Description Logics

Material and Examples from the Book

An Introduction to Description Logics

by

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CS6852: Theory and Applications of Ontologies (TAO)

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Introduction

Knowledge of a domain - Essential

- To infer new information from given information
  - John is a research scholar - given
  - John is guided/advised by a professor - inferred
- Take necessary actions as part of an application

Description Logics (DLs)

- Logic-based knowledge representation (KR) and reasoning - 1980’s onwards
- A family of languages for KR
  - Important notions in the domain - represented by “concept descriptions”
  - Concept descriptions - expressions built from atomic concepts and roles
    - Using the “constructors” provided by the particular DL
    - DLs have mathematical logic based semantics
      - Unlike the previous approaches - semantic nets and frames
    - Roughly the same as the semantics of first-order logic

Components of domain knowledge

Description Logic Approach

- Divide the domain knowledge into two components
  - Terminological part -- called the TBox
  - Assertional part -- called the ABox
  - TBox and ABox - together called as the Knowledge Base (KB)
- TBox
  - Knowledge about the “structure” of the domain - generic nature
  - Constraints that hold in the domain
  - Like the DB Scheme
- ABox
  - Knowledge about a concrete situation concerning the domain
  - Like a DB instance
- DL KBs - also referred as Ontologies

University Domain

Example statements in the TBox

- Teacher is a person who teaches a course
- Student is a person who attends a course
- Students do not teach

Example statements in the ABox

- Mary is a person
- CS600 is a course
- Mary teaches CS600

DL statements have logic-based semantics

Example sentences in first-order logic:

$$\forall x\,(\mathsf{Teacher}(x) \Leftrightarrow \mathsf{Person}(x) \wedge \exists y\,(\mathit{teaches}(x,y) \wedge \mathsf{Course}(y))),$$
$$\forall x\,(\mathsf{Student}(x) \Leftrightarrow \mathsf{Person}(x) \wedge \exists y\,(\mathit{attends}(x,y) \wedge \mathsf{Course}(y))),$$
$$\forall x\,((\exists y\,\mathit{teaches}(x,y)) \Rightarrow \neg\mathsf{Student}(x)),$$
$$\mathsf{Person}(\mathsf{Mary}),$$
$$\mathsf{Course}(\mathsf{CS600}),$$
$$\mathit{teaches}(\mathsf{Mary},\mathsf{CS600}).$$

Description Logic Notation

Same sentences in DL notation: compact and variable-free

TBox:

$$\mathsf{Teacher} \equiv \mathsf{Person} \sqcap \exists \mathit{teaches}.\mathsf{Course},$$

$$\mathsf{Student} \equiv \mathsf{Person} \sqcap \exists \mathit{attends}.\mathsf{Course},$$

$$\exists \overset{\textit{teaches}}{\cancel{\mathit{attends}}}.\top \sqsubseteq \neg\mathsf{Student},$$

ABox:

$$\mathsf{Mary} : \mathsf{Person},$$

$$\mathsf{CS600} : \mathsf{Course},$$

$$(\mathsf{Mary}, \mathsf{CS600}) : \mathit{teaches}.$$

What can you infer about Mary?

$$\forall x\,(\mathsf{Teacher}(x) \Leftrightarrow \mathsf{Person}(x) \wedge \exists y\,(\mathit{teaches}(x,y) \wedge \mathsf{Course}(y))),$$
$$\forall x\,(\mathsf{Student}(x) \Leftrightarrow \mathsf{Person}(x) \wedge \exists y\,(\mathit{attends}(x,y) \wedge \mathsf{Course}(y))),$$
$$\forall x\,((\exists y\, \mathit{teaches}(x,y)) \Rightarrow \neg\mathsf{Student}(x)),$$
$$\mathsf{Person}(\mathsf{Mary}),$$
$$\mathsf{Course}(\mathsf{CS600}),$$
$$\mathit{teaches}(\mathsf{Mary}, \mathsf{CS600}).$$

Semantics of DLs

- Based on Mathematical Logic
- Well-defined and *shared* understanding of
  - When a new statement is ***entailed*** by a KB
  - Example KB entails - Mary is a teacher
- Automated tools compute the entailments
  - Support development of Knowledge-Based applications

- Common reasoning tasks
  - Concept satisfiability
  - Consistency of KBs
  - Whether one concept is more specific than the other - subsumption
    - Is every Surgeon also a Doctor?
  - And Database-style queries
    - Is Jane a Teacher?
    - Who is Sara’s advisor?

A trade-off

DL-based systems

- Powerful and perform reasoning wrt to the whole KB - TBox and ABox
- Reasoning power comes with a cost
- Computational cost of reasoning
  - Depends on the specific DL
  - DL constructs - constrained to ensure reasoning tasks are “decidable”
- Trade-off
  - Highly expressive DLs have high computational complexity of reasoning
  - A large variety of DLs are proposed - aiming to get decidable, tractable and useful systems
  - Careless combinations of DL extensions - may lead to either undecidability or intractability
- Practical Systems
  - Efficient implementations and optimizations need to be explored

Applications of DL-based systems

DL systems are used in a variety of applications

- Semantic Web
  - Web Ontology Language (OWL) is based on DLs
- Databases
  - Schema and Data Integration
  - Query Answering
- NLP - Word-problem solving/generation
- Biological Sciences
  - Gene, protein ontologies - in widespread use
- Software
  - Static Analysis, Semantic Debugging
- ...

Typical Application Development

Formalise the domain into an appropriate TBox / Ontology

- Choose the relevant concept names / role names
    - The vocabulary of the domain
- Formulate axioms of the TBox

Ontology Engineering

- Manual or semi-automatic process
- Reasoners
    - Consistency of the ontology
    - Ontology editors - Protege
- KBs may layer on top of DBs also

Applications - interface with KB through APIs

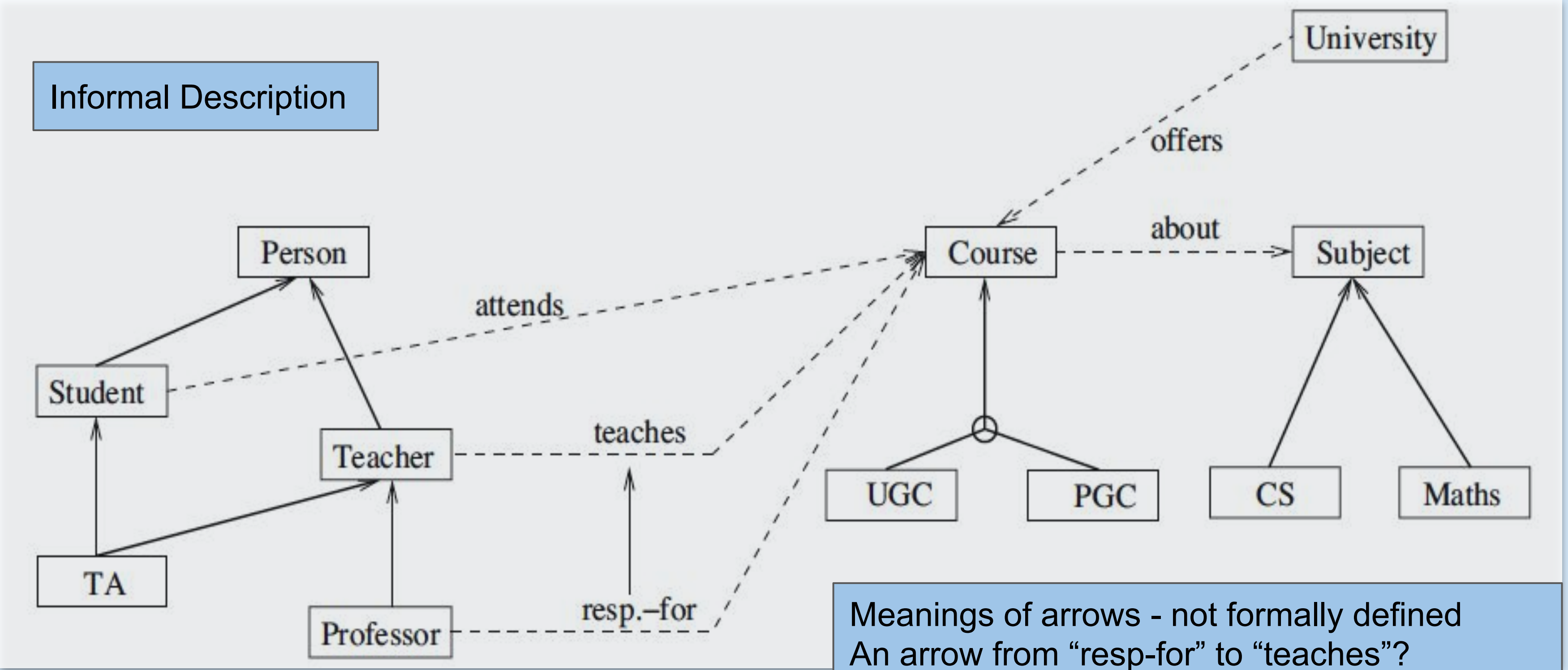
- And carry out tasks

Description Logics - The Specifics

- We start with an concrete example domain
  - The University Domain
    - Consisting of students, professors, courses, etc
    - Running example
  - An informal conceptualization
    - Graphical representation
      - Ambiguous
      - Different people may interpret the arrows in different ways...

University Domain

Informal Description



Meanings of arrows - not formally defined
An arrow from “resp-for” to “teaches”?
Dotted arrows vs Solid arrows ?

Domains and Their Descriptions

Domain

- An abstract ***set*** that contains things of interest - called the "elements"
  - May have a variety of elements - e.g. students, courses, professors, class rooms etc
- Captures / represents an application scenario

Description of the domain

- Three Building blocks - Concept Names, Role Names, Concept Language
- Concepts - *sets* of elements that belong together in some way
- Roles - *binary* relationships among elements
- Concept Language - set of concept-forming operators
  - To build concept expressions - representing compound concepts
  - Using concept names, role names and operators

The $\mathcal{ALC}$ Concept Language (Syntax)

$\mathcal{ALC}$ - Attributive Language with Complements

C - set of concept names ***R*** - set of role names; ***C*** and ***R*** are disjoint

$\mathcal{ALC}$ *concept descriptions* are inductively defined as:

- Every concept name is an $\mathcal{ALC}$ concept description
- $\top$ and $\bot$ are $\mathcal{ALC}$ concept descriptions (aka *top* and *bottom* concepts)
- If C and D are $\mathcal{ALC}$ concept descriptions and r is a role name, the following are also $\mathcal{ALC}$ concept descriptions
  $C \sqcap D$ (conjunction); $C \sqcup D$ (disjunction); $\neg C$ (negation);
  $\exists r.C$ (existential restriction); $\forall r.C$ (value restriction).

Concept Names and Expressions

Concept - A ***set*** of elements that belong together in some way

-An element may belong to multiple concepts

Concept Names

Represent atomic concepts

Student - elements of the domain who are students

Professor - elements of the domain who are faculty of the university

Staff - employees who are not professors

Concept Expressions / Descriptions

Represent compound concepts

Woman professors - Woman $\sqcap$ Professor

University employees - Staff $\sqcup$ Professor

Male professors - Professor $\sqcap$ $\neg$Woman

Role Names

Role Names

Represent ***binary*** relations among elements of the domain

Examples

‘teaches’ - “Prof John” teaches the course “Discrete Mathematics”

‘attends’ - “Sara” attends course “Discrete Mathematics”

Use of *role names* to define new sets of elements (concepts)

We often use this in our day-to-day activities..

Students-in-Core-Courses

- Elements of the domain who have ‘attends’ relation to any of the core courses.

Existential Restriction

Role Fillers

r , s - role names; e_1, e_2 and e_3 - elements

$(e_1, e_2) \in r$ -- e_2 is called an r-filler for e_1

$(e_1, e_3) \in s$ -- e_3 is called an s-filler for e_1

Existential Restriction:

Notation: $\exists r.C$ Here, r is a role name and C is a concept description

Set of elements that have at least one r-filler belonging to C

Examples:

$\exists$ *attends*.CoreCourse -- persons who attend a core course

$\exists$ *teaches*.CoreCourse -- persons who teach a core course

$\exists$ *hasChild*.Woman -- persons who have a girl child

Value Restrictions

Notation: $\forall r.C$ -- Here, *r* is a role name and *C* is a concept description

Set of elements that have ***all*** of their *r*-fillers in *C*

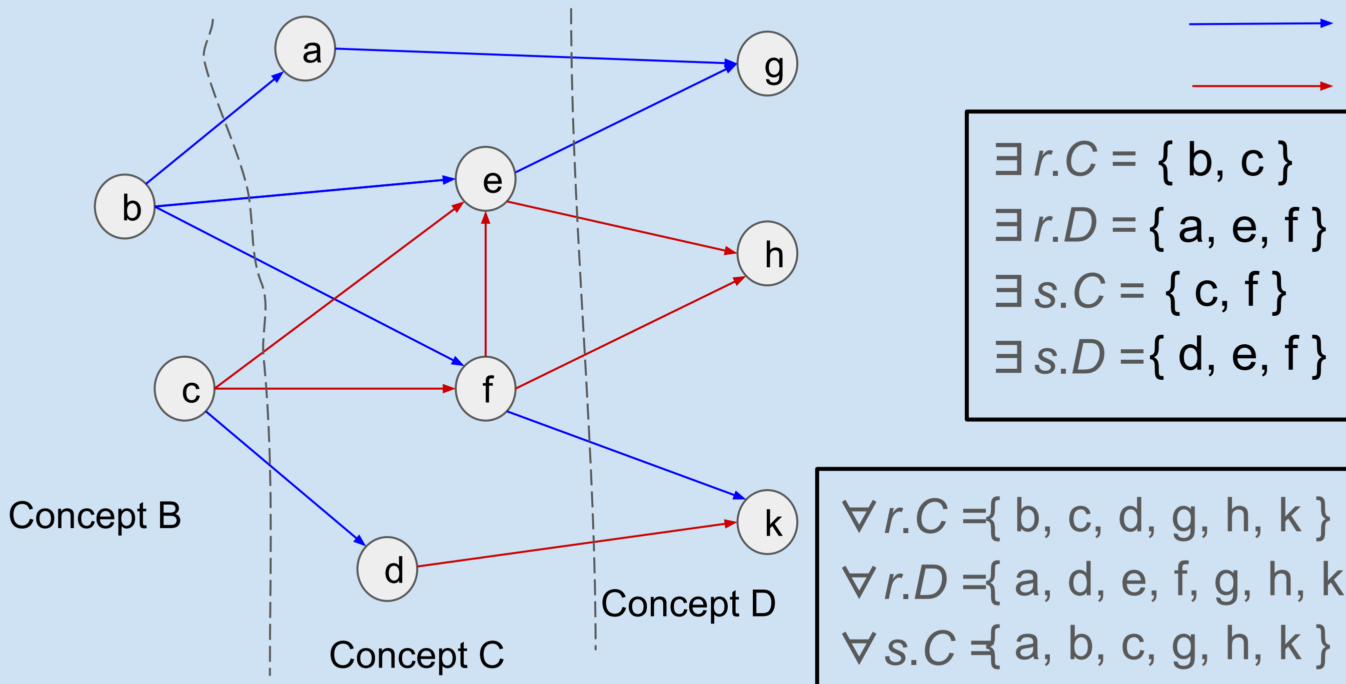
Note that elements with *no* *r*-fillers also belong to this set!

- consequence of the mathematical definition
- to be seen later

Examples:

- $\forall$*hasChild.Doctor* -- persons whose children are all doctors!
- $\forall$*teaches.CoreCourse* -- persons who teach only core courses
- $\forall$*attends.ElectiveCourse* -- persons who attend only elective courses

Examples - existential / value restrictions



$\exists r.C = \{ b, c \}$
$\exists r.D = \{ a, e, f \}$
$\exists s.C = \{ c, f \}$
$\exists s.D = \{ d, e, f \}$

$\forall r.C = \{ b, c, d, g, h, k \}$
$\forall r.D = \{ a, d, e, f, g, h, k \}$
$\forall s.C = \{ a, b, c, g, h, k \}$
$\forall s.D = \{ a, b, d, e, g, h, k \}$

Interpretation Functions

- Used to fix / assign meanings
  - for concept names
  - for role names
  - and by extension, for concept descriptions
- Consists of
  - a domain set,
  - a function that assigns
    - subsets of the domain to concept names, and
    - binary relations on the domain to role names

$\mathcal{I}$(A) = a subset of $\Delta^{\mathcal{I}}$

$\mathcal{I}$(r) = a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

- Definition: An interpretation $\mathcal{I} = \{ \Delta^{\mathcal{I}}, .^{\mathcal{I}} \}$ consists of a non-empty set $\Delta^{\mathcal{I}}$, called the interpretation domain and a mapping $.^{\mathcal{I}}$ that maps
  - every concept name $A \in C$ to a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - every role name $r \in R$ to a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Notes and Notations

- Domain set $\Delta^{\mathcal{I}}$ -- a non-empty set; -- can be finite or infinite also
- A, B -- represent concept names (sometimes with subscripts)
- C, D -- represent compound concepts (sometimes with subscripts)
- *r*, *s* -- represent role names (sometimes with subscripts)

- Domain of the interpretation function: the set of concept descriptions
- $A^{\mathcal{I}}$ -- result of applying the interpretation function $\mathcal{I}$ to A
  - Can be empty or the whole domain or something in-between
- $r^{\mathcal{I}}$ -- result of applying the interpretation function $\mathcal{I}$ to r
  - Can be empty or the whole cross-product or something in-between
- $(X)^{\mathcal{I}}$ -- result of applying the interpretation function $\mathcal{I}$ to X

Interpretation of Concept Descriptions

The mapping $\cdot^{\mathcal{I}}$ is extended to $\top$, $\bot$ and compound concepts as follows:

$$
\begin{aligned}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}}, \\
\bot^{\mathcal{I}} &= \emptyset, \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}}, \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}}, \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}, \\
(\exists r.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \text{there is an } e \in \Delta^{\mathcal{I}} \text{ with } (d,e) \in r^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\}, \\
(\forall r.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \text{for all } e \in \Delta^{\mathcal{I}},\ \text{if } (d,e) \in r^{\mathcal{I}},\ \text{then } e \in C^{\mathcal{I}}\}.
\end{aligned}
$$

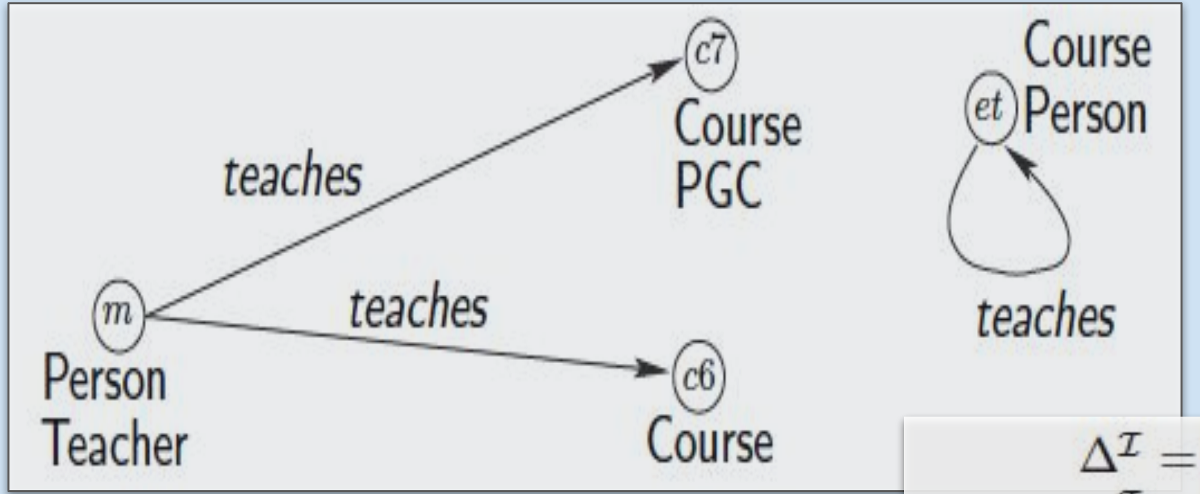
We call: $C^{\mathcal{I}}$ the *extension of* C in $\mathcal{I}$,

$b \in \Delta^{\mathcal{I}}$ an *r-filler of a in* $\mathcal{I}$ if $(a,b) \in r^{\mathcal{I}}$

An example interpretation function $\mathcal{I}$

$$
\begin{aligned}
\Delta^{\mathcal{I}} &= \{m, c6, c7, et\}, \\
\mathsf{Teacher}^{\mathcal{I}} &= \{m\}, \\
\mathsf{Course}^{\mathcal{I}} &= \{c6, c7, et\}, \\
\mathsf{Person}^{\mathcal{I}} &= \{m, et\}, \\
\mathsf{PGC}^{\mathcal{I}} &= \{c7\}, \\
\mathit{teaches}^{\mathcal{I}} &= \{(m, c6), (m, c7), (et, et)\}.
\end{aligned}
$$

Interpretation function as a graph



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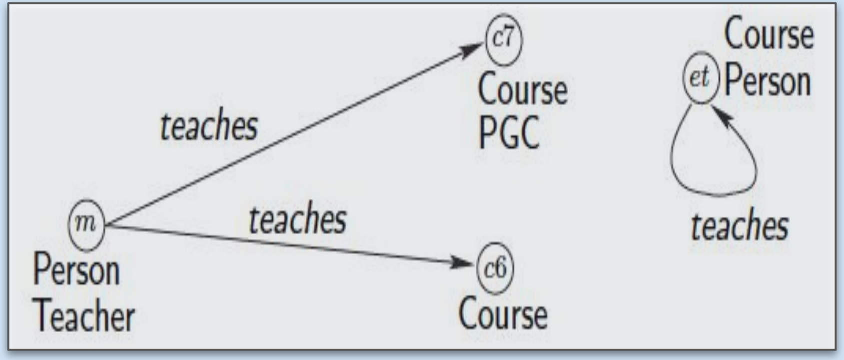
\mathsf{Course}^{\mathcal{I}} &= \{c6, c7, et\},\\
\mathsf{Person}^{\mathcal{I}} &= \{m, et\},\\
\mathsf{PGC}^{\mathcal{I}} &= \{c7\},\\
\mathit{teaches}^{\mathcal{I}} &= \{(m, c6), (m, c7), (et, et)\}.
\end{aligned}
$$

Examples

- $(\text{Person} \sqcap \text{Teacher})^{\mathcal{I}} = \{ m \}$
- $(\text{Course} \sqcap \neg\text{Person})^{\mathcal{I}} = \{ c6, c7 \}$
- $(\exists\, \text{teaches.Course})^{\mathcal{I}} = \{ m, et \}$
- $(\forall\, \text{teaches.Course})^{\mathcal{I}} = \{ m, c6, c7, et \}$
- $(\exists\, \text{teaches.}(\text{Course} \sqcap \neg\text{Person}\,)^{\mathcal{I}} = \{ m \}$
- $(\text{Person} \sqcap \forall\, \text{teaches.}(\text{Course} \sqcap \text{PGC}))^{\mathcal{I}} = \{\ \}$
- $(\text{Person} \sqcap \exists\, \text{teaches.}(\text{Course} \sqcap \neg\text{PGC}))^{\mathcal{I}} = \{ m, et \}$
- ...



$$
\begin{aligned}
\Delta^{\mathcal{I}} &= \{m, c6, c7, et\},\\
\text{Teacher}^{\mathcal{I}} &= \{m\},\\
\text{Course}^{\mathcal{I}} &= \{c6, c7, et\},\\
\text{Person}^{\mathcal{I}} &= \{m, et\},\\
\text{PGC}^{\mathcal{I}} &= \{c7\},\\
\textit{teaches}^{\mathcal{I}} &= \{(m, c6), (m, c7), (et, et)\}.
\end{aligned}
$$

$\mathcal{ALC}$ syntax is generous

Lemma 2.3. *Let $\mathcal{I}$ be an interpretation, C, D concepts, and r a role. Then*

$$
\begin{array}{lrcl}
\text{(i)} & \top^{\mathcal{I}} & = & (C \sqcup \neg C)^{\mathcal{I}}, \\
\text{(ii)} & \bot^{\mathcal{I}} & = & (C \sqcap \neg C)^{\mathcal{I}}, \\
\text{(iii)} & (\neg\neg C)^{\mathcal{I}} & = & C^{\mathcal{I}}, \\
\text{(iv)} & \neg(C \sqcap D)^{\mathcal{I}} & = & (\neg C \sqcup \neg D)^{\mathcal{I}}, \\
\text{(v)} & \neg(C \sqcup D)^{\mathcal{I}} & = & (\neg C \sqcap \neg D)^{\mathcal{I}}, \\
\text{(vi)} & (\neg(\exists r.C))^{\mathcal{I}} & = & (\forall r.\neg C)^{\mathcal{I}}, \\
\text{(vii)} & (\neg(\forall r.C))^{\mathcal{I}} & = & (\exists r.\neg C)^{\mathcal{I}}.
\end{array}
$$

Proof of part (vi)

(vi) $(\neg(\exists r.C))^{\mathcal{I}} = (\forall r.\neg C)^{\mathcal{I}}$

$(\neg(\exists r.C))^{\mathcal{I}} = \Delta^I \setminus \{ d \in \Delta^I \mid \text{there is an } e \in \Delta^I \text{ s.t } (d,e) \in r^I \text{ and } e \in C^I \}$

$= \{ d \in \Delta^I \mid \text{there is no } e \in \Delta^I \text{ s.t } (d,e) \in r^I \text{ and } e \in C^I \}$

$= \{ d \in \Delta^I \mid \text{for all } e \in \Delta^I , \neg[(d,e) \in r^I \text{ and } e \in C^I] \}$

$= \{ d \in \Delta^I \mid \text{for all } e \in \Delta^I, [\neg(d,e) \in r^I \lor \neg(e \in C^I)] \}$

$= \{ d \in \Delta^I \mid \text{for all } e \in \Delta^I, [(d,e) \in r^I \rightarrow \neg(e \in C^I)] \}$

$= (\forall r.\neg C)^{\mathcal{I}}$

$\mathcal{ALC}$ Knowledge Bases

We can do several things with $\mathcal{ALC}$

- Represent domain knowledge // Terminological Knowledge - *TBox*
  a. Set-up new terms in the domain

     UG-Student $\equiv$ Student $\sqcap$ $\forall$attends.UG-Course

     CS-Faculty $\equiv$ Teacher $\sqcap$ $\exists$teaches.(Course $\sqcap$ $\exists$about.CS-Topic)

  b. State domain constraints / background knowledge

     PGC $\sqsubseteq$ $\neg$UGC

     Teacher $\sqsubseteq$ $\exists$teaches.UG-Course

- State specific knowledge // Assertional Knowledge - *ABox*
  a. Membership of elements in concepts

     E.g., John is a Teacher

  b. Role relationships

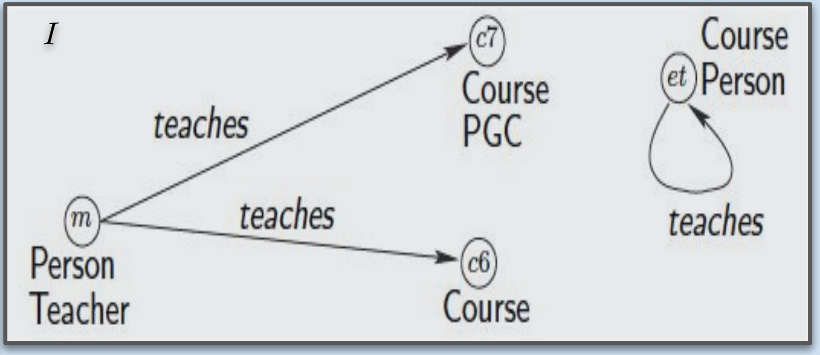
     E.g., John "teaches" Discrete Mathematics"

$\mathcal{ALC}$ KB - TBoxes

- General Concept Inclusion (GCI) statement / axiom
  - C ⊑ D   where C, D are (possibly compound) concepts
  - Examples: Teacher ⊑ Person;   ∃teaches.Course ⊑ Teacher
- Equivalence axiom
  - C ≡ D is same as C ⊑ D and D ⊑ C
  - Example: ∃teaches.Course ≡ Teacher
- An $\mathcal{ALC}$ TBox
  - A finite set of GCIs
- An interpretation $\mathcal{I}$ ***satisfies*** a GCI C ⊑ D if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- An interpretation $\mathcal{I}$ satisfies a TBox $\mathcal{T}$ if it satisfies ***all*** the GCIs in $\mathcal{T}$
- An interpretation $\mathcal{I}$ that satisfies a TBox $\mathcal{T}$ is called a ***model*** of $\mathcal{T}$

TBox - An example

$$
\begin{aligned}
\mathcal{T}_1 = \{\text{Teacher} &\sqsubseteq \text{Person},\\
\text{PGC} &\sqsubseteq \neg\text{Person},\\
\text{Teacher} &\sqsubseteq \exists \textit{teaches}.\text{Course},\\
\exists \textit{teaches}.\text{Course} &\sqsubseteq \text{Person}\}
\end{aligned}
$$



$\mathcal{I}$ satisfies all the GCIs of $\mathcal{T}_1$

$\mathcal{I}$ is a ***model*** of $\mathcal{T}_1$

Focus: Design a TBox such that only the interpretations that capture the essence of the domain are its ***models***

$\mathcal{I}$ does *not* satisfy the GCIs:

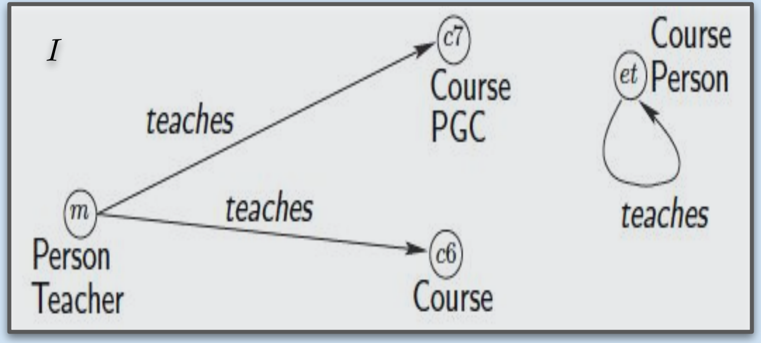
$\exists$ teaches.Course $\sqsubseteq$ Teacher

Course $\sqsubseteq \neg$ Person

Lemma:
$\mathcal{T}_1 \subseteq \mathcal{T}_2 \Rightarrow$ any model of $\mathcal{T}_2$ is also a model of $\mathcal{T}_1$

TBox $\mathcal{T}_{\text{ex}}$

$$
\begin{array}{rcll}
\mathcal{T}_{ex} = \{\mathsf{Course} & \sqsubseteq & \neg\mathsf{Person}, & (\mathcal{T}_{ex}.1) \\
\mathsf{UGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.2) \\
\mathsf{PGC} & \sqsubseteq & \mathsf{Course}, & (\mathcal{T}_{ex}.3) \\
\mathsf{Teacher} & \equiv & \mathsf{Person} \sqcap \exists \mathit{teaches}.\mathsf{Course}, & (\mathcal{T}_{ex}.4) \\
\exists \mathit{teaches}.\top & \sqsubseteq & \mathsf{Person}, & (\mathcal{T}_{ex}.5) \\
\mathsf{Student} & \equiv & \mathsf{Person} \sqcap \exists \mathit{attends}.\mathsf{Course}, & (\mathcal{T}_{ex}.6) \\
\exists \mathit{attends}.\top & \sqsubseteq & \mathsf{Person}\ \} & (\mathcal{T}_{ex}.7)
\end{array}
$$



$\mathcal{T}_{\text{ex}}$ partially captures our university domain.

$\mathcal{I}$ is ***not a model*** of $\mathcal{T}_{\text{ex}}$ as $\mathcal{I}$ does not satisfy $\mathcal{T}_{\text{ex}}.1$, $\mathcal{T}_{\text{ex}}.4$

$\mathcal{ALC}$ KB - ABoxes

I - set of individual names -- disjoint with concept names ***C***, role names ***R***

For a,b $\in$ ***I***, C $\in$ ***C*** and r $\in$ ***R*** :

An expression of the form a:C is called a ***concept assertion***

An expression of the form (a,b):r is called a ***role assertion***

An $\mathcal{ALC}$ ABox : a finite set of concept and role assertions

Additional work for the interpretation function:

Map individual names to elements of the domain: $a \mapsto a^{\mathcal{I}}$

$\mathcal{I}$ ***satisfies*** a:C if $a^{\mathcal{I}} \in C^{\mathcal{I}}$

$\mathcal{I}$ ***satisfies*** (a,b):r if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

If $\mathcal{I}$ satisfies all the assertions in an ABox $\mathcal{A}$, then $\mathcal{I}$ is a ***model*** for $\mathcal{A}$

An example ABox - $\mathcal{A}_{ex}$

$$
\begin{array}{rll}
\mathcal{A}_{ex} = \{\mathsf{Mary} : \mathsf{Person}, & & (\mathcal{A}_{ex}.1) \\
\mathsf{CS600} : \mathsf{Course}, & & (\mathcal{A}_{ex}.2) \\
\mathsf{Ph456} : \mathsf{Course} \sqcap \mathsf{PGC}, & & (\mathcal{A}_{ex}.3) \\
\mathsf{Hugo} : \mathsf{Person}, & & (\mathcal{A}_{ex}.4) \\
\mathsf{Betty} : \mathsf{Person} \sqcap \mathsf{Teacher}, & & (\mathcal{A}_{ex}.5) \\
(\mathsf{Mary}, \mathsf{CS600}) : \mathit{teaches}, & & (\mathcal{A}_{ex}.6) \\
(\mathsf{Hugo}, \mathsf{Ph456}) : \mathit{teaches}, & & (\mathcal{A}_{ex}.7) \\
(\mathsf{Betty}, \mathsf{Ph456}) : \mathit{attends}, & & (\mathcal{A}_{ex}.8) \\
(\mathsf{Mary}, \mathsf{Ph456}) : \mathit{attends} \ \} & & (\mathcal{A}_{ex}.9)
\end{array}
$$

$\mathcal{I}$ is a model for $\mathcal{A}_{ex}$

Unique Names Assumption (UNA) is *not* made

UNA : a ≠ b → $a^{\mathcal{I}} \neq b^{\mathcal{I}}$

Betty and Hugo are both mapped to *h*

Interpretation $\mathcal{I}$

$$
\begin{array}{rcl}
\Delta^{\mathcal{I}} & = & \{h, m, c6, p4\}, \\
\mathsf{Mary}^{\mathcal{I}} & = & m, \\
\mathsf{Betty}^{\mathcal{I}} & = & \mathsf{Hugo}^{\mathcal{I}} = h, \\
\mathsf{CS600}^{\mathcal{I}} & = & c6, \\
\mathsf{Ph456}^{\mathcal{I}} & = & p4, \\
\mathsf{Person}^{\mathcal{I}} & = & \{h, m, c6, p4\}, \\
\mathsf{Teacher}^{\mathcal{I}} & = & \{h, m\}, \\
\mathsf{Course}^{\mathcal{I}} & = & \{c6, p4\}, \\
\mathsf{PGC}^{\mathcal{I}} & = & \{p4\}, \\
\mathsf{UGC}^{\mathcal{I}} & = & \{c6\}, \\
\mathsf{Student}^{\mathcal{I}} & = & \emptyset, \\
\mathit{teaches}^{\mathcal{I}} & = & \{(m, c6), (h, p4)\}, \\
\mathit{attends}^{\mathcal{I}} & = & \{(h, p4), (m, p4)\}.
\end{array}
$$

An example ABox - $\mathcal{A}_{ex}$

$$
\begin{aligned}
\mathcal{A}_{ex} = \{ & \mathsf{Mary} : \mathsf{Person}, && (\mathcal{A}_{ex}.1)\\
& \mathsf{CS600} : \mathsf{Course}, && (\mathcal{A}_{ex}.2)\\
& \mathsf{Ph456} : \mathsf{Course} \sqcap \mathsf{PGC}, && (\mathcal{A}_{ex}.3)\\
& \mathsf{Hugo} : \mathsf{Person}, && (\mathcal{A}_{ex}.4)\\
& \mathsf{Betty} : \mathsf{Person} \sqcap \mathsf{Teacher}, && (\mathcal{A}_{ex}.5)\\
& (\mathsf{Mary}, \mathsf{CS600}) : \mathit{teaches}, && (\mathcal{A}_{ex}.6)\\
& (\mathsf{Hugo}, \mathsf{Ph456}) : \mathit{teaches}, && (\mathcal{A}_{ex}.7)\\
& (\mathsf{Betty}, \mathsf{Ph456}) : \mathit{attends}, && (\mathcal{A}_{ex}.8)\\
& (\mathsf{Mary}, \mathsf{Ph456}) : \mathit{attends} \ \} && (\mathcal{A}_{ex}.9)
\end{aligned}
$$

Interpretation $\mathcal{I}$

$$
\begin{aligned}
\Delta^{\mathcal{I}} &= \{h, m, c6, p4\},\\
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\mathsf{Betty}^{\mathcal{I}} &= \mathsf{Hugo}^{\mathcal{I}} = h,\\
\mathsf{CS600}^{\mathcal{I}} &= c6,\\
\mathsf{Ph456}^{\mathcal{I}} &= p4,\\
\mathsf{Person}^{\mathcal{I}} &= \{h, m, c6, p4\},\\
\mathsf{Teacher}^{\mathcal{I}} &= \{h, m\},\\
\mathsf{Course}^{\mathcal{I}} &= \{c6, p4\},\\
\mathsf{PGC}^{\mathcal{I}} &= \{p4\},\\
\mathsf{UGC}^{\mathcal{I}} &= \{c6\},\\
\mathsf{Student}^{\mathcal{I}} &= \emptyset,\\
\mathit{teaches}^{\mathcal{I}} &= \{(m, c6), (h, p4)\},\\
\mathit{attends}^{\mathcal{I}} &= \{(h, p4), (m, p4)\}.
\end{aligned}
$$

$\mathcal{I}$ is a model for $\mathcal{A}_{ex}$

$\mathcal{I}$ puts $c6$, $p4$ in Person$^{\mathcal{I}}$ and m in Teacher$^{\mathcal{I}}$.
Also, $\mathcal{I}$ interprets UGC.
The ABox does not demand these !

$\mathcal{T}_{ex}$ and interpretation I

$$
\begin{aligned}
\mathcal{T}_{ex} = \{\mathsf{Course} &\sqsubseteq \neg\mathsf{Person}, & (\mathcal{T}_{ex}.1)\\
\mathsf{UGC} &\sqsubseteq \mathsf{Course}, & (\mathcal{T}_{ex}.2)\\
\mathsf{PGC} &\sqsubseteq \mathsf{Course}, & (\mathcal{T}_{ex}.3)\\
\mathsf{Teacher} &\equiv \mathsf{Person} \sqcap \exists \mathit{teaches}.\mathsf{Course}, & (\mathcal{T}_{ex}.4)\\
\exists \mathit{teaches}.\top &\sqsubseteq \mathsf{Person}, & (\mathcal{T}_{ex}.5)\\
\mathsf{Student} &\equiv \mathsf{Person} \sqcap \exists \mathit{attends}.\mathsf{Course}, & (\mathcal{T}_{ex}.6)\\
\exists \mathit{attends}.\top &\sqsubseteq \mathsf{Person}\ \} & (\mathcal{T}_{ex}.7)
\end{aligned}
$$

I does not satisfy $\mathcal{T}_{ex}$

- $\mathcal{T}_{ex}.1$ is not satisfied
- $\mathcal{T}_{ex}.6$ is not satisfied

Interpretation I

$$
\begin{aligned}
\Delta^{\mathcal{I}} &= \{h, m, c6, p4\},\\
\mathsf{Mary}^{\mathcal{I}} &= m,\\
\mathsf{Betty}^{\mathcal{I}} &= \mathsf{Hugo}^{\mathcal{I}} = h,\\
\mathsf{CS600}^{\mathcal{I}} &= c6,\\
\mathsf{Ph456}^{\mathcal{I}} &= p4,\\
\mathsf{Person}^{\mathcal{I}} &= \{h, m, c6, p4\},\\
\mathsf{Teacher}^{\mathcal{I}} &= \{h, m\},\\
\mathsf{Course}^{\mathcal{I}} &= \{c6, p4\},\\
\mathsf{PGC}^{\mathcal{I}} &= \{p4\},\\
\mathsf{UGC}^{\mathcal{I}} &= \{c6\},\\
\mathsf{Student}^{\mathcal{I}} &= \emptyset,\\
\mathit{teaches}^{\mathcal{I}} &= \{(m, c6), (h, p4)\},\\
\mathit{attends}^{\mathcal{I}} &= \{(h, p4), (m, p4)\}.
\end{aligned}
$$