Assumptions about individual names

Unique Names Assumption (UNA)

Distinct names represent distinct elements of the domain set.

$a \neq b \rightarrow a^{I} \neq b^{I}$

Sometimes it is convenient to adopt this.

Non-Unique Names Assumption (Non-UNA)

Distinct names may represent same element of the domain set

(I.e., More than one name is mapped to an element of the domain set)

More realistic assumption..

When needed, one has to explicitly state : $a \neq b$

$\mathcal{ALC}$ Knowledge Bases

An $\mathcal{ALC}$ Knowledge Base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$

where $\mathcal{T}$ is an $\mathcal{ALC}$ TBox and $\mathcal{A}$ is an $\mathcal{ALC}$ ABox

An interpretation $\mathcal{I}$ is a *model* of $\mathcal{K}$ if it satisfies both $\mathcal{T}$ and $\mathcal{A}$

In a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$,

If $\mathcal{A}$ is empty then $\mathcal{K}$ contains domain knowledge only

- an ontology

If $\mathcal{T}$ is empty then $\mathcal{K}$ has only situation-specific knowledge

- a knowledge graph

Example $\mathcal{ALC}$ KB $\mathcal{K}_{ex}$ = ($\mathcal{T}_{ex}$, $\mathcal{A}_{ex}$) and $\mathcal{I}'$

$$
\begin{array}{rcll}
\mathcal{T}_{ex} = \{\mathsf{Course} & \sqsubseteq & \neg\mathsf{Person}, & (\mathcal{T}_{ex}.1) \\
\mathsf{UGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.2) \\
\mathsf{PGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.3) \\
\mathsf{Teacher} & \equiv & \mathsf{Person} \sqcap \exists \mathit{teaches}.\mathsf{Course}, & (\mathcal{T}_{ex}.4) \\
\exists \mathit{teaches}.\top & \sqsubseteq & \mathsf{Person}, & (\mathcal{T}_{ex}.5) \\
\mathsf{Student} & \equiv & \mathsf{Person} \sqcap \exists \mathit{attends}.\mathsf{Course}, & (\mathcal{T}_{ex}.6) \\
\exists \mathit{attends}.\top & \sqsubseteq & \mathsf{Person}\ \} & (\mathcal{T}_{ex}.7)
\end{array}
$$

$$
\begin{array}{rll}
\mathcal{A}_{ex} = \{\mathsf{Mary} : & \mathsf{Person}, & (\mathcal{A}_{ex}.1) \\
\mathsf{CS600} : & \mathsf{Course}, & (\mathcal{A}_{ex}.2) \\
\mathsf{Ph456} : & \mathsf{Course} \sqcap \mathsf{PGC}, & (\mathcal{A}_{ex}.3) \\
\mathsf{Hugo} : & \mathsf{Person}, & (\mathcal{A}_{ex}.4) \\
\mathsf{Betty} : & \mathsf{Person} \sqcap \mathsf{Teacher}, & (\mathcal{A}_{ex}.5) \\
(\mathsf{Mary}, \mathsf{CS600}) : & \mathit{teaches}, & (\mathcal{A}_{ex}.6) \\
(\mathsf{Hugo}, \mathsf{Ph456}) : & \mathit{teaches}, & (\mathcal{A}_{ex}.7) \\
(\mathsf{Betty}, \mathsf{Ph456}) : & \mathit{attends}, & (\mathcal{A}_{ex}.8) \\
(\mathsf{Mary}, \mathsf{Ph456}) : & \mathit{attends}\ \} & (\mathcal{A}_{ex}.9)
\end{array}
$$

$$
\begin{array}{rcl}
\Delta^{\mathcal{I}'} & = & \{h, m, b, c6, p4, c5\}, \\
\mathsf{Mary}^{\mathcal{I}'} & = & m, \\
\mathsf{Betty}^{\mathcal{I}'} & = & b, \\
\mathsf{Hugo}^{\mathcal{I}'} & = & h, \\
\mathsf{CS600}^{\mathcal{I}'} & = & c6, \\
\mathsf{Ph456}^{\mathcal{I}'} & = & p4, \\
\mathsf{Person}^{\mathcal{I}'} & = & \{h, m, b\}, \\
\mathsf{Teacher}^{\mathcal{I}'} & = & \{h, m, b\}, \\
\mathsf{Course}^{\mathcal{I}'} & = & \{c6, p4, c5\}, \\
\mathsf{PGC}^{\mathcal{I}'} & = & \{p4\}, \\
\mathsf{UGC}^{\mathcal{I}'} & = & \{c6\}, \\
\mathsf{Student}^{\mathcal{I}'} & = & \{h, m, b\}, \\
\mathit{teaches}^{\mathcal{I}'} & = & \{(m, c6), (h, p4), (b, c5)\}, \\
\mathit{attends}^{\mathcal{I}'} & = & \{(h, p4), (m, p4), (b, p4)\}.
\end{array}
$$

$\mathcal{I}'$ is a model of $\mathcal{K}_{ex}$

$\mathcal{ALC}$ KB $\mathcal{K}_{ex} = (\mathcal{T}_{ex}, \mathcal{A}_{ex})$ and model $\mathcal{I}'$ - Notes - 1

$$
\begin{array}{rcll}
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\mathsf{PGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.3) \\
\mathsf{Teacher} & \equiv & \mathsf{Person} \sqcap \exists \mathit{teaches}.\mathsf{Course}, & (\mathcal{T}_{ex}.4) \\
\exists \mathit{teaches}.\top & \sqsubseteq & \mathsf{Person}, & (\mathcal{T}_{ex}.5) \\
\mathsf{Student} & \equiv & \mathsf{Person} \sqcap \exists \mathit{attends}.\mathsf{Course}, & (\mathcal{T}_{ex}.6) \\
\exists \mathit{attends}.\top & \sqsubseteq & \mathsf{Person}\ \} & (\mathcal{T}_{ex}.7)
\end{array}
$$

$$
\begin{array}{rll}
\mathcal{A}_{ex} = \{\mathsf{Mary} : & \mathsf{Person}, & (\mathcal{A}_{ex}.1) \\
\mathsf{CS600} : & \mathsf{Course}, & (\mathcal{A}_{ex}.2) \\
\mathsf{Ph456} : & \mathsf{Course} \sqcap \mathsf{PGC}, & (\mathcal{A}_{ex}.3) \\
\mathsf{Hugo} : & \mathsf{Person}, & (\mathcal{A}_{ex}.4) \\
\mathsf{Betty} : & \mathsf{Person} \sqcap \mathsf{Teacher}, & (\mathcal{A}_{ex}.5) \\
(\mathsf{Mary}, \mathsf{CS600}) : & \mathit{teaches}, & (\mathcal{A}_{ex}.6) \\
(\mathsf{Hugo}, \mathsf{Ph456}) : & \mathit{teaches}, & (\mathcal{A}_{ex}.7) \\
(\mathsf{Betty}, \mathsf{Ph456}) : & \mathit{attends}, & (\mathcal{A}_{ex}.8) \\
(\mathsf{Mary}, \mathsf{Ph456}) : & \mathit{attends}\ \} & (\mathcal{A}_{ex}.9)
\end{array}
$$

$$
\begin{array}{rcl}
\Delta^{\mathcal{I}'} & = & \{h, m, b, c6, p4, c5\}, \\
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\mathsf{Betty}^{\mathcal{I}'} & = & b, \\
\mathsf{Hugo}^{\mathcal{I}'} & = & h, \\
\mathsf{CS600}^{\mathcal{I}'} & = & c6, \\
\mathsf{Ph456}^{\mathcal{I}'} & = & p4, \\
\mathsf{Person}^{\mathcal{I}'} & = & \{h, m, b\}, \\
\mathsf{Teacher}^{\mathcal{I}'} & = & \{h, m, b\}, \\
\mathsf{Course}^{\mathcal{I}'} & = & \{c6, p4, c5\}, \\
\mathsf{PGC}^{\mathcal{I}'} & = & \{p4\}, \\
\mathsf{UGC}^{\mathcal{I}'} & = & \{c6\}, \\
\mathsf{Student}^{\mathcal{I}'} & = & \{h, m, b\}, \\
\mathit{teaches}^{\mathcal{I}'} & = & \{(m, c6), (h, p4), (b, c5)\}, \\
\mathit{attends}^{\mathcal{I}'} & = & \{(h, p4), (m, p4), (b, p4)\}.
\end{array}
$$

Unnamed element !

$\mathcal{A}_{ex}$ has Betty: Teacher,
but (Betty, XX): teaches does not exist in $\mathcal{A}_{ex}$!
Its OK in KBs; but DBs may crib!
KBs can handle incomplete knowledge.

$\mathcal{ALC}$ KB $\mathcal{K}_{ex} = (\mathcal{T}_{ex}, \mathcal{A}_{ex})$ and model $\mathcal{I}'$ - Notes - 2

$$
\begin{array}{rcll}
\mathcal{T}_{ex} = \{\mathsf{Course} & \sqsubseteq & \neg\mathsf{Person}, & (\mathcal{T}_{ex}.1)\\
\mathsf{UGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.2)\\
\mathsf{PGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.3)\\
\mathsf{Teacher} & \equiv & \mathsf{Person} \sqcap \exists \mathit{teaches}.\mathsf{Course}, & (\mathcal{T}_{ex}.4)\\
\exists \mathit{teaches}.\top & \sqsubseteq & \mathsf{Person}, & (\mathcal{T}_{ex}.5)\\
\mathsf{Student} & \equiv & \mathsf{Person} \sqcap \exists \mathit{attends}.\mathsf{Course}, & (\mathcal{T}_{ex}.6)\\
\exists \mathit{attends}.\top & \sqsubseteq & \mathsf{Person}\ \} & (\mathcal{T}_{ex}.7)
\end{array}
$$

$$
\begin{array}{rl}
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\mathsf{CS600} : \mathsf{Course}, & (\mathcal{A}_{ex}.2)\\
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\mathsf{Hugo} : \mathsf{Person}, & (\mathcal{A}_{ex}.4)\\
\mathsf{Betty} : \mathsf{Person} \sqcap \mathsf{Teacher}, & (\mathcal{A}_{ex}.5)\\
(\mathsf{Mary}, \mathsf{CS600}) : \mathit{teaches}, & (\mathcal{A}_{ex}.6)\\
(\mathsf{Hugo}, \mathsf{Ph456}) : \mathit{teaches}, & (\mathcal{A}_{ex}.7)\\
(\mathsf{Betty}, \mathsf{Ph456}) : \mathit{attends}, & (\mathcal{A}_{ex}.8)\\
(\mathsf{Mary}, \mathsf{Ph456}) : \mathit{attends}\ \} & (\mathcal{A}_{ex}.9)
\end{array}
$$

Extra fact: (Hugo, Ph456) : *attends*

Not enforced by $\mathcal{K}_{ex}$

That is, other models w/o this exist.

In I$'$, drop (h, $p4$) from *attends* and h from Student

$$
\begin{array}{rcl}
\Delta^{\mathcal{I}'} & = & \{h, m, b, c6, p4, c5\},\\
\mathsf{Mary}^{\mathcal{I}'} & = & m,\\
\mathsf{Betty}^{\mathcal{I}'} & = & b,\\
\mathsf{Hugo}^{\mathcal{I}'} & = & h,\\
\mathsf{CS600}^{\mathcal{I}'} & = & c6,\\
\mathsf{Ph456}^{\mathcal{I}'} & = & p4,\\
\mathsf{Person}^{\mathcal{I}'} & = & \{h, m, b\},\\
\mathsf{Teacher}^{\mathcal{I}'} & = & \{h, m, b\},\\
\mathsf{Course}^{\mathcal{I}'} & = & \{c6, p4, c5\},\\
\mathsf{PGC}^{\mathcal{I}'} & = & \{p4\},\\
\mathsf{UGC}^{\mathcal{I}'} & = & \{c6\},\\
\mathsf{Student}^{\mathcal{I}'} & = & \{h, m, b\},\\
\mathit{teaches}^{\mathcal{I}'} & = & \{(m, c6), (h, p4), (b, c5)\},\\
\mathit{attends}^{\mathcal{I}'} & = & \{(h, p4), (m, p4), (b, p4)\}.
\end{array}
$$

$\mathcal{ALC}$ KB $\mathcal{K}_{ex} = (\mathcal{T}_{ex}, \mathcal{A}_{ex})$ and model $\mathcal{I}'$ - Notes - 3

$$
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\exists \mathit{teaches}.\top & \sqsubseteq & \mathsf{Person}, & (\mathcal{T}_{ex}.5)\\
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\exists \mathit{attends}.\top & \sqsubseteq & \mathsf{Person}\ \} & (\mathcal{T}_{ex}.7)
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$$

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\mathsf{Hugo} : & \mathsf{Person}, & (\mathcal{A}_{ex}.4)\\
\mathsf{Betty} : & \mathsf{Person} \sqcap \mathsf{Teacher}, & (\mathcal{A}_{ex}.5)\\
(\mathsf{Mary}, \mathsf{CS600}) : & \mathit{teaches}, & (\mathcal{A}_{ex}.6)\\
(\mathsf{Hugo}, \mathsf{Ph456}) : & \mathit{teaches}, & (\mathcal{A}_{ex}.7)\\
(\mathsf{Betty}, \mathsf{Ph456}) : & \mathit{attends}, & (\mathcal{A}_{ex}.8)\\
(\mathsf{Mary}, \mathsf{Ph456}) : & \mathit{attends}\ \} & (\mathcal{A}_{ex}.9)
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\mathsf{Hugo}^{\mathcal{I}'} & = & h,\\
\mathsf{CS600}^{\mathcal{I}'} & = & c6,\\
\mathsf{Ph456}^{\mathcal{I}'} & = & p4,\\
\mathsf{Person}^{\mathcal{I}'} & = & \{h, m, b\},\\
\mathsf{Teacher}^{\mathcal{I}'} & = & \{h, m, b\},\\
\mathsf{Course}^{\mathcal{I}'} & = & \{c6, p4, c5\},\\
\mathsf{PGC}^{\mathcal{I}'} & = & \{p4\},\\
\mathsf{UGC}^{\mathcal{I}'} & = & \{c6\},\\
\mathsf{Student}^{\mathcal{I}'} & = & \{h, m, b\},\\
\mathit{teaches}^{\mathcal{I}'} & = & \{(m, c6), (h, p4), (b, c5)\},\\
\mathit{attends}^{\mathcal{I}'} & = & \{(h, p4), (m, p4), (b, p4)\}.
\end{array}
$$

Add: PG-Student ≡ Student ⊓ ∀attends.PGC

Is Betty a PG-Student in every model of $\mathcal{K}_{ex}$?

NO, $\mathcal{K}_{ex}$ does not enforce it!

Get model J: add (b,c6) to *attends* of $\mathcal{I}'$ - OWA

Restricted TBoxes and Concept *Definitions*

An equivalence $A \equiv C$, where	An axiom $A \sqsubseteq C$, where
A is a concept name and	A is a concept name and
C is a concept description	C is a concept description
is called a *concept definition* of A	is called a *primitive concept definition* of A

Lemma 2.8. Suppose $A \sqsubseteq C$ is a primitive concept definition of A. Say, B is a name that *does not occur* in C. Then,
(1) Any model of $A \sqsubseteq C$ can be extended such that it is model of $A \equiv B \sqcap C$ and (2) any model of $A \equiv B \sqcap C$ is a model of $A \sqsubseteq C$.

Proof: (1) Let I be a model of $A \sqsubseteq C$ and hence $A^I \subseteq C^I$.
Extend I and set $B^I = A^I$.
Now, $(B \sqcap C)^I = B^I \cap C^I = A^I \cap C^I = A^I$.
Thus the extended I is a model of $A \equiv B \sqcap C$.

Restricted TBoxes and Concept *Definitions*

An equivalence $A \equiv C$, where A is a concept name and C is a concept description is called a *concept definition* of A	An axiom $A \sqsubseteq C$, where A is a concept name and C is a concept description is called a *primitive concept definition* of A

Lemma 2.8. Suppose $A \sqsubseteq C$ is a primitive concept definition of A. Say, B is a name that does not occur in C. Then,
(1) Any model of $A \sqsubseteq C$ can be extended such that it is model of $A \equiv B \sqcap C$ and (2) any model of $A \equiv B \sqcap C$ is a model of $A \sqsubseteq C$.

Proof : (2) Let I be a model of $A \equiv B \sqcap C$.
Since $A^I = B^I \cap C^I$ and hence $A^I \subseteq C^I$, I is a model of $A \sqsubseteq C$.

Cyclic Definitions - 1

$$Happy \equiv Person \sqcap \forall likes.Happy$$

Say, $Person^I = \{ p,m \}$ and $likes^I = \{ (p,m), (m,p) \}$

Now, say $Happy^I = \{ p, m \}$

Is I a model?

YES

Now, say $Happy^I = \Phi$

Is I a model?

YES !

Unique model is not there…

Cyclic Definitions - 2

Cyclic definitions

Sometimes are meaningful and also useful

“Man who has only male descendents” (abbreviated as Momd)

Say, we have “Man” concept and “hasChild” role

$$\text{Momd} \equiv \text{Man} \sqcap \forall \text{hasChild.Momd}$$

Current semantics - insufficient to deal with cyclic definitions

Fixpoints-based semantics needed

Maybe later….(refer to the book: DL Handbook, if interested)

Acyclic TBoxes

$\mathcal{T}$ - a finite set of concept *definitions*

A *directly uses* B

if A ≡ C is in $\mathcal{T}$ and B occurs as part of C

“Uses” is the transitive closure of “directly uses”

$\mathcal{T}$ is called *acyclic* if

- There is no concept name in $\mathcal{T}$ that uses itself and
- No concept name appears on the LHS of more than one definition

An Example (taken from the DL Handbook)

Acyclic TBox defining concepts about family relationships

$$\begin{aligned}
\text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\
\text{Man} &\equiv \text{Person} \sqcap \neg\text{Woman} \\
\text{Mother} &\equiv \text{Woman} \sqcap \exists\,\text{hasChild}.\text{Person} \\
\text{Father} &\equiv \text{Man} \sqcap \exists\,\text{hasChild}.\text{Person} \\
\text{Parent} &\equiv \text{Father} \sqcup \text{Mother} \\
\text{GrandMother} &\equiv \text{Mother} \sqcap \exists\,\text{hasChild}.\text{Parent}
\end{aligned}$$

Base Symbols/ Primitive Concepts/roles: atomic names that appear only on RHS

-- Person, Female, hasChild

Defined Symbols / Defined Concepts: atomic names that appear on LHS

Deriving the meaning of defined concepts

- Let $\mathcal{T}$ be an acyclic TBox and
  Let *I* be an interpretation that interprets the primitive symbols only.

- There exists a *model J* of $\mathcal{T}$ that is an extension of *I*
  That is, *J* coincides with *I* on the interpretation of primitive symbols

- Given the “meaning” of primitive symbols,
  we can automatically derive the “meaning” of defined concepts.

- In the example, if *I* interprets Person, Female and haschild,
  We can extend it into a model *J* of $\mathcal{T}$

An Example (taken from the DL Handbook)

Acyclic TBox defining concepts about family relationships

$$\begin{aligned}
\text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\
\text{Man} &\equiv \text{Person} \sqcap \neg\, \text{Woman} \\
\text{Mother} &\equiv \text{Woman} \sqcap \exists\, \text{hasChild.Person} \\
\text{Father} &\equiv \text{Man} \sqcap \exists\, \text{hasChild.Person} \\
\text{Parent} &\equiv \text{Father} \sqcup \text{Mother} \\
\text{GrandMother} &\equiv \text{Mother} \sqcap \exists\, \text{hasChild.Parent}
\end{aligned}$$

Given Person^I, Female^I and hasChild^I,

We can proceed to “compute” Woman, Man, …

Using the semantics of the operators.