CS & IT ENGINEERING

Theory of Computation

DFA

Lecture No.- 07

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Recap of Previous Lecture

Topic — ~~Minimization~~ of DFA

Topic — ~~DFA~~ for binary numbers.

length of the string

Topics to be Covered

- **Topic** Finite Automaton & Regular Languages.
- **Topic** Pushdown Automata & Context free Languages.
- **Topic** Turing Machine & Recursive Enumerable Languages.
- **Topic** Undecidability.

Topic : DFA

$\Sigma = \{a, b\}$

Construct the minimal DFA that accept all string a's and b's where

1. Length of string exactly 4. → 6
2. Number of a's length of string atleast 4. → 5
3. Length of string atmost 4. → 6
4. Length of string divisible by 4. → 4
5. Number of a's exactly 5. → 7
6. Number of b's exactly 2. → 4
7. Number of a's divisible by 3. → 3
8. Number of b's not divisible by 4. → 4
9. Length of the string even.

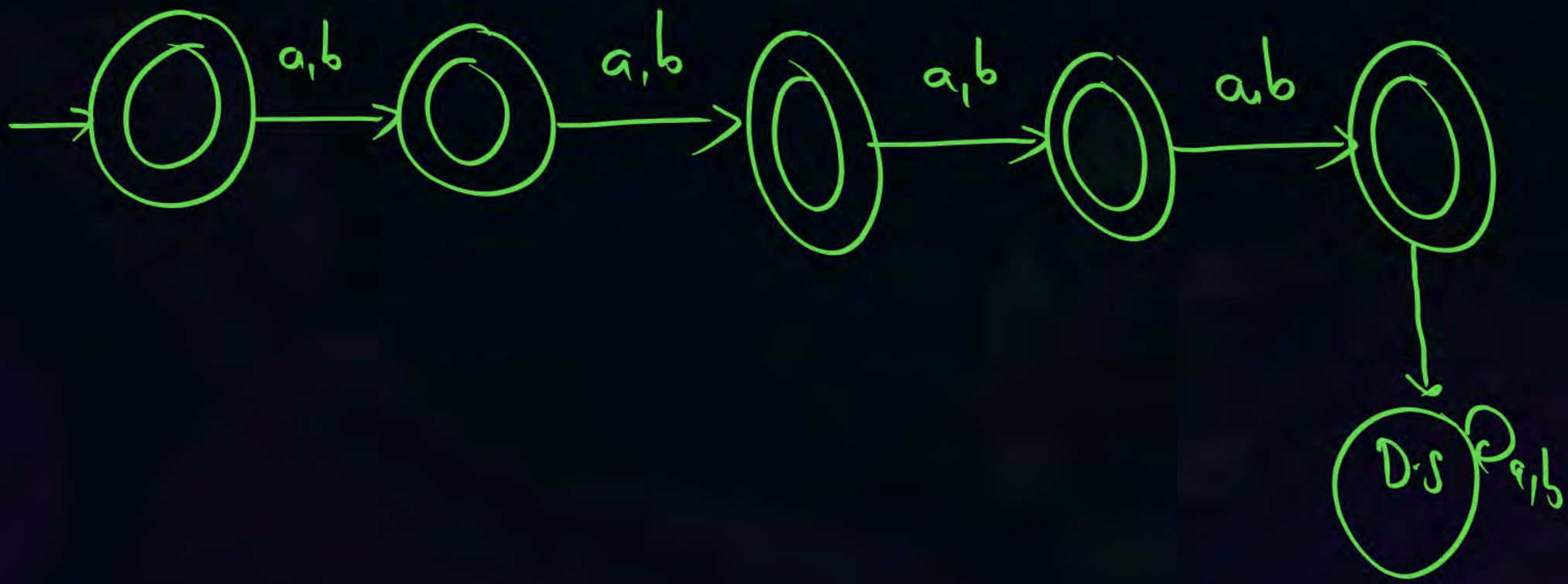
Topic : DFA

#Q. Length of string atleast 4.

Topic : DFA

{0, 1, 2, 3, 4}

#Q. Length of string atmost 4. → 6



Topic : DFA

$\{0, 4, 8, 12, 16 \ldots\}$

#Q. Length of string ~~x~~ not divisible by 4.

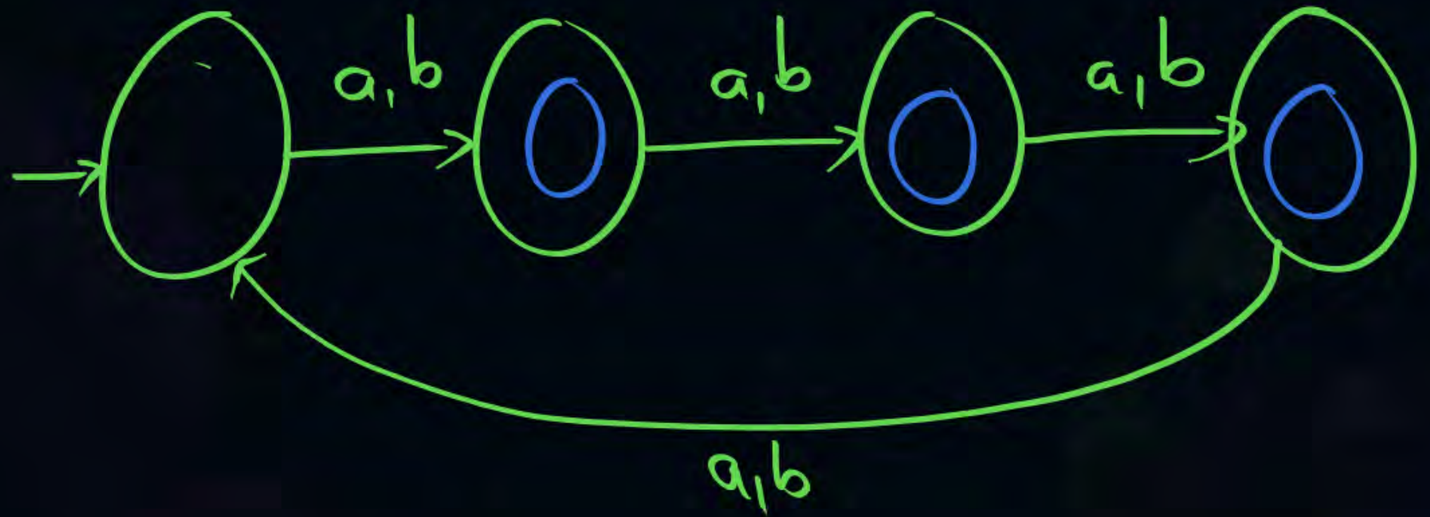
even

$4 \to 4$

$5 \to 5$

$\vdots$

$\eta \to \eta$



Topic : DFA

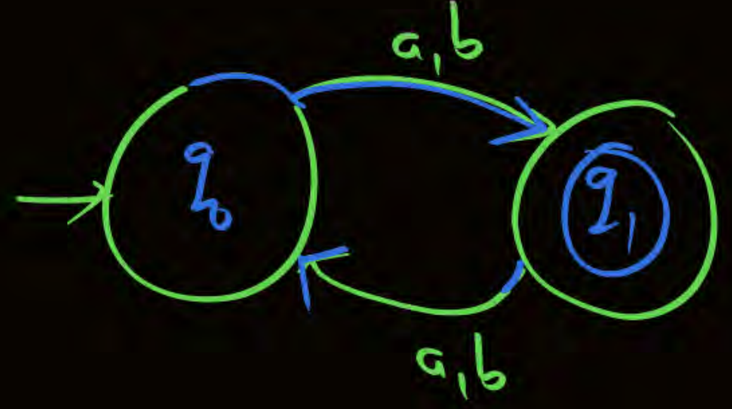
NOTE:

→ Minimal DFA that accept exactly N length string requires (N + 2) states includes dead state.

→ Minimal DFA that accept atleast N length string requires (N + 1) states.

→ Minimal DFA that accept atmost N length string requires (N + 2) states includes dead states.

→ The minimal DFA that accept length of the string divisible by N then requires N states.


a, b
q_0
q_1
a, b

Topic : DFA

$\Sigma = \{a, b\}$

Construct a minimal DFA that accept all string a's and b's. where number of a's divisible by 2 and number of b's divisible by 3

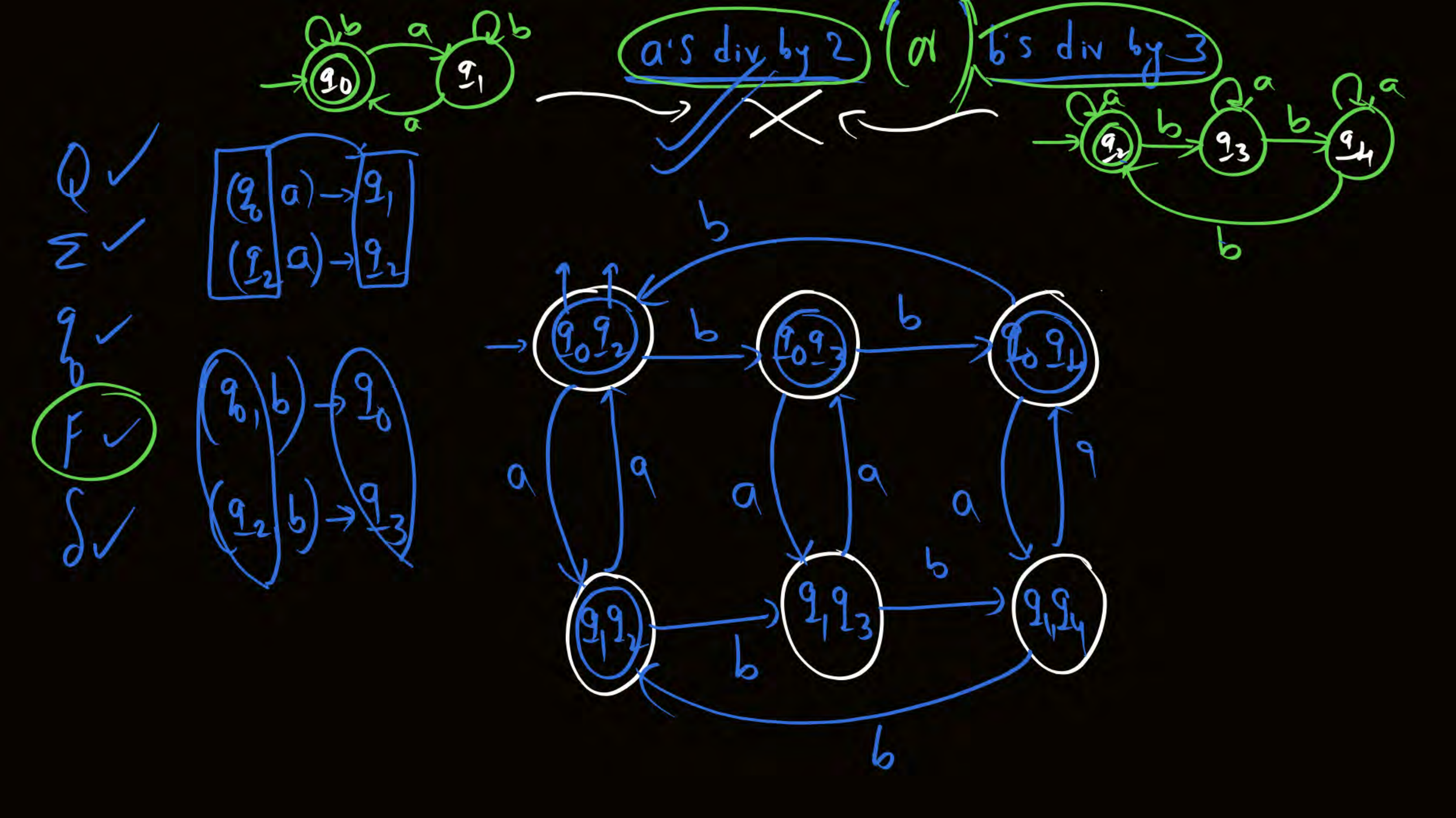
2 (and) 3

(a) 6

(b) 5

(c) 3

(d) 2


a's div by 2
(a)
b's div by 3
Q ✓
Σ ✓
q0 ✓
F ✓
δ ✓
(q0, a) → q1
(q1, a) → q2
(q0, b) → q0
(q2, b) → q3
q0q2
q0q3
q0q4
q1q2
q1q3
q1q4
a
b

DFA minimal states

① #a's even (2) (and)* #b's odd (2) → 4 ✓

② #a's odd (2) (and) #b's odd (2) → 4

③ #a's even (2) ((a|b)^2)^* #b's odd (2) → 4

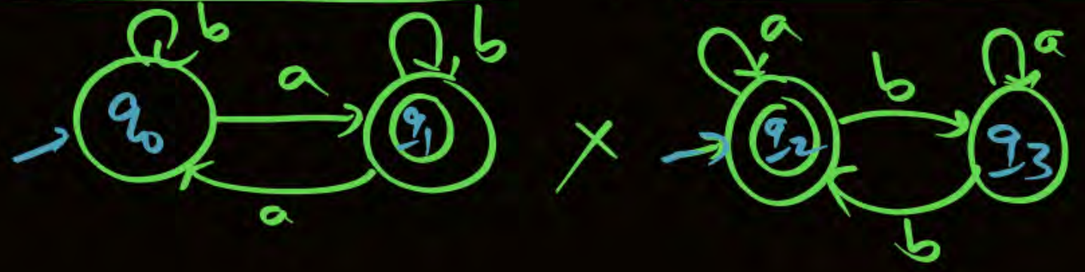
$\Sigma = \{0,1\}$

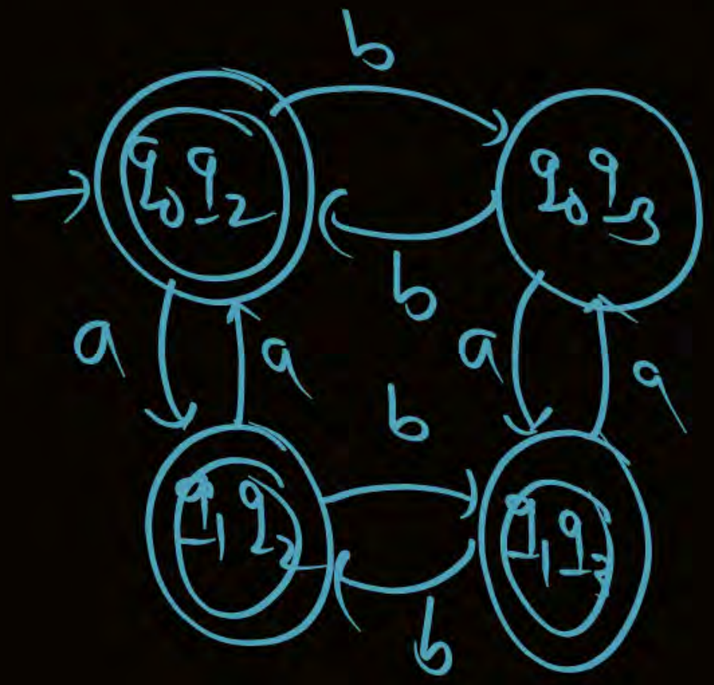
④ #0's exactly 4 and #1's atleast 2 →

$5 * 3 = (15+1) = 16$ ✓

(Q) How many final states are in DFA for the given Lang

no. of a's odd (or) no. of b's even





(a) 1

(b) 2

(c) 3

(d) 4

$\Sigma = \{a, b\}$

LCM(4,6) = 12

(Q) How many states in min DFA that accepts length of the string div by 4 and div by 6.

(Q) no. of a's is div by 4 (and) no. of b's is div by 6

$4 \times 6 = 24$

(a) 24

(b) 12

(c) 10

(d) 6

2 × 4 → { min DFA }

(1) # a's div by 2 (or) # b's div by 4 ⇒ 8 ✓

(2) length of the string div by 2 (or) div by 4 ⇒ 2 ✓

GCD(2,4) ⇒

{0,2,4,6,8---} ∪ {0,4,8,12,16---}

{0,2,4,6,8---} → even → 2 ✓

Topic : DFA

$\Sigma = \{a, b\}$

How many number of state are there with minimum DFA for the following language.

a) Number of a's divisible by 2 and number of b's not divisible by 3 → 6

b) Number of a's divisible by 2 and number of b's atleast 3. → 2 * 4 → 8

c) Number of a's atleast 2 and number of b's atleast 3 → 3 * 4 → 12

d) Number of a's exactly 2 and number b's atleast 2. → 3 * 3 → 9 + 1 → 10

e) Number of b's atmost 3 and number b's exactly 3. →

f) Number of a's not divisible by 2 or number of b's exactly 3. → 4 * 2 = 8 + 1 = 9

4 * 4 = 16 + 1 = 17

a's exactly 2 (and) b's atleast 2

4 * 3

1, 3 * 3

9 + 1 → 10

LCM(2,4) → 4 ✓

length of the string div by 2 (and) 4 → 4 ✓

" " " 2 (or) 4 → 2 ✓

GCD(2,4)

2 and 4 → LCM

2 (and) 4 → GCD

↑

length of the string div by (3) and (4) → MUL

" " " " (4) and (6) → LCM

(and), (or) $\xrightarrow{\text{MUL}}$ GCD(3,4) = 1 → multiplication

(and) (or) $\xrightarrow{\text{LCM}}$ 1 ≠ GCD(4,6) = 2 } LCM(4,6)

Topic : DFA

NOTE:

Number of States of DFA on length conditions

(i) Then in the given condition on length if one number divide other number then number of states of minimal DFA for "and" automata is LCM of given condition.

(ii) Number of states of minimal DFA for "OR" automata is GCD of given condition.

(iii) In the given length condition one number not divide other number then

→ If GCD of given condition is 1 then number of states of 'and' automata OR automata is multiplication of given condition.

Topic : DFA

(iv) The given condition on length one number not divides other and GCD of given condition is not equal to 1 then number of states of 'and' automata, number of states of 'OR' automata is LCM of given condition.

Find the number of stage of minimal DFA for the following matrix.

(i) (Length of the string divisible by 3 or divisible by 6)

GCD (3, 6) = 3

(ii) Length of the string divisible by 4 and divisible by 6

(iii) LCM (4, 6) = 12

~~(iv)~~ (v) Number of a's divisible by 4 AND number of b's divisible by 6.

$4 \times 6 = 24$

Topic : DFA

Find the number of stage of minimal DFA for the following matrix.

4

Length of the string divisible by 3 or divisible by 6

GCD (3, 6) = 3

Length of the string di is by 4 and di by 6

LCM (4, 6) = 12

Number of a's divisible by 4 AND number of b's divisible by 6.

$4 \times 6 = 24$

Topic : DFA

1. Length of the string divisible by 2 and divisible by 3 → LCM (2,6) → 6
2. Length of string divisible by 2 OR divisible by 4. → GCD (2,4) → 2
3. Length of string divisible by 3 and divisible by 4 → {GCD(3,4)=1} 3 * 4 → 12
4. Length of string divisible by 3 OR divisibly by 4 → GCD (3,4)=1 } 3 * 4 → 12
5. Length of string divisible by 6 ~~OR~~ and divisibly by 8 → GCD(6,8)≠1 } LCM(6,8)=24
6. Number of a's divisible by 6 and number of divisible by 8.

6 * 8 → 48

$\Sigma = \{a, b\}$

How many states in min DFA that accepts 8^{th} input symbol from from Right Hand Side is a.

THANK - YOU