Honor code: I pledge on my honor that: I have completed all steps in the below homework on my own, I have not used any unauthorized materials while completing this homework, and I have not given anyone else access to my homework.

N Kausik
Name and Signature

1. (1 point) Have you read and understood the honor code?

> **Solution:** Yes

Count, Count, Count!

2. (1 point) In how many ways can 10 people be seated:

   (a) in a row such that Motu and Patlu sit next to each other (there is only one boy named Motu and only one boy named Patlu in the group)

   > **Solution:**
   >
   > = 2 x 9 x 8! = 725760

   (b) in a row such that there are 5 engineers and 5 doctors and no two doctors or no two engineers can sit next to each other

   > **Solution:**
   >
   > Since no 2 doctors or engineers can sit together, they must sit alternatively. Hence they can be seated as BGBGBGBGBG or GBGBGBGBGB where B denotes any boy and G denotes any girl.
   >
   > Hence number of ways = 2 x 5! x 5! = 28800

(c) in a row such that there are 3 engineers, 3 doctors and 4 lawyers and all people of the same profession should sit in consecutive positions.

> **Solution:**
>
> $= 3! \text{ x } 3! \text{ x } 4! \text{ x } 3! = 5184$

(d) in a row such that there are 5 married couples and each couple must sit together.

> **Solution:**
>
> $= 2^5 \text{ x } 5! = 3840$

3. (½ point) How many unique 9 letter words can you form using the letters of the word MANMOHANA (the words can be gibberish)?

> **Solution:**
>
> $= \frac{9!}{2!3!2!} = \frac{362880}{2.6.2} = \frac{362880}{24} = 15120$

4. (½ point) Suppose you have a class of 7 students (A,B,C,D,E,F,G) who need to be arranged in a line with the following restrictions:

   1. A has to be in one of the first 3 slots
   2. B and A are very good friends and insist on being next to each other
   3. B doesn't want to stand immediately behind C

   In how many different ways can you arrange them?

> **Solution:**

Here, A can be in one of 3 positions and B can either be before A or after A. Hence the possible configurations are, ABXXXXX, XABXXXX, XXABXXX, BAXXXXX, XBAXXXX, where X can denote any of the other students.

In the AB cases, B is immediately after A and hence cant be immediately behind C. In BA cases, C must be at any position other than before B.

= 5! + 5! + 5! + 5! + 4x4! = 480 + 96 = 576

The boring questions are done. I hope you find the rest of the assignment to be interesting!

The birthday problem

5. (3 points) The days of the year can be numbered 1 to 365 (ignore leap days). Consider a group of n people, of which you are not a member. Any of the 365 days is equally likely to be the birthday of any member of this group. An element of the sample space Ω will be a sequence of n birthdays (one for each person).

   (a) How many elements are there in the sample space?

   Solution:

   $= 365^n$

   (b) Let A be the event that at least one member of the group has the same birthday as you. What is the probability of this event A?

   Solution:

   $P(A) = 1 - P(B)$, where B is event that noone in the group has same birthday as me.
   $= 1 - (\frac{364}{365})^n$

   (c) What is the minimum value of n such that $P(A) \geq 0.5$?

   Solution:

$P(A) \geq 0.5$
$\implies 1 - (\frac{364}{365})^n \geq 0.5$
$\implies (\frac{364}{365})^n \leq 0.5$
Since n is a positive integer, for n = 253, $\implies (\frac{364}{365})^{253} = 0.4995 \leq 0.5$
and for n = 252 $\implies (\frac{364}{365})^{252} = 0.5009 > 0.5$.
Hence minimum n is 253.

(d) Let B be the event that at least two members of the group share the same birthday. What is the probability of this event B?

Solution:

P(B) = 1 - P(C), where C is event that all of the people have different birthdays.
P(B) = 1 - $(\frac{365!}{(365-n)!(365)^n})$

(e) What is the minimum value of n such that $P(B) \geq 0.5$?

Solution:

$P(B) \geq 0.5$
$\implies 1 - (\frac{365!}{(365-n)!(365)^n}) \geq 0.5$
$\implies \frac{365!}{(365-n)!(365)^n} \leq 0.5$
Since n is a positive integer, for n = 23, $\implies (\frac{365!}{(342)!(365)^{23}} = 0.4927 \leq 0.5$
and for n = 22 $\implies (\frac{365!}{(343)!(365)^{22}} = 0.5243 > 0.5$.
Hence minimum n is 23.

(f) **[Ungraded question]** Why is there a big gap between the answers to part (c) and part (e)? (although at "first glance" they look very similar problems)

Solution:

Thought both the problems seem similar, in reality they have very different probabilities since in (c), we are fixing the birthday which needs to be matched by someone in the group as our birthday. However, in (e) the common birthday is not fixed to any particular date and hence there is more probability for (e) occuring than (c). Hence minimum n for (c) comes out to be much larger than for (e).

A biased coin

6. (1 point) Your friend Chaman has a coin which is biased (i.e., $P(H) \neq P(T)$). He proposes that he will toss the coin twice and asks you to bet on one of these events: A: both the tosses will result in the same outcome or B: both the tosses will result in a different outcome. Which event will you bet on to maximize your chance of winning the bet. (I am looking for a precise mathematical answer. No marks for answers which do not have an explanation).

Solution:

Let for the coin, $P(H) = p$. Hence, $P(T) = 1 - p$, since there are only 2 outcomes.

Now, event A happens when if we toss a coin twice and get the result as HH or TT.

Therefore, P(A) = P(H $\cap$ H) + P(T $\cap$ T) = $p^2 + (1-p)^2 = 2p^2 - 2p + 1 = 1 - 2p(1-p)$.

Similarly, event B happens when if we toss a coin twice and get the result as HT or TH.

Therefore, P(B) = P(H $\cap$ T) + P(T $\cap$ H) = $2p(1-p)$.

Here p is probability, $p \in [0, 1]$.

Since we want to compare P(A) and P(B),

LHS = $1 - 2p(1-p)$ and RHS = $2p(1-p)$. If we add $2p(1-p)$ to LHS and RHS,

$\implies LHS = 1$ and $RHS = 4p(1-p)$ If we divide LHS and RHS by 4,

$\implies LHS = \frac{1}{4}$ and $RHS = p(1-p)$

Now since $p \in [0, 1]$, to find maxima, derivative of RHS wrt p is $1 - 2p = 0$

$\implies p = 0.5$.

Hence RHS $\leq 0.5(1 - 0.5) => 0.25 = LHS$.

Hence, LHS $\geq$ RHS. $\implies P(A) \geq P(B)$.

Hence we should always bet on event A since its probability is always greater than or equal to probability of event B.

Alice in Wonderland

7. (1 point) A bag contains one ball which could either be green on red. You take another red ball and put it in this pouch. You now close your eyes and pull out a ball from the pouch. It turns out to be red. What is the probability that the original ball in the pouch was red?

Solution:

Let G_1 be event that the original ball was green and R_1 be event that the original ball was red.

Let R be the event that a picked ball is red.

Since original ball can only either be red or green $P(G_1) = P(R_1) = \frac{1}{2}$.

$P(R) = P(G_1)P(R/G_1) + P(R_1)P(R/R_1)$,

$\implies P(R) = (\frac{1}{2})(\frac{1}{2}) + \frac{1}{2}(1) = \frac{3}{4}$

$P(R_1/R) = P(R/R_1)\frac{P(R_1)}{P(R)}$.

Hence, $P(R_1/R) = 1.\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$

Hence the probability that the original ball was red given that the drawn ball was red is $\frac{2}{3}$.

Rock, paper and scissors

8. (2 points) Your friend Chaman has 3 strange dice: red, yellow and green. Unlike a standard die whose 6 faces are the numbers 1,2,3,4,5,6 these 3 dice have the following faces: red: 3,3,3,3,3,6, yellow: 5,5,5,2,2,2 and green: 4,4,4,4,4,1. Chaman suggests the following game: (i) You pick any one die (ii) Chaman then "carefully" picks one of the remaining two dice. Each of you will then roll your own die a 100 times. If on a given roll, the score of your die is higher than the score of Chaman's die then you get 1 INR else Chaman gets 1 INR. You play this game for many days and realise that you lose more often than Chaman.

   (a) Why are you losing more often? or What is Chaman's "carefully" planned strategy? (the key thing to note is that he lets you choose first)

   Solution:

For Red dice, Expected value is $\frac{5}{6}.3 + \frac{1}{6}.6 = \frac{21}{6} = \frac{7}{2} = 3.5$
For Yellow dice, Expected value is $\frac{3}{6}.5 + \frac{3}{6}.2 = \frac{7}{2} = 3.5$
For Green dice, Expected value is $\frac{5}{6}.4 + \frac{1}{6}.1 = \frac{21}{2} = 3.5$
Let, Y_n be event that you get a number n in your dice and C_m be the event that Chaman gets a number m in his dice.
Strategy is,
1) If you pick Red dice
If Chaman picks yellow dice,
P(You win) = P($Y_3 \cap C_2$) + P($Y_6 \cap C_2$) + P($Y_6 \cap C_5$) = $\frac{5}{6}\frac{3}{6} + \frac{1}{6}\frac{3}{6} + \frac{1}{6}\frac{3}{6} = \frac{21}{36} = \frac{7}{12}$
If Chaman picks green dice,
P(You win) = P($Y_3 \cap C_1$) + P($Y_6 \cap C_4$) + P($Y_6 \cap C_1$) = $\frac{5}{6}\frac{1}{6} + \frac{1}{6}\frac{5}{6} + \frac{1}{6}\frac{1}{6} = \frac{11}{36}$
Hence Chaman will pick the Green dice if you pick the Red dice so that he has a higher probability of winning.

2) If you pick Yellow dice
If Chaman picks red dice,
P(You win) = P($Y_5 \cap C_3$) = $\frac{3}{6}\frac{5}{6} = \frac{15}{36} = \frac{5}{12}$
If Chaman picks green dice,
P(You win) = P($Y_2 \cap C_1$) + P($Y_5 \cap C_4$) + P($Y_5 \cap C_1$) = $\frac{3}{6}\frac{1}{6} + \frac{3}{6}\frac{5}{6} + \frac{3}{6}\frac{1}{6} = \frac{21}{36} = \frac{7}{12}$
Hence Chaman will pick the Red dice if you pick the Yellow dice so that he has a higher probability of winning.

3) If you pick Green dice
If Chaman picks red dice,
P(You win) = P($Y_4 \cap C_3$) = $\frac{5}{6}\frac{5}{6} = \frac{25}{36}$
If Chaman picks yellow dice,
P(You win) = P($Y_4 \cap C_2$) = $\frac{5}{6}\frac{3}{6} = \frac{15}{36} = \frac{5}{12}$
Hence Chaman will pick the Yellow dice if you pick the Green dice so that he has a higher probability of winning.

Since using this strategy, Chaman always has a higher probability of winning than you, you lose more often.

(b) You realise what is happening and decide to turn the tables on Chaman. You buy 3 dice which are identical to Chaman's red, yellow and green dice. You now propose that instead of rolling a single die each of you will roll two dice of the same color. The rest of the rules remain the same (i) You pick any one color (ii) Chaman then uses his original strategy to carefully pick a different color (he is overconfident and simply uses the same strategy that he used when you were rolling only one die) (iii) If on a given roll, the sum of your two dice is greater than the sum of Chaman's two dice then you get 1 INR else Chaman gets 1 INR. To his horror Chaman realises that now he is loosing more often. Explain why?

Solution:

Since Chaman follows his original strategy,

1) If you pick Red, Chaman picks Green. But now,
P(You Win) = $P(Y_6 \cap C_5)$ + $P(Y_6 \cap C_2)$ + $P(Y_9)$ + $P(Y_12)$ = $\frac{5}{6}\frac{5}{6}\frac{5}{6}\frac{1}{6}.2 + \frac{5}{6}\frac{5}{6}\frac{1}{6}\frac{1}{6} + \frac{5}{6}\frac{1}{6}.2 + \frac{1}{6}\frac{1}{6} = \frac{250}{1296} + \frac{25}{1296} + \frac{10}{36} + \frac{1}{36} = \frac{671}{1296} > 0.5$

2) If you pick Yellow, Chaman picks Red. But now,
P(You Win) = $P(Y_7 \cap C_6)$ + $P(Y_10 \cap C_6)$ + $P(Y_10 \cap C_9)$ = $2.\frac{3}{6}\frac{3}{6}\frac{5}{6}\frac{5}{6} + \frac{3}{6}\frac{3}{6}\frac{5}{6}\frac{5}{6} + \frac{3}{6}\frac{3}{6}\frac{5}{6}\frac{1}{6}.2 = \frac{450}{1296} + \frac{225}{1296} + \frac{90}{1296} = \frac{765}{1296} > 0.5$

3) If you pick Green, Chaman picks Yellow. But now,
P(You Win) = $P(Y_5 \cap C_4)$ + $P(Y_8 \cap C_4)$ + $P(Y_8 \cap C_7)$ = $2.\frac{5}{6}\frac{1}{6}\frac{3}{6}\frac{3}{6} + \frac{5}{6}\frac{5}{6}\frac{3}{6}\frac{3}{6} + \frac{5}{6}\frac{5}{6}\frac{3}{6}\frac{3}{6}.2 = \frac{90}{1296} + \frac{225}{1296} + \frac{450}{1296} = \frac{765}{1296} > 0.5$

Since in all cases, you have a greater probability of winning than Chaman, he ends up losing more.

Sitting under an apple tree

9. (1 point) Which of the following has a greater chance of success?

   A. Six fair dice are tossed independently and at least one "6" appears.

   B. Twelve fair dice are tossed independently and at least two "6"s appear.

   C. Eighteen fair dice are tossed independently and at least three "6"s appear.

   Explain your answer.

Solution:

P(A) = 1 - P(no 6) = 1 - $(\frac{5}{6})^6$ = 1 - 0.3349 = 0.6651

P(B) = 1 - P(no 6) - P(one 6) = 1 - $(\frac{5}{6})^{12}$ - 12.$(\frac{5}{6})^{11}(\frac{1}{6})$ = 1 - 0.1122 - 0.2692 = 0.6186

P(C) = 1 - P(no 6) - P(one 6) - P(two 6) = 1 - $(\frac{5}{6})^{18}$ - 18.$(\frac{5}{6})^{17}(\frac{1}{6})$ - 153.$(\frac{5}{6})^{16}(\frac{1}{6})^2$ = 1 - 0.0376 - 0.1352 - 0.2299 = 0.5973

Here, clearly, greatest chance of success is for A.

With love from Poland

10. (1 point) A chain smoker carries two matchboxes - one in his left pocket and another in his right pocket. Every time he wants to light a cigarette he randomly selects a matchbox from one of the two pockets and then uses a matchbox from that box to light his cigarette. Suppose he takes out a matchbox and sees for the first time that it is empty, what is the probability that the matchbox in the other pocket has exactly one matchstick left?

Solution:

Let the matchboxes originally contain n matches.

Let the empty matchbox be called M_1 and the other matchbox be called M_2.

Hence, for M_1 to be empty and M_2 to have exactly 1 matchstick, he must have taken n matchsticks from M_1 and n-1 matchsticks from M_2.

Hence, number of trials is 2n-1.

We need to find $P(A|B)$ where A is event that M_2 has 1 matchstick left and B is event that M_1 has 0 matchsticks left.

$P(A|B)$ is probability that out of the 2n-1 trials, n matchsticks belong to M_1.

$P(A|B) = \binom{2n-1}{n}(\frac{1}{2})^n(\frac{1}{2})^{2n-1-n} = \binom{2n-1}{n}(\frac{1}{2})^{2n-1}$

A paradox

11. (1 point) Suppose there are 3 boxes:

    1. a box containing two gold coins,

2. a box containing two silver coins,
3. a box containing one gold coin and one silver coin.

You select one box at random and draw a coin from it. The coin turns out to be a gold coin. You remove this coin and draw another coin from the same box. What is the probability that the second coin is also a gold coin?

Solution:

For the first coin,

P(Coin 1 is Gold | Coin 1 is from Box 1) = 1

P(Coin 1 is Gold | Coin 1 is from Box 2) = 0

P(Coin 1 is Gold | Coin 1 is from Box 3) = $\frac{1}{2}$

Since the coin must be from one of these boxes, P(Coin 1 is from Box 1) = P(Coin 1 is from Box 2) = P(Coin 1 is from Box 3) = $\frac{1}{3}$

P(Coin 1 is Gold) = $\frac{1}{3}.1 + \frac{1}{3}.0 + \frac{1}{3}.\frac{1}{2} = \frac{1}{2}$

For the second coin,

If Coin 1 was Gold and was from Box 1, Box 1 will now contain only 1 Gold coin. So, since we pick coin 2 also from Box 1, it will always be Gold.

If Coin 1 was Gold, it cannot be from Box 2 as its probability is 0. Hence Coin 2 cant be Gold.

If Coin 1 was Gold and was from Box 3, Box 3 will now contain no Gold coins. Hence, since we pick Coin 2 also from Box 3, we cannot get a Gold coin.

Hence Coin 2 and Coin 1 can be Gold only if we picked Box 1. Hence, P(Coin 2 is Gold $\cap$ Coin 1 is Gold) = $\frac{1}{3}$

P(Coin 2 is Gold | Coin 1 is Gold) = $\frac{P(Coin2isGold \cap Coin1isGold)}{P(Coin1isGold)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

Once upon a time in Goa

12. (1 point) You are in one of the famous casinos in Goa[1]. You are observing the game of roulette. A roulette has 36 slots of which 18 are red and the remaining 18 are black. Each slot is equally likely. The manager places a ball on the roulette and then spins the roulette. When the roulette stops spinning, the ball lands in one of the 36 slots. If it lands in a slot which has the same color as what you bet on then you win. You do not

[1] I know about casinos in Goa purely out of academic interest.

believe in gambling but you are a student of probability[2]. You observe that the ball has landed in a black slot for the 26 consecutive rounds. Based on what you have learned in CS6015 you predict that there is a much higher chance of the ball landing in a red slot in the next round (since the probability of 27 consecutive black slots is very very low). You bet all your life's savings on red. What is the probability that you will win?

Solution:

Since for the past 26 rounds we got black and this has already happened and each round is independent of each other,

P(We win) = P(Red in 27th round | Black in all prev 26 rounds) = P(Red in 27th round) = $\frac{18}{36} = \frac{1}{2}$

Hence since the past 26 rounds already happened, the answer is NOT (1 - the probability of getting 27 blacksin a row) and instead it is simply $\frac{1}{2}$.

Oh Gambler! Thy shall be ruined!

13. (2 points) You play a game in a casino[3] where your chance of winning the game is p. Every time you win, you get 1 rupee and every time you lose the casino gets 1 rupee. You have i rupees at the start of the game and the casino has $N - i$ rupees (obviously, $N >> i$). The game ends when you go bankrupt or the casino goes bankrupt. In either case, the winner will walk away with a total of N rupees.

    (a) Find the probability p_i of winning when you start the game with i rupees.

    Solution:

[2] Ah! That's why you are in a casino! That makes perfect sense!

[3] Again, my interest in casinos in purely academic

Here, if i = 0, casino has already won and hence $p_0 = 0$ and if i = N, you have won and hence $p_N = 1$.

Since at the first bet we can win with prob p and lose with prob 1-p,

p_i = P(win first bet)P(win with initial Rs. i | won first bet)

+ P(lose first bet)P(win with initial Rs. i | lost first bet)

= p(p_{i+1}) + (1-p)(p_{i-1})

as if we win the first bet, we now have i+1 rupees and probability that we will win with this i+1 rupees is p_{i+1} and similarly, if we lose the first bet, we now have i-1 rupees and probability that we will win with this i-1 rupees is p_{i-1}.

Hence, $p_i = pp_{i+1} + (1-p)p_{i-1}$.

$\implies p_{i+1} - p_i = (\frac{1-p}{p})(p_i - p_{i-1})$

If we give i = 1, $p_2 - p_1 = (\frac{1-p}{p})(p_1 - p_0) = \frac{1-p}{p}p_1 \implies$ Equation 1

If we give i = 2, $p_3 - p_2 = (\frac{1-p}{p})(p_2 - p_1) \implies$ Equation 2

Substitute Eq 1 in 2, $p_3 - p_2 = (\frac{1-p}{p})(\frac{1-p}{p})p_1 = (\frac{1-p}{p})^2 p_1$

Generalising, $p_{j+1} - p_j = (\frac{1-p}{p})^j p_1$.

Adding all equations after substituting j as 1 to i,

$p_{i+1} - p_1 = \sum_{k=1}^{i}(\frac{1-p}{p})^k p_1 \implies p_{i+1} = p_1 + p_1\sum_{k=1}^{i}(\frac{1-p}{p})^k$

Using Geometric series sum, $\implies p_{i+1} = p_1\frac{1-(\frac{1-p}{p})^{i+1}}{1-\frac{1-p}{p}}$

If we take i = N-1, $p_N = 1 = p_1\frac{1-(\frac{1-p}{p})^N}{1-\frac{1-p}{p}} \implies p_1 = \frac{1-\frac{1-p}{p}}{1-(\frac{1-p}{p})^N}$

Hence, $p_i = p_1\frac{1-(\frac{1-p}{p})^i}{1-\frac{1-p}{p}} = \frac{1-(\frac{1-p}{p})^i}{1-(\frac{1-p}{p})^N}$

Hence, $p_i = \frac{1-(\frac{1-p}{p})^i}{1-(\frac{1-p}{p})^N}$

(b) What happens if $p = \frac{1}{2}$?

Solution:

$p_{i+1} = p_1 + p_1 \sum_{k=1}^{i} (\frac{1-p}{p})^k$ and hence, $p_i = p_1 + p_1 \sum_{k=1}^{i-1} (\frac{1-p}{p})^k$
if p = 1-p = $\frac{1}{2}$,
$p_i = p_1 + p_1 \sum_{k=1}^{i-1} (\frac{\frac{1}{2}}{\frac{1}{2}})^k = p_1 + p_1 \sum_{k=1}^{i-1} (1) = p_1 + p_1(i-1) = ip_1$
If we take i = N-1, $p_N = 1 = Np_1 \implies p_1 = \frac{1}{N}$
Hence, $p_i = \frac{i}{N}$.

(c) **[Ungraded question]** Can you reason why it does not make sense to take on a casino ($N >> i$)? Will you always go bankrupt in the long run?

Solution:

If N >> i, $\frac{i}{N} \simeq 0$ and hence you will lose in the long run always.

The disappointed professor

14. (1 point) A particular class has had a history of low attendance. The dejected professor decides that he will not lecture unless at least k of the n students enrolled in the class are present. Each student will independently show up with probability p if the weather is good, and with probability q if the weather is bad. Given that the probability of bad weather on a given day is r, obtain an expression for the probability that the professor will teach his class on that day. [Bertsekas and Tsitsikilis, Introduction to Probability, 2nd edition.]

Solution:

P(Prof will teach) = P(T)

= P(T | Weather = Bad)P(Weather = Bad) + P(T | Weather = Good)P(Weather = Good)

If Weather is Bad,

Prob that atleast k students come = $\binom{n}{k}q^k$

If Weather is Good,

Prob that atleast k students come = $\binom{n}{k}p^k$

Hence, P(T) = $\binom{n}{k}q^k r + \binom{n}{k}p^k(1-r) = \binom{n}{k}(rq^k + (1-r)p^k)$

The John von architecture

15. (1 point) Suppose you have a biased coin ($P(H) \neq P(T)$). How will you use it to make unbiased decision. (hint: you can toss the coin multiple times)

Solution:

Let P(H) = p, hence P(T) = 1-p.

Now, even though the coin is biased (p $\neq$ 1-p), we can get a fair decision by doing the following,

Step 1) Toss the coin 2 times.

Step 2) If we get same on both tosses, i.e. HH or TT, go back to Step 1.

Step 3) If we get different result on both tosses, assign the decision as the result of the first toss. I.e. if we got HT, result is H and if we got TH, result is T.

Here, P(HT) = p(1-p) and P(T) = (1-p)p.

Clearly, P(HT) = P(TH) and hence this method is fair.

Pascal to the rescue

16. (1 point) A six-side die is rolled three times independently. What is more likely: a sum of 11 or 12?

Solution:

Since it is rolled 3 times,

For a sum of 11 $\implies$ (1, 4, 6), (1, 5, 5), (1, 6, 4), (2, 3, 6), (2, 4, 5), (2, 5, 4), (2, 6, 3), (3, 2, 6), (3, 3, 5), (3, 4, 4), (3, 5, 3), (3, 6, 2), (4, 1, 6), (4, 2, 5), (4, 3, 4), (4, 4, 3), (4, 5, 2), (4, 6, 1), (5, 1, 5), (5, 2, 4), (5, 3, 3), (5, 4, 2), (5, 5, 1), (6, 1, 4), (6, 2, 3), (6, 3, 2), (6, 4, 1).

We have 27 possiblities.

For a sum of 12 $\implies$ (1, 5, 6), (1, 6, 5), (2, 4, 6), (2, 5, 5), (2, 6, 4), (3, 3, 6), (3, 4, 5), (3, 5, 4), (3, 6, 3), (4, 2, 6), (4, 3, 5), (4, 4, 4), (4, 5, 3), (4, 6, 2), (5, 1, 6), (5, 2, 5), (5, 3, 4), (5, 4, 3), (5, 5, 2), (5, 6, 1), (6, 1, 5), (6, 2, 4), (6, 3, 3), (6, 4, 2), (6, 5, 1)

We have 25 possiblities.

Hence sum of 11 is more likely.

Enemy at the gates

17. (1 point) There are 41 soldiers surrounded by the enemy. They would rather die than get captured. They sit around in a circle and devise the following plan. Each soldier will kill the person to his immediate left. They will continue this till only one soldier remains who would then commit suicide. For example, if there are 7 soldiers numbered 1, 2, 3, 4, 5, 6, 7 sitting in a circle then they proceed as follows: 1 kills 2, 3 kills 4, 5 kills 6, 7 kills 1, 3 kills 5, 7 kills 3, 7 commits suicide.

    (a) In how many ways can 41 soldiers be arranged around a circle?

    Solution:

    Number of ways of arranging 41 soldiers in a circle = $\frac{41!}{41} = 40!$

    (b) If you were one of the 41 soldiers and the soldiers were randomly arranged in the circle, what is the probability that you would survive?

    Solution:

Since it is random arrangement, we can be placed in any position. However only one position among the 41 ensures that we will survive.
Hence, P(survive) = $\frac{1}{41}$.

(c) **[Ungraded question]** Is there a specific position in which you can sit so that you are the last surviving soldier?

Solution:

At Step 1, 1 kills 2, 3 kills 4 ... 39 kills 40 and 41 kills 1.
Remaining are 3, 5, ... 39, 41.
At Step 2, 3 kills 5, 7 kills 9, ... 39 kills 41.
Remaining are 3, 7, 11, 15, 19, 23, 27, 31, 35, 39.
At Step 3, 3 kills 7, ... 35 kills 39.
Remaining are 3, 11, 19, 27, 35.
At Step 4, 3 kills 11, 19 kills 27, 35 kills 3.
Remaining are 19, 35.
At Step 5, 19 kills 35.
Final remaining position is 19.
Hence we can be the last surviving soldier if we sit at 19th position.