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# FE/Insem./APR-1

**SEAT No.:** [Total No. of Pages: 2

## F.E.

## 107008: ENGINEERING MATHEMATICS - II (2019 Pattern) (Semester - II)

Time: 1 Hour]

IMax. Marks: 30

Instructions to the candidates:

- Attempt Q1 or Q2 and Q3 or Q4.
- Use of electronic pocket calculator is allowed. *2*)
- Assume suitable data, if necessary. *3*)
- Neat diagram must be drawn wherever necessary. **4**)
- Figures to the right indicate full marks. 5)

Q1) a) Solve: 
$$\frac{dy}{dx} = \frac{x - 2y + 5}{2x + y - 1}$$
 [5]  
b) Solve:  $(x^2y^2 + 5xy + 2)ydx + (x^2y^2 + 4xy + 2)xdy = 0$  [5]

b) Solve: 
$$(x^2y^2 + 5xy + 2)ydx + (x^2y^2 + 4xy + 2)xdy = 0$$
 [5]

c) Solve: 
$$\tan y \cdot \frac{dy}{dx} + \tan x = \cos y \cdot \cos^2 x$$
 [5]

**Q2**) a) Solve: 
$$\frac{dx}{dy} = xy + x^2y$$

b) Solve: 
$$x^2 \frac{dy}{dx} = 3x^2 + 2xy + 1$$

[5]

c) Solve: 
$$[2x \ln x - xy] dy + [2y] dx = 0$$

- **Q3**) a) A body is heated to 110 °C and placed in air at 10 °C. After one hour its temperature is 60 °C. How much time is required for it to cool to 30 °C? [5]
  - A constant electromotive force E volt is applied to a circuit containing a b) constant resistance Rohm in series with a constant inductance t henry. If the initial current is zero, show that the current builds upto half its theoretical

maximum in 
$$\frac{L}{R}(\ln 2)$$
 seconds. [5]

P.T.O.

c) A particle of mass m is projected upwards with velocity  $V_0$ . Assuming the air resistance k times its velocity, write the equation of motion. Show

that it will reach maximum height in time 
$$\left(\frac{m}{k}\right).\ln\left(1+\frac{kV_0}{mg}\right)$$
. [5]

OR

- **Q4**) a) Find orthogonal trajectories of the family of curves given by xy = C [5]
  - b) A circuit consists of resistance Rohm and a condenser of C farad connected to a constant electromotive force E volt. If  $\frac{Q}{C}$  is the voltage of the condenser at time t after closing the circuit, show that the voltage at time t is  $E(1-e^{-t/RC})$ . [5]
  - A pipe 10cm in diameter contains steam at 100 °C. It is covered with asbestos 5cm thick for which K=0.0006 and the outside surface is at 30 °C. Find the amount of heat lost per second from a centimeter length pipe. Also find heat lost per hour from a meter length pipe. [5]

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Total No. of Questions: 9]

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[5868]-109

[Total No. of Pages: 4

**SEAT No.:** 

# First Year Engineering

## **ENGINEERING MATHEMATICS-II** (2019 Pattern) (Semester - I & III) (107008)

Time: 2½ Hours]

[Max. Marks: 70

Instructions to the candidates:

- Q.No. 1 is compulsory. 1)
- Solve Q.2 on Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8, or Q.9. 2)
- Neat diagrams must be drawn whenever necessary. 3)
- Figures to the right indicate full marks. 4)
- Use of electronic pocket calculator is allowed. *5)*
- Assume suitable data if necessary.

Q1) Write the correct option for the following multiple choice questions.

a) 
$$\int_{0}^{\frac{\pi}{2}} \cos^{6} x =$$
i) 
$$\frac{5}{16}$$
ii) 
$$\frac{5\pi}{32}$$
iii) 
$$\frac{16\pi}{10}$$
iv) 
$$\frac{5\pi}{48}$$
b) The curve  $y^{2}(x-a) = x^{2}(2a-x)$  is
i) Symmetric about X - axis and net passing through origin
ii) Symmetric about Y - axis and passing through origin
iii) Symmetric about X - axis and passing through origin
iv) Symmetric about Y - axis and passing through origin
iv) Symmetric about Y - axis and passing through origin
iv) The value of double integral 
$$\int_{0}^{1} \frac{1}{\sqrt{1-x^{2}}\sqrt{1-y^{2}}} dx dy \text{ is}$$
[2]

- b)
  - Symmetric about X axis and net passing through origin
  - Symmetric about Y axis and net passing through origin
  - Symmetric about X axis and passing through origin
  - Symmetric about Y axis and passing through origin

c) The value of double integral 
$$\int_{0}^{1} \int_{0}^{1} \frac{1}{\sqrt{1-x^2}\sqrt{1-y^2}} dx dy \text{ is}$$
 [2]

i)

iii)

d) The Centre (C) and radius (r) of the sphere 
$$x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$$
 are [2]

- $C \equiv (0,1,2); r = 4$ **i**)
- $C \equiv (0,2,4); r = 4$

e) The number of loops in the rose curve 
$$r = a \cos 4\theta$$
 are [1]

f) 
$$\iint_{\mathbb{R}} dxdy$$
 represents [1]

- ii) Centre of gravity
- Moment of inertia
- iv) Area of region R

**Q2)** a) If 
$$I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta \ d\theta$$
 prove that  $I_n = \frac{1}{1 - 1} - I_{n-2}$ . [5]

b) Show that 
$$\int_{0}^{1} x^{m-1} (1-x^2)^{n-1} dx = \frac{1}{2} \beta \left( \frac{m}{2}, n \right)$$
. [5]

b) Show that 
$$\int_{0}^{1} x^{m-1} (1-x^{2})^{n-1} dx = \frac{1}{2} \beta \left(\frac{m}{2}, n\right)$$
. [5]

c) Prove that  $\int_{0}^{1} \frac{x^{a}-1}{\log x} dx = \log(b+a), a \ge 0$ .

OR

OR

$$Q3) \text{ a) If } I_n = \int_0^{\pi/2} x^n \sin x \, dx \text{ then prove that } I_n = n \left(\frac{\pi}{2}\right)^{n-1} - n(n-1) I_{n-2}. \quad [5]$$

b) Show that 
$$\int_{0}^{\infty} e^{-h^2 x^2} dx = \frac{\sqrt{\pi}}{2h}$$
. [5]

If 
$$I_n = \int_0^\infty x^n \sin x \, dx$$
 then prove that  $I_n = n \left(\frac{\pi}{2}\right) - n(n-1)$ .  
Show that 
$$\int_0^\infty e^{-h^2 x^2} dx = \frac{\sqrt{\pi}}{2h}.$$
Show that
$$\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \left[ erf(b) - erf(a) \right]$$
OR
$$2$$

<b>04)</b> a)	Trace the curve $x^2y^2 = a^2(y^2 - x^2)$	[5]

- Trace the curve  $r = a(1 \sin \theta)$ . b) [5]
- Find the whole length of the loop of the curve  $3y^2 = x(x-1)^2$ . c) [5]

Q5) a) Trace the curve 
$$y^2(2a-x)=x^3$$
. [5]  
b) Trace the curve  $r=a\cos 2\theta$ .

- b) [5]
- Trace the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ . c) [5]
- Prove that the two spheres  $x^2 + y^2 + z^2 = 2x + 4y 4z = 0$ **Q6)** a)  $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$  touch each other and find the co-ordinates of the point of contact [5]
  - Find the equation of right circular cone whose vertex is (1,-1,2), axis is b) the line  $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{2}$  and the semi-vertical angle 45°. [5]
  - Find the equation of right circular cylinder of radius a whose axis passes c) through the origin and makes equal angles with the co-ordinate axes [5]

- Show that the plane x-2y-2z-7=0 touches the sphere **Q7**) a)  $x^{2} + y^{2} + z^{2} - 10y - 10z - 31 = 0$ . Also find the point of contact. [5]
  - Find the equation of right circular cone with vertex at origin, axis the b) Y-axis and semi-vertical angle 30°. [5]
  - Find the equation of right circular cylinder of radius  $\sqrt{6}$  whose axis is c) the line  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ . [5]

- Change the order of integration and evaluate  $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dx dy$ . **Q8)** a) [5]
  - Find the area of one loop of  $r = a \sin 2\theta$ . b) [5]
  - Find the moment of inertia of one loop of the lemniscate  $r^2 = a^2 \cos 2\theta$ c) about initial line. Given that  $\rho = \frac{2m}{a^2}$ , m is the mass of loop of lemniscate.

[5]

- Evaluate  $\iint y dx dy$  over the region enclosed by the parabola  $x^2 = y$ , and **Q9**) a) the line y = x + 2. [5]
  - Evaluate  $\iiint x^2 yz dx dy dz$ , throughout the volume bounded by the plane x = 0, y = 0, z = 0  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ [5]
  - Find the y coordinate of the centre of gravity of the area bounded by c) by these states of the states  $r = a \sin \theta$  and  $r = 2a \sin \theta$ . Given that the area bounded by these curves is  $\frac{3\pi a^2}{\Delta}$ . [5]

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F.E.							
ENGINEERING MATHEMATICS-II							
	ern) (Semester - I/II						
Time: 2½ Hours] Instructions to the candidates:		[Max. Marks: 70					
1) Q. No.1 is compulsory	2/1						
	r Q.5, Q.6 or Q.7, Q.8 or Q	9.9.					
<ul><li>3) Neat diagrams must be</li><li>4) Figures to the right ind</li></ul>	drawn wherver necessary. licate full marks.						
5) Use of electronic pocke	•	290					
6) Assume suitable data, i	f necessary.						
Q1) Write the correct option	for the following multip	le choice questions					
a) $\int_{0}^{\pi/2} \sin^4 + dt =$		[2]					
)0	0,0						
i) $\frac{3\pi}{16}$	$\frac{3}{8}$						
iii) $\frac{3}{16}$	$\frac{3\pi}{8}$						
	U' 250						
b) The equation of the	tangent to the curve $y(1)$	$(1+x^2) = x$ at origin, if exist is					
i) X=0	ii) $Y=0$ iv) $y = 1$						
iii) $x=1, x=-1$	iv) $y = 3$						
c) The value of double	e integration $\iint \frac{1}{1} \frac{1}{1}$	$\frac{\partial}{\partial x} dx dy = $ [2]					

c) The value of double integration 
$$\int_{0}^{1} \int_{0}^{1} \frac{1}{1+x^{2}} \frac{1}{1+y^{2}} dxdy = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$
i) 
$$\frac{\pi}{2}$$
ii) 
$$\frac{\pi}{4}$$
iv) 
$$\frac{\pi^{2}}{8}$$
P.T.O.

i) 
$$\frac{\pi}{2}$$

ii) 
$$\frac{\pi^2}{2}$$

iii) 
$$\frac{\pi}{4}$$

iv) 
$$\frac{\pi^2}{8}$$

d) Centre (C) of sphere 
$$x^2 + y^2 + z^2 - 2z = 4$$
 is [2]

i) 
$$C \equiv (0,0,0)$$

i) 
$$C \equiv (0,0,1)$$

iii) 
$$C \equiv (0,1,0)$$

iv) 
$$C = (1,0,0)$$

e) The curve 
$$r = 2a \sin \theta$$
 is symmetrical about  
i) Pole ii)  $\theta = 0$   
iii)  $\theta = \frac{\pi}{4}$ 

ii) 
$$\theta = 0$$

iii) 
$$\theta = \frac{\pi}{2}$$

iv) 
$$\theta = \frac{\pi}{4}$$

f) 
$$\iiint_{V} dxdydz \text{ represents}$$

[1]

**Q2**) a) If 
$$I_n = \int_0^{\frac{\pi}{4}} sec^n \theta \ d\theta$$
, then prove that  $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$  [5]

b) Evaluate 
$$\int_{2}^{5} (x-2)^{3} (5-x)^{2} dx$$

i) Area ii) Mass iv) Volume Prove that 
$$I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$$
 [5]

b) Evaluate  $\int_0^5 (x-2)^3 (5-x)^2 dx$  [5]

c) Using DUIS, prove that  $\int_0^5 \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log \left(\frac{a^2 + 1}{2}\right), a > 0$  [5]

OR

Q3) a) Evaluate

i)  $\int_0^{2\pi} \sin^2 \frac{\theta}{2} \cos^{10} \frac{\theta}{2} d\theta$  [3]

ii)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 t \ dt$  [2]

i) 
$$\int_{0}^{2\pi} \sin^{2}\frac{\theta}{2} \cos^{10}\frac{\theta}{2} d\theta$$

ii) 
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 t \ dt$$

b) Evaluate: 
$$\int_{0}^{1} (x \log x)^{4} dx$$
 [5]  
c) Prove that: 
$$\frac{1}{x} \frac{d}{da} erf_{c}(ax) = \frac{1}{a} \frac{d}{dx} erf(ax)$$
 [5]

c) Prove that: 
$$\frac{1}{x} \frac{d}{da} erf_c(ax) = \frac{1}{a} \frac{d}{dx} erf(ax)$$
 [5]

**Q4**) a) Trace the curve 
$$x^2y^2 = a^2(y^2 - x^2)$$
. [5]

b) Trace the curve 
$$r = a(1 + \cos \theta)$$
. [5]

c) Find the are length of Astroid 
$$x^{2/3} + y^{2/3} = a^{2/3}$$
 [5]

Q5) a) Trace the curve 
$$x^3 + y^3 = 3axy$$
. [5]

b) Trace the curve 
$$r = a\cos 2\theta$$
 [5]

c) Trace the curve 
$$x = a(t + \sin t)$$
,  $y = a(1 + \cos t)$ . [5]

- **Q6)** a) Show that the plane x-2y-2z touches the sphere  $x^2+y^2+z^2-$ 10y - 10z - 31 = 0. Also find the point of contact. [5]
  - Find the equation of right circular cone whose vertex is at origin, whose b) axis is the line  $\frac{x}{1} = \frac{y}{2} = \frac{8}{3}$  and which has a semi-vertical angle of 60°. [5]
  - Find the equation of right circular cylinder of radius 3 and axis is the line  $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$
- Show that the two spheres:  $x^2 + y^2 + z^2 = 25$  and  $x^2 + y^2 + z^2 = -18x 24y 40z + 225 = 0$  touches externally. Also find the point of **Q7**) a) contact.
  - Find the equation of right circular cone whose vertex is at (0,0,10), axis b) is the Z-axis and the semi-vertical angle is cos 1 [5]
  - Find the equation of right circular cylinder of radius  $\sqrt{6}$ , whose axis c) passes through the origin and has direction cosines  $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ . [5]

<b>Q</b> 8) a)	Evaluate $\iint xy  dx  dy$ ,	where R is $x^2 = y$ , $y^2 = -x$ .	[5]
	R		

Find area of cardioide  $r = a(1 \cos \theta)$  using double integration. b) [5]

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Find the moment of inertia of one loop of the lemniscate c)  $r^2 = a^2 \cos 2\theta$  about initial line. Given that density  $\rho = \frac{2m}{a^2}$ , m is a mass of the area. [5]

**Q9**) a)

- Change order of integration  $\int_{0}^{5} \int_{2-x}^{2+x} f(x,y) dx dy$ . [5] Find the volume bounded by the cone  $x^2 + y^2 = z^2$  and paraboloid [5] [5]
- Find the x co-ordinate of centre of gravity of one loop of  $r = a\cos 2\theta$ , which is in the first quadrant, given that area of loop is  $A = \frac{\pi a^2}{8}$ . that the state of the state of

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