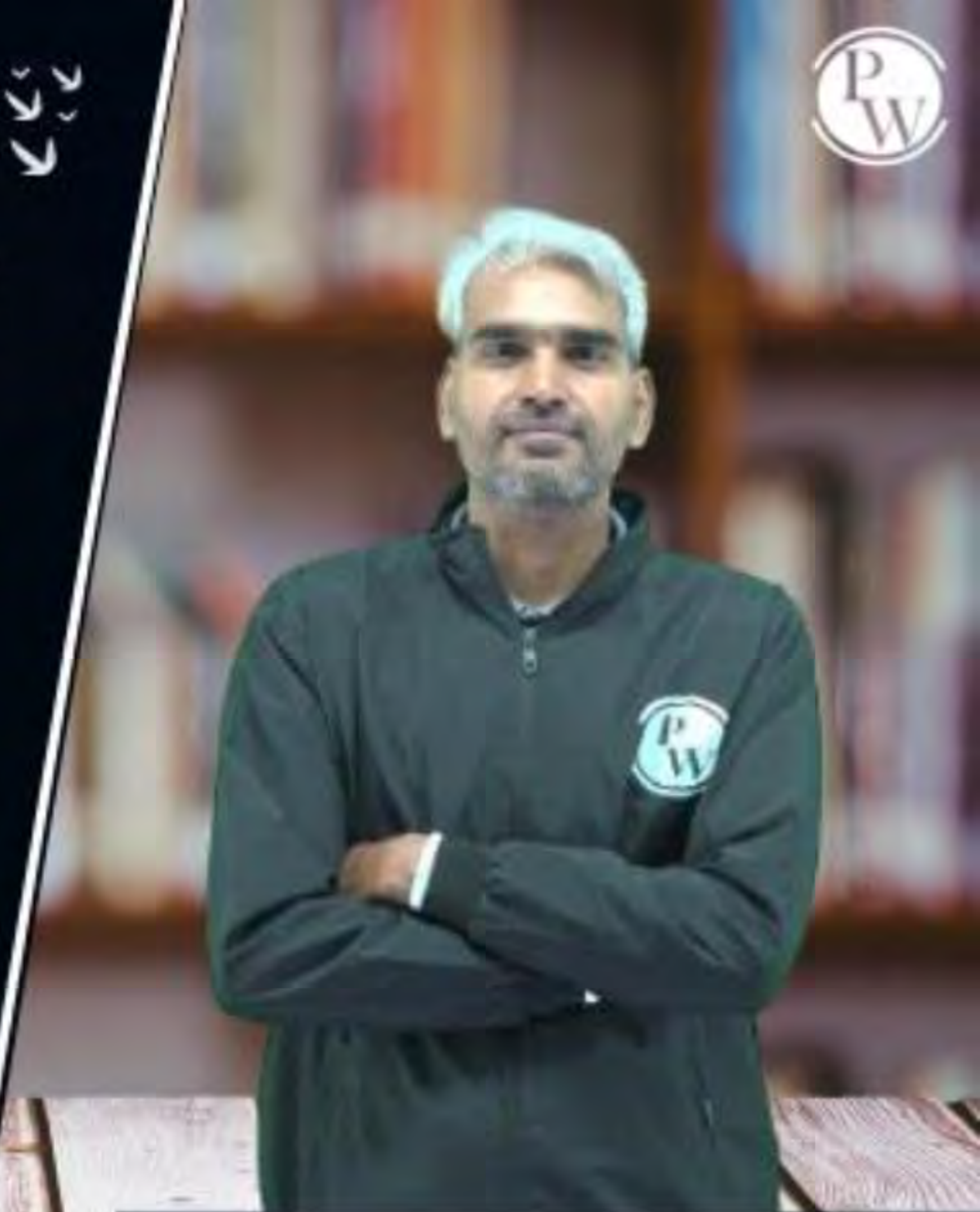
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 02



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Recap of Previous Lecture

Topic Sets and representation of sets

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Topics to be Covered

- **Topic** Types of sets
- **Topic** Venn diagram
- **Topic** Set operations and properties of set operations

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Topic : Cardinality of a set

* $|A|$ = No. of elements in set A.

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Topic : Types of sets

Empty Set: A set with no elements in it

denoted by ϕ or $\{ \}$

$$\boxed{\{ \} = \phi}$$

* $|\phi| = 0$

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Singleton Set: A set with exactly one element in it

eg: $A = \{1\}$

$B = \{a\}$

$C = \{\text{January}\}$

$D = \{\underbrace{\{1, 2, \{3\}\}}_{\text{One single element}}\}$

All are singleton sets

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Topic : Types of sets

Finite Set: Any set with finite number of element in it is called a finite set.

eg: $A = \{1, 2, 3\}$

$B = \{1, a, b, 2, d\}$

$C = \{\ \}$ ← Empty set is also a finite set.

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Topic : Types of sets

Infinite Set: A set with infinite number of elements in it is called an infinite set.

eg: $A = \{x \mid x \in R \text{ and } 0 \le x \le 1\}$

$B =$ Set of all Natural Numbers $= N$

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Another Classification of sets

Countable Set

Finite Set

Finite sets are always Countable

Infinit Set

eg: Set of all Natural Numbers

Uncountable Set

Uncountable Sets are always infinite

Set $\{x \mid x \in R \text{ and } 0 \le x \le 1\}$

Topic : Types of sets

Equal Sets: Two sets A & B are said to be equal if they have exactly same elements

eg: $A = \{1, 2, 3\}$ & $B = \{3, 2, 1\} \Rightarrow \boxed{A = B}$

$A = \{1, 2, 3\}$ & $B = \{1, a, 2\} \Rightarrow \boxed{A \neq B}$

$A = \{1, 2, 3, 4\}$ & $B = \{1, 2, 3\} \Rightarrow \boxed{A \neq B}$

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Topic : Types of sets

Equivalent Sets: Two sets A & B are said to be equivalent if their cardinality is same

eg: $A = \{1, 2, 3\}$ & $B = \{1, 2, 3\}$

$|A| = 3$ & $|B| = 3$ $\Longrightarrow$ $\therefore A \cong B$

$A = \{1, 2, 3\}$ & $B = \{1, a, 2\}$

$|A| = 3$ & $|B| = 3$ $\Longrightarrow$ $\therefore A \cong B$

$A = \{1, 2, 3, 4\}$ & $B = \{1, 2, 3\}$

$|A| = 4$ & $|B| = 3$ $\Longrightarrow$ $\therefore A \ncong B$

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- If $A = B$ then $A \cong B$
- <u>But</u> if $A \cong B$ <u>then</u> A may <u>or</u> may not be equal to B.
- if $A \ncong B$ <u>then</u> $A \neq B$

Topic : Types of sets

Universal Set: A set of all elements w.r.t. problem under Consideration is called universal set.

It is generally denoted by "U"

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Topic : Types of sets

Subset & Superset:

$\subseteq$ $\supseteq$

Let A & B are two sets,

If every element of set A is also a member of set B, then we say A is subset of B {i.e $A \subseteq B$}

If A is a subset of B then B is a superset of A. {i.e. $B \supseteq A$}

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eg: let $A = \{1, 2, 3, a, b\}$ $B = \{1, a, b\}$

$B \subseteq A$ but $\underbrace{A \nsubseteq B}$ $\Rightarrow$ $\boxed{A \neq B}$

elements 2 & 3 of A are not member of B

eg: let $A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$

$A \subseteq B$ & $B \subseteq A$

we also know $A = B$

$A = B$ if and only if $A \subseteq B$ & $B \subseteq A$

$$A = \{1, 2, \underbrace{\{1, 2\}}\} \qquad B = \{\underbrace{\{1\}}\}$$

$$\boxed{1 \notin \{\{1\}\} \qquad A \nsubseteq B}$$

$$\boxed{\{1\} \in \{\{1\}\}}$$

element of set B

$$\{1\} \notin A$$

$$\therefore \boxed{B \nsubseteq A}$$

Note: ① For any set A, ϕ is always a subset of set A

i.e. ϕ is a subset of every set

② For any set A, set A itself is always a subset.

i.e. Every set is a subset of itself

Topic : Types of sets

"$\subset$"

Proper Subset:

Let A & B are two sets such that every element of set A is a member of set B, but Every element of set B is not a member of set A then A is a proper subset of B.

i.e. if $A \subseteq B$ & $B \nsubseteq A$ then $A \subset B$

Every subset of a set 'A' except the set itself are called proper subsets of set A.

eg. let $A = \{1, 2, 3\}$

Subsets of set A are $= \{\ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

Proper subsets of A $= \{\ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \cancel{\{1,2,3\}}$

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Q: Let A is a set of size = 3, then how many subsets of set A are possible?

let $A = \{1, 2, 3\}$ ⇒ Subsets of set A

of size = 0 — $\{\ \}$

of size = 1 — $\{1\}, \{2\}, \{3\}$

of size = 2 — $\{1,2\}, \{1,3\}, \{2,3\}$

of size = 4 — $\{1, 2, 3\}$

Total 8 subsets

# subsets of size = 0		# subsets of size = 1		# subsets of size = 2		# subsets of size = 3	

$$\boxed{\text{Total No. of subsets} = 1 + 3 + 3 + 1 = 8}$$

$$\text{Total No. of subsets} = {}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3$$

Topic : Number of subsets of a set 'A' of cardinality 'n'

let $A = \{1, 2, 3, \ldots n\}$

No. of Subsets of set A of

size = 0, size = 1, Size = 2, Size = k $\{k < n\}$, Size = n

Total No. of subsets $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_k \ldots + {}^nC_n = 2^n$

No. of subsets of A of size atmost 'k' i.e. $\le k$.

Number of subsets of size exactly 'k'

No. of subsets of A of size at least k i.e $\ge k$

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$$(1+x)^n = {}^nC_0 \cdot x^0 + {}^nC_1 \cdot x^1 + {}^nC_2 \cdot x^2 + {}^nC_3 \cdot x^n + \cdots + {}^nC_n \cdot x^n$$

Put $\underline{x = 1}$

$$(1+1)^n = {}^nC_0 \cdot (1)^0 + {}^nC_1 \cdot (1)^1 + {}^nC_2 \cdot (1)^2 + \cdots + {}^nC_n \cdot (1)^n$$

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$$

Q:- No. of subsets of set

$$A = \{1, 2, 3, 4, 5, \ldots\ldots, (n-1), n\}$$

No. of subsets of A $= \underbrace{2 * 2 * 2 * 2 * 2 * \ldots\ldots * 2 * 2}_{\text{"n times 2"}} = 2^n$

Q: Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $\{a\}$

How many subsets of set A are possible such that all elements of the subset are even and every element of the subset is greater than '4'.

$A : \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

No. of subsets satisfying the Condⁿ $= 1 * 1 * 1 * 1 * 1 * 2 * 1 * 2 * 1 * 2 = 2^3 = 8$

only one choice i.e. don't select

odd ∴ don't select

Q: let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

and $A = \{1, 3, 5\}$ is a subset of universal set U

find the no of supersets of set A.

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

No of supersets of $A = \{1, 3, 5\}$ $= 1 \times 2 \times 1 \times 2 \times 1 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

$= 128$

Ans

In every superset of A, all elements of set A must be present.

Must be present.

$\therefore$ Only one choice

Note: Let A is a set with 'n' elements,

(i) Number of subsets of $A = 2^n$

(ii) Number of proper subsets of $A = 2^n - 1$

Subset

$\subseteq$

Equal sets are also considered

Proper subset

$\subset$

Equal not considered

Topic : Power Set

Power Set: Let 'A' is a finite set, then Power set of set A $\underbrace{\text{is a set}}$ containing all subsets of set A

- Power set of set 'A' is denoted by $P(A)$ or 2^A

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eg: $A = \{1, 2, 3\}$

$$2^A \equiv P(A) = \{\underbrace{\{\ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}}_{\text{all subsets of set } A}\}$$

set of $\rightarrow$ all subsets of set A

Topic : Cardinality of Power Set

Let $P(A)$ is the power set of set A.
then Cardinality of set $P(A)$ is denoted by $|P(A)|$

$$|P(A)| = \text{Number of } \underbrace{\text{elements in } P(A)}_{\text{Elements of } P(A) \text{ are subsets of set } A}$$

$$= \text{Number of subsets of set } A.$$

$$|P(A)| = 2^{|A|}$$

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Q: let $\emptyset$ be an empty set, then

$$|P(P(\emptyset))| = ?$$

$$|P(P(\emptyset))| = 2^{|P(\emptyset)|}$$

$$|P(\emptyset)| = \left(2^{|\emptyset|}\right), \quad |\emptyset| = 0$$

$$= 2^{\left(2^{|\emptyset|}\right)}$$

$$= (2)^{\left(2^{0}\right)} = 2^{1}$$

$$= 2$$

- given set is $\emptyset = \{\ \}$
- the only subset of $\emptyset$ is $\emptyset$ itself.

$\therefore P(\emptyset)$ = Set of $\underbrace{\text{subsets of } \emptyset}$

$$= \{\ \{\ \}\ \}$$

$$\boxed{P(\emptyset) = \{\{\ \}\} = \{\emptyset\}}$$

$$|\{\{\ \}\}| = 1$$

$$\underbrace{\{\ \}}_{\text{Empty set}} \neq \underbrace{\{\{\ \}\}}_{\text{set Containing an element}}$$

element is an empty set

$P(\emptyset) = \{ \{ \} \}$

Subsets of $P(\emptyset) = \underbrace{\{ \}}_{}, \quad \underbrace{\{ \{ \} \}}_{}$

empty set is subset of every set

every set is a subset of itself

$P(P(\emptyset))$ = Set of subsets of $P(\emptyset)$

$= \{ \{ \}, \{ \{ \} \} \}$

$= \{ \emptyset, \{ \emptyset \} \}$

Q: Let $A = \{1, 2, \{1,2\}, \{\{2,3\}\}\}$

which of the following is/are true.

✓ (a) $1 \in A$

✗ (b) $1 \subseteq A$

✗ (c) $\emptyset \in A$

✓ (d) $\emptyset \subseteq A$

$\{\ \} \subseteq A$

✗ (e) $\{1\} \in A$

✓ (f) $\{1\} \subseteq A$

✓ (g) $\{1,2\} \in A$

✓ (h) $\{1,2\} \subseteq A$

✗ (i) $\{\{1,2\}\} \in A$

✓ (j) $\{\{1,2\}\} \subseteq A$

✗ (k) $\{2,3\} \in A$

✗ (l) $\{2,3\} \subseteq A$

✓ (m) $\{\{2,3\}\} \in A$

✗ (n) $\{\{\{2,3\}\}\} \subseteq A$

Subset of A = $\{\underbrace{\qquad\qquad}_{\text{elements from set}}\}$

set

Q1. Let $P(S)$ denote the power set of a set S. Which of the following is always true?

A. $P(P(S)) = P(S)$

B. $P(S) \cap P(P(S)) = \{\emptyset\}$

C. $P(S) \cap S = P(S)$

D. $S \notin P(S)$

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Q2. For a set A, the power set of A is denoted by 2^A. If $A = \{5, \{6\}, \{7\}\}$ which of the following options are true?

1. $\emptyset \in 2^A$
2. $\emptyset \subseteq 2^A$
3. $\{5, \{6\}\} \in 2^A$
4. $\{5, \{6\}\} \subseteq 2^A$

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Topic : Venn Diagram

Venn diagram is used to represent the relationship among the sets pictorially.

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Topic : Set Operations

- Complement of a set
- Union of two sets
- Intersection of two sets
- Set difference
- Symmetric difference of two sets

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Topic : Properties of Set Operations

1. Idempotent:

   a. $A \cap A = A$

   b. $A \cup A = A$

2. Identity:

   a. $A \cup \emptyset = A$

   b. $A \cap U = A$

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Topic : Properties of Set Operations

3. Domination:

   a. $A \cap \emptyset = \emptyset$

   b. $A \cup U = U$

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Topic : Properties of Set Operations

4. Complementation:

   a. $A \cup A^{c} = U$

   b. $A \cap A^{c} = \emptyset$

5. Double Complement:

   a. $(A^{c})^{c} = A$

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Topic : Properties of Set Operations

6. Commutative

   a. $A \cup B = B \cup A$

   b. $A \cap B = B \cap A$

7. Associative

   a. $A \cup (B \cup C) = (A \cup B) \cup C$

   b. $A \cap (B \cap C) = (A \cap B) \cap C$

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Topic : Properties of Set Operations

8. Absorption

   a. $A \cup (A \cap B) = A$

   b. $A \cap (A \cup B) = A$

9. DeMorgan's

   a. $(A \cup B)^C = A^c \cap B^c$

   b. $(A \cap B)^C = A^c \cup B^c$

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Topic : Properties of Set Operations

10. Distributive

    a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

    b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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2 mins Summary

Topic Types of sets

Topic Venn diagram

Topic Set operations and properties of set operations

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THANK - YOU

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