Chapter 12

PROJECTIONS OF PLANES

12-0. INTRODUCTION

Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

In this chapter, we shall discuss the following topics:

1. Types of planes and their projections.
2. Traces of planes.

12-1. TYPES OF PLANES

Planes may be divided into two main types:

(1) Perpendicular planes.

(2) Oblique planes.

(1) Perpendicular planes: These planes can be divided into the following sub-types:

(i) Perpendicular to both the reference planes.

(ii) Perpendicular to one plane and parallel to the other.

(iii) Perpendicular to one plane and inclined to the other.

(i) *Perpendicular to both the reference planes* (fig. 12-1)**:** A square *ABCD* is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to *xy*.

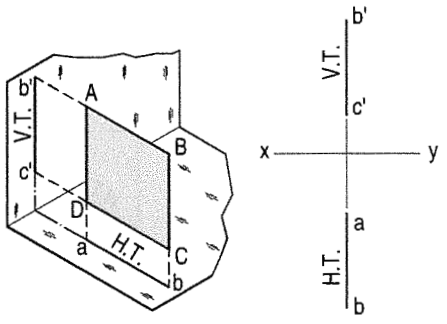


FIG. 12-1

The front view *b'c'* and the top view *ab* of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) *Perpendicular to one plane and parallel to the other plane:*

(a) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 12-2(i)].

A triangle *PQR* is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to *xy*. It has no V.T.

The front view *p'q'r'* shows the exact shape and size of the triangle. The top view *pqr* is a line parallel to *xy*. It coincides with the H.T.

(b) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 12-2(ii)].

A square *ABCD* is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to *xy*. It has no H.T.

The top view *abcd* shows the true shape and true size of the square. The front view *a'b'* is a line, parallel to *xy*. It coincides with the V.T.

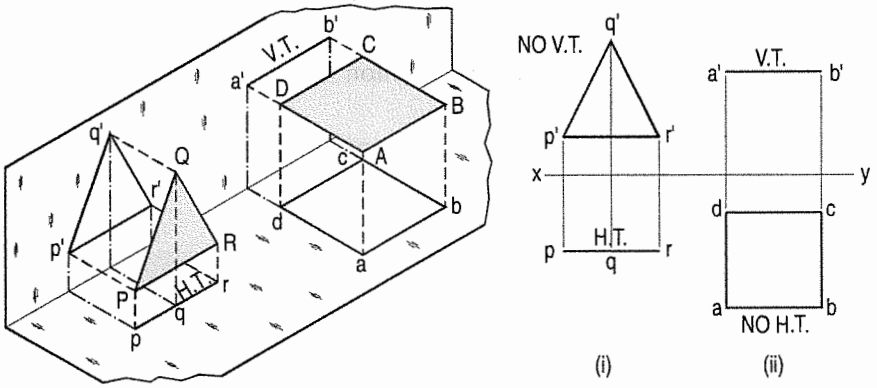


FIG. 12-2

(iii) *Perpendicular to one plane and inclined to the other plane:*

(a) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 12-3).

A square *ABCD* is perpendicular to the H.P. and inclined at an angle ø to the V.P. Its V.T. is perpendicular to *xy*. Its H.T. is inclined at ø to xy.

Its top view *ab* is a line linclined at ø to *xy*. The front view *a'b'c'd'* is smaller than *ABCD*.

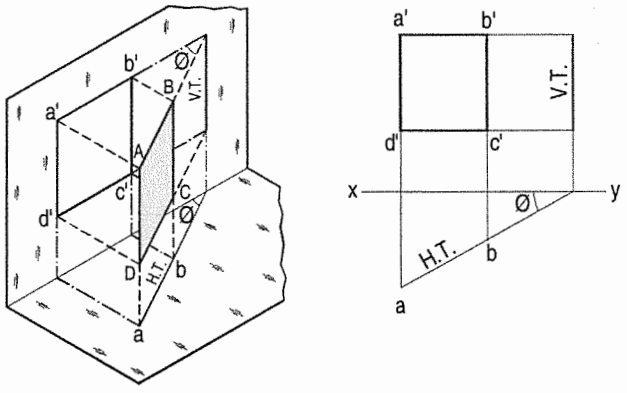


FIG. 12-3

(b) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 12-4).

A square *ABCD* is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to *xy*. Its V.T. makes the angle θ with *xy*. Its front view *a'b'* is a line inclined at θ to *xy*. The top view *abcd* is a rectangle which is smaller than the square *ABCD*.

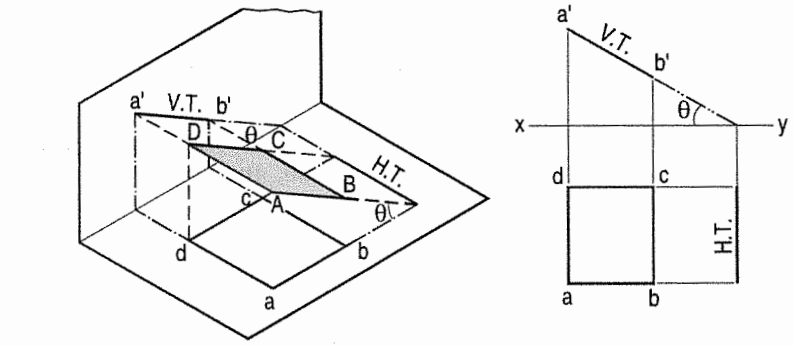


FIG. 12-4

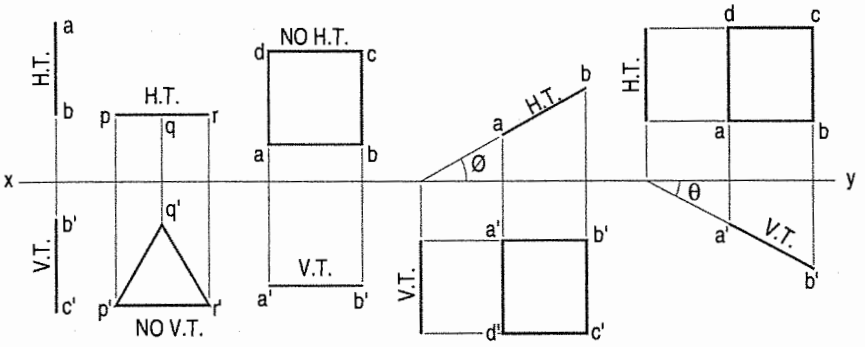


FIG. 12-5

Fig. 12-5 shows the projections and the traces of all these perpendicular planes by third-angle projection method.

(2) Oblique planes: Planes which are inclined to both the reference planes are called *oblique planes*. Representation of oblique planes by their traces is too advanced to be included in this book.

A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

12-2. TRACES OF PLANES

A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them.

These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

12-3. GENERAL CONCLUSIONS

(1) Traces:

(a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to *xy*.

(b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to *xy* (except when it is parallel to the other plane).

(c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to *xy*.

(d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with *xy*. When it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with *xy*.

(e) When a plane has two traces, they, produced if necessary, intersect in *xy* (except when both are parallel to *xy* as in case of some oblique planes).

(2) Projections:

(a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.

(b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

(c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy*. Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 12-1. *Show by means of traces, each of the following planes:*

(a) *Perpendicular to the H.P. and the V.P.*

(b) *Perpendicular to the H.P. and inclined at 30° to the V.P.*

(c) *Parallel to and 40 mm away from the V.P.*

(d) *Inclined at 45° to the H.P. and perpendicular to the V.P.*

(e) *Parallel to the H.P. and 25 mm away from it.*

Fig. 12-6 and fig. 12-7 show the various traces.

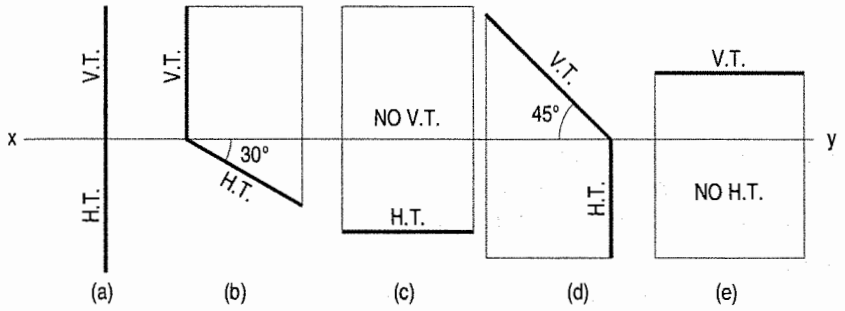
(a) The H.T. and the V.T. are in a line perpendicular to xy.

(b) The H.T. is inclined at 30° to *xy*; the V.T. is normal to *xy*; both the traces intersect in *xy*.

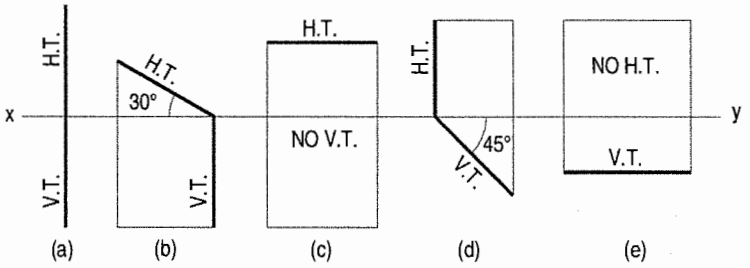
(c) The H.T. is parallel to and 40 mm away from *xy*. It has no V.T.

(d) The H.T. is perpendicular to *xy*; the V.T. makes 45° angle with *xy*; both intersect in *xy*.

(e) The V.T. is parallel to and 25 mm away from *xy*. It has no H.T.



(First-angle projection)
FIG. 12-6



(Third-angle projection)
FIG. 12-7

12-4. PROJECTIONS OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES

The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(1) When the plane is parallel to the H.P.: The top view should be drawn first and the front view projected from it.

Problem 12-2. (fig. 12-8): *An equilateral triangle of 50 mm side has its V.T. parallel to and 25 mm above xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.*

As the V.T. is parallel to *xy* and as there is no H.T. the triangle is parallel to the H.P. Therefore, begin with the top view.

(i) Draw an equilateral triangle *abc* of 50 mm side, keeping one side, say *ac*, inclined at 45° to *xy*.

(ii) Project the front view, parallel to and 25 mm *above* *xy*, as shown.

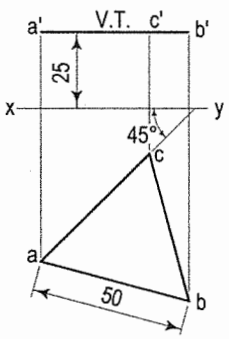


FIG. 12-8

(2) When the plane is parallel to the V.P.: Beginning should be made with the front view and the top view projected from it.

Problem 12-3. (fig. 12-9): *A square ABCD of 40 mm side has a corner on the H.P. and 20 mm in front of the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.*

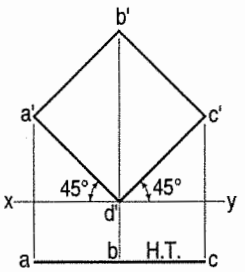


FIG. 12-9

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The front view will show the true shape and position of the square.

(i) Draw a square *a'b'c'd'* in the front view with one corner in *xy* and all its sides inclined at 45° to *xy*.

(ii) Project the top view keeping the line *ac* parallel to *xy* and 30 mm below it. The top view is its H.T. It has no V.T.

12-5. PROJECTIONS OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER

When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(1) Plane, inclined to the H.P. and perpendicular to the V.P.: When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its top view will show the true shape. The front view will be a line parallel to *xy*. The plane is then tilted so that it is inclined to the H.P. The new front view will be inclined to *xy* at the true inclination. In the top view the corners will move along their respective paths (parallel to *xy*).

Problem 12-4. (fig. 12-10): *A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.*

Assuming it to be parallel to the H.P.

(i) Draw the pentagon in the top view with one side perpendicular to *xy* [fig. 12-10(i)]. Project the front view. It will be the line *a'c'* contained by *xy*.

(ii) Tilt the front view about the point *a'*, so that it makes 45° angle with *xy*.

(iii) Project the new top view $ab_1c_1d_1e$ upwards from this front view and horizontally from the first top view. It will be more convenient if the front view is reproduced in the new position separately and the top view projected from it, as shown in fig. 12-10(ii). The V.T. coincides with the front view and the H.T. is perpendicular to *xy*, through the point of intersection between *xy* and the front view-produced.

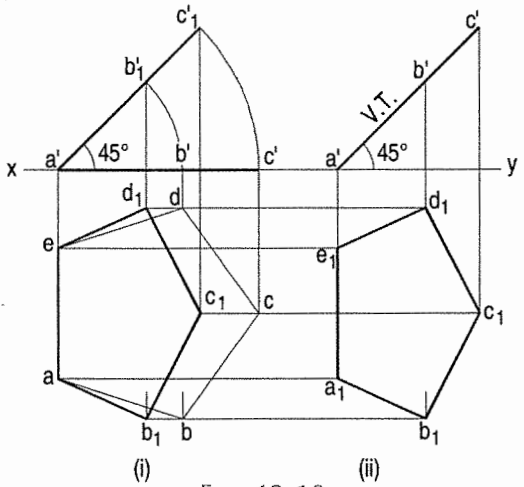


FIG. 12-10

(2) Plane, inclined to the V.P. and perpendicular to the H.P.: In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 12-5. (fig. 12-11): *Draw the projections of a circle of 50 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 30 mm above the H.P. and 20 mm in front of the V.P. Show also its traces.*

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

(i) Assuming the circle to be parallel to the V.P., draw its projections. The front view will be a circle [fig. 12-11(i)], having its centre 30 mm above *xy*. The top view will be a line, parallel to and 20 mm below *xy*.

(ii) Divide the circumference into twelve equal parts (with a 30°-60° set-square) and mark the points as shown. Project these points in the top view. The centre *O* will coincide with the point 4.

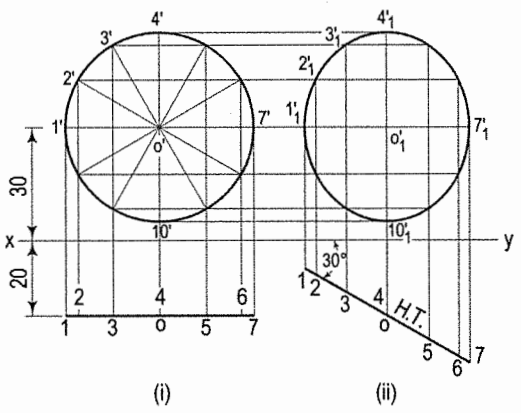


FIG. 12-11

(iii) When the circle is tilted, so as to make 30° angle with the V.P., its top view will become inclined at 30° to *xy*. In the front view all the points will move along their respective paths (parallel to *xy*). Reproduce the top view keeping the centre *o* at the same distance, viz. 20 mm from *xy* and inclined at 30° to *xy* [fig. 12-11(ii)].

(iv) For the final front view, project all the points upwards from this top view and horizontally from the first front view. Draw a freehand curve through the twelve points $1'_1$, $2'_1$ etc. This curve will be an ellipse.

12-6. PROJECTIONS OF OBLIQUE PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to that plane and inclined to the other plane, its projections are drawn in three stages.

(1) If the surface of the plane is inclined to the H.P. and an edge (or a diameter or a diagonal) is parallel to the H.P. and inclined to the V.P.,

(i) in the initial position the plane is assumed to be parallel to the H.P. and an edge perpendicular to the V.P.

(ii) It is then tilted so as to make the required angle with the H.P. As already explained, its front view in this position will be a line, while its top view will be smaller in size.

(iii) In the final position, when the plane is turned to the required inclination with the V.P., only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distances of all the corners from *xy* will remain the same as in the second front view.

If an edge is in the H.P. or on the ground, in the initial position, the plane is assumed to be lying in the H.P. or on the ground, with the edge perpendicular to the V.P. *If a corner is in the H.P. or on the ground, the line joining that corner with the centre of the plane is kept parallel to the V.P.*

(2) Similarly, if the surface of the plane is inclined to the V.P. and an edge (or a diameter or a diagonal) is parallel to the V.P. and inclined to the H.P.,

(i) in the initial position, the plane is assumed to be parallel to the V.P. and an edge perpendicular to the H.P.

(ii) It is then tilted so as to make the required angle with the V.P. Its top view in this position will be a line, while its front view will be smaller in size.

(iii) When the plane is turned to the required inclination with the H.P., only the position of the front view will change. Its shape and size will not be affected. In the final top view, the corresponding distances of all the corners from *xy* will remain the same as in the second top view.

If an edge is in the V.P., in the initial position, the plane is assumed to be lying in the V.P. with an edge perpendicular to the H.P. *If a corner is in the V.P., the line joining that corner with centre of the plane is kept parallel to the H.P.*

Problem 12-6. (fig. 12-12): *A square ABCD of 50 mm side has its corner A in the H.P., its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections.*

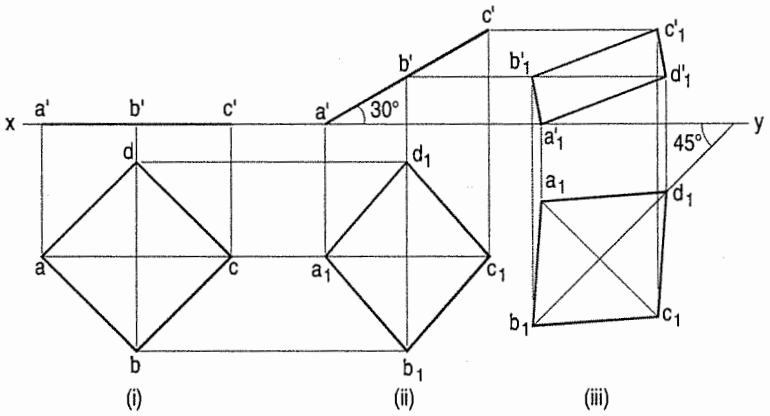


FIG. 12-12

In the initial stage, assume the square to be lying in the H.P. with *AC* parallel to the V.P.

(i) Draw the top view and the front view. When the square is tilted about the corner *A* so that *AC* makes 30° angle with the H.P., *BD* remains perpendicular to the V.P. and parallel to the H.P.

(ii) Draw the second front view with *a'c'* inclined at 30° to *xy*, keeping *a'* or *c'* in *xy*. Project the second top view. The square may now be turned so that *BD* makes 45° angle with the V.P. and remains parallel to the H.P. Only the position of the top view will change. Its shape and size will remain the same.

(iii) Reproduce the top view so that b_1d_1 is inclined at 45° to *xy*. Project the final front view upwards from this top view and horizontally from the second front view.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 27 for the following problem.

Problem 12-7. (Fig. 12-13): *A rectangular plane surface of size L × W is positioned in the first quadrant and is inclined at an angle of 60° with the H.P. and 30° with the V.P. Draw its projections.*

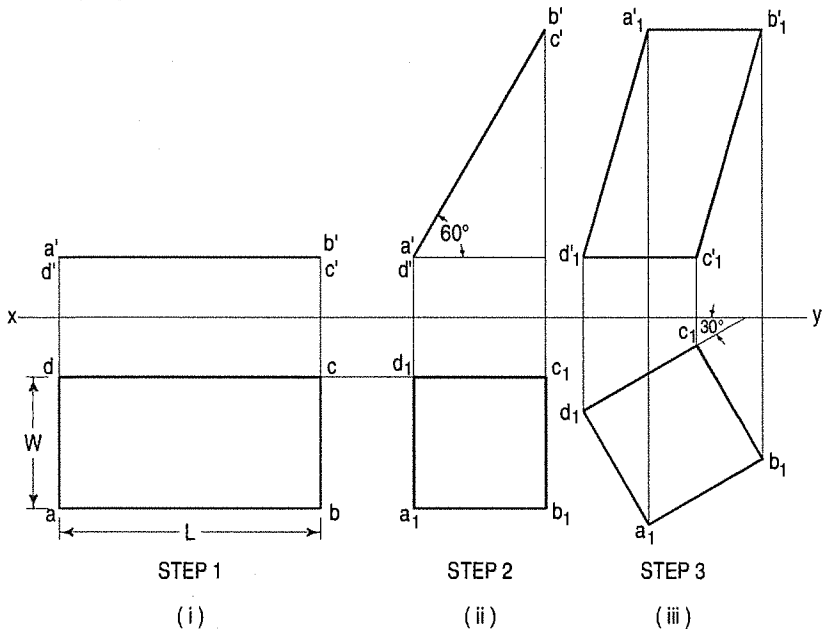


FIG. 12-13

(i) The plane is first assumed to be parallel to H.P. with its shorter edge perpendicular to V.P. In this position, true shape and size of the plane is given by its projection on H.P. The front view will be a true line parallel to the reference line *xy*.

(ii) Rotate the front view projection by 60° (the angle of inclination of plane with H.P.) as shown in Step 2 of fig. 12-13(ii). Draw vertical lines from the ends of line *a'd'* and *b'c'* to intersect horizontal lines drawn from the top view *abcd* (step 1) at points b_1, c_1, d_1 and a_1. Join $a_1b_1c_1d_1$ to obtain the top view of the plane in this inclined position.

(iii) Now rotate the edge d_1c_1 of the top view (step 2) by 30° (the angle of inclination of plane with V.P.) and reproduce it as shown in step 3 of the fig. 12-13(iii). Draw projections from a_1, b_1, c_1 and d_1 to intersect the horizontal projections from *a'd'* and *b'c'* to get the points a'_1, b'_1, c'_1 and d'_1. Join the lines $a'_1b'_1c'_1d'_1$ to obtain the final front view of the given plane surface.

Problem 12-8. (fig. 12-14): *Draw the projections of a regular hexagon of 25 mm side, having one of its sides in the H.P. and inclined at 60° to the V.P., and its surface making an angle of 45° with the H.P.*

(i) Draw the hexagon in the top view with one side perpendicular to *xy*. Project the front view *a'c'* in *xy*.

(ii) Draw *a'c'* inclined at 45° to *xy* keeping *a'* or *c'* in *xy* and project the second top view.

(iii) Reproduce this top view making a_1f_1 inclined at 60° to *xy* and project the final front view.

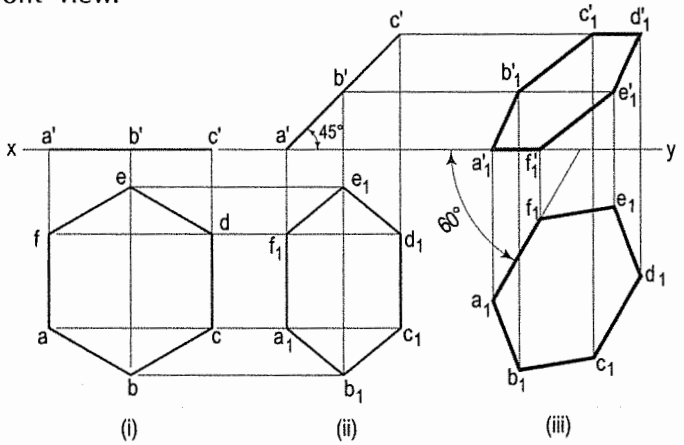


FIG. 12-14

Problem 12-9. (fig. 12-15): *Draw the projections of a circle of 50 mm diameter resting in the H.P. on a point A on the circumference, its plane inclined at 45° to the H.P. and*

(a) the top view of the diameter AB making 30° angle with the V.P.;

(b) the diameter AB making 30° angle with the V.P.

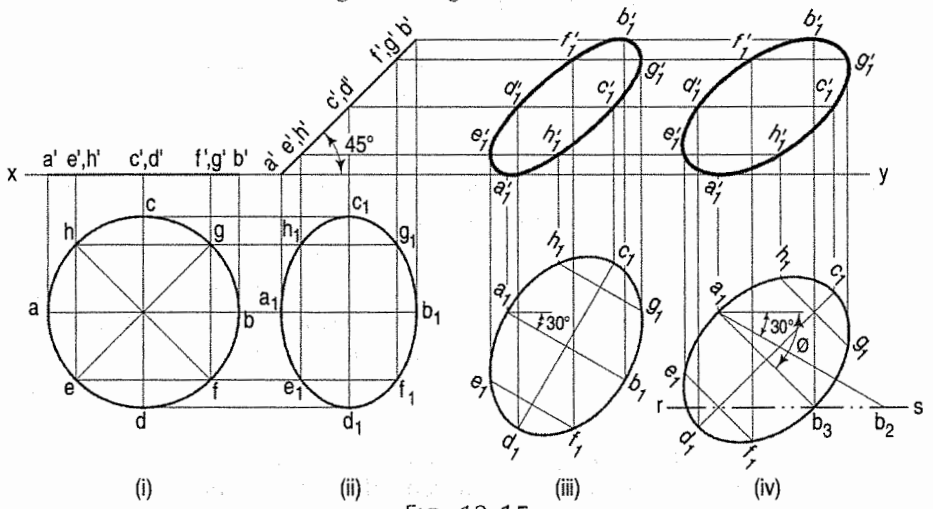


FIG. 12-15

Draw the projections of the circle with *A* in the H.P. and its plane inclined at 45° to the H.P. and perpendicular to the V.P. [fig. 12-15(i) and fig. 12-15(ii)].

(a) In the second top view, the line a_1b_1 is the top view of the diameter *AB*. Reproduce this top view so that a_1b_1 makes 30° angle with *xy* [fig. 12-15(iii)]. Project the required front view.

(b) If the diameter *AB*, which makes 45° angle with the H.P., is inclined at 30° to the V.P. also, its top view a_1b_1 will make an angle greater than 30° with *xy*. This apparent angle of inclination is determined as described below.

Draw any line a_1b_2 equal to *AB* and inclined at 30° to *xy* [fig. 12-15(iv)]. With a_1 as centre and radius equal to the top view of *AB*, viz. a_1b_1, draw an arc cutting *rs* (the path of *B* in the top view) at b_3. Draw the line joining a_1 with b_3, and around it, reproduce the second top view. Project the final front view. It is evident that a_1b_3 is inclined to *xy* at an angle ø which is greater than 30°.

Problem 12-10. (fig. 12-16): *A thin 30°-60° set-square has its longest edge in the V.P. and inclined at 30° to the H.P. Its surface makes an angle of 45° with the V.P. Draw its projections.*

In the initial stage, assume the set-square to be in the V.P. with its hypotenuse perpendicular to the H.P.

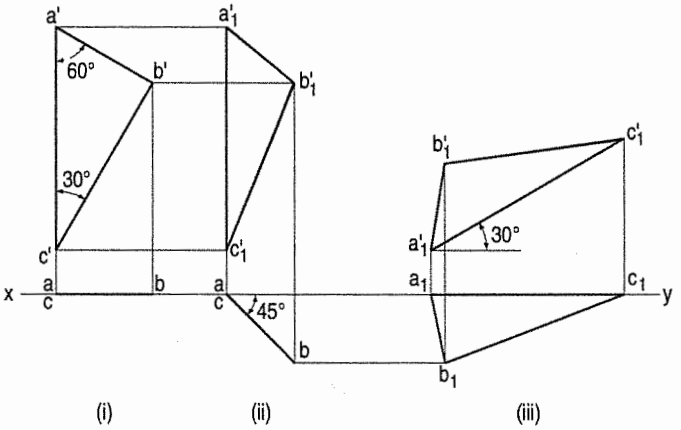


FIG. 12-16

(i) Draw the front view *a'b'c'* and project the top view *ac* in *xy*.

(ii) Tilt *ac* around the end *a* so that it makes 45° angle with *xy* and project the front view $a'_1b'_1c'_1$.

(iii) Reproduce the second front view $a'_1b'_1c'_1$ so that $a'_1b'_1$ makes an angle of 30° with *xy*. Project the final top view $a_1b_1c_1$.

Problem 12-11. (fig. 12-17): *A thin rectangular plate of sides 60 mm × 30 mm has its shorter side in the V.P. and inclined at 30° to the H.P. Project its top view if its front view is a square of 30 mm long sides.*

As the front view of the plate is a square, its surface must be inclined to the V.P. Hence, assume the plate to be in the V.P. with its shorter edge perpendicular to the H.P.

(i) Draw the front view *a'b'c'd'* and project the top view *ab* in *xy* [fig. 12-17(i)].

(ii) The line *ab* should be so inclined to *xy* that the front view becomes a square. Therefore, draw the square $a'_1b'_1c'_1d'_1$ of side equal to *a'd'*. With *a* as centre and radius equal to *ab* draw an arc cutting the projector through b'_1 at *b*. Then *ab* is the new top view.

(iii) Reproduce the second front view in such a way that $a'_1d'_1$ makes 30° angle with *xy*. Project the final top view as shown.

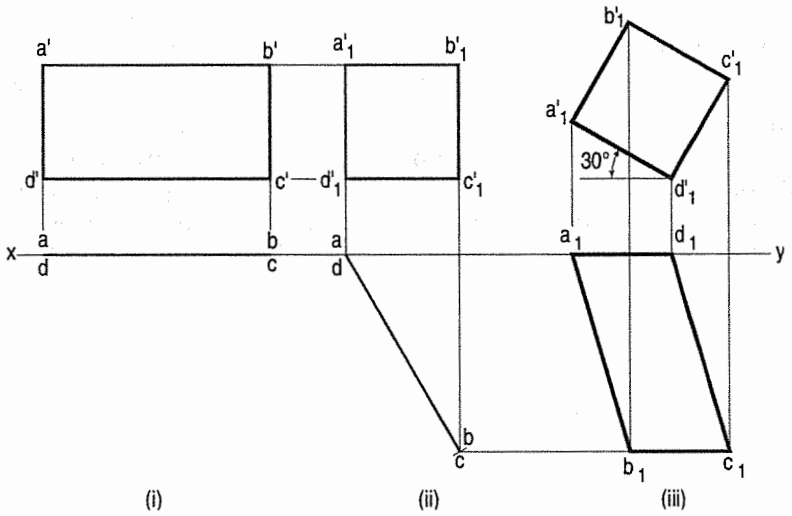
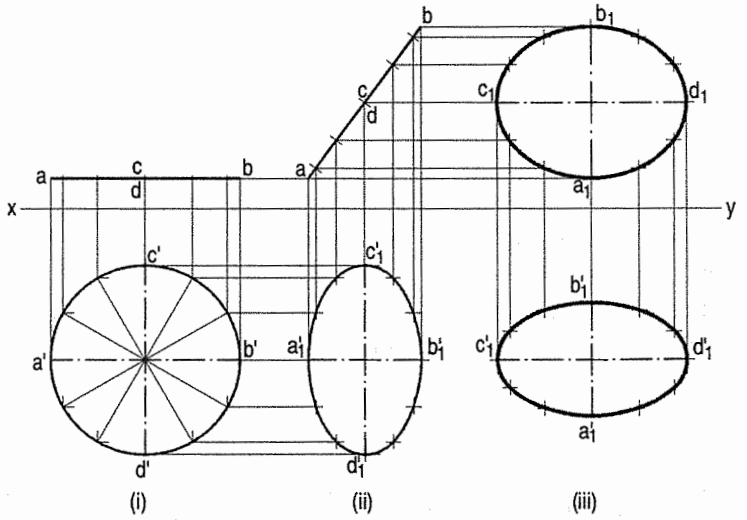


FIG. 12-17

Problem 12-12. (fig. 12-18): *A circular plate of negligible thickness and 50 mm diameter appears as an ellipse in the front view, having its major axis 50 mm long and minor axis 30 mm long. Draw its top view when the major axis of the ellipse is horizontal.*

As the plate is seen as an ellipse in the front view, its surface must be inclined to the V.P.



(Third-angle projection)

FIG. 12-18

(i) Therefore, assume it to be parallel to the V.P. and draw its front view and the top view.

(ii) Turn the line *ab* so that its length in the front view becomes 30 mm, and project the front view. It will be an ellipse whose major axis is vertical.

(iii) Reproduce this view so that the major axis $c'_1d'_1$ is horizontal, and project the required top view.