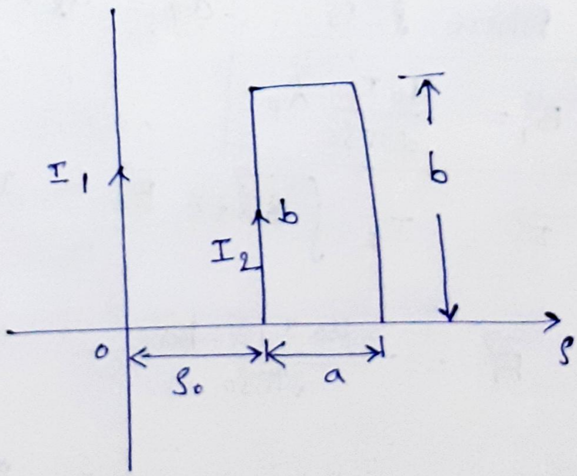
Conduction current : conduction current requires a conductor

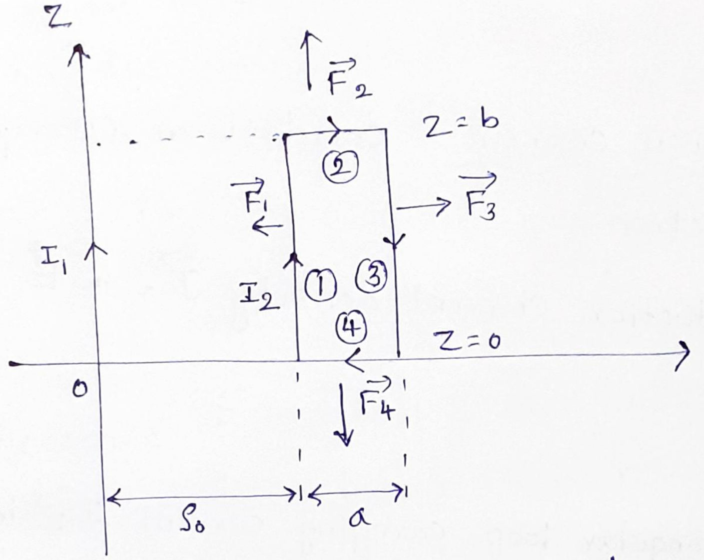
Conduction current density $\vec{J} = \sigma \vec{E}$

1) A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in figure. Show that the force experienced by the loop is given by

$$\vec{F} = -\frac{\mu_0 I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a}\right]\hat{a}_\rho \text{ N}$$

Solution :





Let the force on the loop be

$$\vec{F}_l = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$\vec{B}_1$ = Magnetic flux density due to infinitely long conductor carrying current I_1

$$\vec{B}_1 = \mu_0 \vec{H}_1 = \cancel{\frac{\mu_0 I_1 \hat{a}_\phi}{2\pi\rho_0}} = \frac{\mu_0 I_1}{2\pi\rho}\hat{a}_\phi$$

where $\rho = \rho_0$ $\qquad \hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = \hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi$

$$\boxed{\vec{B}_1 = \frac{\mu_0 I_1}{2\pi\rho_0}\hat{a}_\phi}$$

$$\vec{F}_1 = I_2 \int d\vec{l}_2 \times \vec{B}_1 = I_2 \int_{z=0}^{z=b} dz\,\hat{a}_z \times \frac{\mu_0 I_1}{2\pi\rho_0}\hat{a}_\phi$$

$$\vec{F}_1 = -\frac{\mu_0 I_1 I_2 b\,\hat{a}_\rho}{2\pi\rho_0} \qquad \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\hat{a}_\rho$$

$$\vec{F}_3 = I_2 \int d\vec{l}_2 \times \vec{B}_1 = I_2 \int_{z=b}^{0} dz\,\hat{a}_z \times \frac{\mu_0 I_1}{2\pi(\rho_0 + a)}\hat{a}_\phi$$

$$\vec{F}_3 = \frac{\mu_0 I_1 I_2 b}{2\pi(\rho_0 + a)}\hat{a}_\rho$$

$$\vec{F_2} = I_2 \int_{\rho=\rho_0}^{\rho=\rho_0+a} \vec{dl_2} \times \vec{B_1}$$

$$= I_2 \int_{\rho=\rho_0}^{\rho=\rho_0+a} d\rho\, \hat{a}_\rho \times \frac{\mu_0 I_1 \hat{a}_\phi}{2\pi\rho}$$

$$\begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{a}_z$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \int_{\rho=\rho_0}^{\rho=\rho_0+a} \frac{1}{\rho}\, d\rho\, \hat{a}_z$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln \rho \Big|_{\rho=\rho_0}^{\rho=\rho_0+a} \hat{a}_z \quad \text{(Parallel)}$$

$$\vec{F_2} = \frac{\mu_0 I_1 I_2}{2\pi} \ln\left[\frac{\rho_0+a}{\rho_0}\right] \hat{a}_z \quad \text{(Parallel)}$$

$$\vec{F_4} = I_2 \int_{\rho=\rho_0+a}^{\rho=\rho_0} \vec{dl_2} \times \vec{B_1}$$

$$= I_2 \int_{\rho=\rho_0+a}^{\rho=\rho_0} d\rho\, \hat{a}_\rho \times \frac{\mu_0 I_1 \hat{a}_\phi}{2\pi\rho}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{\rho_0}{\rho_0+a}\right) \hat{a}_z$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{\rho_0+a}{\rho_0}\right) \hat{a}_z$$

Total Force $\therefore \vec{F_l} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

$$= \frac{\mu_0 I_1 I_2 b}{2\pi} \left[\frac{1}{\rho_0} - \frac{1}{\rho_0+a}\right] (-\hat{a}_\rho)$$

1. A charged particle moves with a uniform velocity $4\hat{a}_x$ m/s in a region where $\vec{E} = 20\hat{a}_y$ V/m and ~~Bo~~ $\vec{B} = B_o\hat{a}_z$ Wb/m^2. Determine B_o such that the velocity of the particle remains constant

Solution:

If the particle moves with a constant velocity, it is implied that its acceleration is zero. In other words, the particle experiences no net ~~force.~~

$$\vec{F} = m\vec{a} = m(0) \rightarrow (1)$$

constant velocity

$$\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B})$$

$$0 = Q(\vec{E} + 4\hat{a}_x \times B_o\hat{a}_z)$$

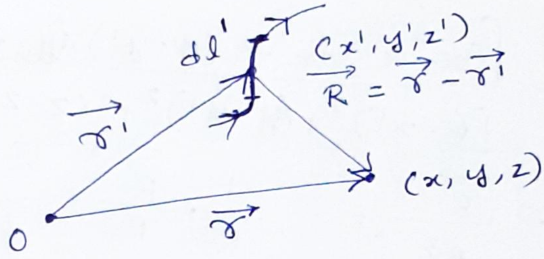
$$-\cancel{Q}\vec{E} = -\cancel{Q}4B_o\hat{a}_y$$

$$-\cancel{Q} \times 20\hat{a}_y = -\cancel{Q}4B_o\hat{a}_y$$

$$20 = 4B_o$$

$$B_o = 5$$

Derivation of Biot-Savart's law and Ampere's law



Both Biot-Savart's law and Ampere's law may be derived by using the concept of magnetic Vector potential.

We know $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 A$

consider $\vec{B} = \nabla \times \vec{A} = \nabla \times \oint_L \frac{\mu_0 I \vec{dl'}}{4\pi R}$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint_L \nabla \times \frac{1}{R} \vec{dl'} \rightarrow (1)$$

We know from vector identity

$$\vec{F} \times \nabla f = f \nabla \times \vec{F} - \nabla \times f \vec{F}$$

$$\therefore \nabla \times f \vec{F} = f \nabla \times \vec{F} - \vec{F} \times \nabla f$$

$$\nabla \times \frac{1}{R} \vec{dl'} = \frac{1}{R} \nabla \times \vec{dl'} - \vec{dl'} \times \nabla\left(\frac{1}{R}\right) \rightarrow (2)$$

where $\vec{F} = \vec{dl'}$, $f = \frac{1}{R}$

using (2) in (1)

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint_L \left[\frac{1}{R} \nabla \times \vec{dl'} - \vec{dl'} \times \nabla\left(\frac{1}{R}\right)\right] \rightarrow (3)$$

Since ∇ operator, operates with respect (x, y, z) and $\vec{dl'}$ is a function of (x', y', z')

$$\therefore \nabla \times \vec{dl'} = 0 \rightarrow (4)$$

$$|\vec{R}| = R = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{\frac{1}{2}}$$

$$\frac{1}{R} = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{-\frac{1}{2}}$$

$$\nabla\left(\frac{1}{R}\right) = \frac{\partial}{\partial x}(1/R)\hat{a}_x + \frac{\partial}{\partial y}(1/R)\hat{a}_y + \frac{\partial}{\partial z}\left(\frac{1}{R}\right)\hat{a}_z$$

$$\therefore \nabla\left(\frac{1}{R}\right) = -\frac{\left[(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z\right]}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

$$\nabla\left(\frac{1}{R}\right) = -\frac{\vec{R}}{R^3} = -\frac{1}{R^2}\cdot\frac{\vec{R}}{R} = -\frac{1}{R^2}\hat{a}_R \rightarrow (5)$$

where $\hat{a}_R = \frac{\vec{R}}{R}$ = unit vector

using (4) and (5) in (3) ~~$\vec{B} = \frac{\mu_0 I}{4\pi}\oint \vec{dl}\times a_R$~~

$$-\frac{\hat{a}_R}{R^2} = -\frac{\vec{R}}{R^3}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint_L \left[\cancel{\nabla \times \vec{dl}'}^{\,0} - \vec{dl}' \times \nabla\left(\frac{1}{R}\right)\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint_L \left[-\vec{dl}' \times \left(-\frac{\hat{a}_R}{R^2}\right)\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\oint_L \frac{\vec{dl}' \times \hat{a}_R}{R^2} \quad \text{or} \quad \vec{B} = \frac{\mu_0 I}{4\pi}\oint_L \frac{\vec{dl}' \times \vec{R}}{R^3}$$

~~which is the Biot-Savart's law~~

~~If we~~ If we drop prime in $\vec{dl}'$, it becomes $\vec{dl}$

$$\therefore \vec{B} = \frac{\mu_0 I}{4\pi}\oint_L \frac{\vec{dl} \times \hat{a}_R}{R^2} \quad \text{or} \quad \vec{B} = \frac{\mu_0 I}{4\pi}\oint_L \frac{\vec{dl} \times \vec{R}}{R^3}$$

which is Biot-Savart's law

$$\nabla \times \vec{B} = \nabla \times \nabla \times \vec{A}$$

$$\nabla \times \vec{B} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 A$$

$\boxed{\nabla \cdot \vec{A} = 0}$ which is called coulomb's guage

$$\nabla \times \vec{B} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\nabla \times \mu_0 \vec{H} = \nabla(\cancel{\nabla \cdot \vec{A}}^{0}) - \nabla^2 \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \nabla \times \vec{H}$$

$\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$ where $\nabla \times \vec{H} = \vec{J}$

$\nabla^2 \vec{A} = -\mu_0 \vec{J}$ is known as vector Poisson equation

$$\nabla^2 A_x = -\mu_0 J_x$$

$$\nabla^2 A_y = -\mu_0 J_y$$

$$\nabla^2 A_z = -\mu_0 J_z$$

which may be regarded as the scalar Poisson equa-tions

$$\oint_L \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds}$$

$$= \int_S \left(\nabla \times \frac{\vec{B}}{\mu_0}\right) \cdot \vec{ds}$$

$$= \frac{1}{\mu_0} \int_S \nabla \times (\nabla \times \vec{A}) \cdot \vec{ds}$$

$$\nabla \times \nabla \times \vec{A} = -\nabla^2 \vec{A} = \mu_0 \vec{J}$$

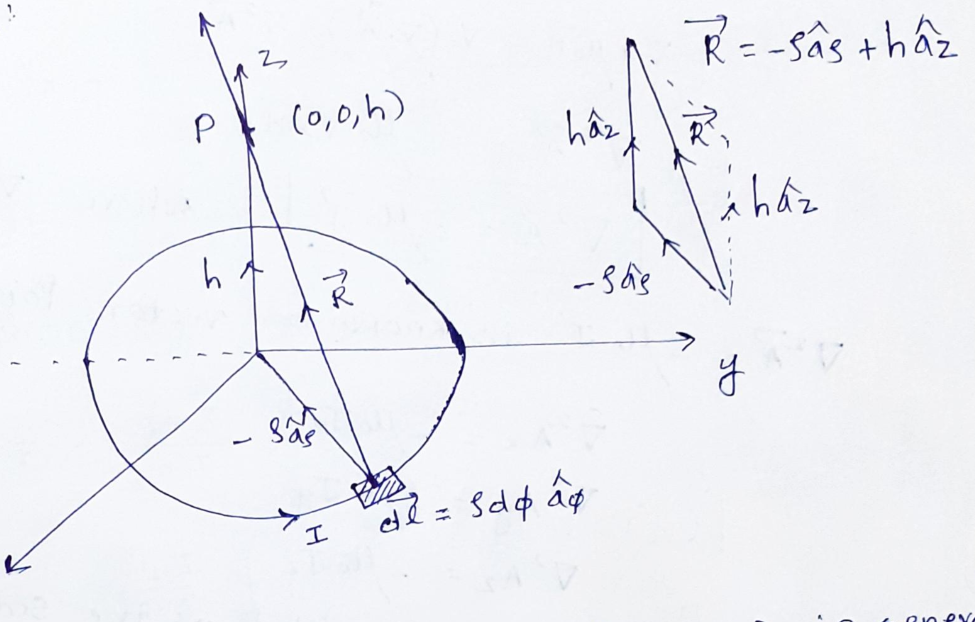
$$= \frac{1}{\cancel{\mu_0}} \int_S \cancel{\mu_0} \vec{J} \cdot \vec{ds}$$

$\oint_L \vec{H} \cdot \vec{dl} = \int_S \vec{J} \cdot \vec{ds} = I$ which is

Ampere's circuit law

1) A circular loop located on $x^2+y^2=9$, $z=0$ carries a direct current of 10A along $\hat{a}_\phi$. Determine $\vec{H}$ at $(0,0,4)$ and $(0,0,-4)$

Solution :



(a) Case(i): Let point of interest P (0,0,4) in general it is taken as (0,0,h)

$$\vec{dH} = \frac{I\,\vec{dl}\times\vec{R}}{4\pi R^3}, \quad \text{Now } \vec{dl} = \rho d\phi\, \hat{a}_\phi$$

$$\vec{R} = \rho(-\hat{a}_\rho) + h\hat{a}_z \Rightarrow R = |\vec{R}| = \sqrt{\rho^2+h^2}$$

$$\vec{dl}\times\vec{R} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & h \end{vmatrix} = \hat{a}_\rho[\rho h d\phi] - \hat{a}_\phi[0-0] + \hat{a}_z(+\rho^2 d\phi)$$

$$= h\rho d\phi\, \hat{a}_\rho + \rho^2 d\phi\, \hat{a}_z$$

Due to symmetry the contribution along $\hat{a}_\rho$ components add up to zero. Because the radial components produced by the current element pairs are seperated by 180°. Hence they cancel.

$$\vec{dH} = \frac{I\,\vec{dl}\times\vec{R}}{4\pi R^3} = \frac{I\,\rho^2 d\phi\,\hat{a}_z}{4\pi\left(\sqrt{\rho^2+h^2}\right)^3}$$

$x^2+y^2=\rho^2$, given $x^2+y^2=9 \Rightarrow \rho=3$

$P(0,0,4)$ $\therefore$ $h=4$

$$\vec{H} = \int_{\phi=0}^{2\pi} \frac{I\,\rho^2 d\phi\,\hat{a}_z}{4\pi\left(\sqrt{\rho^2+h^2}\right)^3} = \frac{I\,\rho^2\,\cancel{2\pi}\,\hat{a}_z}{\cancel{4\pi}_{2}\left(\sqrt{\rho^2+h^2}\right)^3}$$

$$\vec{H} = \frac{I\,\rho^2\,\hat{a}_z}{2\left(\sqrt{\rho^2+h^2}\right)^3} = \frac{(10)(3^2)(\hat{a}_z)}{2\left(\sqrt{3^2+4^2}\right)^3} = 0.36\,\hat{a}_z$$

(b) case (ii) At $(0,0,-4)$

$$\vec{dH} = \frac{I\,\vec{dl}\times\vec{R}}{4\pi R^3}$$

$$\vec{dl} = \rho\,d\phi\,\hat{a}_\phi$$

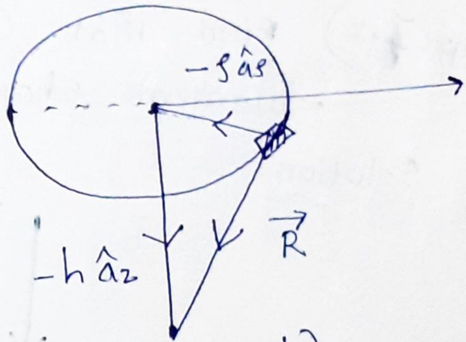
$$\vec{R} = -\rho\hat{a}_\rho - h\hat{a}_z$$

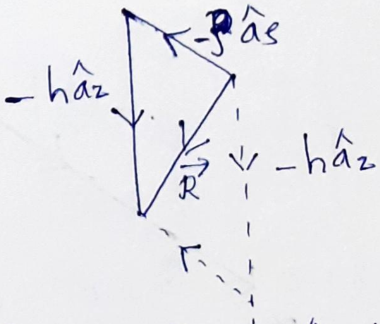
$$\vec{H} = \int_{\phi=0}^{2\pi} \frac{I\,\rho^2 d\phi\,\hat{a}_z}{4\pi\left(\rho^2+h^2\right)^{3/2}}$$

$$\vec{H} = \frac{I\,\rho^2\,\cancel{2\pi}\,\hat{a}_z}{\cancel{4\pi}_{2}\left(\rho^2+h^2\right)^{3/2}}$$

$\rho=3$, $h=-4$, $I=10$

$\therefore \vec{H} = 0.36\,\hat{a}_z$





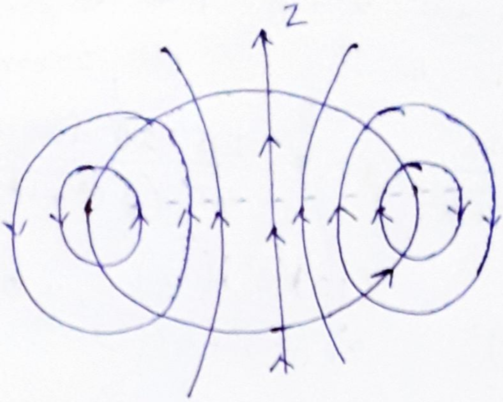
$\vec{dl} = \rho d\phi\hat{a}_\phi$ $\vec{R} = -\rho\hat{a}_\rho - h\hat{a}_z$

$$\vec{dl}\times\vec{R} = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & \rho d\phi & 0 \\ -\rho & 0 & -h \end{vmatrix}$$

$= \hat{a}_\rho(-h\rho d\phi) - \hat{a}_\phi(0)$
$+ \hat{a}_z(\rho^2 d\phi)$

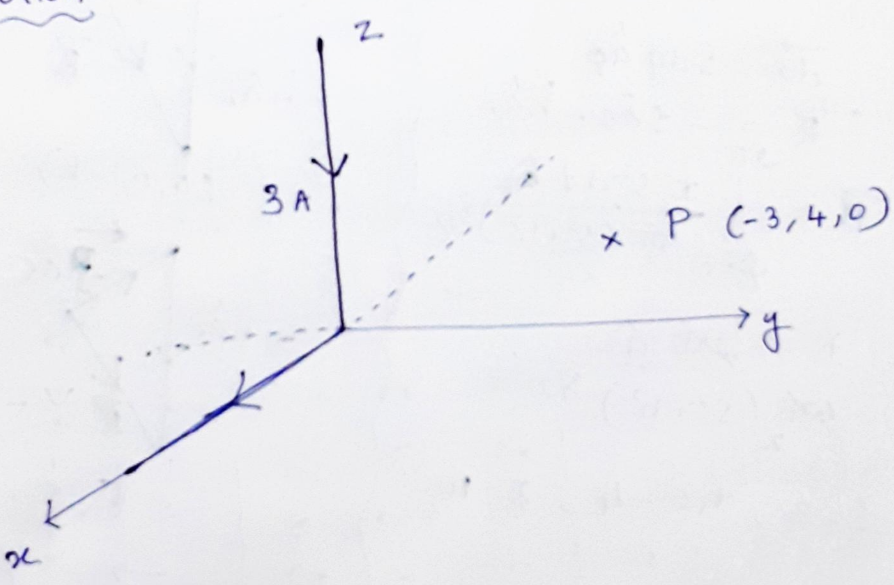
$\Rightarrow$ $\hat{a}_\rho$ component concel due to symmetry

The flux lines due to current loop



Example 7.2) Find $\vec{H}$ at $(-3, 4, 0)$ due to current filament shown in figure

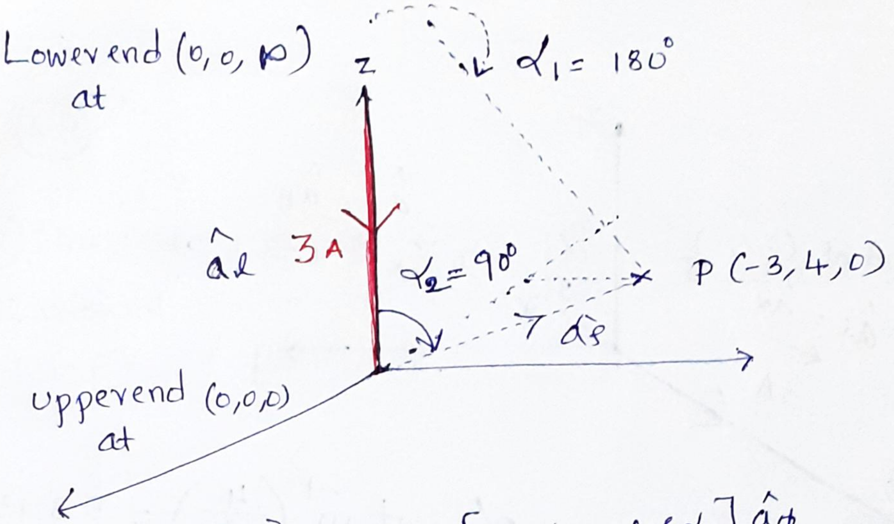
Solution:



In the given problem point $P(-3,4,0)$ is in xy-plane $\therefore$ $\vec{H} = \vec{H_1} + \vec{H_2}$

$\vec{H_1}$ = Magnetic field intensity at 'P' due to the portion of the filament along x-axis

$\vec{H_2}$ = Magnetic field intensity at 'P' due to the portion of the filament along Z-axis



We know $\vec{H}_2 = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$

$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$

$\alpha_2 = 90^\circ \qquad \alpha_1 = 180^\circ$

$\hat{a}_l = -\hat{a}_z \qquad \hat{a}_\rho = \frac{-3\hat{a}_x + 4\hat{a}_y + 0\hat{a}_z}{\sqrt{3^2+4^2}}$

$\hat{a}_\rho = -\frac{3}{5}\hat{a}_x + \frac{4}{5}\hat{a}_y + 0\hat{a}_z$

$I = 3 \qquad \rho = \sqrt{3^2+4^2} = 5$

$\vec{H}_2 = \frac{3}{4\pi(5)}\left[\cos 90^\circ - \cos 180^\circ\right]\hat{a}_\phi$

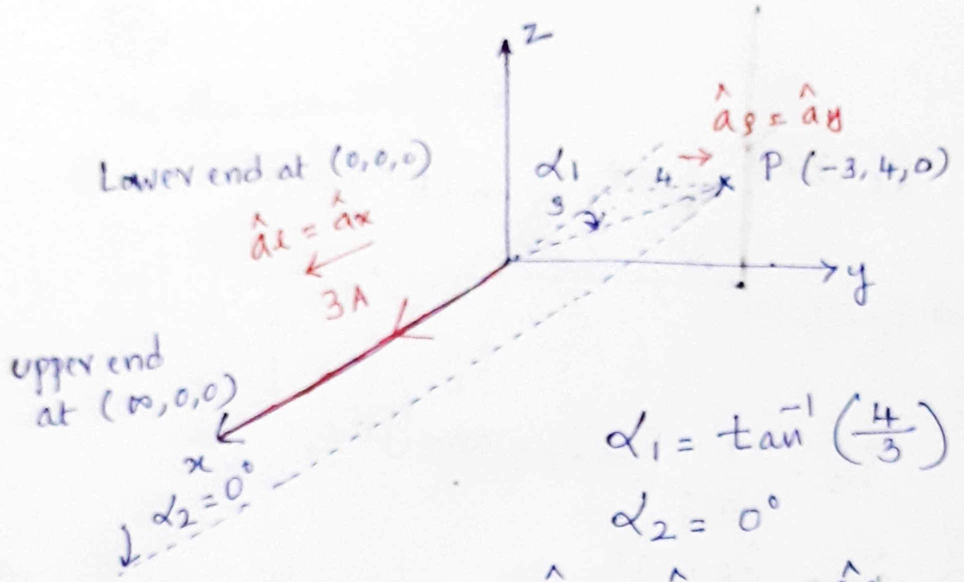
$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & -1 \\ -\frac{3}{5} & \frac{4}{5} & 0 \end{vmatrix}$

$= (-\hat{a}_z) \times \left(-\frac{3}{5}\hat{a}_x + \frac{4}{5}\hat{a}_y\right)$

$= \hat{a}_x\left(\frac{4}{5}\right) - \hat{a}_y\left(-\frac{3}{5}\right) + \hat{a}_z(0) = 0.8\,\hat{a}_x + 0.6\,\hat{a}_y$

$\vec{H}_2 = \frac{3}{4\pi(5)}\left[0 - (-1)\right]\left[0.8\,\hat{a}_x + 0.6\,\hat{a}_y\right]$

$\vec{H}_2 = 38.1971\times10^{-3}\,\hat{a}_x + 28.6478\times10^{-3}\,\hat{a}_y$



$$\alpha_1 = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

$$\alpha_2 = 0°$$

$$\hat{a}_l = \hat{a}_x \qquad \hat{a}_\rho = \frac{4\hat{a}_y}{\sqrt{4^2}} = \hat{a}_y$$

$$\rho = 4$$

$$\vec{H_1} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

$$\hat{a}_\phi = \hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\vec{H_1} = \frac{3}{4\pi(4)}\left[\cos 0° - \cos 53.13°\right]\hat{a}_z$$

$$\vec{H_1} = \frac{3}{4\pi(4)}\left[1 - 0.6\right]\hat{a}_z$$

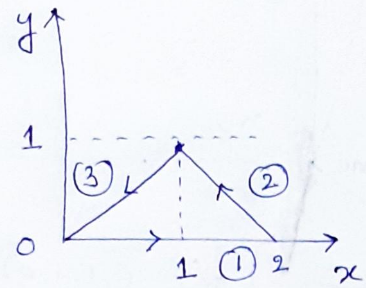
$$\vec{H_1} = 23.8732\times10^{-3}\,\hat{a}_z$$

$$\vec{H_1} = 23.8732\times10^{-3}\,\hat{a}_z \qquad \vec{H_2} = 38.1971\times10^{-3}\,\hat{a}_x + 28.6478\times10^{-3}\,\hat{a}_y$$

$$\vec{H} = \vec{H_1} + \vec{H_2}$$

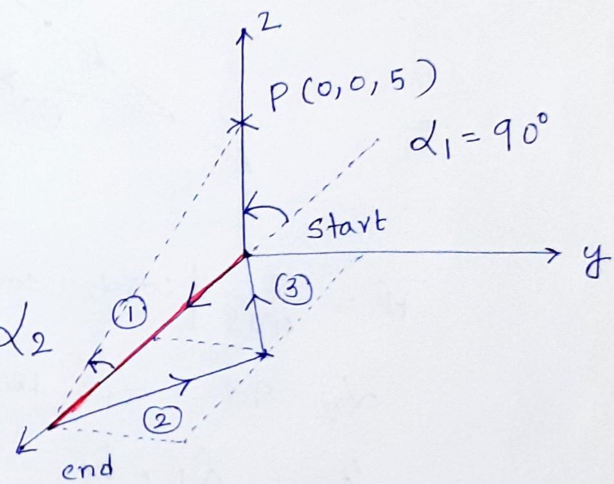
$$\vec{H} = 38.1971\times10^{-3}\,\hat{a}_x + 28.6478\times10^{-3}\,\hat{a}_y + 23.8732\times10^{-3}\,\hat{a}_z \ \text{A/m}$$

7.1) The conducting loop triangular loop shown in figure carries a current of 10 A. Find $\vec{H}$ at (0,0,5) due to side ① of the loop.



$\alpha_2 = \tan^{-1}\left(\frac{5}{2}\right)$

$\alpha_2 = 68.19^\circ$



$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

$$\hat{a}_\phi = \hat{a}_\ell \times \hat{a}_\rho$$

$\hat{a}_\ell = \hat{a}_x$ $\quad\quad$ $\hat{a}_\rho = \hat{a}_z$

$I = 10$ $\quad\quad$ $\rho = 5$

$$\vec{H} = \frac{10}{4\pi(5)}\left[\cos 68.19^\circ - \cos 90^\circ\right](\hat{a}_x \times \hat{a}_z)$$

$$\vec{H} = -59.108 \times 10^{-3}\,\hat{a}_y \text{ A/m}$$

Practice Exercise 7.1

7.1) The conducting triangular loop shown in figure carries a current of 10 A. Find $\vec{H}$ at (0,0,5) due to side ③ of the loop

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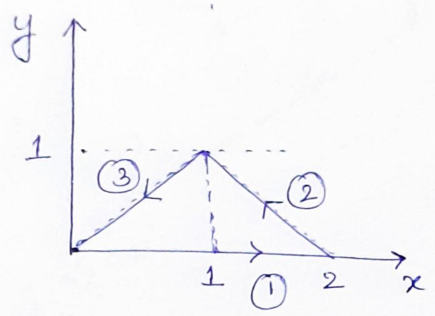
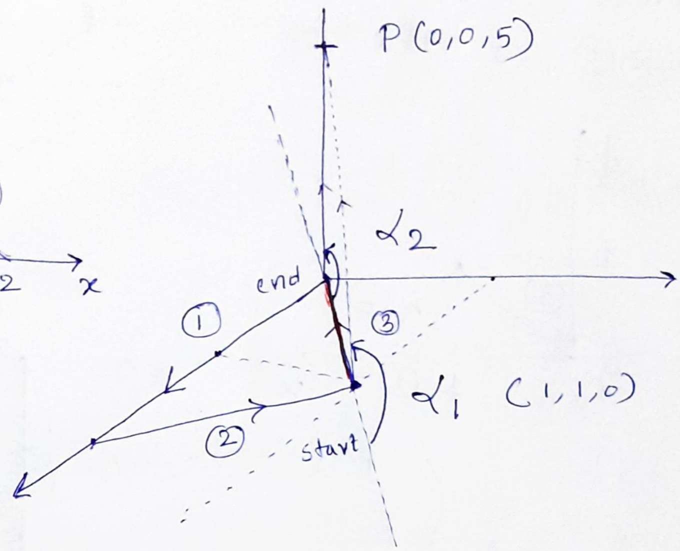




$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

$\alpha_2 = 90°$ $\quad \alpha_1 = 180 - \tan^{-1}\left(\frac{5}{\sqrt{2}}\right) = 180° - \tan^{-1}(3.535)$

$= 180° - 74.20$

$= 105.8°$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho$$

$$\hat{a}_l = \frac{-\hat{a}_x - \hat{a}_y + 0}{\sqrt{2}} \qquad \hat{a}_\rho = \hat{a}_z$$

$$\hat{a}_l \times \hat{a}_\rho = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \hat{a}_x\left(-\frac{1}{\sqrt{2}}\right) - \hat{a}_y\left(-\frac{1}{\sqrt{2}}\right) + \hat{a}_z(0)$$

$$= -\frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y$$

$\rho = 5$

$$\vec{H} = \frac{10}{4\pi(5)}\left[\cos 90^\circ - \cos(105.8)\right]\left[-\frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y\right]$$

$$= \frac{10}{4\pi(5)}\left[0 - (-0.2722)\right]\left[-\frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y\right]$$

$$= \frac{10}{4\pi 5}\left[0.2722\right]\left[-\frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y\right]$$

$$= -\underline{30.628 \times 10^{-3}}\,\hat{a}_x + \underline{30.628 \times 10^{-3}}\,\hat{a}_y \text{ A/m}$$

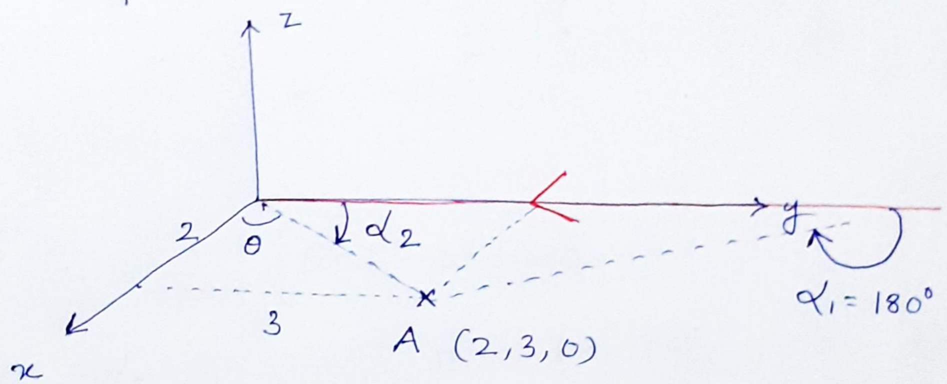
Practice Exercise 7.2

(46)

(1) The positive y-axis (semi-infinite line with respect to the origen) carries a filamentary current 2A in the $-\hat{a}_y$ direction. Assume it is part of a large circuit. Find $\vec{H}$ at

(a) A (2, 3, 0) (b) B (3, 12, -4)

(a) Field point A (2,3,0)



$$\alpha_1 = 180° \qquad \alpha_2 = 90 - \tan^{-1}\left(\frac{3}{2}\right)$$

$$\alpha_2 = 90 - 56.30° = 33.7°$$

$$\rho = 2$$

$$a_l = -\hat{a}_y \qquad \hat{a}_\rho = \hat{a}_x$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = \hat{a}_z (1)$$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = -\hat{a}_y \times \hat{a}_x = \hat{a}_z$$

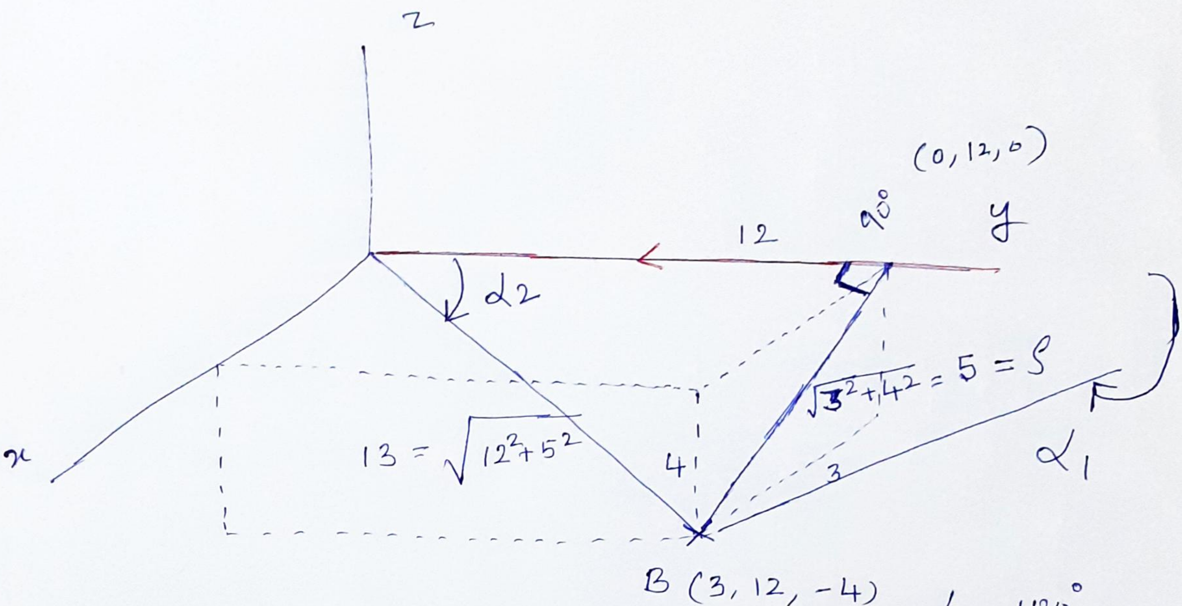
$$H = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\cancel{(\hat{a}_l \times \hat{a}_\rho)}\ \hat{a}_\phi$$

$$= \frac{\cancel{2}}{4\pi(\cancel{2})}\left[\cos 33.7° - \cos 180°\right]\hat{a}_z$$

$$= \frac{1}{4\pi}\left[0.8319 + 1\right]\hat{a}_z = 0.1457\,\hat{a}_z$$

$$= 145.7\times10^{-3}\,\hat{a}_z \text{ A/m}$$

(b) Field point B (3, 12, −4)



$\alpha_1 = 180°$

$\cos \alpha_2 = \frac{12}{13}$ or $\alpha_2 = 22.69°$

or $\alpha_2 = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$

$\rho = \sqrt{3^2+4^2} = 5$

$a_l = -\hat{a}_y$ $\quad \hat{a}_\rho = \frac{3\hat{a}_x + 0\hat{a}_y - 4\hat{a}_z}{\sqrt{3^2+4^2}}$

$\hat{a}_l = -\hat{a}_y$ $\quad \hat{a}_\rho = 0.6\hat{a}_x - 0.8\hat{a}_z$

$$\hat{a}_l \times \hat{a}_\rho = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & -1 & 0 \\ 0.6 & 0 & -0.8 \end{vmatrix} = \hat{a}_x(0.8) + \hat{a}_z(0.6)$$

$$\vec{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1](\hat{a}_x(0.8) + \hat{a}_z(0.6))$$

$$= \frac{2}{4\pi(5)}[\cos 22.62° - \cos 180°][0.8\hat{a}_x + 0.6\hat{a}_z]$$

$$= \frac{2}{4\pi 5}[0.9230 - (-1)][0.8\hat{a}_x + 0.6\hat{a}_z]$$

$$= \underline{48.96 \times 10^{-3}}\,\hat{a}_x + \underline{36.72 \times 10^{-3}}\,\hat{a}_z \text{ A/m}$$