2. $\begin{bmatrix} c & c & c \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix} = A$ (say)

Since # indep rows = # indep columns,
for A to have atleast 1 dep column, it must have 1 dep row.
i.e. row_1 should be linear combination of other 2 rows.

$\therefore [c\ c\ c] = p[2\ 1\ 5] + q[3\ 3\ 6]$

$2p + 3q = c$ (1)   (1) − (2), $p = 0$

$p + 3q = c$ (2) $\Rightarrow$ (3) − 2×(2), $3p = -c$

$5p + 6q = c$ (3) $\Rightarrow c = -3p = 0$.

$\therefore$ For matrix to have dependent columns,

$\boxed{c = 0.}$

3 $A = \begin{bmatrix} a & b & c \\ 1 & e & f \\ 0 & i & i \end{bmatrix}$

If we do Gaussian Elimination of A,

$\Rightarrow Row_2 = Row_2 - \frac{1}{a} Row_1 \Rightarrow \begin{bmatrix} a & b & c \\ 0 & e-\frac{b}{a} & f-\frac{c}{a} \\ 0 & i & i \end{bmatrix}$

$\Rightarrow Row_3 = Row_3 - i \cdot \left(\frac{a}{ae-b}\right) Row_2 \Rightarrow \begin{bmatrix} a & b & c \\ 0 & \frac{ae-b}{a} & \frac{af-c}{a} \\ 0 & 0 & i - i\left(\frac{af-c}{a}\right)\left(\frac{a}{ae-b}\right) \end{bmatrix}$

as $a, e \neq 0$, Row_1 has pivot $a \neq 0$.

Row_2 has pivot $= \frac{ae-b}{a}$

For dependent columns, pivot (any 1) can be 0,

If $\frac{ae-b}{a} = 0 \Rightarrow$ as $a \neq 0 \Rightarrow ae-b = 0 \Rightarrow b = ae$

Row_3 has pivot $= i\left(1 - \frac{af-c}{ae-b}\right) = i\left(\frac{ae-b-af+c}{ae-b}\right)$

For pivot $= 0$, $i\left(\frac{ae-b-af+c}{ae-b}\right) = 0$

either $i = 0$, (or) if $ae-b \neq 0$, [Row_2 can't have pivot 0]

$\Rightarrow ae-b-af+c = 0 \Rightarrow \boxed{ae-b = af-c}$

4. System is

$$\begin{array}{ccc} A & x & b \\ \begin{bmatrix} 4 & 7 & 1 \\ 8 & d & 1 \\ 0 & 1 & -1 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} & = \begin{bmatrix} 0 \\ 2 \\ t \end{bmatrix} \end{array}$$

For 0 solutions,

any one row in A should be linear comb of other 2 rows but its corresponding b should not be formed with the same weights.

$\therefore$ Let $[8 \ d \ 1] = p[4 \ 7 \ 1] + q[0 \ 1 \ -1]$

$\Rightarrow 4p = 8 \Rightarrow p = 2$

$p - q = 1 \Rightarrow q = p - 1 = 1$

$\therefore d = 7p + q = 14 + 1 = 15$

for same weights $p=2$ and $q=1$,

$0 \times p + q \times t \neq 2$

$t \neq 2$

$\therefore$ For $d = 15$ and $t = 3$,

system has 0 solutions.

5. $A \neq B$, $A \neq I$, $B \neq I$

But $AB = B$ and $BA = B$.

Since B can be O matrix,

If $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$,

$A \neq B$, $A \neq I$, $B \neq I$,

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = B$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = B,$$

6. A) Always Invertible

as Permutation matrix is invertible.

as given A is invertible,

PA = matrix mul of 2 invertible matrices

$\Rightarrow$ invertible.

7. We need A such that

$C(A) = N(A)$

This is not possible

8. an $A = LU$

$U = L^{-1}A$

$$L^{-1}_{k} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 \\ -3 & -\frac{11}{2} & \frac{1}{3} & 1 & 0 \\ -5 & \frac{-9}{2} & \frac{22}{30} & -\frac{247}{311} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -5 & 3 & 5 \\ 2 & 2 & -8 & -5 & 1 \\ -5 & -8 & -7 & 6 & 7 \\ 3 & -5 & 6 & -3 & -1 \\ 5 & 7 & 7 & -1 & -2 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 & -5 & 3 & 5 \\ 0 & -2 & 2 & -11 & -9 \\ & & & & \\ & & & & \end{bmatrix}$$

9. $A = LDU$ we need U as symmetric

If we take $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4$,

LDU factorisation is

$A = LDU = I_4 \cdot I_4 \cdot I_4$

$U = I_4 =$ symmetric.