PRML Quiz 1

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CS21M037

1) $P(Y=1 \mid X=x) = \dfrac{1}{1+e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2)}}$

$= \dfrac{1}{1+e^{-(x_1+\beta_2 x_2)}}$

$P(Y=1 \mid X=[1, \frac{3}{2}]) = 0.5 = \frac{1}{2} = \dfrac{1}{1+e^{-(1+\beta_2/2)}}$

$\Rightarrow e^{-\left(\frac{\beta_2}{2}+1\right)} = 1 \Rightarrow \frac{\beta_2}{2}+1 = 0 \Rightarrow \beta_2 = -\frac{1}{2}$

$P(Y=1 \mid X=[1, \frac{3}{2}]) = \dfrac{1}{1+e^{-(1-\frac{1}{2}+\frac{3}{2})}} = \dfrac{1}{1+e^{-(1-\frac{3}{2})}}$

$= \dfrac{1}{1+e^{-1/4}} = 0.562176$

2) $X = [-3, 5, 4]$ $\quad Y = [-1, 10, 10]$

$\lambda = 50$

$\hat{w}_{MLE} = (X^T X)^{-1} X^T Y$ $\quad \hat{w}_{Ridge} = (X^T X + \lambda I)^{-1} X^T Y$

$\frac{\hat{w}_{Ridge}}{\hat{w}_{MLE}} \Rightarrow$ $X^T X = \begin{bmatrix} -3 \\ 5 \\ 4 \end{bmatrix} [-3 \; 5 \; 4] = \begin{bmatrix} 9 & -15 & -12 \\ -15 & 25 & 20 \\ -12 & 20 & 16 \end{bmatrix}$

$X^T X + \lambda I = \begin{bmatrix} 59 & -15 & -12 \\ -15 & 75 & 20 \\ -12 & 20 & 66 \end{bmatrix}$

4) For class A,

pts are, (1,1), (2,3), (2,4), (5,3)

For class B,

pts are, (8,6), (9,6), (11,7), (8,8)

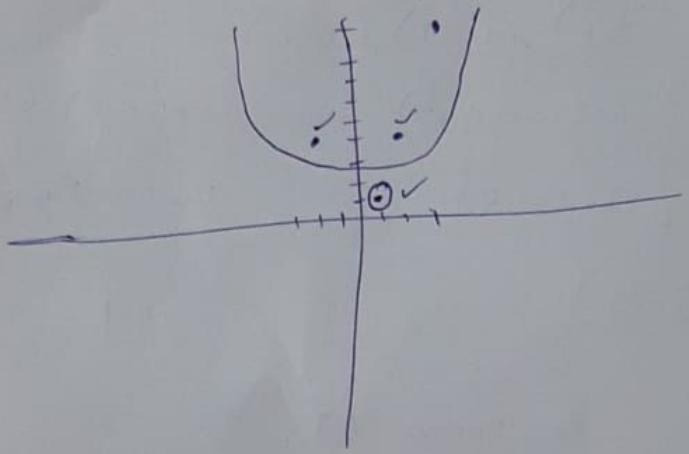
Covariance Matrix = $\begin{bmatrix} \text{Var(class A pts)} & \text{Cov}(A,B) \\ \text{Cov}(A,B) & \text{Var (class B pts)} \end{bmatrix}$

$\mu_A = [2.5, 2.75]$ $\mu_B = [9, 6.75]$

Var (A) $\simeq$ 2.5 So option (A).

5) values, $(-2,-1)$, $(2,-1)$, $(3,-1)$, $(1,1)$

Transformed $\Rightarrow$ $((-2,4),-1)$, $((2,4),-1)$, $((3,9),-1)$, $((1,1),1)$



3 support vectors

Upward curve

Coeff must be +ve.

Rough

5.5

7.75

6) as x_i ~~and~~ x_j are close for x and z_2,

$k(x, z_2) \Rightarrow \|x_i - x_j\|^2 \sim 0$

$\hookrightarrow = -0$

$e \approx 1$

as x and z_1 are far,

$k(x, z_1) \Rightarrow \|x_i - x_j\|^2$ is high, $k(x, z_1) \sim e^{-\infty}$

~ 0

8) $P(\text{set 1} \mid 1, 2, \text{not 3}) = \dfrac{P(\text{~~set 1~~} 1 \mid \text{set 1})\, P(2 \mid \text{set 1})\, P(\text{not 3} \mid \text{set 1})\, P(\text{set 1})}{P(1)\, P(2)\, P(\text{not 3})}$

$= \dfrac{\frac{25}{40} \times \frac{10}{40} \times \frac{15}{40} \times \frac{40}{120}}{\frac{60}{120} \times \frac{45}{120} \times \frac{70}{120}} = \dfrac{25 \times 15 \times 4 \times 3 \times 3}{40 \times 4 \times 45 \times 7}$

$= \dfrac{25}{140} = 0.178$

9)