1. (1 point) Honor code.

Count, Count, Count!

2. (1 point) In how many ways can 10 people be seated:

   (a) in a row such that Motu and Patlu sit next to each other (there is only one boy named Motu and only one boy named Patlu in the group)

   Solution: $9!2!$

   (b) in a row such that there are 5 engineers and 5 doctors and no two doctors or no two engineers can sit next to each other

   Solution: $5!5!2!$

   (c) in a row such that there are 3 engineers, 3 doctors and 4 lawyers and all people of the same profession should sit in consecutive positions.

   Solution: $3!3!4!3!$

   (d) in a row such that there are 5 married couples and each couple must sit together.

   Solution: $5!(2!)^5$

3. (½ point) How many unique 9 letter words can you form using the letters of the word MANMOHANA (the words can be gibberish)?

   Solution: If there would have been unique letters in MANMOHANA, the answer would be 9!. But, since M repeats twice, A repeats thrice and N repeats twice, according to the division principle, the answer would be $\frac{9!}{2!3!2!}$.

4. (½ point) Suppose you have a class of 7 students (A,B,C,D,E,F,G) who need to be arranged in a line with the following restrictions:

   1. A has to be in one of the first 3 slots
   2. B and A are very good friends and insist on being next to each other
   3. B doesn't want to stand immediately behind C

   In how many different ways can you arrange them?

Solution: There are 5 cases possible:
(i)A in the first slot and B in the second slot: 5!.
(ii)A in the second slot and B in the third slot: 5!.
(iii)A in the third slot and B in the fourth slot: 5!.
(iv)B in the first slot and A in the second slot: 5!.
(iv)B in the second slot and A in the third slot:
Here, C can't stay in the first slot according to the restrictions. Now, C can take 1 of the 4 slots and then D, E, F, G can arrange in 4! ways for each of the 4 slots C can take and hence: $4 * 4!$.
Hence, according to addition principle, total arrangements =3D $4 * 5! + 4 * 4! =$.

The boring questions are done. I hope you find the rest of the assignment to be interesting!

The birthday problem

5. (3 points) The days of the year can be numbered 1 to 365 (ignore leap days). Consider a group of n people, of which you are not a member. Any of the 365 days is equally likely to be the birthday of any member of this group. An element of the sample space Ω will be a sequence of n birthdays (one for each person).

   (a) How many elements are there in the sample space?

   Solution: Each person can have birthday on any 1 of the 365 days of the year, and hence, 365 choices for each person. Therefore, the total number of elements in the sample space:
   $= 3D(365) * (365) * \ldots n$ times$= 3D(365)^n$.

   (b) Let A be the event that at least one member of the group has the same birthday as you. What is the probability of this event A?

   Solution: The probability of the event that no member of the group has the same birthday as mine $= 3D(\frac{364}{365})^n$.
   Hence, the probability of the event that at least one member of the group will have the same birthday $= 3D1 - (\frac{364}{365})^n$.

   (c) What is the minimum value of n such that $P(A) \geq 0.5$?

   Solution: $P(A) \geq 0.5$

   $1 - (\frac{364}{365})^n \geq 0.5$

$(\frac{364}{365})^n \leq 0.5$————(1)

For $n \geq 253$, (1) is satisfied.
Hence, required n =3D 253.

A biased coin

6. (1 point) Your friend Chaman has a coin which is biased (i.e., $P(H) \neq P(T)$). He proposes that he will toss the coin twice and asks you to bet on one of these events: A: both the tosses will result in the same outcome or B: both the tosses will result in a different outcome. Which event will you bet on to maximize your chance of winning the bet. (I am looking for a precise mathematical answer. No marks for answers which do not have an explanation).

Solution: Let p be the probability that the toss will result in head. Hence, $(1-p)$ will be the probability that the toss will result in tail.
So, $P(A) = 3Dp.p + (1-p)(1-p) = 3D2p^2 - 2p + 1 = 3D1 - (2p - 2p^2)$.
Probability of event B $= 3Dp(1-p) + (1-p)p = 3D2p - 2p^2$.
Now, since the coin is biased, we have two cases:

Case 1: When $0 \leq p < 0.5$
For any p, $P(A) > P(B)$, i.e., for every possible value of p in the above range, the probability of event A is higher than B.

Case 2: When $0.5 < p \leq 1$
For any p, $P(A) > P(B)$, i.e., for every possible value of p in the above range, the probability of event A is higher than B.

Hence, I will bet on event A, to maiximize my choice of winning.

Alice in Wonderland

7. (1 point) A bag contains one ball which could either be green on red. You take another red ball and put it in this pouch. You now close your eyes and pull out a ball from the pouch. It turns out to be red. What is the probability that the original ball in the pouch was red?

Solution: Let the event that the first ball is red and green be R_1 and G_1 respectively, and the event that the second ball is red be R_2.
Then, according to the given information in the question, we have:

$P(R_1) = 3DP(G_1) = 3D0.5$
$P(R_2/R_1) = 3D1$
$P(R_2/G_1) = 3D0.5$

Calculating total probability of R_2:
$P(R_2) = 3DP(R_2/R_1)P(R_1) + P(R_2/G_1)P(G_1) = 3D1 * 0.5 + 0.5 * 0.5 = 3D0.5 + 0.25 = 3D0. = 75$

Now, we have to calculate $P(R_1/R_2)$ which can be done bayes' formula as:
$P(R_1/R_2) = 3D\frac{P(R_2/R_1)P(R_1)}{P(R_2)} = 3D\frac{1*0.5}{0.75} = 3D\frac{=2}{3}$.

Rock, paper and scissors

8. (2 points) Your friend Chaman has 3 strange dice: red, yellow and green. Unlike a standard die whose 6 faces are the numbers 1,2,3,4,5,6 these 3 dice have the following faces: red: 3,3,3,3,3,6, yellow: 5,5,5,2,2,2 and green: 4,4,4,4,4,1. Chaman suggests the following game: (i) You pick any one die (ii) Chaman then "carefully" picks one of the remaining two dice. Each of you will then roll your own die a 100 times. If on a given roll, the score of your die is higher than the score of Chaman's die then you get 1 INR else Chaman gets 1 INR. You play this game for many days and realise that you lose more often than Chaman.

   (a) Why are you losing more often? or What is Chaman's "carefully" planned strategy? (the key thing to note is that he lets you choose first)

   Solution: I wonder what is the connection to rock, paper and scissors!
   Since Chaman let me choose first, there are three possible scenarios:

   (i) I choose red: 3,3,3,3,3,6
   Chaman would then choose green: 4,4,4,4,4,1 and in this way, probability of Chaman winning(25/36) is more than the probability that I win(11/36). Hence, in this case, Chaman will win more often.

   (ii) I choose yellow: 5,5,5,2,2,2
   Chaman would then choose red: 3,3,3,3,3,6 and thus, probability of Chaman winning(7/12) is more than the probability that I win(5/12). Hence, in this case as well, Chaman will win more often.

(iii) I choose green: 4,4,4,4,4,1
Chaman would then choose yellow: 5,5,5,2,2,2 and thus, probability of Chaman winning(7/12) is more than the probability that I win(5/12). Hence, in this case too, Chaman will win more often.

(b) You realise what is happening and decide to turn the tables on Chaman. You buy 3 dice which are identical to Chaman's red, yellow and green dice. You now propose that instead of rolling a single die each of you will roll two dice of the same color. The rest of the rules remain the same (i) You pick any one color (ii) Chaman then uses his original strategy to carefully pick a different color (he is overconfident and simply uses the same strategy that he used when you were rolling only one die) (iii) If on a given roll, the sum of your two dice is greater than the sum of Chaman's two dice then you get 1 INR else Chaman gets 1 INR. To his horror Chaman realises that now he is loosing more often. Explain why?

Solution: Chaman's strategy from the solution of 8(a):
1. If I choose red die, Chaman chooses green.
2. If I choose green die, Chaman chooses yellow.
3. If I choose yellow die, Chaman chooses red.

case 1 : I choose red die, Chaman chooses green.

Possible sum of red die outcomes when thrown twice: $\{6, 9, 12\}$.
$P(sum = 3D6/die = 3Dred)$ =3D $\frac{25}{36}$
$P(sum = 3D9/die = 3Dred)$ =3D $\frac{10}{36}$
$P(sum = 3D12/die = 3Dred)$ =3D $\frac{1}{36}$

Possible sum of green die outcomes when thrown twice: $\{2, 5, 8\}$.
$P(sum = 3D2/die = 3Dgreen)$ =3D $\frac{1}{36}$
$P(sum = 3D5/die = 3Dgreen)$ =3D $\frac{10}{36}$
$P(sum = 3D8/die = 3Dgreen)$ =3D $\frac{25}{36}$

P(I win) =3D P(sum on my die > Chaman's) =3D
$P(sum = 3D6/die = 3Dred)P(sum = 3D2||sum = 3D5/die = 3Dgreen) +$
$P(sum = 3D9/die = 3Dred)P(sum = 3D2||sum = 3D5||sum = 3D8/die = 3Dgreen) +$
$P(sum = 3D12/die = 3Dred)P(sum = 3D2||sum = 3D5||sum = 3D8/die = 3Dgreen)$ =3D

=3D $\frac{25}{36}$ $[\frac{1}{36} + \frac{10}{36}] + [\frac{1}{36} + \frac{10}{36} + \frac{25}{36}] \times [\frac{10}{36} + \frac{1}{36}] = 3D\frac{25}{36}$ $[\frac{11}{36}] + [\frac{10}{36} + \frac{1}{36}]$ =3D $\frac{671}{1296}$

P(I win) =3D $\frac{671}{1296}$ =3D> P(Chaman wins) =3D 1 - $\frac{671}{1296}$ =3D $\frac{625}{1296}$.

Clearly, in this case, P(I win) > P(Chaman wins).

case 2 :2. I choose green die, Chaman chooses yellow.

Possible sum of green die outcomes when thrown twice: $\{2, 5, 8\}$.
$P(sum = 3D2/die = 3Dgreen)$ =3D $\frac{1}{36}$
$P(sum = 3D5/die = 3Dgreen)$ =3D $\frac{10}{36}$
$P(sum = 3D8/die = 3Dgreen)$ =3D $\frac{25}{36}$

Possible sum of yellow die outcomes when thrown twice: $\{4, 7, 10\}$.
$P(sum = 3D4/die = 3Dyellow)$ =3D $\frac{1}{4}$
$P(sum = 3D7/die = 3Dyellow)$ =3D $\frac{1}{2}$
$P(sum = 3D10/die = 3Dyellow)$ =3D $\frac{1}{4}$

P(I win) =3D P(sum on my die > Chaman's) =3D
$P(sum = 3D5/die = 3Dgreen)P(sum = 3D4/die = 3Dyellow) +$
$P(sum = 3D8/die = 3Dgreen)P(sum = 3D4||sum = 3D7/die = 3Dyellow)$
=3D

=3D $\frac{10}{36}$ $[\frac{1}{4}]$ + $\frac{25}{36}$ $[\frac{1}{2} + \frac{1}{4}]$ =3D $\frac{85}{144}$.

P(I win) =3D $\frac{85}{144}$ =3D> P(Chaman wins) =3D 1 - $\frac{85}{144}$ =3D $\frac{59}{144}$.

Clearly, in this case as well, P(I win) > P(Chaman wins).

case 3 : I choose yellow die, Chaman chooses red.

Possible sum of yellow die outcomes when thrown twice: $\{4, 7, 10\}$.
$P(sum = 3D4/die = 3Dyellow)$ =3D $\frac{1}{4}$
$P(sum = 3D7/die = 3Dyellow)$ =3D $\frac{1}{2}$
$P(sum = 3D10/die = 3Dyellow)$ =3D $\frac{1}{4}$

Possible sum of red die outcomes when thrown twice: $\{6, 9, 12\}$.
$P(sum = 3D6/die = 3Dred)$ =3D $\frac{25}{36}$
$P(sum = 3D9/die = 3Dred)$ =3D $\frac{10}{36}$
$P(sum = 3D12/die = 3Dred)$ =3D $\frac{1}{36}$

P(I win) =3D P(sum on my die > Chaman's) =3D
$P(sum = 3D7/die = 3Dyellow)P(sum = 3D6/die = 3Dred) +$
$P(sum = 3D10/die = 3Dyellow)P(sum = 3D6||sum = 3D9/die = 3Dred)$

=3D

=3D $\frac{1}{2}$ [$\frac{25}{36}$] + $\frac{1}{4}$ [$\frac{25}{36}$ + $\frac{10}{36}$)] =3D $\frac{85}{144}$

P(I win) =3D $\frac{85}{144}$ =3D> P(Chaman wins) =3D 1 - $\frac{85}{144}$ =3D $\frac{59}{144}$.

Clearly, in this case too, P(I win) > P(Chaman wins).

From **cases 1, 2, and 3**, we can conclude that I win more number of times if Chaman sticks to his strategy here as well.

Sitting under an apple tree

9. (1 point) Which of the following has a greater chance of success?

   A. Six fair dice are tossed independently and at least one =E2= =80=9C6=E2=80=9D appears.

   B. Twelve fair dice are tossed independently and at least two =E2=80=9C6=E2=80=9Ds appear.

   C. Eighteen fair dice are tossed independently and at least three =E2=80=9C6=E2=80=9Ds appear.

   Explain your answer.

Solution: Let us find the probability of each case:

A. The probability of getting at least one "6", when six fair dice are tossed independently $= 3D1 - \binom{6}{0}(\frac{1}{6})^0(\frac{5}{6})^6 = 3D1 - (\frac{5}{6})^6 = 3D0.66 = 5$

B. The probability of getting at least two "6", when twelve fair dice are tossed independently $= 3D1 - \binom{12}{0}(\frac{1}{6})^0(\frac{5}{6})^{12} - \binom{12}{1}(\frac{1}{6})^1(\frac{5}{6})^{11} = 3D0.619$

C. The probability of getting at least three "6", when eighteen dice are tossed independently $= 3D1 - \binom{18}{0}(\frac{1}{6})^0(\frac{5}{6})^{18} - \binom{18}{1}(\frac{1}{6})^1(\frac{5}{6})^{17} - \binom{18}{2}(\frac{1}{6})^2(\frac{5}{6})^1 =^6 = 3D0.597$

Therefore, getting at least one "6", when six fair dices are tossed independently has the highest chance of success among all the three.

With love from Poland

10. (1 point) A chain smoker carries two matchboxes - one in his left pocket and another in his right pocket. Every time he wants to light a cigarette he randomly selects a matchbox from one of the two pockets and then uses a matchbox from that box to light his cigarette. Suppose he takes out a matchbox and sees for the first time that it is empty, what is the probability that the matchbox in the other pocket has exactly one matchstick left?

> **Solution:** Let both of the matchboxes contain n matchsticks initially.
> Hence, Total number of matchsticks =3D 2n.
> The probability of choosing one of these two matchboxes =3D $\frac{1}{2}$
> Now, according to the question, one of the matchboxes is found empty and we need to calculate the probability of finding exactly 1 matchstick in the other matchbox.
> Total matchsticks drawn until now =3D 2n-1, out of which n matchsticks woul= d have been drawn from the matchbox which has been found to be empty, and n-1 matchsticks would have been drawn from the matchbox which now contains exactly 1 matchstick.
> Hence, Required probability =3D $\binom{2n-1}{n-1}(\frac{1}{2})^{n-1}(\frac{1}{2})^{n} = 3D\binom{2n-1}{n-=1}(\frac{1}{2})^{2n-1}$

A paradox

11. (1 point) Suppose there are 3 boxes:

    1. a box containing two gold coins,
    2. a box containing two silver coins,
    3. a box containing one gold coin and one silver coin.

    You select one box at random and draw a coin from it. The coin turns out to be a gold coin. You remove this coin and draw another coin from the same box. What is the probability that the second coin is also a gold coin?

> **Solution:** Let
> $P(box = 3D1) = 3D1/3$
> $P(box = 3D2) = 3D1/3$
> $P(box = 3D3) = 3D1/3$
> $P(coin1 = 3Dgold/box = 3D1) = 3D1$,
> $P(coin1 = 3Dgold/box = 3D2) = 3D0$, and
> $P(coin1 = 3Dgold/box = 3D3) = 3D1/2$
>
> According to the question, the first coin turned out to be gold when a coin was drawn from a box chosen at random. Now, we removed this coin and we're removing

the last coin in that chosen box as well. This coin will be gold as well only if the box chosen at random was box=3D1.

Hence, required probability is $P(box = 3D1/coin1 = 3Dgold) = 3D$ $\frac{P(box=3D1)P(coin1=3Dgold/box=3D1)}{P(box=3D1)P(coin1=3Dgold/box=3D1)=+P(box=3D2)P(coin1=3Dgold/box=3D2)+P(box=3D3)P(coin1=3Dgold/box=3D3)}$

$= 3D\frac{1/3*1}{1/3*1+1/3*0+1/3*1/2} = 3D\frac{2}{3}$.

Once upon a time in Goa

12. (1 point) You are in one of the famous casinos in Goa[1]. You are observing the game of roulette. A roulette has 36 slots of which 18 are red and the remaining 18 are black. Each slot is equally likely. The manager places a ball on the roulette and then spins the roulette. When the roulette stops spinning, the ball lands in one of the 36 slots. If it lands in a slot which has the same color as what you bet on then you win. You do not believe in gambling but you are a student of probability[2]. You observe that the ball has landed in a black slot for the 26 consecutive rounds. Based on what you have learned in CS6015 you predict that there is a much higher chance of the ball landing in a red slot in the next round (since the probability of 27 consecutive black slots is very very low). You bet all your life's savings on red. What is the probability that you will win?

Solution: The answer should of course be (1 - the probability of getting 27 blacks in a row). Right?
No, it is wrong since the events are independent, i.e., outcome of an experiment doesn't depend upon the previous outcomes.
Let A and B denote the outcome of present and past experiments respectively.
Then, $P(A/B) = 3DP(A)$.
So, $P(red) = 3D18/36 = 3D1/2$.

Oh Gambler! Thy shall be ruined!

13. (2 points) You play a game in a casino[3] where your chance of winning the game is p. Every time you win, you get 1 rupee and every time you lose the casino gets 1 rupee. You have i rupees at the start of the game and the casino has $N - i$ rupees (obviously, $N >> i$). The game ends when you go bankrupt or the casino goes bankrupt. In either case, the winner will walk away with a total of N rupees.

[1] I know about casinos in Goa purely out of academic interest.
[2] Ah! That's why you are in a casino! That makes perfect sense!
[3] Again, my interest in casinos in purely academic

(a) Find the probability p_i of winning when you start the game with i rupees.

Solution: Given: p is the probability of winning one game.
Also, p_i is the probability of winning completely when we have i rupees.
So, we have thus a recurrence relation:
$p_i = 3Dp * p_{i+1} + (1-p) * p_{i-1}.......(i)$

At the start, we have i rupees, hence probability of winning =3D
$p_i = 3Dp * p_{i+1} + (1-p) * p_{i-1}$

Putting i=3D(i-1) in equation(i), we get:
$p_{i-1} = 3Dp * p_i + (1-p) * p_{i-2}$
$p * p_i - p_{i-1} + (1-p) * p_{i-2} = 3D0$
Putting $p_i = 3Dx^2$, $p_{i-1} = 3Dx^1$, $p_{i-2} = 3Dx^0$, we get:
$p * x^2 - x + (1-p) = 3D0$

Solving the above equation, we get:
$x = 3D\frac{1+\sqrt{1-4(1-p)*p}}{2p}, \frac{1-\sqrt{1-4(1-p)*p}}{2p}$

$= 3D\frac{1+\sqrt{4p^2-4p+1}}{2p}, \frac{1-\sqrt{4p^2-4p+1}}{2p}$

$= 3D\frac{1+\sqrt{(2p-1)^2}}{2p}, \frac{1-\sqrt{(2p-1)^2}}{2p}$

$= 3D\frac{1+(2p-1)}{2p}, \frac{1-(2p-1)}{2p}$

$= 3D > x = 3D1, \frac{1-p}{p}$

$p_i = 3DC_1(1)^i + C_2(\frac{1-p}{p})^i......(A)$

$p_0 = 3D0$
Putting i=3D0 in equation(A), we get:
$p_0 = 3DC_1(1)^0 + C_2(\frac{1-p}{p})^0 = 3DC_1 + C_2 = 3D > C_1 = 3D - C_2$

$p_N = 3D1$
Putting i=3DN in equation(A), we get:
$p_N = 3DC_1(1)^N + C_2(\frac{1-p}{p})^N = 3DC_1 + C_2(\frac{1-p}{p})^N = 3D1$

Putting $C_1 = 3D - C_2$, we get:

$-C_2 + C_2(\frac{1-p}{p})^N = 3D1$

$C_2 = 3D\frac{p^N}{(1-p)^N - p^N}$

$C_1 = 3D - C_2 = 3D - \frac{p^N}{(1-p)^N - p^N}$

Putting values of C_1 and C_2 in equation (A), we get:

$p_i = 3D - \frac{p^N}{(1-p)^N - p^N} + \frac{p^N}{(1-p)^N - p^N} * (\frac{1-p}{p})^{i-}$
$= 3D\frac{p^N}{(1-p)^N - p^N}[-1 + (\frac{1-p}{p})^i]$

$= 3D\frac{p^N}{(1-p)^N - p^N}[\frac{-p^i + (1-p)^i}{p^i}]$

$= 3Dp^{N-i}[\frac{(1-p)^i - p^i}{(1-p)^N - p^N}]$

(b) What happens if $p = 3D\frac{1}{2}$?

Solution: When $p = 3D\frac{1}{2}$, then both roots of the equation $px^2 - x + (1-p) = 3D0 =$ turns out to be 1. Hence, solution of p_i is given as:

$p_i = 3DC_1 * i * 1^N + C_2 * 1^N = 3DC_1 * i + C_2$

$p_0 = 3D0$ and hence, $C_2 = 3D0$
$p_N = 3D1$ and hence, $C_1 = 3D\frac{1}{N}$

Therefore, $p_i = 3D\frac{1}{N} * i + 0 = 3D\frac{i}{N}$.

(c) **[Ungraded question]** Can you reason why it does not make sense to take on a casino ($N >> i$)? Will you always go bankrupt in the long run?

Solution: Note that in a casino $p < \frac{1}{2}$, i.e, the odds are always in favour of the casino (How does a casino do this without you realising it? We will see this when we discuss the game of roulette!)

The disappointed professor

14. (1 point) A particular class has had a history of low attendance. The dejected professor decides that he will not lecture unless at least k of the n students enrolled in the class are present. Each student will independently show up with probability p if the weather is good, and with probability q if the weather is bad. Given that the probability of bad weather on a given day is r, obtain an expression for the probability that the professor will teach his class on that day. [Bertsekas and Tsitsikilis, Introduction to Probability, 2nd edition.]

Solution: Let S be the event of teacher teaching and B be the event that the weather is bad.
Using Total probability theorem, we have;
$P(S) = 3DP(B)P(S/B) + P(B^C)P(S/B^C)$
Since, $P(B) = 3Dr$, $P(B^C) = 3D1 - r$.
Now, according to the question, the professor will teach only k out of n registered students attend the class.
Therefore, let X denote the random variable denoting the number of students attending the class.
Hence, $P(S/B^C) = 3D\sum_{i=3Dk}^{n}\binom{n}{i}p^i(1-p)^{n-i}$.

And, $P(S/B) = 3D\sum_{i=3Dk}^{n}\binom{n}{i}q^i(1-q)^{n-i}$.

Now, $P(S) = 3D(1-r)\sum_{i=3Dk}^{n}\binom{n}{i}p^i(1-p)^{n-i} + r\sum_{i=3Dk}^{n} \equiv inomniq^i(1-q)^{n-i}$.

The John von architecture

15. (1 point) Suppose you have a biased coin ($P(H) \neq P(T)$). How will you use it to make unbiased decision. (hint: you can toss the coin multiple times)

Solution: Let $P(H) = 3Dp$, then $P(T) = 3D(1-p)$.
We'll toss the coin twice.
Total number of outcomes possible =3D 4.
$P(HH) = 3Dp^2, P(TT) = 3D(1-p)^2, P(HT) = 3DP(TH) = 3Dp(1-p)$.

Consider HT as 0 and TH as 1, and discard HH and TT.
So, we can then have outcome as either $0(HT)$ or $1(TH)$ or invalid outcome(HH, TT).
If we get an invalid outcome, toss the coin twice until you get HT or TH.

Pascal to the rescue

16. (1 point) A six-side die is rolled three times independently. What is more likely: a sum of 11 or 12?

Solution: When a die is rolled thrice;
A sum of 11 is possible in 27 ways:
(1,4,6),(1,5,5),(1,6,4)
(2,3,6),(2,4,5),(2,5,4),(2,6,3)

(3,2,6),(3,3,5),(3,4,4),(3,5,3),(3,6,2)
(4,1,6),(4,2,5),(4,3,4),(4,4,3),(4,5,2),(4,6,1)
(5,1,5),(5,2,4),(5,3,3),(5,4,2),(5,5,1)
(6,1,4),(6,2,3),(6,3,2),(6,4,1)
Therefore, P(sum=3D11) =3D $\frac{27}{216}$

A sum of 12 is possible in 25 ways:
(1,5,6),(1,6,5)
(2,4,6),(2,5,5),(2,6,4)
(3,3,6),(3,4,5),(3,5,4),(3,6,3)
(4,2,6),(4,3,5),(4,4,4),(4,5,3),(4,6,2)
(5,1,6),(5,2,5),(5,3,4),(5,4,3),(5,5,2),(5,6,1)
(6,1,5),(6,2,4),(6,3,3),(6,4,2),(6,5,1)
Therefore, P(sum=3D12) =3D $\frac{25}{216}$

Hence, getting a sum of 11 is more likely.

Enemy at the gates

17. (1 point) There are 41 soldiers surrounded by the enemy. They would rather die than get captured. They sit around in a circle and devise the following plan. Each soldier will kill the person to his immediate left. They will continue this till only one soldier remains who would then commit suicide. For example, if there are 7 soldiers numbered 1, 2, 3, 4, 5, 6, 7 sitting in a circle then they proceed as follows: 1 kills 2, 3 kills 4, 5 kills 6, 7 kills 1, 3 kills 5, 7 kills 3, 7 commits suicide.

    (a) In how many ways can 41 soldiers be arranged around a circle?

    Solution: n non-identical things can be arranged circularly in $(n-1)!$ ways and since, the soldiers are 41 in number people can be arranged in $(41-1)! = 3D(40)!$ ways.

    (b) If you were one of the 41 soldiers and the soldiers were randomly arranged in the circle, what is the probability that you would survive?

    Solution: The soldier at position 19 would always survive. So, the number of arrangements such that I'll always get position 19 $= 3D(40)!$, since remaining 40 soldiers can arrange without restriction.
    And total number of arrangements $= 3D(40)!$.
    Now, the soldier number 1 can be placed at any one of the 41 positions and therefore, required probability $= 3D\frac{1}{41}\frac{(40)!}{(40)!} = 3D\frac{1}{41}$.

(c) **[Ungraded question]** Is there a specific position in which you can sit so that you are the last surviving soldier?

Solution: Yes, at position 19.