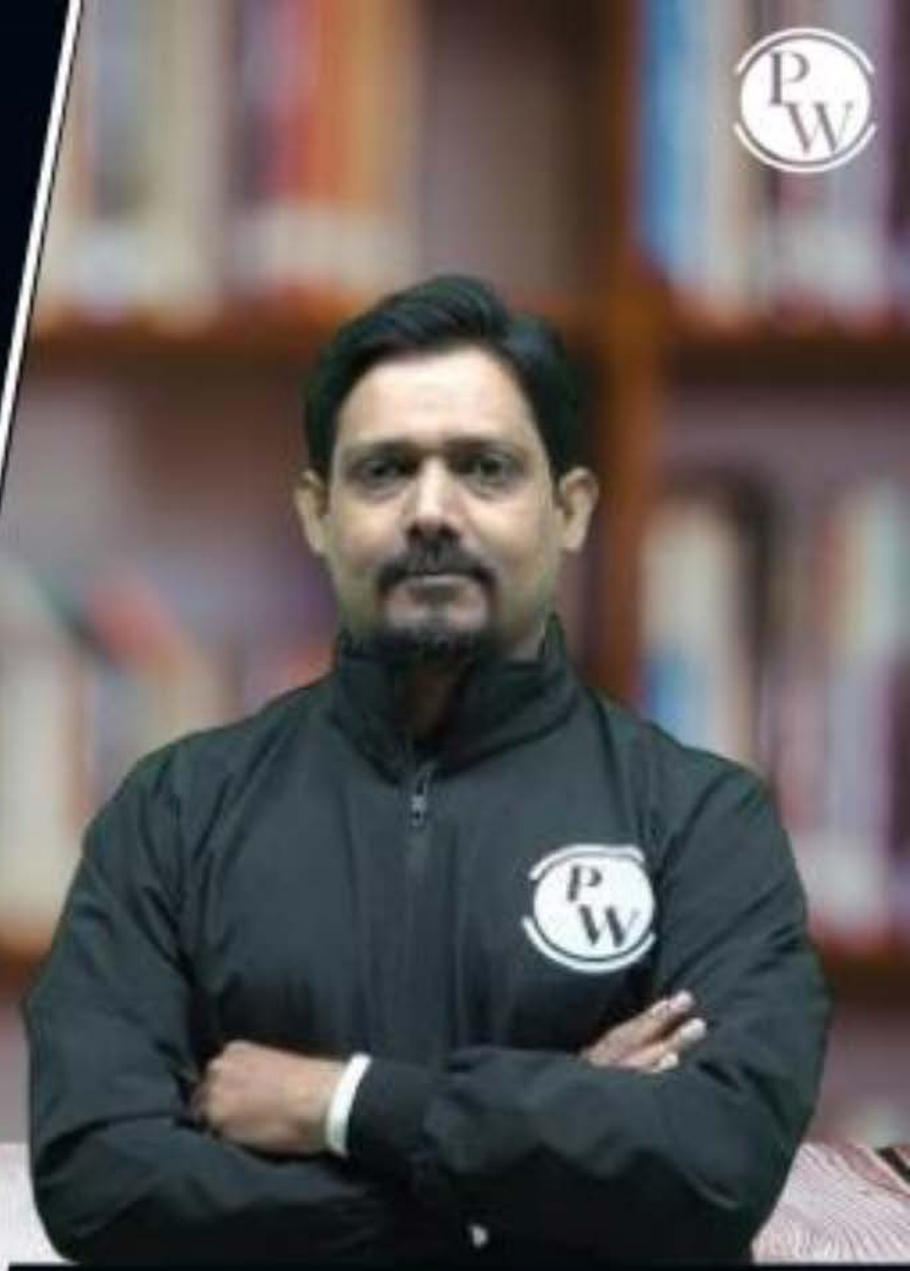
Computer Science & IT

Data Structure & Programming

Tree

Lecture No. 02



By- Abhishek Sir

Recap of Previous Lecture

- Topic
- Topic
- Topic
- Topic
- Topic

Tree Terminology

forest with n node p components

No. of edges = n - p

k-ary tree $L = I(k-1) + 1$

Slide

Topics to be Covered

Topic	Binary tree
Topic	Binary tree Height
Topic	Binary tree maximum & minimum Nodes
Topic	Binary tree Theorem
Topic	Counting unlabelled tree

Slide

Topic : Tree

K ary tree
Binary tree
Height of binary tree
Binary tree theorem
Special tree
Counting unlabeled node

Slide

Topic : Tree

$L = 41 \quad I = 10 \quad K = ?$

$$L = I(K-1)+1$$

$$41 = 10(K-1)+1$$

$$10(K-1) = 40$$

$$K-1 = 4$$

$$K = 5$$

$$L \times 1 + I(K+1) - 1$$

$$= 2(1+I-1)$$

$$L + IK + I - 1$$

$$= 2L + 2I - 2$$

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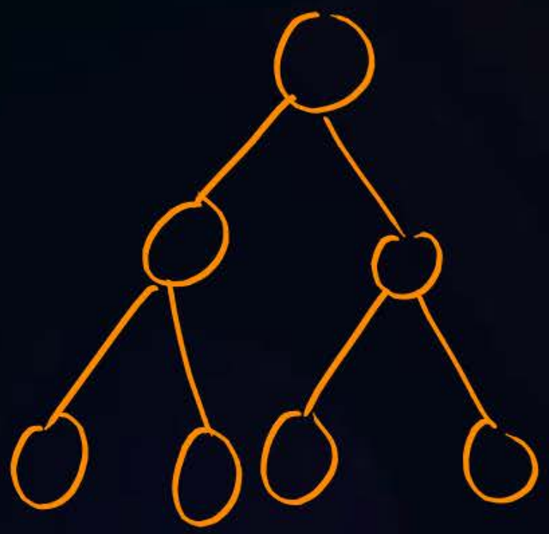
Topic : Tree

Binary tree : Binary tree is a tree in which each node is having 0, 1 or 2 children (atmost 2)



unlabelled tree

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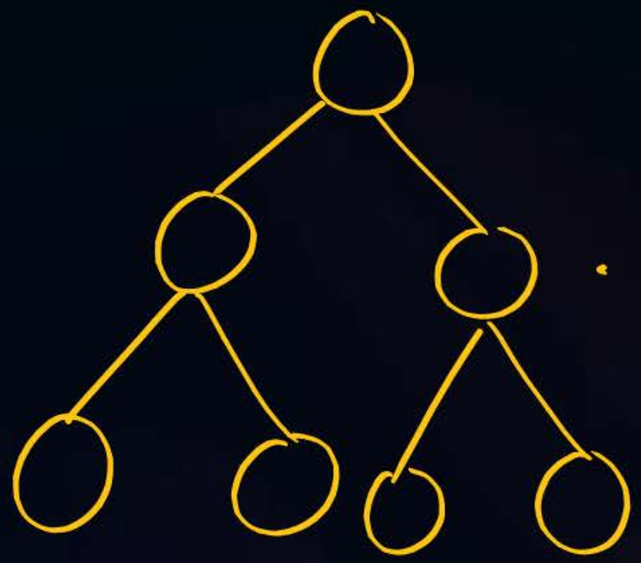
Topic : Tree

Full Binary tree (2019)



All levels are full
and all Leaves at same Level.

Every FBT is 2-ary tree

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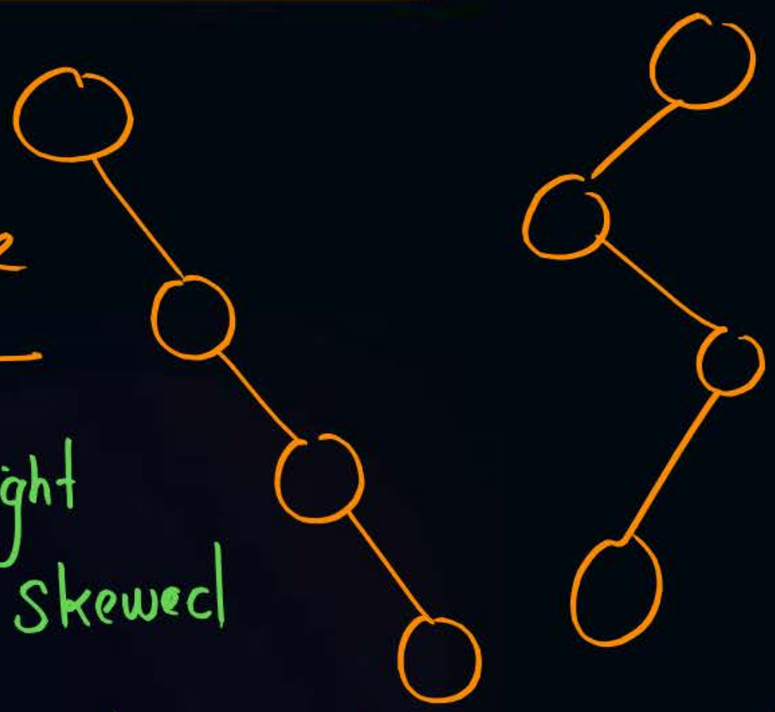
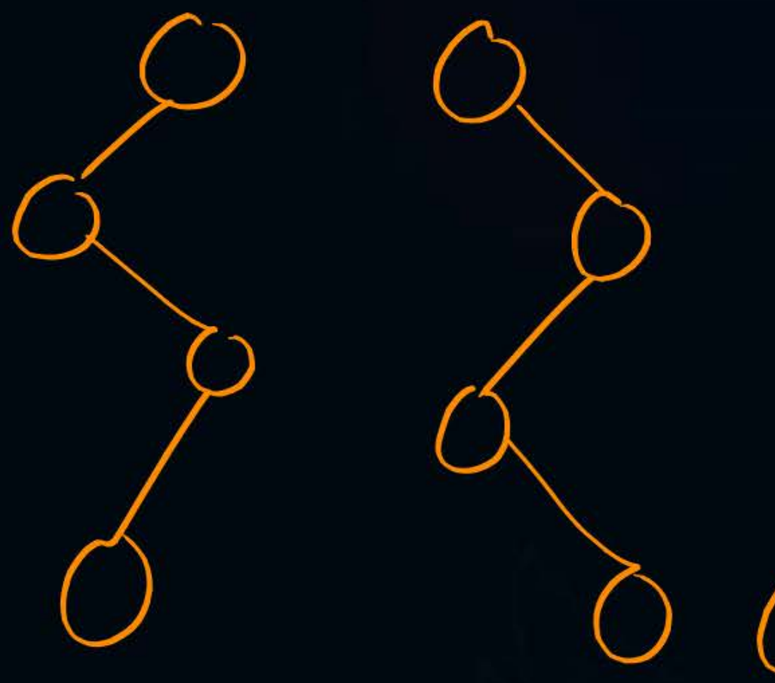
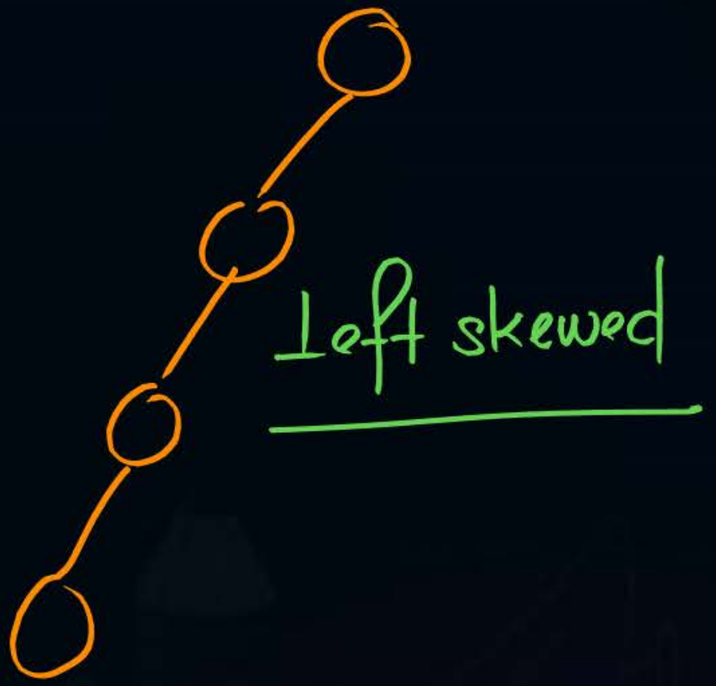
Topic : Tree

Binary tree

Right skewed







Left skewed

Skewed Binary tree : A Binary tree in which each Node has one children except 1 (which has zero children)

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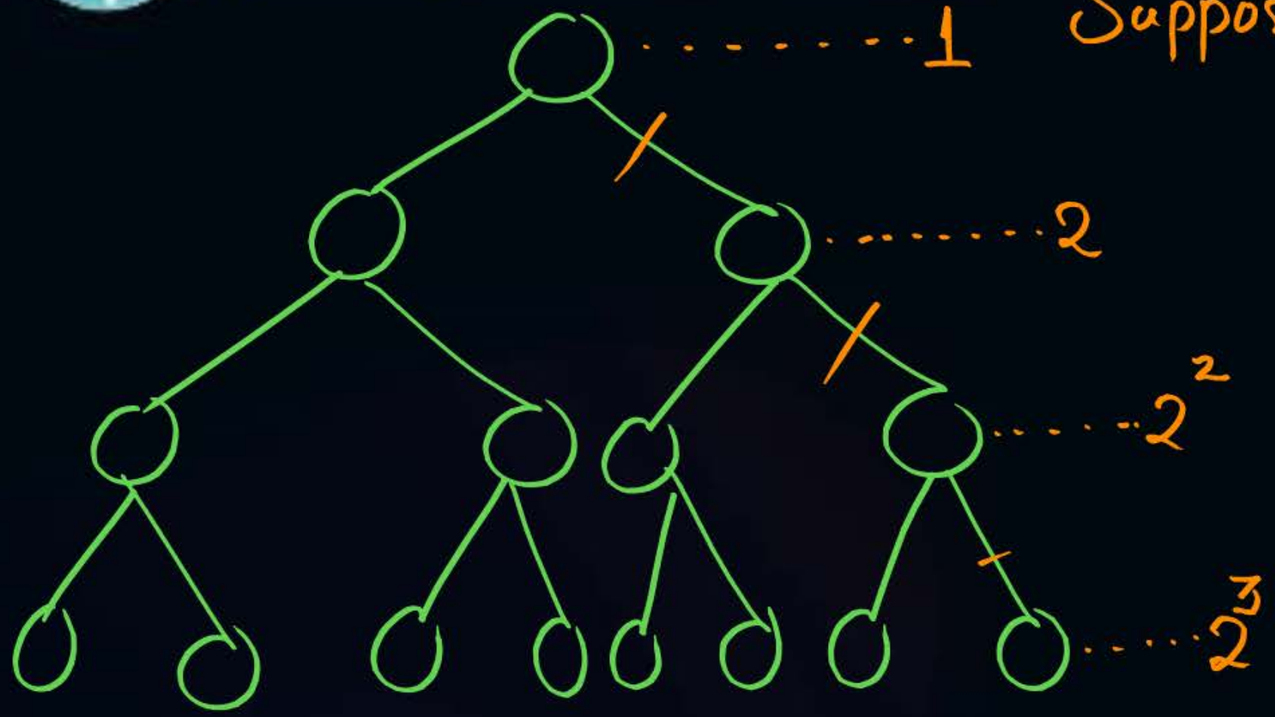
Topic : Tree

height of tree

Suppose tree contain n elements



FBT with heigh h

the No. of leaves $= 2^h$

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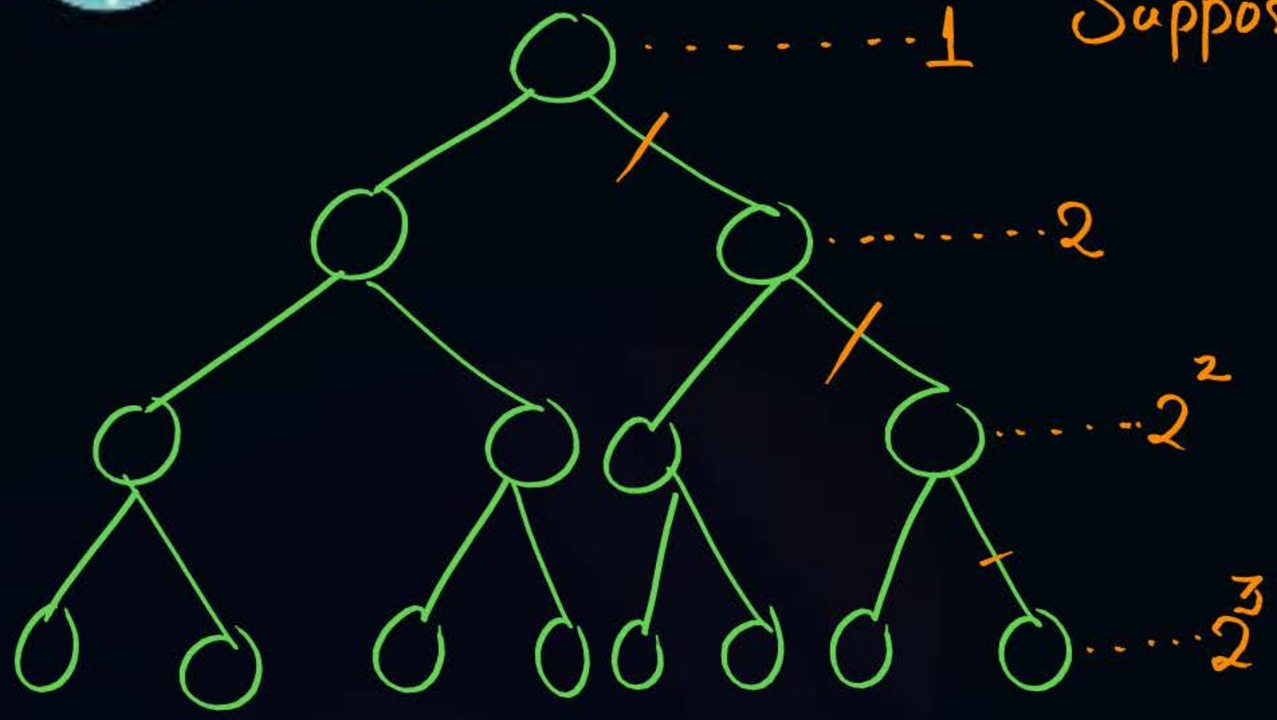
Topic : Tree



$$\frac{a(r^n-1)}{r-1}$$

height of tree

Suppose tree contain n elements

$$2^0+2^1+2^2+\cdots+2^h = n$$

$$\frac{1(2^{h+1}-1)}{2-1} = n$$

$$2^{h+1}-1 = n$$

$$2^{h+1} = n+1$$

$$2^h = \frac{n+1}{2}$$

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Topic : Tree

$$2^h = \frac{n+1}{2}$$

$$\text{No. of leaves} = \frac{n+1}{2}$$

take $\log_2$ both side

$$\log_2 2^h = \log_2\left(\frac{n+1}{2}\right)$$

$$h \cdot \log_2 2 = \log_2(n+1) - \log_2 2$$

$$h = \log_2(n+1) - 1$$

← FBT

$$h = \lfloor \log n \rfloor$$

Slide

Topic : Tree

$$h = \log_2(n+1) - 1$$

$$= \log_2(15+1) - 1$$

$$\log_2 16 - 1$$

$$\log_2 2^4 - 1$$

$$4 - 1 = 3$$

$$\lfloor \log_2 n \rfloor$$

$$\lfloor \log_2 15 \rfloor$$

$\lfloor 3.\sim \rfloor$ floor value

$= 3$

Slide

Topic : Tree

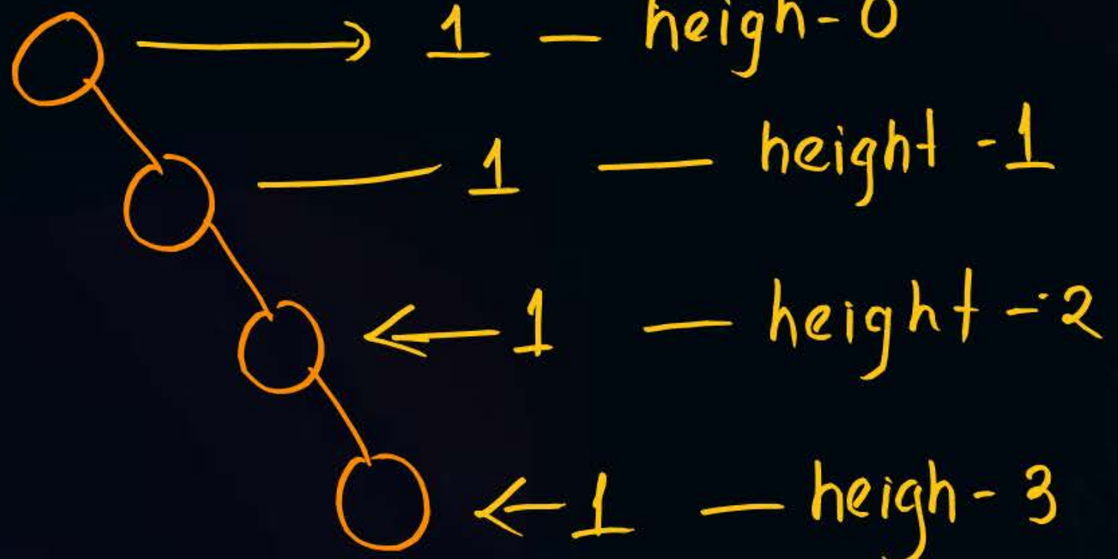
n is total No. of Nodes

$\underbrace{1+1+1+1\ldots 1}_{h+1} = n$

$h + 1 = n$

$h = n - 1$

Skewed Binary tree $h = 3$



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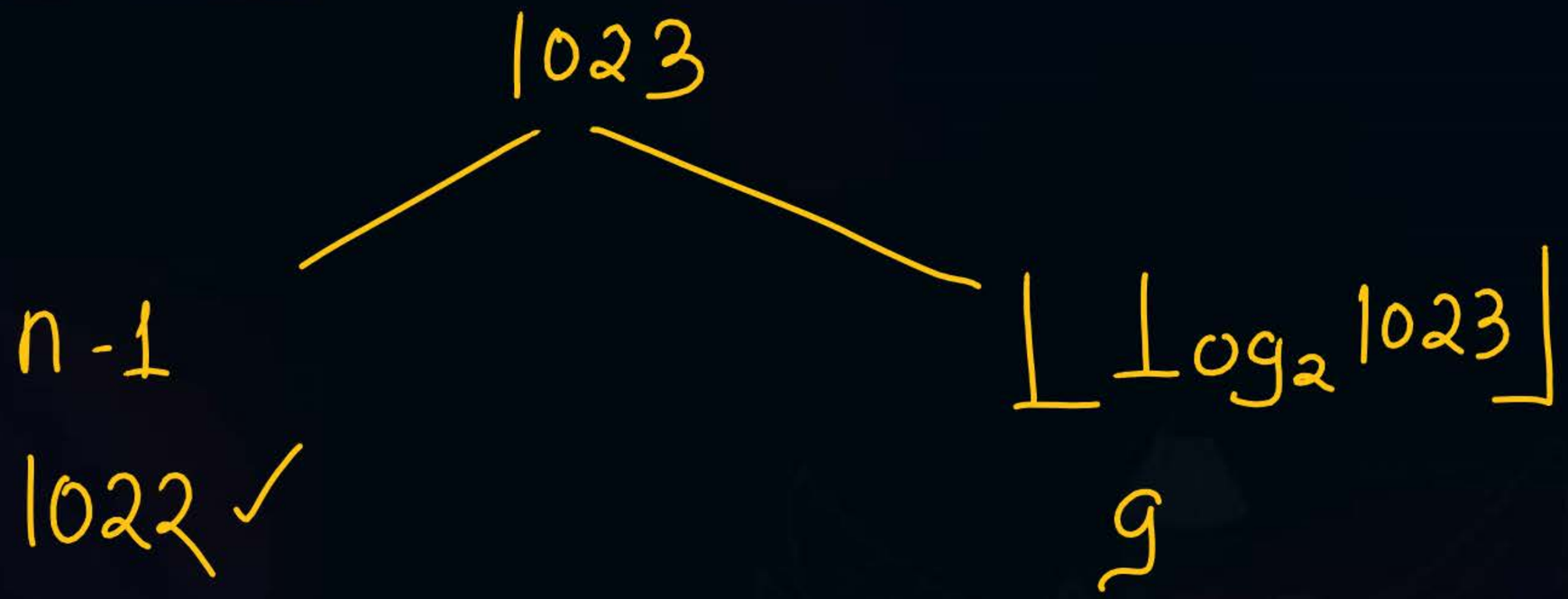
Topic : Tree



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Topic : Tree

Maximum No. of Nodes in k-ary tree with height h is $\frac{k^{h+1}-1}{k-1}$

Minimum No. of Nodes in 4-ary tree with Height 10 is 41.

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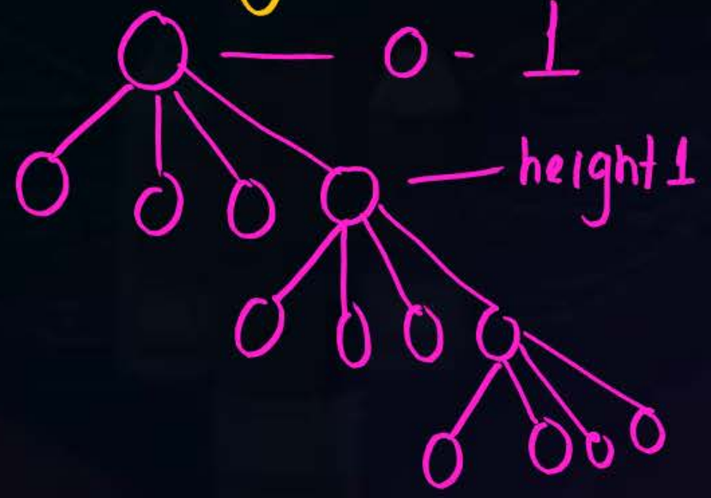
$1+(4+4+4+1\cdots+4)$

$1+40=41$



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Topic : Tree

For k-ary tree of Height h

Minimum No. of Nodes $1 + k \times (h)$

$= 1 + kh$

Slide

Topic : Tree

Binary tree : In a binary tree if n_0 is No. of leaf Nodes n_1 is No. of Node with 1 children n_2 is No. of Node with 2 children.

$$n_0 = n_2 + 1$$

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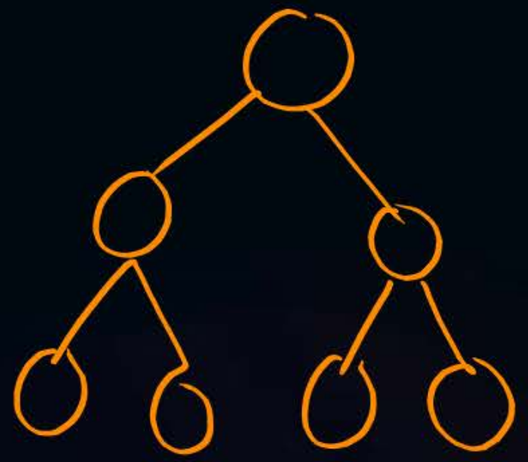
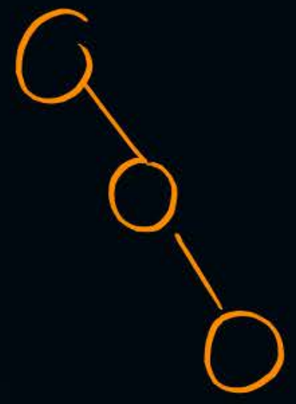
Topic : Tree



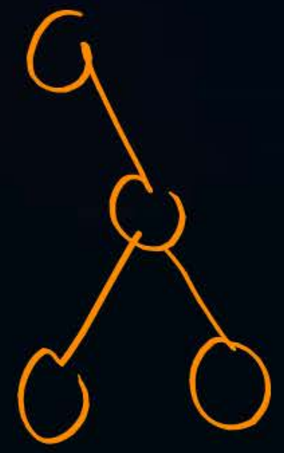
$n_0 = 4$

$n_2 = 3$



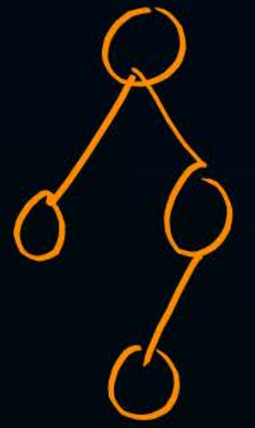
$n_0 = 1$

$n_2 = 0$



$n_0 = 2$

$n_2 = 1$



$n_0 = 2$

$n_2 = 1$

$$n_0 = n_2 + 1$$

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Topic : Tree

1. degree of n_0 — 1
2. degree of n_1 — 2
3. degree of n_2 — 3

Handshaking with adjustment for root.

4. Total No. of Nodes (n_0, n_1, n_2) : $n_0 + n_1 + n_2$
5. Total No. of edges : $n_0 + n_1 + n_2 - 1$

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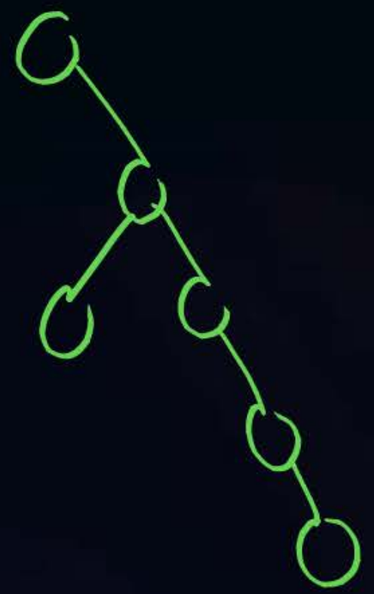
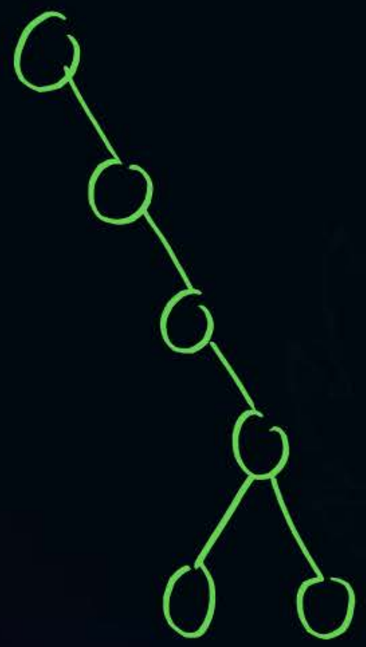
Topic : Tree

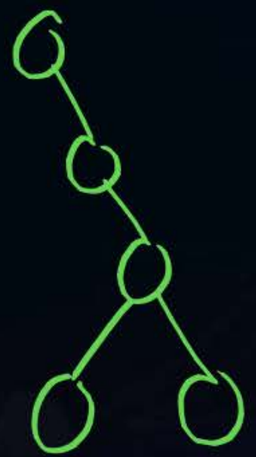
$n_0 = 2, n_2 = 1$

$n_1 = 3$





$n_1 = 2$



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Topic : Tree

Counting unlabelled trees

(1) $n = 1$

(2) $n = 2$

$n = 2$ 2 structure possible

$n = 3$. 5 structure possible

(3) $n = 3$

2 Node to distribute left and right

L	R
2	0
1	1
0	2

Slide

Topic : Tree

(4) $n = 4$

3 Left.

L	R
3	0 — 5
2	1 — 2
1	2 — 2
0	3 — 5

$2 \times 2 = 4$

$2 \times 5 = 10$

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Topic : Tree

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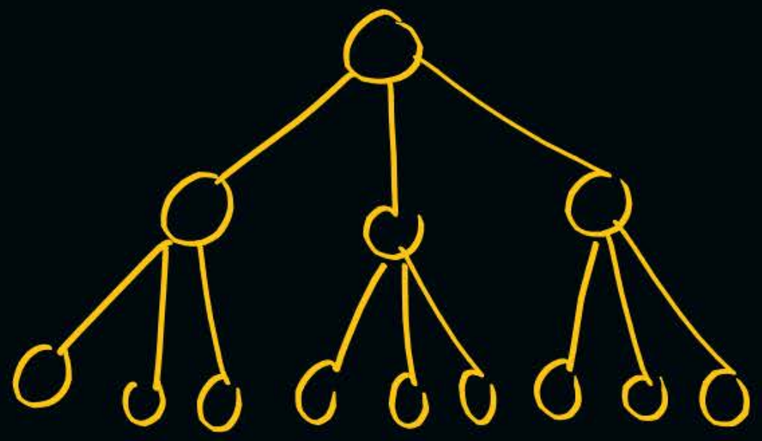
Q The maximum No. of Nodes in 3-ary tree of Height 6 is ____?



$$3^0 + 3^1 + 3^2 + \cdots + 3^6$$

$$= \frac{1(3^7 - 1)}{3 - 1} = \frac{(2187 - 1)}{2}$$

$$= \frac{2186}{2} = 1093$$

The maximum No. of Node for k any tree of height h ——————

$$\frac{(k^{h+1} - 1)}{k - 1}$$

Topic : Question

#Q The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is:

$n = 2^{h+1} - 1$

(A) $2^h - 1$

(B) $2^{h-1} - 1$

(C) $2^{h+1} - 1$ ✓

(D) 2^{h+1}

Topic : Question

$h = n - 1 = 126$

#Q Let T be a binary tree with 127 nodes. The minimum and maximum possible heights of T are:

if minimum height is x and 2 maximum Height is y

$\lfloor \log_2 127 \rfloor$

$\lfloor 6.- \rfloor$

$= 6$

$\log_2(127+1) - 1$

$7 - 1 = 6$

6

then $2^x + y$ is 190 ?

$64 + 126$

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2 mins Summary

Topic — Binary tree Height

Topic — Max- Min Nodes in Binary tree & k-ary tree

Topic — Theorem

Topic — Counting of unlabelled trees.

Topic

Slide

THANK - YOU