ADSA End Sem

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ROLL NO: CS21M037

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Honour Code: I have not engaged in any dishonest means to answer this question paper. I have maintained a high level of decency and dignity in attempting this question paper. I understand that my quest for knowledge is life-long, while this exam will end in just 180 minutes. I understand with pride, dignity and decency that I do not have to resort to cheating in exams.

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9:00 am 20/11/2021

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1) For Quick Sort,

Worst case is when at each iteration (recursion), the array of size 'n' is partitioned into 2 subarrays of sizes $n-1$ and 0 (excluding pivot element).

eg. If we take pivot element as first element, for an array which is in descending order, quick sort always partitions into $n-1$ and 0.

Worst case T.C $\Rightarrow$ $T(n) = T(n-1) + T(0) + \Theta(n)$

By expanding, $T(n) = n \cdot \Theta(n) = \Theta(n^2)$.

For Randomised Quick Sort,

Here, even though we pick a random element as pivot, still the worst case is that at each iteration (recursion) we pick the pivot such that array is partitioned as $n-1$ and 0.

$\therefore$ Worst case T.C $\Rightarrow$ $\Theta(n^2)$

Worst Case T.C of Quick Sort and Randomised Quick Sort is <u>Same</u> and is $\Theta(n^2)$.

For expected time of Randomised Quick Sort, during each partition, on average, the array is split into 2 parts where both have a fraction of elements of full array. (eg. $\frac{n}{2}$, $\frac{n}{2}$).

$\therefore$ the recursion tree will have a depth (expected) of $\log n$. To partition, time taken is $\Theta(n)$.

Hence Expected T.C $= \Theta(n \log n)$.

2) $f_1(n) = \begin{cases} n & , \text{ if } n \text{ is even} \\ 2n & , \text{ if } n \text{ is odd} \end{cases}$

$f_2(n) = \begin{cases} 2n+1 & , \text{ if } n \text{ is even} \\ n & , \text{ if } n \text{ is odd} \end{cases}$

Suppose n is even,

$f_1(n) = n \quad , \quad f_2(n) = 2n+1$

For $c_1 = \frac{1}{4}$, $c_2 = 1$, $n_0 = 1$,

$c_1 f_2(n) = \frac{n}{2} + \frac{1}{4} \leq f_1(n) = n \leq 2n+1 = c_2 f_2(n)$
$\forall n \geq n_0$.

$\therefore f_1(n) = \Theta(f_2(n))$ if n is even.

Suppose n is odd,

$f_1(n) = 2n \quad , \quad f_2(n) = n$

For $c_1 = 1$, $c_2 = 4$, $n_0 = 1$,

$c_1 f_2(n) = n \leq f_1(n) = 2n \leq 4n = c_2 f_2(n) \quad \forall n \geq n_0$

$\therefore f_1(n) = \Theta(f_2(n))$ if n is odd.

$\therefore$ Asymptotic relationship is that,

$$f_1(n) = \Theta(f_2(n))$$

3) Rahul Baadal Priya Jose Ameen
2 5 4 2 5

Init array $C = [0\ 0\ 0\ 0\ 0]$
and records $R = [$ Null Null Null Null Null $]$

For Looping through records,

→ Rahul : 2 $\quad C = [0, 1, 0, 0, 0]$
$R = [\cdot, (\text{Rahul}), \cdot, \cdot, \cdot]$ $[\cdot \to \text{Null}]$

→ Baadal : 5 $\quad C = [0, 1, 0, 0, 1]$
$R = [\cdot, (\text{Rahul}), \cdot, \cdot, (\text{Baadal})]$

→ Priya : 4 $\quad C = [0, 1, 0, 1, 1]$
$R = [\cdot, (\text{Rahul}), \cdot, (\text{Priya}), (\text{Baadal})]$

→ Jose : 2 $\quad C = [0, 2, 0, 1, 1]$
$R = [\cdot, (\text{Rahul}, \text{Jose}), \cdot, (\text{Priya}), (\text{Baadal})]$

→ Ameen : 5 $\quad C = [0, 2, 0, 1, 2]$
$R = [\cdot, (\text{Rahul}, \text{Jose}), \cdot, (\text{Priya}), (\text{Baadal}, \text{Ameen})]$

Output Sorted Order,
Rahul, Jose, Priya, Baadal, Ameen
2 2 4 5 5

4) A universal class of hash functions is defined as a collection of hash functions that map a universe U of keys to $\{0, 1, 2, \dots m-1\}$ that satisfies, for every pair of distinct keys $k, l \in U$, number of hash functions h belonging to the class of hash functions where $h(k) = h(l)$ is AT MOST $\frac{|\mathcal{H}|}{m}$ where $\mathcal{H}$ is the given class of hash functions.

Example,

$$\mathcal{H} = \{ h_{ab} : a \in Z'_p , b \in Z_p \}$$

where $Z_p = \{0, 1, \dots p-1\}$, $Z'_p = \{1, 2, \dots p-1\}$

$$h_{ab}(k) = ((ak+b) \bmod p) \bmod m.$$

For example we can take $p = 37$ for $m = 12$.

Then $h_{7,8}(11) = ((7 \times 11 + 8) \bmod 37) \bmod 12$

$= (85 \bmod 37) \bmod 12 = 11 \bmod 12$

$= 11$ //

This example is indeed universal as,

if we consider 2 keys k, l, $k \neq l$,

Let,

$r = (ak+b) \bmod p$

$s = (al+b) \bmod p$ $\Rightarrow r - s = a(k-l) \bmod p$

Since $k \neq l$, $k - l \neq 0$, $a \neq 0$ and as p is prime,

$r - s \neq 0 . \Rightarrow r \neq s$.

$\therefore$ Solving, $a = ((r-s)(k-l)^{-1} \bmod p) \bmod p$.

$b = (r - ak) \bmod p$.

Since we choose a and b uniformly at random,
$\therefore$ r and s are uniformly likely to be any pair of values $(r \neq s)$.

# of possiblities $= p(p-1)$

For k and l to collide, $r \neq s$ but $r = s \pmod{m}$.
If we take any r, there are $p-1$ possiblities for s for $r \neq s$. For $s = r \pmod{m}$,

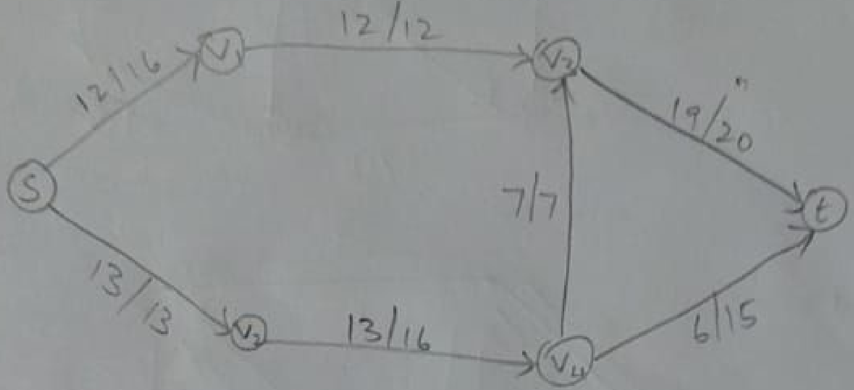
$\# = \lceil p/m \rceil - 1 \leq \frac{p-1}{m}$.

$\therefore$ Prob that s collides with $r \leq \frac{\left(\frac{p-1}{m}\right)}{p-1} = \frac{1}{m}$.

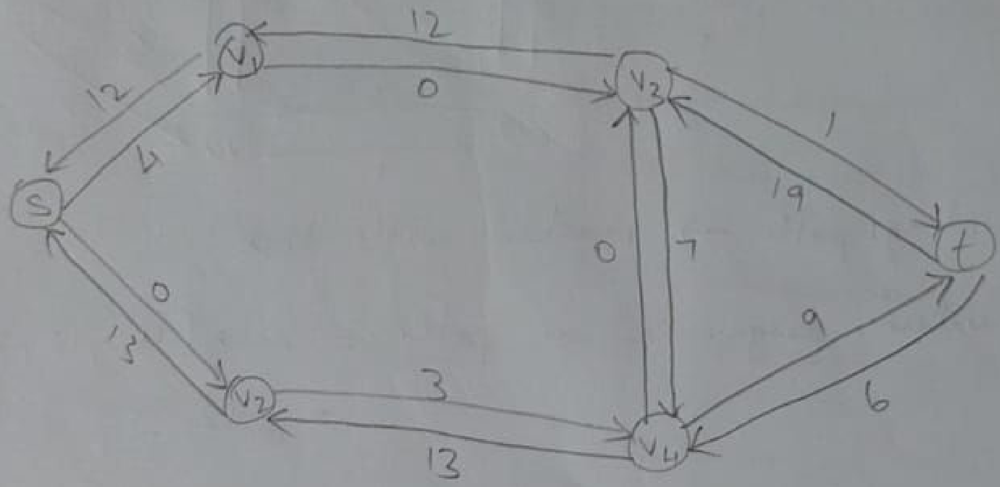
$\therefore$ The condition is satisfied, given example is a universal family of hash functions.

5) a) 25

b)



c)



d) Min s-t cut in residual network ⇒

$S = \{s, v_1, v_2\}$ $T = \{v_3, v_4, t\}$

net flow = $3 - 12 - 13 = -22$

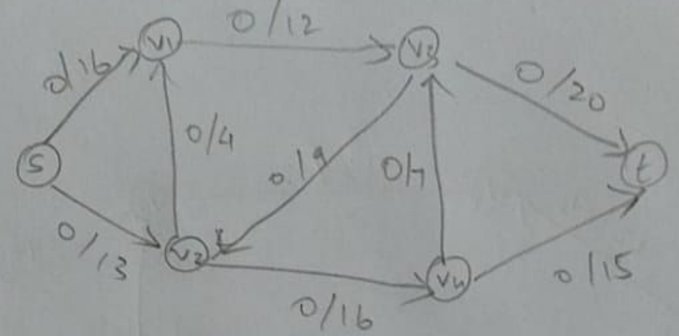
Min s-t cut in given flow network ⇒

$S = \{s, v_1, v_2\}$ $T = \{v_3, v_4, t\}$

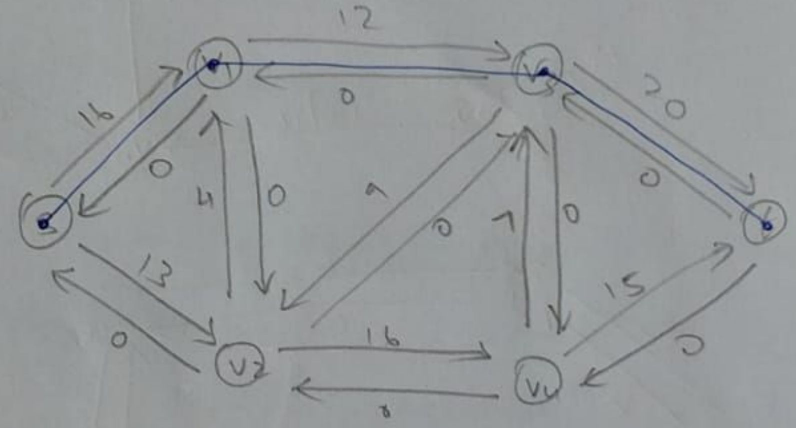
net flow = $12 + 11 - 4 = 19$

Both cuts are same.

c) Initial flow,



Residual Network,



Augmenting path → marked with Pen

flow value augmented on path = min(16, 12, 20)

= 12.

6) i) $u.d < u.f$

ii) Possiblities,

$u.d < u.f < v.d < v.f$

$u.d < v.d < v.f < u.f$

Other 2 cases with u and v reversed.

iii) Edges, (u, v)

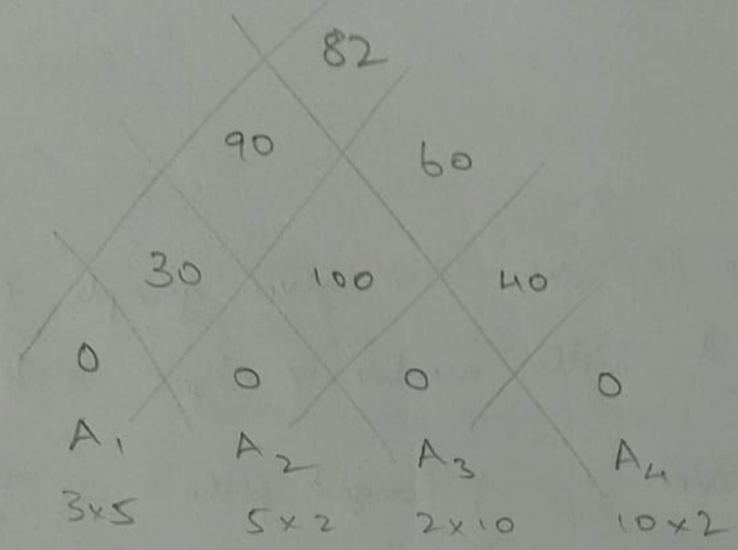
-> Tree Edge : Edge present in the DFS tree

-> Forward Edge : If v is a descendent of u but (u, v) is not part of DFS tree

-> Back Edge : If u is a descendent of v but (u, v) is not part of DFS tree

-> Cross Edge : u and v are not related as descendent of one another.

7)



8)

9) a)

b) yes instance is when the given graph G has a path in it with number of edges $\geq k$.

c) witness function = $\begin{cases} \text{Longest path}, & \text{if \#edges of longest path} \geq k \\ \text{any path}, & \text{otherwise} \end{cases}$

Verifier ⇒ i) Verify if witness gives a valid path in G (all edges in path are in G)

ii) check #edges in path $\geq k$.

Return 1 if both are satisfied, else 0.

∴ This is NP.

10. 2 - SAT,

i) It is in P.

ii) 2-SAT can be converted into a graph problem and Solved using BFS / DFS. T.C ⇒ $O(V+E)$ and So it is solved in poly time.
Hence P.

3 - SAT,

i) It is NP hard and is in NP

ii) ∴ It is in NP-Complete

iii) It doesnt have a deterministic poly time algo.

11)

Part II

1) Selection Problem,

Input: Array of values A, i - Select i^{th} minimum.

Output: Value of i^{th} minimum.

A) If we divide into groups of 3,

the time comp becomes,

$$T(n) = T\left(\left\lceil \frac{n}{3} \right\rceil\right) + T\left(\frac{4n}{6}\right) + O(n) \geq T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + O(n)$$

$$T(n) \geq c\left(\frac{n}{3}\right)\log\left(\frac{n}{3}\right) + c\left(\frac{2n}{3}\right)\log\left(\frac{2n}{3}\right) + O(n)$$

If we assume for $n\log n$.

$$\geq cn\log n + O(n)$$

$\therefore T(n)$ is NOT linear.

B) If divide by 7,

$$T(n) \leq T\left(\frac{n}{7}\right) + T\left(\frac{10n}{14}\right) + O(n) \leq cn\left(\frac{1}{7} + \frac{5}{7}\right) + O(n)$$

$$\leq O(n)$$

$\therefore T(n)$ is linear.

2) No, it will not give optimum number of mult.

Example,

$(2\times1)\cdot(1\times2)\cdot(2\times5)$
$A_1 \quad A_2 \quad A_3$

This algorithm,

does $(A_1 \cdot A_2) A_3 \Rightarrow$ # mult $= 2\times1\times2 + 2\times2\times5$
$= 24$

But optimum is

$A_1 (A_2 \cdot A_3) \Rightarrow$ # mult $= 1\times2\times5 + 2\times1\times5$
$= 20$.

This is a greedy algorithm and it does not guarentee optimum solution for MCM.
This is because if we choose based on min mult, it might result in a larger matrix which causes large # of mult in later stages.

3) A) The greedy strategy selects,

$\{a_1, a_2, a_3, a_4\}$, it rejects a_5 since it's deadline is 1 and its penalty is least among $\{a_1 \dots a_5\}$.

It also rejects a_6 as it's deadline is 4 but there are $\{a_1, a_2, a_3, a_4\} \Rightarrow$ 4 tasks with higher penalties before it.

It accepts a_7.

Schedule $\Rightarrow \{a_2, a_4, a_1, a_3, a_7, a_5, a_6\}$

Penalty $\Rightarrow w_5 + w_6 = 30 + 20 = 50$

B)

a_i	1	2	3	4	5	6	7
d:	4	2	4	3	1	4	6
wi	10	20	30	40	50	60	70

By greedy strategy,

a_1 and a_2 are rejected as they have least weights.

All other tasks are accepted.

$\therefore$ Schedule $\Rightarrow \{a_5, a_3, a_4, a_6, a_7, a_1, a_2\}$

Penalty $\Rightarrow w_1 + w_2 = 10 + 20 = 30$

7) a) P - Polynomial time algorithm exists
Deterministic

NP - Non-deterministic Polynomial time algorithm exists

b) P is set of all decision problems that can be solved by a deterministic polynomial time algorithm.

NP is set of all decision problems that can be solved by a non-deterministic polynomial time algorithm.

$P \subseteq NP$

c) Decision Problem, M is max bipartite matching,

$\forall M'$: $|M| \geq |M'|$ is True
M' is a matching

This belongs to NP.

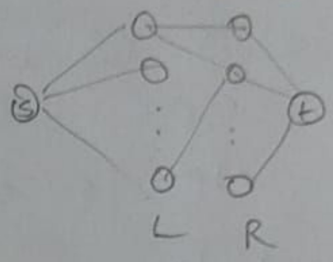
d) Decision Problem, f is max flow

$\forall f'$: $|f| \geq |f'|$ is True
f' is a valid flow

Since this can be solved using Ford Fulkerson Method, and it is poly time algo, it is in P.

e) To reduce the max bipartite graph problem to a max flow problem,

i) Add a source s and terminal t vertices to the bipartite graph.
Connect s to all vertices in L
t to all vertices in R.



ii) Give unit capacity to every edge.
Also we restrict that flow is always integer valued.
So, any edge can be either 0 or 1.

iii) Then we solve maximum flow and the resultant without s and t is the maximum bipartite graph.

8) 3 - Colouring problem

It is ~~where~~ we give colours (any one of 3 colours) to the nodes of a graph such that no 2 vertices connected by an edge have the same colour.

A node can be assigned any 1 colour among 3.

It is in NP as we can verify if any adjacent nodes have same colour in poly time.

It is in NP hard as ~~it can~~ we can do a reduction from 3-SAT problem which is NP-hard.

∴ It is NP-Complete.