

Master Thesis

Architecture Development to Replace Finite Element Methods with Deep Neural Networks

Examiners:

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Presented by:

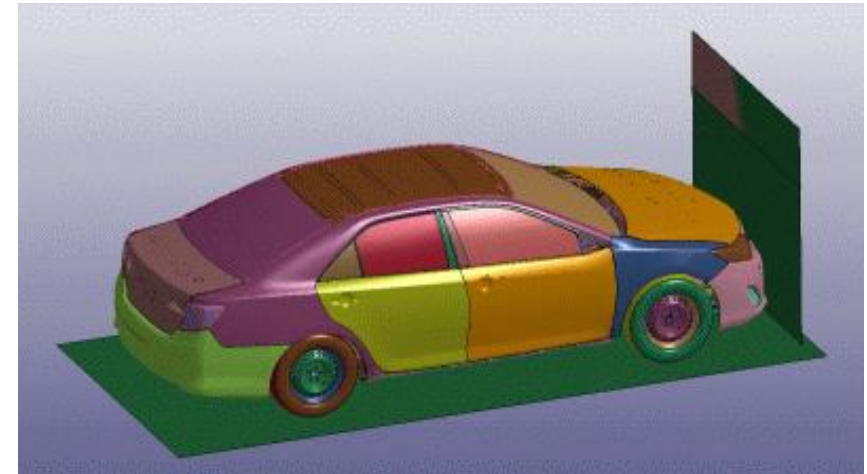
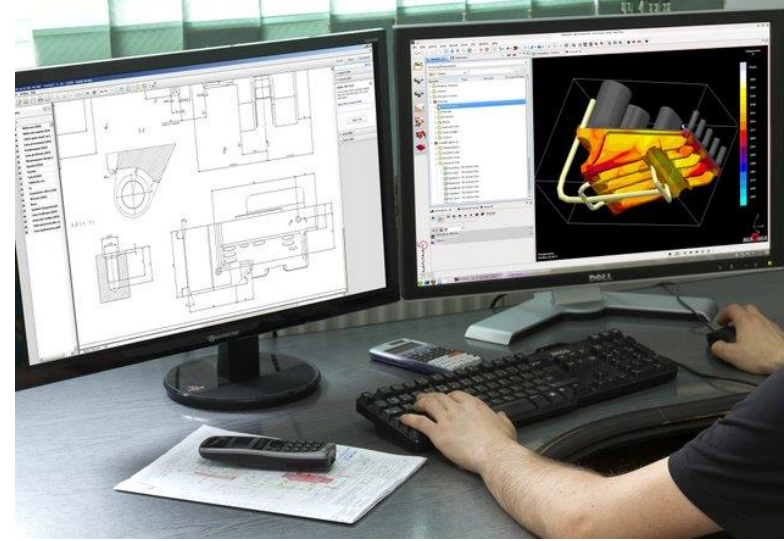
**Kaushal Narendra Tare
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1 Introduction

Computer Aided Engineering (CAE)

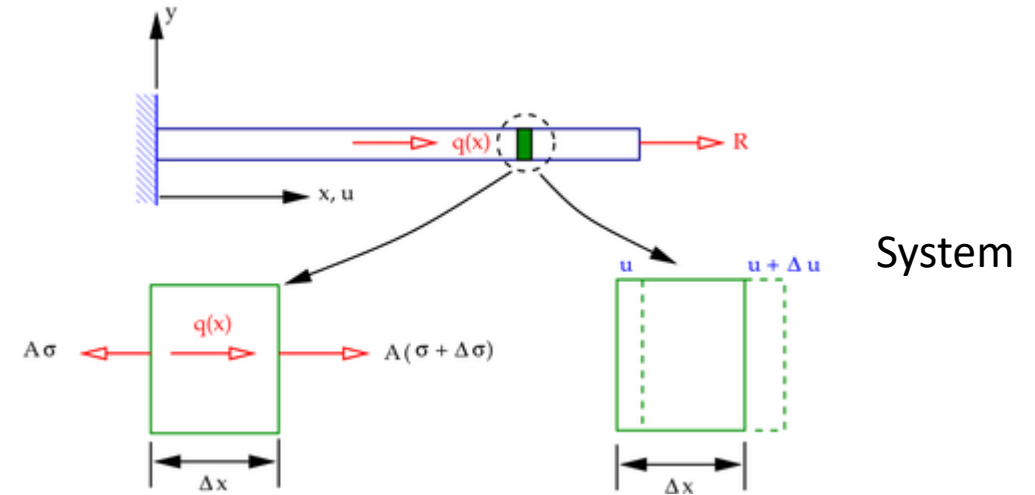
The use of computer software to simulate performance in order to -

- Improve product design
- Assist in resolution of engineering problems



Finite Element Method (FEM)

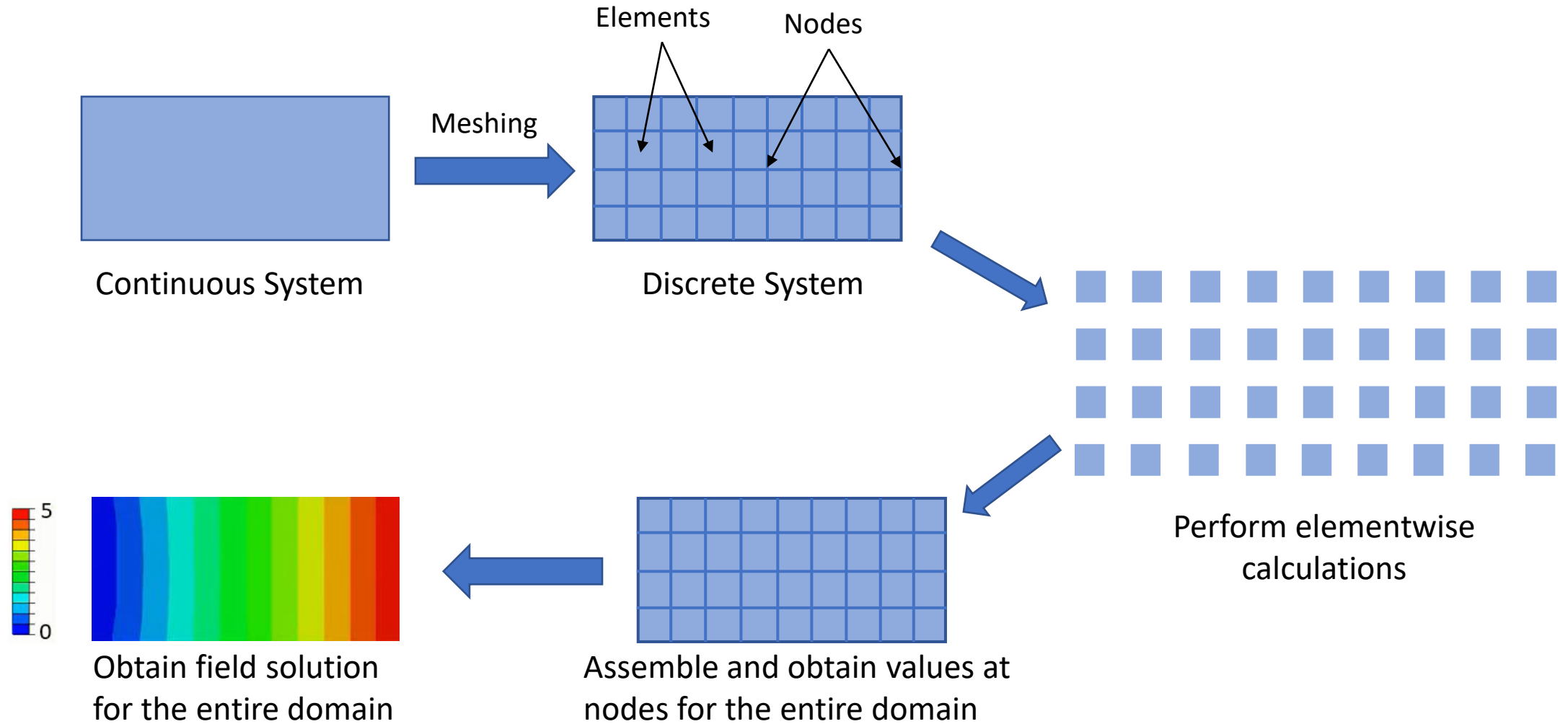
- FEM is a numerical method used to obtain the approximate solution in CAE
- Most systems can be mathematically represented by differential equations
- Analytical solution is easy to calculate for simple geometries and difficult for complex geometry



$$\begin{aligned}
 [K]\{u\} &= \{F\} \\
 \frac{AE}{L} \frac{d^2 u}{dx^2} &= -q(x) \\
 u(0) &= 0 \\
 \frac{du}{dx} \Big|_{x=L} &= \frac{R}{AE}
 \end{aligned}$$

$$\frac{AE}{h} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix} = \frac{a}{h} \begin{bmatrix} \left(\frac{x_2^3 - x_1^3}{3} - \frac{x_1(x_2^2 - x_1^2)}{2} \right) + \left(\frac{x_3(x_3^2 - x_2^2)}{2} - \frac{x_3^3 - x_2^3}{3} \right) \\ \left(\frac{x_3^3 - x_2^3}{3} - \frac{x_2(x_3^2 - x_2^2)}{2} \right) + \left(\frac{x_4(x_4^2 - x_3^2)}{2} - \frac{x_4^3 - x_3^3}{3} \right) \\ \left(\frac{x_4^3 - x_3^3}{3} - \frac{x_3(x_4^2 - x_3^2)}{2} \right) + \frac{R}{a} \end{bmatrix}$$

The FEA Procedure (Practical visualization)

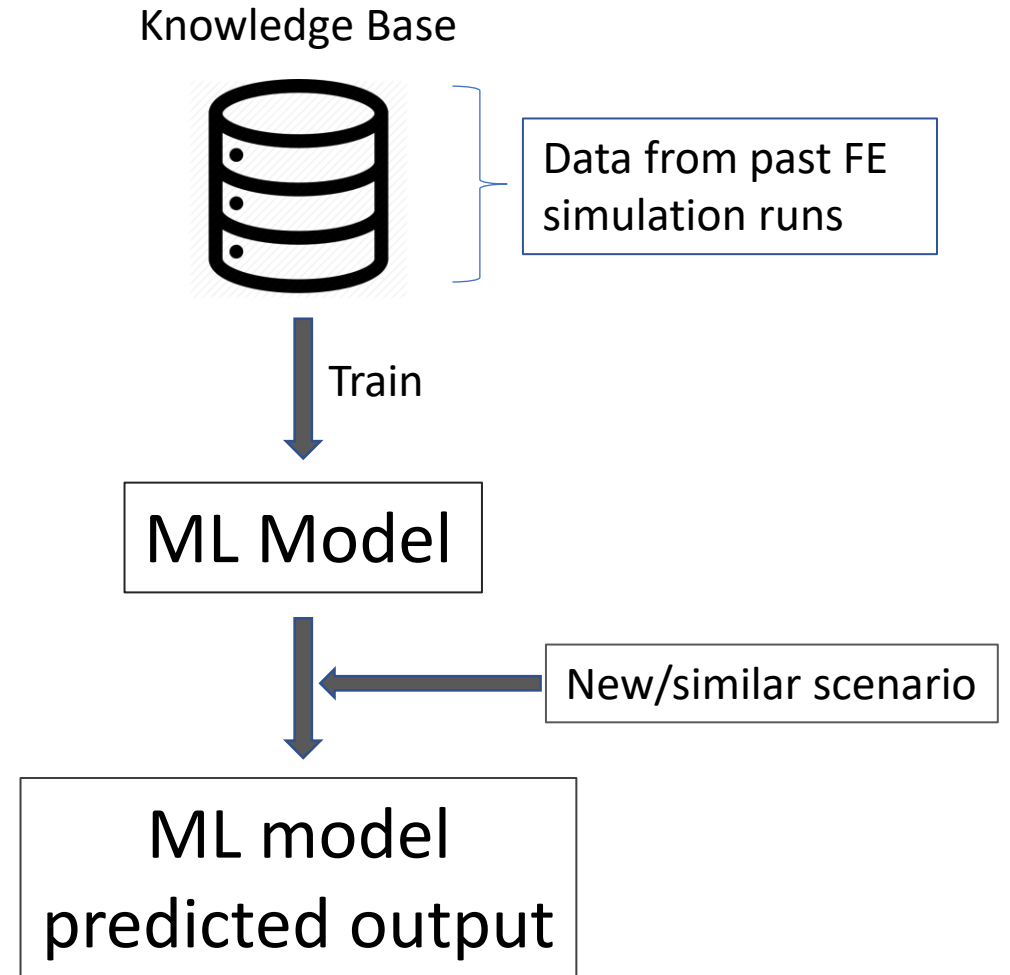


Limitations of FEA

- Re-running the complete scenario for slightest change in the inputs (time consuming)
- Highly complex scenarios are computationally expensive

Machine Learning as a Solution

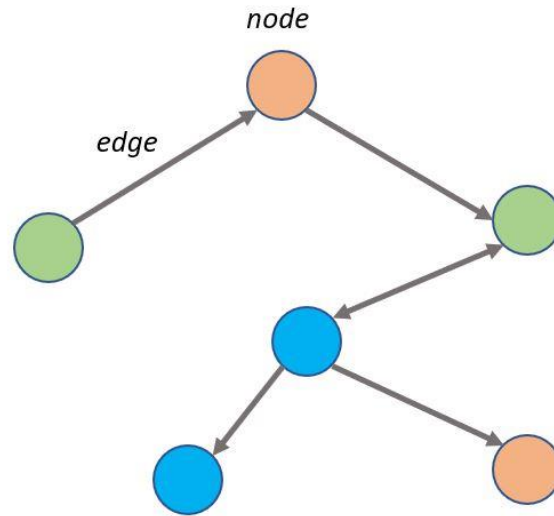
- ML model learns relation between inputs and outputs from historical simulation data
- Predicts the new simulation scenario based on learned relations



3 Deep Learning

Graph Neural Networks

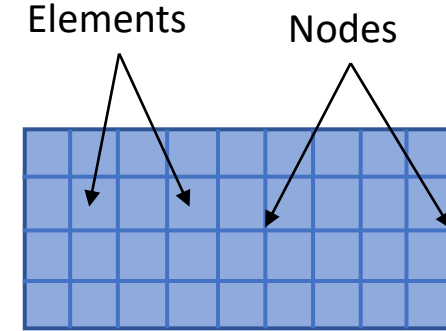
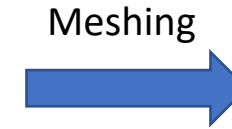
- Data structure consisting of nodes and edges
- Structural analogy with discrete system in FEM
- A discrete system can be represented as a graph network



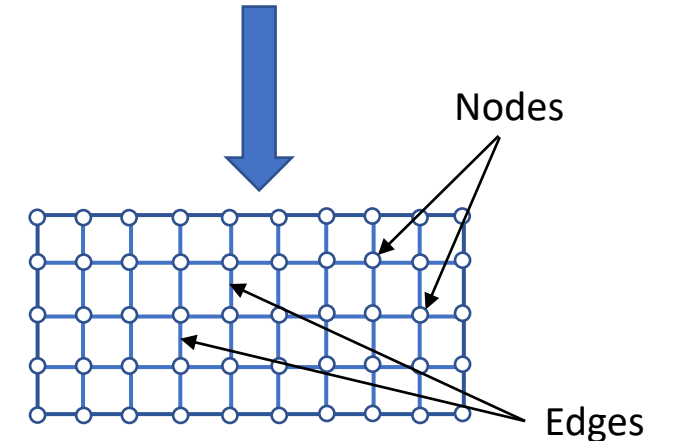
$$G = (V, E)$$



Continuous System



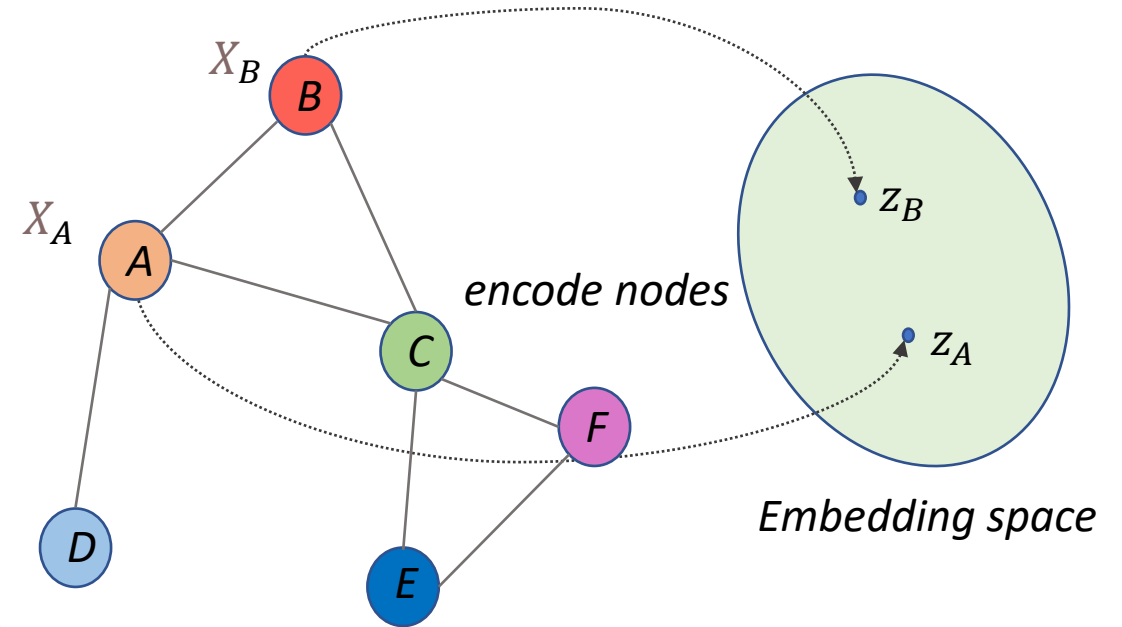
Discrete System



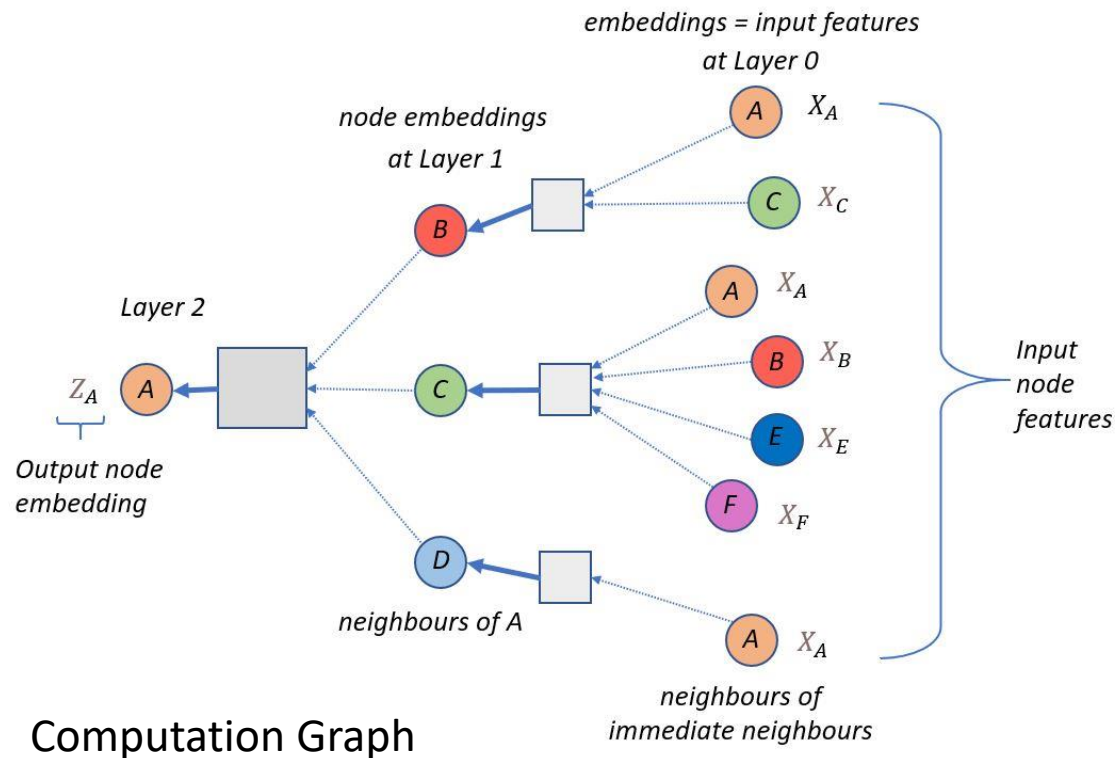
Graph Network

Working of GNNs

- Gather locality information
- Aggregate and compute the information through multiple layers
- Obtain the final node embeddings



$$\text{similarity}(A, B) \approx z_A^T z_B$$

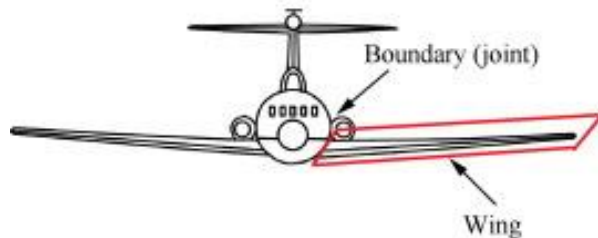
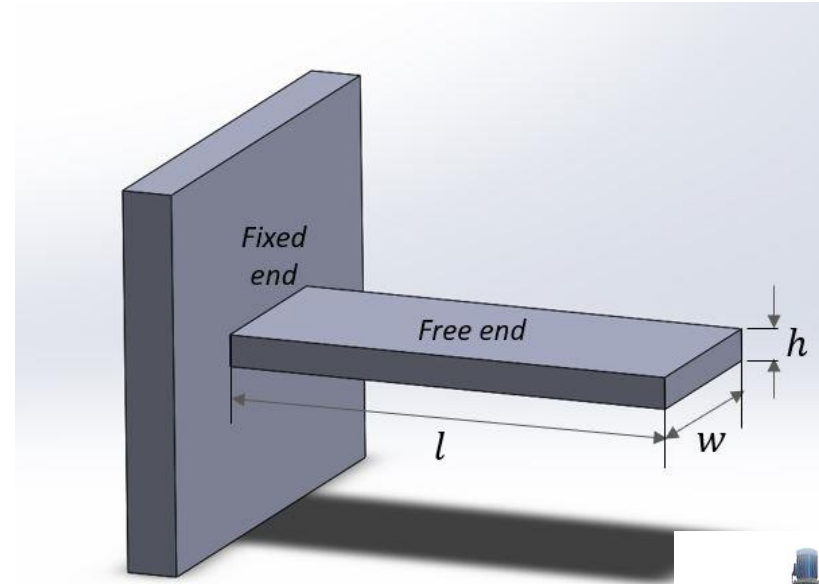


Case Study Scenario: Cantilever Beam

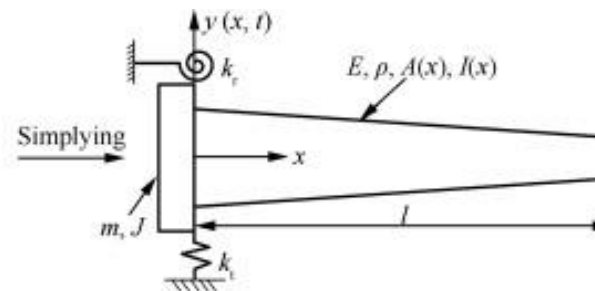
- A rigid structural element supported at one end and free at other

Applications:

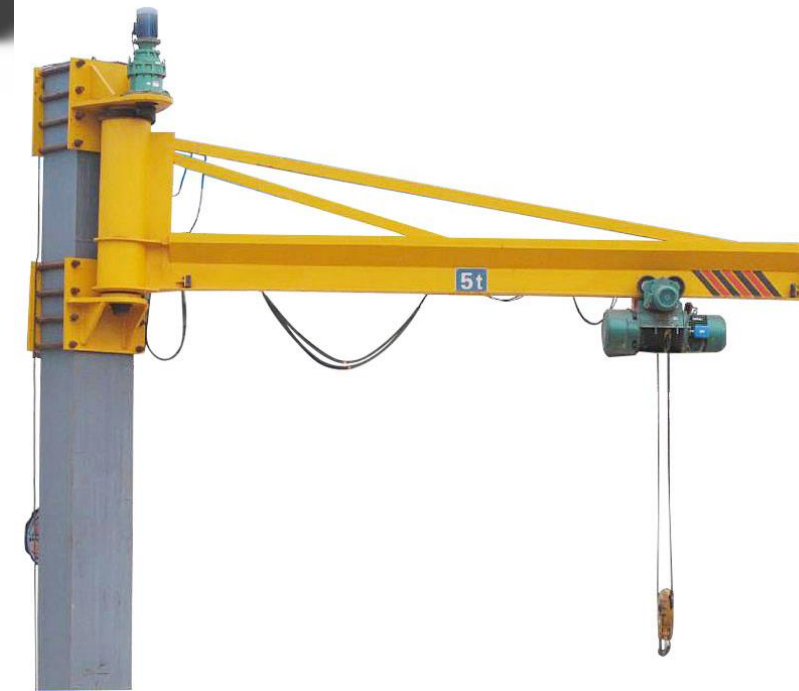
- Aesthetic roofs
- Cranes
- Span of bridges
- Aircraft wing



(a) Schematic diagram of aircraft

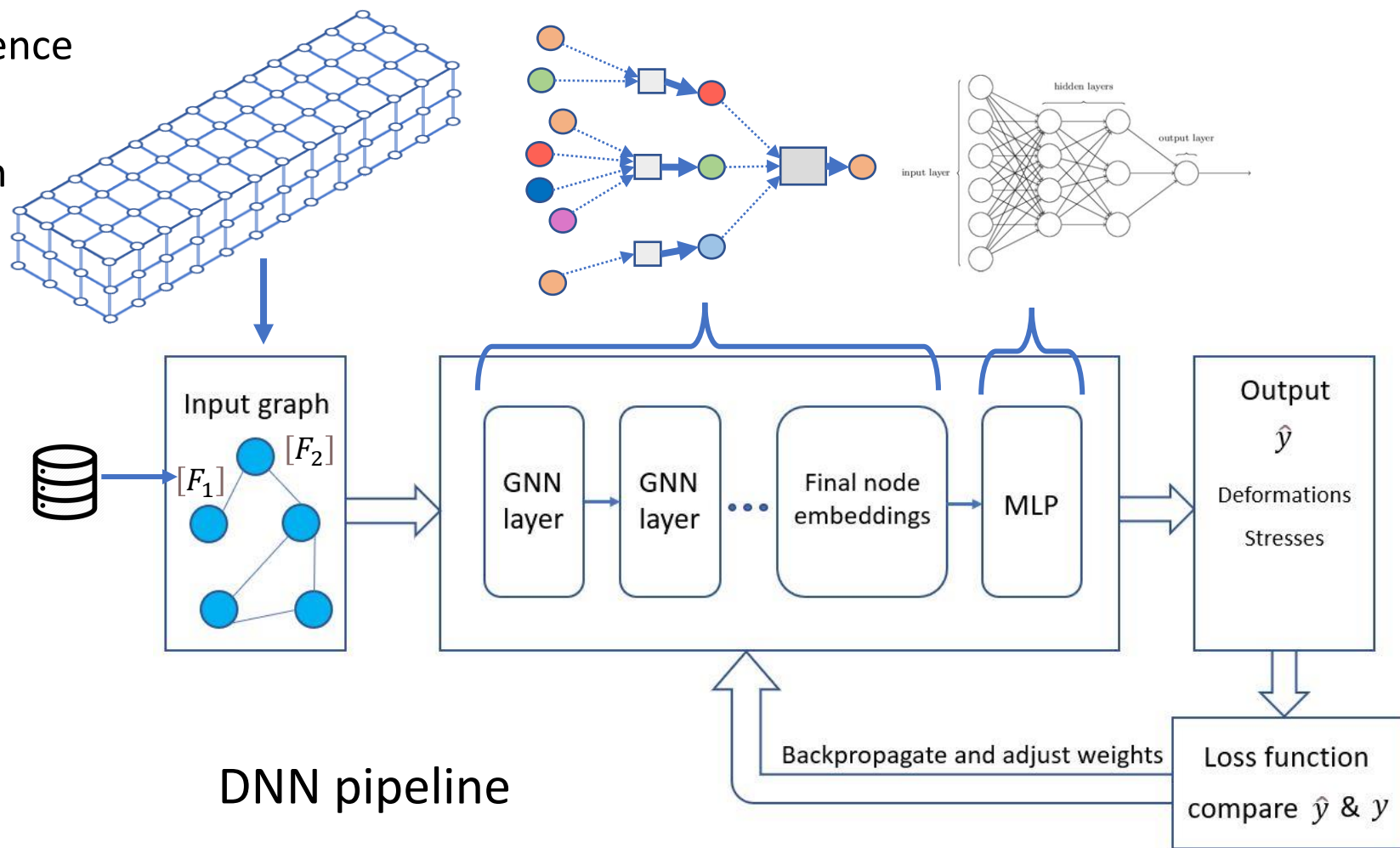


(b) Cantilever beam with end constraint



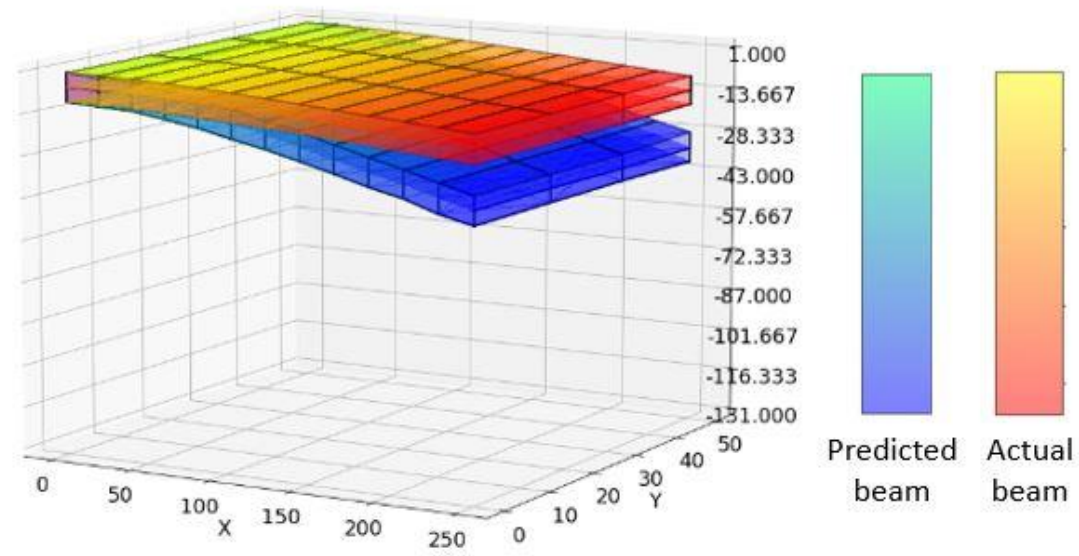
Building the DNN Model

- Model the discrete system to GNN
- Input node features as sequence of forces
- Graph computations through multiple GNN layers
- Pass the final node embeddings to MLP
- Obtain output deformation and stress



3D Deformation Plots

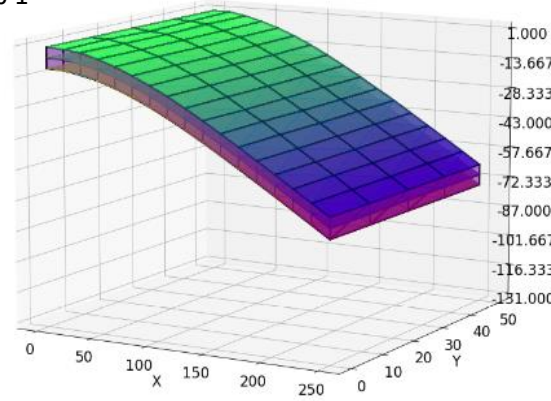
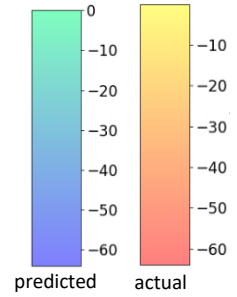
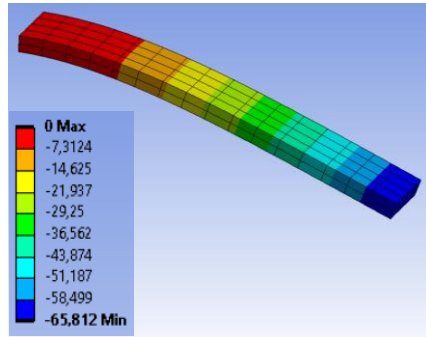
Bad prediction



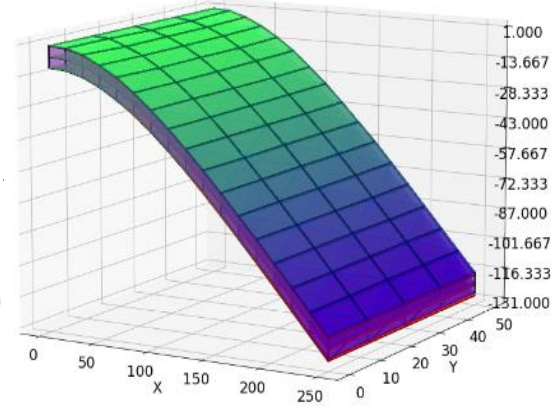
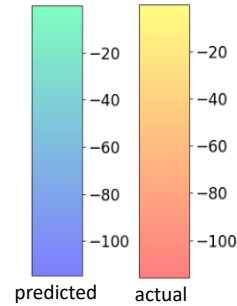
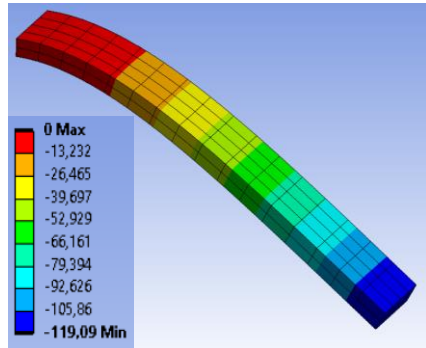
Experiment Results

3D Deformation Plots

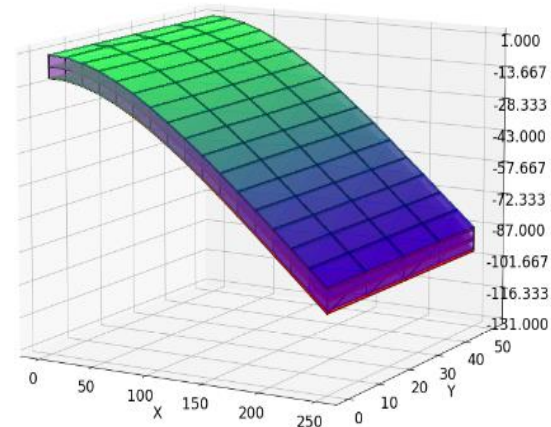
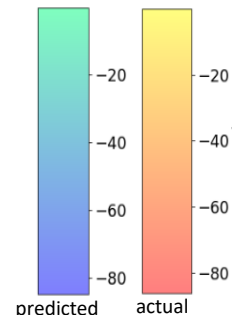
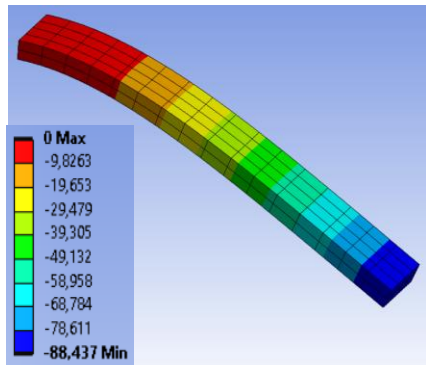
Deformation (mm) at time step 1



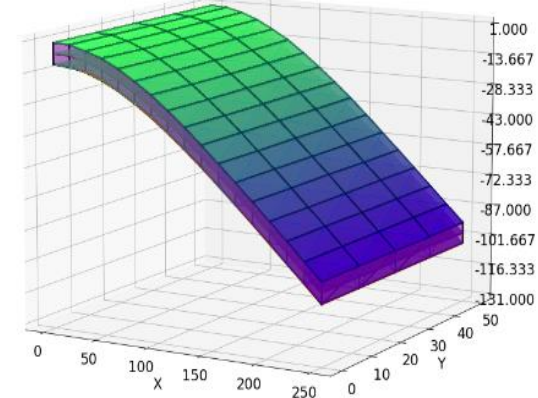
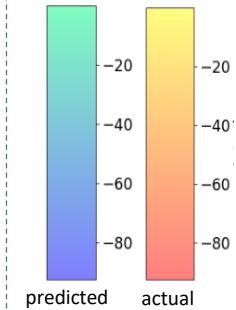
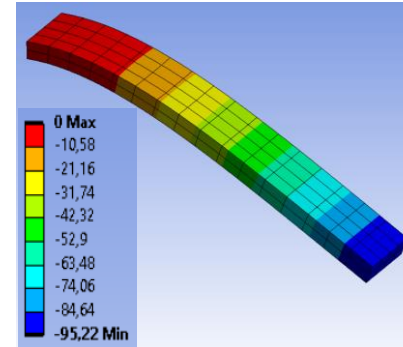
Deformation (mm) at time step 2



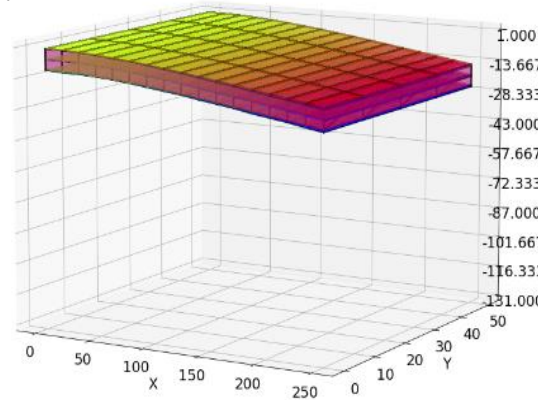
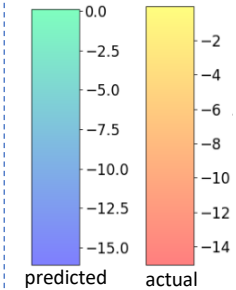
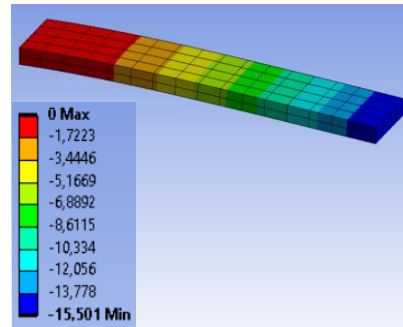
Deformation (mm) at time step 3



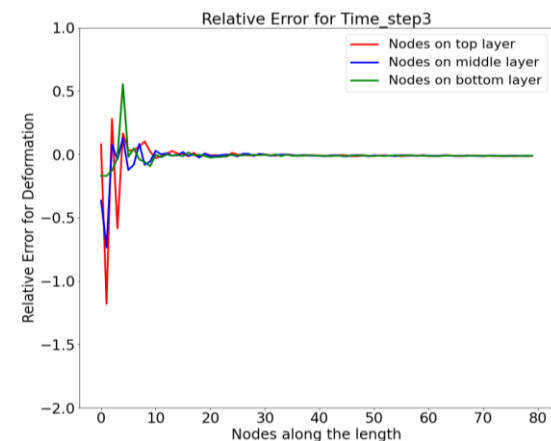
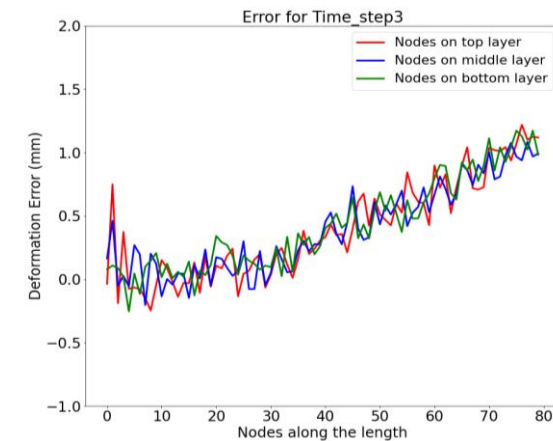
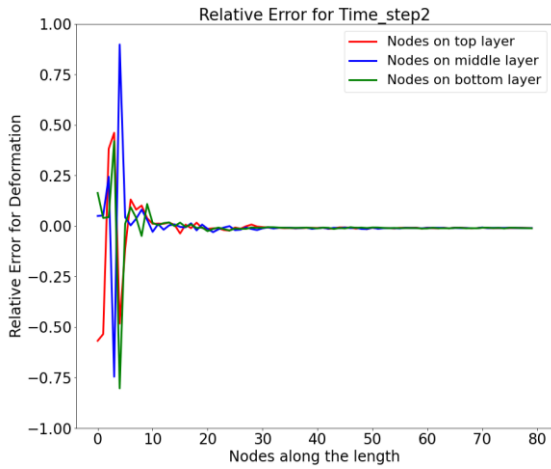
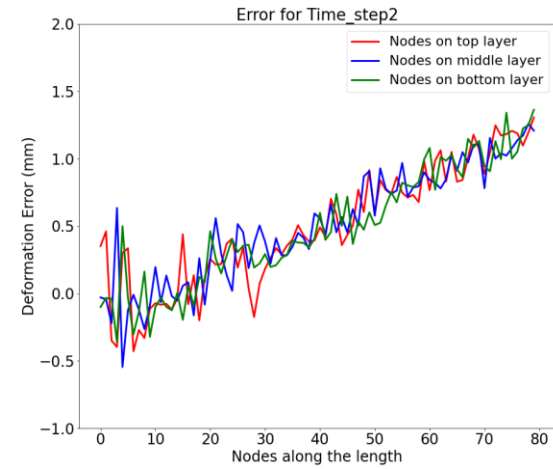
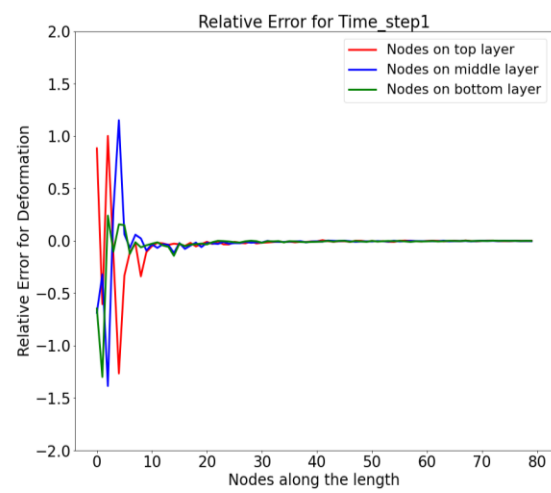
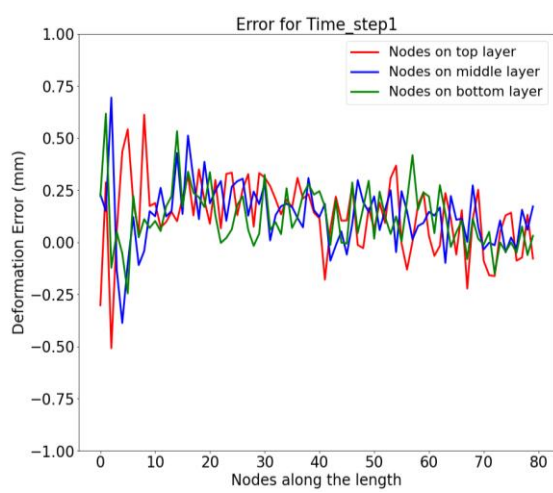
Deformation (mm) at time step 4



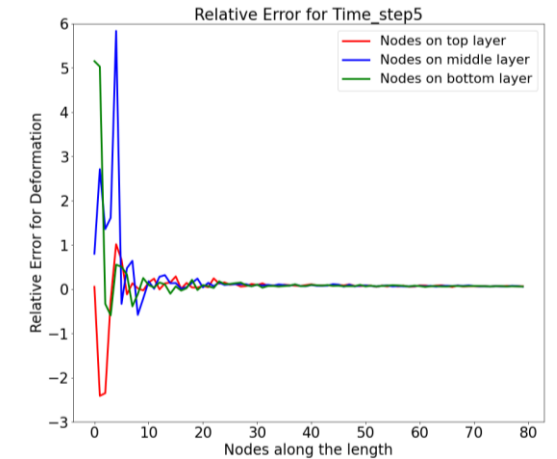
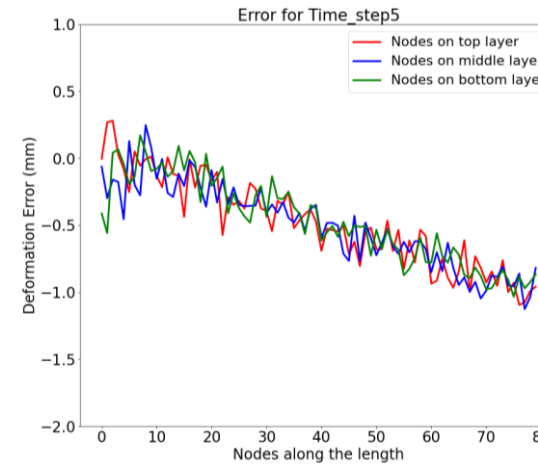
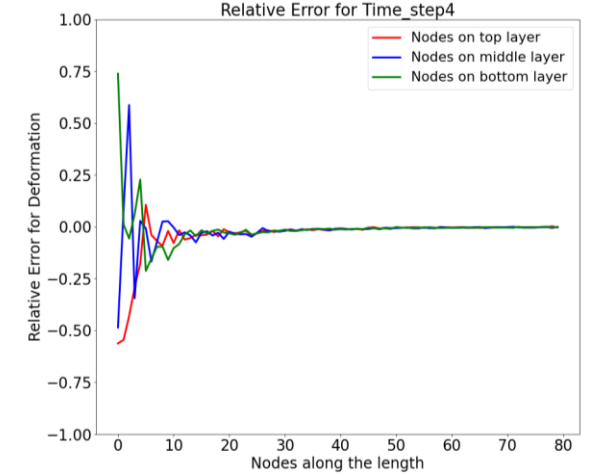
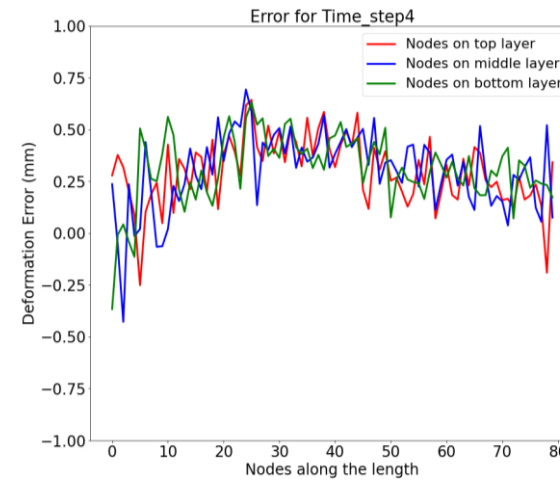
Deformation (mm) at time step 5



- Excellent deformation prediction



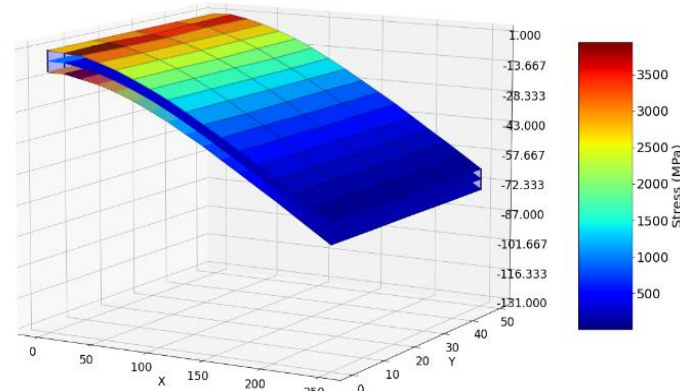
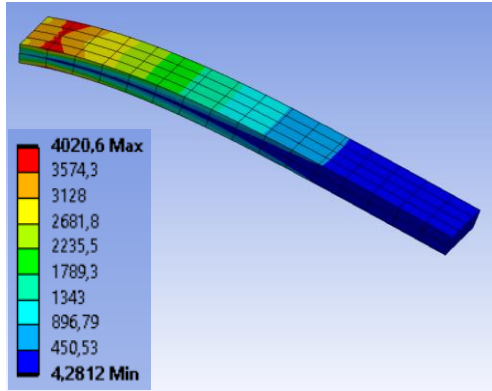
Deformation Error Visualization



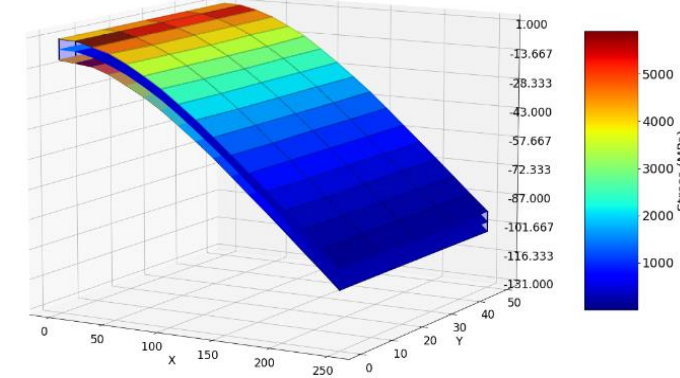
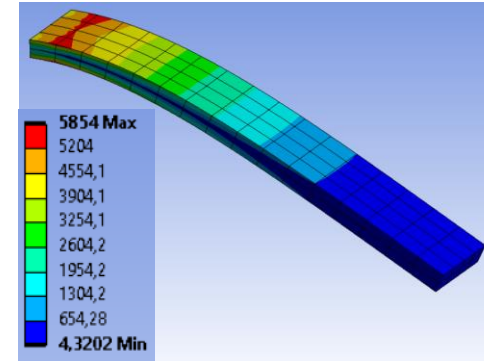
- Deformation error varies between -1 to +1.5 mm
- Relative error is close to zero for higher deformation values at the free end

Stress Distribution Plots

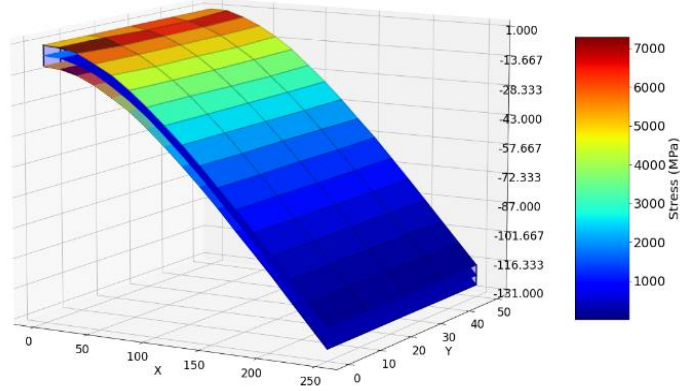
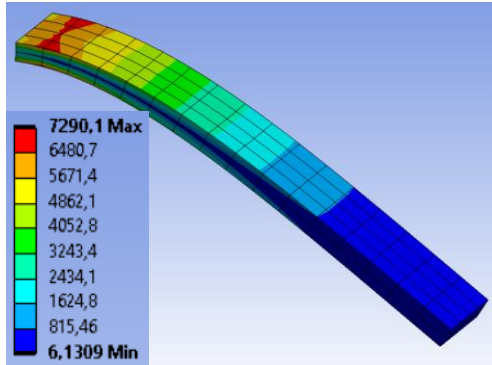
Stress (MPa) at time step 1



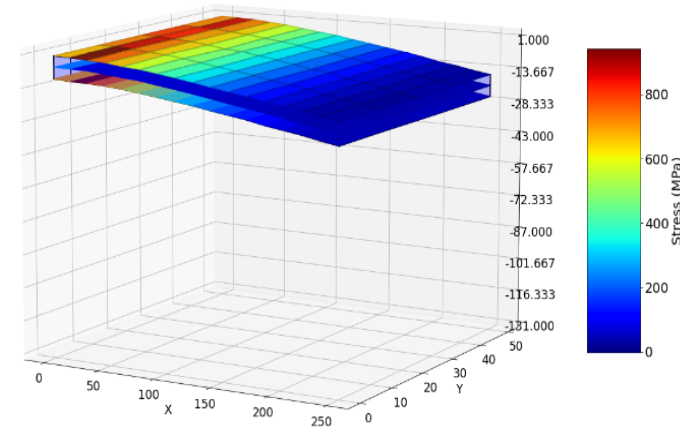
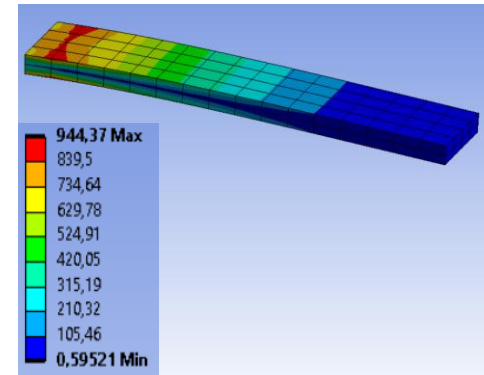
Stress (MPa) at time step 4



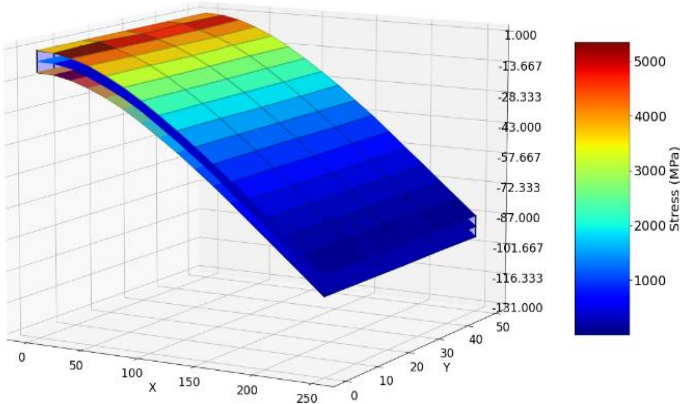
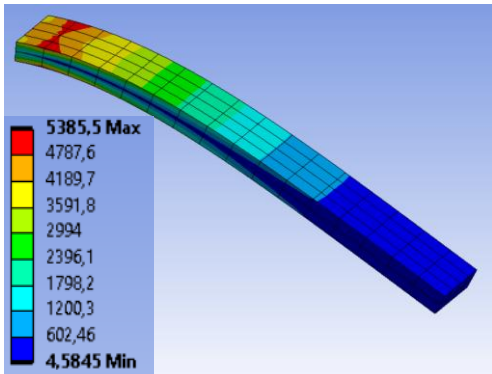
Stress (MPa) at time step 2



Stress (MPa) at time step 5

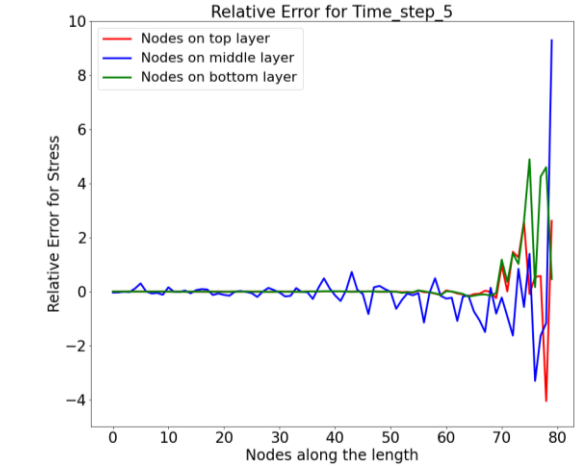
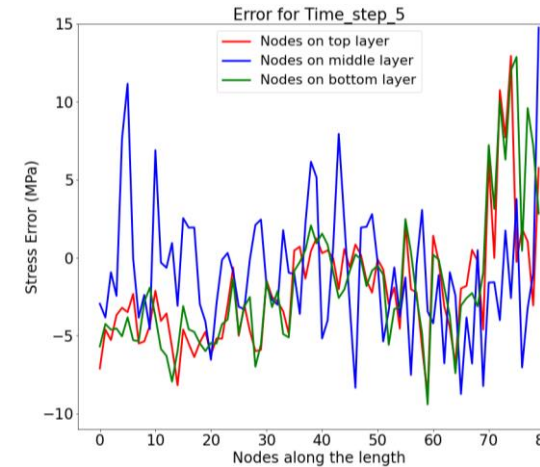
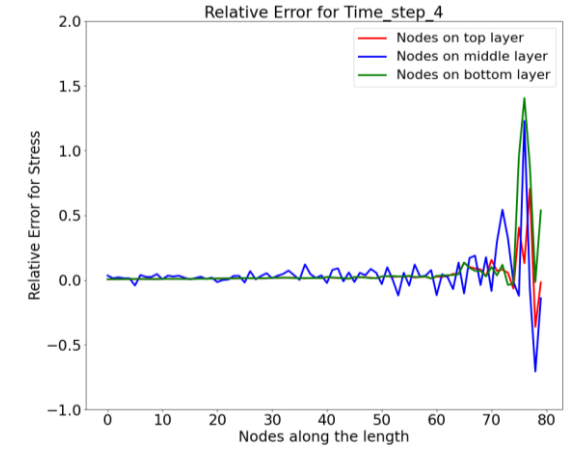
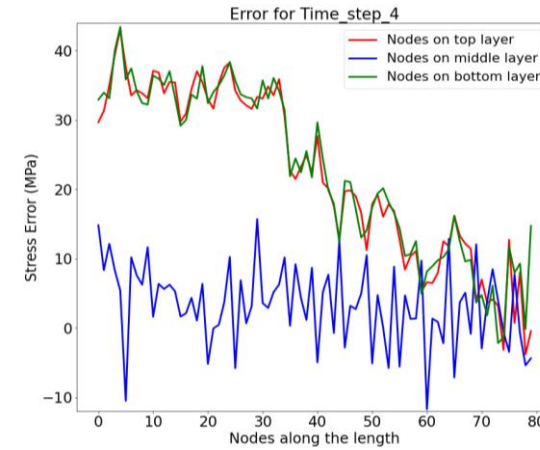
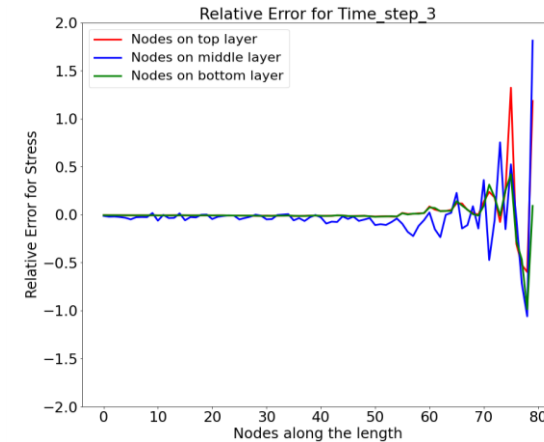
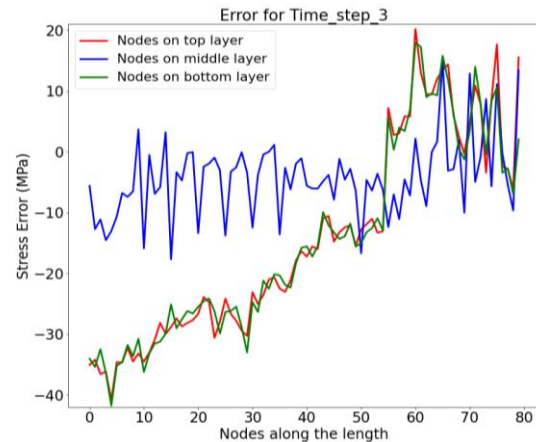
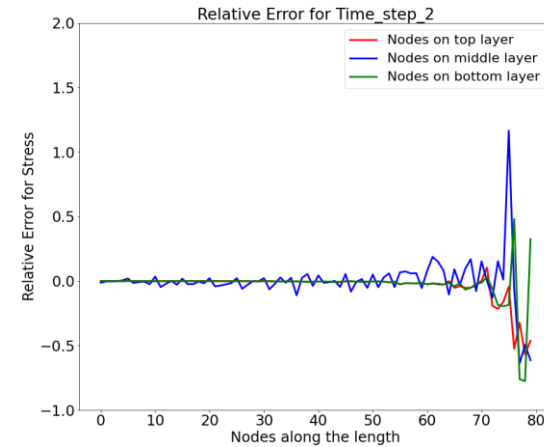
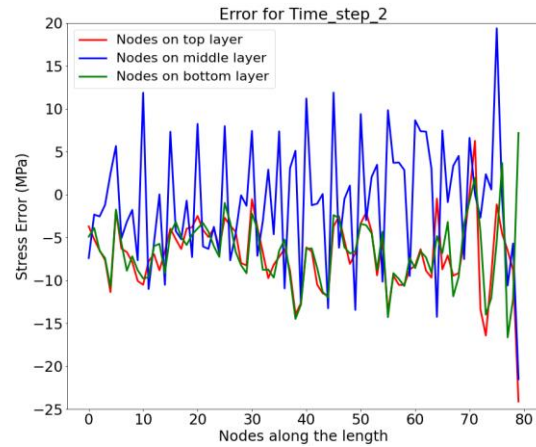
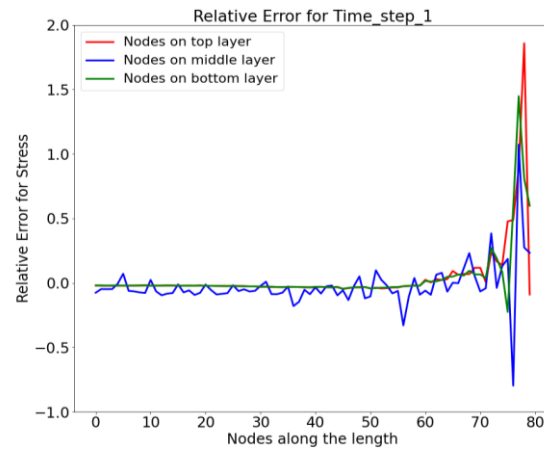
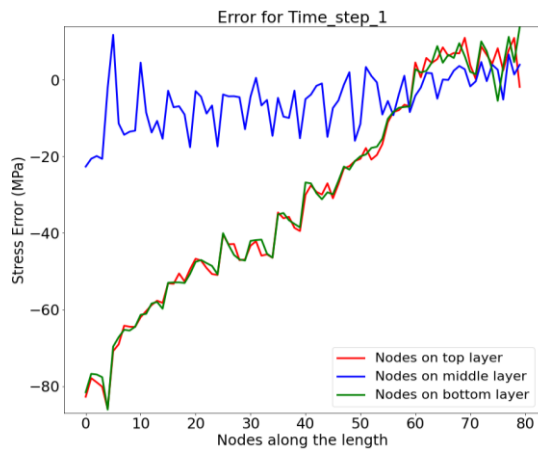


Stress (MPa) at time step 3



- The stress concentration regions are rightly predicted by the model

Stress Error Visualization



- Stress error varies between -40 to +40 MPa
- Relative stress error is close to zero for higher stress values

Time Analysis

FE Simulation time for
300 rows



112.5 hrs

DNN computation time for
300 rows



9 min



Conclusion

Conclusion

- Model with GCN architecture replicates the FEM results obtained from the simulation software, thus capable of replacing FEM based simulation
- Required computation time for DNN model is very less as compared to FE simulation software

Thank You!

Any questions?