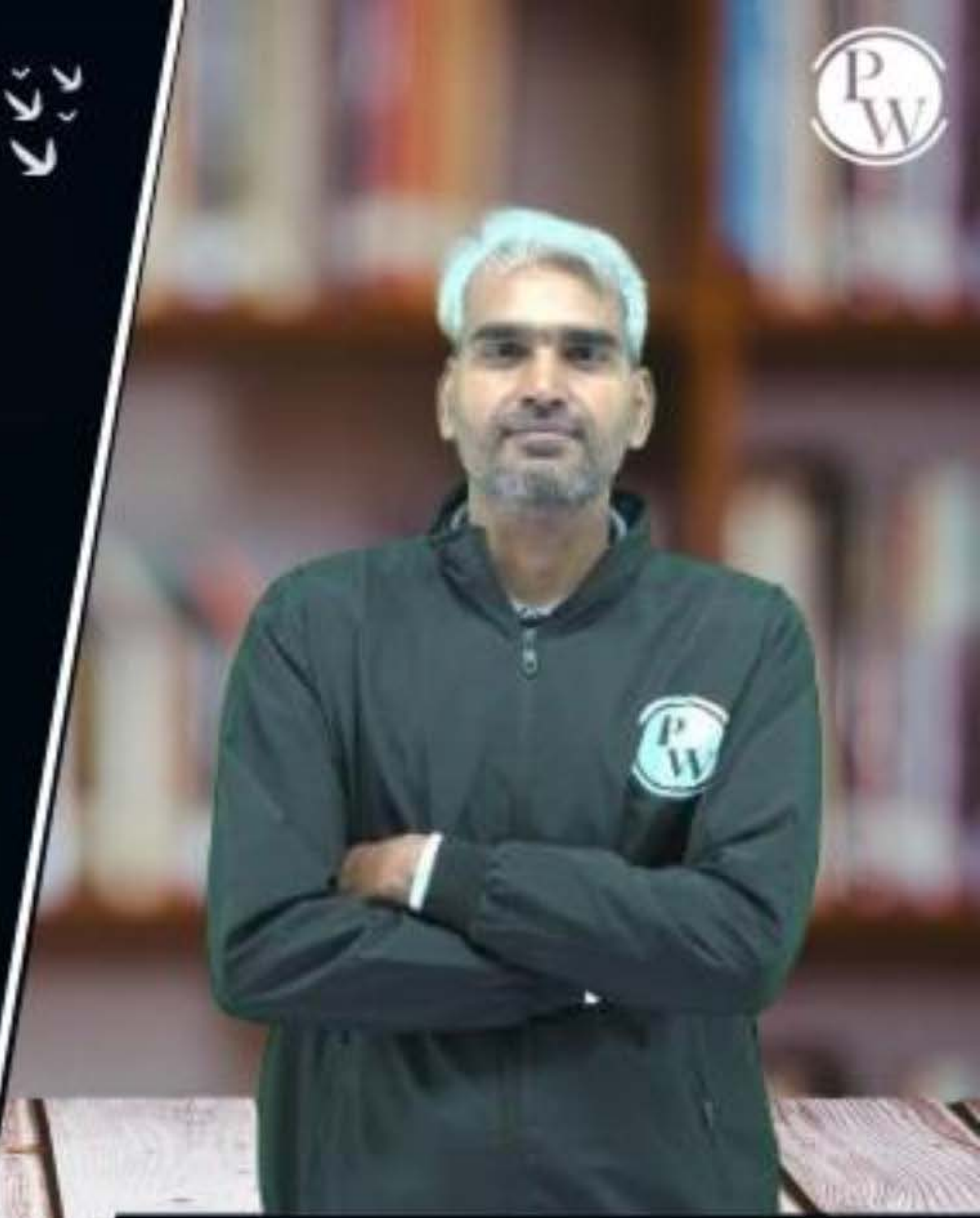
Computer Science & IT

Database Management System

Relational Model & Normal Forms

Lecture No. 07



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Recap of Previous Lecture

- Topic: Closure of an attribute set
- Topic: Super key
- Topic: Identification of Candidate key

Slide

Topics to be Covered

- **Topic** Identification of candidate key w.r.t. FD set
- **Topic** Membership test
- **Topic** Relation between two FD sets
- **Topic** FD set of a sub-relation

Slide

Topic : Closure of an attribute set

Closure of an attribute set X (i.e., X^+) can be defined as set of all the attributes which can be functionally determined from attribute of set X.

#e.g. Assume a relation R (A,B,C,D) that has the following functional dependencies:

$$\left\{\begin{array}{l} A\rightarrow B \\ B\rightarrow C \\ C\rightarrow D \end{array}\right.$$

$(A)^+ = \{A, B, C, D\}$

$(AB)^+ =$ — // —

$(AC)^+ =$ — // —

$(AD)^+ =$ — // —

$(ABC)^+ =$ — // —

$(ABD)^+ =$ — // —

$(ACD)^+ =$ — // —

$(ABCD)^+ =$ — // —

$(B)^+ = \{B, C, D\}$

$(BC)^+ = \{B, C, D\}$

$(BD)^+ = \{B, D, C\}$

$(BCD)^+ = \{B, C, D\}$

$(C)^+ = \{C, D\}$

$(CD)^+ = \{C, D\}$

$(D)^+ = \{D\}$

#e.g. Consider the following FD set

$$F = \{ AB \rightarrow CD, AF \rightarrow D, DE \rightarrow F, C \rightarrow G, F \rightarrow E, G \rightarrow A\}$$

find the closure of following set of attributes.

(i) $\{C,F\}^+ = \{C, F, G, E, A, D\}$

(ii) $\{B,G\}^+ = \{B, G, A, C, D\}$

(iii) $\{A,F\}^+ = \{A, F, D, E\}$

(iv) $\{A,B\}^+ = \{A, B, C, D, G\}$

Topic : Super key

Let R be the relational schema, and let X be some attribute set over relation R. If X^+ determines all attributes of relation R, then X is called super key of relation R.

#e.g. Assume a relation R (A,B,C,D) that has the following functional dependencies:

$$\left\{\begin{array}{l} A \to B \\ B \to C \\ C \to D \end{array}\right.$$

$(A)^+ = \{A, B, C, D\}$

$(AB)^+ =$ — " —

$(AC)^+ =$ — " —

$(AD)^+ =$ — " —

$(ABC)^+ =$ — " —

$(ABD)^+ =$ — " —

$(ACD)^+ =$ — " —

$(ABCD)^+ =$ — " —

All attributes of relation R

∴ all are Super keys of relation R.

$(B)^+ = \{B, C, D\}$

$(BC)^+ = \{B, C, D\}$

$(BD)^+ = \{B, D, C\}$

$(BCD)^+ = \{B, C, D\}$

Not all attributes

$(C)^+ = \{C, D\}$

$(CD)^+ = \{C, D\}$

Not all attributes

$(D)^+ = \{D\}$

Not all attributes

* **Proper subset :-** For a given set A, any subset of set 'A' except set 'A' itself are called Proper subsets of set A.

eg. let $A = \{a, b, c\}$ and let $\{a, b, c\}^+ = \underbrace{\text{all attributes of relation}}$

$\therefore \{a, b, c\}$ is a key.

$\Downarrow$

Proper subsets of set A = $\left\{ \begin{array}{l} \{a\} \\ \{b\} \\ \{c\} \\ \{a, b\}^+ = \text{all attributes of Rel}^n \\ \{a, c\} \\ \{b, c\} \\ \{\ \} \end{array} \right.$

then $\{a, b\}$ will be key $\implies$ Even if we delete 'c' from $\{a, b, c\}$ it does not loose its property of being a key

$\therefore \{a, b, c\}$ is not a minimal key.

Topic : Candidate key (Minimal Super key)

Let R be the relational schema, and let X be the super key of relation R.

i.e., $(X)^+ = \{$All attributes of relation R$\}$

If no proper subset of X is a super key, then X is minimal super key i.e., X is Candidate key

eg. let $(AB)^+$ Contains all attributes of relation R

∴ (AB) is a Super key of relation R.

Proper subsets of AB =

- {A}, if $(A)^+$ = Not all attributes of Relⁿ
  ∴ A is not a S.K. of relⁿ R.
- {B}, if $(B)^+$ = Not all attributes of relⁿ R
  ∴ B is not a S.K. of relⁿ R
- $\emptyset$ ← We don't need to check w.r.t. proper subset '$\emptyset$'.

→ i.e., No proper subset of {A,B} is a Super key

Hence {A,B} will be the Candidate key.

#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

AB→C,

B→E,

C→D

Find the Candidate key of R.

Note: The attributes that are not present in R.H.S. part of any FD of given FD set are called essential attributes.

- Every essential attribute must be present in Every Key of the relation

In the given question A & B are essential attributes.

$$(AB)^+ = \{\underbrace{A, B, C, E, D}_{\text{all attributes}}\}$$

∴ AB is a Super Key

A & B both are essential

∴ No one can be removed from the key

Hence AB is also the minimal Super Key

i.e. $\boxed{AB}$ is a C.K.

Closure of Proper Subsets of AB

$(A)^+ = \{A\}$ = Not all attributes

$(B)^+ = \{B, E\}$ = Not all attributes

No proper subset is a Super key

∴ AB is minimal

i.e. $\boxed{AB}$ is a C.K.

In the above example $\boxed{AB}$ is a Candidate key

$\therefore$ Prime attributes = { A, B }

* There is no FD in the FD set of above relation in which any of the prime attribute appears in the R.H.S. part of that FD.

→ $\therefore$ Relation will have only one C.K.

i.e, $\boxed{AB}$ is the only C.K. of the above relation.

#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

AB→C,

B→E,

C→D

E→A

Find the Candidate key of R.

Hit & trial Method:-

$(AB)^+ = \{\underbrace{A, B, C, D, E}_{\text{all attributes}}\}$

∴ AB is a Super Key

Check for minimal

Closure w.r.t. Proper subsets of AB = $\{A\}^+ = \{A\}$ Not all attributes ∴ 'A' is not a S.K.

$\{B\}^+ = \{\underbrace{B, E, A, C, D}_{\text{all attributes}}\}$ ∴ B is a S.K.

→ B is a new Super Key

Check for minimal

it is a key with a single attribute ∴ always minimal

Hence $\boxed{B}$ is a C.K.

one proper subset of {A, B} is a S.K

∴ AB is not a minimal S.K.

i.e., AB is not a C.K.

'B' is the essential attribute,

$(B)^+ = \{\underbrace{B, E, A, C, D}_{\text{all attributes}}\}$

∴ B is a Super Key

A key with a single attribute is always minimal, ∴ Candidate Key

$\boxed{B}$ is a Candidate Key.

* In the above eg. "B" is a C.K.

∴ Prime Attributes = {B}

- No prime attribute is present in the R.H.S. part of any FD of FD set.

Hence, only one C.K.

i.e $\boxed{B}$ is the only C.K.

Topic : Note

If there exist any non-trivial FD of the form $X \longrightarrow Y$ where 'Y' is any prime attribute of the relation, then that relation will have more than one candidate key.

Reason,

$X \longrightarrow Y$

$\therefore$ Whatever that can be determined by Y can also be determined by X

$\therefore$ We can replace Y by X in the corresponding candidate key, in order to obtain a new Super Key

#Q. Assume a relation R (A, B, C, D) that has the following functional dependencies:

AB→CD, ≡ $AB \to C$ & $AB \to D$

D→A

Find all the Candidate keys of R.

∴ B is the essential attribute.

$(B)^+ = \{B\}$

Not all attributes

∴ Not a Super Key.

$(AB)^+ = \{A, B, C, D\}$ = all attributes.

∴ AB is S.K

$(A)^+ = \{A\}$, $(B)^+ = \{B\}$ } No proper subset of AB is a S.K.

∴ $\boxed{AB}$ is a Candidate Key

Prime attributes = {A, B}

We have D → $\underbrace{A}_{\text{Prime Attri.}}$

∴ Replace A by D

DB is another S.K

$(D)^+ = \{D, A\}$

$(B)^+ = \{B\}$

$\boxed{DB}$ is a C.K ⇒ Prime Attributes = {A, B, D}

D is a Prime attribute & AB → D

∴ Replace D by AB in C.K = $\boxed{DB}$

H.W.

#Q. Assume a relation R (M, N, O, P, Q) that has the following functional dependencies:

MNO → PQ and ≡ MNO → Q, MNO → P

P → MN ≡ P → M, P → N

Find the Candidate keys of R.

O is essential

$(MNO)^+ = \{\underbrace{M, N, O, P, Q}_{all}\}$ ∴ MNO is S.K.

$(MO)^+ = \{M, O\}$

$(NO)^+ = \{N, O\}$

∴ $\boxed{MNO}$ is a C.K.

MNO is a C.K

P → M

Prime Attribute

∴ Replace M by P

PNO is S.K.

$(NO)^+ = \{N, O\}$

$(PO)^+ = \{\underbrace{P, O, M, N, Q}_{all}\}$

∴ PNO is not a C.K.

PO is a Super key as well as a C.K

Prime attributes = {M, N, O}

P → MN

Prime attributes

∴ Replace MN by P

PO is a S.K. as well as a Candidate Key

Now Prime attributes = {M, N, O, P}

MNO & PO are the only two Candidate Keys

H.W.

#Q. Assume a relation R (A, B, C, D) that has the following functional dependencies:

$$\left\{\begin{array}{l} AB \to CD \\ C \to A \\ D \to B \end{array}\right.$$

Find the Candidate keys of R.

$(AB)^+ = \{\underbrace{A, B, C, D}_{\text{all}}\}$

$\left.\begin{array}{l}(A)^+ = \{A\} \\ (B)^+ = \{B\}\end{array}\right\} \therefore$ AB is a C.K.

① $\boxed{AB}$ is a C.K. Prime Attributes $= \{A, B\}$

$C \to A$ Prime

$\therefore$ Replace A by C

CB is a S.K

$(C)^+ = \{C, A\}$

$(B)^+ = \{B\}$

② $\therefore \boxed{CB}$ is a C.K

Prime Attribute $= \{A, B, C\}$

$C \to A$ & $D \to B$

Replace A by C & B by D

$D \to B$ Prime

$\therefore$ Replace B by D

AD is a S.K

$(A)^+ = \{A\}$

$(D)^+ = \{D, B\}$

$\therefore$ AD is C.K. P.A. $= \{A, B, C, D\}$

$C \to A$

$\therefore$ Replace A by C

$D \to B$

$\therefore$ Replace B by D

CD is a S.K

$(C)^+ = \{C, A\}$, $(D)^+ = \{D, B\}$

$\therefore \boxed{CD}$ is C.K. P.A $= \{A, B, C, D\}$

$AB \to CD$

$\therefore$ Replace CD by AB

H.W.

#Q. Assume a relation R (A, B, C, D, E, H) that has the following functional dependencies:

$A \rightarrow B$

$BC \rightarrow D$

$E \rightarrow C$

$D \rightarrow A$

Find the Candidate keys of R.

E & H are essential attributes

$(EH)^+ = \{\underbrace{E, H, C}_{\text{Not all attributes}}\}$

(EH) can not be a S.K

but E & H will be part of every C.K.

$A \rightarrow B$

$\therefore$ Replace B by A

$(AEH)^+ = \{\underbrace{A, E, H, B, C, D}_{\text{all attributes}}\}$

$\boxed{AEH}$ is a Super key as well as a Candidate key

Prime Attributes = {A, E, H}

$D \rightarrow A$ ← Prime

$\therefore$ Replace A by D

$\boxed{DEH}$ is a Super key as well as a Candidate key

Prime Attributes = {A, E, H, D}

$BC \rightarrow D$

$\therefore$ Replace D by BC

BCEH is a S.K. but Not a C.K

$(CEH)^+ = \{C, E, H\}$

$\boxed{BEH}$ is a S.K as well as a Candidate key

$(BEH)^+ = \{\underbrace{B, E, H, C, D, A}_{\text{all}}\}$ $\therefore$ BEH is a S.K

PA = {A, E, H, D, B}

Topic : Membership test

- Membership test is used to check whether a given FD is a member of given FD set or not.

- To check whether $X \rightarrow Y$ is a member of FD set F or not (i.e., $F \vDash X \rightarrow Y$ or not) F yields $X \rightarrow Y$ or not

  We first obtain X^+ (closure of X) w.r.t. FD set F.

  If $Y \in X^+$, then $X \rightarrow Y$ is a member of FD set F otherwise not a member of FD set F

Note: If FD $X \rightarrow Y$ is a member of FD set F, then we say that

$X \rightarrow Y$ is implied in FD set F

#Q. Let FD set $F = \{A \to B, B \to C\}$

Check whether $A \to C$ is a member of F or not?

$A \longrightarrow C$ is a member of F or not

$(A)^+$ w.r.t $F = \{A, B, C\}$

$C \in (A)^+$ w.r.t. $F = \{A, B, C\}$

$\therefore A \to C$ is a member of F.

#Q. Let FD set F = { AB→C, BC→D}

Check whether AB→D is a member of F or not?

$(AB)^+$ w.r.t F = {A, B, C, D}

$D \in (AB)^+$ w.r.t. F.

$\therefore$ $\boxed{AB \rightarrow D}$ is implied in F.

#Q. Let FD set F = { AB→C, C→A}

Check whether C→B is a member of F or not?

$(C)^+$ w.r.t. $F = \{C, A\}$, $B \notin (C)^+$ w.r.t. F

$\therefore$ C→B is not a member of F.

$F_1 = \{ \underbrace{\quad}_{fd_1}, \underbrace{\quad}_{fd_2}, \ldots \}$

$F_2 = \{ \underbrace{\quad}_{fd_{x_1}}, \underbrace{\quad}_{fd_{x_2}}, \ldots \}$

1. $F_1 = F_2$
2. $F_1 \subset F_2$
3. $F_2 \subset F_1$
4. No relationship

$A \subseteq B$

$B \subseteq A$

$A = B$

$\underbrace{\{1, 2, 3\}}_{A} = \underbrace{\{2, 3, 1\}}_{B}$

$A = B$

$\underset{X}{\{1, 2, 3, 4\}}$ $\quad$ $\underset{Y}{\{1, 2, 3\}}$

$Y \subseteq X$, but $X \not\subseteq Y$

$\therefore Y \subset X$

$\underset{P}{\{1, 2, 3\}}$ $\quad$ $\underset{Q}{\{1, 2, 3, 4\}}$

$P \subseteq Q$, $\quad Q \not\subseteq P$

$\therefore P \subset Q$

Topic : Relationship between two FD sets

- Let F and G are any two FD sets.

- If all the FDs of FD set F are member of FD set G, then F ⊆ G
  if G covers F (or) if All FDs of F are implied in G.
- If all the FDs of FD set G are member of FD set F, then G ⊆ F
  (or) F Covers G (or) All the FDs of G are implied in F.
- If both F ⊆ G and G ⊆ F are true, then F = G

#Q. Consider the following FD sets

$F1 = \{A \to B, B \to C, AB \to D\}$ and $F2 = \{A \to B, B \to C, A \to C, A \to D\}$

Find the relationship between FD sets F1 and F2

#Q. Consider the following FD sets

$F1 = \{A \to B, B \to C, AB \to D\}$ and $F2 = \{A \to B, B \to C, A \to C, A \to D\}$

Find the relationship between FD sets F1 and F2

Check if F_1 Covers F_2. {i.e. Check if all FDs of F_2 are member of F_1}

FDs of F_2	Closure w.r.t. F_1	Remark	Conclusion
$A \to B$	$(A)^+$ w.r.t. $F_1 = \{A, B, C, D\}$	Directly available in F_1 ∴ Member of F_1	all FDs of F_2 are member of F_1
$B \to C$	$(B)^+$ w.r.t. $F_1 = \{B, C\}$	$C \in (B)^+$ ∴ $B \to C$ Member of F_1	∴ $F_2 \subseteq F_1$ — eq^n (1)
$A \to C$	$(A)^+$ w.r.t $F_1 = \{A, B, C, D\}$	$C \in (A)^+$ ∴ $A \to C$ member of F_1	
$A \to D$	— // — $\{A, B, C, D\}$	$D \in (A)^+$ ∴ $A \to D$ is Member of F_1	

#Q. Consider the following FD sets

$F1 = \{A \to B, B \to C, AB \to D\}$ and $F2 = \{A \to B, B \to C, A \to C, A \to D\}$

Find the relationship between FD sets F1 and F2

Check if F_2 covers F_1 {i.e. Check if all FDs of F_1 are member of F_2}

FDs of F_1	Closure w.r.t. F_2	Remark
$A \to B$	$(A)^+ = \{A, B, C, D\}$	$B \in (A)^+$ ∴ Member of F_2
$B \to C$	$(B)^+ = \{B, C\}$	$C \in (B)^+$ ∴ Member of F_2
$AB \to D$	$(AB)^+ = \{A, B, C, D\}$	$D \in (AB)^+$ ∴ Member of F_2

Conclusion

all FDs of F_1 are member of F_2

∴ $F_1 \subseteq F_2$ — eqn (2)

By eq^n 1 & eq^n 2

$$\boxed{F_1 = F_2}$$

#Q. Consider the following FD sets

$F1 = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$ and

$F2 = \{A \rightarrow BC, D \rightarrow AE\}$

Find the relationship between FD sets F1 and F2

- Check if F_1 covers F_2, {Check if FDs of F_2 are member of F_1}

FDs of F_2	Closure w.r.t. F_1	Remark	Conclusion
$A \rightarrow BC$	$(A)^+$ w.r.t $F_1 = \{A, B, C\}$	$BC \in (A)^+$ $\therefore A \rightarrow BC$ is Member of F_1	All FDs of F_2 are member of F_1 $\therefore F_2 \subseteq F_1$ — (1)
$D \rightarrow AE$	$(D)^+$ w.r.t. $F_1 = \{D, A, C, E, B\}$	$AE \in (D)^+$ $\therefore D \rightarrow AE$ is Member of F_1	

#Q. Consider the following FD sets

$F1 = \{A \to B, AB \to C, D \to AC, D \to E\}$ and

$F2 = \{A \to BC, D \to AE\}$

Find the relationship between FD sets F1 and F2

- Check if F_2 covers F_1 {Check if FDs of F_1 are member of F_2}

FDs of F_1	Closure w.r.t. F_2	Remark	Conclusion
$A \to B$	$(A)^+ = \{A, B, C\}$	$B \in (A)^+$ ∴ Member of F_2	All FDs of F_1 are member of F_2 ∴ $F_1 \subseteq F_2$ — (2)
$AB \to C$	$(AB)^+ = \{A, B, C\}$	$C \in (AB)^+$ ∴ Member of F_2	
$D \to AC$	$(D)^+ = \{D, A, E, B, C\}$	$AC \in (D)^+$ ∴ Member of F_2	By eq^n (1) & (2)
$D \to E$		$E \in (D)^+$ ∴ Member of F_2	$F_1 = F_2$

Topic : FD set of a sub-relation

- Let R be a relation with FD set F. We can use the concept of membership test to obtain the FD set of a sub-relation of relation R.

#Q. Consider a relational schema R(A,B,C,D,E) with FD set

$F = \{A \to B, AB \to C, D \to AC, D \to E\}$

Find the FD set F1 for sub-relation R1(A,B,E) of R(A,B,C,D,E).

Also find candidate keys for the sub-relation R1(A,B,E).

Candidate Key of a relation is identified by the FD set of that relation

∴ To identify C.Ks of $R_1(A,B,E)$ We need FDs of set F_1

We are interested only in non-trivial FDs.

Only two non-trivial functional dependencies exist in $R_1(A,B,E)$

$\therefore F_1 = \{A \to B, AE \to B\}$

∴ C.K. of R_1 = (AE)

$A \to ?$

$(A)^+$ w.r.t $F = \{\underbrace{A, B}_{\text{trivial}}, \underbrace{C}_{\text{Not in } R_1}\}$ = $\boxed{A \to B}$

$B \to ?$

$(B)^+$ w.r.t $F = \{\underbrace{B}_{\text{trivial}}\}$

$E \to ?$

$(E)^+$ w.r.t $F = \{\underbrace{E}_{\text{trivial}}\}$

$AB \to ?$

$(AB)^+$ w.r.t. $F = \{\underbrace{A, B}_{\text{trivial}}, \underbrace{C}_{\text{Not in } R_1}\}$

$AE \to ?$

$(AE)^+$ w.r.t. $F = \{\underbrace{A, E}_{\text{trivial}}, B, \underbrace{C}_{\text{Not in } R_1}\}$ $\boxed{AE \to B}$

$BE \to ?$

$(BE)^+ = \{\underbrace{B, E}_{\text{trivial}}\}$

H.W.

#Q. Consider a relational schema R(A,B,C,D,E,F) with FD set

$F = \{AB \rightarrow C,\ B \rightarrow D,\ BC \rightarrow A,\ D \rightarrow EF\}$

Find the FD set F1 for sub-relation R1(A,B,C,D) of R(A,B,C,D,E,F).

Also find candidate keys for the sub-relation R1(A,B,C,D).

2 mins Summary

Topic Identification of candidate key w.r.t. FD set

Topic Membership test

Topic Relation between two FD sets

Topic FD set of a sub-relation

Slide

THANK - YOU

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