Proceedings of the 8th
World Congress on Intelligent Control and Automation
July 6-9 2010, Jinan, China

Coordinated Motion Planning of Multiple Mobile Robots in Formation

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Abstract—**This paper presents an approach to coordinating a group of mobile robots that move along the designed paths. Subject to the constraints of velocity/acceleration bounds and collision avoidance, the robots are required to maintain the formation relationship while moving. The coordination is realized by posing the coordination problem as a velocity optimization problem in motion planning and then properly planning robots' velocities along the paths. The linear interactive and general optimizer (lingo) is utilized to achieve the velocity optimization goal. Simulations are performed on a group of mobile robots to demonstrate the effectiveness of the proposed approach in coordinated motion planning with the formation requirement.**

I. Introduction

Multirobots have been widely used in industrial plants and warehouses. In many multirobot applications, robots are required to form formations to accomplish complex tasks such as transportation of large awkward objects, mapping, search, and rescue. An important issue in these applications lies in optimal motion planning of the robots, which becomes more important especially when the task is executed repeatedly or resources must be conserved [1][2]. This paper discusses the optimal motion planning of multiple mobile robots. The robots are required to follow the designed paths, while moving in a desired formation.

Existing methodologies of formation forming may be distinguished by their level of constraint. In a common topic of formation forming, each robot has a predefined role with its start and goal position [3-5]. For instance, in [6], a group of robots are placed in a cluttered environment and required to move to their individual goals that are assigned in a formation. The formation is formed when all the robots achieve their goals.

In a slight constrained problem, one of the robot's path has been already designed, and the other robots are required to maintain formation with this robot [7][8]. A popular category of the methods dealing with this problem is leader-follower method, where the path-constrained robot is usually defined as leader, and the other robots are defined as followers [9]. The leader robot plans the path of the group itself, and the followers keep formation with the leader. When the leader reaches its goal and the desired formation between the leader and followers are formed, the formation task is completed.

If the problem is further constrained so that the paths of all the robots are specified, one obtains a path coordination problem [10][11]. Recently, Sun et al [12] proposed a synchronization approach control approach to solve this kind of problem. The robots are required to follow their designed paths to switch amongst a set of desired formation. A generalized superellipse with varied parameters is used to represent the different kinds of formation curves. In this paper, this problem is extended to a common one, in which the formation relationships are represented by constraint network, and the optimal motion planning to reduce the formation errors is proposed. We propose to formulate this motion planning problem as a velocity optimization problem, which is further solved in mathematical optimal tools. A related work in the formulation has been proposed in [13], in which the problem of path coordination is formulated as a Mixed Integer Non-Linear Programming problem, and the formulation was further extended subject to communication constraint in [14]. However, these existing methods do not consider formation requirement. In this paper, the constraints of the velocity/acceleration bounds and collision avoidance are considered. An objective function is established based on velocity optimization goal, to measure the formation performance with the generated velocity profiles. Through the use of Linear Interactive and General Optimizer (Lingo) [15], the velocity optimization problem can be solved.

In some other problems of formation planning, robots do not have individual identities or assignments [16]. The robots decide their motions based on their current locations, and the locations of whatever other robots are nearby. Generally, there is no specific goal configuration; instead robots move until certain local constraints are met [17]. Behavior-based methods are used in these problems, in which some simple actions are defined, and the individual robot performs these low-level actions to make the group behavior accomplished [18][19].

The remainder of this paper is organized as follows. Section II presents the coordinated problem and the robot model. Section III formulates the problem of coordinated motion planning of robots with the formation requirement. In Section IV, simulations are provided to show the effectiveness of the proposed approach. Finally, conclusions of this work are given in Section V.

This work was supported in part by a grant from Research Grants Council of the Hong Kong Special Administrative Region, China (Refer-ence no. CityU 119907), and a grant from City University of Hong Kong (Reference no. 7002461).

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II. Overview of the Problem

Consider that a group of mobile robots follow the designed paths while meeting the formation requirement in a dynamic environment, as shown in Fig.1. The robots are required to move along the paths to gradually form the target formation as labeled in the dashed circle. The problem is to design velocity profiles of the robots to avoid collisions with moving obstacles or other robots, while trying to form and maintain the required formationship.

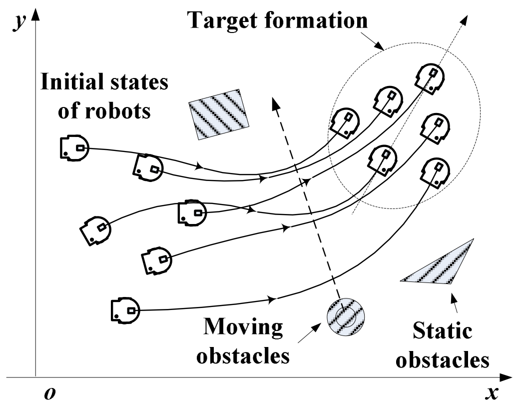

Fig. 1. Motion planning for multiple robots with formation goal

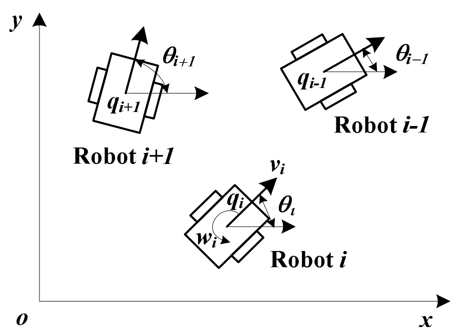

Fig. 2. A group of two-wheel mobile robots

Consider n two-wheel mobile robots in x-y plane, as shown in Fig. 2. Let x_i, y_i, and θ_i be the coordinates of robot i, where $i = 1,2,...,n$. Denote the location of the ith robot as $q_i = [x_i, y_i, \theta_i]^T$. Similar to [6][9], the kinematics of robot i can be represented by a unicycle model as follows

$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = w_i \tag{1}$$

where v_i and w_i denote the linear and angular velocities with respect to the center of the mass of robot i, respectively. The bounds of the velocity and acceleration are given as follows

$$0 \le v_i \le V_{\max} \tag{2}$$

$$|\dot{v}_i| \le a_{\max} \tag{3}$$

where $V_{\max}$ and $a_{\max}$ are the maximum bounds of the linear velocity and acceleration, respectively. The angular velocity w_i can be derived by the linear velocity v_i according to the path curvature as follows

$$w_i(s_i) = k_i(s_i) v_i(s_i) \tag{4}$$

where the parameter s_i denotes the arc length along the path, and $k_i(s_i)$ denotes the curvature of the path at the position s_i on the path. Generally, it is assumed that the curvature of the designed path is small enough such that the angular speed corresponding to the optimal speed is always achievable [14]. Note that both the angular and linear velocities are presented as functions of the the distance s_i in (4).

III. Coordinated Motion Planning with Formation Requirement

The motion planning aims to generate proper velocity profiles to coordinate the robots in following the designed paths while maintaining the desired formations, subject to the constraints of velocity/acceleration bounds and mutual collision avoidance. Since the positions of the robots in their designed paths are closely related to the moving velocities of the robots, the formation can be achieved by coordinating the robots' velocities. Thus, the coordinated motion planning is posed as a velocity optimization problem as detailed below.

A. Formation constraint

The required formation relationship during the motions is described by the relative position of each robot with respect to its neighboring robots. Fig. 3 illustrates the formation relationship of multiple mobile robots, where the arrows represent that the neighboring two robots have a desired relative position in the required formation. These formation relationships construct a constraint net in the formation.

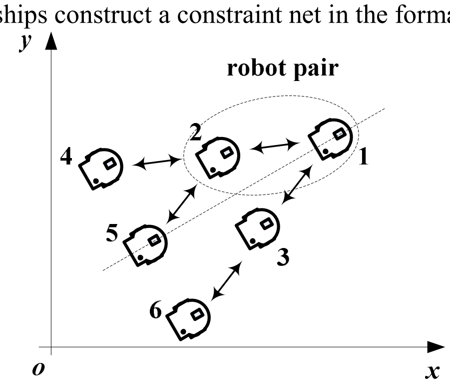

Fig. 3. Formation position determined by neighboring robots.

The formation relationship between robots i and j is measured by offsets in x and y axes, denoted by $\Delta x_{i,j}$ and $\Delta y_{i,j}$, as

$$\begin{cases} \Delta x_{i,j} = x_i - x_j \\ \Delta y_{i,j} = y_i - y_j \end{cases} \tag{5}$$

Divide time t into m intervals, and each interval is denoted by Δt, i.e., $t = m\Delta t$, $\forall m = 0,1,...,M$, where $M\Delta t$ is the maximum time for any robot to complete the task. The formation error between robot i and j in each time interval can then be represented as follows

$$q_{i,j}^e(t) = \begin{bmatrix} \Delta x_{i,j}^e(t) \\ \Delta y_{i,j}^e(t) \end{bmatrix} = \begin{bmatrix} \Delta x_{i,j}(t) - \Delta x_{i,j}^d \\ \Delta y_{i,j}(t) - \Delta y_{i,j}^d \end{bmatrix} \tag{6}$$

where $\Delta x_{i,j}^d$ and $\Delta y_{i,j}^d$ denote the desired formation relationship of $\Delta x_{i,j}$ and $\Delta y_{i,j}$.

We then introduce an objective function Q^e, which can be used to guide the motion planning to meet the formation requirement.

$$Q^e = \sum_{t=0}^{M\Delta t} \sum_{i,j}^{n} \left\| q_{i,j}^e(t) \right\|^2 \tag{7}$$

where the term $\sum_{i,j}^{n} \left\| q_{i,j}^e(t) \right\|^2$ integrates the variations of the relative distances of all the robot pairs in each time interval, and Q^e integrates them during the whole motions. When the function Q^e equals zero, the robots move with the desired formation perfectly.

B. Boundary conditions

To meet the constraints (2) and (3), $v_i(t)$ must be subject to the following conditions

$$v_i(t) - a_{\max}\Delta t \le v_i(t+\Delta t) \le v_i(t) + a_{\max}\Delta t \tag{8}$$

$$0 \le v_i(t) \le V_{\max} \tag{9}$$

Without loss of generality, all the robots are assumed to have zero velocity at the initial and final times. So the boundary conditions of velocities at time $t = 0$ and the final time $t = M\Delta t$ are given by

$$v_i(0) = 0\,,\quad v_i(M\Delta t) = 0 \tag{10}$$

The travel distance $s_i(t)$ can be further determined by $v_i(t)$ and $\dot{v}_i(t)$ in each time interval.

$$s_i(t+\Delta t) = s_i(t) + \frac{1}{2}\Delta t\big(v_i(t) + v_i(t+\Delta t)\big) \tag{11}$$

Accordingly, to arrive in the target formation within the prescribed time limit $M\Delta t$, the boundary conditions of the travel distances at time $t = 0$ and $t = M\Delta t$ should be given by

$$s_i(0) = 0\,,\; s_i(M\Delta t) = L_i \tag{12}$$

where L_i is the total length of the designed path of robot i when it reaches the goal. The conditions in (12) describe that the robots start at their initial positions and ends at the final positions along the paths. Note that the robots may finish the task with a time less than $M\Delta t$.

Since the paths of the robots are known *a priori*, the location of robot i at each particular time can be parameterized by the travel distance $s_i(t)$ as an explicit function of $f_i(s_i(t))$. Hence, the robot location can be derived by the travel distance $s_i(t)$ as follows

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$$q_i(t) = \left[x_i(s_i(t)),\, y_i(s_i(t)),\, \theta_i(s_i(t))\right]^T = f_i(s_i(t)) \tag{13}$$

As a result, the objective function Q^e can be represented by travel distance $s_i(t)$, and hence determined by velocity $v_i(t)$.

C. Collision avoidance

Although the paths are designed such that the robots can avoid collisions with those static obstacles, it is still possible that the robots have mutual collisions or collisions with some moving obstacles. Hence, the velocities of the robots should be coordinated to ensure that the distance between any two robots i and j at each time interval should be larger than a minimum safe distance D_{safe}. The collision-free condition is then formulated as

$$\left\| (q_i(s_i(t)) - q_j(s_j(t)) \right\| \ge D_{safe} \tag{14}$$

The safe distance is dependent on many factors such as the minimum circumcircle radius of the robot, fault tolerance of control errors, and localization problem [20], etc.

In case that there exists a moving obstacle with predicted motion, the obstacle can be treated as a moving robot with state coordinates of $q_{obs}(t) = [x_{ob}(t),\, y_{obs}(t),\, \theta_{obs}(t)]^T$. The following condition is to guarantee the collision avoidance between the robots and the moving obstacle

$$\left\| (q_i(s_i(t)) - q_{obs}(t) \right\| \ge D_{safe} \tag{15}$$

D. Motion Optimization

Now the motion coordination problem has been formulated as a velocity optimization problem, since all the formations with constraints of velocity/acceleration bounds and collision avoidance are dependent on the velocities of the robots. To obtain an optimal solution in velocity optimization, the objective function (7) derived for formation should be minimized while meeting both the velocity/acceleration bounds and collision avoidance. This optimization can be achieved by a mathematical programming technique, i.e., linear interactive and general optimizer (Lingo) [15], for the following nonlinear optimization problem:

Minimize: $$Q^e = \sum_{t=0}^{M\Delta t}\left(\sum_{i,j}^{n} (\Delta x_{i,j}^e(s_i(t)))^2 + (\Delta y_{i,j}^e(s_i(t)))^2 \right)$$

Subject to: $i = 1,2,\ldots,n$;

$t = 0, \Delta t, 2\Delta t, \ldots, M\Delta t$;

$0 \le v_i(t) \le V_{\max}$,

$v_i(0) = 0$, $v_i(M\Delta t) = 0$;

$s_i(0) = 0$, $s_i(M\Delta t) = L_i$;

$s_i(t+\Delta t) = s_i(t) + \frac{1}{2}\Delta t[v_i(t) + v_i(t+\Delta t)]$;

$[x_i(s_i(t)), y_i(s_i(t)), \theta_i(s_i(t))] = f_i(s_i(t))$;

$\forall i, j$, $\left\| (q_i(s_i(t)) - q_j(s_i(t)) \right\| \ge D_{safe}$;

$\forall i$, $\left\| (q_i(s_i(t)) - q_{obs}(s_i(t)) \right\| \ge D_{safe}$;

IV. SIMULATIONS

To demonstrate the effectiveness of the proposed approach, simulations were performed on a group of mobile robots. The paths of the robots in the simulations were generated by using Matlab function Spline(). The paths were parameterized by arc length in Matlab such that the path function $f_i(s_i(t))$ of each robot in eq. (13) could be obtained. The optimization problem was modeled and solved in Lingo 9.0 [15], which was implemented in windows XP system with 2GB of main memory and a 2.2GHz clock speed. The simulations were performed in Matlab 7.

The formation relationship is represented by a constraint net, labeled by the double arrows as shown in Fig. 4. When all the desired relationships are formed, the desired formation is formed. It is seen that the constraint net can easily represent a common formationship.

The parameters of the desired formation relationship are listed in Table.1. In the simulations, other parameters are $\Delta t = 1$ (s), $V_{\max} = 130$ (mm/s), $a_{\max} = 130$ (mm/s^2), $D_{safe} = 100$ (mm) and $M = 15$.

The formation errors $q^e_{i,j}(t)$ can be obtained from (6), and the optimization goal Q^e can be represented by (7) as follows

$$q^e_{i,j}(t) = \begin{bmatrix} \Delta x^e_{i,j}(t) \\ \Delta y^e_{i,j}(t) \end{bmatrix} = \begin{bmatrix} \Delta x_{i,j}(t) - \Delta x^d_{i,j} \\ \Delta y_{i,j}(t) - \Delta y^d_{i,j} \end{bmatrix} \tag{16}$$

$$Q^e = \sum_{t=0}^{M\Delta t} \sum_{(i,j)} \left((\Delta x^e_{i,j}(t))^2 + (\Delta y^e_{i,j}(t))^2 \right) \tag{17}$$

where $(i,j) = (1,2),(1,3),(2,4),(2,5),(3,6)$.

Fig. 5 shows the simulation results at six different times. The robots are marked by the circles. The robots' paths are designed to avoid the static obstacles and form the target formation as shown in the right hand side. A moving obstacle exists and follows its path with a constant velocity, represented by a solid box. As shown in Fig. 5, after starting at 0 second, robots 1 and 2 moved at low speed and other robots moved at high speed to maintain the desired formation at the time of 2 seconds. When the robots were close to the moving obstacle, they all slowed down to wait the path clear. At 6 seconds, the paths of robots 1 and 2 were clear, but they did not speed up to move towards their goals individually, because the other robots were still waiting. So they moved at a low speed to maintain the formation and wait for robots 3 and 6. Until the path of robot 6 was clear at 10 seconds, all the robots speeded up and moved as a whole. It is seen that the formation relationship was maintained well during the motions by optimizing the motions. At 12 and 14 seconds, all the robots moved in the formation and finally reached the goal position at the same time.

The simulation demonstrates that the proposed method can optimize the motions of robots to maintain desired formation while moving along the designed path with collision avoidance.

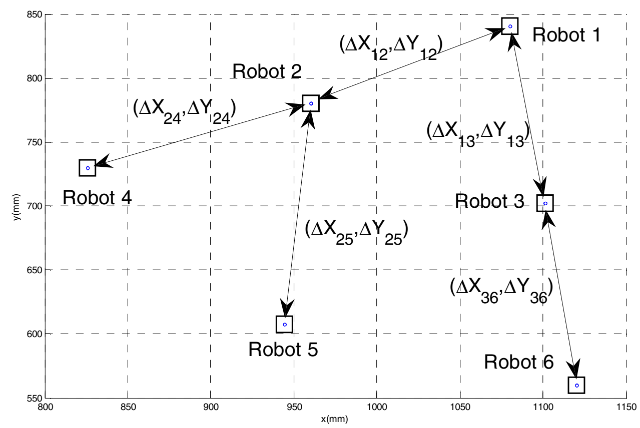
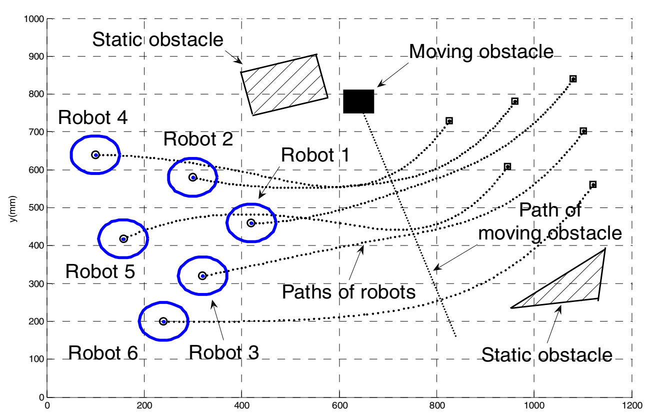


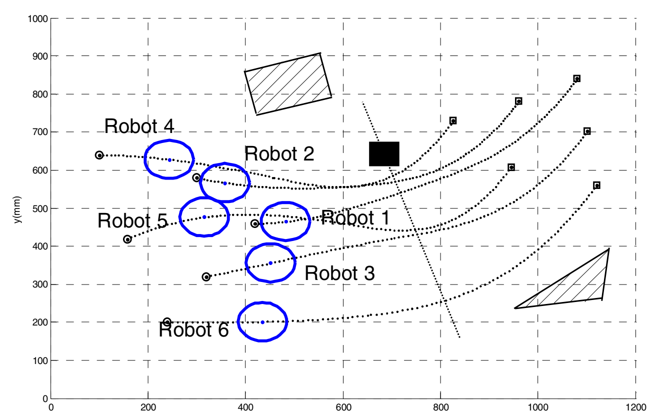
Fig. 4. Formation relationship.

Table 1. Desired formation relationship (mm)

$\Delta x^d_{1,2}$	120	$\Delta x^d_{2,4}$	135	$\Delta x^d_{3,6}$	-20
$\Delta y^d_{1,2}$	60	$\Delta y^d_{2,4}$	50	$\Delta y^d_{3,6}$	140
$\Delta x^d_{1,3}$	-20	$\Delta x^d_{2,5}$	15		
$\Delta y^d_{1,3}$	140	$\Delta y^d_{2,5}$	172		



0 second



2 seconds

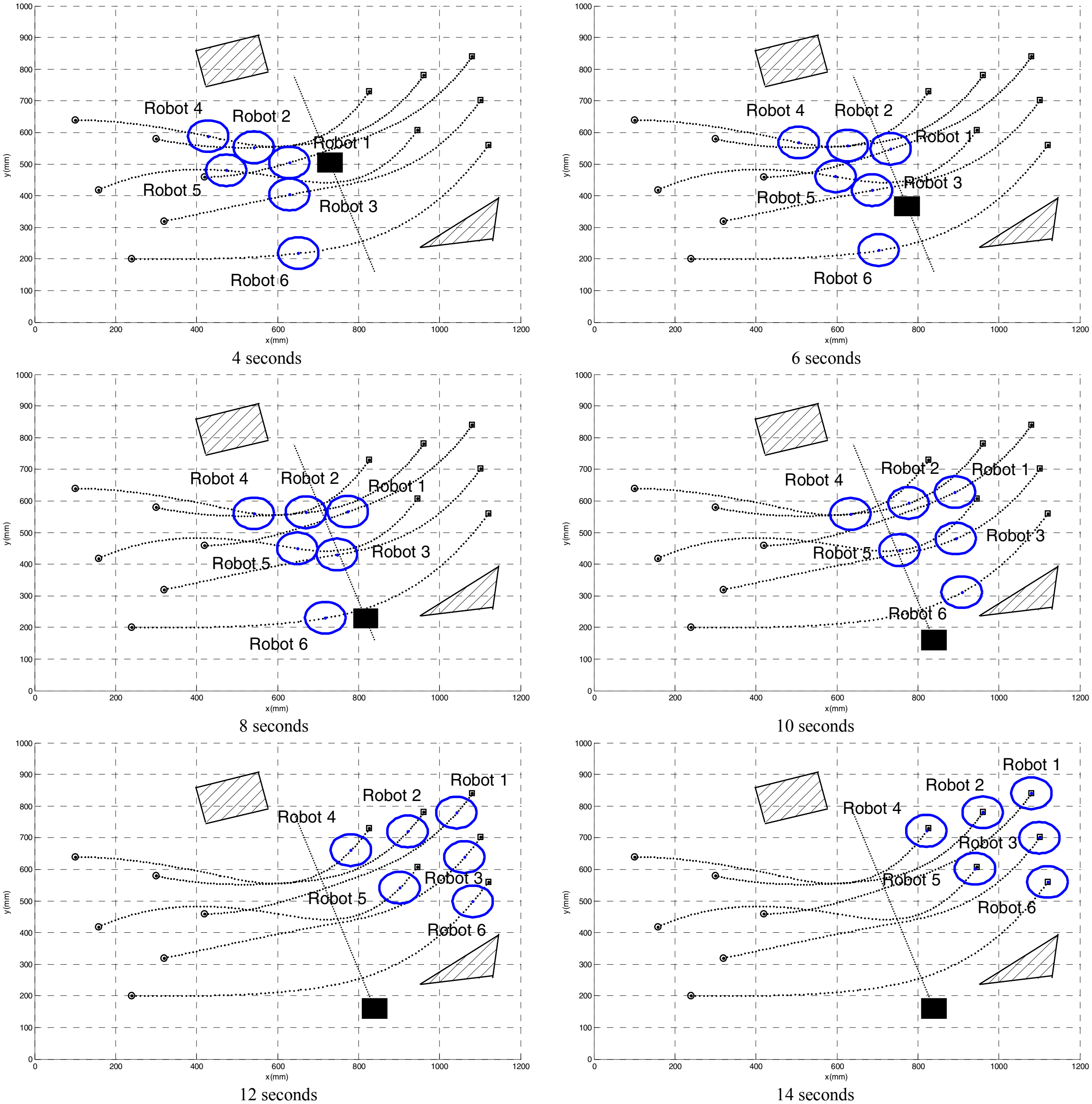


Fig. 5. Simulation with formation constraint.

To show the performance of the proposed approach in reducing the formation errors, simulation with the optimization goal in [14] was further performed for comparison. The following optimization goal [14] was used.

$$\text{Minimize: } \sum_{t=0}^{M\Delta t} \left(L_i - s_i(t) \right) \tag{18}$$

This goal forces on the reduction of the distance between the robots' current locations and the goal positions [14]. However, formation constraint was not considered in [14].

The formation errors $q_{1,2}^e(t)$, $q_{1,3}^e(t)$, $q_{2,4}^e(t)$, $q_{2,5}^e(t)$, and $q_{3,6}^e(t)$ in the two simulations were compared in Fig. 6. It is seen that by optimizing the velocity profiles to reduce the indicator Q^e, the formation errors of the robots during the motions are obviously reduced, which implies that the formation relationship is better maintained through the coordinated motion planning.

V. CONCLUSIONS

In this paper, the coordinated motion planning problem of multiple mobile robots with formation requirement is discussed. The robots are not only required to follow the specified path, but also required to keep formation relationship during the motions. The problem is formulated as a velocity optimization problem, which can be solved as a nonlinear programming problem in the linear interactive and general optimizer (Lingo). Simulations are performed on a group of mobile robots to demonstrate the effectiveness of the proposed approach to coordinate the robots' motions, while keeping the formation relationship during the motions.

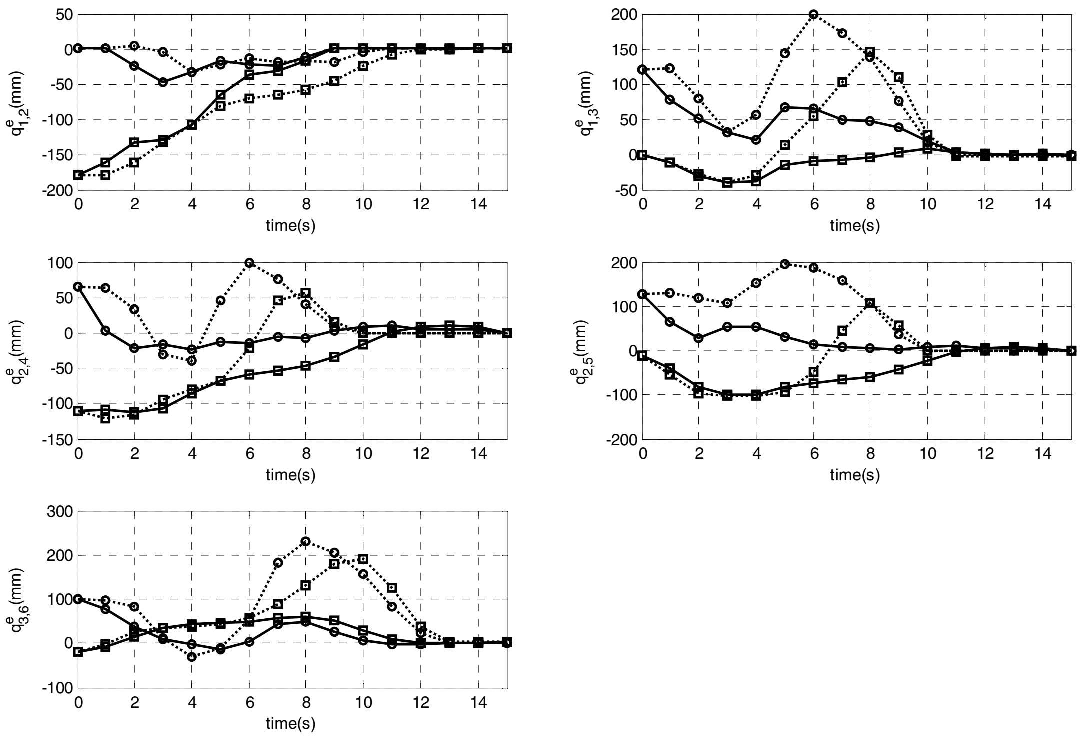


Fig. 6. Formation errors of robots.

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