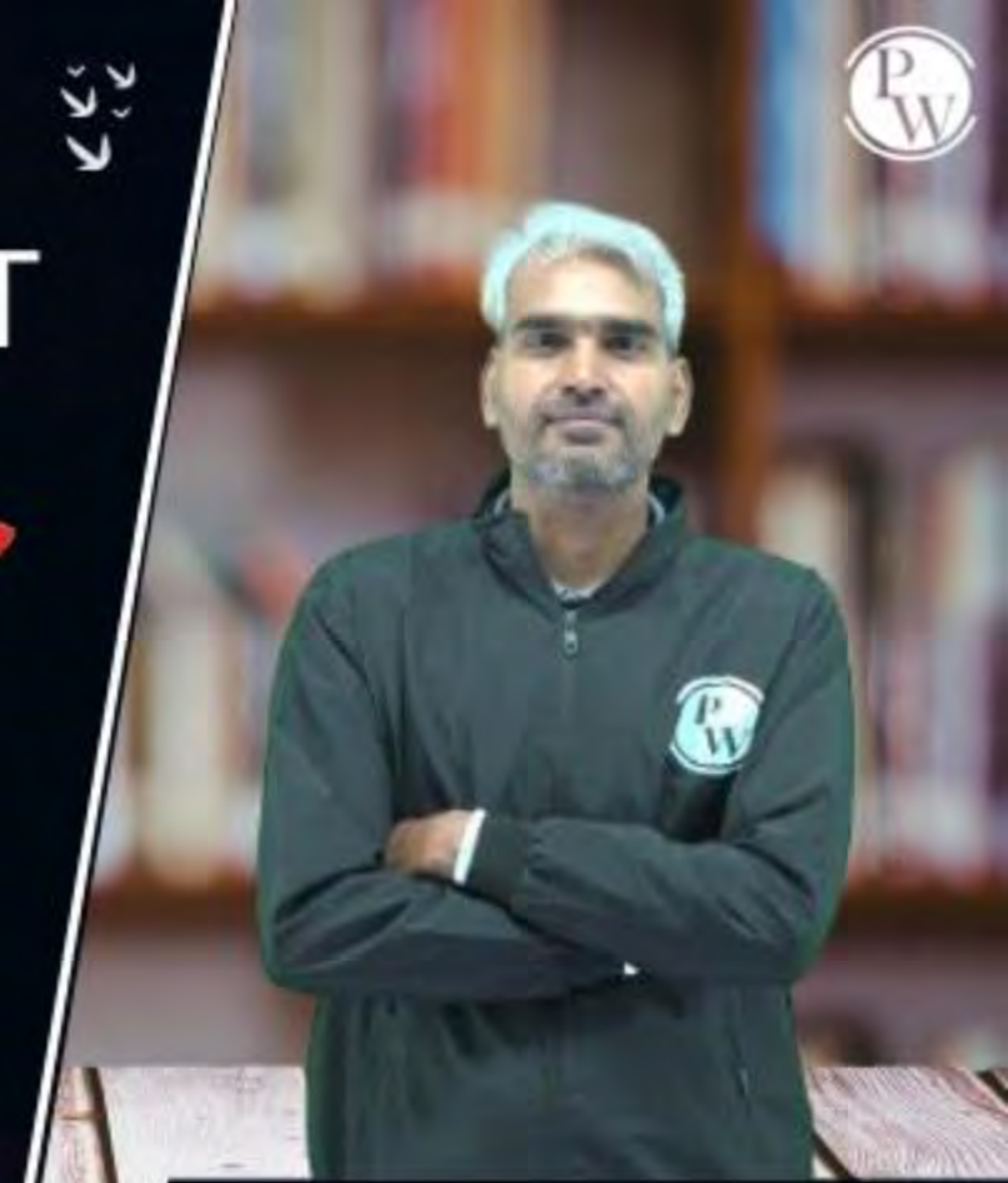
Computer Science & IT

Database Management System

Relational Model & Normal Forms

Lecture No. 06



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Recap of Previous Lecture

- **Topic** Different types of keys in RDBMS
- **Topic** Candidate key
- **Topic** Super key

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Topics to be Covered

- **Topic** Closure of an attribute set
- **Topic** Super key
- **Topic** Identification of Candidate key

Slide

Topic : Super key

{May or may not be minimal}

A set of attributes that can determine all the attributes of a relation is called a Super key

{For super key, it need not be minimal}

* Every Candidate key is a Super key, but Every Super key need not be a Candidate key.

eg1: Consider the following relation

Student(Sid, Sname, fee)

Student

Sid	Sname	fee
S1	A	500
S2	A	400
S3	B	700
S4	C	500
S5	C	500
S6	D	600

Let FDs that exist in the relation are

$$\boxed{\begin{aligned} Sid &\rightarrow Sname \\ Sid &\rightarrow fee \end{aligned}}$$

* Values of Sid will always be unique in the student table. ∴ Sid is a Key
* The values of (Sid, Sname) together will also be uniques in all the tuples.
  ∴ (Sid, Sname) is also a Key

In eg 1:

"Sid" is the Only Candidate Key of the relation

Q: Find all the Superkeys of the relation!

'Sid' is a C.K.

∴ all the values in the relation w.r.t. 'Sid' are unique.

If we take any Super-set of 'Sid', then values w.r.t. that set of attributes will also be unique.

⇒ ∴ Superkeys of the relation Student are,

{Sid} ← Every set is a Superset of itself.

{Sid, Sname}

{Sid, fee}

{Sid, Sname, fee}

Total '4' Super Keys

Hence, Every Super-set of 'Sid' is a Super Key.

Topic : Super key

Note :- At least one subset of set of attributes of a super key is a candidate key.

Note :

Super key attribute set = All attributes of any one Candidate key + 0 or more attributes out of remaining attributes of the relation

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(i) When attribute 'A_1' is the only Candidate key of relation R

Super Key = Attribute "A_1" must be present (and) 0 or more attributes out of remaining '$n-1$' attributes

$\therefore$ # Super Keys = $1 * \left\{ {}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \cdots + {}^{(n-1)}C_{(n-1)} \right\}$

Only one way to select 'A_1'

$= 1 * 2^{n-1}$

$= 2^{n-1}$ Ans

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n}$$

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(i) When attribute 'A_1' is the only Candidate key of relation R

Super key = Attribute A_1 must be present (and) no constraint on remaining $(n-1)$ attributes

$$\therefore \text{ \# Super keys } = \begin{array}{c} A_1 \\ \downarrow \\ 1 \end{array} * \underbrace{\begin{array}{c} A_2 \\ \downarrow \\ 2 \end{array} * \begin{array}{c} A_3 \\ \downarrow \\ 2 \end{array} * \begin{array}{c} A_4 \\ \downarrow \\ 2 \end{array} * \cdots * \begin{array}{c} A_n \\ \downarrow \\ 2 \end{array}}_{(n-1) \text{ time } 2} = 2^{n-1}$$

Ans

Only one way to choose 'A_1'

for every other attribute we have two choices, either take it or leave it

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(ii) when $(A_1 A_2)$ together is the only candidate key of relation R.

Super key = Both A_1 & A_2 must be present (and) no constraint on remaining $(n-2)$ attributes

$$\begin{array}{ccccccc} A_1 & & A_2 & A_3 & A_4 & \cdots & A_n \\ \downarrow & & \downarrow & \downarrow & \downarrow & & \downarrow \end{array}$$

$$\#\text{ Super keys} = \underbrace{1 \times 1}_{\text{Select both } A_1 \& A_2} \; \underbrace{2 \times 2 \times \cdots \times 2}_{(n-2) \text{ times '2'}} = 2^{n-2}$$

Ans

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(iii) When 'A_1' and 'A_2' are the only two candidate keys of relation R.

$A_1 \quad A_2 \quad A_3 \quad A_4 \quad \ldots \quad A_n$

In a super key of R:

A1 is taken, A2 is not taken

$$1 \times 1 \times \underbrace{2 \times 2 \times \ldots \times 2}_{(n-2)\text{ times}} = 2^{n-2}$$

(or)

A1 is not taken & A2 is taken

$$1 \times 1 \times \underbrace{2 \times 2 \times \ldots \times 2}_{(n-2)\text{ times}} = 2^{n-2}$$

(or)

Both A1 & A2 are taken

$$1 \times 1 \times \underbrace{2 \times 2 \times \ldots \times 2}_{(n-2)\text{ times}} = 2^{n-2}$$

Total No. of Superkeys $= 2^{n-2} + 2^{n-2} + 2^{n-2} = 3 \cdot 2^{n-2}$

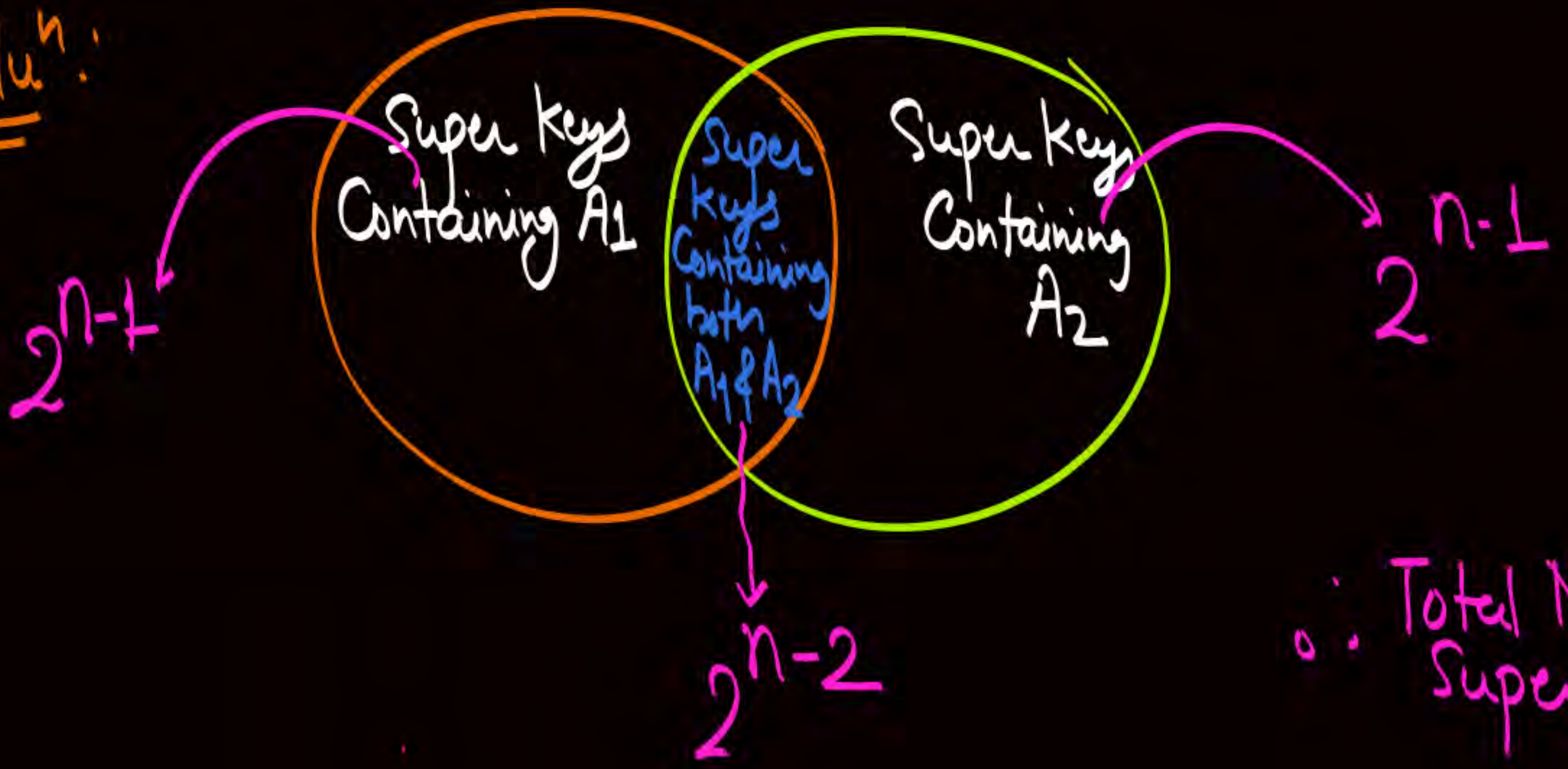
Ans

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(iii) when 'A_1' and 'A_2' are the only two candidate keys of relation R.

Solu^n:



$\therefore$ Total No. of Super Keys $= 2^{n-1} + 2^{n-1} - 2^{n-2}$

$= 3 \cdot 2^{n-2}$ **Ans**

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(iii) When 'A_1' and 'A_2' are the only two candidate keys of relation R.

Solu^n:

Total no. of Super Keys = Total no. of subsets of n-attributes − No. of subsets of attributes in which neither A_1 is Present nor A_2 is present

$$= 2^n - 2^{n-2}$$

$$= 4 \cdot 2^{n-2} - 2^{n-2}$$

$$= 3 \cdot 2^{n-2}$$

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(iv) When (A_1A_2) & (A_2A_3) are the only two candidate keys of relation R.

$$A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \; - - - \; A_n$$

$$1 \times 1 \times 1 \times 2 \times 2 \times \cdots \times 2 = 2^{n-3}$$

$$1 \times 1 \times 1 \times 2 \times 2 \times - - - \times 2 = 2^{n-3}$$

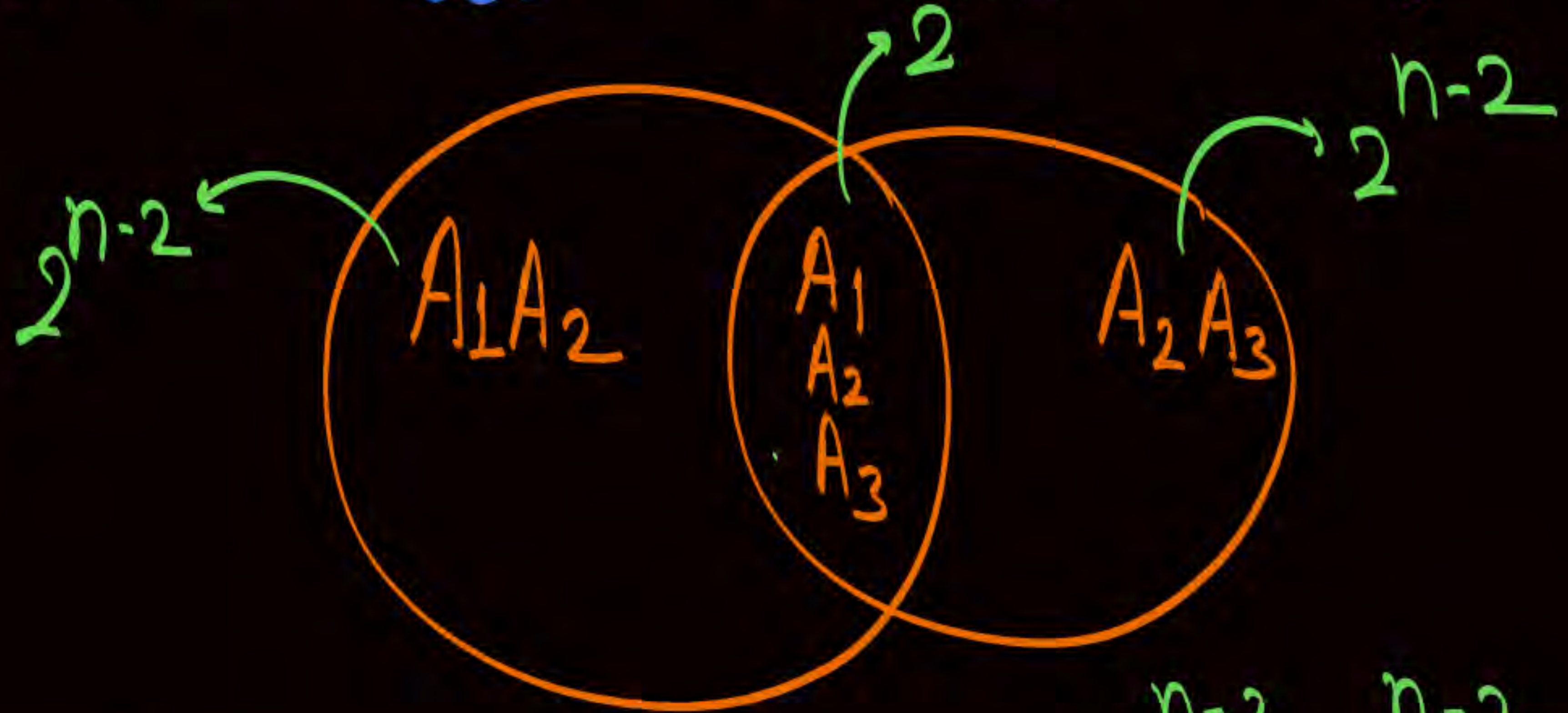
$$1 \times 1 \times 1 \times 2 \times 2 \times - - \times 2 = 2^{n-3}$$

$$\underline{3 \cdot 2^{n-3}}$$

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(iv) When $(A_1 A_2)$ & $(A_2 A_3)$ are the only two candidate keys of relation R.



Total No. of Super Keys $= 2^{n-2} + 2^{n-2} - 2^{n-3}$

$= 3 \cdot 2^{n-3}$

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$
Find the total number of super keys in relation R.

(V) When (A_1A_2) & (A_3A_4) are the only two candidate keys of relation R.

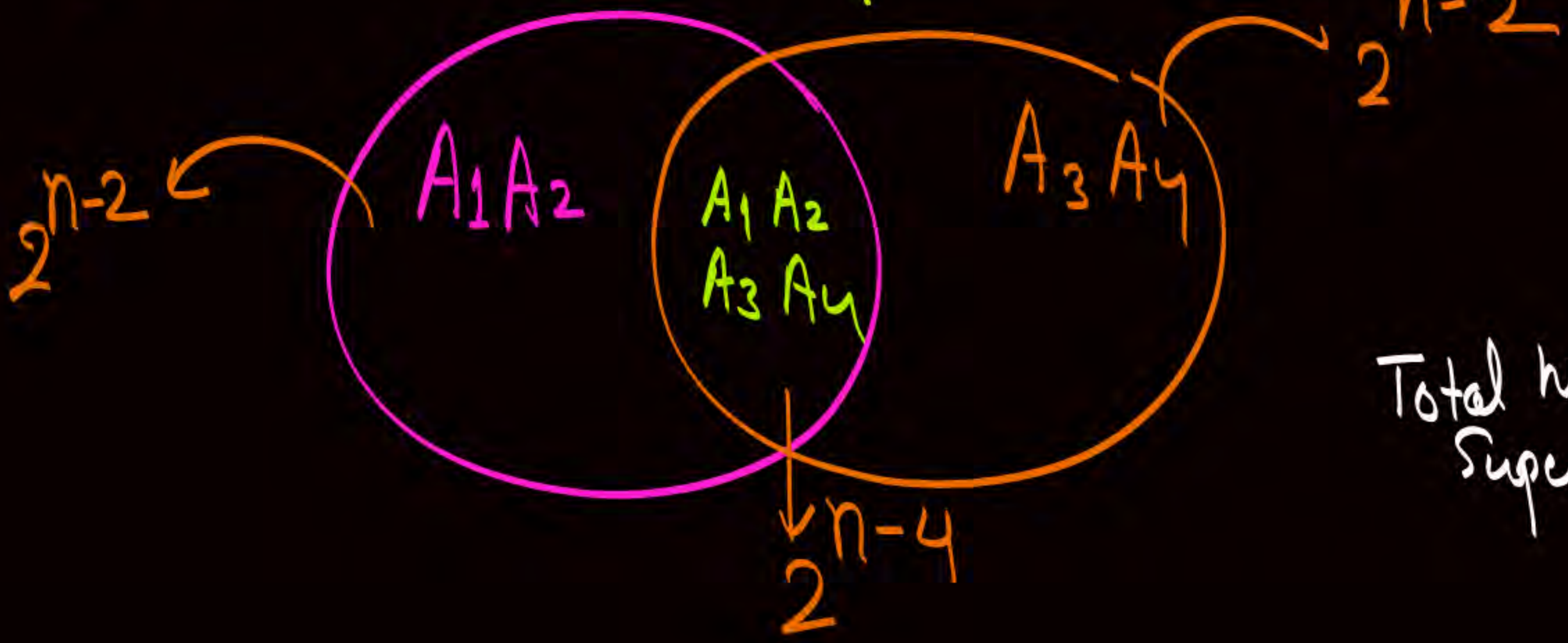
	A_1	A_2	A_3	A_4	A_5 A_6 . . . . A_n	
1	✓	✓	×	×	$2 \times 2 \times \ldots 2$	$= 2^{n-4}$
2	✓	✓	✓	×	$2 \times 2 \times \ldots 2$	$= 2^{n-2}$
3	✓	✓	×	✓		$= 2^{n-4}$
4	×	×	✓	✓		$= 2^{n-4}$
5	✓	×	✓	✓		$= 2^{n-4}$
6	×	✓	✓	✓		$= 2^{n-4}$
7	✓	✓	✓	✓		$= 2^{n-4}$

Total $7 \cdot 2^{n-4}$ super keys

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

Find the total number of super keys in relation R.

(V) When (A_1A_2) & (A_3A_4) are the only two candidate keys of relation R.



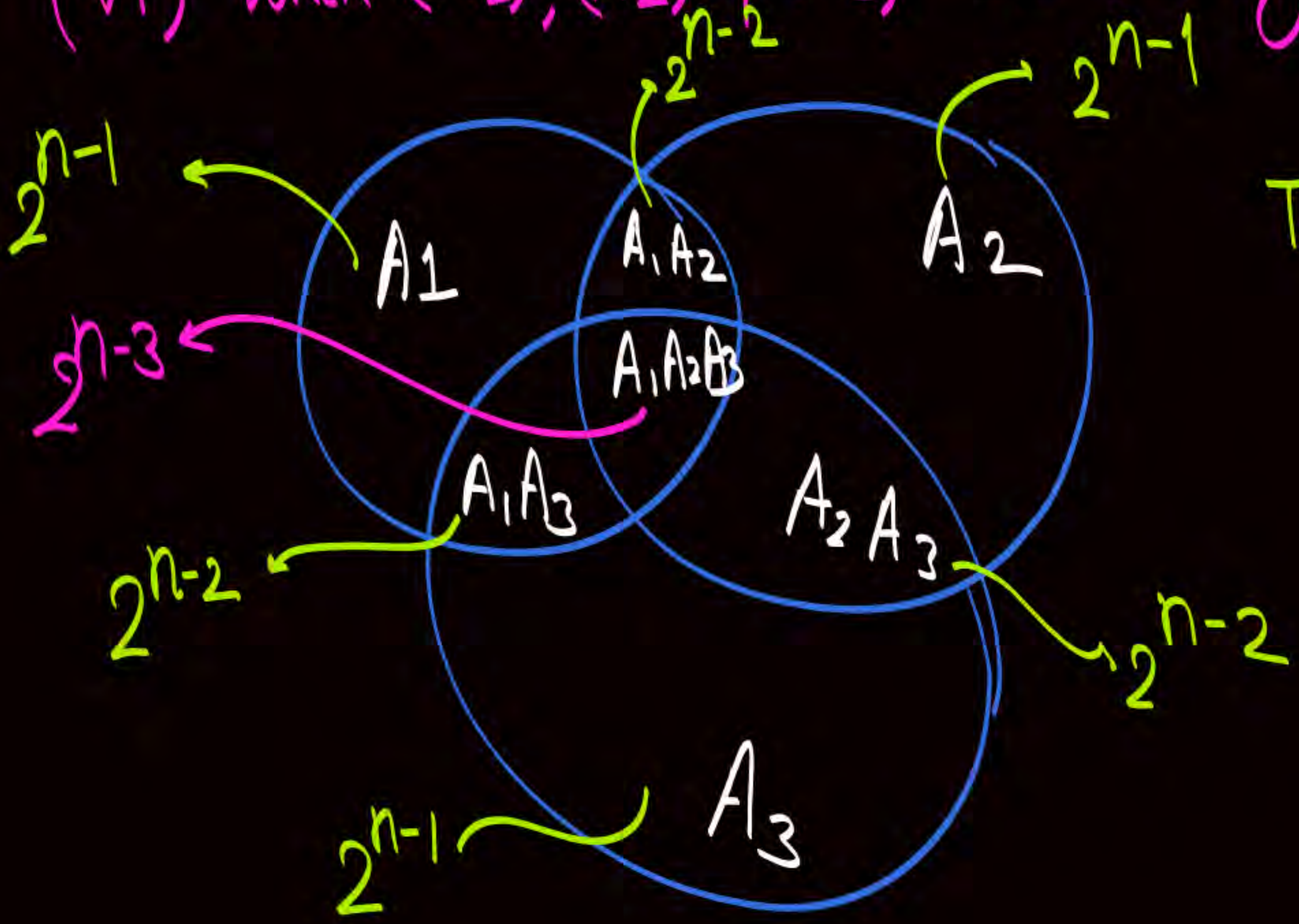
Total no. of super keys $= 2^{n-2} + 2^{n-2} - 2^{n-4}$

$= 7 \cdot 2^{n-4}$

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$

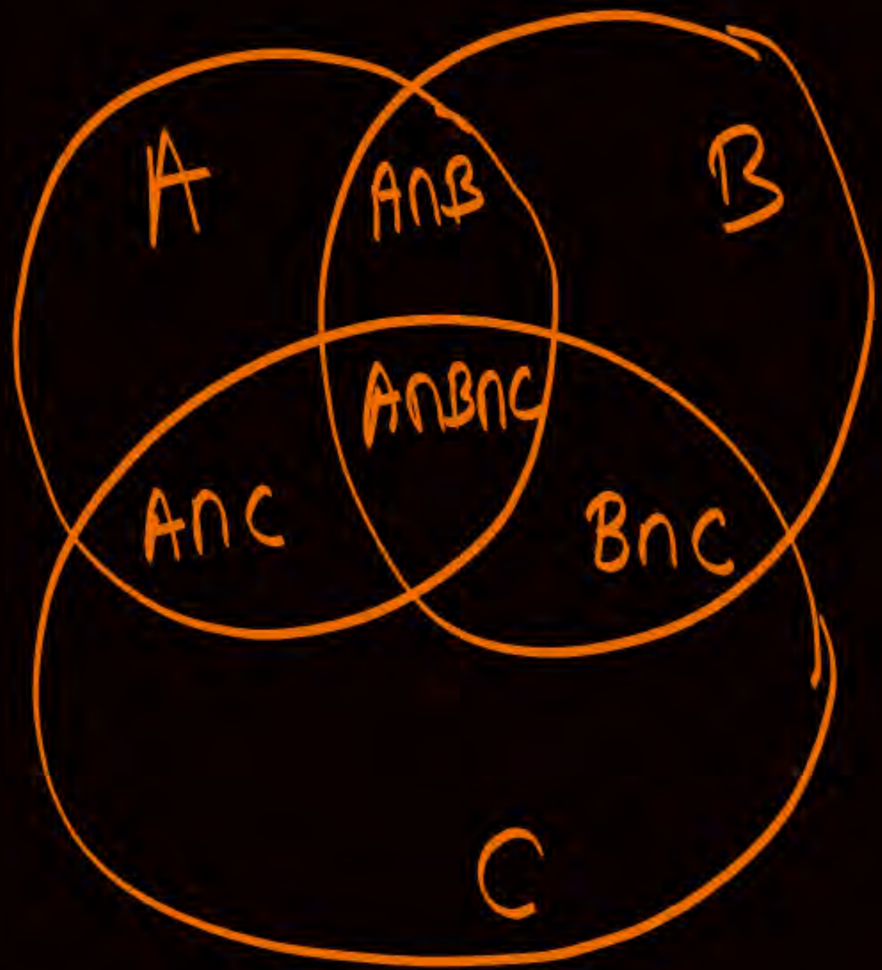
Find the total number of super keys in relation R.

(vi) When (A_1), (A_2) & (A_3) are the only three candidate keys of relation R.



Total No. of Super keys $= 2^{n-1} + 2^{n-1} + 2^{n-1} - 2^{n-2} - 2^{n-2} - 2^{n-2} + 2^{n-3}$

$= \boxed{7 \cdot 2^{n-3}}$



$$n(A \cup B \cup C) = \underline{n(A) + n(B) + n(C)} - \underline{n(A \cap B) - n(A \cap C) - n(B \cap C)} + n(A \cap B \cap C)$$

Q: Consider the following relational schema $R(A_1, A_2, A_3, \ldots, A_n)$ Find the total number of super keys in relation R. when each attribute of relation "R" is a Candidate key

Total no. of Super Key = Total no. of subset $- 1$

$= 2^n - 1$

2^n also include the Empty subset

Empty subset can not be a super key of any relation

$\therefore$ Subtract Empty subset

Note :- No two tuples of a relation can be exactly same.

∴ All attributes taken together will always form a superkey of that relation

Topic : Closure of an attribute set

Closure of an attribute set X (i.e., X^+) can be defined as set of all the attributes which can be functionally determined from attribute of set X.

#e.g. Assume a relation R (A,B,C,D) that has the following functional dependencies:

$$\left\{\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow D \end{array}\right.$$

$(A)^+ = \{A, B, C, D\}$

$(AB)^+ =$ — // —

$(AC)^+ =$ — // —

$(AD)^+ =$ — // —

$(ABC)^+ =$ — // —

$(ABD)^+ =$ — // —

$(ACD)^+ =$ — // —

$(ABCD)^+ =$ — // —

$(B)^+ = \{B, C, D\}$

$(BC)^+ = \{B, C, D\}$

$(BD)^+ = \{B, D, C\}$

$(BCD)^+ = \{B, C, D\}$

$(C)^+ = \{C, D\}$

$(CD)^+ = \{C, D\}$

$(D)^+ = \{D\}$

#e.g. Consider the following FD set

$$F = \{AB \rightarrow CD, AF \rightarrow D, DE \rightarrow F, C \rightarrow G, F \rightarrow E, G \rightarrow A\}$$

find the closure of following set of attributes.

(i) $\{C,F\}^+ = \{C, F, G, E, A, D\}$

(ii) $\{B,G\}^+ = \{B, G, A, C, D\}$

(iii) $\{A,F\}^+ = \{A, F, D, E\}$

(iv) $\{A,B\}^+ = \{A, B, C, D, G\}$

Topic : Super key

Let R be the relational schema, and let X be some attribute set over relation R. If X^+ determines all attributes of relation R, then X is called super key of relation R.

#e.g. Assume a relation R (A,B,C,D) that has the following functional dependencies:

$$\left\{\begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ C \rightarrow D \end{array}\right.$$

$(A)^+ = \{A, B, C, D\}$

$(AB)^+ =$ — " —

$(AC)^+ =$ — " —

$(AD)^+ =$ — " —

$(ABC)^+ =$ — " —

$(ABD)^+ =$ — " —

$(ACD)^+ =$ — " —

$(ABCD)^+ =$ — " —

All attributes of relation R

∴ all are Super keys of relation R.

$(B)^+ = \{B, C, D\}$

$(BC)^+ = \{B, C, D\}$

$(BD)^+ = \{B, D, C\}$

$(BCD)^+ = \{B, C, D\}$

Not all attributes

$(C)^+ = \{C, D\}$

$(CD)^+ = \{C, D\}$

Not all attributes

$(D)^+ = \{D\}$

Not all attributes

* **Proper subset :-** For a given set A, any subset of set 'A' except set 'A' itself are called Proper subsets of set A.

eg. let $A = \{a, b, c\}$ and let $\{a, b, c\}^+ = \underbrace{\text{all attributes of relation}}$

$\therefore \{a, b, c\}$ is a key.

$\Downarrow$

Proper subsets of set A $=$
$\{a\}$
$\{b\}$
$\{c\}$
$\{a, b\}^+ =$ all attributes of Relⁿ then $\{a, b\}$ will be key $\Longrightarrow$ Even if we delete 'c' from $\{a, b, c\}$ it does not loose its property of being a key
$\{a, c\}$
$\{b, c\}$
$\{\ \}$

$\therefore \{a, b, c\}$ is not a minimal key.

Topic : Candidate key (Minimal Super key)

Let R be the relational schema, and let X be the super key of relation R.

i.e., $(X)^+ = \{$All attributes of relation R$\}$

If no proper subset of X is a super key, then X is minimal super key i.e., X is Candidate key

eg. let $(AB)^+$ Contains all attributes of relation R

$\therefore$ (AB) is a Super key of relation R.

Proper subsets of AB =

- $\{A\}$, if $(A)^+$ = Not all attributes of Rel^n
  $\therefore$ A is not a S.K. of rel^n R.
- $\{B\}$, if $(B)^+$ = Not all attributes of rel^n R
  $\therefore$ B is not a S.K. of rel^n R
- ϕ ← We don't need to check w.r.t. proper subset 'ϕ'

→ i.e., No Proper-subset of $\{A,B\}$ is a Super key

Hence $\{A,B\}$ will be the Candidate key.

#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

AB→C,

B→E,

C→D

Find the Candidate key of R.

Note: The attributes that are not present in R.H.S. part of any FD of given FD set are called essential attributes.

* Every essential attribute must be present in every key of the relation

In the given question A & B are essential attributes.

$$(AB)^+ = \{\underbrace{A, B, C, E, D}_{\text{all attributes}}\}$$

∴ AB is a Super key

A & B both are essential

∴ No one can be removed from the key

Hence AB is also the minimal Super key

i.e. $\boxed{AB}$ is a C.K.

Closure of Proper subsets of AB

$(A)^+ = \{A\}$ = Not all attributes

$(B)^+ = \{B, E\}$ = Not all attributes

No proper subset is a Super key

∴ AB is minimal

i.e. $\boxed{AB}$ is a C.K.

In the above example $\boxed{AB}$ is a Candidate key

$\therefore$ Prime attributes = { A, B }

* There is no FD in the FD set of above relation in which any of the prime attribute appears in the R.H.S. part of that FD.

$\therefore$ Relation will have only one C.K.

i.e, $\boxed{AB}$ is the only C.K. of the above relation.

#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

AB→C,

B→E,

C→D

E→A

Find the Candidate key of R.

'B' is the essential attribute,

$(B)^+ = \underbrace{\{B, E, A, C, D\}}_{\text{all attributes}}$

∴ B is a Super Key.

$\boxed{B}$ is a Candidate Key.

{ A key with a single attribute is always minimal, ∴ Candidate Key }

Hit & trial Method:-

$(AB)^+ = \{\underbrace{A, B, C, D, E}_{\text{all attributes}}\}$

∴ AB is a Super Key

Check for minimal

Closure w.r.t. Proper subsets of AB =

$\{A\}^+ = \{A\}$ Not all attributes ∴ 'A' is not a S.K.

$\{B\}^+ = \underbrace{\{B, E, A, C, D\}}_{\text{all attributes}}$ ∴ B is a S.K.

→ B is a new Super Key

Check for minimal

it is a key with a single attribute ∴ always minimal

Hence $\boxed{B}$ is a C.K.

one proper subset of {A, B} is a S.K.

∴ AB is not a minimal S.K.

i.e., AB is not a C.K.

* In the above eg. "B" is a C.K.

∴ Prime Attributes = {B}

- No prime attribute is present in the R.H.S. part of any FD of FD set.

Hence, only one C.K.

i.e $\boxed{B}$ is the only C.K.

Topic : Note

If there exist any non-trivial FD of the form $X \rightarrow Y$

where 'Y' is any prime attribute of the relation.,

then that relation will have more than one Candidate Key.

<u>Reason</u>,

$X \rightarrow Y$

∴ Whatever that can be determined by Y Can also be determined by X

∴ We Can replace Y by X in the Corresponding Candidate Key, in order to obtain a new Super Key

#Q. Assume a relation R (A, B, C, D) that has the following functional dependencies:

$AB \to CD$, $\equiv$ $AB \to C$ & $AB \to D$

$D \to A$

Find all the Candidate keys of R.

- B is the essential attribute.

$(B)^+ = \{B\}$

Not all attributes

$\therefore$ Not a Super Key.

$(AB)^+ = \{A, B, C, D\}$ = all attributes.

$\therefore$ AB is S.K

$(A)^+ = \{A\}$

$(B)^+ = \{B\}$

No proper subset of AB is a S.K.

$\therefore$ $\boxed{AB}$ is a Candidate Key

Prime attributes $= \{A, B\}$

We have $D \to A$ (Prime Attri.)

$\therefore$ Replace A by D

DB is another S.K

$(D)^+ = \{D, A\}$

$(B)^+ = \{B\}$

$\boxed{DB}$ is a C.K $\Rightarrow$ Prime Attributes $= \{A, B, D\}$

D is a Prime attribute & $AB \to D$

$\therefore$ Replace D by AB in C.K $= \boxed{DB}$

H.W.

#Q. Assume a relation R (M, N, O, P, Q) that has the following functional dependencies:

$MNO \rightarrow PQ$ and

$P \rightarrow MN$

Find the Candidate keys of R.

H.W.

#Q. Assume a relation R (A, B, C, D) that has the following functional dependencies:

$AB \rightarrow CD$

$C \rightarrow A$

$D \rightarrow B$

Find the Candidate keys of R.

H.W.

#Q. Assume a relation R (A, B, C, D, E, H) that has the following functional dependencies:

$A \rightarrow B$

$BC \rightarrow D$

$E \rightarrow C$

$D \rightarrow A$

Find the Candidate keys of R.

2 mins Summary

- **Topic** Closure of an attribute set
- **Topic** Super key
- **Topic** Identification of Candidate key

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THANK - YOU

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