PRML End Sem

2. P_3, P_4 combined first. 0.8421

Next P_5, P_6 0.8105

Next P_1, P_2 0.7895

Next $(P_3, P_4), (P_5, P_6)$ as dist bw ~~P_2, P_5~~ (max dist, single link) = 0.3840 (min.)

3. $$\text{Specificity} = \frac{TN}{TN + FP}$$

$N = 30$, ~~$P = 15$~~, ~~$N = 15$~~

$FN = 2$ $FP = 3$ $TP = P - FP = 12$, $TN = N - FN = 13$

$$\text{Specificity} = \frac{15}{15 + 3} = \frac{5}{6}$$

$$= \frac{13}{13+3} = \frac{13}{16}$$

closest answer = $\frac{12}{15}$

5. (1,2) (1,4) (1,0) (1,1) → pts

Centroids $(0,0)$ $(1,1)$
C_1 C_2

It 1 :-

Pt	Dist with $(0,0)$	Dist with $(1,1)$
(1,2)	$\sqrt{5}$	(1)
(1,4)	$\sqrt{17}$	(3)
(1,0)	(1)	1
(1,1)	$\sqrt{2}$	(0)

Closest centroid,

C_2 pts ⇒ (1,2) (1,4) (1,1)

C_1 pts ⇒ (1,0)

New $C_2 \Rightarrow (1, 7/3)$ New $C_1 \Rightarrow (1,0)$

It 2 :-

Pt	Dist with $(1, 7/3)$ C_2	$(1,0)$ C_1
Closest centroid, $(1,2)$	(0.33)	2
$(1,4)$	(1.66)	4
$(1,0)$	2.33	(0)
$(1,1)$	1.33	(1)

C_1 pts $\Rightarrow$ $(1,0)$ $(1,1)$

C_2 pts $\Rightarrow$ $(1,2)$ $(1,4)$

New $C_1 \Rightarrow (1, 1/2)$

New $C_2 \Rightarrow (1, 3)$

It 3 :-

Pt	Dist with $(1, 1/2)$ C_1	$(1,3)$ C_2
$(1,2)$	1.5	(1)
$(1,4)$	3.5	(1)
$(1,0)$	(0.5)	3
$(1,1)$	(0.5)	2

No change so Stop. Converged.

The 2 centroids are, $(1, 0.5)$, $(1, 3)$

12. $e = 0.35 \qquad N = 5$

For ensemble to make wrong prediction, majority of the 5 classifiers must make wrong prediction.

i.e. either 3, 4 or all 5 must give wrong prediction.

$\therefore P(\text{wrong pred}) = \sum_{i=3}^{5} \binom{5}{i} e^{i} (1-e)^{5-i}$

$= \frac{4 \times 5}{3 \times 2} \times (0.35)^3 (0.65)^2 + 5 \times (0.35)^4 (0.65) + (0.35)^5$

$= 0.1811 + 0.04877 + 0.0052 = 0.235$ //

7. $f(x) = \sigma(\sigma(x_1 w_1) w_2 + x_2) \qquad \eta = \text{learning rate} = 0.5$

$\sigma(x) = \frac{1}{1+e^{-x}} \qquad$ Initially $w_1 = w_2 = 0$

For $x_1 = 0, \; x_2 = 1, \; y = 5,$

Forward Pass,

$f(x) = \sigma(\sigma(0 \times 0) \times 0 + 1) = \sigma(1) = 0.73106$ ~~0.76159~~

$y = 5.$

$\therefore$ Taking ~~Mean~~ Square Error,

$E = \frac{1}{2}(5 - 0.73106)^2 = 18.22$ ~~9.11~~

Now, $\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial f} \cdot \frac{\partial f}{\partial w_2} = 2 \times (0.73106 - 5) \cdot \frac{\partial f}{\partial w_2}$

$= -4.26894 \cdot \frac{\partial f}{\partial w_2} \times 2$

$$\frac{\partial f}{\partial w_2} = f(n) \cdot (1 - f(n)) \cdot r(x, w_1)$$

$$= 0.73106 \times 0.26894 \times \sigma(0 \times 0)$$

$$= 0.1966 \times \frac{1}{1 + e^0} = 0.1966 \times \frac{1}{1+1}$$

$$= 0.0983$$

$$\frac{\partial E}{\partial w_2} = -0.83927$$

$$\therefore w_2^{new} = w_2 - \eta \times \frac{\partial E}{\partial w_2} = 0 - \frac{1}{2} \times (-0.83927)$$

$$= 0.4196 \text{ //}$$

4. Here, $P(\text{Pass}) = \frac{8}{13}$ $P(\text{Fail}) = \frac{5}{13}$

$\therefore$ Entropy (Result) $= -\frac{8}{13}\log\frac{8}{13} - \frac{5}{13}\log\frac{5}{13} = 0.96123$

If Root node is,

a) Course Attended.

For No, $P(\text{Pass}/\text{No}) = \frac{3}{6} = \frac{1}{2}$ $P(\text{Fail}/\text{No}) = \frac{1}{2}$

$P(\text{No}) = \frac{6}{13}$

For Yes, $P(\text{Pass}/\text{Yes}) = \frac{5}{7}$ $P(\text{F}/\text{No}) = \frac{2}{7}$

$P(\text{Yes}) = \frac{7}{13}$

Info gain $= 0.96123 - \frac{6}{13}\left(-\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2}\right)$

$- \frac{7}{13}\left(-\frac{5}{7}\log\frac{5}{6} - \frac{2}{7}\log\frac{1}{6}\right)$

$= 0.96123 - 0.4615 - 0.55 = 0.14973$

$0.46475 \qquad 0.035$

b) Entrance Grade.

For A, $P(A) = \frac{4}{13}$ $P(\text{Pass}/A) = \frac{2}{4}$