EE2703: Applied Programming Lab
End Semester Examination

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Contents

1 Introduction

This assignment deals with finding the current distribution in a **half-wave dipole antenna**. We are given an antenna of half length l which is radiating at the wavelength of $4l$.
To tackle the problem, we use two important concepts from physics, namely **Ampere's Law** and **Magnetic Vector Potential**. We try to compute it numerically by representing the data in matrices and doing matrix algebra. Finally, we show the **estimated plot** with the **true plot** and explain any found discrepancies.
The main flow of the assignment is as follows:

- Split the antenna into segments of same size and store the location of each point in a vector, say z. By boundary condition, we know the currents at two end points and the middle of the wire, so we construct another vector for unknown current locations u.
- Next, make a diagonal matrix M obtained from **Ampere's Law** using a function.
- Then, from carrying out the **Vector Potential** computation, we get matrices Q and Q_B which when finally put into the relation $(M - Q)J = Q_B I_m$, would give us J vector, which is the unknown current vector. I current vector can be obtained by putting in the boundary condition values.
- Finally, plot this **estimated value** with the **true value** obtained from the actual relation and analyze the differences in the plot.

2 Problem Statements

2.1 Statement 1

We have to divide the antenna into equal number of segments and define currents at those segments. The boundary conditions which we have to take care in the case of currents are as follows;

$$I[-l/2] = 0 = I[l/2], I[center] = I_m$$

The pseudocode for the above algorithm is as follows:

Algorithm 1 **Defining points and currents on wire**

1: $z \leftarrow i \times dz$
2: $I \leftarrow$ Values of currents corresponding to z
3: $u \leftarrow$ Position for unknown currents
4: $J \leftarrow$ Values of currents corresponding to u

2.2 Statement 2

We need an equation for each unknown current. These equations are obtained by calculating the **Magnetic field** in two different ways. From Ampere's Law, we have for $H_\phi(z, r = a)$:

$$2\pi a H_\phi(z_i) = I_i$$

The matrix equation is as follows:

$$\begin{pmatrix} H_\phi[z_1] \\ ... \\ H_\phi[z_{N-1}] \\ H_\phi[z_{N+1}] \\ ... \\ H_\phi[z_{2N-1}] \end{pmatrix} = \frac{1}{2\pi a} \begin{pmatrix} 1 & ... & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & ... & 1 & 0 & ... & 0 \\ 0 & ... & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & ... & 0 & 0 & ... & 1 \end{pmatrix} \begin{pmatrix} J_1 \\ ... \\ J_{N-1} \\ J_{N+1} \\ ... \\ J_{2N-1} \end{pmatrix}$$

The pseudocode for the alogrithm is as follows:

Algorithm 2 **Computing M matrix from Ampere's Law**

1: $H_\phi \leftarrow$ Magnetic Field corresponding to z
2: $I \leftarrow$ Identity Matrix
3: $M \leftarrow \frac{1}{2\pi a} \times I$
4: $H_\phi \leftarrow M \times J$

The computed value of M vector is as follows:

$$\begin{pmatrix} 15.92 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.92 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15.92 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15.92 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15.92 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.92 \end{pmatrix}$$

2.3 Statement 3

The second computation involves the calculation of the vector potential

$$\vec{A}(r, z) = \frac{\mu_0}{4\pi} \int \frac{I(z')\, \hat{z} e^{-jkR} dz'}{R}$$

This can be reduced to the sum:

$$
\begin{aligned}
A_{z,i} &= \frac{\mu_0}{4\pi}\sum_j \frac{I_j \exp\left(-jkR_{ij}\right) dz'_j}{R_{ij}} \\
&= \sum_j I_j \left(\frac{\mu_0}{4\pi}\frac{\exp\left(-jkR_{ij}\right)}{R_{ij}} dz'_j\right) \\
&= \sum_j P_{ij} I_j + P_B I_N
\end{aligned}
$$

where P_B is the contribution to the vector potential due to current I_N, and is given by:

$$
P_B = \frac{\mu_0}{4\pi}\frac{\exp\left(-jkR_{iN}\right)}{R_{iN}} dz'_j
$$

We have to compute R_z and R_u which are distances from observer at $\vec{r}+z_i\hat{\boldsymbol{z}}$ and source $z'_j\hat{\boldsymbol{z}}$. The difference between R_z and R_u is that the former computes distances including distances to known currents, while Ru is a vector of distances to unknown currents.

Note that the value of k taken here is π as $k = \frac{\omega}{c}$ which comes out as π from the given ω and c

The pseudocode for the algorithm is as follows:

Algorithm 3 Solving for R and P matrices

1: $Z_j, Z_i \leftarrow$meshgrid(z,z)
2: $U_j, U_i \leftarrow$meshgrid(u,u)
3: $r \leftarrow$a
4: $R_z \leftarrow \sqrt{r^2 + (Z_i - Z_j)^2}$
5: $R_u \leftarrow \sqrt{r^2 + (U_i - U_j)^2}$
6: $P \leftarrow \frac{\mu_0}{4\pi}\frac{\exp(-jkR_{ij=})}{R_{ij}} dz'_j$
7: $P_B \leftarrow \frac{\mu_0}{4\pi}\frac{\exp(-jkR_{iN})}{R_{iN}} dz'_j$

The computed value of R_z is as follows:

$$
\begin{pmatrix}
0.01 & 0.13 & 0.25 & 0.38 & 0.5 & 0.63 & 0.75 & 0.88 & 1.00 \\
0.13 & 0.01 & 0.13 & 0.25 & 0.38 & 0.5 & 0.63 & 0.75 & 0.88 \\
0.25 & 0.13 & 0.01 & 0.13 & 0.25 & 0.38 & 0.5 & 0.63 & 0.75 \\
0.38 & 0.25 & 0.13 & 0.01 & 0.13 & 0.25 & 0.38 & 0.5 & 0.63 \\
0.5 & 0.38 & 0.25 & 0.13 & 0.01 & 0.13 & 0.25 & 0.38 & 0.5 \\
0.63 & 0.5 & 0.38 & 0.25 & 0.13 & 0.01 & 0.13 & 0.25 & 0.38 \\
0.75 & 0.63 & 0.5 & 0.38 & 0.25 & 0.13 & 0.01 & 0.13 & 0.25 \\
0.88 & 0.75 & 0.63 & 0.5 & 0.38 & 0.25 & 0.13 & 0.01 & 0.13 \\
1.00 & 0.88 & 0.75 & 0.63 & 0.5 & 0.38 & 0.25 & 0.13 & 0.01
\end{pmatrix}
$$

The computed value of R_u is as follows:

$$\begin{pmatrix} 0.01 & 0.13 & 0.25 & 0.5 & 0.63 & 0.75 \\ 0.13 & 0.01 & 0.13 & 0.38 & 0.5 & 0.63 \\ 0.25 & 0.13 & 0.01 & 0.25 & 0.38 & 0.5 \\ 0.5 & 0.38 & 0.25 & 0.01 & 0.13 & 0.25 \\ 0.63 & 0.5 & 0.38 & 0.13 & 0.01 & 0.13 \\ 0.75 & 0.63 & 0.5 & 0.25 & 0.13 & 0.01 \end{pmatrix}$$

The computed value of P is as follows:

$$\begin{pmatrix} 124.94-3.93j & 9.2-3.83j & 3.53-3.53j & -2.5j & -0.77-1.85j & -1.18-1.18j \\ 9.2-3.83j & 124.94-3.93j & 9.2-3.83j & 1.27-3.08j & -2.5j & -0.77-1.85j \\ 3.53-3.53j & 9.2-3.83j & 124.94-3.93j & 3.53-3.53j & 1.27-3.08j & -2.5j \\ -2.5j & 1.27-3.08j & 3.53-3.53j & 124.94-3.93j & 9.2-3.83j & 3.53-3.53j \\ -0.77-1.85j & -2.5j & 1.27-3.08j & 9.2-3.83j & 124.94-3.93j & 9.2-3.83j \\ -1.18-1.18j & -0.77-1.85j & -2.5j & 3.53-3.53j & 9.2-3.83j & 124.94-3.93j \end{pmatrix}$$

The computed value of P_b is as follows:

$$\begin{pmatrix} 1.27-3.08j & 3.53-3.53j & 9.2-3.83j & 9.2-3.83j & 3.53-3.53j & 1.27-3.08j \end{pmatrix}$$

2.4 Statement 4

Now, we only want the ϕ component of $\vec{H}$ and $\vec{A}$ only has the z component.
So the equation becomes:

$$H_\phi(r,z) = -\frac{1}{\mu}\frac{\partial A_z}{\partial r} = -\sum_j \frac{\mu_0}{4\pi}\frac{dz'_j}{\mu}\frac{\partial}{\partial r}\left(\frac{\exp\left(-jkR_{ij}\right)}{R_{ij}}\right) I_j$$

Note that all the currents contribute to H_ϕ, and so the current vector is I.
Now, $\vec{R} = r\hat{r} + (z-z')\,\hat{z}$. So, $R = \sqrt{r^2+(z-z')^2}$. The derivative becomes:

$$\frac{\partial}{\partial r}R_{ij} = \frac{1}{2R_{ij}}2r = \frac{r}{R_{ij}}$$

So,

$$\begin{aligned} H_\phi\left(r,z_i\right) &= -\sum_j \frac{dz'_j}{4\pi}\left(\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^2}\right)\exp\left(-jkR_{ij}\right)\frac{rI_j}{R_{ij}} \\ &= -\sum_j P_{ij}\frac{r}{\mu_0}\left(\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^2}\right) I_j + P_B\frac{r}{\mu_0}\left(\frac{-jk}{R_{iN}} - \frac{1}{R_{iN}^2}\right) I_m \\ &= \sum_j Q'_{ij}I_j \end{aligned}$$

We now have a second expression for $H_\phi(r, z_i)$:

$$\begin{aligned} H_\phi(r, z_i) &= \sum_j Q'_{ij} I_j \\ &= \sum_j Q_{ij} J_j + Q_{Bi} I_m \end{aligned}$$

The Q'_{ij} in the equation is over all the currents. However this needs to be split into the unknown currents, J_j and the boundary currents. Only one of the boundary currents is non-zero, namely the feed current at $i = N$. The matrix corresponding to J_j we call Q_{ij}, and corresponding to boundary current we call $Q_B = Q'_{iN}$
The pseudocode for the algorithm is as follows:

Algorithm 4 Solving for Q and Q$_B$

1: $Q \leftarrow -P_{ij} \frac{r}{\mu_0} \left(\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^2} \right)$
2: $Q_B \leftarrow -P_B \frac{r}{\mu_0} \left(\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^2} \right)$

The computed value of Q is as follows:

$$\begin{bmatrix} 99.5 & 0.05 & 0 & 0 & 0 & 0 \\ 0.05 & 99.5 & 0.05 & 0 & 0 & 0 \\ 0 & 0.05 & 99.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 99.5 & 0.05 & 0 \\ 0 & 0 & 0 & 0.05 & 99.5 & 0.05 \\ 0 & 0 & 0 & 0 & 0.05 & 99.5 \end{bmatrix}$$

The computed value of Q_b is as follows:

$$\begin{pmatrix} 0 & 0 & 0.05 & 0.05 & 0 & 0 \end{pmatrix}$$

2.5 Statement 5

Our final equation is

$$\begin{aligned} MJ &= QJ + Q_B I_m \\ (M - Q)J &= Q_B I_m \end{aligned}$$

This can be easily inverted to obtain J and I. The pseudocode for the above algorith is as follows:

Algorithm 5 Solving for J and I_m

1: $MJ \leftarrow QJ + Q_B I_m$
2: $(M - Q)J \leftarrow Q_B I_m$
3: $J \leftarrow (M - Q)^{-1} Q_B$
4: $I \leftarrow$J including the boundary conditions

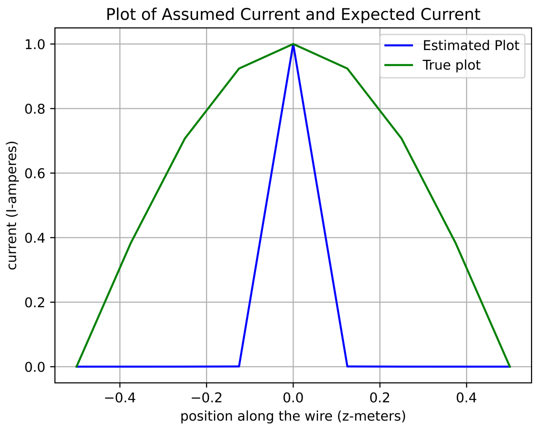
The computed value of J (without rounding off) is as follows:

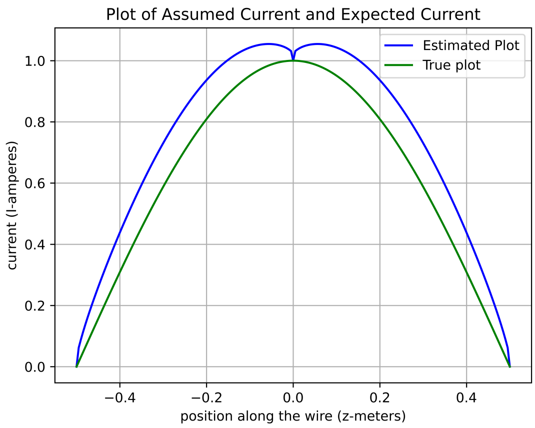
$$J = \begin{pmatrix} -3.30e-05+1.064e-05j \\ -9.55e-05+1.15e-05j \\ -6.48e-04+1.21e-05j \\ -6.48e-04+1.21e-05j \\ -9.55e-05+1.15e-05j \\ -3.30e-05+1.06e-05j \end{pmatrix} \tag{1}$$

The computed value of I (without rounding off) is as follows:

$$I = \begin{pmatrix} 0.00+0.00j \\ -3.30e-05+1.064e-05j \\ -9.55e-05+1.15e-05j \\ -6.48e-04+1.21e-05j \\ 1.00+0.00j \\ -6.48e-04+1.21e-05j \\ -9.55e-05+1.15e-05j \\ -3.30e-05+1.06e-05j \\ 0.00+0.00j \end{pmatrix} \tag{2}$$

The plot (for N = 4) former and (N = 100) latter is as follows:





3 Conclusion

Based on the data and the plots seen above, we can make following conclusions:

- When we keep N value quite small, we observe very high deviation in the estimated and true (assumed) plot as depicted in plot 1.
- But as we keep on increasing N, the plot becomes smoother and tries to converge with the true plot.
- The reason for this is that we have converted the integral into summation in the **Magnetic Vector Potential** computation, because of which we have deviation in the expected plots.
- Hence, at lower values of N, the algorithm's estimation is not good.