COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No: 05

Miscellaneous Topics

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Recap of Previous Lecture

Number System

PI, EPI

Question Discussion

Q. A quadratic equation mentioned in two different number system is as given below:

a. $$(x^2 - 14x + 43)_r = (x^2 - 11x + 32)_{r_1}$$

Then the value of $(r_1 + r)$ ___19___.

b. $$(x^2 + 5x - 124)_r = (x^2 + 11x - 1110)_{r_1}$$

Value of $(r_1 + r)$ ___4 + 8 = 12___.

$(5)_r = (11)_{r_1} \Rightarrow (5)_{10} = (r_1 + 1)_{10}$

$r_1 = 4$

$(124)_r = (1110)_{r_1}$

$(r^2 + 2r + 4)_{10} = r_1^3 + r_1^2 + r_1 = 84$

$r^2 + 2r - 80 = 0 \Rightarrow (r+10)(r-8) = 0 \Rightarrow r = -10, 8$

$(a_1)_r, (b_1)_r$ $\quad$ $(a_2)_{r_1}, (b_2)_{r_1}$

$(a_1)_r + (b_1)_r = (14)_r$

$(a_2)_{r_1} + (b_2)_{r_1} = (11)_{r_1}$

$(14)_r = (11)_{r_1} \Rightarrow (r+4)_{10} = (r_1+1)_{10}$

$(43)_r = (32)_{r_1}$ $\quad$ $r_1 - r = 3$

$(4r+3)_{10} = (3r_1+2)_{10}$ $\quad$ $3r_1 - 4r = 1$

$r = 8$

$r_1 = 11$

Q. A relationship b/w two no. is as given below:

$$(15)_r^2 = (13)_{r_1}^2 \Rightarrow \boxed{r \geq 6, \quad r_1 \geq 4}$$

then minimum value of $(r + r_1)$ <u>14</u>.

$$(r_1)_{min} = 8$$
$$(r)_{min} = 6$$

$(6, 8), (7, 9), (8, 10)$
$(9, 11)$
$(10, 12)$

$$\overset{r^1 r^0}{(15)_r} = (r \times 1 + r^0 \times 5)_{10} = (r+5)_{10}$$
$$(13)_{r_1} = (r_1 + 3)_{10}$$

$$(r+5)^2 = (r_1+3)^2 \Rightarrow r+5 = \pm\sqrt{(r_1+3)^2}$$
$$r+5 = \pm(r_1+3)$$
$$\Rightarrow r+5 = r_1+3$$
$$\boxed{r = r_1 - 2} \checkmark$$

Q. A relationship b/w two no. is given as:

$$(62)_x = (y2)_9 \Rightarrow x \geq 7, \quad 0 \leq y \leq 8$$

Where x and y are integers.

- Then minimum value of $(x+y)$ ___15___.
- Maximum value of $(x+y)$ ___20___.
- No. of solution of this equation ___2___.

$$(6x+2)_{10} = (9y+2)_{10}$$

$$6x = 9y$$

$$\boxed{x = 1.5y}$$

$$\boxed{\begin{array}{l} y=6, \quad x=9 \\ y=8, \quad x=12 \end{array}}$$

b. $(63)_x = (y9)_{12} \Rightarrow x \geq 7, \quad 0 \leq y \leq 11$

No. of solution exist for this equation ___9___.

Minimum value of $(x+y)$ ___10___. $\Rightarrow (6x+3)_{10} = (12y+9)_{10}$

Maximum value of $(x+y)$ ___34___.

$$6x = 12y + 6 \Rightarrow \boxed{x = 2y+1}$$

$$\boxed{\begin{aligned} y &= 3 \\ x &= 7 \end{aligned}}$$

$3 \leq y \leq 11 \longrightarrow 7 \leq x \leq 23$

$\Rightarrow$ Total 9 solutions $\Rightarrow (3,7), (4,9), (5,11), (6,13), (7,15)$

$(8,17), (9,19), \underset{A}{(10,21)}, \underset{B}{(11,23)}$

Q. $(\overset{16}{x}\overset{4}{2}\overset{1}{y})_4 = (52)_z$ $\quad 0 \leq x \leq 3, \quad 0 \leq y \leq 3, \quad z \geq 6$

Minimum value of $(x+y+z)$ 12.

Maximum value of $(x+y+z)$ 15.

No. of solutions of given equation 2.

$$(16x + 8 + y)_{10} = (5z + 2)_{10}$$

$$16x + y = 5z - 6$$

$$5z = 16x + y + 6$$

$x = 2, y = 2 \Rightarrow z = 8$

$x = 3, y = 1 \Rightarrow z = 11$

#Q. A number is written in decimal no system as:

$(3 \times 512 + 2 \times 128 + 5 \times 8 + 7)_{10}$ then no. of 1's in its binary representation will be $\underline{\ 8\ }$.

b. $(5 \times 2^{12} + 6 \times 2^{5} + 3 \times 2^{3} + 4)_{10}$, no. of 1's in its binary representation $\underline{\ 7\ }$.

c. $(7 \times 2^{12} + 13 \times 2^{8} + 11)_{10}$, no. of 1's in its binary representation $\underline{\ 9\ }$.

d. A decimal no. is given as $(101 \times 2^{7})_{10}$ then no. of 1's in its binary representation ____.

$$101 \times 2^{7} = (2^{6} + 2^{5} + 2^{2} + 2^{0}) \times 2^{7} = 2^{13} + 2^{12} + 2^{9} + 2^{7}$$

$$3 \times 512 + 2 \times 128 + 5 \times 8 + 7$$

$$= 3 \times 2^9 + 2 \times 2^7 + 5 \times 2^3 + 7$$

$$= 3 \times 8^3 + \underline{\underline{4 \times 8^2}} + 5 \times 8^1 + \underline{\underline{7}} \times 8^0$$

$$= (\underline{\underline{3}}\,\underline{\underline{4}}\,\underline{\underline{5}}\,\underline{\underline{7}})_8 = (011\ 100\ 101\ 111)_2$$

$$\Rightarrow 2 + 1 + 2 + 3 = 8$$

$$(234)_{16} = 2 \times 16^2 + 3 \times 16^1 + 4 \times 16^0$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^4 = 16 \qquad 2^6 = (2^3)^2 = 8^2$$

$$2^8 = 16^2 \qquad 2^9 = (2^3)^3 = 8^3$$

$$2^{12} = 16^3 \qquad 2^{12} = (2^3)^4 = 8^4$$

$$(234)_8 = (2 \times \underline{8}^2 + 3 \times \underline{8} + 4 \times \underline{\underline{8^0}})_{10}$$

$$5 \times 2^{12} + 6 \times 2^{5} + 3 \times 2^{3} + 4$$

$$5 \times 8^{4} + 3 \times 8^{2} + 3 \times 8^{1} + 4 \times 8^{0}$$

$$= (50334)_8$$

$$\Rightarrow \quad 2, 2, 2, 1 = 7$$

- $7 \times 2^{12} + 13 \times 2^{8} + 11$

$$7 \times 16^{3} + 13 \times 16^{2} + 11 \times 16^{0}$$

$$= (7D0B)_{16} \Rightarrow \quad 3, 3, 3 \qquad = (0111\ 1101\ 0000\ 1011)_2$$

$$\left(2^{13}+2^{12}+2^{9}+2^{7}\right)_{10} = 1\ \underline{\quad 13-\text{'}0\text{'} \quad}$$

$$= \left(11001010000000\right)_2$$

$$(100)_2 = (2^2)_{10}$$

$$(2^6)_{10} = 1000000$$

$$(2^n)_{10} = \left(1\ \underline{\quad n \text{ times } 0 \quad}\right)_2$$

Q. A binary no is multiplication of two binary no as given

$$B = (101101)_2 \times (111010)_2 \quad = (2^5 + 2^3 + 2^2 + 2^0)$$

$$(2^5 + 2^4 + 2^3 + 2^1)$$

then no. of 1's in B $\underline{\ \ 5\ \ }$.

	5	3	2	0
5	10	8	7	5
4	9	7	6	4
3	8	6	5	3
1	6	4	3	1

$$2^{11} + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^1$$

$$+ 2^6$$

$$= 2^{11} + 2^9 + 2^5 + 2^4 + 2^1$$

5

[NAT]

$2^{-1}\,2^{-2}\,2^{-3}\,2^{-4}$

If x and y are two decimal digits and $(0.1101)_2 = (0.8xy5)_{10}$, the decimal value of $x + y$ is ___3___.

[GATE-2021-CS: 1M]

$0 \leq x \leq 9$

$0 \leq y \leq 9$

$\Rightarrow \quad 0.5 + 0.25 + 0.0625 = (0.8xy5)_{10}$

$$\begin{array}{r} 0.7500 \\ 0.0625 \\ \hline 0.8125 \\ \hline \end{array} = (0.8xy5)_{10}$$

$x = 1$

$y = 2$

[NAT]

Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then b = 8.

[GATE-2017-CS: 1M]

$$(5)_b + (6)_b = (13)_8$$

$$\begin{array}{r} (5)_8 \\ (6)_8 \\ \hline (13)_8 \end{array}$$

$$(13)_b = (b+3)_{10}$$

$$(36)_b = (3b+6)_{10}$$

$$x^2 - (b+3)x + (3b+6) = 0$$

$$x = (5)_b = (5)_{10}$$

$$25 - (b+3) \times 5 + 3b + 6 = 0$$

$$2b = 16$$

$$b = 8$$

[NAT]

Consider the euqation $(\overset{x}{4}\overset{x^0}{3})_x = (\overset{8^1}{y}\overset{8^0}{3})_8$ where x and y are unknown. The number of possible solution is ___5___.

[GATE-2015-CS: 2M]

$\Longrightarrow x \geq 5, \quad , \quad 0 < y \leq 7$

$\Rightarrow (4x+3)_{10} = (8y+3)_{10}$

$\boxed{x = 2y} \longrightarrow \quad 3 \leq y \leq 7 \longrightarrow 6 \leq x \leq 14$

$(3,6), (4,8), (5,10), (6,12), (7,14)$

[NAT]

The base (or radix) of the number system such that the following equation holds is 5. 312/20 = 13.1

[GATE-2014-CS: 1M]

$$\frac{(312)_r}{(20)_r} = (13.1)_r \qquad \boxed{r \geq 4}$$

$$(312)_r = (20)_r \, (13.1)_r$$

$$3r^2 + r + 2 = (2r)(r + 3 + r^{-1}) = 2r^2 + 6r + 2$$

$$r^2 - 5r = 0$$

$$(r-5)(r) = 0$$

$$r = 0 \ \times$$

$$r = 5$$

[NAT]

Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is <u>3</u>.

[GATE-2014-CS: 1M]

$$\boxed{0 \leq x \leq y-1}$$

$$\boxed{y \geq 9}$$

$$(25 + 10 + 3)_{10} = (xy + 8)_{10}$$

$$xy = 30$$

$$\boxed{\begin{array}{l} x = 3,\ y = 10 \\ x = 2,\ y = 15 \\ x = 1\ \ y = 30 \end{array}}$$

[MCQ]

Let r denote number system radix. The only value(s) of r that satisfy the equation $\sqrt{121_r} = 11_r$ is/are $\rightarrow \boxed{r \geq 3} \rightarrow r > 2$ [GATE-2008-CS: 1M]

$$\sqrt{(r^2+2r+1)_{10}} = (r+1)_{10}$$

$$\underline{r^2+2r+1} = (r+1)^2 = \underline{\underline{r^2+2r+1}} \rightarrow \text{infinite solution}$$

$$\boxed{r \geq 3}$$

(A) Decimal 10

(B) Decimal 11

(C) Decimal 10 and 11

(D) Any value > 2

[MCQ]

Consider a system that uses 5 bits for representing signed integers in 2's complement format. In this system, two integers A and B are represented as A=01010 and B = 11010 . Which one of the following operations will result in either an arithmetic overflow or an arithmetic underflow? [GATE-2024-CS: 1M]

(A) A + B ✗ $= 4$ $\Rightarrow$ $n = 5$ bit $\Rightarrow$ $-(2^4)$ to $+(2^4 - 1)$

(B) ✓ A – B $= +16$ $A = 01010 = (10)_{10}$ -16 to $\underline{\underline{15}}$

(C) ✗ B – A $= -16$ $B = 11010 = (-6)_{10}$

(D) ✗ 2 * B $= -12$

[MCQ]

In 16-bit 2's complement representation, the decimal number -28 is:

[GATE-2019-CS: 1M]

(A) 1111 1111 0001 1100

(B) 0000 0000 1110 0100

(C) 1111 1111 1110 0100

(D) 1000 0000 1110 0100

$-(2^{n-1})$ to $+(2^{n-1}-1)$

6 bit $\rightarrow (100100)_2 = (-28)_{10}$

$(-44)_{10} = (1010100)_2$

[NAT]

Two numbers are chosen independently and uniformly at random from the set $\{1, 2, \ldots, 13\}$. The probability (rounded off to 3 decimal places) that their 4-bit (unsigned) binary representations have the same most significant bit is 0.461.

[GATE-2019-CS: 1M]

$\Rightarrow$

0001
0010
0011
0100
0101
0110
0111

1000
1001
1010
1011
1100
1101

Total combinations $= {}^{13}C_2$

favourable $= {}^{7}C_2 + {}^{6}C_2$

$$P = \frac{{}^{7}C_2 + {}^{6}C_2}{{}^{13}C_2} = \frac{\frac{\lfloor 7}{\lfloor 5 \times \lfloor 2} + \frac{\lfloor 6}{\lfloor 2 \lfloor 4}}{\frac{\lfloor 13}{\lfloor 2 \lfloor 11}}$$

$$= \frac{7 \times 3 + 5 \times 3}{13 \times 6} = \frac{36}{13 \times 6} = \frac{6}{13}$$

$= 0.461$

0.461538

$= 0.462$

[MCQ]

Consider Z = X - Y, where X, Y and Z are all in sign-magnitude form. X and Y are each represented in n bits . To avoid overflow , the representation of Z would require a minimum of : [GATE-2019-CS: 1M]

A n bits

B n-1 bits

C n+1 bits

D n+2 bits

$n = 3$ bits $\rightarrow$ $-(3)$ to $+(3)$

$Z \rightarrow -6$ to 6

$n=4$, -7 to 7 $\boxed{n \geq 4}$

2's $-(4)$ to 3, -7 to 7

[MCQ]

Consider the unsigned 8-bit fixed point binary number representation below,

$$b_7\, b_6\, b_5\, b_4\, b_3 \cdot b_2\, b_1\, b_0$$

where the position of the binary point is between b_3 and b_2. Assume b_7 is the most significant bit. Some of the decimal numbers listed below cannot be represented exactly in the above representation:

(i) 31.500 (ii) 0.875 (iii) 12.100 (iv) 3.001

Which one of the following statements is true? [GATE-2018-CS: 1M]

(A) None of (i), (ii), (iii), (iv) can be exactly represented H.W.

(B) Only (ii) cannot be exactly represented

(C) Only (iii) and (iv) cannot be exactly represented

(D) Only (i) and (ii) cannot be exactly represented

[MCQ]

When two 8-bit numbers $A_7 \ldots A_0$ and $B_7 \ldots B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a ripple-carry adder, the sum bits obtained are $S_7 \ldots S_0$ and the carry bits are $C_7 \ldots C_0$. An overflow is said to have occurred if [GATE-2017-CS: 1M]

(A) the carry bit C_7 is 1

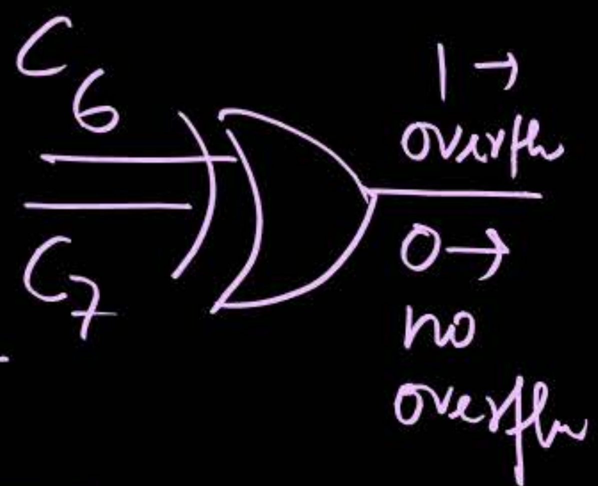
(B) all the carry bits ($C_7, \ldots, C_0$) are 1

(C) $(A_7 \cdot B_7 \cdot \overline{S_7} + \overline{A_7} \cdot \overline{B_7} \cdot S_7)$ is 1.

(D) $(A_0 \cdot B_0 \cdot \overline{S_0} + \overline{A_0} \cdot \overline{B_0} \cdot S_0)$ is 1.

$$
\begin{array}{rcccc}
 & A_7 & A_6 & \cdots & A_0 \\
+ & B_7 & B_6 & \cdots & B_0 \\
 & & C_6 & \cdots\; C_1 & C_0 \\
\hline
C_7 & S_7 & S_4\, S_3\, S_2 & S_1 & S_0
\end{array}
$$

discard



$\overline{A_7}\,\overline{B_7} \cdot S_7 + A_7 B_7 \cdot \overline{S_7}$

[NAT]

Let X be the number of distinct 16-bit integers in 2's complement representation. Let Y be the number of distinct 16-bit integers in sign magnitude representation. Then X-Y is ___1___. [GATE-2016-CS: 1M]

n bit $\longrightarrow \underline{\underline{2^n}} \rightarrow 2^n = X$

$Y = (2^n - 1) \longrightarrow$ sign magnitude

$2^{16} - (2^{15} - 1)$

$= 2^{15} + 1$

$2^{16} - 2^{15} = 2^{15}(2-1)$

[NAT]

The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101; its decimal representation is (-11)

[GATE-2016-CS: 1M]

$= (10101)_2 = -(11)_{10}$

[MCQ]

The smallest integer that can be represented by an 8-bit number in 2's complement form is

[GATE-2013-CS: 1M]

(A) -256

(B) -128

(C) -127

(D) 0

$-(2^{n-1})$ to $+(2^{n-1}-1)$

$-2^{8-1} = -2^7 = -(128)_{10}$

[MCQ]

P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of $8\times P$ is [GATE-2010-CS: 1M]

A. $(C3D8)_{16}$

B. $(187B)_{16}$

C. $(F878)_{16}$

D. $(987B)_{16}$

$$(1111\ 1000\ 0111\ 1011)_2 \times 2^3$$

$$=(C3D8)_{16}$$

$$=$$

$$8\times P = 11111\ 1000\ 0111\ 1011\ 000$$

$$2^1 \times 2^3 = 2^4$$

$$10 \times 2^3 = 10000$$

$$(101110)_2 = (11101110)_2 \rightarrow 2's$$

$$=(EE)_{16}$$

$$(00101110)_2 = (2E)_{16} \rightarrow \text{unsigned.}$$

[MCQ]

Which of the following is/are EQUAL to 224 in radix-5 (i.e., base-5) notation?

[GATE-2014-CS: 1M]

(A) 64 in radix-10

(B) 100 in radix-8

(C) 121 in radix-7

(D) 50 in radix-16

H.W.

[NAT]

The total number of prime implicants of the function $f(w,x,y,z) = \Sigma(0, 2, 4, 5, 6, 10)$ is $\underline{\ 3\ }$.

[GATE-2022-CS: 1M]

	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1			1
$\bar{w}x$	1	1		1
wx				
$w\bar{x}$				1

PI $\bar{w}\bar{z}$, $\bar{w}x\bar{y}$

$\bar{x}y\bar{z} \rightarrow 3$

EPI - 3

[NAT]

Consider the minterm list form of a Boolean function F given below.

$$F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is ___3___. [GATE-2018-CS: 2M]

PI → $\bar{Q}\bar{S}$, $P\bar{Q}$, $\bar{P}QS$

$R\bar{Q}$

EPI → $\bar{Q}\bar{S}$, $P\bar{Q}$, $\bar{P}QS$

	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
$\bar{P}\bar{Q}$	1		X	1
$\bar{P}Q$		1	1	
PQ	X			X
$P\bar{Q}$	X	1	1	X

10to12

Thank you

GW Soldiers !