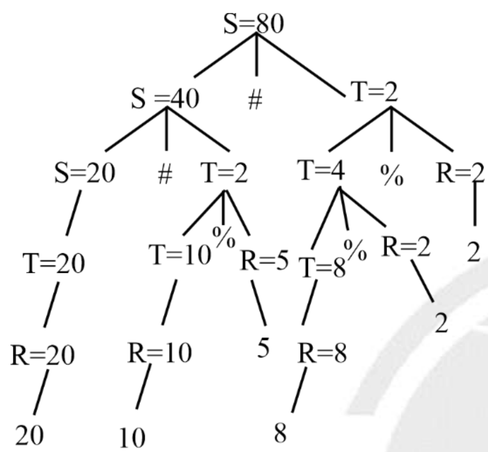
4. **(80 to 80)**

Parse tree for the given expression:



Scan for Video solution

5. **(b)**

Given,

A → PQ

A → XY

Rule 1 is ⇒ L. attributed

Rule 2 is ⇒ Not L-attributed.

So, option b is correct.

Scan for Video solution

6. **(a)**

If a production is of type S → XY, this means Y can inherit values from S or X, and similarly X can inherit value from S, and S can depend on both X and Y.

In the given productions,

T → int, Here T gets integer type using child.

D → TL, Here L gets type from T.

L → L_1, id, Here L_1 can inherit the type from L.

This means X_3 must be L_1 and X_4 must be L.

So, option A is the correct answer.

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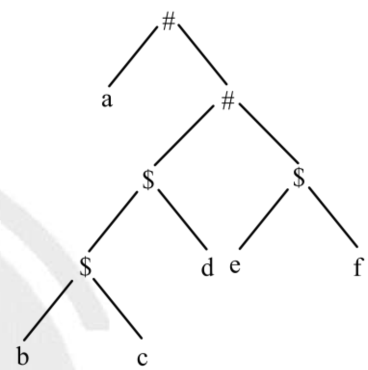
7. **(a)**



$ has highest precedence. $ must be evaluated before other operators in the above tree.

$ is left associative. If we see bcd, first left $ will be evaluated.

is right associative. We can see right # will be evaluated first when both times # happens in the expression.

So, option a is correct.

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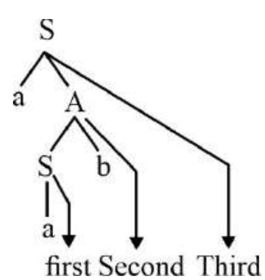
8. **(c)**

Input : aab

Output: 231



First will print: 2

Second will print: 3

Third will print: 1

Output: 231

So, option C is the correct answer.

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9. **(9 to 9)**

Given,

2 – 5 + 1 – 7 * 3

⇒ 2 – (5 + 1) – 7 * 3 // + has higher precedence

⇒ (2 – (6 – 7)) * 3 // - has higher precedence and it is right associative.

⇒ 3 * 3 = 9

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10. **(c)**

One of the purposes of using intermediate code in compilers is to increase the chances of reusing the machine-independent code optimizer in other compilers.

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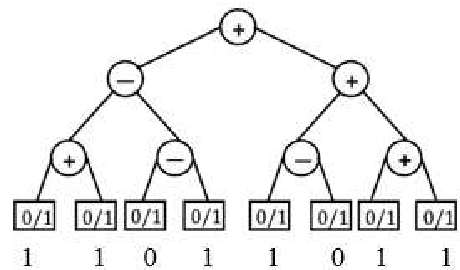
11. **(6 to 6)**

Putting the 0 and 1 accordingly to get maximum possible value.



Here, both left and right subtree can give value 3 and the resulting value will be 3+3 = 6

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12. **(b)**

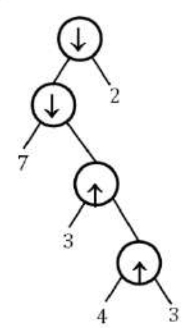
Given,

1. Precedence of operator ↓ being lower than that of the operator ↑
2. Operator ↑ is right associative while operator ↓ is left associative.
3. Expression: (7 ↓ 3 ↑ 4 ↑ 3 ↓ 2)

   So,

   ((7 ↓ (3 ↑ 4 ↑ 3)) ↓ 2)

Therefore, the tree corresponding to the above expression is



Therefore, option B is the correct answer.

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CHAPTER

5

INTERMEDIATE CODE GENERATION

Basics of ICG

1. [MCQ] [GATE-2021 : 1M]

In the context of compilers, which of the following is/are NOT an intermediate representation of the source program?

(a) Three address code

(b) Abstract Syntax Tree (AST)

(c) Control Flow Graph (CFG)

(d) Symbol table

2. [NAT] [GATE-2017 : 2M]

Consider the expression (a-1)*(((b+c)/3)+d).

Let X be the minimum number of registers required by an optimal code generation (without any register spill) algorithm for a load/store architecture, in which

(i) only load and store instructions can have memory operands and

(ii) arithmetic instructions can have only register or immediate operands.

The value of X is______.

3. [MCQ] [GATE-2017 : 1M]

Consider the following intermediate program in three address code

p = a – b

q = p * c

p = u * v

q = p + q

Which one of the following corresponds to a static single assignment form of the above code?

(a) $p_1 = a - b$
$q_1 = p_1 * c$
$p_1 = u * v$
$q_1 = p_1 + q_1$

(b) $p_3 = a - b$
$q_4 = p_3 * c$
$p_4 = u * v$
$q_5 = p_4 + q_4$

(c) $p_1 = a - b$
$q_1 = p_2 * c$
$p_3 = u * v$
$q_2 - p_4 + q_3$

(d) $p_1 = a - b$
$q_1 = p * c$
$q_2 = u * v$
$q_2 = p + q$

4. [NAT] [GATE-2016 : 1M]

Consider the following code segment.

x = u − t;

y = x ∗ v;

x = y + w;

y = t − z;

y = x ∗ y;

The minimum number of total variables required to convert the above code segment to static single assignment form is ______.

5. [MCQ] [GATE-2014 : 2M]

For a C program accessing $X[i][j][k]$, the following intermediate code is generated by a compiler. Assume that the size of an integer is 32 bits and the size of a character is 8 bits.

t0 = i∗1024

t1 = j∗32

t2 = k∗4

t3 = t1+t0

t4 = t3+t2

t5 = X[t4]

Which one of the following statements about the source code for the C program is CORRECT?

(a) X is declared as "int X[32][32][8]"".

(b) X is declared as "X[4][1024][32]".

(c) X is declared as "char X[4][32][8]".

(d) X is declared as "char X[32][16][2]".

Three Address Code

6. [NAT] [GATE-2021 : 1M]

The least number of temporary variables required to create a three-address code in static single assignment form for the expression

q+r/3+s−t∗5+u∗v/w is _______.

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ANSWER KEY

1.	(d)	**2.**	(2 to 2)	**3.**	(b)	**4.**	(10 to 10)
5.	(a)	**6.**	(8 to 8)				

SOLUTIONS

1. **(d)**

In the context of compilers, symbol table is not an intermediate representation of the source program. Symbol table is a table used to store attributes information and it is used in all the phases of compilers. Three address code, AST, Control Flow Graph, and DAG are intermediate representations.

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2. **(2 to 2)**

X is minimum number of registers without any register spill.

So,

$R_1 \leftarrow b$

$R_2 \leftarrow c$

$R_1 \leftarrow R_1 + R_2$

$R_1 \leftarrow R_1 / 3$

$R_2 \leftarrow d$

$R_1 \leftarrow R_1 + R_2$

$R_2 \leftarrow a$

$R_2 \leftarrow R_2 - 1$

$R_1 \leftarrow R_2 \times R_1$

Memory operands a, b, c, and d are used only with load/store instructions. In all arithmetic instructions, we have used only register or immediate operand(constant). Registers R1 and R2 are enough to covert given expression into assembly code without any additional registers. So, Minimum number of registers required is 2.

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3. **(b)**

The corresponding single static assignment is

$p_3 = a - b$

$q_4 = p_3 * c$

$p_4 = u * v$

$q_5 = p_4 + q_4$

All of a, b, c, u, v, p_3, p_4, q_4, and q_5 have single static assignment.

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4. **(10 to 10)**

$y = x * y$

$= (y + w) * (t - z)$

$= ([x * v] + w) * (t - z)$

$y = ([u - t] * v + w) * (t - z)$

$u_1 = u - t$

$u_2 = u_1 - v$

$u_3 = u_2 - w$

$t_1 = t - z$

$u_4 = u_3 - t_1$

Total variables, u, v, t, w, z, u_1, u_2, u_3, u_4, t_1.

So, total 10 variables are required for static single assignment form.

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5. **(a)**

t5 $= X[t_4]$

$= X[t_3 + t_2]$

$= X[t_1 + t_0 + k \times 4]$

$= X[j \times 32 + i \times 1024 + k \times 4]$

t5 = $X[i \times 1024 + j \times 32 + k \times 4]$

Note: & X[i][j][[k] = & X[0][0][0] + (i×qr + j×r + k) × size of elements

t5 $= X[i \times 256 + j \times 8 + k] \times 4 = X[i \times 32 \times 8 + j \times 8 + k] \times 4$

Comparing it with options, we can say X is declared as "int X[32][32][8]"". Therefore, option A is the correct answer.

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6. **(8 to 8)**

For the given expression: q+r/3+s−t∗5+u∗v/w, the corresponding three address code is:

r = r/3
t = t × 5
u = u × v
u = u/w
q = q + r
q = q + s
q = q − t
q = q + 4

Now, this optimized three address code have 7 variables (r, t, u, v, w, q, s).

To produce the SSA form, we will get one new temporary variable in each instruction.

SSA code will have 15 total variables (r, t, u, v, w, q, s, r_1, t_1, u_1, u_2, q_1, q_2, q_3, q_4). So, we have 7 variables already defined outside expression and 8 temporary variables used in the expression.

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CHAPTER

6 CODE OPTIMIZATION

Basics of Code Optimization

1. [MCQ] [GATE-2022 : 2M]

For a statement S in a program, in the context of liveness analysis, the following sets are defined

USE(S) : the set of variables used in S

IN (S) : the set of variables that are live at the entry of S

OUT (S) : the set of variables that are live at the exit of S

Consider a basic block that consists of two statements, S_1 followed by S_2.

Which one of the following statements is correct?

(a) OUT (S_1) = IN(S_2)

(b) OUT (S_1) = IN(S_1) ∪ USE(S_1)

(c) OUT (S_1) = IN(S_2) ∪ OUT(S_2)

(d) OUT (S_1) = USE(S_1) ∪ IN(S_2)

2. [MCQ] [GATE-2021 : 1M]

Consider the following ANSI C code segment:

```
z = x + 3 + y →f1 + y → f2;
for (i = 0; i < 200; i = i + 2) {
  if (z > i) {
    p = p + x + 3;
    q = q + y → f1;
  } else {
    p = p + y → f2;
    q = q + x + 3;
  }
}
```

Assume that the variable y points to a struct (allocated on the heap) containing two fields f1 and f2, and the local variables x, y, z, p, q, and i are allotted registers. Common sub-expression elimination (CSE) optimization is applied on the code. The number of addition and dereference operations (of the form y → f1 or y → f2) in the optimized code, respectively, are:

(a) 403 and 102 (b) 203 and 2

(c) 303 and 102 (d) 303 and 2

3. [MCQ] [GATE-2009 : 1M]

Which of the following statements are TRUE?

I: There exist parsing algorithms for some programming languages whose complexities are less than $\theta(n^3)$.

II: A programming language which allows recursion can be implemented with static storage allocation.

III: No L-attributed definition can be evaluated in the framework of bottom-up parsing.

IV: Code improving transformations can be performed at both source language and intermediate code level.

(a) I and II (b) I and IV

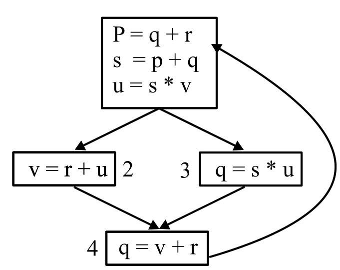
(c) III and IV (d) I, III and IV

4. [MCQ] [GATE-2015 : 2M]

A variable x is said to be live at a statement S_i in a program if the following

1. three conditions hold simultaneously:
2. There exists a statement S_j that uses x
3. There is a path from S_i to S_j in the flow graph corresponding to the program

The path has no intervening assignment to x including at S_i and S_j



The variables which are live both at the statement in basic block 2 and at the statement in basic block 3 of the above control flow graph are:

(a) p, s, u (b) r, s, u
(c) r, u (d) q, v

5. [MCQ] [GATE-2014 : 2M]

Consider the basic block given below.

a = b + c
c = a + d
d = b + c
e = d − b
a = e + b

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively are

(a) 6 and 6 (b) 8 and 10
(c) 9 and 12 (d) 4 and 4

6. [MCQ] GATE-2014 : 1M]

Which one of the following is FALSE?

(a) A basic block is a sequence of instructions where control enters the sequence at the beginning and exits at the end.
(b) Available expression analysis can be used for common subexpression elimination.
(c) Live variable analysis can be used for dead code elimination.
(d) $x = 4 \times 5 \Rightarrow x = 20$ is an example of common subexpression elimination.

7. [NAT] [GATE-2014 : 2M]

The minimum number of arithmetic operations required to evaluate the polynomial $P(X) = X^5 + 4X^3 + 6X + 5$ for a given value of X, using only one temporary variable is_________.

Common Data for Next two Questions:

The following code segment is executed on a processor which allows only register operands in its instructions. Each instruction can have atmost two source operands and one destinations operand. Assume that all variables are dead after this code segment.

```
c = a + b;
d = c * a;
e = c + a;
x = c * c;
if (x > a) {
   y = a * a;}
 else {
 d = d * d;
 e = e * e;
 }
```

8. [MCQ] [GATE-2013 : 2M]

Suppose the instruction set architecture of the processor has only two registers. The only allowed compiler optimization is code motion, which moves statements from one place to another while preserving correctness. What is the minimum number of spills to memory in the compiled code?

(a) 0 (b) 1
(c) 2 (d) 3

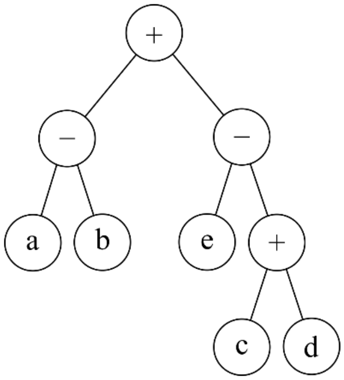
9. [MCQ] [GATE-2013 : 2M]

What is the minimum number of registers needed in the instruction set architecture of the processor to compile this code segment without any spill to memory? Do not apply any optimization other than optimizing register allocation.

(a) 3 (b) 4
(c) 5 (d) 6

10. [MCQ] [GATE-2011 : 2M]

Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables a, b, c, d and e are initially stored in memory. The binary operators used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register. If no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(a) 2 (b) 9
(c) 5 (d) 3

11. [MCQ] [GATE-2008 : 1M]

Some code optimizations are carried out on the intermediate code because

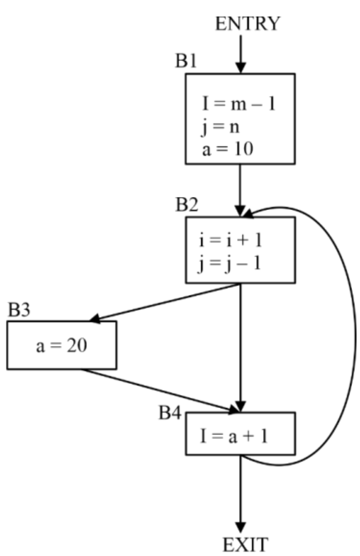
(a) They enhance the portability of the compiler to the target processors
(b) Program analysis is more accurate on intermediate code than on machine code
(c) The information from data flow analysis cannot otherwise be used for optimization
(d) The information from the front end cannot otherwise be used for optimization

Data Flow Analysis

12. [MCQ] [GATE-2023 : 2M]

Consider the control flow graph shown.

Which one of the following choices lists the set of live variable at the exit point of each basic block?



(a) B1: { }, B2: {a}, B3: {a}, B4: {a}
(b) B1: {i, j}, B2: {a}, B3: {a}, B4: {i}
(c) B1: { a, i, j }, B2: {a, i , j}, B3: {a, i}, B4: {a}
(d) B1: { a, i , j }, B2: {a, j}, B3: {a, j}, B4: {a, i, j}

13. [NAT] [GATE-2021 : 2M]

Consider the following C code segment:

a = b + c;
e = a + 1;
d = b + c;
f = d + 1;
g = e + f;

In a compiler, this code segment is represented internally as a directed acyclic graph (DAG). The number of nodes in the DAG is _______.

14. [NAT] [GATE-2017 : 2M]

Consider the following grammar:

Stmt → if expr then expr else expr; stmt | ε
expr → term relop term | term
term → id | number
id → a | b | c
number→[0-9]

where relop is a relation operator (e.g., <,>,…), 0 refers to the empty statement, and if, then, else are terminals.

Consider a program P following the above grammar containing ten if terminals. The number of control flow paths in P is ______. if e_1 then e_2 else e_3 has 2 control flow paths, $e_1 \rightarrow e_2$ and $e_1 \rightarrow e_3$.

15. [MCQ] [GATE-2015 : 2M]

Consider the intermediate code given below:

1. i=1
2. j=1
3. t1=5∗i
4. t2=t1+j
5. t3=4∗t2
6. t4=t3
7. a[t4]=−1
8. j=j+1

9. if j<=5 goto (3)
10. i=i+1
11. if i<5 goto (2)

The number of nodes and edges in the control-flow-graph constructed for the above code, respectively, are

(a) 5 and 7 (b) 6 and 7
(c) 5 and 5 (d) 7 and 8

16. [MCQ] [GATE-2015 : 1M]

In the context of abstract-syntax-tree (AST) and control-flow-graph (CFG), which one of the following is True?

(a) In both AST and CFG, let node N_2 be the successor of node N_1. In the input program, the code corresponding to N_2 is present after the code corresponding to N_1
(b) For any input program, neither AST nor CFG will contain a cycle.
(c) The maximum number of successors of a node in an AST and a CFG depends on the input program.
(d) Each node in AST and CFG corresponds to at most one statement in the input program.

17. [MCQ] [GATE-2009 : 1M]

Match all items in **Group 1** with correct options from those given in **Group 2.**

Group 1	Group 2
P: Regular expression	1. Syntax analysis
Q: Pushdown automata	2. Code generation
R: Dataflow analysis	3. Lexical analysis
S: Register allocation	4. Code optimization

(a) P-4, Q-1, R-2, S-3
(b) P-3, Q-1, R-4, S-2
(c) P-3, Q-4, R-1, S-2
(d) P-2, Q-1, R-4, S-3

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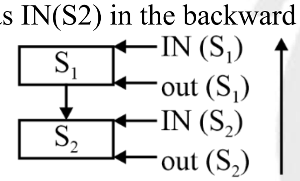
ANSWER KEY

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(c)
5.	(a)	**6.**	(d)	**7.**	(7 to 7)	**8.**	(b)
9.	(b)	**10.**	(d)	**11.**	(a)	**12.**	(d)
13.	(6 to 6)	**14.**	(1024 to 1024)	**15.**	(b)	**16.**	(c)
17.	(b)						

SOLUTIONS

1. (a)

For two statements S1 and S2, If S2 executes after S1 then we will first compute IN and OUT sets for S2 and then we will do it for S1. Here OUT(S1) will be same as IN(S2) in the backward analysis.



OUT (S1) purely depends on IN(S2).

Therefor, OUT(S1) = IN(S2).

Option A is the correct answer.

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2. (d)

Computing common sub expressions outside the loop:

$t_1 = x + 3$; // Here 1 addition happens

$t_2 = y \rightarrow f_1$; // 1 dereference

$t_3 = y \rightarrow f_2$; // 1 dereference

$z = t_1 + t_2 + t_3$; // 2 additions required to perform this instruction

for (i = 0; i < 200; i = i + 2) // 100 additions will happen, i=i+2 executed 100 times

```
{
    if (z > i)
    {
        p = p + t1;
        q = q + t2;
    }
    else
    {
        p = p + t3;
        q = q + t1;
    }                    // 2 additions will
```

happen for each iteration of loop whether "if block" or "else block" is executed.

}

100 *2 +100 + 3 = 303 additions

2 dereference operations

In the above code, total 303 additions and 2 dereference operations performed.

So, option D is the correct answer

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3. (b)

I. "There exist parsing algorithms for some programming languages whose complexities are less than $\theta(n^3)$" is **TRUE.** For example, CYK algorithm.

II. "A programming language which allows recursion can be implemented with static storage allocation" is **FALSE.** A programming language which allows recursion can be implemented with stack storage allocation.

III. "No L-attributed definition can be evaluated in the framework of bottom-up parsing" is **FALSE.** Every S-attributed definition is also L-attributed definition and hence they can be evaluated in the framework of bottom-up parsing.

IV. "Code improving transformations can be performed at both source language and intermediate code level" is **TRUE.**

So, statement I and IV are true, and hence option b is the correct amswer.

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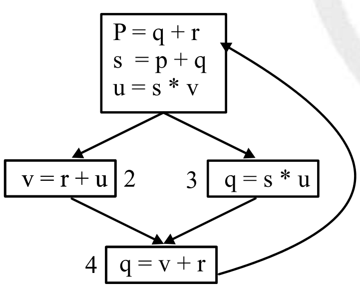
4. (c)

In the given CFG:



We have p, q, r, s, v, and u variables in the above CFG

Variable live at Basic Block 2: {r, u}

Variable live at Basic Block 3: {r, s, u}

Variable live at both Basisc Blocks 2 and 3: {r, u}

So, option C is correct.

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5. (a)

a = e + b

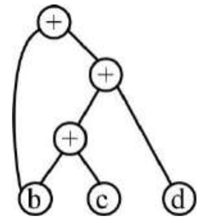
= (d – b) + b

= ([b + c] – b) + b

= ([b + (a + d)] – b) + b

= ([b + (b + c] + d) – b) + b

a = b + ([b + c] + d)



Corresponding DAG representation:

There are total 6 nodes and 6 edges. So, option a is correct answer.

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6. (d)

(a) "A basic block is a sequence of instructions where control enters the sequence at the beginning and exits at the end" is TRUE. In a basic block control only enters at the beginning and exits at the end. Control never enters in the middle of the instruction and never exits from the middle of the sequence of instructions.

(b) "Available expression analysis can be used for common subexpression elimination" is TRUE

(c) "Live variable analysis can be used for dead code elimination" is TRUE, with backward analysis we can use live variable for dead code elimination.

(d) "$x = 4 \times 5 \Rightarrow x = 20$ is an example of common subexpression elimination" is FALSE. This is an example of constant folding.

So, option D is correct answer.

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7. (7 to 7)

Given, $P(X) = X^5 + 4X^3 + 6X + 5$

$= X^5 + 4X^3 + 6X + 5$

$= X[X^4 + 4X^2 + 6] + 5$

$$x\left[x\left[x\left(x(x)+4\right)\right]+6\right]+5$$

So, there are total 4 multiplication and 3 additions, so total 7 arithmetic operation are required.

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8. (b)

Sol. Spill to memory means how many extra registers/memory/variables are required.

Code motion can be performed as follows:

c = a + b; //a and b in register and b replaced by the result c after the instruction

x = c * c; //x replaces c in register and c is spilled (moved to memory)

if (x > a) { //x and a in memory

y = a * a;

d = c * a; //spilled c taken from memory and replaces x in register.

e = c + a;

}

else {

d = c * a; //spilled c taken from memory and replaces x in register. d replaces a in register

d = d * d; //c and d are in register

e = c + a; //a is taken from memory and e replaces c in the register (a taken from memory is not a spill, it is a fill)

e = e * e;

}

So, minimum 1 spill is required in the compiled code.

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9. (b)

It is said not to apply any code optimisation other than register allocation.

Allocation with 3 registers:

c = a + b; //a, c in register

d = c * a; //a, c, d in register

e = c + a; //a, c, e in register, d spilled.

Allocation with 4 registers:

c = a + b; //a, c in register

d = c * a; //a, c, d in register

e = c + a; //a, c, d, e in register

x = c * c; //a, x, d, e in register

if (x > a) {

y = a * a;

}

else {

d = d * d;

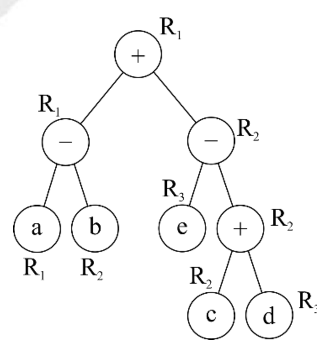
e = e * e;

}

With 4 registers, no spilling is there, so option B is the correct answer.

Scan for Video solution

10. (d)



From the above diagram we can see, three registers are required R_1, R_2, and R_3. Therefore, option d is the correct answer.

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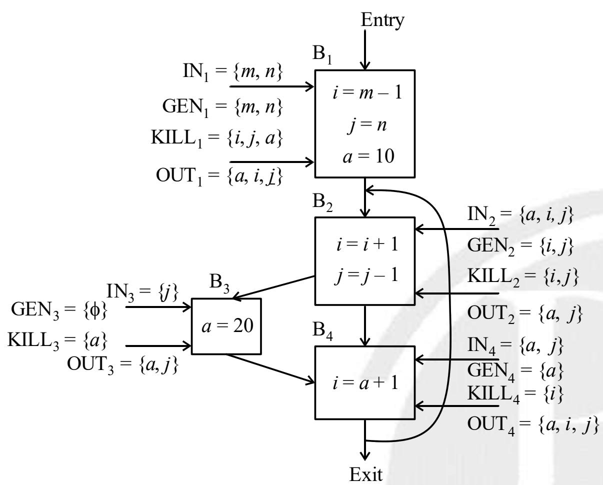
11. (a)

Some code optimizations are carried out on the intermediate code because they enhance the portability of the compiler to the target processors.

Scan for Video solution

12. (d)



Live variables at the end of each basic block are:

B1: { a, i , j }

B2: {a, j}

B3: {a, j}

B4: {a, i, j}

So, option D is the correct answer.

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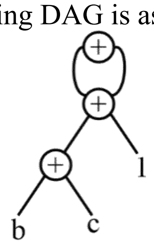
13. (6 to 6)

g = e +1;

$\Rightarrow$ (a+ 1) + (d + 1)

$\Rightarrow$ ((b + c) + 1) + ((b + c) + 1)

The corresponding DAG is as follows:



Total 6 nodes and 6 edges are present in the given DAG.

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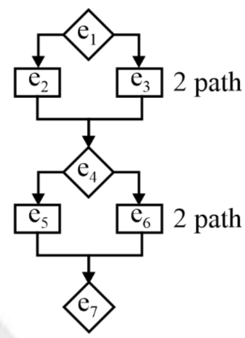
14. (1024 to 1024)

if e_1 then e_2

else e_3

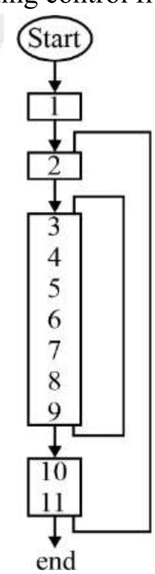


$2 \times 2 \times$ ….. 10 times = 2^{10} = 1024 paths

Scan for Video solution

15. (b)

The corresponding control flow graph is as follows:



Total, 6 nodes and 7 edges. So, option b is correct.

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16. (c)

(a) "In both AST and CFG, let node N_2 be the successor of node N_1. In the input program, the code corresponding to N_2 is present after the code corresponding to N_1" is FALSE. If loop is present than control can move from any statement to any statement.

(b) "For any input program, neither AST nor CFG will contain a cycle" is FALSE. CFG will have a cycle.

(c) "The maximum number of successors of a node in an AST and a CFG depends on the input program" is TRUE.

(d) "Each node in AST and CFG corresponds to at most one statement in the input program" is FALSE.

Only option (c) is correct.

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17. (b)

Regular expression is used in lexical analysis phase.

Pushdown automata is used in syntax analysis phase.

Dataflow analysis is a part of code optimisation.

Register allocation is used in code generation while generating assembly code.

So, option (b) is the correct answer.

Scan for Video solution

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CHAPTER

7 RUNTIME ENVIRONMENT

Basics of Runtime Environment

1. [MCQ] [GATE-2020 : 1M]

Consider the following statements:

I. Symbol table is accessed only during lexical analysis and syntax analysis.

II. Compilers for programming languages that support recursion necessarily need heap storage for memory allocation in the run-time environment.

III. Errors violating the condition 'any variable must be declared before its use' are detected during syntax analysis.

Which of the above statements is/are TRUE?

(a) I only
(b) I and III only
(c) II only
(d) None of I, II and III

2. [MCQ] [GATE-2018 : 1M]

Which one of the following statements is FALSE?

(a) Context-free grammar can be used to specify both lexical and syntax rules.
(b) Type checking is done before parsing.
(c) High-level language programs can be translated to different Intermediate Representations.
(d) Arguments to a function can be passed using the program stack

3. [MCQ] [GATE-2015 : 1M]

Match the following:

	List-I	List-II
a.	Lexical analysis	1. Graph coloring
b.	Parsing	2. DFA minimization
c.	Register allocation	3. Post-order traversal
d.	Expression evaluation	4. Production tree

Code:

	a.	b.	c	d.
(a)	2	3	1	4
(b)	2	1	4	3
(c)	2	4	1	3
(d)	2	3	4	1

4. [MCQ] [GATE-2008 : 2M]

Which of the following are true?

I. A programming language which does not permit global variables of any kind and has no nesting of procedures/functions, but permits recursion can be implemented with static storage allocation.

II. Multi-level access link (or display) arrangement is needed to arrange activation records only if the programming language being implemented has nesting of procedures/functions

III. Recursion in programming languages cannot be implemented with dynamic storage allocation

IV. Nesting of procedures/functions and recursion require a dynamic heap allocation scheme and cannot be implemented with a stack-based allocation scheme for activation records

V. Programming languages which permit a function to return a function as its result cannot be implemented with a stack-based storage allocation scheme for activation records

(a) II and V only
(b) I, II and IV only
(c) I, II and V only
(d) I, III and V only

Static And Dynamic Scoping

5. [MCQ] [GATE-2014 : 1M]

Which of the following statements are CORRECT?

1. Static allocation of all data areas by a compiler makes it impossible to implement recursion.
2. Automatic garbage collection is essential to implement recursion.
3. Dynamic allocation of activation records is essential to implement recursion.
4. Both heap and stack are essential to implement recursion.

(a) 1 and 2 only (b) 2 and 3 only

(c) 3 and 4 only (d) 1 and 3 only

6. [MCQ] [GATE-2014 : 1M]

Which one of the following is NOT performed during compilation?

(a) Dynamic memory allocation

(b) Type checking

(c) Symbol table management

(d) Inline expansion

7. [MCQ] [GATE-2014 : 1M]

Which languages necessarily need heap allocation in the runtime environment?

(a) Those that support recursion

(b) Those that use dynamic scoping

(c) Those that allow dynamic data structure

(d) Those that use global variables

8. [MCQ] [GATE-2012 : 2M]

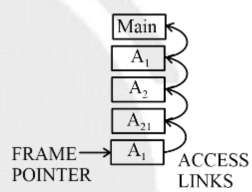
Consider the program given below, in a block-structured pseudo-language with lexical scoping and nesting of procedures permitted.

```
Program main;
Var ...
Procedure A1;
        Var ...
        Call A2;
End A1
Procedure A2;
        Var ...
Procedure A21;
            Var ...
            Call A1;
        End A21
        Call A21;
End A2
Call Al;
End main
```

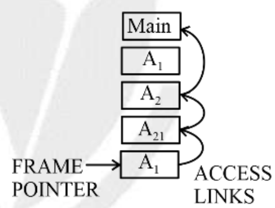
Consider the calling chain: Main → A1 → A2 → A21 → A1

The correct set of activation records along with their access links is given by

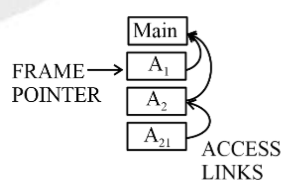
(a)



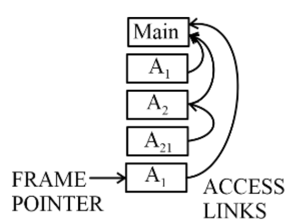
(b)



(c)



(d)



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ANSWER KEY

1.	(d)	**2.**	(b)	**3.**	(c)	**4.**	(a)
5.	(d)	**6.**	(a)	**7.**	(c)	**8.**	(d)

SOLUTIONS

1. (d)

I. Symbol table is used by all the phases of compiler, but here it is saying that only accessed during lexical and syntax analysis. So, this staatement is incorrect.

II. Compilers do not need heap storage they need stack storage to support the recursion. During recursion lots of activation records are form for the same function, and we need stack to maintain this. So, this statement is incorrect.

III. Errors violating the condition 'any variable must be declared before its use' are detected during semantic analysis not during syntax analysis. So, this statement is also false.

Therfore, option (d) is True.

Scan for Video solution

2. (b)

(a) Context-free grammar can be used to specify both lexical and syntax rules. TRUE

(b) Type checking is done before parsing. FALSE. It is done after parsing.

(c) High-level language programs can be translated to different Intermediate Representations. TRUE.

(d) Arguments to a function can be passed using the program stack. TRUE. Program stack is used during runtime.

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3. (c)

Lexical analysis uses DFA minimization, to implement or do design lexical analyser, we need minimized DFA.

Production tree helps us in constructing the parse tree. During parsing we need production tree support.

Register allocation uses graph coloring, when registers are allocated graph coloring is used in order to optimise the use of registers.

Expression can be evaluated with the help of post-order traversals.

So, option (c) is the correct answer.

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4. (a)

I. A programming language which does not permit global variables of any kind and has no nesting of procedures/functions, but permits recursion can be implemented with static storage allocation. False. It can implemented with stack allocation not with static storage allocation.

II. Multi-level access link (or display) arrangement is needed to arrange activation records only if the programming language being implemented has nesting of procedures/functions .True

III. Recursion in programming languages cannot be implemented with dynamic storage allocation. False

IV. Nesting of procedures/functions and recursion require a dynamic heap allocation scheme and cannot be implemented with a stack-based allocation scheme for activation records. False

V. Programming languages which permit a function to return a function as its result cannot be implemented with a stack-based storage allocation scheme for activation records. True

So, option (a) is correct answer.

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5. (d)

1. Static allocation of all data areas by a compiler makes it impossible to implement recursion. TRUE. Static allocation makes it impossible to implement recursion.
2. Automatic garbage collection is essential to implement recursion. FALSE. For recursion we do not require automatic garbage collection.
3. Dynamic allocation of activation records is essential to implement recursion. TRUE. Dynamic allocation are of two types heap and stack.
4. Both heap and stack are essential to implement recursion. FALSE.

So, option (d) is the correct answer.

Scan for Video solution

6. (a)

Dynamic memory allocation is done during runtime and not during compilation. Therefore, option a is the correct answer.

Scan for Video solution

7. (c)

Heap allocation/ dynamic allocation; the language that allows dynamic data structure necessarily needs heap/ dynamic allocation in runtime enviornment.

Languages that support recursion require stack.

Languages that support global variables need static memory allocation. So, option (c) is the correct answer.

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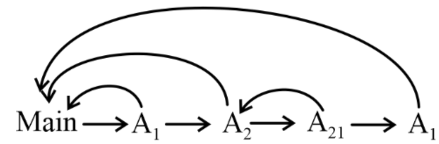
8. (d)

Program's flow can be seen as:

```
Program main;
    Var …
  Procedure A1;
      Var …
      Call A2;
      End A1
  Procedure A2;
      Var …
    Procedure A21;
        Var …
        Call A1;
      End A21
      Call A21;
  End A2
  Call A1;
End main
```

The correct set of activation records along with their access links can be given as:



So, option D is the correct answer.

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INDEX

Theory of Computation

Theory of Computation

Syllabus

Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and context-free languages, pumping lemma. Turing machines and undecidability.

Chapter wise Weightage Analysis

Chapter / Paper Year	Ch.1	Ch.2	Ch.3	Ch.4	Ch.5	Ch.6
2008	7	3	2	12	6	1
2009	4	2	0	2	2	0
2010	2	2	0	2	1	0
2011	3	0	0	1	3	0
2012	4	1	0	0	0	1
2013	2	1	0	0	2	2
2014 (P1)	1	2	0	1	2	0
2014 (P2)	0	0	3	0	0	3
2014 (P3)	0	1	0	2	1	2
2015 (P1)	2	0	0	2	1	0
2015 (P2)	0	2	4	0	0	1
2015 (P3)	0	1	0	1	0	0
2016 (P1)	0	1	1	4	0	3
2016 (P2)	0	1	4	4	1	2
2017 (P1)	0	1	0	7	2	0
2017 (P2)	3	0	0	3	2	2
2018	1	0	2	2	1	2
2019	2	0	2	2	2	0
2020	2	1	1	3	0	2
2021 (P1)	2	0	0	4	1	2
2021 (P2)	4	2	0	2	0	2
2022	0	1	0	4	1	2
2023	1	1	1	2	1	0

CHAPTER 1

FINITE AUTOMATA AND TRANSDUCER

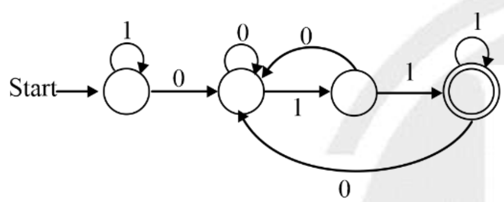
Deterministic Finite Automata

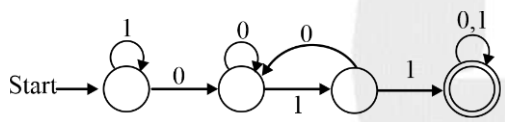
1. [MCQ] **[GATE-2021 : 2M]**

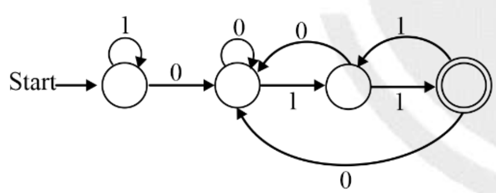
Consider the following language:

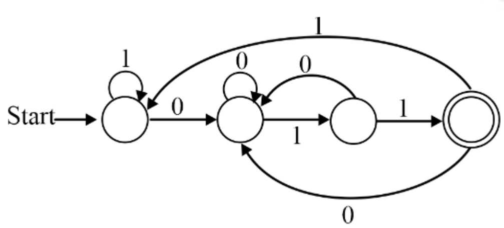
$L = \{w \in \{0,1\}^* \mid w$ ends with the substring $011\}$

Which one of the following deterministic finite automata accepts L?

(a) 

(b) 

(c) 

(d) 

2. [MCQ] **[GATE-2021 : 1M]**

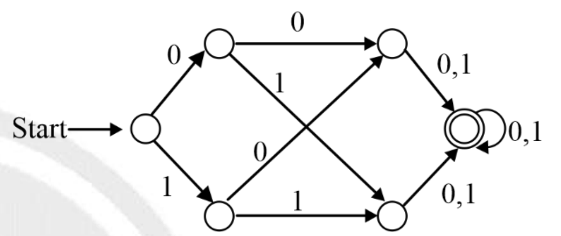
Let $L \subseteq \{0, 1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

(a) $L - \{01\}$ (b) $L \cup \{01\}$

(c) $\{0, 1\}^* - L$ (d) $L . L$

3. [NAT] **[GATE-2021 : 1M]**

Consider the following deterministic finite automaton (DFA)



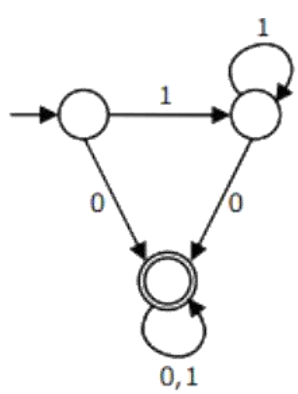
The number of strings of length 8 accepted by the above automaton is ___.

4. [MCQ] **[GATE-2013 : 2M]**

Consider the DFA A given below.

Which of the following are FALSE?

1. Complement of L(A) is context-free.
2. $L(A) = L((11^*0 + 0)(0 + 1)^*0^*1^*)$
3. For the language accepted by A, A is the minimal DFA
4. A accepts all strings over {0, 1} of length at least 2.

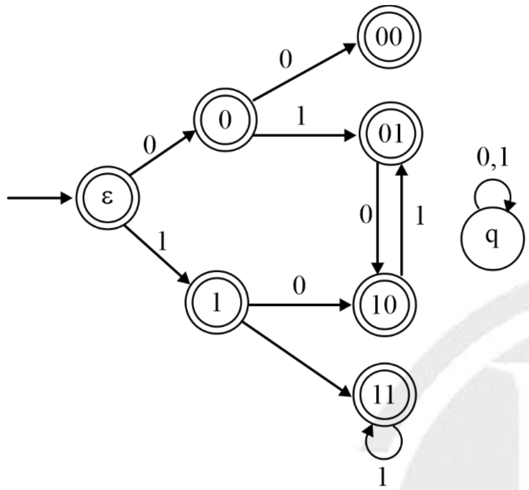


(a) 1 and 3 only (b) 2 and 4 only

(c) 2 and 3 only (d) 3 and 4 only

5. [MCQ] [GATE-2012 : 2M]

Consider the set of strings on {0,1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below. The missing arcs in the DFA are



(a)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(b)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(c)

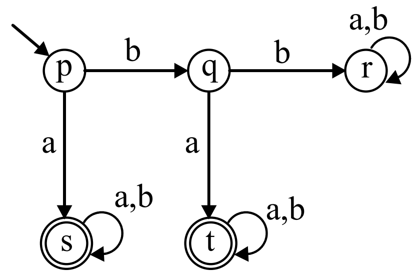
	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

(d)

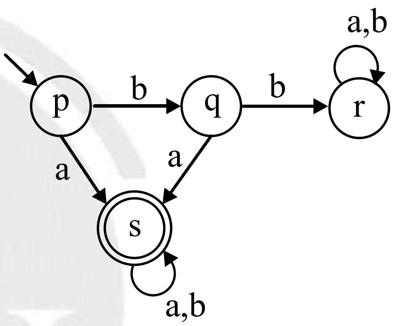
	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

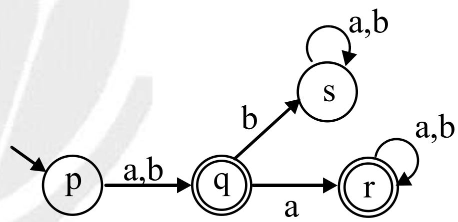
6. [MCQ] [GATE-2011 : 1M]

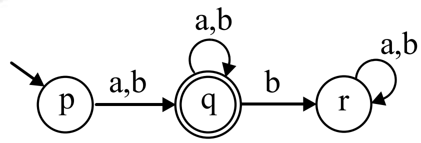
A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is given below:

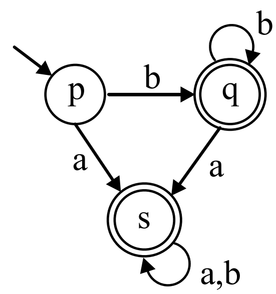


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

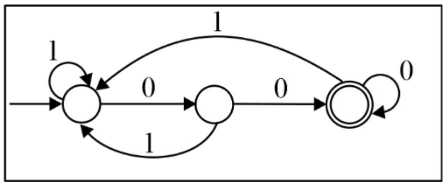
(a) 

(b) 

(c) 

(d) 

7. [MCQ] [GATE-2009 : 2M]



The above DFA accepts the set of all strings over {0,1} that

(a) Begin either with 0 or 1.

(b) End with 0.

(c) End with 00.

(d) Contain the substring 00.

Non-Deterministic Finite Automata

8. [MCQ] [GATE-2017 : 2M]

Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the $\in$-NFA whose transition table is given below:

δ	$\in$	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

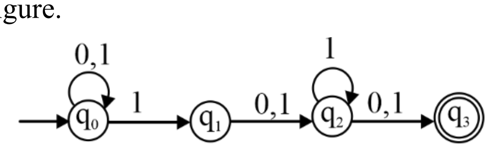
The $\hat{\delta}$ (q_2, aba) is

(a) Φ (b) $\{q_0, q_1, q_3\}$

(c) $\{q_0, q_1, q_2\}$ (d) $\{q_0, q_2, q_3\}$

9. [MCQ] [GATE-2014 : 1M]

Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011 ?

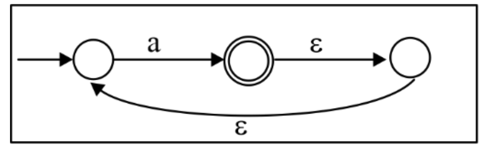
(a) $\{q_0, q_1, q_2\}$ (b) $\{q_0, q1\}$

(c) $\{q_0, q_1, q_2, q_3\}$ (d) $\{q_3\}$

10. [MCQ] [GATE-2012 : 2M]

What is the complement of the language accepted by the NFA shown below?

Assume $\Sigma = \{a\}$ and ε is the empty string.



(a) ϕ (b) $\{\varepsilon\}$

(c) a* (d) $\{a, \varepsilon\}$

11. [MCQ] [GATE-2008 : 2M]

Given below are two finite state automata ($\rightarrow$ indicates the start and F indicates a final state)

Y:

	a	b
$\rightarrow$1	1	2
2(F)	2	1

Z:

	a	b
$\rightarrow$1	2	2
2(F)	1	1

Which of the following represents the product automaton Z × Y ?

(a)

	a	b
$\rightarrow$P	S	R
Q	R	S
R(F)	Q	P
S	Q	P

(b)

	a	b
$\rightarrow$P	S	Q
Q	R	S
R(F)	Q	P
S	P	Q

(c)

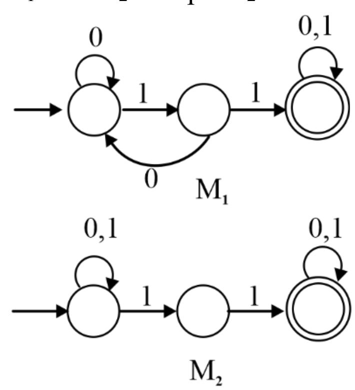
	a	b
$\rightarrow$P	Q	S
Q	R	S
R(F)	Q	P
S	Q	P

(d)

	a	b
$\rightarrow$P	S	Q
Q	S	R
R(F)	Q	P
S	Q	P

12. [MCQ] [GATE-2008 : 2M]

Consider the following two finite automata. M_1 accepts L_1 and M_2 accepts L_2.



Which one of the following is TRUE?

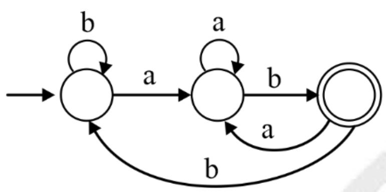
(a) $L_1 = L_2$ (b) $L_1 \cap L_2 = \phi$

(c) $L_1 \cap L_2^C = \phi$ (d) $L_1 \cup L_2 \neq \phi$

Finite Automata Interconversion

13. [MCQ] [GATE-2008 : 2M]

If the final states and non-final states in the DFA below are interchanged, then which of the following languages over the alphabet {a, b} will be accepted by the new DFA?



(a) Set of all strings that do not end with ab

(b) Set of all strings that begin with either an a or ab

(c) Set of all strings that do not contain the substring ab.

(d) The set described by the regular expression b*aa*(ba)*b*

Minimization

14. [NAT] [GATE-2023 : 1M]

Consider the language L over the alphabet {0, 1}, given below:

L = {w ∈ {0, 1}* | w does not contain three or more consecutive 1's}.

The minimum number of states in a Deterministic Finite-state Automaton (DFA) for L is _____.

15. [NAT] [GATE-2020 : 2M]

Consider the following language:

L = {x ∈ {a, b} *| number of a's in x is divisible by 2 but not divisible by 3}

The minimum number of states in a DFA that accepts L is ______.

16. [NAT] [GATE-2019 : 2M]

Let Σ be the set of all bijections from {1, …, 5} to {1, …, 5}, where id denotes the identity function, i.e. id(j) = j, ∀j. Let ° denote composition on functions. For a string $x = x_1x_2 \ldots x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \ldots \circ x_n$.

Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$. The minimum number of states in any DFA accepting L is ______.

17. [MCQ] [GATE-2018 : 1M]

Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?

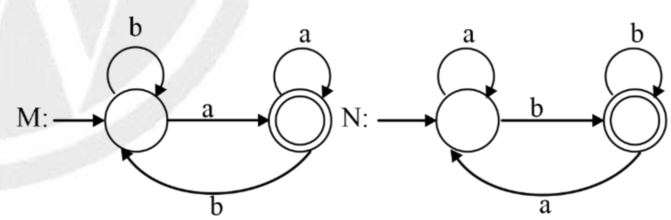
(a) $k \geq 2^n$ (b) $k \geq n$

(c) $k \leq n^2$ (d) $k \leq 2^n$

18. [NAT] [GATE-2017 : 1M]

The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1|=2, |w_2| \geq 3\}$ is___________.

19. [NAT] [GATE-2015 : 2M]



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the language $L(M) \cap L(N)$ is _______.

20. [MCQ] [GATE-2011 : 2M]

Definition of a language L with alphabet {a} is given as following:

$L = \{a^{nk} \mid k > 0$, and n is a positive integer constant}.

What is the minimum number of states needed in a DFA to recognize L?

(a) k + 1 (b) n + 1

(c) 2^{n+1} (d) 2^{k+1}

21. **[MCQ]** **[GATE-2010 : 2M]**

Let w be any string of length n in {0,1}*. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L?

(a) $n - 1$ (b) n

(c) $n + 1$ (d) 2^{n-1}

22. **[MCQ]** **[GATE-2008: 1M]**

Let N be an NFA with n states and let M be the minimized DFA with m states recognizing the same language. Which of the following is NECESSARILY true?

(a) $m \le 2^n$

(b) $n \le m$

(c) M has one accept state

(d) $m = 2^n$

Mealy Machine

23. **[MCQ]** **[GATE-2021 : 2M]**

Suppose we want to design a synchronous circuit that processes a string of 0's and 1's. Given a string, it produces another string by replacing the first 1 in any subsequence of consecutive 1's by a 0. Consider the following example.

Input sequence: 00100011000011100

Output sequence: 00000001000001100

A mealy Machine is a state machine where both the next state and the output are functions of the present state and the current input.

The above-mentioned circuit can be designed as a two-state Mealy machine. The states in the Mealy machine can be represented using Boolean values 0 and 1. We denote the current state, the next state, the next incoming bit, and the output bit of the Mealy machine by the variables s, t, b and y respectively.

Assume the initial state of the Mealy machine is 0.

What are the Boolean expressions corresponding to t and y in terms of s and b?

(a) $t = s + b$
$y = sb$

(b) $t = b$
$y = sb$

(c) $t = b$
$y = s\bar{b}$

(d) $t = s + b$
$y = s\bar{b}$

24. **[MCQ]** **[GATE-2009 : 2M]**

Given the following state table of an FSM with two states A and B, one input and one output:

Present State A	Present State B	Input	Next State A	Next State B	Output
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	0	1	0
0	1	1	0	0	1
1	0	1	0	1	1
1	1	1	0	0	1

If the initial state is A = 0, B = 0, what is the minimum length of an input string which will take the machine to the state A = 0, B = 1 with output = 1?

(a) 3 (b) 4

(c) 5 (d) 6

❑❑❑

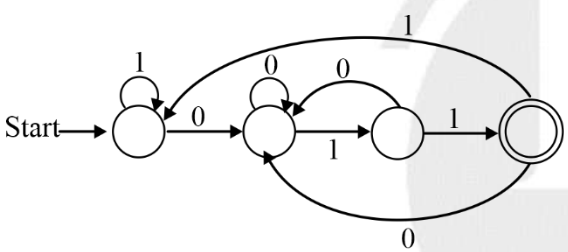
ANSWER KEY

1.	(d)	**2.**	(c)	**3.**	(256 to 256)	**4.**	(d)
5.	(d)	**6.**	(a)	**7.**	(c)	**8.**	(c)
9.	(a)	**10.**	(b)	**11.**	(Marks to All)	**12.**	(a, c)
13.	(a)	**14.**	(4 to 4)	**15.**	(6 to 6)	**16.**	(120 to 120)
17.	(d)	**18.**	(8 to 8)	**19.**	(1 to 1)	**20.**	(b)
21.	(c)	**22.**	(a)	**23.**	(b)	**24.**	(a)

SOLUTIONS

1. (d)

DFA for ending with 011:



Hence, option (d) is correct.

Scan for Video solution

2. (c)

$\overline{L} = \Sigma^* - L$

$= \{0, 1\}^* - L$

If L is a DFA with K states than in $\overline{L}$ DFA also having K states because number of states in L and $\overline{L}$ are same.

Scan for Video solution

3. (256 to 256)

Every symbol is having two choices

$\frac{0/1}{2}\ \frac{0/1}{2}\ \frac{0/1}{2}\ \frac{0/1}{2}\ \frac{0/1}{2}\ \frac{0/1}{2}\ \frac{0/1}{2}\ \frac{0/1}{2}$

Total strings with length 8 = 2^8

Total strings with length 8 = 256

Scan for Video solution

4. (d)

1. L(A) = Regular

   Complement of L(A) = $\overline{\text{Regular}}$ = Regular

   Every Regular language is CFL

2. L(A) = $11^*0(0 + 1)^* + 0(0 + 1)^*$

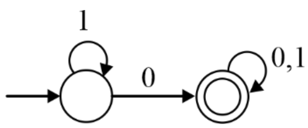
   $= (11^*0 + 0)\ (0 + 1)^*$

   $= (1^+\ 0 + 0)\ (\ 0 + 1)^*$

   $= (1^+ + \in)\ 0\ (0 + 1)^*$

   $= 1^*0\ (0 + 1)^*$

3. Minimal **DFA for [1*0 (0 + 1)*]:**



Number of states in minimal DFA = 2

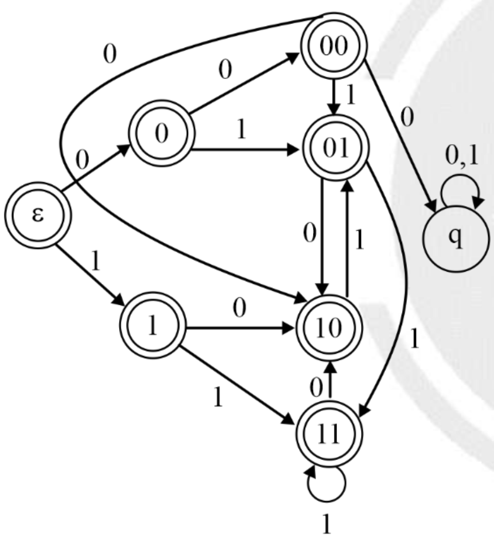
4. Given DFA accepts all strings containing 0 as substring.

5. (d)

From state 00 to read input 0 ⇒ q(dead state)

From state 00 to read input 1 ⇒ 01

From state 01 to read input 1 ⇒ 11



Hence, option (d) is correct.

6. (a)

Partition algorithm:

(i) {p, q, r} {s, t}

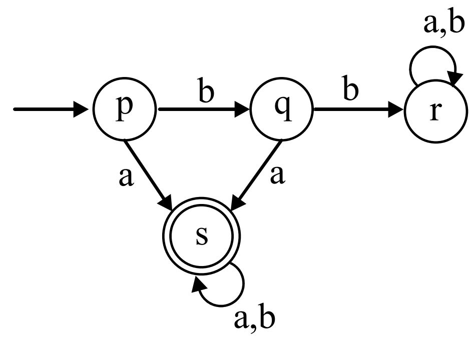
Non-final Final

(ii) {p, q}, {r}, {s, t}

(iii) {p}, {q}, {r}, {s, t}

Number of states = 4.

Here s and t are equivalent.



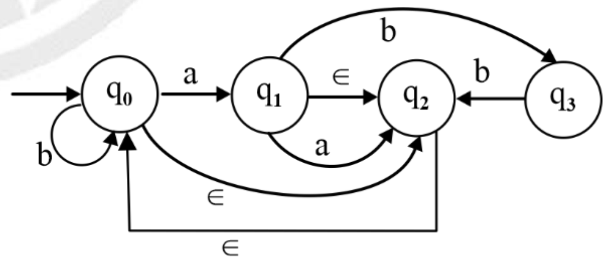
7. (c)

Minimal accepted string = 00

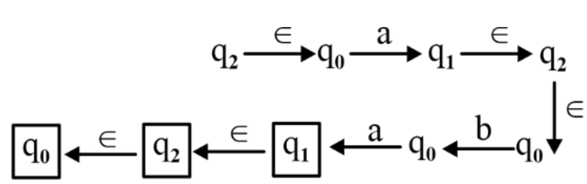
It will accept all the strings end with 00.

8. (c)

∈-NFA:



Transition with (q_2, aba):



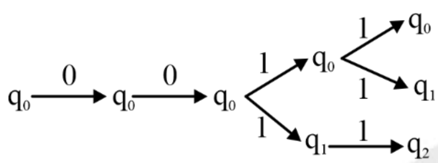
$\boxed{\hat{\delta}(q_2, aba) = \{q_1, q_2, q_0\}}$

Hence, option (c) is correct.

Scan for Video solution

9. **(a)**

Reachable states for the input string 0011:

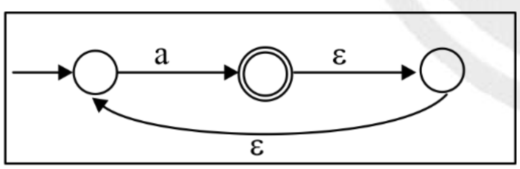


$\hat{\delta}(q_0, 0011) = \{q_0, q_1, q_2\}$

Hence, option (a) is correct.

Scan for Video solution

10. **(b)**



$\Rightarrow L = a^+$

$\bar{L} = \Sigma^* - L$

$= a^* - a^+$

$= \{\varepsilon\}$

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11. **(Marks to All)**

$Z \times Y$:

$\rightarrow Z_1Y_1$	Z_2Y_1	Z_2Y_2
Z_1Y_2	Z_2Y_2	Z_2Y_1
Z_2Y_1	Z_1Y_1	Z_1Y_2
Z_2Y_2	Z_1Y_2	Z_1Y_1

	a	b
$\rightarrow P$	S	R
Q	R	S
S	P	Q
R	Q	P

We don't have correct option

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12. **(a, c)**

L_1 (M_1) is DFA and it accepts $(0 + 1)^*\ 11\ (0 + 1)^*$

L_2 (M_2) is NFA and it accepts $(0 + 1)^*\ 11\ (0 + 1)^*$

Both L_1 and L_2 are equal and $L_1 \cap L_2^C = \phi$ is also correct.

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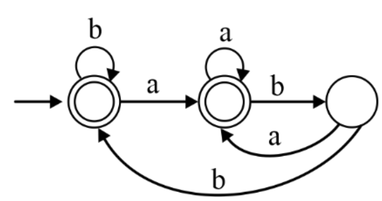
13. **(a)**

Complement of given machine:



It will accept set of all strings that do not end with ab.

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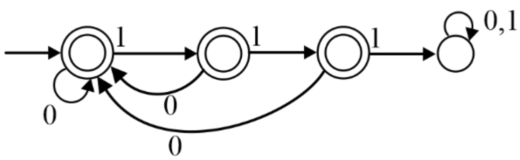
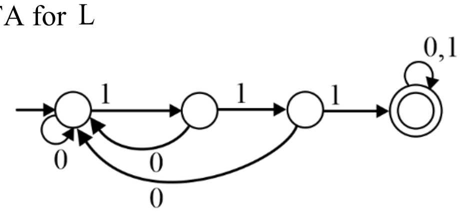
14. (4 to 4)

DFA for L:



DFA for $\overline{L}$



Number of states will be same in both L and $\overline{L}$.

Number of states = 4

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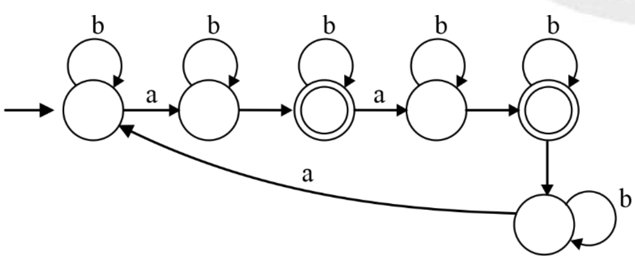
15. (6 to 6)

Divisible by 2 strings = ∈, aa, aaaa, aaaaaa, …..

Divisible by 3 strings = ∈, aaa, aaaaaa, aaaaaaaaa, …..

Divisible by 2 but not 3 strings = $a^2, a^4, a^8, a^{10}, a^{12}$, …..

DFA design for number of a's divisible by 2 but not 3:



Number of states = 6

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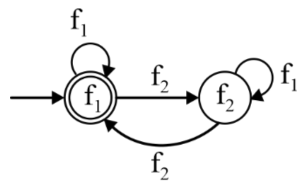
16. (120 to 120)

If Σ is set of all bijections from {1, 2} to {1, 2}

Number of bijective functions = 2! = 2

Here, Language L = {∈, f_1, f_1f_2, f_2f_2 ….}

Number of states =2



If set has 2 elements, then 2! bijective functions and hence 2! States.

Each bijective function requires one state to represent as result of some composition.

If set has 5 elements, then number of bijective functions = 5! and number of states = 5!

Number of states = 5 × 4 × 3 × 2 × 1 = 120

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17. (d)

If NFA (N) having 'n' states and DFA having 'k' states.

Then, relation between k and n is using subset construction algorithm is:

$\boxed{k \le 2^n}$

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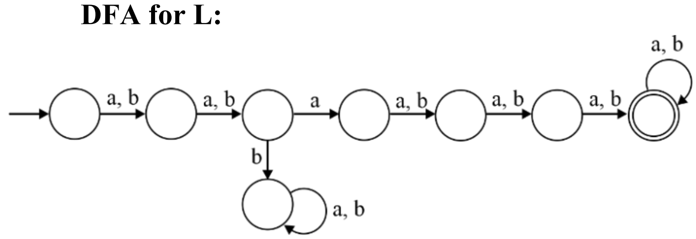
18. (8 to 8)

$L = \{w_1aw_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$

$= (a + b)^2\ a\ (a + b)^3\ (a + b)^*$

DFA for L:



Number of states in DFA for L = 8.

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19. (1 to 1)

M accepts all strings ending in a.

L(M) = (a + b)* a

N accepts all strings ending in b.

L (N) = (a + b)* b

$L(M) \cap L(N) = \phi$

To represent empty language, only one state present in minimal DFA.

DFA:

To design DFA for ϕ minimum 1 state is needed.

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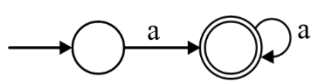
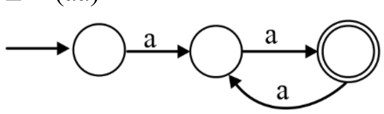
20. (b)

L = $\{a^{nk} \mid k > 0$, and n is a positive integer constant}.

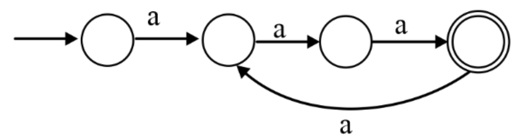
- If $n = 1 \Rightarrow L = \{a^k \mid k > 0\}$
  $L = a^+$
  FA:



  2 states are needed.
- If $n = 2 \Rightarrow L = \{a^{2k} \mid k > 0\}$
  $L = (aa)^+$



  3 states are needed.
- If $n = 3 \Rightarrow L = \{a^{3k} \mid k > 0\}$
  $L = (aaa)^+$



4 states are needed.

For L = $\{a^{nk} \mid k > 0$, n is constant}, n + 1 states are needed.

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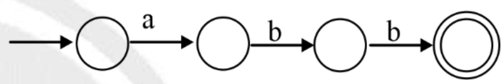
21. (c)

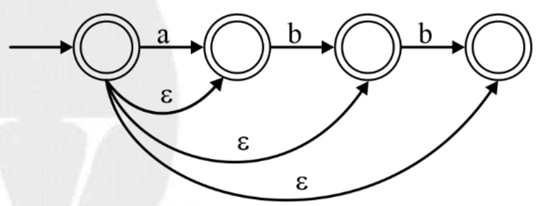
w = abb(n = 3) $\Rightarrow$ L = {ε, a, b ab, bb, abb}

DFA for abb:



Accept w = abb

DFA for L:



For 3 length 4 states are needed. So, for n length n + 1 states are needed.

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22. (a)

NFA(N) $\Rightarrow$ n states

Min DFA(M) $\Rightarrow$ m States

Relation between m & n:

DFA having $\leq 2^n$ states

$\boxed{m \leq 2^n}$

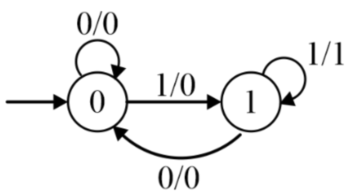
Scan for Video solution

23. **(b)**

Input = 001011110

Output = 000001110

Mealy Machine:



Transition Table:

s	b	
	0	1
0	0, 0 t, y	1, 0 t, y
1	0, 0 t, y	1, 1 t, y

t = b

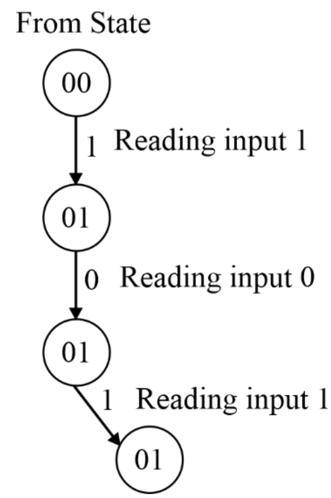
y = sb

Hence, option (b) is correct.

Scan for Video solution

24. **(a)**

A = 0, B = 1 and output = 1



Input = 101

From state 00, by reading 3 length minimum input it takes to 01.

Scan for Video solution

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CHAPTER

2 REGULAR EXPRESSION

Construction of Regular Expression

1. **[MSQ]** **[GATE-2021 : 2M]**
Which of the following regular expressions represent(s) the set of all binary numbers that are divisible by three? Assume that the string $\in$ is divisible by three.
(a) (0 + 1(01* 0)* 1)*
(b) (0 + 11 + 10(1 + 00)* 01)*
(c) (0*(1(01* 0)* 1)*)*
(d) (0 + 11 + 11(1 + 00)* 00)*

2. **[MCQ]** **[GATE-2020 : 1M]**
Which one of the following regular expressions represents the set of all binary strings with an odd number of 1 's?
(a) (0* 10* 10*) * 0* 1
(b) 10*(0* 10* 10*)*
(c) ((0 + 1)* 1(0 + 1)* 1)* 10*
(d) (0* 10* 10*)* 10*

3. **[MCQ]** **[GATE-2016 : 1M]**
Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's?
(a) (0 + 1) * 0011(0 + 1)* + (0 + 1)* 1100(0 + 1) *
(b) (0 + 1) *(00(0 + 1) * 11 + 11(0 + 1) * 00) (0 + 1) *
(c) (0 + 1) * 00(0 + 1) * + (0 + 1) * 11(0 + 1) *
(d) 00(0 + 1) * 11+11(0 + 1) * 00

4. **[MCQ]** **[GATE-2013 : 1M]**
Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1L_2^* \cup L_1^*$?
(a) $\{\in\}$
(b) ϕ
(c) a*
(d) $\{\in, a\}$

5. **[MCQ]** **[GATE-2012 : 1M]**
Given the language L = {ab, aa, baa}, which of the following strings are in L*?
1. abaabaaabaa
2. aaaabaaaa
3. baaaaabaaaab
4. baaaabaa

(a) 1, 2 and 3
(b) 2, 3 and 4
(c) 1, 2 and 4
(d) 1, 3 and 4

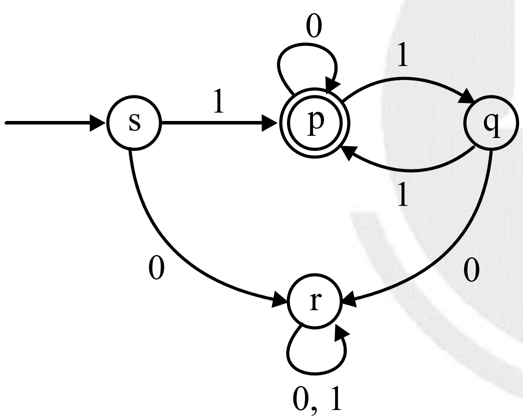
6. **[MCQ]** **[GATE-2010 : 2M]**
Let L = {w$\in$ (0 + 1)* | w has even number of 1s}, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L ?
(a) (0* 10* 1)*
(b) 0*(10* 10*)*
(c) 0*(10* 1)* 0*
(d) 0* 1(10* 1)* 10*

7. **[MCQ]** **[GATE-2009 : 2M]**
Which one of the following languages over the alphabet {0,1} is described by the regular expression: (0 + 1)* 0(0 + 1)* 0(0 + 1)* ?
(a) The set of all strings containing the substring 00
(b) The set of all strings containing at most two 0's
(c) The set of all strings containing at least two 0's
(d) The set of all strings that begin and end with either 0 or 1

8. [MCQ] [GATE-2008 : 1M]
Which of the following regular expressions describes the language over {0, 1} consisting of strings that contain exactly two 1's?
(a) (0 + 1) * 11(0+1) *
(b) 0 * 110*
(c) 0 * 10* 10*
(d) (0 + 1) * 1(0 + 1) * 1 (0 + 1) *

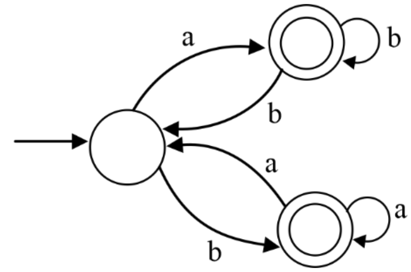
Interconversion Regular Expression to Finite Automata

9. [MCQ] [GATE-2023 : 1M]
Consider the Deterministic Finite-state Automaton (DFA) A shown below. The DFA runs on the alphabet {0, 1}, and has the set of states {s, p, q, r}, with s being the start state and *p* being the only final state.



Which one of the following regular expressions correctly describes the language accepted by A?
(a) 1(0*11)*
(b) 0(0 + 1)*
(c) 1(0 + 11)*
(d) 1(110*)*

10. [MCQ] [GATE-2022 : 1M]
Which one of the following regular expressions correctly represents the language of the finite automaton given below?



(a) a b* b a b* + b a* a b a*
(b) (a b* b)* a b* + (b a* a)* b a*
(c) (a b* b + b a* a)*(a* + b*)
(d) (b a*a + a b* b)*(a b* + b a*)

11. [NAT] [GATE-2017 : 1M]
Consider the language L given by the regular expression (a + b)* b(a + b) over the alphabet {a, b}. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is________.

12. [NAT] [GATE-2016 : 1M]
The number of states in the minimum sized DFA that accepts the language defined by the regular expression (0 + 1)*(0 + 1)(0 + 1) * is______.

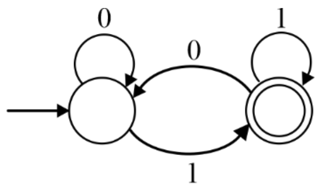
13. [MCQ] [GATE-2015 : 1M]
Let L be the language represented by the regular expression Σ* 0011 Σ* where Σ = {0, 1}.
What is the minimum number of states in DFA that recognizes $\overline{L}$ (complement of L)?
(a) 4 (b) 5
(c) 6 (d) 8

14. [NAT] [GATE-2015 : 2M]
The number of states in the minimal deterministic finite automaton corresponding to the regular expression (0 + 1)*(10) is ______.

15. [NAT] [GATE-2014 : 1M]
The length of the shortest string NOT in the language (over Σ = {a, b}) of the following regular expression is ________.
a* b* (ba)* a*

16. [MCQ] [GATE-2014 : 2M]

Which of the regular expressions given below represent the following DFA?



I. $0^*1(1+00^*1)^*$

II. $0^*1^*1+11^*0^*1$

III. $(0+1)^*1$

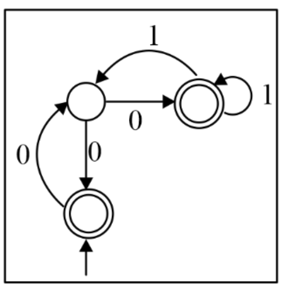
(a) I and II only (b) I and III only
(c) II and III only (d) I, II and III

17. [MCQ] [GATE-2008 : 2M]

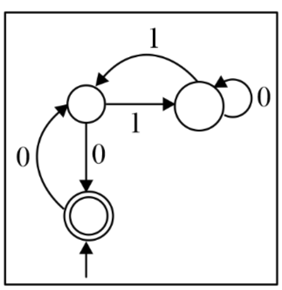
Match List - I with List - II and select the correct answer using the codes given below the lists:

List – I

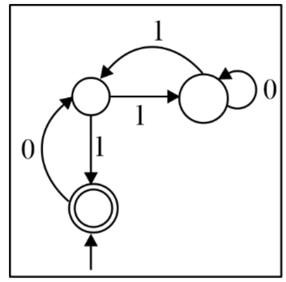
P



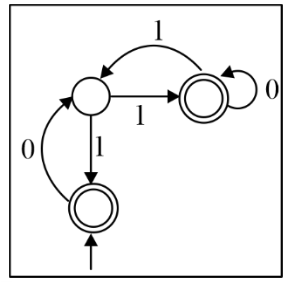
Q



R



S



List - II

1. $\varepsilon + 0(01^*1 + 00)^*01^*$
2. $\varepsilon + 0(10^*1 + 00)^*0$
3. $\varepsilon + 0(10^*1 + 10)^*1$
4. $\varepsilon + 0(10^*1 + 10)^*10^*$

	P	Q	R	S
(a)	2	1	3	4
(b)	1	3	2	4
(c)	1	2	3	4
(d)	3	2	1	4

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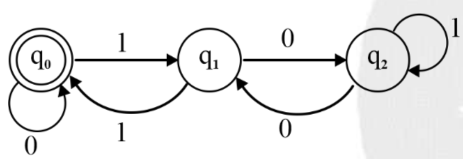
ANSWER KEY

1.	(a, b, c)	**2.**	(Marks to All)	**3.**	(b)	**4.**	(a)
5.	(c)	**6.**	(b)	**7.**	(c)	**8.**	(c)
9.	(c)	**10.**	(d)	**11.**	(4 to 4)	**12.**	(2 to 2)
13.	(b)	**14.**	(3 to 3)	**15.**	(3 to 3)	**16.**	(b)
17.	(c)						

SOLUTIONS

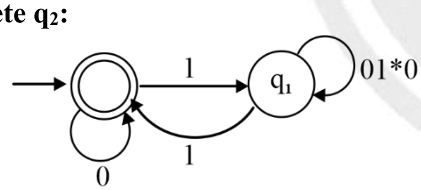
1. (a, b, c)

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

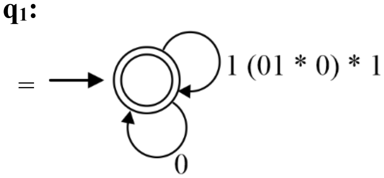


Approach 1: First delete q_2 then q_1.

Delete q_2:



Delete q_1:

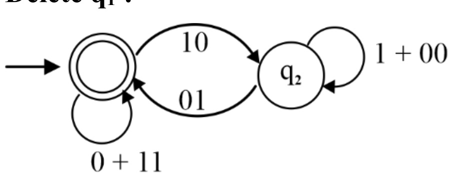


R = (0 + 1 (01 * 0) * 1)* = (0*(1(01* 0)* 1)*)*

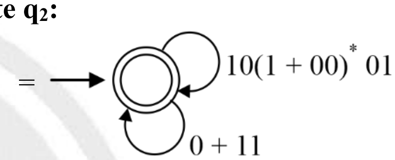
Compare with (a + b)* = (a*b*)*

Approach 2: First delete q_1 then q_2.

Delete q_1 :



Delete q_2:



Regular expression = (0 + 11 + 10 (1 + 00)*01)*

Scan for Video solution

2. (Marks to All)

(a) (0* 10* 10*) * 0* 1
This regular expression always ending with 1.
It can't generate 10

(b) 10*(0* 10* 10*)*
This regular expression always starting with 1.
It can't generate 01

(c) ((0 + 1)* 1(0 + 1)* 1)* 10*
It will generate even number of 1 also.

(d) (0* 10* 10*)* 10*
This regular expression can not generate 01 string.

Scan for Video solution

3. **(b)**

Regular expression that generates all strings having 00 and 11.

$= \underbrace{X\,00\,X\,11\,X} + \underbrace{X\,11\,X\,00\,X}$

Here, Assume X = (0 + 1)*

= X [00 X 11 + 11X00] X

= (0 + 1)*[00(0 + 1)*11 + 11(0 + 1)*00](0 + 1)*

It is matched with option (b).

Scan for Video solution

4. **(a)**

$L_1 = \phi$

$L_2 = \{a\}$

$L_1 L_2^* \cup L_1^* = ?$

$= \phi.a^* \cup \phi^* = \phi \cup \{\in\} = \{\in\}$

Note: $\phi^* = \{\in\}$

Scan for Video solution

5. **(c)**

L = ab + aa + baa

L* = (ab + aa + baa)*

1. $\underline{ab}\ \underline{aa}\ \underline{baa}\ \underline{ab}\ \underline{aa} \in L^5$
2. $\underline{aa}\ \underline{aa}\ \underline{baa}\ \underline{aa} \in L^4$
3. $\underline{baa}\ \underline{aa}\ \underline{ab}\ \underline{aa}\ \underline{aa}\ b \notin L^*$
4. $\underline{baa}\ \underline{aa}\ \underline{ab}\ \underline{aa} \in L^4$

Scan for Video solution

6. **(b)**

Even number of 1's = 0* ($\underline{0}$*1 0* 10*)0*

= (0* 1 0* 1)* 0* = 0*(10* 10*)*

There are many ways two represent the regular expression for even number of 1's.

Matched with option (b).

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7. **(c)**

a. X00X is containing 00.

b. 1* (0 + ε) 1* (0 + ε) 1* is containing atmost two 0's.

c. X0X0X is containing at least two 0's.

d. $(0 + 1)^+$ is begin and end with either 0 or 1.

Hence, option (c) is correct.

Scan for Video solution

8. **(c)**

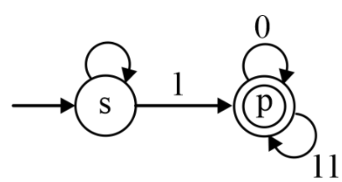
R.E that contain exactly two 1's = 0*10*10*

Hence, option (c) is correct.

Scan for Video solution

9. **(c)**

In given machine, r is dead state. After deleting state q, the resultant transition graph is:



Above transition graph represents R = 1 (0 + 11) *

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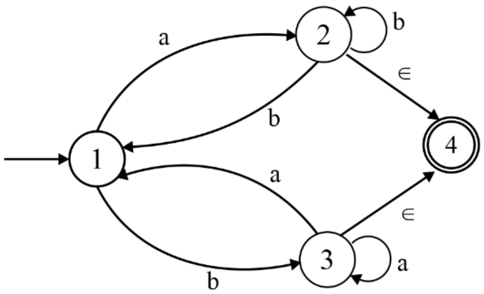
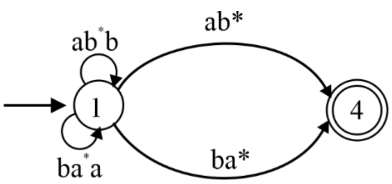
10. (d)





Expression = (b a*a + a b* b)*(a b* + b a*)

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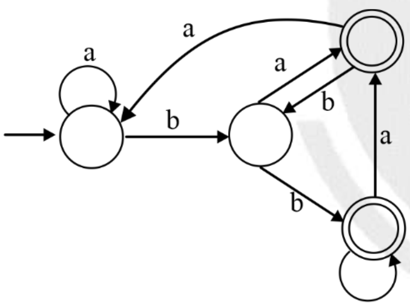
11. (4 to 4)

DFA:



$\boxed{2^k = 2^2 = 4}$ where k is the position of the symbol.

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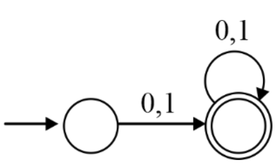
12. (2 to 2)

$(0 + 1)^* (0 + 1) (0 + 1)^* = (0 + 1)^+$



Here number of states are two for $L = (0 + 1)^+$

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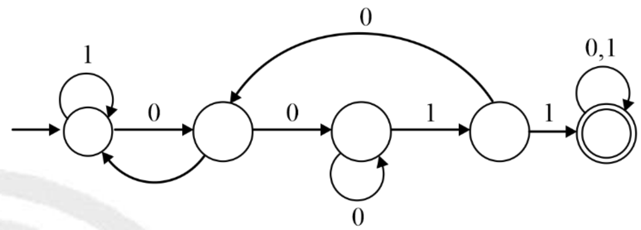
13. (b)

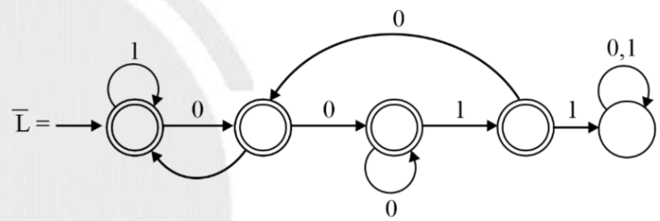
$L = \Sigma^* 0011\Sigma^*$

$L = (0 + 1)^* 0011(0 + 1)^*$

DFA for L:



DFA for $\overline{L}$:



Number of states are always same in both L and $\overline{L}$.

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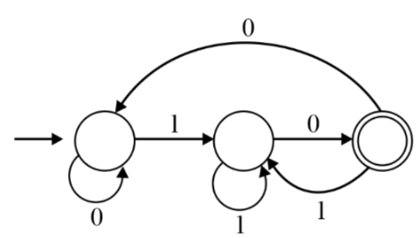
14. (3 to 3)

Minimal string for regular expression = 10

DFA Design for (0 + 1)*(10):



Number of states = 3

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15. (3 to 3)

a* b* (ba)* a*

It will produce all 0 length, 1 length and 2 length string. But it will produce all 3 length string.

∈	Possible
a	Possible
b	Possible
aa	Possible
ab	Possible
ba	Possible
bb	Possible
bab	Not Possible

The shortest string, which is NOT present in the language = bab (3 length).

Scan for Video solution

16. (b)

I. 0*1(1 + 00*1)*

0*1 ((∈ + 00*)1)*

$\underline{0^*1}$ $\underline{(0^*1)}^*$

$(0^*1)^+$

II. 0* 1* 1 + 11* 0* 1 it will not generate 0101 string.

III. (0 + 1)* 1 is same as $(0^*1)^+$

Hence, I and III regular expressions are correct with respective DFA.

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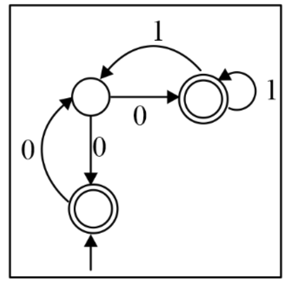
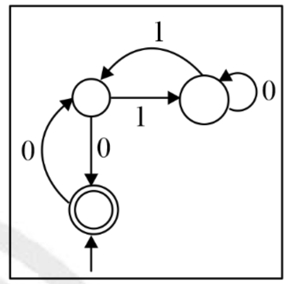
17. (c)

P



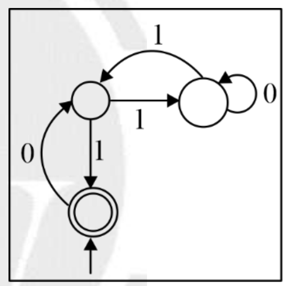
Regular expression = ε + 0(01*1 + 00)*01*

Q



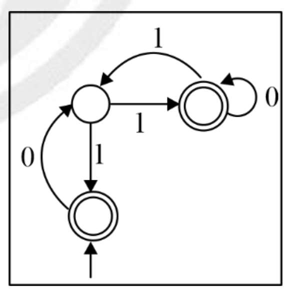
Regular expression = ε + 0(10*1 + 00)*0

R



Regular expression = ε + 0(10*1 + 10)*1

S



Regular expression = ε + 0(10*1 + 10)*10*

Hence, opiton (c) is correct.

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CHAPTER

3

REGULAR LANGUAGE AND GRAMMAR

Regular Grammar and Pumping Lemma

1. [MCQ] [GATE-2023 : 1M]

Consider the context-free grammar G below

$S \rightarrow aSb \mid X$

$X \rightarrow aX \mid Xb \mid a \mid b$

Where S and X are non-terminals, and a and b are terminal symbols. The starting non-terminal is S.

Which one of the following statements is CORRECT?

(a) The language generated by G is $(a + b)^*$

(b) The language generated by G is $a^*(a + b)b^*$

(c) The language generated by G is $a^*b^*(a + b)$

(d) The language generated by G is not a regualar language.

2. [MCQ] [GATE-2019 : 1M]

For $\Sigma=\{a, b\}$, let us consider the regular language

$L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$.

Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?

(a) 3 (b) 5

(c) 9 (d) 24

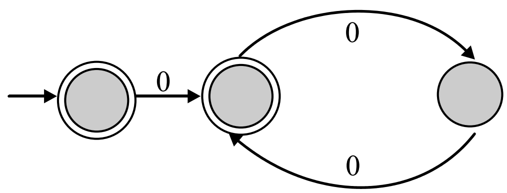
3. [NAT] [GATE-2018 : 2M]

Given a language L, define L^i as follows:

$L^0 = \{\in\}$

$L^1 = L^{i-1}.\ L$ for all $i > 0$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.



Consider the language L_1 (over alphabet 0) accepted by the following automaton.

The order of L_1 is _____________.

4. [MCQ] [GATE-2016 : 2M]

Language L_1 is defined by the grammar:

$S_1 \rightarrow aS_1b \mid \varepsilon$

Language L_2 is defined by the grammar:

$S_2 \rightarrow abS_2 \mid \varepsilon$

Consider the following statements:

P: L_1 is regular

Q: L_2 is regular

Which of the following is TRUE?

(a) Both P and Q are true.

(b) P is true and Q is false.

(c) P is false and Q is true.

(d) Both P and Q are false.

5. [MCQ] [GATE-2016 : 1M]

Which of the following languages is generated by the given grammar?

$S \rightarrow aS \mid bS \mid \in$

(a) $\{a^n b^m \mid n, m \geq 0\}$

(b) $\{w \in \{a, b\}^* \mid$ w has equal number of a's and b's$\}$

(c) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$

(d) $\{a, b\}^*$

6. [MCQ] [GATE-2015 : 2M]

Consider alphabet $\Sigma = \{0,1\}$, the null/empty string λ and the sets of strings X_0, X_1, and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows:

$X_0 = 1\ X_1$

$X_1 = 0\ X_1 + 1\ X_2$

$X_2 = 0\ X_1 + \{\lambda\}$

Which one of the following choices precisely represents the strings in X_0?

(a) $10(0^* + (10)^*)\ 1$

(b) $10(0^* + (10)^*)^*\ 1$

(c) $1(0 + 10)^*\ 1$

(d) $10(0 + 10)^*\ 1 + 110(0 + 10)^*\ 1$

Regular Language Identification

7. [MCQ] [GATE-2019 : 1M]

If L is a regular language over $\Sigma = \{a, b\}$, which one of the following languages is NOT regular?

(a) $L.\ L^R = \{x\ y \mid x \in L, y^R \in L\}$

(b) $\{w\ w^R \mid w \in L\}$

(c) Prefix $(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^*$ such that $xy \in L\}$

(d) Suffix $(L) = \{y \in \Sigma^* \mid \exists x \in \Sigma^*$ such that $xy \in L\}$

8. [MCQ] [GATE-2015 : 2M]

Which of the following languages is/are regular?

L_1: $\{w\ x\ w^R \mid w, x \in \{a, b\}^*$ and $|w|, |x| > 0\}$, w^R is the reverse of string w.

L_2: $\{a^n\ b^m \mid m \neq n$ and $m, n \geq 0\}$

L_3: $\{a^p\ b^q\ c^r \mid p, q, r \geq 0\}$

(a) L_1 and L_3 only (b) L_2 only

(c) L_2 and L_3 only (d) L_3 only

9. [MCQ] [GATE-2014 : 2M]

Let $L_1 = \{w \in \{0, 1\}^* \mid w$ has at least as many occurrences of (110)'s as (011)'s$\}$. Let $L_2 = \{w \in \{0, 1\}^* \mid w$ has at least as many occurrences of (000)'s as (111)'s$\}$. Which one of the following is TRUE?

(a) L_1 is regular but not L_2

(b) L_2 is regular but not L_1

(c) Both L_1 and L_2 are regular

(d) Neither L_1 nor L_2 are regular

10. [MCQ] [GATE-2014 : 1M]

If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 . L_2$ is a regular language

II. $L_1 . L_2 = \{a^n\ b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

11. [MCQ] [GATE-2008 : 2M]

Which of the following are regular sets?

I. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\}$

II. $\{a^n b^m \mid n = 2m\}$

III. $\{a^n b^m \mid n \neq m\}$

IV. $\{xcy \mid x, y \in \{a, b\}^*\}$

(a) I and IV only (b) I and III only

(c) IV only (d) IV only

Closure Properties of Regular Language

12. [MCQ] [GATE-2020 : 1M]

Consider the following statements:

I. If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular.

II. The class of regular languages is closed under infinite union.

Which of the above statements is/are TRUE?

(a) Both I and II (b) II only

(c) I only (d) Neither I not II

13. [MCQ] [GATE-2016 : 2M]

Consider the following two statements:

I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^*.

II. There exists a regular language A such that for all languages B, $A \cap B$ is regular.

Which one of the following is CORRECT?

(a) Only I is true

(b) Only II is true

(c) Both I and II are true

(d) Both I and II are false

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ANSWER KEY

1.	(b)	**2.**	(d)	**3.**	(2 to 2)	**4.**	(c)
5.	(d)	**6.**	(c)	**7.**	(b)	**8.**	(a)
9.	(a)	**10.**	(a)	**11.**	(a)	**12**	(d)
13.	(b)						

SOLUTIONS

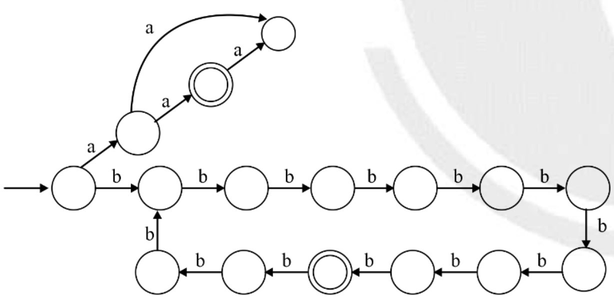
1. (b)

$X = a^*(a + b)b^*$

$S = a^n\, b^n = a^n\, a^* \,(a + b)\, b^*\, b^n = a^*\, (a + b)\, b^*$

Scan for Video solution

2. (d)



$L = \{a^{2 + 3k}\} \cup \{b^{10 + 12k}\}$

$= aa\,(aaa)^* + b^{10}.\,(b^{12})^*$

Number of states = 16

So, pumping length must be greater than 16.

Hence, option (d) is correct.

Scan for Video solution

3. (2 to 2)

$L = \in + 0\,(00)^*$ for given FA.

$L^0 = \{\in\}$

$L^1 = L^0.\, L = \in.(\in + 0(00)^*) = L$

$L^2 = L^1.\, L = L.L = [\in + 0(00)^*]\, [\in + 0(00)^*] = 0^*$

$L^3 = L^2.\, L = 0^*.\, (\in + 0\,(00)^*) = 0^*$

So, O(L) = 2

Scan for Video solution

4. (c)

- $S_1 \rightarrow aS_1b \mid \in$

  $L_1 = \{a^n b^n \mid n \geq 0\}$ is CFL

- $S_2 \rightarrow abS_2 \mid \in$

  $L_2 = (ab)^*$ is Regular

Hence, L_1 is CFL and L_2 is Regular.

So, option (c) is correct.

Scan for Video solution

5. (d)

$S \rightarrow aS \mid bS \mid \in$

$L = (a + b)^*$

$= \{a, b\}^*$

Hence, option (d) is correct.

Scan for Video solution

6. (c)

$X_2 \rightarrow 0X_1 \mid \in$

$X_2 = 0X_1 + \in$

$X_1 \rightarrow 0X_1 \mid 1X_2$

$X_1 = 0X_1 \mid 10X_1 \mid 1$

$X_1 = (0 + 10)^*1$

$X_0 = 1X_1$

$X_0 = \mathbf{1(0 + 10)^*1}$

Hence, option (c) is correct.

Scan for Video solution

7. (b)

(a) If L is Regular then $L.L^R$ is Regular.

(b) May/may not be Regular over $\Sigma = \{a, b\}$.

If $L = a^*$ then $ww^R = (aa)^*$ is regular

If $L = (a+b)^*$ then ww^R is not regular

(c) Prefix(Regular) is always Regular

(d) Suffix(Regular) is always Regular

Hence, option (b) is correct.

Scan for Video solution

8. (a)

L_1: $\{wxw^R \mid w, x \in \{a, b\}^+\}$

$L_1 = axa + bxb$

$= a\,(a + b)^+\,a + b(a + b)^+\,b$ = Regular

L_2: $\{a^n b^m \mid n \neq m\}$= Not Regular

L_3: $a^*b^*c^*$ = Regular

Hence, L_1 and L_3 are Regular.

Scan for Video solution

9. (a)

$L_1 : n_{110}$ and n_{011}

We can construct DFA for L_1 with the help of cycle to count number of occurrences of both patterns 110 and 011. While counting one pattern, other pattern will be either equal or one more or less compared to other pattern. This can be remembered with the few states.

$L_2 : n_{000}$ and n_{111}

We can not make DFA for L_2. While counting 000, there is no way to remember the count using DFA.

Hence, option (a) is correct.

Scan for Video solution

10. (a)

$L_1.L_2 = \{a^{n_1} b^{n_2} \mid n_1, n_2 \geq 0\}$

$= a^*b^*$ Regular language

Scan for Video solution

11. (a)

I. $\{a^n b^{2m} \mid n \geq 0, m \geq 0\} \Rightarrow a^*(bb)^*$

II. $\{a^n b^m \mid n = 2m\} \Rightarrow$ CFL

III. $\{a^n b^m \mid n \neq m\} \Rightarrow$ CFL

IV. $\{xcy \mid x, y \in \{a, b\}^*\} \Rightarrow (a + b)^*\ c\ (a + b)^*$

Hence, option (a) is correct.

Scan for Video solution

12. (d)

I. If $L_1 \cup L_2$ is Regular then, L_1 and L_2 may/may not be regular.

$a^n b^n$ (DCFL) $\cup$ $a^* b^*$(Regular) = Regular

$a^n b^n$ (DCFL) $\cup$ $(a + b)^*$ (Regular) = Regular

II. $\{\varepsilon\} \cup \{ab\} \cup \{a^2 b^2\} \cup \ldots = \{a^n b^n\}$ is not regular

Infinite union is not closed under regular language.

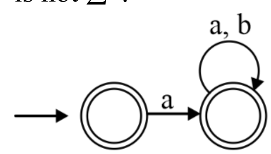
Hence, option (d) is correct.

Scan for Video solution

13. (b)

I. FALSE

Consider the following NFA. Here all states are accepting states, but language accepted by NFA is not Σ^*.



II. TRUE

$A \cap B$ is Regular, when A is empty language.

$\phi \cap$ Any $= \phi$ (Regular)

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CHAPTER

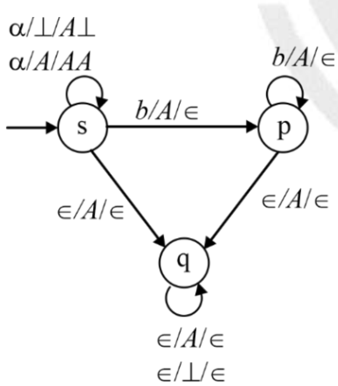
4

CONTEXT FREE LANGUAGE AND CONTEXT FREE GRAMMAR

Pushdown Automata

1. [MCQ] [GATE-2023 : 2M]

Consider the pushdown automaton (PDA) P below, which runs on the input alphabet {a, b}, has stack alphabet {⊥, A}, and has three states {s, p, q}, with s being the start state. A transition from state u to state v, labelled c/X/γ, where c is an input symbol or ∈, X is a stack symbol, and γ is a string of stack symbols, represents the fact that in state u, the PDA can read c from the input, with X on the top of its stack, pop X from the stack, push in the string γ on the stack, and go to state v. In the initial configuration, the stack has only the symbol ⊥ in it. The PDA accepts by empty stack.



Which one of the following options correctly describes the language accepted by P?

(a) $\{a^m b^n \mid 1 \le m \text{ and } n < m\}$

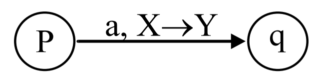
(b) $\{a^m b^n \mid 0 \le n \le m\}$

(c) $\{a^m b^n \mid 0 \le m \text{ and } 0 \le n\}$

(d) $\{a^m \mid 0 \le m\} \cup \{b^n \mid 0 \le n\}$

2. [NAT] [GATE-2021 : 2M]

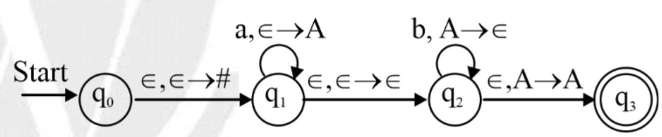
In a pushdown automaton P = (Q, Σ, Γ, δ, q_0, F), a transition of the form,



where p, q ∈ Q, a ∈ Σ ∪ {∈}, and X, Y ∈ Γ ∪ {∈}, represents

(q, Y)∈ δ (p,a,X)

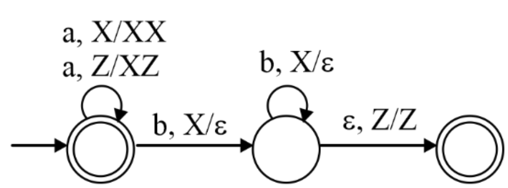
Consider the following pushdown automaton over the input alphabet Σ = {a, b} and stack alphabet Γ = {#, A}.



The number of strings of length 100 accepted by the above pushdown automaton is _____.

3. [MCQ] [GATE-2016 : 2M]

Consider the transition diagram of a PDA given below with input alphabet Σ = {a, b}and stack alphabet Γ = {X, Z}. Z is the initial stack symbol. Let L denote the language accepted by the PDA

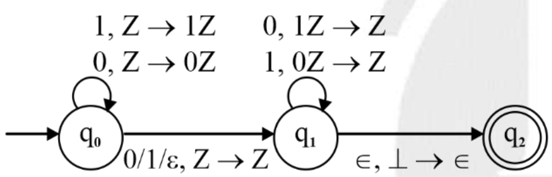


Which one of the following is TRUE?

(a) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata
(b) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA
(c) L is not accepted by any Turing machine that halts on every input
(d) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context – free

4. [MCQ] [GATE-2015 : 2M]

Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \sum = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\} \rangle$ where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:



Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

(a) 10110 (b) 10010
(c) 01010 (d) 01001

Context Free Grammar

5. [MCQ] [GATE-2017 : 2M]

If G is a grammar with productions

$S \to SaS \mid aSb \mid bSa \mid SS \mid \in$

Where S is the start variable, then which one of the following strings is not generated by G?

(a) abab (b) aaab
(c) abbaa (d) babba

6. [MCQ] [GATE-2017 : 1M]

Identify the language generated by the following grammar, where S is the start variable.

$S \to XY$

$X \to aX \mid a$

$Y \to aYb \mid \in$

(a) $\{a^m b^n \mid m > n, n > 0\}$
(b) $\{a^m b^n \mid m > n, n \geq 0\}$
(c) $\{a^m b^n \mid m \geq n, n \geq 0\}$
(d) $\{a^m b^n \mid m \geq n, n > 0\}$

7. [MCQ] [GATE-2017 : 1M]

Consider the following context - free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol:

$S \to abScT \mid abcT$

$T \to bT \mid b$

Which one of the following represents the language generated by the above grammar

(a) $\{(ab)^n (cb)^n \mid n \geq 1\}$
(b) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots\ldots cb^{m_n} \mid n, m_1, m_2 \ldots\ldots m_n \geq 1\}$
(c) $\{(ab)^n (cb^m)^n \mid m, n \geq 1\}$
(d) $\{(ab)^n (cb^n)^m \mid m, n \geq 1\}$

8. [MCQ] [GATE-2017 : 2M]

Consider the context-free grammars over the alphabet {a, b, c} given below. S and T are non-terminals.

G_1: $S \to aSb \mid T$, $T \to cT \mid \in$

G_2: $S \to bSa \mid T$, $T \to cT \mid \in$

The language $L(G_1) \cap L(G_2)$ is

(a) Finite.
(b) Not finite but regular.
(c) Context - Free but not regular.
(d) Recursive but not context-free.

9. [MCQ] [GATE-2016 : 2M]

Which one of the following grammars is free from left recursion?

(a) $S \to AB$
$A \to Aa \mid b$
$B \to c$

(b) $S \to Ab \mid Bb \mid c$
$A \to Bd \mid \in$
$B \to e$

(c) $S \to Aa \mid B$
$A \to Bb \mid Sc \mid \in$
$B \to d$

(d) $S \to Aa \mid Bb \mid c$
$A \to Bd \mid \in$
$B \to Ae \mid \in$

10. [MCQ] [GATE-2016 : 2M]

Consider the following context-free grammars:

G_1: $S \to aS \mid B$

$B \to b \mid bB$

G_2: $S \to aA \mid bB$

$A \to aA \mid B \mid \varepsilon$

$B \to bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2 respectively?

(a) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$

(b) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$

(c) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$

(d) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$

11. [MCQ] [GATE-2009 : 1M]

$S \to aSa \mid bSb \mid a \mid b$

The language generated by the above grammar over the alphabet $\{a, b\}$ is the set of

(a) All palindromes

(b) All odd length palindromes

(c) Strings that begin and end with the same symbol

(d) All even length palindromes

Common data for question 12 and 13.

A CFG G is given with the following productions where S is the start symbol, A is a non-terminal and a and b are terminals.

$S \to aS \mid A$

$A \to aAb \mid bAa \mid \varepsilon$

12. [MCQ] [GATE-2008 : 2M]

Which of the following strings is generated by the grammar above?

(a) aabbaba (b) aabaaba

(c) abababb (d) aabbaab

13. [MCQ] [GATE-2008 : 2M]

For the correct answer in above Question, how many steps are required to derive the string "aabbaab" and how many parse trees are there?

(a) 6 and 1 (b) 6 and 2

(c) 7 and 2 (d) 4 and 2

14. [MCQ] [GATE-2008 : 2M]

Consider a CFG with the following productions.

$S \to AA \mid B$

$A \to 0A \mid A0 \mid 1$

$B \to 0B00 \mid 1$

S is the start symbol, A and B are non-terminals and 0 and 1 are the terminals. The language generated by this grammar is

(a) $\{0^n 1 0^{2n} \mid n \geq 1\}$

(b) $\{0^i 1 0^j 1 0^k \mid i, j, k \geq 0\} \cup \{0^n 1 0^{2n} \mid n \geq 0\}$

(c) $\{0^i 1 0^j \mid i, j \geq 0\} \cup \{0^n 1 0^{2n} \mid n \geq 0\}$

(d) The set of all strings over $\{0, 1\}$ containing at least two 0's

15. [MCQ] [GATE-2008 : 2M]

Which of the following statements are true?

1. Every left – recursive grammar can be converted to a right – recursive grammar and vice-versa
2. All $\in$ - productions can be removed from any context – free grammar by suitable transformations
3. The language generated by a context-free grammar all of whose productions are of the form $X \to w$ or wY (where, w is a string of terminals and Y is a non-terminal), is always regular
4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees

(a) 1, 2, 3 and 4

(b) 2, 3 and 4 only

(c) 1, 3 and 4 only

(d) 1, 2 and 4 only

Context Free Language Identification

16. [MSQ] [GATE-2022 : 2M]

Consider the following languages:

$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$

$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$

Note that w^R is the reversal of the string w. Which of the following is/are TRUE?

(a) L_1 and L_2 are regular
(b) L_1 and L_2 are context – free
(c) L_1 is regular and L_2 is context – free
(d) L_1 and L_2 are context – free but not regular

17. [MSQ] [GATE-2022 : 2M]

Consider the following languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$

$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$

$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$

Which of the following statements is/are FALSE?

(a) L_1 is not context – free but L_2 and L_3 are deterministic context – free.
(b) Neither L_1 nor L_2 is context – free.
(c) L_2, L_3 and $L_2 \cap L_3$ all are context - free.
(d) Neither L_1 nor its complement is context – free.

18. [MSQ] [GATE-2021 : 2M]

For a string w, we define w^R to be the reverse of w. For example, if $w = 01101$ then $w^R = 10110$.

Which of the following languages is/are context-free?

(a) $\{w\, x\, w^R\, x^R \mid w, x \in \{0, 1\}^*\}$
(b) $\{w\, w^R\, x\, x^R \mid w, x \in \{0, 1\}^*\}$
(c) $\{w\, x\, w^R \mid w, x \in \{0, 1\}^*\}$
(d) $\{w\, x\, x^R w^R \mid w, x \in \{0, 1\}^*]$

19. [MCQ] [GATE-2020 : 1M]

Consider the language $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and the following statements

I. L is deterministic context-free
II. L is context-free but not deterministic context-free
III. L is not LL(k) for any k.

Which of the above statements is/are TRUE?

(a) I only
(b) II only
(c) I and III only
(d) III only

20. [MCQ] [GATE-2020 : 2M]

Consider the following languages.

$L_1 = \{wxyx \mid w, x, y \in (0 + 1)^+\}$

$L_2 = \{xy \mid x, y \in (a + b)^*, |x| = |y|, x \neq y\}$

Which one of the following is TRUE?

(a) L_1 is regular and L_2 is context-free.
(b) L_1 is context-free but L_2 is not context-free.
(c) Neither L_1 nor L_2 is context-free.
(d) L_1 is context-free but not regular and L_2 is context-free.

21. [MCQ] [GATE-2019 : 2M]

Which one of the following language over $\Sigma = \{a, b\}$ is NOT context – free?

(a) $\{ww^R \mid w \in \{a, b\}^*\}$
(b) $\{wa^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$
(c) $\{wa^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$
(d) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$

22. [MCQ] [GATE-2018 : 2M]

Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q$, where $m, n, p, q \geq 0\}$
II. $\{a^m b^n c^p d^q \mid m = n$ and $p = q$, where $m, n, p, q \geq 0\}$
III. $\{a^m b^n c^p d^q \mid m = n = p$ and $p \neq q$, where $m, n, p, q \geq 0\}$
IV. $\{a^m b^n c^p d^q \mid mn = p + q$, where $m, n, p, q \geq 0\}$

Which of the languages above are context-free?

(a) I and IV only
(b) I and II only
(c) II and III only
(d) II and IV only

23. [MCQ] [GATE-2016 : 2M]

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$ Which one of the following is TRUE?

(a) Both L_1 and L_2 are context-free.
(b) L_1 is context-free while L_2 is not context-free.
(c) L_2 is context-free while L_1 is not context-free.
(d) Neither L_1 nor L_2 is context-free.

24. [MCQ] [GATE-2015 : 1M]
Which of the following languages are context-free
$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$
$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$
$L_3 = \{a^m b^n \mid m = 2n + 1\}$
(a) L_1 and L_2 only (b) L_1 and L_3 only
(c) L_2 and L_3 only (d) L_3 only

25. [MCQ] [GATE-2014 : 1M]
Which of the following is TRUE?
(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.
(b) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular
(c) The language $L = \{w \mid w$ has $3k + 1$ b' s for some $k \in N$ with $\Sigma = \{a, b\}\}$ is regular.
(d) The language $L = \{w w \mid w \in \Sigma^*$ with $\Sigma = \{0, 1\}\}$ is regular.

26. [MCQ] [GATE-2014 : 2M]
Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:
$L_1 = \{0^n 1^n \mid n \geq 0\}$
$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$
$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$
Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?
(a) None of the languages
(b) Only L_1
(c) Only L_1 and L_2
(d) All the three languages

27. [MCQ] [GATE-2010 : 2M]
Consider the language
$L_1 = \{0^i 1^j \mid i \neq j\}$
$L_2 = \{0^i 1^j \mid i = j\}$
$L_3 = \{0^i 1^j \mid i = 2j + 1\}$
$L_4 = \{0^i 1^j \mid i \neq 2j\}$
Which one of the following statements is true?
(a) Only L_2 is context free
(b) Only L_2 and L_3 are context free
(c) Only L_1 and L_2 are context free
(d) All are context free

28. [MCQ] [GATE-2008 : 2M]
Consider the following languages.
$L_1 = \{a^i b^j c^k \mid i = j, k \geq 1\}$
$L_2 = \{a^i b^j \mid j = 2i, i \geq 0\}$
Which of the following is true?
(a) L_1 is not a CFL but L_2 is
(b) $L_1 \cap L_2 = \phi$ and L_1 is non-regular
(c) $L_1 \cup L_2$ is not a CFL but L_2 is
(d) There is a 4 state PDA that accepts L_1 but there is no DPDA that accepts L_2

29. [MCQ] [GATE-2008 : 2M]
Which of the following languages is (are) non – regular?
$L_1 = \left\{0^m 1^n \mid 0 \leq m \leq n \leq 10000\right\}$
L_2 = {reads the same forward and backward}
L_3 = {contains an even number of 0's and an even number of 1's}
(a) L_2 and L_3 only (b) L_1 and L_2 only
(c) L_3 only (d) L_2 only

Closure Properties of Context Free Language

30. [MSQ] [GATE-2021 : 1M]
Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?
(a) $L_1 \cap \overline{L}_2$
(b) $\overline{\overline{L_1} \cup \overline{L}_2}$
(c) $L_1 \cup (L_2 \cup \overline{L}_2)$
(d) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$

31. **[MCQ]** **[GATE-2021 : 1M]**
Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages as NOT necessarily context-free?
(a) $L_1 \cap L_2$ (b) $L_1 . L_2$
(c) $L_1 - L_2$ (d) $L_1 \cup L_2$

32. **[MCQ]** **[GATE-2017 : 1M]**
Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?
I. $L_1 \cup L_2$ is context − free
II. $\overline{L_1}$ is context − free
III. $L_1 - R$ is context − free
IV. $L_1 \cap L_2$ is context − free
(a) I, II and IV only (b) I and III only
(c) II and IV only (d) I only

33. **[MCQ]** **[GATE-2017 : 2M]**
Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$
Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and
$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$.
Which of the following are context-free languages?
I. $L_1 \cup L_2$ **II.** $L_1 \cap L_2$
(a) I only (b) II only
(c) I and II (d) Neither I nor II

34. **[MCQ]** **[GATE-2011 : 1M]**
Let P be a regular language and Q be a context free language such that $Q \subseteq P$. (For example, let P be the language represented by the regular expression p*q* and Q be $\{p^n q^n \mid n \in N\}$). Then which of the following is ALWAYS regular?
(a) $P \cap Q$ (b) $P - Q$
(c) $\Sigma^* - P$ (d) $\Sigma^* - Q$

35. **[MCQ]** **[GATE-2009 : 1M]**
Which one of the following is FALSE?
(a) There is a unique minimal DFA for every regular language.
(b) Every NFA can be converted to an equivalent PDA.
(c) Complement of every context-free language is recursive.
(d) Every nondeterministic PDA can be converted to an equivalent deterministic PDA.

❑❑❑

ANSWER KEY

1.	(a)	**2.**	(50 to 50)	**3.**	(d)	**4.**	(b)
5.	(d)	**6.**	(b)	**7.**	(b)	**8.**	(b)
9.	(b)	**10.**	(d)	**11.**	(b)	**12.**	(d)
13.	(a)	**14.**	(b)	**15.**	(c)	**16.**	(a, b, c)
17.	(b, c, d)	**18.**	(b, c, d)	**19.**	(c)	**20.**	(a)
21.	(c)	**22.**	(b)	**23.**	(b)	**24.**	(b)
25.	(c)	**26.**	(c)	**27.**	(d)	**28.**	(b)
29.	(d)	**30.**	(b, c, d)	**31.**	(c)	**32.**	(b)
33.	(a)	**34.**	(c)	**35.**	(d)		

SOLUTIONS

1. **(a)**

State s can push A for each input a, and whenever b comes as input, it will pop A and then changes to state p. If end of the input happens from state s, then it will change to state q and will pop all A's to make empty stack.

State p can pop A for each input b, and whenever end of input reaches, it will change to state q and pop all A's from stack to make empty stack.

Case1: a* will be accepted if s and q are involved.

Case2: $a^m b^n$ will be accepted ($m > n > 0$) if s, p, and q are involved.

$L = \{a^m b^n \mid m > n \geq 0\} = \{a^m b^n \mid m > n, m \geq 1\}$

Therefore, option (a) is correct.

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2. **(50 to 50)**

Language by PDA = $\{a^m b^n \mid m > n\}$

- Sum of a's and b's must be 100
- Possible strings are: $a^{100} b^0$, $a^{99} b^1$, $a^{92} b^2$, …., $a^{51} b^{49}$.
- Total number of 100 length strings

  $= 100 - 51 + 1 = 50$

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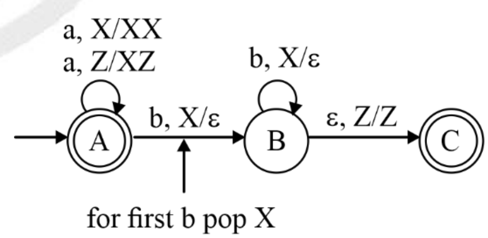
3. **(d)**



State **A:** For every input a push X (A is final state).

State **B:** For every input b pop X.

$L = \{a^n b^n \mid n \geq 0\} \cup \{a^n \mid n \geq 0\}$

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4. (b)

q_0:

$0, z \rightarrow 0z$; Push 0 for input 0

$1, z \rightarrow 1z$; Push 1 for input 1

$0/1/\varepsilon, z \rightarrow z$; no operation

q_1:

$1, 0z \rightarrow z$; pop 0 for input 1

$0, 1z \rightarrow z$; pop 1 for input 0

- String = 101100

Skip

1 0 1 1 0 $\boxed{0}$ _ _ _ _ _

1 0 1 1 0 **0 <u>1</u> <u>0</u> <u>0</u> <u>1</u> <u>0</u>**

0	→ To pop 0, 1 should be come
1	→ To pop 1, 0 should be come
1	→ To pop 1, 0 should be come
0	→ To pop 0, 1 should be come
1	→ To pop 1, 0 should be come
1	→ To pop 1, 0 should be come
⊥	

Last 5 bits = 10010

Hence, option (b) is correct.

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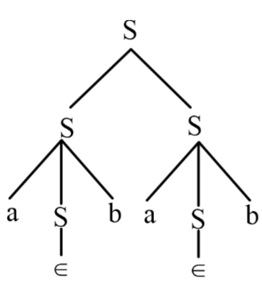
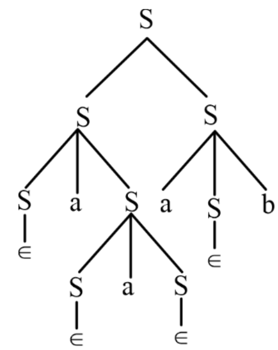
5. (d)

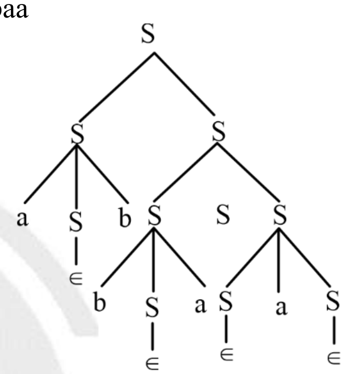
(a) abab



(b) aaab



(c) abbaa



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6. (b)

$Y = a^n b^n$

$X = a^+$

$S = a^+ a^n b^n \mid n \geq 0$

Hence, $L = \{a^m b^n \mid m, n \geq 0 \text{ and } m > n\}$

$= \{a^m b^n \mid m > n \geq 0\}$

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7. (b)

$T \rightarrow bT \mid b$

$T = b^+$

$S \rightarrow abScT \mid abcT$

$S = abS\, cb^+ \mid abcb^+$

$= \{ab^n\, abcb^+\, (cb^+)^n \mid n \geq 0\}$

Minimal string = $abcb^+$

$L = \{(ab)^n (cb^+)^n \mid n > 0\}$

$= \{(ab)^n cb^{m_1} cb^{m_2} \ldots\ldots n \text{ times}\}$

$= \{(ab)^n cb^{m_1} cb^{m_2} cb^{m_3} \ldots . cb^{m_n} \mid n, m_1, m_2 \ldots . m_n \geq 1\}$

Scan for Video solution

8. (b)

G1: $S \to aSb \mid T$

$T \to c\,T \mid \in = c^*$

$L(G1) = \{a^n c^* b^n \mid n \geq 0\}$

G2: $S \to bSa \mid T$

$T \to c\,T \mid \in = c^*$

$L(G2) = \{b^n c^* a^n \mid n \geq 0\}$

$L(G1) \cap L(G2) = \{c^*\}$

It is a regualr but infinite(Not finite).

Hence, option (b) is correct.

Scan for Video solution

9. (b)

(a) $S \to AB$

$A \to Aa \mid b$

$B \to c$

$A \to A$ there is a left recursion A to A.

(b) $S \to Ab \mid Bb \mid c$

$A \to Bd \mid \in$

$B \to e$

There is not left recursion.

(c) $S \to Aa \mid B$

$A \to Bb \mid Sc \mid \in$

$B \to d$

$S \to A \to S$ indirectly there is a left recursion S to S.

(d) $S \to Aa \mid Bb \mid c$

$A \to Bd \mid \in$

$B \to Ae \mid \in$

$A \to B \to A$ indirectly there is a left recursion A to A.

Scan for Video solution

10. (d)

- **G_1:** $S \to aS \mid B$

  $B \to b \mid bB$

  $B = b^+$

  $S = a^*b^+$

  $L_1 = \{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$

- **G_2:** $S \to aA \mid bB$

  $A \to aA \mid B \mid \varepsilon$

  $B \to bB \mid \varepsilon$

  $B = b^*$

  $A = a^*b^*$

  $S = aa^*b^* \mid bb^*$

  $L_2 = a^+ b^* \cup b^+$

  $L_2 = \{a^m b^n \mid m > 0 \text{ or } n > 0\}$

Hence, option (d) is correct.

Scan for Video solution

11. (b)

- $S \to aSa \mid bSb \mid a \mid b$

  This grammar will generate all odd length palindrome strings.

Scan for Video solution

12. (d)

$S \to aS \mid A$

$A \to aAb \mid b\,A\,a \mid \varepsilon$

(a) aabbaba $\to$ **Not Possible.**

(b) aabaaba $\to$ **Not Possible.**

(c) abababb $\to$ **Not Possible.**

(d) aabbaab $\to$ **Possible.**

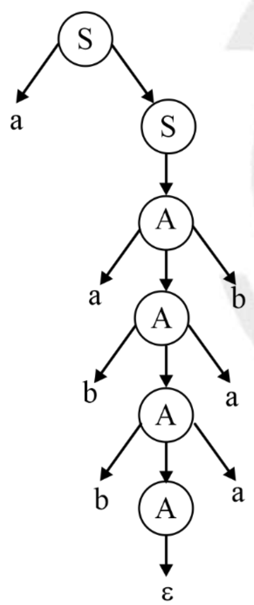
Hence, option (d) is correct.

Scan for Video solution

13. (a)

$S \to aS \mid A$

$A \to aAb \mid bAa \mid \varepsilon$



$S \to aS \to aA \to aaAb \to aabAab \to aabbAaab$
$\to aabbaab$

Total number of steps = 6 and 1 Parse Tree.

Scan for Video solution

14. (b)

$B \to 0B00 \mid 1$

$B = 0^n10^{2n}$

$A \to 0A \mid A0 \mid 1$

$A = 0^*10^*$

$S \to AA \mid B$

$S = \{0^*10^*0^*10^*\} \cup \{0^n10^{2n}\}$

$= \{0^*10^*10^*\} \cup \{0^n10^{2n}\}$

- $0^*10^*10^* = \{0^i10^j10^k \mid i, j, k \ge 0\}$
- $0^n10^{2n} = \{0^n10^{2n} \mid n \ge 0\}$

Scan for Video solution

15. (c)

1. Every left – recursive grammar can be converted to a right – recursive grammar and vice-versa. **TRUE**
2. All $\in$ - productions can be removed from any context – free grammar by suitable transformations. **FALSE**
3. The language generated by a context-free grammar all of whose productions are of the form $X \to w$ or wY (where, w is a string of terminals and Y is a non-terminal), is always regular. **TRUE**
4. The derivation trees of strings generated by a context-free grammar in Chomsky Normal Form are always binary trees. **TRUE**

Scan for Video solution

16. (a, b, c)

- $L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$

  Assume n = 0

  $L_1 = \in.w.\in$

  $= w = \{a, b\}^*$

  $= (a + b)^*$ Regular

- $L_2 = \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$

OR

$= \{wxw^R \mid w, x \in \{a, b\}^+\}$

It is well known regular language.

Every regular language is also CFL.

Hence, option (a, b, c) are correct.

Scan for Video solution

17. (b, c, d)

$L_1 = \{ww \mid w \in \{a, b\}^*\} \Rightarrow CSL$

$L_2 = \{a^n b^n c^m \mid m, n \in 0\} \Rightarrow a^n b^n c^* \Rightarrow DCFL$

$L_3 = \{a^m b^n c^n \mid m, n \in 0\} \Rightarrow a^* b^n c^n \Rightarrow DCFL$

- Every DCFL language is CFL also
- DCFL $\cap$ DFCL

  $a^n b^n c^* \cap a^* b^n c^n$

  $= a^n b^n c^n = CSL$

$L_1 = \overline{ww} = CFL$

Scan for Video solution

18. (b, c, d)

(a) $w\ x\ w^R\ x^R$ Not CFL

(b) $w\ w^R\ x\ x^R$ CFL

(c) $\in.x.\in = x = (0+1)^* =$ Regular

Regular means CFL also

(d) $w\ x\ x^R\ w^R$ CFL

Hence, option (b, c, d) are correct.

Scan for Video solution

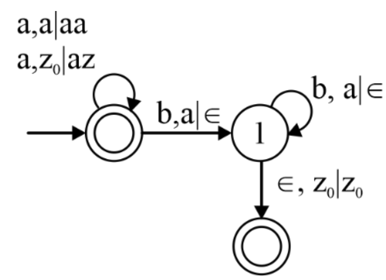
19. (c)

I. L is deterministic context-free **(True)**

II. L is context-free but deterministic context-free **(False)**

Regular $\cup$ DCFL = DCFL

DPDA:



III. L is not LL(k) for any k. **(True)**

Scan for Video solution

20. (a)

$L_1 = wxwy$

Put minimum string in x = 0 or 1

- $L_1 = w0y0 + w1y1$

  $(0+1)^+ 0 (0+1)^+ 0 + (0+1)^+ 1 (0+1)^+ 1$ is Regular
- L_2 = CFL language

Scan for Video solution

21. (c)

(a) $\{ww^R \mid w \in \{a, b\}^*\}$. **CFL**

(b) $\{wa^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$. **CFL**

(c) $\{wa^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$. **Not CFL**

(d) $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$. **CFL**

Scan for Video solution

22. (b)

I. $\{a^m b^n c^p d^q \mid m + p = n + q\}$ is CFL

II. $L = \underbrace{a^m\ b^n}\ \underbrace{c^b\ d^q}$ is CFL.

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q\}$. PDA not exist. So,it is not CFL.

IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$. It is also not CFL.

Hence, option (b) is correct.

Scan for Video solution

23. (b)

- $L_1 = \{a^n b^m c^{n+m} \mid m, n \geq 1\}$

  $= \{a^n b^m c^m c^n \mid m, n \geq 1\}$

  L_1 is CFL.

- $L_2 = \{a^n b^n c^{2n} \mid n \geq 1\}$

  = CSL but Not CFL.

  Hence, option (b) is correct.

Scan for Video solution

24. (b)

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$= CFL

$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$ = Not CFL

$L_3 = \{a^m b^n \mid m = 2n + 1\}$ = CFL

Scan for Video solution

25. (c)

(a) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular. **False**

(b) The language $L = \{a^n \mid n \text{ is prime }\}$ is regular. **False**

(c) The language $L = \{w \mid w$ has $3k + 1$ b' s for some $k \in N$ with $\Sigma = \{a, b\}\}$ is regular. **True**

(d) The language $L = \{w w \mid w \in \Sigma^*$ with $\Sigma = \{0, 1\}\}$ is regular. **False**

Scan for Video solution

26. (c)

$\Sigma = \{0, 1, c\}$

$L_1 = \{0^n 1^n \mid n \geq 0\} \Rightarrow$ DCFL

$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\} \Rightarrow$ DCFL

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\} \Rightarrow$ CFL but not DCFL

Scan for Video solution

27. (d)

$L_1 = \{0^i 1^j \mid i \neq j\}$ = **DCFL**

$L_2 = \{0^i 1^j \mid i = j\}$ = **DCFL**

$L_3 = \{0^i 1^j \mid i = 2j + 1\}$= **DCFL**

$L_4 = \{0^i 1^j \mid i \neq 2j\}$= **DCFL**

All are DCFLs and every DCFL is CFL.

Scan for Video solution

28. (b)

$L_1 = \{a^i b^j c^k \mid i = j, k \geq 1\} \Rightarrow a^n b^n c^+$ = **DCFL**

$L_2 = \{a^i b^j \mid j = 2i, i \geq 0\} \Rightarrow a^k b^{2k}$ = **DCFL**

Every DCFL is CFL.

- $L_1 \cap L_2 = \phi$
- $L_1 \cup L_2$ = CFL
- L_2 is a DCFL. So, DPDA possible.

Hence, option (b) is correct.

Scan for Video solution

29. (d)

$L_1 = \left\{0^m 1^n \mid 0 \le m \le n \le 10000\right\}$ **Finite**

L_2 = {reads the same forward and backward}

Set of all palindrome (CFL)

L_3 = {contains an even number of 0's and an even number of 1's} **Regular**

L_2 is not a regular.

Hence, option (d) is correct.

Scan for Video solution

30. (b, c, d)

L_1 = Regular, L_2 = CFL

(a) $\text{Regular} \cap \overline{\text{CFL}}$ = may/may not be CFL.

(b) $\overline{\overline{\text{Regular}} \cup \overline{\text{CFL}}} = \text{Regular} \cap \text{CFL} = \text{CFL}$

(c) $\text{Regular} \cup (\text{CFL} \cap \overline{\text{CFL}}) = \text{Regular} \cup \Sigma^* = \Sigma^*$

(d) $(\text{Regular} \cup \text{CFL}) \cup (\overline{\text{Regular}} \cap \text{CFL})$

$= \text{CFL} \cup \text{CFL} = \text{CFL}$

- Every Regular is CFL

Hence, option (b, c, d) are correct.

Scan for Video solution

31. (c)

$L_1 \Rightarrow$ Regular

$L_2 \Rightarrow$ CFL

(i) $L_1 \cap L_2 \Rightarrow \text{Reg} \cap \text{CFL} \Rightarrow \text{CFL}$

(ii) $L_1 . L_2 \Rightarrow \text{Reg CFL} \Rightarrow \text{CFL}$

(iii) $L_1 - L_2 \Rightarrow \text{Reg} - \text{CFL} \Rightarrow \text{Reg} \cap \overline{\text{CFL}} \Rightarrow \text{CSL}$

(iv) $L_1 \cup L_2 \Rightarrow \text{Reg} \cup \text{CFL} \Rightarrow \text{CFL}$

Scan for Video solution

32. (b)

L_1 = CFL, L_2 = CFL and R = Regular language

I. $\text{CFL}_1 \cup \text{CFL}_2 \Rightarrow \text{CFL}$

II. $\overline{\text{CFL}} \Rightarrow \text{CSL}$

III. $\text{CFL} - \text{Regular} = \text{CFL} \cap \overline{\text{Regular}}$

$= \text{CFL} \cap \text{Regular} = \text{CFL}$

IV. $\text{CFL}_1 \cap \text{CFL}_2 \Rightarrow \text{CSL}$

- CFL is not closed under compliment and intersection.

Scan for Video solution

33. (a)

$L_1 = \{a^n b^n c^m \mid m, n \ge 0\}$ **= CFL**

$L_2 = \{a^m b^n c^n \mid m, n \ge 0\}$ **= CFL**

I. $L_1 \cup L_2 \Rightarrow \text{CFL} \cup \text{CFL} = \textbf{CFL}$

II. $L_1 \cap L_2 = \{a^n b^n c^* \cap a^* b^n c^n\}$

$= \{a^n b^n c^n \mid n \ge 0\} \Rightarrow \textbf{CSL}$

Scan for Video solution

34. (c)

A. $\text{Regular} \cap \text{CFL} \Rightarrow \text{CFL}$

B. Regular – CFL

$\text{Regular} \cap \overline{\text{CFL}} \Rightarrow$ need not be regular

C. Σ^* – Regular $\Rightarrow$ Regular

D. Σ^* – CFL $\Rightarrow$ need not be regular

Hence, option (c) is correct.

Scan for Video solution

35. (d)

- For Regular language minimal DFA is unique.
- Every NFA can be converted to an equivalent PDA.
- $\overline{\text{CFL}} = \text{CSL}$

  Every CSL is recursive.
- Every nondeterministic PDA can be converted to an equivalent deterministic PDA.

Scan for Video solution

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CHAPTER

5

TURING MACHINE (RECURSIVE AND RELs)

Turing Machine

1. [MCQ] [GATE-2019 : 2M]

Consider the following sets:

S_1: Set of all recursively enumerable languages over the alphabet {0, 1}

S_2: Set of all syntactically valid C programs

S_3: Set of all languages over the alphabet {0, 1}

S_4: Set of all non-regular languages over the alphabet 0, 1}

Which of the above sets are uncountable?

(a) S_1 and S_2 (b) S_3 and S_4

(c) S_2 and S_3 (d) S_1 and S_4

2. [MCQ] [GATE-2017 : 2M]

Let A and B be finite alphabets and let # be a symbol outside both A and B. Let f be a total function from A* to B*. We say f is computable if there exists a Turing machine M which given an input x in A*, always halts with f(x) on its tape. Let L_f denote the language $\{x \# f(x) | x \in A^*\}$. Which of the following statements is true?

(a) f is computable if and only if L_f is recursive.

(b) f is computable if and only if L_f is recursively enumerable.

(c) If f is computable then L_f is recursive, but not conversely.

(d) If f is computable then L_f is recursively enumerable, but not conversely.

3. [MCQ] [GATE-2014 : 1M]

Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is TRUE?

(a) Both 2^{Σ^*} and Σ^* is countable

(b) 2^{Σ^*} is countable and Σ^* is uncountable

(c) 2^{Σ^*} is uncountable and Σ^* is countable

(d) Both 2^{Σ^*} and Σ^* are uncountable

4. [MCQ] [GATE-2011 : 1M]

Which of the following pairs have DIFFERENT expressive power

(a) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA)

(b) Deterministic push down automata (DPDA) and Non-deterministic push down automata (NPDA)

(c) Deterministic single-tape Turing machine and Non-deterministic single – tape Turing machine

(d) Single-tape Turing machine and multi-tape Turing machine

5. [MCQ] [GATE-2011 : 2M]

Consider the languages L_1, L_2 and L_3 are given below:

$L_1 = \{0^p 1^q \mid p, q \in N\}$,

$L_2 = \{0^p 1^q \mid p, q \in N \text{ and } p = q\}$ and

$L_3 = \{0^p 1^q 0^r \mid p, q, r \in N \text{ and } p = q = r\}$

Which of the following statements is NOT TRUE?

(a) Push Down Automata (PDA) can be used to recognize L_1 and L_2

(b) L_1 is a regular language

(c) All the three languages are context free

(d) Turing machines can be used to recognize all the languages

6. [MCQ] [GATE-2008 : 1M]

Which of the following is true for the language $\{a^p \mid p \text{ is a prime number}\}$?

(a) It is not accepted by a Turning Machine

(b) It is regular but not context-free

(c) It is context-free but not regular

(d) It is neither regular nor context-free, but accepted by a Turing machine

Recursive and RE Language Identification

7. [MSQ] [GATE-2023 : 1M]

Which of the following statements is/are correct?

(a) The intersection of two redular languages is regular.

(b) The intersection of two context-free languages is context-free.

(c) The intersection of two recursive languages is recursive.

(d) The intersection of two recusively enumerable languages is recursively enumerable.

8. [MCQ] [GATE-2017 : 2M]

Consider the following languages:

$L_1 = \{a^P \mid p \text{ is a prime number}\}$

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$

$L_4 = \{a^n b^n \mid n \geq 1\}$

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 deterministic context-free.

(a) I, II and IV only

(b) II and III only

(c) I and IV only

(d) III and IV only

9. [MCQ] [GATE-2013 : 2M]

Consider the following languages

$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$

$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0\ p \neq r\}$

Which one of the following statements is FALSE?

(a) L_2 is context-free

(b) $L_1 \cap L_2$ is context-free

(c) Complement of L_2 is recursive

(d) Complement of L_1 is context-free but not regular

10. [MCQ] [GATE-2008 : 2M]

Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I

A. Checking that identifier are declared before their use

B. Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function

C. Arithmetic expressions with matched pairs of parentheses

D. Palindromes

List-II

1. $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$
2. $X \rightarrow XbX \mid XcX \mid dXf \mid g$
3. $L = \{wcw \mid w \in (a|b)^*\}$
4. $X \rightarrow bXb \mid cXc \mid \in$

Codes:

	a	b	c	d
(a)	1	3	2	4
(b)	3	1	4	2
(c)	3	1	2	4
(d)	1	3	4	2

Closure Properties of Recursive and Rels

11. [MSQ] [GATE-2022 : 1M]

Which of the following statements is / are TRUE?

(a) Every subset of a recursively enumerable language is recursive.

(b) If a language L and its complement $\overline{L}$ are both recursively enumerable, then L must be recursive.

(c) Complement of a context-free language must be recursive.

(d) If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be deterministic context - free.

12. [MCQ] [GATE-2021 : 1M]

Let <M> denote an encoding of an automaton M. Suppose that $\Sigma = \{0, 1\}$. Which of the following languages is/are NOT recursive?

(a) L = {<M> | M is a DFA such that $L(M) = \emptyset$}

(b) L = {<M> | M is a DFA such that $L(M) = \Sigma^*$}

(c) L = {<M> | M is a PDA such that $L(M) = \emptyset$}

(d) L = {<M> | M is a PDA such that $L(M) = \Sigma^*$}

13. **[MCQ]** **[GATE-2018 : 1M]**
The set of all recursively enumerable languages is
(a) Closed under complementation.
(b) Closed under intersection.
(c) A subset of the set of all recursive language.
(d) An uncountable set.

14. **[MCQ]** **[GATE-2016 : 1M]**
Consider the following types of languages:
L_1: Regular, L_2: Context-free, L_3: Recursive,
L_4: Recursively enumerable.
Which of the following is/are TRUE?
I. $\overline{L_3} \cup L_4$ is recursively enumerable.
II. $\overline{L_2} \cup L_3$ is recursive.
III. $L_1^* \cap L_2$ is context-free.
IV. $L_1 \cup \overline{L_2}$ is context-free.
(a) I only (b) I and III only
(c) I and IV only (d) I, II and III only

15. **[MCQ]** **[GATE-2015 : 1M]**
For any two languages L_1 and L_2 such that L_1 is context free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true?
I. $\overline{L_1}$ (complement of L_1) is recursive
II. $\overline{L_2}$ (complement of L_2) is recursive
III. $\overline{L_1}$ is context-free
IV. $\overline{L_1} \cup L_2$ is recursively enumerable
(a) 1 only (b) 3 only
(c) 3 and 4 only (d) 1 and 4 only

16. **[MCQ]** **[GATE-2014 : 2M]**
Let L be a language and $\overline{L}$ be its complement. Which one of the following is NOT a viable possibility?
(a) Neither L nor $\overline{L}$ is recursively enumerable (r.e.).
(b) One of L and $\overline{L}$ is r.e. but not recursive; the other is not r.e.
(c) Both L and $\overline{L}$ are r.e. but not recursive.
(d) Both L and $\overline{L}$ are recursive.

17. **[MCQ]** **[GATE-2010 : 1M]**
Let L_1 be a recursive language. Let L_2 and L_3 be language that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?
(a) $L_2 - L_1$ is recursively enumerable
(b) $L_1 - L_3$ is recursively enumerable
(c) $L_2 \cap L_3$ is recursively enumerable
(d) $L_2 \cup L_3$ is recursively enumerable

18. **[MCQ]** **[GATE-2009 : 2M]**
Let $L = L_1 \cap L_2$, where L_1 and L_2 are languages as defined below:
$L_1 = \{a^m b^m ca^n b^n \mid m, n \geq 0\}$
$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$
Then L is
(a) Not recursive
(b) Regular
(c) Context-free but not regular
(d) Recursively enumerable but not context-free

19. **[MCQ]** **[GATE-2008 : 2M]**
Which of the following statements is false?
(a) Every NFA can be converted to an equivalent DFA.
(b) Every non-deterministic Turing machine can be converted to an equivalent deterministic Turing machine.
(c) Every regular language is also a context-free language.
(d) Every subset of a recursively enumerable set is recursive.

20. **[MCQ]** **[GATE-2008 : 1M]**
If L and $\overline{L}$ are recursively enumerable then L is
(a) regular (b) context-free
(c) context-sensitive (d) recursive

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ANSWER KEY

1.	(b)	**2.**	(a, d)	**3.**	(c)	**4.**	(b)
5.	(c)	**6.**	(d)	**7.**	(a, c, d)	**8.**	(d)
9.	(d)	**10.**	(c)	**11.**	(b, c, d)	**12.**	(d)
13.	(b)	**14.**	(d)	**15.**	(d)	**16.**	(c)
17.	(b)	**18.**	(c)	**19.**	(d)	**20.**	(d)

SOLUTIONS

1. (b)

S_1: Set of all recursively enumerable languages over the alphabet {0, 1}
Set of RELs is **Countable**.

S_2: Set of all syntactically valid C programs
All C, C^{++}, Java, etc. are **Countable**.

S_3: Set of all languages over the alphabet {0, 1}
Set of all languages = 2^{Σ^*} is **Uncountable**.

S_4: Set of all non-regular languages over the alphabet {0, 1}

Set of all regular languages is **Countable** but set of all non-regular language is **Uncountable**.

Scan for Video solution

2. (a, d)

$A^* \xrightarrow{f} B^*$

$a_1 \rightarrow f(a_1) = b_1$

$a_2 \rightarrow f(a_2) = b_2$

$a_3 \rightarrow f(a_3) = b_3$

$a_4 \rightarrow f(a_4) = b_4$

$\vdots \quad \vdots \quad \vdots$

$L_f = \{a_1\#\ f(a_1),\ a_2\ \#,\ f(a_2)\ldots\ldots\}$

- Lexicographical order is possible. So, L_f is recursive.
- If L is REL then f may/may not be computable.
- If f is computable then L_f is recursively enumerable, but not conversely.

Scan for Video solution

3. (c)

- Σ^* = Set of all strings = Regular
  Regular is countable.
- 2^{Σ^*} = Set of all languages
  Set of all languages is uncountable.
  Hence, option (c) is correct.

Scan for Video solution

4. (b)

- DFA ≡ NFA
- DPDA ≠ NPDA
- DTM ≡ NTM whether it is a single tape or multi tape.

Scan for Video solution

5. (c)

$L_1 = \{0^p1^q \mid p, q \in N\} \Rightarrow 0^* 1^* \Rightarrow$ Regular

$L_2 = \{0^p1^q \mid p, q \in N \text{ and } p = q\}$ and $\Rightarrow$ DCFL

$L_3 = \{0^p1^q0^r \mid p, q, r \in N \text{ and } p = q = r\} \Rightarrow$ CSL

Hence, option (c) is correct.

Scan for Video solution

6. (d)

- $\{a^p \mid p$ is a prime number$\}$ it is a CSL language.
  Every CSL is recursive and RE.
  Hence, option (d) is correct.

Scan for Video solution

7. (a, c, d)

(a) The intersection of two redular languages is regular.
Reg $\cap$ Reg $\Rightarrow$ Reg **True**

(b) The intersection of two context-free languages is context-free.
CFL $\cap$ CFL $\Rightarrow$ CSL **False**

(c) The intersection of two recursive languages is recursive.
Rec $\cap$ Rec $\Rightarrow$ Rec **True**

(d) The intersection of two recusively enumerable languages is recursively enumerable.
REL $\cap$ REL $\Rightarrow$ REL **True**

Scan for Video solution

8. (d)

$L_1 = \{a^P \mid p \text{ is a prime number}\} \Rightarrow$ Not CFL

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\} \Rightarrow$ DCFL

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\} \Rightarrow$ Not CFL

$L_4 = \{a^n b^n \mid n \geq 1\} \Rightarrow$ DCFL

Scan for Video solution

9. (d)

- $L_1 = \{0^p1^q0^r \mid p, q, r \geq 0\}$
  $= 0^*1^*0^*$ is Regular language
- $L_2 = \{0^p1^q0^r \mid p, q, r \geq 0\ p \neq r\}$
  = DCFL language
- $L_1 \cap L_2$ = Regular $\cap$ DCFL = DCFL
- $\overline{\text{DCFL}} = \text{DCFL}$
- $\overline{\text{Regular}} = \text{Regular}$

Scan for Video solution

10. (c)

List-I	List-II
A. Checking that identifier are declared before their use	**3.** $L = \{wcw \mid w \in (a \mid b)^*\}$
B. Number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of that function	**1.** $L = \{a^n b^m c^n d^m \mid n \geq 1, m \geq 1\}$
C. Arithmetic expressions with matched pairs of parentheses	**2.** $X \rightarrow XbX \mid XcX \mid dXf \mid g$
D. Palindromes	**4.** $X \rightarrow bXb \mid cXc \mid \in$

Hence, option (c) is correct.

Scan for Video solution

11. (b, c, d)

(a) Every subset of a recursively enumerable language is recursive. **FALSE**
Because subset is not closed under any language.

(b) If a language L and its complement $\overline{L}$ are both recursively enumerable, then L must be recursive. **TRUE**

(c) $\overline{CFL} = CSL$

Every CSL is recursive and recursive enumerable.

(d) Regular $\cap$ Regular = Regular

Every Regular language is DCFL, CFL, Recursive and RE.

Scan for Video solution

12. (d)

(a) Emptiness for DFA is decidable (Recursive).

(b) Totality for DFA is decidable (Recursive).

(c) Emptiness for PDA is decidable (Recursive).

(d) Totality for PDA is undecidable (Not Recursive).

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13. (b)

The set of all recursively enumerable language is **Countable** and closed under **Intersection** but not closed under **Complementation**.

The set of all RELs is superset of Set of all recursive languages.

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14. (d)

- $\overline{\text{Recursive}} \cup RE = \text{Recursive} \cup RE = RE$.
- $\overline{CFL} \cup \text{Recursive} = CSL \cup \text{Recursive} = \text{Recursive}$.
- $(\text{Regular})^* \cap CFL = \text{Regular} \cap CFL = CFL$.
- $\text{Regular} \cup \overline{CFL} = \text{Regular} \cup CSL = CSL$

Hence, option (d) is correct.

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15. (d)

L_1 : CFL

L_1 : RE but not Recursive

I. $\overline{CFL} = CSL$

Every CSL is recursive and recursive enumerable.

II. $\overline{\text{RE but not Recursive}} = \text{NOT RE}$

III. $\overline{CFL} = CSL$ (may/may not be CFL) but it will be CSL

IV. $\overline{CFL} \cup RE = CSL \cup RE = RE$

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16. (c)

(a) Neither L nor $\overline{L}$ is recursively enumerable (r.e.).

It is possible that both L and $\overline{L}$ are not RELs.

(b) One of L and $\overline{L}$ is r.e. but not recursive; the other is not r.e.

If L is RE but not REC, then $\overline{L}$ is always NOT RE.

(c) It is impossible to have both L and $\overline{L}$ are r.e. but not recursive.

(d) Both L and $\overline{L}$ are recursive.

If L is recursive, $\overline{L}$ must be recursive.

Hence, option (c) is correct.

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17. (b)

L_1 = Recursive

L_2 = RE but not Recursive

L_3 = RE but not Recursive

- $L_2 - L_1 = L_2 \cap \overline{L_1}$

  = RE but not Recursive $\cap$ $\overline{\text{Recursive}}$

  = RE
- $L_1 - L_3 = L_1 \cap \overline{L_3}$

  = $L_1 \cap$ NOT REL

  = Need not be RE
- RE but not Rec $\cap$ RE but not Rec = RE.
- RE but not Rec $\cup$ RE but not Rec = RE.

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18. (c)

- $L_1 = \{a^m b^m c a^n b^n \mid m, n \geq 0\}$

  = CFL
- $L_2 = \{a^i b^j c^k \mid i, j, k \geq 0\}$

  $= a^*b^*c^*$ = Regular
- $L = L_1 \cap L_2$

  $= a^m b^m c$ = **DCFL**

Hence, option (c)is correct

Scan for Video solution

19. (d)

Subset is not closed under any language.

Hence, option (d) is correct.

Scan for Video solution

20. (d)

L and $\overline{L}$ are recursively enumerable if and only if L is recursive.

Hence, option (d) is correct.

Scan for Video solution

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CHAPTER

6

UNDECIDABILITY AND REDUCIBILITY

Undecidability

1. [MSQ] [GATE-2022 : 2M]

Which of the following is/are undecidable?

(a) Given two Turing machines M_1 and M_2, decide if $L(M_1) = L(M_2)$.

(b) Given a Turing machine M, decide if L(M) is regular.

(c) Given a Turing machine M, decide if M accepts all strings.

(d) Given a Turing machine M, decide if M takes more than 1073 steps on every string.

2. [MCQ] [GATE-2021 : 2M]

Consider the following two statements about regular languages:

S_1: Every infinite regular language contains an undecidable language as a subset.

S_2: Every finite language is regular.

Which one of the following choices is correct?

(a) Only S_1 is true

(b) Only S_2 is true

(c) Both S_1 are S_2 are true

(d) Neither S_1 nor S_2 is true

3. [MCQ] [GATE-2021 : 2M]

For a Turing machine M, $\langle M\rangle$ denotes an encoding of M. Consider the following two languages.

$L_1 = \{\langle M\rangle | M$ takes more than 2021 steps on all inputs$\}$.

$L_2 = \{\langle M\rangle | M$ takes more than 2021 steps on some input$\}$.

Which one of the following options is correct?

(a) Both L_1 and L_2 are decidable.

(b) L_1 is decidable and L_2 is undecidable.

(c) L_1 is undecidable and L_2 is decidable.

(d) Both L_1 and L_2 are undecidable.

4. [MCQ] [GATE-2020 : 2M]

Which of the following languages are undecidable? Note that $\langle M\rangle$ indicates encoding of the Turing machine M.

$L_1 = \{\langle M\rangle | L(M) = \phi\}$

$L_2 = \{\langle M, w, q\rangle | M$ on input w reaches state q in exactly 100 steps$\}$

$L_3 = \{\langle M\rangle | L(M)$ is not recursive$\}$

$L_4 = \{\langle M\rangle | L(M)$ contains at least 21 members)

(a) L_1, L_3 and L_4 only
(b) L_2, L_3 and L_4 only
(c) L_1 and L_3 only
(d) L_2 and L_3 only

5. [MCQ] [GATE-2018 : 2M]

Consider the following problems. L(G) denotes the language generated by a grammar G. L(M) denotes the language accepted by a machine M.

I. For an unrestricted grammar G and a string w, whether $w \in L(G)$.

II. Given a Turing Machine M, whether L(M) is regular.

III. Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$.

IV. Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct?

(a) Only I and II are undecidable
(b) Only III is undecidable
(c) Only II and IV are undecidable
(d) Only I, II and III are undecidable

6. [MCQ] [GATE-2017 : 2M]

Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L(M) be the language accepted by a Turing machine M.

Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $w \in L(R)$?

II. Given a context-free grammar G, is $L(G) = \phi$?

III. Given a context-free grammar G, is $L(G) = \Sigma^*$ for some alphabet Σ?

IV. Given a Turing machine M and a string w, is $w \in L(M)$?

(a) I and IV only (b) II and III only
(c) II, III and IV only (d) III and IV only

7. [MCQ] [GATE-2016 : 1M]

Which of the following decision problems are undecidable

I. Given NFAs N_1 & N_2, is $L(N_1) \cap L(N_2) = \Phi$?

II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?

III. Given CFGs G_1 and G_2 is $L(G1) = L(G_2)$?

IV. Given a TM M, is $L(M) = \Phi$?

(a) I and IV only
(b) II and III only
(c) III and IV only
(d) II and IV only

8. [MCQ] [GATE-2016 : 2M]

Consider the following languages:

$L_1 = \{\langle M\rangle | M$ takes at least 2016 steps on some input$\}$.

$L_2 = \{\langle M\rangle | M$ takes at least 2016 steps on all inputs$\}$.

$L_3 = \{\langle M\rangle | M$ accepts $\varepsilon\}$.

Where for each Turning machine M, $\langle M\rangle$ denotes a specific encoding of M. Which one of the following is TRUE?

(a) L_1 is recursive and L_2, L_3 are not recursive.
(b) L_2 is recursive and L_1, L_3 are not recursive.
(c) L_1, L_2 are recursive and L_3 is not recursive.
(d) L_1, L_2, L_3 are recursive .

9. [MCQ] [GATE-2015 : 1M]

Consider the following statements:

1. The complement of every Turning decidable language is Turning decidable.
2. There exists some language which is in NP but is not Turing decidable.
3. If L is a language in NP, L is Turing decidable.

Which of the above statements is/are True?

(a) Only 2 (b) Only 3
(c) Only 1 and 2 (d) Only 1 and 3

10. [MCQ] [GATE-2014 : 2M]

Which one of the following problems is undecidable?

(a) Deciding if a given context-free grammar is ambiguous.
(b) Deciding if a given string is generated by a given context-free grammar.
(c) Deciding if the language generated by a given context-free grammar is empty.
(d) Deciding if the language generated by a given context-free grammar is finite.

11. [MCQ] [GATE-2014 : 2M]

Let $\langle M\rangle$ be the encoding of a Turing machine as a string over $\Sigma = \{0, 1\}$. Let $L = \{\langle M\rangle |$ M is a Turing machine that accepts a string of length 2014$\}$. Then, L is

(a) decidable and recursively enumerable
(b) undecidable but recursively enumerable
(c) undecidable and not recursively enumerable
(d) decidable but not recursively enumerable

12. [MCQ] [GATE-2013 : 2M]

Which of the following is/are undecidable?

1. G is CFG. Is $L(G) = \phi$?
2. G is a CFG. Is $L(G) = \Sigma^*$?
3. M is a Turing machine. Is L(M) regular?
4. A is a DFA and N is an NFA. Is $L(A) = L(N)$?

(a) 3 Only
(b) 3 and 4 only
(c) 1, 2 and 3 only
(d) 2 and 3 only

13. [MCQ] [GATE-2013 : 1M]

Which of the following statements is/are FALSE?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine.
2. Turing recognizable languages are closed under union and complementation.
3. Turing decidable languages are closed under intersection and complementation.
4. Turing recognizable languages are closed under union and intersection.

(a) 1 and 4 only (b) 1 and 3 only
(c) 2 only (d) 3 only

14. [MCQ] [GATE-2012 : 1M]

Which of the following problems are decidable?

1. Does a given program ever produce an output?
2. If L is a context-free language, then, is $\bar{L}$ also context-free?
3. If L is a regular language, then, is $\bar{L}$ also regular?
4. If L is a recursive language, then, is $\bar{L}$ also recursive?

(a) 1, 2, 3, 4 (b) 1, 2
(c) 2, 3, 4 (d) 3, 4

15. [MCQ] [GATE-2008 : 1M]

Which of the following are decidable?

1. Whether the intersection of two regular languages is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

(a) 1 and 2 (b) 1 and 4
(c) 2 and 3 (d) 2 and 4

Reducibility

16. [MCQ] [GATE-2016 : 2M]

Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduces to W, and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

(a) W can be recursively enumerable and Z is recursive.
(b) W can be recursive and Z is recursively enumerable.
(c) W is not recursively enumerable and Z is recursive.
(d) W is not recursively enumerable and Z is not recursive.

17. [MCQ] [GATE-2014 : 1M]

Let $A \le_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(a) If $A \le_m B$ and B is recursive then A is recursive.
(b) If $A \le_m B$ and A is undecidable then B is undecidable.
(c) If $A \le_m B$ and B is recursively enumerable then A is recursively enumerable.
(d) If $A \le_m B$ and B is not recursively enumerable then A is not recursively enumerable.

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ANSWER KEY

1.	(a, b, c)	**2.**	(c)	**3.**	(a)	**4.**	(a)
5.	(d)	**6.**	(d)	**7.**	(c)	**8.**	(c)
9.	(d)	**10.**	(a)	**11.**	(b)	**12.**	(d)
13.	(c)	**14.**	(d)	**15.**	(b)	**16.**	(c)
17.	(d)						

SOLUTIONS

1. (a, b, c)

- Equivalence problem for Turing machine is **Undecidable**.
- Regularity problem for Turing machine is **Undecidable**.
- Totality (Completeness) problem for Turing machine is **Undecidable**.
- Given a Turing machine M, decide if M takes more than 1073 steps on every string is **Decidable.**

Scan for Video solution

2. (c)

Both Statements S_1 and S_2 are correct.

Scan for Video solution

3. (a)

$L_1 = \{\langle M\rangle | M$ takes more than 2021 steps on all inputs$\}$ **Decidable**.
Verify every string in length 0 to 2021.
$L_2 = \{\langle M\rangle | M$ takes more than 2021 steps on some input$\}$ **Decidable**.
Verify each string upto 2021 length. If any of these strings takes more than 2021 steps, it is valid M.

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4. (a)

$L_1 = \{\langle M\rangle | L(M) = \phi\}$
Emptiness problem for Turing machine is **Undecidable**.
$L_2 = \{\langle M, w, q\rangle | M$ on input w reaches state q in exactly 100 steps$\}$
For given M, given w, run exactly 100 steps to check whether q is reachable or not **Decidable**.
$L_3 = \{\langle M\rangle | L(M)$ is not recursive$)\} \Rightarrow$ **Undecidable**.
$L_4 = \{\langle M\rangle | L(M)$contains at least 21 members$)\}$
$|L(M)| \geq 21$ non trivial property is **Undecidable**.

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5. (d)

I. Membership problem for an unrestricted grammar is **Undecidable**.

II. Given a Turing Machine M, whether L(M) is regular. Non-trivial problem for Turing machine is **Undecidable**.

III. Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$. Grammar is not given assume unrestricted grammar for an unrestricted grammar equivalence problem is **Undecidable**.

IV. Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same language. **Decidable**

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6. (d)

I. Given a regular expression R and a string w, is $w \in L(R)$?

Membership problem for Regular is **Decidable**.

II. Given a context-free grammar G, is $L(G) = \phi$? Emptines?

Membership, finiteness and emptiness problems for CFL are **Decidable**.

III. Given a context-free grammar G is $L(G) = \Sigma^*$ for some alphabet Σ?

Totality problem CFL is **Undecidable**.

IV. Given a Turing machine M and a string w, is $w \in L(M)$?

Membership problem for Turing machine is **Undecidable**.

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7. (c)

I. Dis-jointness problem for FA is Decidable.
II. Membership problem for CFG is Decidable.
III. Equivalence problem for CFG is Undecidable.
IV. Emptiness problem for TM is Undecidable.

Hence, option (c) is correct.

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8. (c)

- $L_1 = \{\langle M\rangle \mid$ M takes at least 2016 steps on some input$\}$. **Decidable**
- $L_2 = \{\langle M\rangle \mid$ M takes at least 2016 steps on all inputs$\}$. **Decidable**
- $L_3 = \{\langle M\rangle \mid$ M accepts $\varepsilon\}$.
  Membership problem for Turing machine is **Undecidable.**

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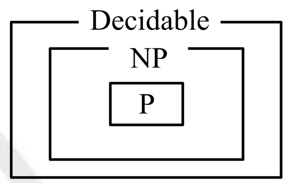
9. (d)



- Decidable complement is always Decidable.
- Every NP problem is always Decidable.

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10. (a)

- Checking ambiguity for grammar is always **Undecidable.**
- Membership problem for CFL is **Decidable** (using CYK algorithm).
- Emptiness problem for CFL is **Decidable.**
- Finiteness problem for CFL is **Decidable.**

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11. (b)

- L is undecidable but partially decidable (RE).

Hence, option (b) is correct.

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12. **(d)**

- Emptiness problem for CFL is **Decidable.**
- Totality problem for CFL is **Undecidable.**
- Regularity problem for TM is **Undecidable.**
- Equivalence problem for Regular is **Decidable.**

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13. **(c)**

- NTM is equivalent to DTM
- RELs are not closed under complement.
- Recursive languages are closed under both intersection and complement.
- RELs are closed under both union and intersection.

Hence, option (c) is correct.

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14. **(d)**

1. These kind of problems are non-trivial. So, it is **Undecidable**.
2. If L is a context-free language, then, is $\overline{L}$ also context-free. It is **Undecidable.**
3. If L is a regular language, then, is $\overline{L}$ also regular. Regular is closed under complement. So, it is **Decidable**.
4. If L is a recursive language, then, is $\overline{L}$ also recursive. It is **Decidable.**

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15. **(b)**

1. Whether the intersection of two regular languages is infinite.
   Infiniteness problem for Regular is **Decidable**.
2. Whether a given context-free language is regular. **Undecidable**
3. Whether two push-down automata accept the same language.
   Equivalence problem for PDA(CFL) is **Undecidable.**
4. Whether a given grammar is context-free.
   Algorithm exists to identify the context-free grammar. So this problem is **Decidable**.

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16. **(c)**

X: Recursive

Y: RE but not Recursive

- $\overline{Y} \le W$

  $= \overline{\text{RE but not Recursive}} \le W$

  = NOT RE ≤ W

  So, W can not be RE
- $Z \le \overline{X}$

  $= Z \le \overline{\text{Recursive}}$

  = Z ≤ Recursive

  So, Z must be Recursive

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17. **(d)**

Given $A \le_m B$

- If $A \le_m$ recursive then, A will be recursive.
- If Undecidable $\le_m$ B then, B must be Undecidable.
- If $A \le_m$ RE then, A will be RE.
- If $A \le_m$ NOT RE then, A may/may not be NOT RE.

Hence, option (d) is false statement in given data.

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INDEX

Digital Logic

Digital Logic

Syllabus

Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and computer arithmetic (fixed and floating point).

Chapter wise Weightage Analysis

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2008	2	4		
2009	1	1	1	
2010		2	4	2
2011		1	1	5
2012	1	3	1	
2013	1	1	3	
2014 (P1)	1	1	2	
2014 (P2)	3	1		1
2014 (P3)		3	1	2
2015 (P1)		2		3
2015 (P2)		2	2	
2015 (P3)	1	2	2	
2016 (P1)		2	2	3
2016 (P2)		2	1	1
2017 (P1)	1	1	1	2
2017 (P2)	2	4		2
2018	2	3	1	1
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2021 (P2)	4	3		
2022	3		2	
2023	1		3	2

CHAPTER

1

NUMBER SYSTEM AND BINARY CODES

Conversion

1. **[NAT]** **[GATE-2021: 1M]**

If x and y are two decimal digits and $(0.1101)_2 = (0.8xy5)_{10}$, the decimal value of x + y is_________.

2. **[MCQ]** **[GATE-2017: 1M]**

The representation of the value of a 16-bit unsigned integer X in hexadecimal number system is BCA9. The representation of the value of X in octal number system is

(a) 136251 (b) 736251
(c) 571247 (d) 571244

3. **[NAT]** **[GATE-2017: 1M]**

Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then b =_____.

4. **[NAT]** **[GATE-2015: 2M]**

Consider the euqation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solution is______.

5. **[NAT]** **[GATE-2014: 1M]**

The base (or radix) of the number system such that the following equation holds is____________.

312/20 = 13.1

6. **[NAT]** **[GATE-2014: 1M]**

Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is ____.

7. **[MCQ]** **[GATE-2009: 1M]**

$(1217)_8$ is equivalent to

(a) $(1217)_{16}$ (b) $(028F)_{16}$
(c) $(2297)_{10}$ (d) $(0B17)_{16}$

8. **[MCQ]** **[GATE-2008: 1M]**

Let r denote number system radix. The only value(s) of r that satisfy the equation $\sqrt{121_r} = 11_r$ is/are

(a) decimal 10 (b) decimal 11
(c) decimal 10 and 11 (d) any value > 2

Signed Binary Numbers

9. **[MCQ]** **[GATE-2019: 1M]**

In 16-bit 2's complement representation, the decimal number -28 is:

(a) 1111 1111 0001 1100
(b) 0000 0000 1110 0100
(c) 1111 1111 1110 0100
(d) 1000 0000 1110 0100

10. **[NAT]** **[GATE-2019: 1M]**

Two numbers are chosen independently and uniformly at random from the set {1, 2, ... , 13}. The probability (rounded off to 3 decimal places) that their 4-bit (unsigned) binary representations have the same most significant bit is ______.

11. [MCQ] [GATE-2019: 1M]

Consider Z = X - Y, where X, Y and Z are all in sign-magnitude form. X and Y are each represented in n bits . To avoid overflow , the representation of Z would require a minimum of :

(a) n bits (b) n – 1 bits
(c) n + 1 bits (d) n + 2 bits

12. [MCQ] [GATE-2018: 2M]

Consider the unsigned 8-bit fixed point binary number representation below, $b_7\ b_6\ b_5\ b_4\ b_3 \cdot b_2\ b_1\ b_0$ where the position of the binary point is between b_3 and b_2 . Assume b_7 is the most significant bit. Some of the decimal numbers listed below cannot be represented exactly in the above representation:

(i) 31.500 (ii) 0.875 (iii) 12.100 (iv) 3.001

Which one of the following statements is true?

(a) None of (i), (ii), (iii), (iv) can be exactly represented
(b) Only (ii) cannot be exactly represented
(c) Only (iii) and (iv) cannot be exactly represented
(d) Only (i) and (ii) cannot be exactly represented

13. [MCQ] [GATE-2017:1M]

When two 8-bit numbers $A_7 \ldots A_0$ and $B_7 \ldots B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a ripple-carry adder, the sum bits obtained are $S_7 \ldots S_0$ and the carry bits are $C_7 \ldots C_0$.

An overflow is said to have occurred if

(a) The carry bit C_7 is 1
(b) All the carry bits ($C_7, \ldots, C_0$) are 1
(c) $(A_7 \cdot B_7 \cdot \overline{S_7} + \overline{A_7} \cdot \overline{B_7} \cdot S_7)$ is 1.
(d) $(A_0 \cdot B_0 \cdot \overline{S_0} + \overline{A_0} \cdot \overline{B_0} \cdot S_0)$ is 1.

14. [NAT] [GATE-2016:1M]

Consider an eight-bit ripple-carry adder for computing the sum of A and B, where A and B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is _________.

15. [NAT] [GATE-2016:1M]

Let X be the number of distinct 16-bit integers in 2's complement representation. Let Y be the number of distinct 16-bit integers in sign magnitude representation. Then X-Y is _________.

16. [NAT] [GATE-2016: 1M]

The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is___.

17. [MCQ] [GATE-2013: 1M]

The smallest integer that can be represented by an 8-bit number in 2's complement form is

(a) – 256 (b) – 128
(c) – 127 (d) 0

18. [MCQ] [GATE-2010: 1M]

P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of $8 \times P$ is

(a) $(C3D8)_{16}$ (b) $(187B)_{16}$
(c) $(F878)_{16}$ (d) $(987B)_{16}$

❑❑❑

ANSWER KEY

1.	(3 to 3)	**2.**	(a)	**3.**	(8 to 8)	**4.**	(5 to 5)
5.	(5 to 5)	**6.**	(3 to 3)	**7.**	(b)	**8.**	(d)
9.	(c)	**10.**	(0.502 to 0.504)	**11.**	(c)	**12.**	(c)
13.	(c)	**14.**	(–1 to –1)	**15.**	(1 to 1)	**16.**	(– 11 to –11)
17.	(b)	**18.**	(a)				

SOLUTIONS

1. (3)

0.5   0.25   0.125   0.0625

$(0.1\ 1\ 0\ 1)_2 = (\ )_{10}$

$= 1 \times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4}$

$= (0.5 + 0.25 + 0 + 0.625)$

$= (0.8125)_{10}$

$(0.8125)_{10} = (0.8xy5)_{10}$

x = 1   4 = 2,   x + 4 = 1 + 2 = 3

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2. (a)

Given X = BCA9 in Hexadecimal format

we know in Binary system B = 1011, C = 1100,

A = 1010, 9 = 1001

$(B\ C\ A\ 9)_{16} \rightarrow (\ ?\ )_2 \rightarrow (\ ?\ )_8$

Therefore, convert hexadecimal number system first in binary then octal system.

$$\underbrace{0\,0\,1}_{1}\ \underbrace{0\,1\,1}_{3}\ \underbrace{1\,1\,0}_{6}\ \underbrace{0\,1\,0}_{2}\ \underbrace{1\,0\,1}_{5}\ \underbrace{0\,0\,1}_{1}$$

$(BCA\ 9)_{16} = (136251)_8$

Hence correct option is (a).

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3. (8)

Given the base of quadratic equation is b.

$(1)_b\, x^2 - (13)_b\, x + (36)_b = (0)_b$

$x^2 - (b + 3)x + 3b + 6 = 0$

Let $x = 5$

$25 - (b + 3)5 + 3b + 6 = 0$

$25 - 5b - 15 + 3b + 6 = 0$

$16 - 2b = 0$

$b = 8$

Let $x = 6$

$36 - (b + 3)6 + 3b + 6 = 0$

$36 - 6b - 18 + 3b + 6 = 0$

$24 - 3b = 0$

$b = 8$

Hence correct answer is (8).

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4. (5)

$(43)_x = (y3)_8$ $(43)_x = (y3)_8$

$4x + 3 = 8y + 3$ $x > 4 \quad y < 8$

$4x = 8y$

$\boxed{x = 2y}$

$\boxed{y < 8}$

$\boxed{x > 4}$

X	Y	Condition
2	1	(×)
4	2	(✓)
6	3	(✓)
8	4	(✓)
10	5	(✓)
12	6	(✓)
14	7	(✓)

Total 5 possible solution

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5. **(5)**

Let x be the base of the given number

$$\frac{(312)_x}{(20)_x} = (13.1)_x$$

The above number in Decimal will be given as

$$\frac{3 \times x^2 + 1 \times x^1 + 2 \times x^0}{2 \times x^1 + 0 \times x^0} = 1 \times x^1 + 3 \times x^0 + 1 \times x^{-1}$$

$$\frac{3x^2 + x + 2}{2x} = x + 3 + \frac{1}{x}$$

$$\Rightarrow \frac{3x^2 + x + 2}{2x} = \frac{x^2 + 3x + 1}{x}$$

$$\Rightarrow 3x^2 + x + 2 = 2x^2 + 6x + 2$$

$$\Rightarrow x^2 - 5x = 0$$

$$\Rightarrow (x - 5)x = 0$$

$$x = 0 \quad x = 5$$

Therefore, $x = 5$

Hence, the base of the given number will be 5.

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6. **(3)**

Given equation

$(123)_5 = (x\ 8)_y$

By Converting the above equation into decimal form

$(1\times5^2 + 2\times5^1 + 3\times5^0) = xy + 8$

$38 = xy + 8$

$xy = 30$

and $(x < y)$ $(y > 8)$

The possible solutions are

$x = 1 \qquad y = 30 \quad$ (✓)

$x = 2 \qquad y = 15 \quad$ (✓)

$x = 3 \qquad y = 10 \quad$ (✓)

$x = 5 \qquad y = 6 \quad$ (X)

From the above solution only 3 satisfies the given condition.

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7. **(b)**

$$(1217)_8 = (\quad)_{16}$$

$$(\quad)_2$$

First write the given number in binary representation

Write each digit 3 bits

$(1217)_8 = (001010001111)_2$

Now, to represent the above number in hexadecimal form, make the group of 4 bits.

$(0010\ 1000\ 1111)_2 = (028F)_{16}$

Hence, correct answer is option (b).

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8. (d)

The given equation is

$$\sqrt{(121)_r} = (11)_r$$

Let r be any base

$$\sqrt{1\times r^2 + 2\times r^1 + 1\times r^0} = 1\times r^1 + 1\times r^0$$

$$\sqrt{r^2 + 2r + 1} = r + 1$$

$$\sqrt{(r+1)^2} = r + 1$$

$\pm (r + 1) = r + 1$

Take negative sign

$-(r + 1) = r + 1$

$2r + 2 = 0$

$r = -1$ (radix cannot be negative)

Take positive sign

$r + 1 = r + 1$

for all value which is greater than 2.

Hence, correct answer is option (d).

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9. (c)

To find 2's complement of -28 first we convert +28 in binary system and then take 2's complement

$(28)_{10} = (0000\ 0000\ 0001\ 1100)_2$

Now taking 2's complement of $(0000\ 0000\ 0001\ 1100)_2 = 1111\ 1111\ 111\ 0100$

Hence

$-28 \rightarrow 1111\ 1111\ 1110\ 0100$

Therofre correct answer is option (C)

10. (0.502)

Decimal	Unsigned representation
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
4	0111
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101

From above table 7 number with MSB 'O'

6 number with MSB '1'

Two number are selected indepednely with same MSB

Probability (P)=P (MSB = 0) + P(MSB = 1)

Probability (P)=$\frac{7}{13}\times\frac{7}{13}+\frac{6}{13}\times\frac{6}{13}$

Probability (P)=$\frac{49}{169}+\frac{36}{169} = 0.5029$

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11. (c)

Overflow occurs when two same sign number are add/sub And result we get in different sign

Example

3 bit sign number

(+3) + (1.2) = +5

$$\begin{array}{r} 011 \\ +\ 010 \\ \hline 101 \end{array} \rightarrow \text{in signed it is } -1$$

Extension of bit required to avoid overflow

$$\begin{array}{c} Z = X - Y \\ \quad\ \ \downarrow \quad \downarrow \\ \quad \text{n-bit} \ \ \text{n-bit} \end{array}$$

To avoid overflow z should be (n + 1) bits

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12. (c)

Given $b_7\ b_6\ b_5\ b_4\ b_3 \cdot b_2\ b_1\ b_0$

before . 5 bit that means we can represent at max 31_{10}

after . 3 bit that means total 8 comibanations are

Binary	Decimal
000	0.000
001	0.125
010	0.250
011	0.375
100	0.500
101	0.625
110	0.750
111	0.875

From above analysis we can't represent $(0.100)_{10}$ and $(0.001)_{10}$

From above analysis only (iii) and (iv) cannot be exactly same represented.

13. (c)

Overflow:

When two same sign number are added and result we will get in different sign.

To avoid overflow bit extension required

$$\begin{array}{l} A = \boxed{A_7} \ldots\ldots A_0 \\ B = \boxed{B_7} \ldots\ldots B_0 \\ \hline S = \boxed{S_7} \ldots\ldots S_0 \end{array} \quad \text{or} \quad \begin{array}{l} A_7 = B_7 = 1\ S_7 = 0 \\ A_7 = B_7 = 0\ \ S_7 = 1 \end{array}$$

$$\Rightarrow \underbrace{\underline{\underline{A_7\, B_7\, \bar{S}_7 + \bar{A}_7\, \bar{B}_7\, S_7}}}_{\text{Over flow}}$$

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14. (–1)

Given:

A = 1 in decimal form

binary form of A = 0000 0001

For longest latency, we must have C_{in} = 1 at every stage of ripple carry adder.

Maximum delay will be given when we, take B = –1 in decimal and (1111 1111) in binary

$$\begin{array}{rl} B = 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 & \Rightarrow -1 \\ A = 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1 & \rightarrow +1 \\ \text{Carry out} = 1\ \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 & \end{array}$$

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15. (1)

If x is in 2's then range of x will be

-2^{n-1} to $2^{n-1} - 1$

Here x is 16 bits number, n = 16

-2^{16-1} to $2^{16-1} - 1$

x = 65536

If y is in sign magnitude representation, then range of y will be

$-(2^{n-1}-1)$ to $+(2^{n-1}-1)$ n = 16

$-(2^{16-1}-1)$ to $+(2^{16-1}-1)$

y = 65535

Hence, x – y = 1

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16. (–11)

Given 16-bit number is 1111 1111 1111 0101

Here MSB bit is 1 that means the given number is negative signed number

11

1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1

MSB = 1 means it is a negative number

0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1

2'S

+ 11

2's complement of given number is 0000 0000 0000 1011, which is equivalent to + 11 but MSB here is 1 hence number is negative i.e - 11 and correct answer is -11.

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17. (b)

For 2's Complement form

The range will be given by

$-(2^{n-1})$ to $+(2^{n-1}-1)$

For n = 8 (Given 8 bit number)

The minimum will be

$\Rightarrow -(2^{8-1})$

$\Rightarrow -(2^{7})$

$\Rightarrow -128$

The maximum number will be given as

$(2^{n-1}) = 127$

Hence, correct answer is option (b).

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18. (a)

Method 1:

The given number is

$P = (F87B)_{16}$

$P = 1111100001111011 = (-1925)$

2's complement of the above number will be given as

$= 0000011110000101$

Its decimal representation will be

$\Rightarrow 1 \times 2^{10} + 1 \times 2^{9} + 1 \times 2^{8} + 1 \times 2^{7} + 4 + 2$

$\Rightarrow 1024 + 512 + 256 + 128 + 4 + 1 = 1925$

Now,

$8 \times P = 8 \times (-1925) = -15400$

Its binary representation will be 0011110000101000

Now, 2's complement of the above number will be given as

1100001111011000

$8P \rightarrow (C3D8)_{16}$

Hence, correct answer is option (a)

Method 2:

$P = (F87B)_{16}$

$= 1111100001111011$

By multiplying 2^1, number will be shifted to left by one position, and by multiplying 2^3, number will be shifted by 3 position to the left.

$\Rightarrow 11\ 00\ 00\ 1111011000$

$8P \Rightarrow (C3D8)_{16}$

Hence, correct answer is option (a)

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CHAPTER

2

BOOLEAN ALGEBRA, LOGIC GATES AND K-MAPS

Boolean Algebra

1. [MCQ] **[GATE-2022: 1M]**

The dual of a Boolean function $F(x_1, x_2, \ldots, x, +, \cdot, ')$, written as F^D, is the same expression as that of F with + and · swapped. F is said to be self-dual if $F = F^D$. The number of self-dual functions with n Boolean variables is

(a) 2n
(b) 2^{n-1}
(c) 2^{2^n}
(d) $2^{2^{n-1}}$

2. [MCQ] **[GATE-2017: 1M]**

If w, x, y, z are Boolean variables, then which one of the following is INCORRECT?

(a) $wx + w(x + y) + x(x + y) = x + wy$
(b) $\overline{w\bar{x}(y + \bar{z})} + \bar{w}x = \bar{w} + x + \bar{y}z$
(c) $(w\bar{x}(y + x\bar{z}) + \bar{w}\bar{x})y = x\bar{y}$
(d) $(w + y)(wxy + wyz) = wxy + wyz$

3. [MCQ] **[GATE-2015: 1M]**

The binary operator $\neq$ is defined by the following truth table

P	Q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator $\neq$?

(a) Both commutative and associative
(b) Commutative but not associative
(c) Not commutative but associative
(d) Neither commutative nor associative

4. [MCQ] **[GATE-2015: 1M]**

The number of min-terms after minimizing the following Boolean expression is ______ .

[D' + AB' + A'C + AC'D + A'C'D] '

(a) 1
(b) 2
(c) 3
(d) 4

5. [MCQ] **[GATE-2015: 2M]**

Given the function F = P' + QR, wehre F is a function in three Boolean variables P,Q and R and P' = !P, consider the following statements.

(S1) $F = \sum(4, 5, 6)$

(S2) $F = \sum(0,1,2,3,7)$

(S3) $F = \prod(4, 5, 6)$

(S4) $F = \sum(0,1,2,3,7)$

Which of the following is true?

(a) (S1) – False, (S2) – True, (S3) – True, (S4) -False
(b) (S1) – True, (S2) – False, (S3) – False, (S4) -True
(c) (S1) – False, (S2) – False, (S3) – True, (S4) -True
(d) (S1) – True, (S2) – True, (S3) – False, (S4) -False

6. [MCQ] **[GATE-2012: 1M]**

The truth table

X	Y	F(X,Y)
0	0	0
0	1	0
1	0	1
1	1	1

represents the Boolean function

(a) x
(b) x + y
(c) $x \oplus y$
(d) y

7. [MCQ] [GATE-2012: 1M]

The amount of ROM needed to implement a 4 bit multiplier is

(a) 64 bits (b) 128 bits

(c) 1 Kbits (d) 2 Kbits

8. [MCQ] [GATE-2011: 1M]

The simplified SOP (sum of product) form of the boolean expression

$(P+\bar{Q}+\bar{R})\cdot(P+\bar{Q}+R)\cdot(P+Q+\bar{R})$

(a) $(\bar{P}.Q+\bar{R})$ (b) $(P+Q'.R')$

(c) $(P'.Q+R)$ (d) $(P.Q+R)$

9. [MCQ] [GATE-2011: 1M]

The minterm expansion of $f(P, Q, R) = PQ + Q\bar{R} + P\bar{R}$ is

(a) $m_2 + m_4 + m_6 + m_7$ (b) $m_0 + m_1 + m_3 + m_5$

(c) $m_0 + m_1 + m_6 + m_7$ (d) $m_2 + m_3 + m_4 + m_5$

10. [MCQ] [GATE-2008: 1M]

If P, Q, R are Boolean variables, then $(P + Q')(P.Q' + P.R)(P'.R' + Q')$ Simplifies to

(a) $P.\bar{Q}$

(b) $P.\bar{R}$

(c) $P.\bar{Q}+R$

(d) $P.\bar{\bar{R}}+Q$

Logic Gates

11. [MCQ] [GATE-2022: 1M]

Let, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator.

Which one of the following must always be TRUE?

(a) $x_1x_2x_3x_4 = 0$

(b) $x_1x_3+x_2 = 0$

(c) $\overline{x_1} \oplus \overline{x_3} = \overline{x_2} \oplus \overline{x_4}$

(d) $x_1 + x_2 + x_3 + x_4 = 0$

12. [MCQ] [GATE-2022: 1M]

Consider the Boolean operator # with the following properties:

$x \# 0 = x, x \# 1 = \bar{x}, x \# x = 0$ and $x \# \bar{x} = 1$

Then x # y is equivalent to

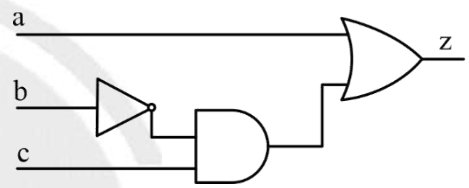
(a) $x\bar{y} + \bar{x}y$ (b) $x\bar{y} + \bar{x}\bar{y}$

(c) $\bar{x}y + xy$ (d) $xy + \bar{x}\bar{y}$

13. [MCQ] [GATE-2020: 2M]

Consider the Boolean function z(a,b,c).

Which one of the following minterm lists represents the circuit given above?



(a) Z = Σ (0, 1, 3, 7)

(b) Z = Σ (2, 4, 5, 6, 7)

(c) Z = Σ (1, 4, 5, 6, 7)

(d) Z = Σ (2, 3, 5)

14. [MCQ] [GATE-2019: 1M]

Which one of the following is NOT a valid identity?

(a) $(x + y) \oplus z = x \oplus (y + z)$

(b) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

(c) $x \oplus y = x + y$, if $xy = 0$

(d) $x \oplus y = (xy + x'y')'$

15. [MCQ] [GATE-2019: 2M]

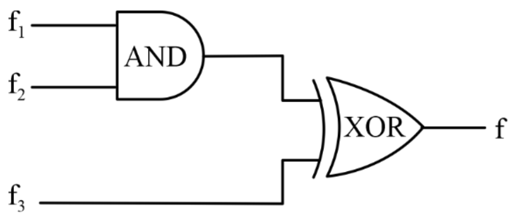
What is the minimum number of 2-input NOR gates required to implement a 4-variable function expressed in sum-of-minterms form as f = Σ(0, 2, 5, 7, 8, 10, 13, 15)? Assume that all the inputs and their complements are available.________

16. [MCQ] [GATE-2019: 2M]

Consider three 4-variable functions f1, f2 and f3, which are expressed in sum-of-minterms as

$f_1 = \Sigma(0, 2, 5, 8, 14)$, $f_2 = \Sigma(2, 3, 6, 8, 14, 15)$, $f_3 = \Sigma(2, 7, 11, 14)$

For the following circuit with one AND gate and one XOR gate, the output function f can be expressed as:



(a) Σ (2, 14)

(b) Σ (7, 8, 11)

(c) Σ (2, 7, 8, 11, 14)

(d) Σ (0, 2, 3, 5, 6, 7, 8, 11, 14, 15)

17. [MCQ] [GATE-2018: 1M]

Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT?

(a) $P \oplus Q = P \odot Q$

(b) $\overline{P} \oplus Q = P \odot Q$

(c) $P \oplus \overline{Q} = P \oplus Q$

(d) $\left(P \oplus \overline{P}\right) \oplus Q = \left(P \odot \overline{P}\right) \odot \overline{Q}$

18. [MCQ] [GATE-2014: 1M]

Let $\oplus$denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q:

$F(P, Q) = \big((1 \oplus P) \oplus (P \oplus Q)\big) \oplus \big((P \oplus Q) \oplus (Q \oplus 0)\big)$

The equivalent expression for F is

(a) $P + Q$ (b) $(P + Q)'$

(c) $P \oplus Q$ (d) $(P \oplus Q)'$

19. [MCQ] [GATE-2013: 1M]

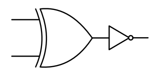
Which one of the following expressions does NOT represent exclusive NOR of x and y?

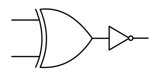
(a) $xy + x'y'$ (b) $x \oplus y'$

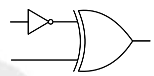
(c) $x' \oplus y$ (d) $x' \oplus y'$

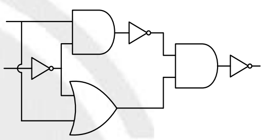
20. [MCQ] [GATE-2011: 1M]

Which one of the following circuits is NOT equivalent to a 2-input X-NOR (exclusive NOR) gate?

(a) 

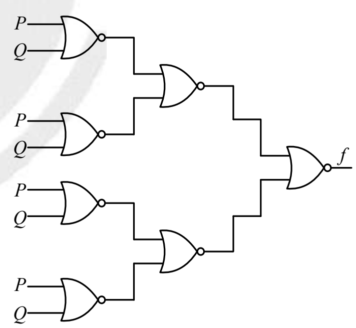
(b) 

(c) 

(d) 

21. [MCQ] [GATE-2010: 1M]

What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below?



(a) $\overline{Q+R}$ (b) $\overline{P+Q}$

(c) $\overline{P+R}$ (d) $\overline{P+Q+R}$

22. [MCQ] [GATE-2009: 1M]

What is the minimum number of gates required to implement the Boolean function (AB + C) if we have to use only 2-input NOR gates?

(a) 2 (b) 3

(c) 4 (d) 5

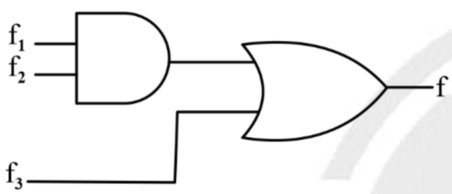
23. [MCQ] [GATE-2008: 1M]

A set of Boolean connectives is functionally complete if all Boolean functions can be synthesized using those. Which of the following sets of connectives is NOT functionally complete?

(a) EX-NOR (b) implication, negation
(c) OR, negation (d) NAND

24. [MCQ] [GATE-2008: 1M]

Given f1,f3 and f in canonical sum of products from (in decimal) for the circuit.



$F_1 = \sum m\ (4,5,6,7,8)$

$F_3 = \sum m\ (1, 6, 15)$

$F\ = \sum m\ (1, 6, 8, 15)$

Then f_2 is

(a) $\sum m\ (4, 6)$ (b) $\sum m\ (4, 8)$
(c) $\sum m\ (6, 8)$ (d) $\sum m\ (4, 6, 8)$

K-Maps

25. [NAT] [GATE-2022: 1M]

The total number of prime implicants of the function *f(w,x,y,z) = Σ(0, 2, 4, 5, 6, 10)* is ______.

26. [NAT] [GATE-2018: 2M]

Consider the minterm list form of a Boolean function F given below.

F(P, Q, R, S) = Σm(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is _______ .

27. [NAT] [GATE-2017: 1M]

Consider the Karnaugh map given below, where X represents "don't care" and blank represents 0.

dc \ ba	00	01	11	10
00		×	×	
01	1			×
11	1			1
10		×	×	

Assume for all inputs(a, b, c, d) , the respective complements $(\bar{a}, \bar{b}, \bar{c}, \bar{d})$ are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is _________.

28. [MCQ] [GATE-2017: 1M]

Given *f(w, x, y, z) = Σm(0, 1, 2, 3, 7, 8, 10) + Σd(5, 6, 11, 15)*, where d represents the don't-care condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of *f(w, x, y, z)*?

(a) $f = (\bar{w} + \bar{z})(\bar{x} + z)$

(b) $f = (\bar{w} + z)(x + z)$

(c) $f = (w + z)(\bar{x} + z)$

(d) $f = (w + \bar{z})(\bar{x} + z)$

29. [MCQ] [GATE-2014: 1M]

Consider the following Boolean expression for F:

F(P, Q, R, S) = PQ + P'QR + P'QR'S

The minimal sum-of-products form of F is

(a) PQ +QR +QSz

(b) P + Q + R + S

(c) $\overline{P} + \overline{Q} + \overline{R} + \overline{S}$

(d) $\overline{P}R + \overline{PR}S + P$

30. [MCQ] **[GATE-2014: 1M]**

Consider the following minterm expression for F:

F (P,Q,R,S) = Σ 0, 2, 5, 7, 8,10,13,15

The minterms 2, 7, 8 and 13 are 'do not care' terms.

The minimal sum-of-products form for F is

(a) $Q\bar{S} + \bar{Q}S$ (b) $\bar{Q}\bar{S} + QS$

(c) $\bar{Q}\bar{R}\bar{S} + \bar{Q}R\bar{S} + Q\bar{R}S + QRS$

(d) $\bar{P}\bar{Q}\bar{S} + \bar{P}QS + PQS + P\bar{Q}\bar{S}$

31. [MCQ] **[GATE-2012: 1M]**

What is the minimal form of the karnaugh map shown below? Assume that X denotes a don't care term

cd \ ab	00	01	11	10
00	1	×	×	1
01	×			1
11				
10	1			×

(a) $\bar{b}\bar{d}$ (b) $\bar{b}\bar{d} + \bar{b}\bar{c}$

(c) $\bar{b}\bar{d} + a\bar{b}\bar{c}d$ (d) $\bar{b}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{d}$

32. [MCQ] **[GATE-2008: 1M]**

In the Karnaugh map shown below, X denotes a don't care term. What is the minimal form of the function represented by the Karnaugh map?

cd \ ab	00	01	11	10
00	1	1		1
01	×			
11	×			
10	1	1		×

(a) $\bar{b}.\bar{d} + \bar{a}.\bar{d}$

(b) $\bar{a}.\bar{b} + \bar{b}.\bar{d} + \bar{a}.b.\bar{d}$

(c) $\bar{b}.\bar{d} + \bar{a}.b.\bar{d}$

(d) $\bar{a}.\bar{b} + \bar{b}.\bar{d} + \bar{a}.\bar{d}$

33. [MCQ] **[GATE-2008: 1M]**

Consider the following Boolean function of four variables f(A, B, C, D) = Σ(2, 3, 6, 7, 8, 9, 10, 11, 12, 13) The function is

(a) independent of one variable

(b) independent of two variables

(c) independent of three variable

(d) dependent on all the variables

❑❑❑

ANSWER KEY

1.	(d)	**2.**	(c)	**3.**	(b)	**4.**	(a)
5.	(a)	**6.**	(a)	**7.**	(d)	**8.**	(b)
9.	(a)	**10.**	(a)	**11.**	(c)	**12.**	(a)
13.	(c)	**14.**	(a)	**15.**	(3)	**16.**	(b)
17.	(d)	**18.**	(d)	**19.**	(d)	**20.**	(d)
21.	(a)	**22.**	(b)	**23.**	(a)	**24.**	(c)
25.	(3)	**26.**	(3)	**27.**	(1)	**28.**	(a)
29.	(a)	**30.**	(b)	**31.**	(b)	**32.**	(a)
33.	(a)						

SOLUTIONS

1. (d)

For $n = 1$, Total number of functions = 4

Total number of self-dual functions = 2

For $n = 2$, Total number of functions = 16

Total number self-dual functions = 4

Now, if we have 'n' variable with us

Then total number of function = 2^{2^n}

Among then number of self-dual functions

$$= 2^{2^{n-1}}$$

Self -Dual:

$F = F^D$

$F = AB$

$F^D = A+B$

$A \xrightarrow{\text{Dual}} A$

$\bar{A} \xrightarrow{\text{Dual}} \bar{A}$

Ex-

$F = AB + BC + AC$

$F^D = (A + B) \cdot (B + C) (A + C)$

$F^D = AB + BC + AC$

Hence, correct answer is option (d)

Scan for Video solution

2. (c)

(a) $wx + w(x + y) + x(x + y) = x + wy$

$wx + wx + wy + x + xy$

$x[w + w + 1 + y] + wy$

$= x + wy$

Therefore, option a is correct

(b) $\overline{w\bar{x}(y+\bar{z})} + \bar{w}x = \bar{w} + x + \bar{y}z$

$\overline{w\bar{\bar{x}}} + \overline{y+\bar{\bar{z}}} + \bar{w}x$

$\bar{w} + x + \bar{y}z + \bar{w}x$

$\bar{w} + x(1+\bar{w}) + \bar{y}z = \bar{w} + x + \bar{y}z$

Therefore, option b is correct

(c) $\left(w\bar{x}(y + x\bar{z}) + \bar{w}\bar{x}\right)y = x\bar{y}$

$w\bar{x}y + w\bar{x}x\bar{z} + \bar{w}\bar{x}y$

$\left[w\bar{x}y + \bar{w}\bar{x}y\right]$

$$\overline{x}y\left(w+\overline{w}\right)$$

$$\overline{x}y$$

Hence correct answer is option (c)

(d) (w + y) (wxy + wyz) = wxy +wyz

wxy +wyz + wxy + wyz

wxy + wyz

Therefore, option d is correct.

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3. **(b)**

We can see that the given truth table is the output of X - OR gate and we know that X-OR logic follows both commutative and associative operation.

$A \oplus B = B \oplus A \rightarrow$ follow commutative law

$(A \oplus B) \oplus C = A \oplus (B \oplus C)$ Follow associative law.

Hence, correct answer is option (a).

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4. **(a)**

The given Boolean expression is

$$= \overline{\overline{D}+A\overline{B}+\overline{A}C+A\overline{C}D+\overline{A}\,\overline{C}D}$$

$$= \overline{\overline{D}+A\overline{B}+\overline{A}C+\overline{C}D\left(A+\overline{A}\right)}$$

$$= \overline{\overline{D}+A\overline{B}+\overline{A}C+\overline{C}D}$$

$$= \overline{\overline{D}+\overline{C}D+A\overline{B}+\overline{A}C}$$

$$= \overline{\left(\overline{C}+\overline{D}\right)\left(\overline{D}+D\right)+A\overline{B}+\overline{A}C}$$

$$\Rightarrow \overline{\left(\overline{C}+\overline{D}\right)+\left(A\overline{B}\right)+\overline{A}C}$$

$$\Rightarrow \overline{\overline{\overline{C}}+\overline{\overline{D}}}\cdot\overline{A\overline{B}}\cdot\overline{\overline{A}C}$$

$$\Rightarrow CD\left[\overline{A}+B\right]\cdot\left[A+\overline{C}\right]$$

$$\Rightarrow CD\left[\overline{A}\cdot A+\overline{A}\,\overline{C}+AB+B\overline{C}\right]$$

$$\Rightarrow CD\left[\overline{A}\,\overline{C}+AB+B\overline{C}\right]$$

$$= \overline{A}\,\overline{C}\cdot CD+ABCD+B\overline{C}CD$$

$$\Rightarrow ABCD$$

Hence, correct answer is option (a).

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5. **(a)**

Standard mininmal from

$$F = \overline{P}+QR$$

$$F = \overline{P}\left(\overline{Q}+Q\right)\left(\overline{R}+Q\right)+\left(\overline{P}+P\right)QR$$

$$= \overline{P}\,\overline{Q}\,\overline{R}+\overline{P}\,\overline{Q}R+\overline{P}Q\overline{R}+\overline{P}QR+\overline{P}QR+\overline{P}QR+PQR$$

$$= \sum m(0,1,2,3,7)$$

$$= \prod M\,(4,5,6)$$

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6. **(a)**

The equation of the given truth table will be given as

$$F(x,y) = x\overline{y}+xy$$

$$= x\left(\overline{y}+y\right) = x$$

Hence, correct answer is option (a)

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7. **(d)**

Let us see the 4-bit number multiplication.

$$A = A_3\ A_2\ A_1\ A_0 \longrightarrow 2^4$$

$$B = B_3\ B_2\ B_1\ B_0 \longrightarrow 2^4$$

$$A\times B \longrightarrow \underline{M_7\ M_6\ M_5\ M_4\ M_3\ M_2\ M_1\ M_0}$$

Input = $2^4 \times 2^4$ **output** = 8

The amount of ROM needed to implement a 4 bit multiplier is

ROM $= 2^4 \times 2^4 \times 8$
$= 2^8 \times 2^3$
$= 2^{11}$
$= 2 \times 2^{10}$
= 2k bits

Hence, correct answer is option (d).

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8. (b)

The given expression is

$(P + \bar{Q} + \bar{R}) \cdot (P + \bar{Q} + R) \cdot (P + Q + \bar{R})$

Let $P + \overline{Q} = x$

$\Rightarrow \left(x + \overline{R}\right) \cdot \left(x + R\right) \left(P + Q + \overline{R}\right)$

(By distribution Theorem $x + \overline{R} . R = (x + \overline{R})(x + R)$)

$\Rightarrow x.(P + Q + \overline{R})$

$\Rightarrow \left(P + \overline{Q}\right) \cdot \left(P + Q + \overline{R}\right)$

$\Rightarrow P + PQ + P\overline{R} + P\overline{Q} + \overline{Q} \cdot Q + \overline{Q}\overline{R}$

$= P\left[1 + Q + \overline{R} + \overline{Q}\right] + \overline{Q}\,\overline{R}$

$= P + \overline{Q}\,\overline{R}$

Hence, correct answer is option (b).

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9. (a)

The given expression is

$F\left(P,Q,R\right) = PQ + Q\overline{R} + P\overline{R}$

The Standard canonical form of the above expression will be given as:

$F\left(P,Q,R\right) = PQ\left(\overline{R} + R\right) + \left(\overline{P} + P\right)Q\overline{R} + P\left(\overline{Q} + Q\right)\overline{R}$

$= PQ\overline{R} + PQR + \overline{P}Q\overline{R} + PQ\overline{R} + P\overline{Q}\overline{R} + PQ\overline{R}$

$= m_6 + m_7 + m_2 + m_6 + m_4 + m_6$

$= m_2 + m_4 + m_6 + m_7$

Hence, correct answer is option (a).

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10. (a)

The given expression is

(P + Q′) (P.Q′ + P.R) (P′.R′ + Q′)

$\Rightarrow \left(P\overline{Q} + PR + P\overline{Q} + P\overline{Q}R\right) \cdot \left(\overline{P}\overline{R} + \overline{Q}\right)$

$\Rightarrow \left(P\overline{Q} + PR\right) \cdot \left(\overline{P}\overline{R} + \overline{Q}\right)$

$= P\left(\overline{Q} + R\right) \cdot \left(\overline{P}\overline{R} + \overline{Q}\right)$

$= \left(\overline{Q} + R\right)\left(P\overline{P}\overline{R} + P\overline{Q}\right)$

$= \left(\overline{Q} + R\right)\left(P\overline{Q}\right)$

$= P\overline{Q} + P\overline{Q}R$

$= P\overline{Q}\left(1 + R\right)$

$= P\overline{Q}$

Hence, correct answer is option (a).

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11. (c)

We will put $X_1 = 1, X_2 = 1, X_3 = 1, X_4 = 1$

$1 \oplus 1 \oplus 1 \oplus 1 = 0$

By taking the above values ,we will see all the options one-by-one.

→ $X_1 . X_2 . X_3 . X_4 = 1.1.1.1 = 0$ (wrong)

→ $X_1 X_3 + X_2 = 1.1 + 1 = 0$ (wrong)

→ $\overline{X} \oplus \overline{X}_3 = \overline{X}_2 \oplus \overline{X}_4 = 0 \oplus 0 = 0 \oplus 0 = 0$ (correct)

→ $X_1 + X_2 + X_3 + X_4 = 0$ (wrong)

Alternate Method:

If $A \oplus B = 0$

Then $A \oplus 0 = B$ … (i)

Given, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$

$A = x_1 \oplus x_2$, $B = x_3 \oplus x_4$

Substitute values of A, B in equation (i)

$x_1 \oplus x_2 \oplus 0 = x_3 \oplus x_4$

$x_1 \oplus x_2 = x_3 \oplus x_4$

We know that $\overline{A \oplus B} = \bar{A} \oplus B$

$\overline{\overline{x_1 \oplus x_2}} = \overline{\overline{x_3 \oplus x_4}}$

$\overline{x_1} \oplus \overline{x_2} = \overline{x_3} \oplus \overline{x_4}$

Hence, correct answer is option (c)

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12. (a)

Given

$x \# 0 = x$, $x \# 1 = \bar{x}$,

$x \# x = 0$, $x \# \bar{x} = 1$

We can see from the given condition that the logic operator # resembles the logic of EX-OR gate. Therefore,

$$x \# y = \bar{x}y + x\bar{y}$$

Hence correct answer is option (a)

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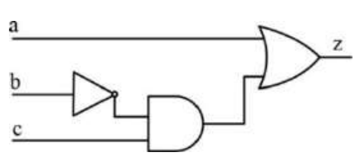
13. (c)



$z = a + \bar{b}c$

standard canonical form

$z = a(\bar{b} + b)(\bar{c} + c) + (\bar{a} + a)\bar{b}c$

$z = a\bar{b}\bar{c} + a\bar{b}c + ab\bar{c} + abc + \bar{a}\bar{b}c + a\bar{b}c$

$z = \Sigma m(1, 4, 5, 6, 7)$

Hence correct answer is option (c).

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14. (a)

Option (a)

$(x + y) \oplus z = x \oplus (y + z)$

$(x + y) \oplus z = (x + y)\bar{z} + \overline{(x + y)}z$

$(x + y) \oplus z = x\bar{z} + y\bar{z} + \bar{x}\bar{y}z$

$x \oplus (y+z) = x\overline{(y + z)} + \bar{x}(y + z)$

$x \oplus (y+z) = x\bar{y}\bar{z} + \bar{x}y + \bar{x}z$

$(x + y) \oplus z \neq x \oplus (y + z)$

Option (b)

$(x \oplus y) \oplus z = x \oplus (y \oplus z)$

X-OR gate follows associative law

Option (c)

$x \oplus y = x + y$, if $x.y = 0$

$x \oplus y = \bar{x}y + x\bar{y}$

$= (x + y)(\bar{x} + \bar{y})$

$= (x + y) = 1 = x+y$

$\overline{xy} = 1$

$\bar{x} + \bar{y} = 1$

Option (d)

$x \oplus y = (xy + \bar{x}\bar{y})$

$x \oplus y = \overline{x \odot y}$

$x \oplus y = \overline{xy + \bar{x}\bar{y}}$

Hence option (a) is correct answer.

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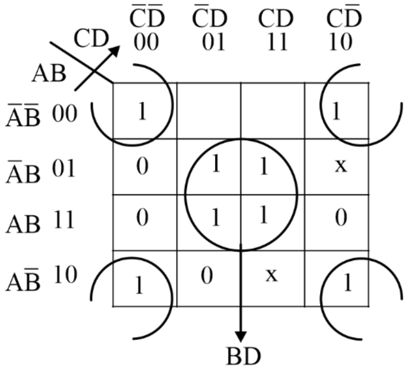
15. (3)

$F = \Sigma m\ (0, 2, 5, 7, 8, 10, 13, 15)$



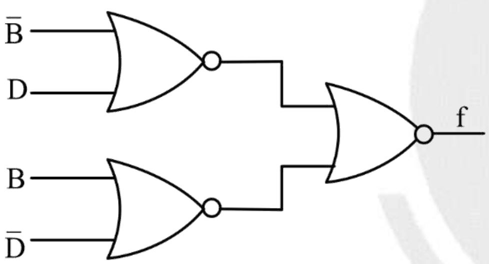
Simplified expression from above K-Map

In SOP

$F = \bar{B}\bar{D} + BD$

In POS

$F = (\bar{B} + D)\ (B + \bar{D})$



From above logic circuit minimum number NOR gate required is 3.

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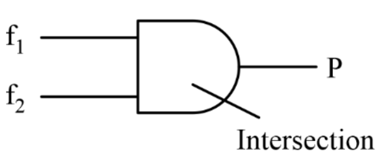
16. (b)

Given

$f_1 = \Sigma(0, 2, 5, 8, 14)$

$f_2 = \Sigma(2, 3, 6, 8, 14, 15)$

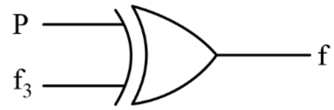
$f_3 = \Sigma\ (2, 7, 11, 14)$



$P = f_1 \cap f_2$

$P = (0, 2, 5, 8, 14) \cap (2, 3, 6, 8, 14, 15)$

$P = (2, 8, 14)$



$f = P \oplus f_3$

$f = P\bar{f}_3 + \bar{P} f_3$

$\bar{P} f_3 = (2, 8, 14) \cdot (0, 1, 3, 5, 6, 8, 9, 10, 12, 13, 15)$

$P\bar{f}_3 = (8)$

$\bar{P} f_3 = (0, 1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 15) \cdot (2, 7, 11, 14)$

$\bar{P} f_3 = (7, 11)$

$f = (8) \oplus (7, 11)$

$f = (7, 8, 11)$

$f = \Sigma\ (7, 8, 11)$

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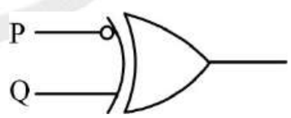
17. (d)

Option (b)

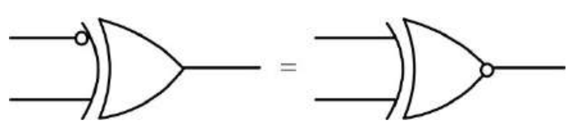


$f = \bar{P} \oplus Q$

Let $\bar{P} = Z$

$f = Z \oplus Q = \quad \bar{Z}Q + Z\bar{Q}$

$f = \bar{\bar{P}}Q + \bar{P}\bar{Q} = PQ + \bar{P}\bar{Q} = P \odot Q$



Hence option (b) is correct

Option (c)

$$\bar{P} \oplus \bar{Q} = \bar{P}\,\bar{Q} + \bar{P}\,\bar{Q}$$

$$= P\bar{Q} + \bar{P}\,Q$$

$$= P \oplus Q$$

Hence option (c) is correct

Option (d)

$$(P \oplus \bar{P}) \oplus Q = (P \odot \bar{P}) \odot \bar{Q}$$

$$1 \oplus Q = \bar{Q} \neq 0 \odot \bar{Q} = Q$$

Hence option (d) is incorrect.

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18. (d)

Let $P \oplus Q = Z$

$$F = (1 \oplus P) \oplus (Z) \oplus (Z) \oplus (Q \oplus 0)$$

$$= \left[\bar{P} \oplus P \oplus Q\right] \oplus \left[P \oplus Q \oplus Q\right]$$

$$= [1 \oplus Q] \oplus [P \oplus 0]$$

$$= \bar{Q} \oplus P$$

$$= \overline{P \oplus Q}$$

Note: $\begin{cases} P \oplus 1 = \bar{P} \\ Q \oplus 0 = Q \\ \bar{P} \oplus Q = P \odot Q = \overline{P \oplus Q} \\ Z \oplus Z = O \end{cases}$

Hence, correct answer is option (d).

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19. (d)

We know that Exclusive NOR of x and y

$$= \left(\overline{x \oplus y}\right)$$

$$= \left(\overline{x \oplus y}\right) = (xy' + x'y)'$$

$$= ((x' + y)(x + y'))$$

$$= x'y' + xy$$

$\rightarrow$ $xy + x'y' = x \odot y$ [Exclusive OR]

$\rightarrow$ $x \oplus y' = x \odot y$ [Exclusive OR]

$\rightarrow$ $x' \oplus y = x \odot y$ [Exclusive OR]

$\rightarrow$ $x' \oplus y' = x \oplus y$ [Exclusive NOR]

Hence, correct answer is option (d).

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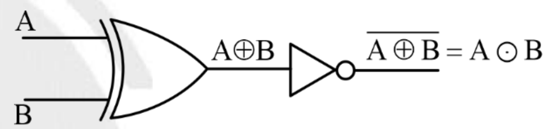
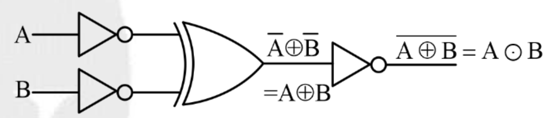
20. (d)

Let us see the output of all the given options we get

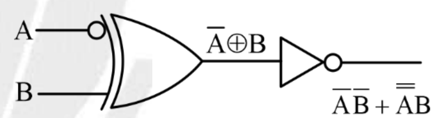
(a)



(b)



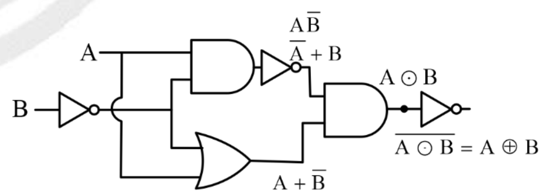
(c)



$$\bar{A}\bar{B} + \bar{\bar{A}}B$$

$$\bar{A}B + AB = A \odot B$$

(d)



$$(\bar{A} + B) \bullet (A + \bar{B})$$

$$\bar{A}\bar{B} + AB$$

Hence, correct answer is option (d).

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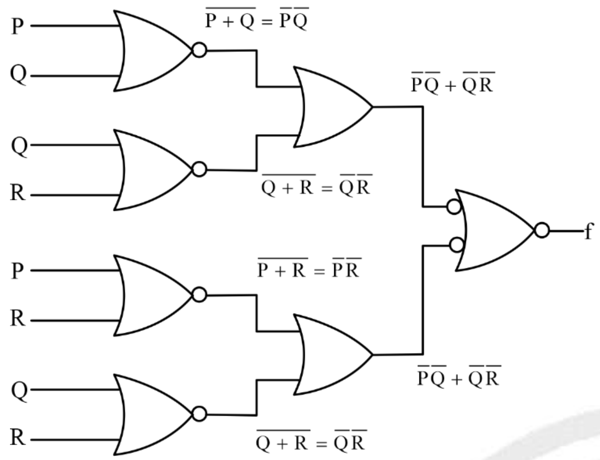
21. (a)

Let us see the output of the given logic circuit at each stage, we get



The output of the above logic circuit will be given by

$$f=\left(\overline{P}\,\overline{Q}+\overline{Q}\,\overline{R}\right)\left(\overline{P}\,\overline{R}+\overline{Q}\,\overline{R}\right)$$

$$f=\overline{P}\,\overline{Q}\,\overline{R}+\overline{P}\,\overline{Q}\,\overline{R}+\overline{P}\,\overline{Q}\,\overline{R}+\overline{Q}\,\overline{R}$$

$$f=\overline{P}\,\overline{Q}\,\overline{R}+\overline{Q}\,\overline{R}$$

$$f=\overline{Q}\,\overline{R}\left(\overline{P}+1\right)$$

$$f=\overline{Q+R}$$

Hence, correct answer is option (a).

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22. (b)

The given expression is

f = (AB + C)

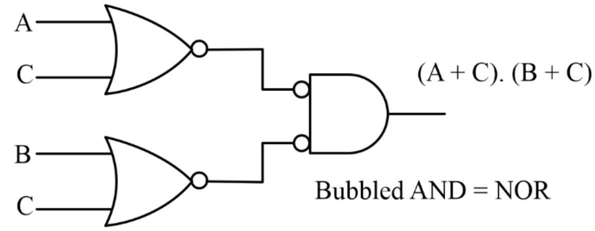
By using distribution theorem, we can write above expression as

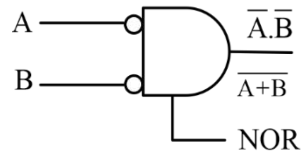
f = (A + C) · (B + C)

Note: Whenever in the problem, NOR gates are asked, write the function in POS.

Now, the above expression is in POS form.

Hence, three NOR gates will be required to implement f as shown below





Hence, correct answer is option (b).

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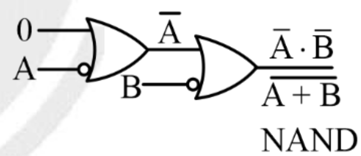
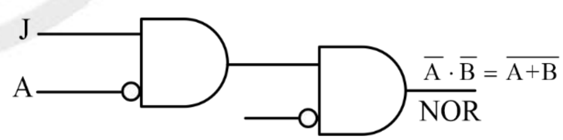
23. (a)

We now, that

NAND/NOR is functionally complete.



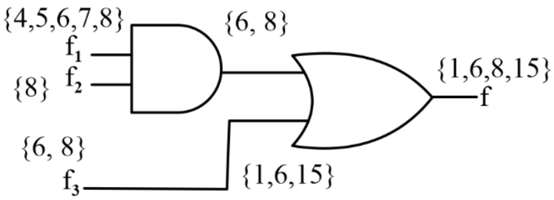


And from the above options only EX-NOR is not functionally complete

Hence, correct answer is option (a).

Scan for Video solution

24. (c)



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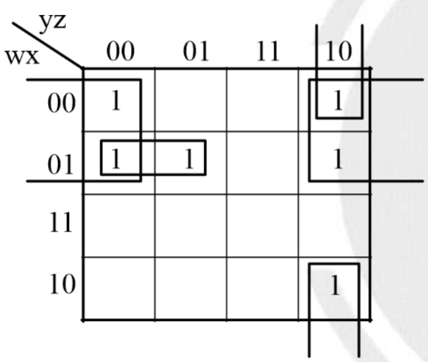
25. (3)

The given function is

f(w,x,y,z) = Σ(0, 2, 4, 5, 6, 10)

The k-map of the above function will be given as below



From the above k-map

$F = \overline{W}\,\overline{Z} + \overline{W}X\overline{Y} + \overline{X}Y\overline{Z}$

∴ Total prime implicants = 3

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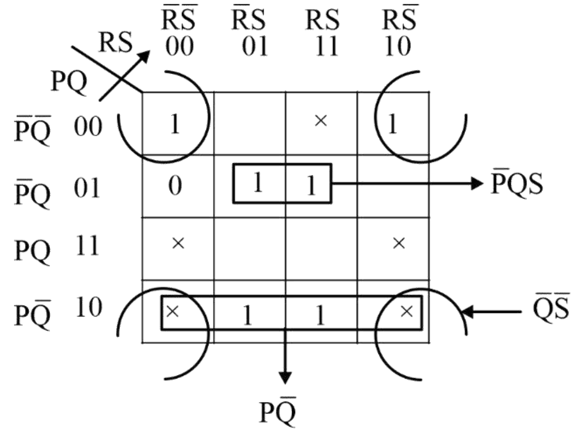
26. (3)

K-Map of given boolean function F



From above K-Map number of Essential prime implicats is 3.

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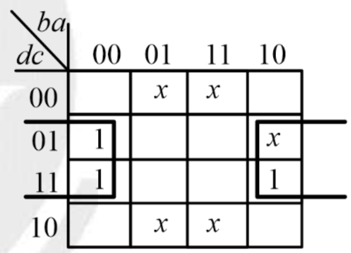
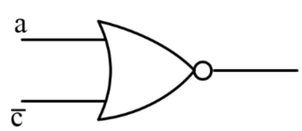
27. (1)

The given k-map is shown below



here only one quad group is possible then therefore,representation of given k map is

$F = \bar{a}\,c = \overline{a + \bar{c}} = \bar{a}\,c$



Hence only one 2-input NOR gate required.

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28. (a)

$f(w, x, y, z) = \Sigma m(0, 1, 2, 3, 7, 8, 10) + \Sigma d(5, 6, 11, 15)$

wx \ yz	$y+z$ 00	$y+\bar{z}$ 01	$\bar{y}+\bar{z}$ 11	$\bar{y}+z$ 10
$w+x$ 00	1	1	1	1
$w+\bar{x}$ 01	0	x	1	x
$\bar{w}+\bar{x}$ 11	0	0	x	0
$\bar{w}+x$ 10	1	0	x	1

$(\bar{x}+z)$

$(\bar{w}+\bar{z})$

from above K-Map 2 quad is possible

Therefore $f(w,x,y,z) = (\bar{x} + z)(\bar{w}+\bar{z})$

Hence option answer is option (a).

Scan for Video solution

29. (a)

The given function is

$F(P, Q, R, S) = PQ + \bar{P}Q\bar{R}S$

Its standard canonical form will be

$F(P, Q, R, S) = PQ(\bar{R}+R)(\bar{S}+S) + \bar{P}QR(\bar{S}+S) + \bar{P}Q\bar{R}S$

$$F(P, Q, R, S) = PQ(\bar{R}+R)(\bar{S}+S) + \bar{R}QR(\bar{S}+S) + \bar{P}Q\bar{R}S$$

$$= PQ\bar{R}\bar{S} + PQ\bar{R}S + PQR\bar{S} + PQRS + \bar{P}QR\bar{S} + \bar{P}QRS + \bar{P}Q\bar{R}S$$

$\sum m = (12, 13, 14, 15, 6, 7, 5)$

Now, the k-map of the above function will be given as below

PQ \ RS	00	01	11	10
00				
01		1	1	1
11	1	1	1	1
10				

$= PQ + QS + QR$

Hence, correct answer is option (a).

Scan for Video solution

30. (b)

The given function is

$F(P,Q,R,S) = \Sigma\ 0, 2, 5, 7, 8, 10, 13, 15$

The k-map of the above functions is given below

PQ \ RS	00	01	11	10
00	1			×
01		1	×	
11		×	1	
10	×			1

From the above k-map

$F = \bar{Q}\bar{S} + QS$

Hence, correct answer is option (b).

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31. (b) The given k-map is as shown below

cd \ ab	00	01	11	10
00	1	X	X	1
01	X			1
11				
10	1			X

From the given k-map ,function f will be given as

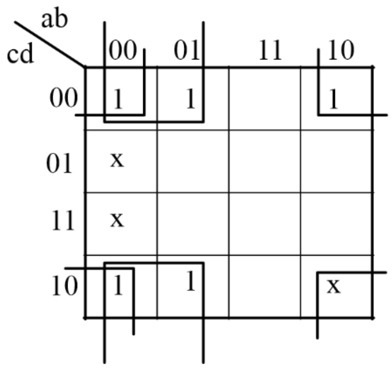
$f = \bar{b}\bar{c} + \bar{b}\bar{d}$

Hence correct answer is option (b).

Scan for Video solution

32. (a)

Given truth table is



From the above k-map the function f will be given as

$f = \bar{a}\bar{d} + \bar{b}\bar{d}$

Hence, correct answer is option (a).

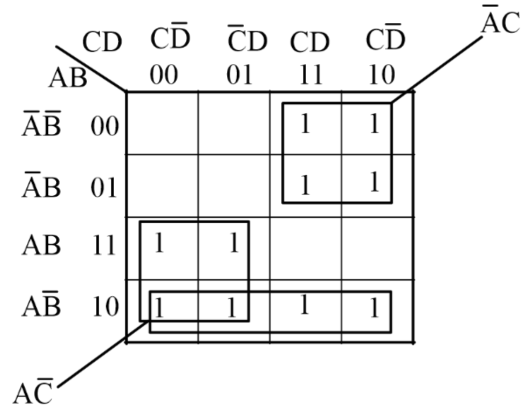
Scan for Video solution

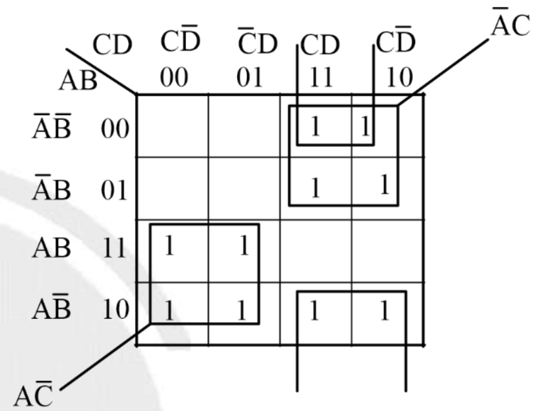
33. (a)

The given expression is

$f(A, B, C, D) = \Sigma m(2, 3, 6, 7, 8, 9, 10, 11, 12\ 13)$

k-map of the above expression will be given as





$f = \bar{A}C + A\bar{C} + A\bar{B}$

$f = \bar{A}C + A\bar{C} + \bar{B}\ C$

We can see from the above expression that f is independent of D.

Hence, correct answer is option (a).

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CHAPTER 3

COMBINATIONAL LOGIC CIRCUITS

BCD Adder and Subtractor

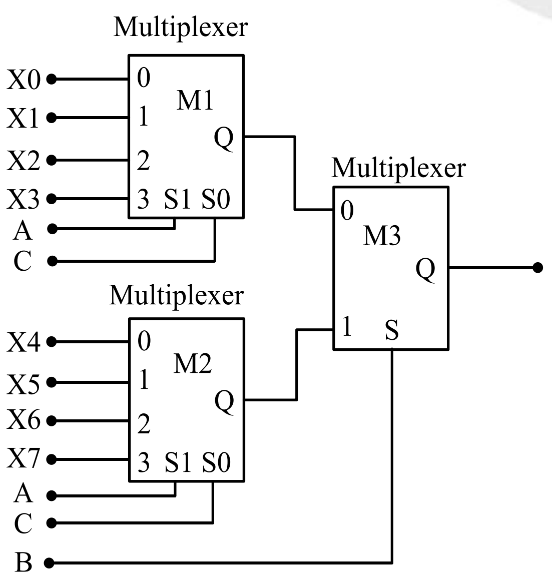
1. [NAT] [GATE-2016: 1M]

Consider an eight-bit ripple-carry adder for computing the sum of A and B where A and B are integers represented in 2's complement from. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is__________.

Multiplexer

2. [MCQ] [GATE-2023: 2M]

A Boolean digital circuit is composed using two 4-input multiplexers (M1 and M2) and one 2-input multiplexer (M3) as shown in the figure. X0-X7 are the inputs of the multiplexers M1 and M2 and could be connected to either 0 or 1. The select lines of the multiplexers are connected to Boolean variables A, B and C as shown.



Which one of the following set of values of (X0, X1, X2, X3, X4, X5, X6, X7) will realise the Boolean function $\bar{A} + \bar{A}.\bar{C} + A.\bar{B}.C$?

(a) (1, 1, 0, 0, 1, 1, 1, 0)

(b) (1, 1, 0, 0, 1, 1, 0, 1)

(c) (1, 1, 0, 1, 1, 1, 0, 0)

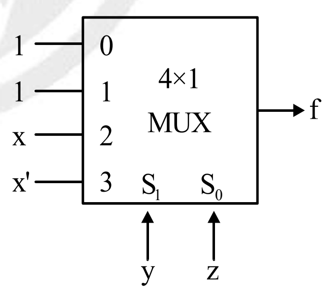
(d) (0, 0, 1, 1, 0, 1, 1, 1)

3. [MCQ] [GATE-2021: 1M]

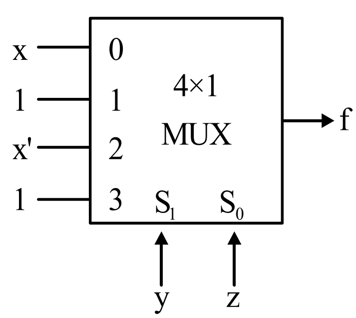
Which one of the folloiwng circuits implements the Boolean function given below?

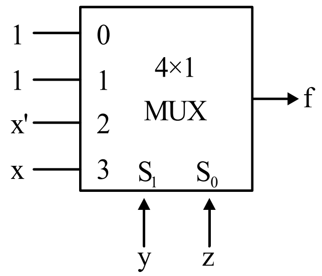
$f(x,y,z,) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6$ where m_t is the i^{th} minterm.

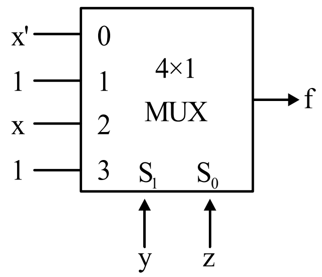
(a)



(b)



(c) 

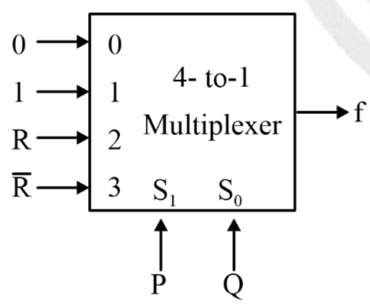
(d) 

4. [NAT] [GATE-2020: 1M]

A multiplexer is placed between a group of 32 registers and an accumulator to regulate data movement such that at any given point in time the content of only one register will move to the accumulator. The number of select lines needed for the multiplexer is _____.

5. [MCQ] [GATE-2014: 2M]

Consider the 4-to-1 multiplexer with with two select S_1 and S_0 given below.



The minimal sum of products from the Boolean expression for the output F of the multiplexer is-

(a) $\overline{P}Q + Q\overline{R} + P\overline{Q}R$

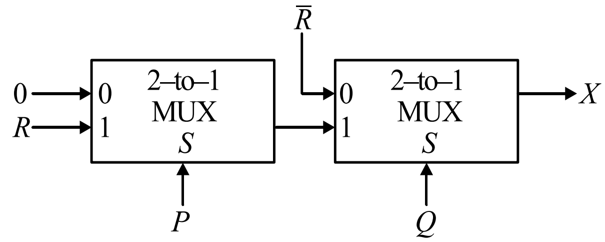
(b) $\overline{P}Q + \overline{P}Q\overline{R} + PQ\overline{R} + P\overline{Q}R$

(c) $\overline{P}QR + \overline{P}Q\overline{R} + Q\overline{R} + P\overline{Q}R$

(d) $PQ\overline{R}$

6. [MCQ] [GATE-2016: 1M]

Consider the two cascaded 2 to 1 multiplexers as shown in the figure.



The minimal sum of products form of the output X is

(a) $\bar{P}\bar{Q} + PQR$ (b) $\bar{P}$Q+QR

(c) PQ+ $\bar{P}\bar{Q}$R (d) $\bar{Q}\bar{R}$ +PQR

7. [MCQ] [GATE-2014: 1M]

Consider the following combinational function block involving four Boolean variables x, y, a, b where x, a, b are inputs and y is the output.

f (x, y, a, b)
{
if (x is 1) y = a;
else y = b;
}.

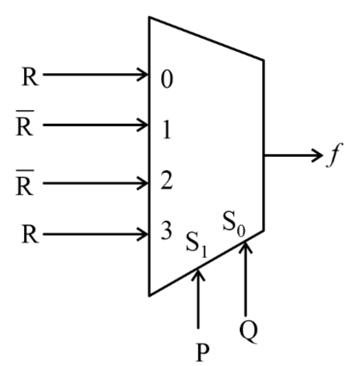
Which one of the following digital logic blocks is the most suitable for implementing this function?

(a) Full adder (b) Priority encoder

(c) Multiplexer (d) Flip-flop

8. [MCQ] [GATE-2010: 1M]

The Boolean expression of the output f of the multiplexer shown below is



(a) $\overline{P \oplus Q \oplus R}$ (b) $P \oplus Q \oplus R$

(c) $P + Q + R$ (d) $\overline{P + Q + R}$

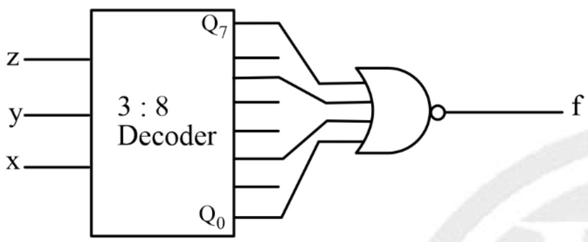
Decoder/De-MUX

9. [NAT] [GATE-2020: 1M]

If there are m input lines and n output lines for a decoder that is used to uniquely address a byte addressable 1 KB RAM, then the minimum value of m + n is ____.

10. [MCQ] [GATE-2008: 1M]

What Boolean function does the circuit below realize?



(a) $xz + \bar{x}\,\bar{z}$ (b) $x\bar{z} + \bar{x}z$

(c) $\bar{x}\,\bar{y} + yz$ (d) $xy + \bar{y}\,\bar{z}$

Encoder and Priority Encoder

11. [MCQ] [GATE-2013: 1M]

In the following truth table, V = 1 if and only if the input is valid. What function does the truth table represent?

Inputs				Outputs		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	x	1	0	1	0	1
X	x	x	1	1	1	1

What function does the truth table represent?

(a) Priority encoder (b) Decoder

(c) Multiplexer (d) Demultiplexer

❑❑❑

ANSWER KEY

1.	(–1)	**2.**	(c)	**3.**	(a)	**4.**	(5)
5.	(a)	**6.**	(d)	**7.**	(c)	**8.**	(b)
9.	(1034)	**10.**	(b)	**11.**	(a)		

SOLUTIONS

1. (–1)

B = 1 1 1 1 1 1 1 1 $\Rightarrow -1$

0 0 0 0 0 0 0 1

+1

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2. (c)

Given function $f(ABC) = \overline{A} + \overline{A}\,\overline{C} + A\overline{B}\,C$

- First, we minimize the given function

  $f(ABC) =$

  $\overline{A}\left(B+\overline{B}\right)\left(C+\overline{C}\right)+\overline{A}\left(B+\overline{B}\right)\overline{C}+A\overline{B}\,C$

$f(ABC) =$

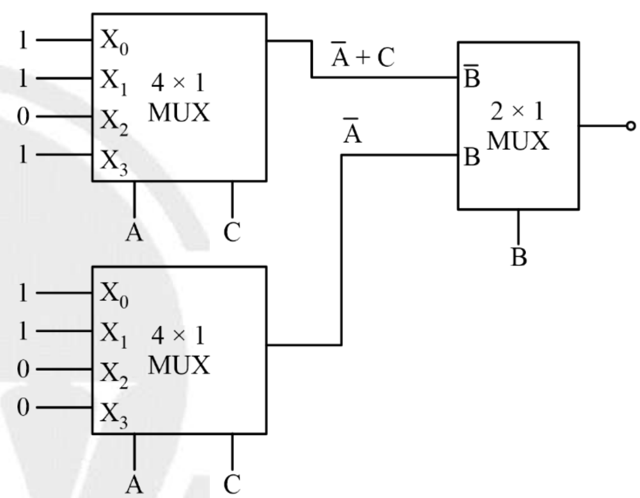
$\overline{A}\,\overline{B}\,\overline{C}+\overline{A}\,\overline{B}\,C+\overline{A}B\overline{C}+\overline{A}BC+\overline{A}\,\overline{B}\,\overline{C}+\overline{A}B\overline{C}+A\overline{B}C$

$f(ABC) = \overline{A}\,\overline{B}\,\overline{C}+\overline{A}\,\overline{B}\,C+\overline{A}B\overline{C}+\overline{A}BC+\overline{A}B\overline{C}+A\overline{B}C$

$f(ABC) = \Sigma m\,(0, 1, 2, 3, 5)$

Now implementing on 2 × 1 mux.

	$\overline{B}$	B
$\overline{A}\,\overline{C}$	0	2
$\overline{A}\,C$	1	3
$A\,\overline{C}$	4	6
$A\,C$	5	7
	$\overline{A}+C$	$\overline{A}$



$\therefore (X_0\,X_1\,X_2\,X_3\,X_4\,X_5\,X_6\,X_7) = (11011100)$

Hence correct answer is option (c).

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3. (a)

$F = \left(\overline{X}+X\right)\overline{Y}\,\overline{Z}+\left(\overline{X}+X\right)\overline{Y}Z+XY\overline{Z}+\overline{X}YZ$

$= \overline{X}\,\overline{Y}\,\overline{Z}+X\overline{Y}\,\overline{Z}+\overline{X}\,\overline{Y}Z+X\overline{Y}Z+XY\overline{Z}+\overline{X}YZ$

$= \sum m\left(0,1,3,4,5,6\right)$

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4. (5)

Given:

Number of registers = 32

Mux $\Rightarrow 32 \times 1$

Inputs = 32

Number of select line = log2(number of inputs)

Number of select line $= \log_2 32 = 5$

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5. (a)

$F = \overline{P}Q + P\overline{Q}R + PQ\overline{R}$

Standard cononcial from

$F = \overline{P}Q\left(\overline{R} + R\right) + P\overline{Q}R + PQ\overline{R}$

$= \overline{P}Q\overline{R} + \overline{P}QR + P\overline{Q}R + PQ\overline{R} = \sum m(2,3,5,6)$

P \ QR	00	01	11	10
0			1	1
1		1		1

$\boxed{P\overline{Q}R + \overline{P}Q + Q\overline{R}}$

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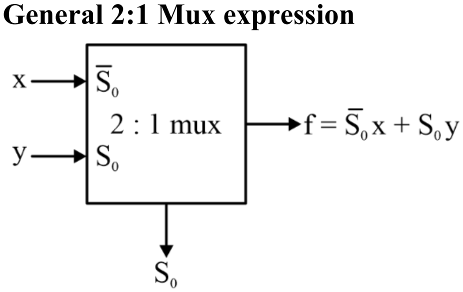
6. (d)

General 2:1 Mux expression



If S_0 is enabled and x, y are inputs then output

$f = \overline{S}_0\, x + S_0\, y$

Here in given problem output of MUX-1 is input to MUX -2

output MUX -1

$f_1 = \overline{P}.0 + P.R = P\,R$

MUX -2 output

$X = \overline{Q}.\,\overline{R} + Q.\,f_1$

$X = \overline{Q}.\,\overline{R} + Q.P.R$

Its minimal sum of product form output

$X = \overline{Q}.\overline{R} + PQR$

Hence, correct answer is option (d).

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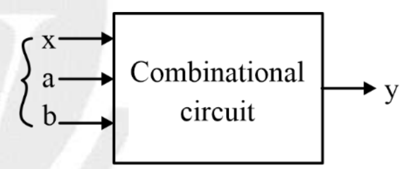
7. (c)

Then given circuit will be as follows



From the given condition if

x =1 y = a

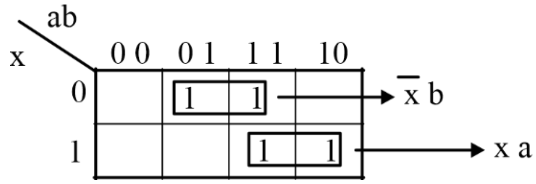
Otherwise, y = b

The truth table of the above circuit will be given as

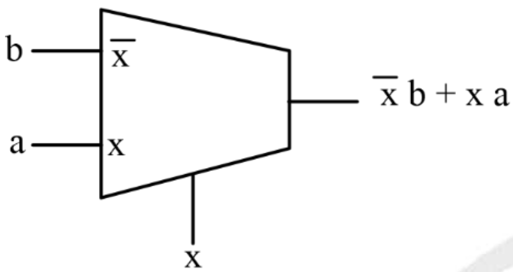
x	a	b	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

y (x, a, b) = $\Sigma m(1, 3, 6, 7)$

The k-map of the above function will be



$y = x\,a + \bar{x}\,b$



Hence, correct answer is option (c).

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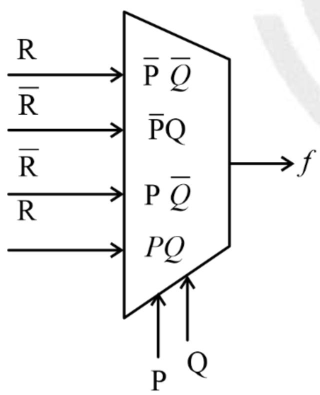
8. **(b)**

Given multiplexer diagram redrawn as



The output of the given multiplexer will be given as

$f = \bar{P}\bar{Q}R + \bar{P}Q\bar{R} + P\bar{Q}\bar{R} + PQR$

$f = m_1 + m_2 + m_4 + m_7$

$f = \sum m\,(1,2,4,7)$

By using k-map we get

P \ QR	0 0	0 1	1 1	10
0	0	1	0	1
1	1	0	1	0

$f = \bar{P}\bar{Q}R + \bar{P}Q\bar{R} + P\bar{Q}\bar{R} + PQR$

$f = P \oplus Q \oplus R$

Hence, correct answer is option (b).

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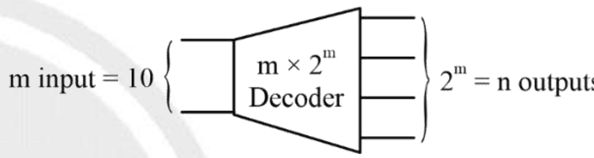
9. **(1034)**



1 kB RAM=1024 = 2^{10}

m = 10

n = 2^{10} = 1024

m + n = 10 + 1024= 1034

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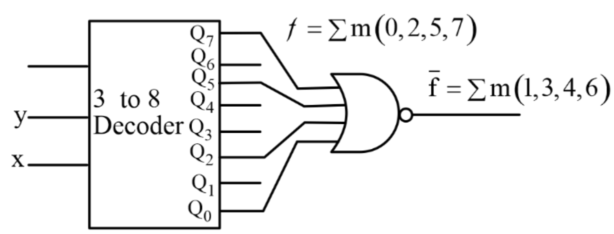
10. **(b)**



We can see that the given circuit is 3 to 8 decoder and its output will be given as.

$\bar{f} = \sum m(1,3,4,6)$

By using k-map we get

x \ yz	00	01	11	10
0		1	1	
1	1			1

$f = \bar{x}z + x\bar{z}$

Hence, correct answer is option (b).

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11. (a)

We know that

$D_0 \rightarrow$ MSB

$D_3 \rightarrow$ LSB

we can see that the given truth table is the truth table of priority encoder.

V=0 is invalid output.

V=1 is valid output

Hence, correct answer is option (a).

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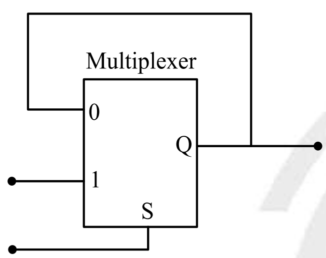
CHAPTER

4 SEQUENTIAL CIRCUITS

Flip Flops

1. [MCQ] [GATE-2023: 1M]

The output of a 2-input multiplexer is connected back to one of its inputs as shown in the figure.

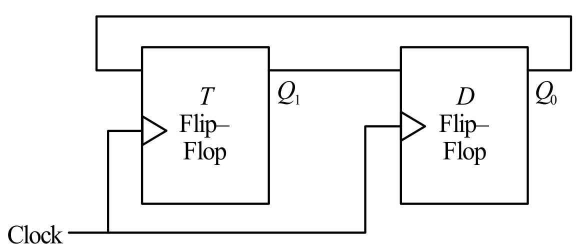


Match the function equivalence of this circuit to one of the following options.

(a) D Flip-flop (b) D Latch
(c) Half-adder (d) Demultiplexer

2. [MCQ] [GATE-2017: 1M]

Consider a combination of T and D flip-flops connected as shown below. The output of the D flip-flop is connected to the input of the T flip-flop and the output of the T flip-flop is connected to the input of the D flip-flop.



Initially, both Q_0 and Q_1 are set to 1 (before the 1st clock cycle). The outputs

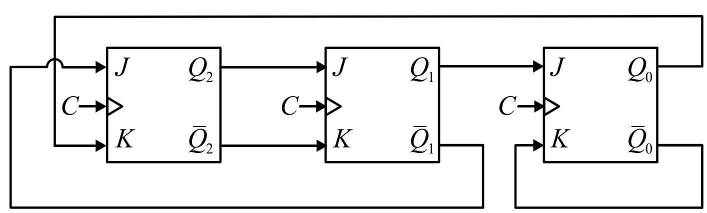
(a) Q_1Q_0 after the 3^{rd} cycle are 11 and after the 4^{th} cycle are 00 respectively
(b) Q_1Q_0 after the 3^{rd} cycle are 11 and after the 4^{th} cycle are 01 respectively
(c) Q_1Q_0 after the 3rd cycle are 00 and after the 4^{th} cycle are 11 respectively
(d) Q_1Q_0 after the 3rd cycle are 01 and after the 4^{th} cycle are 01 respectively.

3. [MCQ] [GATE-2015: 1M]

A positive edge-triggered D flip-flop is connected to a positive edge-triggered JK flip-flop as follows. The Q output of the D flip-flop is connected to both the J and K inputs of the JK flip-flop, while the Q output of the JK flip-flop is connected to the input of the D flip-flop. Initially, the output of the D flip-flop is set to logic one and the output of the JK flip-flop is cleared. Which one of the following is the bit sequence (including the initial state) generated at the Q output of the JK flip-flop when the flip-flops are connected to a free-running common clock? Assume that J = K = 1 is the toggle mode and J = K = 0 is the state-holding mode of the JK flip-flop. Both the flip-flops have non-zero propagation delays.

(a) 0110110... (b) 0100100...
(c) 011101110... (d) 011001100...

4. [MCQ] [GATE-2014: 1M]



The above synchronous sequential circuit built using JK flip-flops is initialized with $Q_2Q_1Q_0 = 000$. The state sequence for this circuit for the next 3 clock cycles is

(a) 001, 010, 011 (b) 111, 110, 101
(c) 100, 110, 111 (d) 100, 011, 001

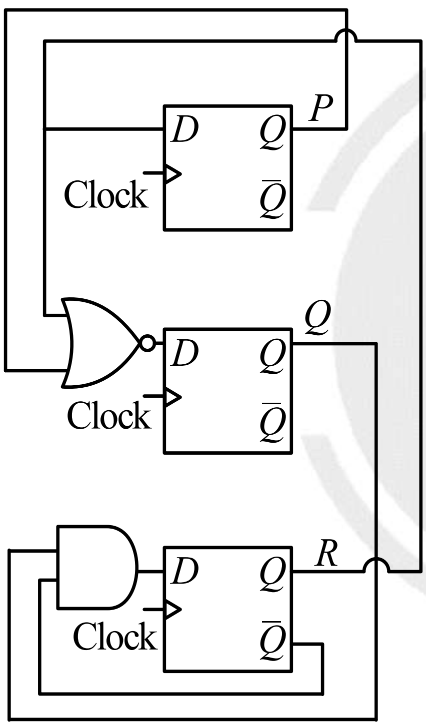
5. **[MCQ]** **[GATE-2011: 1M]**

The minimum number of D flip-flops needed to design a mod-258 counter is

(a) 9 (b) 8
(c) 512 (d) 258

6. **[MCQ]** **[GATE-2011: 1M]**

Consider the following circuit involving three D-type flip-flops used in a certain type of counter configuration. If all the flip-flops were reset to 0 at power on, what is the total number of distinct outputs (states) represented by PQR generated by the counter?

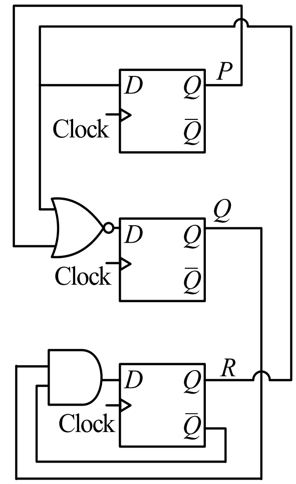


(a) 3 (b) 4
(c) 5 (d) 6

7. **[MCQ]** **[GATE-2011: 1M]**

Consider the following circuit involving three D-type flip-flops used in a certain type of counter configuration.

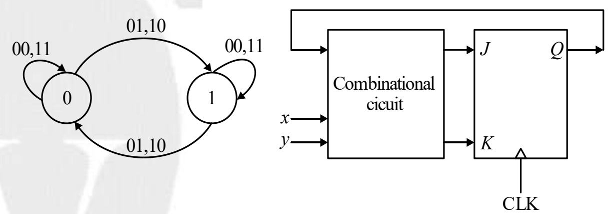
If at some instance prior to the occurrence of the clock edge, P, Q and R have a value 0, 1 and 0 respectively, what shall be the value of PQR after the clock edge?



(a) 000 (b) 001
(c) 010 (d) 011

8. **[MCQ]** **[GATE-2008: 1M]**

Consider the following state diagram and its realization by a JK flip flop



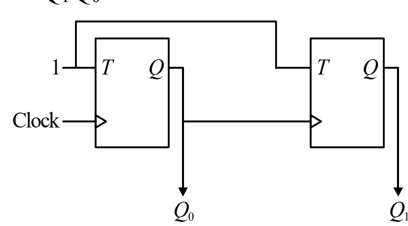
The combinational circuit generates J and K in terms of x, y and Q. The Boolean expressions for J and K are:

(a) $(x\oplus y)'$ and $(x\oplus y)'$ (b) $(x\oplus y)'$ and $x\oplus y$
(c) $x\oplus y$ and $(x\oplus y)'$ (d) $x\oplus y$ and $x\oplus y$

Asynchronous Counter

9. **[MCQ]** **[GATE-2010: 1M]**

In the sequential circuit shown below, if the initial value of the output Q_1Q_0 is 00, what are the next four values of Q_1Q_0?

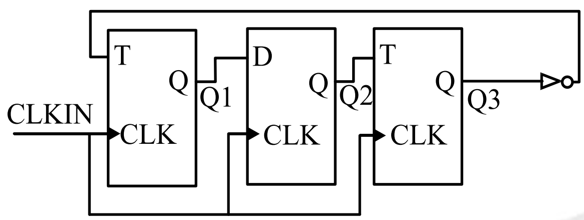


(a) 11, 10, 01, 00 (b) 10, 11, 01, 00
(c) 10, 00, 01, 11 (d) 11, 10, 00, 01

Synchronous Counter

10. [MCQ] [GATE-2023: 2M]

Consider a sequential digital circuit consisting of T flip-flops and D flip-flops as shown in the figure. CLKIN is the clock input to the circuit. At the beginning, Q1, Q2 and Q3 have values 0, 1 and 1, respectively.

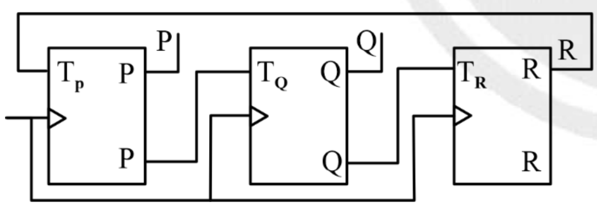


Which one of the given values of (Q1, Q2, Q3) can NEVER be obtained with this digital circuit?

(a) (0, 0, 1) (b) (1, 0, 0)
(c) (1, 0, 1) (d) (1, 1, 1)

11. [MCQ] [GATE-2021: 1M]

Consider a 3-bit counter, designed using T flip-flops, as shown below.



Assuming the intial state of the counter given by PQR as 000. What are the next three states?

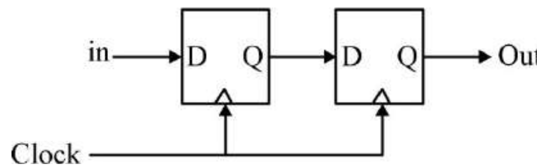
(a) 011,101,000 (b) 001,010,111
(c) 011,101,111 (d) 001,010,000

12. [MCQ] [GATE-2021: 2M]

The minimum number of JK flip-flops requried to construct a synchronous counter with the count sequence (0,0,1,1,2,2,3,3,0,0….)is_______.

13. [NAT] [GATE-2018: 1M]

Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.



The number of states in the state transition diagram of this circuit that have a transition back to the same state on some value of "in" is ______

14. [MCQ] [GATE-2017: 1M]

The next state table of a 2-bit saturating up-counter is given below.

Q_1	Q_0	Q_1^+	Q_0^+
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

The counter is built as a synchronous sequential circuit using T flip-flops.

The expressions for T_1 and T_0 are

(a) $T_1 = Q_1Q_0$, $T_0 = \overline{Q_1}\,\overline{Q_0}$

(b) $T_1 = \overline{Q_1}Q_0$, $T_0 = \overline{Q_1} + \overline{Q_0}$

(c) $T_1 = Q_1 + Q_0$, $T_0 = \overline{Q_1} + \overline{Q_0}$

(d) $T_1 = \overline{Q_1}Q_0$, $T_0 = Q_1 + Q_0$

15. [MCQ] [GATE-2015: 1M]

Conisder a 4-bit Johnson counter an initial value of 0000. The counting sequence of this counter is-

(a) 0,1,3,7,15,14,12,8,0
(b) 0,1,3,5,7,9,11,13,15,0
(c) 0,2,4,6,8,10,12,14,0
(d) 0,8,12,14,15,7,3,1,0

16. [MCQ] [GATE-2014: 1M]

Let $k = 2^n$. A circuit is built by giving the output of an n-bit binary counter as input to an n-to-2^n bit decoder. This circuit is equivalent to a

(a) k-bit binary up counter.

(b) k-bit binary down counter.

(c) k-bit ring counter.

(d) k-bit Johnson counter.

17. [MCQ] [GATE-2009: 1M]

How many 32K × 1 RAM chips are needed to provide a memory capacity of 256K-bytes?

(a) 8 (b) 32

(c) 64 (d) 128

Memoery

18. [MCQ] [GATE-2012: 00M]

The amount of ROM needed to implement a 4 bit multiplier is

(a) 64 bits (b) 128 bits

(c) 1 Kbits (d) 2 Kbits

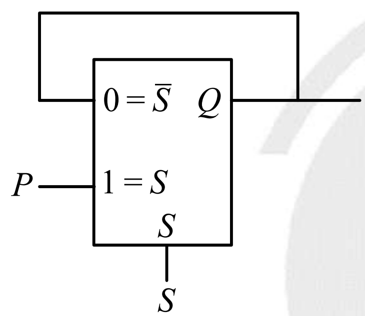
❑❑❑

ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(c)
5.	(a)	**6.**	(c)	**7.**	(d)	**8.**	(d)
9.	(a)	**10.**	(a)	**11.**	(a)	**12.**	(3)
13.	(2)	**14.**	(b)	**15.**	(d)	**16.**	(c)
17.	(c)	**18.**	(d)				

SOLUTIONS

1. (b)



$Q_{n+1} = Q\,\bar{S} + DS$

$Q_{n+1} = Q.0+D.1$

$Q_{n+1} = D$

Q_{n+1} is generated after delay

Now in a flip flop we always consider edge but in the circuit, it is level sensitive, therefore the circuit is equivalent to a D-latch.

Scan for Video solution

2. (b)

Given present state of both flip flop before apply clock pulse is High (i.e. 1)

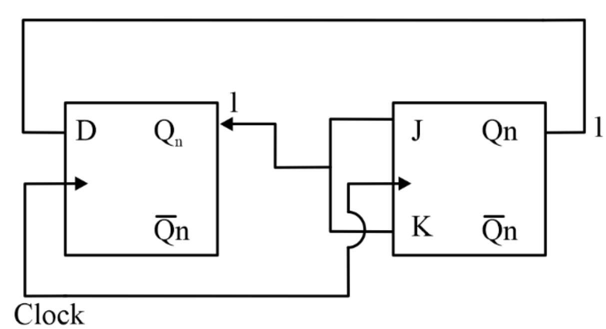
Clock	Q_1	Q_0
0	1	1
1	0	1
2	1	0
3	1	1
4	0	1
5	1	0
6	1	1

from above table the output state after 3rd and 4th clock pulse are 11 (Q_1Q_0) & 01 (Q_1Q_0) respectively hence correct option is (b).

Scan for Video solution

3. (a)

According to the question, the given sequential circuit will look like as follows.



CLK	Q_n of JK Flip Flop
0	0
1	1
2	1
3	0
4	1
5	1

From the above truth table the Output of JK Flip Flop will be

$0 \rightarrow 1 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 1$

= 011011...

Hence, correct answer is option (a).

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4. **(c)**

The truth table of the given flip-flop will be

JK-flip flop

CLK	Q_2	Q_1	Q_0
0	0	0	0
1	1	0	0
2	1	1	0
3	1	1	1

From the above truth table, the state sequence for next 3 clock cycles are 100, 110 and 111.

Hence, correct answer is option (c).

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5. **(a)**

The number of Flip Flops needed to design Mod-M counter,

$$M \le 2^n \quad \ldots(i)$$

where n = number of flip flops

to solve (i) by taking log on both sides we get,

$n \ge \log_2 M$

$n \ge \log_2 258 \qquad [\because \ M = 258]$

$n \ge 8.01122$

$\boxed{n \cong 9}$

Hence, correct answer is option (a).

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6. **(c)**

As per the given conditions, the truth table will be given as

CLOCK	P	Q	R
0	0	0	0
1	0	1	0
2	0	1	1
3	1	0	0
4	0	0	0

We can see that the above truth table is the output of a mod- 4 counter.

Hence total number of distinct states = 4

Hence, correct answer is option (b)

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7. **(d)**

In the circuit PQR = 010

Before applying the clock, input of the Flip-flop =011

So, in the D-FF whatever the inputs applied that will come at the output along with clock.

So, output of counter = 011

Hence, correct answer is option (d).

Scan for Video solution

8. (d)

We know that truth table of JK flip-flop is given by

Q_n	Q_{n+1}	J	K
0	1	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Now the truth table of given circuit is given by

x	y	Q_n	Q_{n+1}	J	K
0	0	0	0	0	×
0	0	1	1	×	0
0	1	0	1	1	×
0	1	1	0	×	1
1	0	0	1	1	×
1	0	1	0	×	1
1	1	0	0	0	×
1	1	1	1	×	0

Solving J & K using k-map, we get

x \ Q_ny	00	01	11	10
0	0	×	×	1
1	1	×	×	

$J = x\bar{y} + \bar{x}y = x \oplus y$

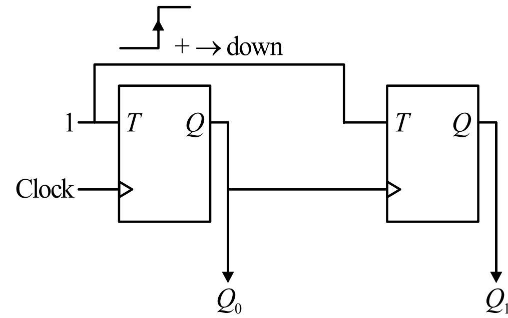
x \ Q_ny	00	01	11	10
0	×		1	×
1	×	1		×

$K = \bar{x}y + x\bar{y} = x \oplus y$

Hence, correct answer is option (d).

Scan for Video solution

9. (a)



Asynchronous down counter

The truth table of the above sequential circuit is given as follows.

CLK	Q_1	Q_0
0	0	0
1	1	1
2	1	0
3	0	1
4	0	0

From the above truth table, we can see that it is an output of an Asynchronous down counter.

Hence, correct answer is option (a).

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10. (a)

The characteristics table of given sequential circuit

Clock	$T = \bar{Q}_3$	$D = Q_1$	$T = Q_2$	Q_1	Q_2	Q_3
0	-	-	-	0	1	1
1	0	0	1	0	0	0
2	1	0	0	1	0	0
3	1	1	0	0	1	0
4	1	0	1	1	0	1

5	0	1	0	1	1	1
6	0	1	1	1	1	0
7	1	1	1	0	1	1

From above table states involved 011, 000, 100, 010, 101, 111, 110. The only state 001 can never be achieved.

Hence correct answer is option (a).

Scan for Video solution

11. (a)

According to question , the initial state of counter is PQR = 000

CLK	P	Q	R
0	0	0	0
1	0	1	1
2	1	0	1
3	0	0	0

From the truth table the next three states are

$011 \rightarrow 101 \rightarrow 000$

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12.. (3)

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1
0	0	0
0	0	1
0	1	0
0	1	1

If we ignore the value of 'C'

$00 \rightarrow 00 \rightarrow 01 \rightarrow 01 \rightarrow 10 \rightarrow 10 \rightarrow 11 \rightarrow 11$

$0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow 2 \rightarrow 3 \rightarrow 3 \rightarrow 0 \rightarrow 0$

Design the up counter of 3bit ignore the LSB.

So, numebr of FF's are = 3

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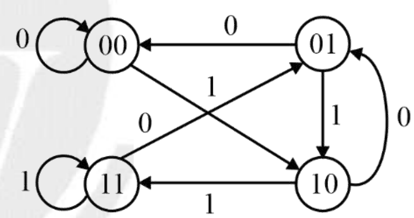
13. (2)

The characteristics table of given Flip-Flop

Input	D_1	D_2	Q_1^+	Q_2^+(Out)
0	0	0	0	0
1	0	0	1	0
0	0	1	0	0
1	0	1	1	0
0	1	0	0	1
1	1	0	1	1
0	1	1	0	1
1	1	1	1	1

From above chracteristics table state transistion diagram of the given positive edge-triggered D flip flop is



From above state transistion diagram after 2 state it come back to some value.

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14. (b)

Q_1	Q_0	Q^+	Q_0^+	T_1	T_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	0	0

For T_0

$Q_1 \backslash Q_0$	0	1
0	1	1
1	1	0

$T_0 = \overline{Q}_1 + \overline{Q}_0$

For T_1

$Q_1 \backslash Q_0$	0	1
0	0	1
1	0	0

$T_1 = \overline{Q}_1 Q_0$

Hence correct answer is option (b).

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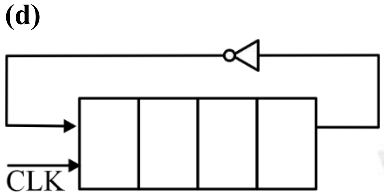
15. (d)



0 0 0 0→ 0

1 0 0 0→ 8

1 1 0 0→ 12

1 1 1 0→ 14

1 1 1 1→ 15

0 1 1 1→ 7

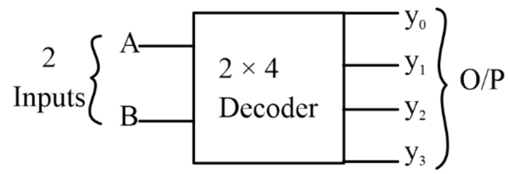
0 0 1 1→ 3

0 0 0 1→ 1

Scan for Video solution

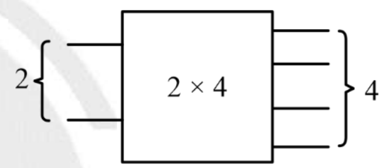
16. (c)

Let 2 × 4 Decoder as shown below, be at the output

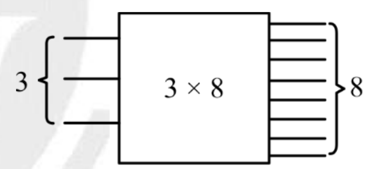


Now, according to question 2-bit binary counter is connected in the input of the above Decoder. The output of the combination will be given as

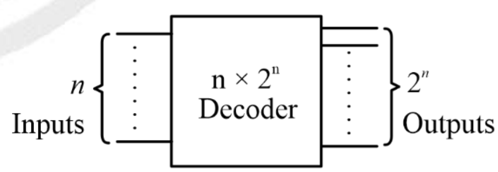
A	B	y_0	y_1	y_2	y_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1



4 bit Ring counter



8 bit Ring counter



2^n bit Ring counter

k-bit Ring counter

Ring counter of 2^n bits i.e. k-bit ring counter.

Hence, correct answer is option (c)

Scan for Video solution

17. (c)

Number of required chips for memory capacity

$= 256 \times 2^{10} \times 8$

Number of available chips for RAM capacity

$= 32 \times 2^{10} \times 1$

Now,

Number of chips needed to build 256K-bytes will be

$$\text{Number of chips} = \frac{\text{Required}}{\text{Avaliable}} = \frac{256 \times 2^{10} \times 8}{32 \times 2^{10}} = 64$$

Hence, correct answer is option (c).

Scan for Video solution

18. (d)

$\boxed{4}$ $A = A_3\ A_2\ A_1\ A_0 \longrightarrow 2^4$

$\boxed{4}$ $B = B_3\ B_2\ B_1\ B_0 \longrightarrow 2^4$

$\Rightarrow M_7\ M_6\ M_5\ M_4\ M_3\ M_2\ M_1\ M_0$

$\rightarrow$ 8 bits 8 o/p

$$\text{ROM} = \overbrace{2^4 \times 2^4}^{\text{Input}} \times \overset{\text{output}}{2}$$

$= 2^8 \times 2^3$

$= 2^{11}$

$= 2 \times 2^{10}$

$= 2\text{k bits}$

Scan for Video solution

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NOTE :

INDEX

Discrete Mathematics

Discrete Mathematics

Syllabus

Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Chapter wise Weightage Analysis

Paper Year \ Chapter	Ch.1	Ch.2	Ch.3	Ch.4
2008	3	2	0	4
2009	4	3	1	0
2010	0	2	3	0
2011	0	0	0	0
2012	1	2	3	0
2013	4	1	1	0
2014 (P1)	1	2	4	2
2014 (P2)	2	0	5	1
2014 (P3)	3	2	4	0
2015 (P1)	3	2	2	4
2015 (P2)	2	9	2	1
2015 (P3)	1	0	3	1
2016 (P1)	1	1	0	5
2016 (P2)	3	2	0	0
2017 (P1)	4	0	1	0
2017 (P2)	1	1	2	2
2018	0	1	3	2
2019	0	0	0	0
2020	2	2	2	2
2021 (P1)	2	2	1	0
2021 (P2)	2	3	0	0
2022	0	1	14	0
2023	2	5	2	3

]

CHAPTER

1 PROPOSITIONAL LOGIC

Propositional Logic

1. [MCQ] [GATE-2021 : 2M]

Let p and q be two propositions. Consider the following two formulae in propositional logic

S_1: $(\neg p \wedge (p \vee q)) \rightarrow q$

S_2: $q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct?

(a) Both S_1 and S_2 are tautologies.

(b) S_1 is a tautology but S_2 is not a tautology.

(c) S_1 is not a tautology but S_2 is a tautology.

(d) Neither S_1 nor S_2 is a tautology.

2. [MCQ] [GATE-2021 : 2M]

Choose the correct choice(S) regarding the following proportional logic assertion S:

S: $((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$

(a) S is neither a tautology nor a contradiction.

(b) S is a tautology.

(c) S is a contradiction.

(d) The antecedent of S is logically equivalent to the consequent of S.

3. [MCQ] [GATE-2017 : 1M]

Let p, q, r denote the statements "It is raining", "It is cold", and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by

(a) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

(b) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$

(c) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$

(d) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$

4. [MCQ] [GATE-2017 : 2M]

Let p, q and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is

(a) a tautology

(b) a contradiction

(c) always TRUE when p is FALSE

(d) always TRUE when q is TRUE

5. [NAT] [GATE-2016 : 1M]

Let p, q, r, s represent the following propositions.

p : $x \in \{8, 9, 10, 11, 12\}$

q : x is a composite number

r : x is a perfect square

s : x is a prime number

The integer $x \geq 2$ which satisfies $\neg ((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is __________.

6. [MCQ] [GATE-2015 : 2M]

The binary operator ≠ is defined by the following truth table.

p	q	p ≠ q
0	0	0
0	1	1
1	0	1
1	1	0

Which of the following is true about the binary operator ≠ ?

(a) Both commutative and associative

(b) Commutative but not associative

(c) Not commutative but associative

(d) Neither commutative nor associative

7. **[MCQ]** **[GATE-2015 : 1M]**
Let # be a binary operator defined as X # Y = X′ + Y′ where X and Y are Boolean variables.
Consider the following two statements:
S_1 : (P # Q)# R=P # (Q # R)
S_2 : Q # R = R # Q
Which of the following is/are true for the Boolean variables P, Q and R?
(a) Only S_1 is True
(b) Only S_2 is True
(c) Both S_1 and S_2 are True
(d) Neither S_1 nor S_2 are True

8. **[MCQ]** **[GATE-2014 : 2M]**
Which one of the following Boolean expressions is NOT a tautology?
(a) $((a \to b) \wedge (b \to c)) \to (a \to c)$
(b) $(a \leftrightarrow c) \to (\sim b \to (a \wedge c))$
(c) $(a \wedge b \wedge c) \to (c \vee a)$
(d) $a \to (b \to a)$

Logical Equivalence

9. **[MCQ]** **[GATE-2017 : 1M]**
The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below?
I. $p \Rightarrow q$ II. $q \Rightarrow p$
III. $(\neg q) \vee p$ IV. $(\neg p) \vee q$
(a) I only (b) I and IV only
(c) II only (d) II and III only

10. **[MCQ]** **[GATE-2015 : 1M]**
Which one of the following is Not equivalent to $p \leftrightarrow q$?
(a) $(\neg p \vee q) \wedge (p \vee \neg q)$
(b) $(\neg p \vee q) \wedge (q \to p)$
(c) $(\neg p \wedge q) \vee (p \wedge \neg q)$
(d) $(\neg p \wedge \neg q) \vee (p \wedge q)$

11. **[MCQ]** **[GATE-2009 : 2M]**
The binary operation □ is defined as follows:

P	Q	P □ Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to $P \vee Q$?
(a) $\neg Q$ □ $\neg P$ (b) P □ $\neg Q$
(c) $\neg P$ □ Q (d) $\neg P$ □ $\neg Q$

12. **[MCQ]** **[GATE-2008 : 2M]**
P and Q are two propositions, which of the following logical expressions are equivalent?
1. $P \vee \sim Q$
2. $\sim(\sim P \wedge Q)$
3. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$
4. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$
(a) Only 1 and 2 (b) Only 1, 2 and 3
(c) Only 1, 2 and 4 (d) All of 1, 2, 3, and 4

Inference Rule

13. **[NAT]** **[GATE-2016 : 1M]**
Consider the following expressions:
I. False II. Q
III. True IV. $P \vee Q$
V. $\neg Q \vee P$
The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is _____.

14. **[MCQ]** **[GATE-2015 : 2M]**
Consider the following two statements:
S_1: If a candidate is known to be corrupt, then he will not be elected.
S_2: If a candidate is kind, he will be elected.
Which one of the following statements follows from S_1 and S_2 as per sound inference rules of logic?
(a) If a person is known to be corrupt, he is kind
(b) If a person is not known to be corrupt, he is not kind
(c) If a person is kind, he is not known to be corrupt
(d) If a person is not kind, he is not known to be corrupt

15. **[MCQ]** **[GATE-2014 : 1M]**
Consider the following statements:
P: Good mobile phones are not cheap
Q: Cheap mobile phones are not good
L: P implies Q
M: Q implies P
N: P is equivalent to Q
Which one of the following about L, M, and N is CORRECT?
(a) Only L is TRUE
(b) Only M is TRUE
(c) Only N is TRUE
(d) L, M and N are TRUE

16. **[MCQ]** **[GATE-2012 : 1M]**
Consider the following logical inferences.
I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.
I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.
Which of the following is TRUE?
(a) Both I_1 and I_2 are correct inferences
(b) I_1 is correct but I_2 is not a correct inference
(c) I_1 is not correct but I_2 is a correct inference
(d) Both I_1 and I_2 are not correct inferences

Predicate logic

17. **[MSQ]** **[GATE-2023: 1M]**
Geetha has a conjecture about integers, which is of the form
$\forall x(P(x) \Rightarrow \exists yQ(x, y)),$
where P is a statement about integers, and Q is a statement about pairs of integers. Which of the following (one or more) option(s) would imply Geetha's conjecture?
(a) $\exists x(P(x) \wedge \forall yQ(x, y))$
(b) $\forall x \forall yQ(x, y))$
(c) $\exists y \forall y (P(x) \Rightarrow Q(x, y))$
(d) $\exists x(P(x) \wedge \exists yQ(x, y))$

18. **[MCQ]** **[GATE-2014 : 2M]**
The CORRECT formula for the sentence, "not all rainy days are cold" is
(a) $\forall d$ (Rainy (d) $\wedge \sim$ Cold (d))
(b) $\forall d$ ($\sim$Rainy (d) $\rightarrow$ Cold (d))
(c) $\exists d$ ($\sim$Rainy (d) $\rightarrow$ Cold (d))
(d) $\exists d$ (Rainy (d) $\wedge \sim$ Cold (d))

19. **[MCQ]** **[GATE-2014 : 1M]**
Consider the statement:
"Not all that glitters is gold"
Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement?
(a) $\forall x$: glitters(x) $\Rightarrow \neg$gold(x)
(b) $\forall x$: gold(x) $\Rightarrow$ glitters(x)
(c) $\exists x$: gold(x) $\wedge \neg$glitters(x)
(d) $\exists x$: glitters(x) $\wedge \neg$gold(x)

20. **[MCQ]** **[GATE-2013 : 2M]**
What is the logical translation of the following statements?
"None of my friends are perfect."
(a) $\exists x (F(x) \wedge \neg P(x))$
(b) $\exists x(\neg F(x) \wedge P(x))$
(c) $\exists x (\neg F(x) \wedge \neg P(x))$
(d) $\neg\exists x(F(x) \wedge P(x))$

Quantifier

21. **[MCQ]** **[GATE-2020 : 2M]**
Which one of the following predicate formulae is NOT logically valid?
Note that W is a predicate formula without any free occurrence of x.
(a) $\exists x(p(x) \wedge W) \equiv \exists x\, p(x) \wedge W$
(b) $\exists x(p(x) \rightarrow W) \equiv \forall x\, p(x) \rightarrow W$
(c) $\forall x (p(x) \rightarrow W) \equiv \forall x\, p(x) \rightarrow W$
(d) $\forall x (p(x) \vee W) \equiv \forall x\, p(x) \vee W$

22. [MCQ] [GATE-2017 : 1M]

Consider the first-order logic sentence.

F: $\forall x(\exists y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F?

I. $\exists y(\exists x\ R(x, y))$
II. $\exists y(\forall x\ R(x, y))$
III. $\forall y(\exists x R(x, y))$
IV. $\neg\exists x(\forall y \neg R(x, y))$

(a) IV only
(b) I and IV only
(c) II only
(d) II and III only

23. [MCQ] [GATE-2016 : 2M]

Which one of the following well-formed formulae in predicate calculus is NOT valid?

(a) $(\forall x\ p(x) \Rightarrow \forall x\ q(x)) \Rightarrow (\exists x\ \neg p(x) \vee \forall x\ q(x))$
(b) $(\exists x\ p(x) \vee \exists x\ q(x)) \Rightarrow \exists x(p(x) \vee q(x))$
(c) $\exists x(p(x) \wedge q(x)) \Rightarrow (\exists x\ p(x) \wedge \exists x\ q(x))$
(d) $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x\ p(x) \vee \forall x\ q(x))$

24. [MCQ] [GATE-2013 : 2M]

Which one of the following is NOT logically equivalent to $\neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$?

(a) $\forall x(\exists z(\neg\beta) \rightarrow \forall y(\alpha))$
(b) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg\alpha))$
(c) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg\beta))$
(d) $\forall x(\exists y(\neg\alpha) \vee \exists z(\neg\beta))$

25. [MCQ] [GATE-2009 : 2M]

Consider the following well-formed formulae:

I. $\neg\forall x(P(x))$
II. $\neg\exists x(P(x))$
III. $\neg\exists x(\neg P(x))$
IV. $\exists x(\neg P(x))$

Which of the above are equivalent?

(a) I and III
(b) I and IV
(c) II and III
(d) II and IV

26. [MCQ] [GATE-2008 : 1M]

Which of the following is the negation of $[\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, \gamma))]$?

(a) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \gamma))]$
(b) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \neg \gamma))]$
(c) $[\forall x, \neg \alpha \rightarrow (\exists y, \neg \beta \rightarrow (\forall u, \exists v, \neg\gamma))]$
(d) $[\exists x, \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall v, \neg\gamma))]$

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ANSWER KEY

1.	(b)	**2.**	(b, d)	**3.**	(a)	**4.**	(11 to 11)
5.	(a)	**6.**	(b)	**7.**	(b)	**8.**	(d)
9.	(c)	**10.**	(b)	**11.**	(b)	**12.**	(d)
13.	(4 to 4)	**14.**	(c)	**15.**	(d)	**16.**	(b)
17.	(b, c)	**18.**	(d)	**19.**	(d)	**20.**	(d)
21.	(c)	**22.**	(b)	**23.**	(d)	**24.**	(a, d)
25.	(b)	**26.**	(d)				

SOLUTIONS

1. (b)

S_1: $(\neg p \wedge (p \vee q)) \to q$

$$\left(\underset{T}{\neg p} \vee (p \wedge q)\right) \to q \quad \begin{matrix} p = F \\ q = F \end{matrix}$$

$$\downarrow \quad \downarrow$$

$$\underbrace{T \wedge (F \vee F)}_{F} \to F$$

$\to F \Rightarrow \text{True}$

S_2: $q \to (\neg p \wedge p \vee q)$

$$\underset{T}{q} \to \underset{F}{\neg p} \wedge \left(\underset{T \vee T}{p \vee q}\right) \quad \begin{matrix} p = T \\ q = T \end{matrix}$$

Scan for Video solution

2. (b, d)

Option d: Correct

$A \to B = \neg A \vee B$

$(P \wedge Q) \to R$

$\neg (p \wedge q) \vee R$

$\neg p \vee \neg q \vee R$

$(P \wedge Q) \to (Q \to R)$

$\neg (p \wedge q) \vee (q \to R)$

$(\neg p \vee \neg q) \vee (\neg q \vee R)$

$\neg p \vee \neg q \vee \neg q \vee R$

$\neg p \vee \neg q \vee R$

Option b: Correct

$$\underline{T} \qquad T \quad T \qquad T \quad F$$

$((P \wedge Q) \to R) \to ((P \wedge Q) \to (Q \to R))$

$\underline{(T \wedge T) \to F}$ $\qquad P = T$

$\underline{F \quad \to \qquad}$ $\qquad Q = T$

T $\qquad R = F$

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3. (a)

$(\neg P \wedge R) \wedge [\neg R \to (\neg p \wedge q)]$

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4. (11 to 11)

$\neg[(P \to q) \wedge (\neg r \vee \neg s)]$

$\neg[(\neg P \vee q) \wedge \neg(R \wedge S)]$

$(\neg[(\neg P \vee q))] \vee (R \vee S)$

$(P \wedge \neg q) \vee (R \wedge S)$

$(P \wedge \neg q) \vee (R \wedge S)$

P.S: Perfect Square

P.N: Prime number

$$\left((8,9,10,11,12) \wedge \frac{\text{x is not composite}}{\text{no.}}\right) \vee [\text{P.S.V.P.N})$$

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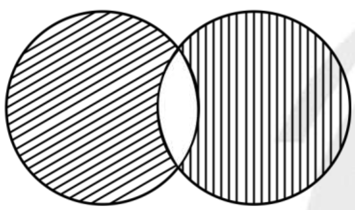
5. (a)

A∆B = B∆A

A∠B = B∠A



$P \neq (q \neq R) \equiv (P \neq q) \neq R$

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6. (b)

(P#Q)#R = P # (Q # R)

(P′+Q′)#R = P # (Q′+R′)

↓ ↓

(P′+Q′)+R′ = P′ + (Q′+R′)

((PQ′))′ +R′ = P′ + ((QR)) ′

PQ + R′= P′ + QR

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7. (b)

(a) $$\left(\frac{T}{(a \to b) \wedge (b \to c)}\right) \to \frac{F}{\left(\underset{T}{\underset{\downarrow}{a}} \to \underset{F}{\underset{\downarrow}{c}}\right)}$$

$$\frac{F \qquad \to}{F \to}$$

(c) $$\frac{T}{(a \wedge b \wedge \underset{F}{c})} \to \frac{F}{(\underset{\downarrow}{c} \vee \underset{\downarrow}{a})}$$

$$\frac{\dfrac{F}{F} \to \dfrac{(F \vee F)}{F}}{T}$$

(d) $$\frac{T}{a \to} \frac{F}{(b \to a)}$$

$$\begin{matrix} \downarrow & \downarrow \\ T & F \end{matrix}$$

$$\frac{T}{(a \leftrightarrow c) \to} \quad \frac{F}{\underline{\neg b} \to (a \wedge c)}$$

$\downarrow$ $\qquad\downarrow \quad \downarrow \quad \downarrow$

$\dfrac{(F \leftrightarrow F)}{T}$ $\qquad T \to (F \wedge F)$

$\dfrac{T \to F}{F}$

$\neg b = T$

$b \equiv F$

	a	b	c	
a = F c = F b = F	F	F	F	F

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8. (d)

$\neg p \to \neg q \equiv q \to p$

$\equiv \neg q \vee p$

Scan for Video solution

9. (c)

$(P \to q) \wedge (\neg q \vee P)$

$P \leftrightarrow q \equiv (P \to q) \wedge (q \to P)$

$\underbrace{(\neg P \vee q) \wedge (\neg q \vee P)}_{A.}$

$\underbrace{(\neg P \vee q) \wedge (q \to P)}_{B.}$

$P \leftrightarrow q \equiv (P \to q) \wedge (q \to P)$

$\underbrace{(\neg P \vee q)}_{A} \wedge (\neg q \vee P)$

$= A \wedge (\neg q \vee P)$

$\equiv (A \wedge \neg q) \vee (A \wedge P)$

$\equiv (\neg q \wedge A) \vee (P \wedge A)$

$(\neg q \wedge (\neg P \vee q)) \vee (P \wedge (\neg P \vee q))$

$\equiv (\neg q \wedge \neg P) \vee (\neg q \wedge q) \vee (p \wedge \neg P) \vee (P \wedge q)$

$\equiv ((\neg q \wedge \neg P) \vee F) \vee (F \vee (P \wedge q))$

$\equiv (\neg q \wedge \neg P) \vee (P \wedge q)$

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10. (b)

(a) $P = F$ $\quad P = F \quad Q = F$

$\neg Q□\neg P$

$T□T = T$

(b) $P□\neg Q$

$F□T \equiv F$

(c) $\neg P□Q$

$T□F \equiv T$

(d) $\neg P□\neg Q$

$T□T \equiv T$

Scan for Video solution

11. (b)

$1 \equiv 2 \equiv 3$

1. $P \vee \neg Q$

2. $\neg[\neg P \wedge Q]$

$P \vee \neg Q$

3. $(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

$(P \wedge Q) \vee [(P \vee \neg P) \wedge \neg Q]$

$(P \wedge Q) \vee [T \wedge \neg Q]$

$\neg Q \vee [P \wedge Q]$

$(\neg Q \vee P) \wedge [\neg Q \vee Q] \equiv \neg Q \vee P$

4. $[(P \wedge Q) \vee (P \wedge \neg Q)] \vee (\neg P \wedge Q)$

$P \wedge (Q \wedge \neg Q)$

$\frac{P \wedge T}{P}$

$P \vee (\neg P \wedge Q)$

$(P \vee \neg P) \wedge (P \vee Q)$

$T \wedge (P \vee Q) \equiv P \vee Q$

Scan for Video solution

12. (d)

$\frac{T}{(p \to q)} \to \frac{F}{R}$

$R = F$

$(R \to P) \to q$

$(F \to) \to q$

$T \to q$

Scan for Video solution

13. (4 to 4)

$[P \wedge (P \to Q)] \to \text{True}$

$Q \to \text{False}$

$\left[P \wedge (P \to Q)\right] Q \to P \vee Q.$

$$\left\{\begin{array}{l} P \to Q \\ \underline{P} \\ \therefore \quad Q \end{array}\right.$$

$$\overset{T}{\overline{\left(P \wedge (P \to Q)\right)}} \to \left(\overset{F}{\neg Q} \vee \overset{F}{P}\right)$$

$\underline{F \quad \wedge \quad}$ $\quad Q = T$

F $\quad P = F$

Scan for Video solution

14. (c)

$C \to \neg E \equiv E \to \neg C$

$K \to E$

$K \to \neg C$

Scan for Video solution

15. (d)

$$\frac{A}{\text{Good}} \to \frac{\neg B}{\text{Cheap}}$$

$$\frac{\text{Cheap} \to \neg\text{Good}}{B \to \neg A}$$

L: P implies Q (T)

M: Q implies P (T)

N: P is equivalent to Q (T)

$A \to \neg B \equiv B \to \neg A$

$A \equiv B$

$A \to B \wedge B \to A$

Scan for Video solution

16. (b)

$R \to \neg P \equiv P \to \neg R$

$$\frac{P}{\therefore \neg R} \qquad \frac{P}{\therefore \neg R} \text{(Modus pones)}$$

$R \to \neg P \equiv P \to \neg R$

$$\frac{\neg R}{P} \qquad \frac{\neg R}{\therefore P} \text{(Not valid)}$$

Scan for Video solution

17. (b, c)

L : $\forall x[P(x) \to \exists y\, Q(x,y)]$

"For every x if P(x) is true then there exists some y such that Q(x,y) will be true.

Option :

(i) $\exists x(P(x) \wedge \forall y\, Q(x,y))$

"For some x. P(x) is true and for all y Q(x.y) is true which is not implies L"

(ii) $\forall x \forall y Q(x,y)$

"For every x and every yQ(x,y) is true which implies L"

(iii) $\exists y\ \forall x[P(x) \to Q(x,y)]$

"These exists some y such that for every x if P(x) is true then Q(x,y) is also true which implies L"

(iv) $\exists x P(x) \wedge \exists y Q(x,y)$

"There exists some x for which P(x) is true and also for some y Q(x,y) is true which cannot implies L"

Scan for Video solution

18. (d)

Not (all rainy days are cold)

$\neg\forall d\left(\text{Rainy} \to \text{Cold}\right)$

$\neg\forall d\left[\neg\text{Rainy}(d) \vee \text{cold}(d)\right]$

$\exists d\left[\text{Rainy}(d) \wedge \neg\text{cold}(d)\right]$

Scan for Video solution

19. (d)

$\neg\forall x\left[\text{Glitter }(x) \rightarrow \text{Gold}(x)\right]$

$\neg\forall x\left[\neg\text{Glitter }(x) \vee \text{Gold}(x)\right]$

$\exists x\left[\text{Glitter }(x) \wedge \neg\text{Gold}(x)\right]$

Scan for Video solution

20. (d)

All my friends are not perfect

$\forall x\left[F(x) \rightarrow \neg P(x)\right]$

$\forall x\left[\neg F(x) \vee \neg P(x)\right]$

$(\neg)(\neg)\left[\forall x\left[\neg F(x) \vee \neg P(x)\right]\right]$

$\neg\exists x\left[F(x) \wedge P(x)\right]$

$A \rightarrow B \equiv \neg A \vee B$

$\neg\neg A \equiv A$

Scan for Video solution

21. (c)

(a) $\exists x[P(x) \wedge w]$

$\boxed{P(1) \wedge w}$

$\vee$

$\boxed{P(2) \wedge w}$

$(P(1) \wedge w) \vee (P(2) \wedge w)$

$(P(1) \vee P(2)) \wedge w$

$\exists x[P(x) \wedge w]$

$\boxed{\begin{matrix}P(1)\\ \vee\\ P(2)\end{matrix}} \wedge w\ 0$

(b) $\exists x[P(x) \rightarrow w] \equiv \forall x P(x) \rightarrow w$

$\exists x[\neg P(x) \vee w] \begin{pmatrix}P1\\ \wedge\\ P2\end{pmatrix} \rightarrow w$

$(\neg P(1) \vee w) \quad \equiv \quad (\neg P(1) \wedge P(2) \vee w)$

$\vee$

$(\neg P(2) \vee w) \quad \equiv \quad (\neg P1 \vee \neg P2 \vee w)$

$(\neg P(1) \vee w) \vee (\neg P(2) \vee w)$

$(\neg P(1) \vee \neg P(2)) \vee w$

(c) $\forall x[P(x) \rightarrow w] \equiv \forall x P(x) \rightarrow w \equiv \neg[\forall x P(x)\ ] \rightarrow w$

$\forall x[\neg P(x) \vee w] \equiv$

$(\neg P(1) \vee w) \quad\quad \neg[P(1) \wedge P(2)] \vee w)$

$\wedge$

$(\neg P(2) \vee w) \quad\quad (\neg P1 \vee \neg P2 \vee w)$

$(\neg P(1) \vee w) \wedge (\neg P(2) \vee w)$

$(\neg P(1) \wedge \neg P(2)) \vee w$

Scan for Video solution

22. (b)

IV. $\neg\exists x\left[\forall y \neg R(x, y)\right]$

$\left[\neg\exists x \neg\exists y\ R(x, y)\right]$

$(\neg)(\neg)\forall x\exists y R(x, y) \leq \forall x\exists y R(x, y)$

$$\boxed{\begin{array}{cc}\forall x\forall y & \equiv \forall y\forall x\\ \downarrow & \downarrow\\ \exists y\forall x & \exists x\forall y\\ \downarrow & \downarrow\\ \forall x\exists y & \forall y\exists x\\ \downarrow & \downarrow\\ \exists x\exists y & \equiv\ \exists y\exists x\end{array}}$$

$\exists x\ \exists y(x + y = 10)$

$x = 1 \quad y = 1$

$\forall x\ \exists y(x + y = 10)$

$x \quad\quad y$

$1 \to 9$
$2 \to 8$
$3 \to 7$

Scan for Video solution

23. (d)

(c) $\dfrac{\text{D: 1, 2}}{\exists x[P(x) \wedge q(x)]} \to \exists x(x) \wedge \exists x Q(x)$

$$\left\{\begin{array}{c} \boxed{P(1) \wedge Q(1)} \\ \vee \\ \boxed{P(\underset{T}{2}) \wedge Q(\underset{T}{2})} \end{array}\right. \quad \boxed{\begin{array}{c} P(1) \\ \vee \\ T \\ P(2) \end{array}} \wedge \boxed{\begin{array}{c} Q(2) \\ \vee \\ T \\ Q(2) \end{array}}$$

$T \qquad \dfrac{T \wedge T}{T}$

$P(2) = T$, $Q(2) = T$

$\forall x[P(x) \wedge Q(x)] \equiv \forall x P(x) \wedge \forall x Q(x)$

$\forall x[P(x) \vee Q(x)] \leftarrow \forall x P(x) \vee \forall x Q(x)$

$\exists x[P(x) \vee Q(x)] \equiv \exists x P(x) \vee \exists x Q(x)$

$\exists x[P(x) \wedge Q(x)] \to \exists x P(x) \vee \exists x Q(x)$

$\forall x[P(x) \to Q(x)] \to \forall x P(x) \to \forall x Q(x)$

$\forall x[P(x) \leftrightarrow Q(x)] \to \forall x P(x) \leftrightarrow \forall x Q(x)$

Scan for Video solution

24. (a, d)

$\neg\exists x[\forall y(\alpha) \wedge \forall z(\beta)]$

$\forall x[\neg\forall y(\alpha) \forall \neg\forall z(\beta)]$

$\forall x[\forall y(\alpha) \to \neg\forall z(\beta)]$

$\forall x\,[\,\forall y(\alpha) \to \exists z\,(\neg\beta)]$

$\forall x[\neg\forall z(\beta) \vee \neg\forall y(\alpha)]$

$\forall x[\forall z(\beta) \to \neg\forall y(\alpha)]$

$\forall x[\forall z(\beta) \to \exists y(\neg\alpha)]$

Scan for Video solution

25. (b)

I. $\neg\forall x(P)$

$\exists x\ \neg P(x)$

II. $\neg\exists x\ P(x)$

$\forall x \neg\ P(x)$

$\neg\exists x\ \neg P(x)$

$\forall x \neg\neg\ P(x)$

$\forall x P(x)$

Scan for Video solution

26. (d)

$A \to B \equiv \neg A \vee B$

$\forall x \alpha \to (\exists y, \beta \to (\forall u, \exists u, \gamma))$

$\neg\{\forall x \neg\alpha \vee (\exists y, \neg\beta \vee (\forall u, \exists u, \gamma))$

$\exists x \alpha \wedge (\forall y, \beta \wedge (\exists u, \forall u \neg\gamma))$

Scan for Video solution

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CHAPTER

2

SETS, RELATIONS AND FUNCTIONS

Sets, Relations and Functions

1. [MSQ] [GATE-2023: 2M]

Let $f : A \rightarrow B$ be an onto (or surjective) function, where A and B are nonempty sets. Define an equivalence relation ~ on the set A as

$a_1 \sim a_2$ if $f(a_1) = f(a_2)$,

Where $a_1, a_2 \in A$. Let $\varepsilon = \{|x| : x \in A\}$ be the set of all the equivalence classes under ~. Define a new mapping $F : \varepsilon \rightarrow B$ as

$F([x]) = f(x)$, for all the equivalence classes $[x]$ in ε.

Which of the following statements is/are TRUE?

(a) F is NOT well – defined.

(b) F is an onto (or surjective) function.

(c) F is a one-to-one (or injective) function.

(d) F is a bijective function.

2. [NAT] [GATE-2023: 1M]

Let U = {1, 2, 3}. Let 2^U denote the powerset of U. Consider an undirected graph G whose vertex set is 2^U. For any A.B $\in 2^U$, (A,B) is an edge in G if and only if (i) $A \neq B$, and (ii) either $A \subseteq B$ or $B \subseteq A$. For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by B(A).

If ϕ denotes the empty set, then the cardinality of B (ϕ) is ?

3. [MSQ] [GATE-2021 : 1M]

Consider the following sets, where $n \geq 2$:

S_1: Set of all $n \times n$ matrices with entries from the set {a, b, c}

S_2: Set of all functions from the set $\{0, 1, 2, \ldots, n^2 - 1\}$ to the set {0, 1, 2}

Which of the following choice(s) is/are correct?

(a) There does not exist a bijection from S_1 to S_2.

(b) There exists a surjection from S_1 to S_2.

(c) There exists a bijection from S_1 to S_2.

(d) There does not exist an injection from S_1 to S_2.

4. [NAT] [GATE-2021 : 2M]

Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B subsets of S, and $A \subseteq B$ is _____.

5. [NAT] [GATE-2020 : 1M]

Let R be the set of all binary relations on the set{1, 2, 3}. Suppose a relation is chosen from R at random. The probability that the chosen relation is reflexive (round off to 3 decimal places) is ________

6. [MCQ] [GATE-2016 : 2M]

A binary relation R on $\mathbb{N} \times \mathbb{N}$ is defined as follows: (a, b) R (c, d) if $a \leq c$ or $b \leq d$. Consider the following propositions:

P: R is reflexive

Q: R is transitive

Which one of the following statements is TRUE?

(a) Both P and Q are true

(b) P is true and Q is false

(c) P is false and Q is true

(d) Both P and Q are false

7. [NAT] [GATE-2016 : 1M]

A function f: $\mathbb{N}^+ \to \mathbb{N}^+$, defined on the set of positive integers $\mathbb{N}^+$, satisfies the following properties:

$f(n) = f(n/2)$ if n is even

$f(n) = f(n+5)$ if n is odd

Let $R = \{i \mid \exists j: f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is _______ .

8. [MCQ] [GATE-2015 : 2M]

Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p - s = q - r$. Which one of the following is true about R?

(a) Both reflexive and symmetric
(b) Reflexive but not symmetric
(c) Not reflexive but symmetric
(d) Neither reflexive nor symmetric

9. [NAT] [GATE-2015 : 2M]

Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X and Y. Let f be randomly chosen from F. The probability of f being one-to-one is ____________.

10. [NAT] [GATE-2015 : 2M]

The number of onto functions (surjective functions) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is _____.

11. [NAT] [GATE-2015 : 1M]

The cardinality of the power set of {0, 1, 2 …, 10} is ________.

12. [MCQ] [GATE-2015 : 1M]

Let R be the relation on the set of positive integers such that aRb if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is True?

(a) R is symmetric and reflexive but not transitive
(b) R is reflexive but not symmetric and not transitive
(c) R is transitive but not reflexive and not symmetric
(d) R is symmetric but not reflexive and not transitive

13. [MCQ] [GATE-2015 : 1M]

For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True?

1. $\emptyset \in 2^A$
2. $\emptyset \subseteq 2^A$
3. $\{5, \{6\}\} \in 2^A$
4. $\{5, \{6\}\} \subseteq 2^A$

(a) 1 and 3 only
(b) 2 and 3 only
(c) 1, 2 and 3 only
(d) 1, 2 and 4 only

14. [MCQ] [GATE-2015 : 1M]

If $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$ then, $\frac{g(h(x))}{h(g(x))}$ is.

(a) $\frac{h(x)}{g(x)}$
(b) $\frac{-1}{x}$
(c) $\frac{g(x)}{h(x)}$
(d) $\frac{x}{(1-x)^2}$

15. [NAT] [GATE-2014 : 2M]

Let S denote the set of all functions $f : \{0,1\}^4 \to$ 0, 1}. Denote by N the number of functions from S to the set {0, 1}. The value of $\log_2 \log_2 N$ is ________.

16. [MCQ] [GATE-2012 : 2M]

How many onto (or surjective) functions are there from an n-element ($n \geq 2$) set to a 2-element set?

(a) 2^n
(b) $2^n - 1$
(c) $2^n - 2$
(d) $2(2^n - 2)$

17. [MCQ] [GATE-2010 : 1M]

What is the possible number of reflexive relations on a set of 5 elements?

(a) 2^{10}
(b) 2^{15}
(c) 2^{20}
(d) 2^{25}

18. [MCQ] [GATE-2009 : 1M]

Consider the binary relation: R = {(x, y), (x, z), (z, x), (z, y)} on the set {x, y, z}. Which one of the following is TRUE?

(a) R is symmetric but NOT antisymmetric
(b) R is NOT symmetric but antisymmetric
(c) R is both symmetric and antisymmetric
(d) R is neither symmetric nor antisymmetric

19. [MCQ] [GATE-2008 : 1M]

If P, Q, R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P^c \cap Q \cap R) \cup Q^c \cup R^c$ is

(a) $Q^c \cup R^c$ (b) $P \cup Q^c \cup R^c$
(c) $P^c \cup Q^c \cup R^c$ (d) U

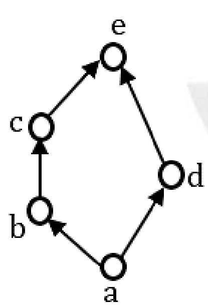
Lattice

20. [NAT] [GATE-2017 : 1M]

Consider the set X = {a, b, c, d, e} under the partial ordering:

R = {(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, c), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)}

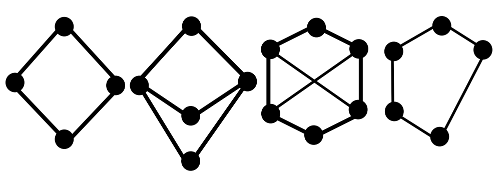
The Hasse diagram of the partial order (X, R) is shown below:



The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is _______.

21. [MCQ] [GATE-2008 : 1M]

Consider the following Hasse diagrams.



(i) (ii) (iii) (iv)

Which all of the above represent a lattice?

(a) (i) and (iv) only (b) (ii) and (iii) only
(c) (iii) only (d) (i), (ii) and (iv) only

Groups

22. [MSQ] [GATE-2023: 2M]

Let X be a set and 2^X denote the powerset of X.

Define a binary operation Δ on 2^X as follows:
$A \Delta B = (A - B) \cup (B - A)$.

Let H = (2^X, Δ). Which of the following statements about H is/are correct?

(a) H is a group.
(b) Every element in H has an inverse, but H is NOT a group.
(c) For every $A \in 2^X$, the inverse of A is the complement of A.
(d) For every $A \in 2^X$, the inverse of A is A.

23. [MSQ] [GATE-2022 : 1M]

Which of the following statements is/are TRUE for a group G?

(a) If for all x, y ∈ G, (x y) 2 = x2 y2, then G is commutative.
(b) If for all x ∈G, x2 = 1, then G is commutative. Here, 1 is the identity element of G.
(c) If the order of G is 2, then G is commutative.
(d) If G is commutative, then a subgroup of G need not be commutative.

24. [MCQ] [GATE-2021 : 2M]

Let G be a group of order 6, and H be a subgroup of G such that 1<|H|<6. Which one of the following options is correct?

(a) Both G and H are always cyclic.
(b) G may not be cyclic, but H is always cyclic.
(c) G is always cyclic, but H may not be cyclic.
(d) Both G and H may not be cyclic.

25. [NAT] [GATE-2020 : 1M]

Let G be a group of 35 elements. Then the largest possible size of a subgroup of G other than G itself is ______.

26. [MCQ] [GATE-2018 : 1M]

Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is _______.

27. [NAT] [GATE-2014 : 2M]

Let G be a group with 15 elements. Let L be a subgroup of G. It is known that $L \neq G$ and that the size of L is at least 4. The size of L is _______.

28. [MCQ] [GATE-2013 : 1M]

A binary operation $\oplus$ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which of the following statements if TRUE about $\oplus$?

(a) Commutative but not associative
(b) Both commutative and associative
(c) Associative but not commutative
(d) Neither commutative nor associative

29. [MCQ] [GATE-2010 : 1M]

Consider the set $S = \{1, \omega, \omega^2\}$, where ω and ω^2 are cube roots of unity. If * denotes the multiplication operation, the structure {S, *} forms

(a) a group (b) a ring
(c) an integral domain (d) a field

30. [MCQ] [GATE-2009 : 2M]

For the composition table of a cyclic group shown below:

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Which one of the following choices is correct?

(a) a, b are generators (b) b, c are generators
(c) c, d are generators (d) d, a are generators

❑❑❑

ANSWER KEY

1.	(b, c, d)	**2.**	(5040)	**3.**	(b, c)	**4.**	(59049 to 59049)
5.	(0.125 to 0.125)	**6.**	(b)	**7.**	(2 to 2)	**8.**	(c)
9.	(0.95 to 0.95)	**10.**	(36 to 36)	**11.**	(2048 to 2048)	**12.**	(b)
13.	(c)	**14.**	(a)	**15.**	(16)	**16.**	(c)
17.	(c)	**18.**	(d)	**19.**	(d)	**20.**	(0 to 0)
21.	(a)	**22.**	(a, b)	**23.**	(a, b, c)	**24.**	(b)
25.	(7 to 7)	**26.**	(42 to 42)	**27.**	(5 to 5)	**28.**	(a)
29.	(a)	**30.**	(c)				

SOLUTIONS

1. (b, c, d)

Because each class of x that follows an equivalence relation is uniquely mapped to one element of x in a function, Option A is false.

f. Every function has a clear definition.

If no two elements of x are mapped to any element of y, the function is said to be one-one.[X], [Y] are distinct equivalence, and as a result, they adhere to the one-to-one property.

A function is onto if it has a relationship between each element of domain E and an element of co-domain B.

F is a bijective function if it is one-one and onto.

Therefore opions (b), (c), and (d) are correct choices.

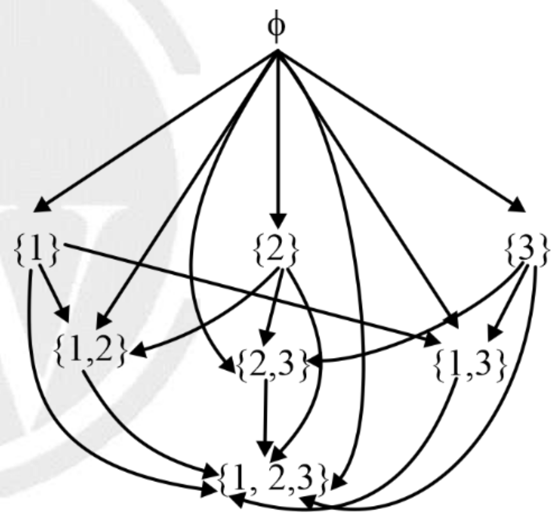
Scan for Video solution

2. (5040)

The powerset of U is denoted by 2^U.

V= { ϕ,1,2,3 (1,2),(1,3),(2,3),(1,2,3)}

A, B$\in 2^U$



A, B is an edge in G if and only if

(i) A$\neq$ B

(ii) Either A $\subseteq$ B or B $\subseteq$ A

As ϕ connects to all the node in the graph Cardinality of B(ϕ) = 7! = 5040

Scan for Video solution

3. **(b, c)**

a/ b/ c

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{n\times n}$$

$\underset{3}{\downarrow}\ \underset{3}{\downarrow}\ \downarrow .....n^2$

3^{n^2}

Scan for Video solution

4. **(59049 to 59049)**

S = {1,2,….10}

(A, B) $A \subseteq B$ $A \subseteq B$

$\{1\} \subseteq \{1, 2\}$ $\{1\} \subseteq \{1, 2\}$

$\phi \subseteq \{1, 2\}$ $\phi \subseteq \{1, 2\}$

$\{2\} \subseteq \{1, 2\}$

Scan for Video solution

5. **(0.125 to 0.125)**

n ×n

non-diagonal = $n^2 - n$

Reflexive = $2n^2 - n$

A = {1,2,3} = n

$A \times B = n^2$

Total number of relation $= 2^{n^2} = 2^9$

Nondiagonal = $n^2 - n$

Reflexive = 2^{n^2-n}

n = 3

$$\text{Probity} = \frac{2^{n^2}-1}{2^{n^2}} = \frac{2^{3^2-3}}{2^{3^2}} = \frac{2^6}{2^9} = \frac{1}{2^3} = \frac{1}{8} = 0.125$$

Scan for Video solution

6. **(b)**

$\overset{23}{ab}\ R\ \overset{31}{cd}$ $\wedge$ $\overset{31}{cd}\ R\ \overset{11}{xy}$ $ab\ R\ xy$

$a \le c(\tau)$ $\wedge$ $c \le n(f)$ $c \le n(f)$

$\vee$ $\wedge$ $\vee$ $\vee$ False

$b \le d(f)$ $\wedge$ $c \le n(f)$ $c \le n(f)$

$2 \le 3(\tau)$ $\wedge$ $3 \le 1$ $2 \le 1$

$\vee$ $\wedge$ $\vee$ $\vee$

$b \le 1$ $1 \le 1$ $3 \le 1$

Scan for Video solution

7. **(2 to 2)**

n =4

f(1) = a n = 5

f(5) =b f(5) = f(5+5) =fc(10) = f(10/2) =f(5) =b

n =6

n = 4

f(7) =f(7+5) =f(12) f(12)=f(12/12) = f(16) =f(6/2)

=f(3)

n = 2	n = 3
f(2) =f(2/2)=f(1)	f(3) = f(3 + 5)
	= f(18)
	= f(8/2) = f(4)
	= f(4/2) = f(2) = f(2/2) = f(1) = a

Scan for Video solution

8. **(c)**

Relexive:

a R a

$\left(\underset{1}{p}\ ,\ \underset{2}{q}\right) R \left(\underset{3}{p}\ ,\ \underset{4}{q}\right)$

1 – 4 = 2–3

p – q = q – p

$p - q \ne (p - q)$

aRb $\rightarrow$ bRa

(p, q) R (R, S) $\rightarrow$ (R, S) R (p, q)
1 2 3 4 1 2 3 4
1–4 = 2–3 1–4 = 2–3
p– s q–R R–q S–P
–(q–R) = –(P–S)
q–R P–S

Scan for Video solution

9. (0.95 to 0.95)

F: x$\rightarrow$y

|x| =2 |y| = 20

Total number of function = $(R.S)^{LS}$

$= 20^2$

1:1 Functions = RS P_{LS} = 20P_2 = 20.19

$= \frac{20.19}{20.20} = \frac{19}{20} = 0.95$

$\frac{A \cap S}{\text{total}} = \frac{1.1}{\text{Total}}$

Scan for Video solution

10. (36 to 36)

F: x$\rightarrow$y

|x| = 4 |y | =3

|x| = m |y| =m

$\sum_{i=0}^{n} (-1) i \times n_{c_j} (n-i)^m$

$3_{c_0}(3-0)^4 - 3_{c_1}(3-1)^4 + 3_{c_2}(3-2)^4 - 3_{c_3}(3-3)^4$

$= 3^4 - 3\times2^4 + 3 \times 1^4 - 0 = 81 - 3\times16 + 3$

$= 84 - 48 = 36$

Scan for Video solution

11. (2048 to 2048)

A : {0…..10}

|A| = 11

P (A) $=2^A = 2^{11} = 2048$

Scan for Video solution

12. (b)

(1) Not reflexive

(2) (3,6) $\in$R$\rightarrow$(6, 3) $\in$R

(3) (2,6) $\wedge$ (6, 9) $\rightarrow$ (2, 9)

$\downarrow$ $\downarrow$

2 3 acd(2, 9) = 1

Scan for Video solution

13. (c)

A = {5} ,{6} {7}

$2^A = \left\{ \overset{e1}{\phi}, \overset{e2}{\{5\}}, \overset{e3}{\{6\}}, \overset{e4}{\{7\}} \right\} \left\{ 5, \overset{e5}{\{6\}} \right\}$

$e5 \in 2^A$

$\{5, \{6\}\} \in 2^A$

Scan for Video solution

14. (a)

$I(x) = 1 - x \quad h(x) = \frac{x}{x-1}$

$\frac{g(h(x)}{h(g(x)} = \frac{1}{x-1} \times - \frac{-x}{1-x}$

g(h(x) h(g(x) $\frac{1}{x-1} \times \frac{x}{1-x}$

$= 1 - \frac{x}{x-1} \quad = \frac{1-x}{1-x-1} \quad = \frac{x}{x-1} \times \frac{1}{1-x} = \frac{h(x)}{g(x)}$

Scan for Video solution

15. **(16)**

$S: \{0, 1\}^4 \to \{0, 1\}$

$S: 2^4 \to 2$

Total function = $(R.S.)^{L.S}$

$= 2^{2^4} = 2^{16}$

$N : S \to \{0, 1\}$

$N : 2^{16} \to 2$

Total function = $(R.S.)^{L.S}$

$= (2)^{2^{16}}$

$109_2 \quad 109_2 \quad N$

$109_2 \quad 109_2^{2^{2^{16}}}$

$109_2^{2^{2^{16}}} = 16$

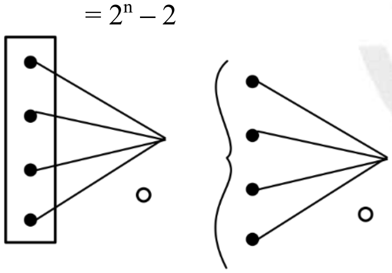
Scan for Video solution

16. **(c)**

F: A→B

Anto = Total function – Total non-onto

$= 2^n - 2$



Scan for Video solution

17. **(c)**

n = 3

nondiagonal $= \mathbf{n^2 - n}$

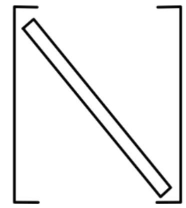
Q: n = 5

$2^{n^2-n} = 2^{5^2-5}$

$= 2^{25-5}$

$= 2^{20}$

n =3



2^{n^2-n}

Scan for Video solution

18. **(d)**

$\{(x,y)(x,z)(z,x)(z,y)\}$

$\forall a \forall b((a,b) \in R \to (b,a) \in R)$

Anti: $\forall a \forall b((a,b) \in R \wedge (b,a) \in R \to a = b)$

$(x^2) \in R \wedge (z,x) \in R \to x = z(\text{False})$

Scan for Video solution

19. **(d)**

$(P \cap Q \cap R) \cup (\overline{P} \cap Q \cap R) \cup (\overline{Q} \cup \overline{R})$

$(P \cup \overline{P}) \cup (Q \cap R) \cup \neg(Q \cap R)$

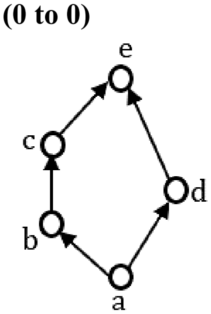
$\cup \cap (Q \cap R)$

$(Q \cap R) \cup (Q \cap R) = \cup$

$A \cup .\overline{A} \equiv \cup$

Scan for Video solution

20. **(0 to 0)**



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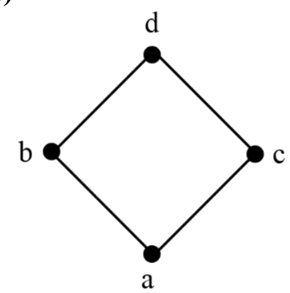
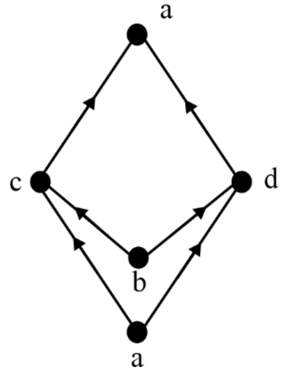
21. (a)



a|b (b, c) = a
|ub| (b, c) = d



g|b (c, d) = N.A.
|b of (c, d) = {ab}
ab ≤ a | ab ≤ b

Scan for Video solution

22. (a, d)

Let x be a set and 2^x is a power set of x

$A\Delta B = (A — B) \cup (B — A)$ for $A, B \in 2^x$

$H = (2^x, \Delta)$

Check H for each property.

(i) H satisfies the closure property under Δ

(ii) H satisfies the associative property under a such that Δ

$A\Delta (B\Delta C) = (A\Delta B) \Delta C$

(iii) H satisfies the identity property such that 'ϕ', is the identity element. $A\Delta e = \phi$

(iv) H satisfies inverse property such that inverse of A = A

$A\Delta\phi = \phi = (A - \phi) \text{ u } (\phi - A)$

$= A \cup \phi$

$= A$

So, H is group and inverse of A is A.

Scan for Video solution

23. (a,b,c)

$x^2 = 1$

$x \cdot x = 1$

$x^{-1}.x.x = x^{-1}$ $\qquad x = x^{-1}$

$(xy)^2 = x^2 y^2$

$xy \cdot xy = xy\, yy$

$x^{-1} \cdot y \cdot x \cdot y . y^{-1} \cdot xy \cdot y^{-1}$

$yx = xy$

Scan for Video solution

24. (b)

$1 < H < 6$

↓

2 , 3

↓

Prime

Scan for Video solution

25. (7 to 7)

$H \subseteq G \ \dfrac{G}{H}$

35

1, 5, 7, $\cancel{35}$

Scan for Video solution

26. **(42 to 42)**

$G \neq H$

$H \subseteq G \rightarrow \frac{G}{H}$ 84.

$\downarrow$

$1, \ldots\ldots 42, \cancel{84}$

Scan for Video solution

27. **(5 to 5)**

$|G| = 15$

L subgroup G

$\downarrow$ $\times$

1 3, 5, 15

$\times$ $\times$ $\uparrow$

Scan for Video solution

28. **(a)**

Assaciabve : $a \times (b \times c) = (a \times b) \times c$

$x \oplus (y \oplus 2) = (x \oplus y) \oplus 2$

$x \oplus (y^2 + z^2) =$

$x^2 + (y^2 + z^2) = (x^2 + y^2) \oplus z.$

$= \left(x^2 + y^2\right)^2 + z^2$

Commutable: $a \# b$ $b \times a$

$x \oplus y$ $y \oplus x$

$x^2 + y^2$ $y^2 + x^2$

Scan for Video solution

29. **(a)**

$$\begin{array}{c|ccc} \times & \boxed{1 \quad w \quad w^2} \\ 1 & \boxed{1 \quad w \quad w^2} \\ w & w & w^2 & 1 \\ w^2 & w^2 & 1 & w \end{array}$$

$S = \{! \; w \cdot w^2\}$ $= (1 \cdot w) \cdot w^2$

$1 \cdot w^3$ $= w \cdot w^2$

$1 \cdot 1$ $= w^3$

1 $= 1$

Scan for Video solution

30. **(c)**

$b = b$

$b \times b \equiv a$

$b \times b^2 = b \times a \equiv b$

$c^1 = c$

$c^2 = c \times c = b$

$c = c^2 \times c$

$= b \times c \equiv d$

$c^4 = c \times 4$

$\equiv a$

$c^5 = c^4 \times c$

$= a \times c \equiv c$

$<c> = \{c, b, d, a\}$

$c \times d = a$

Scan for Video solution

❑❑❑

CHAPTER

3 GRAPH THEORY

Degree Sequence

1. **[NAT]** **[GATE-2017 : 2M]**

G is an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is _____.

2. **[NAT]** **[GATE-2017 : 1M]**

Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is ______.

3. **[MCQ]** **[GATE-2014 : 2M]**

An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$, is called graphic if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, \ldots, d_n$ respectively. Which of the following 6-tuples is NOT graphic?

(a) (1, 1, 1, 1, 1, 1)
(b) (2, 2, 2, 2, 2, 2)
(c) (3, 3, 3, 1, 0, 0)
(d) (3, 2, 1, 1, 1, 0)

4. **[MCQ]** **[GATE-2010 : 2M]**

The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1
II. 6, 6, 6, 6, 3, 3, 2, 2
III 7, 6, 6, 4, 4, 3, 2, 2
IV 8, 7, 7, 6, 4, 2, 1, 1

(a) I and II
(b) III and IV
(c) IV only
(d) II and IV

5. **[MCQ]** **[GATE-2010 : 1M]**

Let, G =(V, E) be a graph. Define (G) = $\Sigma_d i_d \times d$, where whre i_d is the number of vertices of degree d in G. If S and T are two different tree with (S) = (T) then.

(a) |S| = 2 |T|
(b) |S| = |T|–1
(c) |S| = |T|
(d) |S| = |T| +1

Types of Graphs

6. **[MCQ]** **[GATE-2022 : 2M]**

Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?

(a) $\frac{1}{8}$
(b) 1
(c) 7
(d) 8

7. **[NAT]** **[GATE-2015 : 2M]**

Let G be connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ________.

8. **[NAT]** **[GATE-2015 : 1M]**

Consider a binary tree T that has 200 leaf nodes. Then, the number of nodes in T that have exactly two children are _______.

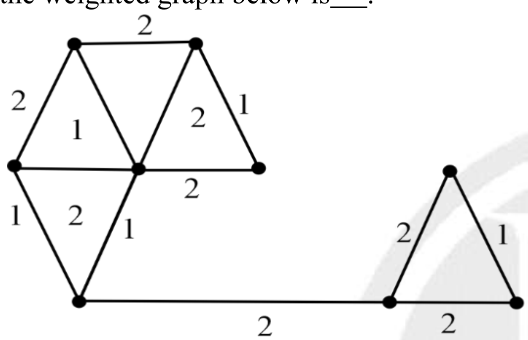
9. [MCQ] [GATE-2015 : 2M]

A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is

(a) A multiple of 4
(b) Even
(c) Odd
(d) Congruent to 0 mod 4, or 1 mod 4

10. [NAT] [GATE-2014 : 2M]

The number of distinct minimum spanning trees for the weighted graph below is___.



11. [NAT] [GATE-2014 : 2M]

A cycle on n vertices is isomorphic to its complement.
The value of n is_______.

12. [NAT] [GATE-2014 : 1M]

The maximum number of edges in a bipartite graph on 12 vertices is_____

13. [MCQ] [GATE-2013 : 1M]

Which of the following statements is/are TRUE for undirected graph?

P : Number of odd degree vertices is even.
Q: Sum of degrees of all vertices is even.

(a) P only
(b) Q only
(c) Both P and Q
(d) Neither P nor Q

14. [NAT] [GATE-2014 : 1M]

Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i, j): 1 \le i \le 12, 1 \le j \le 12)$. There is an edge between (a, b) and (c, d) if $|a - c| \le 1$ and $|b - d| \le 1$.
The number of edges in this graph is_______

15. [MCQ] [GATE-2012 : 2M]

Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to

(a) 15
(b) 30
(c) 90
(d) 360

Connectivity

16. [NAT] [GATE-2022 : 2M]

Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is _____.

17. [MCQ] [GATE-2022 : 2M]

Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of

(a) A^3
(b) A^3 divided by 2
(c) A^3 divided by 3
(d) A^3 divided by 6

18. [MCQ] [GATE-2022 : 2M]

Which of the properties hold for the adjacency matrix A of a simple undirected unwieghted graph having n vertices?

(a) The diagonal entries of A^2 are the degrees of the vertices of the graph.
(b) If the graph is connected, then none of the entries of $A^{n-1} + I_n$ can be zero.
(c) If the sum of all the elements of A is at most 2(n-1), then the graph must be acyclic.
(d) If there is at least a 1 in each of A's rows and columns, then the graph must be connected.

19. [NAT] [GATE-2022 : 2M]

Let G(V,E) be a directed graph, where V={1,2,3,4,5} is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A. $A[i][j] = \begin{cases} 1, & 1 \le j \le i \le 5 \\ 0, & \text{otherwise} \end{cases}$

$A[i][j] = 1$ indicates a directed edge from node i to node j. A directed spanning tree of G, rooted at r $\in$ V, is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V. The number of such directed spanning trees rooted at vertex 5 is ____.

20. [NAT] [GATE-2018 : 2M]

Let G be a graph with 100! vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, … 100. There is an edge between vertices u and υ if and only if the label of u can be obtained by swapping two adjacent numbers in the label of υ. Let y denote the degree of a vertex in G and z denote the number of connected components in G.

Then, y + 10z = _______.

21. [MCQ] [GATE-2014 : 2M]

If G is a forest with n-vertices and k connected components, how many edges does G have?

(a) $\lfloor n/k \rfloor$ (b) $\lceil n/k \rceil$

(c) n –1 (d) n– k +1

22. [MCQ] [GATE-2014 : 1M]

Let G = (V, E) be a directed graph where V is the set of vertices and E the set of edges. Then which one of the following graphs has the same strongly connected components as G?

(a) $G_1 = (V, E_1)$ where $E_1 = \{(u, v)|(u, v) \notin E\}$.

(b) $G_2 = (V, E_2)$ where $E_2 = \{(u, v)|(v, u) \in E\}$.

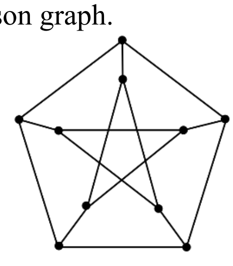
(c) $G_3 = (V, E_3)$ where $E_3 = \{(u, v)|$ there is a path of length ≤ 2 from u to v in E}.

(d) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated.

Graph Number

23. [MSQ] [GATE-2022 : 2M]

The following simple undirected graph is referred to as the Peterson graph.



Which of the following statements is/are TRUE?

(a) The chromatic number of the graph is 3.

(b) The graph has a Hamiltonian path.

(c) The following graph is isomorphic to the Peterson graph.

(d) The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

24. [MCQ] [GATE-2022 : 2M]

What is the size of the smallest MIS Maximal Independent set of a chain of nine nodes?

(a) 5 (b) 4

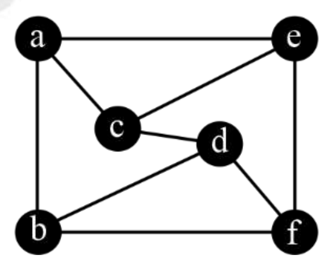
(c) 3 (d) 2

25. [NAT] [GATE-2020 : 2M]

Graph G is obtained by adding vertex s to $K_{3,4}$ and making s adjacent to every vertex of $K_{3,4}$. The minimum number of colours required to edge-colour G is _________.

26. [NAT] [GATE-2018 : 1M]

The chromatic number of the following graph is_____.



27. [MCQ] [GATE-2009 : 1M]

What is the chromatic numebr of an n-vertex simaple connected graph which not contain any length cycle? Assume n ≥ 2.

(a) 2 (b) 3

(c) n –1 (d) n

Planarity

28. [MSQ] [GATE-2023: 2M]

Let G be a simple, finite, undirected graph with vertex set $\{v_1,...,v_n\}$. Let $\Delta(G)$ denote the maximum degree of G and let $\mathbb{N} = \{1, 2,...\}$ denote the set of all possible colors. Color the vertices of G using the following greedy strategy:

for $i = 1,...,n$

color(v_i) $\leftarrow$ min{$j \in \mathbb{N}$: no neighbour of v_i is colored j}

Which of the following statements is/are TRUE?

(a) This procedure results in a proper vertex coloring of G.
(b) The number of colors used in at most Δ (G) + 1.
(c) The number of colors used is at most Δ (G).
(d) The number of colors used is equal to the chromatic number of G.

29. [NAT] [GATE-2021 : 1M]

In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is ________.

30. [NAT] [GATE-2015 : 2M]

Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is _______.

31. [MCQ] [GATE-2014 : 2M]

Let δ denote the minimum degree of a vertex in a graph. For all palner graphs on n vertices with $\delta \geq 3$ which one of the following is TRUE?

(a) In any planner embedding, the number of faces is at least $\frac{n}{2}+2$
(b) In any planner embedding, the number of faces is less than $\frac{n}{2}+2$
(c) There is a planner embedding in which the number of faces is less than $\frac{n}{2}+2$
(d) There is a planner embedding in which the number of faces is at most $\frac{n}{\delta+1}$

32. [MCQ] [GATE-2012 : 1M]

Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

(a) 3 (b) 4
(c) 5 (d) 6

❑❑❑

ANSWER KEY

1.	(16 to 16)	**2.**	(18 to 18)	**3.**	(c)	**4.**	(d)
5.	(c)	**6.**	(c)	**7.**	(995 to 995)	**8.**	(199 to 199)
9.	(d)	**10.**	(6 to 6)	**11.**	(5 to 5)	**12.**	(36 to 36)
13.	(c)	**14.**	(506 to 506)	**15.**	(45)	**16.**	(36 to 36)
17.	(d)	**18.**	(a)	**19.**	(24 to 24)	**20.**	(109 to 109)
21.	(c)	**22.**	(b)	**23.**	(a, b, c)	**24.**	(3)
25.	(7 to 7)	**26.**	(3 to 3)	**27.**	(a)	**28.**	(a, b)
29.	(11 to 11)	**30.**	(24 to 24)	**31.**	(a)	**32.**	(d)

SOLUTIONS

1. (16 to 16)

e = 25

Minimum degree = 3

δ (G) = 3

$$\delta(G) < \frac{2e}{n} \le \Delta(G) \le n-1$$

n =16

$$\delta(G) \le \frac{2e}{n}$$

$$3 \le \frac{2.25}{n}$$

$$n \le \frac{2.25}{3}$$

$n \le 16.6..$

↓

$(\ldots\ldots 16,16) \le 16.66$

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2. (18 to 18)

Tree

n =10

e (Tree) = n –1

n = 10–1 = 9

$\sum d(vi) = 2e$

$= 2.9 = 18$

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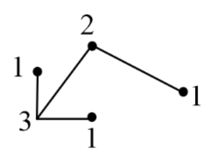
3. (c)

Not graphical

$\cancel{3}$ 3 3 1 0 0

2 2 0 0 0



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4. (d)

Graphical

$\cancel{7}$ 6 5 4 4 3 2 1

$\cancel{5}$ 4 3 3 2 1 0

$\cancel{3}$ 2 2 1 0 0

1 1 0 0 0

$\cancel{6}$ 6 6 6 3 3 2 2

$\cancel{5}$ 5 5 2 2 1 2

$\cancel{5}$ 5 5 2 2 2

$\cancel{4}$ 4 1 1 1 1

3 0 0 0 1

31000 (not graphical)

$\cancel{7}$ 6 6 4 4 3 2 2

$\cancel{5}$ 5 3 3 2 1 1

4 2 2 1 0 1

$\cancel{4}$ 2 2 1 1 0

1 1 0 0 0

Graphical

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5. (c)

$$\begin{array}{cc} S & T \\ \downarrow & \downarrow \\ n-1 & n-1 \end{array}$$

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6. (c)

$$8_{C_3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$\frac{5.76}{32.1} \times \frac{1}{8} = 7$$

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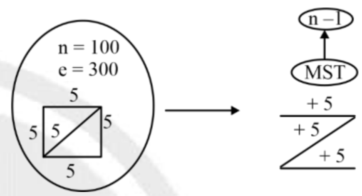
7. (995 to 995)

n = 100

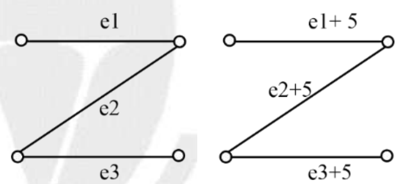
e = 100



= 500 + 5× (n –1)

= 500 + 5 ×99

= 995



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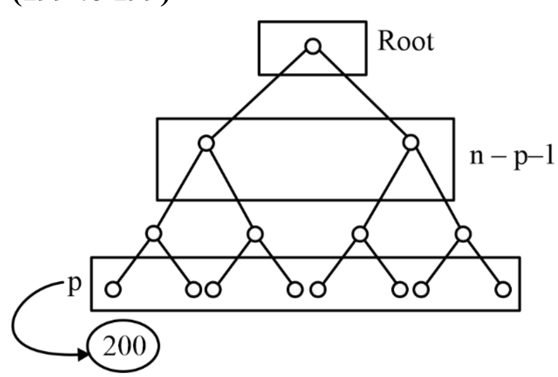
8. (199 to 199)



$\Sigma d(iv) = 22$

$2 + P + 3\,(n - P - 1) = 2e = 2(n - 1)$

$2 + 200 + 3(n - 200 - 1) = 2(n - 1) = 2n - 2$

$2 + 200 + 3n - 600 - 3 = 2n - 2$

$3n - 2n = 600 - 200 \;\; - 2 + 3 - 2$

$n = 400 - = 399$

$= 399 - 200$

$= 199$

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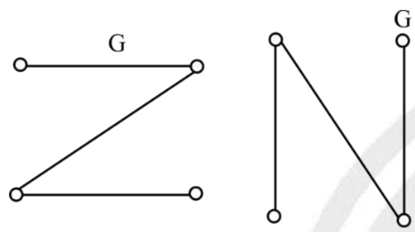
9. (d)



$\dfrac{n-0}{4}$ *or* $\dfrac{n-1}{4}$

$\dfrac{a-b}{n} \in z$

$a \equiv b(\text{mod}n)$

$n \equiv (\text{mode}4)$ or $n \equiv 1(\text{mode})$

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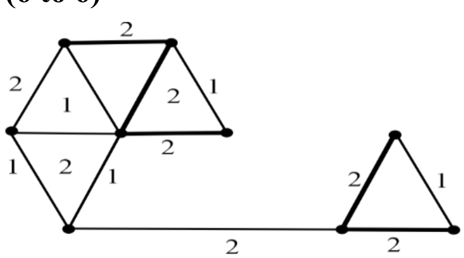
10. (6 to 6)



$3 \times 2 = 6$ ways.

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11. (5 to 5)

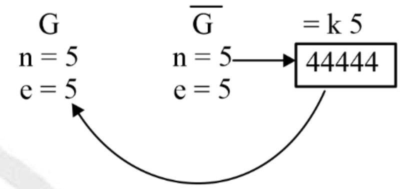
$$e(G) + e(\overline{G}) = \frac{n(n-1)}{2}$$

$$e(G) + e(\overline{G}) = \frac{n(n-1)}{2}$$

$$e = \frac{n(n-1)}{4}$$

$n = 4 \quad e = \dfrac{4.3}{4} = 3$

$n = 4 \quad e = \dfrac{5.3}{4} = 5$



$n = 6 \quad e = \dfrac{6.5}{4}$ X

$n = 7 \quad e = \dfrac{7.6}{4}$ X

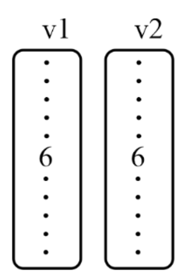
$n = 8 \quad e = \dfrac{8.7}{4} = 14$

C 3
C 4
C 5
C 6
C 7
C 8 e = 8

Scan for Video solution

12. (36 to 36)

$n = 12$



K6, 6 $\quad e = 6 \times 6 = 36$

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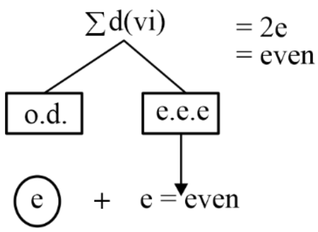
13. (c)



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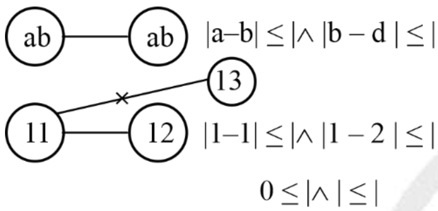
14. (506 to 506)



$\Sigma d(vi) = 2e$

$4 \times 3 + 40 \times 5 + 100 \times 8 = 2e$

$12 + 200 + 800 = 2e$

$6 + 100 + 400 = e$

$e = 506$

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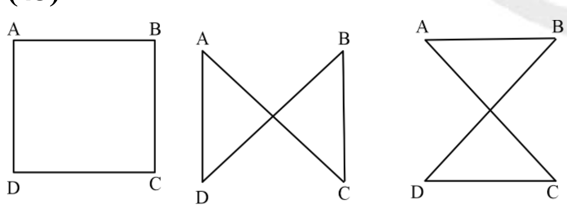
15. (45)



3×2 × 1 = 3!

$6_{C_3} = 15 \text{ways}$

$$15 \times \frac{(n-1)!}{2} = 15 \times \frac{6}{2} = 15 \times 3 = 45$$

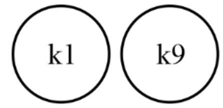
Scan for Video solution

16. (36 to 36)

$n = 10$

$$e = \frac{(n-k)(n-k+1)}{2}$$

$k = 2$



$\frac{9 \times 8}{2} = 36$

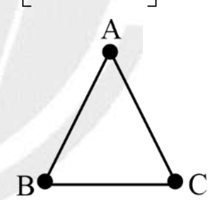
$$\begin{cases} k = 2 \\ k = 5 \\ k = 4 \end{cases}$$

Scan for Video solution

17. (d)

$$A = \begin{array}{c} a \\ b \\ c \end{array}\begin{bmatrix} a & b & c \\ & & \\ & & \end{bmatrix} \qquad A^2 = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$A^3 = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$



$$\begin{cases} A\text{–}C\text{–}B\text{–}A \\ A\text{–}B\text{–}C\text{–}A \end{cases}$$

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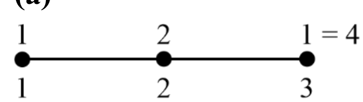
18. (a)



$$\begin{array}{c} \\ A = \end{array} \begin{array}{ccc} 1 & 2 & 3 \end{array}$$

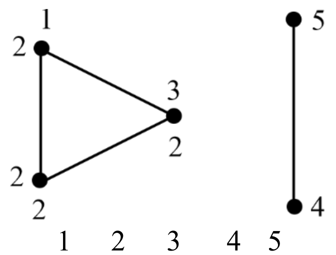
$$A = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^2 = \begin{bmatrix} & & \\ 1 & 0 & 1 \\ & & \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$A^2 = 2\begin{bmatrix} & 2 & \\ & \downarrow & \\ & 2 & \\ & & 1 \end{bmatrix}$$

(n =5)

$\Rightarrow 2\,(n-1)$

$= 2(4) = 8$



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & 0 & 0 & 0 & 0 \\ 3 & & & & \\ 4 & & & & \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & & & & & \\ 2 & & & & & 0 \\ 3 & & & & & 0 \\ 4 & & & & & 1 \\ 5 & & & & 1 & 0 \end{array}$$

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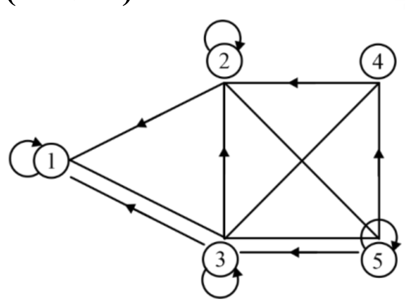
19. (24 to 24)



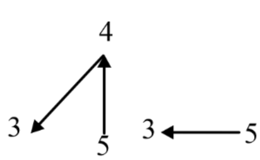
St

$5 \rightarrow 4$ (1 ways)

$5 \rightarrow 3$ (2 ways)

$5 \rightarrow 2$ (3 ways)

$5 \rightarrow 1$ (4 ways)



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20. (109 to 109)

$99 + 10.1 = 109$

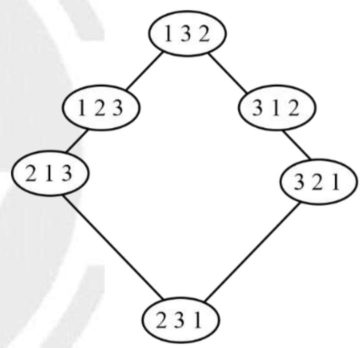
$y = 99$

$z = 1$

$n = 3$

Total vertices = 3! = 6

$$\left\{\begin{matrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{matrix}\right. \quad 104 \left\{ \vphantom{\begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix}} \right.$$



Degree Fetch event = 99

$\boxed{1, 2, \ldots\ldots\ldots.100}$

Scan for Video solution

21. (c)

Total vertices = n   k = 4

$e = n_{1-1}$	$e = n_{2-1}$	$e = n_{3-1}$	$e = n_{4-1}$
Tree n_1	Tree n_2	Tree	Tree

$E \;= e_1 + e_2 + e_3 + e_4$

$\quad = n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1$

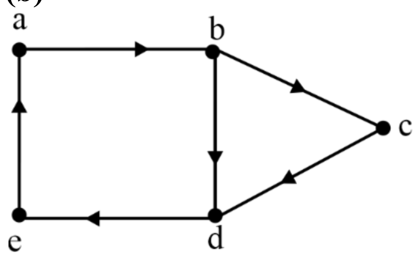
$= (n_1 + n_2 + n_3 + n_4) - 4$

$= n - 4$

$= n - k$

Scan for Video solution

22. (b)



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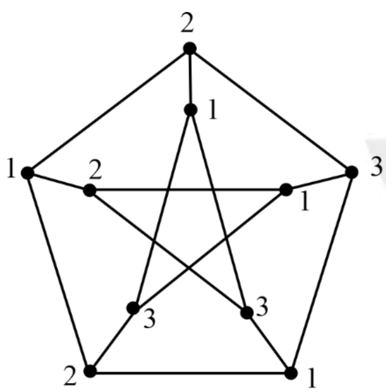
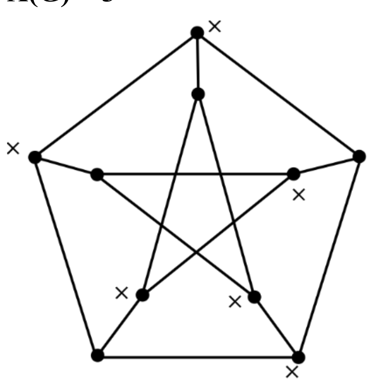
23. (a, b, c)



X(G) = 3



B(G) = 4

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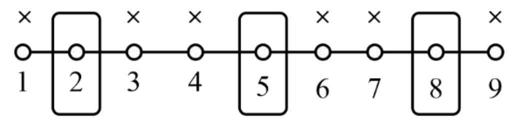
24. (3)

Mis $\{1, 3, 5, 7, 9\}$

Mis $\{2, 4, 6, 8\}$



MIS = $\{2, 5, 8\}$

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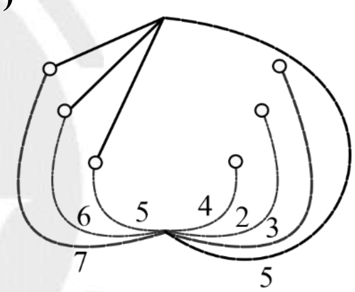
25. (7 to 7)



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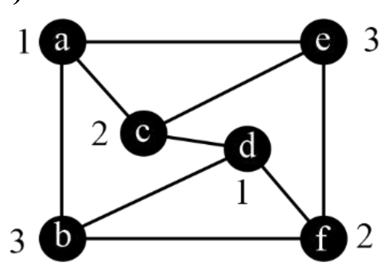
26. (3 to 3)



X(G) = 3

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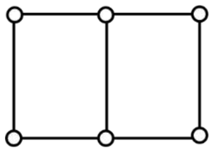
27. (a)

→ no cycle X (G) = 2

→ even length
X (G) = 2



B. PG
X (BPG) = 2

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28. (a, b)

(i) This procedure results in a proper vertex coloring of G.

(ii) The number of colors used in at most Δ (G) + 1.

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29. (11 to 11)

n = 8
f = 5

n – e + f = 2
8 – e + 5 = 2
13 – e = 2
13 – 2 = e
11 = e

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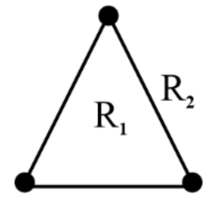
30. (24 to 24)



n = 10
e = 3n – 6
= 3.10 – 6
= 30 – 6
= 24

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31. (a)

$$\delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1$$

$$\delta(G) \le \frac{2e}{\pi}$$

↓

$$3 \le \frac{2(n+f-2)}{n}$$

$3n \le 2^{nd} + 2f - 4$

$n - e + f = 2$

$n + f - 2 = e$

$3n - 3n + 4 \le f$

$n + 4 \le 2f$

$$\frac{n}{2} + 2 \le f$$

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32. (d)

$n = 10 \quad e = 15$

$n - e + f = 2$

$10 - 15 + f = 2$

$-5 + f = 2$

$f = 7$

bounded = Total –1

= 7 – 1

= 6

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CHAPTER

4 COMBINATORICS

Permutation

1. [MCQ] [GATE-2023: 2M]

Let U = {1, 2,...,n}, where n is a large positive integer greater than 1000. Let k be a positive integer less than n. Let A, B be subsets of U with |A| = |B| = k and $A \cap B = \phi$. We say that a permutation of U separates A from B if one of the following is true.

All members of A appear in the permutation before any of the members of B.

All members of B appear in the permutation before any of the members of A.

How many permutations of U separate A from B?

(a) n!

(b) $\binom{n}{2k}(n-2k)!$

(c) $\binom{n}{2k}(n-2k)!(k!)^2$

(d) $2\binom{n}{2k}(n-2k)!(k!)^2$

2. [NAT] [GATE-2020: 2M]

The number of permutations of the characters in LILAC so that no character appears in its original position, if the two L's are indistinguishable, is ____.

Combinations

3. [NAT] [GATE-2015 : 1M]

The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set {1, 2, 3} is ______.

4. [NAT] [GATE-2014 : 1M]

Each of the nine words in the sentence "The quick brown fox jumps over the lazy dog" is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the world drawn is ___. (The answer should be rounded to one decimal place).

5. [NAT] [GATE-2014 : 2M]

A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennants. The set of all possible 1-pennant is {(1)}, the set of all possible 2-pennants is {(2), (1,1)} and the set of all 3-pennants is {(2,1), (1,1,1,), (1,2)}. Note that the pennant (1,2) is not the same as the pennant (2,1). The number of 10-pennants is ________.

6. [MCQ] [GATE-2008 : 2M]

In how many ways can b blue balls and r red balls be distributed in n distinct boxes?

(a) $\frac{(n+b-1)!(n+r-1)}{(n-1)!b!(n-1)!r!}$

(b) $\frac{(n+(b+r)-1!)}{(n-1)!(n-1)!(b-r)!}$

(c) $\frac{n!}{b!r!}$

(d) $\frac{(n+(b+r)-1)!}{n!(b+r-1)!}$

Binomial Identities

7. [NAT] [GATE-2015 : 1M]
The number of divisors of 2100 is _______.

8. [NAT] [GATE-2015 : 2M]
$\sum_{x=1}^{99} \frac{1}{x(x+1)} =$ ____

9. [NAT] [GATE-2014 : 2M]
The number of distinct positive inetegral factors of 2014 is _____.

10. [NAT] [GATE-2008 : 2M]
Let, $P = \sum_{\substack{1\le i2\le k \\ i \text{ odd}}}$ and $Q = \sum_{\substack{1\le i\le 2k \\ i \text{ even}}} i$, where k is positive integer. Then
(a) P = Q – k
(b) P = Q + k
(c) P = Q
(d) P = Q + 2k

Generating Functions

11. [MCQ] [GATE-2018-: 2M]
Which one of the following is a closed form expression for the generating function of the sequence {an}, where an = 2n + 3 for all n = 0, 1, 2, …?
(a) $\frac{3}{(1-x)^2}$
(b) $\frac{3x}{(1-x)^2}$
(c) $\frac{2-x}{(1-x)^2}$
(d) $\frac{3-x}{(1-x)^2}$

12. [NAT] [GATE-2017 : 2M]
If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^3}$, then $a_3 - a_0$ is equal to

Recurrence Relation

13. [MCQ] [GATE-2023: 1M]
The Lucas sequence Ln is defined by the recurrence relation:
$L_n = L_{n-1} + L_{n-2}$, for $n \ge 3$,
with $L_1 = 1$ and $L_2 = 3$.
Which one of the options given is TRUE?
(a) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$
(b) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$
(c) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$
(d) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

14. [NAT] [GATE-2016 : 2M]
Consider the recurrence relation $a_1 = 8$, $a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$.
The value of K is ______.

15. [NAT] [GATE-2016 : 2M]
The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + \ldots)^3$ is .

16. [MCQ] [GATE-2016 : 1M]
Let a_n be the number of n-bit strings that do NOT contain two consecutive 1's. Which one of the following is the recurrence relation for a_n?
(a) $a_n = a_{n-1} + 2a_{n-2}$
(b) $a_n = a_{n-1} + a_{n-2}$
(c) $a_n = 2a_{n-1} + a_{n-2}$
(d) $a_n = 2a_{n-1} + 2a_{n-2}$

17. [MCQ] [GATE-2015 : 2M]
Let a_n represent the number of bit strings of length n containing two consecutive 1s. What is the recurrence relation for a_n?
(a) $a_{n-2} + a_{n-1} + 2^{n-2}$
(b) $a_{n-2} + 2a_{n-1} + 2^{n-2}$
(c) $2a_{n-2} + a_{n-1} + 2^{n-2}$
(d) $2a_{n-2} + 2a_{n-1} + 2^{n-}$

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ANSWER KEY

1.	(d)	**2.**	(12 to 12)	**3.**	(15 to 15)	**4.**	(3.8 to 3.9)
5.	(88.9 to 89.1)	**6.**	(a)	**7.**	(36 to 36)	**8.**	(0.99 to 0.99)
9.	(8 to 8)	**10.**	(a)	**11.**	(d)	**12.**	(15 to 15)
13.	(a)	**14.**	(197.9 to 198.1)	**15.**	(10 to 10)	**16.**	(b)
17.	(a)						

SOLUTIONS

1. (d)

$u = \{1, 2, 3.......n\} \forall n > 2000$

- $|A| = |B| = k$ & $A \cap B = \phi$
- $|A| = k$ such that $A \subseteq u$
- $|B| = k$ such that $B \subseteq u$
- K is positive integer less than n.
- Number of permutation for every element of A that appear before element of B.

  $^nC_{2k} * (n - 2k)! \times k! \times k!$

- Number of permutations for all element of B appear before element of A.

  $^nC_{2k} * (n - 2k)! \times k! \times k!$

Total permutation $= 2 \times {}^nC_{2k}$ $(n - 2k)! \times k! \times k!$.

Therefore option (d) is the right choice.

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2. (12 to 12)

$\underline{1\,2\,3\,4\,5}$ 1   3

L I L A C   L   L

1	2	3	4	5		4
_	L	_	L	_		
_	_	_	L	L		
_	L	_	_	L		

$\underline{2}$ $\underline{4}$ $\underline{5} \rightarrow {}^3C_2$ ways = 3 ways.

$\downarrow$ $\downarrow$ $\downarrow$ $\rightarrow$ 2 ways

$= 3 \times 2 \times 2$ $\rightarrow 2!$

$=$ 12 ways .

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3. (15 to 15)

{1, 2, 3}

$$\begin{cases} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 3 \end{cases}$$

$$\begin{cases} 1 & 2 & 2 & 3 & 2 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 3 & 3 \\ 1 & 2 & 3 & 3 & 2 & 3 & 3 & 3 \end{cases}$$

$\rightarrow$ 3 3 3 3

10 + 4 + 1 = 15

$$\begin{cases}1 & 1 & 1 & 2\\ 1 & 1 & 2 & 3 \rightarrow \{1\ \ 3\ \ 3\ \ 3\ 2\ 2\ 2\ 3\\ 1 & 1 & 3 & 3\end{cases}$$

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4. (3.8 to 3.9)

$$3\times\frac{1}{9}+5\times\frac{1}{9}+3\frac{1}{9}+5\times\frac{1}{9}+4\times\frac{1}{9}+3\times\frac{1}{9}+4\times\frac{1}{9}+3\times\frac{1}{9}$$

$$=\frac{3+5+5+3+5+4+3+4+3}{9}=\frac{35}{9}=3.9$$

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5. (88.9 to 89.1)

a_n	1	2	3
	{1}	{2}	{1, 2}
	$a_1 = 1$	{1, 1}	{2, 1}
		$a_2 = 2$	{1, 1, 1}
			$a_3 = 3$

$a_4 = 5$

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, a_{10}

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6. (a)

$$\frac{(b+n-1)!}{(n-1)!\times b!}\times\frac{(R+n-1)!}{R!\times(n-1)}$$

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7. (36 to 36)

$N = P_1^a.P_2^b.P_3^c$ .....

$= \quad (a + 1) \times (b + 1) \times (c + 1)$

$2100 = 3 \times 700 = 3 \times 7 \times 100$

$= \quad 3 \times 7 \times 2 \times 50$

$2100 = 2^2 \times 3^1 \times 5^2 \times 7^1$

$= \quad (2 + 1) \times (1 + 1) \times (2 + 1) \times (1 + 1)$

$= \quad 3 \times 2 \times 3 \times 2 = 6 \times 6 = 36$

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8. (0.99 to 0.99)

n = 1 x = 2 x = 3

$$\frac{1}{0.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\frac{1}{99.100}$$

$$\left(1-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)\ldots..+\left(\cancel{\frac{1}{99}}-\frac{1}{100}\right)$$

$$=1-\frac{1}{100}=\frac{100-1}{100}=\frac{99}{100}=0.99$$

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9. (8 to 8)

$n \quad = P_1^a \times P_2^b \times P_3^c$

$(a + 1) \times (b + 1) \times (c + 1)$

$2014 = 2 \times 19 \times 53$

$= \quad 2^1 \times 19^1 \times 53^1$

$(1 + 1) \times (1 + 1) \times (1 + 1) = 2.2.2 = 2^3 = 8$

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10. (a)

$P = 1 + 3 + 5 + 7 \ldots.(2k - 1)$

$= (2 - 1) + (4 - 1) + (6 - 1) \ldots.(2k - 1)$

$= (2 + 4 + 6 + 8) + (-1, -1, -1, -1\ldots.-k)$

$= Q - K$

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11. (d)

$an = 2n = 3$

$$= \sum_{n=0}^{\infty} an.x^n = \sum_{n=0}^{\infty}(2n+3).x^n$$

$$= \sum_{n=0}^{\infty} 2n.x^n + \sum_{n=0}^{\infty} 3.x^n$$

$$= 2(1.x + 2x^2 + 3x^3 \ldots.) + 3(1 + x + x^2 = x^3 \ldots)$$

$$G(x) = \sum_{n=0}^{\infty} an.x^n$$

$$= 2x(1+2x+3x^2+4x^3\ldots)+3\left(\frac{1}{1-x}\right)$$

$$= 2x\frac{1}{(1-x)^2}+\frac{3}{1-x}$$

$$= \frac{2x}{(1-x)^2}+\frac{3}{(1-x)}\frac{(1-x)}{(1-x)}$$

$$= \frac{2x+3(1-x)}{(1-x)^2}=\frac{2x+3-3x}{(1-x)^2}=\frac{3-x}{(1-x)^2}$$

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12. (15 to 15)

$$\frac{1}{(1-x)^n}=1+\frac{n.x}{1!}+\frac{n(n+1)}{2!}x^2$$

$$+\frac{n(n+1)(n+2)}{3!}x^3\ldots$$

$$n=3\frac{1}{(1-z)^3}=1+\frac{3\cdot 2}{1!}+\frac{3\cdot 4}{2!}2^2+\frac{3\cdot 4\cdot 5}{3!}\ldots$$

$$= 1+3z+6z^2+10z^3\ldots.$$

$$\frac{1}{(1-z)^3}=1+3z+6z^2+10z^3.$$

$$\frac{1+z}{(1-z)^3}=(1+z)\left[1+3z+6z^2+10z^3\ldots\right]$$

$$= 1 + 3z + 6z^2 + 10z^3 \ldots z + 3z^2 + 6z^3 + 10z^4\ldots$$

$$\frac{1+z}{(1-z)^3}=1+4z=9z^2+16z^3+\ldots.$$

$$a_0z^0+a_1z1+a_2z^2+a_3z^3+\ldots.$$

$a_0 = 1 \quad a_2 = 9$

$a_1 = 4 \quad a_3 = 16$

$a_0 = !$

$a_3 = 16 = 16 - 1 = 15$

$a_3 - a_0$

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13. (a)

$L_n = L_{n-1} + L_{n-2}$, for $n \geq 3$

$L_1 = 1$ and $L_2 = 3$

Putting L = 1 and 2 in option one, we get

$$L_1 = \left(\frac{1+\sqrt{5}}{2}\right)^1 + \left(\frac{1-\sqrt{5}}{2}\right)^1 = \frac{1}{2}+\frac{1}{2} = 1$$

$$L_2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 + \left(\frac{1-\sqrt{5}}{2}\right)^2 = (a + b)^2 + (a - b)^2$$

$$= 2(a^2 + b^2)$$

$$= 2\left(\frac{1}{4}+\frac{5}{4}\right) = 2\times\frac{6}{4} = 3$$

Therefore option (a) is correct.

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14. (197.9 to 198.1)

$an = 6n^2 + 2n + a_{n-1}$.

$= 6n^2 = 2n + 6(n-1)^2 + 2(n-1) + a_{n-2}$.

$= 6n^2 + 2n + 6(n-1)2 + 2(n-1) + 6(n-2)^2 + 2(n- \quad 2) \ldots..$

$= 6\,(n^2 + (n-1)^2 + (n-2)^2 + \ldots + 2[n + (n-1) + (n-2)\ldots 1]$

$$= 6\left[\frac{n(n+1)(2n+1)}{6}\right]+2\left[\frac{n(n+1)}{2}\right]$$

$= n(n + 1)\,(2n + 1) + n(n + 1)$

$= n(n + 1)\,[2n + 1 + 1] = n(n + 1)\,(2n + 2)$

$= 2n(n + 1)\,(n + 1) = 2n(n + 1)^2$

$a_n = 2n(n+1)^2, n = 99$
$a_{99} = 2 \times 99\ (99+1)^2$
$= 198\ (10^2)^2 = 198 \times 10^4.$

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15. (10 to 10)

$5_{C_3} = 5_{C_2} = 10\ (x^3 + x^4 + x^5 + x^6 + \ldots)^3$

$(x^3)^3\ [1 + x + x^2 + x^3 + \ldots]^3$

$x^9\left(\frac{1}{1-x}\right)^3 = x^9(1-x)^{-3} = x^9\left[\ldots. - 3_{C_3}\left(-x^3\right)..\right]$

$= x^9\left[\ldots. - 1 \times 5_{C_3}(x)^3(x)^3\right] = 5_{C_3}x^{12}$

$\frac{1}{x-1} = 1 + x + x^2 + x^3 + x^4 + \ldots.$

$-3_{C_3} = (-1)^3\, 3 + 3 - 1_{C_3} = {}^5C_3 = {}^5C_2 = 10$

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16. (b)

a_1	2 bit	3 bit
0	00	000
1	01	001
$a_1 = 2$	10	010
	11	011
	$a_2 = 3$	100
		101
		110
		111
		$a_3 = 5$

$a_n = a_{n-1} + a_{n-2}$

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17. (a)

Do not 11

1 bit	2 bit	3bit
0	00	000
1	01	001
$a_1 = 2$	10	010
	11	011
	$a_2 = 2$	100
		101
		110
		111
		$a_3 = 6$

$a_2 = 2$

$a_1 = 2$

$a_3 = 6$

$a_1 = a_2 = 3\ a_3 = 6$

$a_n = a_{n-1} + a_{n-2}$

contain 11

$2^n - a_n = 2^{n-1} - a^{n-1} + 2^{n-2} - a_{n-2}$

$(2^n - 2^{n-1} - 2^{n-2}) + a_{n-1} + a_{n-2} = a_n$

$2^{n-2}(2^2 - 2^1 - 1) + a_{n-1} + a_{n-2} = a_n$

$2^{n-2} + a_{n-1} + a_{n-2} = an$

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