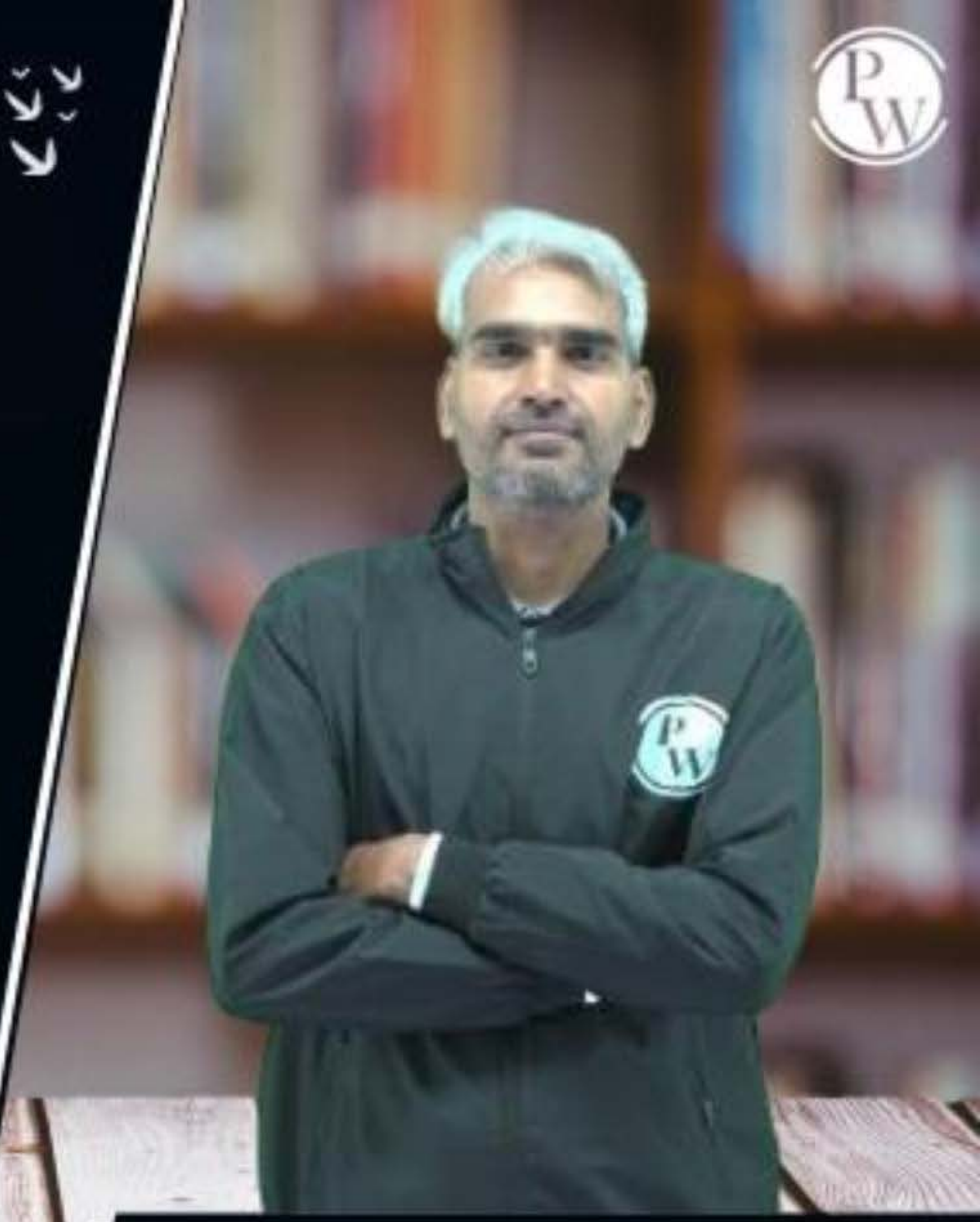
# Computer Science & IT

## Discrete Mathematics

**Set Theory & Algebra**

**Lecture No. 04**



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# Recap of Previous Lecture

- **Topic** Venn diagram
- **Topic** Set operations and properties of set operations
- **Topic** Multi-set

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# Topics to be Covered

- **Topic** Cartesian product
- **Topic** Different types of relations
- **Topic** Diagonal relation (Identity relation)
- **Topic** Reflexive relation and irreflexive relation

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$S = \{1\}$ $\quad \cap = \phi$

$P(S) = \{\phi, \{1\}\}$

$S = \{1, \{1\}\}$ $\quad \cap = \{1\}$

$P(S) = \{\phi, \{1\}, \{\{1\}\}, \{1, \{1\}\}\}$

$P(P(S)) = \{\phi, \{\phi\}, \{\{1\}\}, \ldots\}$

Q1. Let $P(S)$ denote the power set of a set $S$. Which of the following is always true?

A. ~~$P(P(S)) = P(S)$~~ $\quad \boxed{P(\phi) \neq P(P(\phi))}$

$\cap \neq \phi$

$\neq \{\phi\}$

B. ~~$P(S) \cap P(P(S)) = \{\emptyset\}$~~

C. ~~$P(S) \cap S = P(S)$~~ $\quad$ (E) ~~$P(S) \cap S = \phi$~~ | (F) $P(S) \cap S = S$

D. ~~$S \notin P(S)$~~ $\quad \boxed{S \in P(S)}$

is always true

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$A = \{a, b, c\}$

$a \in A$ < True / False

Q2. For a set $A$, the power set of $A$ is denoted by $2^A$. If $A = \underbrace{\{5, \{6\}, \{7\}\}}$ which of the following options are true?

1. $\emptyset \in 2^A$ { Always true }
2. $\emptyset \subseteq 2^A$
3. $\{5, \{6\}\} \in 2^A$
4. $\{5, \{6\}\} \subseteq 2^A$

$P(A) = 2^A = \{ \{\ \},$
$\{5\}, \{\{6\}\}, \{\{7\}\},$
$\{5, \{6\}\}, \{5, \{7\}\}, \{\{6\}, \{7\}\},$
$\{5, \{6\}, \{7\}\} \}$

it is also a set
∴ $\emptyset$ will be a subset of this.

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H.W.

Q: Let $A = \{1, 2, 3, 4, 5, ----, n\}$

How many multisets are possible using the elements of set A.

{ Each element of set A can appear any number of times }

$M_1 = \{1\}$ $\quad M_1 \neq M_2$

$M_2 = \{1, 1\}$ $\quad M_2 \neq M_3$

$M_3 = \{1, 1, 1\}$

⋮

$M_{100} = \{\underbrace{1, 1, 1, -----, 1}_{100 \text{ times}}\}$

⋮

$M_\infty = \{\underbrace{1, 1, 1, 1, ---------}_{\infty \text{ times}}\}$

If we do not restrict the size of multiset, then with a single element infinite multisets are possible.

∴ Correct answer will be $\infty$

H.W.

Q: Let $A = \{1, 2, 3, 4, 5, \ldots, n\}$

How many multisets of size = 4 are possible using the elements of set A such that at least one element appears exactly twice.

{ Each element of set A can be used any number of times }

Total elements = n

Based on given Cond$^n$
two types of Multiset are possible

$a, b, c$
- $a, a, b, c$
- $b, b, a, c$
- $c, c, a, b$

Case ① $\{a, a, b, c\}$

Case ② $\{a, a, b, b\}$

1st Concept

$$^nC_1 * 1 * {}^{(n-1)}C_1 * 1 * {}^{(n-2)}C_1 * 1 \over 2!$$

$$= \frac{n * (n-1) * (n-2)}{2}$$

$$\frac{^nC_1 * 1 * {}^{n-1}C_1 * 1}{2!} = \frac{n * (n-1)}{2}$$

2nd Concept

$$^nC_1 * 1 * {}^{(n-1)}C_2 * 1 = n * \frac{(n-1)(n-2)}{2}$$

$$^nC_2 * 1 = \frac{n(n-1)}{2}$$

3rd Concept

$$^nC_3 * {}^3C_1 * 1 * 1$$

$$= \frac{n(n-1)(n-2)}{3!} * 3 = \frac{n(n-1)(n-2)}{2}$$

Final answer:

Required Multisets can be formed using Case 1 or Case 2

$$\therefore \text{No. of Multisets Possible} = \frac{n(n-1)(n-2)}{2} + \frac{n(n-1)}{2}$$

$$= \frac{n(n-1)(n-2) + n(n-1)}{2} = \frac{n(n-1)^2}{2}$$

(Cross Product)

# Topic : Cartesian Product

Let A & B are two sets, Cartesian product of A & B is denoted by "$A \times B$" and it is defined as,

$$A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}$$

Order pair

$\therefore$ Order in which elements appear is important.

i.e; $(a, b) \neq (b, a)$

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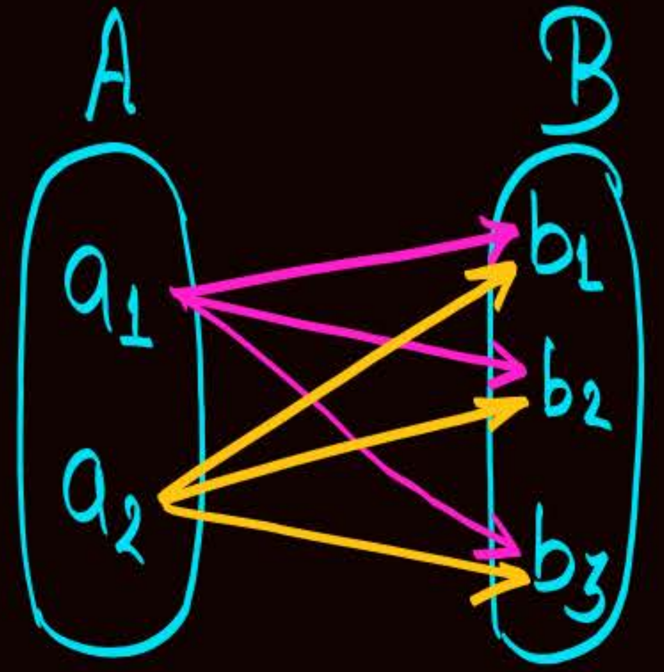
Let $A = \{a_1, a_2\}$

$B = \{b_1, b_2, b_3\}$

$\therefore A \times B = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2), (a_2, b_3)\}$




$A \times B$

$B \times A = \{(b_1, a_1), (b_1, a_2), (b_2, a_1), (b_2, a_2), (b_3, a_1), (b_3, a_2)\}$

We know,

$(a_1, b_1) \neq (b_1, a_1)$

$\therefore A \times B \neq B \times A$

- In general,

$$A \times B \neq B \times A$$

- If $A \times B = B \times A$, then

either $A = B$

<u>or</u> at least one of $A$ or $B$ is an empty set.

- If $A$ <u>or</u> $B$ is an empty set

then $A \times B = B \times A = \emptyset = \{\ \}$

- In $A \times B$, every element of set $A$ relates with every element of set $B$.

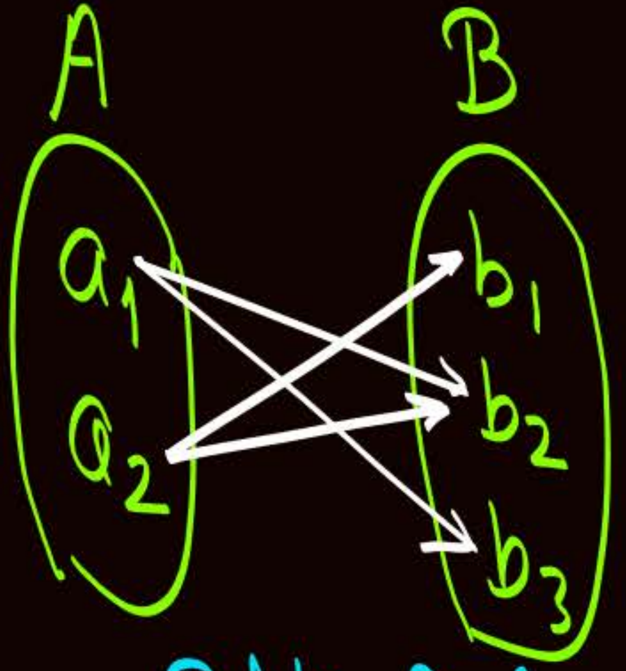
$\boxed{\begin{aligned} &\bullet \text{ If } |A| = m \ \& \ |B| = n \\ &\text{then } |A \times B| = |A| \cdot |B| \\ &\qquad |A \times B| = m \cdot n \end{aligned}}$

A relation from set A to set B defines that how exactly elements of set A relates with elements of set B

Note: Every relation from set A to set B is a subset of '$A \times B$'.

eg: let $A = \{a_1, a_2\}$, $B = \{b_1, b_2, b_3\}$

and let $R_1$ is relation which defines that how exacty elements of set A relates with set B.




$\Rightarrow$ $R_1 = \{(a_1, b_2), (a_1, b_3), (a_2, b_1), (a_2, b_2)\}$

we can observe that it is a subset of '$A \times B$'

Relation $R_1$ from A to B.

$R_1 : A \to B$

Q: Let $|A| = m$ & $|B| = n$, then
how many different relation are possible from set A to set B.

Solu$^n$ :- Every relation from A to B is a subset of $A \times B$

$\therefore$ Number of relation from A to B = Number of subsets of $A \times B$

$$= 2^{|A \times B|} = 2^{|A| \cdot |B|}$$

$$= 2^{m \cdot n}$$

Note :- One of the subset of '$A \times B$' is an Empty set, that Empty set is also a relation from A to B, and that relation is called "Empty relation"

* A relation from set A to set A is called a relation on set A.

- If $|A| = m$, then

No. of relations possible on set $A = 2^{|A \times A|} = 2^{|A| \cdot |A|} = 2^{m \cdot m}$

$= 2^{(m^2)}$

# Topic : Types of Relations

All this relations are defined from set A to same set A {i.e. on set A}

1. Diagonal Relation (Identity relation)
2. Reflexive Relation
3. Irreflexive Relation
4. Symmetric Relation
5. Anti-symmetric Relⁿ
6. Asymmetric Relⁿ
7. Transitive Relation

8. Complement of a relation
9. Inverse of a relation
10. Composite relation

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# Topic : Diagonal Relation

(Identity Relation)

Diagonal relation on set A is denoted by $\Delta_A$.
and it is defined as,

$$\Delta_A = \{(a,a) \mid a \in A\}$$

it is definition of a set

eg. let $A = \{1,2,3\}$

$R_1 = \{(1,1),(2,2)\}$ $(3,3) \notin R_1$ $\therefore R_1$ is not a diagonal relation on set A.

$R_2 = \{(1,1),(2,2),(3,3),(1,2)\}$ ∵ order pair (1,2) can never be an element of diagonal relⁿ
$\therefore R_2$ is not a diagonal relⁿ.

$R_3 = \Delta_A = \{(1,1),(2,2),(3,3)\}$

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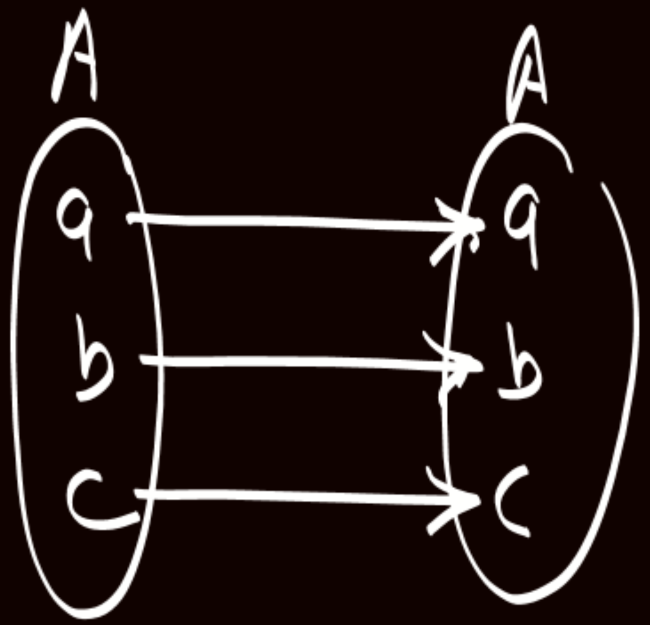


(Diagonal Relⁿ)

# THANK - YOU

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