



# CS & IT ENGINEERING

**THEORY OF COMPUTATION**

Grammar

Lecture - 02



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# Recap of Previous Lecture



Topic

Grammar Def  
?????

Grammar Construction

Terminology

- Derivation
- L.M.D
- R.M.D
- Parse tree





# Topics to be Covered



Topic

Types of Grammar (0,1,2,3)

Topic

??

Ambiguous Grammar

Topic

??

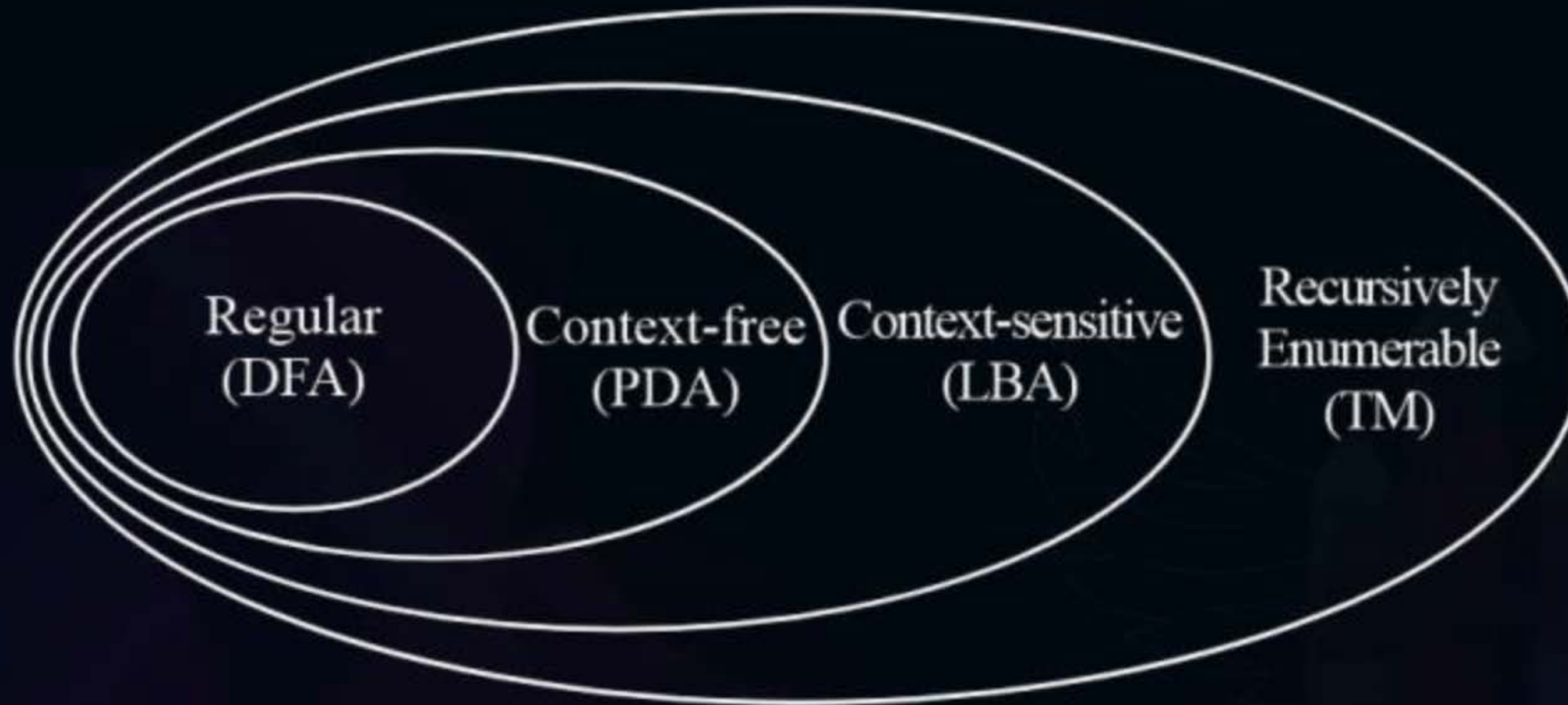
Regular Grammar

Topic

??



## Topic : Regular expression







## Topic : Grammar

- Set of rules used to describe strings of a language is known as grammar.
- Formal definition of grammar is

$G = (N, T, P, S)$

- **N** :- non terminals (or) variables
- **T** :- Terminals
- **P** :- no. of productions
- **S** :- Starting symbol



## Topic : Grammar

only one



- For every language grammar exist & every grammar generates one language.
- All grammars is of a form  $\alpha \rightarrow \beta$ , where  $\beta$  is replacement of  $\alpha$

$$\begin{array}{c} \alpha \rightarrow \beta \\ \alpha = \beta \end{array}$$





## Topic : Derivation

- The process of deriving strings from the given grammar known as derivation.
- The derivation can be either left most derivation or right most derivation
- **Left most derivation:**  
It is the derivation in which left most non terminal is replaced by its R.H.S part at every step.
- **Right most derivation:**  
It is a derivation in which right most non terminal is replaced by its R.H.S part at every step.

## Derivation Tree (or )Parse Tree

- Tree representation of the derivation is known as derivation tree.
- All leaf node of the parse tree is known as yield of parse tree .
- while reading yield from left to right sentence of the grammar can be generate.



## Sentential form

- Each step in the derivation is one sentential form.
- Hence sentential form is combination of terminals & non terminals (sentence also can be included)
- If the derivation is left most then sentential form is left sentential form.
- If the derivation is right most then sentential is right sentential form
- Every grammar represents only one language but for one language more than one grammar may exist.
- For regular languages there exist a grammar known as regular grammar.

- Context free language there exist a grammar known as context free grammar.
- Context sensitive language there exist a grammar known as context sensitive grammar.
- For recursive enumerable language there exist a grammar known as unrestricted grammar.



# Types of Grammar

Non Chomsky

Type	Language (Grammers)	From of Productions	Accepting Dived
3 ✓	Regular ✓	$A \rightarrow aB, A \rightarrow A$	Finite Automaton
2 ✓	Context-free ✓	$A \rightarrow \alpha$	Pushdown Automaton
1 ✓	Context sensitive ✓	$A \rightarrow \beta$ With $ \alpha  \geq  \beta $	LBA
0 ✓	Unrestricted	$\alpha \rightarrow \beta$	Turing machine

(R.E.L)

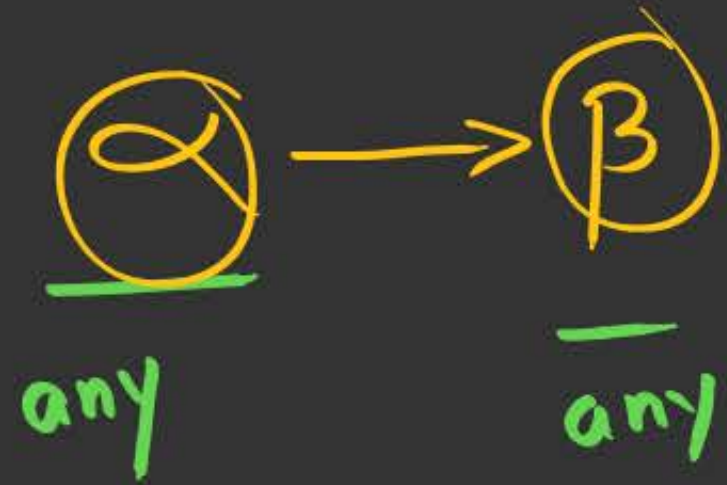
$$\alpha \longrightarrow \beta$$

$$|\alpha| \geq 1$$

$\alpha$  should contain atleast  
one Nonterminal.



# Type 0 Grammar (or) Unrestricted Grammar (or) Recursive Enumerable Grammar



$$\alpha \in (V+T)^+$$
$$\beta \in (V+T)^*$$

Ex:

$$\begin{aligned} S &\rightarrow aAB \\ \underline{aA} &\rightarrow \underline{a} \\ \underline{bB} &\rightarrow \underline{baa} \end{aligned}$$

# Type 1 Grammar (or) Context sensitive Grammar

$$\alpha \rightarrow \beta$$
$$|\alpha| \leq |\beta|$$

$$\alpha, \beta \in (V+T)^+$$

Ex:

$$S \rightarrow aAB$$

$$\underline{aA} \rightarrow \underline{ba}$$

$$B \rightarrow b$$

Type 1



## Type 2 Grammar (or) Contextfree Grammar

$$\begin{array}{l} A \rightarrow \alpha \\ \vdash \\ 1 \rightarrow \text{any} \end{array}$$

$$\alpha \in (v + T)^*$$

2

 $C \rightarrow b$

# Type 3 Grammar (a) Regular Grammar

$$A \rightarrow x \underline{B}$$

$$A \rightarrow xB|x$$

{ Right linear Grammar }

Ex: ①

$$\begin{aligned} S &\rightarrow aS \\ S &\rightarrow bS \\ S &\rightarrow a \end{aligned}$$

$$A \rightarrow Bx$$

$$A \rightarrow Bx|x$$

{ Left linear Grammar }

Ex: ②

$$\begin{aligned} S &\rightarrow Sa \\ S &\rightarrow Sb \\ S &\rightarrow a \end{aligned}$$

$$\alpha \rightarrow \beta$$

$$①$$

almost  $1 N^+$

$$③$$

$$S \rightarrow aSb|a$$



$$S \rightarrow aSb \mid ab$$

linear

but not Regular

Regular?

④

$$S \rightarrow aA$$
$$A \rightarrow bB$$
$$A \rightarrow a$$
$$B \rightarrow c$$

Regular?



$$S \rightarrow \underline{aS} / \underline{bS} / \epsilon$$

Regular?  
Grammar ✓

②

$$S \rightarrow \underline{aS} \mid Sb \mid a$$

R.L.G. | L.L.G.

→ Not regular

Ⓐ Yes

Ⓑ No

No

$$\begin{array}{l} S \rightarrow aS \\ S \rightarrow Sb \\ S \rightarrow a \end{array}$$

Regular Grammar?

Linear Grammar but not Regular grammar



## Linear Grammar:

Any grammar having left hand side exactly one nonterminal and R.H.S having atmost one N.T it is called as Linear Grammar.



{ Every Regular Grammar  
of Linear Grammar }

①  $S \rightarrow \underline{A}\underline{B}$   
 $A \rightarrow a$   
 $B \rightarrow b$

not Regular  
CFG

3 x

2 ✓  $\Rightarrow$  C.F.G.

1

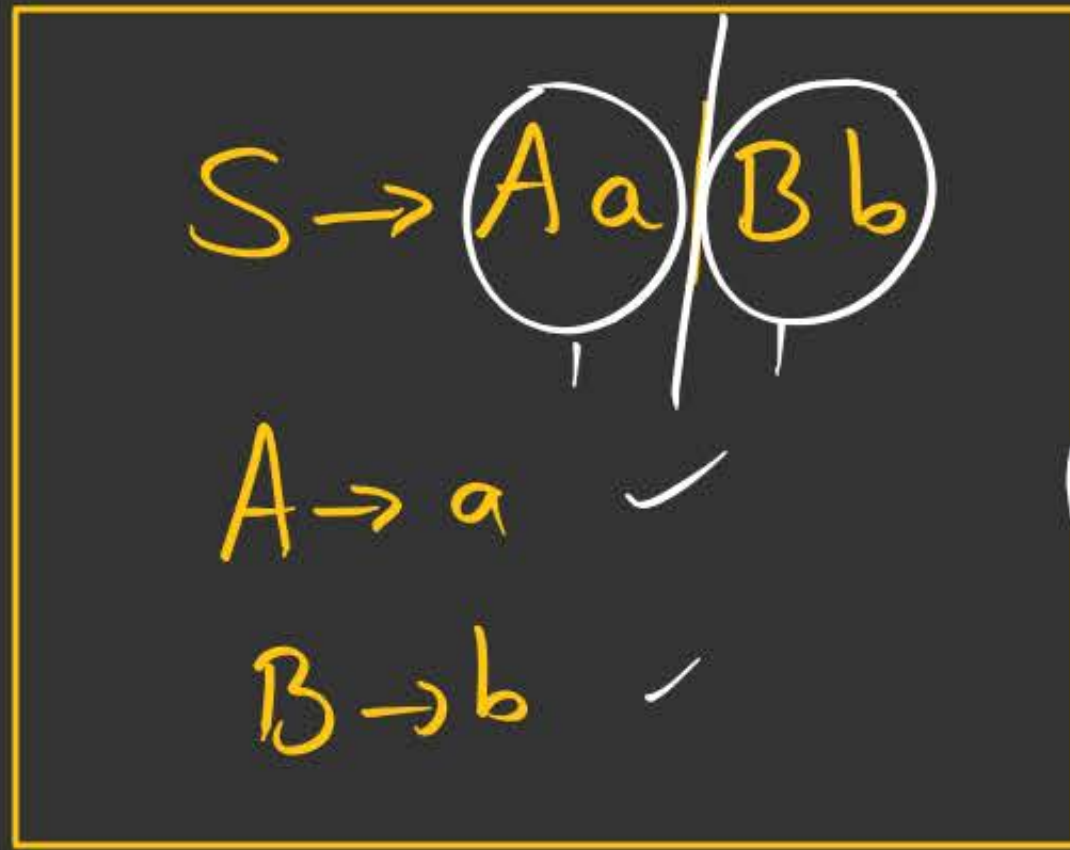
0



②  $S \rightarrow \underline{a}S / \underline{S}b / \epsilon$

3  $\rightarrow$  x  
2  $\rightarrow$  ✓  $\rightarrow$  (CFG)  
1  
0

②



L.L.G

3, 2, 1, 0

Regular Grammar

3  $\rightarrow$  Regular Grammar

2

1

0



$$\textcircled{3} \quad |S| \xrightarrow{4} aAB$$

$$\underline{\underline{aA}} \xrightarrow{2} \underline{\underline{aa}}$$

$$\underline{\underline{bB}} \xrightarrow{2} \underline{\underline{bb}}$$

$$\begin{array}{l} 3 \downarrow \times \\ 2 \downarrow \times \\ 1 \rightarrow \checkmark \\ 0 \rightarrow \end{array}$$

C.S.G ✓

④

$$S \rightarrow aAB$$

$$\underline{aA} \rightarrow a$$

2      1

$$B \rightarrow b$$

3 x

2 x

1 x

0 ✓ → Unrestricted Grammar



what are the possible type number for this grammar

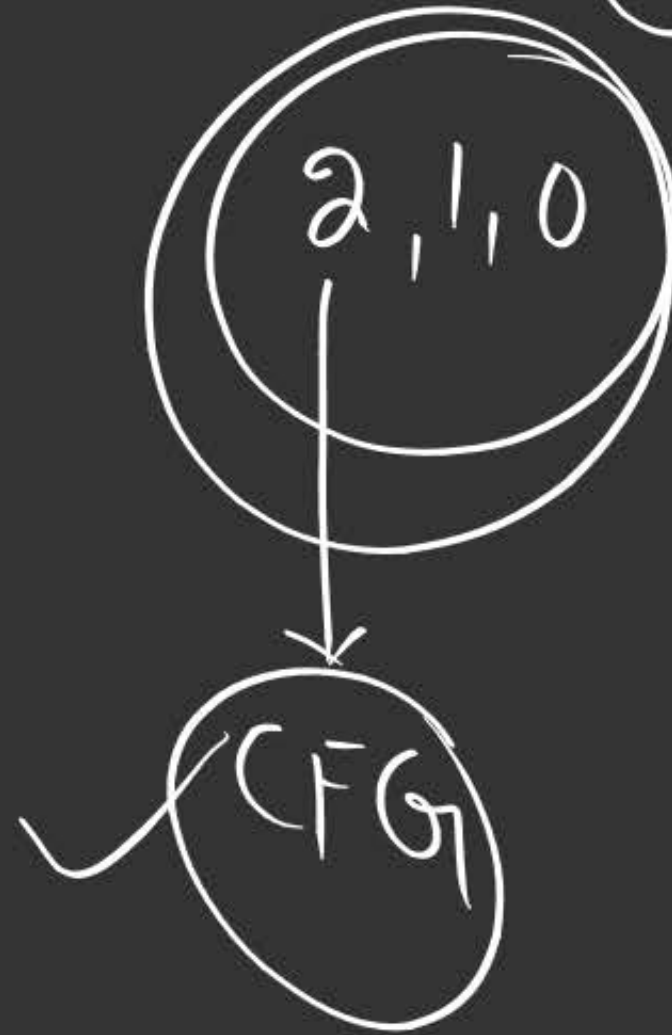
⑤

$S \rightarrow aS / Sb / a$

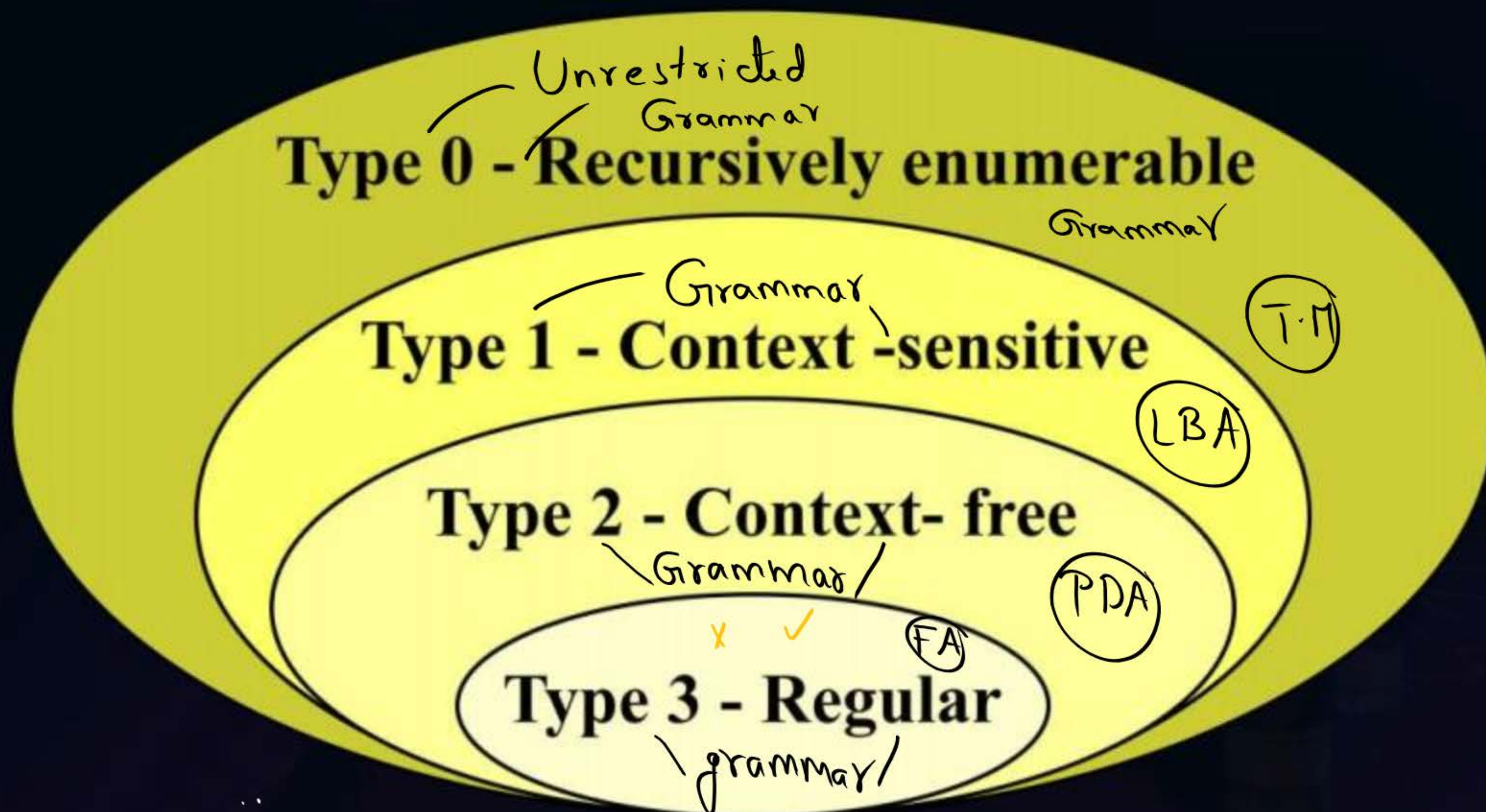
CFG

3 ×

2 ✓



# Chomsky Hierarchy





F.A  $\rightarrow$  Regular

PDA  $\rightarrow$  Regular, CFG <sup>Context free</sup>

LBA  $\rightarrow$  CSG, CFG, Regular

T.M  $\rightarrow$  REL, CSG, CFG, Regular.

Expressive Power :-

No. of languages accepted by Automata

$$\textcircled{T.M.} > LBA > PDA > \textcircled{FA}$$



3, 2, 1, 0

$S \rightarrow aS \mid a$

3 ✓

2 ✓

1 ✓

0 ✓

Ambiguous Grammar: Grammar is ambiguous if

→ There exist a string from grammar that has more than 1 L.M.D

(or)

→ There exist a string from grammar that has more than 1 RMD

(or)

\*\*\*

→ There exist a string from grammar that has more than 1 Parse tree



①

$$E \rightarrow E + E \mid E * E \mid \eta$$

$$\eta + \eta * \eta$$

$$2 + 3 * 4 = 14$$

$$2 + 12 = 14$$

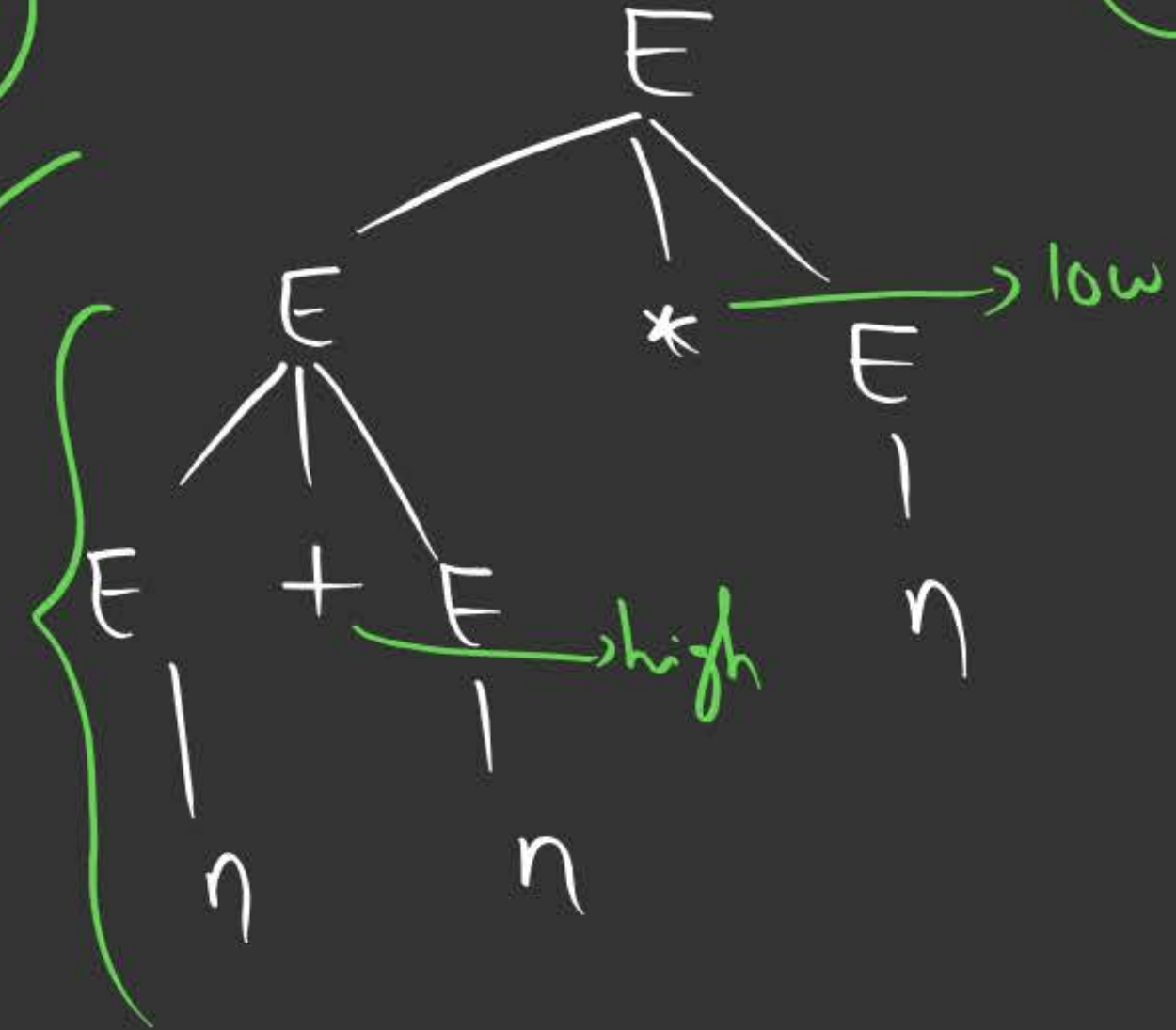
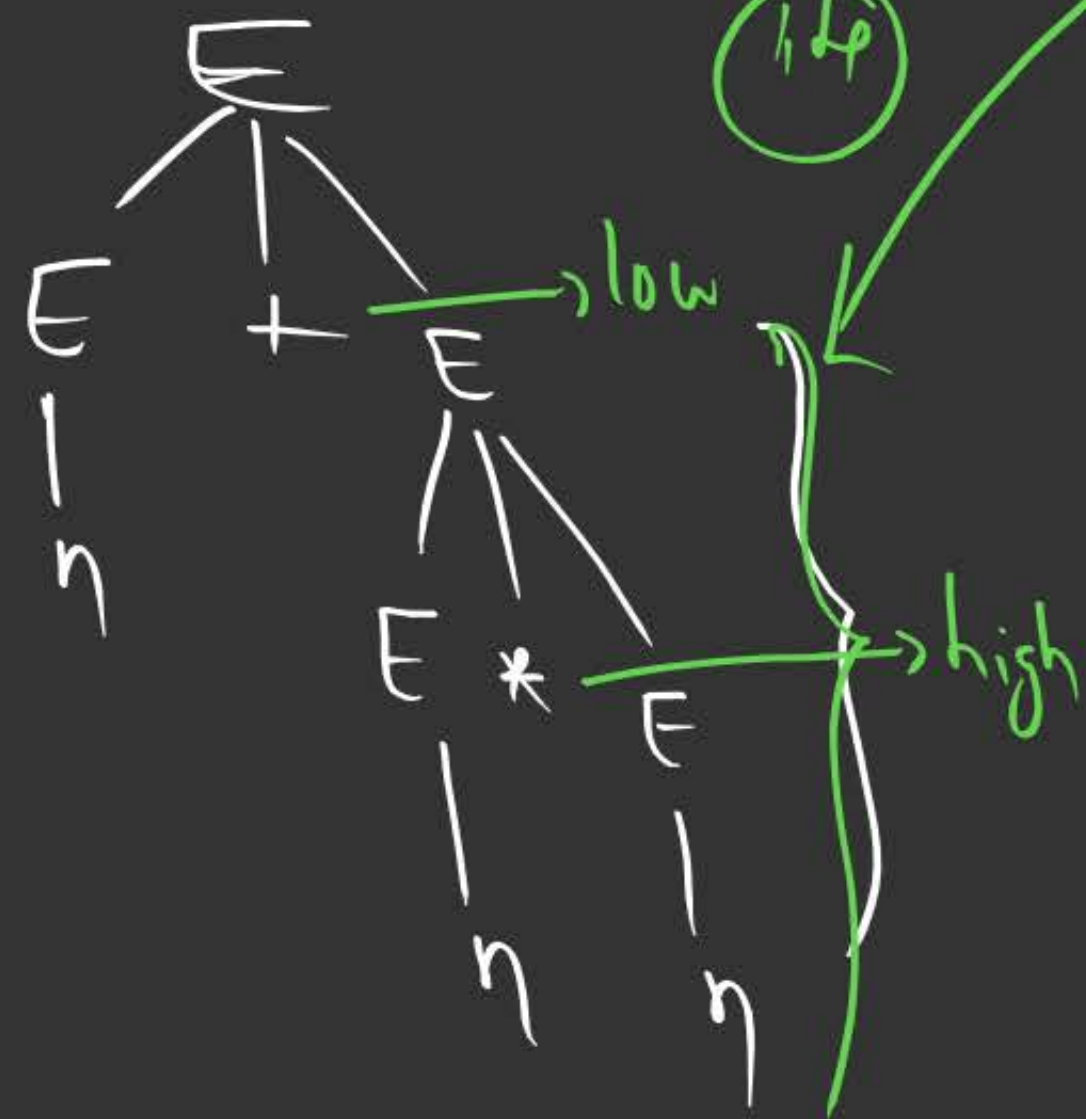
$$14$$

$$\begin{matrix} \eta \\ \eta + \eta \\ \eta * \eta \end{matrix}$$

$$\eta + \eta * \eta$$

$$\begin{matrix} 5 * 4 \\ (2 + 3) * 4 = 20 \end{matrix}$$

$$20$$



②

$S \rightarrow AA$

$A \rightarrow aA$

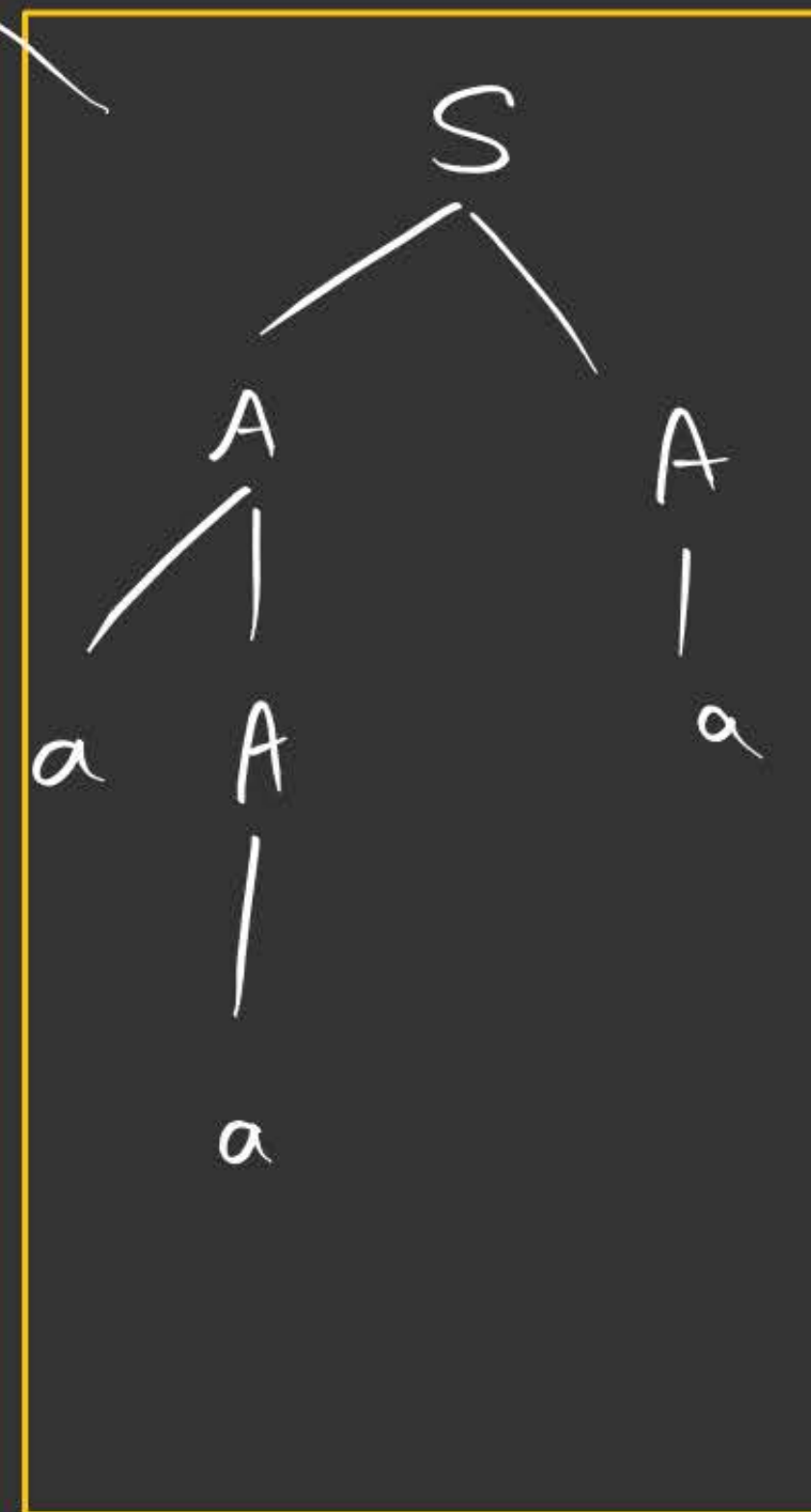
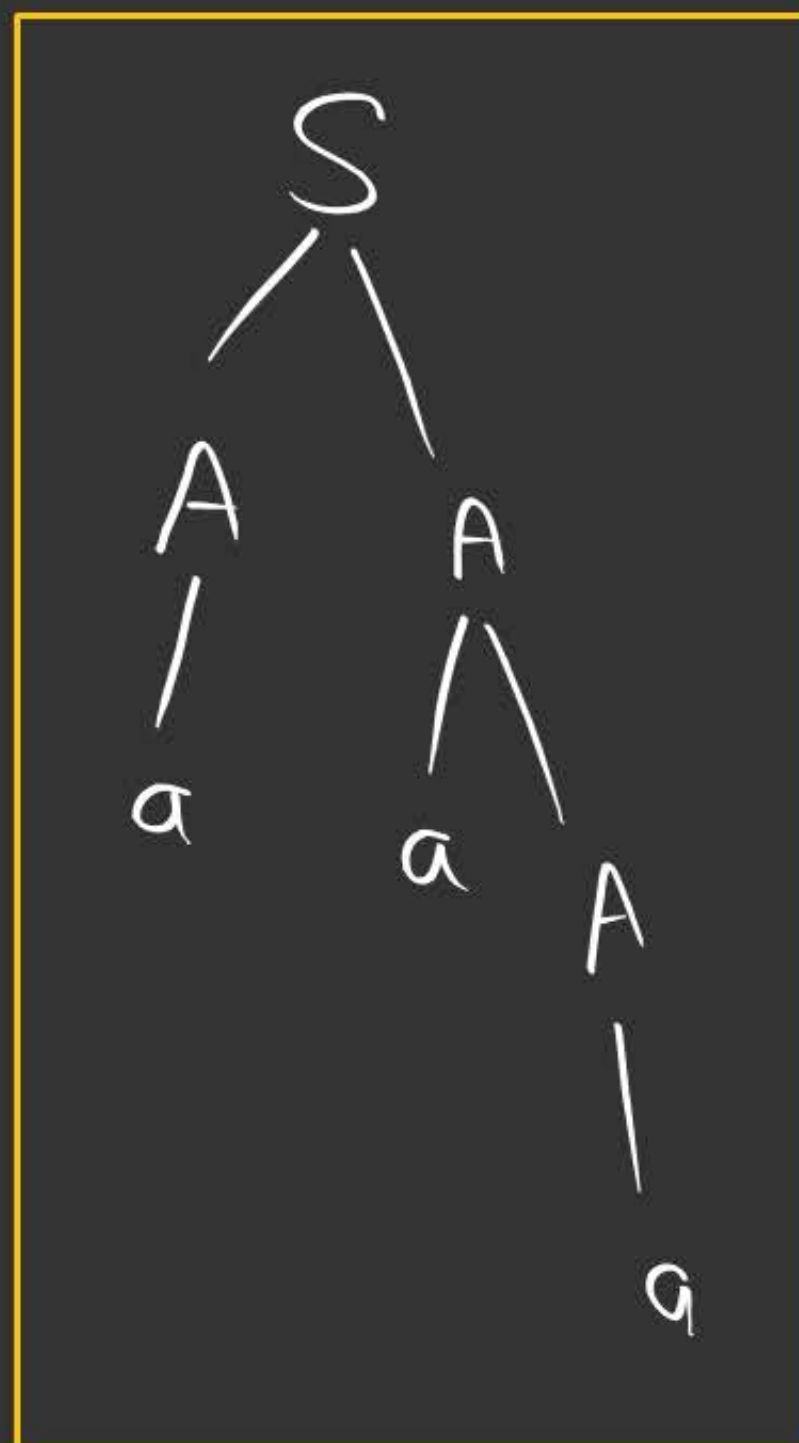
$A \rightarrow a$

{ Ambiguous Grammar }

This grammar is Ambiguous (or) not?

aaa

aaa

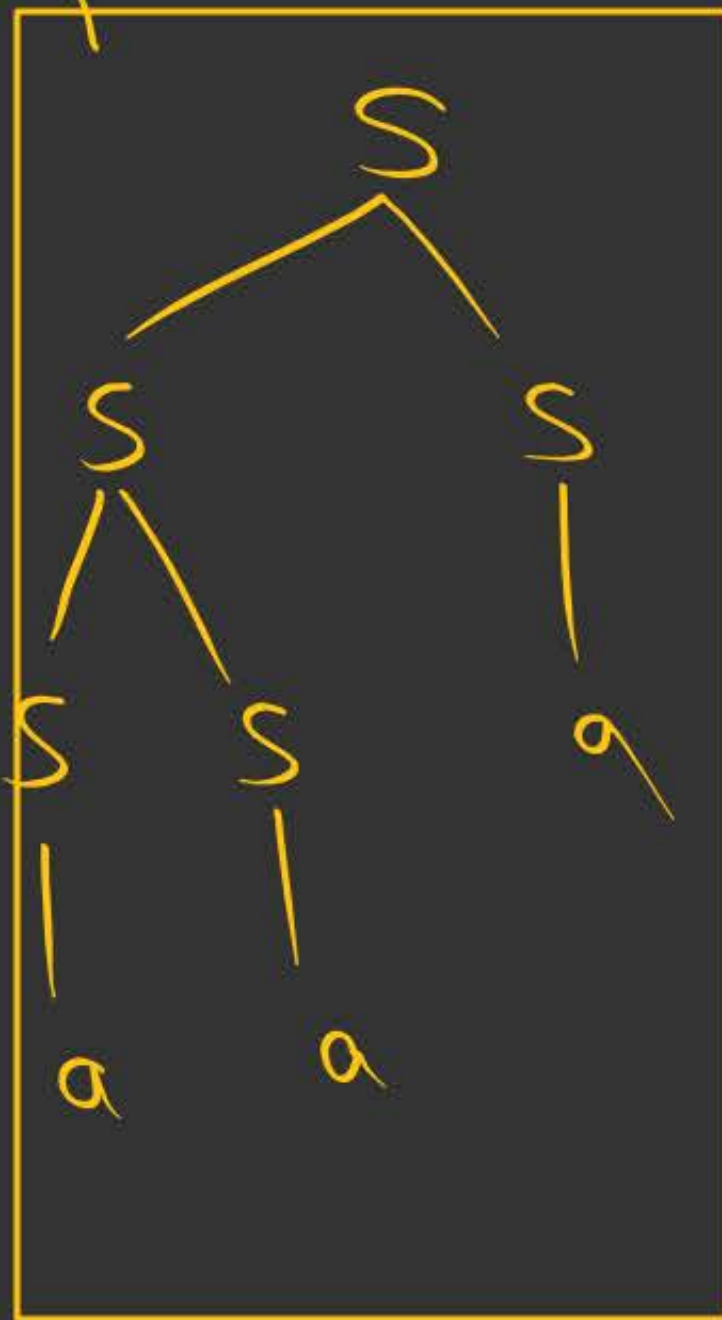




$a, \underline{aa}, \underline{aaaa}$

③  $S \rightarrow \underline{SS} / a$  } Ambiguous

aaaa



① Ambiguous

② Un Ambiguous



(4)

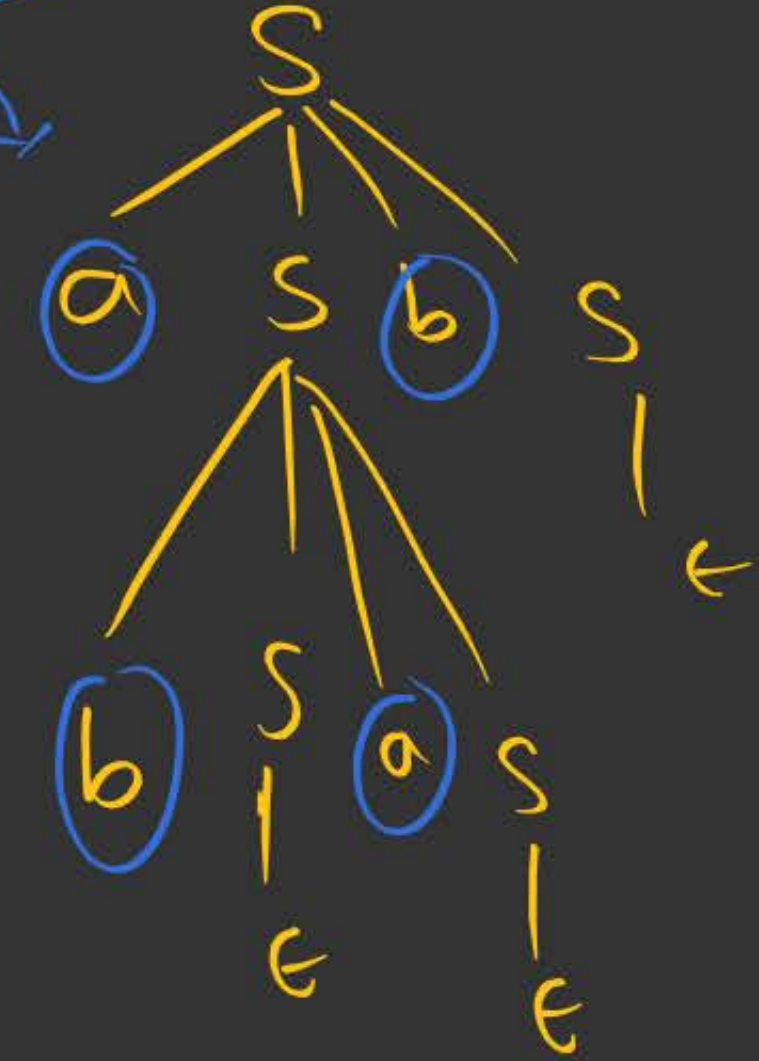
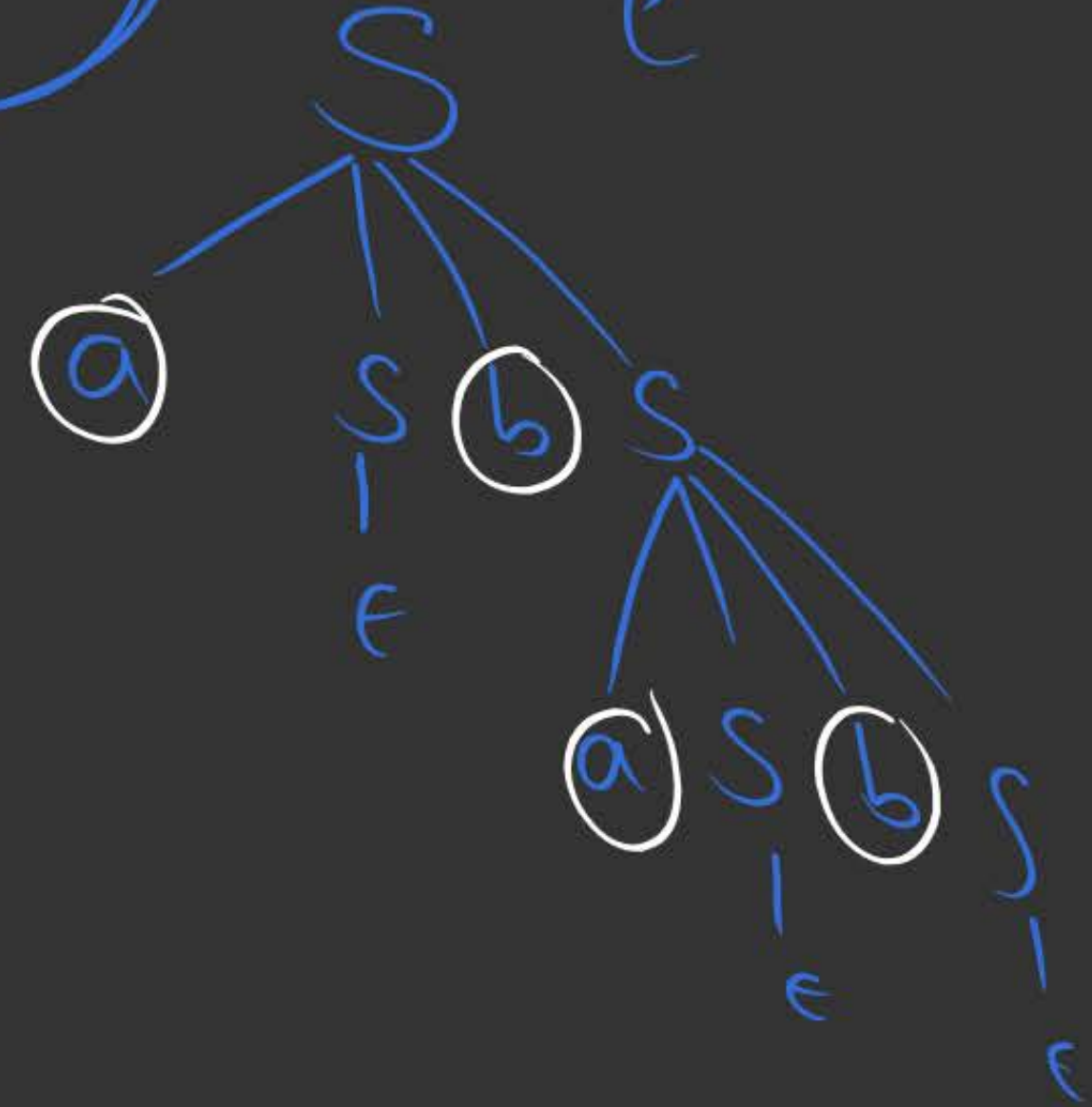
$$S \rightarrow aSbS / bSaS / \epsilon$$

(2, 1, 0)

abab

Ambiguous Grammar

abab





## Elimination of Ambiguity

① Possible

② not possible

⑤

$$S \rightarrow AaAb \mid BbBa$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

$\{\underline{ab}, \underline{ba}\}$

ab



ba



① Ambiguous

~~② Unambiguous~~

⑥

$S \rightarrow AB|BC$

$A \rightarrow BA|a$

$B \rightarrow CC|b$

$C \rightarrow AB|a$

Home work



⑦

$$S \rightarrow aSa / bSb / c$$

Home Work

Ambiguity problem is undecidable problem

No Algorithm exist to check Ambiguity of a  
grammar

# Elimination of Ambiguity from Grammar

Undecidable problem

no solution exist

no algorithm exist



## Unambiguous Grammar

→ Every string derived from grammar has exactly one LMD

(A)

"

"

"

"

"

"

one RMD

(A)

"

"

"

"

"

one parse tree



## 2 mins Summary



Topic

One

Topic

Two

Topic

Three

Topic

Four

Topic

Five

$$L = \{ a^n b^m \mid (n+m) \text{ is odd} \}$$
$$\underline{a(aa)^*(bb)^*} + (aa)^* b(bb)^*$$

$$S \rightarrow aAB \mid AbB$$

$$A \rightarrow aaA \mid \epsilon \quad (aa)^*$$

$$B \rightarrow bbB \mid \epsilon \quad (bb)^*$$



**THANK - YOU**