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Assignment 1

Exercise 1

① $T(n) = \begin{cases} 1 & , \text{if } n = 1. \\ T(n-1) + \frac{1}{n} & , \text{if } n > 1. \end{cases}$

$$T(n) = T(n-1) + \frac{1}{n}$$

Substituting $T(n-1) = T(n-2) + \frac{1}{n-1}$

$$T(n) = T(n-2) + \frac{1}{n-1} + \frac{1}{n}$$

$$T(n) = T(n-3) + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n}$$

Performing substitution n time

$$T(n) = T(n-k) + \frac{1}{n-(k-1)} + \frac{1}{n-(k-2)} + \dots + \frac{1}{n}$$

$$T(1) = 1$$

$$n-k = 1 \Rightarrow k = n-1$$

$$T(n) = T(1) + \frac{1}{n-(n-2)} + \frac{1}{n-(n-3)} + \dots + \frac{1}{n}$$

$$T(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

This is a harmonic series.

Therefore, Time Complexity $= O(\log n)$.

② $T(n) = \begin{cases} 1 & , \text{if } n = 0 \\ T(n-2) + n^2 & , \text{if } n > 0. \end{cases}$

$$T(n) = T(n-2) + n^2.$$

$$T(n-2) = T(n-4) + (n-2)^2$$

$$T(n-4) = T(n-6) + (n-4)^2.$$

$$T(n) = T(n-4) + (n-2)^2 + n^2$$
$$= T(n-6) + (n-4)^2 + (n-2)^2 + n^2$$
$$= T(n-8) + (n-6)^2 + (n-4)^2 + (n-2)^2 + n^2$$

k times

$$= T(n-2k) + (n-2k+2)^2 + (n-2k+4)^2 + (n-2k+6)^2 + \cdots + n^2$$

$$n - 2k = 0 \Rightarrow k = \frac{n}{2}$$

$$T(n) = T\left(n - 2\cdot\frac{n}{2}\right) + \left(n - 2\cdot\frac{n}{2} + 2\right)^2 + \left(n - 2\cdot\frac{n}{2} + 4\right)^2 + \cdots + n^2$$

$$= T(0) + 2^2 + 4^2 + 6^2 + \cdots + n^2$$
$$= 1 + 4 + 16 + 36 + \cdots + n^2$$
$$= 1 + 4(1 + 4 + 9 + 16 + \cdots + n^2)$$
$$= 1 + 4(1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2)$$
$$= 1 + 4\sum_{k=1}^{n} k = 1 + 4\left(\frac{n(n+1)(2n+1)}{6}\right)$$
$$= \frac{2n^3 + n^2 + 2n^2 + n}{6}$$

Time Complexity $= O(n^3)$

(3) $T(n) = \begin{cases} 10, & \text{if } n = 0 \\ T(n-2) + \log n, & \text{if } n > 0 \end{cases}$

$$T(n) = T(n-2) + \log n$$
$$T(n-2) = T(n-4) + \log(n-2)$$
$$T(n) = T(n-4) + \log(n-2) + \log n$$
$$= T(n-6) + \log(n-4) + \log(n-2) + \log n$$

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$$T(n) = T(n-8) + \log(n-6) + \log(n-4) + \log(n-2) + \log n$$

k times

$$T(n) = T(n-2k) + \log(n-2k+2) + \log(n-2k+4) + \log(n-2k+6) + \dots + \log n$$

$$n - 2k = 0 \Rightarrow k = n/2$$

$$T(n) = T\left(n - 2.\frac{n}{2}\right) + \log\left(n - 2.\frac{n}{2} + 2\right) + \log\left(n - 2.\frac{n}{2} + 4\right) + \dots + \log n$$

$$T(n) = T(0) + \log 2 + \log 4 + \log 6 + \dots + \log n.$$

$$= 10 + \log(2.4.6 \dots n).$$

$$= 10 + \log(n+1)^n$$

Time Complexity) $= O(n)$.

Exercise 2

$$f(n) = n \quad \text{and} \quad g(n) = n^{(1+\sin n)}.$$

Statement 1

$$f(n) = O(g(n))$$

$$f(n) \leq c_1 . (g(n))$$

$$n \leq c_1 . n^{(1+\sin n)}.$$

$$\text{Or} \quad c_1 \geq n^{1-1-\sin n}.$$

$$c_1 \geq \frac{1}{n^{\sin n}} \rightarrow \text{Dependent on } n$$

$$f(n) \neq O(g(n))$$

Statement 2

$$f(n) = \Omega(g(n))$$

$$f(n) \geq C_2 \cdot (g(n))$$

$$n \geq C_2 \cdot n^{(1+\sin n)}$$

$$C_2 \leq \frac{1}{n^{\sin n}} \rightarrow \text{Dependent on } n$$

$$f(n) \neq \Omega(g(n))$$

None of the staments are correct.

Exercise 3

$f_1(n) = 2^n$, $f_2(n) = n^{\frac{3}{2}}$, $f_3(n) = n \log_2 n$,
$f_4(n) = n^{\log_2 n}$

$$n \log_2 n << n^{\frac{3}{2}} << n^{\log_2 n} << 2^n$$

$$f_3(n) << f_2(n) << f_4(n) << f_1(n)$$