Foundations of Quantum Information Theory

Project Report: Phase 1

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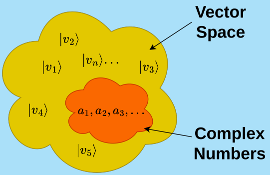
State Space and Time Evolution

State Space:

An isolated physical system is associated with a Hilbert Space consisting of vectors and complex numbers, this is called the State Space.



Hilbert Space

Time Evolution:

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \qquad \text{where } U(t) = e^{\frac{-i\hat{H}t}{\hbar}}$$

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Projective Measurements

Key Characteristics:

- It is always done with respect to set of orthogonal bases.
- The state after measurement collapses to Eigen States.
- The possible outcomes of the measurement are Eigen values of the observable.

Formulation:

$$M = \sum_m mP_m, \qquad where\ M = M^\dagger \quad \& \quad \sum_i M_i^\dagger M_i = I$$

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Projective Measurements

Formulation:

Once measurement is done, state will collapse to

$$|\psi'\rangle = \frac{P_m|\psi\rangle}{\sqrt{p(m)}}$$

with the probability

$$p(m) = \langle\psi|P_m|\psi\rangle$$

Once the state collapses, it will remain in the same state with probability 1 forever.

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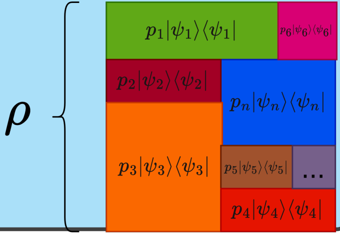
Density Operator and its Properties

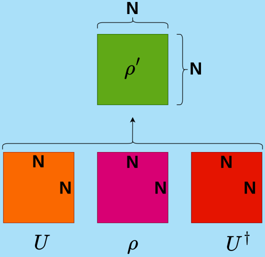
Ensemble:

$$\rho = \sum_{i=1}^{n} p_i |\psi_i\rangle\langle\psi_i|$$



Time Evolution:

$$\rho \to \rho' : \rho' = U\rho U^\dagger$$



Density Operator and its Properties

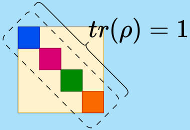
Measurement:

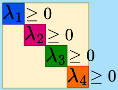
$$\rho_m = \frac{M_m \rho {M_m}^\dagger}{tr({M_m}^\dagger M_m \rho)}, \qquad p_m = tr({M_m}^\dagger M_m \rho)$$

Properties:

$$tr(\rho) = 1$$

$$\lambda_i \geq 0$$





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1. Postulates of QM
2. Density Operator
   - Density Operator and its Properties
   - Reduced Density Matrix
   - Kraus Representation
3. Matrix Decomposition
4. Teleportation
5. EPR Experiment
6. Summary

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Reduced Density Matrix

Requirement:

- Descriptive tool for Subsystem.
- Useful in Composite Quantum Systems.
- Valid for both pure and mixed states.

Partial Trace (Pure State):

$$\rho_{AB} = |a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|, \qquad \textit{here A and B are pure states.}$$

$$\rho_A = tr_B(\rho^{AB}) \qquad tr_B(\rho^{AB}) = |a_1\rangle\langle a_2| tr(|b_1\rangle\langle b_2|)$$

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Kraus Representation

Requirement:

- Descriptive tool for the evolution of Complete System.
- Includes both unitary and non-unitary transformations.

Kraus Operators:

$$If\ \rho(0) = \rho_S \otimes \rho_E \quad , then\ \rho_S(t) = \sum_{i=1}^{n} E_a(t)\rho_S E_a(t)^\dagger$$

$$where\ E_a(t) = (I \otimes \langle \epsilon_a |) U(t) (I \otimes | \epsilon_0 \rangle)$$

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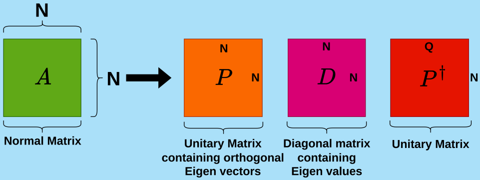
Spectral Decompostion

Requirement for Spectral Decomposition:

- The matrix A must be a Normal matrix.
- It decomposes matrix into Eigen values and Eigen vectors.

$$A = PDP^{\dagger}$$



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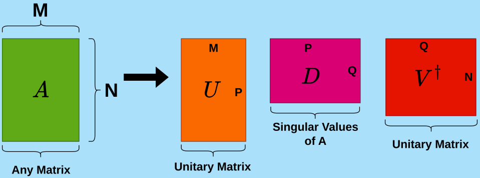
Singular Value Decompostion

Speciality of SVD:

- It can be applied to any matrix.
- It can be applied to real or complex matrices.

$$A = UDV^{\dagger}$$



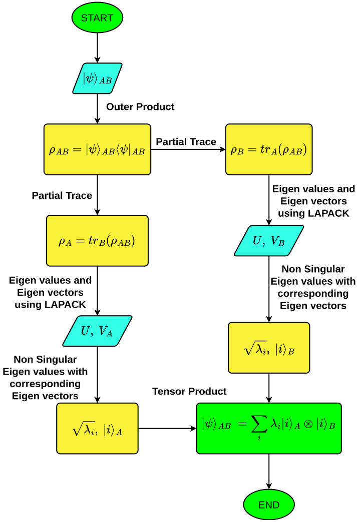
Schmidt Decompostion

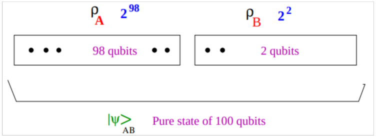
Characteristics of Schmidt Decomposition:

- It is only applicable to the Bipartite pure state.
- It is closely related to Singular Value Decomposition.
- Rank of the matrix gives the degree of entanglement.

$$|\psi\rangle_{AB} = \sum_i \lambda_i |i\rangle_A \otimes |i\rangle_B$$

Flowchart





$$|\psi\rangle_{AB} = \underbrace{\sum_{i}^{2^{100}} c_{AB}|\alpha_i\rangle_A|\beta_i\rangle_B}_{2^{100}\ terms} = \underbrace{\sum_{i}^{2^{2}} \sqrt{\lambda_i}|\psi_{98}\rangle_i|\psi_2\rangle_i}_{Only\ 2^{2}\ terms}$$

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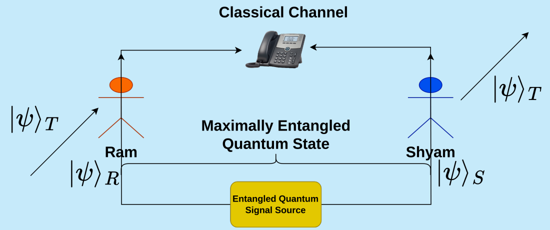
Teleportation

Requirements for Teleportation:

- Both parties should share an entangled state.
- One channel of classical communication is allowed.



1. Postulates of QM
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5. **EPR Experiment**
6. Summary
7. Future Work
8. References

EPR Experiment

EPR Paper:

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

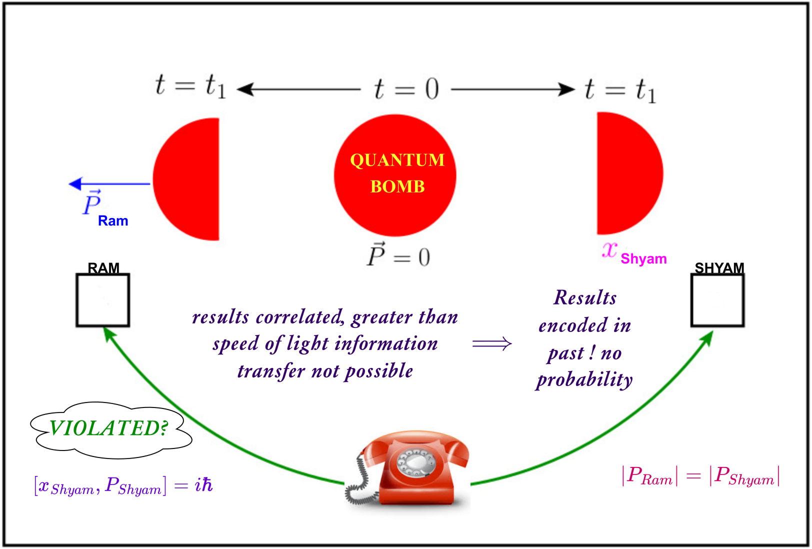
A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

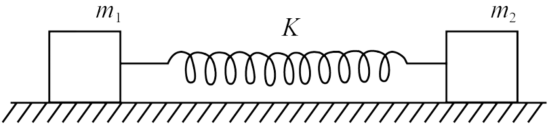
For a Bipartite system correlated in the past, even if the systems are spatially separated now, we can define the state of the 2nd system by doing the measurement in the 1st system locally.

Quantum Mechanical Approach



Classical Mechanics Analogy

$t = 0$



$t = t_1$

q_1 $\qquad Q = q_1 - q_2 \qquad P = p_1 + p_2 \qquad$ q_2

p_1 $\qquad [P, Q] = 0 \qquad$ p_2

$$[p_1, q_1] = -i\hbar \qquad [p_1, q_2] = 0 \qquad [p_2, q_1] = 0 \qquad [p_2, q_2] = -i\hbar$$

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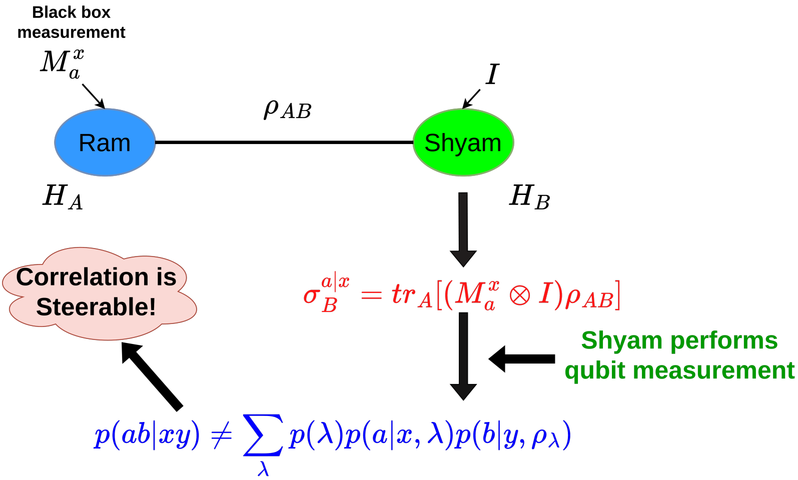
Work done so far

- Literature review for basics of Quantum Information Theory.
- Literature review for the basics of Linear Algebra.
- Literature review of Teleportation.
- EPR paper review.
- Fortran/ Python codes (i.e., Inner Product, Outer Product, Tensor Product, Schmidt Decomposition, Used Lapack to calculate the Eigen values and Eigen vectors for matrix).
- Made code for 1-D Ising model and plotted the eigenstates.

EPR Quantum Steering in Many-Body Systems



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Thanks!