Proceedings of the 41st IEEE
Conference on Decision and Control
Las Vegas, Nevada USA, December 2002

TuA11-1

Stabilization of Nonholonomic Systems in Chained Form based on Sampled Data Control

Manabu Yamada*, Shinichi Ohta**, Toshiaki Morinaka** and Yasuyuki Funahashi**

* Research Center for Micro-Structure Devices, Nagoya Institute of Technology, Nagoya, 4668555, JAPAN,
e-mail: *yamam@eine.mech.nitech.ac.jp*

** Department of Mechanical Engineering, Nagoya Institute of Technology, Nagoya, 4668555, JAPAN

Abstract

This paper proposes a new feedback control system based on sampled data control for a class of nonholonomic systems in chained form. The problem of finding a controller to stabilize the nonholonomic systems is reduced to the well-known pole assignment problem for linear time-invariant discrete-time systems. As a result, a simple and explicit design method of stabilizing controller is obtained.

1. Introduction

This paper addresses a feedback stabilization problem for a class of nonholonomic systems described in chained form. The major obstruction to the stabilization problem was the fact that there exists no continuous time-invariant state feedback controller to stabilize chained systems [1]. During the last few years, the stabilization problem has been studied by many researchers, for example, the methods based on discontinuous feedback control, on time-varying one, on hybrid one [2,3,4] and so on.

This paper proposes a new hybrid controller based on sampled data control to achieve asymptotically stabilization. The key idea is the introduction of a coordinate transformation to transform chained systems discretized with a zero order hold and a sampler into *linear time-invariant discrete-time systems*. As a result, the problem of finding a controller to stabilize chained systems is reduced to the well-known discrete-time pole placement problem. Accordingly, a simple design method of the stabilizing controller is obtained in an explicit form. The advantage of our approach is to allow easily extending to many interesting control problems for chained systems, for example, deadbeat control, optimal control, output regulation and so on.

2. Problem formulation

The class of nonholonomic systems to be studied in this paper is described in the following chained form.

$$\begin{cases} \frac{d}{dt}x_0(t) = u_1(t) \\ \frac{d}{dt}x_1(t) = u_2(t) \\ \frac{d}{dt}x_i(t) = x_{i-1}(t)\,u_1(t),\ i \in \{2, \cdots, n\}, \end{cases} \tag{2.1}$$

where $x_i \in \boldsymbol{R}$, $i = 0, 1, \cdots, n$ is the state to be regulated and $u_i \in \boldsymbol{R}$, $i = 1, 2$ are two control inputs. Equation (2.1) can be represented as the following two subsystems of Σ_1 and Σ_2.

$$\Sigma_1:\ \frac{d}{dt}x_0(t) = u_1(t) \tag{2.2}$$

$$\Sigma_2:\ \frac{d}{dt}\boldsymbol{x}(t) = A(u_1(t))\boldsymbol{x}(t) + \boldsymbol{b}u_2(t), \tag{2.3}$$

where $\boldsymbol{x} = [x_n, \cdots, x_1]^T \in \boldsymbol{R}^n$, $\boldsymbol{b} = [0, \cdots, 1]^T \in \boldsymbol{R}^n$ and

$$A(u_1(t)) = \begin{bmatrix} 0 & u_1(t) & & 0 \\ \vdots & \ddots & \ddots & \\ \vdots & & \ddots & u_1(t) \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \in \boldsymbol{R}^{n \times n}. \tag{2.4}$$

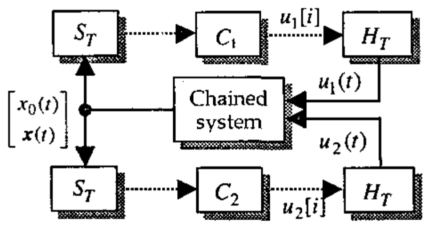

Fig.1 Proposed control system for chained systems

Figure 1 shows a sampled data feedback control system proposed in this paper. S_T is a sampler with the sampling period $T \in \boldsymbol{R}$, and the states of both $x_0(t)$ and $\boldsymbol{x}(t)$ are sampled at the sampling instants. H_T is a zero order hold with the sampling period $T \in \boldsymbol{R}$. Note that S_T and H_T are synchronized. In the sampled data controllers, the control inputs $u_1[i]$ and $u_2[i]$ are determined from the sampled states, $x_0(iT)$ and $\boldsymbol{x}(iT)$ at the sampling instants. Then the control inputs are piecewise constants as follows:

$$u_j(t) = u_j[i] \in \boldsymbol{R} \quad (iT \le t < (i+1)T) \tag{2.5}$$

for $j = 1, 2$. In this paper, we consider the following problem.

Stabilization Problem

For any $x_0(0) \ne 0$ and any $\boldsymbol{x}(0) \in \boldsymbol{R}^n$, find feedback controllers

$$u_j[i] = C_j(i, x_0(iT), \boldsymbol{x}(iT)), \quad j = 1, 2 \tag{2.6}$$

such that $\lim_{t\to\infty} |x_0(t)| = 0$ and $\lim_{t\to\infty} \|\boldsymbol{x}(t)\| = 0$, where $\|\ \|$ denotes the Euclidean norm.

3. Stabilizing controllers for chained systems

The purpose of this section is to present controllers satisfying the above stabilization problem. First, let us make clear an interesting property on the chained system under sampled data control. From eq.(2.5), the system of eqs.(2.2) and (2.3) can be written as

$$\Sigma_1:\ \frac{d}{dt}x_0(t) = u_1[i] \tag{3.1}$$

$$\Sigma_2:\ \frac{d}{dt}\boldsymbol{x}(t) = A(u_1[i])\boldsymbol{x}(t) + \boldsymbol{b}u_2[i] \tag{3.2}$$

during the sampling intervals $I_i = [iT, (i+1)T)$. As can be seen, the subsystem of eq.(3.2) is a controllable piecewise linear time-invariant system as long as $u_1[i]$ is nonzero. We focus on the subsystem Σ_2 of eq.(3.2). Then the response of the state $\boldsymbol{x}(t)$ can be expressed in a simple form as follows.

$$\boldsymbol{x}(t) = e^{A(u_1[i])(t-iT)}\boldsymbol{x}(iT) + \int_0^{t-iT} e^{A(u_1[i])\sigma}\boldsymbol{b}\,d\sigma\,u_2[i], \qquad t \in I_i. \tag{3.3}$$

Especially, at the sampling instants, we obtain a linear time-varying discrete-time state equation

0-7803-7516-5/02/$17.00 ©2002 IEEE

$$x((i+1)T)=\hat{A}(i)x(iT)+\hat{b}(i)u_2[i] \ , \tag{3.4}$$

where

$$\hat{A}(i)=e^{A(u_1[i])T}\in R^{n\times n}, \quad \hat{b}(i)=\int_0^T e^{A(u_1[i])\sigma} b\, d\sigma \in R^n . \tag{3.5}$$

It is interesting that eq.(3.5) has the following decomposition.

$$\hat{A}(i)=T[i]e^{JT}T[i]^{-1}\in R^{n\times n}, \quad \hat{b}(i)=T[i]\int_0^T e^{J\sigma} b\, d\sigma \in R^n , \tag{3.6}$$

where

$$T[i]=diag\ [u_1[i]^{n-1},\cdots,u_1[i],1]\in R^{n\times n} \tag{3.7}$$

and

$$J=\begin{bmatrix} 0 & 1 & & 0 \\ \vdots & \ddots & \ddots & \\ \vdots & & \ddots & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}\in R^{n\times n}. \tag{3.8}$$

Let

$$\lambda_i=\frac{u_1[i+1]}{u_1[i]} \ . \tag{3.9}$$

Then

$$T[i+1]=T[i]\ diag\ [\ \lambda_i^{n-1},\cdots,\lambda_i,1\ ] \ . \tag{3.10}$$

We introduce a piecewise constant of coordinate transformation as follows

$$\bar{x}(t)=T[i]^{-1}x(t)\in R^n , \quad t\in I_i . \tag{3.11}$$

This transforms time-varying discrete-time state equation of eq.(3.4) into a time-invariant one as shown in the following lemma.

Lemma 3.1

Consider the case that $\lambda_i=\lambda\in R, \forall i$. Under the proposed sampled data control system, the state sequences $x(iT)$ of the chained system at the sampling instants can be expressed in both the following linear, time-invariant, discrete-time state equation

$$\bar{x}((i+1)T)=\bar{A}\,\bar{x}(iT)+\bar{b}\,u_2[i] \tag{3.12}$$

and the following coordinate transformation

$$x(iT)=T[i]\ \bar{x}(iT), \tag{3.13}$$

where

$$\begin{aligned}\bar{A}&=diag[\ \lambda^{-(n-1)},\cdots,\lambda^{-1},1\ ]\ e^{JT}\in R^{n\times n}\\ \bar{b}&=diag[\ \lambda^{-(n-1)},\cdots,\lambda^{-1},1\ ]\ \int_0^T e^{J\sigma} b\, d\sigma\in R^n .\end{aligned} \tag{3.14}$$

Moreover, the discrete-time state equation $(\bar{A},\bar{b})$ of eq.(3.12) is controllable for any sampling periods $T\in R$ and any $\lambda\neq 0,-1$.

Proof: It is easy to check that the relationship between $(\hat{A}(i),\hat{b}(i))$ and $(\bar{A},\bar{b})$ is given as follows.

$$\hat{A}(i)=T[i+1]\ \bar{A}\ T[i]^{-1}, \quad \hat{b}(i)=T[i+1]\,\bar{b} \ . \tag{3.15}$$

By using eq.(3.11), the discrete-time state equation of eq.(3.4) is transformed into that of eq.(3.12). The proof of the controllability is similar to [4], and it is omitted. Q.E.D.

The following theorem presents a simple and explicit design method of stabilizing controller. For the sake of simplicity, it is assumed that $x_0(0)\neq 0$.

Theorem 3.2

Consider the following state feedback controllers

$$u_1[i]=k_1 x_0(iT) \ , \tag{3.16}$$

$$u_2[i]=\bar{k}_2 T[i]^{-1}x(iT) \ , \tag{3.17}$$

where

$$k_1=\frac{\lambda-1}{T} \ , \tag{3.18}$$

λ is a nonzero constant such that $|\lambda|<1$. $\bar{k}_2\in R^n$ is a constant state feedback gain such that $(\bar{A}+\bar{b}\bar{k}_2)$ is stable, i.e.,

$$\max_j \left|\lambda_j(\bar{A}+\bar{b}\,\bar{k}_2)\right|<1 \ . \tag{3.19}$$

Then it follows that $\lim_{t\to\infty}|x_0(t)|=0$ and $\lim_{t\to\infty}\|x(t)\|=0$ for any $x_0(0)\neq 0$ and any $x(0)\in R^n$.

Proof: First, we focus on the subsystem of Σ_1. From eqs.(3.1) and (3.16), the response of the state $x_0(t)$ is obtained by

$$x_0(t)=(1+k_1(t-iT))\,\lambda^i x_0(0), \quad t\in I_i \ . \tag{3.20}$$

Therefore $\lim_{t\to\infty}|x_0(t)|=0$ for $\forall x_0(0)\in R$.

Secondly, we consider the subsystem of Σ_2. From eqs.(3.16), note that

$$\frac{u_1[i+1]}{u_1[i]}=\lambda\in R, \quad \forall i \tag{3.21}$$

holds. By using eq.(3.17) and Lemma 3.1, the state sequences of eq.(3.12) at the switching instants can be expressed as

$$\bar{x}((i+1)T)=(\bar{A}+\bar{b}\,\bar{k}_2\ )\,\bar{x}(iT) \ . \tag{3.22}$$

Then the response of $x(t)$ can be expressed as follows.

$$\begin{aligned}x(t)=T[i]&\left(e^{J(t-iT)}+\int_0^{t-iT}e^{J\sigma}b\,d\sigma\,\bar{k}_2\right)\\ &(\bar{A}+\bar{b}\,\bar{k}_2)^i\,T[0]^{-1}x(0), \quad t\in I_i\end{aligned} \tag{3.23}$$

Note that

$$\lim_{i\to\infty}\ \|T[i]\|=1 \tag{3.24}$$

and there exists a positive constant $\bar{c}_1$ such that

$$\left\|e^{J(t-iT)}\right\|\leq\bar{c}_1 \ \ \forall t\in I_i \ . \tag{3.25}$$

Therefore $\lim_{t\to\infty}\|x(t)\|=0$ for $\forall x(0)\in R^n$. Q.E.D.

Remark: From Lemma 3.1, since the discrete-time state equation $(\bar{A},\bar{b})$ is controllable, the design method of finding $\bar{k}_2\in R^n$ satisfying eq.(3.19) is provided easily by the well-known linear control theory.

4. Conclusion

This paper has proposed a new feedback control system based on a sampled data control for nonholonomic chained systems.

This work was supported in part by Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan.

References

[1] R.W.Brockett: Differential Geometric Control Theory, pp.181-191, Boston: Birkhauser (1983)

[2] C.Canudas de Wit, H.Berhuis and H.Nijmeijer: Proc. of IEEE CDC, pp.3475-3479 (1994)

[3] S.Monaco and D.N-Cyrot: Proc. of IEEE CDC, pp.1780-1785, (1992)

[4] M.Yamada, S.Ohta, Y.Syumiya and Y.Funahashi: Trans. of the Society of Instrument and Control Engineers, Vol.38, No.4, pp.369-378 (2002)