THEORY OF DUALITY

Associated with every LPP (Maximisation/Minimisation) There always exists another LPP which is based upon the same data and having the same solution. The original problem is called the Primal problem while the Associated one is called dual problem.

Consider a LPP of the form

$$\left.\begin{aligned} \text{Max } Z &= c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{s.t. } & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\ & \cdots\cdots\cdots \\ & \cdots\cdots\cdots \\ & a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \end{aligned}\right\} \quad (1)$$

and $x_1 \geq 0,\ x_2 \geq 0, \ldots x_n \geq 0$

Simillarly consider an LPP.

$$\left.\begin{aligned} \text{Min } Z &= b_1w_1 + b_2w_2 + \cdots + b_mw_m \\ \text{s.t. } & a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m \geq c_1 \\ & a_{12}w_1 + a_{22}w_2 + \cdots + a_{m2}w_m \geq c_2 \\ & \cdots\cdots\cdots \\ & \cdots\cdots\cdots \\ & a_{1n}w_1 + a_{2n}w_2 + \cdots + a_{mn}w_m \geq c_n \end{aligned}\right\} \quad (2)$$

and $w_1 \geq 0,\ w_2 \geq 0, \ldots w_m \geq 0$

The LPP given in (1) is refered to as PRIMAL LPP and LPP given in (2) is refered as ~~dual~~ DUAL LPP.

The Theory of duality deals with establishing Theoritical relationship between (1) and (2).

List of changes between Primal and Dual problem.

PRIMAL PROBLEM	DUAL PROBLEM.
1. Solution vector $\rightarrow X$	1. Solution vector – W.
2. Maximisation Problem.	2. Minimisation Problem
3. Cost vector $\rightarrow C$	3. Cost vector $\rightarrow B'$
4. '$\leq$ type' constraints	4. "$\geq$ type" constraints
5. Requirement vector $\rightarrow B$	5. Requirement vector $\rightarrow C'$
6. objective function. $Z = CX$.	6. objective function. $Z = B'W$.

DEFINITION (Primal problem).

A LPP of determining X^T so as to ~~maximi~~

$$\text{Maximize } Z = CX.$$

$$\text{s.t. } AX \leq B$$

$$\text{and } X \geq 0$$

Where A is an $m \times n$ real matrix, is called a primal problem.

DEFINITION (Dual problem).

A LPP of determining W^T so as to

$$\text{Minimize } Z = B'W.$$

$$\text{s.t. } A'W \geq C'$$

$$\text{and } W \geq 0$$

Where A' is the transpose of A, is called a Dual problem.

The variable W are called dual variables. The constraints of the dual problem are called dual constraints.

NOTE :-

1. If the first primal variable is unrestricted in sign then the first dual constraint is an EQUALITY

2. If the first constraint of the primal is an EQUALITY then the first dual variable is UNRESTRICTED in SIGN.

3. Identify the variables to be used in the dual problem, The number of these variables equals the Number of constraints in the primal problem.

4. Dual of the Dual is PRIMAL.

1. Write the dual of the following LPP.

$$\text{Maximize } z = 4x_1 + 2x_2$$
$$\text{s.t. } x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 2$$
$$\text{and } x_1, x_2 \geq 0.$$

Soln:- The standard primal problem is given by

$$\text{MAX. } z = 4x_1 + 2x_2$$
$$\text{s.t. } -x_1 - x_2 \leq -3$$
$$-x_1 + x_2 \leq -2$$
$$\text{and } x_1, x_2 \geq 0$$

∴ The dual of the given problem is given by

$$\text{Min } z = -3\omega_1 - 2\omega_2$$
$$\text{s.t. } -\omega_1 - \omega_2 \geq 4$$
$$-\omega_1 + \omega_2 \geq 2$$
$$\text{and } \omega_1, \omega_2 \geq 0.$$

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2. Write the dual of the following LPP

Maximize (MAX ~~Minimize~~) $z = 2x_1 - 3x_2 + x_4$

$$\text{s.t. } x_1 + 2x_2 - x_3 \leq 7$$
$$x_1 + 4x_2 - x_4 = 5$$
$$x_2 + x_3 + 5x_4 \geq 3$$
$$\text{and } x_1, x_2, x_3, x_4 \geq 0.$$

Soln:- In this problem the IInd primal constraint is an Equality.

∴ The IInd dual variable will be unrestricted in sign.

The standard primal problem is given by

$$\text{Max.}\ \cancel{\text{Min}}\ z = 2x_1 - 3x_2 + x_4$$

$$\text{s.t.}\quad x_1 + 2x_2 - x_3 \le 7$$
$$x_1 + 4x_2 - x_4 \le 5$$
$$-x_1 - 4x_2 + x_4 \le -5$$
$$-x_1 - x_3 - 5x_4 \le -3$$

and $x_1, x_2, x_3, x_4 \geq 0$.

The dual ~~of the given~~ problem is given by

$$\text{Min}\ \cancel{\text{Max}}\ z = 7w_1 + 5w_2 - 5w_3 - 3w_4$$

$$\text{s.t}\quad w_1 + w_2 - w_3 - w_4 \geq 2$$
$$2w_1 + 4w_2 - 4w_3 \geq -3$$
$$-w_1 - w_4 \geq 0$$
$$-w_2 + w_3 - 5w_4 \geq 1$$

and $w_1, w_2, w_3, w_4 \geq 0$

$$\Rightarrow \text{Min}\ \cancel{\text{Max}}\ z = 7w_1 + 5(w_2 - w_3) - 3w_4$$

$$\text{s.t.}\quad w_1 + (w_2 - w_3) - w_4 \geq 2$$
$$2w_1 + 4(w_2 - w_3) \geq -3$$
$$-w_1 - w_4 \geq 0$$
$$-(w_2 - w_3) - 5w_4 \geq 1$$

and $w_1, w_2, w_3, w_4 \geq 0$

Let $w' = w_2 - w_3$, $\quad \text{Min}\ \cancel{\text{Max}}\ z = 7w_1 + 5w' - 3w_4$

$$\text{s.t.}\quad w_1 + w' - w_4 \geq 2$$
$$2w_1 + 4w' \geq -3$$
$$-w_1 - w_4 \geq 0$$
$$-w' - 5w_4 \geq 1$$

and $w_1, w_4 \geq 0$ and w' is unrestricted in sign.

3. Write the dual of the following LPP.

$$\text{Max } Z = 3x_1 + 5x_2 + 7x_3$$
$$\text{s.t. } x_1 + x_2 + 3x_3 \leq 10$$
$$4x_1 - x_2 + 2x_3 \geq 15$$

and $x_1, x_2 \geq 0$, x_3 is unrestricted in sign.

Soln:- In this problem III^{rd} primal variable is unrestricted in sign, So the III^{rd} Dual constraint will be an Equality. Let $x_3 = x_3^+ - x_3^-$

The standard Primal problem is given by

$$\text{Max } Z = 3x_1 + 5x_2 + 7(x_3^+ - x_3^-)$$
$$\text{s.t. } x_1 + x_2 + 3(x_3^+ - x_3^-) \leq 10$$
$$-4x_1 + x_2 - 2(x_3^+ - x_3^-) \leq -15$$

and $x_1, x_2 \geq 0$, $x_3^+, x_3^- \geq 0$.

$$\Rightarrow \text{Max } Z = 3x_1 + 5x_2 + 7x_3^+ - 7x_3^-$$
$$\text{s.t. } x_1 + x_2 + 3x_3^+ - 3x_3^- \leq 10$$
$$-4x_1 + x_2 - 2x_3^+ + 2x_3^- \leq -15$$

and $x_1, x_2 \geq 0$, $x_3^+, x_3^- \geq 0$.

The dual problem is given by

$$\text{Min } Z = 10\omega_1 - 15\omega_2$$
$$\text{s.t. } \omega_1 - 4\omega_2 \geq 3$$
$$\omega_1 + \omega_2 \geq 5$$
$$3\omega_1 - 2\omega_2 \geq 7$$
$$-3\omega_1 + 2\omega_2 \geq -7$$

and $\omega_1, \omega_2 \geq 0$.

$$\Rightarrow \quad \text{Min } z = 10\omega_1 - 15\omega_2$$

$$\text{s.t.} \quad \omega_1 - 4\omega_2 \geq 3$$
$$\omega_1 + \omega_2 \geq 5$$
$$3\omega_1 - 2\omega_2 \geq 7$$
$$3\omega_1 - 2\omega_2 \leq 7$$
$$\text{and} \quad \omega_1, \omega_2 \geq 0$$

$$\Rightarrow \quad \text{Min } z = 10\omega_1 - 15\omega_2$$

$$\text{s.t.} \quad \omega_1 - 4\omega_2 \geq 3$$
$$\omega_1 + \omega_2 \geq 5$$
$$3\omega_1 - 2\omega_2 = 7$$
$$\text{and} \quad \omega_1, \omega_2 \geq 0.$$

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4. Verify the statement dual of the dual is primal for the following LPP.

$$\text{Max } z = 4x_1 + 10x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$\text{and} \quad x_1, x_2 \geq 0.$$

Soln:- The dual problem is given by

$$\text{Min } z = 50\omega_1 + 100\omega_2 + 90\omega_3$$
$$\text{s.t.} \quad 2\omega_1 + 2\omega_2 + 2\omega_3 \geq 4$$
$$\omega_1 + 5\omega_2 + 3\omega_3 \geq 10$$
$$\text{and} \quad \omega_1, \omega_2, \omega_3 \geq 0.$$

The dual of the dual is given by

$$\text{Max } z = 4x_1 + 10x_2$$
$$\text{s.t} \quad 2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$\text{and} \quad x_1, x_2 \geq 0.$$

Write the dual of the following. LPP.

$$\text{Maximize} \quad Z = 4x_1 + 2x_2$$
$$\text{s.t.} \quad x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 2$$
$$x_1 \geq 0, \; x_2 \geq 0.$$

<u>Soln</u>:- The standard primal problem is.

$$\text{Max.} \; Z = 4x_1 + 2x_2$$
$$\text{s.t.} \; -x_1 - x_2 \leq -3$$
$$-x_1 + x_2 \leq -2$$
$$\text{and} \quad x_1 \geq 0, \; x_2 \geq 0.$$

The dual of the given problem is given by

$$\min \; Z = -3w_1 - 2w_2$$
$$\text{s.t.} \; -w_1 - w_2 \geq 4$$
$$-w_1 + w_2 \geq 2$$
$$\text{and} \quad w_1, w_2 \geq 0.$$

2) Write the dual of the following L.P.P.

$$\text{Minimize} \quad Z = 2x_1 - 3x_2 + x_4$$
$$\text{s.t.} \quad x_1 + 2x_2 - x_3 \leq 7$$
$$x_1 + 4x_2 - x_4 = 5$$
$$x_2 + x_3 + 5x_4 \geq 3$$
$$\text{and} \quad x_1, x_2, x_3, x_4 \geq 0.$$

<u>Soln</u>:- In this problem the second primal constraint is an Equality Therefore the second dual variable will be unrestricted in sign.

The standard primal problem is

$$\text{Max.} \; Z = 2x_1 - 3x_2 + x_4$$
$$\text{s.t.} \quad x_1 + 2x_2 - x_3 \leq 7$$
$$x_1 + 4x_2 - x_4 \leq 5$$
$$-x_1 - 4x_2 + x_4 \leq -5$$
$$-x_1 - x_3 - 5x_4 \leq -3$$
$$\text{and} \quad x_1, x_2, x_3, x_4 \geq 0.$$

The dual of the given problem is given by

Min $Z = \cancel{7w_1 - 3x_2 + x_4}\ 7w_1 + 5w_2 - 5w_3 - 3w_4$

s.t. $w_1 + w_2 - w_3 - w_4 \geq 2$

$2w_1 + 4w_2 - 4w_3 \geq -3$

$-w_1 - w_4 \geq 1$ and $w_1, w_2, w_3 \geq 0$

Min $Z = 7w_1 + 5(w_2 - w_3) - 3w_4$

s.t. $w_1 + (w_2 - w_3) - w_4 \geq 2$

$2w_1 + 4(w_2 - w_3) \geq -3$

$-w_1 - w_4 \geq 1$ & $w_1, w_2, w_3 \geq 0$
$w_4,$

Let $w' = w_2 - w_3$

Min $Z = 7w_1 + w' - 3w_4$

s.t. $w_1 + w' - w_4 \geq 2$

$2w_1 + 4w' \geq -3$

$-w_1 - w_4 \geq 1$ and $w_1, w_4 \geq 0$ and w' is unrestricted in sign.

③. Write the dual of the following LPPs

1) Max. $Z = 3x_1 - 5x_2 + x_3$

s.t. $-x_1 + 4x_2 - 5x_3 \leq 18$

$-2x_2 + x_3 \leq -10$

$6x_1 + x_2 \leq 15$

$-6x_1 - x_2 \leq -15$

$x_1 - x_2 + 3x_3 \leq -20$ & $x_1, x_3 \geq 0$, x_2 is unrestricted in sign.

Sol:- In this problem the second primal variable is unrestricted in sign Therefore 2^{nd} dual constraint will be an Equality.

Let $x_2 = x_2^+ - x_2^-$

The standard primal problem is given by

Max. $Z = 3x_1 - 5(x_2^+ - x_2^-) + x_3$

s.t. $-x_1 + 4(x_2^+ - x_2^-) - 5x_3 \leq 18$

$$-2(x_2^+ - x_2^-) + x_3 \le -10$$

$$6x_1 + x_2^+ - x_2^- \le 15$$

$$-6x_1 - (x_2^+ - x_2^-) \le -15$$

$$x_1 - (x_2^+ - x_2^-) + 3x_3 \le -20$$

and $x_1, x_2^+, x_2^-, x_3 \geq 0$.

Max $z = 3x_1 - 5x_2^+ + 5x_2^- + x_3$

s.t. $-x_1 + 4x_2^+ - 4x_2^- - 5x_3 \le 18$

$$-2x_2^+ + 2x_2^- + x_3 \le -10$$

$$6x_1 + x_2^+ - x_2^- \le 15$$

$$-6x_1 - x_2^+ + x_2^- \le -15$$

$x_1 - x_2^+ + x_2^- + 3x_3 \le -20$ and $x_1, x_2^+, x_2^-, x_3 \geq 0$

The dual of the given problem is given by

Min $z = 18w_1 - 10w_2 + 15w_3 - 15w_4 - 20w_5$.

s.t. ~~$w_1 - 2w_2 + 6w_3 - 6w_4 + w_5 \geq 3$~~

$$w_1 + 6w_3 - 6w_4 + w_5 \geq 3$$

$$4w_1 - 2w_2 + w_3 - w_4 - w_5 \geq -5$$

$$-4w_1 + 2w_2 - w_3 + w_4 + w_5 \geq 5$$

$-5w_1 + w_2 + 3w_5 \geq -20$, $w_1, w_2, w_3, w_4, w_5 \geq 0$.

Min $z = 18w_1 - 10w_2 + 15w_3 - 15w_4 - 20w_5$

s.t. $-w_1 + 6w_3 - 6w_4 + w_5 \geq 3$

$$-4w_1 + 2w_2 - w_3 + w_4 + w_5 \le 5$$

$$-4w_1 + 2w_2 - w_3 + w_4 + w_5 \geq 5$$

$-5w_1 + w_2 + 3w_5 \geq -20$, $w_i \geq 0, i = 1, \dots 5$

Min $z = 18w_1 - 10w_2 + 15w_3 - 15w_4 - 20w_5$

s.t. $-w_1 + 6w_3 - 6w_4 + w_5 \geq 3$

$$-4w_1 + 2w_2 - w_3 + w_4 + w_5 = 5$$

$-5w_1 + w_2 + 3w_5 \geq -20$, $w_1, w_2, w_3, w_4, w_5 \geq 0$.

DUAL SIMPLEX METHOD

Dual Simplex method is applicable to those LPP that start with Infeasible solution but otherwise Optimum solution

Step1 : Write the given LPP in its standard form and obtain a starting basic solution.

Step2 : ⟨i⟩ If the current basic solution is Feasible, use Simplex method to obtain optimum solution.

⟨ii⟩ If the current basic solution is Infeasible, go to the next step. [values of Basic variables ≤ 0]

Step3 : Check whether the solution is optimum.

⟨i⟩ If the solution is NOT optimum, add an artificial constraint in such a way that the condition of optimality is satisfied.

⟨ii⟩ If the solution is optimum, go to next step.

Step4 : Select the basic variable having the most Negative value. This basic variable becomes the leaving variable and the row corresponding to it becomes the Key row.

Step 5 : Obtain the ratios Left hand side coefficients of the Last row $[z_j - c_j]$ to the corresponding coefficients in the Key row. Corresponding to the smallest absolute value of the ratio will enter the basic variable, and the corresponding column becomes the Key column

Step 6 : Reduce the Key element into UNITY and all other entries of key column to zero by elementary row operations.

Step 7 : Go to Step 2 and repeat the procedure until an optimum basic feasible solution is obtained.

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NOTE : (i) Ignore the ratio's associated with positive / zero denominators.

(ii) The main advantage of dual simplex method over the usual simplex method is that we do not require any artificial variables in the dual simplex method.

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Problems

II Use dual simplex method to Solve the following LPP.

Maximize $Z = -3x_1 - x_2$

s.t. $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

and $x_1, x_2 \geq 0$

Soln: The standard form of LPP is given by

Max. $Z = -3x_1 - x_2 + 0S_1 + 0S_2$

s.t $-x_1 - x_2 + S_1 \leq -1$

$-2x_1 - 3x_2 + S_2 \leq -2$

Table-1 and $x_1, x_2 \geq 0$, $S_1, S_2 \geq 0$

	-3	-1	0	0		
B.V.	x_1	$\downarrow x_2$	S_1	S_2	Soln	~~Ratio~~
S_1	-1	-1	1	0	-1	
$\leftarrow S_2$	-2	$\boxed{-3}$	0	1	$-2 \leftarrow$	
$Z_j - C_j$	3	1 $\uparrow$	0	0	0	

Ratio $= \min \left\{ \left|\frac{3}{-2}\right|, \left|\frac{1}{-3}\right| \right\} = \left|\frac{1}{-3}\right|$

Here $x_2 \rightarrow$ enters the Basis

$S_2 \rightarrow$ Leaves the Basis

$\therefore$ operating $R_2 = -\frac{1}{3} R_2$

$R_1 = R_1 +$ New R_2

$R_3 = R_3 -$ New R_2.

$\rightarrow$

Table - 2

B.V.	x_1	x_2	s_1	s_2	soln
s_1	$-1/3$	0	1	$\boxed{-1/3}$	$-1/3$
x_2	$2/3$	1	0	$-1/3$	$2/3$
$z_j - c_j$	$7/3$	0	0	$1/3$	$-2/3$

$$\text{Ratio} = \text{Min}\left\{\left|\frac{7/3}{-1/3}\right|, -\,-, \left|\frac{1/3}{-1/3}\right|\right\} = \left|\frac{1/3}{-1/3}\right|$$

Here $s_1 \rightarrow$ Leaves the Basis
$s_2 \rightarrow$ Enters the Basis,

operating $R_1 = -3R_1$
$R_2 = R_2 + \frac{1}{3}(\text{new } R_1)$
$R_3 = R_3 - \frac{1}{3}(\text{new } R_1)$

B.V.	x_1	x_2	s_1	s_2	soln
s_2	1	0	-3	1	1
x_2	1	1	-1	0	1
$z_j - c_j$	2	0	1	0	-1

Since the current Basic soln is feasible [soln column > 0]

$\therefore$ The optimal soln is reached.

optimum soln is

$$\text{Max } Z = -1$$

when $x_1 = 0$ & $x_2 = 1$

[2] Solve the following LPP by dual simplex method.

$$\text{Min. } z = 2x_1 + x_2$$

s.t. $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

and $x_1, x_2 \geq 0$.

Sol: Since, the problem is of 'Minimization type'. So Converting into 'maximization type'. The Standard form of LPP becomes

$$\text{Max } z' = -2x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 \quad ; \; z' = -z$$

s.t. $-3x_1 - x_2 + s_1 = -3$

$-4x_1 - 3x_2 + s_2 = -6$

$-x_1 - 2x_2 + s_3 = -3$

and $x_1, x_2 \geq 0$, $s_1, s_2, s_3 \geq 0$.

c_j :	-2	-1	0	0	0	
B.V.	x_1	↓ x_2	s_1	s_2	s_3	Soln
s_1	-3	-1	1	0	0	-3
← s_2	-4	$\boxed{-3}$	0	1	0	-6 ←
s_3	-1	-2	0	0	1	-3
$z_j' - c_j$	2	1 ↑	0	0	0	0

$$\text{Ratio} = \text{Min} \left\{ \left|\frac{2}{-4}\right|, \left|\frac{1}{-3}\right|, - - - \right\} = \left|\frac{1}{-3}\right|$$

operating $R_2 = -\frac{1}{3} R_2$

$R_1 = R_1 + (\text{New } R_2)$

$R_3 = R_3 + 2\,(\text{New } R_2)$

$R_4 = R_4 - (\text{New } R_2)$

B.V.	x_1	x_2	s_1	s_2	s_3	Soln
s_1	$\boxed{-5/3}$	0	1	$-1/3$	0	-1
x_2	$4/3$	1	0	$-1/3$	0	2
s_3	$5/3$	0	0	$-2/3$	1	1
$z_j^{0}-c_j$	$2/3$	0	0	$1/3$	0	-2

$$\text{Ratio} = \min\left\{\left|\frac{2/3}{-5/3}\right|, - - \left|\frac{1/3}{-1/3}\right| - \right\} = \left|\frac{2/3}{-5/3}\right|$$

operating $R_1 = -\frac{3}{5}R_1$

$R_2 = R_2 - \frac{4}{3}(\text{New}R_1)$

$R_3 = R_3 - \frac{5}{3}(\text{New}R_1)$

$R_4 = R_4 - \frac{2}{3}(\text{new}R_1)$

B.V.	x_1	x_2	s_1	s_2	s_3	Soln
x_1	1	0	$-\frac{3}{5}$	$1/5$	0	$3/5$
x_2	0	1	$4/5$	$-3/5$	0	$6/5$
s_3	0	0	1	-1	1	0
$z_j^{1}-c_j$	0	0	$2/5$	$1/5$	0	$-\frac{12}{5}$

Here the soln Column (≥ 0), so the optimum sln is reached.

optimum soln $\quad \text{Max } z' = -\frac{12}{5}$

when $x_1 = 3/5, \; x_2 = \frac{6}{5}$.

But the solution of original LPP is given by

$$\text{Min } z = -\text{Max } z' = -\left(-\frac{12}{5}\right) = \frac{12}{5}$$

when $x_1 = \frac{3}{5}, \; x_2 = \frac{6}{5}$

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PROBLEMS

1

$$\text{Max } z = -2x_1 - x_3$$

s.t. $x_1 + x_2 - x_3 \geq 5$

$x_1 - 2x_2 + 4x_3 \geq 8$

and $x_1, x_2, x_3 \geq 0$

Ans:

$\text{Max } z = -9$

$x_1 = 0,\ x_2 = 14,\ x_3 = 9.$

2

$$\text{Max } z = -2x_1 - 2x_2 - 4x_3$$

s.t. $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

and $x_1, x_2, x_3 \geq 0$

Ans:

$\text{Max } z = \frac{4}{3}$

$x_1 = 0,\ x_2 = 2/3,\ x_3 = 0.$

3

$$\text{Min } z = 80x_1 + 60x_2 + 80x_3$$

s.t. $x_1 + 2x_2 + 3x_3 \geq 4$

$2x_1 + 3x_3 \geq 3$

$2x_1 + 2x_2 + x_3 \geq 4$

$4x_1 + x_2 + x_3 \geq 6$

and $x_1, x_2, x_3 \geq 0$

Ans:

$\text{Min } z = \frac{2280}{13}$

$x_1 = \frac{16}{13},\ x_2 = \frac{6}{13}$

$x_3 = \frac{8}{13}.$