Electrodynamics-I

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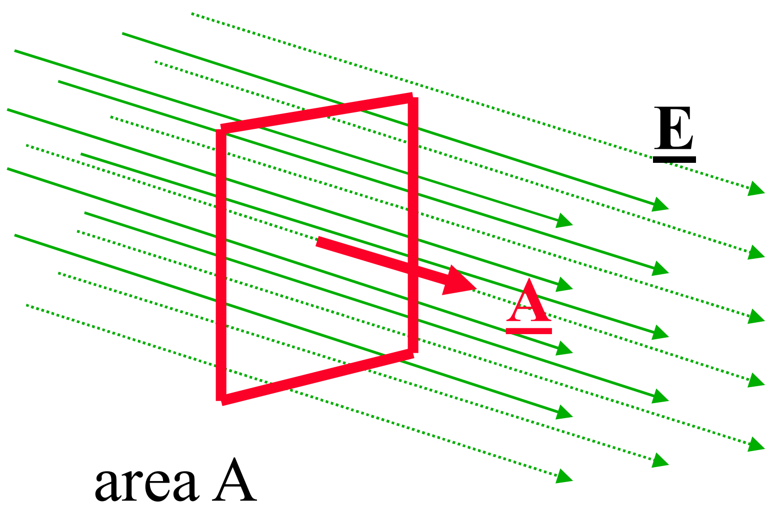
Electric Flux



We define the electric flux Φ, of the electric field E, through the surface A, as:

$$\boxed{\Phi = \mathbf{E} \cdot \mathbf{A}}$$

$$\Phi = E\,A\cos(\theta)$$

Where:

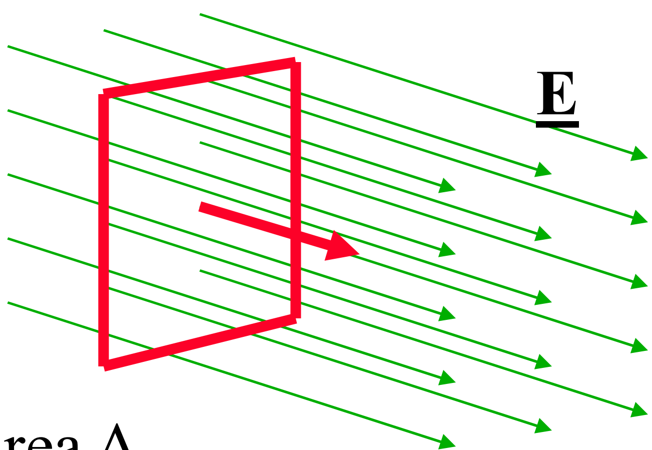
A is a vector normal to the surface (magnitude A, and direction normal to the surface).

θ is the angle between E and A

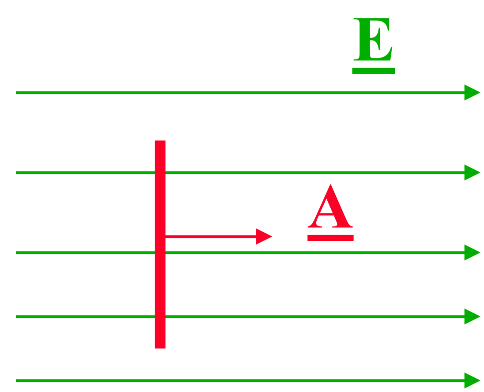
Electric Flux

You can think of the flux through some surface as a measure of the number of field lines which pass through that surface.

Flux depends on the strength of $\underline{\mathbf{E}}$, on the surface area, and on the relative orientation of the field and surface.



area A



Normal to surface, magnitude A

Here the flux is

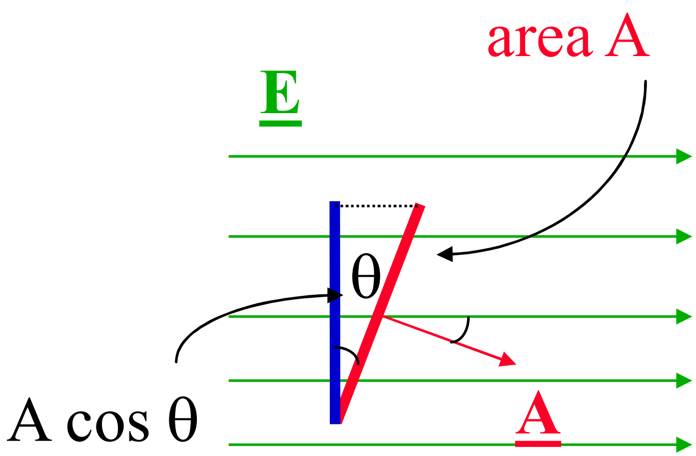
$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$

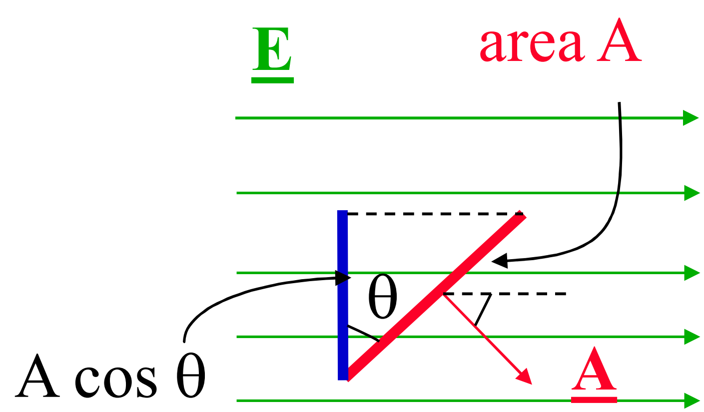
$$\underline{\mathbf{A}} \equiv \longrightarrow$$

Electric Flux

The flux also depends on orientation

$$\Phi = \underline{\mathbf{E}} \,.\, \underline{\mathbf{A}} = \mathbf{E\ A\ cos}\ \theta$$





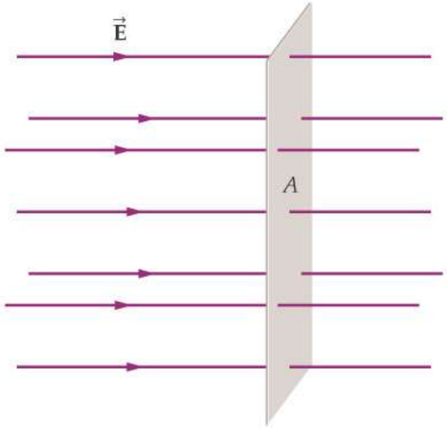
The number of field lines through the tilted surface equals the number through its projection . Hence, the flux through the tilted surface is simply given by the flux through its projection: E (A cosθ).

Calculate the flux of the electric field E, through the surface A, in each of the three cases shown:

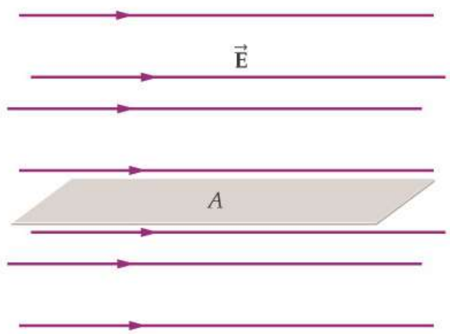
a) $\Phi =$

b) $\Phi =$

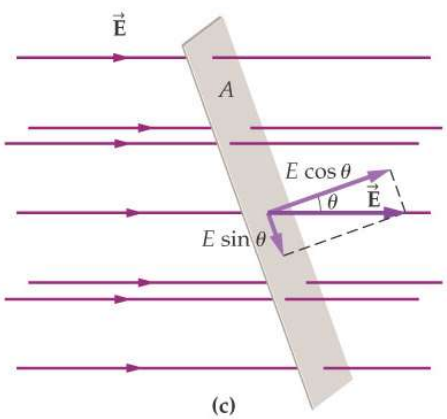
c) $\Phi =$



(a)



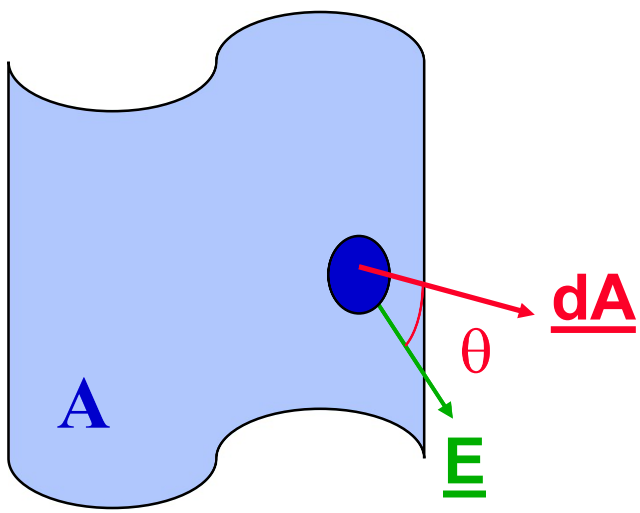
(b)



(c)

What if the surface is curved, or the field varies with position ??

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$



1. We divide the surface into small regions with area dA

2. The flux through dA is

$$d\Phi = E\, dA \cos\theta$$

$$\mathbf{d\Phi = \underline{E} \cdot d\underline{A}}$$

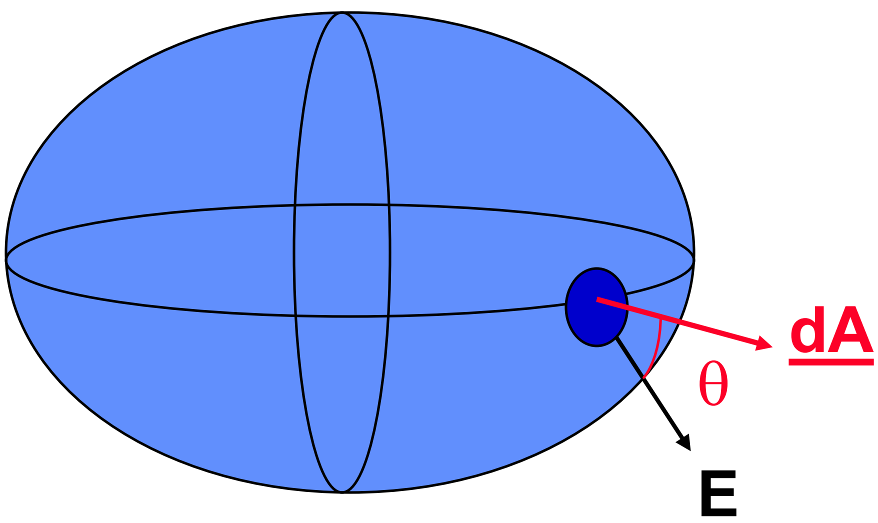
3. To obtain the total flux we need to integrate over the surface A

$$\boxed{\Phi = \int d\Phi = \int \underline{\mathbf{E}} \cdot \underline{\mathbf{dA}}}$$

In the case of a closed surface

$$\Phi = \oint d\Phi = \oint \underline{E} \bullet \underline{dA}$$

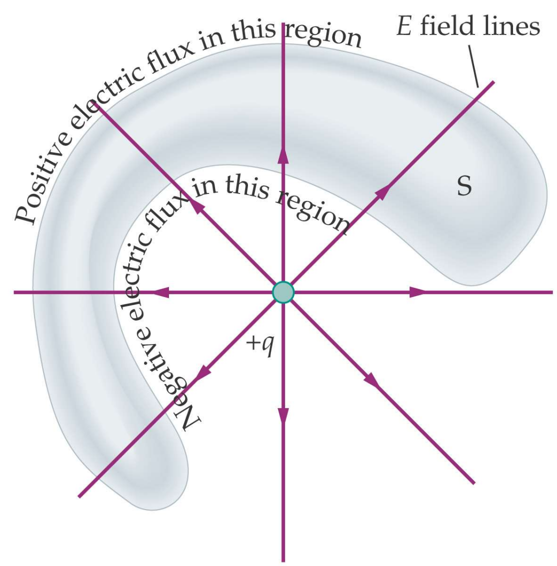
The loop means the integral is over a closed surface.



For a closed surface:

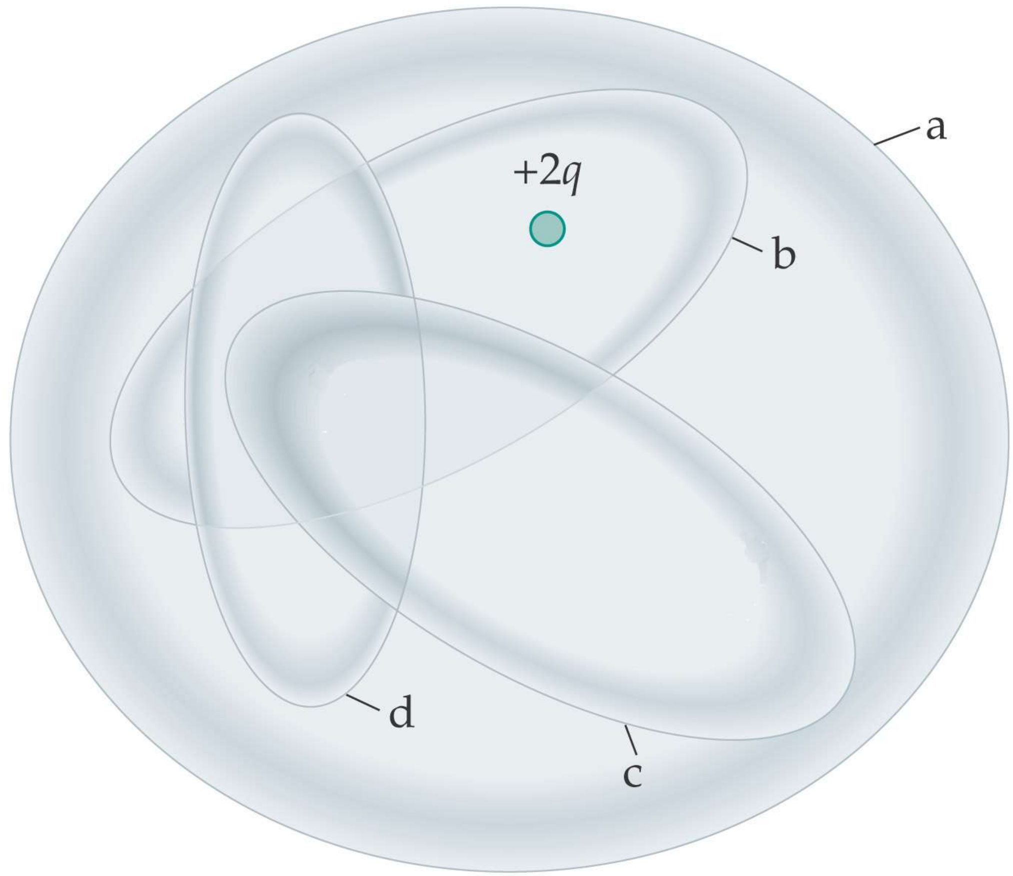
The flux is positive for field lines that leave the enclosed volume

The flux is negative for field lines that enter the enclosed volume



If a charge is outside a closed surface, the net flux is zero. As many lines leave the surface, as lines enter it.

For which of these closed surfaces (a, b, c, d) the flux of the electric field, produced by the charge +2q, is zero?



Spherical surface with point charge at center

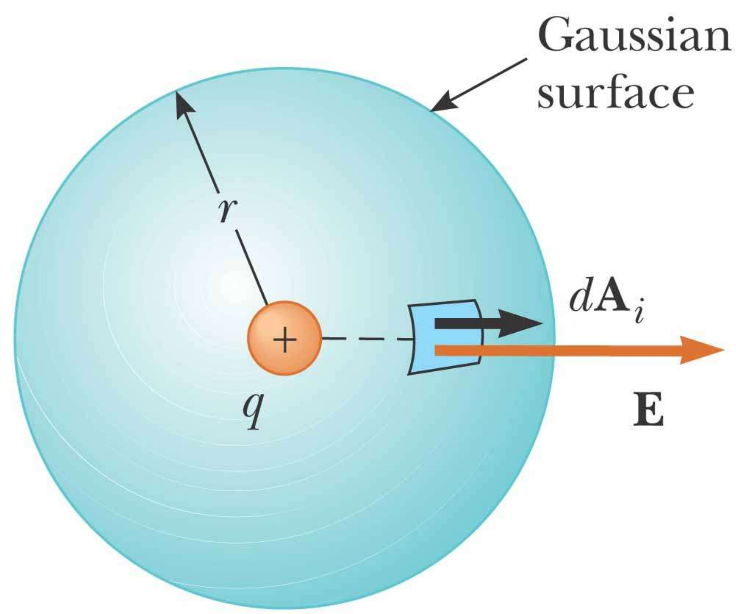
Flux of electric field:

$$\Phi = \oint d\Phi = \oint \vec{E} \cdot \vec{dA}$$

$$\Phi = \oint E\, dA \cos\theta = \oint \frac{1}{4\pi\varepsilon_0} \frac{q\, dA}{r^2}$$

but $\dfrac{dA}{r^2} = d\Omega$, then:

$$\Phi = \frac{q}{4\pi\varepsilon_0} \oint d\Omega = \frac{q}{4\pi\varepsilon_0} 4\pi = \frac{q}{\varepsilon_0}$$



Gauss's Law

$$\boxed{\oint \vec{E} \cdot \vec{dA} = \frac{q_{enclosed}}{\varepsilon_0}}$$

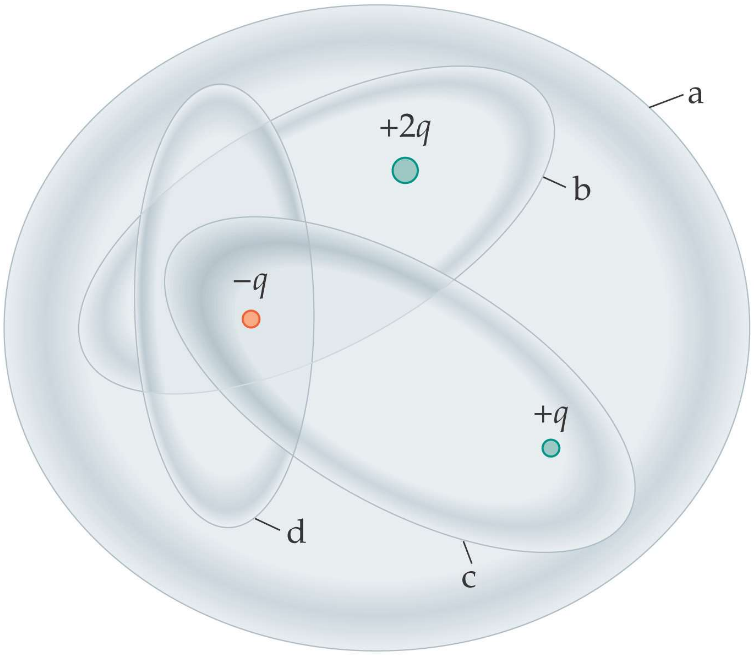
Gauss's Law

The electric flux
through any closed surface
equals □ enclosed charge / ε_0

$$\boxed{\oint \underline{E} \bullet \underline{dA} = \frac{\sum_{inside} q}{\varepsilon_0}}$$

This is always true.
Occasionally, it provides a very easy way
to find the electric field
(for highly symmetric cases).

Calculate the flux of the electric field Φ for each of the closed surfaces a, b, c, and d



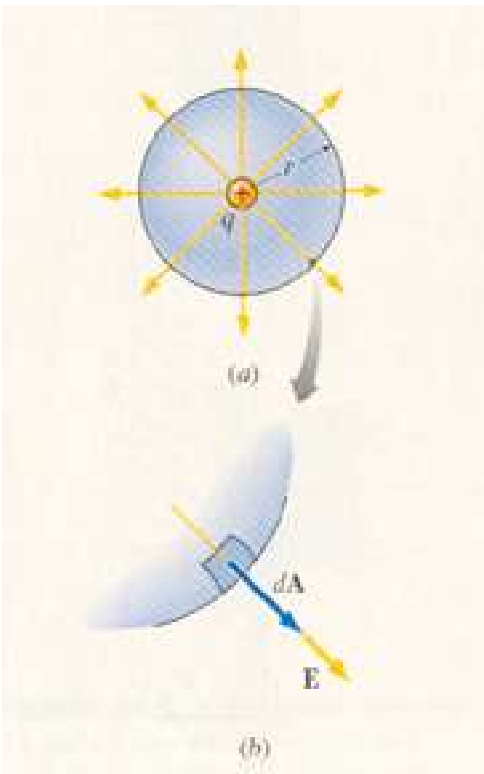
Surface a, $\Phi_a =$

Surface b, $\Phi_b =$

Surface c, $\Phi_c =$

Surface d, $\Phi_d =$

Calculate the electric field produced by a point charge using Gauss Law



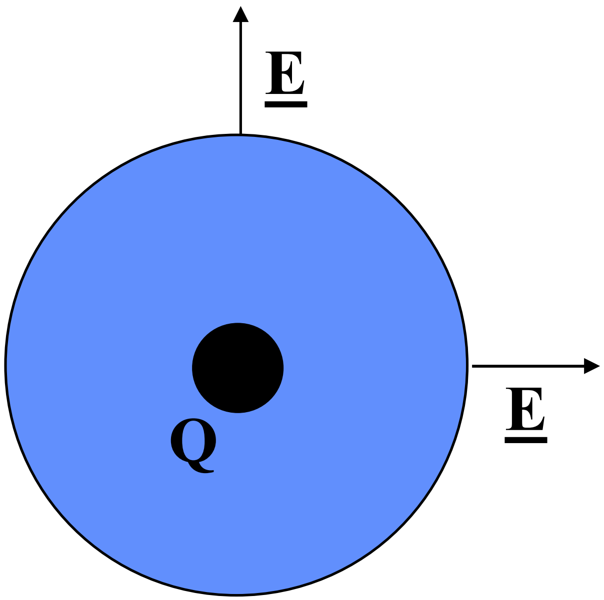
We choose for the gaussian surface a sphere of radius **r**, centered on the charge **Q**.

Then, the electric field **E**, has the same magnitude everywhere on the surface (radial symmetry)

Furthermore, at each point on the surface, the field **E** and the surface normal **dA** are parallel (both point radially outward).

$$\mathbf{\underline{E}} \cdot \mathbf{\underline{dA}} = \mathbf{E\,dA} \quad [\cos\theta = 1]$$

Electric field produced by a point charge



$$\int \underline{\mathbf{E}} \cdot \underline{\mathbf{dA}} = Q / \varepsilon_0$$

$$\int \underline{\mathbf{E}} \cdot \underline{\mathbf{dA}} = E \int dA = E\,A$$

$$A = 4\,\pi\,r^2$$

$$E\,A = E\;4\,\pi\,r^2 = Q / \varepsilon_0$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$k = 1 / 4\,\pi\,\varepsilon_0$

ε_0 = permittivity

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

→ **Coulomb's Law !**

Ampère's Law

- A Useful law that relates the net magnetic field along a closed loop to the electric current passing through the loop

- First discovered by **André-Marie Ampère** in 1826.

- André Ampère formulated a law based on Oersted's as well as his own experimental studies. This law can be regarded as an alternative expression of Biot-savert's law which also relates the magnetic field and current produced. But it needed an exclusive calculation of the curl of **B** . And that calculation, leads to the limitation of the usual form of this law i.e. its validity holding only for steady currents.

- After four decades later, the James Clerk Maxwell realized that the equation provided by the Ampere was incomplete, and extended his law by including that the magnetic field arises due to the electric current by giving a mathematical formulation.

Ampere's circuital law states: The line integral of the magnetic field, over a closed path, or loop, equals times the total current enclosed by that closed loop. We express this law through the mathematical expression:

$$\oint \vec{B}.\overrightarrow{dl} = \mu_o i_{inside}$$

where ,i is the net current enclosed by the loop
μ_o = permeability of free space = $4\pi\times10^{-15}$N/A^2

Faraday's Law

Experimentally, if the flux through N loops of wire changes by $d\Phi_B$ in a time dt, the induced emf is

$$\varepsilon = -N\frac{d\Phi_B}{dt}.$$

*Faraday's Law of Magnetic Induction

$$\varepsilon_{average} = -N\frac{\Delta\Phi_B}{\Delta t}.$$

Faraday's law of induction is one of the fundamental laws of electricity and magnetism.

I wonder why the – sign…

In the equation $\varepsilon = -N\dfrac{d\Phi_B}{dt}$,

Faraday's Law of Magnetic Induction

$\Phi_B = \int \vec{B}\cdot d\vec{A}$ is the magnetic flux.

This is another expression of Faraday's Law:

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

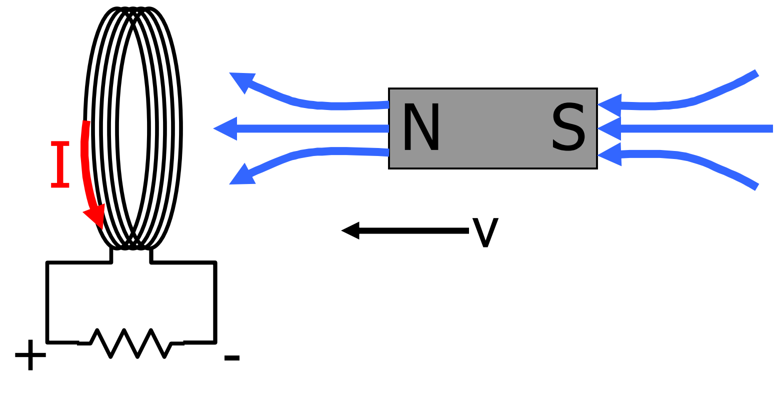
We'll use this version in the next lecture.

The fine print, put here for me to ponder, and not for students to worry about. A magnetic force does no work on a moving (or stationary) charged particle. Therefore the magnetic force cannot change a charged particle's potential energy or electric potential. But electric fields can do work. This equation shows that a changing magnetic flux induces an electric field, which can change a charged particle's potential energy. This induced electric field is responsible for induced emf. During this lecture, we are mostly going to examine how a changing magnetic flux induces emf, without concerning ourselves with the "middleman" induced electric field.

Example: move a magnet towards a coil of wire.

N=5 turns A=0.002 m^2

$$\frac{dB}{dt} = 0.4 \text{ T/s}$$



$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d\int \vec{B}\cdot d\vec{A}}{dt}$$

$$\varepsilon = -N\frac{d(BA)}{dt}$$

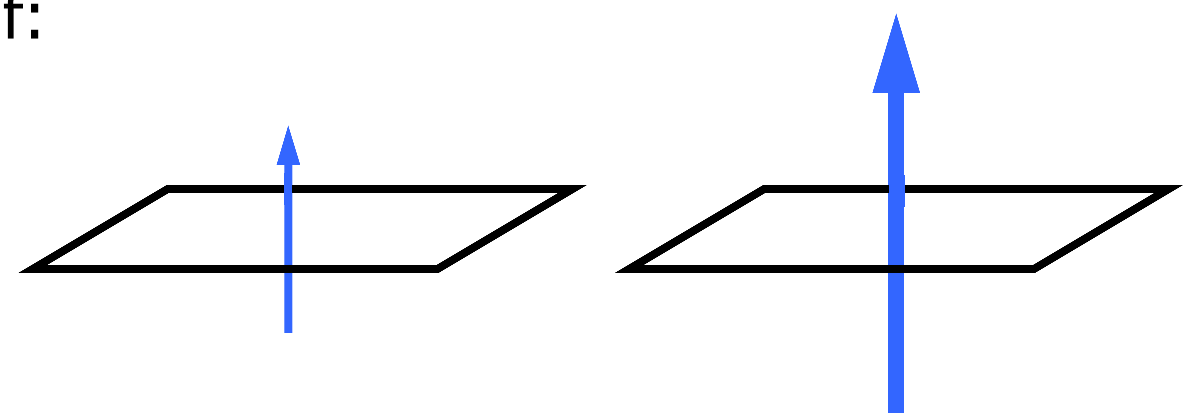
(what assumptions did I make here?)

$$\varepsilon = -NA\frac{dB}{dt}$$

$$\varepsilon = -5\left(0.002\,m^2\right)\left(0.4\frac{T}{s}\right) = -0.004 \text{ V}$$

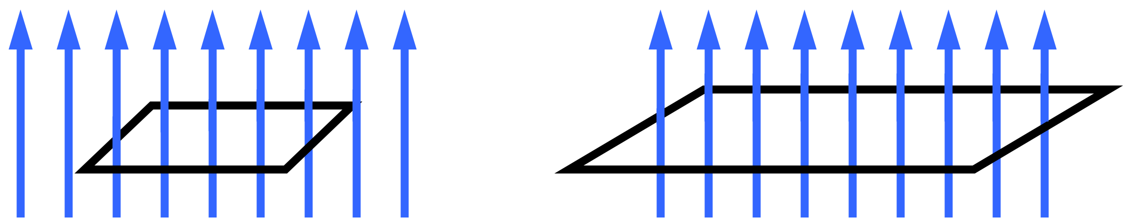
Ways to induce an emf:

- change B



Possible homework hint: $\Phi_B = \int d\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B(t)\, dA$ if B varies but loop $\perp \vec{B}$.

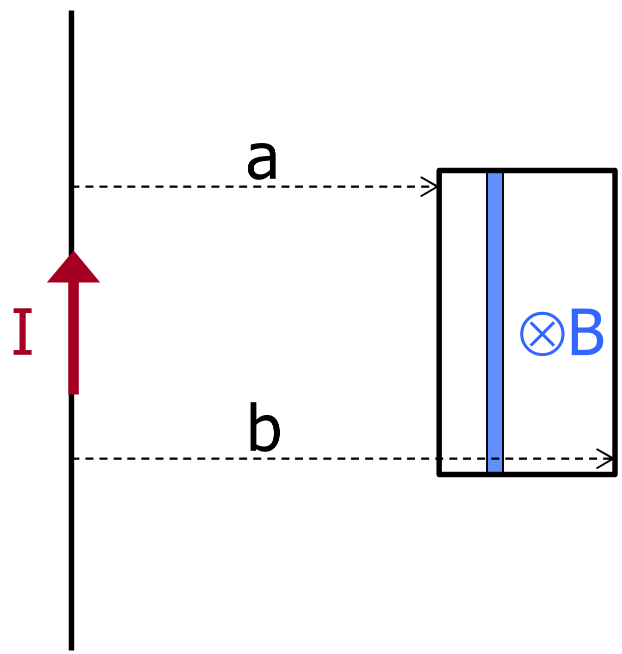
- change the area of the loop in the field



Possible homework hint: for a circular loop, $C=2\pi R$, so $A=\pi r^2=\pi(C/2\pi)^2=C^2/4\pi$, so you can express $d(BA)/dt$ in terms of dC/dt.

Ways to induce an emf (continued):

- changing current changes B through conducting loop

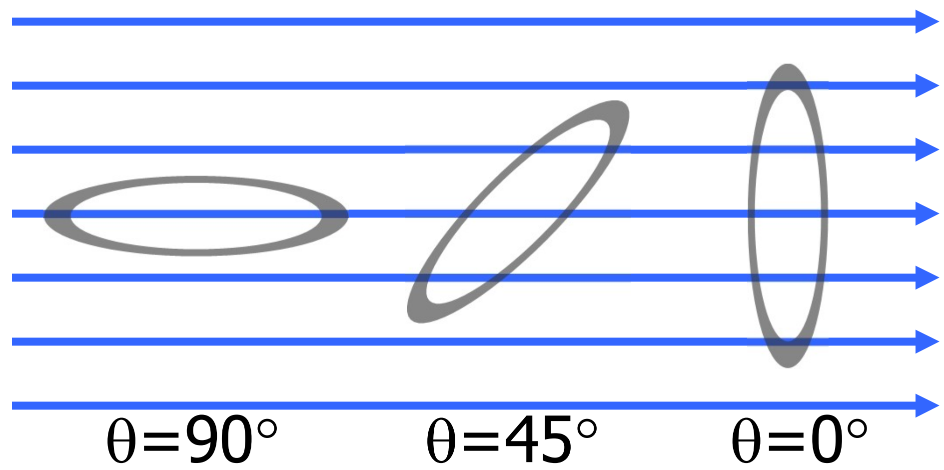


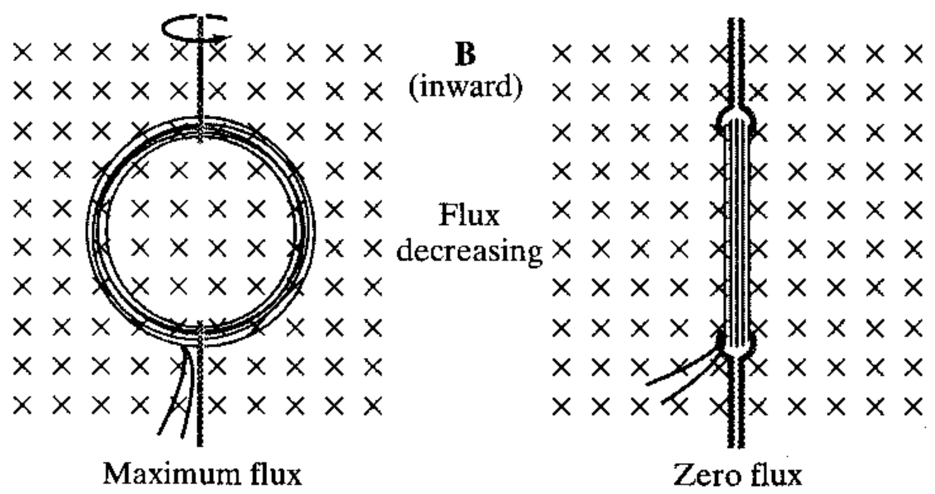
Possible Homework Hint.

The magnetic field is not uniform through the loop, so you can't use BA to calculate the flux. Take an infinitesimally thin strip. Then the flux is $d\Phi = BdA_{strip}$. Integrate from a to b to get the flux through the strip.

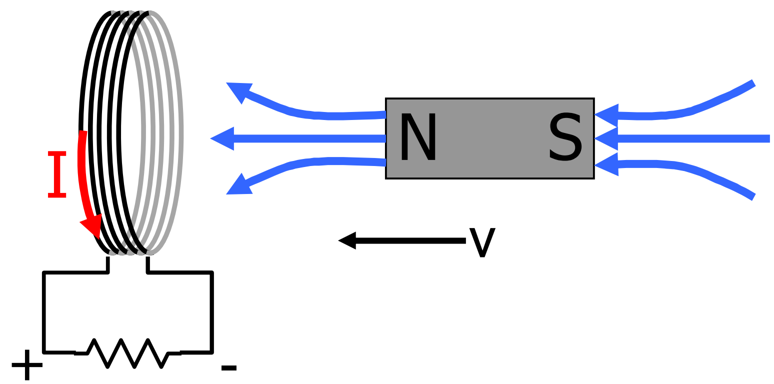
Ways to induce an emf (continued):

- change the orientation of the loop in the field





Lenz's law—An induced emf always gives rise to a current whose magnetic field opposes the change in flux.*



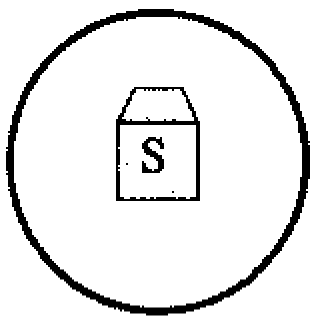
If Lenz's law were not true—if there were a + sign in Faraday's law—then a changing magnetic field would produce a current, which would further increase the magnetic field, further increasing the current, making the magnetic field still bigger…

*Think of the current resulting from the induced emf as "trying" to maintain the status quo—to prevent change.

…violating conservation of energy and ripping apart the very fabric of the universe…

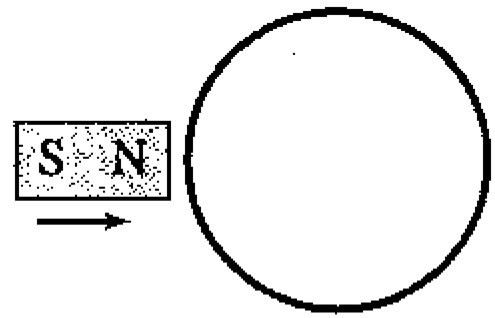
Practice with Lenz's Law.

In which direction is the current induced in the coil for each situation shown? Practice on your own. In lecture, skip to here.



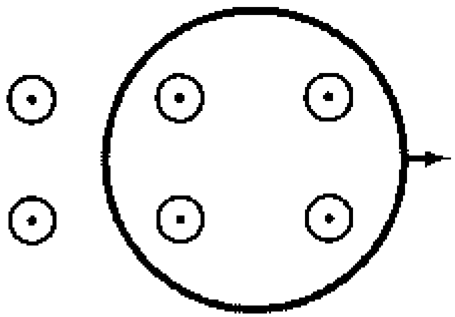
(a)
N magnetic pole moving toward coil from above the page

(counterclockwise)



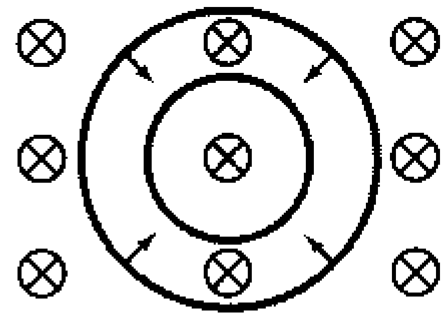
(b)
N magnetic pole moving toward the coil in the plane of the page

(no current)



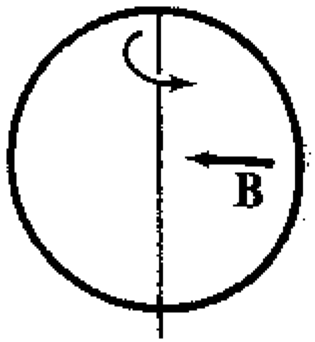
(c)
Pulling the coil to the right out of a magnetic field that points out of the page

(counterclockwise)



(d)
Shrinking a coil in a magnetic field pointing into the page

(clockwise)



Rotating the coil about the vertical diameter by pulling the left side toward the reader and pushing the right side away from the reader in a magnetic field that points from right to left in the plane of the page.

(counterclockwise)

Faraday's Law $\quad \varepsilon = -N \frac{d\Phi_B}{dt}$

You can use Faraday's Law (as written above) to calculate the magnitude of the emf (or whatever the problem wants). Then use Lenz's Law to figure out the direction of the induced current (or the direction of whatever the problem wants).

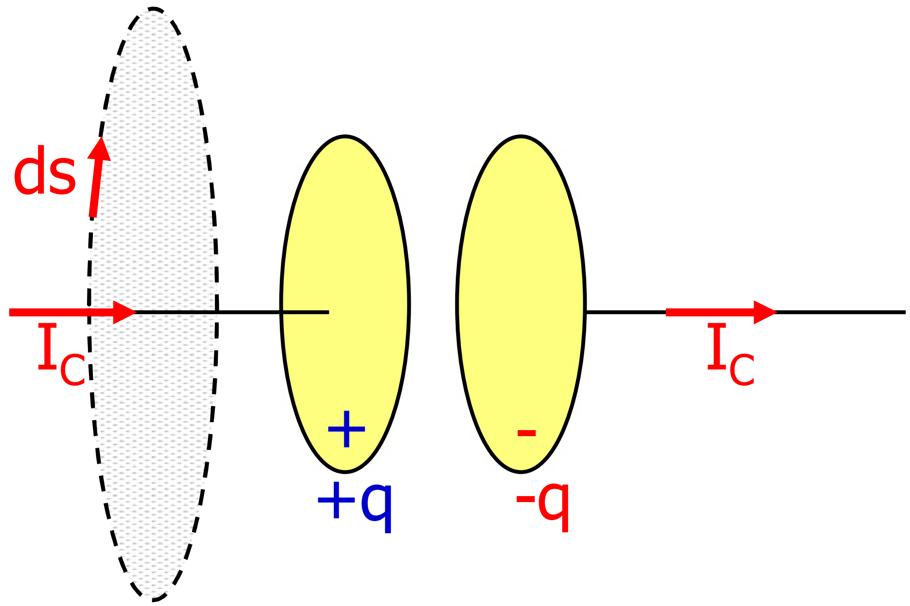
The direction of the induced emf is in the direction of the current that flows in response to the flux change. We usually ask you to calculate the magnitude of the induced emf ($|\varepsilon|$) and separately specify its direction.

Magnetic flux is not a vector. Like electrical current, it is a scalar. Just as we talk about current direction (even though it is not a vector), we often talk about flux direction (even though it is not a vector). Keep this in mind if your recitation instructor talks about the direction of magnetic flux.

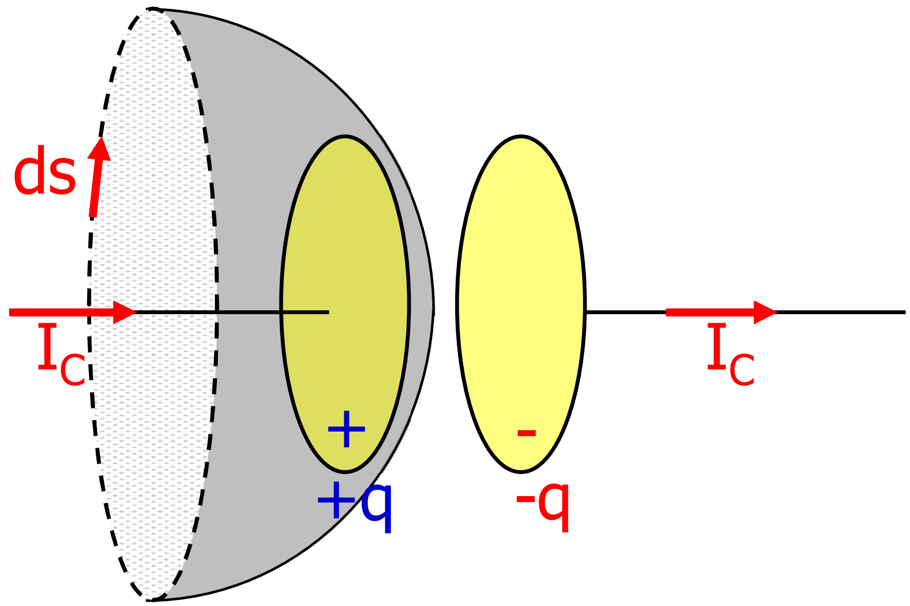
Displacement Current

Apply Ampere's Law to a charging capacitor.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_C$$



The shape of the surface used for Ampere's Law shouldn't matter, as long as the "path" is the same.



Imagine a soup bowl surface, with the + plate resting near the bottom of the bowl.

Apply Ampere's Law to the charging capacitor.

$$\oint \vec{B} \cdot d\vec{s} = 0$$

The integral is zero because no current passes through the "bowl."

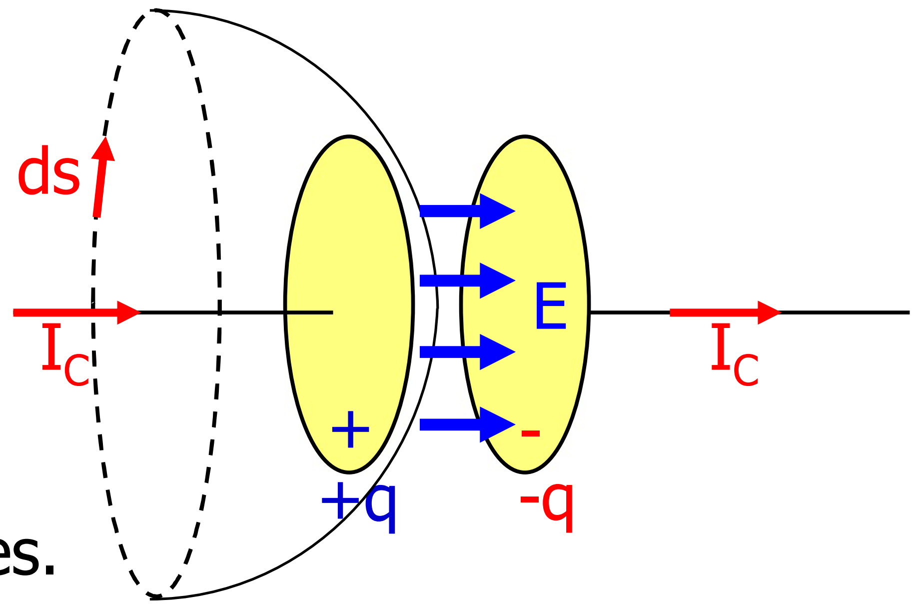
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_C \qquad \oint \vec{B} \cdot d\vec{s} = 0$$

Hold it right there pal! You can’t have it both ways. Which is it that you want? (The equation on the right is actually incorrect, and the equation on the left is incomplete.)

As the capacitor charges, the electric field between the plates changes.

$$q = C\Delta V = \left(\kappa\varepsilon_0 \frac{A}{d}\right)(Ed)$$

$$= \kappa\varepsilon_0 EA = \kappa\varepsilon_0 \Phi_E$$



As the charge and electric field change, the electric flux changes.

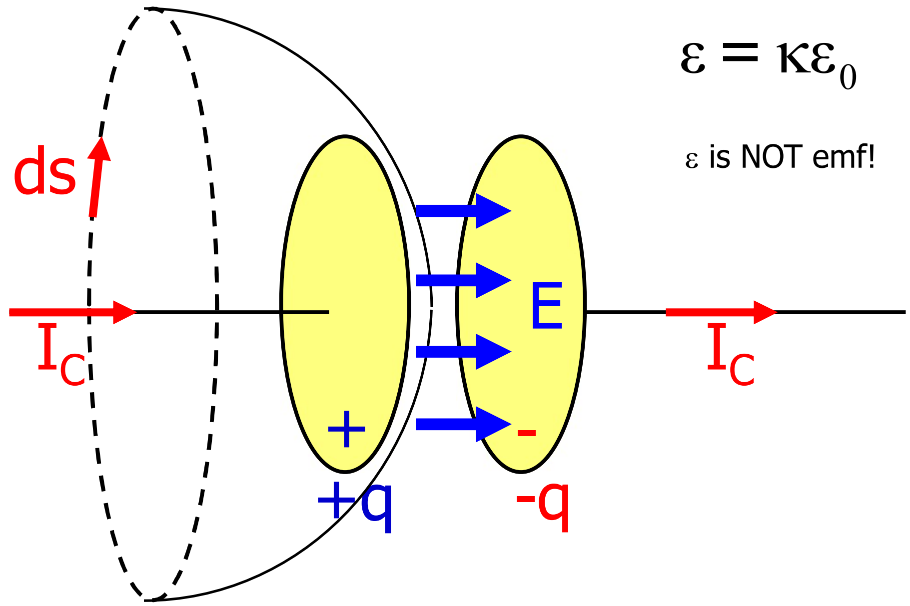
$$\frac{dq}{dt} = \frac{d}{dt}\left(\kappa\varepsilon_0 \Phi_E\right) = \underbrace{\kappa\varepsilon_0 \frac{d}{dt}\left(\Phi_E\right)}$$

This term has units of current.

κ Is the dielectric constant of the medium in between the capacitor plates. In the diagram, with an air-filled capacitor, κ = 1.

We define the displacement current to be

$$I_D = \kappa\varepsilon_0 \frac{d}{dt}\left(\Phi_E\right).$$



$\varepsilon = \kappa\varepsilon_0$

ε is NOT emf!

The changing electric flux through the "bowl" surface is equivalent to the current I_C through the flat surface.

The generalized ("always correct") form of Ampere's Law is then

$$\boxed{\oint \vec{B}\cdot d\vec{s}} = \mu_0\left(I_C + I_D\right)_{encl} = \boxed{\mu_0 I_{encl} + \mu_0\varepsilon\frac{d\Phi_E}{dt}}.$$

Magnetic fields are produced by both conduction currents and time varying electric fields.

The "stuff" inside the gray boxes serves as your official starting equation for the displacement current I_D.

$$\boxed{\oint \vec{B}\cdot d\vec{s}} = \mu_0\left(I_C + I_D\right)_{encl} = \boxed{\mu_0 I_{encl} + \mu_0 \varepsilon \underbrace{\frac{d\Phi_E}{dt}}_{I_D}}.$$

ε is the relative dielectric constant; not emf.
In a vacuum, replace ε with ε_0.

Why "displacement?" If you put an insulator in between the plates of the capacitor, the atoms of the insulator are "stretched" because the electric field makes the protons "want" to go one way and the electrons the other. The process of "stretching" the atom involves displacement of charge, and therefore a current.

Homework

Q 1. Proof displacement current: $$I_D = \varepsilon \frac{d\Phi_E}{dt}$$

where $\varepsilon = \kappa\varepsilon_0$ ε is the relative dielectric constant; not emf

Q. 2 Displacement current density is calculated the same as conventional current density $$J_D = \frac{I_D}{A}$$

Q. 3 Proof $I_c = I_d$

Maxwell Equation

You have now learned Gauss's Law for both electricity and magnetism, Faraday's Law of Induction,.and the generalized form of Ampere's Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

These equations can also be written in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{d\vec{E}}{dt} + \mu_0 \vec{J}$$

$$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$