STAT 410, Homework 4

Due Sunday, March 17 at 11:59 PM EDT on Gradescope

Instructions: Scan and upload it to Gradescope. **If you have never used Gradescope, there is a short video in the Files section explaining how to use it. MAKE SURE TO MARK YOUR PAGES.** You can use CamScanner or other phone apps to do the scan.

The file must be uploaded by 11:59 pm EDT on March 17.

Unless otherwise stated, you may leave your answer as a product/sum of terms using factorials, binomial coefficients $\binom{n}{k}$, and permutations $P(n,k)$. Do NOT reduce your answer to a decimal or a big number. However, your answers should be simple e.g. it should not contain a summation sign.

1. Let X denote the development time of a Polaroid photo. Suppose X is normally distributed with a mean of 25 seconds and standard deviation 1.3 seconds. **For the following, put your answer in terms of the CDF $\Phi(x)$ as defined in class. Then use a table online/in the text to get the value. Make sure to express it using Φ first so it is easier to see if your answer is correct (in case of rounding errors). You may use negative values as long as you can find the value through a table online, which you do NOT do need to cite in your homework. Round to the nearest hundredth or thousandth.**

   (a) What is the probability a photo takes more than 26.5 seconds to develop?

   (b) What is the probability the time it takes to develop differs from the mean by more than 2.5 seconds?

2. Suppose random variable X has pmf

$$p(x) = \begin{cases} \frac{1}{4}\left(\frac{3}{4}\right)^x & \text{for } x = 0, 1, 2... \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

   Find the pmf of the random variable $Y = \frac{X}{X+1}$.

3. Suppose random variable X has pdf

$$f_X(x) = \begin{cases} \frac{x}{2} & \text{for } 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

   Find the pdf of the random variable $Y = X^2 + 3$.

4. Let X be the uniform random variable for $x \in (0, 1)$.

   (a) Determine the pdf of the random variable $Y = -2\ln(X)$.

(b) What distribution is this? State the parameters.

5. If X is the uniform random variable for $x \in (0, 1)$, determine the pdf of $Y = e^X$ in two ways:

   (a) By the definition of the cdf of $F_Y(y)$ and integrating the pdf of X.

   (b) The theorem from class, observing that e^x is a continuous, increasing function.

6. Suppose the joint pmf of X and Y is

$$f(x, y) = k(x^2 + y^2) \qquad \text{for } x = -1, 0, 1, 3; \;\; y = -1, 2, 3.$$

   (a) Determine the value of k.

   (b) Compute $P(X \leq 1, Y > 2)$.

   (c) Compute $P(X + Y > 3)$.

7. Let the joint pdf of X and Y be

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{for } x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

   (a) Find $P(X \leq 1/2, Y \leq 1/2)$.

   (b) Find $P(X > 2Y)$.

8. Let the joint **cdf** of X and Y be

$$F_{X,Y}(x, y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & \text{for } x > 0, y > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

   (a) Determine the joint pdf using the cdf.

   (b) Find $P(X + Y > 10)$.

Extra Practice from *A First Course in Probability* by Sheldon Ross, Tenth Edition (does not have to be submitted)

Chapter 6: *Problems:* # 1, 2, 4, 8 (just know what to do), 9, 10, 19, 46

Extra Practice

1. If $X \sim \Gamma(\alpha, \beta)$, prove that $E(X) = \frac{\alpha}{\beta}$ (apply a u-sub on the first step of the integral similar to computing $E(X^2)$ in class, and then it should convert into the gamma function).

2. Let the joint pdf of X and Y be

$$f_{X,Y}(x, y) = \begin{cases} e^{-(x+y)} & \text{for } x > 0, y > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

   Find the joint cdf of $f(x, y)$. (Hint: You should get the cdf in problem 8.)