

Total No. of Questions : 9]

SEAT No. :

P6485

[Total No. of Pages : 4

[5868]-101

F.E. (Semester- I & II)

ENGINEERING MATHEMATICS - I

(2019 Pattern) (107001)

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Q. 1 is compulsory.
- 2) Attempt Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.
- 3) Neat diagrams must be drawn wherever necessary.
- 4) Figures to the right indicate full marks.
- 5) Use of electronic pocket calculator is allowed.
- 6) Assume suitable data, if necessary.

Q1) Write the correct option for the following multiple choice questions.

a) If eigen value of a square matrix A is zero then. [1]

- i) A is non-singular
- ii) A is orthogonal
- iii) A is singular
- iv) None of these

b) If  $u = y^x$  then  $\frac{\partial u}{\partial x}$  is equal to [1]

- i) 0
- ii)  $xy^{x-1}$
- iii)  $y^x \log y$
- iv) None of these

c) The orthogonal transformation  $x = py$  transforms the quadratic form  $Q = x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$  to the canonical form  $Q' = y_1^2 + 2y_2^2 + y_3^2$ . The rank of quadratic form is [2]

- i) 2
- ii) 3
- iii) 1
- iv) 0

d)  $u = \sec^{-1} \left[ \frac{x^2 + y^2}{xy^2} \right]$ . Find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  [2]

- i)  $-\tan u$
- ii)  $-\cot u$
- iii)  $\tan u$
- iv)  $\cot u$

P.T.O.

e) If  $u = x^2 - y^2$  and  $v = 2xy$  then the value of  $\frac{\partial(u, v)}{\partial(x, y)}$  is [2]

- i)  $4(x^2 + y^2)$                       ii)  $-4(x^2 + y^2)$   
 iii)  $4(x^2 - y^2)$                       iv)  $0$

f) A system of linear equations  $Ax = B$ , where  $B$  is a null (zero) matrix is [2]

- i) Always consistent  
 ii) Consistent only if  $|A| = 0$   
 iii) Consistent only if  $|A| \neq 0$   
 iv) In consistent if  $\rho(A) < \text{No. of variables}$

**Q2)** a) If  $z = \tan(y + ax) + (y - ax)^{3/2}$  find value of  $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$ . [5]

b) If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$  then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u) \sin 2u \quad [5]$$

c) If  $u = f(x^2 - y^2; y^2 - z^2; z^2 - x^2)$  find value of  $\frac{1}{x} \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial u}{\partial y} + \frac{1}{z} \frac{\partial u}{\partial z}$  [5]

OR

**Q3)** a) If  $u = ax + by; v = bx - ay$  find value of  $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$  [5]

b) If  $u = \sin^{-1}(\sqrt{x^2 + y^2})$  then find value of  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$  [5]

c) If  $u = f(r, s)$  where  $r = x^2 + y^2; S = x^2 - y^2$  then show that

$$y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 4xy \frac{\partial u}{\partial r} \quad [5]$$

**Q4) a)** If  $x = uv$  and  $y = \frac{u+v}{u-v}$ , find  $\frac{\partial(u,v)}{\partial(x,y)}$ . [5]

b) Examine for functional dependence  $u = \frac{x-y}{1+xy}$ ,  $v = \tan^{-1} x - \tan^{-1} y$  and if dependent find the relation between them. [5]

c) Discuss maxima and minima of  $f(x, y) = x^2 + y^2 + 6x + 12$  [5]

OR

**Q5) a)** Prove  $JJ' = 1$  for  $x = u \cos v$ ,  $y = u \sin v$ . [5]

b) In calculating the volume of a right circular cone, errors of 2% and 1% are made in measuring the height and radius of base respectively find the error in the calculated volume. [5]

c) Find maximum value of  $u = x^2 y^3 z^4$  such that  $2x + 3y + 4z = a$  by Lagrange's method. [5]

**Q6) a)** Investigate for what values of  $\mu$  &  $\lambda$  the equations  $x+y+z = 6$ ,  $x+2y+3z = 10$ ,  $x+2y+\lambda z = \mu$  have i) No solution ii) Infinitely many solutions. [5]

b) Examine for linear dependence and independence the vectors  $(1,1,3)$ ,  $(1,2,4)$ ,  $(1,0,2)$ . If dependent, find the relation between them. [5]

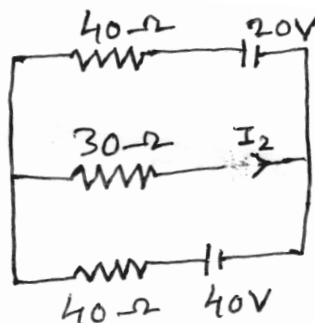
c) Verify whether matrix  $A = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$  is orthogonal or not. [5]

OR

**Q7) a)** Solve the system of equations  $x+y+2z = 0$ ,  $x+2y+3z=0$ ,  $x+3y+4z=0$ . [5]

b) Examine following vectors for linear dependence and independence  $(1,-1,1)$ ,  $(2,1,1)$ ,  $(3,0,2)$ . If dependent, find the relation between them. [5]

c) Determine the currents in the network given in the figure. [5]



- Q8) a)** Find the eigen values of the matrix  $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ . [5]

Find eigen vector corresponding to the highest eigen value.

- b)** Verify cayley-Hamilton theorem for  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ . Hence find  $A^{-1}$  if it exists. [5]

- c)** Find the modal matrix  $p$  which diagonalises  $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$ . [5]

OR

- Q9) a)** Find the eigen values of  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ . [5]

Find eigen vector corresponding to the highest eigen value.

- b)** Verify cayley-Hamilton theorem for  $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  [5]

- c)** Reduce the quadratic form  $Q = x_1^2 + 2x_2^2 + x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$  to canonical form by congruent transformations. [5]



Total No. of Questions : 9]

SEAT No. :

P-3926

[Total No. of Pages : 5

[60011-4001

F.E.

**ENGINEERING MATHEMATICS - I**  
**(2019 Pattern) (Semester - I) (107001)**

Time : 2½ Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Question No. 1 is compulsory.
- 2) Solve Q. No. 2 or Q. No. 3, Q. No. 4 or Q. No. 5, Q. No. 6 or Q. No. 7, Q. No. 8 or Q. No. 9.
- 3) Neat diagrams must be drawn wherever necessary.
- 4) Figures to the right indicate full marks.
- 5) Electronic pocket calculator is allowed.
- 6) Assume suitable data, if necessary.

Q1) Write the correct option for the following multiple choice questions :

a) If  $u = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{x^2 + y^2}$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to [2]

i)  $2u$

ii)  $-2u$

iii)  $0$

iv)  $\text{None}$

b) If  $u = x^y$  then  $\frac{\partial u}{\partial y}$  is equal to [1]

i)  $0$

ii)  $yx^{y-1}$

iii)  $x^y \log x$

iv)  $x^{y-1}$

c) If  $x = uv$ ,  $y = \frac{u}{v}$  then the value of  $\frac{\partial(u,v)}{\partial(x,y)}$  is [2]

i)  $\frac{-2u}{v}$

ii)  $uv$

iii)  $\frac{v}{2u}$

iv)  $\frac{-v}{2u}$

P.T.O.

- d) A is orthogonal matrix then  $A^{-1}$  equal to [1]  
 i) A ii)  $A^T$   
 iii)  $A^2$  iv) 1
- e) For what value of K the homogeneous system  $x + 2y - z = 0$ ,  $3x + 8y - 3z = 0$ ;  $2x + 4y + (k-3)z = 0$  has infinitely many solution. [2]  
 i)  $K = 0$  ii)  $K = 1$   
 iii)  $K = 2$  iv)  $K = 3$
- f) Using Cayley Hamilton theorem  $A^{-1}$  for the matrix  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$  is calculated from [2]  
 i)  $\frac{1}{5}(-A - 4I)$  ii)  $\frac{1}{5}(A - 4I)$   
 iii)  $\frac{1}{5}(A + 4I)$  iv)  $\frac{1}{5}(4I - A)$

Q2) a) If  $u = \ln(x^2 + y^2)$ , show that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ . [5]

b) If  $e^{2u} = y^2 - x^2$ ,  $\operatorname{cosec} v = \frac{y}{x}$  then find the value of [5]

$$\left( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) \cdot \left( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right)$$

c) If  $u = f(x - y, y - z, z - x)$  then find the value of  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ . [5]

OR

Q3) a) If  $u = ax + by$ ,  $v = bx - ay$  find the value of  $\left( \frac{\partial u}{\partial x} \right)_y \cdot \left( \frac{\partial x}{\partial u} \right)_v$ . [5]

b) If  $T = \sin\left(\frac{xy}{x^2 + y^2}\right) + \sqrt{x^2 + y^2}$ , find the value of  $x \frac{\partial T}{\partial x} + y \frac{\partial T}{\partial y}$ . [5]

c) If  $u = f(r, s)$  where  $r = x^2 + y^2$ ,  $s = x^2 - y^2$  then show that  

$$y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 4xy \frac{\partial u}{\partial r}$$
 [5]

**Q4) a)** If  $x = u + v$ ,  $y = v^2 + w^2$ ,  $z = u^3 + w^3$  then find  $\frac{\partial u}{\partial x}$ . [5]

b) In calculating resistance R of a circuit by using the formula :

$$R = \frac{V}{I}$$

errors of 3% and 1% are made in measuring Voltage V and current I respectively. Find the % error in the calculated resistance. [5]

c) Discuss the maxima and minima of : [5]

$$f(x, y) = x^2 + y^2 + xy + x - 4y + 5$$

OR

**Q5) a)** If  $u + v^2 = x$ ,  $v + w^2 = y$ ,  $w + u^2 = z$  find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$  [5]

b) Examine for functional dependence : [5]

$$u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2$$

c) A space probe in the shape of the ellipsoid  $4x^2 + y^2 + 4z^2 = 16$  enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point  $(x, y, z)$  on the surface of the probe is

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600.$$

Find the hottest point on the surface of the probe, by using Lagrange's method. [5]

**Q6) a)** Examine for consistency and if consistent then solve it [5]

$$2x + 3y + 5z = 1 ; 3x + y - z = 2 ; x + 4y - 6z = 1$$

b) Examine whether the vectors [5]

$$X_1 = (1, 1, -1, 1); X_2 = (1, -1, 2, -1); X_3 = (3, 1, 0, 1)$$

are linearly independent or dependent. If dependent find relation between them.

c) If  $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$  is orthogonal [5]

Find a, b, c.

OR

Q7) a) Investigate for what values of  $k$ , the equations [5]

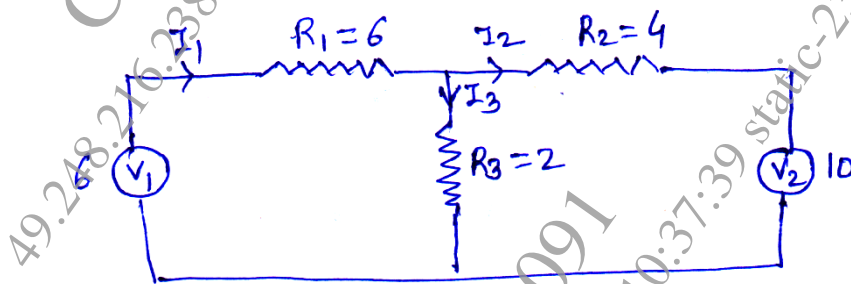
$x + y + z = 1$ ;  $2x + y + 4z = k$ ;  $4x + y + 10z = k^2$  have infinite number of solution? Hence find solution.

b) Examine whether the vectors. [5]

$X_1 = (2, 3, 4, -2)$ ;  $X_2 = (-1, -2, -2, 1)$ ;  $X_3 = (1, 1, 2, -1)$

are linearly independent or dependent. If dependent find relation between them.

c) Find the current  $I_1$ ;  $I_2$ ;  $I_3$  in the circuit shown in the figure [5]



Q8) a) Find eigen values and eigen vectors of the following matrix [5]

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

b) Verify Cayley-Hamilton theorem for  $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$  and use it to find  $A^{-1}$ . [5]

c) Find the modal matrix  $p$  which transform the matrix  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$  to the diagonal form. [5]

OR



**Q9)** a) Find eigen values and eigen vectors of the following matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .  
[5]

b) Verify Cayley-Hamilton theorem for  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$  and use it to find  $A^{-1}$  [5]

c) Reduce the following quadratic form to the "sum of the squares form". [5]

$$Q(x) = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz$$



Total No. of Questions—8]

[Total No. of Printed Pages—4+1

Seat No.	
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**[5667]-1001**

**F.E. (I Semester) EXAMINATION, 2019**

**ENGINEERING MATHEMATICS—I**

**(Phase-II)**

**(2019 PATTERN)**

**Time : 2½ Hours**

**Maximum Marks : 70**

- N.B. :—** (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,  
Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
- (ii) Use of electronic pocket calculator is allowed.
- (iii) Assume suitable data, if necessary.
- (iv) Neat diagrams must be drawn wherever necessary.
- (v) Figures to the right indicate full marks.

1. (a) If  $z = \tan (y + ax) + (y - ax)^{3/2}$ , find the value of  $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$ . [6]

(b) If  $T = \sin \left( \frac{xy}{x^2 + y^2} \right) + \sqrt{x^2 + y^2}$ , by using Euler's theorem find  $x \frac{\partial T}{\partial x} + y \frac{\partial T}{\partial y}$ . [6]

(c) If  $u = x^2 - y^2$ ,  $v = 2xy$  and  $z = f(u, v)$ , then show that  $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2} \frac{\partial z}{\partial u}$ . [6]

P.T.O.

Or

2. (a) If  $x = u \tan v$ ,  $y = u \sec v$ , prove that : [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x.$$

- (b) If  $u = \sin^{-1} \left( \frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ , by using Euler's theorem.

prove that : [6]

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u).$$

- (c) If  $x = \frac{\cos \theta}{u}$ ,  $y = \frac{\sin \theta}{u}$  and  $z = f(x, y)$ , then show that : [6]

$$u \frac{\partial z}{\partial u} - \frac{\partial z}{\partial \theta} = (y-x) \frac{\partial z}{\partial x} - (y+x) \frac{\partial z}{\partial y}.$$

3. (a) If  $u = x + y + z$ ,  $v = x^2 + y^2 + z^2$ ,  $w = xy + yz + zx$   
find  $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ . [6]

- (b) Examine whether  $u = \frac{x-y}{1+xy}$ ,  $v = \tan^{-1} x - \tan^{-1} y$  are functionally dependent, if so find the relation between them. [5]

- (c) Find the extreme values of  $x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$ . [6]

Or

4. (a) If  $u = x + y^2$ ,  $v = y + z^2$ ,  $w = z + x^2$ , using Jacobian  
find  $\frac{\partial x}{\partial u}$ . [6]

(b) A power dissipated in a resistor is given by  $P = \frac{\varepsilon^2}{R}$ . If errors of 3% and 2% are found in  $\varepsilon$  and  $R$  respectively, find the percentage error in  $P$ . [5]

(c) Using Lagrange's method find extreme value of  $xyz$  if  $x + y + z = a$ . [6]

5. (a) Examine for consistency of the system of linear equations and solve if consistent : [6]

$$\begin{aligned}x_1 + x_2 + x_3 &= 0 \\-2x_1 + 5x_2 + 2x_3 &= 1 \\8x_1 + x_2 + 4x_3 &= -1\end{aligned}$$

(b) Examine for linear dependence or independence the vectors  $(1, 1, 1, 3)$ ,  $(1, 2, 3, 4)$ ,  $(2, 3, 4, 7)$ . Find the relation between them if dependent. [6]

(c) Determine the values of  $a$ ,  $b$ ,  $c$  when  $A$  is orthogonal where : [5]

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}.$$

Or

6. (a) Investigate for what values of  $a$  and  $b$ , the system of equations  $2x - y + 3z = 2$ ,  $x + y + 2z = 2$ ,  $5x - y + az = b$  have :

(1) No solution

(2) A unique solution

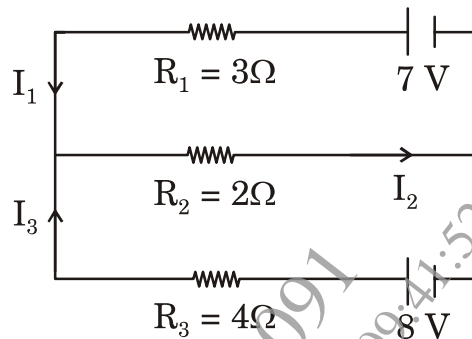
(3) An infinite number of solutions. [6]

- (b) Examine for linear dependence or independence the vectors

$$x_1 = (2, 3, 4, -2), x_2 = (1, 1, 2, -1), x_3 = \left(\frac{-1}{2}, -1, -1, \frac{1}{2}\right).$$

Find the relation between them if dependent. [6]

- (c) Determine the currents in the network given in figure below : [5]



7. (a) Find the eigen values and the corresponding eigen vectors for the following matrix : [6]

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

- (b) Verify Cayley-Hemilton theorem for  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$  and use it to find  $A^{-1}$ . [6]

- (c) Find a matrix P that diagonalizes the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}. \quad [6]$$

Or

8. (a) Find the eigen values and the corresponding eigen vectors for the following matrix : [6]

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}.$$

- (b) Verify Cayley-Hamilton theorem for  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$  and use it to find  $A^{-1}$ . [6]

- (c) Reduce the following quadratic form to the sum of the squares form : [6]

$$Q = 2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6xz.$$

**OCT/FE/INSEM-1**  
**F.E. (Phase - I)**  
**ENGINEERING MATHEMATICS - I**  
**(2019 Pattern)**

Time : 1 Hour]

[Max. Marks : 30

Instructions to the candidates:

- 1) Attempt Q.1 or Q.2 and Q.3 or Q.4.
- 2) Use of electronic pocket calculator is allowed.
- 3) Assume suitable data, if necessary.
- 4) Neat diagrams must be drawn wherever necessary.
- 5) Figures to the right indicate full marks.

Q1) a) For  $0 < a < b$ , show that

[5]

$$\left( \frac{b-a}{b} \right) < \log \left( \frac{b}{a} \right) < \left( \frac{b-a}{a} \right)$$

Hence show that  $\frac{1}{4} < \log \left( \frac{4}{3} \right) < \frac{1}{3}$

b) By using Taylor's theorem, expand  $f(x) = e^x$  in powers of  $(x-2)$ .

[5]

c) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x}{2} \right)^{\frac{1}{x}}$

[5]

OR

Q2) a) Prove that  $\log(1 + \tan x) = x - \frac{x^2}{2} + \frac{2}{3}x^3 - \dots$

[5]

b) Expand  $7 + (x+1) + 3(x+1)^3 + (x+1)^4$  in ascending powers of  $x$  by using Taylor's theorem.

[5]

c) Find  $a$  and  $b$  if

$$\lim_{x \rightarrow 0} \left[ \frac{a \cos x - a + bx^2}{x^4} \right] = \frac{1}{12}$$

[5]

P.T.O.

- Q3)** a) Find fourier series to represent the function  
 $f(x) = x$  for  $-\pi < x < \pi$  and  $f(x) = f(x + 2\pi)$ . [5]
- b) Find half range cosine series for  $f(x) = x^2$ ,  $0 < x < 2$ . [5]
- c) Obtain constant term and coefficients of the first sine and cosine terms in the Fourier expansion of y as given in the following table. [5]

(Given  $f(x) = f(x + 2\pi)$ )

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\pi$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	1.0	1.4	1.9	1.7	1.5	1.2

OR

- Q4)** a) Find Fourier series for the function  $f(x) = x^2 - 2$ ,  $-2 \leq x \leq 2$  and  $f(x) = f(x + 4)$ . [5]
- b) Find half-range sine series for  $f(x) = \pi x - x^2$  where  $0 < x < \pi$ . [5]
- c) Find first three terms in cosine series to represent y as given in the following table. [5]

x	0	1	2	3	4	5
y	4	8	15	7	6	2





Total No. of Questions : 4]

SEAT No. :

**PA-1678**

[Total No. of Pages : 2

[5931]-1901

**F.E.**

**ENGINEERING MATHEMATICS-I**  
**(2019 Pattern) (Semester-I) (107001)**

*Time : 1 Hour]*

*[Max. Marks : 30*

*Instructions to the candidates:*

- 1) Attempt Q1 or Q2 and Q3 or Q4.
- 2) Figures to the right indicate full marks.
- 3) Assume suitable data wherever necessary.
- 4) Use of electronic pocket calculator is allowed.

**Q1) a)** If  $f(x) = \sin^{-1}x$  then show that  $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$  where  $0 < a < b < 1$ . [5]

**b)** Using Taylor's theorem, expand  $1+2x+3x^2+4x^3$  in powers of  $x+1$  [5]

**c)** Evaluate  $\lim_{x \rightarrow \frac{\pi}{2}} (\cos x)^{\cos x}$  [5]

OR

**Q2) a)** Expand  $\sqrt{1 + \sin x}$  upto  $x^4$  in ascending powers of  $x$  [5]

**b)** Expand  $\log \cos x$  in ascending powers of  $(x - \frac{\pi}{3})$  upto the term in  $(x - \frac{\pi}{3})^2$  by using Taylor's theorem. [5]

**c)** Find the values of  $a$  and  $b$  if  $\lim_{x \rightarrow 0} \frac{\sin x + ax + bx^3}{x^3} = 0$  [5]

**Q3) a)** Find Fourier series for  $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 < x < 2\pi$  and  $f(x) = f(x+2\pi)$  [5]

**b)** Find half-range sine series for  $f(x) = 2x - 1, 0 < x < 1$  [5]

**P.T.O.**

- c) Obtain the constant term and the coefficients of the first sine and cosine term in the fourier series of  $f(x)$  as given in the following table. [5]

x	0	1	2	3	4	5
y	9	18	24	28	26	20

OR

- Q4) a) Find the fourier series to represent [5]

$$f(x) = \begin{cases} -3, & -1 < x < 0 \\ 3, & 0 < x < 1 \end{cases}, f(x) = f(x+2)$$

- b) Find half-range consine series for  $f(x) = x^2, 0 < x < \pi$  [5]

- c) Find half-range sine series for  $f(x) = 1, 0 < x < \pi$ . Hence using parsevals identify, deduce that [5]

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$$

