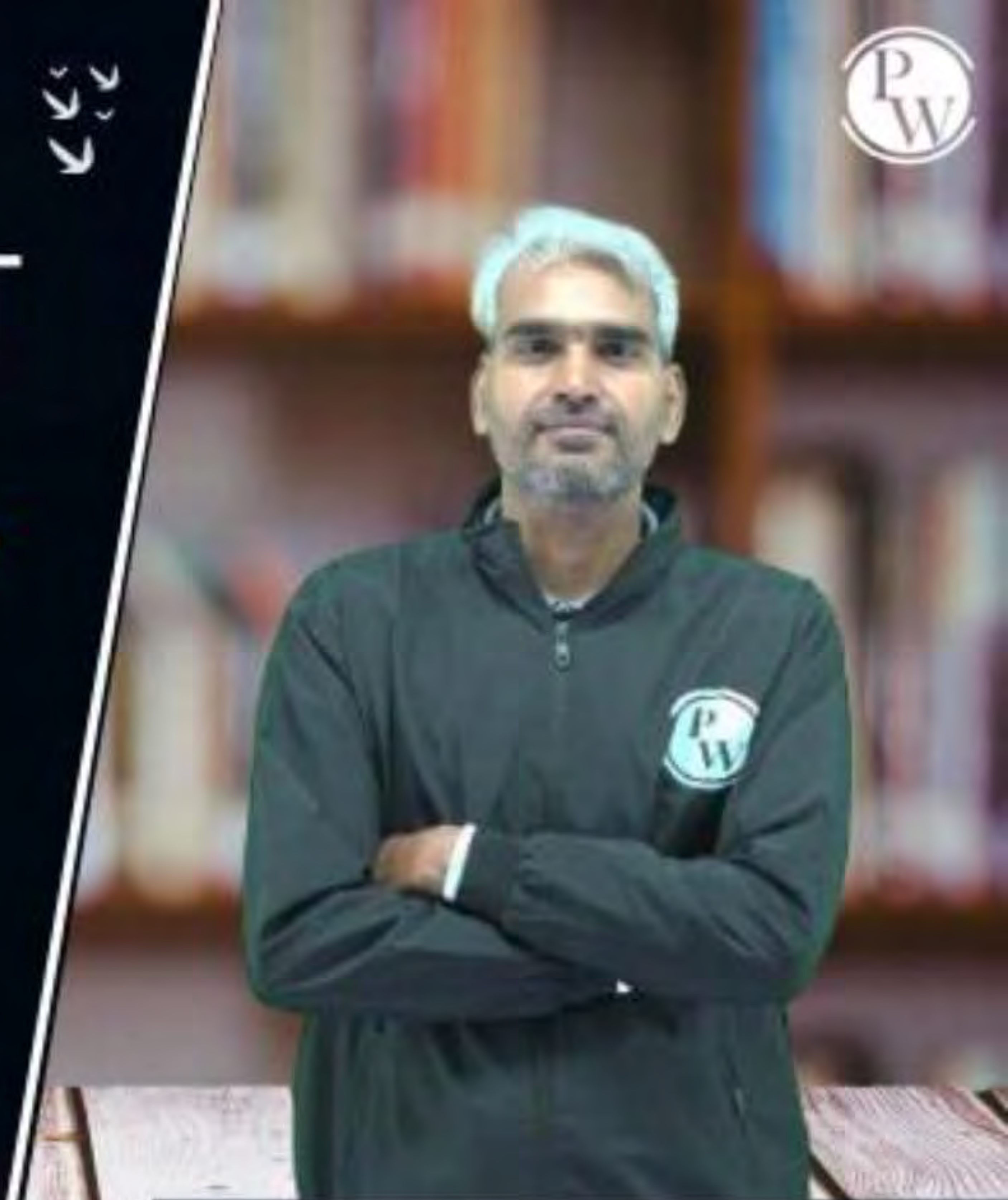
Computer Science & IT

Discrete Mathematics

Graph Theory

Lecture No. 09



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Recap of Previous Lecture

k-chromatic

Topic Vertex Coloring and Chromatic Number

Topic Welsh Powell's Algorithm

Uses 'm' Colors

$\chi(G) \ngtr m$

$\chi(G) \leq m$

Slide

Topics to be Covered

- **Topic** Matching
- **Topic** Maximal matching & maximum matching
- **Topic** Perfect matching
- **Topic** Matching number

Slide

Topic : Matching

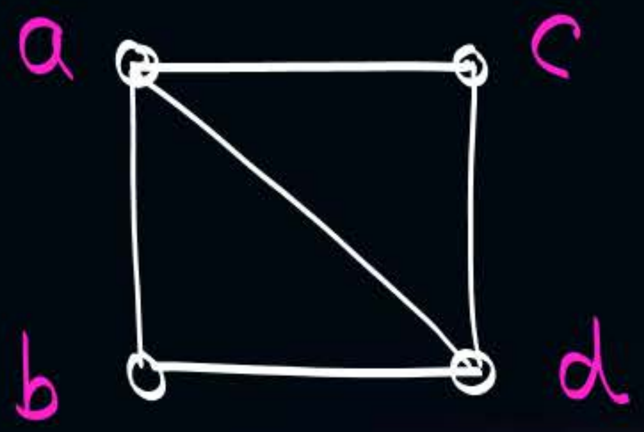
must contain all vertices of graph G.

A subgraph M of graph G is called a matching of graph G, if all the vertices of graph G are incident with at most one edge in subgraph M,

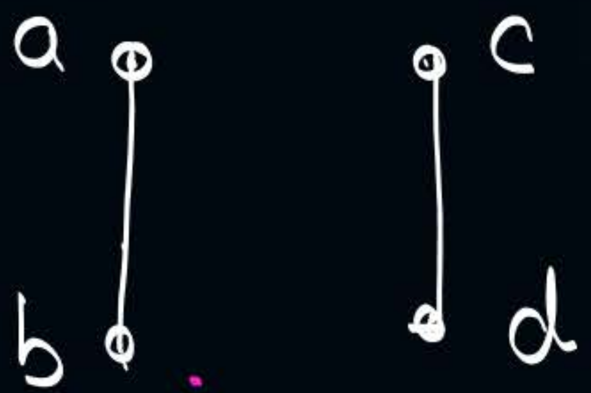
i.e. in a matching $deg(v) \leq 1, \forall v \in G$

Topic : Matching

eg:



G



M_1

M_2

M_3

M_4

M_5

M_1, M_2, M_3, M_4 & M_5. all are matching of graph G.

These may be some more matchings

Note:

① In a matching of graph G if degree of a vertex V is = 1 then vertex 'V' is called a matching vertex.

② In a matching of graph G if degree of a vertex V is = 0 then vertex 'V' is called a non-matching vertex

③ In a matching no two edges should be adjacent.

* Maximal & Minimal, are generally used w.r.t. Property

* Maximum & minimum are generally used w.r.t. number

Topic : Maximal Matching

A matching of graph G is said to be maximal if no other edge of graph G can be added to the matching without destroying its property of being a matching

In the above example M_1, M_2 & M_3 are maximal matching

Related to Number

Topic : Maximum Matching

Largest Maximal Matching

* A matching of graph G with maximum number of edges is called maximum matching of graph G.

{ There may be more than one maximum matching for a graph G }

* In the above example M_1 & M_2 are maximum matching

Note: ① Every maximum matching is also a maximal matching but every maximal matching need not be a maximum matching { eg: M_3 }

Topic : Matching Number

Matching No. of graph G = Number of edges in any one of the Maximum matching of graph G.

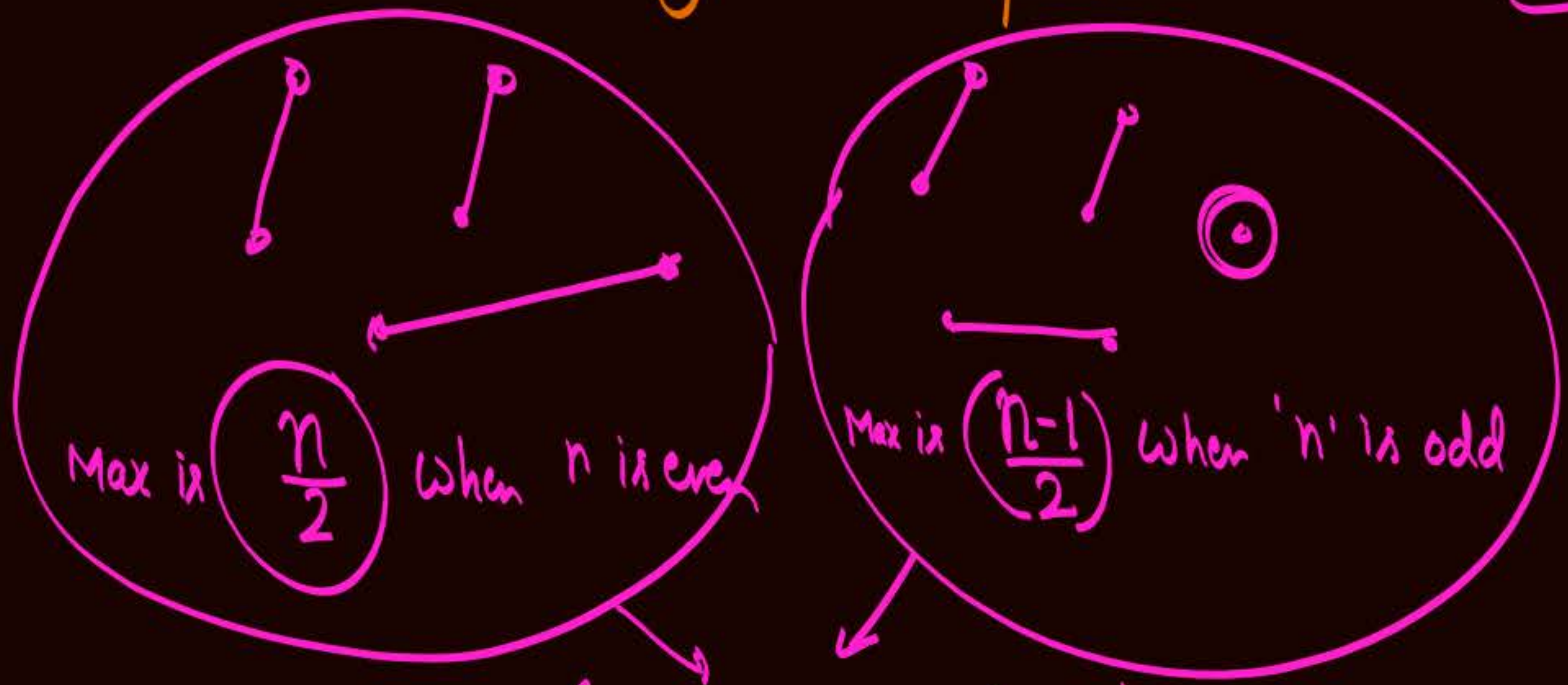
e.g. In above example,

Matching no. of graph G = 2

Note: In a matching no two edges can be adjacent,

$\therefore$ In a graph G with 'n' vertices

Matching number of graph G $\leq \underbrace{\left\lfloor \frac{n}{2} \right\rfloor}_{\text{it is the upperbound on Matching No.,}}$

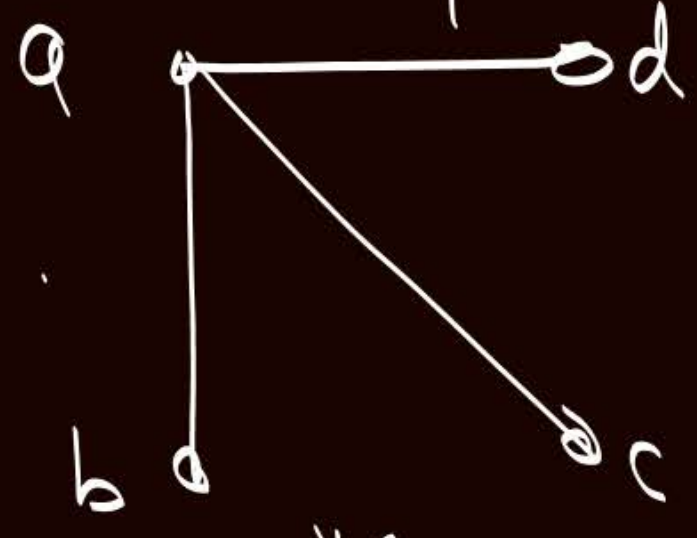


for any random 'n'

Max possible will be $\left\lfloor \frac{n}{2} \right\rfloor$

* For a graph G with 'n' vertices, matching number need not be $\lfloor \frac{n}{2} \rfloor$ {i.e it may be less than $\lfloor \frac{n}{2} \rfloor$}

eg. Consider the following graph



"G"

$n = 4$

$\lfloor \frac{n}{2} \rfloor = 2$

{ Matching no. of the graph $= 1 < \lfloor \frac{4}{2} \rfloor$

{We can never choose two or more edges in the given graph such that they are not adjacent to each other} }

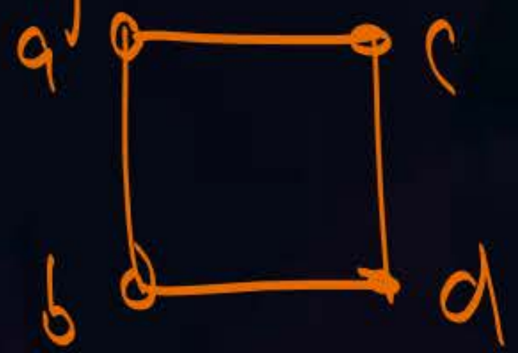
Topic : Perfect Matching

* A matching of graph G is said to be perfect matching if degree of every vertex in that matching is '1'

i.e. $\deg(v) = 1, \forall v \in G$

{ i.e. in a perfect matching of graph G $\deg(v) = 1, \forall v \in G$ }

For the graph



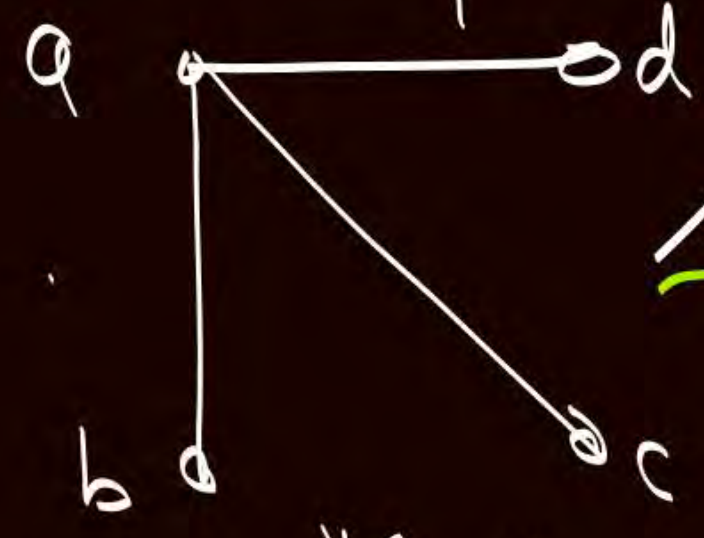
of above eg. M_1 & M_2 are perfect matching

Note :-

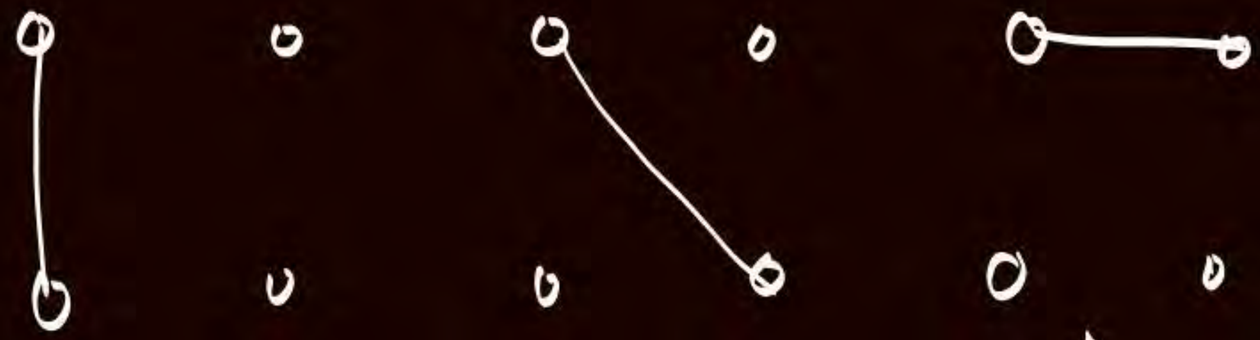
1. Every Perfect matching is a maximum matching, but every maximum matching need not be Perfect matching

2. Perfect matching need not exist for every graph

Consider the following graph



"G"



M_1 M_2 M_3

all are maximum matching of graph G

No perfect matching exist for this graph

Topic : Perfect Matching

(1) In a graph G, if the number of vertices are odd then perfect matching can never exist for graph G.

(2) If perfect matchings exists for graph G, then number of vertices in graph G are even, but converse of the statement need not be true

(3) If no. of vertices in graph G are even, then perfect matching may or may not exist for graph G.

Note:

If perfect matching exists for a graph G with 'n' vertices,

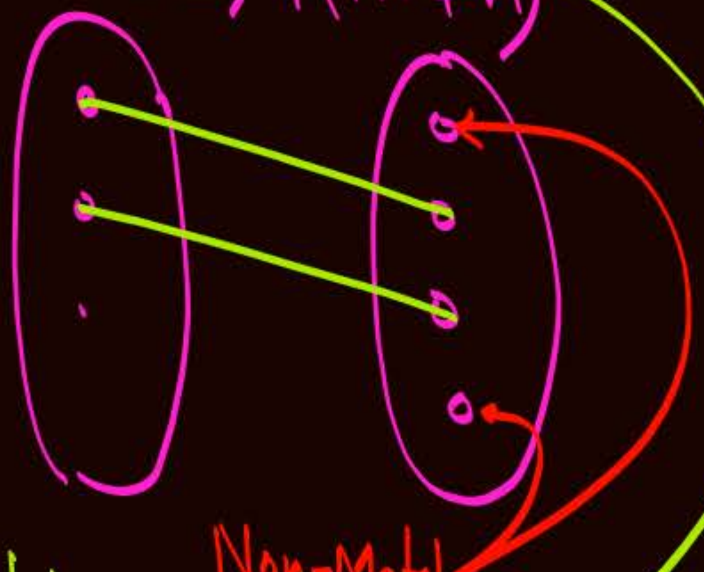
then Matching No. of graph $G = \frac{n}{2}$

Note: ① For a complete graph K_n perfect matchings exist if and only if 'n' is even

{ i.e. for K_{2m} perfect matching always exist }

Note ②: In a complete bipartite graph $K_{m,n}$ perfect matching exists if and only if $m = n$

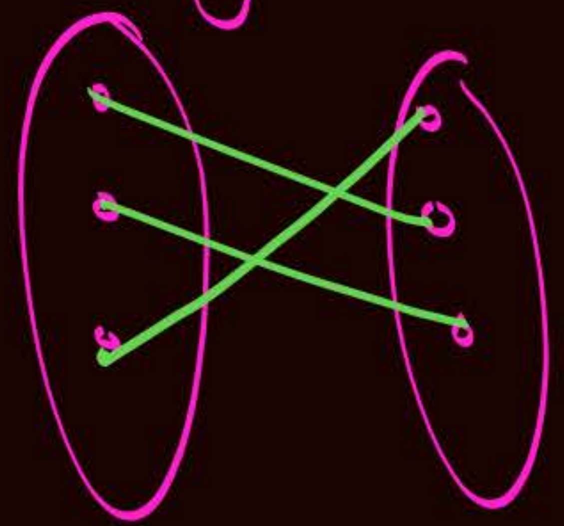
Matching w.r.t. $K_{2,4}$ $(m \neq n)$



Non-Matching vertices.

It is a Maximum Matching w.r.t $K_{2,4}$ but not a Perfect Matching

Matching w.r.t. $K_{3,3}$ $\{m = n\}$



{ Every vertex is a matching vertex, ∴ Perfect Matching }

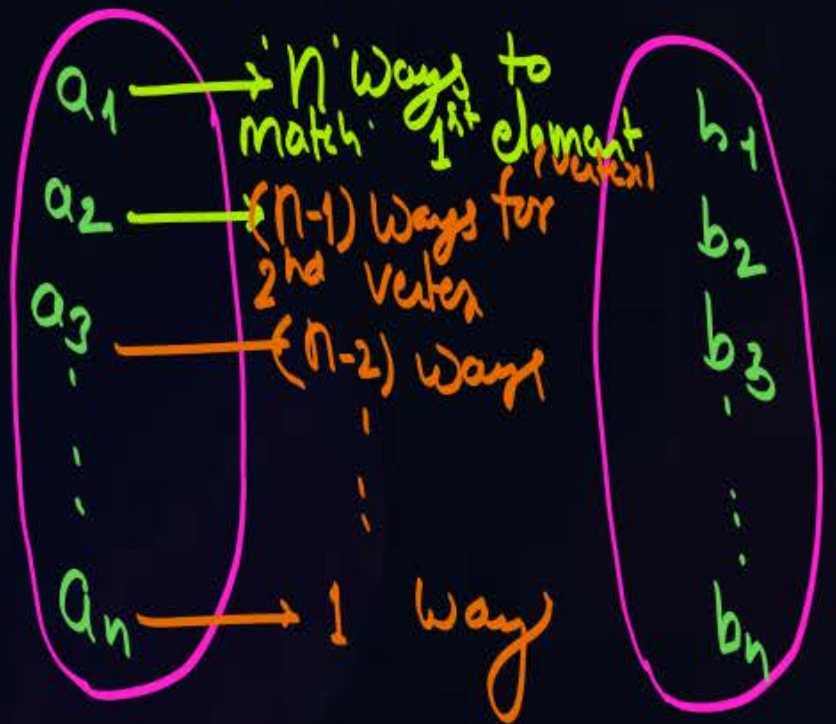
Topic : Perfect Matching

Find the number of perfect matching in a complete bipartite graph $\underbrace{K_{n,n}}$ $(n=n)$ ∴ perfect matching exist

↳ Total No. of vertices $= 2n$

$K_{n,n}$

V_1 $\{|V_1|=n\}$ V_2 $\{|V_2|=n\}$



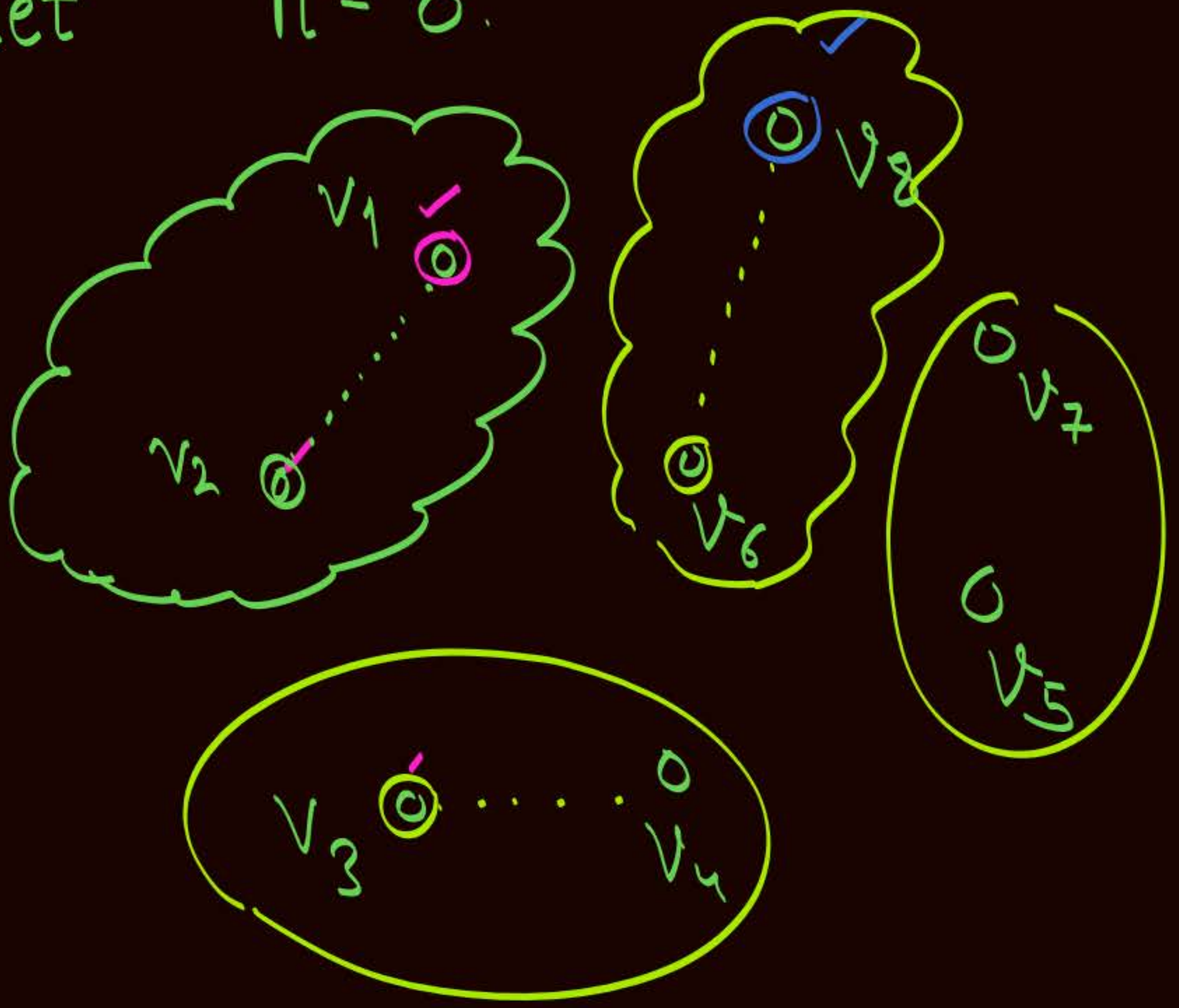
$\boxed{\therefore \text{ No. of Perfect matching } = n\cdot(n-1)\cdot\ldots 3\cdot 2\cdot 1 = n!}$

$\boxed{\text{Matching No. w.r.t. } K_{n,n} = n}$

Topic : Perfect Matching

Find the number of perfect matching in a complete graph Kn (n is even)

Let $n = 8$.



No. of Perfect matching for Complete graph K_8

$$= 7 * 5 * 3 * 1 = 105$$

7 choices to match 1^{st} vertex

No. of choices w.r.t. 3^{rd} vertex after matching 2 vertices

$$(8-1) * (8-3)\,(8-5)\,(8-7)$$

→ No. of perfect matching of Complete graph K_n {where n = even}

$$= (n-1) * (n-3) * (n-5) * \cdots 5 * 3 * 1$$

$$= (n-1) * (n-3) * (n-5) * \cdots 5 * 3 * 1 * \frac{n * (n-2) * (n-4) * \cdots 6 \cdot 4 \cdot 2}{n * (n-2) * (n-4) * \cdots 6 \cdot 4 \cdot 2}$$

$$= \frac{n * (n-1) * (n-2) * \cdots 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(n-0) * (n-2) * (n-4) * \cdots 6 \cdot 4 \cdot 2}$$

$$= \frac{n!}{\left\{2 * \left(\frac{n}{2}-0\right)\right\} * \left\{2 * \left(\frac{n}{2}-1\right)\right\} * \left\{2 * \left(\frac{n}{2}-3\right)\right\} * \cdots \left(2 * \frac{6}{2}\right) * \left(2 * \frac{4}{2}\right) * \left(2 * \frac{2}{2}\right)}$$

$$= \frac{n!}{(2)^{n/2} * \left\{\left(\frac{n}{2}\right) \cdot \left(\frac{n}{2}-1\right) \cdot \left(\frac{n}{2}-2\right) \cdots 3 \cdot 2 \cdot 1\right.} = \frac{n!}{(2)^{n/2} \cdot \left(\frac{n}{2}\right)!}$$

Topic : Perfect Matching

Find the number of perfect matching in a complete graph Kn (n is even)

$$= \frac{n!}{(2)^{n/2} \cdot \left(\frac{n}{2}\right)!}$$

* No. of Perfect matching for Complete graph $K_{2n} = \dfrac{(2n)!}{(2)^{n} \cdot n!}$

H.W. Q. Find the matching numbers of the following graphs

(i) Complete graph K_n

(ii) Cycle graph C_n

(iii) Wheel graph W_n

(iv) Complete bipartite graph $K_{m,n}$

(v) Star graph with n-vertices

2 mins Summary

Topic Matching

Topic Maximal matching & maximum matching

Topic Perfect matching

Topic Matching number

THANK - YOU

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