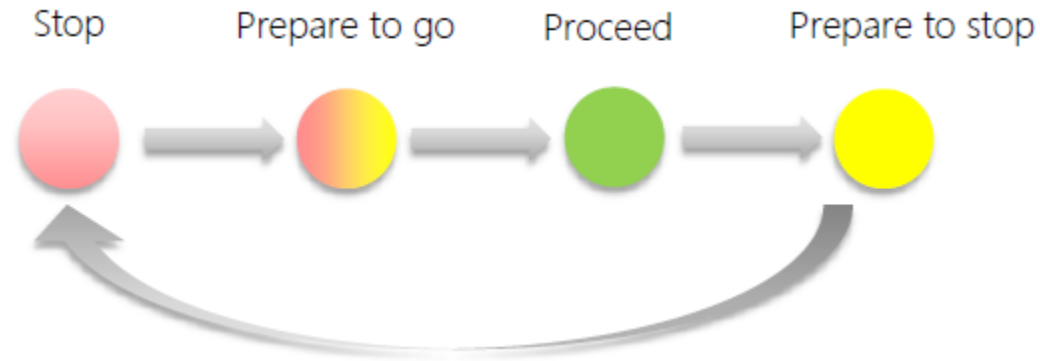


Hidden Markov Models

Hidden Markov Models

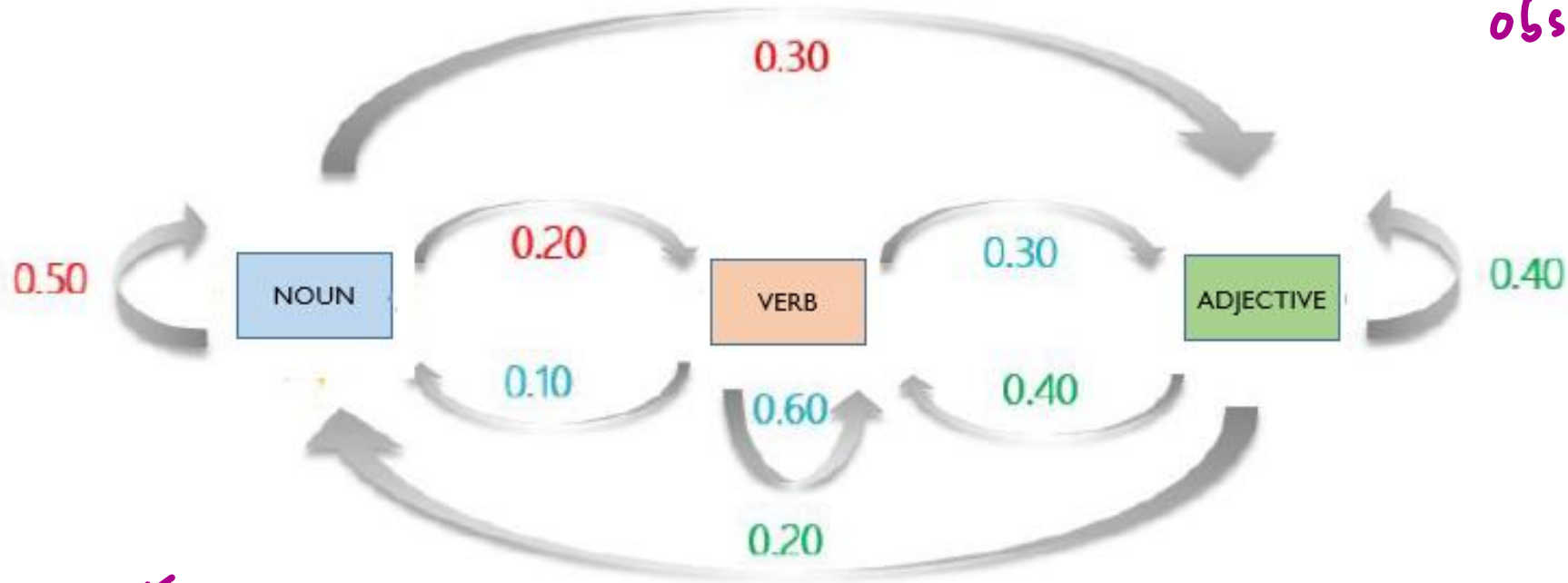
- Foundation: Markov Processes
- Observable Markov Process: Traffic Light example



Hidden Markov Model: an example

WSD }
POST }

hidden state : POS
tag
observer : words.

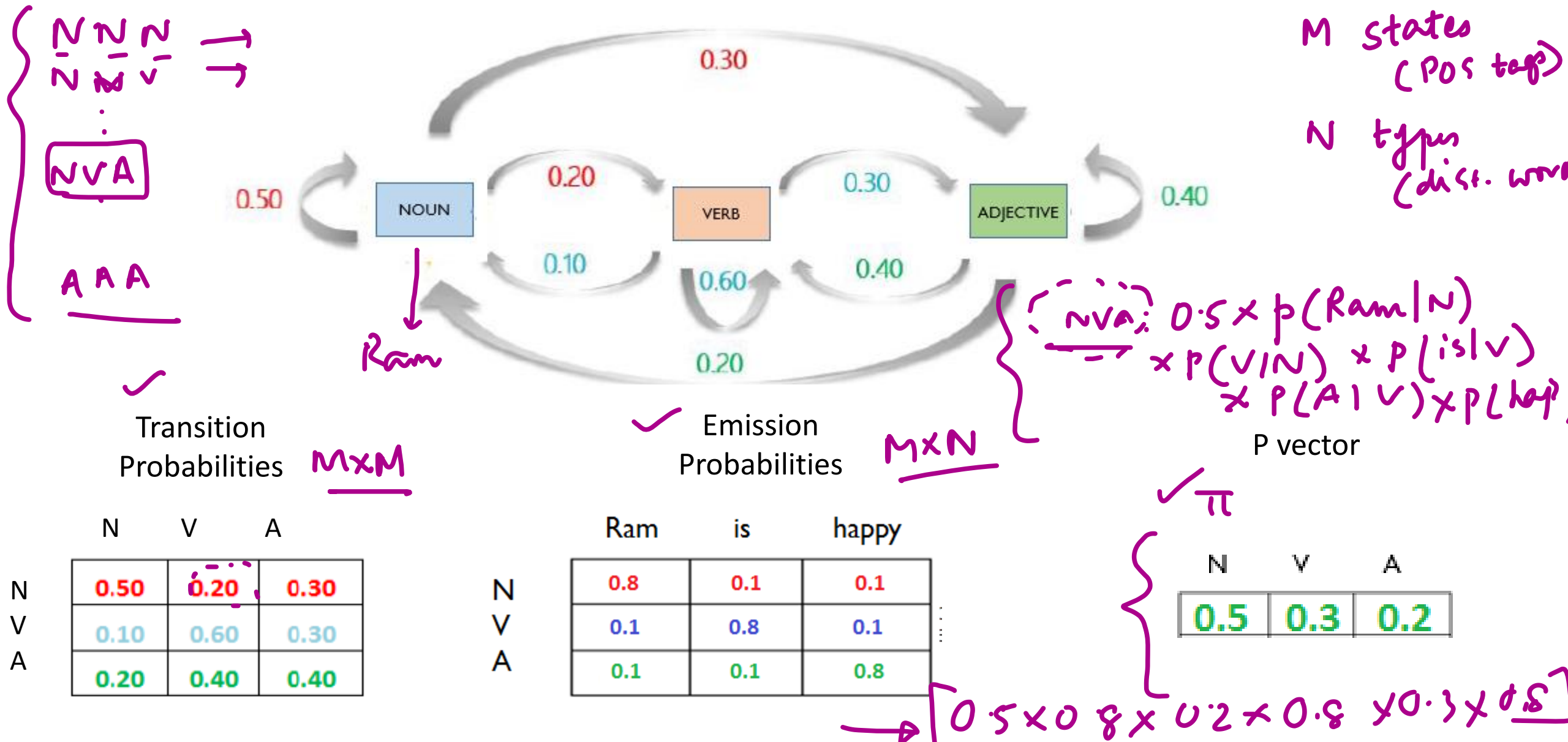


Rain is happy

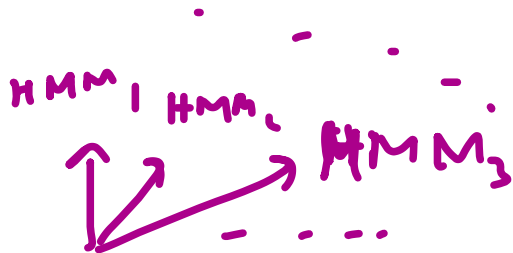
Transition
Probabilities

	N	V	A
N	0.50	0.20	0.30
V	0.10	0.60	0.30
A	0.20	0.40	0.40

Hidden Markov Model: example (contd)



Three Problems



- ✓ • Evaluation: Given a HMM, find probability of an observed sequence

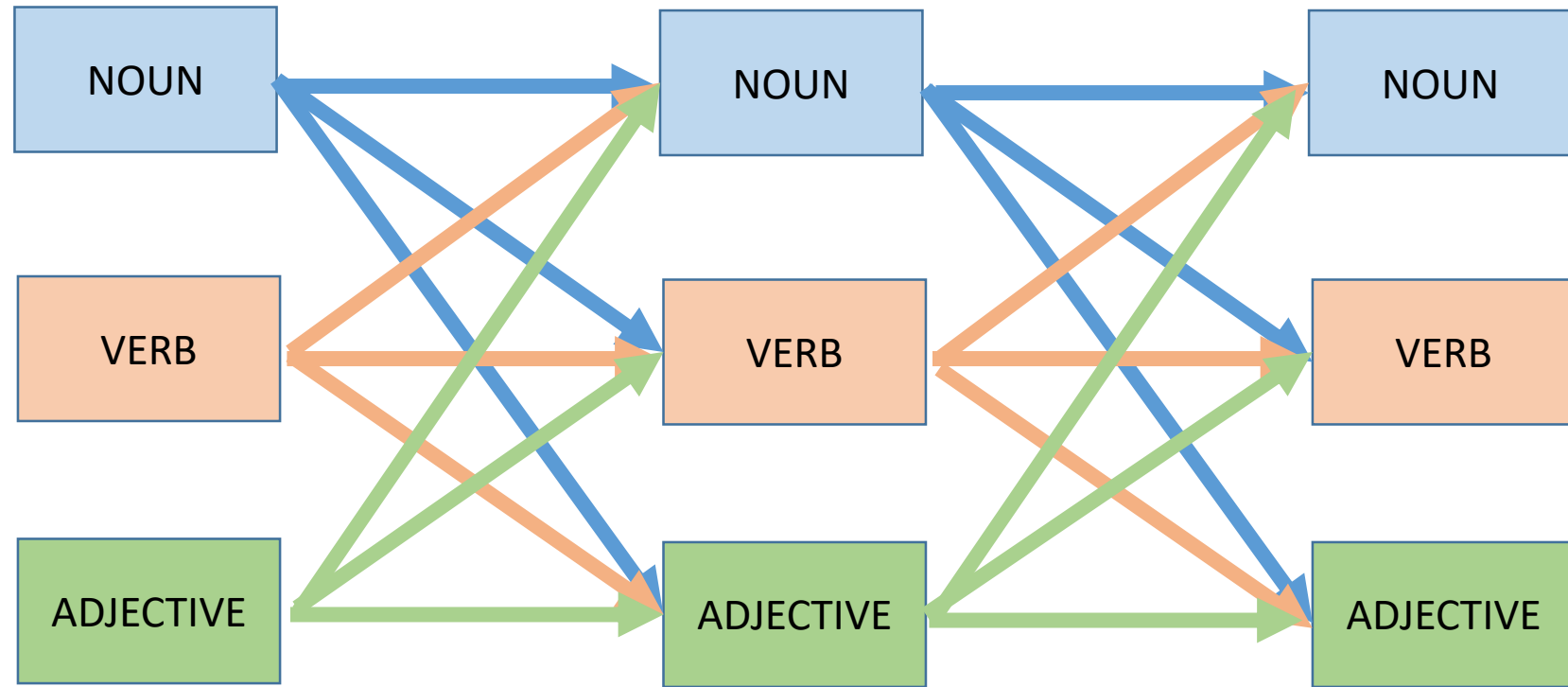
POST

- Decoding: Given a HMM, find most probable sequence of states that could have given rise to an observation.

- { • Learning: Generate a HMM that best explains a given set of observations

The forward algorithm

[Evaluation problem.]



Observation

Ram

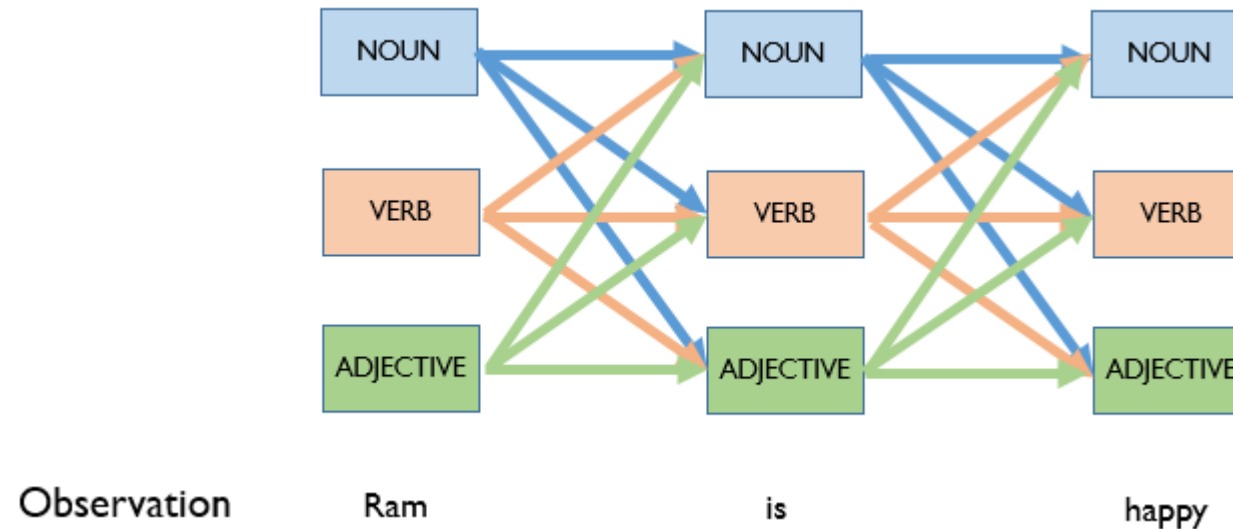
is

happy

Goal : We want to find the probability of the observed sequence, given an HMM whose parameters are known.

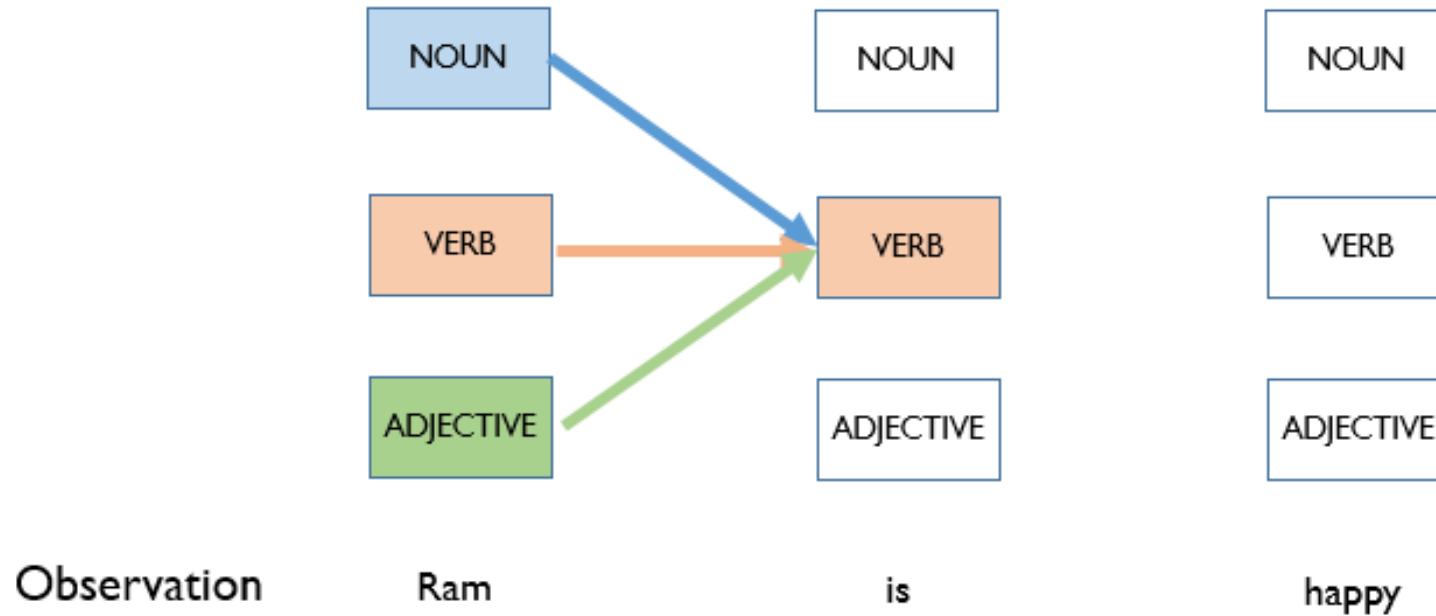
Brute Force Approach

Dynamic Programming



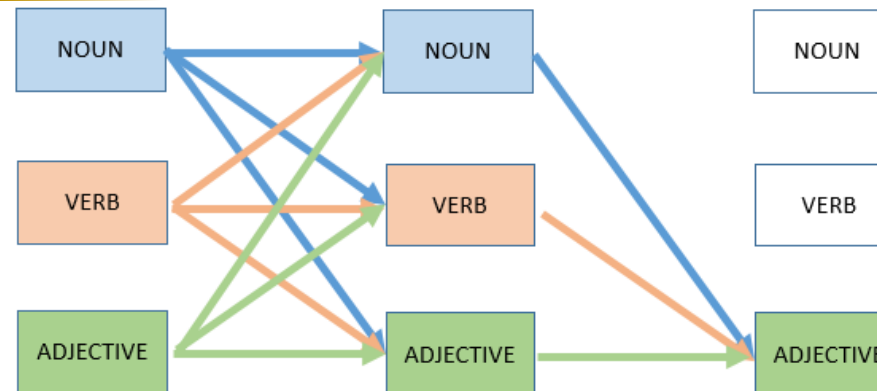
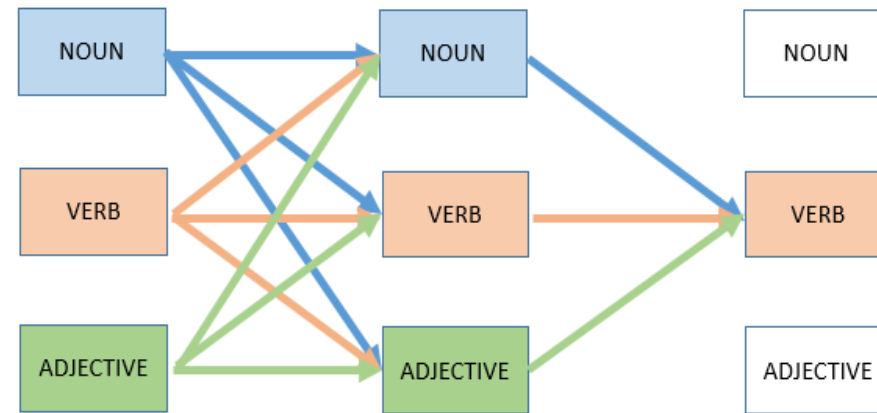
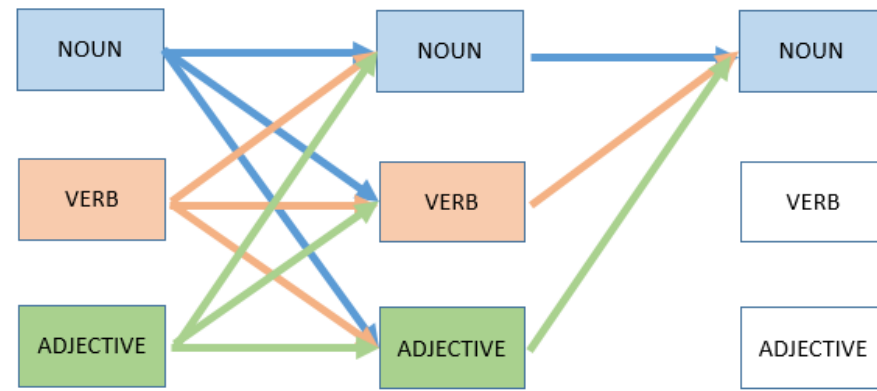
Sum over probabilities of each possible sequence of state transitions that could have given rise to the observed sequence, there are 27 such possibilities in this simple example.

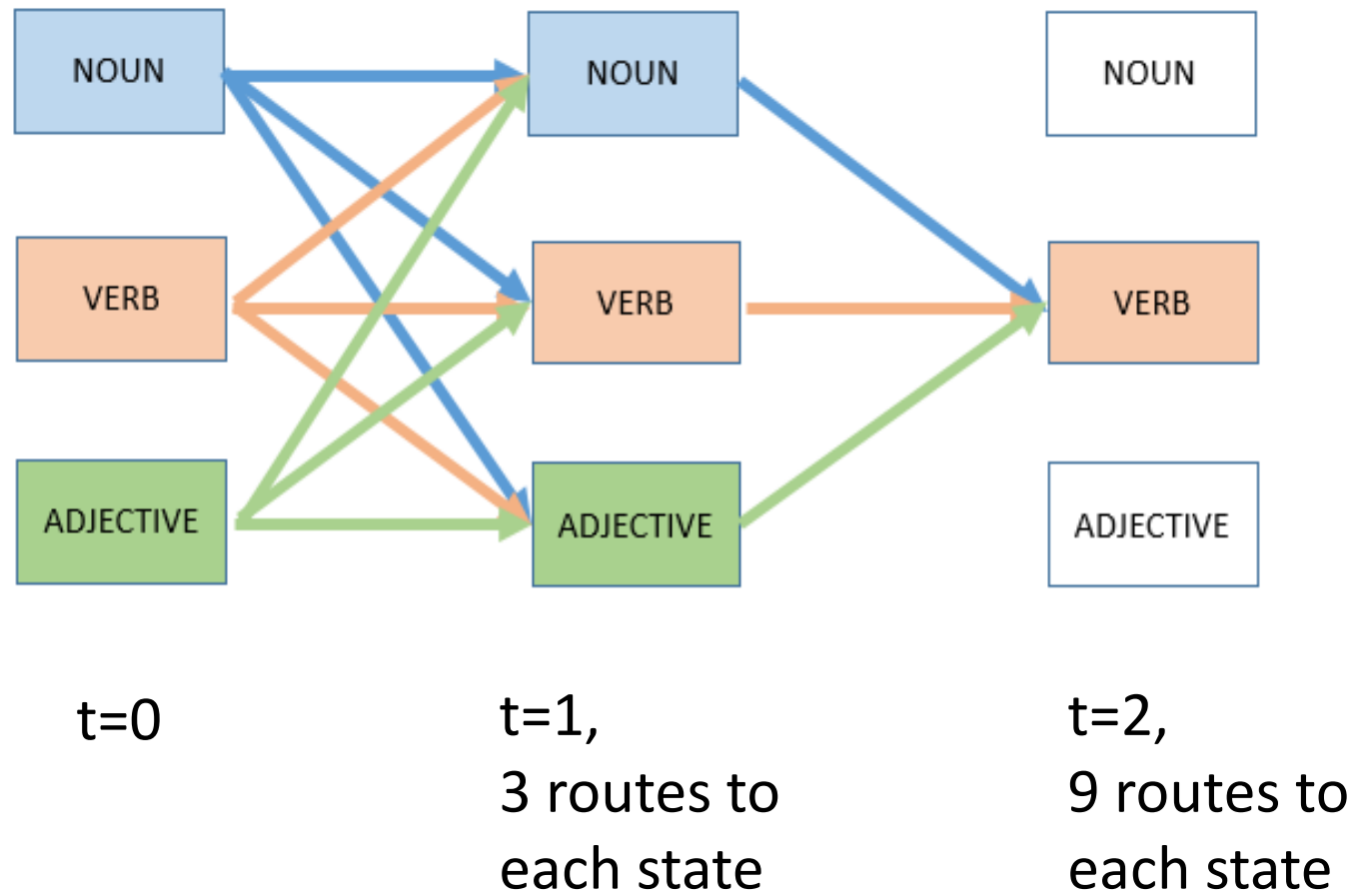
Observation

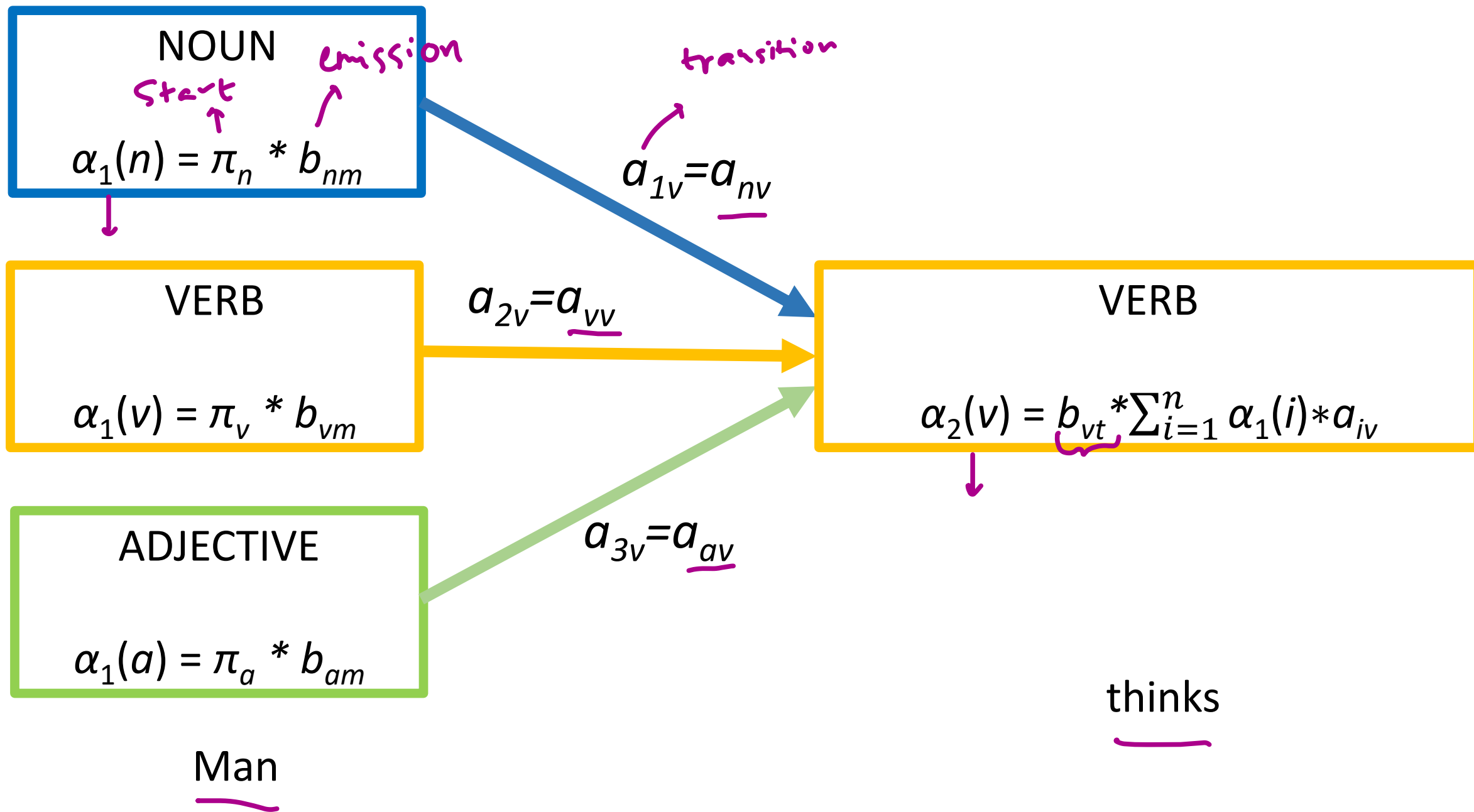


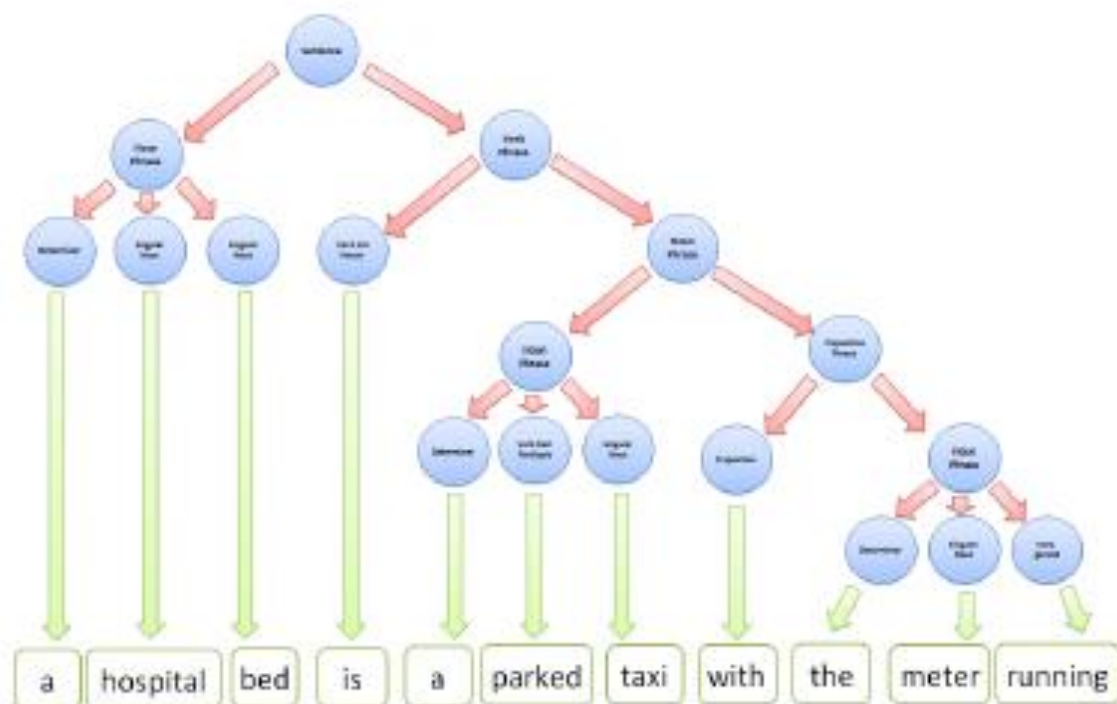
We can calculate the probability of reaching an intermediate state in the trellis as the sum of all possible paths to that state.

For example, the probability of being at a verb at $t = 2$ is calculated from the paths.









Key insight

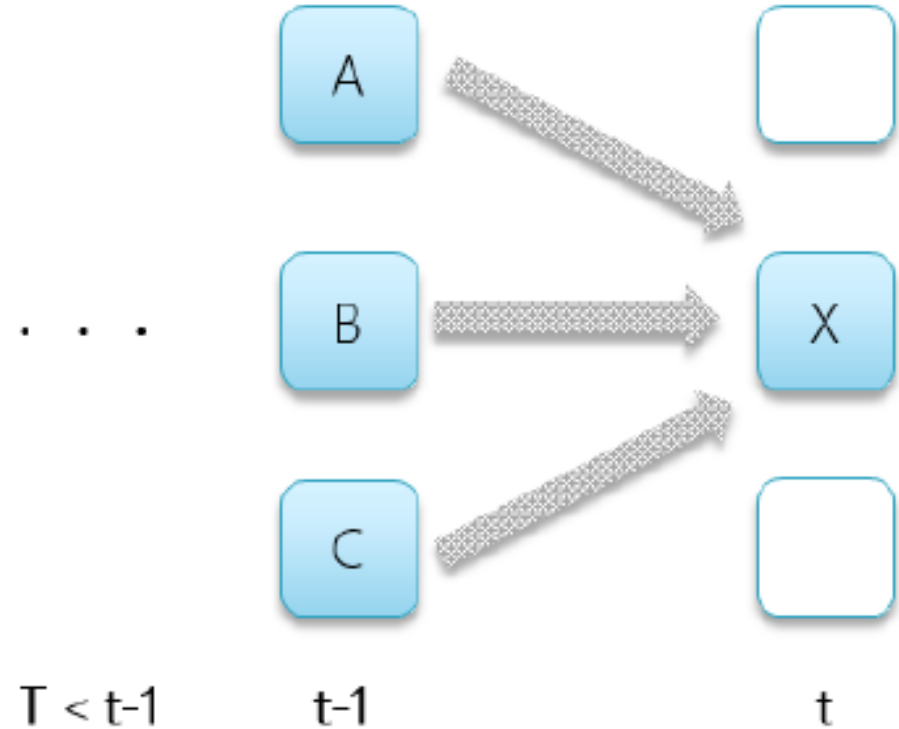
We need to find the most probable path to the state X at time t

Consider the trellis to the right

The most probable path through X must pass through one of A, B or C at time t-1

Therefore the most probable path to X will be one of

- sequence of states , ... ,A,X
- sequence of states , ... ,B,X
- sequence of states , ... ,C,X

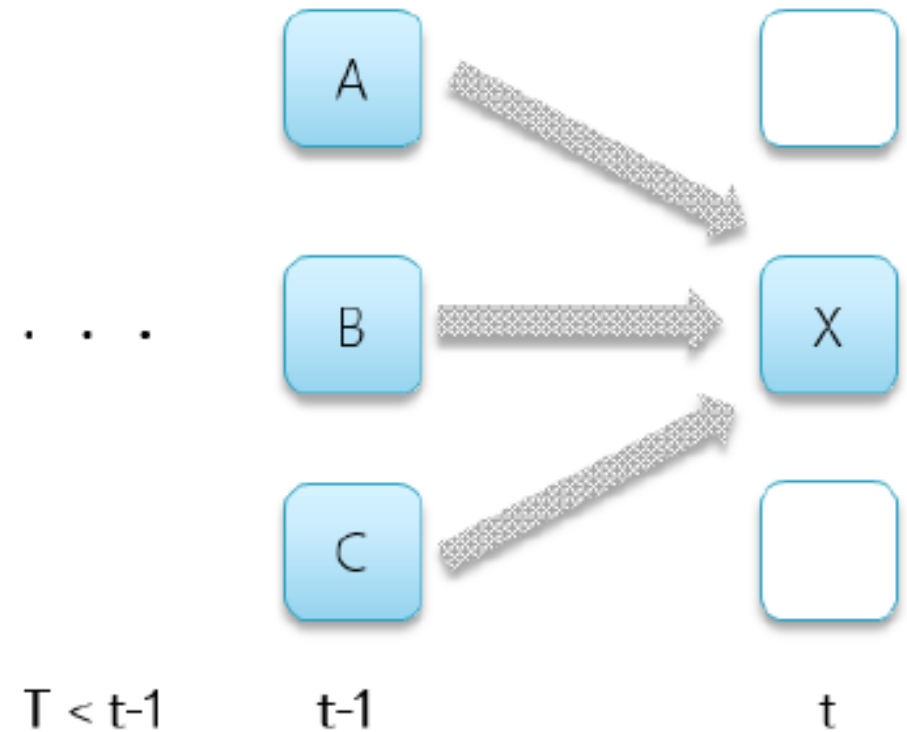


Key insight


The most probable path to X will be one of
sequence of states , ... ,A,X
sequence of states , ... ,B,X
sequence of states , ... ,C,X

1st order Markov assumption: probability of X occurring after a sequence depends only on the previous state

$$P(\text{most probable path to A}) = P(X|A).P(\text{Obs}|X)$$



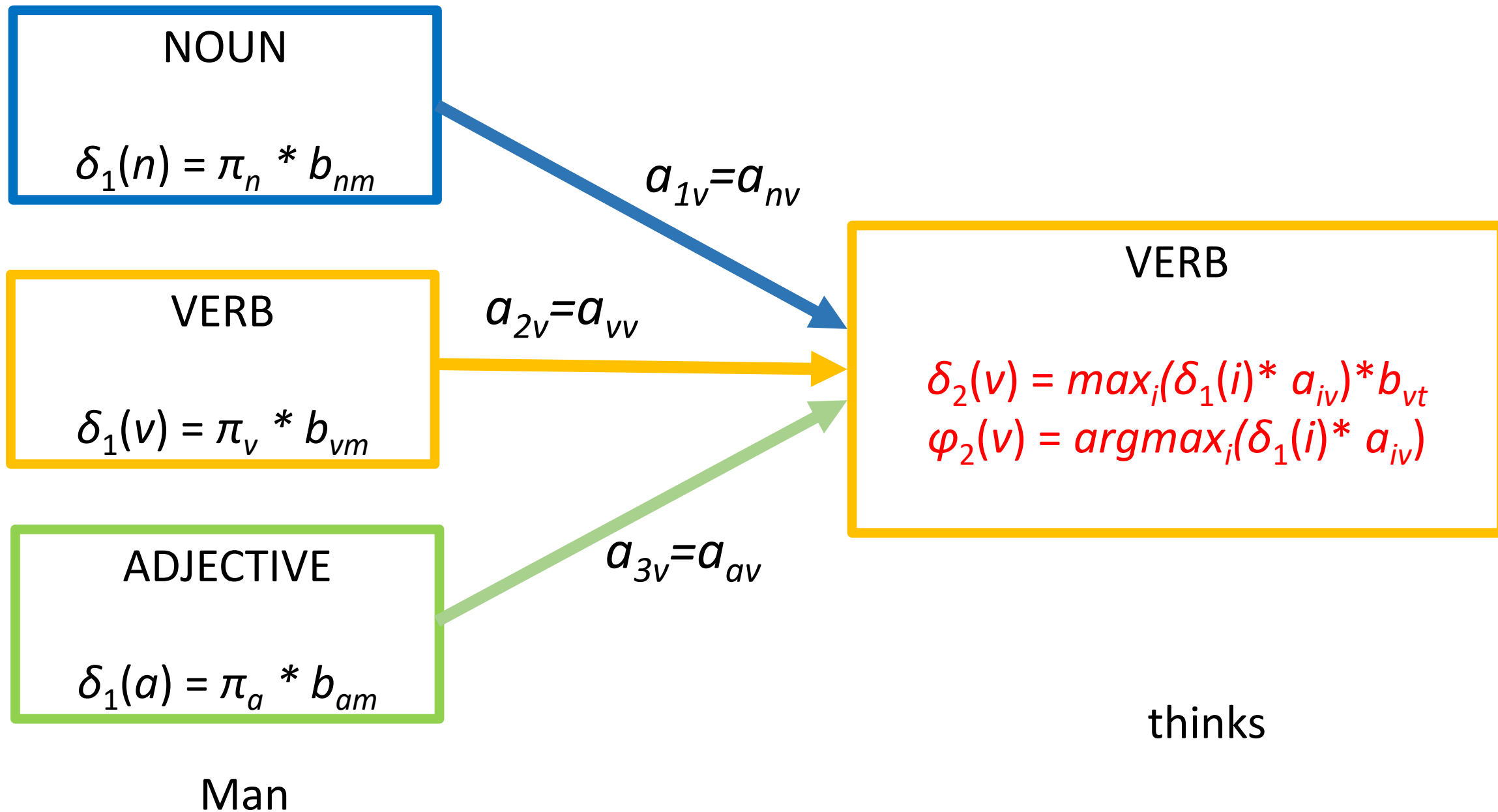
Generalizations

$$\Pr(X \text{ at time } t) = \max_{i=A_i B_i C_i} \Pr(i \text{ at time } (t-1)) * \Pr(X|i) * \Pr(\text{obs. at time } t | X)$$


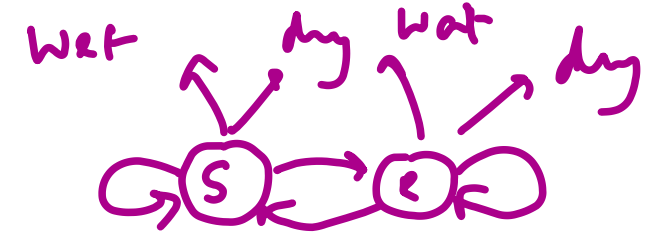
$$\delta_t(i) = \max_j (\delta_{t-1}(j) a_{ij} b_{ik_t})$$

The idea of back pointers

Viterbi Algorithm



Baum Welch Initialization



\sim ~~Sunny~~ \checkmark ~~Rainy~~
 \checkmark ~~Sunny~~ \sim ~~Rainy~~

	Sunny	Rainy
\sim Sunny	0.50	0.50
\checkmark Rainy	0.40	0.60

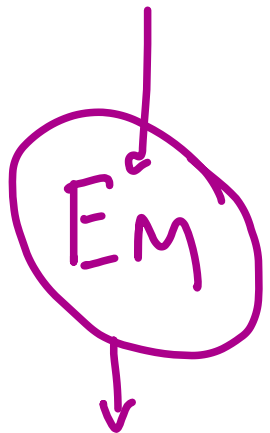
Man thinks

\sim ~~Sunny~~ \checkmark ~~Rainy~~
 \checkmark ~~Sunny~~ \sim ~~Rainy~~

	Wet	Dry
\sim Sunny	0.20	0.80
\checkmark Rainy	0.90	0.10

Complete Data	Incomplete Data
$\sim \quad \checkmark \quad \sim$	— — —
— — —	— — —
— — —	— — —

do not
come here
POSTs



\sim ~~p(Sunny) = 0.3~~ and \checkmark ~~p(Rainy) = 0.7~~
 \checkmark ~~p(Sunny) = 0.3~~ and \sim ~~p(Rainy) = 0.7~~

Mo:

E step \rightarrow Preparatory \rightarrow {Forward model}

Ram is happy

Symptom

Ram is happy

disturbance

NNN
NNN
⋮
NVA
⋮
AAA

NOISY
CHANNEL

B.i

E:

A
B → [] → $\bar{H}\bar{T}\bar{T}\bar{H}$

$P(NNN | \text{Ram is happy})$

FM
Part 2
Group

$P(S|O)$

$$\frac{P(O|S) \cdot P(S)}{\sum_{i=1}^n P(O|S_i) \cdot P(S_i)}$$

generative
process

Preparation

FM
Part 1

$P(A|O) =$
 $P(B|O) =$

Baum Welch

Observations:

WW, WW, WW, WW, WD, DD, DW, WW,
WW

W = Wet

D = Dry

If WW came from Sunny → Rainy sequence, the probability would be:

$$0.3 * 0.2 * 0.5 * 0.9 = 0.027$$

Handwritten annotations for the calculation:

- π_s points to 0.3
- $P(W|S)$ (labeled emission) points to 0.2
- $P(R|S)$ (labeled state) points to 0.5
- $\frac{P(W|R)}{\text{emission}}$ points to 0.9

	Sunny	Rainy
Sunny	0.50	0.50
Rainy	0.40	0.60

	Wet	Dry
Sunny	0.20	0.80
Rainy	0.90	0.10

$$p(\text{Sunny}) = 0.3 \text{ and } p(\text{Rainy}) = 0.7.$$

Homework: complete the E_1 and M_1 steps

Baum Welch

- The revised parameters feed into the next E step, which in turn leads to fresh estimation of parameters
- The process is repeated till parameters converge