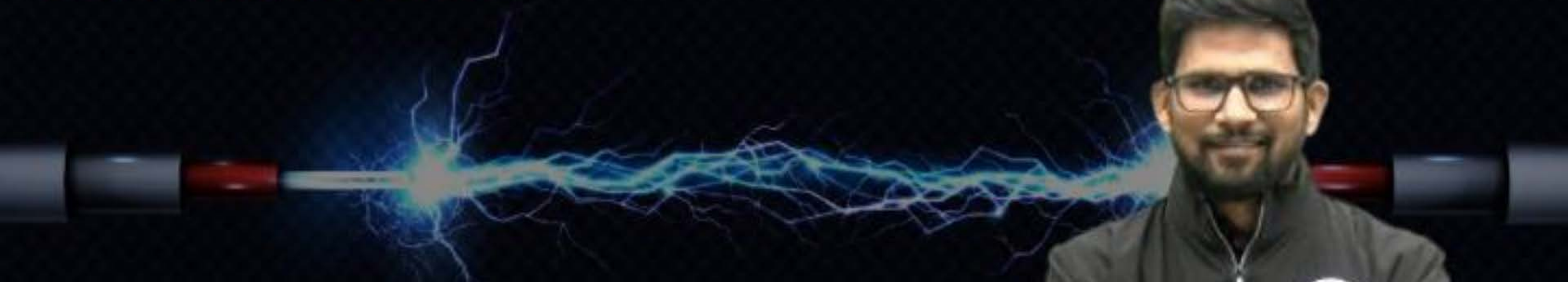




# COMPUTER SCIENCE & IT

## DIGITAL LOGIC



Lecture No. 12

Combinational Circuit



By- Chandan Gupta Sir

A circular arrangement of various tools including pliers, wire cutters, and剥线钳, with several multi-colored wires protruding from the center. A dark blue circle is overlaid on the tools.

# Recap of Previous Lecture

MUX & DEMUX

A diagram showing a vertical line on the left with four horizontal arrows pointing to the right. The top arrow is labeled 'MUX & DEMUX'.



A collection of various tools and wires arranged in a circular pattern around a central dark blue circle. The tools include pliers, wire cutters, a screwdriver, and a pair of scissors. The wires are of various colors (red, blue, green, yellow, orange) and are bundled together.

# Topics to be Covered

Question Practice

[NAT]



What is the minimum number of 2-input NOR gates required to implement a 4-variable function expressed in sum-of-minterms form as  $f = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$ ? Assume that all the inputs and their complements are available. 3

$$R \rightarrow \overline{B+D} \rightarrow 1$$

$$\overline{R+B} \rightarrow 1 \rightarrow P$$

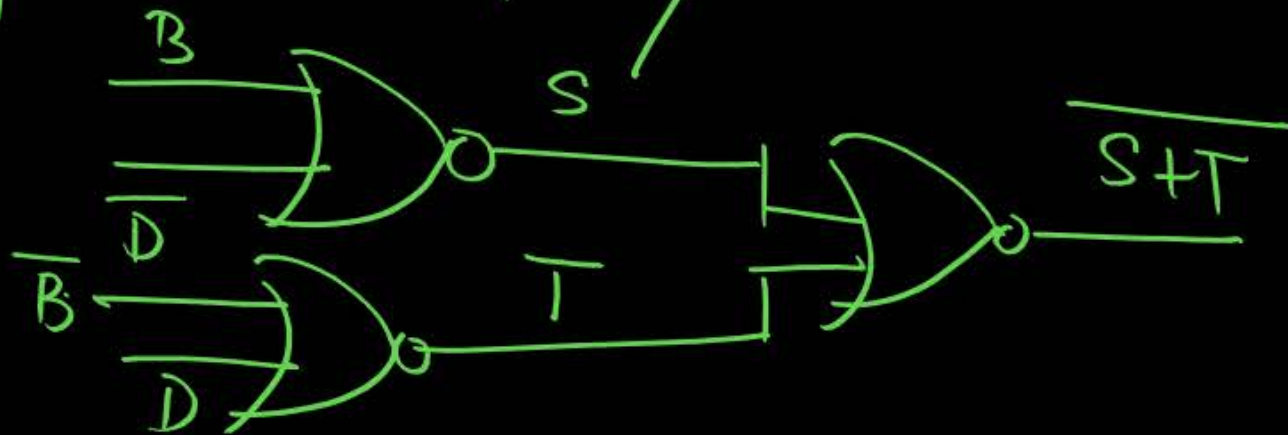
$$\overline{R+D} \rightarrow 1 \rightarrow Q$$

$$\overline{P+Q} \rightarrow 1$$

$$\overline{f} = \overline{B+D} + \overline{B+D}$$

$$\overline{f} = S + T$$

$$f = \overline{S+T}$$



	$\overline{C}\overline{D}$	$\overline{C}D$	$CD$	$C\overline{D}$
$\overline{A}\overline{B}$	1			1
$\overline{A}B$		1	1	
$AB$		1	1	
$A\overline{B}$	1			1

[GATE-2019-CS: 2M]

$$\overline{A\overline{B} + \overline{A}B} = (\overline{A+B})(\overline{A+B})$$

$$= \overline{BD} + \overline{B\overline{D}} = \overline{B \odot D} = (\overline{B+D})(\overline{B+D})$$

$$= \overline{BD} + BD = \overline{B+D} + BD$$

$$f = R + BD = (R+B)(R+D)$$

$$\overline{f} = \overline{R+B} + \overline{R+D}$$

$$f = \overline{\overline{R+B} + \overline{R+D}} = \overline{P+Q}$$



## [MCQ]

Let  $\oplus$  denote the Exclusive OR (XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q:

$$F(P, Q) = ((1 \oplus P) \oplus (P \oplus Q)) \oplus ((P \oplus Q) \oplus (Q \oplus 0))$$

The equivalent expression for F is

[GATE-2014-CS: 1M]

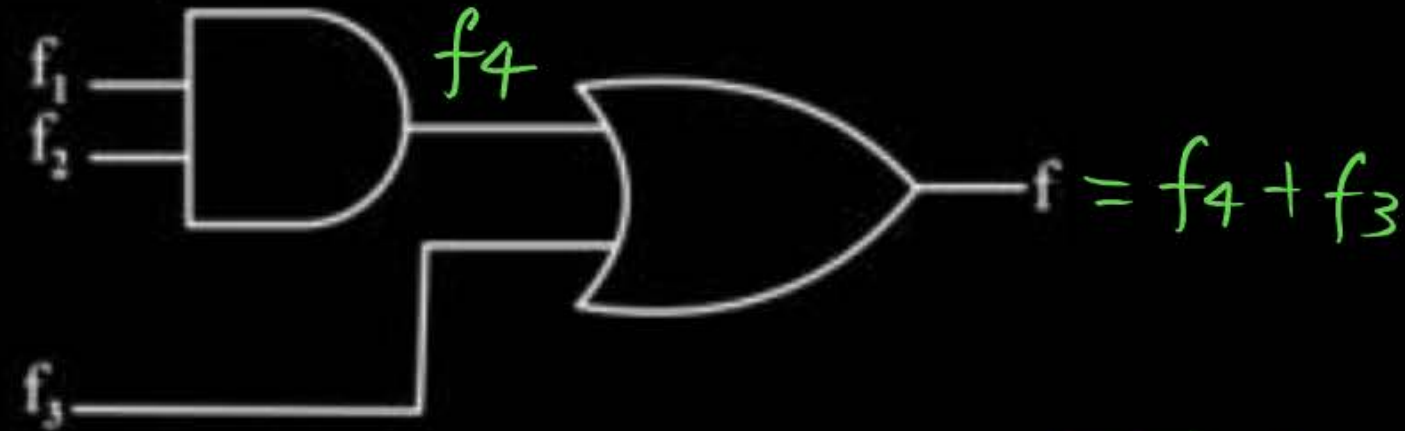
$$\overline{Q} \oplus P = P \odot Q = \overline{P \oplus Q}$$

- ☐ A  $P + Q$
- ☐ B  $(P + Q)'$
- ☐ C  $P \oplus Q$
- ☒ D  $(P \oplus Q)'$

# [MCQ]



Given  $f_1, f_3$  and  $f$  in canonical sum of products form (in decimal) for the circuit.



[GATE-2008-CS: 1M]

$$F_1 = \sum m(4, 5, 6, 7, 8)$$

$$F_3 = \sum m(1, 6, 15)$$

$$F = \sum m(1, 6, 8, 15)$$

Then  $f_2$  is

☐ A  $\sum m(4, 6)$  ✗

☐ B  $\sum m(4, 8)$  ✗

☒ C  $\sum m(6, 8)$

☐ D  $\sum m(4, 6, 8)$  ✗

$$f_4 = f_1 \cdot f_2$$

$$= \sum 1, 6, 15$$

$$f_2 = \sum \dots, 8$$

$$2^3 \rightarrow 8$$

$$\sum 1, 8, \sum 1, 6, 8, \sum (1, 8, 15)$$

$$f_4 \Rightarrow \frac{1, 6, 15}{8 \rightarrow \text{common}}$$

$$f_2 \rightarrow 8 \text{ common}$$

1, 6, 15 option



## [ Question ]

A logical function  $f(x, y, z)$  is given as

$$f(x, y, z) = x\bar{y} + yz$$

H.W.

Another logical function

$f_1(x, y, z) = f[\bar{f}, f, z]$  will be

a.  $\bar{x} + y + z$

b.  $\bar{x}y + yz$

c.  $(x + \bar{y})(\bar{y} + \bar{z})$

d.  $(x + \bar{y})(\bar{y} + \bar{z})(y + z)$

## [ Question ]



A 3-input majority gate is defined by the logic function

$M(a, b, c) = ab + bc + ca$ . Which one of the following gate is

$M(\overline{M(a, b, c)}, M(a, b, \bar{c}), c)$ ?

(a) 3-input NAND gate

(b) ✓ 3-input XOR gate

(c) 3-input NOR gate

(d) ✓ 3-input XNOR gate

$$M(a, b, c) = ab + bc + ca = \sum(3, 5, 6, 7)$$

$$f_1 = \overline{M(a, b, c)}, \quad f_2 = M(a, b, \bar{c})$$

$$M(f_1, f_2, c) = f_1 f_2 + f_2 c + f_1 c$$

$$f_1(a, b, c) = \sum(0, 1, 2, 4) \quad f_2(a, b, c) = M(a, b, \bar{c}) = \sum(2, 4, 6, 7)$$

$$ab\bar{c} + \bar{a}bc + a\bar{b}c = f(a, b, c)$$

$$f(a, b, \bar{c}) = f(a, b, d) =$$

$$d = \bar{c}$$

$$\bar{d} = c$$



$$M(f_1, f_2, c) = f_1 f_2 + f_1 \cdot c + f_2 \cdot c$$

$$= \Sigma(2, 4) + \Sigma(1) + \Sigma(7) = \Sigma(1, 2, 4, 7) = a \oplus b \oplus c$$

$$f_1 = \Sigma(0, 1, 2, 4) \cdot c = \Sigma(1)$$

$$f_2 = \Sigma(2, 4, 6, 7) \cdot c = \Sigma(7)$$

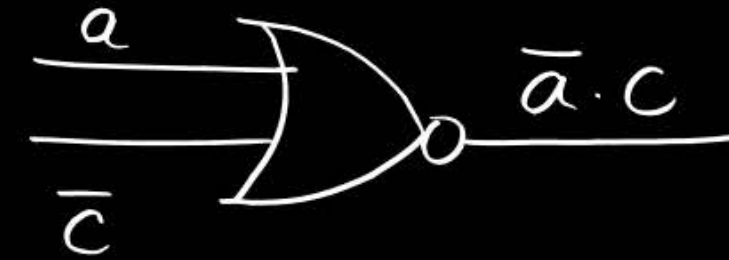
[NAT]



Consider the Karnaugh map given below, where X represents "don't care" and blank represents 0.

$ba \backslash dc$	00	01	11	10
00		x	x	
01	1			x
11	1			1
10		x	x	

$$c\bar{a} = \bar{a} \cdot c = \overline{a + \bar{c}}$$



Assume for all inputs(a, b, c, d) , the respective complements ( $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ ) are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is 1. [GATE-2017-CS: 1M]



# [MCQ]

Consider the following Boolean expression for

$$F: F(P, Q, R, S) = PQ + P'QR + P'QR'S$$

The minimal sum-of-products form of F is

[GATE-2014-CS: 1M]

- ☒ A  $PQ + QR + QS$
- ☐ B  $\underline{P} + \underline{Q} + \underline{R} + \underline{S}$  X
- ☐ C  $\bar{P} + \bar{Q} + \bar{R} + \bar{S}$  X
- ☐ D  $\underline{\bar{P}R} + \underline{\bar{P}RS} + P$  X

	$\bar{R}\bar{S}$	$\bar{R}S$	$RS$	$R\bar{S}$
$\bar{P}\bar{Q}$				
$\bar{P}Q$		1	1	1
$PQ$	1	1	1	1
$P\bar{Q}$				

$$PQ + QR + RS$$

# [MCQ]

Consider the following minterm expression for

$$F: F(P, Q, R, S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$$

The minterms 2, 7, 8 and 13 are 'do not care' terms.

The minimal sum-of-products form for F is

[GATE-2014-CS: 1M]

- ☐ A  $Q\bar{S} + \bar{Q}S$
- ☒ B  $\bar{Q}\bar{S} + QS$
- ☐ C  $\bar{Q}\bar{R}\bar{S} + \bar{Q}R\bar{S} + Q\bar{R}S + QRS$  ✗
- ☐ D  $\bar{P}\bar{Q}\bar{S} + \bar{P}QS + PQS + P\bar{Q}\bar{S}$  ✗

$$= \bar{Q}\bar{S} + QS$$

	$\bar{R}\bar{S}$	$\bar{R}S$	$RS$	$R\bar{S}$
$\bar{P}\bar{Q}$	1			X
$\bar{P}Q$		1	X	
$PQ$		X	1	
$P\bar{Q}$	X			1



# [MCQ]

What is the minimal form of the karnaugh map shown below? Assume that X denotes a don't care term

[GATE-2012-CS: M]

- A**  $\bar{b}\bar{d}$
- B**  $\bar{b}\bar{d} + \bar{b}\bar{c}$
- C**  $\bar{b}\bar{d} + a\bar{b}\bar{c}d$
- D**  $\bar{b}\bar{d} + \bar{b}\bar{c} + \bar{c}\bar{d}$

cd \ ab	00	01	11	10
00	1	X	X	1
01	X			1
11				
10	1			X

H.W.

# [MCQ]

In the Karnaugh map shown below, X denotes a don't care term. What is the minimal form of the function represented by the Karnaugh map?

[GATE-2008-CS: M]

- A**  $\bar{b}. \bar{d} + \bar{a}. \bar{d}$
- B**  $\bar{a}. \bar{b} + \bar{b}. \bar{d} + \bar{a}. b. \bar{d}$
- C**  $\bar{b}. \bar{d} + \bar{a}. b. \bar{d}$
- D**  $\bar{a}. \bar{b} + \bar{b}. \bar{d} + \bar{a}. \bar{d}$

		ab			
		00	01	11	10
cd	00	1	1		1
	01	X			
	11	X			
	10	1	1		X

H.W.





## [MCQ]

Consider the following Boolean function of four variables  $f(A, B, C, D) = \Sigma(2, 3, 6, 7, 8, 9, 10, 11, 12, 13)$  The function is [GATE-2008-CS: 1M]

- ☐ A independent of one variable
- ☐ B Independent of two variables
- ☐ C Independent of three variables
- ☐ D Dependent on all the variables

H.W.

# [MCQ]



Given  $f(w, x, y, z) = \sum m(0, 1, 2, 3, 7, 8, 10) + \sum d(5, 6, 11, 15)$ , where d represents the don't-care condition in Karnaugh maps. Which of the following is a minimum product-of-sums (POS) form of  $f(w, x, y, z)$ ? [GATE-2017-CS: 1M]

- ☒ A  $f = (\bar{w} + \bar{z})(\bar{x} + z)$
- ☒ B  $f = (\bar{w} + \underline{z})(x + \underline{z})$   $z + \bar{w}x$
- ☒ C  $f = (w + z)(\bar{x} + z)$   $z + w\bar{x}$
- ☒ D  $f = (\bar{w} + \bar{z})(\bar{x} + z)$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$yz$
$\bar{w}\bar{x}$	1	1	1	1
$\bar{w}x$	0	X	1	X
$w\bar{x}$	0	0	X	0
$wx$	1	0	X	1

$$(\bar{w}z + \bar{x}\bar{z})$$
$$(\bar{x} + \bar{y})(\bar{x} + z)$$
$$= \bar{x}\bar{z} + \bar{x}y$$

$$(\bar{w} + \bar{z})(\bar{x} + z) = \bar{w}\bar{x} + \bar{w}z + \bar{x}\bar{z}$$

$\downarrow$   
redundant



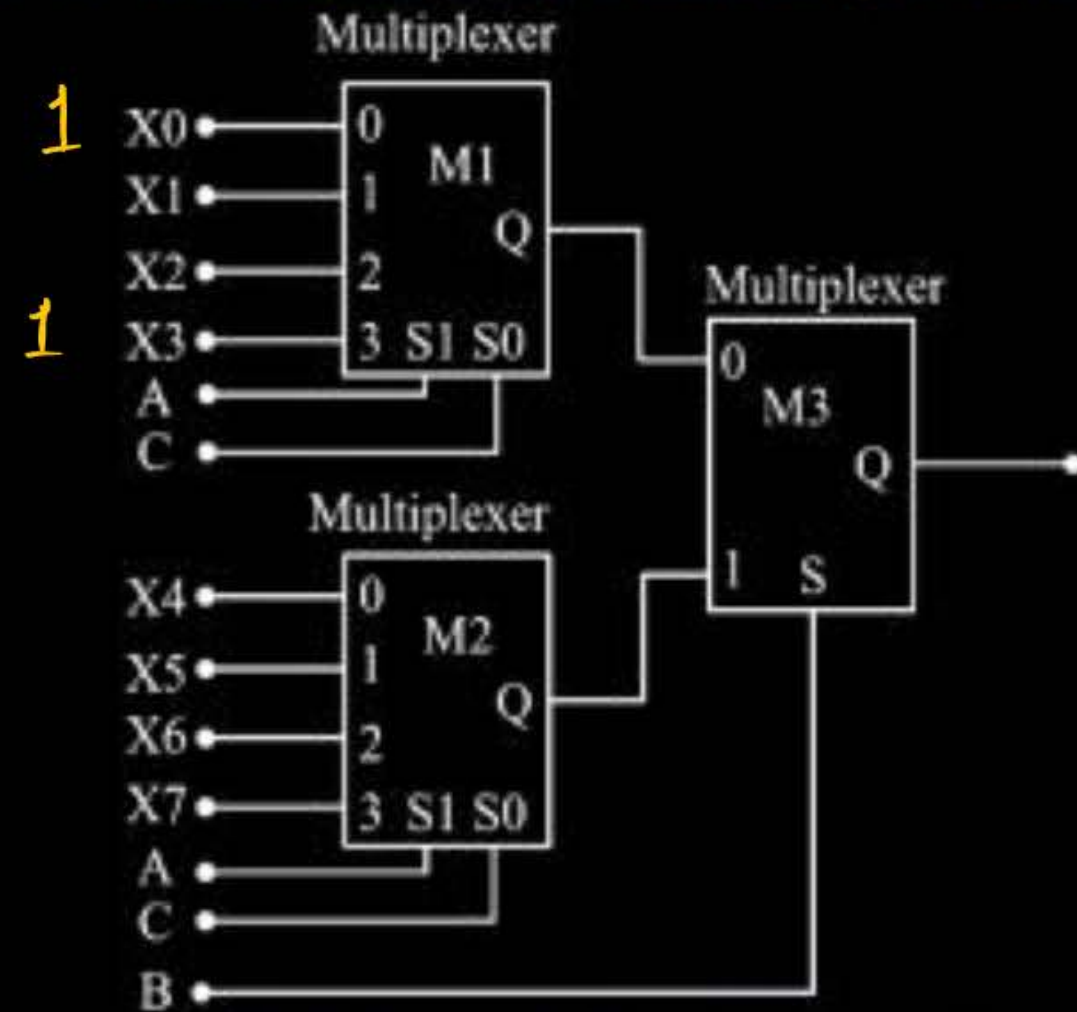
# [MCQ]



A Boolean digital circuit is composed using two 4-input multiplexers (M1 and M2) and one 2-input multiplexer (M3) as shown in the figure. X0-X7 are the inputs of the multiplexers M1 and M2 and could be connected to either 0 or 1. The select lines of the multiplexers are connected to Boolean variables A, B and C as shown.

$$8:1 \xrightarrow{4:1} 2(4:1) + 1(2:1)$$

↓  
 $S_2 S_1 S_0$   
A B C



$$f(A, B, C) = \bar{A} + \bar{A}\bar{C} + A\bar{B}C$$

$$f(0, 0, 0) = 1, \quad X_0 = 1$$

$$f(1, 0, 1) = 1, \quad X_3 = 1$$

Which one of the following set of values of (X0, X1, X2, X3, X4, X5, X6, X7) will realise the Boolean function  $\bar{A} + \bar{A}\bar{C} + A\bar{B}C$ .

[GATE-2023-CS: 2M]

$$x_1 = 1$$

$$x_3 = 1$$

**A**  $(1, 1, 0, 0, 1, 1, 1, 0)$  ✗

**B**  $(1, 1, 0, 0, 1, 1, 0, 1)$  ✗

**C** ✓  $(1, 1, 0, 1, 1, 1, 0, 0)$

**D** ✗  $(0, 0, 1, 1, 0, 1, 1, 1)$

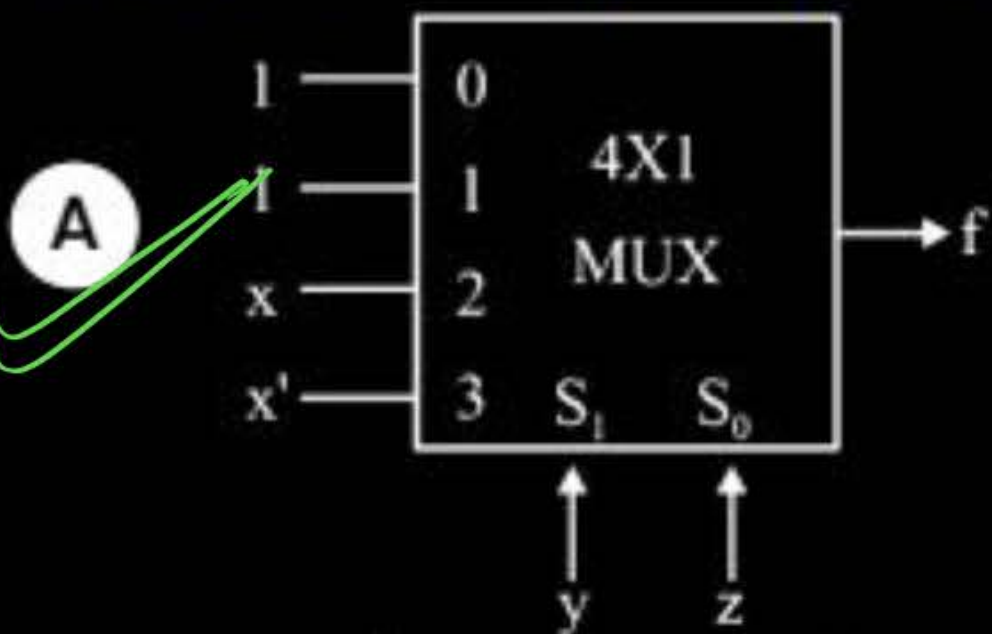


# [MCQ]

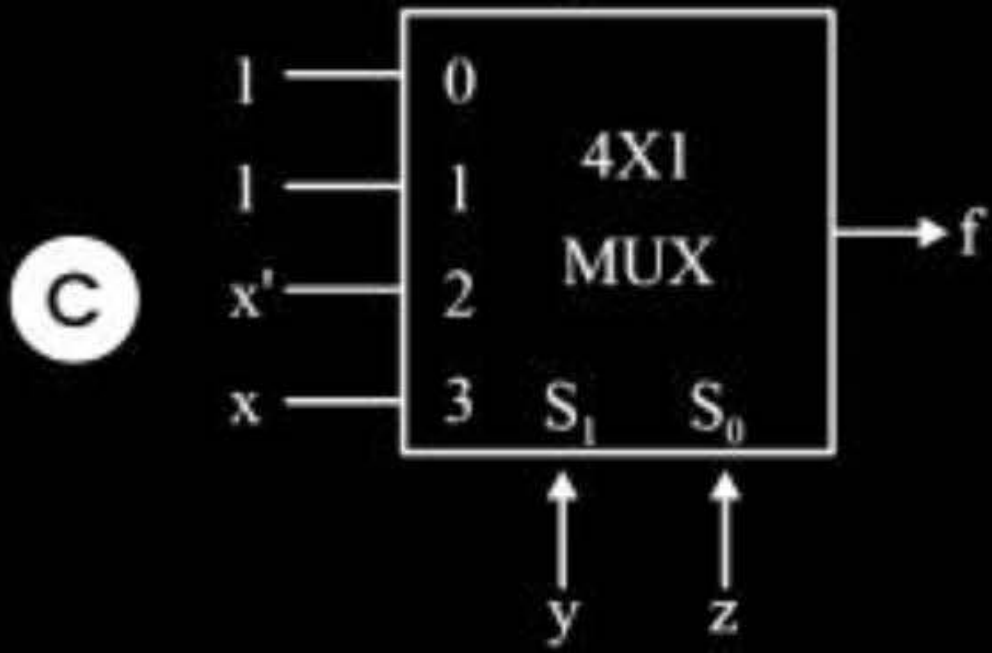
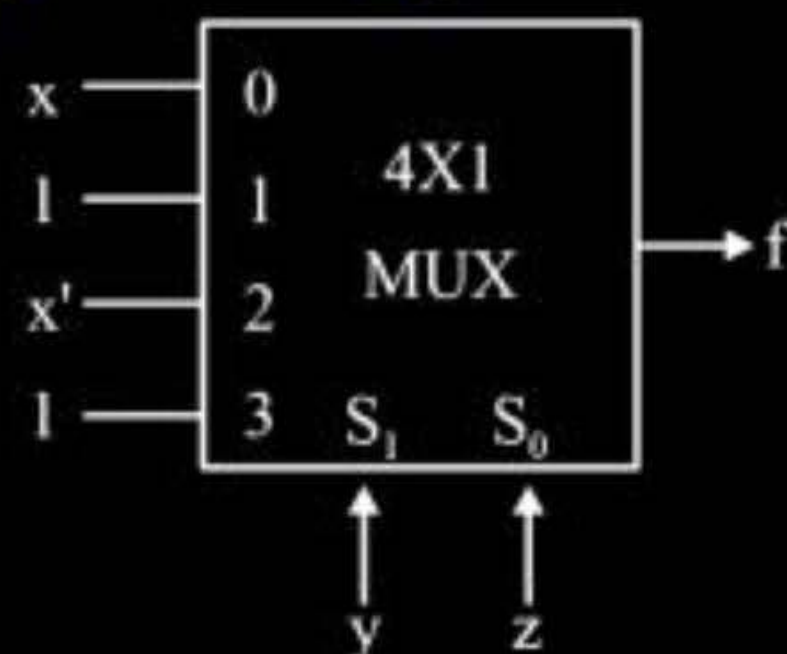
Which one of the following circuits implements the Boolean function given below?

$$f(x, y, z) = m_0 + m_1 + m_3 + m_4 + m_5 + m_6 \text{ where } m_i \text{ is the } i^{\text{th}} \text{ minterm.}$$

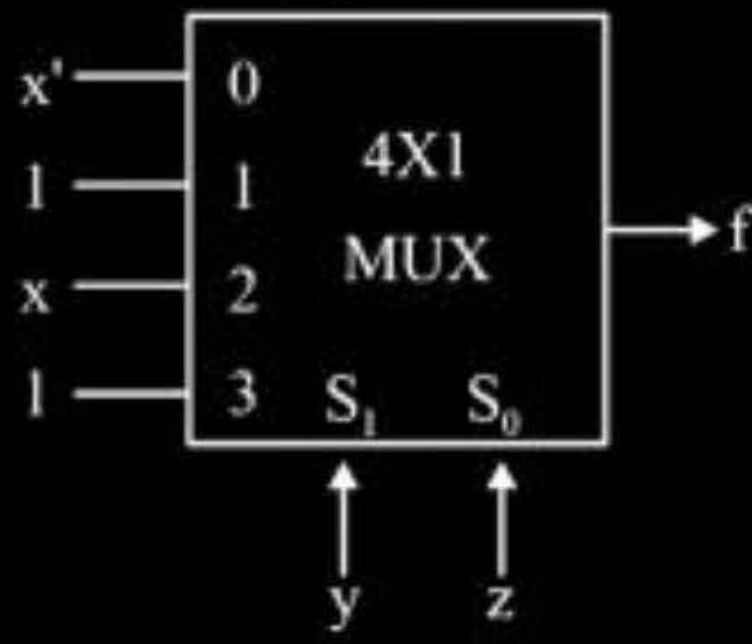
[GATE-2021-CS: 1M]



**B**



**D**



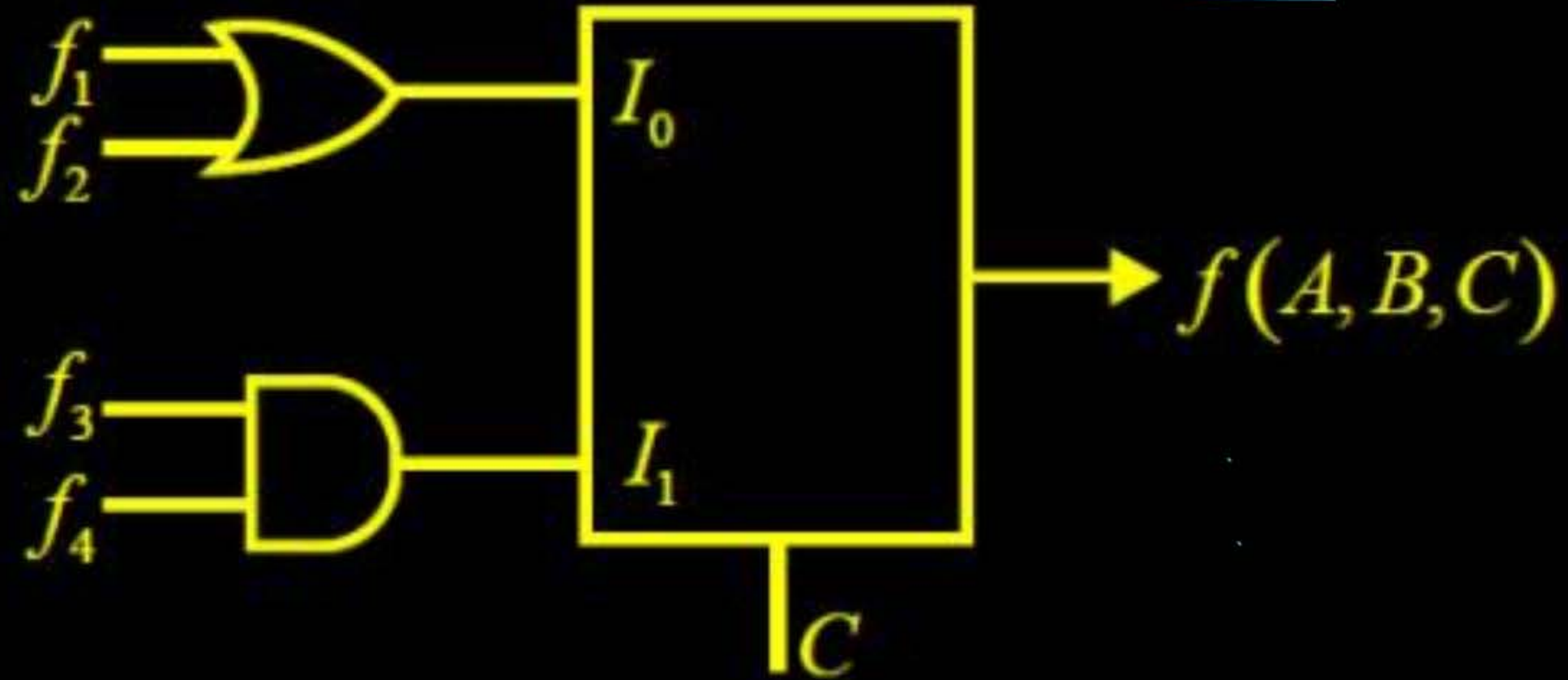
		$I_0$	$I_1$	$I_2$	$I_3$
0	$\bar{x}$	0	1	2	3
1	$x$	4	5	6	7
		1	1	$x$	$\bar{x}$

## [ Question ]

A digital circuit is as shown below:

When  $f_1(A, B) = \Sigma(0, 1)$   $f_2(A, B) = \Sigma 1 = f_3(A, B) = \Sigma(0, 2, 3)$ ,  
 $f_4(A, B) = \Sigma(1, 2, 3)$  then  $f(A, B, C)$  will be

- (a)  $A \oplus C$
- (b)  $A \odot C$
- (c)  $A \oplus B \oplus C$
- (d)  $A \odot B \oplus C$







**[NAT]**

A multiplexer is placed between a group of 32 registers and an accumulator to regulate data movement such that at any given point in time the content of only one register will move to the accumulator. The number of select lines needed for the multiplexer is \_\_\_\_.

[GATE-2020-CS: 1M]

H.W.

# [ Question ]

The Boolean function realized by the logic circuit shown is

- (a)  ~~$F = \Sigma m(0,1,3,5,9,10,14)$~~
- (b)  ~~$F = \Sigma m(2,3,5,7,8,12,13)$~~
- (c)  ~~$F = \Sigma m(1,2,4,5,11,14,15)$~~
- (d)  $F = \Sigma m(2,3,5,7,8,9,12)$

$\bar{A}\bar{B}C$  ✓  
 $0010 \rightarrow 2$   
 $0011 \rightarrow 3$

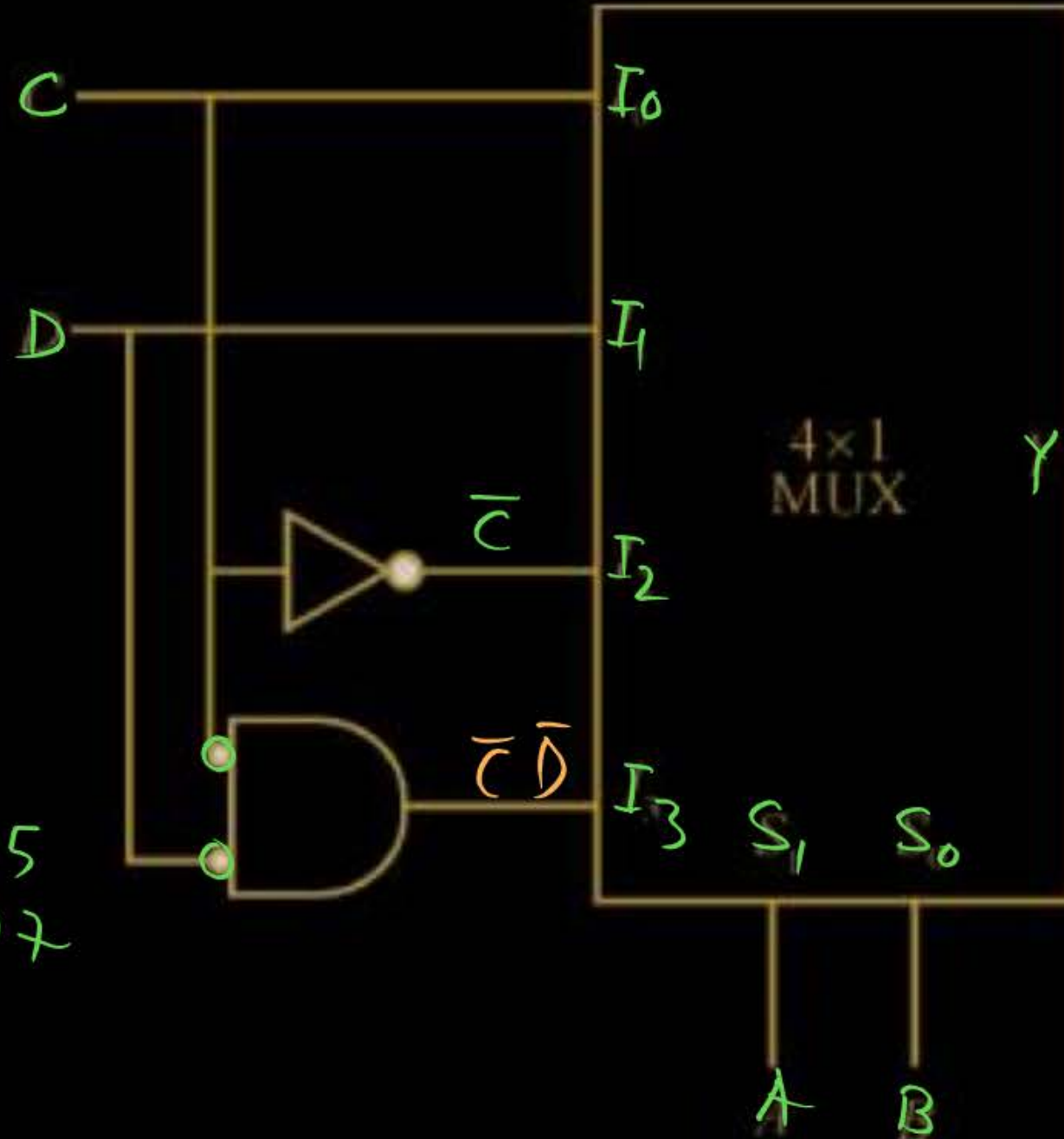
$\bar{A}\bar{B}\bar{C}$   
 $1000 \rightarrow 8$   
 $1001 \rightarrow 9$

$\bar{A}B\bar{D}$   
 $0101 \rightarrow 5$   
 $0111 \rightarrow 7$

$\bar{A}\bar{B}$   
 $000 \rightarrow 0$   
 $001 \rightarrow 1$   
 $100 \rightarrow 4$   
 $101 \rightarrow 5$

$\bar{A}B$   
 $010 \rightarrow 2$   
 $011 \rightarrow 3$

$AB$   
 $1000 \rightarrow 8$   
 $1001 \rightarrow 9$



$$F(A, B, C, D) = \bar{A}\bar{B}C + \bar{A}B\bar{D} + A\bar{B}\bar{C}$$



# [MCQ]



Consider the 4 – to – 1 multiplexer with two select  $S_1$  and  $S_0$  given below.

The minimal sum of products from the Boolean expression for the output  $F$  of the multiplexer is -

[GATE-2014-CS: 2M]

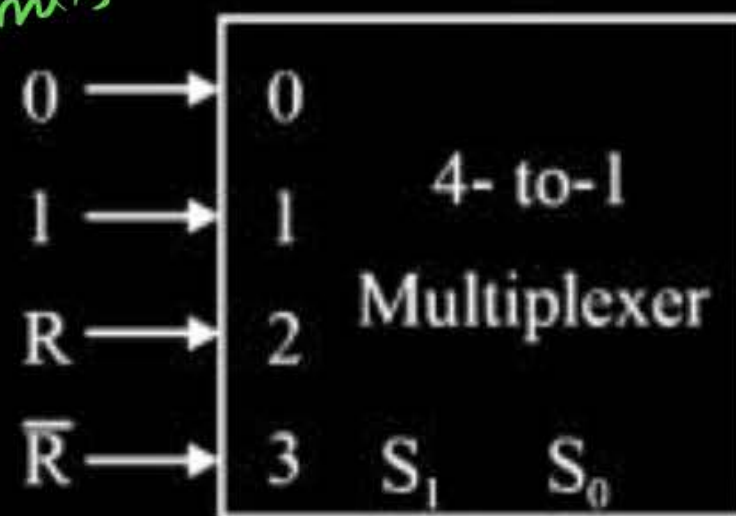
☒ A  $\bar{P}Q + Q\bar{R} + P\bar{Q}R$

☒ B  $\bar{P}Q + \bar{P}Q\bar{R} + PQR + P\bar{Q}R$

☒ C  $\bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$

☒ D  $PQ\bar{R}$

partially minimised solution



$$f = \bar{P}\bar{Q} \cdot 0 + \bar{P}Q \cdot 1 + P\bar{Q} \cdot R + P\bar{Q} \cdot \bar{R}$$

$$= \bar{P}Q + P[\bar{Q}R + \bar{Q}\bar{R}]$$

$$= \bar{P}Q + P[\bar{Q}(R + \bar{R})]$$

$$= \bar{P}Q + P\bar{Q}$$

$$= \Sigma(2, 3, 5, 6)$$

	$\bar{Q}\bar{R}$	$\bar{Q}R$	$QR$	$Q\bar{R}$
$\bar{P}$			1	1
$P$		1		1

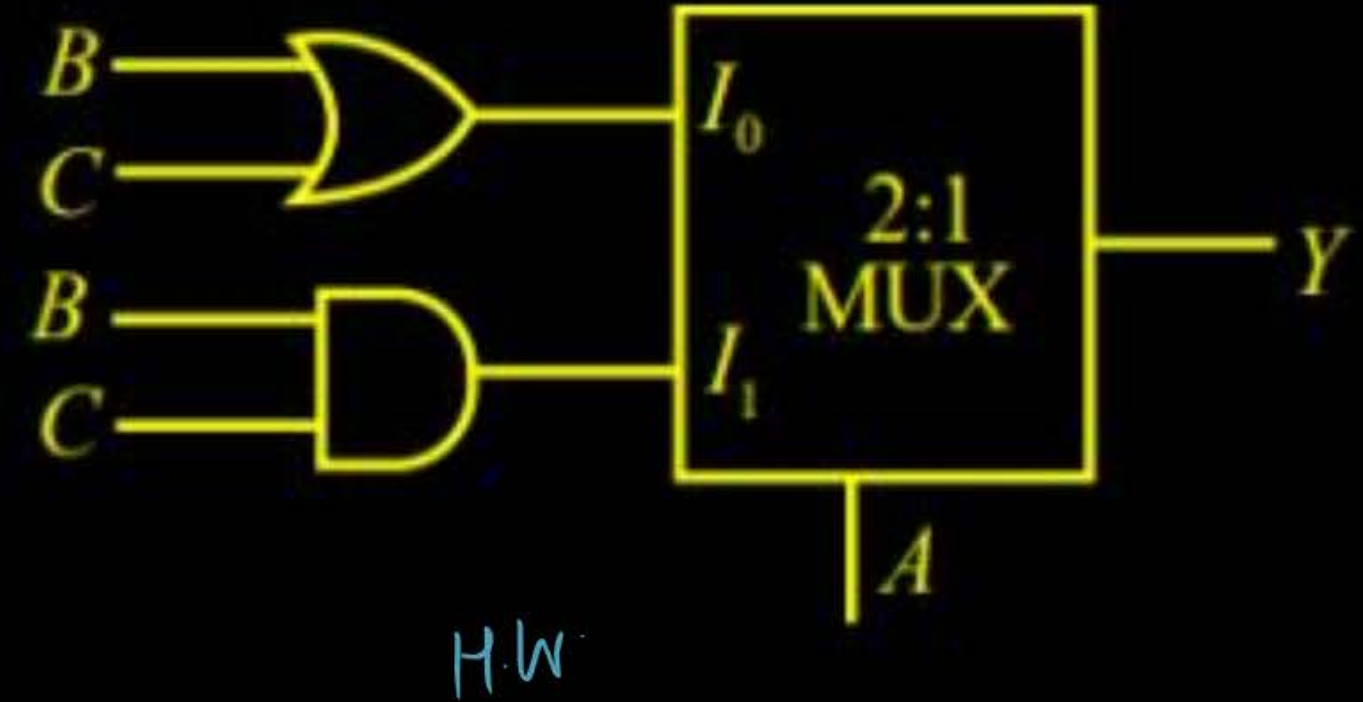
$$= P\bar{Q}R + \bar{P}Q + Q\bar{R}$$

## [ Question ]

Logical circuit is implemented as

$Y$  represents

- (A) Borrow output of full subtractor
- (B) Carry output of full adder
- (C) 3-input XOR gate
- (D) 3-input NOR gate





## [MCQ]

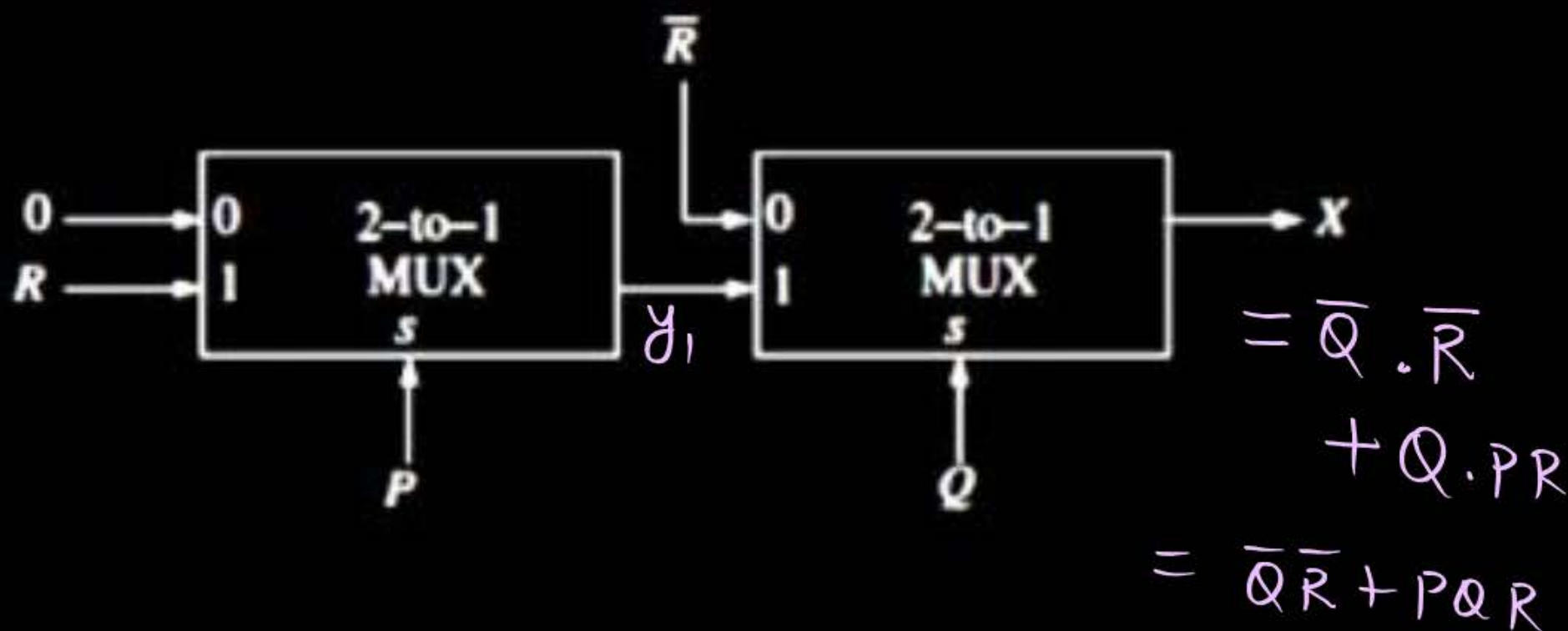


Consider the two cascaded 2 to 1 multiplexers as shown in the figure.

The minimal sum of products form of the output  $X$  is : [GATE-2016-CS: 1M]

$$y_1 = \bar{P} \cdot 0 + P \cdot R$$

- ☐ A  $\bar{P}\bar{Q} + PQR$
- ☐ B  $\bar{P}Q + QR$
- ☐ C  $PQ + \bar{P}\bar{Q}R$
- ☒ D  $\bar{Q}\bar{R} + PQR$

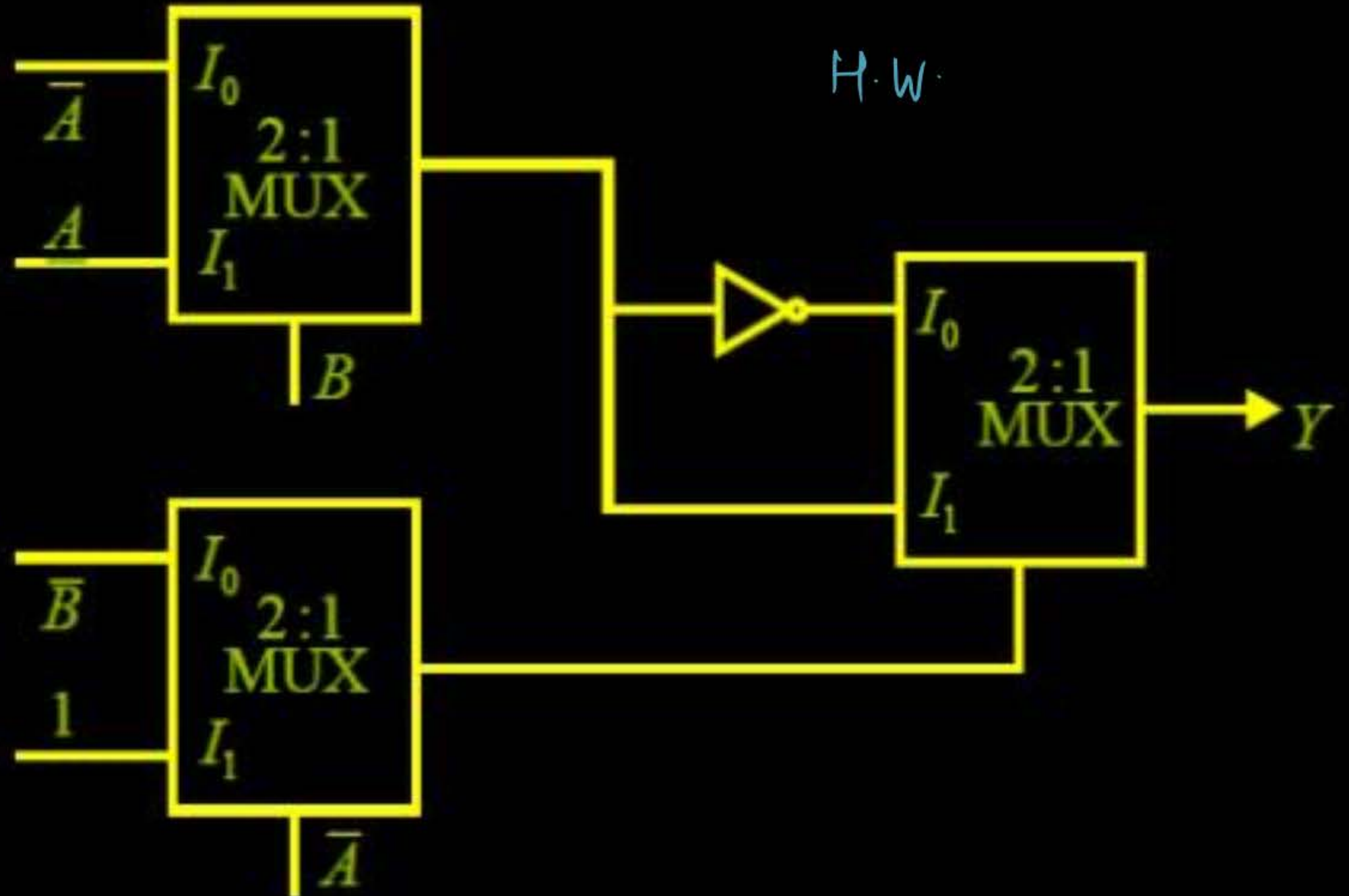


## [ Question ]

Combination circuit is designed as shown below:-

Output Y is

- (a) 2 input NOR gate
- (b) 2 input AND gate
- (c) 2 input OR gate
- (d) 2 input NAND gate





## [ Question ]

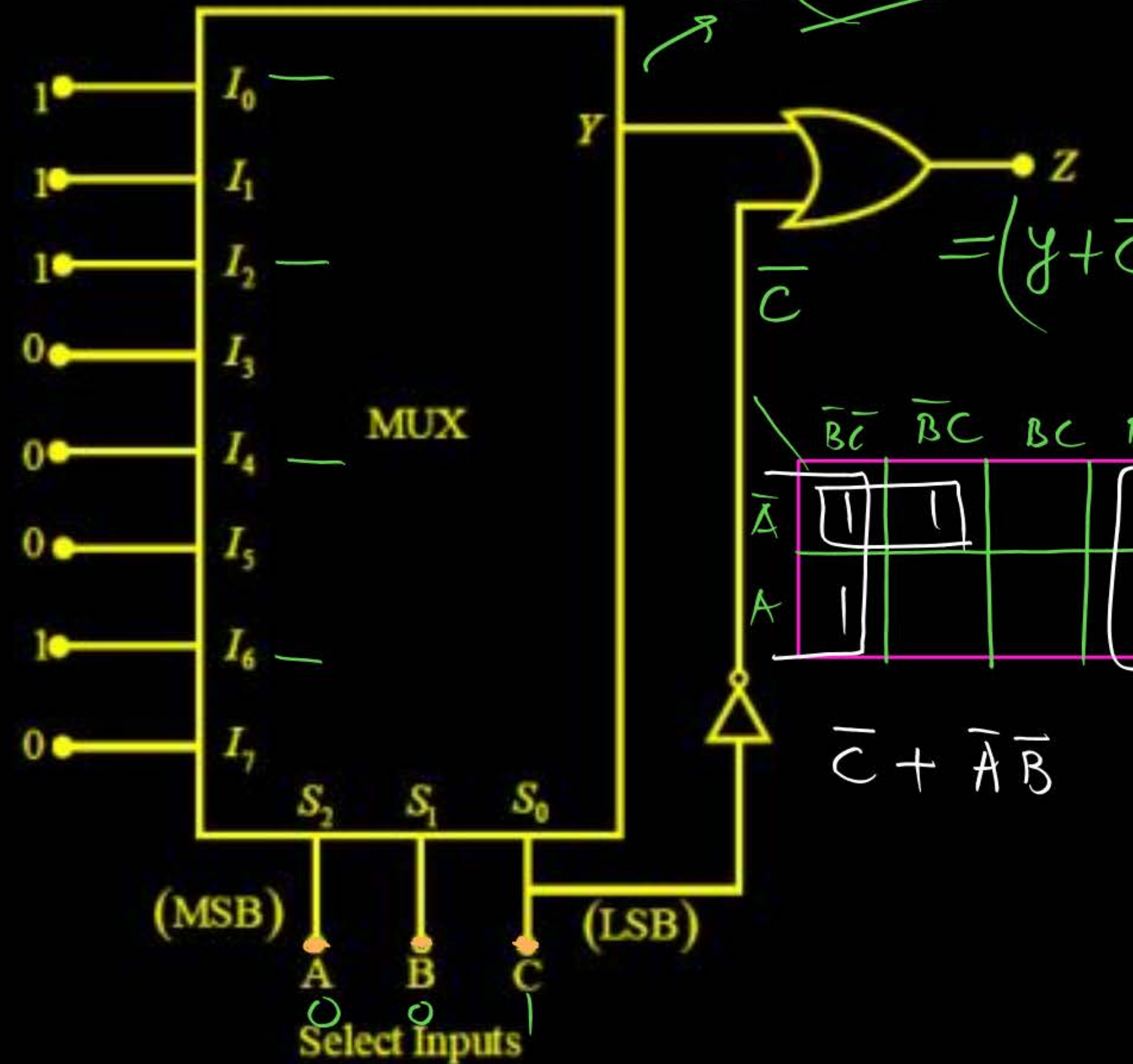
A combinational circuit using an 8 to 1 multiplexer is shown in the following figure. The minimized expression for the output (Z) is

(a)  $C(\bar{A} + \bar{B})$  ✗ 0

(b)  $C(A + B)$  ✗ 6

(c)  $\bar{C} + \bar{A}\bar{B}$  ✓ 1  $0 + 1 \cdot 1 = 1$

(d)  $\bar{C} + AB$  ✓ 1  $0 + 0 \cdot 0 = 0$



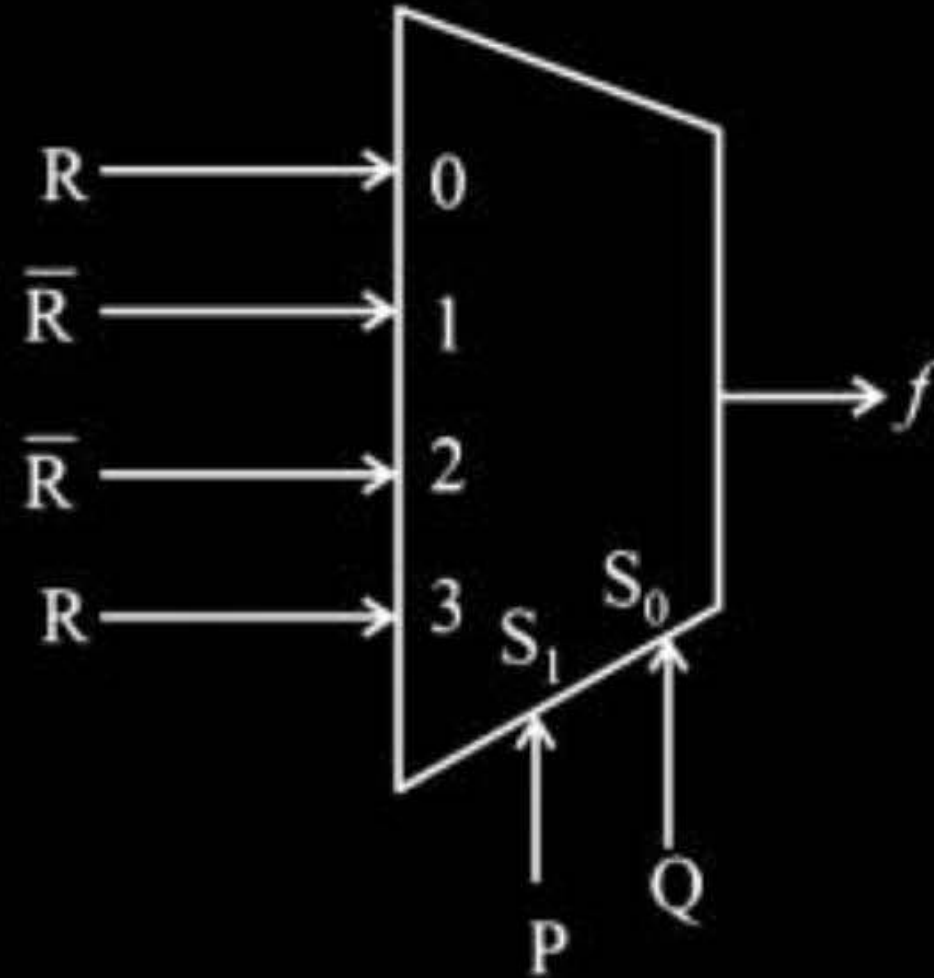
# [MCQ]



The Boolean expression of the output  $f$  of the multiplexer shown below is :

[GATE-2010-CS: 1M]

- A**  $\overline{P \oplus Q \oplus R}$
- B**  $P \oplus Q \oplus R$
- C**  $P + Q + R$
- D**  $\overline{P + Q + R}$



H.W.



# **Question** ~~\*\*\*~~

logical function  $f(A, B, C)$  is given as

$$f(A, B, C) = \overline{AB + BC + CA}$$

H.W.

Then  $f_1(A, B, C) = f[AB, BC, CA]$

$f_1$  implemented using 2 : 1 MUX with C as select input then input  $I_0$  &  $I_1$  of 2: 1 MUX will be

- (a)  $I_0 = 1, I_1 = \bar{A}$
- (b)  $I_0 = 0, I_1 = \bar{A} + \bar{B}$
- (c)  $I_0 = 1, I_1 = \bar{A} + \bar{B}$
- (d)  $I_0 = 0, I_1 = \bar{A} + B$

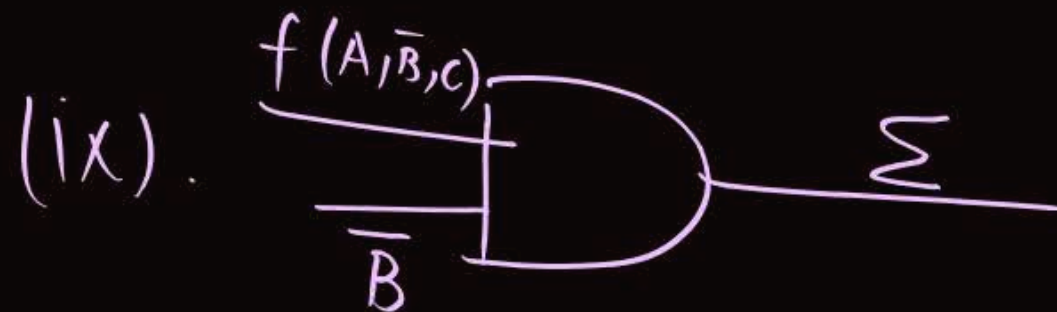
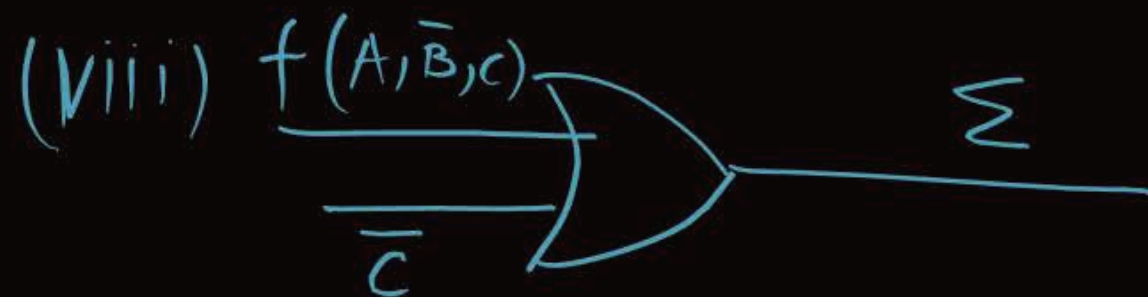
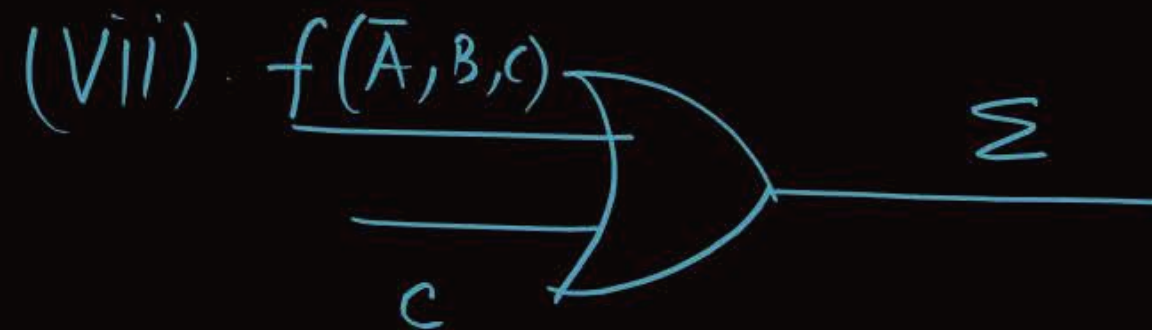
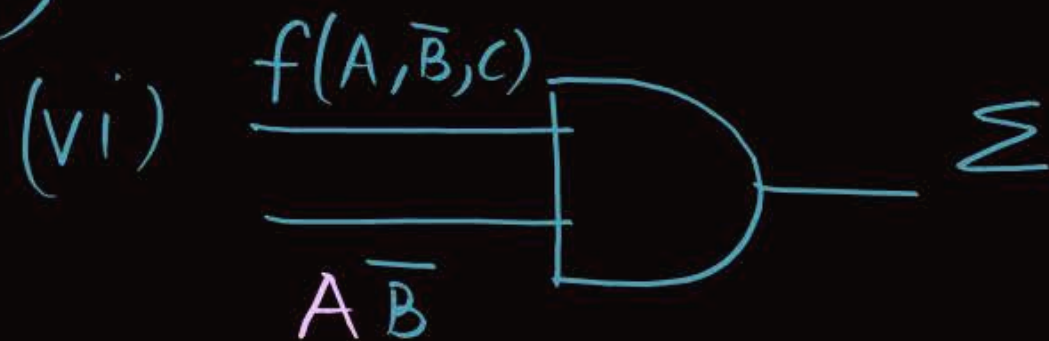
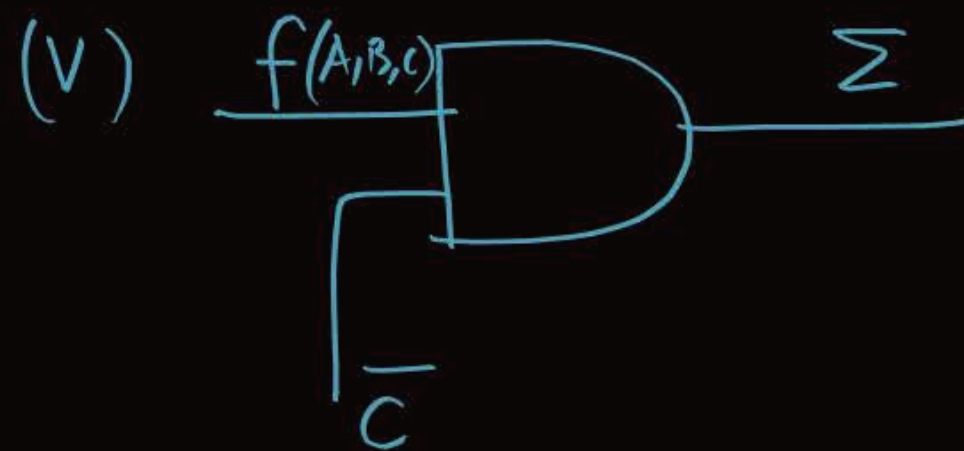
# Q.  $f(A, B, C) = \Sigma(1, 2, 3, 5, 7)$

(i)  $f(\bar{A}, B, C) = ?$

(ii)  $f(\bar{A}, \bar{B}, C) = ?$

(iii)  $f(A, \bar{B}, C) = ?$

(iv)  $f(A, \bar{B}, \bar{C}) = ?$







## 2 Minute Summary



Questions Practice

Thank you

**GW**  
*Soldiers!*

