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Formation Tracking Control of Nonholonomic Chained Form Systems

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Abstract— **Formation tracking control problem for multiple nonholonomic chained form systems are considered in this paper. In the framework of virtual structure, the formation keeping problem between chained-form agents are treated as tracking problem between the virtual agent and the actual agent. Theories from nonautonomous cascaded systems are introduced to simplify the design of formation controllers after carefully studying the structure of the error dynamic systems and global exponential controllers are constructed for the the above cooperative control problems in the end. Simulation results using Matlab show the feasibility of solving the formation tracking problems using methodology presented in this paper.**

I. INTRODUCTION

The advent of powerful embedded systems and communication networks have caused intensive interests in the cooperative motion control of multi robotic systems on the land [15], in the air [26][27] or even in the water [10]. Due to multi-agents' potentiality of accomplishing complex tasks and single agent's simplicity and inexpensiveness, multi-agent robotic systems have great potential to be used in rescue mission, large object moving, troop hunting, formation control and satellites clustering which is difficult or impossible for one single robot to accomplish.

Previous work mainly focused on decentralized control of integrator systems and nominal linear system. In [11], cooperative control laws based on nearest neighbor rules were presented for single integrator systems in discrete form. Consensus problem for double-integrator systems were considered in [23] where conditions of both undirected graph and directed graph were given. Previous consensus results of single or double integrator systems were generalized in [25] to high order integrator system with consensus to desired reference model. Decentralized dynamic output feedback were introduced in [33], [34] to deal with the consensus problem of nominal linear systems. Cooperative consensus and formation control problems for nominal linear systems were also studied in [22] utilizing results of irreducible and reducible matrices. While cooperative target tracking control of single integrator robots are considered in [30] using distributed Kalman filter. In the view of input and output, the powerful tool of passivity was adopted in [17] to solve the cooperative control of multiple passive systems. But the passivity-based method may not applied to nonholonomic system as shown in [2]. Compared with linear systems, control of multiple nonholonomic systems is much more difficult. As pointed in [3], there are no smooth (or even continuous) time-invariant static state feedback laws to asymptotically stabilize such systems. This kind of nonlinear systems becomes uncontrollable when it is linearized about some equilibria due to existence of nonholonomic constraints. Therefore, control of such systems will inevitably involves discontinuous, time-varying or hybrid control laws due to Brockett's necessary conditions. Based on previous results for multiple linear systems, how to control multiple nonholonomic chained form systems is the question to be answered in this paper.

Recent years have seen a lot of work on nonholonomic mobile robots. Circular motion of multiple nonholonomic robots around a virtual reference beacon were studied in [6] under the assumption that all the robots rotate around the beacon in the same direction. Due to the existence of nonholonomic constraints, the averaging method was adopted in [31] to solve the formation and hunting problem. But each robot in [31] was required to keep rotating or oscillating during transient process in order to fulfill the condition of controllability. This method was also used in [15] for studying the cooperative position control of multiple nonholonomic mobile robots whose direction angles were left to rotate freely. Some other methods such as additional path parameters and backstepping were adopted in [7] to consider the formation control of unicycle mobile robots and model predictive control schemes were also utilized in formation control problems for nonholonomic robots to follow virtual leader [9]. One thing should be pointed out is in that some of the previous papers such as [6], [15], [31], only position of the nonholonomic mobile robots are synchronized and the control of direction angle was open-loop or rotated freely. And it is preferred to find some better control methods to solve the zigzag phenomenon.

Using state and input transformation stated in [19], any kinematic model of first-order nonholonomic system with three states and two inputs can be converted into the chained form systems such as unicycles or cars with tailers. While some mechanical systems can be transformed into second-order chained form system, e.g. underactuated surface vessel, the planar V/STOL (vertical/short take-off and landing) aircraft in the absence of gravity or a hovercraft type vehicle [1]. As a canonical form of nonholonomic systems, cooperative control of multiple nonholonomic chained form systems has been an active area. By introducing a complex variable transformation, [8] considered the formation control of nonholonomic chained-form systems based on the results of multi linear-agent systems. The output synchronization problem of

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nonholonomic chained form system is recently considered in [16] but the synchronization of internal states is obtained by stabilization using agent's own states. Consensus problems and formation control problems for multiple nonholonomic chained form systems are solved in this paper.

Based on our previous study on tracking control of nonholonomic mobile robots [4] and [29], coupling parameters are introduced in the design of cooperative formation controllers to balance the tracking control of single agent system and formation keeping of group system. Our previous research [5] on formation control of nonholonomic mobile robots is enhanced in this paper. Theoretical proofs and simulation results illustrate that the cooperative control problem of multiple nonholonomic chained form systems can be solved using decentralized time-varying control law which is much simpler than those obtained in previous papers [8].

The rest of the paper is organized as follows. System model and problem statements are first given in Section II. Some preliminary results are also included in Section II. Our main results on consensus and formation control problems are presented in Section III. Section IV presents simulation results of the proposed controllers and conclusions are given in Section V.

II. Problem Formulation

Many mechanical systems in the world such as unicycle mobile robots or UAVs with fixed wings can be transformed into chained form by input and state transformation [18]. In this paper, we consider the following widely used canonical form of nonholonomic systems

$$\begin{aligned} \dot{x}_{i1} &= u_{i1}, \\ \dot{x}_{i2} &= u_{i2}, \\ \dot{x}_{i3} &= x_{i2}u_{i1}, \\ &\vdots \\ \dot{x}_{in} &= x_{i(n-1)}u_{i1}, \end{aligned} \tag{1}$$

where $i = 1, 2, \cdots, N$ is the number of systems, $u = (u_{i1}, u_{i2})$ and $x = (x_{i1}, \cdots, x_{in})$ are the control input and state of ith agent system, respectively.

As done in previous papers, it is natural to model information exchanges among all agents by directed/undirected graphs. The interaction topology of the N mobile robots can be described using a directed graph $G = (\mathcal{V}, E)$ with the set of nodes $V = \{1, 2, \cdots, N\}$ and edges $E \subseteq V \times V$. Neighbors of robot i are denoted by $\mathcal{N}_i = \{j \in V : (i, j) \in E\}$. The $n \times n$ adjacency matrix is defined as $A = A(G) = (a_{ij})$, where $a_{ij} = 1$ if there exist one edge from node i to node j in E otherwise $a_{ij} = 0$. Then the graph Laplacian matrix $L = [l_{ij}]$ can be written as:

$$l_{ij} = \begin{cases} a_{ij}, & j \in N_i, \\ \sum_{j\neq i} a_{ij}, & j = i. \end{cases} \tag{2}$$

Both consensus problem with reference target and formation moving along desired trajectory for nonholonomic chained form system are to be considered in this paper. We assume that the desired reference trajectory is described in the following chained form:

$$\begin{aligned} \dot{x}_{d1} &= u_{d1}, \\ \dot{x}_{d2} &= u_{d2}, \\ \dot{x}_{d3} &= x_{d2}u_{d1}, \\ &\vdots \\ \dot{x}_{dn} &= x_{d(n-1)}u_{d1}. \end{aligned} \tag{3}$$

In the formation control problem, we first complete every agent's estimate of the group trajectory using the obtained consensus protocol. Then every agent's desired dynamics can be easily obtained using the methodology of virtual structure [24]. Then formation control problem can be treated as a cooperatively tracking control problem in this framework.

Formation Control Problem: Assume that the desired states of every agent is denoted as $(x_{i1}^d, x_{i2}^d, \cdots, x_{in}^d)^T$ which evolves according to dynamics (3). Design a controller for each chained form system (1) based on its and its neighbors' states such that the group of all chained systems comes into the desired formation and moving along the desired trajectory (3), i.e.

$$\lim_{t\to\infty} (x_{ik} - x_{ik}^d) = 0, \tag{4}$$

where $i = 1, 2, \cdots, N$ and $k = 1, 2, \cdots, n$.

Now, we recall some definitions and lemmas that will be used in the next Section.

Definition 2.1: [14] ω_d is persistently exciting, i.e., there exist positive constants α_1, α_2 and δ such that the following condition holds for all $t > 0$:

$$\alpha_1 I \leq \int_t^{t+\delta} \omega_d(\tau)\omega_d^T(\tau)d\tau \leq \alpha_2 I.$$

Lemma 2.1: [28] Consider a linear time-varying system of the form

$$\dot{x} = (A_0 + A_1(t))x,$$

where A_0 is a constant and Hurwitz matrix, $A_1(t)$ is time-varying and satisfies

$$A_1(t) \to 0 \text{ as } t \to \infty, \qquad \int_0^\infty \|A_1(t)\|\, dt < \infty,$$

then the systems is globally uniformly exponentially stable (GUES).

Lemma 2.2: [14] Consider a linear time-varying system

$$\dot{x} = A(\phi(t))x + Bu, \tag{5}$$

where $A(\phi)$ is continuous and $\phi : \mathbf{R} \to \mathbf{R}$ continuous. Assume that for all $s \neq 0$ the pair $(A(s), B)$ is controllable. If $\phi(t)$ is bounded, Lipschitz and there exist constants $\delta_c > 0$ and $\epsilon > 0$ such that

$$\forall t \geq 0, \exists s : t - \delta_c \leq s \leq t \text{ such that } |\phi(s)| \geq \epsilon,$$

then the system (5) is uniformly completely controllable.

Consider a time-varying cascaded system $\dot{z} = f(t, z)$ that can be written as [21]

$$\begin{aligned} \dot{z}_1 &= f_1(t, z_1) + g(t, z_1, z_2)z_2, \\ \dot{z}_2 &= f_2(t, z_2). \end{aligned} \tag{6}$$

Actually, system (6) can be regarded as the system

$$\Sigma_1: \quad \dot{z}_1 = f_1(t, z_1) \tag{7}$$

perturbed by the output of the system

$$\Sigma_2: \quad \dot{z}_2 = f_2(t, z_2). \tag{8}$$

Lemma 2.3: [14], [20] The cascade time varying system (6) is globally $\mathcal{K}$-exponentially stable if the following conditions are satisfied:

1) The subsystem (7) is globally uniformly exponential stable;
2) The function $g(t, z_1, z_2)$ satisfies the following condition for all $t \geq t_0$:
$$g(t, z_1, z_2) \leq \theta_1(\|z_2\|) + \theta_2(\|z_2\|)\|z_1\|,$$
where $\theta_1 : R^+ \to R^+, \theta_2 : R^+ \to R^+$ are continuous functions;
3) The subsystem (8) is globally $\mathcal{K}$-exponentially stable.

III. Formation Control Problems For Multiple Chained Form Systems

Based on related consensus protocol results in book [24], the formation control problems for nonholonomic chained form system are considered in this section. Firstly, every agent system reach consensus on the estimate of the reference target system (3) using consensus protocol in previous section. Then each agent system's desired dynamics can be easily computed with the method of virtual structure. For example, suppose the obtained reference target is (x_d, y_d) and the desired position of ith agent system can be described as

$$\begin{bmatrix} x_i^d \\ y_i^d \end{bmatrix} = \begin{bmatrix} x_d \\ y_d \end{bmatrix} + \begin{bmatrix} \cos\theta_d & -\sin\theta_d \\ \sin\theta_d & \cos\theta_d \end{bmatrix} \begin{bmatrix} x_{iF}^d \\ y_{iF}^d \end{bmatrix}, \tag{9}$$

where (x_{iF}^d, y_{iF}^d) is the desired relative position to the reference target (x_d, y_d) and θ_d is the relative angel between the virtual coordinate frame that located on the reference target and the initial coordinate frame. Denote $\theta_i^d = \theta_d$ and it is easy to find that the dynamics of $(x_i^d, y_i^d, \theta_i^d)$

$$\begin{aligned} \dot{x}_i^d &= v_d \cos\theta_i^d - (x_{iF}^d \sin\theta_i^d + y_{iF}^d \cos\theta_i^d), \\ \dot{y}_i^d &= v_d \sin\theta_i^d + (x_{iF}^d \cos\theta_i^d - y_{iF}^d \sin\theta_i^d), \\ \dot{\theta}_i^d &= \omega_d \end{aligned}$$

also satisfy the nonholonomic constraints

$$\dot{x}_i^d \sin\theta_i^d - \dot{y}_i^d \cos\theta_i^d = x_{iF}^d.$$

Under the following coordinate transformation brought out in ([19])

$$\begin{aligned} x_1 &= \theta_d, \\ x_2 &= x_i \cos\theta_d + y_i \sin\theta_d, \\ x_3 &= -x_i \sin\theta_d + y_i \cos\theta_d, \end{aligned}$$

the desired dynamics of the ith agent can be described in nonholonomic chained form (3).

Generally suppose the obtained desired dynamics for ith chained form system is described as

$$\begin{aligned} \dot{x}_{i1}^d &= u_{i1}^d, \\ \dot{x}_{i2}^d &= u_{i2}^d, \\ \dot{x}_{i3}^d &= x_{i2}^d u_{i1}^d, \\ &\vdots \\ \dot{x}_{in}^d &= x_{i(n-1)}^d u_{i1}^d. \end{aligned} \tag{10}$$

A. Controller Design

The error system between the ith chained form system (1) and its desired reference target (10) is

$$\begin{aligned} \dot{e}_{i1} &= u_{i1} - u_{i1}^d, \\ \dot{e}_{i2} &= u_{i2} - u_{i2}^d, \\ \dot{e}_{i3} &= e_{i2}u_{i1}^d + (e_{i2} + x_{i2}^d)(u_{i1} - u_{i1}^d), \\ &\vdots \\ \dot{e}_{in} &= e_{i(n-1)}u_{i1}^d + (e_{i(n-1)} + x_{i(n-1)}^d)(u_{i1} - u_{i1}^d), \end{aligned} \tag{11}$$

where $e_{i1} = x_{i1} - x_{i1}^d, i = 1, 2, \cdots, N$.

The error dynamics of the system (11) can be treated as the following subsystem

$$\begin{aligned} \dot{e}_{i2} &= u_{i2} - u_{i2}^d, \\ \dot{e}_{i3} &= e_{i2}u_{i1}^d, \\ &\vdots \\ \dot{e}_{in} &= e_{i(n-1)}u_{i1}^d \end{aligned} \tag{12}$$

cascaded by

$$\dot{e}_{i1} = u_{i1} - u_{i1}^d, \tag{13}$$

and the cascaded term is

$$\begin{bmatrix} 0 \\ (e_{i2} + x_{i2}^d)(u_{i1} - u_{i1}^d) \\ \vdots \\ (e_{i(n-1)} + x_{i(n-1)}^d)(u_{i1} - u_{i1}^d) \end{bmatrix}. \tag{14}$$

1) Control design for u_{i1}: Under the control law

$$u_{i1} = u_{i1}^d - k_1 e_{i1} - \sum_{j \in \mathcal{N}_i} a_{ij}(e_{i1} - e_{j1}), \tag{15}$$

where a_{ij} is defined in the adjacent matrix A and $k_1 > 0$, the closed loop systems obtained by stacking all the subsystems (13) is defined as

$$\dot{X}_1 = -(k_1 I + L)X_1, \tag{16}$$

where $X_1 = [e_{11}, e_{21}, \cdots, e_{N1}]^T$.

The eigenvalues of matrix $k_1 I + L$ is $k_1 + \mu_r$, where μ_r is the rth eigenvalues of the Laplacian Matrix of the communication topology graph. If the communication topology graph has a spaning tree then $\mu \geq 0$ and then all eigenvalues of matrix $-(k_1 I + L)$ will have negative real parts under condition that $k_1 > 0$. Then the global exponential stability of the system (16) is obtained.

2) Control design for u_{i2}: Before presenting the controller of u_{i2}, we fist give the following lemma that will be used in the proof.

Lemma 3.1: The system

$$\dot{\rho}=\begin{bmatrix} -k_2(I+L) & -k_3(I+L)u_{i1}^d & -k_4(I+L) & k_5(I+L)u_{i1}^d & \cdots \\ u_{i1}^d & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & u_{i1}^d & 0 \end{bmatrix}\rho$$

is globally uniformly exponentially stable (GUES) if $u_{i1}^d(t)$ is persistently exciting and the $k_2, k_3, \cdots, k_n$ are parameters such that the polynomial

$$\lambda^{(n-1)} + k_2\lambda^{(n-2)} + \cdots + k_{n-1}\lambda + k_n = 0$$

is Hurwitz, where $\rho = \begin{bmatrix}\rho_2 & \rho_3 & \cdots & \rho_n\end{bmatrix}^T \in R^{(n-1)\times N}$, $\rho_i \in R^N, i = 2, 3, \cdots, n$ and L is Laplacian matrix of communication topology graph that having a spanning tree.

Remark 3.1: The only different between Theorem 2.3.7 of [14] and Lemma 3.1 is that ρ in Lemma 3.1 has dimension of $(n-1)\times N$ while x in Theorem 2.3.7 of [14] has dimension of $n-1$. Similar proof is omitted due to the space limitation.

As stated above, the remaining error dynamics systems of the i error systems can be considered as the following system (17)

$$\begin{bmatrix}\dot{e}_{i2} \\ \dot{e}_{i3} \\ \vdots \\ \dot{e}_{in}\end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ u_{i1}^d & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & u_{i1}^d & 0 \end{bmatrix}\begin{bmatrix} e_{i2} \\ e_{i3} \\ \vdots \\ e_{in}\end{bmatrix} + \begin{bmatrix}1 \\ 0 \\ \vdots \\ 0\end{bmatrix}\begin{bmatrix}u_{i2} - u_{i2}^d\end{bmatrix} \quad (17)$$

cascaded by the system (13) and the cascaded term (14).

In order to cooperatively solve the multiple tracking control problem of nonholonomic chained form systems such that they could keep desired formation and move along the desired trajectory, we consider the following controller of u_{i2}

$$\begin{aligned} u_{i2} &= u_{i2}^d - k_2 e_{i2} - k_3 u_{i1}^d e_{i3} - k_4 e_{i4} - k_5 u_{i1}^d e_{i5} - \cdots \\ &\quad - k_2 \sum_{j\in\mathcal{N}_i} a_{ij}(e_{i2} - e_{j2}) - k_3 u_{i1}^d \sum_{j\in\mathcal{N}_i} a_{ij}(e_{i3} - e_{j3}) \quad (18) \\ &\quad - k_4 \sum_{j\in\mathcal{N}_i} a_{ij}(e_{i4} - e_{j4}) - k_5 u_{i1}^d \sum_{j\in\mathcal{N}_i} a_{ij}(e_{i5} - e_{j5}) - \cdots, \end{aligned}$$

where $k_2, k_3, \cdots, k_n$ are parameters such that the polynomial

$$\lambda^{(n-1)} + k_2\lambda^{(n-2)} + \cdots + k_{n-1}\lambda + k_n = 0$$

is Hurwitz.

B. Main Results

Assumption 3.1: u_{d1} is persistently exciting signal that defined in Definition 2.1.

Theorem 3.1: For the formation tracking control problem between multiple nonholonomic chained form systems (1) and (3), if the Assumption 3.1 is satisfied and trajectories of the target system (3) is bounded, then control laws (15) and (18) cooperatively solve the formation tracking problem if there is a spaning tree in the communication topology graph whose root node is the reference systems.

Proof:

A. As shown in Section III-A.1, the stacking of subsystems (13) is globally exponentially stable under the control law (15).

The third condition of Lemma 2.3 is satisfied.

B. Let $X_i = [e_{1i}, e_{2i}, \cdots, e_{Ni}]^T$, where $i = 2, 3, \cdots, n$. The remaining system can be written as

$$\begin{bmatrix}\dot{X}_2 \\ \dot{X}_3 \\ \vdots \\ \vdots \\ \dot{X}_n\end{bmatrix} = \begin{bmatrix} -k_2(I+L) & -k_3(I+L)u_{i1}^d & -k_4(I+L) & k_5(I+L)u_{i1}^d & \cdots \\ \Delta(t)I_N & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \Delta(t)I_N & 0 \end{bmatrix}\begin{bmatrix}X_2 \\ X_3 \\ \vdots \\ \vdots \\ X_n\end{bmatrix}$$

where L is the Laplacian matrix and $\Delta(t) = diag\{u_{11}^d, u_{21}^d, \cdots, u_{N1}^d\}$.

By lemma 3.1, the above subsystem is globally uniformly exponentially stable (GUES) under the control law (18).

Then the first condition of Lemma 2.3 is satisfied.

C. Under the above control law (15)(18), the cascaded term of (14) between the above system and system (13) can be written as following after rearranging the order of variables as that of X_i.

$$\begin{bmatrix} 0_{N\times 1} \\ (e_{12} + x_{12}^d)(u_{11} - u_{11}^d) \\ \vdots \\ (e_{N2} + x_{N2}^d)(u_{N1} - u_{N1}^d) \\ \vdots \\ (e_{1(n-1)} + x_{1(n-1)}^d)(u_{11} - u_{11}^d) \\ \vdots \\ (e_{N(n-1)} + x_{N(n-1)}^d)(u_{N1} - u_{N1}^d) \end{bmatrix}$$

$$= -(k_1 I + L)Z\left\{\begin{bmatrix} 0_{N\times 1} \\ e_{12} \\ \vdots \\ e_{N2} \\ \vdots \\ e_{1(n-1)} \\ \vdots \\ e_{N(n-1)}\end{bmatrix} + \begin{bmatrix} 0_{N\times 1} \\ x_{12}^d \\ \vdots \\ x_{N2}^d \\ \vdots \\ x_{1(n-1)}^d \\ \vdots \\ x_{N(n-1)}^d\end{bmatrix}\right\},$$

$$
=-(k_1 I+L)Z\left\{\begin{bmatrix} 0_{N\times 1}\\ X_2\\ X_3\\ \vdots\\ \vdots\\ \vdots\\ X_{(n-1)}\end{bmatrix}+\begin{bmatrix} 0_{N\times 1}\\ x_{12}^d\\ \vdots\\ x_{N2}^d\\ \vdots\\ \vdots\\ x_{N(n-1)}^d\end{bmatrix}\right\}. \quad (19)
$$

where $Z=[e_{11},e_{11},\cdots,e_{N1}]^T$.

Since the desired reference trajectory (3) of the group is bounded, it is easy to see that the cascaded term (19) satisfy the second condition of Lemma 2.3 with respect to vector $[X_2,\cdots,X_n]^T$. Thus, the second condition of Lemma 2.3 is also satisfied.

Therefore, by Lemma 2.3 we conclude that all the stacking error systems (11) formed from (17) cascaded by (13) is globally $\mathcal{K}$-exponentially stable under linear decentralized control laws (15) and (18). ■

Remark 3.2: Different to previous results as shown in [15], all the state variables of the chained form system are controlled with presence of nonholonomic constraints.

As done in [24], both control law (15) and (18) are distributed in the sense that only information exchange among neighbors is required. The terms in the second line and third line of (18) can be considered as coupling terms and its role is maintaining the formation while the terms in the first line of (18) is to guarantee that the group should tracking some desired target.

IV. Simulation Results

Formation control problem are studied using Matlab simulation to illustrate the effectiveness of our methodology .

In this case, the desired linear velocity and angular velocity are chosen as $\omega_d(t) = -1rad/s$ and $v_d(t) = 10m/s$. Thus the entire formation will move along a circular path constantly. The desired triangular formation pattern between the agents is shown in Figure 1 and we assume that there is a spanning tree in the communication topology.

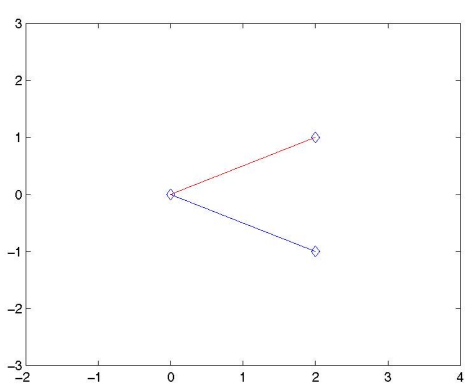

Fig. 1. Desired Formation

Simulation results under the formation tracking controllers (15) and (18) are listed in Figure 2 to 4. Formation trajectories on the two-dimensional plane are shown in Figure 2 and control inputs with respect to time are given in Figure 3 and Figure 4. Obtained results show the effectiveness of the proposed controllers in Theorem 3.1.

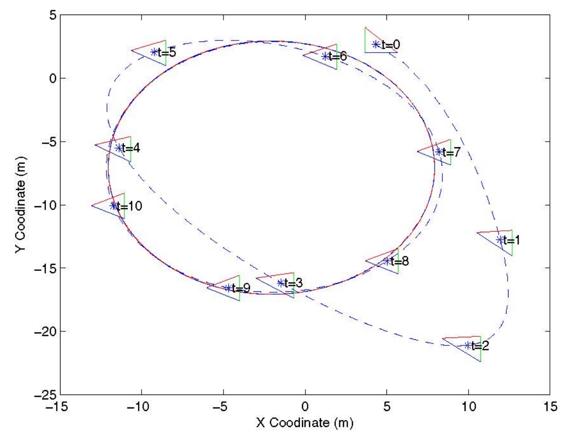

Fig. 2. Trajectory of tracking of a circle in triangular formation

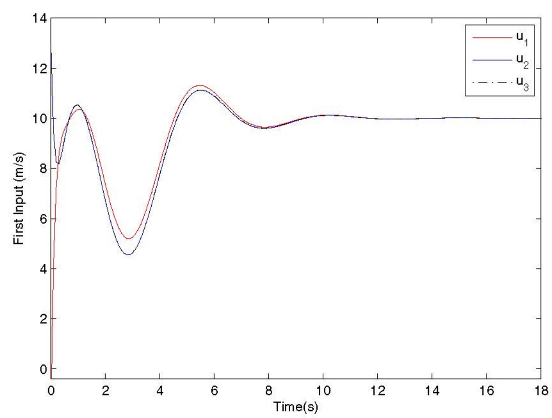

Fig. 3. First control input of the mobile robots

V. Conclusions

Formation tracking control problems for nonholonomic chained form systems are considered in this paper. Different to previous results all of the state variables of multiple nonholonomic chained form systems are synchronized. Cascaded theory of nonautonomous systems is adopted to transform the original problem into two stabilization problems involving simpler dynamical subsystems. Globally exponentially cooperative controls are achieved by our methods and the effectiveness of the proposed controllers is demonstrated by Matlab simulations.

Based on our obtained results on fault detection and tolerance on hybrid systems[12], [13], [32], fault detection and isolation problem and fault tolerant control problem for systems with nonholonomic or underactuated constrains will be considered in the future.

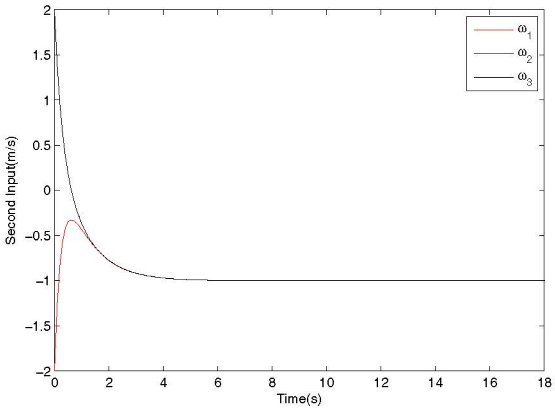

Fig. 4. Second control input of the mobile robots

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