

**Instructions**

- All answers are to be written on one side of a single A4 sheet (could be a single page from your notebook also). We will refer to this as the **answer sheet**.
  - All Qs are either MCQs or fill-in-the-blanks.
  - Each Q carries 2 marks.
  - For MCQ Qs, write down the Q number followed by the option (A,B,C,D)
  - For fill-in-the-blanks Qs just write down the Q number followed by the answer (the answer could be a scalar, a vector or a matrix)
  - On the **answer sheet**, we only expect you to write the answer and not provide any explanations
  - You can write the explanations separately in rough sheets
  - By 9:50 am, you will take a photo of your **answer sheet** and upload them on grade-scope.
  - By 10:00 am, you need to upload the **rough sheets**.
1. Suppose  $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$ . From the triangular law of inequality we know that  $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$ . Give an example of  $\mathbf{u}, \mathbf{v}$  such that  $\mathbf{u} \neq \mathbf{0}, \mathbf{v} \neq \mathbf{0}$  and  $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u} - \mathbf{v}\|$
  2. Write down all values of  $c$  for which the columns of this matrix will be dependent.  
$$\begin{bmatrix} c & c & c \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix}$$
  3. Consider the following matrix  $A = \begin{bmatrix} a & b & c \\ 1 & e & f \\ 0 & i & i \end{bmatrix}$  If  $a, e \neq 0$  then  $A$  would have dependent columns if
    - A.  $ae - 2b = ac - 2f$
    - B.  $ae + b = ac + f$
    - C.  $ae - b = ac - f$
    - D.  $ae + 2b = ac + 2f$
  4. Write down some value of  $d$  and  $t$  for which this system of equations will have 0 solutions.
$$\begin{aligned} 4x + 7y + z &= 0 \\ 8x + dy + z &= 2 \\ y - z &= t \end{aligned}$$

5. Write down two matrices  $A$  and  $B$  such that  $A, B \in \mathbb{R}^{2 \times 2}$  and  $A \neq B$ ,  $A \neq I$ ,  $B \neq I$  and  $AB = B$ ,  $BA = B$ .
6. If  $A$  is an invertible  $n \times n$  matrix such that  $n = 2^k$  and  $P$  is a permutation matrix then  $PA$  is
  - A. always invertible
  - B. invertible only if it exchanges exactly one pair of rows in  $A$
  - C. invertible only if it exchanges more than one pair of rows in  $A$
7. Write down a  $4 \times 4$  matrix  $A$  whose column space is the same as its nullspace or explain why this is not possible.
8. Consider the following matrix

$$A = \begin{bmatrix} 1 & 2 & -5 & 3 & 5 \\ 2 & 2 & -8 & -5 & 1 \\ -5 & -8 & -7 & 6 & 7 \\ 3 & -5 & 6 & -3 & -1 \\ 5 & 7 & 7 & -1 & -2 \end{bmatrix}$$

I performed  $LDU$  factorisation on the above matrix and got

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ -5 & -1 & 1 & 0 & 0 \\ 3 & \frac{11}{2} & -\frac{1}{3} & 1 & 0 \\ 5 & \frac{9}{2} & -\frac{23}{30} & \frac{247}{311} & 1 \end{bmatrix}$$

Write down  $U$ .

9. Write down a  $4 \times 4$  matrix  $A$  such that the matrix  $U$  obtained after the  $LDU$  factorisation of  $A$  is a symmetric matrix.
10. Which of the following statements are true? (select all statements that are true)
  - A. Any rank-1 matrix  $A(m \times n)$  can **always** be written as  $\mathbf{u}\mathbf{v}^\top$  where  $\mathbf{u} \in \mathbb{R}^m$  and  $\mathbf{v} \in \mathbb{R}^n$ .
  - B. If  $A$  and  $B$  are two rank-1 matrices then the rank of their product  $AB$  can **never** be greater than 1.
  - C. If  $A$  and  $B$  are two rank-1 matrices then the rank of their sum  $A + B$  can **never** be greater than 1.
  - D. If  $A$  is a  $m \times p$  matrix and  $B$  is a  $p \times n$  matrix then  $\text{rank}(A) \leq p$  (**always**) and  $\text{rank}(B) \leq p$  (**always**) but the rank of  $AB$  can be greater than  $p$ .