N. KAUSIK
CS21M037

LARP Quiz 3

1. $n = 240$ $\cancel{p = \frac{2}{6}}$ $\cancel{\lambda = \frac{240}{60} = 4}$ cycles/ per min

In 60 mins, $\lambda = 240$

A) Poisson dist

$$P_x(x) = \frac{e^{-6} \cdot 6^x}{x!}$$

For 3 cycle in $1\frac{1}{2}$ min $= \frac{3}{2} = 1.5$ min,

In 1.5 min, ~~cycles $= 3 \times \frac{2}{3} = 2$~~,

$\lambda = 4 \times \frac{3}{2} = 6$

$$P_x(3) = \frac{e^{-6} \cdot 6^3}{3 \times 2} = \cancel{8e^{-4} < 0.134}$$

$$= e^{-6} \times 36 = 0.089$$

B) $E(x) = \lambda = 4$ cycles per 1 min

In 2 mins, exp $= 8$

c) $\cancel{P_x(8)}$ In 2 mins, 8 cycles. $\lambda = 8$

~~In 1 min, 4 cycles.~~

$$\therefore \cancel{P_x(4) = \frac{e^{-4} \, 4^4}{4 \times 3 \times 2} = \frac{e^{-4} \times 16 \times 2}{3} = e^{-4} \times \frac{32}{3}}$$

$$P_x(8) = \frac{e^{-8} \, 8^8}{8!} = 0.1395$$

2. i) $f_{XY}(x,y) = \begin{cases} 2k^2(x+y) & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$

$\int_x \int_y f_{XY}(x,y)\,dx\,dy = 1 = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy$

$= 0 + \int_0^2 \int_0^2 2k^2(x+y)\,dy\,dx$

$= 2k^2 \int_0^2 \left[\int_0^2 x\,dy + \int_0^2 y\,dy \right] dx = 2k^2 \int_0^2 \left[x(2) + \left(\frac{2^2}{2}\right) \right] dx$

$= 4k^2 \int_0^2 (x+1)\,dx = 4k^2 \left(\frac{2^2}{2} + 2 - 0 \right) = 1$

$= 16k^2 = 1 \Rightarrow k = \frac{1}{4}$

ii) $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$

$= 0 + \int_0^2 2k^2(x+y)\,dy = 2k^2 \left[\int_0^2 x\,dy + \int_0^2 y\,dy \right]$

$= 2k^2 \left(2x + \frac{2^2}{2} \right) = 4k^2(x+1)$

at $x = 1$,

$f_X(1) = 4k^2(1+1) = 4 \times \frac{1}{16} \times 2 = \frac{1}{2}$

3. $P(\text{Head}) = p = 0.4$

For 9 tosses,

For getting coin,

9^{th} toss = Head

In 8 tosses, we need exactly 2 heads.

$\therefore P(\text{getting coin}) = P(9^{th}\text{ toss} = \text{Head}) \cdot P(2 \text{ heads in } 8 \text{ tosses})$

$= 0.4 \times \left({}^8C_2 (0.4)^2 (0.6)^6 \right)$

$\Rightarrow P(\text{not getting coin}) = 1 - P(\text{getting}) = 0.9163$

4. X = Sum

Y = diff.

$E(XY) = E(X)\,E(Y)$ [Indep]

$E(X) = 2 \times \frac{1}{36} + \dots 12 \times \frac{1}{36} = \frac{252}{36}$

$E(Y) \Rightarrow$ here for every y, there is a $-y$ with equal prob

$= 0$

$\therefore E(XY) = 0$.

5. $p = 0.1$ $n = 4$. Binomial dist

$P(X = 3 / Y = 1) = \frac{P(Y = 1 (X = 3)\, P(X = 3)}{P(Y = 1)}$

$= \frac{\left({}^4C_3 (0.1)^3 (0.9) \times \right.}{0 + {}^4C_1 (0.1)(0.9)^3 + {}^4C_2 (0.1)^2 (0.9)^2 + {}^4C_3 (0.1)^3 (0.9)}$

6. $$\int_1^3\int_1^4\int_1^5 \frac{1}{k}(2a+b+4c)\,dc\,db\,da = 1$$

$$\Rightarrow k = \int_1^3\int_1^4\int_1^5 (2a+b+4c)\,dc\,db\,da$$

$$= \int_1^3\int_1^4 (10a+5b+50-2a-b-2)\,db\,da$$

$$= \int_1^3\int_1^4 (8a+4b+48)\,db\,da = 4\int_1^3\int_1^4 (2a+b+12)\,db\,da$$

$$= 4\int_1^3 \left[8a+8+48-2a-\frac{1}{2}-12\right] da$$

$$= 4\int_1^3 (6a+43.5)\,da = 4\left[27+130.5-3-43.5\right]$$

$= k$

$= 444.$

7. 1) $P(x^2+y^2 < \frac{1}{4}) \Rightarrow$ area of circle of radius $\sqrt{x^2+y^2} = \pi(x^2+y^2)$

for $< \frac{1}{4}$, area ratio $\Rightarrow \frac{\pi}{4^2} = \frac{\pi}{16} = \text{Prob}.$

2) Since indep,

$P(X<\frac{1}{2}) = \frac{1}{2}$, $P(1-Y < \frac{1}{2}) = P(Y > \frac{1}{2}) = \frac{1}{2}$

$P = \frac{1}{4}$

8. $Y = c\left(\frac{1}{4}\right)^y$

$$\sum_{y=1}^{\infty} c\left(\frac{1}{4}\right)^y = 1 \Rightarrow c\sum_{y=1}^{\infty} \frac{1}{4^y} = 1$$

$$c\left[\frac{\frac{1}{4}}{1-\frac{1}{4}}\right] = 1 \Rightarrow c\left[\frac{\frac{1}{4}}{\frac{3}{4}}\right] = 1$$

$$\Rightarrow c = 3$$

9. $Var(S) = E(S^2) - (E(S))^2$

$$E(S^2) = \sum_{i=0}^{6} E(x_i^2) + \sum_{j=0}^{6}\sum_{\substack{k=1 \\ j \neq k}}^{6} E(x_j x_k)$$

$$= n \cdot \frac{1}{n} + 2 \times {}^nC_2 \times \frac{(n-2)!}{n!} = 1 + 1 = 2$$

$$E(S) = E\left(\sum^{n} x_i\right) = \sum_0^n E(x_i) = \sum_0^n \left(0 \cdot P(0) + 1 \cdot P(1)\right)$$

$$= \sum_0^n \left(\frac{(n-1)!}{n!}\right) = 1$$

$$Var(S) = 2 - 1^2 = 1$$

10. $P(x=1) = 0.1$

$$P(x=1) + P(x=2) = 0.35 \Rightarrow P(x=2) = 0.25$$

$$P(x=2) + P(x=3) = 0.55 \Rightarrow P(x=3) = 0.3$$

$$P(x=4) = 1 - 0.1 - 0.25 - 0.3 = 1 - 0.65 = 0.35$$

$$E((4+x)^2) = \sum_{i=1}^{4} (4+i)^2 P(x=i)$$

$$= 0.1 \times 25 + 0.25 \times 36 + 0.3 \times 49 + 0.35 \times 64 \simeq 48.6$$

11. $\mu = 47 \quad \sigma^2 = 9 \quad \sigma = 3.$

For ~~pass~~ D grade, Mark $> \mu + \sigma = 50$

For fail, Mark $< \mu - 2\sigma = 41$

S1 $\rightarrow$ # D grade $\sim \left(1 - \frac{34+50}{100}\right) \times 800 = \cancel{272}$ 128

~~256~~

S2 $\rightarrow$ # failed $\sim \left(1 - \frac{50+47.5}{100}\right) \times 800$

$= (20)$

12. P(same coin after 99 tosses) [No. of tails should be even]

$n = 99, \; = {}^nC_0 (0.5)^n + {}^nC_2 (0.5)^2 (0.5)^{n-2} \ldots + \left({}^nC_n (0.5)^n\right)$

$= \frac{1}{2} \times 2^{99} (0.5)^{99} = \frac{1}{2}.$

$\therefore P = 2$

P(same coin after 100 tosses)

If 100^{th} toss is T, we need odd # tails in 99 tosses.

" " " " H, " " even " " " "

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}.$

$Q = 2$

$P - Q = 0$

~~4~~

14. $n = 10000$

$p = 1 - \frac{1}{1600}$ as $\frac{1}{1600}$ = prob that house hit.

$\therefore P(\text{no piece falls}) = \left(\frac{1599}{1600}\right)^{10000} \sim 0.00193$

15. Since you stop after first win,

Win at round 1 $\Rightarrow P(\text{win at } 1) = 0.5$

Final value $= 100 + 1 = 101$

Profit $= 1$

Win at round 2 $\Rightarrow LW,\ P(LW) = (0.5)^2$

Val $= 100 - 1 + 2 = 101$

Profit $= 1$

. . . 3 $\Rightarrow LLW \Rightarrow P = (0.5)^3$

Profit $= -1 - 2 + 4 = 1$

. . . 4 $\Rightarrow LLLW \Rightarrow P = (0.5)^4$

Profit $= -1 - 2 - 4 + 8 = 1$

. . . 5 $\Rightarrow P = (0.5)^5$ Profit $= -1 - 2 - 4 - 8 + 16 = 1$

. . . 6 $\Rightarrow P = (0.5)^6$ Profit $= 1$

Lose at $6^{th} \Rightarrow P = (0.5)^6$ Can't play further

Profit $= -63$.

$E(\cancel{\text{win}}\ \text{Profit}) = \sum_{i=1}^{6} (0.5)^i - 63 \times (0.5)^6 = 0$

16. $f(x) = k(1-3x) \qquad 0 \le x \le 1$

$$\int_0^1 k(1-3x)\,dx = 1 \Rightarrow k\left[1-\frac{3}{2}\right] = 1$$

$$k \times \frac{-1}{2} = 1 \Rightarrow k = -2$$

$$f(x) = 2(3x-1) \qquad 0 \le x \le 1$$

$f(0) = -2$

$f(1) = 4$

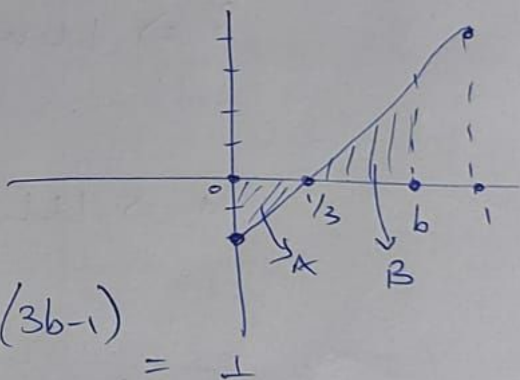
For $0 \le b \le 1$,

$$P(x \le b) = \frac{1}{3} P(x > b)$$

Since, $P(x \le b) + P(x > b) = 1$

$$P(x \le b) + 3P(x \le b) = 1$$

$$P(x \le b) = \frac{1}{4}$$



$$A + B = \frac{1}{4}$$

$$\frac{1}{\cancel{2}} \times \frac{1}{3} \times (-\cancel{2}) + \frac{1}{\cancel{2}} \times \left(b - \frac{1}{3}\right) \times \cancel{2}(3b-1) = \frac{1}{4}$$

$$\frac{-1}{3} + \left(b-\frac{1}{3}\right)(3b-1) = \frac{1}{4}$$

$$(3b-1)^2 = \frac{3}{4} + 1 = \frac{7}{4}$$

$3b-1 = \frac{\sqrt{7}}{2}$ as $0 \le b \le 1$, b can't be -ve.

$\therefore 3b-1 > -1$

can't be $-\frac{\sqrt{7}}{2}$.

$$b = \frac{\frac{\sqrt{7}}{2}+1}{3} = \frac{2+\sqrt{7}}{6} \simeq \underline{0.77}$$

13. $P(H) = 0.3 \quad P(W) = 0.3 \quad P(G) = 0.4$

$P(U|H) = 0.05 \quad P(U|W) = 0.04 \quad P(U|G) = 0.02$

$$P(W|U) = \frac{P(U|W)\,P(W)}{P(U)}$$

$$= \frac{0.04 \times 0.3}{0.3 \times 0.05 + 0.3 \times 0.04 + 0.4 \times 0.02} = \frac{4 \times 3}{3 \times 5 + 3 \times 4 + 4 \times 2}$$

$$= \frac{12}{15 + 12 + 8} = \frac{12}{35} \quad /\!/$$

15. $X \rightarrow$ # of ppl infected out of 4

$Y \rightarrow$ You get infected.

$$P(X=3 \mid Y=1) = \frac{P(Y=1 \mid X=3)\,P(X=3)}{P(Y=1)}$$

$$= \frac{1 \times {}^4C_3 \times (0.1)^3 (0.9)}{0 + {}^4C_1 \times (0.1)(0.9)^3 + {}^4C_2 (0.1)^2 (0.9)^2 + {}^4C_3 (0.1)^3 (0.9) + {}^4C_4 (0.1)^4}$$

$$= \frac{4 \times (0.1)^3 (0.9)}{4 \times (0.1)(0.9)^3 + 6 \times (0.1)^2 (0.9)^2 + \cdots}$$

$$= \frac{4 \times (0.1)^3 (0.9)}{1 - {}^4C_0 (0.9)^4} = \frac{0.0036}{0.3439} = 0.010$$