**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676 - Ans**
4. 0.5
5. 0.6987
6. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
7. More employees at the processing center are older than 44 than between 38 and 44.

**FALSE**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS –** In case of X1 + X2,

Mean(X) = Mean(X1) + Mean(X2)

And Var(X) = Var(X1) + Mean (X2)

Mean(X) = μ + μ = 2 μ

Var(X) = S2 = σ2 + σ2 = 2σ2

Therefore Std Dev = S =1.414 σ

In case of N\*X1,

We can write it as, X1 + X1 + X1 +…………+X1 - N times

Therefore, Mean(N\*X1) = N \* μ

Mean(2X1) = 2 μ

And Var(N\*X1) = N\* σ2

Var(2X1) = 2 σ2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5 - ANS**
6. 90.1, 109.9
7. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
8. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
9. Specify the 5th percentile of profit (in Rupees) for the company
10. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans -** Total Profit(P) = Profit1 + Profit2

Let’s assume Profit1 and Profit2 are normally distributed

Thus, P is also Normally Distributed

Mean(P)= Pm = Mean(Profit1) + Mean(Profit2) = 5 + 7 = 12

Var(P) = S2 = Var(Profit1) + Var(Profit2) = 32 + 42 = 25

Therefore Std Dev = S = 5

1. To contain 95% probability for the annual profit of the company.

Range = Mean +/- 2\*S …………………….from the definition of Normal distribution

Range = 12 +/- 5

Range = (7,17)

Rupee Range = (7,17) \*45 = (315, 765)

1. the 5th percentile of profit (in Rupees) for the company

z val corresponding to 5th percentile = -1.645

thus, value in $ = -1.645\*5+12 = $ 3.775

value in Rupee = 3.775\*45 = Rs. 169.875

1. We know the disribotion is centred at mean profit to make loss for any division value should be less than 0 on the distribution

Thus to answer which division is more likely to make loss, we should compare the probability till 0 value in both distribution

Z value for first distribution = z1 = (0-5)/3 = -1.667

P value corresponding to z1 = 0.0478 ----- 4.78 %

Z value for second distribution = z2 = (0-7)/4 = -1.75

P value corresponding to z2 = 0.04 ------- 4 %

First division is more likely make loss.