Usage of artificial neural networks – The OR example

- we will utilize the McCulloch-Pitt model to train a neural network to learn the logic OR function. The OR function we will use is a two-input binary OR function given in Table 1.

Two inputs, one output

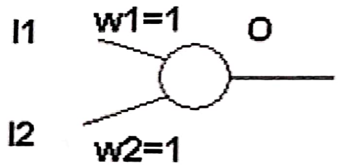
Table 1: OR function

I_1	I_2	Output
0	0	0
0	1	?
1	0	?
1	1	1

a) 0   b) 1

The OR example

First, we will use one neuron with two inputs. Note that the inputs are given equal weights by assigning the weights (w's) to '1'. The threshold, *T*, is set to 0 in this example.



We calculate the output as follows:

1) Compute the total weighted inputs

$$X = \sum_{i=1}^{2} I_i w_i$$

$$X=I_1\, w_1+I_2\, w_2= I_1\, 1+I_2\, 1=I_1+I_2$$

The OR example

2) Calculate the output using the logistic sigmoid activation function

$$O = sig(X - T) = sig(X) = \frac{1}{1+e^{-X}}$$

Now, let's try it for the inputs given in Table 1.
For $I_1=0$ and $I_2=0$; $X=0$,

$$O = \frac{1}{1+e^{0}} = \frac{1}{1+1} = 0.5$$

The OR example (cont.)

For I_1=0 and I_2=1, and I_1=1 and I_2=0; X=1,

$$O = \frac{1}{1+e^{-1}} = \frac{1}{1+0.37} \cong 0.73$$

$$\frac{1}{1+e^{-0.5}}$$

For I_1=1 and I_2=1; X=2,

$$O = \frac{1}{1+e^{-2}} = \frac{1}{1+0.14} \cong 0.88$$

- For all cases the results match with Table 1 assuming that '0.5' and below are considered as '0' and above as '1'.

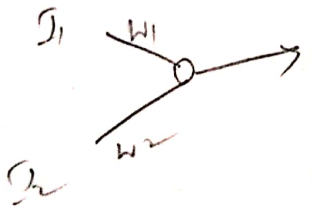
Assessment of Learning Objective #2

1. (Two minute discussion) If the weights were 0.5 rather than 1, will the network still function like OR?

"Yes"

a) Yes b) No

.5	0
.5	0
.5	0
.73	1



$I_1 \cdot I_2$

$W_1 I_1 \cdot W_2 I_2$

00	.5	.5
01	.73	.625
10	.73	.625
11	.88	.73.

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Assessment of Learning Objective #2

2. (5-minute paper) In groups of two students, discuss whether the same network can be used to learn the AND function? (Hint: You may change the threshold(=0.5) if necessary)

Table 2: AND function

I_1	I_2	Output
0	0	0
0	1	?
1	0	?
1	1	1

a) 0 b) 1

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The classic XOR problem

Table 3: XOR function

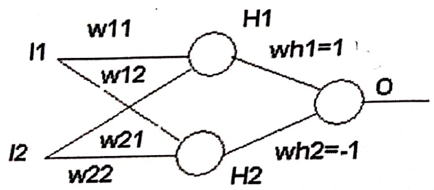
I_1	I_2	Output
0	0	0
0	1	1
1	0	1
1	1	0

If we use the same one-neuron model to learn the XOR (exclusive or) function, the model will fail.

The first three cases will produce correct results; however, the last case will produce '1', which is not correct.

The classic XOR problem (cont.)

The solution is to add a middle (hidden in ANN terminology) layer between the inputs and the output neuron



Choose the weights w11=w12=w21=w22=1. Use a different sigmoid function, which is given with a certain threshold for each neuron:

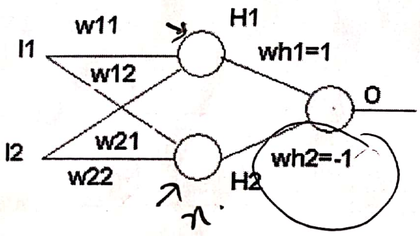
$$sig_{H1}(x) = \frac{1}{1+e^{-(x-0.5)}}$$

$$sig_{H2}(x) = \frac{1}{1+e^{-(x-1.5)}}$$

$$sig_{O}(x) = \frac{1}{1+e^{-(x-0.2)}}$$

Confirm by calculating the neuron outputs for each possible input combinations that this neural network is indeed functioning like an XOR.
(*Hint:* The output equal or below 0.5 is considered '0', otherwise '1')

Neuron calculation



w11=w12=w21=w22=1

$$sig_{H1}(x) = \frac{1}{1+e^{-(x-0.5)}}$$

$$sig_{H2}(x) = \frac{1}{1+e^{-(x-1.5)}}$$

$$sig_{O}(x) = \frac{1}{1+e^{-(x-0.2)}}$$

Sigmoid fn

I1	I2	XOR	X	H1	H2	O	Out
0	0	0	0				
0	1	1	1				
1	0	1	1				
1	1	0	2				

o/p of

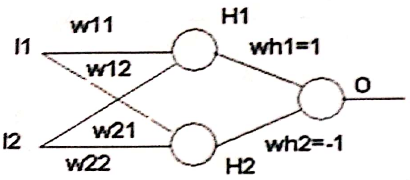
H1 calculation

$$sig_{H1}(0) = \frac{1}{1+e^{-(0-0.5)}} = 0.3775$$

$$sig_{H1}(1) = \frac{1}{1+e^{-(1-0.5)}} = 0.6225$$

$$sig_{H1}(2) = \frac{1}{1+e^{-(2-0.5)}} = 0.8176$$

Neuron calculation (2)



I1	I2	XOR	X	H1	H2	O	Out
0	0	0	0	0.3775			
0	1	1	1	0.6225			
1	0	1	1	0.6225			
1	1	0	2	0.8176			

w11=w12=w21=w22=1

$$sig_{H1}(x) = \frac{1}{1+e^{-(x-0.5)}}$$

$$sig_{H2}(x) = \frac{1}{1+e^{-(x-1.5)}}$$

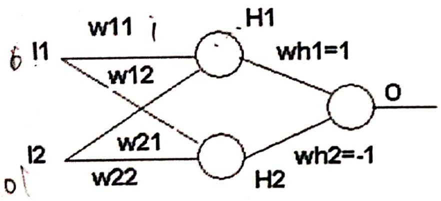
$$sig_{O}(x) = \frac{1}{1+e^{-(x-0.2)}}$$

$$sig_{H2}(0) = \frac{1}{1+e^{-(0-1.5)}} = 0.1824$$

$$sig_{H2}(1) = \frac{1}{1+e^{-(1-1.5)}} = 0.3775$$

$$sig_{H2}(2) = \frac{1}{1+e^{-(2-1.5)}} = 0.6225$$

Neuron calculation (3)



I1	I2	XOR	X	H1	H2	O	Out
0	0	0	0	0.3775	0.1824		
0	1	1	1	0.6225	0.3775		
1	0	1	1	0.6225	0.3775		
1	1	0	2	0.8176	0.6225		

w11=w12=w21=w22=1

$$sig_{H1}(x) = \frac{1}{1+e^{-(x-0.5)}}$$

$$sig_{H2}(x) = \frac{1}{1+e^{-(x-1.5)}}$$

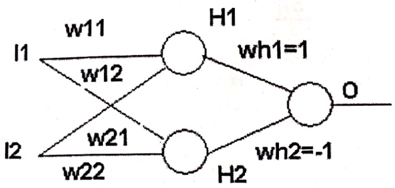
$$sig_{O}(x) = \frac{1}{1+e^{-(x-0.2)}}$$

$$sig_O(H2-H1) = sig_O(0.1951) = \frac{1}{1+e^{-(0.1951-0.2)}} = 0.4988$$

$$sig_O(H2-H1) = sig_O(0.2450) = \frac{1}{1+e^{-(0.2450-0.2)}} = 0.5112$$

1

Neuron calculation (4)



w11=w12=w21=w22=1

$$sig_{H1}(x) = \frac{1}{1+e^{-(x-0.5)}}$$

$$sig_{H2}(x) = \frac{1}{1+e^{-(x-1.5)}}$$

$$sig_{O}(x) = \frac{1}{1+e^{-(x-0.2)}}$$

I1	I2	XOR	X	H1	H2	O	Out
0	0	0	0	0.3775	0.1824	0.4988	**0**
0	1	1	1	0.6225	0.3775	0.5112	**1**
1	0	1	1	0.6225	0.3775	0.5112	**1**
1	1	0	2	0.8176	0.6225	0.4988	**0**

Assuming that '0.5' and below are considered as '0' and above as '1'.