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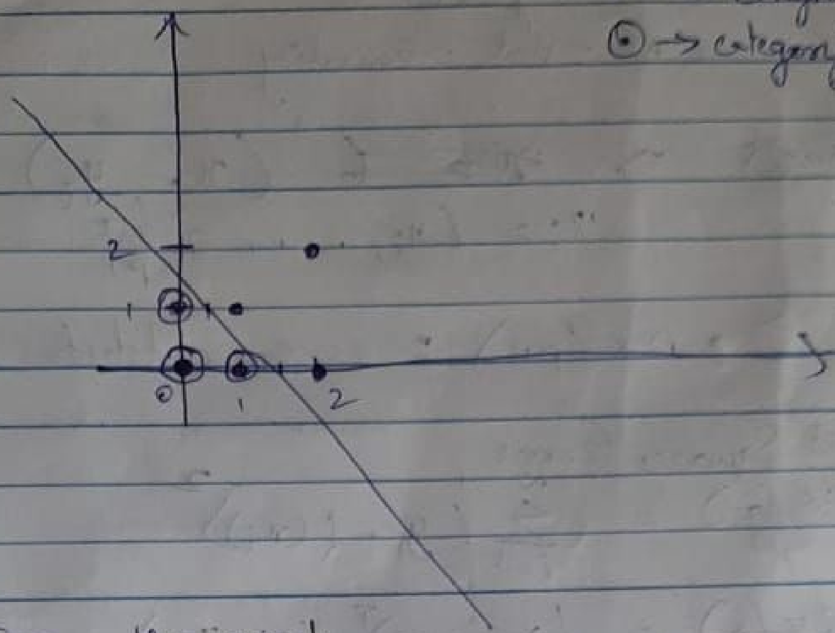
CS21M037

Tutorial 7

1. a)

• → category 1
⊙ → category 2



From the graph,
hyperplane passes through (0.5, 1), (1.5, 0)

$w = [1, 1]$ bias $= -1.5$

b) Support vectors ⇒ (0,1), (1,1), (1,0), (2,0)

c) Separator hyperplane depends only on the support vectors. As D is not a support vector, deleting it wont change solution.

2. a) TRUE
As Gaussian kernel can be expanded using taylor series expansion. It can be seen as basis exp with ∞ dimensions as gaussian using taylor series has ∞ terms.
Hence due to ∞ dimensions it can model any complex separating hyperplane.

2.b) FALSE

For Gaussian kernel, there is no closed form basis expansion.

2. c) TRUE

As separator depends on support vectors, more the num supp vecs, more overfitting.

3. a) $K = \phi^T \phi = \begin{bmatrix} 1 & x^2 \end{bmatrix} \begin{bmatrix} 1 \\ x^2 \end{bmatrix} = 1 + x^4$

3. b) Using $\phi(x) = [1 \ \ x^2]$,

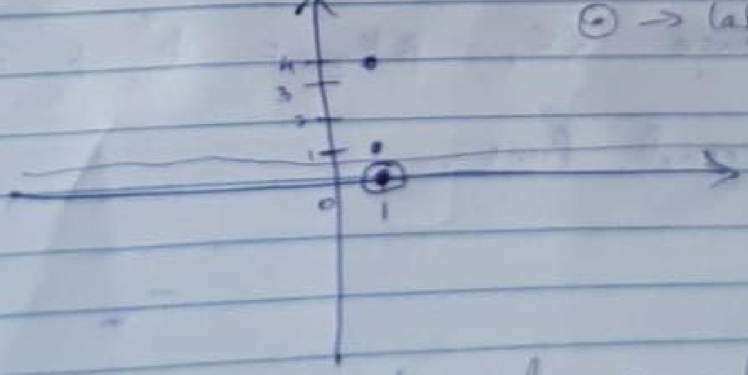
the pts become,

$x_1' = \begin{bmatrix} 1 \\ (-1)^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad x_2' = \begin{bmatrix} 1 \\ 0^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$x_3' = \begin{bmatrix} 1 \\ 2^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

• → label 1

⊙ → label −1



From graph, it is linearly separable.

3.c) Let $K_1 = \phi_1^T \phi_1$ and $K_2 = \phi_2^T \phi_2$

$\therefore \alpha K_1 = (\sqrt{\alpha}\,\phi_1)^T (\sqrt{\alpha}\,\phi_1)$

$\beta K_2 = (\sqrt{\beta}\,\phi_2)^T (\sqrt{\beta}\,\phi_2)$

$\therefore \alpha K_1 + \beta K_2 = (\sqrt{\alpha}\,\phi_1)^T (\sqrt{\alpha}\,\phi_1) + (\sqrt{\beta}\,\phi_2)^T (\sqrt{\beta}\,\phi_2)$

$= \phi_k^T \phi_k$

This is possible by forming ϕ_k by simply putting the elements of $\sqrt{\alpha}\,\phi_1$ and $\sqrt{\beta}\,\phi_2$ together.

If elements of ϕ_1 are $[\phi_{11}, \phi_{12} \ldots \phi_{1n}]$ and ϕ_2 are $[\phi_{21}, \phi_{22} \ldots \phi_{2m}]$,

$\phi_k = [\sqrt{\alpha}\,\phi_{11}, \ldots \sqrt{\alpha}\,\phi_{1n}, \sqrt{\beta}\,\phi_{21}, \ldots \sqrt{\beta}\,\phi_{2m}]$

Hence $\alpha K_1 + \beta K_2 = \phi_k^T \phi_k \Rightarrow$ another Kernel func.