COMPUTER SCIENCE & IT

DIGITAL LOGIC

Lecture No: 03

Miscellaneous Topics

By- Chandan Gupta Sir

Recap of Previous Lecture

- Number System
- Concept of compliment
- Signed number representation

Signed no. representation

[2's Compliment Method]

$n = 4$ bits

$$0000 \longrightarrow +0 \xrightarrow{2's} 0000 \longrightarrow -0$$
$$0001 \longrightarrow +1 \xrightarrow{2's} 1111 \longrightarrow -1$$
$$0010 \longrightarrow +2 \xrightarrow{2's} 1110 \longrightarrow -2$$
$$0011 \longrightarrow +3 \xrightarrow{2's} 1101 \longrightarrow -3$$
$$0100 \longrightarrow +4 \xrightarrow{2's} 1100 \longrightarrow -4$$
$$0101 \longrightarrow +5 \xrightarrow{2's} 1011 \longrightarrow -5$$
$$0110 \longrightarrow +6 \xrightarrow{2's} 1010 \longrightarrow -6$$
$$0111 \longrightarrow +7 \xrightarrow{2's} 1001 \longrightarrow -7$$

$$\boxed{-8 \text{ to } +7}$$

$$\boxed{\begin{aligned} &1000 \\ &= -8\times1+0+0+0 \\ &= (-8)_{10} \end{aligned}}$$

- Total no. of numbers (distinct numbers) that can be represented using n-bits in 2's compliment representation $= 2^n$

- Range : $-[2^{n-1}]$ $\quad +[2^{(n-1)} - 1]$

$$\underset{(-7)}{1001} \xrightarrow{2's} \underset{+7}{(0111)}$$

$$\underset{(-3)}{1101} \xrightarrow{2's} \underset{+3}{(0011)}$$

$$(1001)_2 = -8 + 0 + 0 + 1$$
$$= (-7)_{10}$$
$$(1101)_2 = -8 + 4 + 0 + 1 = (-3)_{10}$$

Imp Discussion regrading 2's compliment representation

-

$$A = (1001)_2 = (-7)_{10} \xrightarrow{2's} (0111)$$

$$+7 = -A$$

$$B = (0101)_2 = (+5)_{10} \xrightarrow{2's} (1011)_2 = (-5)_{10} = -B$$

$$A \xrightarrow{2's} -A$$

Could be +ve or -ve

$$A - B$$

$$= A + (-B)$$

$(1001)_2 = (-7)_{10}$

$(11001)_2 = -16+8+0+0+1 = (-7)_{10}$

$(111001)_2 = -32+16+8+0+0+1 = (-7)_{10}$

$0101 = +5$

$00101 = +5$

$000101 = +5$

$(1111111010)_2$

$= (1010)_2$

$= (-6)_{10}$

$(1110110)_2 = (10110)_2$

$= (-10)_{10}$

$(1001)_2 \xrightarrow[\text{left shift}]{\text{1 bit}} 1001\underline{0} = (-14)_{10} = 2 \times (-7)$

$= (-7)_{10}$

$(10010)_2 \xrightarrow[\text{left shift}]{\text{1 bit}} 10010\underline{0} = (-28)_{10} = 2 \times (-14)$

$= (-14)_{10}$

$(0101)_2 \xrightarrow[\text{left shift}]{\text{1 bit}} 0101\underline{0}$

$(+5)_{10}$ $\quad$ $(+10)_{10}$

$= 2 \times (+5)$

- Imp Points :

- If we copy MSB bit any no. of times to the left of MSB, then no. will not change.

Addition and Substraction in 2's compliment signed no. representation

Lets understand with example :

- $A = 1001 = (-7)_{10}$

  $B = 0101 = (+5)_{10}$

  $+$ ———————

  $(1110)_2 = (-2)_{10}$

- $A = (1111)_2 = (-1)_{10}$

  $B = (0111)_2 = (+7)_{10}$

  $+$ ———————

  $\boxed{1}\,\underline{0110} = (+6)_{10}$

  discard result

- $A = (1010)_2 = (-6)_{10}$

  $B = (0100)_2 = (+4)_{10}$

  $+$ ———————

  $(1110)_2 = (-2)_{10}$

- $A = (1100)_2 = (-4)_{10}$

  $B = (0111)_2 = (+7)_{10}$

  $+\ 1$ ———————

  $1\,\underline{0011} = (+3)_{10}$

  discard

- $A = (0011)_2 = (+3)_{10}$

  $B = (0100)_2 = (+4)_{10}$

  $+\ 0$ ———————

  $0\,(0111)_2 = (+7)_{10}$

- $A = (0001)_2 = (+1)_{10}$

  $B = (0100)_2 = (+4)_{10}$

  $+$ ———————

  $(0101)_2 = (+5)_{10}$

- $A = (0100)_2 = (+4)_{10}$

  $B = (0101)_2 = (+5)_{10}$

  $+\ 1$ ———————

  $0\,(1001)_2 = (-7)_{10}$

- $A = 0011 = (+3)_{10}$
  $B = (0111)_2 = (+7)_{10}$
  $+$
  $(1010)_2 = (-6)_{10}$

- $A = 1001 = (-7)_{10}$
  $B = 1111 = (-1)_{10}$
  $+\ 1$
  $\underbrace{(1)}_{\text{discard}}\ \underbrace{1000}_{\text{result}} = (-8)_{10}$

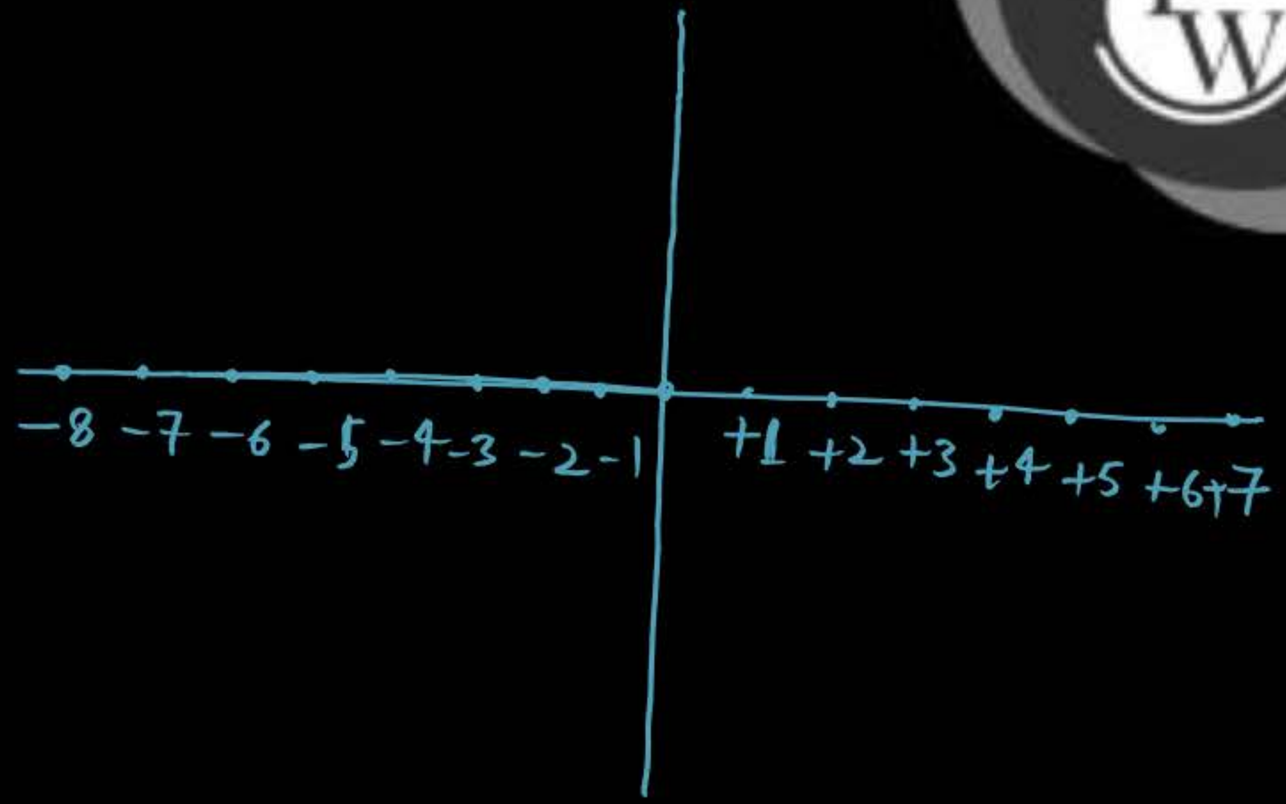
- $A = (1100)_2 = (-4)_{10}$
  $B = (1101)_2 = (-3)_{10}$
  $+$
  $\underbrace{(1)}_{\text{discard}}\ 1001 = (-7)_{10}$

- $A = (1100)_2 = (-4)_{10}$
  $B = (1011)_2 = (-5)_{10}$
  $+$
  $\underbrace{(1)}_{\text{discard}}\ 0111 = (+7)_{10}$

- $A = (1011)_2 = (-5)_{10}$
  $B = (1000)_2 = (-8)_{10}$
  $+\ 0 - C_{in}$
  $\underbrace{(1)}_{\text{discard}}\ 0011 = (+3)_{10}$
  Cout

$4 \text{ bit} \longrightarrow -(2^{4-1}) \text{ to } +(2^3 - 1)$

$(-8 \text{ to } +7)$



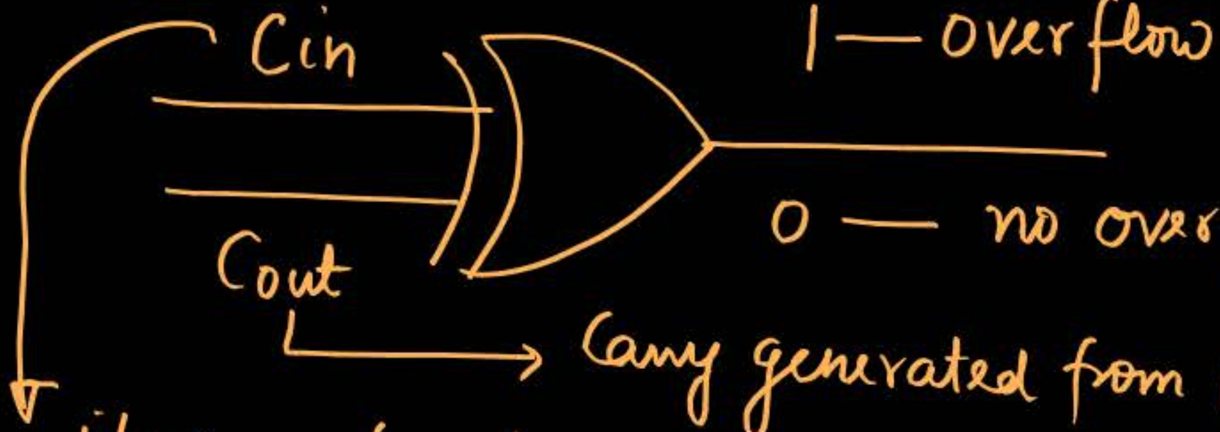
Condition of Overflow

$x \rightarrow$ sign bit of no. A.

$y \rightarrow$ sign bit of no. B

$z \rightarrow$ sign bit of result

$$\boxed{\text{Overflow} = \bar{x}\cdot\bar{y}\cdot z + x\,y\cdot\bar{z}}$$



Cout $\rightarrow$ Carry generated from MSB bit

Cin $\rightarrow$ i/p carry from previous addition to the MSB bit

$$A - B = A + (-B)$$
$$= A + [\text{2's compliment of } B]$$

- $A = (1001) = 9$
  $B = (0111) = 7$
  $-$

[unsigned no. subtraction using 2's compliment method]

$A - B \Rightarrow A + (2\text{'s compliment of } B)^{(2)}$

$$
\begin{array}{r}
1001 = \\
+\ 1001 \\
\hline
1\ 0010 = 2
\end{array}
$$

discard

- $B - A$

$$
= \begin{array}{r}
0111 = B \\
+\ 0111 = -A \\
\hline
1110
\end{array} \rightarrow -\text{ve result}
$$

$$= -(0010) = -(2)_{10}$$

- $A = 1100 = 12$
  $B = 0011 = 3$

$A - B \Rightarrow$

$$
\begin{array}{r}
1100 = A \\
+\ 1101 = -B \\
\hline
1\ 1001 = (9)_{10}
\end{array}
$$

discard

$B - A \Rightarrow$

$$
= \begin{array}{r}
0011 = B \\
+\ 0100 = -A \\
\hline
0111
\end{array} \rightarrow -\text{ve result}
$$

$$-(1001) = -(9)_{10}$$

$A - B = A + (1\text{'s compliment of } B)$

[unsigned no. subtraction using 1's compliment method]

$A = 1001 = +9$

$B = 0111 = +7$

$$
\begin{array}{r}
1001 = A \\
+\ 1000 = -B \\
\hline
1\,0001 \\
1 \\
\hline
0010 = (+2)
\end{array}
$$

$B - A \Rightarrow$

$$
\begin{array}{r}
B = 0111 \\
-A = 0110 \\
\hline
1101
\end{array}
\Rightarrow \text{-ve result}
$$

$\Rightarrow -(0010) = (-2)_{10}$

- $A = 1100 = 12$

  $B = 0011 = 3$

$$
\begin{array}{r}
A = 1100 \\
-B = 1100 \\
\hline
1\,1000 \\
1 \\
\hline
(1001) = (9)
\end{array}
$$

$$
\begin{array}{r}
B = 0011 \\
-A = 0011 \\
\hline
0110
\end{array}
= \text{-ve result}
$$

$= -(1001)$

$= -(9)_{10}$

[**Question**]

Two numbers A and B are represented using 2's compliment representation :

$$A = (1011)_2, = (-5)_{10} \qquad B = (1110)_2 = (-2)_{10}$$

Then the results of (B – A) will be

(a) $(0011)_2$

(b) $(10011)_2$

(c) $(1000)_2$

(d) $(1101)_2$

$B - A = -2 - (-5) = 3$

$(0011)_2$

$A - B = -5 - (-2) = -3$

$(1101)_2 = (-3)_{10}$

$B + (-A) =$

$$\begin{array}{rl} 1110 & = -2 \\ + \; 0101 & = +5 \\ \hline 1\,0011 & = +3 \end{array}$$

discard

$$\begin{array}{rl} 1011 & = A = (-5)_{10} \\ 0010 & = -B = +2 \\ \hline (1101) & = (-3)_{10} \end{array}$$

[**Question**]

Three numbers A, B and C are represented using 2's compliment representation as :

$$A = (1010)_2, \quad B = (0111)_2, \quad C = (1100)_2$$

Then the results of (A + B – C) in 2's compliment representation as :

(a) $(0011)_2$

(b) $(0101)_2$

(c) $(1011)_2$

(d) $(1101)_2$

$A = (1010)_2 = (-6)_{10}$

$B = (0111)_2 = (7)_{10}$

$C = (1100)_2 = (-4)_{10}$

$A+B-C = (+5)_{10} = (0101)_2$

$A+B = 1010 + 0111 = (1)0001$ discard

$A+B-C$

$= 0001 = (A+B)$

$+ 0100 = -C$

$0101 = (+5)_{10}$

$-A-B+C = (-5)_{10} = (1011)_2$

$= 0110 = -A$

$+ 1001 = -B$

$1111 = (-1)_{10}$

$1111 = -A-B = (-1)_{10}$

$+ 1100 = C = -(4)_{10}$

dis $(1)1011 = (-5)_{10}$

[**Question**]

A signed no. is represented using 2's compliment representation as :

$$N_1 = (1001)_2$$

Then its 8-bit representation will be :

(a) $(1001\,1001)_2$

(b) $(1111\,1001)_2$

(c) $(1000\,1001)_2$

(d) $(0000\,1001)_2$

[**Question**]

A 4-bit number in 2's compliment sign no. representation is

$M =$ | S_3 | S_2 | S_1 | S_0 | $= 0001 = (+1)$, $1001 = -7$

and other no. given in 2's compliment representation is

$N =$ | S_3 | S_3 | S_2 | S_1 | S_0 | 1 | 0 | $= 000110 = (+6)_{10}$

Then the relation between M and N is :

(a) $N = 4M$ ✗

(b) $M = 4N + 2$ ✗

(c) $N = 4M + 2$ ✓

(d) $N = 4M + 1$ ✗

$1100110 = (-26)_{10}$

$M = S_3 S_2 S_1 S_0$

$M = S_3 S_3 S_2 S_1 S_0$

$2M = S_3 S_3 S_2 S_1 S_0 0$

$2 \times 2M = S_3 S_3 S_2 S_1 S_0 0 0 = 4M$

$+ \; 0000010 = 2$

$S_3 S_3 S_2 S_1 S_0 1 0 = (4M+2)$

- H.W.

- Q. The numbers A, B, C are represented in 2's compliment representation as:

$$A = (11001)_2$$

$$B = (01001)_2$$

$$C = (11110)_2$$

- Then value of $(A + B - C)$ $(\quad)_{10}$. $(\quad)_2$
- Then Value of $(-A - B - C)$ $(\quad)_{10}$. $(\quad)_2$
- Then value of $(A - B + C)$ $(\quad)_2$

Topic : 2 Min Summary

$\rightarrow$ sign no. representation

Thank you

GW Soldiers !