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Simultaneous tracking and stabilization of mobile robots without velocity measurements

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***Abstract*: We present a time-varying global output-feedback controller that solves both tracking and stabilization for mobile robots simultaneously at the torque level. A coordinate transformation is first derived to cancel the velocity quadratic terms. A passive observer is then designed to globally exponentially estimate the unmeasured velocities. The controller synthesis is based on Lyapunov's direct method and backstepping technique. Simulations illustrate the effectiveness of the proposed controller.**

***Index terms:* Mobile robot, global output-feedback, tracking and stabilization, exponential observer.**

1. Introduction

The main difficulty of solving stabilization and tracking control of mobile robots is due to the fact that the motion of the systems in question to be controlled has more degrees of freedom than the number of control inputs under nonholonomic constraints. Furthermore, the necessary condition of Brockett's theorem [14] shows that any continuous time invariant feedback control law does not make the null solution of the wheeled mobile robots asymptotically stable in the sense of Lyapunov. Over the last decade, a lot of interest has been devoted to stabilization and tracking control of nonholonomic mechanical systems including wheeled mobile robots [1]-[11] to list a few. Tracking and stabilization are studied separately in these papers. Their objectives are mostly kinematic models. Recently several authors focused on the dynamic model [12],[11],[13] using the popular backstepping technique [15].

To our knowledge, output-feedback tracking control of land, air and sea vehicles has been solved for the case of fully actuated, see for example [21], [22] (pp. 311-334). There are no currently available results of output-feedback tracking of mobile robots although some results are proposed for a class of nonholonomic systems in [20]. Some recent results related to the output-feedback control of the single degree of freedom Lagrange systems were addressed in [17],[18] and also in [19] for a nonlinear benchmark system. The main difficulty of designing an observer-based output-feedback for Lagrange systems in general is because of the Coriolis matrix, which results in quadratic cross terms of unmeasured velocities. In addition, the nonholonomic constraints of mobile robots make the output-feedback problem much more challenging. For example, many solutions proposed for robot manipulator control, see [22] and references therein, cannot directly be applied. From the above discussion, an open challenging problem in controlling mobile robots is to find a global output-feedback controller (i.e. the controller uses only position and orientation measurements) that can solve both stabilization and tracking.

This paper contributes a positive answer to the above-mentioned challenging problem. Our new result is carried out by: (1) deriving a coordinate transformation to cancel the velocity cross terms in the mobile robot dynamics to design a global exponential velocity observer; (2) introducing a coordinate transformation based on car-driving practice to transform the tracking errors interpreted in a frame attached to the robot, to a triangular form.

2. Problem statement

In this paper, we consider a mobile robot with two actuated wheels whose equations of motion are given by [13]:

$$\begin{aligned} &\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \\ &\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\dot{\boldsymbol{\eta}})\mathbf{v} + \mathbf{D}\mathbf{v} = \boldsymbol{\tau} \end{aligned} \tag{1}$$

where

$$\boldsymbol{\eta} = [x \quad y \quad \phi]^T, \mathbf{v} = [v_1 \quad v_2]^T, \boldsymbol{\tau} = [\tau_v \quad \tau_w]^T,$$

$$\mathbf{J}(\boldsymbol{\eta}) = \begin{bmatrix} 0.5r\cos(\phi) & 0.5r\cos(\phi) \\ 0.5r\sin(\phi) & 0.5r\sin(\phi) \\ 0.5b^{-1}r & -0.5b^{-1}r \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} n_{11} & n_{12} \\ n_{12} & n_{11} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix},$$

$$\mathbf{C}(\dot{\boldsymbol{\eta}}) = \begin{bmatrix} 0 & 0.5b^{-1}r^2 m_c d\dot{\phi} \\ -0.5b^{-1}r^2 m_c d\dot{\phi} & 0 \end{bmatrix},$$

$$n_{11} = 0.25b^{-2}r^2(mb^2 + I) + I_w, n_{12} = 0.25b^{-2}r^2(mb^2 - I),$$
$$m = m_c + 2m_w, I = m_c d^2 + 2m_w b^2 + I_c + 2I_m.$$

In the above expressions, b is half of the width of the mobile robot and r is the radius of the wheel. d is the distance between the center of mass of the robot and the middle point between the left and right wheels. m_c and m_w are the mass of the body and wheel with a motor, I_c, I_w and I_m are the moment of inertia of the body about the vertical axis through P_c, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. The positive terms $d_{ii}, i=1,2$ are the damping coefficients. (x,y,ϕ) are the position and orientation of the robot, v_1 and v_2 are the angular velocities of the wheels. τ_v and τ_w are the control torques applied to the wheels of the robot. We assume that the reference trajectory is generated by the following virtual robot:

$$\begin{aligned}\dot{x}_d &= \cos(\phi_d)v_d \\ \dot{y}_d &= \sin(\phi_d)v_d \\ \dot{\phi}_d &= w_d\end{aligned} \tag{2}$$

where (x_d, y_d, ϕ_d) are the position and orientation of the virtual robot. v_d and w_d are the linear and angular velocities of the virtual robot, respectively.

Control objective: *Under Assumptions 1 and 2, design the control input vector* $\boldsymbol{\tau}$ *to force the position and orientation,* (x,y,ϕ) *of the real robot (1) to globally asymptotically track* (x_d,y_d,ϕ_d) *generated by (2) with only* (x,y,ϕ) *available for feedback.*

Assumption 1. *The reference velocities* v_d and w_d *satisfy one of the following conditions*

C1. $\int_0^\infty \left(|v_d(t)|+|w_d(t)|+|\dot{v}_d(t)|\right)dt \le \mu_1$, (3)

C2. $\int_{t_0}^t v_d^2(\tau)d\tau \ge \mu_2(t-t_0),\ \forall t\ge t_0 \ge 0$ (4)

C3. $\int_{t_0}^t w_d^2(\tau)d\tau \ge \mu_{31}(t-t_0)$ and $\int_0^\infty |v_d(t)|dt \le \mu_{32}$ $\quad \forall t \ge t_0 \ge 0$ (5)

where μ_1 and μ_{32} *are nonnegative constants, and* μ_2 and μ_{31} *are strictly positive constants*.

Assumption 2. *The reference signals* $v_d, \dot{v}_d, \ddot{v}_d$ and w_d *are bounded.*

The above assumptions imply that the reference trajectory can be a path converging to a set-point, a straight-line, an arc of circle or a combination of these special cases.

3. Observer design

As discussed in Section I, we first remove the quadratic velocity terms in the mobile robot dynamics by introducing the following coordinate change:

$$\mathbf{X} = \mathbf{Q}(\boldsymbol{\eta})\mathbf{v} \tag{6}$$

where $\mathbf{Q}(\boldsymbol{\eta})$ is a globally invertible matrix with bounded elements to be determined. Using (6), we write the second equation of (1) as follows:

$$\dot{\mathbf{X}} = \left[\dot{\mathbf{Q}}(\boldsymbol{\eta})\mathbf{v} - \mathbf{Q}(\boldsymbol{\eta})\mathbf{M}^{-1}\mathbf{C}(\dot{\boldsymbol{\eta}})\mathbf{v}\right] + \mathbf{Q}(\boldsymbol{\eta})\mathbf{M}^{-1}(-\mathbf{D}\mathbf{v}+\boldsymbol{\tau}) \tag{7}$$

Our goal is to cancel the square bracket in the right hand side of (7) for all $(\boldsymbol{\eta},\mathbf{v}) \in \Re^5$. Assuming that $q_{ij}(\boldsymbol{\eta}), i=1,2, j=1,2$ are the elements of $\mathbf{Q}(\boldsymbol{\eta})$. By using the first equation of (1), solving the resulting three partial differential equations yields:

$$\begin{aligned} q_{i1} &= C_{i1}\sin(a\Delta\phi) + C_{i2}\cos(a\Delta\phi), \\ q_{i2} &= \frac{C_{i2}\Delta - C_{i1}m_{12}}{m_{11}}\sin(a\Delta\phi) - \frac{C_{i1}\Delta + C_{i2}m_{12}}{m_{11}}\cos(a\Delta\phi)\end{aligned} \tag{8}$$

where $i=1,2$, $a=0.5r^2b^{-1}m_c d$, $m_{11}=n_{11}(n_{11}^2-n_{12}^2)^{-1}$, $m_{12}=-n_{12}(n_{11}^2-n_{12}^2)^{-1}$, $\Delta=\sqrt{m_{11}^2-m_{12}^2}$. C_{i1} and C_{i2} are arbitrary constants.

A choice of $C_{11}=C_{22}=0$, $C_{12}=C_{21}=m_{11}$ results in

$$\mathbf{Q}(\boldsymbol{\eta}) = \begin{bmatrix} m_{11}\cos(a\Delta\phi) & \Delta\sin(a\Delta\phi) - m_{12}\cos(a\Delta\phi) \\ m_{11}\sin(a\Delta\phi) & -m_{12}\sin(a\Delta\phi) - \Delta\cos(a\Delta\phi)\end{bmatrix} \tag{9}$$

One can directly verify that this matrix is globally invertible and its elements are bounded. Now we write (1) in the $(\boldsymbol{\eta},\mathbf{X})$ coordinates as

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \mathbf{J}(\boldsymbol{\eta})\mathbf{Q}^{-1}(\boldsymbol{\eta})\mathbf{X} \\ \dot{\mathbf{X}} &= -\mathbf{D}_\eta(\boldsymbol{\eta})\mathbf{X} + \mathbf{Q}(\boldsymbol{\eta})\mathbf{M}^{-1}\boldsymbol{\tau}\end{aligned} \tag{10}$$

where $\mathbf{D}_\eta(\boldsymbol{\eta}) = \mathbf{Q}(\boldsymbol{\eta})\mathbf{M}^{-1}\mathbf{D}\mathbf{Q}^{-1}(\boldsymbol{\eta})$. It is seen that (10) has a very nice structure, namely linear in the unmeasured states. Indeed a reduced-order observer can be designed but it is often noise sensitive. We here use the following passive observer:

$$\begin{aligned}\dot{\hat{\boldsymbol{\eta}}} &= \mathbf{J}(\boldsymbol{\eta})\mathbf{Q}^{-1}(\boldsymbol{\eta})\hat{\mathbf{X}} + \mathbf{K}_{01}(\boldsymbol{\eta}-\hat{\boldsymbol{\eta}}) \\ \dot{\hat{\mathbf{X}}} &= -\mathbf{D}_\eta(\boldsymbol{\eta})\hat{\mathbf{X}} + \mathbf{Q}(\boldsymbol{\eta})\mathbf{M}^{-1}\boldsymbol{\tau} + \mathbf{K}_{02}(\boldsymbol{\eta}-\hat{\boldsymbol{\eta}})\end{aligned} \tag{11}$$

where $\hat{\boldsymbol{\eta}}$ and $\hat{\mathbf{X}}$ are the estimates of $\boldsymbol{\eta}$ and $\mathbf{X}$, respectively. The observer gain matrices $\mathbf{K}_{01}$ and $\mathbf{K}_{02}$ are chosen such that $\mathbf{Q}_{01} = \mathbf{K}_{01}^T\mathbf{P}_{01} + \mathbf{P}_{01}\mathbf{K}_{01}$ and $\mathbf{Q}_{02} = \mathbf{D}_\eta^T(\boldsymbol{\eta})\mathbf{P}_{02} + \mathbf{P}_{02}\mathbf{D}_\eta(\boldsymbol{\eta})$ are positive definite, and that

$$\left(\mathbf{J}(\boldsymbol{\eta})\mathbf{Q}^{-1}(\boldsymbol{\eta})\right)^T\mathbf{P}_{01} - \mathbf{P}_{02}\mathbf{K}_{02} = 0 \tag{12}$$

with $\mathbf{P}_{01}$ and $\mathbf{P}_{02}$ being positive definite matrices. It is direct to show that $\mathbf{K}_{01}$ and $\mathbf{K}_{02}$ always exist since $\mathbf{D}_\eta(\boldsymbol{\eta})$ is positive definite. From (11) and (10), we have

$$\begin{aligned}\dot{\tilde{\boldsymbol{\eta}}} &= \mathbf{J}(\boldsymbol{\eta})\mathbf{Q}^{-1}(\boldsymbol{\eta})\tilde{\mathbf{X}} - \mathbf{K}_{01}\tilde{\boldsymbol{\eta}}, \\ \dot{\tilde{\mathbf{X}}} &= -\mathbf{D}_\eta(\boldsymbol{\eta})\tilde{\mathbf{X}} - \mathbf{K}_{02}\tilde{\boldsymbol{\eta}}\end{aligned} \tag{13}$$

where $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \hat{\boldsymbol{\eta}}$ and $\tilde{\mathbf{X}} = \mathbf{X} - \hat{\mathbf{X}}$. It is now seen that (13) is globally exponentially stable at the origin by taking the Lyapunov function $V_0 = \tilde{\boldsymbol{\eta}}^T\mathbf{P}_{01}\tilde{\boldsymbol{\eta}} + \tilde{\mathbf{X}}^T\mathbf{P}_{02}\tilde{\mathbf{X}}$ whose derivative along the solution of (13) and using (12) satisfies $\dot{V}_0 = -\tilde{\boldsymbol{\eta}}^T\mathbf{Q}_{01}\tilde{\boldsymbol{\eta}} - \tilde{\mathbf{X}}^T\mathbf{Q}_{02}\tilde{\mathbf{X}}$, which in turn implies that there exists a strictly positive constant σ_0 such that

$$\|(\tilde{\eta}(t),\tilde{\mathbf{X}}(t))\| \le \|(\tilde{\eta}(t_0),\tilde{\mathbf{X}}(t_0))\| e^{-\sigma_0(t-t_0)},\ \forall t \ge t_0 \ge 0. \quad (14)$$

Define $\hat{\mathbf{v}} = [\hat{v}_1, \hat{v}_2]^T$ being an estimator of the velocity vector $\mathbf{v}$ as

$$\hat{\mathbf{v}} = \mathbf{Q}^{-1}(\eta)\hat{\mathbf{X}}. \quad (15)$$

The velocity estimate error vector, $\tilde{\mathbf{v}} = \mathbf{v} - \hat{\mathbf{v}}$ satisfies

$$\tilde{\mathbf{v}} = \mathbf{Q}^{-1}(\eta)\tilde{\mathbf{X}}. \quad (16)$$

To prepare for the control design in the next section, we convert the angular velocities v_1 and v_2 to the linear, v, and angular, w, velocities of the robot by the relationship:

$$[v, w]^T = \mathbf{B}^{-1}[v_1, v_2]^T, \text{ with } \mathbf{B} = \begin{bmatrix} r^{-1} & br^{-1} \\ r^{-1} & -br^{-1} \end{bmatrix}. \quad (17)$$

By defining $\tilde{v} = v - \hat{v}$, $\tilde{w} = w - \hat{w}$ with $\hat{v}$ and $\hat{w}$ being estimates of v and w, we can see from (16) and (17) that

$$\|(\tilde{v}(t),\tilde{w}(t))\| \le \gamma_0 \|(\tilde{\eta}(t_0),\tilde{\mathbf{X}}(t_0))\| e^{-\sigma_0(t-t_0)},\ \forall t \ge t_0 \ge 0 \quad (18)$$

where γ_0 is a positive constant. We now write (1) in conjunction with (15) and (17) as

$$\begin{aligned}
\dot{x} &= \cos(\phi)\hat{v} + \cos(\phi)\tilde{v} \\
\dot{y} &= \sin(\phi)\hat{v} + \sin(\phi)\tilde{v} \\
\dot{\phi} &= \hat{w} + \tilde{w} \\
\begin{bmatrix} \dot{\hat{v}} \\ \dot{\hat{w}} \end{bmatrix} &= \mathbf{B}^{-1}\mathbf{M}_{\mathbf{Q}}\mathbf{B}\begin{bmatrix} \hat{v} \\ \hat{w} \end{bmatrix}\hat{w} - \mathbf{B}^{-1}\mathbf{M}^{-1}\mathbf{D}\mathbf{B}\begin{bmatrix} \hat{v} \\ \hat{w} \end{bmatrix} + \mathbf{B}^{-1}\mathbf{M}^{-1}\boldsymbol{\tau} + \boldsymbol{\Omega}
\end{aligned} \quad (19)$$

where

$$\boldsymbol{\Omega} = \mathbf{B}^{-1}\mathbf{M}_{\mathbf{Q}}\mathbf{B}\begin{bmatrix} \hat{v} \\ \hat{w} \end{bmatrix}\tilde{w} + \mathbf{B}^{-1}\mathbf{Q}^{-1}(\eta)\mathbf{K}_{02}\tilde{\eta}, \mathbf{M}_{\mathbf{Q}} = a\begin{bmatrix} m_{12} & -m_{11} \\ m_{11} & -m_{12} \end{bmatrix}. \quad (20)$$

4. Control design

As often done in tracking control of mobile robots, we first interpret the tracking errors as

$$\begin{bmatrix} x_e \\ y_e \\ \phi_e \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x - x_d \\ y - y_d \\ \phi - \phi_d \end{bmatrix}. \quad (21)$$

Indeed convergence of (x_e, y_e, ϕ_e) implies that of $(x - x_d, y - y_d, \phi - \phi_d)$. Using (21), (2) and the kinematic part of (19), we have the kinematic tracking errors:

$$\begin{aligned}
\dot{x}_e &= v_e - v_d(\cos(\phi_e) - 1) + y_e(w_e + w_d + \tilde{w}) + \tilde{v} \\
\dot{y}_e &= v_d \sin(\phi_e) - x_e(w_e + w_d + \tilde{w}) \\
\dot{\phi}_e &= w_e + \tilde{w}
\end{aligned} \quad (22)$$

where $v_e = \hat{v} - v_d, w_e = \hat{w} - w_d$. Now if v_e and w_e are considered as virtual controls, we can see directly from (22) that x_e and ϕ_e can be stabilized by v_e and w_e. There are several options to stabilize y_e, namely x_e, w_e or ϕ_e. If x_e or/and w_e are used, then the control design will be extremely complicated since w_e enters both of the first equations of (22). So we use ϕ_e to stabilize y_e. This choice also coincides with the car driving practice. Toward this end, we introduce the following coordinate transformation:

$$z_e = \phi_e + \arcsin\left(\frac{k(t)y_e}{\sqrt{1 + x_e^2 + y_e^2}}\right), k(t) = \lambda_1 v_d + \lambda_2 \cos(\lambda_3 t) \quad (23)$$

where $\lambda_i, i = 1,2,3$ are constants such that $|k(t)| < 1, \forall t$. They will be specified later. It is seen that (23) is well defined and convergence of z_e and y_e implies that of ϕ_e. We now use (23) to write the tracking errors as:

$$\begin{aligned}
\dot{x}_e &= v_e - v_d\varpi_1^{-1}(\varpi_2 - \varpi_1) + y_e(w_e + w_d + \tilde{w}) + \tilde{v} + p_x, \\
\dot{y}_e &= -kv_d\varpi_1^{-1}y_e - x_e(w_e + w_d + \tilde{w}) + p_y, \\
\dot{z}_e &= (1 - k\varpi_2^{-1}x_e)w_e - k\varpi_1^{-2}\varpi_2^{-1}x_e y_e v_e + f_z + p_z + \\
&\quad k\varpi_2^{-1}\left(x_e\tilde{w} + \varpi_1^{-2}x_e\tilde{v}\right), \\
\begin{bmatrix} \dot{v}_e \\ \dot{w}_e \end{bmatrix} &= \mathbf{B}^{-1}\mathbf{M}_{\mathbf{Q}}\mathbf{B}\begin{bmatrix} \hat{v} \\ \hat{w} \end{bmatrix}\hat{w} - \mathbf{B}^{-1}\mathbf{M}^{-1}\mathbf{D}\mathbf{B}\begin{bmatrix} \hat{v} \\ \hat{w} \end{bmatrix} - \begin{bmatrix} \dot{v}_d \\ \dot{w}_d \end{bmatrix} + \\
&\quad \mathbf{B}^{-1}\mathbf{M}^{-1}\boldsymbol{\tau} + \boldsymbol{\Omega}.
\end{aligned} \quad (24)$$

where for simple presentation, we have defined:

$$\begin{aligned}
\varpi_1 &= \sqrt{1 + x_e^2 + y_e^2},\ \varpi_2 = \sqrt{1 + x_e^2 + (1 - k^2)y_e^2}, \\
p_x &= -v_d\left((\cos(z_e) - 1)\varpi_1^{-1}\varpi_2 + \sin(z_e)k\varpi_1^{-1}y_e\right), \\
p_y &= v_d\left(\sin(z_e)\varpi_1^{-1}\varpi_2 - (\cos(z_e) - 1)k\varpi_1^{-1}y_e\right), \\
p_z &= \varpi_2^{-1}\left(kp_y - kv_d\varpi_1^{-2}y_e(x_e p_x + y_e p_y)\right), \\
f_z &= \varpi_2^{-1}\left(\dot{k}y_e - k(kv_d\varpi_1^{-1}y_e + x_e w_d) + k\varpi_1^{-2}y_e \times\right. \\
&\quad \left.(x_e v_d\varpi_1^{-1}(\varpi_2 - \varpi_1) + kv_d\varpi_1^{-1}y_e^2)\right).
\end{aligned} \quad (25)$$

The effort, we have made so far, is to have the term $-kv_d\varpi_1^{-1}y_e$ in the y_e-dynamics, and to put the tracking error dynamics in a triangular form of (24). Furthermore, we observe that p_x, p_y and p_z globally vanish when z_e does. We now design the control input vector $\boldsymbol{\tau}$ to stabilize (24) in two steps.

Step 1. Define the virtual velocity tracking errors $\tilde{v}_e$ and $\tilde{w}_e$ as

$$\tilde{v}_e = v_e - v_e^d,\ \tilde{w}_e = w_e - w_e^d \quad (26)$$

where v_e^d and w_e^d are the virtual controls of v_e and w_e. Based on the first three equations of (24), we design v_e^d and w_e^d as:

$$\begin{aligned}
v_e^d &= -k_1\varpi_1^{-1}x_e + v_d\varpi_1^{-1}(\varpi_2 - \varpi_1), \\
w_e^d &= (1 - k\varpi_2^{-1}x_e)^{-1}(-k_2 z_e - f_z + k\varpi_1^{-2}\varpi_2^{-1}x_e y_e v_e^d - p_z)
\end{aligned} \quad (27)$$

where $k_i, i = 1,2$ are positive constants.

Step 2. Differentiating both sides of (26) along the solution of (24) and (27), the control input vector $\boldsymbol{\tau}$ is designed as

$$\boldsymbol{\tau} = \mathbf{MB}\left(-\mathbf{B}^{-1}\mathbf{M}_{\mathbf{Q}}\mathbf{B}\begin{bmatrix}\hat{v}\\ \hat{w}\end{bmatrix}\hat{w} + \mathbf{B}^{-1}\mathbf{M}^{-1}\mathbf{DB}\begin{bmatrix}\hat{v}\\ \hat{w}\end{bmatrix} + \begin{bmatrix}\dot{v}_d\\ \dot{w}_d\end{bmatrix} + \begin{bmatrix}\tau_{vc}\\ \tau_{wc}\end{bmatrix}\right) \tag{28}$$

where τ_{vc} and τ_{wc} are chosen as

$$\begin{aligned}
\tau_{vc} = & -c_1\tilde{v}_e + \frac{\partial v_e^d}{\partial y_e}\left(-kv_d\varpi_1^{-1}y_e - x_e(w_e + w_d) + p_y\right) + \\
& \frac{\partial v_e^d}{\partial x_e}\left(v_e - v_d\varpi_1^{-1}(\varpi_2 - \varpi_1) + y_e(w_e + w_d) + p_x\right) + \\
& \frac{\partial v_e^d}{\partial v_d}\dot{v}_d + \frac{\partial v_e^d}{\partial t} + k\varpi_1^{-2}\varpi_2^{-1}x_e y_e z_e - \delta_1(x_e^2 + y_e^2)\tilde{v}_e, \\
\tau_{wc} = & -c_2\tilde{w}_e + \frac{\partial w_e^d}{\partial y_e}\left(-kv_d\varpi_1^{-1}y_e - x_e(w_e + w_d) + p_y\right) + \\
& \frac{\partial w_e^d}{\partial x_e}\left(v_e - v_d\varpi_1^{-1}(\varpi_2 - \varpi_1) + y_e(w_e + w_d) + p_x\right) + \\
& \frac{\partial w_e^d}{\partial z_e}\left((1 - k\varpi_2^{-1}x_e)w_e - k\varpi_1^{-2}\varpi_2^{-1}x_e y_e v_e + f_z + p_z\right) + \\
& \frac{\partial w_e^d}{\partial v_d}\dot{v}_d + \frac{\partial w_e^d}{\partial \dot{v}_d}\ddot{v}_d + \frac{\partial w_e^d}{\partial w_d}\dot{w}_d + \frac{\partial w_e^d}{\partial t} - \\
& (1 - k\varpi_2^{-1}x_e)z_e - \delta_2(x_e^2 + y_e^2 + z_e^2)\tilde{w}_e
\end{aligned} \tag{29}$$

where c_i and $\delta_i, i = 1,2$ are positive constants. The terms multiplied by δ_i are nonlinear damping to overcome the effect of the observer errors. Substituting (26), (27), (28) and (29) into (24) yields the closed loop system:

$$\begin{aligned}
\dot{x}_e = & -k_1\varpi_1^{-1}x_e + y_e(w_e + w_d + \tilde{w}) + \tilde{v} + p_x + \tilde{v}_e, \\
\dot{y}_e = & -kv_d\varpi_1^{-1}y_e - x_e(w_e + w_d + \tilde{w}) + p_y, \\
\dot{z}_e = & -k_2 z_e + (1 - k\varpi_2^{-1}x_e)\tilde{w}_e - k\varpi_1^{-2}\varpi_2^{-1}x_e y_e\tilde{v}_e - \\
& k\varpi_2^{-1}\left(x_e\tilde{w} + \varpi_1^{-2}x_e\tilde{v}\right), \\
\dot{\tilde{v}}_e = & -c_1\tilde{v}_e + k\varpi_1^{-2}\varpi_2^{-1}x_e y_e z_e - \frac{\partial v_e^d}{\partial y_e}x_e\tilde{w} - \\
& \frac{\partial v_e^d}{\partial x_e}(y_e\tilde{w} + \tilde{v}) - \delta_1(x_e^2 + y_e^2)\tilde{v}_e + \Omega_v, \\
\dot{\tilde{w}}_e = & -c_2\tilde{w}_e - (1 - k\varpi_2^{-1}x_e)z_e + \frac{\partial w_e^d}{\partial y_e}x_e\tilde{w} + \\
& \frac{\partial w_e^d}{\partial x_e}(y_e\tilde{w} + \tilde{v}) - \frac{\partial w_e^d}{\partial z_e}k\varpi_2^{-1}\left(x_e\tilde{w} + \varpi_1^{-2}x_e\tilde{v}\right) - \\
& \delta_2(x_e^2 + y_e^2 + z_e^2)\tilde{w}_e + \Omega_w
\end{aligned} \tag{30}$$

where Ω_v and Ω_w are the first and second columns of $\boldsymbol{\Omega}$, respectively. To analyze stability of (30), we first consider the $(z_e, \tilde{v}_e, \tilde{w}_e)$-subsystem then move to (x_e, y_e)-subsystem.

4.1 $(z_e, \tilde{v}_e, \tilde{w}_e)$-subsystem

For this subsystem, consider the Lyapunov function

$$V_1 = 0.5\left(z_e^2 + \tilde{v}_e^2 + \tilde{w}_e^2\right) \tag{31}$$

whose derivative along the solution of the last three equations of (30) satisfies

$$\dot{V}_1 \le -k_2 z_e^2 - c_1\tilde{v}_e^2 - c_2\tilde{w}_e^2 + \left(\chi_{11}(\bullet)V_1 + \chi_{12}(\bullet)\right)e^{-\sigma_0(t-t_0)} \tag{32}$$

where $\chi_{11}(\bullet)$ and $\chi_{12}(\bullet)$ are class-K functions of $\|(\tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{X}}(t_0))\|$. From (32), it is direct to show that there exist a positive constant σ_1 and a class-K function $\gamma_1(\bullet)$ of $\|(\mathbf{X}_{1e}(t_0), \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{X}}(t_0))\|$, with $X_{1e} := [z_e, \tilde{v}_e, w_e]^T$, such that $\|\mathbf{X}_{1e}(t)\| \le \gamma_1(\bullet)e^{-\sigma_1(t-t_0)}$, which implies that the $(z_e, \tilde{v}_e, \tilde{w}_e)$-subsystem is globally K-exponentially stable at the origin.

4.2 (x_e, y_e)-subsystem

For this subsystem, we take the Lyapunov function

$$V_2 = \sqrt{1 + x_e^2 + y_e^2} - 1 \tag{33}$$

whose derivative along the solution of the first two equations of (30) satisfies

$$\begin{aligned}
\dot{V}_2 \le & -k_1\varpi_1^{-2}x_e^2 - \left(\lambda_1 v_d^2 - |\lambda_2 v_d \cos(\lambda_3 t)|\right)\varpi_1^{-2}y_e^2 + \\
& \chi_2(\bullet)e^{-\sigma_2(t-t_0)}
\end{aligned} \tag{34}$$

where $\sigma_2 = \min(\sigma_0, \sigma_1)$ and $\chi_2(\bullet)$ is a class-K function of $\|(\mathbf{X}_{1e}(t_0), \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{X}}(t_0))\|$. To analyze stability of (x_e, y_e)-subsystem base on (33) and (34), let us consider each case of Assumption 1.

Case C1. From (34), we have

$$\dot{V}_2 \le -k_1\varpi_1^{-2}x_e^2 + |\lambda_2 v_d| + \chi_2(\bullet)e^{-\sigma_2(t-t_0)}. \tag{35}$$

By integrating both sides of (35) and Barbalat's lemma in [16], it is direct to show that $\lim_{t\to\infty} x_e(t) = 0$. Using (3) and (35), it is seen that $V_2(t) \le \pi_2(\bullet)$ with $\pi_2(\bullet)$ being a class-K function of $\|(\mathbf{X}_{1e}(t_0), \mathbf{X}_{2e}(t_0), \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{X}}(t_0))\|$ with $\mathbf{X}_{2e} := [x_e, y_e]^T$. Therefore $y_e(t)$ is bounded. To prove that $\lim_{t\to\infty} y_e(t) = 0$, applying Lemma 2 in [11] to the first equation of (30) yields:

$$\lim_{t\to\infty}\left(y_e(w_e + w_d + \tilde{w}) + \tilde{v} + p_x + \tilde{v}_e\right) = 0. \tag{36}$$

Since $\lim_{t\to\infty}\left(x_e(t), \tilde{w}_e(t), w_d(t), \tilde{w}(t), \tilde{v}(t), p_x, \tilde{v}_e(t)\right) = 0$, it is direct to show that (36) is equivalent to:

$$\lim_{t\to\infty}\left(\dot{k}y_e^2(1 + (1 - k^2)y_e^2)^{-1}\right) = 0. \tag{37}$$

On the other hand from (35), we have

$$\frac{d}{dt}\left(V_2 - \int_0^t |\lambda_2 v_d(\tau)| d\tau + \sigma_2^{-1}\chi_2(\bullet)e^{-\sigma_2(t-t_0)}\right) \le 0 \tag{38}$$

which means that $V_2 - \int_0^t |\lambda_2 v_d(\tau)| d\tau + \sigma_2^{-1}\chi_2(\bullet)e^{-\sigma_2(t-t_0)})$ is non-increasing. Since V_2 is bounded from below by zero, V_2 tends to a finite nonnegative constant depending on

$\left\|(\mathbf{X}_{1e}(t_0), \mathbf{X}_{2e}(t_0), \tilde{\mathbf{\eta}}(t_0), \tilde{\mathbf{X}}(t_0))\right\|$. This implies that the limit of $|y_e(t)|$ exists and is finite, say l_{y_e}. If l_{y_e} was not zero, there would exist a sequence of increasing time instants $\{t_i\}_{i=1}^{\infty}$ with $t_i \to \infty$, such that both of the limits of $\dot{k}(t_i)$ and $\dot{k}(t_i)y_e^2(t_i)$ are not zero, which is impossible because of (37) for any $\lambda_2 \neq 0$ and $\lambda_3 \neq 0$. Hence l_{y_e} must be zero. Therefore we conclude from (37) that $\lim_{t\to\infty} y_e(t) = 0$.

Case C2. From $V_2(t) \leq \pi_2(\bullet)$ and (34), we have

$$\dot{V}_2 \leq -\frac{k_1 x_e^2}{(1+\pi_2^2)} - \frac{\lambda_1 v_d^2 y_e^2}{(1+\pi_2^2)} + |\lambda_2 v_d| y_e^2 + \chi_2(\bullet)e^{-\sigma_2(t-t_0)} \quad (39)$$

which means that there exist $\sigma_3 > 0$ and a class-K function $\gamma_2(\bullet)$ depending on $\left\|(\mathbf{X}_{1e}(t_0), \mathbf{X}_{2e}(t_0), \tilde{\mathbf{\eta}}(t_0), \tilde{\mathbf{X}}(t_0))\right\|$ such that $\|X_{2e}(t)\| \leq \gamma_2(\bullet)e^{-\sigma_3(t-t_0)}$ as long as

$$\int_{t_0}^{t}\left(\frac{\lambda_1 v_d^2(\tau)}{(1+\pi_2^2)} - |\lambda_2 v_d(\tau)|\right)d\tau \geq \mu_{21}^{*}(t-t_0) \quad (40)$$

where μ_{21}^{*} is a positive constant. It is seen from (4) and (40) that there always exist λ_1 and λ_2 such that (40) holds.

Case C3. This case can be processed similarly to C1.
We have thus proven the following result.

Theorem 1. *Under Assumptions 1 and 2, the global output-feedback control law (28) forces the mobile robot (1) to asymptotically track the virtual vehicle (2) if the constants $\lambda_i, i = 1,2,3$ are chosen such that $\lambda_i \neq 0$, (40) holds, and $|\lambda_1 v_d(t)| + |\lambda_2| < 1, \forall t$.*

5. Simulations

In this section we perform some simulations to illustrate the effectiveness of the proposed controller. The physical parameters are taken from [13]: $b = 0.75, d = 0.3$, $r = 0.15, m_c = 30, m_w = 1, I_c = 15.625, I_w = 0.005, I_m = 0.0025$, $d_{11} = d_{22} = 10$. The reference velocities are chosen as: For case of C1: $v_d = w_d = 0$; for case of C2: $v_d = 2$, $w_d = 0$ for the first 20 seconds and $v_d = 2, w_d = 0.2$ for the rest; for case of C3: $v_d = 0, w_d = 0.2$. The initial conditions are picked as: $(\mathbf{\eta}^T, \mathbf{v}^T) = ((1,1,0.5),(0,0))$, $(\hat{\mathbf{\eta}}^T, \hat{\mathbf{X}}^T) = ((0,0,0),(0,0))$; $(x_d, y_d, \phi_d) = (0,0,0)$.
Based on Theorem 1, control and observer gains are chosen as $k_1 = 1, k_2 = 2$, $c_1 = c_2 = 3, \mathbf{P}_{01} = \mathbf{P}_{02} = diag(1,1)$, $\lambda_1 = 0.4, \lambda_2 = 0.05, \lambda_3 = 4, \mathbf{K}_{01} = diag(1,1), \mathbf{K}_{02} = (\mathbf{J}(\mathbf{\eta})\mathbf{Q}^{-1}(\mathbf{\eta}))^T$ It can be verified that the above choice satisfies requirements in Theorem 1. Results are plotted in Figures 1-3 (robot position in (x,y) plane). The tracking errors in the form of $\sqrt{x_e^2 + y_e^2 + \phi_e^2}$ are plotted in Figure 4.

6. Conclusions

A time-varying global output-feedback controller has been presented to solve both tracking and stabilization for mobile robots simultaneously at the torque level. The keys to success of our proposed control design are the coordinate transformations (6) and (23). Current work is underway to extend our proposed methodology to a class of mechanical systems.

References

[1] A. Astolfi, Discontinuous control of nonholonomic systems, *Systems and Control Letters*, vol. 27, 1996, pp. 37-45.

[2] A. Behal, D.M. Dawson, W.E. Dixon and Y. Fang, Robust tracking and regulation control for mobile robots. *Proceedings of the IEEE Conference on Control and Application*, 1999, pp. 2150-2155.

[3] A.M. Bloch and S. Drakunov, Stabilization and tracking in the nonholonomic integrator via sliding mode, *Systems and Control Letters*, vol. 29, 1996, pp. 91-99.

[4] C. Canudas de Wit, B. Siciliano and G. Bastin (Eds.), *Theory of Robot Control*, Springer, London, 1996.

[5] C. Samson and K. Ait-Abderahim, Feedback control of a nonholonomic wheeled cart in Cartesian space, *Proceedings of IEEE International Conference on Robotics and Automation*, 1991, pp. 1254-1259.

[6] C. Samson, Control of chained systems-Application to path following and time-varying point stabilization of mobile robots, *IEEE Transactions on Automatic Control*, vol. 40, no. 1, 1995, pp. 64-77.

[7] I. Kolmanovsky and N.H. McClamroch, Developments in nonholonomic control problems, *IEEE Control Systems Magazine*, vol. 15, 1995, pp.20-36.

[8] J-M. Yang and J-H. Kim, Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots, *IEEE Transactions on Robotics and Automation*, vol. 15, no. 3, 1999, pp. 578-587.

[9] T.C. Lee, K.T. Song, C.H. Lee and C.C. Teng, Tracking control of unicycle-modelled mobile robots using a saturation feedback controller, *IEEE Transactions on Control Systems Technology*, vol. 9, no. 2, 2001, pp. 305-318.

[10] Z.P. Jiang and H. Nijmeijer, A recursive technique for tracking control of nonholonomic systems in chained form, *IEEE Transactions on Automatic Control*, vol. 44, 1999, pp. 265-279.

[11] Z.P. Jiang and H. Nijmeijer, Tracking control of mobile robots: a case study in backstepping, *Automatica*, vol. 33, 1997, pp. 1393-1399.

[12] R. Fierro and F.L. Lewis, Control of a nonholonomic mobile robot: backstepping kinematics into dynamics, *Proceedings of IEEE/RSJ International Work-*

shop Intelligent Robots and Systems, 1991, pp. 193-198.

[13] T. Kukao, H. Nakagawa and N. Adachi, Adaptive tracking control of nonholonomic mobile robot, *IEEE Transactions on Robotics and Automation*, vol. 16, no. 5, 2000, pp. 609-615.

[14] R.W. Brockett, Asymptotic stability and feedback stabilization, in R.W. Brockett, R.S. Millman and H.J. Sussmann (eds), *Differential geometric control theory*, 1983, pp. 181-191.

[15] M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic, *Nonlinear and adaptive control design*, New York: Wiley, 1995.

[16] H.K. Khalil, *Nonlinear systems*. 3rd ed., Prentice-Hall, NJ, 2002.

[17] Loria A. Global tracking control of one degree of freedom Euler-Lagrange systems without velocity measurement. *European Journal of Control*, 1996, pp. 144-151.

[18] G. Besancon, Global output feedback tracking control for a class of Lagrangian systems. *Automatica*, vol. 36, 2000. pp. 1915-1921.

[19] Z.P. Jiang and I. Kanellakopoulos. Global output-feedback tracking for a benchmark nonlinear system. *IEEE Transactions on Automatic Control*, 2000, pp. 1023-1027.

[20] Z.P. Jiang, Lyapunov design of global state and output feedback trackers for nonholonomic control systems. *International Journal of Control*, vol. 73, no. 9,2000, pp. 744-761.

[21] A. Loria and K. Melhem (2002). Position feedback global tracking control of EL systems: A state transformation approach. *IEEE Transactions on Automatic Control*, vol. 47, 2002, pp. 841-847.

[22] T.I. Fossen and H. Nijmeijer (Eds). *New directions in nonlinear observer design*. Lecture Notes in Control and Information Sciences 244, Springer-Verlag, London, 1999.

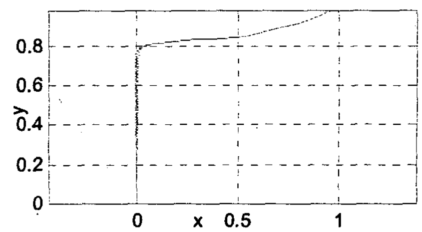

Figure 1. Case of C1: Robot position in (x,y) plane.

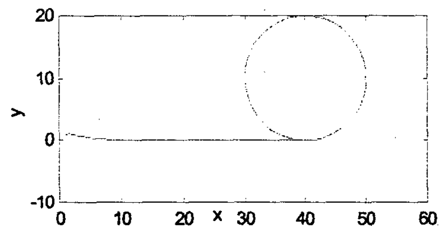

Figure 2. Case of C2: Robot position in (x,y) plane.

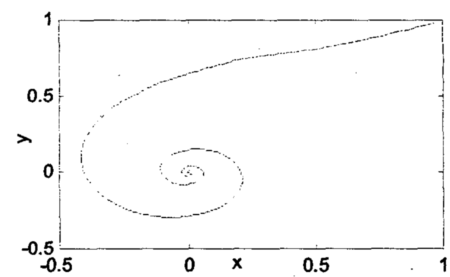

Figure 3. Case of C3: Robot position in (x,y) plane.

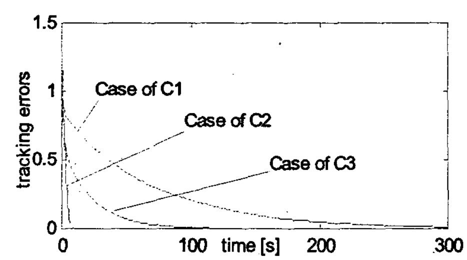

Figure 4. Tracking errors with respect to C1, C2, C3.