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Collision Avoidance and Trajectory Tracking Control based on Approximations of the Maximum Function

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Abstract— **In this paper, convergent over approximations of the maximum function (previously introduced and used for designing strategies for multiple players in pursuit-evasion games) are used to accomplish two objectives for multiple vehicles scenarios: collision avoidance and trajectory tracking. The fact that these approximations are upper approximations of the maximum functions allows us to establish guarantees on the performance (in terms of achieving the objectives) of the vehicles. We combine the approximations of the maximum function with the receding horizon control approach to design control laws. Finally, the method is illustrated by a representative example, where the control laws are designed for nonholonomic vehicles with actuator constraints.**

I. Introduction

Avoidance control was first introduced in the work of Leitmann and Skowronski in late 70s [1]. They proposed a Lyapunov based approach to guarantee collision avoidance which turned out to be very beneficial in terms of obtaining control laws that are not based on solutions of the Hamilton-Jacobi-Isaacs partial differential equations related to multi-player differential games [2]. Later, Leitmann and his collaborators extended this initial work in [1], [3], [4], [5], [6]. It is also important to note that this approach is more general in the sense that this was the first time that a Lyapunov based approach for solving differential games' problems, was proposed.

Some of the attention on collision avoidance has been motivated by the air traffic conflict detection and resolution problem [7]. An inventory that classifies approaches to this problem under different methods (optimization, stochastic and alternative methods), is presented in [8]. Most of the results cited in this survey are quite general and not restricted to aerial vehicles, which opens the possibility of extending the results to a variety of different autonomous vehicles and mobile robotic systems (for specific details refer to [8] and references therein).

In this paper we present a distributed Model Predictive Control (MPC) formulation based on convergent approximations of the maximum function [9] and elements of avoidance control (avoidance functions, [1], [10]). The proposed formulation allows each vehicle to achieve collision avoidance (minimal allowed separation with neighbors) and trajectory tracking objectives, simultaneously. The presented MPC formulation is appealing since it provides a reasonable time interval for solving the proposed MPC formulation without inducing delays in the information exchange and/or mismatches between the achieved solution and the time at which the solution should be applied. The convergent approximations of the maximum function is a scalarization method for multiple objectives based on a parameterized nonlinear transformation. This transformation includes as a special case the linear transformation of multiple cost functions to a single cost function.

The paper is structured as follows. In Section II we introduce the needed preliminaries. Section III presents the proposed MPC optimization formulation. Illustrative simulation results based on the proposed model predictive control formulation are presented in Section IV. Finally, in Section V, conclusions and suggested future research directions are included.

II. Preliminaries

A. Approximations of Max function

It is well known that the complexity of control problems increases significantly if more than one objective have to be accomplished. These problems are known to be extremely difficult even in the case of static optimization as pointed out in [11]. Our approach to address this issue is based on convergent approximations of the minimum and the maximum function as introduced in [9]. The basic idea is to represent multiple objectives (in this paper trajectory tracking and collision avoidance) with corresponding objective functions first, and then use these functions as arguments for the overall goal function. This goal function will be designed using upper-approximations of the maximum function and serves as a Lyapunov-like function.

First, for the completeness of the presentation we start by recalling the approximations of the maximum function as introduced in [9]. Assume that we are given N positive numbers a_i, $i \in \mathbf{N}$.

Following [9], we denote lower and upper maximum function approximations by $\underline{\rho}_\delta(\cdot)$ and $\overline{\rho}_\delta(\cdot)$ and recall their definitions as

$$\underline{\rho}_\delta(a_1,\ldots,a_N) = \sqrt[\delta]{\frac{\sum_{i=1}^N a_i^\delta}{N}} \leq a_M,\ \forall \delta > 0 \tag{1}$$

$$a_M \leq \sqrt[\delta]{\sum_{i=1}^N a_i^\delta} = \overline{\rho}_\delta(a_1,\ldots,a_N),\ \forall \delta > 0 \tag{2}$$

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where $a_M = \max_{i\in \mathbf{N}}\{a_i\}$. M is a variable equal to the index of a maximal a_j element which may not be unique. The maximum approximations are also convergent, that is, $\lim_{\delta\to\infty} \underline{\rho}_\delta = \lim_{\delta\to\infty} \overline{\rho}_\delta = a_M$.

Now, let us assume that each objective for the i-th agent (again, trajectory tracking as well as avoidance of other vehicles) is represented by the corresponding nonnegative function $v_{ij}(t,z(t))$ where $z(t)$ represents the overall state of the system at time t. Without, loss of generality the number of vehicles will be assumed to be M and the number of objectives will be assumed to be $\bar{O}$, implying $i \in \{1,\ldots,M\}$ and $j \in \{1,\ldots,\bar{O}\}$. Then, the goal function for the i-th vehicle is designed as

$$v_i(t,z(t)) = \overline{\rho}_\delta(v_{i1},(t,z(t)),\ldots,v_{i\bar{O}}(t,z(t))) \tag{3}$$

where $\overline{\rho}_\delta(\cdot)$ is an over-approximation of the maximum defined in Equation (2). Notice that in this framework $\delta = 1$ corresponds to a composite Lyapunov function based on a vector Lyapunnov function [12], [13], [14] with weighting coefficients equal to one.

In this this paper only upper-approximations of the maximum function are to be used.

B. Model Predictive Control

Model Predictive Control (MPC) is a widely used methodology for solving constrained infinite horizon optimal control problems. The main idea of MPC is to solve a finite horizon optimal control problem for a system, starting from the current states $z(k)$ over the time interval $[k\Delta t,(k+N)\Delta t]$, where Δt is a sampling time and N is the control horizon length, subject to a set of constraints on the system states and control inputs. After a solution from the optimization problem is obtained, a portion of the computed control actions is applied during the time interval $[k\Delta t,(k+n)\Delta t]$, where n is the receding step, satisfying $n < N$. This process is then repeated as the finite horizon moves by *time steps* of $n\Delta t$ units of time, yielding a state feedback control scheme strategy. This strategy is clearly suboptimal when compared to the optimal infinite horizon control policy but, what we lose in optimality is more than made up for by the tractability of the finite horizon optimization problem.

To formalize some of the key elements in the model predictive control framework, let us consider a general time-invariant nonlinear discrete model of the form:

$$z(k+1) = f(z(k),u(k)), \tag{4}$$

where $z(k) \in \mathbb{R}^p$ defines the state vector, $u(k) \in \mathbb{R}^q$ defines the control inputs at time k and $f(\cdot,\cdot)$ is a continuous, time invariant function, where $f(0,0) = 0$. All control inputs satisfy $u(k) \in \mathbb{U}$, where $\mathbb{U}$ is a convex, compact set which defines the set of all admissible control inputs. Having defined the system model in (4) we can state a general formulation of the receding finite time horizon optimal control problem, denoted as $\mathscr{P}(k)$, starting from initial condition $z(k)$ at time k, as:

$$\begin{aligned}\mathscr{P}(k) : V_N^\circ(z(k)) = \min_{U(z(k))} \{&V_N(z(k+r|k),u(k+r|k)) :\\ &u(k+r|k) \in \mathbb{U}, r \in \{1,\ldots,N\}\}.\end{aligned} \tag{5}$$

The formulation in (5) is subjected to a set of equality constraints $h(\cdot) = 0$ representing the system model in (4) over a finite horizon with N samples and a set of inequality constraints $g(\cdot) \leq 0$ that impose system's input and state constraints. Future predicted states starting from initial conditions $z(k)$ are defined by $z(k+r|k), r \in \{1,\ldots,N\}$, and these are generated by predicted control inputs $u(k+r|k)$, $r \in \{1,\ldots,N\}$. Finally, the sequence of control inputs on the finite horizon staring from initial conditions $z(k)$ is defined as $U(z(k)) = \{u(k+1|k),u(k+2|k),\ldots,u(k+N|k)\}$.

C. MPC Coordination Scheme

Model predictive control is a control framework that may offer advantages when controlling spatially distributed systems by being capable of integrating systems' constraints in an optimization formulation. Two type of constraints that may appear in a spatially distributed setting (for each subsystem) are: (i) *physical constraints* imposed by agents interactions/relations or imposed by the environment and (ii) *information constraints* enforced by the information exchange/flow between subsystems (communication rate, range, and drop-out constraints).

In the present work we consider a spatially distributed system consisting of M physically decoupled subsystems, denoted by $\mathscr{M} := \{1,...,M\}$, where the only coupling arises from shared/sensed information among subsystems. Under these consideration each vehicle $i \in \mathscr{M}$ solves at particular times MPC problems based on the knowledge of its own states and the available shared information from other neighboring agents, aiming to achieve the highest state of coordination. This MPC model points to an important issue regarding time concurrency. Considering that all subsystems are fully cooperative and share their predicted MPC solutions, such information could be used by their neighbors to be incorporated in their own MPC formulations. The problem with this idea is that if at time k a subsystem $i \in \mathscr{M}$ incorporates solutions from neighboring subsystems also obtained at time k, clearly a time concurrency problem arises. A subsystem i can not use solutions from neighboring subsystems at time k since solutions from these subsystems are not available.

Multi-vehicle coordinating schemes based on the MPC framework, in particular for problems associated to collision avoidance and formation control, have being presented in [15], [16], [17], [18], [19]. These appealing results intend to maximize the quality of system output by maximizing the degree of cooperation, that is, to maximize the amount of available/shared useful information at a time where it is of interest/meaningful. A common issue for all the cited results is the presence of time delays in the shared information and mismatch between the times when solutions are computed and when they are supposed to be implemented.

In this paper we present a new MPC coordination framework for spatially distributed systems that adresses the

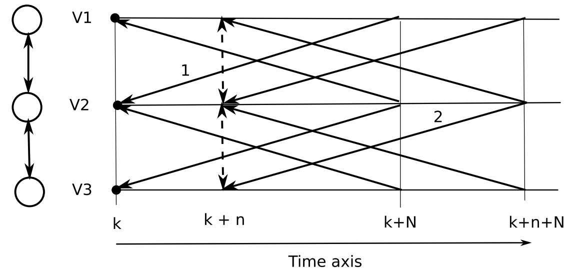


Fig. 1. Schematic showing the communication protocol for a simple linear configuration with three vehicles

information and time mismatches issue. The main idea is that instead of a subsystem $i \in \mathscr{M}$ solving an MPC problem at time k (current time) having as initial condition $z_i(k)$, subsystem i solves at time k an MPC problem having as an initial condition $z_i(k+n|k)$ to generate solutions $z_i(k+n+r|[k+n]_k)$, where $[k+n]_k := k+n|k$, $r \in \{1,\dots,N\}$, and n is the receding step. Then, under the assumption that by the time k all neighboring vehicles $j \in M$, $j \neq i$ have shared their solutions computed based on initial condition $z_j(k|[k]_{k-n})$, $r \in \{1,\dots,N\}$, at time $k-n$ (under the assumption solution times for MPC problems are less than $n\Delta t$ time units), agent i can use for part of this information, namely $z_j(k+r|[k]_{k-n})$, $r \in \{n+1,\dots,N\}$, to be incorporated in its MPC problem formulated at time k, keeping information time consistent. In this framework it is important to keep in mind the following two considerations: (i) At time $k=0$ all agents are supposed to have solutions covering the time interval $[0, N\Delta t]$ to make the proposed scheme consistent. This is not a restriction given that if subsystems start from safe states where neighbors do not pose a conflict or threat, a solution could be found at time $k=0$ without the need of shared information. (ii) The exchanged information are only intended plans and these could change in time. Such observation can be addressed if agents commit to follow/execute at least the first n solution points of their shared solutions. For example, if at time k an agent i shares $z_i(k+r|[k]_{k-n})$, $r \in \{1,\dots,N\}$ it must try to reach/track the predicted states $z_i(k+r|[k]_{k-n})$, $r \in \{1,\dots,n\}$. Figure 1 illustrates the communication aspect of the MPC protocol for a simple linear configuration of three vehicles. The solid lines with arrowheads indicate the information availability of a vehicle at a given time. For example the line numbered 1 indicates that at time k vehicle 2 has information about vehicle 1's predicted trajectory between the time interval $[k, k+N]$. The dotted lines indicate the communication instance between vehicles at time $k+n$. Hence, at time $k+n$ vehicle 3 has information about vehicle 2's predicted trajectory between time interval $[k+n, k+n+N]$. Here, we are assuming that all the MPC computations can be done within a time interval of n time units.

D. Vehicles' Model

Let us consider a vehicle i from a group of M vehicles to be denoted by $\mathbf{V}_i$. For each $\mathbf{V}_i$ let the vehicle's configuration at time t be denoted by $\psi_i(t) = [x_i(t), y_i(t), \theta_i(t)]$, where $x_i(t)$ and $y_i(t)$ are Cartesian coordinates and $\theta_i(t)$ is the heading angle at time t. Each $\mathbf{V}_i$ is modeled as the following nonholonomic kinematic model,

$$\begin{aligned} \dot{x}_i(t) &= s_i(t)\cos(\theta_i(t)) \\ \dot{y}_i(t) &= s_i(t)\sin(\theta_i(t)) \\ \dot{\theta}_i(t) &= \omega_i(t), \end{aligned} \tag{6}$$

where $s_i(t)$ is the vehicle's linear velocity and $\omega_i(t)$ is the vehicle's angular velocity at time t. All considered vehicles are assumed to have on board synchronized clocks. The discrete version of the kinematic model in (6), used in [20], can be summarized as follows:

$$\begin{aligned} x_i(k+1) &= \begin{cases} x_i(k) + \frac{s_i(k)}{\omega_i(k)}\left[\sin\left(\theta_i(k) + \omega_i(k)\Delta t\right)\right. \\ \left. -\sin\left(\theta_i(k)\right)\right], \text{ if } \omega_i(k) \neq 0 \\ x_i(k) + s_i(k)\cos\left(\theta_i(k)\right)\Delta t, \text{ if } \omega_i(k) = 0 \end{cases} \\ y_i(k+1) &= \begin{cases} y_i(k) - \frac{s_i(k)}{\omega_i(k)}\left[\cos\left(\theta_i(k) + \omega_i(k)\Delta t\right)\right. \\ \left. -\cos\left(\theta_i(k)\right)\right], \text{ if } \omega_i(k) \neq 0 \\ y_i(k) + s_i(k)\sin\left(\theta_i(k)\right)\Delta t, \text{ if } \omega_i(k) = 0 \end{cases} \\ \theta_i(k+1) &= \theta_i(k) + \omega_i(k)\Delta t\ , \end{aligned} \tag{7}$$

where $x_i(k)$ and $y_i(k)$ are discrete rectangular coordinates, $\theta_i(k)$ is a discrete heading angle, $s_i(k)$ is the discrete vehicle velocity, $\omega_i(k)$ is the discrete angular turn rate of $\mathbf{V}_i$, Δt is the sampling time, and $k \in \mathbb{Z}_{\geq 0}$ is a discrete time index. The continuous-time path for the described model can be seen as a concatenation of straight lines and arcs with different curvatures.

To allow for a more realistic representation of a vehicle in the considered model in (6) and consequently in (7), let us introduce some constraints in the control inputs for each $\mathbf{V}_i$ [20], such that

$$\begin{bmatrix} s_i^{\min} \\ \omega_i^{\min} \end{bmatrix} \leq \begin{bmatrix} s_i(t) \\ \omega_i(t) \end{bmatrix} \leq \begin{bmatrix} s_i^{\max} \\ \omega_i^{\max} \end{bmatrix}, \forall t. \tag{8}$$

The bounds in inequalities (8) are represented by minimal and maximal velocities s_i^{min} and s_i^{max}, respectively, and minimal and maximal angular velocities ω_i^{min} and ω_i^{max}, respectively. The bounds on angular velocity also satisfy $\omega_i^{max} = -\omega_i^{min}$, implying a symmetry around the point $\omega_i = 0$. The set of feasible control inputs for $\mathbf{V}_i$, described in (8), is denoted by $\mathbb{U}_i$, where no homogeneity condition on sets $\mathbb{U}_i$, $i \in \mathscr{M}$, is assumed.

III. Multi-vehicle Coordination Problems

Let us consider a set of M vehicles gathered in the set $\mathscr{M}$, to be placed in a common environment $\mathbb{W}$ containing no static obstacles, and where the only dynamic obstacles for any vehicle are other vehicles detected within some proximity region. We assume that each vehicle $\mathbf{V}_i$ has a unique tag *id* for inter-vehicle identification. Detected neighbors of $\mathbf{V}_i$ are denoted by $\mathscr{N}(i) \subset \mathscr{M}$. Each $\mathbf{V}_i$ may not be aware of the total number of vehicles in the environment $\mathbb{W}$, but only vehicles within some *detection region* with finite range $D_i \subset \mathbb{R}^2$. The size of the region D_i for each $\mathbf{V}_i$ is constrained by the sensing and/or communication capabilities of the vehicle.

The goal of each $\mathbf{V}_i$ is to track a predetermined discrete trajectory in time, denoted by $\mathscr{T}_i$ and created of desired position samples $[x_i^d(k), y_i^d(k)]^T$, while preserving a predefined safety distance with any other interacting $\mathbf{V}_j,\ j \in \mathscr{N}(i)$, $j \neq i$. The tracking condition imposed on each vehicle clearly implies possible conflicting objectives for the group of vehicles, resulting in potential vehicle collisions. To characterize the safety condition (enforced minimum distance between neighboring vehicles) let us define for each $\mathbf{V}_i$ an *avoidance region* [1], denoted by $A_i \subset D_i \subset \mathbb{R}^p$, such that at any time t any other $\mathbf{V}_j,\ j \in \mathscr{N}(i),\ j \neq i$, must be outside of A_i. Both avoidance regions A_i and detection regions D_i will be considered as bounded circular regions with radius R_{A_i} and R_{D_i}, respectably, such that $0 \leq R_{A_i} \leq R_{D_i}$.

A. Optimization Formulation

To address the considered multi-vehicle coordination problem just described we present an MPC formulation for each $\mathbf{V}_i,\ i \in \mathscr{M}$. The optimization formulation is based on: (i) the minimization of an over approximation of the max function $\overline{\rho}_\delta$ considering as arguments a group of potentially conflicting objectives $v_{ij},\ j \in \mathscr{M}$, and (ii) the information coordination scheme proposed in Subsection II-C. The optimization formulation for each $\mathbf{V}_i,\ i \in \mathscr{M}$, is as follows:

$$\min J_i(k) = \min_{U_i(\psi_i([k+n]_k))} \overline{\rho}_\delta(v_{i1}, \ldots, v_{iM})$$
$$\text{s.t.} \begin{cases} h_i(\psi_i(k+n+r|[k+n]_k), u_i(k+n+r|[k+n]_k)) = 0, \\ \forall r \in \{1, \ldots, N\}; \\ g_i^u(u_i(k+n+r|[k+n]_k)) \leq 0, \forall r \in \{1, \ldots, N\}; \\ g^A(\psi_i(k+n+r|[k+n]_k), \psi_j(k+n+r|[k]_{k-n})) \leq 0, \\ \forall j, j \in \mathscr{N}(i), \forall r \in \{1, \ldots, N-n\}, \end{cases} \tag{9}$$

where

$$v_{ij} = \begin{cases} \sum_{r=1}^{N} \left\| \begin{bmatrix} x_i^d(k+n+r) - x_i(k+n+r|[k+n]_k) \\ y_i^d(k+n+r) - y_i(k+n+r|[k+n]_k) \end{bmatrix} \right\|_2^2, \text{if } i = j; \\ \sum_{r=1}^{N-n} \left(\min\left\{0, \left(\frac{d_{ij}^2(k+n+r) - R_{D_j}^2}{d_{ij}^2(k+n+r) - R_{A_j}^2} \right) \right\} \right)^2, \text{if } i \neq j \end{cases} \tag{10}$$

and

$$d_{ij}^2(k+n+r) = \left\| \begin{bmatrix} x_i(k+n+r|[k+n]_k) - x_j(k+n+r|[k]_{k-n}) \\ y_i(k+n+r|[k+n]_k) - y_j(k+n+r|[k]_{k-n}) \end{bmatrix} \right\|_2^2$$

with $j \in \mathscr{M}$.

In (9) the equality constraints $h_i(\cdot) = 0$ capture the kinematic model in (7), the inequality constraints $g_i^u(\cdot) \leq 0$ capture the input constraint set $\mathbb{U}_i$, and the hard constraints on the avoidance regions projected in time ($d_{ij}(\cdot) - R_{A_j} \leq 0$) are enforced by $g^A(\cdot) \leq 0$. Functions v_{ij} basically describe two different types of objectives: tracking objectives (if $i = j$) and avoidance objectives ($i \neq j$). The tracking objective is to minimize the distances between the sequence of vehicle's predicted positions at some particular times and the corresponding sequence of desired positions belonging to $\mathscr{T}_i$ at these times. The avoidance objectives are based on avoidance functions introduced in [10], where $d_{ij}(\hat{k})$ define the distance function between two vehicles at a time $\hat{k}$. The following avoidance function:

$$\left(\min\left\{0, \left(\frac{d_{ij}^2(k+n+r) - R_{D_j}^2}{d_{ij}^2(k+n+r) - R_{A_j}^2} \right) \right\} \right)^2$$

present in (10), projected in time, relates vehicles $\mathbf{V}_i$ and $\mathbf{V}_j$ from the point of view of $\mathbf{V}_i$. These functions act as detection mechanisms in dealing with potential conflicts. The value of the avoidance functions in (10) tends to increase when a conflicting vehicle $\mathbf{V}_j,\ j \in \mathscr{N}(i)$ predicted position is detected within a distance $R_{A_j} < d_{ij}(k) < R_{D_j}$ of the position of $\mathbf{V}_i$ at the same projected time. In this setting we can make two observations: (i) when $d_{ij}(k) = R_{A_j}$, the value of the function in (10) becomes infinite, and (ii) when $d_{ij}(k) > R_{D_j}$ the value of the function in (10) has a value of zero.

IV. Simulation Results

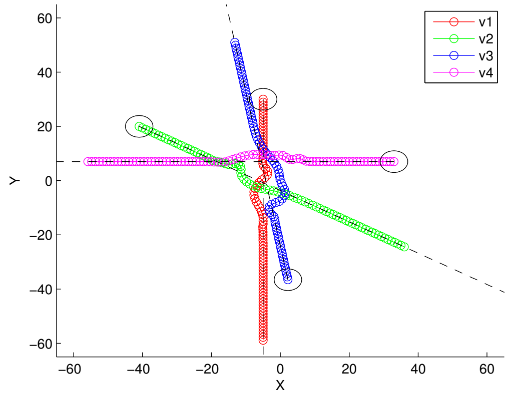
A simulation example resulting from applying the proposed MPC coordinations scheme to a scenario involving four potentially conflicting aircraft vehicles moving in a 2D plane is presented next. In Figures 2(a), 3(a), 4(a), 5(a), collision free trajectories for all vehicles, considering different δ values of $\delta = \{1, 20, 75\}$ and the $\max(\cdot)$ function are depicted. Initial positions for the vehicles in the Figures are marked by big circles and the desired trajectories to be tracked are indicated by dashed lines. Corresponding vehicles distances and computation times for each considered case can be observed in Figures 2(b), 3(b), 4(b), 5(b) and Figures 2(c), 3(c), 4(c), 5(c), respectively.

From Figures 2(a), 3(a), 4(a), 5(a), it can be noticed that after collision avoidance maneuvers have been executed all vehicles return to their original desired trajectories according to the vehicles' tracking scheme while respecting the minimal allowed distance separation. As the value of δ grows, it is possible to observe from Figures 2(b), 3(b), 4(b), 5(b), how the distances between vehicles get tighter and closer to the allowed minimum (in concordance with min over-max approach), as well as how the computation times increase in Figures 2(b), 3(b), 4(b), 5(b), while still respecting the maximum allowed computation time of $n\Delta t$ for each vehicle (which for this example is 12[s]).

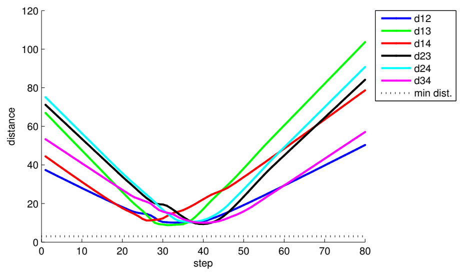
Considered bounds on the aircrafts control inputs $s_i(\cdot)$ and $\omega_i(\cdot)$, and the sampling time Δt are, $s_{\min} = 600$[km/h],$s_{\max} = 750$[km/h], $\omega_{\max} = -\omega_{\min} = 5$[deg/s] and $\Delta t = 6$[s]. Other used parameters are $N = 9$, $n = 2$, $R_{D_i} = 20$[km], $R_{A_i} = 3$[km], $\forall i \in \mathscr{M}$.

V. Conclusions

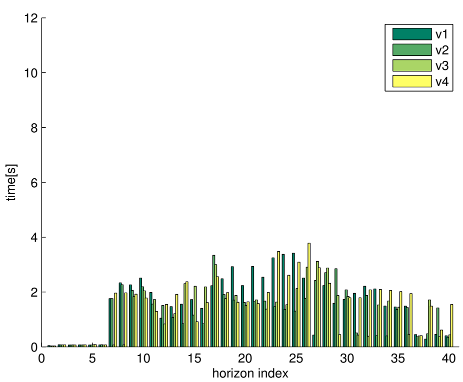
In this paper we have combined the ideas developed in [9] on convergent approximations of the max function and the receding horizon control framework. This approach is shown to be suitable for tackling problems involving two objectives.



(a) Trajectories

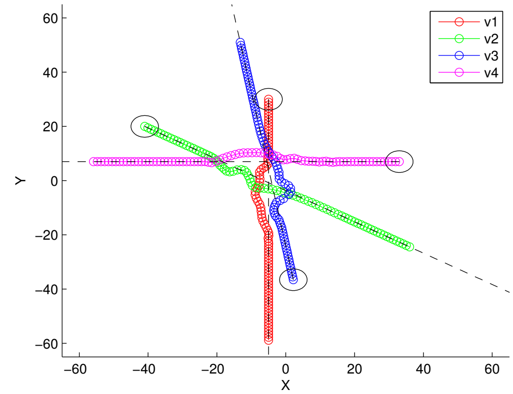


(b) Vehicles' separation

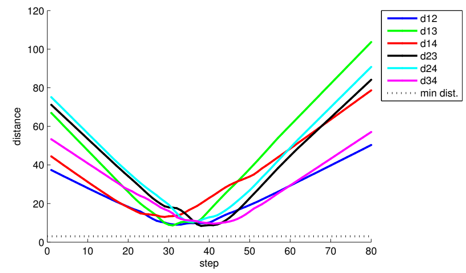


(c) Computation times

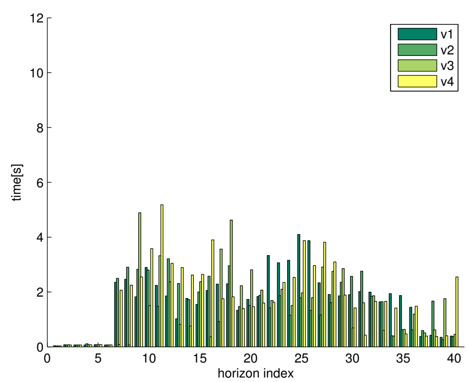
Fig. 2. Simulation example with $\delta = 1$



(a) Trajectories



(b) Vehicles' separation



(c) Computation times

Fig. 3. Simulation example with $\delta = 20$

We provided a comparative study considering different δ values in addressing a joint trajectory tracking and collision avoidance problem and showed that the scheme works in the presence of nonholonomic constraints on the vehicle dynamics and actuator constraints. Further work will focus on considering scenarios with more than two objectives. We will also work on providing Lyapunov based type of guarantees.

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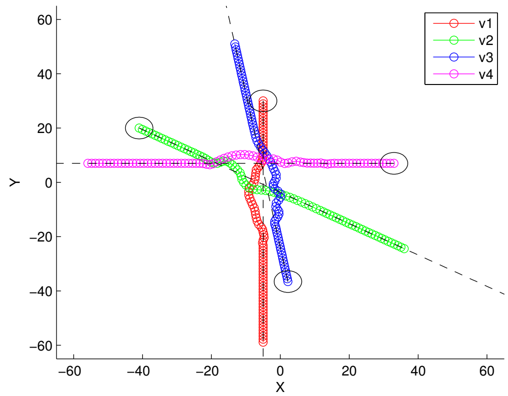
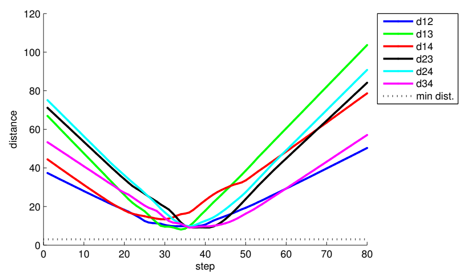
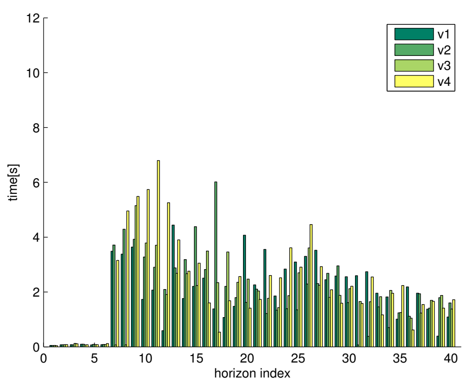


(a) Trajectories

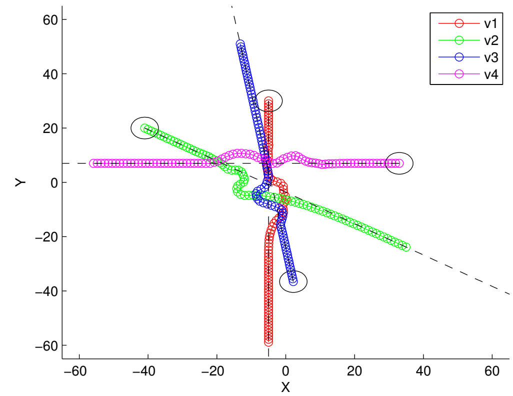


(b) Vehicles' separation

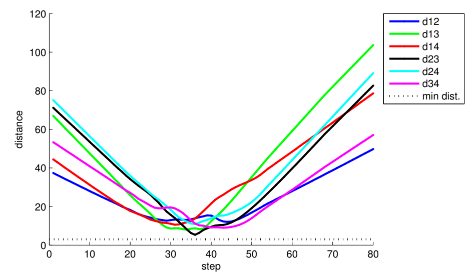


(c) Computation times

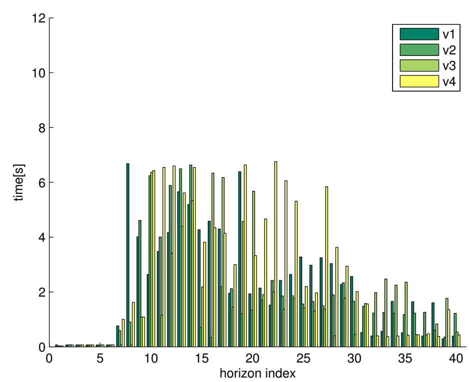
Fig. 4. Simulation example with $\delta = 75$



(a) Trajectories



(b) Vehicles' separation



(c) Computation times

Fig. 5. Simulation example with $\max(\cdot)$ function

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