COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No. 06

BOOLEAN THEOREMS AND GATES

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Arithematic Gates

Question Discussion Cont.

Question

$y = A \oplus [A + B]$ equal to

(a) $A \oplus B$

(b) $y = A \odot B$

(c) $\bar{A}B$

(d) $A\bar{B}$

$A \oplus (A+B) \neq A \oplus A + A \oplus B \neq 0 + A \oplus B = (A \oplus B)$

$A \oplus (A+B) = \bar{A}(A+B) + A \cdot \overline{(A+B)} = \bar{A}B + A \cdot \bar{A} \cdot \bar{B} = \bar{A}B$

$A \oplus (B \cdot C) \neq (A \oplus B) \cdot (A \oplus C)$

Note: XOR & XNOR do not hold distributive law over OR operation or over 'AND' operation

[**Question**]

$y = A \oplus B \oplus AB$ equal to

(a) AB

(b) $(A + B)$

(c) $\overline{AB}$

(d) $\overline{A + B}$

$$A \oplus B \oplus AB = (A+B)$$

$$A \oplus B \oplus (A+B) = A \cdot B$$

$$A \oplus B \oplus AB = A \oplus \left[\overline{B} \cdot AB + B\,\overline{AB}\right] = A \oplus \left[0 + B(\overline{A}+\overline{B})\right] = A \oplus \overline{A}B$$

$$= \overline{A}\,\overline{A}B + A \cdot \overline{\overline{A}B} = \overline{A}B + A \cdot (A+\overline{B}) = \overline{A}B + A = A + \overline{A}B$$

$$= (A+\overline{A}) \cdot (A+B) = (A+B)$$

$$\checkmark\; A\cdot(B\oplus C) = A\cdot B \oplus A\cdot C = \overline{AB}\,AC + AB\,\overline{AC} = (\bar{A}+\bar{B})AC + AB(\bar{A}+\bar{C})$$

$$= A\bar{B}C + AB\bar{C}$$

$$= A[\bar{B}C + B\bar{C}] = A\cdot(B\oplus C)$$

$$A\cdot(B\odot C) \neq A\cdot B \odot A\cdot C$$

$$A + (B\oplus C) \neq (A+B)\oplus(A+C)$$

$$A\oplus B\oplus AB = A\oplus[B\cdot(1\oplus A)] = A\oplus[B\cdot\bar{A}] = \underline{\underline{A}} \oplus \underline{\bar{A}\,B} \neq \overline{A\oplus AB}$$

$$= \overline{\bar{A}\oplus\bar{A}B}$$

$$= \overline{\bar{A}[1\oplus B]} = \overline{\bar{A}\cdot\bar{B}} = (A+B)$$

$$A\oplus P$$

$$A\oplus\overline{AB}$$

$$A\oplus\bar{P}$$

$$\overline{A\oplus P} = \bar{A}\oplus P$$

$$A\odot AB \neq A(1\odot B) \neq AB$$

$$= \bar{A}(\overline{AB}) + AAB$$

$$= \bar{A}(\bar{A}+\bar{B}) + AB = \bar{A} + AB = (\bar{A}+B)$$

$$A \oplus B \oplus \bar{A}B = \overline{\bar{A} \oplus B \oplus \bar{A}B} = \overline{P \oplus Q \oplus P \cdot Q} = \overline{P+Q} = \overline{\bar{A}+B} = A \cdot \bar{B}$$

- $A \oplus B \oplus \bar{A}\bar{B} = \bar{A} \oplus \bar{B} \oplus \bar{A} \cdot \bar{B} = \bar{A} + \bar{B} = \overline{A \cdot B}$

$$= A \oplus B \oplus \overline{A+B} = \overline{A \oplus B \oplus (A+B)} = \overline{A \cdot B}$$

- $\bar{A} \oplus B \oplus \underline{\underline{A\bar{B}}} = \bar{\bar{A}} \oplus \bar{B} \oplus A\bar{B} = P \oplus Q \oplus PQ = (A+\bar{B}) = \overline{\bar{A} \cdot B}$

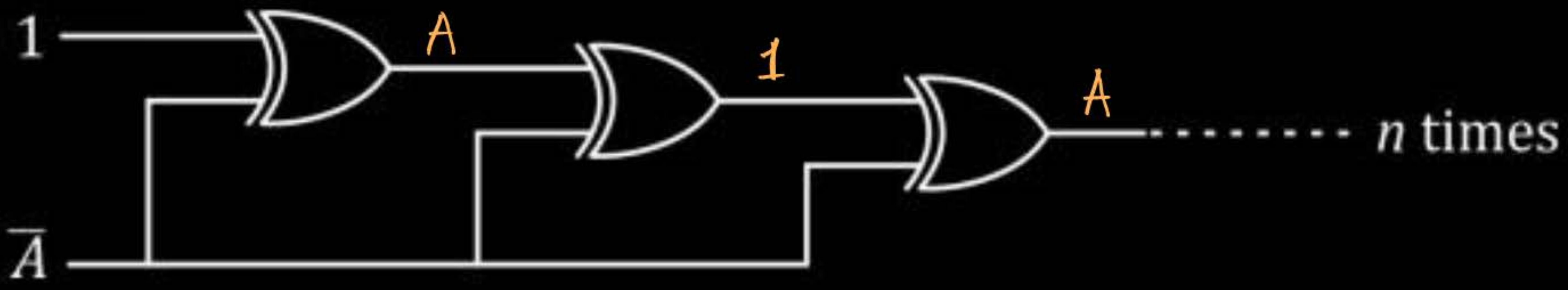
$$= A \oplus \bar{B} \oplus A\bar{B}$$

$$= A \oplus B \oplus \overline{A\bar{B}} = A \oplus B \oplus (\bar{A}+B) = \overline{\bar{A} \oplus B \oplus (\bar{A}+B)} = \overline{\bar{A} \cdot B} = (A+\bar{B})$$

- $A \oplus B \oplus (A+\bar{B}) = \overline{A \oplus \bar{B} \oplus (A+\bar{B})} = \overline{A \oplus B \oplus \overline{(A+\bar{B})}}$

$$= \overline{A \cdot \bar{B}} = (\bar{A}+B)$$

Question



Then which of the following is true :

(a) Output is A if n is even

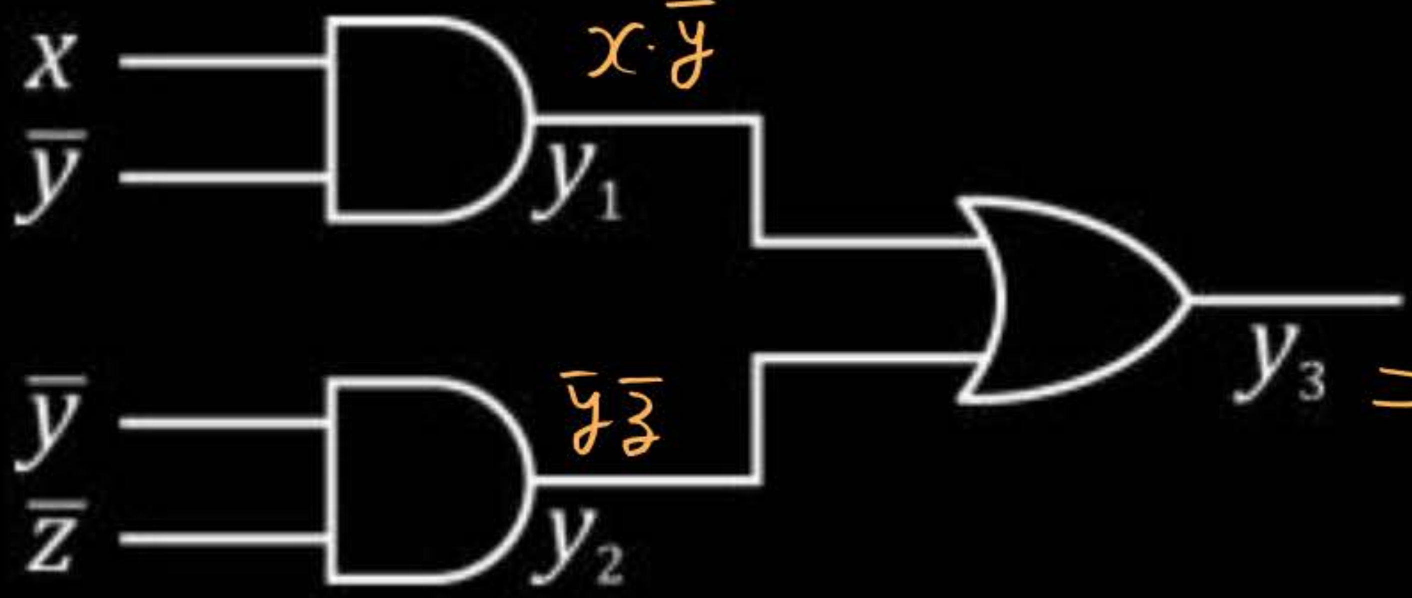
(b) Output is $\bar{A}$ if n is even

(c) Output is A if n is odd

(d) Output is 1 if n is odd

[Question]

A digital circuit is implemented as :



$= x\bar{y} + \bar{y}\bar{z} = (y_1 + y_2)$

Then output $y = y_1 \oplus y_2 \oplus y_3$ will be $= y_1 \oplus y_2 \oplus (y_1 + y_2) = y_1 \cdot y_2 = x\bar{y}\ \bar{y}\bar{z}$

$= x\bar{y}\bar{z}$

(a) $x\bar{y} + \bar{y}\bar{z}$

(b) $x \oplus y \oplus z$

(c) $x\bar{y}\bar{z}$

(d) $x + \bar{y} + \bar{z}$

Q. $\bar{A}(A\bar{B}+\bar{A}+C)+\bar{B}(B\bar{C}+\bar{B}C+\bar{B}\bar{C})+AB\bar{C}(B+\bar{A}B+\bar{A}C)$

$= \bar{A}+\bar{B}+A\bar{C}\cdot B\,(B+\text{anything}) = \bar{A}+\bar{B}+A\bar{C}B = \bar{A}+\underline{\underline{\bar{B}}}+A\underline{\underline{B}}\bar{C}$

$= (\bar{A}+\bar{B})+AB\bar{C}$

$= P+\bar{P}\bar{C} = (P+\bar{P})\cdot(P+\bar{C})$

$= P+\bar{C} = \bar{A}+\bar{B}+\bar{C}$

$= \overline{ABC}$

$P=(\bar{A}+\bar{B})$

$\bar{P}=\overline{(\bar{A}+\bar{B})}=(A\cdot B)$

$= \bar{A}+\bar{B}+B\cdot A\bar{C}$

$= \bar{A}+(\bar{B}+B)\cdot(\bar{B}+A\bar{C})$

$= \bar{A}+\bar{B}+A\bar{C}$

$= \bar{B}+(\bar{A}+A)(\bar{A}+\bar{C})$

$= \bar{B}+\bar{A}+\bar{C}$

$= \overline{A\cdot B\cdot C}$

Q. $AB + BC + \bar{A}B\bar{C}D = B(A+C) + \bar{A}B\bar{C}D = B\left[(A+C) + \bar{A}\bar{C}D\right]$

$= B\left[P + \bar{P}D\right] = B\left[(P+\bar{P})\cdot(P+D)\right] = B\left[A+C+D\right] = AB + BC + BD$

$A + C = P$

$\overline{A+C} = \bar{P}$

$\bar{A}\bar{C} = \bar{P}$

Q. $(A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})(\overline{A}+\overline{B}+\overline{C})$

$= \left[(A+\overline{B}+\overline{C}) + D\cdot\overline{D}\right]\left[\overline{A}+\overline{B}+\overline{C}\right]$

$= (A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+\overline{C})$

$= \left[(\overline{B}+\overline{C}) + A\cdot\overline{A}\right] = \overline{B}+\overline{C} = \overline{B\cdot C}$

Q.

- $(A+\bar{B})(\bar{B}+C)(\bar{A}+\bar{C}) = (\bar{B}+AC)(\bar{A}+\bar{C}) = (\bar{B}+P)\,\bar{P} = \bar{B}\bar{P} = \bar{B}(\bar{A}+\bar{C}) = \bar{A}\bar{B}+\bar{B}\bar{C} = \overline{A+B} + \overline{B+C}$

$= \bar{A}\bar{B}+\bar{B}\bar{C} + 0 + 0$

- $\bar{A}+ABC = (\bar{A}+A)(\bar{A}+BC) = \bar{A}+BC$

- $\underline{\bar{A}\bar{B}} + \bar{B}C + \bar{A}\bar{C} = \bar{B}C+\bar{A}\bar{C}$

redundant

- $A\bar{B}C+BC = C[A\bar{B}+B] = C(B+A\cdot\bar{B}) = C((B+A)\cdot(B+\bar{B})) = BC+AC$

Q. $\bar{A} \oplus \overline{(A+BC)} = \bar{A} \oplus \bar{P} = A \oplus P = A \oplus (A+BC) = \bar{A}\,(A+BC) + A \cdot \overline{(A+BC)}$

$= \bar{A}BC + A \cdot \bar{A} \cdot \overline{BC}$

$= \bar{A}BC$

a. ABC

b. $A\overline{BC}$

c. $\bar{A}BC$ ✓

d. None of them

$$\bar{A} \oplus \bar{A} \cdot \overline{BC} = \bar{A}\left[1 \oplus \overline{BC}\right] = \bar{A}\,\overline{\overline{BC}} = \bar{A}BC$$

Q. $\bar{A}B \oplus BC \oplus \overline{ABC}$

a. $\bar{A}+B+\bar{C}$

b. $A+\bar{B}+C$ ✓

c. $\bar{A}+\bar{B}+\bar{C}$

d. $\bar{A}B+BC$

$$\Rightarrow \overline{\bar{A}B \oplus BC \oplus ABC}$$

$$= \overline{B\left[\bar{A} \oplus C \oplus AC\right]}$$

$$= \overline{B\left[\overline{A \oplus C \oplus AC}\right]}$$

$$= \overline{B\cdot\left[\overline{A+C}\right]} = \overline{B\cdot \bar{A}\cdot\bar{C}} = \bar{B}+A+C$$

$$= \bar{B}+A+C$$

$$= A+\bar{B}+C$$

$$\underline{AB} \oplus \underline{BC} \oplus \underline{ABC} = AB \oplus BC \oplus AB\cdot BC = AB+BC = B(A+C)$$

Q. $AB \oplus BC = y \Rightarrow y = B\cdot(A \oplus C) \longrightarrow 3$ unit

AND $\longrightarrow$ 1 unit ✓

XOR $\longrightarrow$ $\underline{\underline{2}}$ unit ✓

$A \oplus C = R \longrightarrow$ 2 unit

$B\cdot R \longrightarrow$ 1 unit

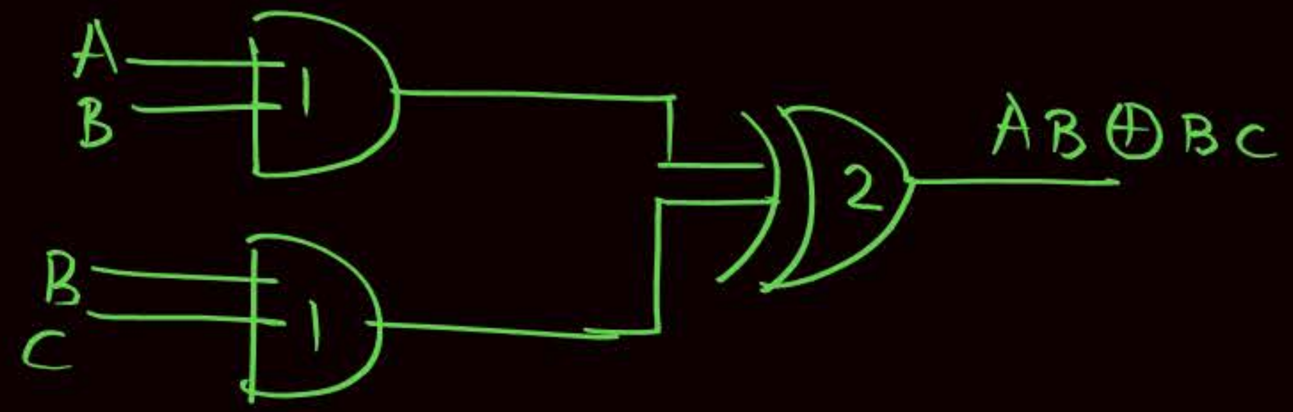
$\}$ 3 unit

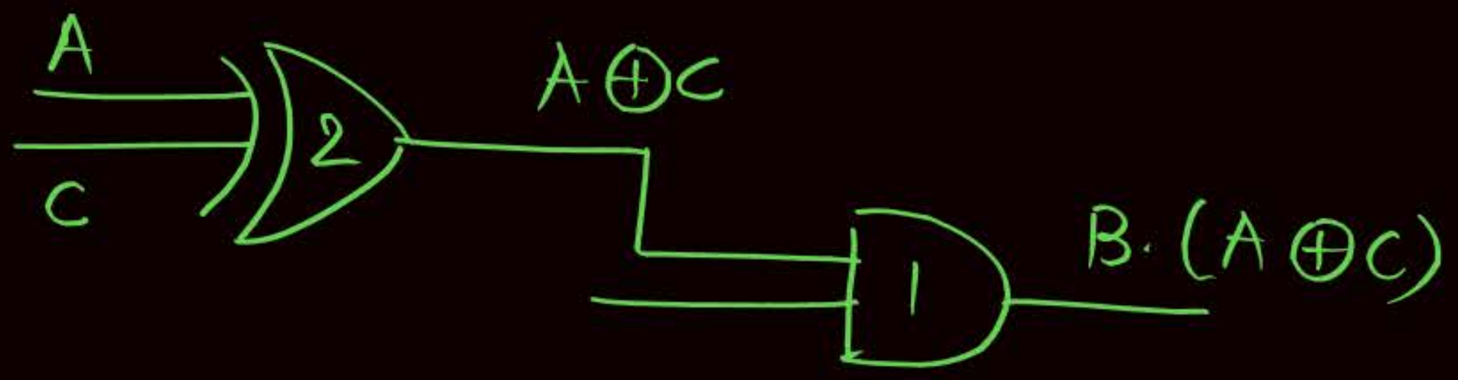
$P = A\cdot B \longrightarrow$ 1 unit

$Q = B\cdot C \longrightarrow$ 1 unit

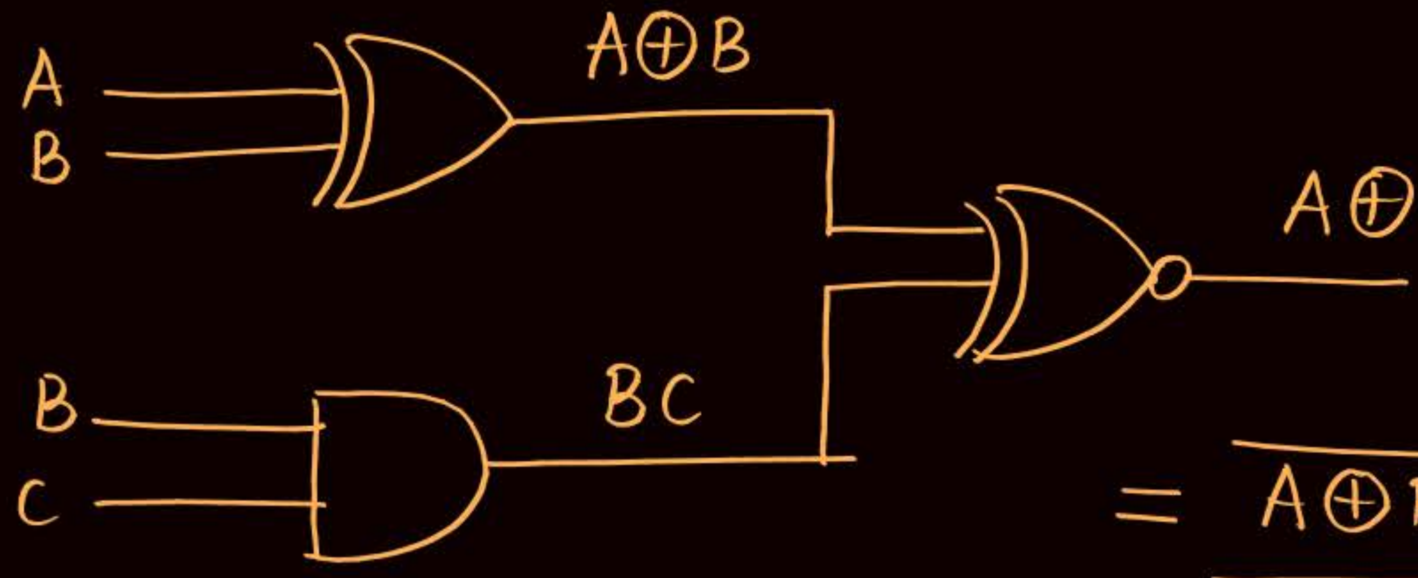
$P \oplus Q \longrightarrow$ 2 unit

$\}$ 4 unit





Q.



$A\oplus B \odot BC = y$

$= \overline{A\oplus B \oplus BC}$

$= \overline{A\oplus [B\oplus BC]}$

$= A\oplus [B\overline{C}]$

4

$A \quad B \quad C \quad B\overline{C} \quad A\oplus B\overline{C}$

$A\oplus B\overline{C}$ → 4 times '1'

⇒ → 4 times '0'

2 Minute Summary

→ Arithematic gates

Thank you

GW

Soldiers !