Leader–Follower Formation and Tracking Control of Mobile Robots Along Straight Paths

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***Abstract*— We address the problem of tracking control of multiple mobile robots advancing in formation along straight-line paths. We use a leader–follower approach, and hence, we assume that only one swarm leader robot has the information of the reference trajectory. Then, each robot receives information from one intermediary leader only. Therefore, the communications graph forms a simple spanning directed tree. As the existence of a spanning tree is necessary to achieve consensus, it is the minimal configuration possible to achieve the formation-tracking objective. From a technological viewpoint, this has a direct impact on the simplicity of its implementation; e.g., less sensors are needed. Our controllers are partially linear time-varying with a simple added nonlinearity satisfying a property of persistency of excitation, tailored for nonlinear systems. Structurally speaking, the controllers are designed with the aim of separating the tasks of position-tracking and orientation. Our main results ensure the uniform global asymptotic stabilization of the closed-loop system, and hence, they imply robustness with respect to perturbations. All these aspects make our approach highly attractive in diverse application domains of vehicles' formations such as factory settings.**

***Index Terms*— Adaptive control, consensus, nonholonomic, stability analysis, synchronisation, time-varying systems.**

I. Introduction

THERE are many situations in which coordinated control of swarms of mobile robots is significant, e.g., in missions which cannot be accomplished by a single agent such as surveillance, recognition, mapping, and rescue operations. Besides, the use of a large group of robots offers increased robustness and flexibility.

In controlling a large group of robots, a decentralized approach becomes rapidly indispensable [1]. One of the most popular control approaches is the leader–follower technique which consists in specifying one or several leader robots and several followers. For instance, there may be one single leader which specifies the trajectory for the formation and the rest are set to follow the leader, modulo a position and orientation offset determined by the physical configuration. Then, following the seminal work [2] on tracking control of mobile robots, one can use a variety of nonlinear control techniques to ensure *individual tracking* control on each follower. Alternatively, one may form a cascade of leader–follower configurations in which each robot follows one leader [3]–[5]. Backstepping control is used in [6] and the problem under additive disturbances is solved via sliding mode in [3]. Another approach is that of virtual structure control, in which the swarm is regarded as a virtual rigid structure advancing as a unit. This approach is tractable for small groups of autonomous robots [7], [8].

In [9] a very simple cascades-based controller was introduced to solve the leader–follower control problem for two robots. The approach was used subsequently, for instance in [10]–[12]. The controller is very simple to implement, it relies on a separation principle by which it is demonstrated that the translational and orientational kinematics may be stabilized independently of each other. The disadvantage of this method is that the controller relies on the assumption that the angular velocity of the leader robot must be different from zero. This rules out straight-line paths. Only very few studies address the problem of formation control along straight-line paths, for example, [12] and [13], where complex nonlinear time-varying controls are designed to allow for reference velocity trajectories that converge to zero. It is worth emphasizing that [13] covers the case when also the forward velocity v_0 may converge to zero, that is, tracking control toward a fixed point. The controller from [12] makes the robot go back and forth on the path.

This brief is the outgrowth of [14].[1] We solve the formation control problem on straight-line paths with time-varying nonlinear controllers which rely on a property of persistency of excitation for nonlinear systems. The stability proofs are constructed using small-gain-type arguments and rely on modern results on nonlinear adaptive control systems.

The rest of this brief is organized as follows. In the following section, we present our main results. For clarity of exposition, we first present a result on leader–follower tracking control (two robots only) and describe the control approach. Then, we present a result for a cascade-like configuration of leader–follower mobile robots. In the communications graph, each robot becomes leader to one robot and follower of another. There is a unique swarm leader robot which receives the information of the reference trajectory and there is a unique tail robot which is leader to none. Simulation results which illustrate our theoretical findings are presented in Section III and we conclude with some remarks in Section IV.

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[1] This conference version does not include any technical proof and the simulation results have been refined.

II. Main Results

A. Leader–Follower Tracking Control

After the seminal paper [2], the tracking control problem for mobile robots may be reformulated as that of controlling a robot in a leader–follower configuration. Hence, for a mobile robot with kinematic model

$$\Sigma_1 : \begin{cases} \dot{x}_1 = v_1 \cos(\theta_1) \\ \dot{y}_1 = v_1 \sin(\theta_1) \\ \dot{\theta}_1 = w_1 \end{cases}$$

with forward velocity v_1 and angular velocity w_1 as control inputs, the tracking control problem consists in following a fictitious vehicle Σ_0 with forward and angular velocity references v_0 and w_0, respectively and coordinates (x_0, y_0, θ_0). From a control viewpoint, the goal is to steer the following quantities to zero:

$$p_{1x} = x_0 - x_1 - d_{x0,1}$$
$$p_{1y} = y_0 - y_1 - d_{y0,1}$$
$$p_{1\theta} = \theta_0 - \theta_1$$

where d_x and d_y are (piecewise-)constant design parameters imposed by the topology and path planner. For the purposes of analysis, we transform the error coordinates $[p_{1x}, p_{1y}, p_{1\theta}]$ of the leader robot from the global coordinate frame to local coordinates fixed on the robot, that is

$$\begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} \\ p_{1y} \\ p_{1\theta} \end{bmatrix}. \quad (1)$$

In the new coordinates, the error dynamics between the virtual reference vehicle and the follower becomes

$$\dot{e}_{1x} = w_1 e_{1y} - v_1 + v_0 \cos e_{1\theta} \quad (2a)$$
$$\dot{e}_{1y} = -w_1 e_{1x} + v_0 \sin e_{1\theta} \quad (2b)$$
$$\dot{e}_{1\theta} = w_0 - w_1. \quad (2c)$$

The tracking control problem is transformed into that of stabilizing the origin for the error dynamics (2). It is commonly assumed that the reference angular velocity w_0 is different from zero. Indeed, otherwise the system may lose controllability in the y coordinate [see (2b)]. For instance, the results in [9], and consequently those of [10] which rely in the former, are based on the assumption that the angular reference velocity satisfies a persistency of excitation condition, that is, $w_0(s) := \psi(s)^2$, where

$$\int_t^{t+T} \psi(s)^2 ds \geq \mu \quad \forall\, t \geq 0 \quad (3)$$

for some positive constants μ and T. In [12] and [13] complex nonlinear time-varying controls are designed to allow for reference velocity trajectories that converge to zero. Furthermore, Lee *et al.* [13] cover the case when also the forward velocity v_0 may converge to zero, that is, tracking control toward a fixed point. In [12] the controller is designed so as to make the robot *converge* to the straight-line trajectory resulting in a path that makes it go back and forth.

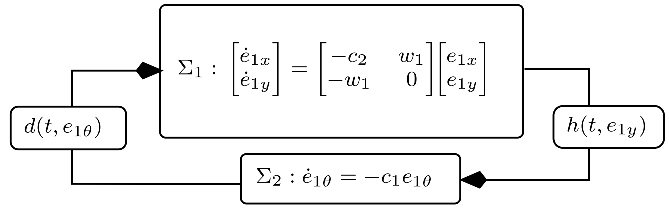

Fig. 1. Small-gain feedback representation of the closed-loop system with a persistently exciting controller.

Our control approach is inspired by the cascade-based controllers originally presented in [9], in which persistency of excitation is used to guarantee exponential stabilization of the origin for the error dynamics. We extend this method to the case in which the reference angular velocity fails to satisfy the persistency of excitation condition. As a matter of fact, we allow for the case in which $w_0 \equiv 0$. Although structurally similar, the control laws are given by

$$v_1 = v_0(t) + c_2 e_{1x}, \quad c_2 > 0 \quad (4a)$$
$$w_1 = h(t, e_{1y}) + c_1 e_{1\theta}, \quad c_1 > 0 \quad (4b)$$

where h is bounded, locally of linear order in e_{1y}, and continuously differentiable. It is the term h above which replaces the zero angular velocity in the controller introduced in [9] which relies on the assumption that w_0 is persistently exciting. In the present context, we impose as condition that $h(t, 0) \equiv 0$ and $\dot{h}$ is persistently exciting for any $e_{1y} \neq 0$; a precise definition is given farther below.

We show that the controller (4) stabilizes the error dynamics globally and uniformly. In order to understand the stabilization mechanism of the controller (4) it is convenient to examine the closed-loop equations, which result from using (4) in (2)

$$\dot{e}_{1x} = w_1 e_{1y} - c_2 e_{1x} + v_0[\cos e_{1\theta} - 1] \quad (5a)$$
$$\dot{e}_{1y} = -w_1 e_{1x} + v_0 \sin e_{1\theta} \quad (5b)$$
$$\dot{e}_{1\theta} = -c_1 e_{1\theta} - h(t, e_{1y}). \quad (5c)$$

This system may be rewritten in compact form as

$$\begin{bmatrix} \dot{e}_{1x} \\ \dot{e}_{1y} \end{bmatrix} = \begin{bmatrix} -c_2 & w_1 \\ -w_1 & 0 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix} + d(t, e_{1\theta}) \quad (6a)$$
$$\dot{e}_{1\theta} = -c_1 e_{1\theta} - h(t, e_{1y}) \quad (6b)$$

where we purposefully dropped the arguments of w_1 and defined the interconnection term

$$d(t, e_{1\theta}) := \begin{bmatrix} v_0(t)(\cos e_{1\theta} - 1) \\ v_0(t) \sin e_{1\theta} \end{bmatrix}. \quad (7)$$

We are interested in establishing uniform global asymptotic stability of the origin of $(e_{1x}, e_{1y}, e_{1\theta}) = (0, 0, 0)$. To that end, we observe that system (6) consists in the feedback interconnection of two systems as shown in Fig. 1. Roughly speaking, after adaptive control systems theory, the system Σ_1, in the center upper block is uniformly asymptotically stable at the origin, provided that $c_2 > 0$ and w_1 is persistently exciting, globally Lipschitz, and bounded. On the other hand, the origin of the system Σ_2, in the lower-center block, is exponentially stable if $c_1 > 0$. As a matter of fact, it may also be established that each of these subsystems is input to state stable (ISS). Moreover, the interconnection terms h and d are both uniformly bounded and satisfy $d(t,0) \equiv 0$, $h(t,0) \equiv 0$. Thus, the interconnected system (6) may be regarded as the feedback interconnection of two ISS systems. Consequently, stability of the origin of (6) may be concluded invoking the small-gain theorem for ISS systems [15].

Although intuitive, the previous arguments hide certain difficulties in the analysis that we intend to clarify next. First, the function w_1 depends on the states and time, and hence, persistency of excitation must be appropriately defined. We use a relaxed notion of persistency of excitation, originally introduced in [16]; the following is a refined definition reported in [17].

Definition 1 (uδ-Persistency of Excitation): Let $f(\cdot,\cdot)$ be such that the system $\dot{x} = f(t,x)$, with state $x = [x_1^\top \; x_2^\top]^\top$ and solution $x(t) = x(t, t_\circ, x_\circ)$ starting at $(t_\circ, x_\circ) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^n$ is forward complete. Let $\phi : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n+m} \to \mathbb{R}^{p\times q}$ be such that $\phi(\cdot, x(\cdot, t_\circ, x_\circ))$ is locally integrable for each solution $x(\cdot, t_\circ, x_\circ)$, e.g., $(t,x) \mapsto \phi(t,x)$ is measurable, locally bounded, and locally Lipschitz in x.

The pair (ϕ, f) is called uniformly δ-persistently exciting (uδ-PE) with respect to x_1 if, for each r and $\delta > 0$, there exist constants $T(r,\delta)$ and $\mu(r,\delta) > 0$ such that, for all $(t_\circ, x_\circ) \in \mathbb{R}_{\geq 0} \times B_r$, all corresponding solutions satisfy, for all $t \geq t_\circ$

$$\min_{s\in[t,\,t+T]} |x_1(s)| \geq \delta \quad \Rightarrow \int_t^{t+T} \phi(\tau, x(\tau))\phi(\tau, x(\tau))^\top d\tau \;\geq \mu I. \tag{8}$$

In words, the pair (ϕ, f) is uδ-PE if the function $\phi(\cdot, x(\cdot))$ is PE in the usual sense of adaptive control, uniformly in initial conditions $(t_\circ, x_\circ) \in \mathbb{R}_{\geq 0} \times B_r$, whenever the trajectory $x(\cdot)$ is away from a δ-neighborhood of the origin. For simplicity, we may also say, with an abuse of terminology, that the *function* ϕ is uδ-PE in the understanding that the *pair* satisfies Definition 1. For instance, the function $\phi(t,x) := \psi(t)|x|$ is uδ-PE if ψ satisfies (3).

There are several properties of uδ-PE functions which are useful in control design for nonholonomic systems; these are reported in [18]. One of them is that if w_1 is uδ-PE then there exists a function $\tilde{w}_1$ which depends only on time and which is persistently exciting in the sense of (3). Moreover, for w_1 in (4b), $\tilde{w}_1$ may be purposefully constructed to satisfy

$$\tilde{w}_1(t) := h(t, e_{1y}(t)) + c_1 e_{1\theta}(t) \quad \forall\, t : |e_{1y}(t)| \geq \delta. \tag{9}$$

Even though the function $\tilde{w}_1$ is parameterized by δ, it is guaranteed that for any $\delta > 0$ there exists $\tilde{w}_1$ satisfying all of the above.

This property is useful because, for any δ and for all t such that $|e_{1y}(t)| \geq \delta$, the trajectories of Σ_1 in Fig. 1 coincide with those of

$$\dot{z}_1 = \tilde{A}(t) z_1,\;\; \tilde{A}(t) := \begin{bmatrix} -c_2 & \tilde{w}_1(t) \\ -\tilde{w}_1(t) & 0 \end{bmatrix},\;\; z_1 := \begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix} \tag{10}$$

which is linear. Therefore, the behavior of the trajectories of (6a) with $d \equiv 0$ may be analyzed as for the linear system (10), at least while the trajectories are away from the origin (strictly speaking away from *any* δ-neighborhood). On the other hand, global exponential stability of the origin of (10) is easily concluded invoking classical results on adaptive control systems [19]. Consequently, one may use the following intuitive contradiction argument to establish uniform global asymptotic stability of (6a) with $d \equiv 0$: assume that the origin is not attractive; then, the trajectories (tend to) remain away from an arbitrary δ-neighborhood of the origin. In that case, since they coincide with those generated by (10) which is exponentially stable, it follows that the trajectories of (6a) must converge to zero. The argument may be repeated for any arbitrarily small δ, and hence, the exponential rate of convergence diminishes but remains uniform in the initial conditions. Precise general statements for nonlinear time-varying systems are reported in [17]. For the purposes of system (6), we proceed by showing the following.

1) The origin is uniformly stable.
2) The solutions are uniformly globally bounded.
3) The origin is uniformly globally attractive.

The first bulleted item comes from the fact that the system corresponds to the feedback interconnection of two *locally* ISS systems. For the first block, Σ_1, the origin is uniformly globally asymptotically stable provided that w_1 is uniformly δ-PE with respect to e_{1y}, bounded, and with bounded derivatives [17]. On the other hand, local input to state stability (also known as total stability) with respect to the additive input d is a direct consequence of uniform global asymptotic stability [20]. For Σ_2, it is evident that the origin is globally exponentially stable and that Σ_2 is ISS with respect to h. Actually, the interconnected system shown in Fig. 1 is (locally) uniformly asymptotically stable.

The boundedness property follows from the fact that the trajectories of (10), for all t such that $|e_{1y}(t)| \geq \delta$, coincide with those of Σ_1 in Fig. 1, which are globally uniformly bounded. To see the latter, we remark that since $\tilde{w}_1$ is persistently exciting the origin of (10) is globally exponentially stable. This implies that, for any δ, there exist positive definite symmetric matrices P_δ and Q_δ such that $-Q_\delta(t) = \tilde{A}_{1_\delta}(t)^\top P_\delta(t) + P_\delta(t)\tilde{A}_{1\delta}(t) + \dot{P}_\delta(t)$ and the total derivative of

$$V_{1\delta}(t, z_1) = z_1^\top P_\delta(t) z_1$$

along the trajectories of (6a) satisfies

$$\dot{V}_{1\delta}(t, z_1) \leq -z_1^\top Q_\delta(t) z_1 + 2 z_1^\top P_\delta(t) d(t, e_{1\theta})$$

for all t such that $|e_{1y}(t)| \geq \delta$. In turn, we have

$$\dot{V}_{1\delta}(t, z_1) \leq -\frac{q_m}{2}|z_1|^2 + \frac{p_M^2}{2q_m}|d(t, e_{1\theta})|^2$$

where we used $p_M I \geq P_\delta(t)$ and $Q_\delta(t) \geq q_m I$. Since $d(t, e_{1\theta}(t))$ is bounded [see (7)], it is clear that if $|z_1(t)| \to \infty$, then $\dot{V}_{1\delta}(t, z_1(t)) \leq 0$ for sufficiently large t. This implies boundedness.

We argue in a similar way for the trajectories of (6b); the total derivative of $V_{2\delta}(e_{1\theta}) := 0.5|e_{1\theta}|^2$ yields

$$\dot{V}_{2\delta}(e_{1\theta}) \leq -\frac{\lambda c_1}{2}|e_{1\theta}|^2 + \frac{|h(t, e_{1y})|^2}{2c_1\lambda}$$

for any $\lambda > 0$. Recall that, by assumption, h is bounded.

Next, we show that the origin of (6) is uniformly globally attractive; that is, we must show that for any r and $\sigma > 0$, there exists T such that

$$|e_1(t_\circ)| \leq r \Longrightarrow |e_1(t)| \leq \sigma \quad \forall\, t \geq t_\circ + T. \tag{11}$$

So let r and σ be arbitrary given positive constants and define $\delta := \sigma$. To establish the convergence property (11), we study the behavior of the solutions of

$$\begin{bmatrix} \dot{e}_{1x} \\ \dot{e}_{1y} \end{bmatrix} = \begin{bmatrix} -c_2 & \tilde{w}_1(t) \\ -\tilde{w}_1(t) & 0 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix} + d(t, e_{1\theta}) \tag{12a}$$

$$\dot{e}_{1\theta} = -c_1 e_{1\theta} - h(t, e_{1y}) \tag{12b}$$

whose trajectories, as we have emphasized, coincide with those of (6) for all t such that $|e_{1y}(t)| \geq \delta$. Therefore, it suffices to establish global exponential stability of the origin of (12). To that end, let

$$\lambda := \sqrt{\frac{5v_0^M}{2} \frac{p_M}{q_m c_1}} \quad \varepsilon := \frac{\lambda c_1}{4} \quad \eta := \frac{2q_m}{p_M^2}\varepsilon \tag{13}$$

and consider the Lyapunov function $V_\delta := \eta V_{1\delta} + V_{2\delta}$. Its total derivative satisfies

$$\dot{V}_\delta(t, z_1, e_{1\theta}) \leq -\left(\frac{q_m^2}{p_M^2}\varepsilon - \frac{v_0^M}{2c_1\lambda}\right)|z_1|^2 - \left(\frac{c_1\lambda}{2} - \varepsilon\right)|e_{1\theta}|^2$$

where we introduced the bound $v_0^M \geq |v_0(t)|$ and we used the assumption that $|h(t, e_{1y})| \leq v_0^M |z_1|$ and $|d(t, e_{1\theta})| \leq |e_{1\theta}|$. In view of the expressions in (13), $\dot{V}_\delta$ is negative definite, actually

$$\dot{V}_\delta(t, z_1, e_{1\theta}) \leq -\alpha|z_1|^2 - \varepsilon|e_{1\theta}|^2, \quad \alpha > 0.$$

We conclude that the trajectories of (6), which coincide with those of (12) for all t such that $|e_{1y}(t)| \geq \delta$, tend to zero exponentially fast as long as the latter inequality holds. In view of this there exists a finite time T such that for any $\delta' \in (0, \delta]$, we have $|e_1(t_\circ + T)| \leq \delta'$. From uniform stability, we have $|e_1(t)| \leq \delta$ for all $t \geq t_\circ + T$. Since $\delta = \sigma$ is arbitrarily given, the statement follows.

Remark 1: Note that even though this reasoning is reminiscent of ultimate boundedness we conclude convergence to zero. This is due to the fact that the previous arguments hold for *fixed* values of the control gains and any given $\delta > 0$.

Lemma 1: The origin of system (6) is uniformly globally asymptotically stable if $c_1 > 0$, $c_2 > 0$, v_0 is bounded, and w_1 is uδ-PE, bounded, and locally Lipschitz in e_{1y} uniformly in t. Moreover, uδ-PE of w_1 is also a necessary condition.

The previous lemma establishes a strong, yet intermediary, convergence result in the pursuit of our main objective: tracking control of nonholonomic robots. It is left to state under which conditions w_1 is uδ-PE. As a matter of fact, this has been established in the context of set-point stabilization, in [18]. The control input w_1 satisfies the differential equation

$$\dot{w}_1 = -c_1 w_1 + \dot{h}(t, e_{1y})$$

which corresponds to the equation of a low-pass filter. That is, a stable strictly proper linear system with input $\dot{h}$. It is well known from adaptive control textbooks that the output of a low-pass filter driven by an input that is persistently exciting is also persistently exciting [19], [21]. Now, for nonlinear functions we have an analogous property [18]. Therefore, w_1 which corresponds to a filtered version of $\dot{h}$ is uδ-PE if so is $\dot{h}$.

Proposition 1: Consider system (2) in closed-loop with controller (4). Let h be bounded, once continuously differentiable and such that $h(t, e_{1y})$ has a unique zero at $e_{1y} = 0$ for each fixed t. Assume further that there exists $c > 0$ such that

$$\sup_{t, e_{1y}} \left\{ |h(t, e_{1y})|, \ \left|\frac{\partial h(t, e_{1y})}{\partial e_{1y}}\right|, \left|\frac{\partial h(t, e_{1y})}{\partial t}\right| \right\} \leq c \tag{14}$$

and, for any $\delta > 0$, there exist positive numbers μ and T such that

$$|e_{1y}| \geq \delta \implies \int_t^{t+T} |\dot{h}(\tau, e_{1y})| d\tau \geq \mu \quad \forall t \geq 0. \tag{15}$$

Then, the origin of the closed-loop system is uniformly globally asymptotically stable.

Remark 2: The function h may be defined as a monotonic locally linear function of e_{1y} and smooth, persistently exciting in t; for instance, $h(t, e_{1y}) = \psi(t)\mathrm{sat}(e_{1y})$, where $\mathrm{sat}(\cdot)$ is a saturation function and ψ is persistently exciting.

Proof of Proposition 1: The closed-loop system is given by (6) and it may be shown, using V_1 and V_2 above, that the system is forward complete. Now, since $\dot{h}$ is a scalar function, (15) implies that

$$\min_{\tau \in [t, t+T]} |e_{1y}(\tau)| \geq \delta \Rightarrow \int_t^{t+T} |\dot{h}(\tau, e_{1y}(\tau))| d\tau \geq \mu$$

holds for all $t \geq 0$ and any trajectory. Therefore, $\dot{h}$ satisfies the properties in Definition 1 and, in view of the filtering property previously mentioned, it follows that w_1 is uδ-PE. The result follows from Lemma 1. ■

B. Leader–Follower Formation Control

Now, we extend the previous result to the case of formation-tracking control. Consider a group of n mobile robots with kinematic models

$$\dot{x}_i = v_i \cos(\theta_i) \tag{16a}$$

$$\dot{y}_i = v_i \sin(\theta_i) \tag{16b}$$

$$\dot{\theta}_i = w_i, \quad i \in [1, n] \tag{16c}$$

where for the ith robot, x_i and y_i determine the position with respect to a globally fixed frame, θ_i defines the heading angle, and the linear and angular velocities are denoted by v_i and w_i, respectively.

The control objective is to make the n robots take specific postures determined by the topology designer and to make the swarm follow a path determined by a virtual reference vehicle labeled R_0. Any physically feasible geometrical configuration may be achieved and one can choose any point in the Cartesian plane to follow the virtual reference vehicle.

We solve the problem using a spanning-tree communication topology and a recursive implementation of the tracking leader–follower controller (4). That is, the swarm has only one leader robot tagged R_1 whose local controller uses knowledge of the reference trajectory generated by the virtual leader R_0. Therefore, in the communications graph, R_1 is the child of the root-node robot R_0 and the other robots are intermediate nodes labeled R_2 to R_{n-1}, that is, R_i acts as leader for R_{i+1} and follows R_{i-1}. The last robot in the communication topology is denoted by R_n and has no followers, that is, it constitutes the tail node of the spanning tree. We remark that the notation R_{i-1} refers to the graph *communication* topology and not to the *formation* topology.

The fictitious vehicle, which serves as reference to the swarm, describes a freely generated reference trajectory; in particular, it produces the desired linear and angular velocities v_0 and w_0 which are communicated to the leader robot R_1 only. According to this communication topology, and following the setting for tracking control, the formation control problem reduces to that of stabilization of the error dynamics between any pair of leader–follower robots, i.e., for all $i \le N$:

$$\dot{e}_{ix} = w_i e_{iy} - v_i + v_{i-1}\cos e_{i\theta} \tag{17a}$$

$$\dot{e}_{iy} = -w_i e_{ix} + v_{i-1}\sin e_{i\theta} \tag{17b}$$

$$\dot{e}_{i\theta} = w_{i-1} - w_i \tag{17c}$$

and for each $i \ge 1$, we define the control inputs v_i and w_i as

$$v_i = v_{i-1} + c_{2i} e_{ix} \tag{18a}$$

$$w_i = w_{i-1} + c_{1i} e_{i\theta} + h_i(t, e_{iy}) \tag{18b}$$

where h_i is once continuously differentiable, bounded, and with bounded derivatives. Then, the closed-loop equations yield

$$\begin{bmatrix}\dot{e}_{ix}\\ \dot{e}_{iy}\end{bmatrix} = \begin{bmatrix}-c_{2i} & w_i\\ -w_i & 0\end{bmatrix}\begin{bmatrix}e_{ix}\\ e_{iy}\end{bmatrix} + \begin{bmatrix}v_{i-1}[1-\cos e_{i\theta}]\\ v_{i-1}\sin e_{i\theta}\end{bmatrix} \tag{19a}$$

$$\dot{e}_{i\theta} = -c_{1i} e_{i\theta} + h_i(t, e_{iy}) \tag{19b}$$

which has the form of (6) and inherits similar properties; actually, similar to Lemma 1, we have the following.

Lemma 2: The origin of system (19) is uniformly globally asymptotically stable, for any $i \le N$, if $c_{1i} > 0$, $c_{2i} > 0$, v_0 is bounded, and w_i is uδ-PE, bounded, and locally Lipschitz in e_{iy} uniformly in t. Moreover, uδ-PE of w_i is also a necessary condition.

The proof of this statement follows *mutatis mutandis* along the proof-lines of Lemma 1 observing that: 1) the function h_i is, by assumption, continuous and bounded; 2) for (19a) with $e_{i\theta} = 0$, the origin is uniformly globally asymptotically stable provided that w_i is uδ-PE; and 3) the interconnection term

$$d_i := \begin{bmatrix}v_{i-1}[1-\cos e_{i\theta}]\\ v_{1-1}\sin e_{i\theta}\end{bmatrix}$$

is also bounded, along trajectories. To see the latter, consider first $i = 2$ and then

$$d_2 := \begin{bmatrix}v_1[1-\cos e_{2\theta}]\\ v_1\sin e_{2\theta}\end{bmatrix}$$

where $v_1 = v_0(t) + c_{21}e_{1x}$ is a function of t and e_{1x}. Hence, the function $\tilde{d}_2$ defined along trajectories as

$$\tilde{d}_2(t, e_{i\theta}) = \begin{bmatrix}v_1(t, e_{1x}(t))[1-\cos e_{2\theta}]\\ v_1(t, e_{1x}(t))\sin e_{2\theta}\end{bmatrix}$$

is also continuous and bounded if so is $v_1(t, e_{1x}(t))$. On the other hand, $e_{1x}(t)$ is part of the solution of (6) whose origin, after Lemma 1, is uniformly globally asymptotically stable. Therefore, $e_{1x}(t)$ is uniformly globally bounded and so is $v_1(t, e_{1x}(t))$. The statement of Lemma 2 for the case $i = 2$ follows, and hence, $v_2(t, \bar{e}_{2x}(t))$, where $\bar{e}_{2x} := [e_{1x}\ e_{2x}]^\top$, is uniformly bounded for any t. Using this and proceeding by induction, we conclude that the result of the lemma holds for any $i \ge 2$. We are ready to present our second main result.

Proposition 2: Consider system (17) in closed loop with controllers (6) and (18). Assume that, for each $i \le N$, $h_i(t, e_{iy})$ has an isolated zero at $e_{iy} = 0$

$$\sup_{t, e_{iy}} \left\{ |h_i(t, e_{iy})|,\ \left|\frac{\partial h_i(t, e_{iy})}{\partial e_{iy}}\right|, \left|\frac{\partial h_i(t, e_{iy})}{\partial t}\right| \right\} \le c \tag{20}$$

$\sum_{j=1}^{i} \dot{h}_j$ is uδ-persistently exciting and the control gains c_{1i}, c_{2i} are positive. Then, the origin of the closed-loop system is uniformly globally asymptotically stable.

Remark 3: The condition of uδ-persistency of excitation holds if we introduce N different harmonics

$$h_j(t, e_{e_y}) = \psi_j(\varpi_j t)\alpha(e_{iy})$$

where, for simplicity only, ψ_j is a periodic function of period $2\pi\varpi_j$.

Proof of Proposition 2: We must establish that under the conditions of the proposition, the control input w_i defined in (18b) is uδ-PE with respect to e_{iy}. We proceed by induction. Let $\bar{e}_{iy} := [e_{1y} \cdots e_{iy}]^\top$; now, for $i = 2$

$$w_2 = w_1 + c_{12}e_{2\theta} + h_2(t, e_{2y})$$

satisfies

$$\begin{aligned}\dot{w}_2 &= -c_{12}w_2 - [c_{11} - c_{12}]w_1 + \dot{h}_1(t, e_{1y}) + \dot{h}_2(t, e_{2y})\\ &=: -c_{12}w_2 + \Phi_2(t, \bar{e}_{2y}).\end{aligned}$$

Under the conditions of Proposition 2 and since w_1 is uδ-PE with respect to e_{1y}, function Φ_2 is uδ-PE with respect to $\bar{e}_{2y}$. Then, in view of the fact that filtered uδ-PE functions are uδ-PE [18], so is w_2. It follows that:

$$\Phi_i(t, \bar{e}_{iy}) = \sum_{j=1}^{i-1}[c_{1j+1} - c_{1j}]w_j + \dot{h}_j(t, e_{jy}) + \dot{h}_i(t, e_{iy})$$

with $i = 3$ is uδ-PE with respect to $\bar{e}_{3y}$ and, consequently, by the filtering property of uδ-PE functions, so is w_3. By induction, it follows that $\Phi_i(t, \bar{e}_{iy})$ is uδ-PE with respect to $\bar{e}_{iy}$ and so is w_i, which satisfies:

$$\dot{w}_i = -c_{1i}w_i + \Phi_i(t, \bar{e}_{iy})$$

for any $i \ge 2$. ■

III. SIMULATION RESULTS

We illustrate our theoretical findings via some simulation results obtained using SIMULINK of MATLAB. We consider a group of five mobile robots. In the first stage of the simulation, the desired formation shape of the mobile robots is linear and they follow a straight line trajectory with initial conditions: $[x_1(0), y_1(0), \theta_1(0)]^\top = [0, -1, \pi/15]$, $[x_2(0), y_2(0), \theta_2(0)]^\top = [20, -4, \pi/12]$,

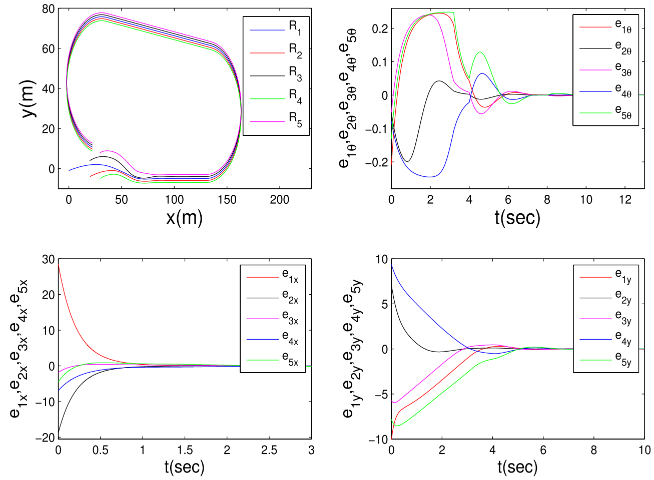

Fig. 2. Described paths and resulting tracking errors for five robots.

$[x_3(0), y_3(0), \theta_3(0)]^\top = [20, 4, \pi/10]$, $[x_4(0), y_4(0), \theta_4(0)]^\top = [30, -5, \pi/8]$, and $[x_5(0), y_5(0), \theta_5(0)]^\top = [30, 8, \pi/6]$. The linear formation shape with a certain desired distance between the robots is obtained by defining $[d_{x1,2}, d_{y1,2}] = [0, 1]$, $[d_{x2,3}, d_{y2,3}] = [0, -2]$, $[d_{x3,4}, d_{y3,4}] = [0, 3]$, and $[d_{x4,5}, d_{y4,5}] = [0, -4]$.

The imposed path by the leader robot has a stadium-circuit shape composed of two straight lines and two half circumferences, as illustrated by the northeast plot in Fig. 2. The forward reference velocity is set to $v_0(t) \equiv 10$ [m/s], while the angular reference velocity is defined as $\omega_0(t) := 0.3$ [rad/s] for all $t \in [10T, 10(T+1))$ for all odd integer values of T and $\omega_0(t) \equiv 0$ otherwise. That is, it switches between 0 and 0.3 [rad/s] every 10 [s]. The total simulation time is set to 40 [s].

The control laws are given by

$$\begin{aligned} v_i &= v_{(i-1)} + c_{2i} e_{ix} \\ \omega_i &= \omega_{(i-1)} + c_{1i} e_{i\theta} + \varphi(t) \tanh(e_{iy}) \end{aligned}$$

with control gains $c_{1i} = 2$ and $c_{2i} = 5$. Function φ is generated as a square-pulse-train signal of an amplitude of 0.5, a period of 4 s, and a pulsewidth of 3.2 [s]. Note that this function is not smooth but it is persistently exciting, and hence, the term $\varphi(t)\tanh(e_{iy})$ induces enough excitation to stabilize the system in the y direction as long as there is an error in this coordinate. The rapid response and excellent performance may be appreciated from the plots of the formation-tracking errors, depicted in Fig. 2.

IV. Conclusion

We presented a very simple decentralized controller for the problem of formation-tracking control of mobile robots in order to follow straight paths. Our approach relies on a simple idea which consists in maintaining the reference angular velocity different from zero by an amount proportional to the translation error. Extensions of this approach to more complex models and under relaxed assumptions, such as time-varying topologies, state-dependent interconnection gains, and the case of force-controlled robots, are currently under study.

References

[1] J. R. T. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Trans. Robot. Autom.*, vol. 19, no. 6, pp. 933–941, Dec. 2003.

[2] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for an autonomous mobile robot," in *Proc. IEEE Int. Conf. Robot. Autom.*, May 1990, pp. 384–389.

[3] M. Defoort, T. Floquet, A. Kokosy, and W. Perruquetti, "Sliding-mode formation control for cooperative autonomous mobile robots," *IEEE Trans. Ind. Electron.*, vol. 55, no. 11, pp. 3944–3953, Nov. 2008.

[4] J. Shao, G. Xie, J. Yu, and L. Wang, "Leader-following formation control of multiple mobile robots," in *Proc. IEEE Int. Symp., Medit. Conf. Control Autom. Intell. Control*, Jun. 2005, pp. 808–813.

[5] L. Consolini, F. Morbidi, D. Prattichizzo, and M. Tosques, "Leader–follower formation control of nonholonomic mobile robots with input constraints," *Automatica*, vol. 44, no. 5, pp. 1343–1349, 2008.

[6] Y.-Y. Chen and Y.-P. Tian, "A backstepping design for directed formation control of three-coleader agents in the plane," *Int. J. Robust Nonlinear Control*, vol. 19, no. 7, pp. 729–745, 2009.

[7] K. D. Do and J. Pan, "Nonlinear formation control of unicycle-type mobile robots," *Robot. Auton. Syst.*, vol. 55, no. 3, pp. 191–204, 2007.

[8] T. H. A. van den Broek, N. van de Wouw, and H. Nijmeijer, "Formation control of unicycle mobile robots: A virtual structure approach," in *Proc. 48th IEEE Conf. Decision Control, 28th Chin. Control Conf. (CDC/CCC)*, Dec. 2009, pp. 8328–8333.

[9] E. Panteley, E. Lefeber, A. Loria, and H. Nijmeijer, "Exponential tracking control of a mobile car using a cascaded approach," in *Proc. IFAC Workshop Motion Control*, Grenoble, France, 1998, pp. 221–226.

[10] A. A. J. Lefeber, "Tracking control of nonlinear mechanical systems," M.S. thesis, Dept. Math., Univ. Twente, Enschede, The Netherlands, 2000.

[11] J. Guo, Z. Lin, M. Cao, and G. Yan, "Adaptive leader-follower formation control for autonomous mobile robots," in *Proc. Amer. Control Conf. (ACC)*, Jun./Jul. 2010, pp. 6822–6827.

[12] K.-C. Cao and Y.-P. Tian, "A time-varying cascaded design for trajectory tracking control of non-holonomic systems," *Int. J. Control*, vol. 80, no. 3, pp. 416–429, 2007.

[13] T.-C. Lee, K.-T. Song, C.-H. Lee, and C.-C. Teng, "Tracking control of unicycle-modeled mobile robots using a saturation feedback controller," *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 2, pp. 305–318, Mar. 2001.

[14] A. Loría, J. Dasdemir, and N. Jarquín-Alvarez, "Decentralized formation tracking control of autonomous vehicles on straight paths," in *Proc. 53rd IEEE Conf. Decision Control*, Los Angeles, CA, USA, 2014, pp. 5399–5404.

[15] H. K. Khalil, *Nonlinear Systems*, 3rd ed. New York, NY, USA: Prentice-Hall, 2002.

[16] A. Loría, E. Panteley, and A. Teel, "A new persistency-of-excitation condition for UGAS of NLTV systems: Application to stabilization of nonholonomic systems," in *Proc. 5th. Eur. Control Conf.*, 1999, pp. 1363–1368, paper 500.

[17] E. Panteley, A. Loría, and A. Teel, "Relaxed persistency of excitation for uniform asymptotic stability," *IEEE Trans. Autom. Control*, vol. 46, no. 12, pp. 1874–1886, Dec. 2001.

[18] A. Loría, E. Panteley, and K. Melhem, "UGAS of skew-symmetric time-varying systems: Application to stabilization of chained form systems," *Eur. J. Control*, vol. 8, no. 1, pp. 33–43, 2002.

[19] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ, USA: Prentice-Hall, 1996.

[20] W. Hahn, *Stability of Motion*. New York, NY, USA: Springer-Verlag, 1967.

[21] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Upper Saddle River, NJ, USA: Prentice-Hall, 1989.