A THEORETICAL APPROACH FOR STIFFNESS INDEXING OF AN INDUSTRIAL ROBOT AND IMPLEMENTATION OF INTELLIGENT SYSTEM

INTELLIGENT SYSTEMS
DIGITAL ASSIGNMENT

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TABLE OF CONTENTS

EXECUTIVE SUMMARY

For machining applications, industrial robots offer a flexible, affordable, and promising substitute. The behavior of the system differs significantly from that of a traditional machine tool because of the kinematics of a vertical articulated robot. With an emphasis on the examination of the system's stiffness and behavior during any operation, this article explains the modeling of the robot structure and the determination of its parameters. As a result, a technique for calculating Cartesian stiffness based on stiffness and the Jacobian matrix is presented. The robot's machining performance is assessed and conclusions are made in light of the identification and experimental validation results.

Keywords: *Robotics, Jacobian Matrix, Robot Stiffness, Fuzzy Inference System.*

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

α	Alpha(Rotation about x axis)
β	Beta(Rotation about z axis)
θ	Joint Angles
L	Link Lengths
[J]	Jacobian Matrix
ω	Rotational velocity
μ	Membership function
∂D	Deformation
K	Stiffness

CHAPTER 1

INTRODUCTION

1.1 Introduction and technical specifications of UR5 robot

The ability of an industrial manipulator to withstand deformation in the face of external forces is known as its stiffness. In numerous applications where precision, accuracy and dependability are critical, it is imperative. It may be impacted by a variety of elements including the robot's control algorithms, design, construction and materials. Pick and place/palletizing operations, material handling, machining, wielding and other critical areas are where stiffness is crucial. Robot with increased stiffness are useful in sectors such as aerospace, automotive and electronics because they help guarantee precise placement of sensors and testing apparatus. Vibration from low stiffness during robot assisted machining degrades the surface quality and precision of the operation. As for example, Pan et al. [2,3] discovered that due to the inadequate stiffness, there was coupling chatter during the robot milling operation in both directions, without having the in depth analysis of the stability conditions. The presence of vibration and impact processes between the tool and workpiece in a robot grinding system was confirmed Hazel et al. [4, 5] using high speed camera observations. The stable impact cutting material removal boundary was obtained by characterizing the dynamic characteristics of regenerative vibration and impact, but the vibration was not suppressed at the root.

It is clear from the research mentioned above that poor stiffness is a common issue with robot based milling systems. Regarding the polishing procedures used in this study, the robot's low rigidity renders it prone to load deformation. This leads to a certain amount of spatial error in the tool center location, which ultimately compromises the polishing quality.

Studies on the stiffness evaluation and optimization of robot based machining systems have garnered a lot of attention in an effort to address the stiffness issue with these systems.

A robots' joint and link flexibility determines its overall flexibility. The main components of the robot manipulator are a motor, controller, mechanical transmission, sensors and additional devices. Electrical energy is converted to mechanical energy by the electric motor. By means of mechanical transmission, a rotor provides mechanical energy to the mechanism of operating elements. An electrical device's rotation drives the actuators movement.

The motion and force characteristics during polishing processes differ from those of drilling and grinding robot systems in the previous research, despite the fact that the assessment model and optimization of the stiffness of robot based machining systems have been extensively explored. As a result, it is important to research an assessment technique that concentrates on the robot polishing systems' operational stiffness. An experimental and theoretical investigation is given and arranged as follows for this goal. This study deals only with UR5 industrial robot. The technical specification of the UR5 robot are provided in table below.

Fig. 1.1 UR5 Industrial Robot

Table 1.1. Technical Specifications of UR5 Robot

Weight	18.4 kg / 40.6 lbs
Payload	5 kg / 11 lbs
Reach	850 mm / 33.5 in
Joint Ranges	+/- 360°
Speed	All joints 180°/s ; Tool 1m/s or 39.4 in/s
Repeatability	+/- 0.1mm or +/- 0.0039 in
Foot Print	0149 mm / 5.9 in
Degrees of freedom	6 rotating joints
I/O Port	Digital IN , Digital OUT , Analog IN , Analog OUT
I/O Power Supply	24V 2A in control box and 12V/24V 600mA in tool
Communication	TCP/IP 100 Mbit : IEEE802.3u, 100BASE-TX Ethernet socket and Modbus TCP

The UR5 robot, manufactured by Universal Robots, has found numerous industrial applications due to its versatility, compact design, and ease of integration into various workflows. In manufacturing, the UR5 is commonly utilized for tasks such as material handling, pick-and-place operations, assembly, and machine tending. For instance, in automotive manufacturing, the UR5 is employed for loading and unloading parts from CNC machines, quality inspection, and small parts assembly. In electronics manufacturing, it assists with PCB handling, soldering, and component placement. The UR5's collaborative features allow it to work safely alongside human operators, facilitating applications in industries like food and beverage, where it can be used for packaging, palletizing, and machine loading. Additionally, the UR5's flexibility makes it well-suited for research and development tasks, such as laboratory automation, prototyping, and testing. Its intuitive programming interface enables rapid deployment and reprogramming for new tasks, making it an ideal solution for industries requiring agile and adaptable automation solutions. Overall, the UR5 robot has proven to be a valuable asset across a wide range of industrial sectors, enhancing productivity, efficiency, and workplace safety.

1.2 IMPORTANCE OF STIFFNESS OF AN INDUSTRIAL ROBOT

In the context of industrial robots, "stiffness" refers to the rigidity or resistance to deformation of the robot's mechanical structure. It measures how much the robot's structure deflects or bends under applied loads or forces. Higher stiffness means less deformation, which is desirable for maintaining accuracy and precision in the robot's movements.

Stability, on the other hand, refers to the ability of the robot to maintain balance and control its motion while performing tasks. Stability is crucial for preventing the robot from tipping over or losing control, especially when handling heavy payloads or operating at high speeds.

Both stiffness and stability are essential characteristics for industrial robots because they directly impact the robot's performance, accuracy, and safety:

1. Stiffness: A stiff robot structure ensures minimal deflection or distortion during movements, which is crucial for precise positioning and manipulation of objects. It helps maintain the accuracy and repeatability of the robot's movements, resulting in consistent performance over time.

2. Stability: Stability is critical for preventing accidents and ensuring safe operation of industrial robots. A stable robot can handle varying loads and external disturbances without tipping over or losing control. It allows the robot to operate reliably in dynamic environments and reduces the risk of damage to equipment or injury to workers.

In summary, stiffness and stability are both fundamental characteristics of industrial robots, and optimizing these factors is essential for achieving high performance, accuracy, and safety in various manufacturing and automation applications.

$\therefore$ Stiffness is crucial for industrial robots primarily because it ensures precision, accuracy, and repeatability in their movements. Here's why stiffness matters:

1. Accuracy: Industrial robots often perform tasks that require precise positioning and manipulation, such as assembly, welding, or painting. A stiff robot structure minimizes deflection and distortion during movements, leading to accurate positioning of the end effector (the tool or device attached to the robot's arm).

2. Repeatability: Repeatability refers to the ability of a robot to consistently perform the same task with high precision. A stiff robot maintains its structural integrity even under varying loads and external disturbances, resulting in consistent performance over time.

3. Dynamic Response: Stiffness contributes to a robot's dynamic response, enabling it to quickly react to changes in its environment or task requirements. This is especially important in applications where speed and agility are necessary, such as pick-and-place operations in manufacturing.

4. Energy Efficiency: Stiff robots typically require less energy to maintain their position and perform tasks compared to less stiff counterparts. This is because they experience less energy loss due to structural flexing or vibration damping.

5. Safety: In industrial environments, safety is paramount. A stiff robot is less likely to experience unexpected movements or vibrations that could pose safety hazards to nearby workers or equipment.

6. Endurance: Stiffness also contributes to the overall durability and longevity of the robot. By minimizing structural fatigue and wear, stiff robots can operate reliably for extended periods without significant degradation in performance.

Overall, stiffness enhances the performance, reliability, and safety of industrial robots, making them more suitable for a wide range of manufacturing and automation tasks.

$\therefore$ The stiffness of the robot is crucial for the following;

1. High-precision machining: In applications such as milling, grinding, or machining, where tight tolerances and surface finishes are essential, the stiffness of the robot affects its ability to maintain stable tool positioning and trajectory tracking. High stiffness minimizes vibrations and deflections, ensuring consistent

material removal rates and surface quality.

2. Assembly and manipulation: During assembly tasks, robots must accurately position and manipulate components with varying shapes and sizes. Stiffness is crucial for maintaining precise contact forces and positioning accuracy, especially when handling delicate or fragile parts. Higher stiffness enables robots to resist external disturbances and maintain tight tolerances during assembly operations.

3. Material handling and palletizing: In logistics and warehousing applications, robots are often tasked with lifting, transporting, and stacking heavy loads or pallets. Stiffness is critical for ensuring the stability and safety of the handling process, preventing excessive sway or deflection that could lead to load instability or collisions with surrounding objects.

4. Welding and joining: In welding operations, robots must accurately control the position and orientation of the welding torch relative to the workpiece. Stiffness is essential for maintaining precise torch alignment and contact with the joint, ensuring consistent weld quality and penetration. High stiffness also helps to minimize distortion and deformation of the workpiece during welding.

5. Inspection and quality control: In applications where robots are used for inspection, metrology, or quality control, stiffness is crucial for maintaining accurate sensor positioning and measurement accuracy. Robots with high stiffness can maintain stable sensor-to-surface contact, enabling precise measurements and detection of defects or deviations from specifications.

6. Human-robot collaboration: In collaborative robotics, where robots work alongside human operators in shared workspaces, stiffness is essential for ensuring safe and reliable interaction. Stiffness control mechanisms allow robots to adjust their compliance and response to external forces, ensuring smooth and safe collaboration with humans while maintaining positional accuracy and stability.

In summary, stiffness is critical for a wide range of industrial robot operations, including machining, assembly, material handling, welding, inspection, and collaboration. High stiffness enables robots to achieve precise motion control, maintain stable positioning, and resist external disturbances, ultimately ensuring efficient and reliable performance in diverse manufacturing and automation applications.

1.3 JACOBIAN AND ANGULAR VELOCITY

In robotics, Jacobians are mathematical matrices that describe the relationship between joint velocities and end effector velocities in a robotic system. They are used to understand how changes in joint positions affect the position and orientation of the robot's end effector / gripper / tool. Jacobians are essential for controlling the robot's movement accurately, enabling tasks like trajectory planning, obstacle avoidance and overall optimization of motion. Understanding the Jacobian allows engineers to design efficient and safe robotic systems by predicting how changes in joint velocities affect the end effector's motion.

The Jacobian matrix is a fundamental tool in robotics, playing a crucial role in various aspects of robot control, motion planning, and analysis. In essence, it represents the relationship between the joint velocities and the end-effector velocities of a robotic manipulator. Understanding the importance of the Jacobian matrix requires delving into its applications and implications within the realm of robotics.

First and foremost, the Jacobian matrix enables the mapping of joint velocities to end-effector velocities, providing valuable insight into how changes in the robot's joints affect its overall motion in the workspace. This relationship forms the basis for inverse kinematics, a fundamental problem in robotics where the goal is to determine the joint configurations required to achieve a desired end-effector pose. By utilizing the Jacobian matrix, robotic systems can compute the necessary joint velocities to drive the end-effector towards a specified target, allowing for precise and efficient control of the robot's motion.

Furthermore, the Jacobian matrix is indispensable in trajectory planning and optimization. Given a desired end-effector trajectory, the Jacobian matrix can be employed to compute the required joint velocities along the trajectory, ensuring smooth and coordinated motion of the robot. Moreover, in tasks such as collision avoidance or obstacle navigation, the Jacobian matrix aids in adjusting the robot's trajectory by modulating the joint velocities while avoiding potential collisions or disturbances in the environment. This capability is particularly valuable in industrial settings where robots operate alongside human workers or other machinery, necessitating safe and adaptive motion planning strategies.

Additionally, the Jacobian matrix plays a crucial role in robot dynamics and control. By incorporating the Jacobian matrix into the equations of motion for the robotic system, engineers can develop sophisticated control algorithms that account for both the kinematic and dynamic characteristics of the robot. This integration facilitates advanced control techniques such as impedance control, where the robot's response to external forces or interactions is modulated based on the Jacobian matrix and the desired end-effector behavior. Consequently, robots equipped with such control schemes can exhibit compliant and adaptive behavior, making them well-suited for tasks requiring interaction with uncertain or dynamic environments.Moreover, the Jacobian matrix enables the analysis of robot manipulability and singularity conditions. Manipulability refers to the ability of a robot to move its end-effector in different directions with varying velocities, while singularity denotes configurations where the Jacobian matrix becomes rank-deficient, resulting in a loss of motion capabilities. Understanding these concepts allows engineers to design robotic systems with optimal kinematic performance and to identify and avoid singularities that may arise during operation. Consequently, the Jacobian matrix serves as a valuable tool for assessing the dexterity and reliability of robotic manipulators across a wide range of tasks and configurations.

In conclusion, the Jacobian matrix stands as a cornerstone of robotics, facilitating essential functionalities such as inverse kinematics, trajectory planning, dynamic control, and manipulability analysis. Its significance lies in its ability to establish the relationship between joint velocities and end-effector motion, enabling precise control, adaptive behavior, and robust performance of robotic systems in diverse applications. As robotics continues to advance, the Jacobian matrix will undoubtedly remain a fundamental tool for engineers and researchers seeking to develop intelligent, agile, and efficient robotic systems.The stiffness of the manipulator is related to Jacobian, for that a through explanation of the Jacobian of a Robot is explained below.

To find the Jacobian of a 6 degree of freedom robot, we can follow these steps mentioned below;

1. Demonstrate the forward kinematics equations to express the position and orientation of the end effector in terms of the robot's joint variables. Let us denote the end effector position as P = $[x\ y\ z]^T$ and the orientation as rotation matrix R.

   $\therefore$ The transformation matrix is denotes as T = $\begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$ and is 4x4 matrix.

2. Differentiate the forward kinematics equations with respect to each joint variable to obtain the linear and angular velocities of the end effector. These velocities are expressed in terms of joint velocities.

3. Combine these velocities into a single vector for the end effector twist ;

V $=\begin{bmatrix} \boldsymbol{V\ linear} \\ \boldsymbol{Vangular} \end{bmatrix}$, where V_{linear} = linear velocity of the end effector and $V_{angular}$ = angular velocity of the end effector.

4. Then we have to define the joint velocities vector,

   $\mathbf{Q = [q_1, q_2, q_3, q_4, q_5, q_6]^T}$

5. Now we can use the chain rule to relate the joint velocities to the end effector's twist as; **V = J.q ,** where **J** = jacobian matrix;

6. Now we need to solve for the jacobian matrix J by rearranging the equation as follows;

$$\mathbf{J = \partial V / \partial q}$$

Each column of the jacobian matrix corresponds to the partial derivative of the end effector's twist with respect to each joint variable. The linear part of the jacobian represents the linear velocity contributions from each joint, while the angular part represents the rotational velocity contributions.

1.4 Procedure for finding the [J] of a 2 link planar robot

The procedure of finding the jacobian matrix [J] of a two link planar robot is explained in this section below.

Let us consider a two link planar robot with link lengths L1 and L2 and joint variables θ1 and θ2. We need to find the transformation matrices for each and every frame to get the position vectors. The four DH parameters are $\boldsymbol{\alpha}_{\mathbf{i-1}}$ = angle between two Z axis, $\boldsymbol{\theta}_{\mathbf{i}}$ = angle between two X axis, $\mathbf{a}_{\mathbf{i-1}}$= distance between two consecutive Z axes and $\mathbf{d}_{\mathbf{i}}$= distance between two X axis . The dh parameters are provided in table 1.1.

Tab. 1.2 DH Parameter Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
0-1	0	0	0	θ_1
1-2	0	L_1	0	θ_2
2-3	0	L_2	0	0

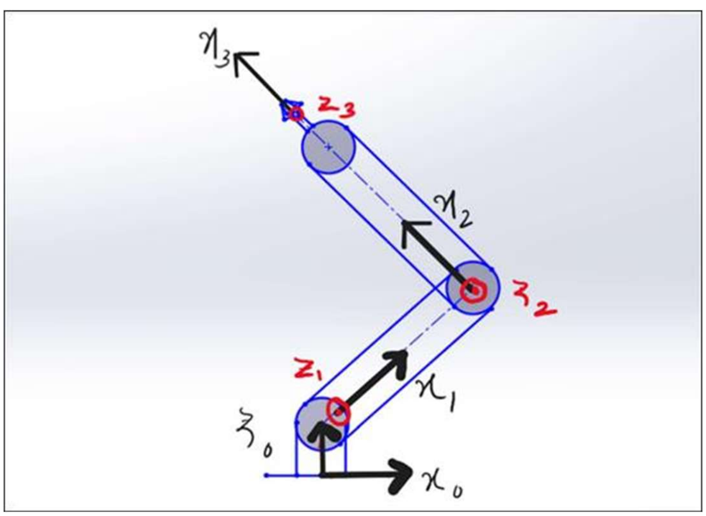


Fig.1.2 DOF Planar Robot

$\therefore$ The transformation matrix from frame {0} to {1} is given by ;

$${}^{0}T_1 = \begin{bmatrix} C1 & -S1 & 0 & 0 \\ S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; C1 = \cos\theta_1,\; C2 = \cos\theta_2,\; S1 = \sin\theta_1,$$

$S2 = \sin\theta_2$;

$\therefore$ Similarly the transformation matrix from {1} to {2}, {2} to {3} are as follows;

$${}^{1}T_2 = \begin{bmatrix} C2 & -S2 & 0 & L1 \\ S2 & C2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; {}^{2}T_3 = \begin{bmatrix} 1 & 0 & 0 & L2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Now the transformation matrix from {0} to {3} is given by;

$${}^{0}T_3 = {}^{0}T_1 \,.\, {}^{1}T_2 \,.\, {}^{2}T_3$$

$$= \begin{bmatrix} C12 & -S12 & 0 & L1C1 + L2C12 \\ S12 & C12 & 0 & L1S1 + L2S12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Now we will find out the velocity matrices of each frame. The two components of velocity are represented as W = angular velocity and V = linear velocity.

$\therefore$ For frame {0}; ${}^{0}W_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and ${}^{0}V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$;

$\therefore$ For frame {1}; ${}^{1}W_1 = {}^{1}R_0 \,.\, {}^{0}W_0 + \theta_1.{}^{1}Z_1$

$$= \begin{bmatrix} 0 \\ 0 \\ d(\theta 1) \end{bmatrix}, \text{ and}$$

$${}^{1}V_1 = {}^{1}R_0({}^{0}V_0 + {}^{0}W_0 \,.\, {}^{0}P_1)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ For frame {2}; ${}^{2}W_2 = {}^{2}R_1 \,.\, {}^{1}W_1 + \theta_2.{}^{2}Z_2$

$$= \begin{bmatrix} 0 \\ 0 \\ d(\theta 1) + d(\theta 2) \end{bmatrix}, \text{ and}$$

$$^{2}V_{2} = {}^{2}R_{1}({}^{1}V_{1} + {}^{1}W_{1}\ .\ {}^{1}P_{2})$$

$$= \begin{bmatrix} L1.S2.d(\theta 1) \\ L1.C2.d(\theta 2) \\ 0 \end{bmatrix}$$

$\therefore$ For frame {3}; $^{3}W_{3} = {}^{3}R_{2}\ .{}^{2}W_{2} + 0.{}^{3}Z_{3}$

$$= \begin{bmatrix} 0 \\ 0 \\ d(\theta 1) + d(\theta 2) \end{bmatrix}, \text{ and}$$

$$^{3}V_{3} = {}^{3}R_{2}({}^{2}V_{2} + {}^{2}W_{2}\ .\ {}^{2}P_{2})$$

$$= \begin{bmatrix} L1.S2.d(\theta 1) \\ L1.C2.d(\theta 1) + L2[d(\theta 1) + d(\theta 2)] \\ 0 \end{bmatrix}$$

$\therefore$ Rearranging the velocities of {3} to find the jacobian;

$$3W3 = \begin{bmatrix} Wx \\ Wy \\ Wz \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d(\theta 1) + d(\theta 2) \end{bmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{vmatrix} \begin{bmatrix} d(\theta 1) \\ d(\theta 2) \end{bmatrix}$$

$$= [J_{angular}] \begin{bmatrix} d(\theta 1) \\ d(\theta 1) \end{bmatrix}, \text{ and}$$

$$3V3 = \begin{bmatrix} Vx \\ Vy \\ Vz \end{bmatrix} = \begin{bmatrix} L1.S2.d(\theta 1) \\ L1.C2 + L2(\theta 1 + \theta 2) \\ 0 \end{bmatrix} = \begin{bmatrix} L1.S2 & 0 \\ L1.C2 + L2 & L2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d(\theta 1) \\ d(\theta 2) \end{bmatrix}$$

$$= [J_{linear}] \begin{bmatrix} d(\theta 1) \\ d(\theta 1) \end{bmatrix}$$

$\therefore$ The final jacobian matrix [J] will be;

$$[J] = \begin{bmatrix} 3W3 \\ 3V3 \end{bmatrix}$$

$$= \begin{bmatrix} L1.S1 & 0 \\ L1.C2 + L2 & L2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} d(\theta 1) \\ d(\theta 1) \end{bmatrix}$$

$\therefore$ By following the same method, we can also find the [J] of 6DOF industrial robot.

1.5 Literature Review

In an effort to increase the application of industrial robots, a great deal of research has been done on robot-assisted polishing systems. One of the main issues that needs to be resolved for any robotic system right now is low working stiffness. An evaluation technique for the working stiffness of robot needs to be investigated in order to resolve this problem. Initially, the RABPS static stiffness model was constructed using the compliance matrix and the Denavit–Hartenberg (D–H) method. The robot's force characteristics during the polishing procedure were then examined. Ultimately, simulation and experimentation were used to validate the NSC of the RABPS's performance. This work offers a crucial theoretical framework for future research on optimizing the working stiffness of the robot.

Studies on the stiffness evaluation and optimization of robot-based machining systems have garnered a lot of attention in an effort to address the stiffness issue with these systems. The majority of the literature currently in publication on robot stiffness focuses on three main areas of research: identifying the robot's joint stiffness, modeling the manipulator's stiffness, and analyzing the robot's overall stiffness. The stiffness of a robot is directly related to its postures, according to stiffness system theory based on robotics, which has been extensively studied to date. However, the majority of the studies have focused on the theoretical derivation of robotic systems . This has led to the application of stiffness theory to the study of practical machining, with some encouraging findings.

The most commonly observed primary static and dynamic performance indicators are force transmission, stiffness, and power consumption. These measurements are widely used and are very important for robots that work in dynamic environments. In this context, three frequently used markers are resolution, accuracy, and repeatability.
The stiffness of industrial robots is a critical factor influencing their performance, accuracy, and reliability in various manufacturing and automation applications. This literature review aims to provide an overview of research efforts and advancements in understanding and enhancing the stiffness characteristics of industrial robots.

Historically, early studies focused on analytical and experimental methods to quantify and improve the stiffness of robotic manipulators. Researchers investigated the structural design, material properties, and joint configurations to minimize deflections and increase rigidity in robot arms. These efforts led to the development of analytical models and simulation techniques for predicting the static and dynamic stiffness of robotic systems, providing valuable insights into the factors affecting their mechanical behavior.

As technology progressed, the advent of finite element analysis (FEA) revolutionized the study of robot stiffness by enabling detailed simulations of complex structural interactions. FEA allowed researchers to explore the effects of various factors, such as link geometry, material properties, and joint clearances, on the overall stiffness of robotic manipulators. Additionally, advances in sensor technology facilitated experimental validation of FEA predictions, leading to more accurate and reliable stiffness assessments in real-world conditions.

In recent years, with the growing demand for high-precision and collaborative robotics, there has been a renewed focus on enhancing the stiffness characteristics of industrial robots. One emerging approach involves the integration of advanced materials, such as carbon fiber composites and metal alloys, into robot structures to achieve lightweight yet stiff designs. These materials offer superior strength-to-weight ratios and damping properties, resulting in enhanced stiffness and reduced vibration in robotic systems. Furthermore, researchers have explored innovative control strategies and adaptive algorithms to actively regulate the stiffness of robots during operation. By dynamically adjusting joint stiffness parameters based on task requirements and environmental conditions, robotic systems can optimize performance, energy efficiency, and safety. Such adaptive stiffness control methods have shown promising results in applications ranging from high-speed machining to human-robot collaboration, where precise force modulation and compliance are essential. Moreover, advancements in additive manufacturing techniques have enabled the design and fabrication of customized robot components with tailored stiffness properties. By leveraging additive manufacturing technologies such as 3D printing, researchers can create intricate geometries and microstructures optimized for specific stiffness requirements, opening new avenues for lightweight, compact, and high-performance robotic designs.

In conclusion, the stiffness of industrial robots plays a crucial role in determining their functionality, accuracy, and reliability across diverse manufacturing and automation tasks. Through interdisciplinary research efforts spanning mechanical engineering, materials science, control theory, and robotics, significant progress has been made in understanding and enhancing the stiffness characteristics of robotic systems. Future directions in this field may involve further integration of advanced materials, control strategies, and manufacturing techniques to develop next-generation robots with unprecedented stiffness, agility, and versatility.

i. Title: Pan, Ri, et al. Research on an evaluation model for the working stiffness of a robot-assisted bonnet polishing system.

Journal and year: Journal of Manufacturing Process, 2021.

Author: Ri et al.

Description: In an effort to increase the application of bonnet polishing technology, a great deal of research has been done on robot-assisted bonnet polishing systems, or RABPSs. One of the main issues that needs to be resolved for RABPSs right now is low working stiffness. An evaluation technique for the working stiffness of RABPSs needs to be investigated in order to resolve this problem. Thus, this paper presents a theoretical and experimental study. Initially, the Denavit–Hartenberg (D–H) approach and the compliance matrix served as the foundation for the construction of the RABPS static stiffness model.
The RABPS's force characteristics during the bonnet polishing procedure were then examined. Subsequently, a new index known as the normal stiffness coefficient (NSC) was proposed to assess the working stiffness of the RABPS by fusing the previously mentioned static stiffness model with the findings of the force analysis. Ultimately, simulation and experimentation were used to validate the NSC of the RABPS's performance. According to these two findings, the NSC is a more accurate

measure of the working stiffness of RABPSs than other widely used indices. This work offers a crucial theoretical framework for future research on optimizing the working stiffness of RABPSs.

ii. Title: Mathematical model and evaluation of dynamic stability of industrial robot manipulator: Universal robot.

Journal: Systems and soft computing, 2024.

Author: Claire et al.

Description: With so many industrial uses, robots are an essential technological advancement in the context of the Industrial 4.0 revolution. The overall quality of a robot is affected by the stability of its manipulator, which is dependent on the associated manipulator settings. Previous research has only looked at a small number of manipulator parameters when evaluating stability, which has resulted in a poor understanding of this phenomenon. Using Lagrangian mechanics and a range of parameters, a mathematical model of a flexible six-link manipulator is developed in this work. For the Universal Robot (UR5), a novel mathematical framework is created to establish a relationship between stability and the motor acceleration and moment of inertia. Moreover, a great deal of research has been done to look at the relationship between deflection, damping, and stiffness in the context of stability. Nevertheless, in order to determine the relative importance of stiffness, damping, and deflection in relation to stability, fuzzy logic inference techniques are used. Several manipulator parameters have their numerical values validated using mathematical techniques. The study's conclusions show that motor acceleration and stability have a positive correlation, meaning that as motor acceleration rises, stability rises as well. On the other hand, stability and moment of inertia have a negative correlation, indicating that stability decreases as moment of inertia increases. The most crucial element when it comes to stiffness, damping, and deflection is stiffness. Stability is increased with stiffness. Stability, however, is low when stiffness is low. It is anticipated that the implications of these findings will increase industrial productivity.

iii. Title: Milling accuracy improvement of a 6-axis industrial robot through dynamic analysis.

Journal: EXAMENSARBETE INOM FARKOSTTEKNIK, AVANCERAD NIVÅ, 2019.

Author: Peter Eriksson.

Description: When paired with a milling head, the industrial robot is an affordable, versatile standard component that can be used to accomplish low-accuracy milling tasks. Researchers and business alike hope to improve milling accuracy in the future so that more high-value added operations can use it. Because of the member and reducer designs, the industrial robot's serial build-up presents non-linear compliance and vibration mitigation challenges. Due to structural modifications made possible by Additive Manufacturing (AM), the conventional cast aluminum structure may be revised, potentially leading to a gain in milling accuracy. The structural adjustments that would increase milling accuracy for a particular trajectory are suggested in this thesis. The robot had to be reverse engineered and a kinematic simulation model had to be created in order to measure the improvement. Subsequently, the kinematic simulation procedure was mechanized to enable the manipulation of several input parameters and the identification of the most advantageous alteration through screening. It was discovered that, without altering the design concept, a mass decrease in any member had no effect on milling accuracy, while a stiffness increase in the second axis member would have the greatest effect. Altering the reducer in axis 1 would decrease the robot's maximum speed while also reducing the mean position error by 7.5% and the mean rotation error by roughly 4.5%. The addition of two support bearings for axes two and three, which reduced the mean positioning error and rotation error by roughly 8% and 13%, respectively, would be the ideal structural modification.

iv. Title: Stiffness performance index based posture and feed orientation optimization in robotic milling process.

Journal and year: Robotics and computer integrated manufacturing g, 2019.

Author: Ri et al.

Description: Industrial robots are competitive and promising replacements for human machining. The main barrier to industrial robots' widespread use in machining applications is their relatively low stiffness. In order to increase the machining accuracy of robotic milling, the stiffness properties of an industrial robot are examined in this study. Additionally, optimization techniques for the robot's posture and tool feed orientation are established. The normal stiffness performance index (NSPI) of the surface, which is derived from the comprehensive stiffness performance index (CSPI), is first proposed to evaluate the stiffness performance of the robot for a given posture based on the relationship between the external force and deformation of the robot end effector (EE). It has been demonstrated that the NSPI depends on the directions of external forces but is not affected by their magnitudes. Next, for a given posture, a distribution rule is proposed for the NSPI with respect to arbitrary directions in the Cartesian space, thereby demonstrating the anisotropic nature of the robot stiffness. An optimization model is established to maximize the NSPI in order to optimize the posture of an industrial robot with six degrees of freedom (DOF) in a milling application. The NSPI is used to determine the optimal tool feed orientation for robot planar milling. Ultimately, a discussion of the robot milling experiment results is presented to demonstrate the viability and efficiency of the suggested optimization techniques.

v. Title: Stiffness-oriented performance indices defined on two-dimensional manifold for 6-DOF industrial robot

Journal and year: Robotics and computer integrated manufacturing, 2021.

Author: Guanhua et al.

Description: This paper presents the modeling of an industrial robot's 6-DOF stiffness, and compares two stiffness-oriented performance indices in order to optimize the two-dimensional manifold. In an earlier study, the curse of dimensionality was avoided by applying the two-dimensional manifold to simplify the workspace for error compensation. A stiffness-oriented manifold is crucial in the real application since stiffness is one of the most important factors in robotic machining to reduce the deflection at the end of the robot arm. To prevent manipulator ill-conditions, a revised two-dimensional index based on the translational compliance matrix's Frobenius-condition-number is presented. A number of matrix theory propositions are used to theoretically demonstrate the superiority of the improved index over the prior one. The results of simulations using sample points on a panel surface suggest that it is possible to successfully prevent robot malfunctions. The outcomes of contrast experiments carried out on KUKA robots demonstrate that the enhanced method continues to guarantee the precision of positioning in robotic machining.

vi. Title: Modeling and identification of an industrial robot for machining applications.

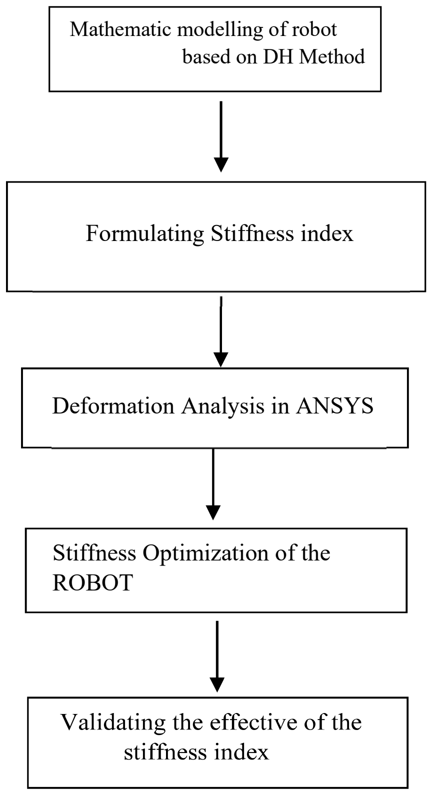
Journal and year: CIRP, 2007.

Author: Abele et al.

Description: For machining applications, industrial robots offer a flexible, affordable, and promising substitute. The behavior of the system differs significantly from that of a traditional machine tool because of the kinematics of a vertical articulated robot. With an emphasis on the analysis of the stiffness of the system and its behavior during the milling process, this article explains the modeling of the robot structure and the determination of its parameters. As a result, a technique for calculating Cartesian stiffness based on polar stiffness and the Jacobian matrix is presented. The robot's machining performance is assessed and conclusions are made in light of the identification and experimental validation results.

CHAPTER 2
METHODOLOGY AND EXPERIMENTAL WORK

2.1 METHODOLOGY



Tab. 2.1 Methodology Flow Chart

The mathematical model of the Jacobean of the robot is studied thoroughly which is based on the Denavit Hartenberg theory. The static structural analysis of the UR5 robot is performed in Ansys Workbench with the same specifications mentioned above and the results are validated with that of the ABB IRB robot which was done in [1]. Five different working postures of the robot are selected and the deformation at the end effector is obtained for each posture from the analysis and the stiffness value is obtained from the deformation value as well. Taking the same deformation and stiffness values a Fuzzy Inference System is developed taking deformation, stiffness and damping as the inputs of the system and stability as the output of the system. The deformation and stiffness values are obtained from the structural analysis and the deformation and stability values are obtained from [1].

2.2 KINEMATIC APPROACH OF THE ROBOTIC SETUP BASED ON DENAVIT HARTENBEG THEORY

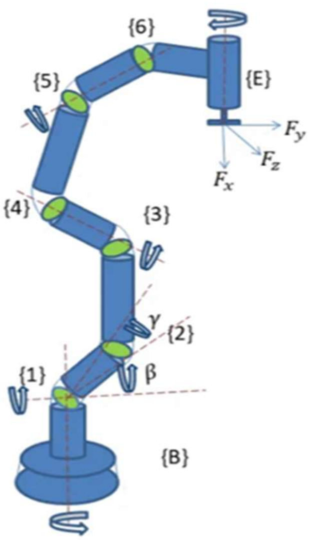


Fig. 2.1 Schematic Diagram of UR5 [2]

As seen in the above fig. 2 the coordinates and frames {0} to {6} based on Denavit Hartenberg model were established. The origin of the reference coordinate system in this case is at the center of the robot's base and its coordinate is {0}: $X_0Y_0Z_0$ and the coordinate {6}is the coordinate of the robot flange with its origin situated at the middle of the robot's six axis flange.

Table 2 lists the D-H parameters of the URG robot used in this study. The joint variable is denoted by **θi**, the link twist angle by **α_i**, the connecting rod length between adjacent joints by **a_i**, and the link offset by **d_i**.

Tab. 2.2 Kinematic Parameters of UR5 [collected from the official website]

Kinematics	Theta(rad)	a(m)	d(m)	Alpha(rad)
Joint 1	0	0	0.162	Π/2
Joint 2	0	-0.425	0	0
Joint 3	0	-0.3922	0	0
Joint 4	0	0	0.1333	Π/2
Joint 5	0	0	0.0997	-Π/2
Joint 6	0	0	0.0996	0

Tab. 2.3 Dynamic Parameters of UR5 [collected from the official website]

Dynamics	Mass(kg)	Centre of mass(m)	Inertia Matrix
Link 1	3.761	[0, -0.02561, 0.00193]	0
Link 2	8.058	[0.2125, 0, 0.11336]	0
Link 3	2.846	[0.15, 0.0, 0.0265]	0
Link 4	1.37	[0, -0.0018, 0.01634]	0
Link 5	1.3	[0, 0.0018, 0.01634]	0
Link 6	0.365	[0, 0, -0.001159]	[0, 0, 0, 0, 0, 0, 0, 0, 0.0002]

∴The coordinate {i} in Fig. 2's individual link-transformation matrices $^{i-1}\mathbf{T}_i$ with respect to coordinate {i − 1} can be expressed as follows, as per DH method;

$$^{i-1}T_i = Rt(X, \alpha i)Trns(X, ai)Rt(Z, \theta i)Trns(Z, di)$$

$$= \begin{bmatrix} C\theta i & -S\theta i & 0 & \alpha i-1 \\ S\theta iC\alpha i-1 & C\theta iS\alpha i-1 & -S\alpha i-1 & -S\alpha i-1.di \\ S\theta iS\alpha i-1 & C\theta iS\alpha i-1 & C\alpha i-1 & C\alpha i-1di \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{- (1)}$$

Where Sθi = Sinθi, Cθi = Cosθi.

∴ Thus, the kinematics equation or the DH Matrix of the UR5 robot employed in this study can be attained by multiplying the transformation matrix ${}^{0}T_{1}$ to ${}^{5}T_{6}$ in turn:

$${}^{0}T_{6} = {}^{0}T_{1}.{}^{1}T_{2}.{}^{2}T_{3}.{}^{3}T_{4}.{}^{4}T_{5}.{}^{5}T_{6}$$

$$= \begin{bmatrix} ax & bx & cx & Px \\ ay & by & cy & Py \\ az & bx & cx & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{-} \quad (2)$$

here $\begin{bmatrix} Px \\ Py \\ Pz \end{bmatrix}$ is the position vector of the end effector of the robot and $\begin{bmatrix} ax \\ ay \\ az \end{bmatrix}, \begin{bmatrix} bx \\ by \\ bz \end{bmatrix}, \begin{bmatrix} cx \\ cy \\ cz \end{bmatrix}$ are the direction vectors respectively.

2.3 MODELING OF THE STIFFNESS OF THE ROBOTIV SYSTEM

Here comes the jacobian matrix [J] of the robot, the kinematic analysis and static stiffness modelling of the robot rely heavily of its [J] matrix. According to [1] , for the joint i (i = 1,2,3,4,5,6) of the robot, if link j (j = 1,2,3,4,5,6) is differentially rotated around the Zj axis of coordinate system {j} with angle dθj, then conferring to the rule of the Cartesian coordinate system and the definitions of ${}^{i-1}T_{i}$ and ${}^{i}T_{n}$ in Eq. 2, The robot end's corresponding differential motion vector can be found in the manner described below:

$$\begin{bmatrix} Tdx \\ Tdy \\ Tdz \\ T\partial x \\ T\partial x \\ T\partial x \end{bmatrix} = \begin{bmatrix} (p*a)z \\ (p*b)z \\ (p*c)z \\ az \\ bz \\ cz \end{bmatrix} \quad \text{- (3) , [1]}$$

∴ The jth column of the [J] matrix of the UR5 robot can be calculated as follows;

$$ {}^{T}\mathbf{J_j} = \begin{bmatrix} (p*a)z \\ (p*b)z \\ (p*c)z \\ az \\ bz \\ cz \end{bmatrix} = \begin{bmatrix} -ax.Py + ny.Px \\ -bx.Py + by.Px \\ -cx.Py + cy.Px \\ az \\ bz \\ cz \end{bmatrix} \quad - (4)\ , [1] $$

∴The following is an expression for the robot's Jacobian matrix in the flange coordinate system, based on the derivation above [1];

$$ {}^{T}J = [{}^{T}J_1.\ {}^{T}J_2.\ {}^{T}J_3.\ {}^{T}J_4.\ {}^{T}J_5.\ {}^{T}J_6] \quad - (5) $$

∴ It is simple to demonstrate that the Jacobian matrix TJ of the robot in the flange system {6} can be used to express the Jacobian matrix J of the robot in the base coordinate {0} as;

${}^{T}J = J_c * J$ – (6), [1] where $J_c = \begin{bmatrix} R & O \\ O & R \end{bmatrix}$ and $R = {}^{0}R_6{}^{T}$ = is the matrix that represents the link-transformation between the base and flange coordinate systems. By combining equations 3-6 we will get the [J] matrix of the robot.

CHAPTER 3

RESULTS AND DISCUSSION

3.1. SIMULATION RESULTS AND DISCUSSION

First, as indicated in Table 4, five arbitrary working postures of the robot with the same end position were chosen. The principles of the validation simulation are composed of the following steps;

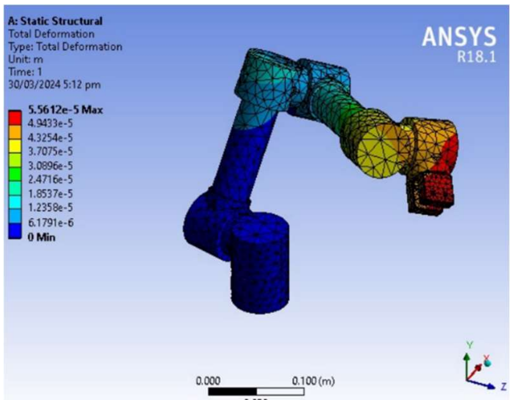
- Select five random working postures of the robotic system with different end position;.
- The same remote force of 50 N along the negative Y-axis in the reference coordinate system is applied to the end flange face of the above five working postures of the robot using ANSYS software, while the bottom face of the robot base is fixed and all the other connditions remain unchanged, to determine the deformations of the terminal of the above five working postures of the robot. Subsequently, based on the deformation results, the working stiffness values of the five postures can be sorted.
- Calculate the NSC values for the above five postures of the robotic system. The working stiffness values of the five postures are then sorted with the corresponding NSC.

The terminal deformations of the five working postures in Table 4 are obtained, as shown in Fig. 12, and the working stiffness of the five postures can be sorted.

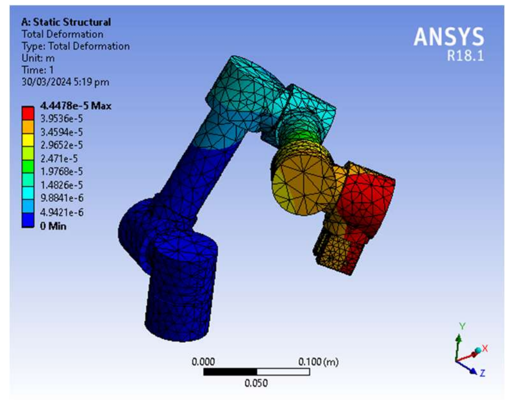
Tab. 3.1 Theta values for different postures

Sl.No.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
1	-4.62	44.72	-2.26	+40.28	+60.17	+12.15
2	+1.5	38.92	-+7.5	-30	+29.68	-7.49
3	-3.47	38.67	+12.19	+48.79	+25.62	-5.29
4	-2.42	45.82	+4.47	-14.39	+54.49	+25.69
5	+0.7	40.52	+5.49	+3.71	+70.41	+5.6

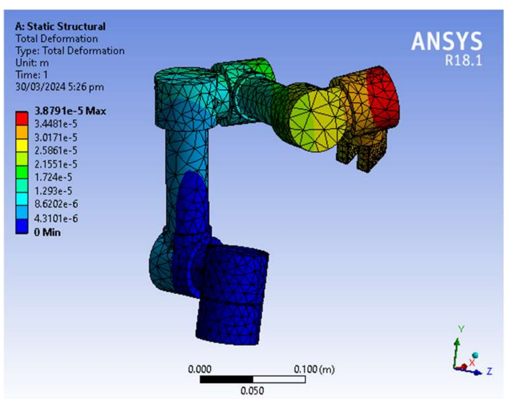
Subsequently, the terminal deformations of the five working postures in Table 4 are obtained, as shown in Fig. 3.1, and the working stiffness of the five postures can be sorted according to the simulation principle in Section 3.1.



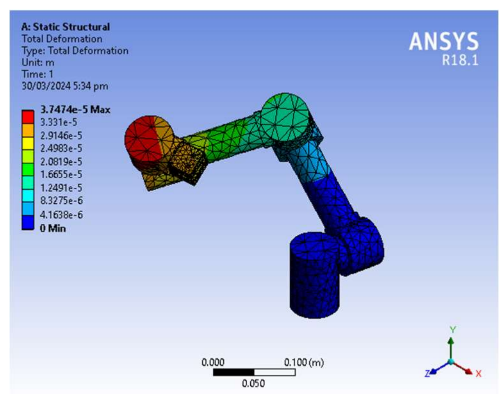
1st Working Posture



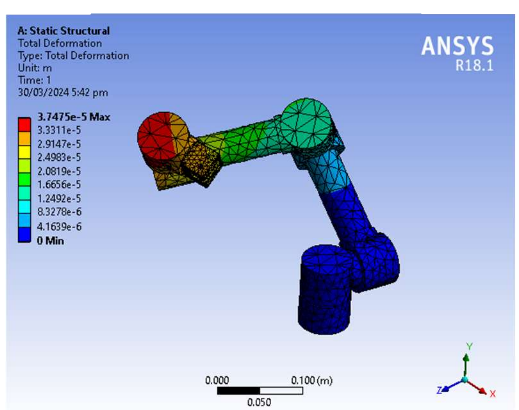
2nd Working Posture



3rd Working Posture



4th Working Posture



5th Working Posture

Fig.3.1 Results of terminal deformation simulation for various working postures

From Hooke's law F = k.∂D we can find out the stiffness for each working posture. The deformation of each posture and the respective stiffness values are presented in the table below.

Tab. 3.2 Deformation and respective stiffness values for different postured

Posture	1st	2nd	3rd	4th	5th
∂D(m)	5.5612×10^{-5}	4.4478×10^{-5}	3.8791×10^{-5}	3.7474×10^{-5}	3.7475×10^{-5}
K(N/m)	8.99×10^{5}	11.24×10^{5}	12.88×10^{5}	13.34×10^{5}	13.342×10^{5}

The ∂D(mm) vs k(N/mm) graph only for the above five working postures is presented below; taking stiffness along vertical axis and deformation along horizontal axis.

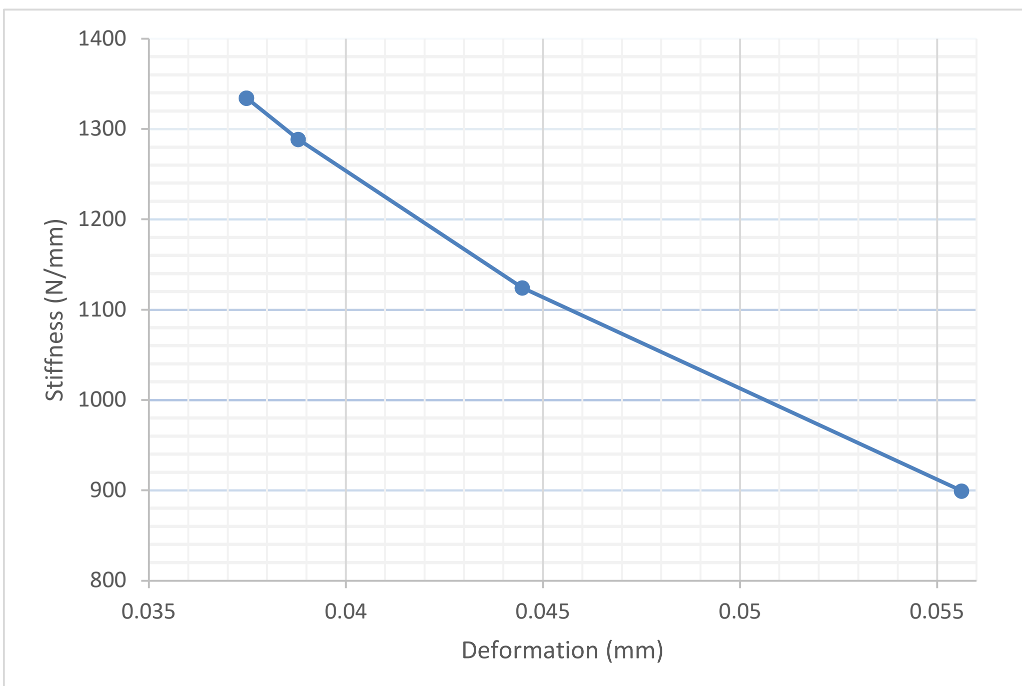


Fig.3.2. Stiffness Vs Deformation Graph

3.3 FUZZY INFERENCE SYSTEM FOR INDUSTRIAL ROBOTS

Fuzzy logic control is indeed used in various industrial applications, including robotics. It is a form of mathematical logic that deals with approximate rather than exact reasoning. In the context of industrial robots, fuzzy logic can be applied in several ways;

1. Path Planning: Fuzzy logic can help robots navigate through complex environments by allowing them to make decisions based on imprecise or uncertain information, such as sensor readings or environmental conditions.
2. Collision Avoidance: Robots equipped with fuzzy logic controllers can dynamically adjust their movements to avoid collisions with obstacles or other robots in their workspace.
3. Force Control: Fuzzy logic can be used to control the force applied by robotic arms during tasks such as assembly or material handling, allowing them to handle delicate objects or adapt to changes in the environment.
4. Adaptive Control: Fuzzy logic controllers can adapt to changes in the environment or the task requirements, making them suitable for applications where precise mathematical models may be unavailable or impractical.
5. Human-Robot Interaction: Fuzzy logic can also be applied to enable more natural interaction between humans and robots, allowing robots to interpret and respond to human commands or gestures in a flexible and intuitive manner.

Overall, fuzzy logic control offers a robust and flexible approach to controlling industrial robots in complex and dynamic environments, where traditional control methods may be inadequate.

3.4 IMPLEMENTATION OF FIS TO ENHANCE THE ROBOT STIFFNESS

A fuzzy inference system (FIS) is a computational model primarily used in fuzzy logic to represent human knowledge and reasoning in a mathematical framework. It is a type of expert system that deals with uncertainty and imprecision in data. FIS is particularly useful when traditional binary logic is inadequate for modeling complex systems due to its ability to handle vague, uncertain or incomplete information.

∴Fuzzy logic can help enhance the stiffness of industrial robots through control strategies that adaptively adjust the robot's behavior based on varying conditions and requirements. Stiffness in industrial robots refers to the ability of the robot to maintain its position accurately and resist external disturbances. Here are several ways fuzzy logic can contribute to improving the stiffness of industrial robots:

1. Adaptive Control: Fuzzy logic controllers can adapt the control parameters of the robot's actuators in real-time based on the sensed environment and task requirements. This adaptability allows the robot to maintain stiffness even in situations where the dynamics of the environment change, such as when interacting with objects of different masses or encountering unexpected disturbances.

2. Compliance Control: Fuzzy logic can be used to design compliant control strategies that enable the robot to exhibit a certain level of flexibility while still maintaining stiffness during interactions with the environment. By adjusting the compliance of the robot's joints or end-effector, it can safely handle delicate tasks or interactions with humans without sacrificing overall stiffness.

3. Force/Torque Control: Fuzzy logic controllers can regulate the forces and torques exerted by the robot during interactions with the environment. By dynamically adjusting these forces based on sensory feedback, the robot can maintain stiffness while ensuring safe and precise manipulation of objects.

4. Error Compensation: Fuzzy logic can be employed to develop error compensation techniques that minimize position or force errors resulting from factors such as friction, wear, or nonlinearities in the robot's mechanisms. By accurately estimating and compensating for these errors, the robot can achieve higher levels of stiffness and accuracy in its movements.

5. Fault Tolerance: Fuzzy logic-based fault detection and tolerance mechanisms can help identify and mitigate issues that could compromise the stiffness of the robot, such as sensor failures, actuator faults, or mechanical wear. By promptly detecting and reacting to such faults, the robot can maintain optimal performance and stiffness over time.

Overall, fuzzy logic provides a flexible and robust framework for designing control strategies that can effectively enhance the stiffness of industrial robots, enabling them to operate with greater precision, reliability, and adaptability in various manufacturing and automation applications.

3.5 DESIGN OF THE FUZZY CONTROLLER

An effective heuristic for designing nonlinear controllers is fuzzy logic control, which integrates important facets of human reasoning with pertinent data. By applying fuzzy control concepts methodically, control objectives can make effective use of qualitative and heuristic aspects that are not taken into account by conventional control theory. In scenarios involving complex, nonlinear, or undefined systems, fuzzy logic controllers often perform better than other controllers, provided that robust variables are mapped to suitable linguistic values. Following the inference rules' evaluation of the linguistic values and their mapping to the fuzzy set, the defuzzification process converts the fuzzy set into a crisp value. The fundamental concept of inference rules, which facilitate the system's computational capabilities, forms the basis of the Mamdani approach. Rules can be developed using historical data, empirical observations, and the wisdom of seasoned individuals. Every fuzzy inference rule consists of both language-specific variables and if-then sentences. The lowest and maximum values obtained from the two operands are displayed, respectively, in the results of the "and" and "or" operations.

$\therefore$ The and / intersection operator can be expressed as follows; $\mu_{A \cap B}$ = minimum[$\mu A(x), \mu B(x)$] and the "or" operator is described as $\mu_{A \cup B}$ = minimum[$\mu A(x), \mu B(x)$] where μA is the membership in class A, μB is for the members of class B and x signifies the degrees of the membership functions.

∴The process of defuzzification involves turning a fuzzy output value—which indicates a state of ambiguity or imprecision—into a clear-cut, crisp value. By evaluating fuzzy rules and then carrying out the necessary computations, a numerical value can be obtained. The membership values that are connected to different output fuzzy sets are correlated with the numerical value.

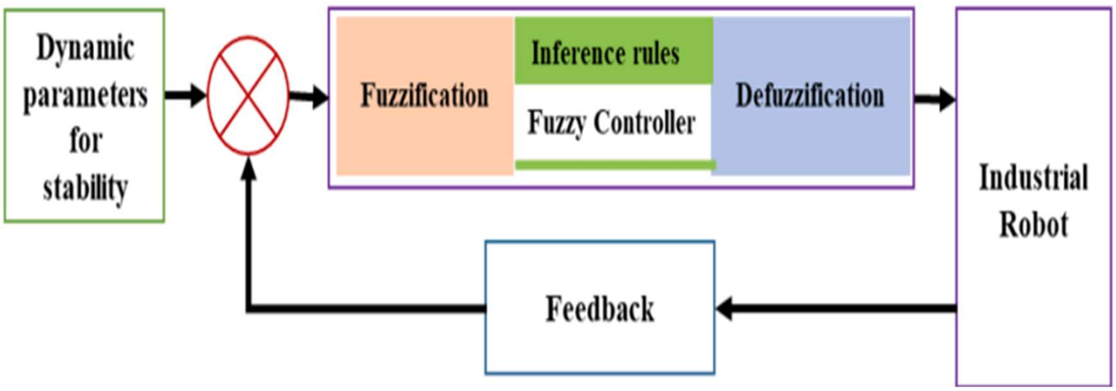


Fig. 3.3 Architecture of Fuzzy Controller [2]

Inference rules have been followed to classify the level of stability as low, medium, and high. The relationships of stability with stiffness, damping, and deflection are also followed by using fuzzy logic. Fig 3.4 shows the different rules of fuzzy logic for stability.

1. If (Stiffness is Low) and (Deformation is Low) and (Damping is Low) then (Stability is High) (1)
2. If (Stiffness is Low) and (Deformation is Low) and (Damping is Medium) then (Stability is Moderate) (1)
3. If (Stiffness is Low) and (Deformation is Low) and (Damping is High) then (Stability is Less) (1)
4. If (Stiffness is Low) and (Deformation is Med) and (Damping is Low) then (Stability is High) (1)
5. If (Stiffness is Low) and (Deformation is Med) and (Damping is Medium) then (Stability is Less) (1)
6. If (Stiffness is Low) and (Deformation is Med) and (Damping is High) then (Stability is Less) (1)
7. If (Stiffness is Low) and (Deformation is High) and (Damping is Low) then (Stability is Less) (1)
8. If (Stiffness is Low) and (Deformation is High) and (Damping is Medium) then (Stability is Less) (1)
9. If (Stiffness is Low) and (Deformation is High) and (Damping is High) then (Stability is Less) (1)
10. If (Stiffness is Med) and (Deformation is Low) and (Damping is Low) then (Stability is High) (1)
11. If (Stiffness is Med) and (Deformation is Low) and (Damping is Medium) then (Stability is Moderate) (1)
12. If (Stiffness is Med) and (Deformation is Low) and (Damping is High) then (Stability is Moderate) (1)
13. If (Stiffness is Med) and (Deformation is Med) and (Damping is Low) then (Stability is High) (1)
14. If (Stiffness is Med) and (Deformation is Med) and (Damping is Medium) then (Stability is Moderate) (1)
15. If (Stiffness is Med) and (Deformation is Med) and (Damping is High) then (Stability is Moderate) (1)
16. If (Stiffness is Med) and (Deformation is High) and (Damping is Low) then (Stability is Less) (1)
22. If (Stiffness is High) and (Deformation is Med) and (Damping is Medium) then (Stability is Moderate) (1)
23. If (Stiffness is High) and (Deformation is Med) and (Damping is High) then (Stability is Moderate) (1)
24. If (Stiffness is High) and (Deformation is High) and (Damping is Low) then (Stability is Moderate) (1)
25. If (Stiffness is High) and (Deformation is High) and (Damping is Medium) then (Stability is Less) (1)
26. If (Stiffness is High) and (Deformation is High) and (Damping is High) then (Stability is Less) (1)
27. If (Stiffness is Med) and (Deformation is Med) and (Damping is Medium) then (Stability is Less) (1)
28. If (Stiffness is High) and (Deformation is Med) and (Damping is Low) then (Stability is Less) (1)

Fig. 3.4 FIS Rules

∴ Ebrahim H. Mamdani first presented the Mamdani fuzzy inference system (FIS), a kind of fuzzy logic controller, in 1975. It is among the most popular and ancient types of fuzzy inference systems. Based on fuzzy logic, a mathematical framework for handling imprecision and uncertainty, the Mamdani FIS functions.
Fuzzy sets are used to define the input and output variables linguistically in a Mamdani fuzzy inference system. Each fuzzy set is characterized by a membership function that allocates degrees of membership to elements of the input or output space, and each fuzzy set represents a linguistic term (e.g., "low," "medium," "high"). The degree of uncertainty present in human language and reasoning is captured by these membership functions. The key components of a Mamdani fuzzy inference system are:

1. Fuzzification: In this step, crisp input values are converted into fuzzy sets using the corresponding membership functions. Each input variable is associated with one or more fuzzy sets, and its value is assigned membership degrees for each fuzzy set based on the shape of the membership functions.

2. Rule Evaluation: The fuzzy rules define the mapping between the fuzzy inputs and fuzzy outputs. Each rule consists of an antecedent (if-portion) and a consequent (then-portion), expressed in terms of fuzzy logic operators (e.g., AND, OR). The antecedent of each rule evaluates the degree to which the input variables satisfy the conditions specified by the fuzzy sets. The consequent of each rule defines the fuzzy output to be activated and its associated membership grade.

3. Aggregation: After evaluating the antecedent of each rule, the activated fuzzy sets from all rules are combined to generate a fuzzy output. This process involves aggregating the membership grades of the activated fuzzy sets using fuzzy logic operators such as MAX or SUM.

4. Defuzzification: In the final step, the aggregated fuzzy output is converted back into a

crisp value. This process involves determining the most representative or "centroid" value of the fuzzy output distribution. Common defuzzification methods include centroid, weighted average, or maximum membership.

Mamdani fuzzy inference systems have been applied in various fields, including control systems, pattern recognition, decision support systems, and expert systems. They provide a flexible and intuitive framework for modeling complex systems in which human reasoning and linguistic expressions play a significant role. Despite their simplicity and interpretability. Mamdani FISs may suffer from computational inefficiency and require careful tuning of parameters and rules to achieve optimal performance. Nonetheless, they remain a valuable tool for handling uncertainty and imprecision in real-world applications.

∴The inputs of the system are stiffness(0-1600mm), deformation(0-0.06N/mm) and damping(0.027-0.07) and the output is stability(0.0361-32.61). The values of daming and stability are collected from [1].

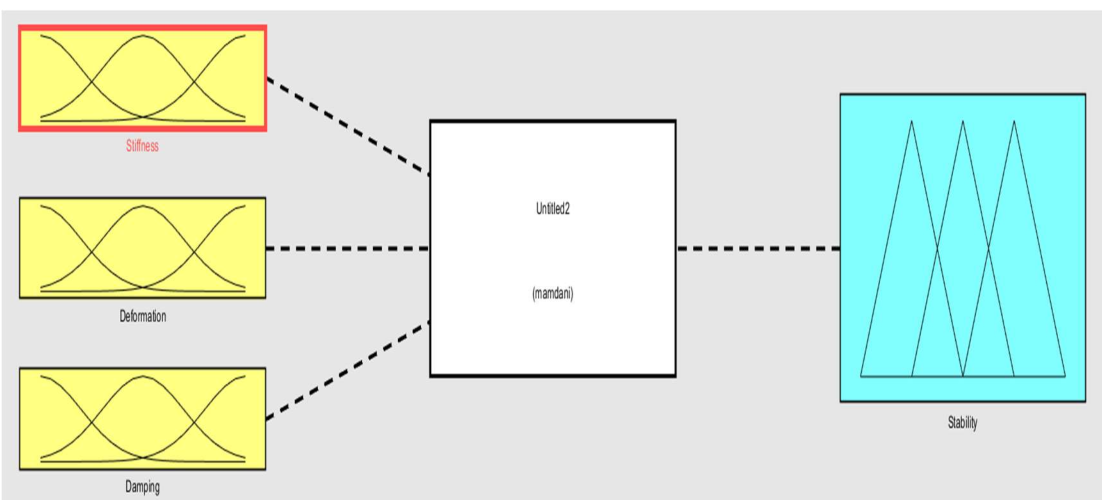
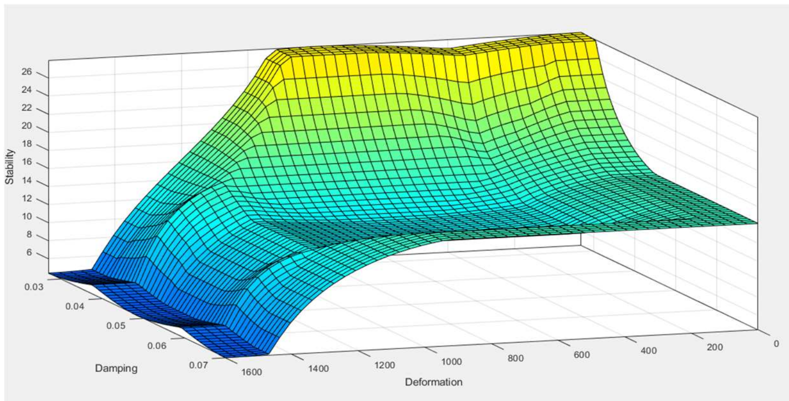


Fig. 3.5 Mamdani FIS

Here we have used the Gaussian membership function for the three inputs and Triangular membership function for the output. The explanation of both the membership functions is provided below.

- Gaussian Membership function: The degree of membership of a given input value to a fuzzy set is represented by the Gaussian membership function, which is a bell-shaped curve. Its symmetrical form and peak value, around which the membership progressively declines, are its defining features. This membership function is mathematically defined by $\mu(x) = e^{(x-c)^{\wedge}2/\sigma^{\wedge}2}$, where x is the input value , c is the center or the peak of the curvature , σ is the standard deviation, which defines the extent of the curve.

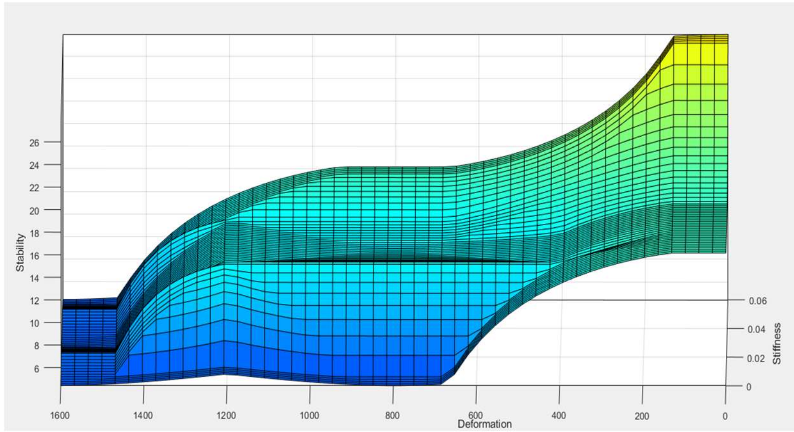


- Triangular Membership function: The degree of membership in the triangular membership function rises linearly from a minimum value to a peak value and then falls linearly to another minimum value. This triangular shape is indicative of the membership function.

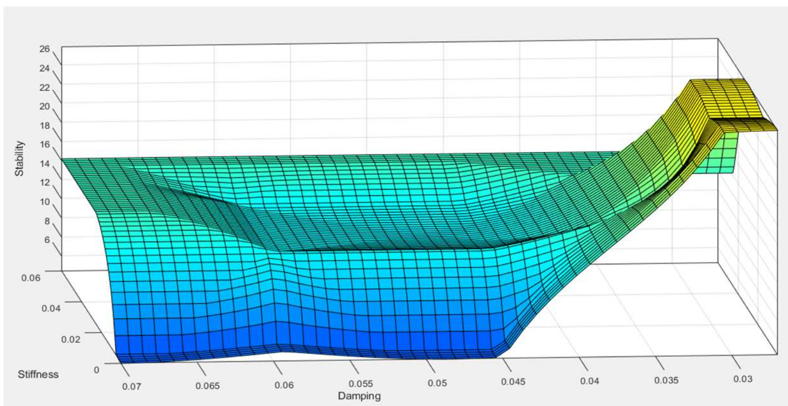
In fuzzy logic systems, Gaussian and triangular membership functions are essential tools for handling and representing uncertainty. They have numerous uses in a variety of industries, including artificial intelligence, pattern recognition, control systems, and decision-making.

In Fig. 3.6 (a, b, c), the graphical presentation is shown and it is observed that the higher stiffness, the higher the stability and the lower the damping and deflection, the higher the stability is found.

(a)



(b)



(c)

Fig. 3.6 Matlab/Fuzzy Toolbox Graphical representation of stability with respect to (1) Deformation-Stiffness (2) Stiffness-Damping (3) Damping-Deformation

CHAPTER 4

CONCLUSIONS

The stability of a robot manipulator has a significant effect on its productivity. The robot manipulator is influenced by the parameters of the motor and manipulator. These parameters are studied for the six-link, six-degree-of-freedom UR5 robot. Using Lagrangian mechanics, the current study creates a mathematical model that takes into consideration all of the parameters related to the motor and manipulator, including acceleration, moment of inertia, stiffness, damping, and deflection. According to a recently developed formula, stability determines the functions of acceleration and moment of inertia, which are connected to other motor parameters and robot manipulator parameters. The numerical values of the robot's parameters are used to validate the mathematical model and formula. This study is unique in that it examines the nonlinear dynamic system while taking different robot motor and manipulator parameters into account. Real data is used to derive the theoretical formulas for acceleration, moment of inertia at capacity, deflection, damping, and stiffness. Based on these variables, the degree of stability is rated as low, medium, or high. The relationship between the stability and the motor and manipulator parameters is illustrated using fuzzy logic. Furthermore, it has been demonstrated that acceleration promotes stability whereas moment of inertia decreases with stability. Stability is correlated with increased stiffness and decreased damping and deflection.

After a working stiffness evaluation model for RABPSs was examined in order to address the issue brought about by the low stiffness characteristics of RABPSs, the following findings were made;

- The robot's static stiffness model was created using the compliance matrix as a basis. The aforementioned model was then coupled with the force parameters of the polishing procedure.
- Experiments with simulation and normal force measurement were carried out.

In an effort to increase the application of bonnet polishing technology, a great deal of research has been done on robot assisted polishing tasks. One of the main issues that needs to be resolved right now is low working stiffness. An evaluation technique for the working stiffness of RABPSs needs to be investigated in order to resolve this problem. Thus, this paper presents a theoretical and experimental study. The Denavit–Hartenberg (D–H) approach served as the foundation for the construction of the RABPS static stiffness model.

Next, the force characteristics of the robot during the polishing process were investigated. The results of the force analysis were combined with the previously mentioned static stiffness model to create a new index called the normal stiffness coefficient (NSC), which was then proposed to evaluate the working stiffness of the robot. Ultimately, the NSC of the robot's performance was verified through experimentation and simulation. These two results suggest that the NSC, compared to other commonly used indices, provides a more accurate measure of the working stiffness of RABPSs. This work provides an important theoretical foundation for further studies aimed at improving the working stiffness of RABPSs.

CHAPTER 6
PROSPECTS

In the future, it will be crucial to put the currently theoretical model for the industrial sector's robot efficacy into practice and validate it. It is anticipated that putting these insights into practice will increase robotics' industrial output and dependability.

Because of their efficiency and flexibility, industrial robots are being used more and more in a variety of manufacturing processes, including polishing operations. Nonetheless, attaining accurate and superior polishing outcomes depends on making sure these robots are designed with the maximum amount of stiffness. In order to optimize the stiffness of an industrial robot designed specifically for polishing tasks, a mathematical model is proposed in this thesis. By reducing deflections and vibrations, the model seeks to improve surface finish quality and polishing accuracy. To evaluate the efficacy of the suggested mathematical framework, this study will combine experimental validation, numerical simulations, and theoretical analysis.

∴ Objectives –

- Gain a thorough understanding of what industrial robots need to have in order to optimize stiffness for polishing tasks.
- Create a quantitative mathematical model that expresses the relationship between stiffness properties and robot design parameters.
- Use numerical simulations to forecast how various robot configurations will perform in terms of stiffness.
- Conduct experimental testing on a prototype industrial robot platform to validate the mathematical model.
- Examine how stiffness optimization affects surface finish quality and polishing accuracy.
- Based on the research findings, provide guidelines for the design and optimization of industrial robots specifically intended for polishing tasks.

∴ Techniques:

- Examine the body of research on the optimization of stiffness in industrial robots as well as pertinent mathematical modeling methods.
- Provide a mathematical framework that combines the needs of polishing operations with the mechanical characteristics of the robot structure.
- Use simulations of finite element analysis (FEA) to forecast the stiffness properties of various robot configurations under various loading scenarios.

- Build a working prototype industrial robot system with sensors to track vibrations and deflections while polishing.
- To verify the accuracy of the mathematical model and evaluate the performance gains made possible by stiffness optimization, conduct experimental testing.
- Examine the experimental data to determine how stiffness optimization affects surface finish quality and polishing accuracy.
- Based on the research, develop standards and suggestions for industrial robot stiffness optimization during polishing operations.

∴ Significance

The manufacturing sector, especially those where precise polishing is crucial for product quality, stands to gain a great deal from the suggested mathematical model for stiffness optimization in industrial robots. Manufacturers can increase the precision, effectiveness, and consistency of polishing operations, resulting in higher-quality final products and higher productivity, by improving the stiffness characteristics of industrial robots. Furthermore, the research findings will advance the theoretical knowledge of stiffness optimization in robotic systems and offer useful information for designing and optimizing robots for a range of industrial applications.

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