A Robust Dynamic Leader-Follower Formation Control with Active Obstacle Avoidance

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Abstract--**This paper presents a novel dynamic formation control strategy for multiple nonholonomic mobile robots. The main idea is converting the leader-follower formation control problem to an equivalent tracking control problem for follower robot based on the states of the relative motion between the robots. Then, the dynamic model of robots is used in formation. A formation controller, consisting of a feedback linearization part and a sliding mode compensator, is designed to stabilize the overall system. The proposed controller generates the commanded torques for the follower robot and makes the formation control system robust to the effect of unknown bounded disturbances in dynamic modeling. Furthermore, on active obstacle avoidance is presented by considering the obstacle as a virtual leader in our proposed model. Simulation results are presented to verify the performance of the proposed scheme.**

Keywords: leader-follower formation, dynamic modeling, Sliding mode controller.

I. INTRODUCTION

There has been a tremendous interest in the coordinated control of multiple autonomous mobile vehicles in recent years. There are several advantages in using a team of robots in tasks such as search and rescue operations, mapping unknown or hazardous environments, and security. Various control methods have been proposed and applied to the formation design of robotic networks, such as behavior-based approach [7, 8], virtual structure approach [9], and the leader-follower approach [1-3]. The leader-follower formation control of mobile robots, one of the main approaches in this field, has been studied by many researchers. In a robot formation with leader-follower configuration, one or more robots are selected as leaders, which are responsible for guiding the formation, and the rest of the robots are controlled to follow the leaders. The control objective is to make the follower robots track the leaders with some prescribed offsets.

Desai *et. al.* [1-3] presented a feedback linearization control method for the formation of nonholonomic mobile robots using the leader-follower approach. In [5], a robust method is presented to keep the follower in formation with leader and absolute acceleration of the leader robot is treated as model uncertainty of the system.

Dierks *et. a.l.* [10] presented a control scheme for a differentially steered robot by using backstepping kinematics into dynamics. An active obstacle avoidance scheme has been given in [4] and [6], although no dynamic model of the robots has been considered in formation.

In this paper, a new formation methodology is presented which is based on dynamic model of the robots. States of The relative motion between the follower robot and the leader robot is used to transform the formation control to an equivalent tracking control for the follower robot in which the follower robot must track a commanded path instead of keeping distance and relative bearing with the leader robot. Similar to [11], we seek to convert a relative pose problem into a tracking control problem, except that the virtual robot used in [11] is not considered. We also seek to bring in the dynamics of the robots themselves thus incorporating the formation dynamics in the controller design. Most of the existing works have proposed many approaches to leader-follower formation considering only the kinematic equations of robots in formation [1,12]. In [13], the inverse dynamics technique is applied to design a centralized formation control. Dynamic model of the follower robot is used in [14] however, an exact knowledge of the robot dynamic model has been assumed. In [10], a combined kinematic/torque control law is developed for leader-follower based formation control using the backstepping technique in order to accommodate the dynamics of the robots and the formation in contrast with kinematic-based formation. However, it uses kinematic equation with an exact knowledge about the dynamics. We also consider the active obstacle avoidance in which the follower robot tracks a path yet has a desired distance with the obstacle.

Based on the proposed dynamic model, feedback linearization is implemented to achieve the objective of the formation control. Furthermore, a sliding mode controller is integrated to the feedback linearization controller to compensate for the uncertainty associated with the unknown dynamics of the follower robot. The Lyapunov method is used to stabilize the overall system.

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Rest of the paper is organized as follows. In Section 2, leader-follower formation control and obstacle avoidance problem are both converted to path following for the follower robot. The dynamic model of robots is given in Section 3. The proposed robust formation controller is presented in Section 4 and Simulation results given in Section 5 are used to verify the effectiveness of the proposed methodology.

II. Problem Formulation

A. Leader-follower formation

The leader-follower setup considered in this paper is presented in Fig. 1. The aim of the formation control is to make the follower robot R_2 track the leader robot R_1 with desired separation l_{12}^d and the desired relative bearing φ_{12}^d between the robots. And a relative motion sensor is mounted at point c on the follower robot R_2.

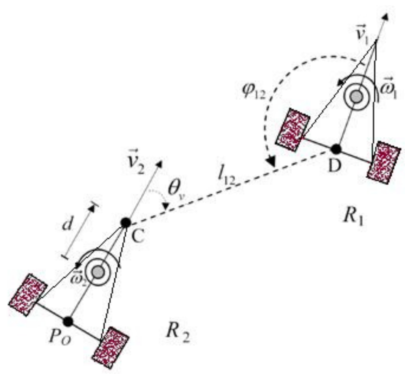


Fig.1 Two robots in leader-follower formation

Where $(x_{ci} \quad y_{ci})$ are the coordinates of the head of robot in the world coordinates system, and θ_i is the heading angle of the robot. As shown in the Fig. 1, $\vec{v}_i$ and $\vec{\omega}_i$ are the linear and angular velocities of the robot R_i .

Let set $\theta_{v2} = \pi - \varphi_{12} - \theta_{12}$ that is the relative bearing between velocity v_2 and line l_{12} in which $\theta_{12} = \theta_1 - \theta_2$.

According to Fig. 1 one can write the following equations:

$$\begin{cases} \theta_v = \pi - \varphi_{12} - \theta_{12} \\ x_1 - x_2 = l_{12} \cos(\theta_2 - \theta_v) \\ y_1 - y_2 = l_{12} \sin(\theta_2 - \theta_v) \end{cases} \tag{1}$$

Remark 1. The follower robot does not know the absolute position of the leader robot and can just measure the relative states of the leader robot with respect to itself (l_{12} ، φ_{12} and θ_v) by relative motion sensor that is mounted at point c on the follower robot.

We can rewrite model (1) in the following form:

$$\begin{cases} x_1 = x_2 + l_{12} \cos(\theta_2 - \theta_v) \\ y_1 = y_2 + l_{12} \sin(\theta_2 - \theta_v) \\ \theta_1 = \pi - \varphi_{12} - \theta_v + \theta_2 \end{cases} \tag{2}$$

The follower robot can estimate position of the leader robot by (2). Equations in (2) represent a path (x_1, y_1, θ_1) that the follower robot must track with the desired distance (l_{12}^d) and desired relative bearing (φ_{12}^d).

So the desired path can be expressed as:

$$\begin{cases} x_2^d = x_1 - l_{12}^d \cos(\theta_2 - \theta_v) \\ y_2^d = y_1 - l_{12}^d \sin(\theta_2 - \theta_v) \\ \theta_2^d = -\pi + \varphi_{12}^d + \theta_v + \theta_1 \end{cases} \tag{3}$$

Using (3) the leader follower formation control problem has been converted to the problem of following the desired path given in (3).

Remark 2. Mobile robots are equipped with incremental encoders that measure the rotation of the wheels, but not directly the position and orientation of the vehicle with respect to a fixed world frame. However, our proposed controller requires the absolute position (x, y, θ) of the follower robot which can be estimated using a *localization procedure*. The kinematic equations of a mobile robot is given by

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i \\ \dot{y}_i &= v_i \sin\theta_i \\ \dot{\theta}_i &= \omega_i \end{aligned} \tag{4}$$

Assume that the robot configuration at time t_k is known, $(t_k) = q_k$, together with the value of the velocity inputs v_k and ω_k applied in the interval $[t_k \;\; t_{k+1})$. The value of the configuration variables q_{k+1} at the next sampling time t_{k+1} can be reconstructed by integration of the kinematic model (39). A simple forward difference scheme can be used for this purpose [19]:

$$\begin{aligned} x_{k+1} &= x_k + v_k T_s \cos\theta_k \\ y_{k+1} &= y_k + v_k T_s \sin\theta_k \\ \theta_{k+1} &= \theta_k + \omega_k T_s \end{aligned} \tag{5}$$

Where $T_s = t_{k+1} - t_k$ is the duration of the sampling interval.

B. Obstacle avoidance formulation

By considering the obstacle as a virtual lead-robot and Similar to the procedure used in Section 2.1, system equations is the form of: (see Fig. 2)

$$\begin{cases} x_2^d = x_3 - l_{23}^d cos(\theta_2 - \theta_v') \\ y_2^d = y_3 - l_{23}^d sin(\theta_2 - \theta_v') \\ \theta_2^d = \theta_2 \end{cases} \quad (6)$$

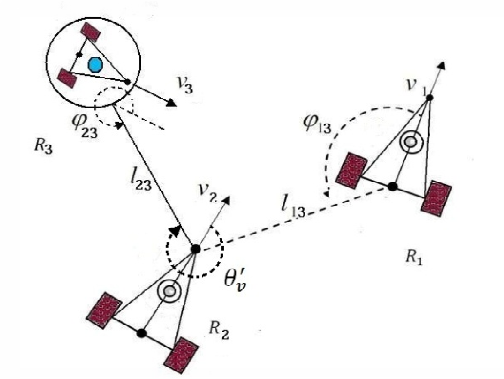


Fig.2 Obstacle avoidance in formation control

When the follower senses an obstacle must track a new desired path and converts its desired trajectory to (3), and when the obstacle run away from the follower and leader, the follower robot should return to primary desired trajectory presented in (2). Notice that it is necessary to keep θ_2 as its previous value to avoid deviation of the follower robot from leader's path.

III. DYNAMIC MODELING

The general coordinates of a robot moving on a plane can be defined as $q = [x, y, \theta]^T$.

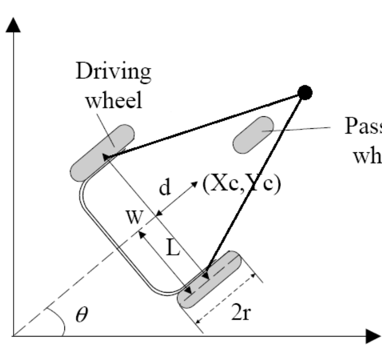


Fig. 3 The two-wheel-driven mobile robot

For a car-like robot shown in figure 3, we take the mass center as the robot's position, then the dynamic equation of robot can be expressed as:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + g(q) + \tau_d = \tau_{in} \quad (7)$$

Where $M(q)\epsilon R^{3\times3}$is a symmetric, positive definite inertial matrix. $V(q,\dot{q})\epsilon R^{3\times3}$is the centripetal and coriolis matrix. $g(q) = -J^T(q)\lambda$ where $J(q)\epsilon R^{3\times1}$ is the matrix associated with the nonholonomic constraints. $\tau_{in} = B(q)\tau$ is the input vector in which $B(q) \in R3\times2$ is the input transformation matrix. All these matrices are given by:

$$M(q) = \begin{bmatrix} m & 0 & mdsin\theta \\ 0 & m & -mdcos\theta \\ mdsin\theta & -mdcos\theta & I_0 + md^2 \end{bmatrix}$$

$$\tau = [\tau_l \quad \tau_r]^T \ , \quad V(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}cos\theta \\ 0 & 0 & md\dot{\theta}sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} cos\theta & cos\theta \\ sin\theta & sin\theta \\ L & -L \end{bmatrix} \ , \quad J(q) = [sin\theta \quad -cos\theta \quad d]$$

τ_d is a torque which represents bounded disturbance and unmodeled dynamics. It is important to highlight the *skew symmetric property* common to robotic systems [16] as$\dot{M}(q) - 2V(q,\dot{q}) = 0$.

IV. THE PROPOSED ROBUST FORMATION CONTROL SCHEME

In this section, a robust controller is designed to stabilize the system in the presence of unmodeled dynamics. The objective is to develop a control law to determine $u = \tau_{in}$ for the formation system such that the follower robot tracks q_d , and then has a predescribed configuration with the leader robot or obstacle. The proposed control scheme consists of a nominal part designed based on the nominal model of the system without the perturbation of modeling uncertainty and a sliding mode robust compensator to stabilize the overall system in the presence of uncertainty. The overall control is defined as

$$\begin{cases} u = u_0 + u_1 \\ u_0 = M\ddot{q}_r + C\dot{q} + G \\ u_1 = -M\eta sgn(s) \end{cases} \quad (8)$$

where u_0 is the nominal control, and u_1 is a sliding mode compensator. Their design is described in the following. The new reference acceleration vector $\ddot{q}_r$ is formed by shifting the desired acceleration $\ddot{q}_r$ according to the position error $\tilde{q}$ and velocity error $\dot{\tilde{q}}$.

$$\ddot{q}_r = \ddot{q}_d - \Lambda_2 \dot{\tilde{q}} - \Lambda_1 \tilde{q} \tag{9}$$

Where $\Lambda_i = diag(\lambda_{i1} \;\; \lambda_{i2} \;\; \lambda_{i3})$ for $i = 1,2,3$ are symmetric positive definite matrices. Thus, the reference trajectory is expressed in terms of the tracking errors. The components of the constant vector η satisfy $\eta_i > [|M^{-1}\delta|]_i + \eta_{0i}$ where η_{0i} is a strictly positive constant.

The nominal control u_0 in (8) leads to a linear closed- loop equation of the nominal system

$$\ddot{\tilde{q}} = -\Lambda_2 \dot{\tilde{q}} - \Lambda_1 \tilde{q} \tag{10}$$

and control law (8) results in the actual closed-loop system of (7) in the form of

$$\ddot{q} = \ddot{q}_r - \boldsymbol{\eta} sgn(s) - M^{-1}\delta \tag{11}$$

Define the tracking errors as $z_i = [\tilde{q}_i \;\; \dot{\tilde{q}}_i]^T$ for $i = 1,2,3$ then, the closed-loop equation can be written as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} - \begin{bmatrix} 0 \\ [\boldsymbol{\eta} sgn(s) + M^{-1}\delta]_1 \\ 0 \\ [\boldsymbol{\eta} sgn(s) + M^{-1}\delta]_2 \\ 0 \\ [\boldsymbol{\eta} sgn(s) + M^{-1}\delta]_3 \end{bmatrix} \tag{12}$$

where

$$A_i = \begin{bmatrix} 0 & 1 \\ -\lambda_{i1} & -\lambda_{i2} \end{bmatrix} \quad , \qquad i = 1,2,3$$

Let P_i be the symmetric positive definite solution of the following Lyapunov equation

$$A_i^T P_i + P_i A_i = -Q_i, \quad i = 1,2,3 \tag{13}$$

where Q_1 , Q_2 and و Q_3 are symmetric positive defnite matrices. Let ¸$\lambda_{min}(Q_i)$ denote the smallest eigenvalue of the matrix Q_i . The stability of the resultant closed-loop (12) can be proved using the Lyapunov theory.

Theorem 1 The closed-loop system (12) under control law (8) is asymptotically stable at the origin with the variable s_i in (8) selected as.

$$s_i = [z_i^T P_i]_2 \quad , \qquad i = 1,2,3 \tag{14}$$

where $[z_i^T P_i]_2$ denotes the second element of the vector $z_i^T P_i$.

Proof. Consider the composite Lyapunov function candidate

$$V(t) = \frac{1}{2}\sum_{i=1}^{2} z_i^T P_i z_i \tag{15}$$

Differentiating (13) with time along the solutions of (10), substituting $z_i^T P_i$
with s_i from (12), we have

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2}\sum_{i=1}^{2}\left(\dot{z}_i^T P_i z_i + z_i^T P_i \dot{z}_i\right) \\ &= \sum_{i=1}^{i=2}\left\{-\frac{1}{2} z_i^T Q_i z_i \right. \\ &\quad \left. - z_i^T P_i \begin{bmatrix} 0 \\ \eta_i sgn(s_i) + (M^{-1}\delta)_i \end{bmatrix}\right\} \\ &\leq -0.5\sum_{i=1}^{2}\lambda_{min}(Q_i)\|z_i\|^2 - \sum_{i=1}^{2}\eta_{0i}|s_i| \end{aligned} \tag{16}$$

From the last equation of (16), we have $V(\dot{t}) < 0$ Thus, Theorem 1 is proved.

V. SIMULATION RESULTS

In this section, the proposed formation control algorithm is verified by the simulations. Simulation results in MATLAB environment confirm the validity of the presented approach. In the first scenario, leader-follower formation is used. Assume that the leader robot is moving along the following path:

$$x_1 = 20\cos(t) + 10$$
$$y_1 = 20\sin(t) + 10$$
$$\theta_1 = 2t + \frac{5\pi}{6}$$

And the desired formation is defined as

$$[\, l_{12}^d \,, \;\; \varphi_{12}^d]^T = [10\sqrt{2} \;, \;\; \frac{2\pi}{3}]^T$$

And also the follower robot measure θ_v as $\frac{\pi}{6}$, so by using (3), the follower robot can determined the following trajectory to follow:

$$q_{des} = [x_2^d \,, y_2^d \,, \theta_2^d] = [20cost \,, 20sint \,, 2t + \tfrac{2\pi}{3}]^T$$

The following robotic parameters are considered for the Follower robot: m=0.5kg, $I_0 = 0.05\ kg^2$, $\lambda = 5$ and $d = 0.1\ m$.

The control parameters are selected as:

$$\Lambda_1 = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 40 \end{bmatrix}, \Lambda_2 = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 40 \end{bmatrix}, \eta = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}$$

Solving the Lyapunov function $A_i^T P_i + P_i A_i = -I$ yields:

$$P_1 = P_2 = P_3 = \begin{bmatrix} 0.3592 & -0.5 \\ -0.5 & 0.775 \end{bmatrix}$$

Then, the sliding variables in (8) are determined as:

$$s_1 = [z_1^T p_1]_2 = -0.5(x_2 - x_2^d) + 0.775(\dot{x}_2 - \dot{x}_2^d)$$

$$s_2 = [z_2^T p_2]_2 = -0.5(y_2 - y_2^d) + 0.775(\dot{y}_2 - \dot{y}_2^d)$$

$$s_3 = [z_3^T p_3]_2 = -0.5(\theta_2 - \theta_2^d) + 0.775(\dot{\theta}_2 - \dot{\theta}_2^d)$$

Under controller (8) x ,y and θ position errors are shown in Fig. 4. All converge to their desired values and the follower robot track the leader with desired configuration.

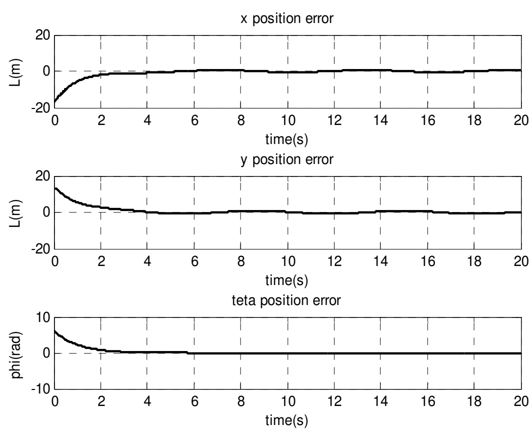


Fig.4 position errors in leader follower formation

The trajectories of the two robots are depicted in Fig. 5. The leader robot has a circular path in simulation. As can be seen, the follower robot tracks the leaders with the desired configuration.

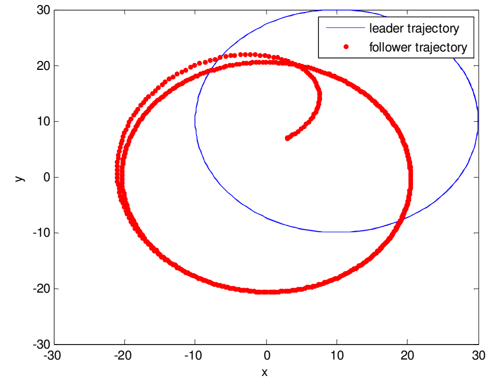


Fig. 5 Leader-follower trajectories

Simulation is repeated in Fig. 5 when the leader has an cosine trajectory.

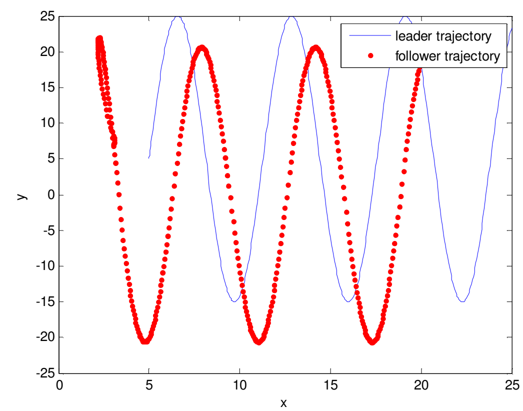
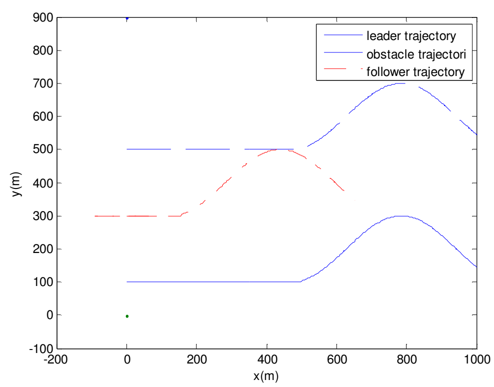


Fig. 6 Leader-follower trajectories

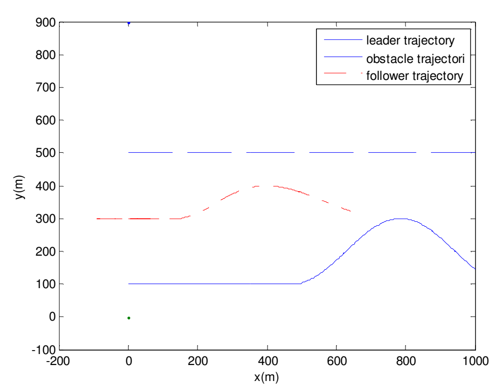
In the second simulation, the leader-obstacle formation is considered. In Fig. 7.a, the leader robot and the obstacle (virtual leader) both have a same trajectory whereas in Fig. 7.b the obstacle has a constant path on 500m and the follower robot changes its cosine trajectory to keep the desired distance from the obstacle . This means when an obstacle is approaching to leader-follower configuration, the follower robot avoids it and has a predetermined distance from the obstacle.

VI. Conclusions

In this paper, a new formation methodology has been presented which is based on dynamic model of robots. Leader-follower formation control problem has been converted to tracking control problem for follower robot based on the relative motion states between the leader robot and the follower robot. Active obstacle avoidance has been solved by considering the obstacle as a virtual leader. Based on the dynamic model of follower robot a robust controller is proposed to control the leader-follower and obstacle avoidance. The proposed controller makes the closed-loop formation system robust to bounded disturbance and unmodeled dynamics in dynamic model. Simulation results demonstrated the effectiveness of the proposed method.



(a)



(b)

Fig. 7 trajectory of leader-obstacle formation

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