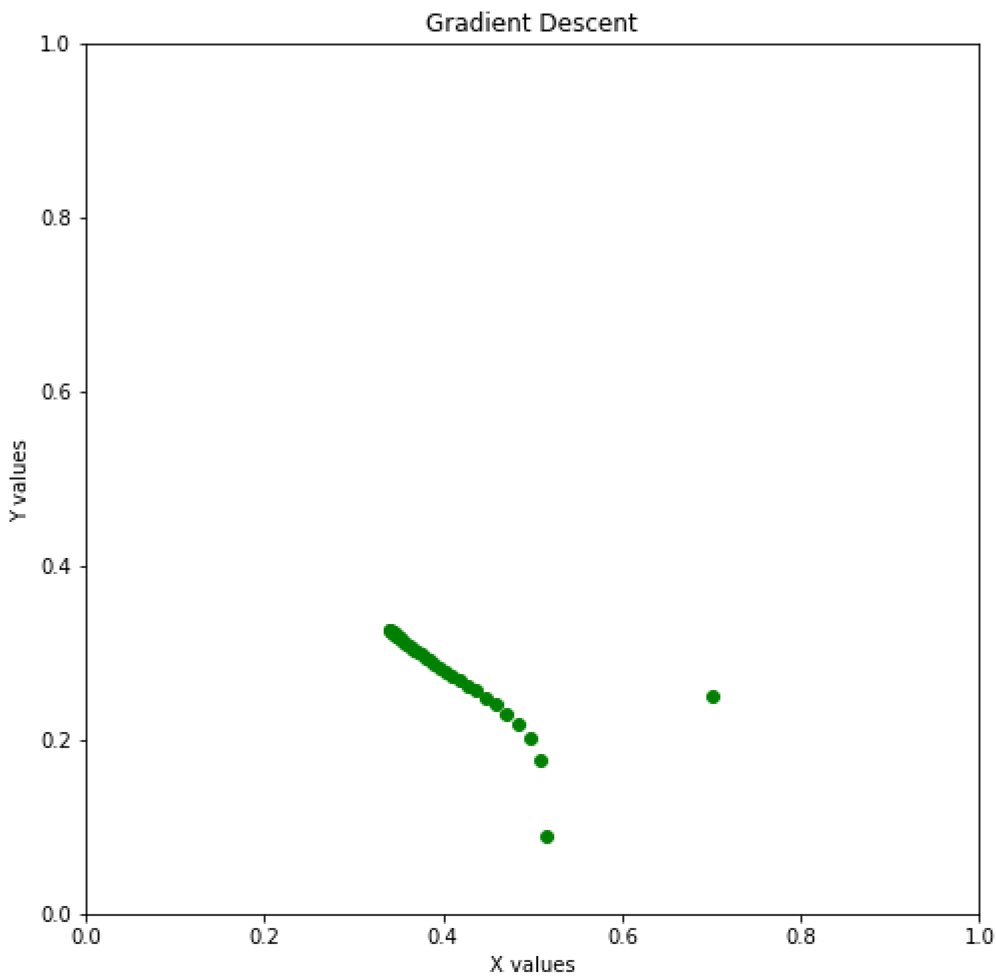
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy.linalg import inv
import time
```

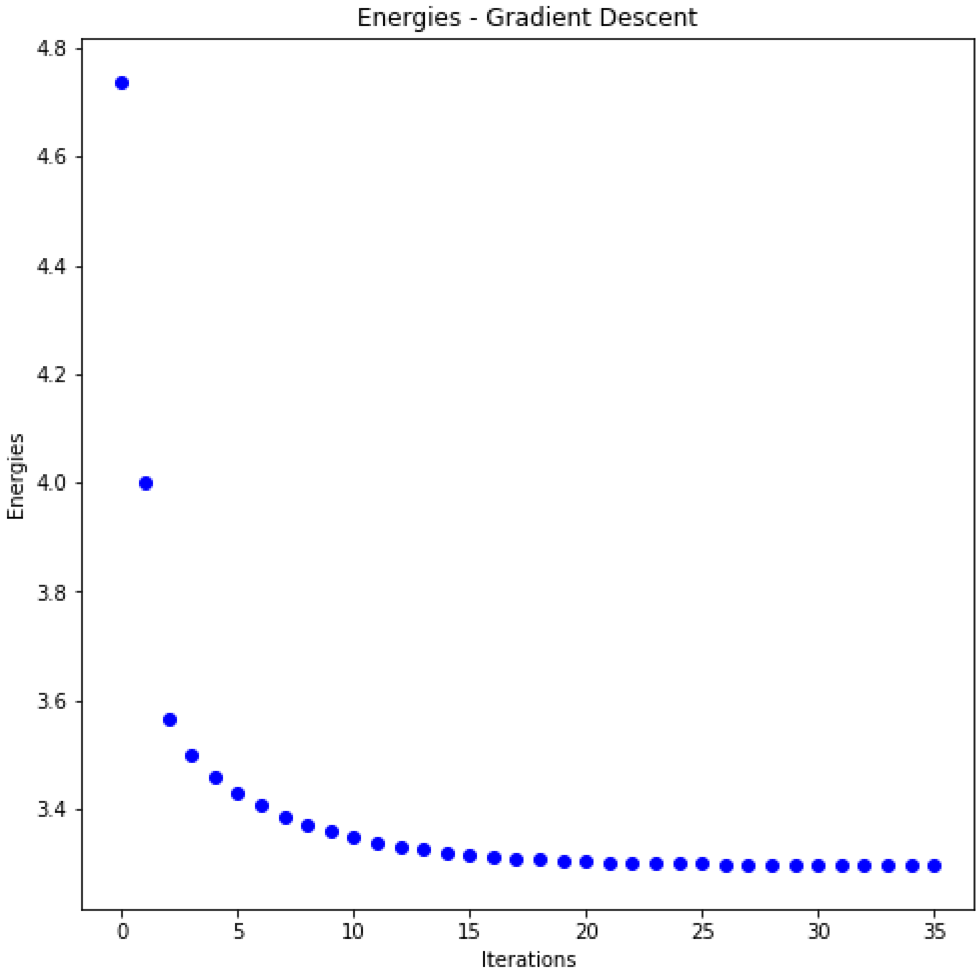
In [2]:
```
start = time.clock()

x = 0.7
y = 0.25
learning_rate = 0.01

w = np.array([x,y])

w_x = []
w_y = []
f = []

while (w[0]+w[1]<1) and (w[0]>0) and (w[1]>0):
    energy = - np.log(1-w[0]-w[1]) - np.log(w[0]) - np.log(w[1])
    f.append(energy)

    w_x.append(w[0])
    w_y.append(w[1])

    grad_x = 1/(1-w[0]-w[1]) - 1/(w[0])
    grad_y = 1/(1-w[0]-w[1]) - 1/(w[1])
    gradient = np.array([grad_x, grad_y])

    update = learning_rate * gradient

    if np.linalg.norm(w - np.subtract(w,update)) < 0.001:
        break
    else:
        w = np.subtract(w,update)

end = time.clock()
print ("Time taken for gradient descent: ", round((end-start), 4))
```

```
Time taken for gradient descent:  0.0062

C:\Users\kaush\Anaconda3\lib\site-packages\ipykernel_launcher.py:1: Deprecati
onWarning: time.clock has been deprecated in Python 3.3 and will be removed f
rom Python 3.8: use time.perf_counter or time.process_time instead
  """Entry point for launching an IPython kernel.
C:\Users\kaush\Anaconda3\lib\site-packages\ipykernel_launcher.py:31: Deprecat
ionWarning: time.clock has been deprecated in Python 3.3 and will be removed
from Python 3.8: use time.perf_counter or time.process_time instead
```

In [3]:
```
fig, ax = plt.subplots(figsize=(8,8))

plt.scatter(w_x, w_y, c = 'green')
plt.ylim([0,1])
plt.xlim([0,1])
plt.ylabel('Y values')
plt.xlabel('X values')
plt.title('Gradient Descent')

plt.show()
```



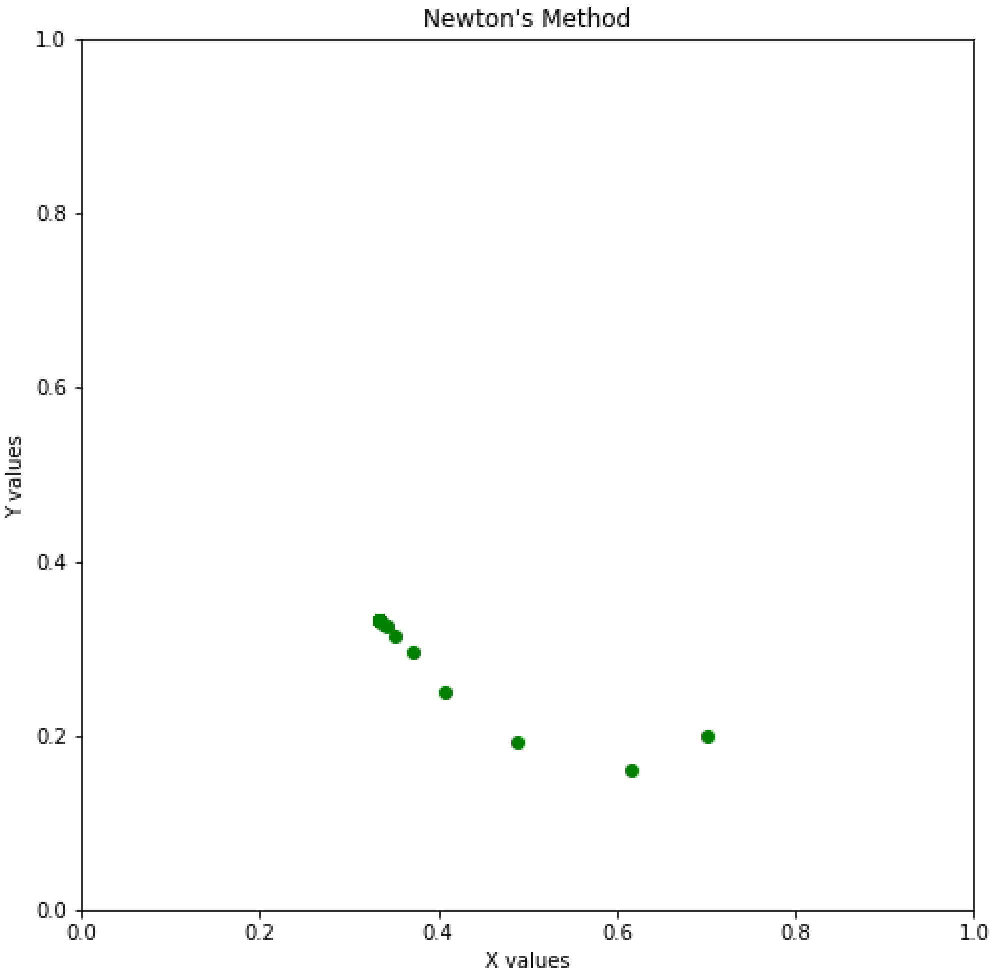
In [4]:
```
fig, ax = plt.subplots(figsize=(8,8))

plt.scatter(range(len(f)), f , c = 'blue')
plt.ylabel('Energies')
plt.xlabel('Iterations')
plt.title('Energies - Gradient Descent')

plt.show()
```



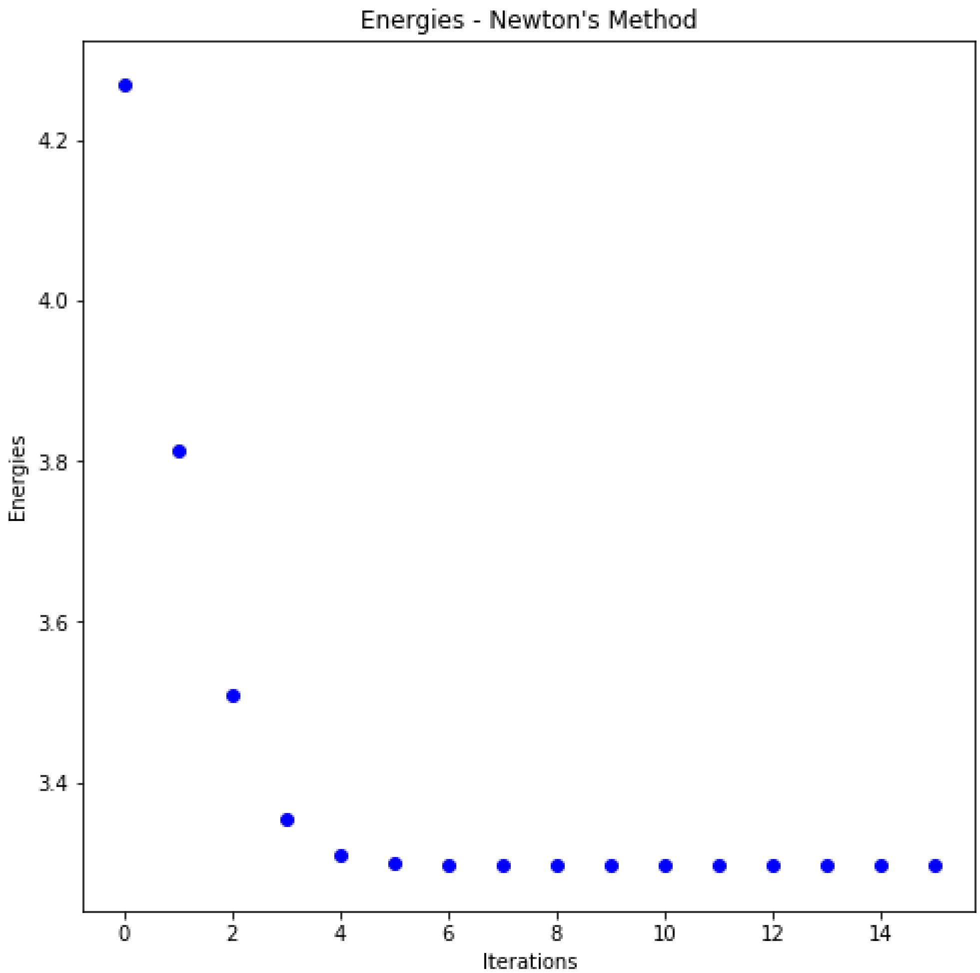
In [5]:
```
start = time.clock()

x = 0.7
y = 0.2
learning_rate = 1

w = np.array([x,y])

w_x = []
w_y = []
f = []

while ((w[0]+w[1])<1) and (w[0]>0) and (w[1]>0):
    energy = - np.log(1-w[0]-w[1]) - np.log(w[0]) - np.log(w[1])
    f.append(energy)

    w_x.append(w[0])
    w_y.append(w[1])

    grad_x = 1/(1-w[0]-w[1]) - 1/(w[0])
    grad_y = 1/(1-w[0]-w[1]) - 1/(w[1])
    gradient = np.array([grad_x, grad_y])

    hessian_x1 = 1/((1-w[0]-w[1])*(1-w[0]-w[1])) + 1/(w[0]*w[0])
    hessian_y2 = 1/((1-w[0]-w[1])*(1-w[0]-w[1])) + 1/(w[1]*w[1])
    hessian_xy = 1/(1- w[0]-w[1]) * (1- w[0]-w[1])
    hessian = np.array([[hessian_x1, hessian_xy],[hessian_xy, hessian_y2]])

    update = learning_rate * np.matmul(inv(hessian), gradient)
    if np.linalg.norm(w - np.subtract(w,update)) < 0.00001:
        break
    else:
        w = np.subtract(w,update)

end = time.clock()
print ("Time taken for Newton's method: ", round((end-start), 4))
```

```
Time taken for Newton's method:  0.0018

C:\Users\kaush\Anaconda3\lib\site-packages\ipykernel_launcher.py:1: Deprecati
onWarning: time.clock has been deprecated in Python 3.3 and will be removed f
rom Python 3.8: use time.perf_counter or time.process_time instead
  """Entry point for launching an IPython kernel.
C:\Users\kaush\Anaconda3\lib\site-packages\ipykernel_launcher.py:35: Deprecat
ionWarning: time.clock has been deprecated in Python 3.3 and will be removed
from Python 3.8: use time.perf_counter or time.process_time instead
```

In [6]:
```
fig, ax = plt.subplots(figsize=(8,8))

plt.scatter(w_x, w_y, c = 'green')
plt.ylim([0,1])
plt.xlim([0,1])
plt.ylabel('Y values')
plt.xlabel('X values')
plt.title('Newton\'s Method')

plt.show()
```



In [7]:
```
fig, ax = plt.subplots(figsize=(8,8))

plt.scatter(range(len(f)), f , c = 'blue')
plt.ylabel('Energies')
plt.xlabel('Iterations')
plt.title('Energies - Newton\'s Method')

plt.show()
```



Newton's method is 0.001 seconds faster than Gradient Descent.

The convergence speed may change from function to function.

Usually, Newton's method has inverse quadratic convergence compared to the inverse linear convergence of gradient descent.

In [ ]: