Longitudinal Control of a Platoon of Vehicles

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Abstract This paper presents a systematic analysis of a longitudinal control law for a platoon of *non-identical* vehicles using a *non-linear* model to represent the vehicle dynamics of each vehicle within the platoon. The basic idea is to take full advantage of recent advances in communication and measurement and using these advances in longitudinal control of a platoon of vehicles: in particular, we assume that for $i = 1, 2, \ldots$ vehicle i knows at all times v_l and a_l (the velocity and acceleration of the lead vehicle) in addition to the distance between vehicle i and the preceding vehicle, $i-1$.

1 Introduction

The subject of design and analysis of various longitudinal control laws has been studied extensively from late 1960's until mid 1970's. Throughout the literature numerous topics such as vehicle-follower controllers, nonlinear vehicle dynamics, entrainment-extrainment maneuvers, and automated guideway transit systems have been reported.[1,2,4,8] Even though much effort has been spent on various control laws for longitudinal control of a platoon of vehicles[3,5,7,9,11], this paper presents a systematic analysis of the longitudinal control for a platoon of *non-identical* vehicles using a *non-linear* model to represent the vehicle dynamics.

The basic concept of this study is: using exact linearization methods[6] to linearize and normalize the input-output behavior of each vehicle in the platoon; taking full advantage of recent advances in communication and measurement[10] and using these advances in longitudinal control of a platoon of vehicles.

To examine the behavior of a platoon of vehicles as a result of a change in the lead vehicle's velocity, simulations for platoons consisting of 16 non-identical vehicles were run. These simulation results show that through the appropriate choice of coefficients in the control law for each vehicle in the platoon the deviations in vehicle spacings from their respective steady-state values do not get magnified from the front to the end of the platoon. An important feature of the design is that such deviations do not exhibit oscillatory time-behavior and their time-variations are well within passengers' comfort limits.

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2 Platoon Configuration

Consider a "platoon" of $N+1$ vehicles traveling in the same lane of a straight stretch of highway and following closely one another. The lead vehicle is labelled "l", the next one is labelled "1", and the last one "N": x_i denotes the abscissa of the rear bumper of the i-th vehicle; each vehicle is alloted a slot of length L; let Δ_i be defined by

$$\Delta_i(t) := x_{i-1}(t) - x_i(t) - L \qquad (2.1)$$

for $i = 2, 3, \ldots$

Δ_i measures the deviation in the assigned distance between vehicle $i-1$ and i. The corresponding kinematic equation for the lead vehicle and the first vehicle are as follows:

$$\Delta_1(t) := x_l(t) - x_1(t) - L \qquad (2.2)$$

Measurements We assume that Δ_i is measured in vehicle i and together with its first and second derivatives, is used in the i-th vehicle's control law. We assume that for each vehicle in the platoon the lead vehicle's velocity (v_l) and acceleration (a_l) are known.(This requires a communication link from lead vehicle to each vehicle of the platoon.)

3 Vehicle Model

In this paper we assume that the road surface is horizontal, there is no wind gust, and the vehicles travel in the same direction at all times. Consequently, the dynamics of the i-th vehicle model ($i = 1, 2, \ldots$) are described by

$$m_i\ddot{x}_i = m_i\xi_i - K_{di}\dot{x}_i^2 - d_{mi} \qquad (3.1)$$

$$\dot{\xi}_i = -\frac{\xi_i}{\tau_i(\dot{x}_i)} + \frac{u_i}{m_i\tau_i(\dot{x}_i)} \qquad (3.2)$$

Equation (3.1) represents Newton's second law for the i-th vehicle: $(K_{di}\dot{x}_i^2)$ specifies the force due to the air resistance, where $K_{di} := \frac{\rho A_i C_{di}}{2}$; ρ denotes the specific mass of air, A_i denotes the cross-sectional area of the i-th vehicle, and C_{di} denotes the i-th vehicle's drag coefficient; $m_i\xi_i$ denotes the (engine) force applied to the i-th vehicle;

the constant d_{mi} denotes the mechanical drag of the i-th vehicle.

Equation (3.2) models the i-th vehicle's engine dynamics: $(\tau_i(\dot{x}_i))$ denotes the i-th vehicle's engine time-constant when the i-th vehicle is traveling with a speed equal to $\dot{x}_i$; u_i denotes the throttle input to the i-th vehicle's engine.

4 Exact Linearization of Vehicle Dynamics

In the following section we will use exact linearization methods[6] to linearize and normalize the input-output behavior of each vehicle in the platoon.

Analysis Substituting the expression for ξ_i from (3.1) in (3.2) gives

$$\dot{\xi}_i = -\frac{1}{\tau_i(\dot{x}_i)}\left[\ddot{x}_i + \frac{K_{di}}{m_i}\dot{x}_i^2 + \frac{d_{mi}}{m_i}\right] + \frac{u_i}{m_i\tau_i(\dot{x}_i)} \qquad (4.1)$$

Differentiating both sides of (3.1) with respect to the time variable and substituting the expression for $\dot{\xi}_i$ from (4.1) we get

$$\dddot{x}_i = -2\frac{K_{di}}{m_i}\dot{x}_i\ddot{x}_i - \frac{1}{\tau_i(\dot{x}_i)}\left[\ddot{x}_i + \frac{K_{di}}{m_i}\dot{x}_i^2 + \frac{d_{mi}}{m_i}\right] + \frac{u_i}{m_i\tau_i(\dot{x}_i)} \qquad (4.2)$$

Linearizing state feedback Using the expression in (4.2), we can write the i-th vehicle's dynamics as follows

$$\dddot{x}_i = b(\dot{x}_i, \ddot{x}_i) + a(\dot{x}_i)u_i \qquad (4.3)$$

where

$$b(\dot{x}_i, \ddot{x}_i) := -2\frac{K_{di}}{m_i}\dot{x}_i\ddot{x}_i - \frac{1}{\tau_i(\dot{x}_i)}(\ddot{x}_i + \frac{K_{di}}{m_i}\dot{x}_i^2 + \frac{d_{mi}}{m_i}) \qquad (4.4)$$

and

$$a(\dot{x}_i) := \frac{1}{m_i\tau_i(\dot{x}_i)} \qquad (4.5)$$

To linearize the i-th vehicle's nonlinear dynamics, we create an exogeneous input c_i which is related to the i-th vehicle throttle input, u_i, by the following equation

$$u_i = \frac{1}{a(\dot{x}_i)}[c_i - b(\dot{x}_i, \ddot{x}_i)] \qquad (4.6)$$

This equation describes a nonlinear state feedback applied to the i-th vehicle's dynamics (4.3).

Substituting (4.6) into (4.3) gives a system of *linear* differential equations representing the dynamics of the i-th vehicle after linearization by state feedback, namely, for $i = 1, 2, \ldots$

$$\frac{d}{dt}x_i = \dot{x}_i \qquad (4.7)$$

$$\frac{d}{dt}\dot{x}_i = \ddot{x}_i \qquad (4.8)$$

$$\frac{d}{dt}\ddot{x}_i = c_i \qquad (4.9)$$

Remark The nonlinear state feedback law (4.6) has achieved two objectives:

1. It linearized the i-th vehicle dynamics;
2. It resulted in dynamics that are independent of m_i, d_{mi}, K_{di}, and $\tau_i(\dot{x}_i)$; i.e., the resulting dynamics of the vehicles are independent of their particular characteristics.

Implementation Issues To compute the linearizing state feedback (4.6), we need to be able to compute the values of the functions $b(.,.)$ and $a(.)$. From (4.4) and (4.5) we note that computation of $b(.,.)$ and $a(.)$ requires sensors to measure the velocity of the i-th vehicle $(\dot{x}_i)$ and the acceleration of the i-th vehicle $(\ddot{x}_i)$. In addition, we need to be able to accurately estimate mass of the i-th vehicle (m_i) and i-th vehicle's mechanical drag (d_{mi}). The vehicle's manufacturer will provide the data regarding engine time constant (the function $\tau_i(.)$), and the vehicle's aerodynamic characteristics ($K_{di} := \frac{\rho A_i C_{di}}{2}$).

5 Platoon Dynamics

In the sequel we will use the linearized vehicle model given in (4.7)-(4.9) for analyzing the platoon dynamics.

Proposed control law Figure 1 shows the linearized model of the i-th vehicle with control input c_i. We propose the following linear control law for longitudinal control of vehicles: for the first linearized vehicle model the control law is

$$\begin{aligned} c_1 &:= c_{p1}\Delta_1(t) + c_{v1}\dot{\Delta}_1(t) + c_{a1}\ddot{\Delta}_1(t) + k_{v1}[v_l(t) - v_0] \\ &+ k_{a1}a_l(t) \end{aligned} \qquad (5.1)$$

where v_0 denotes the steady-state value of the lead vehicle's velocity (v_l);
for linearized vehicle models $2, 3, \ldots$ the control law is

$$\begin{aligned} c_i &:= c_p\Delta_i(t) + c_v\dot{\Delta}_i(t) + c_a\ddot{\Delta}_i(t) + k_v[v_l(t) - v_i(t)] \\ &+ k_a[a_l(t) - a_i(t)] \end{aligned} \qquad (5.2)$$

where $c_{p1}, c_{v1}, c_{a1}, k_{v1}, k_{a1}, c_p, c_v, c_a, k_v$, and k_a are design constants. Note that the control law for the first vehicle differs from the control law for all the other vehicles in the two rightmost terms in (5.1). This is due to the fact that for the first vehicle $v_l - v_1 = \dot{\Delta}_1$ and $a_l - a_1 = \ddot{\Delta}_1$ which are already a part of the first vehicle's control law; whereas, for vehicle i $(i = 2, 3, \ldots)$ $v_l - v_i = \dot{\Delta}_1 + \cdots + \dot{\Delta}_i$ and $a_l - a_i = \ddot{\Delta}_1 + \cdots + \ddot{\Delta}_i$ so that the i-th vehicle's control law contains terms relating to $\dot{\Delta}_1, \ldots, \dot{\Delta}_{i-1}$ and $\ddot{\Delta}_1, \ldots, \ddot{\Delta}_{i-1}$ in addition to terms relating to $\dot{\Delta}_i$ and $\ddot{\Delta}_i$.

Comparison of our control law (5.2) for the i-th vehicle with the control laws in the literature shows that using the lead vehicle's acceleration (a_l) in the i-th vehicle's control law is the new addition to the i-th vehicle's control laws considered in the literature. Shladover had used lead vehicle's velocity (v_l)[8] and $\ddot{\Delta}_i$ [9] in the i-th vehicle's control law.

Implementation Issues The lead vehicle's velocity (v_l) and acceleration (a_l) are transmitted to all the vehicles within the platoon. In addition, sensors on each vehicle, say i, measure the deviation of the i-th vehicle from its assigned position, namely Δ_i. Computation of the first and the second order time derivatives of the i-th vehicle's deviation from its assigned position,namely $\dot{\Delta}_i$ and $\ddot{\Delta}_i$, can be done in two different ways:

1. Communication of the $(i-1)$-st vehicle's velocity $(\dot{x}_{i-1})$ and acceleration $(\ddot{x}_{i-1})$ to the i-th vehicle. Obtaining the i-th vehicle's velocity $(\dot{x}_i)$ and acceleration $(\ddot{x}_i)$ from the sensors on the i-th vehicle, then the computer in this vehicle computes $\dot{\Delta}_i$ $(:= \dot{x}_{i-1} - \dot{x}_i)$ and $\ddot{\Delta}_i$ $(:= \ddot{x}_{i-1} - \ddot{x}_i)$ for use in the i-th vehicle's control law.
2. Direct computation of $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ using the measured values for Δ_i.

The communication of the position,velocity, and acceleration information is unidirectional: from the lead vehicle to each vehicle in the platoon. Communication speed and processing of the measured data should be fast compared to the time constants of the vehicle dynamics.Preliminary studies in [10] suggest that such a requirement is feasible with the present communication and data processing technology.

Initial Conditions Throughout the study of the platoon dynamics we assume the following: for all $t<0$, the platoon is in steady-state; for $t<0$, $\dot{x}_i(t) = \dot{x}_l(t) = v_0$, $\Delta_i(t) = \dot{\Delta}_i(t) = \ddot{\Delta}_i(t) = 0$.

Notation Let w_l denote the increment of velocity of the lead vehicle from its steady-state value (v_0). Thus $w_l(t) := v_l(t) - v_0$.

First vehicle dynamics The linear control law (5.1) applied to the linearized model results in the differential equation (5.4) relating Δ_1 to w_l.

Differentiating both sides of (2.2) three times with respect to the time variable and using the expression for $\dddot{x}_1$ from (4.9) we obtain

$$\dddot{\Delta}_1(t) = \dddot{x}_l(t) - c_1(t) \tag{5.3}$$

Substituting (5.1) in (5.3) and taking Laplace transforms we obtain

$$\begin{aligned}&\left\{s^3 + c_{a1}s^2 + c_{v1}s + c_{p1}\right\}\hat{\Delta}_1(s)\\ &\quad = \left\{s^2 - k_{a1}s - k_{v1}\right\}\hat{w}_l(s)\end{aligned} \tag{5.4}$$

where we use the symbol "^" to distinguish Laplace transforms from the corresponding time-domain functions.

Thus:

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{s^2 - k_{a1}s - k_{v1}}{s^3 + c_{a1}s^2 + c_{v1}s + c_{p1}} \tag{5.5}$$

Equation (5.5) is the first basic design equation. From (5.5), we note that we can independently select all the zeros and all the poles of $\hat{h}_{\Delta_1 w_l}$ by choosing the design parameters $c_{a1}, c_{v1}, c_{p1}, k_{a1}$, and k_{v1}. It is crucial to note that *the selection of zeros and poles are independent of one another.*

Second vehicle dynamics The linear control law (5.2) applied to the linearized model results in the differential equation (5.7) relating Δ_2 to Δ_1 and w_l.

From (4.9) we obtain

$$\dddot{\Delta}_2(t) = c_1(t) - c_2(t) \tag{5.6}$$

Substituting in (5.6) the control laws for the first and the second vehicles, namely (5.1) and (5.2), and taking Laplace transforms we obtain

$$\begin{aligned}&\left\{s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p\right\}\hat{\Delta}_2(s)\\ &= \left\{(c_{a1} - k_a)s^2 + (c_{v1} - k_v)s + c_{p1}\right\}\hat{\Delta}_1(s)\\ &\qquad + \left\{k_{a1}s + k_{v1}\right\}\hat{w}_l(s)\end{aligned} \tag{5.7}$$

Thus:

$$\hat{h}_{\Delta_2\Delta_1}(s) = \frac{(c_{a1} - k_a)s^2 + (c_{v1} - k_v)s + c_{p1}}{s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p} \tag{5.8}$$

From (5.7), we note that in addition to the transfer function from Δ_1 to Δ_2 there is a transfer function from w_l to Δ_2.

i-th vehicle dynamics $(i = 3, 4, \ldots)$ The linear control law (5.2) applied to the linearized model results in the differential equation (5.10) relating Δ_i to Δ_{i-1}.

From (4.9) we obtain

$$\dddot{\Delta}_i(t) = c_{i-1}(t) - c_i(t) \tag{5.9}$$

Substituting the expressions for the proposed linear control laws for the $(i-1)$-st and the i-th vehicles from (5.2) in (5.9) and taking Laplace transforms we obtain

$$\begin{aligned}&\left\{s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p\right\}\hat{\Delta}_i(s)\\ &= \left\{c_a s^2 + c_v s + c_p\right\}\hat{\Delta}_{i-1}(s)\end{aligned} \tag{5.10}$$

From (5.10), we obtain for $i = 3, 4, \ldots$

$$\hat{g}(s) := \hat{h}_{\Delta_i\Delta_{i-1}}(s) = \frac{c_a s^2 + c_v s + c_p}{s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p} \tag{5.11}$$

Let

$$\chi(s) := s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p \tag{5.12}$$

Equation (5.11) is the second basic design equation. From (5.11), we note that we can select independently the poles of $\hat{g}(s)$ (by choosing the appropriate design parameters $(c_a + k_a)$, $(c_v + k_v)$, and c_p) and the zeros of $\hat{g}(s)$ (by choosing the appropriate c_a and c_v).

Furthermore, let us set $c_{a1} = c_a + k_a$, $c_{v1} = c_v + k_v$, and $c_{p1} = c_p$; then (5.5) shows that $\hat{h}_{\Delta_1 w_l}(s)$ has the same poles as $\hat{g}(s)$, and (5.8) shows that $\hat{h}_{\Delta_2\Delta_1}(s)$ has the same poles as $\hat{g}(s)$; in other words, with these choices $\hat{g}(s)$, $\hat{h}_{\Delta_1 w_l}(s)$, and $\hat{h}_{\Delta_2\Delta_1}(s)$ have $\chi(s)$ as denominator polynomial.

Design considerations The main design objectives for the longitudinal control law are as follows: from (5.4), (5.7), and (5.10), we have for $i = 2, 3, \ldots$

$$\hat{h}_{\Delta_i w_l} = (\hat{g}(s))^{i-2}\left[\hat{h}_{\Delta_1 w_l}(s)\hat{g}(s) + \frac{k_{v1} + k_{a1}s}{\lambda(s)}\right] \quad (5.13)$$

1. Since the perturbations in Δ_i due to changes (w_l) in the lead vehicle's velocity from its steady-state value should not get magnified from one vehicle to the next as one goes down the platoon, we require that $|\hat{g}(j\omega)| < 1$ for all $\omega > 0$ and $\omega \mapsto |\hat{g}(j\omega)|$ to be a strictly decreasing function of ω for $\omega > 0$.

2. Since the inverse Laplace transform of $[\hat{g}(s)]^2$ is the convolution of the impulse response of $\hat{g}(s)$ with itself (i.e.,$(g*g)(t)$), to avoid oscillatory behavior down the platoon it is desirable to have $g(t) > 0$ for all t.

6 Simulation Results

To examine the behavior of a platoon of non-identical vehicles under the above control laws, simulations for platoons consisting of 3 different types of vehicles were run using the System Build software package within MATRIXx. We ran simulations for platoons of 4 and 16 vehicles. In all the simulations conducted, all the vehicles were assumed to be initially traveling at the steady-state velocity of $v_0 = 17.9\ m.sec^{-1}$ (i.e., 40 m.p.h.). Beginning at time $t = 0\ sec$, the lead vehicle's velocity was increased from its steady-state value of $17.9\ m.sec^{-1}$ until it reached its final value of $29.0\ m.sec^{-1}$ (i.e., 65 m.p.h.).

Figure 2 shows the lead vehicle's velocity profile as a function of time: the curve $v_l(t)$ corresponds to a maximum jerk of $2.0\ m.sec^{-3}$ and peak acceleration of $3.0\ m.sec^{-2}$ (i.e., roughly $0.3g$).

Simulations were run on a platoon of vehicles assuming different types of physical uncertainties

- Nominal system. Having exact knowledge of all the relevent parameters for applying exact linearization method (4.4)-(4.6) for all of the vehicles within the platoon; assuming no communication delays in transmitting the lead vehicle's velocity (v_l) and acceleration (a_l); assuming no communication delays in using Δ_i in the i-th vehicle's control law (5.1)-(5.2) for $i = 1, 2, \ldots$; assuming no noise in the measurement of Δ_i for $i = 1, 2, \ldots$.
- Perturbed system using push button. Allowing perturbations in the i-th vehicle's mass (m_i) due to passengers' mass and luggage. We assume the driver punches in the number of vehicle occupants using a push button device. The value of the mass parameter used for applying exact linearization method (4.4)-(4.6) is the sum of the vehicle's curb mass and the product of number of vehicle occupants with a pre-assigned mass per passenger. All the assumptions regarding communication delays and measurement noise are identical to the nominal system.
- Perturbed system without push button. Allowing perturbations in the i-th vehicle's mass (m_i) due to passengers' mass and luggage. We assume there are no push button devices. The value of the mass parameter used for applying exact linearization method (4.4)-(4.6) is the vehicle's curb mass. All the assumptions regarding communication delays and measurement noise are identical to the nominal system.
- Perturbed system without push button, including communication delays and noisy measurement. Allowing perturbations in the i-th vehicle's mass (m_i) due to passengers' mass and luggage. We assume there are no push button devices. The value of the mass parameter used for applying exact linearization method (4.4)-(4.6) is the vehicle's curb mass. We assume a constant communication delay in transmitting the lead vehicle's velocity (v_l) and acceleration (a_l); a constant communication delay in using Δ_i in the i-th vehicle's control law (5.1)-(5.2) for $i = 1, 2, \ldots$; and additive Gaussian noise in the measurement of Δ_i for $i = 1, 2, \ldots$.

The following values were chosen for the relevent parameters in the simulation:

$c_{a1} = 15, c_{v1} = 74, c_{p1} = 120, k_{a1} = -3.03, k_{v1} = -0.05$

$c_a = 5, c_v = 49, c_p = 120, k_a = 10, k_v = 25$

Using the above values for the parameters, we obtain

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{(s + 3.01)(s + 0.017)}{(s + 4)(s + 5)(s + 6)}$$

$$\hat{h}_{\Delta_2 w_l}(s) = \frac{s(1.97s^3 + 18.65s^2 + 43.75s - 1.25)}{[(s + 4)(s + 5)(s + 6)]^2}$$

$$\hat{g}(s) = \frac{5(s + 4.9)^2}{(s + 4)(s + 5)(s + 6)}$$

Perturbed system without push button Figure 3 shows the deviations of the first, second, third, and fifteenth vehicles from their pre-assigned positions due to the lead vehicle's velocity profile shown in figure 2 for the perturbed system without a push button device.

Note that the perturbations in the mass parameter range from 8% to 23%.

Simulation results show that the deviations of the vehicles from their pre-assigned positions do not exceed $0.11\ m$ (i.e., 4 inches) and decrease to values which are less than $1\ cm$. Such deviations do not exhibit any oscillatory behavior. The acceleration profiles of the vehicles in the platoon are within the range of acceptable comfort limits and are almost identical to the lead vehicle's acceleration (a_l).

Perturbed system without push button, including communication delays and measurement noise For the perturbed system without a push button device, including communication delays and measurement noise we chose the delay in communicating the lead vehicle's velocity (v_l) and acceleration (a_l) to all the vehicles in the platoon to be 20 $msec$; we chose the communication delay in using Δ to be 5 $msec$; The value of Δ_i used in the i-th vehicle's control law (5.1)- (5.2) was the sum of the actual measured value of Δ_i delayed by 5 $msec$ and some Gaussian noise with zero mean and standard deviation (σ) of 0.05 m.

Figure 4 shows the deviations of the first, second, third, and fifteenth vehicles from their pre-assigned positions due to the lead vehicle's velocity profile shown in figure 2 for the perturbed system without a push button device, including communication delays and measurement noise.

Note that the perturbations in the mass parameter range from 8% to 23%.

Simulation results show that the deviations of the vehicles from their pre-assigned positions do not exceed 0.11 m (i.e., 4 inches) and decrease to values which are less than 1 cm. The acceleration profiles of the vehicles in the platoon are within the range of acceptable comfort limits and are almost identical to the lead vehicle's acceleration (a_l). Note that the non-smooth variations in Δ and $\ddot{x}$ are a result of injecting uncorrelated samples of noise at intervals of 3 $msec$ whereas the linear controller's time constant is on the order of $\frac{1}{6}$ sec;thus, the system does not have enough time to react smoothly to such fast varying inputs.

7 Conclusion

We have shown that through the appropriate choice of design parameters, deviations in the successive vehicle spacings do not get magnified from the front to the back of a platoon of non-identical vehicles as a result of lead vehicle's acceleration from its initial steady-state velocity(v_0) to its final steady-state velocity. Furthermore, the deviations in the successive vehicle spacings do not exhibit any oscillatory time-behavior and the magnitude of such deviations is well within 5 inches for a platoon of 16 vehicles.

Simulation results show that the exact linearization method used performs well in the presence of perturbations in the vehicle's mass (from 8% to 23%), including communication delays and measurement noise; the acceleration profiles of the vehicles in the platoon are within the range of acceptable comfort limits.

8 References

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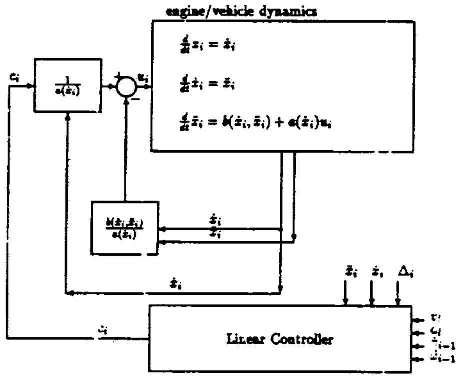

Figure 1: Linearized model of the i-th vehicle with control input c_i, $i = 1, 2, \ldots$

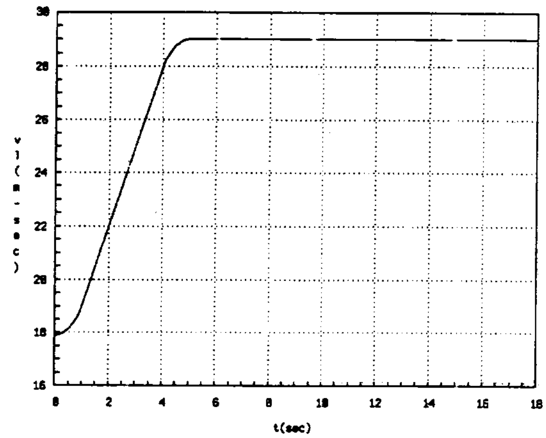

Figure 2: lead vehicle's velocity profile (v_l)

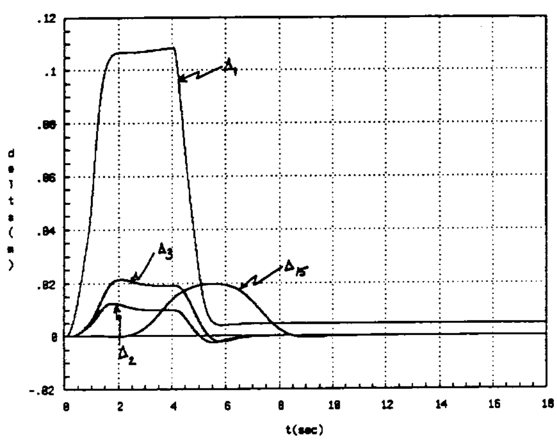

Figure 3: $\Delta_1, \Delta_2, \Delta_3$, and Δ_{15} vs. t: perturbed system, not using push button

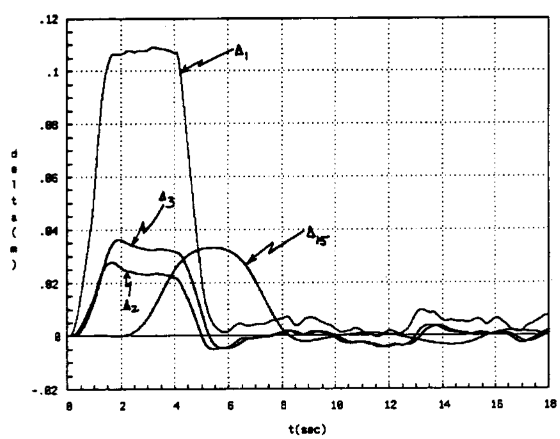

Figure 4: $\Delta_1, \Delta_2, \Delta_3$, and Δ_{15} vs. t: perturbed system with noisy measurement of Δ and communication delay in transmitting lead vehicle's velocity (v_l) and acceleration (a_l); not using push button