Edit Distance

CS6370:NLP Course

Jan-May 2021

What is Edit distance?

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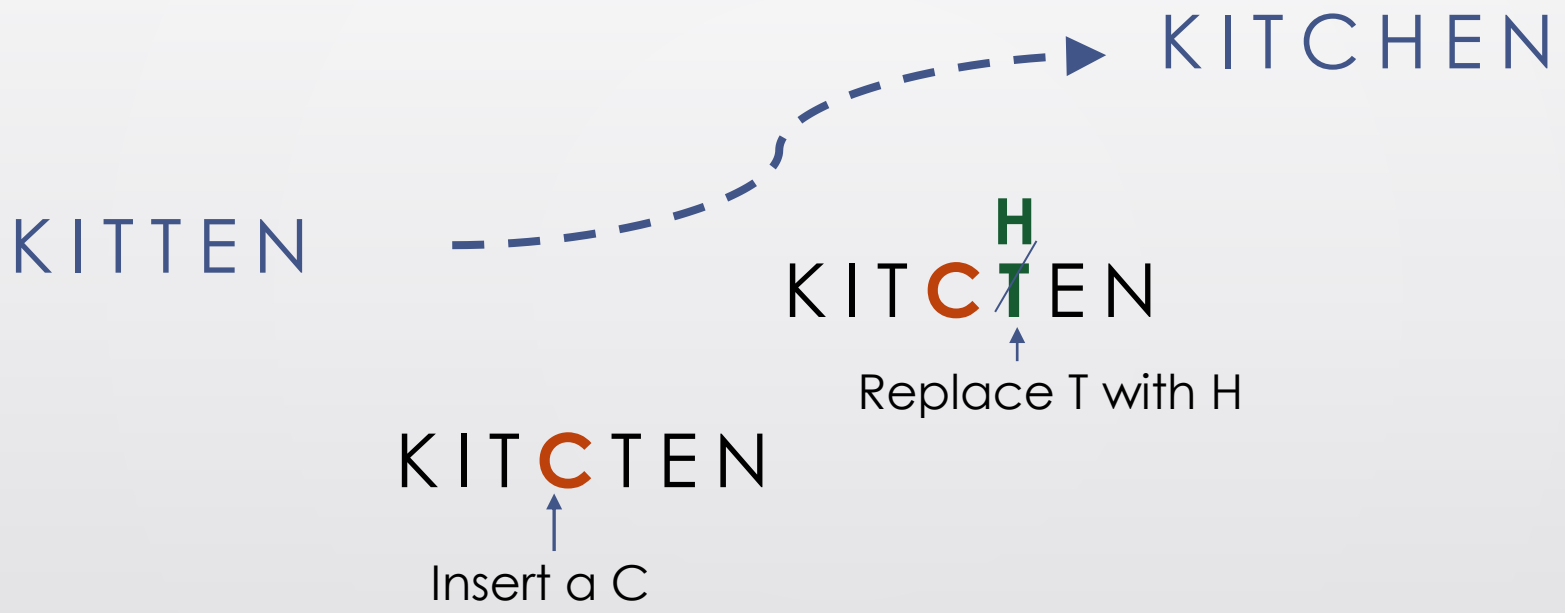
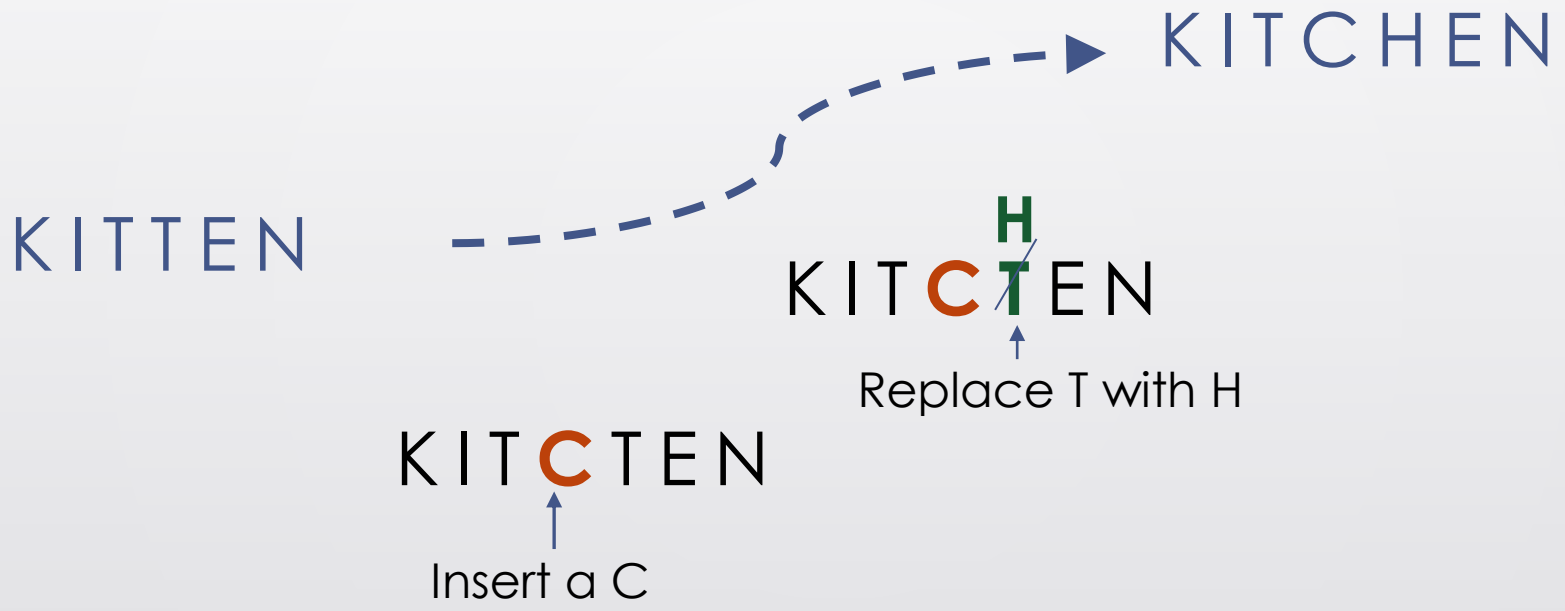
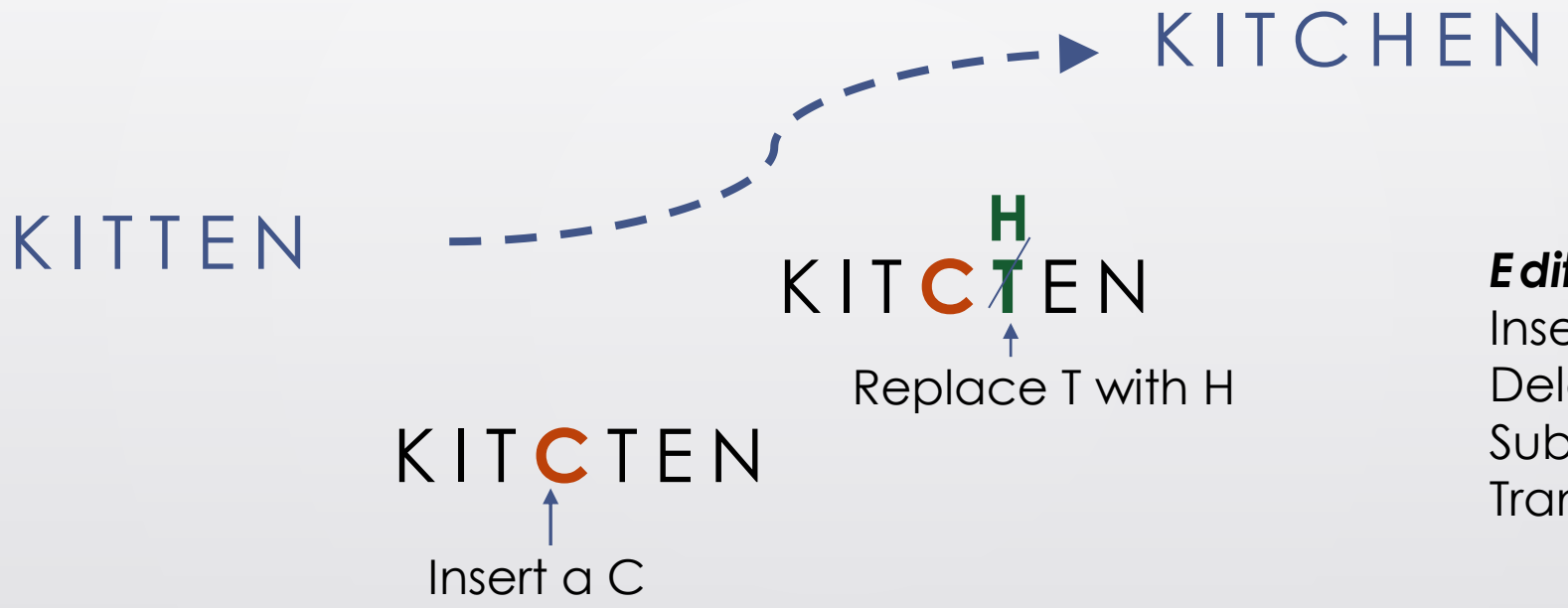


What is Edit distance?



Minimum number of edit operations needed to transform one word into another

What is Edit distance?



Edit operations
Insertion
Deletion
Substitution/Replace
Transposition (ab <-> ba)

Minimum number of edit operations needed to transform one word into another

Where is it used?

- Approximate String Matching
  - Spell Checker
    - Which is the closest candidate to ‘appel’ ?
    - Candidates: apple ; april ; append

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- Approximate String Matching
  - Spell Checker
    - Which is the closest candidate to ‘appel’ ?
    - Candidates: apple ; april ; append
  - Computational Biology
    - Matching of nucleotide sequences

An Example Worked out (Manual)

- **K I T T E N → K I T C H E N**
  - INSERT **C** (K I T C T E N)
  - REPLACE **T** WITH **H** (K I T C ~~T~~ H E N)
  - REPLACE **T** WITH **C** (K I T ~~T~~ C E N)
  - INSERT **H** (K I T C H E N)
  - DELETE **T** (K I T ~~T~~ E N)
  - INSERT **C** (K I T C E N)
  - INSERT **H** (K I T C H E N)

How to do it computationally?

- Enumerate all possible of ways of editing ‘KITTEN’ to ‘KITCHEN’
- Choose the one with minimum edit distance
- Computational complexity?

What is the complexity?

Naïve Recursive Algorithm

- K I T T E N → K I T C H E N

Given: two strings of length m and n

IF the first characters of the two strings are the same, then ignore them and compute the edit distance of the remaining substrings of length m-1 and n-1

Edit Distance (**K** I T T E N , **K** I T C H E N)

= 0 + Edit Distance (**I** T T E N , **I** T C H E N)

= 0 + 0 + Edit Distance (**T** T E N , **T** C H E N)

= 0 + 0 + 0 + Edit Distance(T E N , C H E N)

What is the complexity?

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= 0 + 0 + 0 + Edit Distance(T E N , C H E N)

What is the complexity?

Naïve Recursive Algorithm

T E N → C H E N

Given: two strings of length m and n

IF the first characters of the two don't match, then edit the first string using the three edit operations : insertion, deletion and substitution and compute the minimum edit distance corresponding to all three operations . The minimum of these three distances is returned as the edit distance of the given strings.

Edit Distance (T E N , C H E N)
(m, n)

= min (cost of inserting 'C' in first string + Edit Distance(T E N , H E N)
(m, n-1)

cost of deleting 'T' from first string + Edit Distance(E N , C H E N)
(m-1, n)

cost of substituting 'T' by 'C' in first string + Edit Distance(E N , H E N)
(m-1, n-1)

What is the complexity?

Naïve Recursive Algorithm

a p p l e → b o s s

Given: two strings of length m and n

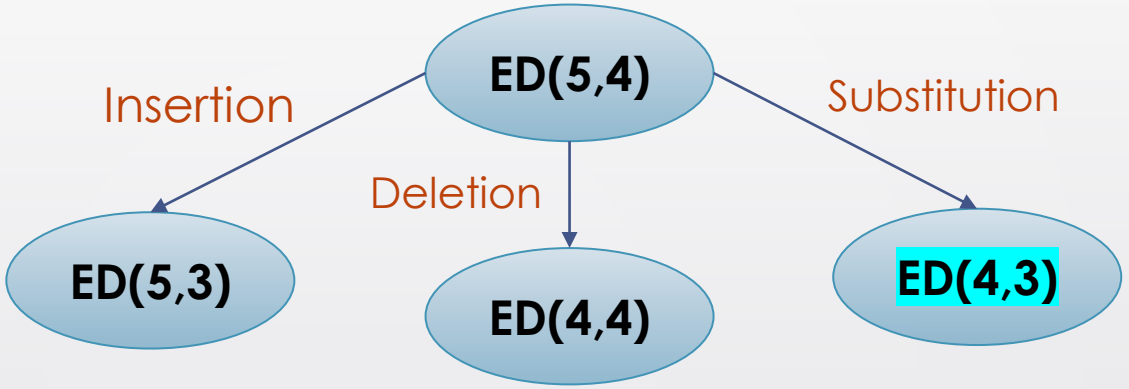
IF the first characters of the two don't match, then edit the first string using the three edit operations : insertion, deletion and substitution and compute the minimum edit distance corresponding to all three operations . The minimum of these three distances is returned as the edit distance of the given strings.

When none of the characters match,

Worst case complexity = $O(3^m)$

Worst Case Complexity

- Apple and Boss
- m=5 n=4



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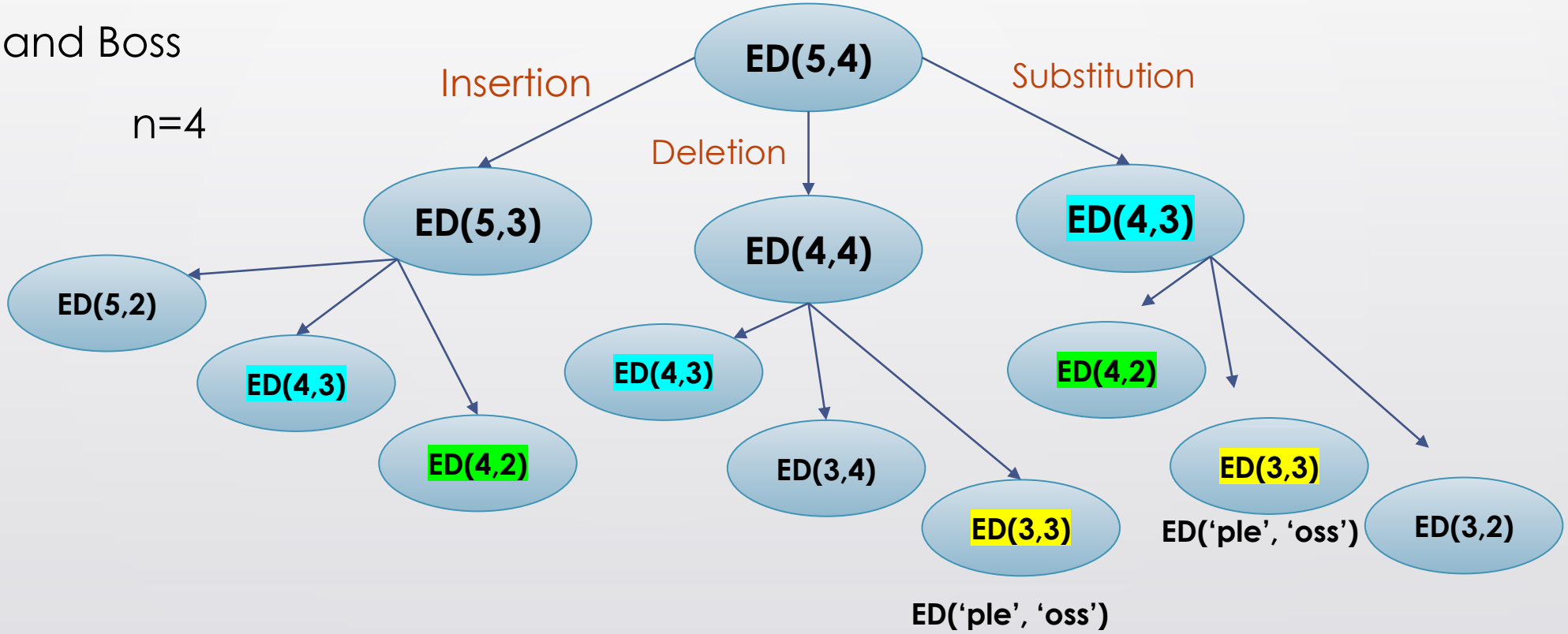

Worst Case Complexity

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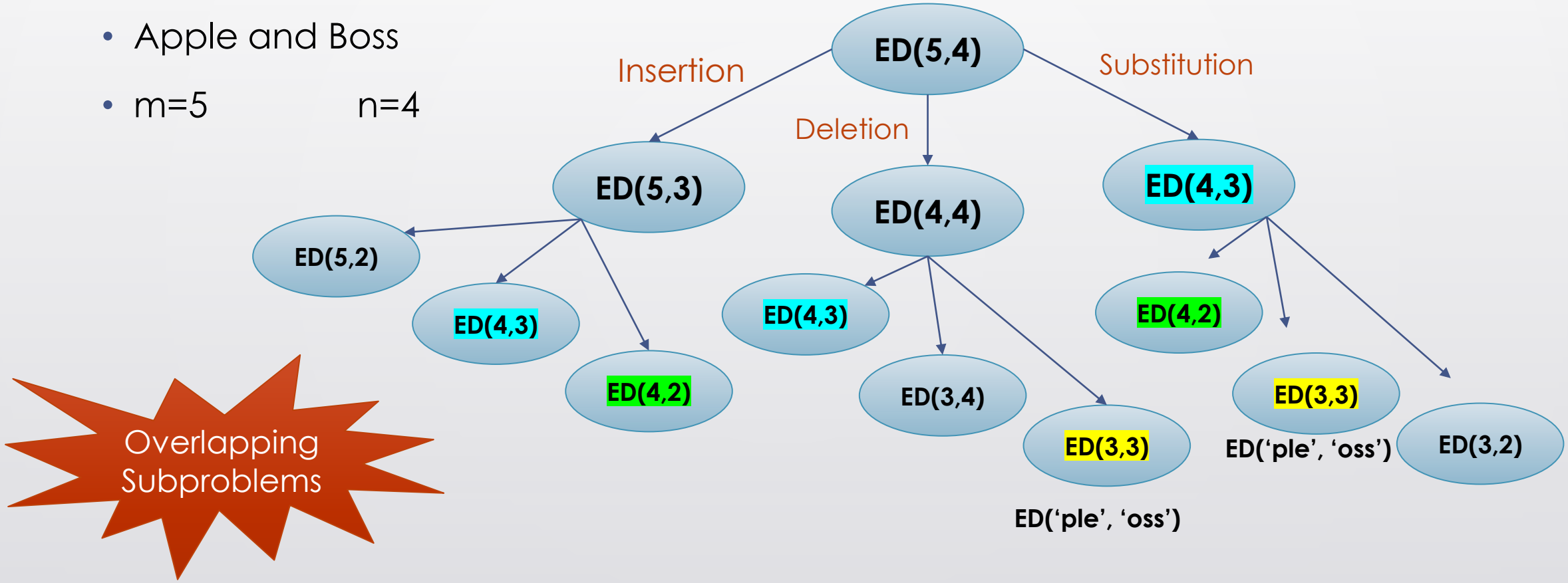

Can we do away with repeated computation of sub-problems?

- How to realize this computationally?

Dynamic Programming

What is it?

- Dynamic Programming (DP) is an algorithmic technique for solving optimization problems
- Mainly an optimization over plain recursion
- Applicable when the following properties hold:
  - Optimal substructure
  - Overlapping Subproblems

Where is it useful?

- Optimal Substructure
  - optimal solution to the problem contains within it optimal solutions to subproblems.

- Overlapping Subproblems
  - When a recursive algorithm revisits the same problem over and over again, the problem has overlapping subproblems.

- Solve each subproblem once and then store the solution in a table and when needed, perform lookup operation.

An Example : Fibonacci number calculation

- Fib (n) = Fib (n-1) + Fib (n-2)

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Plain Recursive solution

```
int fib(int n)
{
    if (n <= 1)
        return n;
    return fib(n-1) + fib(n-2);
}
```

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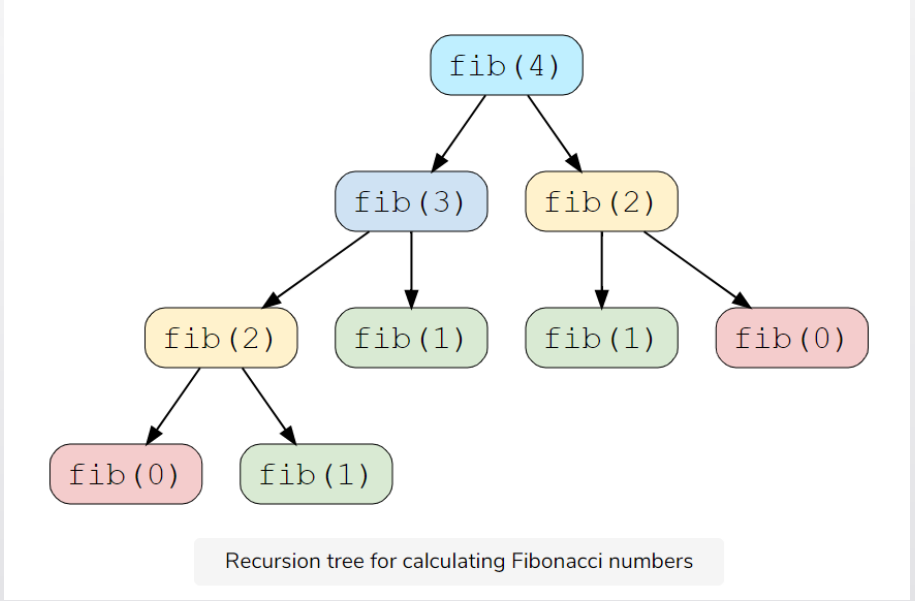

Recursion tree for calculating Fibonacci numbers

Src: https://www.educative.io/courses/grokking-dynamic-programming-patterns-for-coding-interviews/m2G1pAq0OO0

DP solution

```
int fib(int n) {
   if (n==0) return 0;
   int dp[] = new int[n+1];          //table to store solutions to subproblems

   //base cases
   dp[0] = 0;     dp[1] = 1;

   for(int i=2; i<=n; i++)
            dp[i] = dp[i-1] + dp[i-2];
   return dp[n];
}
```

Back to Edit distance

Can we try a DP solution to Edit distance?

- We saw that the problem of edit distance has both the following properties
  - Overlapping subproblems
  - Optimal substructure

Recursive Formula for DP

- Distance b/n two strings P and Q of length M and N is denoted by D(M,N)

```
For each   i = 1…M
      For each   j = 1…N
                          { D(i-1,j) + 1
            D(i,j)= min   { D(i,j-1) + 1
                          { D(i-1,j-1) +    1; { if X(i) ≠ Y(j)
                                            0; { if X(i) = Y(j)
```

[3]Image taken from Coursera NLP lecture(Slightly Modified)

Recursive Formula for DP

- Distance b/n two strings P and Q of length M and N is denoted by D(M,N)

Cost of deletion

```
For each   i = 1…M
      For each   j = 1…N
                          { D(i-1,j) + 1
          D(i,j)= min     { D(i,j-1) + 1
                          { D(i-1,j-1) +    1; { if X(i) ≠ Y(j)
                                            0; { if X(i) = Y(j)
```

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                                            0; if X(i) = Y(j)
```

Cost of insertion

[3]Image taken from Coursera NLP lecture(Slightly Modified)

Recursive Formula for DP

- Distance b/n two strings P and Q of length M and N is denoted by D(M,N)

```
For each   i = 1…M
      For each   j = 1…N
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            D(i,j)= min   { D(i,j-1) + 1
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                                           { 0; if X(i) = Y(j)
```

Cost of substitution

[3] Image taken from Coursera NLP lecture(Slightly Modified)

Recursive Formula for DP

- Base Conditions

$$D(i,0) = i$$
$$D(0,j) = j$$

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 1; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

Costs of Edits

- Let's assume that the cost of all edit operations are the same and equal to 1
- Sometimes, cost may be different for different edit operations

Note:

- When only Insertion, Deletion and Substitution are considered, it is called Levenshtein distance
- When transposition is also considered, it is called Damerau-Levenshtein distance

Distance between two strings PARK and SPAKE

- m =4
- n = 5

			1	2	3	4
		_	P	A	R	K
	_	0	1	2	3	4
1	S	1				
2	P	2				
3	A	3				
4	K	4				
5	E	5				

Base Cases

Distance between two strings PARK and SPAKE

- m =4
- n = 5

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 1; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

		j	1	2	3	4
			P	A	R	K
i		0	1	2	3	4
1	S	1				
2	P	2				
3	A	3				
4	K	4				
5	E	5				

Distance between two strings PARK and SPAKE

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			1	2	3	4
			P	A	R	K
		0	1	2	3	4
1	S	1	1	2	3	4
2	P	2	1	2	3	4
3	A	3	2	1	2	3
4	K	4	3	2	2	2
5	E	5	4	3	3	3

Blue numbers help to keep track of the path via which the minimum edit distance 3 is achieved

Algorithm

```
function MIN-EDIT-DISTANCE(target, source) returns min-distance

  n ← LENGTH(target)
  m ← LENGTH(source)
  Create a distance matrix distance[n+1,m+1]
  distance[0,0] ← 0
  for each column i from 0 to n do
    for each row j from 0 to m do
      distance[i,j] ← MIN( distance[i−1,j] + ins-cost(target_j),
                           distance[i−1,j−1] + subst-cost(source_j, target_i),
                           distance[i,j−1] + ins-cost(source_j))
```

Ref: NLP Textbook of Jurafsky and Martin

- Runtime is O(mn). A polynomial time algorithm.

For Practice

- Compute the edit distance table for KITTEN and KITCHEN

References

- https://www.geeksforgeeks.org/edit-distance-dp-5/
- Slides of Dileep Kumar Pattipati, taken from CS6370:2018