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Nonlinear control of mechatronic systems with permanent-magnet DC motors

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Abstract

The current trends in development and deployment of advanced electromechanical systems have facilitated the unified activities in the analysis and design of state-of-the-art motion devices, electric motors, power electronics, and digital controllers. This paper attacks the motion control problem (stabilization, tracking, and disturbance attenuation) for mechatronic systems which include permanent-magnet DC motors, power circuity, and motion controllers. By using an explicit representation of nonlinear dynamics of motors and switching converters, we approach and solve analysis and control problems to ensure a spectrum of performance objectives imposed on advanced mechatronic systems. The maximum allowable magnitude of the applied armature voltage is rated, the currents are limited, and there exist the lower and upper limits of the duty ratio of converters. To approach design tradeoffs and analyze performance (accuracy, settling time, overshoot, stability margins, and other quantities), the imposed constraints, model nonlinearities, and parameter variations are thoroughly studied in this paper. Our goal is to attain the specified characteristics and avoid deficiencies associated with linear formulation. To solve these problems, an innovative controller is synthesized to ensure performance improvements, robust tracking, and disturbance rejection. One cannot neglect constraints, and a bounded control law is designed to improve performance and guarantee robust stability. The offered approach uses a complete nonlinear mechatronic system dynamics with parameter variations, and this avenue allows one to avoid the conservative results associated with linear concept when mechatronic system dynamics is mapped by a linear constant-coefficient differential equation. To illustrate the reported framework and to validate the controller, analytical and experimental results are presented and discussed. In particular,

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comprehensive analysis and design with experimental verification are performed for an electric drive. A nonlinear bounded controller is designed, implemented, and experimentally tested.

1. Introduction

The problem of controlling mechatronic systems is a very important one in many industrial applications. Nonlinear analysis and control must be researched and solved to improve the steady-state and dynamic characteristics of electric drives and servos which are used in a wide range of motion devices. Intensive research in motion control has been performed by using linear theory, and the developed controllers have been implemented. However, the assumed assumptions limit the generality of results, as well as practical use and applicability of existing control algorithms in high-performance electromechanical systems. As demonstrated in [1–5], augmented nonlinear motor-converter dynamics cannot be linearized, and hard bounds imposed cannot be neglected. Specific performance requirements are assigned, and the absolute limit of motor and converter performances can be placed in the scope of practical design. It is evident that the general line of attack is associated with nonlinear analysis and synthesis. Recognizing the potential difficulties associated with ignoring nonlinearities and constraints, the research has been focused on constrained control. In recent years, a number of new techniques have been developed. Despite this manageable research in electric machinery, fully integrated mechatronic systems must be researched to guarantee the validity of analytical, numerical, and experimental results. In particular, power converters should be incorporated in analysis and design. New features in motion control of mechatronic system arise, and computational and implementation difficulties, associated with classical nonlinear analysis and design, limit the implementation of the existing analytical results as well as a set of the developed controllers. This paper offers an innovative avenue to analyze nonlinear mechatronic systems using the Lyapunov stability theory, and a unified method in nonlinear analysis and design is developed. The Lyapunov concept has been applied in a wide range of applications. Recently, there has been increased interest in the application of the second method to design nonlinear controllers, and the Lyapunov theory has been extended to approach the tracking, disturbance rejection, and stabilization problems [6–9]. A dual analysis-design formulation is developed in this paper. The tradeoff between model accuracy and model simplicity is the central problem of model-based design techniques. It is recognized that the steady-state and dynamic characteristics, stability margins, and performance of mechatronic systems are dependent on the validity of the models used in the design. Nonlinear models of mechatronic systems must be developed augmenting a permanent-magnet DC motor with the converter used (differential equations can be found by using the Kirchhoff laws or

Lagrange equations of motion) and parameters of electromechanical systems (motor parameters and circuit elements) are time-varying. To solve the motion control problem for mechatronic systems, this paper develops a new control law to solve the constrained tracking problem. The bounded control problem is attacked by applying the admissibility framework, and a dual analysis-design formulation is tackled by using the Lyapunov stability theory. A new class of nonquadratic Lyapunov functions is offered, and nonlinear controllers are designed to ensure stability, tracking, and disturbance rejection. The developed theoretical results are verified for an electric drive actuated by a permanent-magnet DC motor.

2. Mechatronic systems models

Electromagnetic theory and classical mechanics form the basis for the development of electromechanical system models. The mathematical model of permanent-magnet DC motors is known; in particular, the motor, shown in Fig. 1, is described by linear differential equations [1,5]

$$\frac{\mathrm{d}i_a}{\mathrm{d}t} = \frac{1}{L_a}(u_a - r_a i_a - k_a \omega_r),$$

$$\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = \frac{1}{J}(k_T i_a - B_m \omega_r - T_L),$$

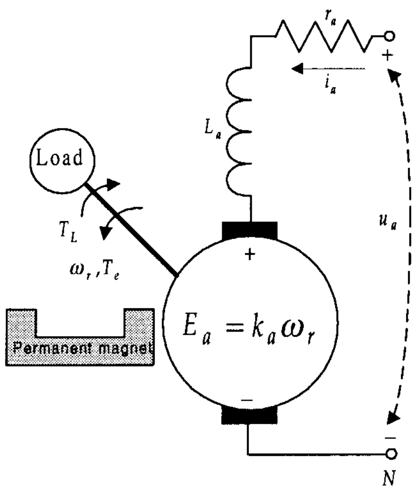

Fig. 1. Permanent-magnet DC motor.

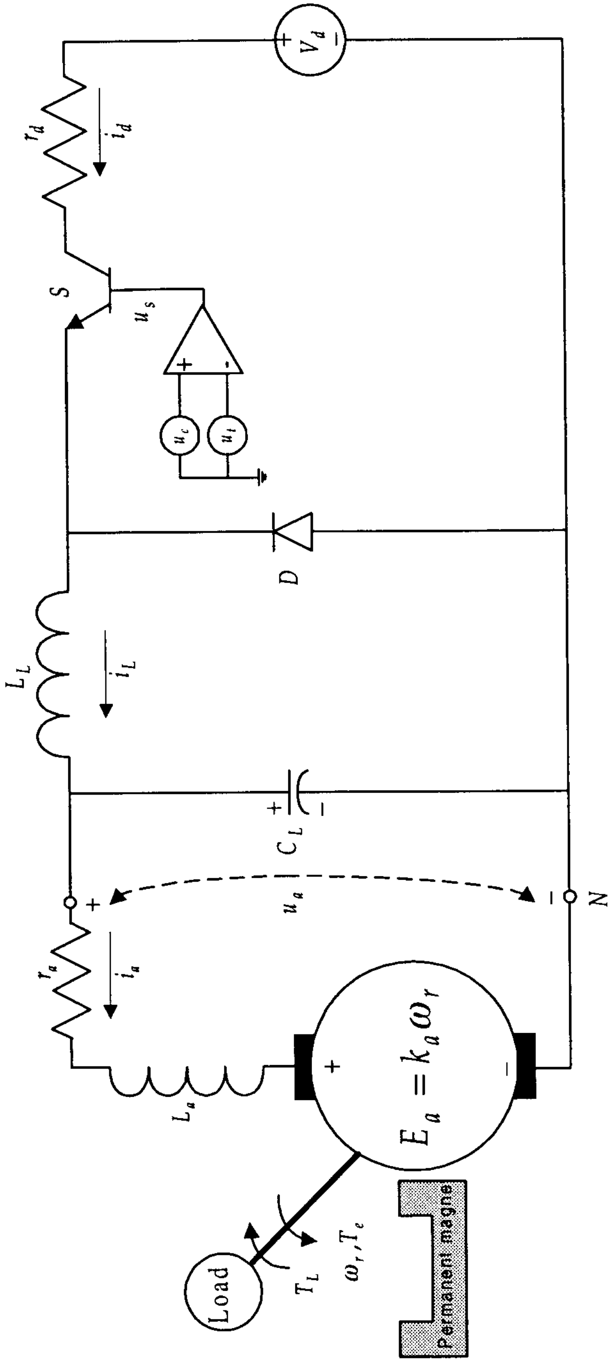

Fig. 2. Regulation of a permanent-magnet DC motor by using a switching principle.

$$\frac{d\theta_r}{dt} = \omega_r, \tag{1}$$

where i_a and u_a are the armature current and voltage; ω_r and θ_r are the rotor angular velocity and rotor displacement; T_L is the load torque; r_a and L_a are the armature resistance and inductance; J is the equivalent moment of inertia; k_a and k_T are the back emf and torque constants; B_m is the viscous friction coefficient. Observe that the permanent-magnet DC motors develop the electromagnetic torque T_e, which is given as $T_e = k_T i_a$, and the load torque T_L is the disturbance.

In (1), the armature voltage u_a can be considered as a control. However, u_a is the converter output. Hence, the power converter must be studied. Control of the motor angular velocity ω_r is established by changing the armature voltage u_a using a pulse-width-modulation (PWM) switching concept. Fig. 2 shows the permanent-magnet motor with a high-frequency step-down switching converter.

The switch S is opened and closed, and the switching frequency is $1/(t_{on} + t_{off})$. Assuming that the switch is lossless, the output voltage is equal to the terminal voltage V_d when the switch is closed, and the output voltage is zero if the switch is open. The output voltage is regulated by controlling the switching on and off durations (t_{on} and t_{off}), and the average voltage, applied to the armature winding, depends on t_{on} and t_{off}. In particular, neglecting r_d, in the steady-state, $u_a = [t_{on}/(t_{on} + t_{off})]V_d = d_D V_d$, where d_D is the duty ratio, which is a function of the switching frequency and the time during which the switch is on, $d_D = [t_{on}/(t_{on} + t_{off})]$; that is, $d_D \in [0\ 1]$. By changing the duty ratio d_D of the switch, which is bounded by [0 1], the average voltage, supplied to the armature winding, is regulated. To establish the PWM switching, the control-triangle concept is used. The switching signal u_s, which drives the switch, is generated by comparing a signal-level control voltage u_c with a repetitive triangular signal u_t. The duration of the output pulses u_s represents the weighted value between the triangular voltage u_t with the assigned switching frequency and control signal u_c. The output voltage of the comparator u_s drives the switch, and on and off durations are obtained by comparing u_c and u_t. A converter has an internal resistance r_d, and a low-pass filter with inductance L_L and capacitance C_L is used to ensure the specified voltage ripple.

By using the averaging concept, Kirchhoff's law, and motor dynamics (1), one obtains the following nonlinear differential equations for the permanent-magnet DC motor with the studied one-quadrant switching converter

$$\frac{du_a}{dt} = \frac{1}{C_L}(i_L - i_a),$$

$$\frac{di_L}{dt} = \frac{1}{L_L}(-u_a - r_d i_L d_D + V_d d_D),$$

$$\frac{di_a}{dt} = \frac{1}{L_a}(u_a - r_a i_a - k_a \omega_r),$$

$$\frac{d\omega_r}{dt} = \frac{1}{J}(k_T i_a - B_m\omega_r - T_L),$$

$$\frac{d\theta_r}{dt} = \omega_r. \tag{2}$$

The duty ratio is regulated by the signal-level control voltage u_c, and u_c is bounded. In particular, $d_D = (u_c/u_{t\,\max}) \in [0\ 1]$, $u_c \in [0\ u_{c\,\max}]$, $u_{c\,\max} = u_{t\,\max}$.

An augmented mechatronic system model has been developed in (2). It is shown that to control DC motors, the signal-level control voltage u_c should be considered as a control, and the converter output (applied armature voltage) u_a is the state variable. Furthermore, by augmenting motor and converter dynamics, a nonlinear model results due to $(r_d/L_L u_{t\,\max}) i_L u_c$, and the hard control limit is imposed because $u_c \in [0\ u_{c\,\max}]$.

It should be emphasized that the degradation of permanent-magnets, which are used in permanent-magnet motors, cannot be neglected because the residual flux density changes over the operating temperature envelope. The temperature sensitivity is characterized by the reversible temperature coefficient which guides one to the percentage change in magnetic flux per unit change in temperature. Hence, the variation of temperature leads to changes of back emf and torque constants, see the values for k_a and k_T which have been identified for different temperatures in Section 4. The armature resistance and inductance also vary due to heating, and the equivalent moment of inertia varies due to load changes. One concludes that k_a, k_T, r_a, L_a and J are time-varying parameters.

As a result of model nonlinearities, control bounds, and parameter variations, we must depart from the conventional concepts in motion control of permanent-magnet DC motors. In particular, nonlinear mechatronic systems with parameter variations should be studied. Basic power converter topologies (for example, buck, boost, buck–boost, Cuk, flyback, resonant, etc.) and filter circuitry configurations have been developed, and nonlinear differential equations can be found by using the Kirchhoff laws or Lagrange equations of motion. A detailed treatment of the deviation of power converter dynamics is reported in [2]. To avoid burdensome notations, which are associated with nonlinear converter dynamics (there exist a great number of converter topologies and filter configurations which are described by high-order nonlinear differential equations), a generic mathematical model of multi-input/multi-output mechatronic systems must be presented in state-space form; in particular,

$$\dot{x}(t) = F_x(x,r,d,z) + B_u(x,p)u, \quad y = H(x), \quad u_{\min} \le u \le u_{\max}\,, \quad x(t_0) = x_0, \tag{3}$$

where $x \in X \subset \mathbb{R}^c$ is the state vector: $u \in U \subset \mathbb{R}^m$ is the control vector; $r \in R \subset \mathbb{R}^b$ and $y \in Y \subset \mathbb{R}^b$ are the measured reference and output vectors; $d \in D \subset \mathbb{R}^s$ is the disturbance vector, $d = T_L$; $z \in Z \subset \mathbb{R}^d$ and $p \in P \subset \mathbb{R}^k$ represent parameter variations with known bounds, functions $z(\cdot)$: $[t_0, \infty) \to Z \subset \mathbb{R}^d$ and $p(\cdot)$: $[t_0, \infty) \to P \subset \mathbb{R}^k$ are Lebesgue measurable, $Z \subset \mathbb{R}^d$ and $P \subset \mathbb{R}^k$ are the known non-empty compact sets; $F_x(\cdot)$: $\mathbb{R}^c \times \mathbb{R}^b \times \mathbb{R}^s \times \mathbb{R}^d \to \mathbb{R}^c$ and $B_u(\cdot)$: $\mathbb{R}^c \times \mathbb{R}^k \to \mathbb{R}^{c \times m}$ are jointly continuous and Lipschitz,

$F_x(0, 0, 0, z) = 0$ and $B_u(0, p) = 0$; $H(\cdot)$: $\mathbb{R}^c \to \mathbb{R}^b$ is the smooth mapping defined in the neighborhood of the origin, $H(0) = 0$.

The output equation $y = H(x)$ is used, and the measured vector is express as

$$e(t) = r(t) - y(t).$$

3. Robust control

Our goal is to synthesize a robust controller which stabilizes nonlinear mechatronic systems and drives the tracking error $e(t) = r(t) - y(t)$ robustly to the specified compact set. A family of bounded control laws is defined as

$$u = -\phi\left(G_e B_e^{\mathrm{T}} \frac{\partial V(e,x)}{\partial e} + \frac{1}{s} G_i B_e^{\mathrm{T}} \frac{\partial V(e,x)}{\partial e} + G_x B^{\mathrm{T}}(x) \frac{\partial V(e,x)}{\partial x}\right), \quad s = \frac{\mathrm{d}}{\mathrm{d}t}, \tag{4}$$

where $\phi(\cdot)$ is the bounded continuous or piecewise continuous function (tanh, erf, sat, sign); $G_e \in \mathbb{R}^{m \times m}$, $G_i \in \mathbb{R}^{m \times m}$ and $G_x \in \mathbb{R}^{m \times m}$ are the positive–definite diagonal weighting matrices; $B_e \in \mathbb{R}^{b \times m}$ and $B(\cdot)$: $\mathbb{R}^c \to \mathbb{R}^{c \times m}$; $V(\cdot)$: $\mathbb{R}^b \times \mathbb{R}^c \to \mathbb{R}_{\geq 0}$ is the continuously differentiable real-analytic function,

$$V(e,x) = \sum_{i=0}^{\zeta} \frac{2\beta + 1}{2(i + \beta + 1)} \left(e^{\frac{i+\beta+1}{2\beta+1}}\right)^{\mathrm{T}} K_{ei} e^{\frac{i+\beta+1}{2\beta+1}} + \sum_{i=0}^{\sigma} \frac{2\mu + 1}{2(i + \mu + 1)} \left(e^{\frac{i+\mu+1}{2\mu+1}}\right)^{\mathrm{T}} K_{eii} e^{\frac{i+\mu+1}{2\mu+1}} + \sum_{i=0}^{\eta} \frac{2\gamma + 1}{2(i + \gamma + 1)} \left(x^{\frac{i+\gamma+1}{2\gamma+1}}\right)^{\mathrm{T}} K_{xi} x^{\frac{i+\gamma+1}{2\gamma+1}} + \frac{1}{2} x^{\mathrm{T}} K x, \tag{5}$$

where non-negative integers $\zeta = 0, 1, 2, \ldots$, $\beta = 0, 1, 2, \ldots$, $\sigma = 0, 1, 2, \ldots$, $\mu = 0, 1, 2, \ldots$, $\eta = 0, 1, 2, \ldots$ and $\gamma = 0, 1, 2, \ldots$ are assigned by the designer to attain the specified performance.

For example, letting $\zeta = 0$, $\beta = 0$, $\sigma = 0$, $\mu = 0$, $\eta = 0$, and $\gamma = 0$, the quadratic Lyapunov candidate $V(e,x) = \frac{1}{2} e^{\mathrm{T}} K_{e0} e + \frac{1}{2} e^{\mathrm{T}} K_{ei0} e + \frac{1}{2} x^{\mathrm{T}} K_{x0} x + \frac{1}{2} x^{\mathrm{T}} K x$ results, and the control law is found to be

$$u = -\phi\left(G_e B_e^{\mathrm{T}} K_{e0} e + \frac{1}{s} G_i B_e^{\mathrm{T}} K_{ei0} e + G_x B^{\mathrm{T}}(x) K_{x0} x + G_x B^{\mathrm{T}}(x) K x\right), \quad \text{see (6).}$$

The unknown diagonal matrices $K_{ei} \in \mathbb{R}^{b \times b}$, $K_{eii} \in \mathbb{R}^{b \times b}$ and $K_{xi} \in \mathbb{R}^{c \times c}$, as well as the symmetric matrix $K \in \mathbb{R}^{c \times c}$, should be found by using conditions imposed on the Lyapunov pair. In particular, upon the second method of Lyapunov, to analyze stability one applies the sufficient criteria as given by $V(e, x) > 0$ and $[\mathrm{d}V(e, x)/\mathrm{d}t] \leq 0$. It is evident that function $V(e, x)$, which is given in nonquadratic form (5), is positive–definite if matrices K_{ei}, K_{eii}, K_{xi} and K are positive–definite. Computing the total derivative of (5), along the solutions of the closed-loop system (3) and (4), the unknown matrices K_{ei}, K_{eii}, K_{xi} and K can be found. In particular, nonlinear inequality $[\mathrm{d}V(e,\, x)/\mathrm{d}t] \leq 0$ must be solved [6–9].

By making use (4) and (5), the resulting bounded controller is found as

$$u = -\phi\left(G_e B_e^{\mathrm{T}} \sum_{i=0}^{\zeta} K_{ei} e^{\frac{2i+1}{2\beta+1}} + \frac{1}{s} G_i B_e^{\mathrm{T}} \sum_{i=0}^{\sigma} K_{eii} e^{\frac{2i+1}{2\mu+1}} + G_x B^{\mathrm{T}}(x) \sum_{i=0}^{\eta} K_{xi} x^{\frac{2i+1}{2\gamma+1}} + G_x B^{\mathrm{T}}(x) K x\right). \tag{6}$$

In controller (6), nonlinear proportional and integral error mappings $G_e B_e^{\mathrm{T}} \Sigma_{i=0}^{\zeta} K_{ei} e^{(2i+1)/(2\beta+1)}$ and $(1/s) G_i B_e^{\mathrm{T}} \Sigma_{i=0}^{\sigma} K_{eii} e^{(2i+1)/(2\mu+1)}$ accomplish the tracking of the bounded reference input $r(t)$, the error vector $e(t)$, and $e(t) = r(t) - y(t)$ is used. The state feedback results due to $G_x B^{\mathrm{T}}(x) \Sigma_{i=0}^{\eta} K_{xi} x^{(2i+1)/(2\gamma+1)} + G_x B^{\mathrm{T}}(x) K x$.

For the given control $u \in U \subset \mathbb{R}^m$, reference $r \in R \subset \mathbb{R}^b$, disturbance $d \in D \subset \mathbb{R}^s$, parameter variations $z \in Z \subset \mathbb{R}^d$ and $p \in P \subset \mathbb{R}^k$, from (3) with (6) one concludes that the mechatronic system with $X_0 = \{x_0 \in \mathbb{R}^c\} \subseteq X \subset \mathbb{R}^c$ and $E_0 = \{e_0 \in \mathbb{R}^b\} \subseteq E \subset \mathbb{R}^b$ evolves in

$$XE(X_0, E_0, U, R, D, Z, P) = \{(x,e) \in X \times E: x_0 \in X_0, e_0 \in E_0, u \in U, r \in R, d \in D, z \in Z, p \in P, t \in [t_0, \infty)\} \subset \mathbb{R}^c \times \mathbb{R}^b.$$

Hence, we consider the system dynamics in $XE \subset \mathbb{R}^c \times \mathbb{R}^b$. Observe that the output equation is $y = H(x)$, and a reference-output map

$$\left\{ \begin{aligned} &\dot{x}(t) = F_x(x,r,d,z) - B_u(x,p)\phi\left(G_e B_e^{\mathrm{T}} \sum_{i=0}^{\zeta} K_{ei} e^{\frac{2i+1}{2\beta+1}} + \frac{1}{s} G_i B_e^{\mathrm{T}} \sum_{i=0}^{\sigma} K_{eii} e^{\frac{2i+1}{2\mu+1}} \right. \\ &\left. \quad + G_x B^{\mathrm{T}}(x) \sum_{i=0}^{\eta} K_{xi} e^{\frac{2i+1}{2\gamma+1}} + G_x B^{\mathrm{T}}(x) K x \right), \\ &y = H(x), e(t) = r(t) - y(t), x_0 \in X_0, e_0 \in E_0, z \in Z, p \in P, t \in [t_0, \infty) \end{aligned} \right\}: \\ R \longrightarrow Y$$

is considered in $XE \subset \mathbb{R}^c \times \mathbb{R}^b$.

In the bounded controller (6), the nonlinear error map $G_e B_e^{\mathrm{T}} \Sigma_{i=0}^{\zeta} K_{ei} e^{(2i+1)/(2\beta+1)} + (1/s) G_i B_e^{\mathrm{T}} \Sigma_{i=0}^{\sigma} K_{eii} e^{(2i+1)/(2\mu+1)}$, as well as the nonlinear proportional state feedback $G_x B^{\mathrm{T}}(x) \Sigma_{i=0}^{\eta} K_{xi} x^{(2i+1)/(2\gamma+1)} + G_x B^{\mathrm{T}}(x) K x$, guarantee the robust tracking, ensure stability, and accomplish disturbance rejection if the criteria, imposed on the Lyapunov pair, are met in $XE \subset \mathbb{R}^c \times \mathbb{R}^b$. It should be emphasized that bounded controllers which guarantee the robust stability in the large of open-loop unstable systems do not exist [6–9]. Therefore, the admissibility framework is used. An invariant domain of stability $S_s \subset \mathbb{R}^c \times \mathbb{R}^b$ can be found as

$$S_s = \{x \in \mathbb{R}^c, e \in \mathbb{R}^b : \|x(t)\| \le \rho_x(t, \|x_0(t)\|) + \rho_u(\|u(t)\|),$$

$$\|e(t)\| \le \rho_e(t, \|e_0(t)\|) + \rho_r(\|r(t)\|) + \rho_d(\|d(t)\|) + \rho_y(\|y(t)\|),$$

$$\forall x \in X(X_0, U, R, D, Z, P), \forall e \in E(E_0, R, D, Y), \forall r \in R, \forall d \in D, \forall y \in Y,$$

$$\forall t \in [t_0, \infty)\} \subset \mathbb{R}^c \times \mathbb{R}^b.$$

Here $\rho_x(\cdot)$: $\mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ and $\rho_e(\cdot)$: $\mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ are the *KL-functions;* $\rho_u(\cdot)$: $\mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$, $\rho_r(\cdot)$: $\mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$, $\rho_d(\cdot)$: $\mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ and $\rho_y(\cdot)$: $\mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ are the *K*-functions.

For a particular class of reference inputs $r \in R$ and disturbances $d \in D$, the formulation above means that the tracking error decays asymptotically to zero. For example if $XE \subseteq S_s$, and $r(t) = \text{const}$, $d(t) = \text{const}$, one obtained $\lim_{t\to\infty} e(t) = 0$.

By using the Lyapunov stability theory, one finds an invariant domain of robust stability as

$$S_s = \Bigg\{ x \in \mathbb{R}^c, e \in \mathbb{R}^b : x_0 \in X_0, e_0 \in E_0, u \in U, r \in R, d \in D, z \in Z, p \in P, t \in [t_0, \infty)$$

$$\mid V(0,0) = 0, V(e,x) > 0, \frac{\mathrm{d}V(e,x)}{\mathrm{d}t} \le 0, \forall x \in X(X_0, U, R, D, Z, P),$$

$$\forall e \in E(E_0, R, D, Y), \forall t \in [t_0, \infty) \Bigg\} \subset \mathbb{R}^c \times \mathbb{R}^b.$$

It is required that $XE \subseteq S_s \subset \mathbb{R}^c \times \mathbb{R}^b$ for all $x_0 \in X_0$, $e_0 \in E_0$, $u \in U$, $r \in R$, $d \in D$, $z \in Z$ and $p \in P$ on $[t_0, \infty)$. Robust stability in $X(X_0, U, R, D, Z, P)$, tracking and disturbance rejection in $E(E_0, R, D, Y)$ can be studied. Under the assumption that X_0, E_0, U, R, D, Z and P are admissible, the robust tracking problem is solvable in XE, and robust convergence and boundedness of $x(\cdot)$ and $e(\cdot)$ can be proven by using results [6–9]. That is, the bounded control law (6) guarantees stability, renders robust tracking, ensures disturbance rejection, and steers the output vector to

$$S_e(\delta) = \{e \in \mathbb{R}^b : e_0 \in E_0, x \in X(X_0, U, R, D, Z, P), u \in U, r \in R, d \in D, t \in [t_0, \infty) \mid$$

$$\|e(t)\| \le \rho_e(t, \|e_0(t)\|) + \rho_r(\|r(t)\|) + \rho_d(\|d(t)\|) + \rho_y(\|y(t)\|) + \delta, \delta \ge 0,$$

$$\forall e \in E(E_0, R, D, Y), \forall r \in R, \forall d \in D, \forall y \in Y, \forall t \in [t_0, \infty)\} \subset \mathbb{R}^b.$$

If $XE \subseteq S_s$, one concludes that trajectories of the resulting closed-loop system do not exceed the admissible set $S_s \subset \mathbb{R}^c \times \mathbb{R}^b$, all solutions are bounded in $XE(X_0, E_0,$

U, R, D, Z, P), and convergence of the tracking error to a compact set $S_e(\delta)$ is guaranteed. Hence, $XE(X_0, E_0, U, R, D, Z, P)$ is a positive–invariant robust domain for the trajectories of mechatronic systems mapped by differential equations (3) with controller (6).

4. Analysis and design of a mechatronic system with experimental verification: a case study

We consider an electric drive actuated by a permanent-magnet motor JDH-2250-BX-1C. The parameters of this motor have been identified using the state-space time-domain nonlinear mapping-based identification method [10] using the measured transient dynamics at different operating conditions and scenarios. The armature resistance and inductance are found to be $r_a(\cdot)\in[2.7_{T=20^\circ\mathrm{C}}\ \ 3.7_{T=140^\circ\mathrm{C}}]$ ohm and $L_a=0.004$ H; back emf k_a and torque k_T constants are $k_a(\cdot)\in[0.11_{T=20^\circ\mathrm{C}}\ \ 0.094_{T=140^\circ\mathrm{C}}]$ V sec/rad and $k_T(\cdot)\in[0.11_{T=20^\circ\mathrm{C}}\ \ 0.094_{T=140^\circ\mathrm{C}}]$ N m/A; moment of inertia is $J=0.0001$ kg m^2; viscous friction coefficient is $B_m=0.00008$ N m sec/rad.

Permanent-magnet motor and step-down converter were shown in Fig. 2 (the switching frequency is 50 kHz). The converter has an internal resistance $r_d=0.05$ ohm, and a low-pass filter ($L_L=0.0007H$ and $C_L=0.003F$) is inserted to ensure the specified 5% voltage ripple.

From (2), we have the following nonlinear state-space model with bounded control

$$\begin{bmatrix} \dfrac{du_a}{dt} \\ \dfrac{di_L}{dt} \\ \dfrac{di_a}{dt} \\ \dfrac{d\omega_r}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{C_L} & -\dfrac{1}{C_L} & 0 \\ -\dfrac{1}{L_L} & 0 & 0 & 0 \\ \dfrac{1}{L_a} & 0 & -\dfrac{r_a}{L_a} & -\dfrac{k_a}{L_a} \\ 0 & 0 & \dfrac{k_T}{J} & -\dfrac{B_m}{J} \end{bmatrix} \begin{bmatrix} u_a \\ i_L \\ i_a \\ \omega_r \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ \left(\dfrac{V_d}{L_L u_{t\ \max}} - \dfrac{r_d}{L_L u_{t\ \max}} i_L\right) \\ 0 \\ 0 \end{bmatrix} u_c - \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{J} \end{bmatrix} T_L,$$

$$u_c \in [0 \quad 10] \text{ V}.$$

By applying the design method developed, a bounded control law should be synthesized. In particular, using (5) with $\zeta=\sigma=1$ and $\beta=\mu=\eta=\gamma=0$, we have

$$V(e,x) = \frac{1}{2}k_{e0}e^2 + \frac{1}{4}k_{e1}e^4 + \frac{1}{2}k_{ei0}e^2 + \frac{1}{4}k_{ei1}e^4 + \frac{1}{2}[u_a \quad i_L \quad i_a \quad \omega_r]\, K_{xo}\begin{bmatrix} u_a \\ i_L \\ i_a \\ \omega_r \end{bmatrix}$$

$$+ \frac{1}{2}[u_a \quad i_L \quad i_a \quad \omega_r] K \begin{bmatrix} u_a \\ i_L \\ i_a \\ \omega_r \end{bmatrix},$$

$$K_{x0} \in \mathbb{R}^{4\times 4} \quad \text{and} \quad K \in \mathbb{R}^{4\times 4}.$$

From (6), one obtains

$$u_c = \begin{cases} 10 & \text{for } u \geq 10, \\ u & \text{for } 0 < u < 10, \\ 0 & \text{for } u \leq 0, \end{cases}$$

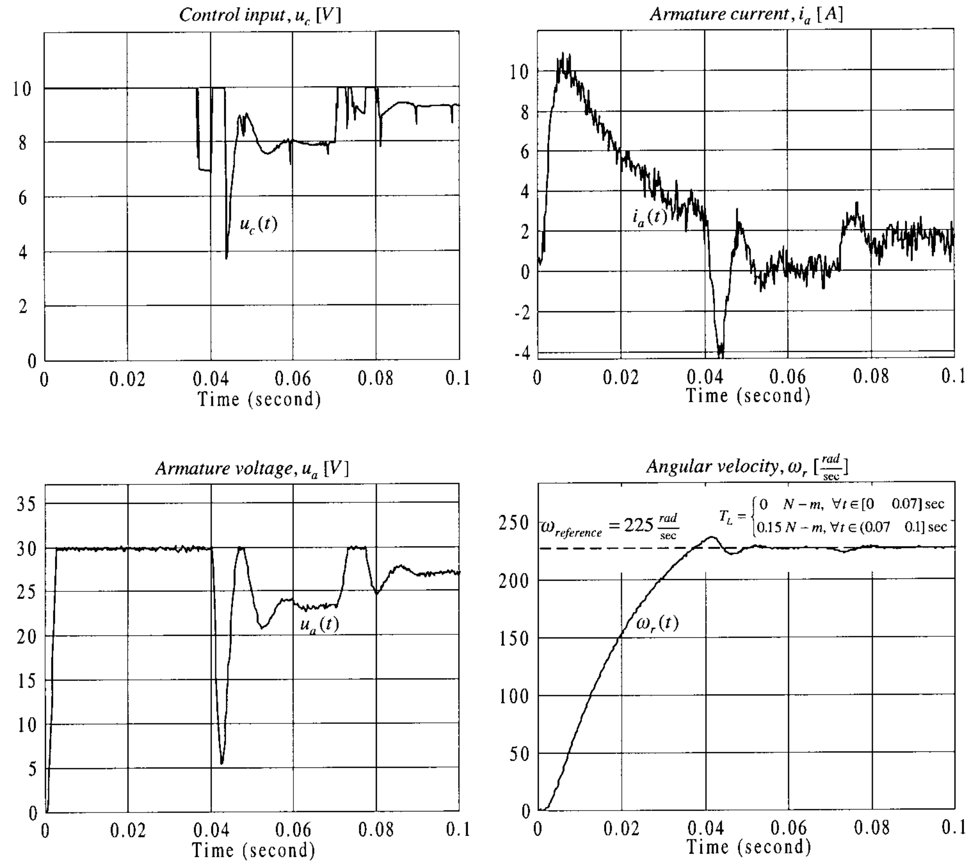

Fig. 3. Transient dynamics of the resulting closed-loop mechatronic system.

$$u = 15e + 0.11e^3 + 6.8\int e\,\mathrm{d}t + 0.49\int e^3\,\mathrm{d}t$$
$$- 0.95u_a - 0.38i_L - 0.62i_a - 0.0041\omega_r - (0.002u_a + 0.0006i_L \tag{7}$$
$$+ 0.001i_a + 0.00001\omega_r)i_L.$$

The feedback gains were found by solving inequality $[\mathrm{d}V(e,\ x)/\mathrm{d}t] \le \|e\|^2 + \|e\|^4 + \|x\|^2$. The criteria, imposed on the Lyapunov pair to guarantee robust stability are satisfied; in particular, $V(e,\ x)>0$ and $[\mathrm{d}V(e,\ x)/\mathrm{d}t]<0$. Hence, the robust bounded control law (7) guarantees stability, ensures tracking and disturbance rejection. Experimental validation of stability, tracking, and disturbance attenuation is needed. The efficacy of controller (7) is verified through comprehensive experiments. Different operating conditions and scenarios were studied to analyze the dynamic performance of a mechatronic system. The worst dynamics are observed in the case the motor temperature is high. For $T=140°\mathrm{C}$ (maxim operating temperature), the stator resistance reaches the maximum value, and the torque constant k_T is minimum. Therefore, the minimum electromagnetic torque, which is given as $T_e=k_Ti_a$, results. One should examine the tracking error $e(t)$, which represents the differences between the actual rotor angular velocity $\omega_r(t)$ and its desired (assigned) reference value $\omega_{\mathrm{reference}}(t)$, as well as disturbance attenuation features (it is required that the angular velocity remains equal to the reference value if the load torque T_L is applied). Fig. 3 depicts the control signal-level voltage $u_c(t)$ as well as the measured transient dynamics for states $u_a(t)$, $i_a(t)$, $\omega_r(t)$ when $\omega_{\mathrm{reference}}=225$ rad/sec and $T=140°\mathrm{C}$. A motor reaches the desired (reference) angular velocity within 0.05 s with overshoot 4.5%, and the steady-state error is zero. Observe that a motor starts from stall, and the plotted states $u_a(t)$, $i_a(t)$, $\omega_r(t)$ have been measured and prefiltered to digitally implement the controller with the sampling time 0.0001 s. The analysis of the experimental results indicates that the tracking error $e(t)=\omega_{\mathrm{reference}}(t)-\omega_r(t)$ converges to zero. Furthermore, the disturbance attenuation has been studied. In particular, the load torque 0.15 N m is applied at $t=0.07$ s. It is evident that the disturbance rejection is achieved, see Fig. 3. By analyzing the angular velocity $\omega_r(t)$, one concludes that the settling time is 0.01 s with 2.5% deflection from $\omega_{\mathrm{reference}}(t)$, and the steady-state error is zero. From the experimental data it follows that the desired performance has been achieved, and the angular velocity precisely follows the reference speed assigned.

Nonlinear controller (7) has been compared to the conventional control of permanent-magnet DC motors. In particular, a high frequency (36 kHZ with 2.5 kHz bandwidth) state-of-the-art PWM servo-amplifier 12A8 (Advance Motion Controls), which integrates motion control features, was used. The analog proportional–integral control algorithm was used to vary the amplitude of the applied armature voltage. This linear control algorithm was designed using the conventional motor model (1), and the linear quadratic regulator concept was used to design a control law with linear state and error mappings [11]. The experimental results are illustrated in Fig. 4 (motor starts from stall, and the load torque is applied at $t = 0.08$ s). Fig. 4 documents the measured dynamics for $\omega_r(t)$. It is easy to see that the settling time is 0.075 s with the overshoot 5.2%. Furthermore, as the load torque 0.15 N m is applied at $t = 0.08$ s, the steady-state error 1.9% results, and the settling time is 0.01 s with 3.4% of maximum deflection. The analysis of two experimentally tested control algorithms shows that the acceleration rate remains the same, and the motor reaches the maximum angular velocity at 0.041 and 0.038 s when the robust controller (7) and the linear control law are used, respectively. This can be easily justified because the same rated voltage 30 V is applied to the armature winding as the motor starts. However, the documented experimental results illustrate that nonlinear controller (7) significantly improves the closed-loop dynamic and steady-state performance due to the use of nonlinear state and error feedback. Hence, compared with conventional controllers, one concludes that dynamic characteristics and stability have been improved, and robust tracking, accuracy, and disturbance attenuation are guaranteed due to the use of nonlinear feedback. The reported analytical and experimental results illustrate that the robust controllers guarantee superior dynamic responses, precise tracking and disturbance attenuation, optimum energy management, as well as enhanced robustness and stability.

5. Conclusions

In this paper, we have addressed the problem of nonlinear analysis and design of mechatronic systems by augmenting permanent-magnet DC motors with power

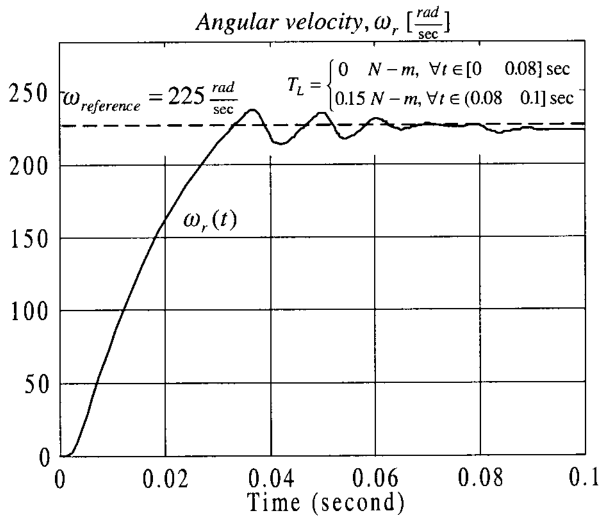

Fig. 4. Transient dynamics of the permanent-magnet DC motor with the 12A8 servo-amplifier.

converters. The application of the Lyapunov theory and the admissibility concept has ensured the solution of the motion control problem. In particular, new features and further enhancements are reported to handle robust stabilization, tracking, and disturbance rejection. A dual formulation of the nonlinear analysis–design problem has been researched by using a new family of nonquadratic positive–definite Lyapunov functions, and innovative bounded controllers were designed. The developed computationally efficient design method is used in conjunction with nonlinear dynamics of motors and converters. This paper researches a new setup of the Lyapunov concept to synthesize constrained controllers for nonlinear mechatronic systems with hard control bounds and parameter variations. The presented analytical and experimental studies for a mechatronic system (controller—switching converter—permanent-magnet DC motor) demonstrate that the control law designed provides superior performance and guarantees requirements imposed on industrial drives (stability and robustness, precise tracking and accuracy, fast dynamics and disturbance rejection, etc.).

References

[1] Krause PC, Wasynczuk O. Electromechanical Motion Devices. New York: McGraw-Hill, 1989.
[2] Mohan N, Undeland TM, Robbins WP. Power Electronics. New York: John Wiley, 1995.
[3] Bose BK. Expert system, fuzzy logic, and neural network in power electronics and drives. In: Power Electronics and Variable Frequency Drives. NJ: IEEE Press, 1997. p. 559–630.
[4] Lyshevski S. Motion control of electromechanical servo-devices with permanent-magnet stepper motors. Int Journal Mechatronics 1997;7(6):521–36.
[5] Lyshevski S, Kravchenko N. Optimization of Dynamics of Precision Drives of Servomechanism Systems. Kiev: The Academy of Sciences of the Ukraine Press, 1991.
[6] Corless M, Leitmann G. Bounded control for robust exponential convergence. Journal Optimization Theory Applications 1993;76(1):1–2.
[7] Khalil HK. Nonlinear Systems. Upper Saddle River, NJ: Prentice-Hall, 1996.
[8] Lyshevski S. Robust nonlinear control of uncertain systems with state and control constraints. Proc Conf Decision and Control 1995;2:1670–5 New Orleans, LA.
[9] Lyshevski S. Constrained optimization and control of nonlinear systems: new results in optimal control. Proc Conf Decision and Control 1996;1:541–6 Kobe, Japan.
[10] Lyshevski S. State-space model identification of deterministic nonlinear systems: nonlinear mapping technology and application of the Lyapunov theory. Automatica 1998;34(5):659–64.
[11] Gezgin C, Heck BS, Bass RM. Control structure optimization of a boost converter: an LQR approach. Proc Power Electronics Specialist Conference 1997;2:901–7 St. Louis.