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A Generalized $\mathcal{H}^\infty$ Control Design Framework for Stable Multivariable Plants subject to Simultaneous Output and Input Loop Breaking Specifications

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Abstract— **In this paper, we present a generalized mixed-sensitivity multivariable framework for linear time invariant (LTI) plants that can handle a broad class of closed loop (e.g. $\mathcal{H}^\infty$, $\mathcal{H}^2$, frequency- and time domain) objectives while being able to directly and systematically address the problem of trading off properties at distinct loop breaking points. This is done by exploiting the Youla-Jabr-Bongiorno-Kucera-Zames (YJBKZ) parameterization, the resulting convexification, and efficient convex solvers that can be applied to smooth as well as non-differentiable problems. Our approach is shown to be particularly useful for ill-conditioned plant having large relative gain array entries - plants that have received considerable attention in the literature without yielding a direct systematic design methodology. Moreover, we also show how our approach can be applied to multivariable infinite-dimensional plants. We specifically show that by suitably approximating the infinite-dimensional plant with a finite-dimensional approximant, a near-optimal finite-dimensional controller can be designed for the infinite-dimensional plant. Illustrative examples are provided.**

I. Introduction and Overview

Motivation. It is well understood that, in general, a good multivariable control system design must possess acceptable properties at distinct loop breaking points [1, 2]; e.g. the plant output and the plant input. Acceptable properties include satisfying nominal performance specifications as well as robustness specifications. It is also well known that tradeoffs can be particularly taxing when the plant is ill-conditioned [1–4] and or has large relative gain array (RGA) entries [5–7]. While much insight has been obtained, there still remains a need for a tool that can be used to systematically address the associated tradeoffs. It is natural, for example, to ask for a tool that can be used to "equilibrate" (to the extent possible) properties at distinct loop breaking points as well as to manage the associated tradeoffs. Moreover, we also want a tool to handle a large set of control objectives and constraints (e.g. peak frequency response, peak overshoot, peak controls, etc.) without undue computational hardship - leveraging (for example) off of the efficient convex optimization solvers [20–23]. Our work is motivated by the above fundamental control-relevant design issues.

Challenges and Related Work. During the past three decades, multiobjective optimization has become a valuable tool to address multiple control performance/robustness specifications. Much has been done in the area of mixed $\mathcal{H}^2/\mathcal{H}^\infty$ optimization [8–11] to address frequency and time-domain specifications that can be conflicting. Many algorithms/approaches based on convex optimization and LMIs have also been investigated [10–14] for multiobjective control design. While these approaches have been successfully apply to address a broad range of specifications, little has appeared in the literature about using these methodologies to address closed loop properties at distinct loop breaking points. This is one of the main points addressed within this paper. Within [15], the author proposed what is called a "generalized" weighted $\mathcal{H}^\infty$ mixed-sensitivity problem subject to convex constraints. Exploiting ideas from convex optimization ideas from [16, pg. 62] by formulating an optimization that directly addresses closed loop maps associated with distinct loop breaking points (e.g. output and input). Within the current paper, we demonstrate the utility of this approach for directly shaping properties at distinct loop breaking points. This is particularly beneficial for multivariable plants that are ill-conditioned and/or have large relative gain array (RGA) entries [1–7]. Our proposed generalized mixed-sensitivity framework provides a tool for directly addressing tradeoffs and equilibration issues associated with distinct loop breaking points.

Moreoever, we also show how our approach can be used to design finite-dimensional controllers for stable multivariable infinite-dimensional (and complex) plants. This is particularly useful given that the more direct design approach [43–50], where one works directly with the infinite-dimensional plant, is often very mathematical and difficult/impossible to apply to problems involving complex control objectives. Despite this issue, it is understood that such an approach (when viable) can lead to fundamental understanding. However, given the above critical shortcoming, we find it practical (for computational viability and tractability) to take an indirect *approximate-and-then-design* approach as developed within [30–42]. This approach, consistent with everyday engineering practice has virtually unlimited potential - particularly when we can leverage off of fast interior point convex optimization solvers [20–23].

Control Methodology. $\mathcal{H}^\infty$ control problems have been used

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extensively as a frequency domain loopshaping technique [17]. In an effort to address loop shaping at distinct loop-breaking points (e.g. plant output and input), we propose a constrained generalized mixed-sensitivity minimization which involves maps associated with breaking the loop at the input as well as at output. The formulated generalized constrained mixed-sensitivity problem relies on the seminal ideas presented within [16, pg. 62] [11].

Goals and Contributions. This paper addresses and provides concrete answers to the following control questions:

1) How can we develop a multivariable control framework to accommodate typical control constraints involving multiple loop breaking points? Such a framework can be particularly useful for ill-conditioned plants which have large relative gain array terms [1–7].
2) How can the framework be applied to infinite-dimensional (or complex) stable plants? Specifically, how can we construct a suitable finite-dimensional plant approximant P_n that can be used to directly design a finite-dimensional controller K_n which is near-optimal for the infinite-dimensional plant P?

Organization. The remainder of the paper is organized in the following manner. Section II describes the proposed generalized mixed-sensitivity design framework. The section also describes the technical approach taken to convexify and solve the problem. The algorithm used is also described. Section III presents results for stable infinite-dimensional plants. The section specifically shows how to approximate the infinite-dimensional plant P by stable approximants P_n so that a near-optimal finite-dimensional controller K_n can be obtained for P. Section IV contains illustrative multivariable control design examples. Finally, Section V summarizes the paper and presents directions for future research.

II. Proposed Design Framework

In this section, we present the proposed generalized $\mathcal{H}^\infty$ mixed-sensitivity control design framework. The main purpose of the framework is to be able to address specifications at distinct loop shaping points (e.g. output and input). More specifically, we want a framework that permits a designer to shape multiple sensitivity transfer function matrices; e.g. the sensitivity $S_o = [I+L_o]^{-1}$ associated with breaking the loop at the plant output ($L_o = PK$) as well as that the sensitivity $S_i = [I + L_i]^{-1}$ associated with breaking the loop at the plant input ($L_i = KP$). Toward this end, we propose the following generalized mixed-sensitivity optimization.

Proposed Generalized $\mathcal{H}^\infty$ Mixed-Sensitivity Optimization. To address the above, we propose the following generalized weighted $\mathcal{H}^\infty$ mixed-sensitivity problem:

$$K = arg\left\{ \min_{\substack{K \\ stabilizing}} \gamma \mid \max\left(\left\| \begin{bmatrix} W_1 S_o \\ W_2 K S_o \\ W_3 T_o \end{bmatrix} \right\|_{\mathcal{H}^\infty}, \left\| \begin{bmatrix} W_4 S_i \\ W_5 P S_i \\ W_6 T_i \end{bmatrix} \right\|_{\mathcal{H}^\infty} \right) < \gamma \right\} \quad (1)$$

subject to convex frequency- and/or time-domain constraints on the closed loop maps. Here, W_1-W_6 are $R\mathcal{H}^\infty$ frequency-dependent weighting matrices that are used to shape the closed properties of S_o, KS_o, T_o, S_i, PS_i and T_i. The above framework captures the traditional mixed-sensitivity at the output problem that has been widely addressed within the controls literature [17–19] ($W_{4,5,6} = 0$ with no constraints) as well the not so broadly addressed mixed-sensitivity at the input problem ($W_{1,2,3} = 0$ with no constraints). The former can be used to systematically achieve desirable properties at the output, while the latter can be used to achieve desirable properties at the input. By combining the two as above, a designer can use the weightings to systematically shape and tradeoff properties at both loop breaking points.

Solution Method. The approach taken in this work is now described.

- Achieving Convexity via YJBKZ Parameterization. The approach relies on using the YJBKZ Q-Parameterization [25–29], to transform the transfer function matrices (S_o, KS_o, T_o, S_i, PS_i & T_i) that depend nonlinearly (linear fractionally) on K into transfer function matrices that depend affinely on the stable YJBKZ parameter Q (stable transfer function matrix). Since the $\mathcal{H}^\infty$ norm is a convex functional [16] and the constraints are convex in the closed loop transfer function matrices, the YJBKZ parameterization results in a convex problem in Q.
- Obtaining a Finite-Dimensional Convex Problem. Because Q can be an arbitrary stable ($\mathcal{H}^\infty$) transfer function matrix, the resulting problem is infinite-dimensional (even for finite-dimensional plants). Fortunately, any real-rational Q (or more generally continuous Q) may be approximated uniformly by a finite linear combination of real-rational stable $R\mathcal{H}^\infty$ transfer function matrices. This transforms the problem in Q to a finite-dimensional convex optimization problem.

It should be noted that many control system performance specifications may be posed as convex constraints [11, 16] (e.g. overshoot, peak magnitude frequency response, etc.).

Convex Optimization Algorithm. Within our design environment we have implemented two convex solvers: (1) Kelley's cutting plane method [24] and (2) Analytic Center Cutting-Plane Method (ACCPM) [22, 23]. Currently, ACCPM is as our main algorithm for solving convex control optimization problems. ACCPM combines the simplicity of a cutting-plane method with the efficiency of an interior point method. The ACCPM, in contrast to most CPMs (e.g. Kelley's CPM), is in practice polynomial-time fast. [20–23]. Both algorithms also permit us to exploit gradient information as well as subgradient information - the latter being critical for non-differentiable convex problems.

III. Results for Stable Infinite-Dimensional Plants

In this section, we present results for addressing stable infinite-dimensional plants $P \in \mathcal{H}^\infty$. In contrast to the more mathematical design-and-approximate approach

[43–50] taken in where one directly designs an infinite-dimensional controller and then approximates it by a finite-dimensional controller, we take a more numerical approximate-and-design approach [18, 30–42] whereby we begin by approximating the infinite-dimensional plant P by finite-dimensional approximants $P_n \in R\mathcal{H}^\infty$. We now present our main result. In short, the result shows how to approximate the infinite-dimensional plant P by a finite-dimensional approximant P_n which we use to solve our generalized mixed-sensitivity problem. The result is a near-optimal finite-dimensional controller K_n.

Definitions and Notation. It is useful to introduce a few definitions and notation.

- Let μ_{opt} denote the *optimal generalized mixed-sensitivity performance* for the infinite-dimensional plant P. We associate with these a near-optimal infinite-dimensional control K_{opt} which, in general, is difficult to obtain.
- Let μ_n denote the performance for the finite-dimensional approximant P_n. We associated with P_n and μ_n a near-optimal finite-dimensional controller K_n. If P_n approximates P closely, then it is reasonable to refer to μ_n as the *expected performance*.
- Let $\tilde{\mu}_n$ denote the *actual performance*; i.e. the performance associated with using K_n with P. This performance, of course, is well defined if and only if K_n internally stabilizes P.

Given the above, we can now state our main result for stable infinite-dimensional plants P.

Theorem 3.1: **Main Result for Stable Infinite-Dimensional Plants.** Assume $W_i, W_2^{-1} \in R\mathcal{H}^\infty$. Let $P \in \mathcal{H}^\infty$ and $\{P_n\}_{n=1}^\infty$ denote a sequence of $R\mathcal{H}^\infty$ matrix-valued functions such that

$$\lim_{n\to\infty} \|P_n - P\|_{\mathcal{H}^\infty} = 0. \tag{2}$$

Given this, it then follows that

$$\lim_{n\to\infty} \mu_n = \mu_{opt} \tag{3}$$

$$\lim_{n\to\infty} \tilde{\mu}_n = \mu_{opt} \tag{4}$$

More specifically, suppose that one is given a desired *a priori* performance tolerance $\epsilon > 0$, however small. Given this, there exists $\delta(\epsilon), M(W_i, P) > 0$ and a positive integer $N(\delta(\epsilon)) \in Z_+$ such that if

$$\|P_n - P\|_{\mathcal{H}^\infty} \le \delta = \frac{\epsilon}{M(W_i, P)} \qquad \forall n \ge N_\epsilon \tag{5}$$

then we have the following for all $n > N$:

$$\mu_{opt} \le \mu_n \le \mu_{opt} + 4\epsilon \tag{6}$$

$$\mu_{opt} \le \tilde{\mu}_n \le \mu_{opt} + 4\epsilon \tag{7}$$

Proof: The outline of the proof is as follows:

1) Get upper bound M_o for μ_{opt} (corresponding to $Q = 0$).
2) Get *a priori* computable bound B_1 on Q_o by exploiting invertibility of W_2 in $R\mathcal{H}^\infty$.
3) *Upper semi-continuity:* Show
$$\mu_n \le \mu_{opt} + 2\epsilon \tag{8}$$
4) Get *a priori* computable upper bound B_2 on Q_n by exploiting invertibility of W_2 in $R\mathcal{H}^\infty$.
5) *Lower semi-continuity:* Show
$$\mu_{opt} \le \mu_n + \epsilon \le \mu_{opt} + 3\epsilon \tag{9}$$
6) *Stability:* Show K_n stabilizes P_n by showing
$$\|(P_n - P)Q_n\|_{\mathcal{H}^\infty} < \epsilon_s < 1 \tag{10}$$
7) Show that the finite-dimensional controller $K_n = Q_n(I - P_nQ_n)^{-1}$ is near-optimal for P; i.e.

$$\begin{aligned}
\mu_{opt} \le \tilde{\mu}_n &\le \frac{\mu_n}{1 - \|(P_n - P)Q_n\|_{\mathcal{H}^\infty}} && (11)\\
&\le \frac{\mu_{opt} + 2\epsilon}{1 - \epsilon_s} && (12)\\
&\le \mu_{opt} + \left[\frac{\mu_{opt}\epsilon_s + 2\epsilon}{1 - \epsilon_s}\right] && (13)\\
&\le \mu_{opt} + 3\epsilon && (14)
\end{aligned}$$

The theorem then follows from the above. ■

IV. Illustrative Examples

Three examples are now presented to illustrate the utility of our generalized mixed-sensitivity approach.

A. Example 1: Finite-Dimensional Ill-Conditioned Plant

In this example, we consider the following finite-dimensional ill-conditioned plant from Freudenberg [2]:

$$P(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 9 & -10 \\ -8 & 9 \end{bmatrix} \tag{15}$$

This plant possesses a large condition number over all frequencies as well as large relative gain array entries [1–7]. Seven designs were generated for this plant:

1) Mixed Sensitivity at Output (MSO).
2) Mixed Sensitivity at Input (MSI).
3) Freudenberg-like design [2] (denoted freu replica) where S_i is emphasized more.
4) Inverse Freudenberg-like design (denoted freu inverse) where S_o is emphasized more.
5) Equilibrated design with no roll-off in KS_o (denoted freu equi) where S_o and S_i are equally emphasized.
6) Equilibrated design with roll-off in KS_o (denoted equilibrated) where S_o and S_i are equally emphasized.
7) Convex Constrained Equilibrated design with roll-off in KS_o (denoted equi constr) where an additional peak control $\mathcal{L}^\infty$ time-constraint has been added.

Table I summarizes the $\mathcal{H}^\infty$ norms for various closed loop transfer function matrices showing fundamental closed loop trade-offs that were made.

Specifically, Table I shows how the framework can be used to trade-off closed loop properties at distinct loop breaking points. Design 1 (MSO) has a good peak S_O but a very bad peak S_i. Design 2 (MSI) has reversed properties. Design 3

Designs	S_o	S_i	T_o	T_i	KS	PS_i
1) MSO	1.58	34.46	0.00	34.46	29.38	12.37
2) MSI	5.04	1.85	3.43	0.00	36.28	2.10
3) Freu Replica	2.08	0.00	5.16	2.86	53.86	-3.798
4) Freu Inverse	0.00	4.40	1.51	6.12	53.80	0.00
5) Freu Equi	0.00	0.00	2.88	2.88	53.70	-1.44
6) Equilibrated	2.35	2.10	0.80	1.53	36.97	6.62
7) Equi-Constr	2.59	2.18	1.40	0.00	35.21	4.96

TABLE I

$\mathcal{H}^\infty$ NORMS FOR CLOSED LOOP MAPS (DB)

replicates the Freudenberg subspace -based design within [2]. It exhibits a very good peak S_i and an very acceptable peak S_o. Design 4 has reversed properties. Design 5 yields S_o and S_i with similar peaks. We refer to this as an equilibrated design. Design 6 represents an equilibrated design with a lower (perhaps more realistic) bandwidth. As claimed, our generalized mixed-sensitivity framework permits designers to systematically address input-output equilibration tradeoffs. Finally, design 7 includes a peal controls constraint. Being able to address such constraints is very important when saturating actuators are a key controls concern.

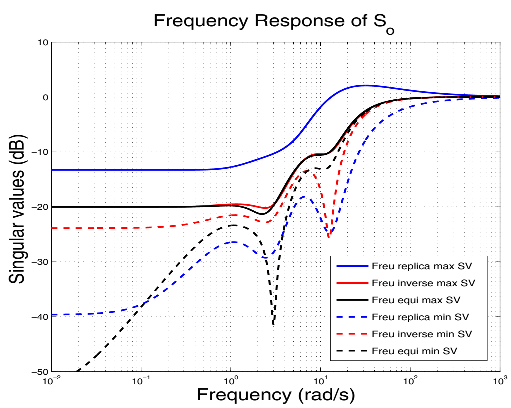

Fig. 1. Maximum Singular Value for S_o for Designs 3-5

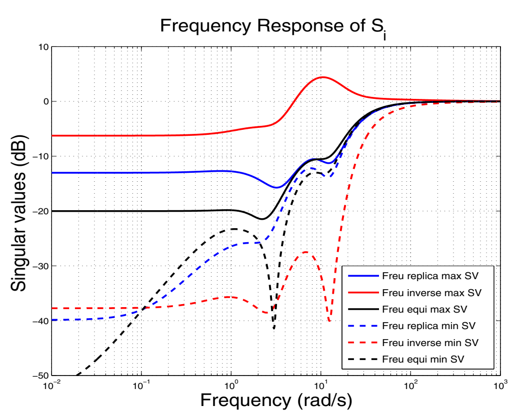

Fig. 2. Maximum Singular Value for S_o for Designs 3-5

Figure 3 shows sensitivity singular values for the equilibrated Design 6. We see that S_o and S_i have similar (equilibrated) peaks. Our framework facilitated equilibration as well as the associated tradeoffs. Hence, the utility of our framework and tool.

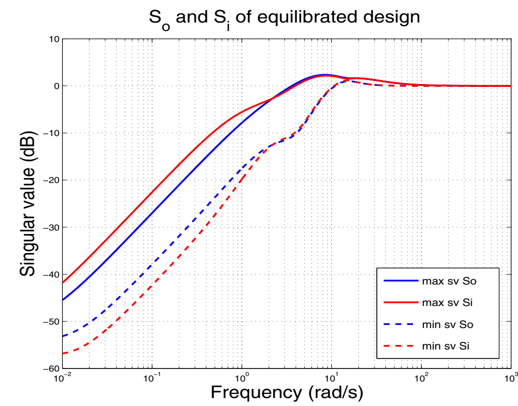

Fig. 3. S_o and S_i for Equilibrated Design 6

Figure 4 shows the control responses to step references commands that result for Design 6 (unconstrained) and Design 7 (constrained).

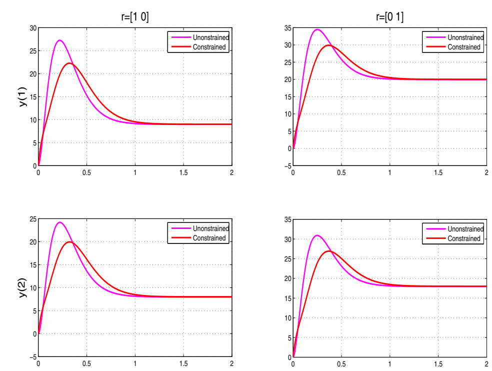

Fig. 4. Control Responses for Designs 6-7; 7 having $|u_2| \leq 30$ constraint

Weighting functions for Design 1: MSO

$$W_1 = \frac{0.714(s+1.4)}{s+0.001} \times I_{2\times 2} \tag{16}$$

$$W_2 = \frac{1000\,(s+1.5)}{s+1e5} \times I_{2\times 2} \tag{17}$$

$$W_3 = \frac{1000\,(s+7.143)}{s+1e4} \times I_{2\times 2} \tag{18}$$

$$W_4 = W_5 = W_6 = 0 \times I_{2\times 2} \tag{19}$$

Weighting functions for Design 2: MSI

$$W_1 = W_2 = W_3 = 0 \times I_{2\times 2} \tag{20}$$

$$W_4 = \frac{0.714(s+1.4)}{s+0.001} \times I_{2\times 2} \tag{21}$$

$$W_5 = \frac{1.4\,(s+2)^2}{(s+4)\,(s+1)} \times I_{2\times 2} \tag{22}$$

$$W_6 = \frac{1000\,(s+7.143)}{s+1e4} \times I_{2\times 2} \tag{23}$$

Weighting functions for Design 3: Freu Replica

$$W_1 = \frac{0.667(s+25.5)}{s+2.55} \times I_{2\times 2} \tag{24}$$

$$W_4 = \frac{0.909\,(s+66)}{s+9.6} \times I_{2\times 2} \tag{25}$$

$$W_2 = W_3 = W_5 = W_6 = 0 \times I_{2\times 2} \tag{26}$$

Weighting functions for Design 4: Freu Inverse

$$W_1 = 0.667(s+25.5)/(s+2.55) \times I_{2\times 2} \quad (27)$$
$$W_2 = 0.133(s+4.5)/(s+2000) \times I_{2\times 2} \quad (28)$$
$$W_3 = W_4 = W_5 = W_6 = 0 \times I_{2\times 2} \quad (29)$$

Weighting functions for Design 5: Equilibrated (no control roll-off)

$$W_1 = W_4 = 0.667(s+25.5)/(s+2.55) \times I_{2\times 2} \quad (30)$$
$$W_2 = W_3 = W_5 = W_6 = 0 \times I_{2\times 2} \quad (31)$$

Weights for Design 6: Equilibrated with control roll-off.

$$W_1 = W_4 = \frac{0.7143(s+1.4)}{s+0.001} \times I_{2\times 2} \quad (32)$$
$$W_2 = \frac{1000(s+4.5)}{s+3e5} \times I_{2\times 2} \quad (33)$$
$$W_3 = W_6 = \frac{1000(s+7.143)}{s+10000} \times I_{2\times 2} \quad (34)$$
$$W_5 = \frac{1.4(s+2)^2}{(s+4)(s+1)} \times I_{2\times 2} \quad (35)$$

For all the designs a basis of the form $\frac{z-s}{s+p}$ where $p = z = 2$ has been used. The number of basis terms was selected so that the objective function value is within 2% of optimal.

B. Example 2: Ill-Conditioned Plant with Time Delay

In this example, we consider the plant addressed in Example 1 with an additional time delay $e^{-s\Delta}$ in the first control channel:

$$P(s) = \begin{bmatrix} \frac{9e^{-s\Delta}}{s+1} & \frac{-10}{s+1} \\ \frac{-8e^{-s\Delta}}{s+2} & \frac{9}{s+2} \end{bmatrix} \quad (36)$$

Like the system considered within Example 1, this system is also highly ill-conditioned with large relative gain array entries [1–7]. Below, the time delay is approximated using an $[n, n]$ Pade approximation. By so doing, we construct finite-dimensional approximants P_n which uniformly approximate P in $\mathcal{H}^\infty$. The following matrix weightings $W_1 - W_6 \in R\mathcal{H}^\infty$ were used:

$$W_1 = W_4 = \frac{0.1\,s+0.11}{s+0.001} \times I_{2\times 2} \quad (37)$$
$$W_2 = \frac{100\,s+250}{s+50000} \times I_{2\times 2} \quad (38)$$
$$W_3 = W_6 = \frac{100\,s+200}{s+2000} \times I_{2\times 2} \quad (39)$$
$$W_5 = \frac{(s+5)^2}{(s+50)(s+0.5)} \times I_{2\times 2} \quad (40)$$

The following all-pass (inner, Blashke) basis element was used: $q_k = \left(\frac{z-s}{s+p}\right)^{k-1}$ where $p = z = 2$ and $k = 1, .., N$. Here, we used $N = n + 3$ where n is the order of the Pade approximation.

Table II shows the convergence of μ and $\tilde{\mu}_n$. Figure 5

n	μ_n	$\tilde{\mu}_n$	n	μ_n	$\tilde{\mu}_n$
1	1.1693	1.9264	9	1.0249	1.0302
2	1.0740	1.8214	10	1.0222	1.0244
3	1.0487	1.7496	11	1.0209	1.0232
4	1.0360	1.0901	12	1.0205	1.0218
5	1.0266	1.0804	13	1.0203	1.0204
6	1.0352	1.0470	14	1.0202	1.0202
7	1.0294	1.0408	15	1.0202	1.0202
8	1.0293	1.0357			

TABLE II

CONVERGENCE OF μ_n AND $\tilde{\mu}_n$ FOR TIME-DELAY PLANT

shows how the closed loop frequency response converge (P with K_n). One can prove (using Arzela-Ascoli and normal family/equicontinuity concepts) that the frequency responses converge uniformly on compact subsets. Figure 5 gives one an idea of what a near-optimal S_i looks like. Figure 6 gives one an idea of what a near-optimal S_o looks like.

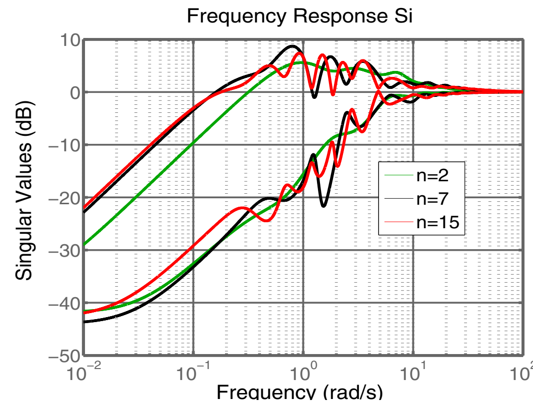

Fig. 5. Convergence of $S_i = [I + L_i]^{-1}$ ($L_i = K_n P$)

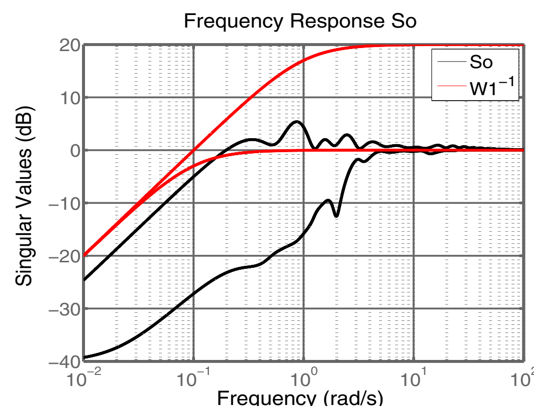

Fig. 6. Near-Optimal $S_o = [I + L_o]^{-1}$ ($L_o = PK_{15}$)

V. SUMMARY & FUTURE WORK

In this paper, we have presented a generalized mixed-sensitivity framework for multivariable control system design. The framework is based on the YJBKZ parameterization and convex optimization. The main utility of the framework is that it permits designers to address specifications at distinct loop breaking points. This is particularly useful for plants that exhibit poor conditioning and/or large relative gain array entries. Moreover, the framework permit designers to accommodate general convex time and frequency domain constraints. The method can also be applied to infinite dimensional plants which are approximated by uniform finite-dimensional approximants. A proof is given for stable infinite-dimensional plants. The main result for infinite-dimensional plants is that we can determine a priori how well the approximants P_n need to approximate the infinite-dimensional plant P in order for the resulting finite-dimensional controllers K_n to approach near-optimal performance for the infinite-dimensional plant. Illustrative examples were provided.

Future work will examine the pros and cons of using different (1) cost functions (e.g. sum or norms versus max of norms, etc.), (2) approximation methods and (3) bases. Future work will also address the pros and cons associated with using weighted constraints versus weighted objectives as well as unstable multivariable infinite-dimensional systems.

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