A Systematic Antiwindup Strategy and the Longitudinal Control of a Platoon of Vehicles with Control Saturations

Sean C. Warnick and Armando A. Rodriguez

***Abstract*—Current methodologies for designing control systems usually ignore the effects of control saturations. For vehicle platooning, this can be especially disastrous since the performance of a platoon of nonidentical vehicles is, in general, severely limited by saturating control signals. This paper presents a systematic design procedure for adapting a nominal controller, designed without regard to control saturation, to a higher performance nonlinear controller that explicitly accounts for the saturating nonlinearities while preserving stability. In particular, the error governor (EG) scheme proposed by Kapasouris *et al.* [10] is extended and applied to the IVHS problem found in [13]. This extension is a less conservative strategy that explicitly accounts for an important class of controller designs, including proportional control and a class of nonlinear feedback designs. Results of the study demonstrate that severe windup found in nominal platooning applications can systematically be eliminated without loss of steady-state performance. This significantly increases the range of maneuvers open to the lead vehicle, thus enhancing the utility of a proposed design. An extensive simulation of a 15-car platoon of nonidentical vehicles with nonlinear vehicle models illustrates these results.**

***Index Terms*—Nonlinear control.**

I. Introduction

THE EFFECTS of multivariable control saturations have been, and continue to be, a topic of extensive investigation in the control systems community (see, for example, [14] and [15] and references therein). This interest is the natural result of tighter performance specifications when all real control actuation devices exhibit saturation. The concern is how saturating nonlinearities can drastically impact the stability and performance of an otherwise well-behaved feedback design.

In general, there are a variety of ways of coping with the control saturations of engineered systems. Since the complexity of analysis of such systems increases dramatically when saturations are explicitly considered, it is quite common in practice to ignore saturations by restricting maneuvers to avoid aggressive control action. Similarly, when system performance is not critical, there are a variety of *ad hoc* antiwindup schemes available that generally improve performance (such as overshoot and settling times) without stability guarantees.

When stability guarantees are important, however, such as when safety is a concern, then there are essentially three approaches for dealing with control saturation. The first simply analyzes a nominal closed-loop system with saturation to discover whether the resulting system is stable. The idea is to develop sufficient conditions that guarantee the stability of the nominal system allowed to saturate, using, for example, integral quadratic constraints [12], [18]. The second, labeled the direct synthesis approach, explicitly accounts for saturation in the problem formulation and deals with the extra complexity accordingly. Examples of this approach include a version of the l_1 optimal control problem [4] or the modified LQG-type problem discussed in [15]. The third approach attempts to avoid the design complexity introduced by saturation by introducing a two-step design. First, a controller is determined for the system ignoring saturation; thus, all the familiar linear design methodologies remain available. Next, the resulting controller is perturbed to account explicitly for the saturating nonlinearities. The error governor (EG) methodology is an example of this approach [10], [11]. This methodology is interesting because it limits design complexity by indirectly addressing saturation, preserves flexibility of the design method, yet provides a systematic approach leading to stability guarantees of the resulting closed-loop system.

Multiple vehicle guidance systems create challenging problems for dealing with the affects of saturation because:

1) safety is critical, implying that stability guarantees are essential;
2) saturations are prevalent at all levels, from turning radius, throttle limits, and braking capability to traffic capacity and highway throughput;
3) expectations for designers are *low-complexity* solutions that address a variety of configurations composed of *nonidentical* vehicles.

Vehicle platooning, for example, naturally implies a saturation problem when nonidentical vehicles are allowed (Fig. 1). Nevertheless, the stability and performance affects of saturation on these systems remain largely unexplored.

This study explores how saturation affects the longitudinal control of a platoon of nonidentical vehicles proposed in [13]. Although the practically of the platooning approach considered in [13] has been criticized at various levels, this problem is generic enough to demonstrate the affects of saturation, and the operation of an antiwindup scheme for coping with these affects, typical for a variety of multiple vehicle guidance problems. In fact, the platooning problem considered here highlights precisely where the EG algorithm "fails," motivating the development of an extension to the EG algorithm. This extension,

Manuscript received November 11, 1996; revised September 19, 1998.

S. C. Warnick is with the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

A. A. Rodriguez is with the Center for Systems Science and Engineering, Arizona State University, Tempe, AZ 85287 USA.

Publisher Item Identifier S 0018-9545(00)03084-X.

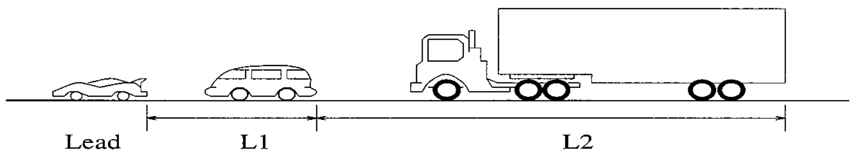


Fig. 1. Platooning nonidentical vehicles implies a saturation problem.

called the extended EG (EEG), recovers performance lost by the EG in the important case where controllers have a feedforward, or proportional control, term. The platoon problem considered here also highlights that the EEG applies to controllers with nonlinear feedforward terms.

The intent of this research is to employ the platooning problem to motivate and develop a new general antiwindup scheme that systematically provides the same benefits as the EG without inadvertently degrading performance. The results here demonstrate that although the EG provides stability guarantees, allowing aggressive platoon maneuvers, steady-state performance degrades significantly. The EEG, on the other hand, recovers virtually the identical performance as the nominal system, allowed to saturate, with the additional bonus of all the benefits of the EG, including the preservation of design flexibility while offering certain guaranteed stability properties. Hence, a broad range of aggressive maneuvers that might not otherwise be safely considered are made available to the lead vehicle.

The paper is outlined as follows. In Section II, the platoon model is discussed focusing on the vehicle model, nominal compensator design, and the platoon dynamics. Section III presents the theory behind the EG, discusses its implementation, and demonstrates the tradeoff between windup management and performance when it is used for antiwindup. Section IV develops the EEG and demonstrates that windup can be eliminated while maintaining steady-state performance. Section V concludes with an outline of future research.

II. Platoon Model

The problem of platooning $N+1$ vehicles is concerned with developing a control design that minimizes the error between the dynamics of the lead vehicle and those of the subsequent N vehicles. This error is measured by monitoring the deviation in nominal separations between vehicles. Specifically, we define

$$\Delta_i(t) \triangleq x_{i-1}(t) - x_i(t) - L_i$$

where x_i denotes the position of the rear bumper of the ith vehicle and L_i is the assigned vehicle slot length.

A platoon, then, is like a train of cars with an established separation control. Although this "train" image is powerful, the platoon model is nevertheless best visualized in the general framework depicted in Fig. 2, where w are exogenous inputs (such as reference commands, e.g., desired speed), z are regulated outputs (deviations in nominal separation between vehicles), y is a measurement vector (e.g., velocities, accelerations, etc.), and u is the presaturation control vector (i.e., the throttle or braking asked for regardless of what is actually physically possible). This general form is useful because it facilitates analysis independent of the number of vehicles in the platoon. In this discussion, "platoon" refers to all vehicles in the train except the lead, which generates the reference commands the "platoon" must follow or track.

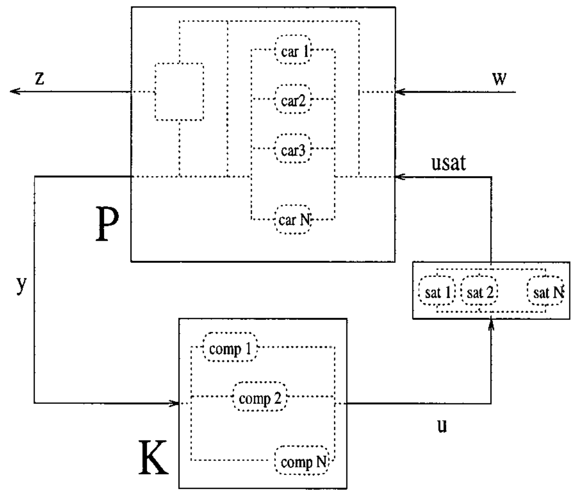


Fig. 2. General platoon model.

The platooning concept has received attention due to its potential in enhancing the safety and efficiency of existing highway systems. These concepts deal with problems ranging from simple longitudinal control to various maneuvers such as lane changes, platoon separation, and platoon merging [1]–[3], [6]–[8]. Nearly all of the resulting control systems are designed ignoring the effects of saturating nonlinearities at the controls—the tacit assumption being that the motion of the lead vehicle will be constrained so that the controls in the platoon remain within allowed limits. In vehicle platooning, for example, it is common to limit the jerk of the lead vehicle to passenger comfort limits—the implication being that these limits are within throttle and braking saturation levels as well. As Fig. 1 suggests, these assumptions and such restrictions can be extremely conservative when platoons of nonidentical vehicles are considered. Clearly, maneuvers that lie well within the passenger comfort limits of a sports coup can easily saturate the controls of a truck, bus, or van. Thus, platoon maneuverability and performance are limited by the *weakest vehicle* in the train unless saturations are explicitly accounted for in the control system design. One contribution of the EEG is to broaden the spectrum of allowable platoon maneuvers.

The platoon model and controller presented in [13] are used as the nominal design for this study. Assumptions made to generate the vehicle model and control laws are as follows. First, a unidirectional communication link from the lead vehicle is needed to pass information about the lead vehicle's dynamics to every other vehicle in the platoon. Also, a horizontal road surface and negligible wind disturbance are assumed, along with the requirement that the platoon vehicles travel in the same direction at all times.

These assumptions, with the appropriate control law, result in platoon behavior such that: 1) the deviations in nominal vehicle separations do not get magnified down the platoon; 2) these deviations do not oscillate; and 3) the time variations of the deviations are well within passenger's comfort limits. Limitations of the nominal control design include extreme simplification of the vehicle dynamics, essentially lumping the engine dynamics and braking system into a single first-order linear ordinary differential equation, and that it does not deliver zero steady-state error to velocity ramp inputs (no internal model). These restrictions are outlined here to indicate that they are features of the nominal platoon controller and not artifacts of the antiwindup strategies motivated in this paper. The methods discussed here are applicable to a very broad range of linear and nonlinear nominal (platoon) controllers.

A. Vehicle Model

The dynamics of the ith $(i = 1, 2, \cdots)$ vehicle model are described by Newton's Second Law and a characterization of the ith vehicle's engine dynamics. These are given by

$$m_i \dot{v}_i = m_i \xi_i - K_{di} v_i^2 - d_{mi} \tag{1}$$

$$\dot{\xi}_i = -\frac{\xi_i}{\tau_i(v_i)} + \frac{u_i}{m_i \tau_i(v_i)} \tag{2}$$

where

$m_i \xi_i$	engine force applied to the ith vehicle;
$K_{di} v^2$	force from air resistance;
d_{mi}	mechanical drag.

Specifically, $K_{di} = \rho A_i C_{di}/2$ with ρ denoting the specific mass of air, A_i denoting the vehicle cross-sectional area, and C_{di} denoting the vehicle's drag coefficient. $\tau_i(v_i)$ specifies the ith vehicle's engine time constant at velocity v (taken here to be constant), m_i is the vehicle's mass, and u_i represents the control for the ith vehicle. The control has units of force, Kg·m/s^2, and thus is negative when the braking system is active.

For convenience, these expressions are combined to yield a single mathematical description of the ith vehicle. Differentiating (1) and then substituting (2) in for ξ, and taking sat$_i$ to be the saturation level of the ith vehicle's control, the following vehicle description is obtained:

$$\begin{aligned}\ddot{v}_i = &-2\frac{K_{di}}{m_i} v_i \dot{v}_i + \frac{u_{\mathrm{sat}_i}}{m_i \tau_i(v_i)} - \frac{1}{\tau_i(v_i)} \\ &\cdot \left[\dot{v}_i + \frac{K_{di}}{m_i} v_i^2 + \frac{d_{mi}}{m_i}\right]\end{aligned} \tag{3}$$

$$u_{\mathrm{sat}_i}(t) \triangleq \begin{cases} \mathrm{sat}_i & u_i(t) > \mathrm{sat}_i \\ u_i(t) & -\mathrm{sat}_i \le u_i(t) \le \mathrm{sat}_i \\ -\mathrm{sat}_i & u_i(t) < -\mathrm{sat}_i. \end{cases}$$

B. Nominal Vehicle Compensator Design

The platoon controller is naturally decentralized so that each vehicle has its own compensator. The control law employed by a vehicle compensator depends on its current position in the platoon. The first vehicle in the platoon (the car immediately following the lead vehicle) has a control law distinct from all other vehicles, which share a common control law structure. The basic concept of each controller is essentially the same, however, dynamically linearize the vehicle model and then introduce dynamics that make for a smooth ride.

The first task, then, of each compensator is to use exact linearization methods [9] to effectively linearize the input–output behavior of the vehicle. This is accomplished by choosing a control of the form

$$u_i = \frac{1}{a(v_i)} \left[c_i - b(v_i, \dot{v}_i)\right] \tag{4}$$

where $a(v_i)$ and $b(v_i, \dot{v}_i)$ are given by

$$\begin{aligned} a(v_i) &\triangleq \frac{1}{m_i \tau_i(v_i)} \\ b(v_i, \dot{v}_i) &\triangleq -2\frac{K_{di}}{m_i} v_i \dot{v}_i - \frac{1}{\tau_i(v_i)} \left[\dot{v}_i + \frac{K_{di}}{m_i} v_i^2 + \frac{d_{mi}}{m_i}\right]. \end{aligned}$$

The vehicle dynamics then become $\dot{x}_i = v_i$, $\ddot{x}_i = \dot{v}_i$, and $\ddot{v}_i = c_i$, and c_i can be chosen appropriately as a control law for the ith vehicle. Note that the exact linearization process requires a nonlinear compensator for each vehicle.

The control law is chosen so that each vehicle, as a closed-loop system, approximately tracks the velocities of the lead and immediately preceding vehicles. Since the immediately preceding vehicle for the first car *is* the lead, its control law differs slightly. For the first car

$$\begin{aligned} c_1 \triangleq\ & c_{p1}\Delta_1(t) + c_{v1}\dot{\Delta}_1(t) + c_{a1}\ddot{\Delta}_1(t) + k_{v1}[v_L(t) - v_o] \\ & + k_{a1} a_L(t) \end{aligned} \tag{5}$$

where v_o denotes the initial (steady-state) value of the lead vehicle's velocity $v_L(0) = V_i(0) = v_o$, $(i = 1, 2, \cdots, N)$. For cars $2, 3, \cdots, N$, the following control law is used:

$$\begin{aligned} c_i \triangleq\ & c_p \Delta_i(t) + c_v \dot{\Delta}_i(t) + c_a \ddot{\Delta}_i(t) + k_v[v_L(t) - v_i(t)] \\ & + k_a[a_L(t) - a_i(t)]. \end{aligned} \tag{6}$$

Note from (6) that Δ_i is the state of the ith compensator, however, we will make a change of notation and call this state x_i for notational simplicity (not to be confused with the bumper position of the ith vehicle); the available measurements are the initial velocity (v_o), velocities of the lead and immediately preceding vehicles (v_L, v_{i-1}), accelerations of the lead and immediately preceding vehicles (a_L, a_{i-1}), and one's own acceleration and velocity (v_i, a_i). The resulting state-space representation of the ith compensator has the form

$$\begin{aligned} \dot{x}_i(t) &= [0]x_i(t) + [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad -1 \quad 0]y_i(t) \\ u_i(t) &= [c_p m_i \tau_i]x_i(t) + f_i(y_i(t)) \end{aligned} \tag{7}$$

where

$$\begin{aligned} &y_i(t) \\ &\quad = [\dot{v}_L(t) \quad \dot{v}_{i-1}(t) \quad \dot{v}_i(t) \quad v_L(t) \quad v_{i-1}(t) \quad v_i(t) \quad v_o]^T \end{aligned}$$

and where f_i is a function related to a, b, and all the terms in c_i except c_p. Recall that since the lead and immediately preceding vehicles are the same for the first car, its measurement vector differs slightly. Important features of the above compensator design are that: 1) it contains integrator dynamics and 2) the nonlinearity resulting from feedback linearization appears strictly as a feedforward term. The constants c_{p1}, c_{v1}, c_{a1}, k_{v1}, k_{a1}, c_p, c_v, c_a, k_v, and k_a are design parameters, and the process of assigning them appropriate values is discussed in the next section.

C. Platoon Dynamics

The platoon dynamics are best understood by developing the transfer function $H(s)$, from δ_{v_L} to Δ_i, where $\delta_{v_L} = v_L - v_o$. From the previous equations it is straightforward to derive $\hat{h}_{\Delta_1\delta_{v_L}}(s)$, from δ_{v_L} to Δ_1, $\hat{h}_{\Delta_2\Delta_1}(s)$, from Δ_1 to Δ_2, and $\hat{g}(s)$, from Δ_{i-1} to Δ_i, $i = 3, 4, \cdots$. These are as follows:

$$\hat{h}_{\Delta_1\delta_{v_L}}(s) = \frac{s^2 - k_{a1}s - k_{v1}}{s^3 + c_{a1}s^2 + c_{v1}s + c_{p1}} \tag{8}$$

$$\hat{h}_{\Delta_2\Delta_1}(s) = \frac{(c_{a1} - k_a)s^2 + (c_{v1} - k_v)s + c_{p1}}{s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p} \tag{9}$$

$$\hat{g}(s) = \frac{c_a s^2 + c_v s + c_p}{s^3 + (c_a + k_a)s^2 + (c_v + k_v)s + c_p}. \tag{10}$$

Comparing these expressions, it is apparent that choosing $c_{a1} = c_a + k_a$, $c_{v1} = c_v + k_v$, and $c_{p1} = c_p$ will result in the same denominator polynomial $\gamma(s)$. The transfer function from Δ_i, $i = 3, 4, \cdots$ to δ_{v_L} is then given by

$$H(s) = (\hat{g}(s))^{i-2}\left[\hat{h}(s)\hat{g}(s) + \frac{k_{a1}s + k_{v1}}{\gamma(s)}\right].$$

To achieve the desired platoon behavior, it is necessary to determine values for the design parameters such that: 1) $|\hat{g}(\jmath\omega)| < 1$ and $\omega \rightarrow |\hat{g}(\jmath\omega)|$ is a strictly decreasing function of $\omega\ \forall\ \omega > 0$. This ensures that perturbations in Δ_i caused by δ_{v_L} do not get magnified from each vehicle down the platoon and 2) $g(t) \triangleq \mathcal{L}^{-1}\{\hat{g}(s)\} > 0\ \forall t$. This will avoid oscillatory behavior in the separations.

These dynamics are investigated through extensive 15-car simulations executed on Simulink 1.3 in the Matlab environment with the following parameter values:

$c_{p1} = 120$	$c_{v1} = 74$	$c_{a1} = 15$	$k_{v1} = -0.05$
$k_{a1} = -3.03$	$c_p = 120$	$c_v = 49$	$c_a = 5$
$k_v = 25$	$k_a = 10$	$m_1 = 5000$	$m_2 = 1000$
$m_3 = 1500$	$m_4 = 2000$	$m_5 = 2500$	$m_6 = 3000$
$m_7 = 3500$	$m_8 = 4000$	$m_9 = 4500$	$m_{10} = 5000$
$m_{11} = 4500$	$m_{12} = 4000$	$m_{13} = 3500$	$m_{14} = 3000$
$m_{15} = 2500$	$\tau_1 = 0.20$	$\tau_2 = 0.05$	$\tau_3 = 0.06$
$\tau_4 = 0.07$	$\tau_5 = 0.08$	$\tau_6 = 0.09$	$\tau_7 = 0.10$
$\tau_8 = 0.12$	$\tau_9 = 0.14$	$\tau_{10} = 0.16$	$\tau_{11} = 0.18$
$\tau_{12} = 0.20$	$\tau_{13} = 0.30$	$\tau_{14} = 0.20$	$\tau_{15} = 0.10$

$$\text{sat}_i = 15\,000,\ d_{mi} = 0 (i = 1, 2, \cdots, N)$$
$$V_L(0) = v_o = 17.9 \text{ m/s}.$$

Figs. 3 and 4 show resulting Bode plots for $\hat{h}(s)$, $\hat{g}(s)$, and $H(s)$, which reveal the closed-loop behavior of any size platoon. The studies presented here simulate a platoon of 15 cars, but since the magnitude plot for H is independent of i for $i \geq 3$, only one curve appears in Fig. 4.

Figs. 5 and 6 show the velocity profile of the lead vehicle and the resulting deviation in nominal separation and corresponding control signals for a platoon without control saturations. Fig. 7 shows the resulting deviation and control for the same maneuver when saturations are present. Note that the saturation levels of these vehicles have been normalized, without loss of generality, to make results of the study more transparent. The lead maneuver is a ramp from 17.9 to 29 m/s in 5 s. Note that although the deviations essentially return to their nominal values in the steady state, windup is considerable. This windup is a result of the slow dynamics in the compensator; in this vehicle platooning problem saturations cause windup because the nominal controller uses integral action. The next section outlines a general procedure for systematically eliminating this windup.

III. Saturation Prevention Methodology: EG

The EG method is a systematic methodology for adapting a nominal, linear controller—designed without accounting for control saturation—to a nonlinear controller that explicitly accounts for saturation [10], [11]. The key idea is to introduce an operator acting at the *input* of the compensator to scale measurements so that controls, at the compensator output, never saturate (Fig. 8). Since it is a two-step procedure, complete flexibility is given in the design of a nominal compensator provided the plant and nominal compensator are stable and the resulting closed-loop system is internally stable.

The objectives of the EG are to: 1) preserve stability; 2) prevent saturation whenever possible; and 3) mimic the behavior of the nominal controller. The first two objectives are always met for linear systems, as shown in the next section. The EG can become conservative in approximating the behavior of the nominal compensator, though, when the nominal compensator is nonlinear or when it has a feedforward term. When a nominal compensator is nonlinear, standard linearization methods can be applied to obtain a linear model of the actual compensator. This linear model is then used in the computation of a scaling operator Λ and the results will hold locally. For the nominal vehicle compensator described above, such a linearization can be obtained from (3) and (4). This results in the same (A, B, C) terms outlined in the state-space description (7), but the nonlinear feedforward term f now becomes a matrix D. For the ith car

$$D_i = \begin{bmatrix} \tau_i m_i k_a \\ \tau_i m_i c_a \\ m_i + \tau_i(2K_{di}v_o - m_i(c_a + k_a)) \\ \tau_i m_i k_v \\ \tau_i m_i c_v \\ 2K_{di}v_o - \tau_i m_i(c_v + k_v) \\ -K_{di}v_o \end{bmatrix}^T. \tag{11}$$

A discussion of the theoretical foundation and issues arising in the implementation of the EG should help make its operation clear.

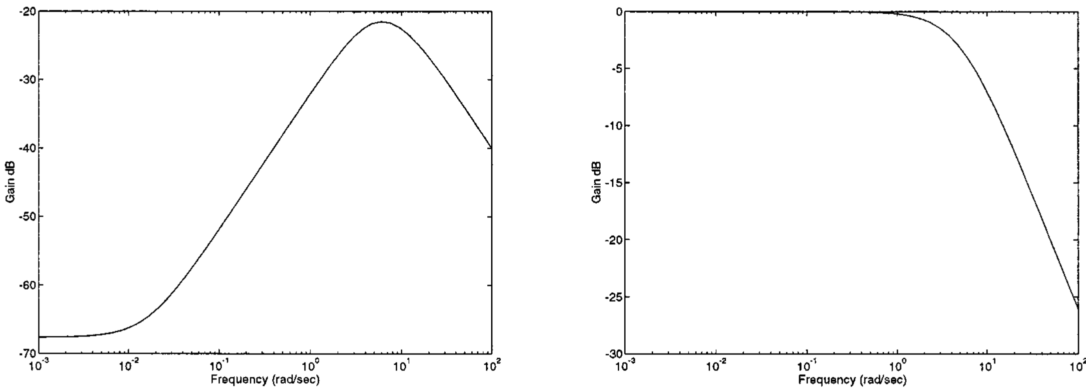


Fig. 3. Bode magnitude plots of $\hat{h}$ and $\hat{g}$.

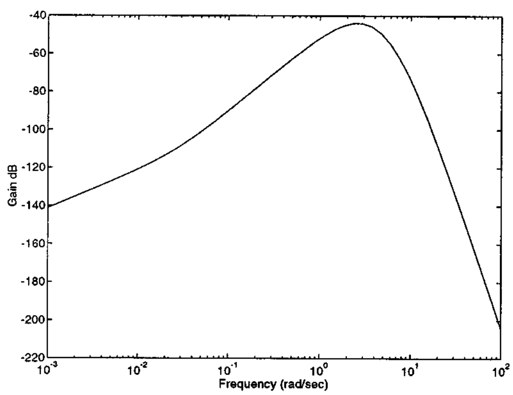


Fig. 4. Bode magnitude plots of $\hat{H}$ for $i \geq 3$.

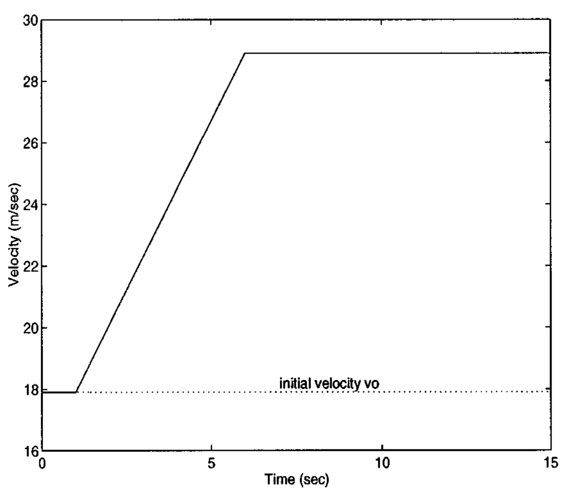


Fig. 5. Lead vehicle velocity profile (platoon reference command).

A. Mathematical Preliminaries

Given the following model of a nominal compensator:

$$\dot{x}(t) = Ax(t) + By(t) \quad x(0) = x_o \tag{12}$$

$$u(t) = Cx(t) + Dy(t). \tag{13}$$

Definition 3.1: The following scalar function can be defined in terms of the homogeneous dynamics:

$$g(x): \mathbb{R}^n \to \mathbb{R},\ g(x) = ||Ce^{At}x||_\infty.$$

Here, the norm is the standard sup-norm defined on vector signals. Note that g is a norm-like function (Fig. 9) that can be redefined to include (exponential) weighting in time.

Definition 3.2: From Definition 2.1, the following set of all states x such that $||u(t)||_\infty \leq 1$ when $y(t) = 0 \forall\ t \geq 0$ can be defined:

$$B_{A,C} = \{x | 0 \leq g(x) \leq 1\}.$$

This set is the unit ball in the metric defined by g and can be thought of as the set of all initial states such that the unforced compensator will never saturate.

Definition 3.3: The upper right Dini derivative is defined as

$$D^+ f(t_o) = \limsup_{t \to t_o^+} \frac{f(t) - f(t_o)}{t - t_o}.$$

We note without proof that the upper right Dini derivative is finite when f is locally Lipschitz, and the function g is locally Lipschitz when the nominal compensator is neutrally stable. It can also be shown that g is nonincreasing on (a, b) iff $D^+ g(x) \leq 0 \forall\ t \in (a, b)$ (proofs and discussion of these points can be found in [10] and its references). This fact reveals how to construct the operator Λ.

Considering Λ to be a scalar λ times the identity I, we can construct λ as follows:

1) $0 \leq \lambda \leq 1$.
2) λ is maximum subject to

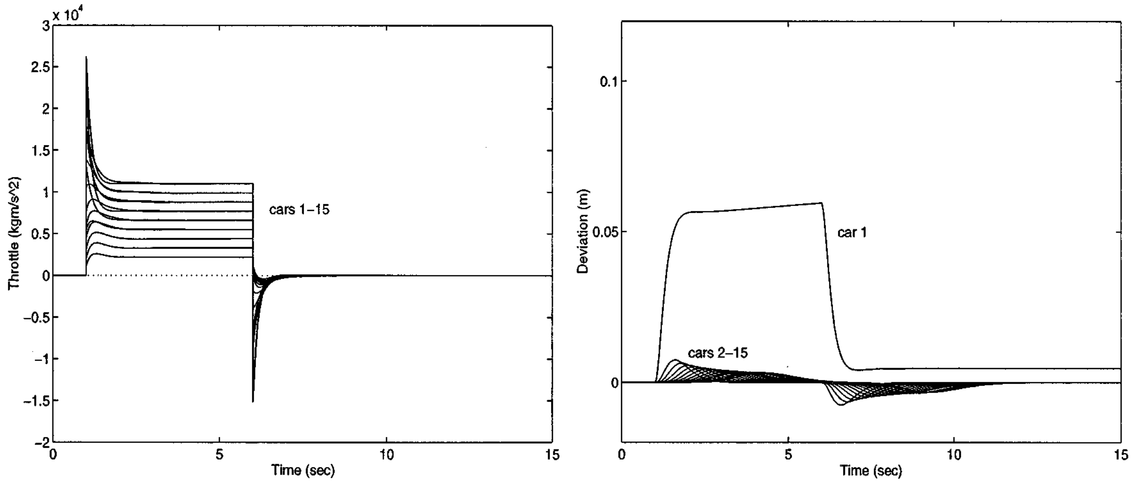
$$||Cx(t) + D\lambda y(t)||_\infty \leq 1.$$

3) If $x(t) \in$ boundary$\{B_{A,C}\}$, then maximize λ subject to

$$\limsup_{\epsilon \to 0} \frac{p(\lambda)}{\epsilon} \leq 0$$

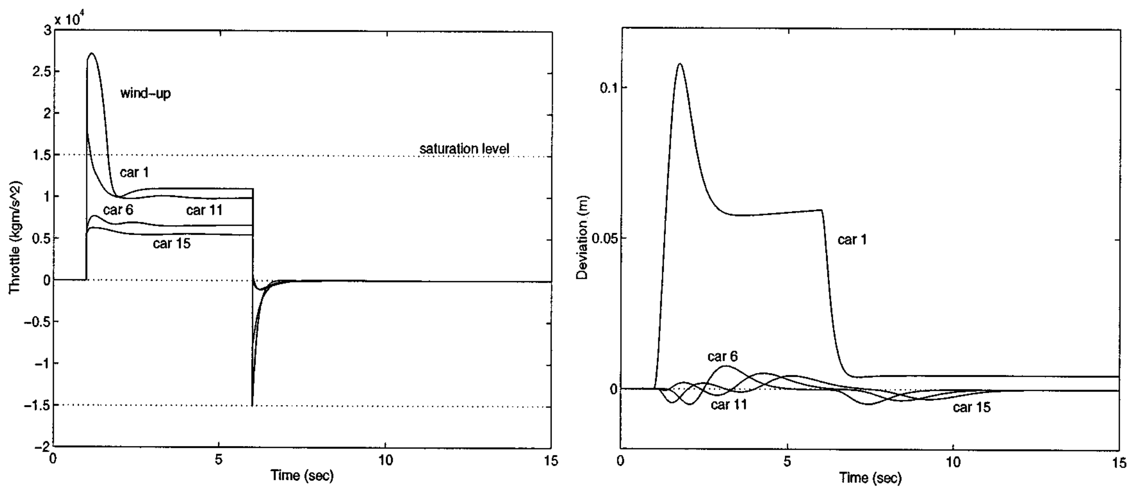
where

$$p(\lambda) \triangleq g(x(t) + \epsilon[Ax(t) + B\lambda y(t)]) - g(x(t)).$$



(a) (b)

Fig. 6. (a) Nominal platoon controls and (b) deviations without saturation.



(a) (b)

Fig. 7. (a) Nominal platoon controls and (b) deviations with saturation.

4) If $x(t) \notin B_{(A,C)}$, then

$$\min_{\lambda} \limsup_{\epsilon \to 0} \frac{p(\lambda)}{\epsilon}.$$

These steps in the computation of λ are easily explained. The first condition demands that the EG not do too much, that it simply scales the nominal control signal between an open-loop response ($\lambda = 0$) or the nominal closed-loop response ($\lambda = 1$). Since the EG can "turn off" feedback by scaling λ to zero, the condition that P be stable should be clear. The second condition reveals our desire for the EG to do as little as possible; it will leave $\lambda = 1$ unless the output, i.e., $Cx(t) + D\lambda y(t)$ will saturate. The third condition reveals how λ is computed. Essentially, the EG knows the set of states for which the output will never saturate (the $B_{A,C}$). If the current state is inside this set, $\lambda = 1$. If it is on the boundary, then the system is on the verge of saturation and a λ is chosen such that it deviates from the nominal compensator as little as possible (i.e., it is as close to one as possible), but such that the state will not leave the $B_{A,C}$ (it is prevented from "sliding up" the cone defined by p). Finally, the last condition simply suggests that if somehow the state of the system starts outside the $B_{A,C}$, then λ is chosen to "slide down" the cone defined by p and bring the state inside the $B_{A,C}$ as quickly as possible.

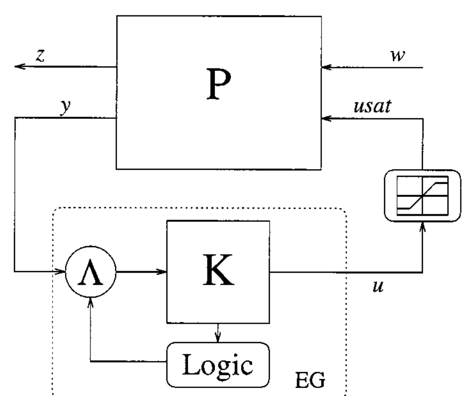


Fig. 8. EG structure.

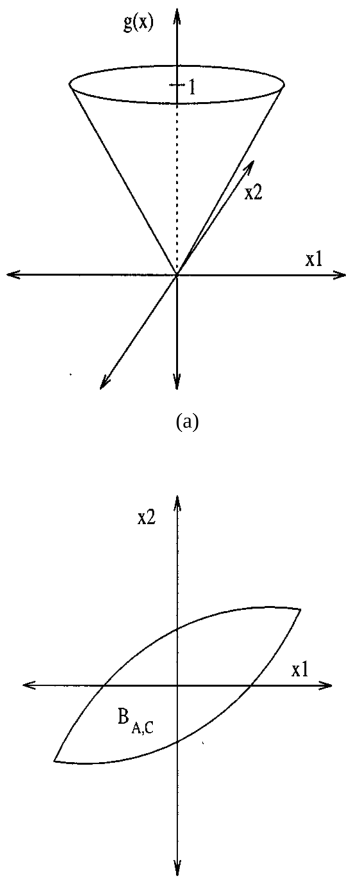


Fig. 9. Visualization of $g(x)$ and $B_{A,\,C}$.

It can be shown that for any state $x \in B_{A,\,C}$ such an operator λ exists, hence, the controls will never saturate for any exogenous input (reference command, disturbance, etc.). Moreover, once the compensator state x is in the $B_{A,\,C}$, it will never leave for any time t. Perhaps the most important result, however, is that such a system is always finite gain stable, i.e., that given the interconnection in Fig. 8, there exists finite constants γ and β such that $||z||_\infty \le \gamma ||w||_\infty + \beta$.

Theorem 3.1: The feedback system with a stable plant (A_p, B_p, C_p) and neutrally stable compensator given by (11) and (12) is finite gain stable.

Proof: $\exists\, r_o \ni ||r||_\infty \le r_o \implies ||u||_\infty \le 1\ \exists\, y_o \ni ||y||_\infty \le y_o\ \forall\, r(t)$ since $P(s)$ is stable with bounded inputs.

CASE 1: $||r||_\infty \le r_o$ then $\lambda = 1\, \forall\, t \ge 0$ and the closed-loop system is finite gain stable as designed.

CASE 2: $||r||_\infty > r_o$ then $||y||_\infty \le y_o \implies ||y||_\infty \le (||r||_\infty / r_o) y_o$ and $||y||_\infty \le (y_o/r_o)||r||_\infty$, thus, for $\gamma = (y_o/r_o)$, $||y||_\infty \le \gamma ||r||_\infty$.

B. EG Implementation

For the vehicle platooning problem, an extension of this theory is made to accommodate the decentralized structure of the platoon controller [16]. The idea is rather than using a scalar gain $\Lambda = \lambda I$, each car should be equipped with its own scalar gain λ_i. This implies that Λ is a diagonal matrix with elements $\lambda_1, \lambda_2, \cdots, \lambda_N$. On a per vehicle basis, then, the resulting decentralized EG reduces to the scalar approach presented above, and all the proofs subsequently hold. Thus, we are guaranteed that once the compensator state is in the $B_{A,\,C}$ the controls will never saturate. Likewise, closed-loop stability follows.

The EG tries to "mimic" the nominal controller in the sense that each λ_i is the largest gain between zero and one that will prevent saturation. When the measurements will not provoke saturation, no action is taken and the EG reduces to the nominal controller.

Implementation issues arise in characterizing g or the set $B_{A,\,C}$. Although there has been research into the computational aspects of this problem [5], it is not an issue in the vehicle platooning problem because $A = [0]$ making the computation of g trivial. Other technicalities include discretization of the compensator model (A, B, C, D) and numerical procedures used in computing λ_i. This study uses standard sample-and-hold techniques (zero-order) to discretize, and employs Runga–Kutta integration procedures.

Fig. 10 shows the result of implementing a decentralized EG on the vehicle platoon. While it is clear that the EG completely eliminates windup, a notable tradeoff in performance is also observed. This degradation in performance is simply the result of λ erroneously scaling the compensator state even though the state is well within the $B_{A,\,C}$.

IV. Performance Enhancement: EEG

The EG becomes conservative when the nominal compensator includes a feedforward term D. This conservatism appears when the compensator state x is internal to the $B_{A,\,C}$, but when large measurements threaten saturation through D. Problems with the EG in such a case are: 1) directional properties of the control vector u are not necessarily preserved and 2) the compensator state x is affected by large measurements—even though it is internal to the $B_{A,\,C}$—since λ scales $\dot{x}$ through B. This is exactly the reason for performance degradation in the vehicle platoon problem (Fig. 11).

An alternative construction for the EG is shown in Fig. 12. This "EEG" rectifies the problems outlined above by introducing a second scalar gain at the *output* of the nominal compensator. This second gain essentially acts as a *radial* saturation that preserves each vehicle's control direction while allowing another degree of freedom in the EG algorithm. Thus, when $x \in B_{A,\,C}$, but large measurements threaten saturation, λ_1 can remain unity—leaving x to evolve undisturbed—while λ_2 scales the resulting control to the edge of saturation (Fig. 13).

Given a compensator of the form

$$\dot{x}(t) = Ax(t) + By(t),\ x(0) = x_o \tag{14}$$

$$u(t) = Cx(t) + f(y(t)) \tag{15}$$

the construction of λ_1 and λ_2 then becomes

- λ_1 is given by the following.
  1) $0 \le \lambda_1 \le 1$.
  2) If $x \in B_{A,\,C}$, then $\lambda_1 = 1$.

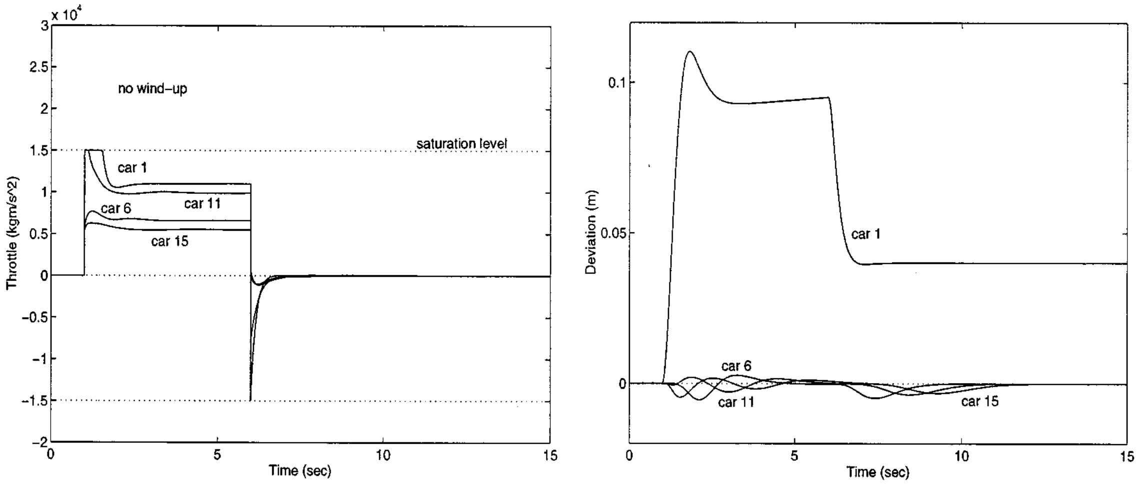
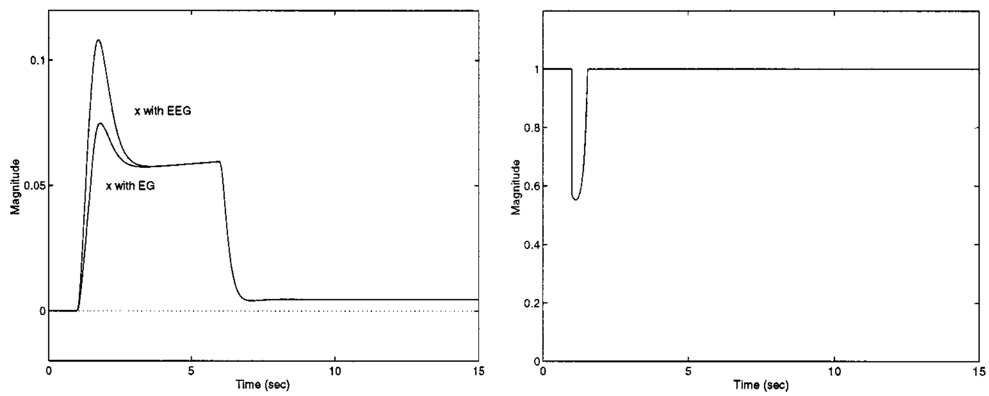


Fig. 10. The EG can eliminate windup at a cost to steady-state performance.



(a) (b)

Fig. 11. (b) λ erroneously affects the (a) EG compensator state for car 1.

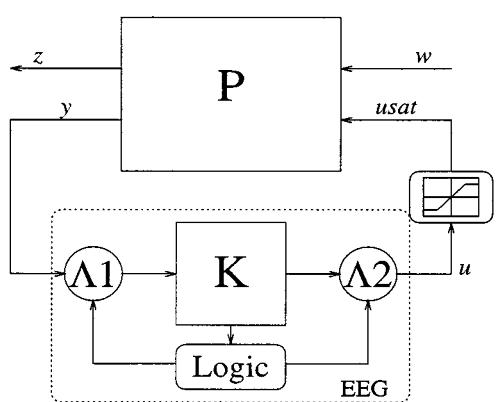


Fig. 12. EEG structure.

3) If $x(t) \in$ boundary $B_{(A,C)}$, then maximize λ_1 subject to

$$\limsup_{\epsilon \to 0} \frac{p(\lambda_1)}{\epsilon} \leq 0$$

where

$$p(\lambda_1) \triangleq g(x(t) + \epsilon[Ax(t) + B\lambda_1 y(t)]) - g(x(t)).$$

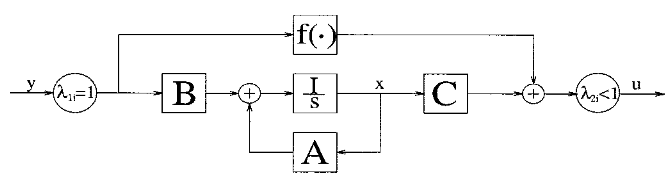


Fig. 13. The EEG can prevent windup without erroneously affecting x by keeping λ_1 unity.

4) If $x(t) \notin B_{(A,C)}$, then

$$\min_{\lambda_1} \limsup_{\epsilon \to 0} \frac{p(\lambda_1)}{\epsilon}.$$

- λ_2 is given by

$$\max_{0 \leq \lambda_2 \leq 1} |\lambda_2|$$

subject to

$$\|\lambda_2(Cx(t) + f(\lambda_1 y(t)))\|_\infty \leq 1.$$

It is easy to see that, just like the EG, the EEG will never allow saturation once $x \in B_{A,C}$. Also, the resulting closed-loop system is finite gain stable since the proof to Theorem 2.1 depends only on the open- and closed-loop stability of P and properties of the saturation. Moreover, the EEG will always pre-

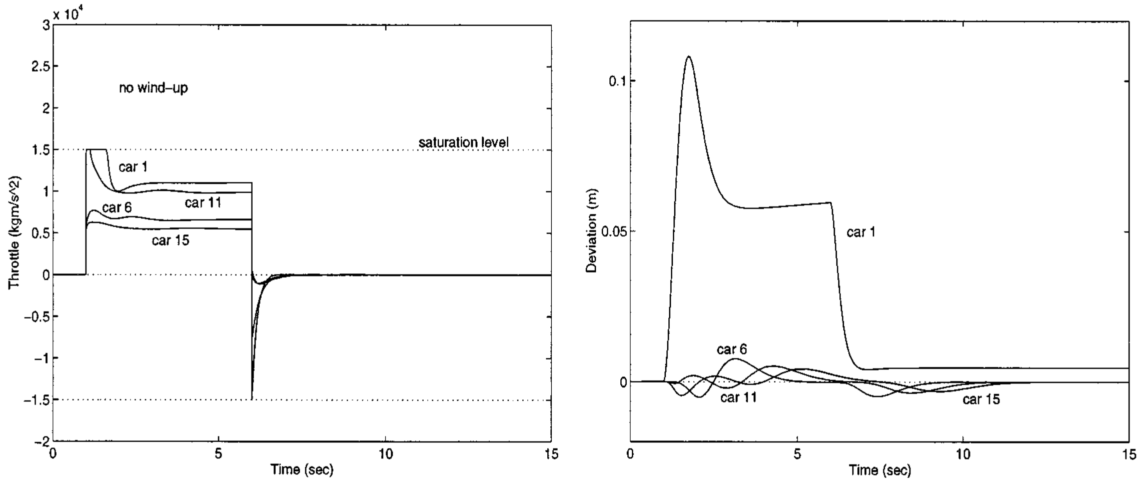


Fig. 14. The EEG eliminates windup while maintaining performance.

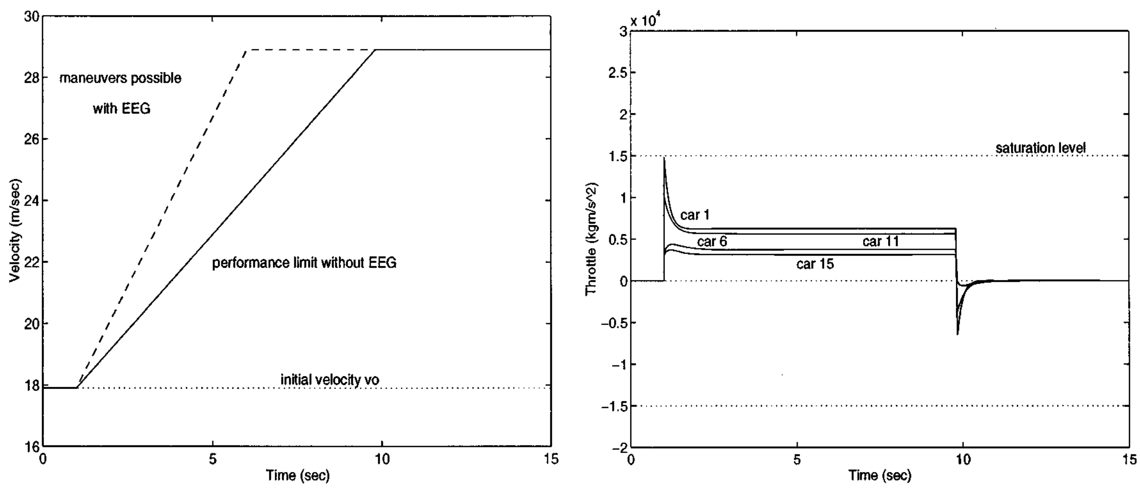


Fig. 15. The EEG allows a broader range of maneuvers.

serve directional properties of the nominal compensator in that measurement and control directions are undisturbed. Also, by construction, the EEG does not erroneously dampen $\dot{x}$ when $x \in B_{A,C}$ and large measurements threaten saturation through D. Finally, note that all these results hold for compensators with linear dynamics but with a nonlinear feedforward term f. Of course, complexity in f will lead to a more difficult computation of λ_2. These properties allow the EEG to maintain key features of the EG while reducing conservatism in its attempt to "mimic" nominal behavior for a class of controllers [17].

Fig. 14 verifies that, in fact, the EEG does recover performance lost by the EG while managing windup. In fact, the *key* properties of the nominal controller, i.e., that deviations do not get magnified down the platoon, etc., are preserved by the EEG to the extent possible. This results from the EEG attempts to "mimic" the nominal platoon behavior by remaining inactive unless it is absolutely necessary to prevent saturation. It is interesting to note that transient performance is lost regardless of the antiwindup strategy. In fact, the deviations in nominal separations for the EEG-corrected platoon are identical to those of the nominal platoon when it is allowed to saturate. In what sense, then, is the EEG a performance enhancing technique, and why can't transient performance be improved in this example?

The value of the EEG as a performance enhancing design strategy, compared to the saturating system, lies in its ability to allow a broader range of windup avoiding maneuvers to the lead vehicle. In the nominal platoon, the lead vehicle's acceleration must be less than 1.25 m/s^2 (Fig. 15) (or some "smoothing" filter on the reference commands must be employed) to prevent windup and guarantee stability. With the EEG, however, windup is prevented and stability is guaranteed *regardless of the maneuver the lead vehicle makes*.

The value of the EEG as a performance enhancing design strategy, compared with the EG-compensated system, lies in the recovery of steady-state performance. The EG is simply not a useful technique for systems with feedforward terms. The platoon problem considered here highlights this point since the windup incurred results entirely from feedforward action, and not from the state leaving the $B_{A,C}$.

Transient performance, however, cannot be uniformly improved by any technique. This is clear since the deviations result from each vehicle's inability to generate large enough controls to "keep up." For this application, the EEG simply recovers the performance lost by the EG algorithm while delivering the same guarantees carried by the EG design.

V. Conclusion

The longitudinal control of a platoon of vehicles, like most IVHS applications, suffer from the various consequences of control saturation. In particular, slow dynamics in the nominal controller provoke arbitrary windup. Moreover, stability can be lost if the controls are simply allowed to saturate.

This research demonstrated a systematic way to deal with the problems posed by saturation. The EG was discussed as an interesting idea for dealing with saturation, primarily because it guarantees stability and windup elimination. Nevertheless, the platoon problem considered here highlights precisely where the EG fails, in situations where saturation is occurring due to large feedforward signals in the compensator and not as a result of the compensator state leaving the set $B_{A,C}$.

The EEG was thus developed as an extension of the EG to cases where the controller has a (possibly nonlinear) feedforward, or proportional control, term. Like the EG, the EEG guarantees stability preservation and windup elimination. Unlike the EG, however, the EEG does not always scale the input of the compensator when compensator outputs are large enough to saturate. Instead, the EEG effectively allows controls to radially-saturate if they are generated by large feedforward terms (rather than the compensator state leaving the "safe" region $B_{A,C}$), as in the platoon example considered here. Hence, the response of the EEG-compensated system and the nominal system allowed to saturate are virtually identical in this study (see Figs. 7 and 14). In this sense, the platoon problem is exceptional; it highlights precisely where the EG fails and commissions the EEG design to "recover" performance of the saturating system, thereby making it's operation transparent. Note that generally, however, the EEG response will deviate from that of both the saturating and EG system when both λ_1 and λ_2 are simultaneously activated.

Nevertheless, there is a critical difference between the EEG-compensated system and simply allowing nominal system to saturate, even for this platoon example where the responses appear identical. That is, the EEG carries the same stability guarantees proven for the EG. The nominal system allowed to saturate carries no such guarantee in general. In this sense, the EEG-compensated system allows a broad range of otherwise prohibited (i.e., potentially unsafe) aggressive maneuvers (Fig. 15).

Future work can expand and compliment the results of this research by studying the effects of saturation on system robustness, or constructing an extension of the reference governor proposed in [10] that parallels the EEG. Also, computational issues related to the characterization of g and $B_{A,C}$ for more complex systems can be studied. In particular, it would be interesting to develop an analogous theory where g is not defined in the sup-norm (that checks if signals will *ever* saturate), but rather uses a less conservative measure to check if things will saturate *soon* (in some appropriate sense). Likewise, low-complexity characterizations of the $B_{A,C}$ using neural networks or other techniques may be critical to make these results available to many complex real-world problems. As better control strategies for IVHS systems are developed, accommodating complex maneuvers such as lane changing, platoon splitting, merging, etc., it is hoped that the design tool given here will be used to accommodate the various consequences of control saturation and yield insight into the performance enhancement of multiple vehicle guidance problems in general.

References

[1] J. Ackermann and T. Buente, "Actuator rate limits in robust car steering control," in *Proc. 36th Conf. Decision and Control*, San Diego, CA, 1997.

[2] M. Broucke and P. Varaiya, "The automated highway system: A transportation technology for the 21st century," in *13th World Congr. IFAC*, San Francisco, CA, 1996.

[3] W. Chee, M. Tomizuka, W. Zhang, and S. N. Patwardhan, "Experimental study of lane change maneuver for AHS applications," in *Proc. American Control Conf.*, Seattle, WA, 1995.

[4] M. Dahleh and I. Bobillo, *Control of Uncertain Systems: A Linear Programming Approach*. Englewood Cliffs, NJ: Prentice-Hall, 1995.

[5] E. Gilbert and K. Tan, "Linear systems with state and control constraints: The theory and application of maximal output admissible sets," *IEEE Trans. Automat. Contr.*, vol. 36, no. 9, 1991.

[6] D. N. Godbole, F. Eskafi, E. Singh, and P. Varaiya, "Design of entry and exit maneuvers of IVHS," in *Proc. American Control Conf.*, Seattle, WA, 1995.

[7] D. N. Godbole, V. Hagenmeyer, R. Sengupta, and D. Swaroop, "Design of emergency maneuvers for automated highway system: Obstacle avoidance problem," in *Proc. 36th Conf. Decision and Control*, San Diego, CA, 1997.

[8] H. Holzmann, Ch. Halfmann, S. Germann, M. Wurtenberger, and R. Isermann, "Longitudinal and lateral control and supervision of autonomous intelligent vehicles," in *13th World Congr. IFAC*, San Francisco, CA, 1996.

[9] A. Isidori, *Nonlinear Control Systems*, 2nd ed. New York: Springer-Verlag, 1989.

[10] P. Kapasouris, "Design for performance enhancement in feedback control systems with multiple saturating nonlinearities," Ph.D. dissertation, MIT, Cambridge, MA, 1988.

[11] P. Kapasouris, M. Athans, and G. Stein, "Design of feedback control systems for stable plants with saturating actuators," in *Proc. 27th Conf. Decision and Control*, Austin, TX, 1988, pp. 469–479.

[12] A. Megretski and A. Rantzer, "System analysis via integral quadratic constraints," *IEEE Trans. Automat. Contr.*, vol. 42, pp. 819–830, June 1997.

[13] S. Sheikholeslam and C. Desoer, "Longitudinal control of a platoon of vehicles," in *Proc. American Control Conf.*, vol. 1, San Diego, CA, 1990, pp. 291–297.

[14] A. R. Teel, "A nonlinear small gain theorem for the analysis of control systems with saturation," *IEEE Trans. Automat. Contr.*, vol. 41, Sept. 1996.

[15] F. Tyan and D. Bernstein, "Antiwindup compensator synthesis for systems with saturating actuators," in *Proc. 33rd Conf. Decision and Control*, Lake Buena Vista, FL, 1994.

[16] S. Warnick and A. Rodriguez, "Longitudinal control of a platoon of vehicles with multiple saturating nonlinearities," in *Proc. American Control Conf.*, vol. 1, Baltimore, MD, 1994.

[17] ——, "Performance enhancement for a class of saturating systems," in *Proc. American Control Conf.*, Seattle, WA, 1995.

[18] G. Zames and P. Falb, "Stability conditions for systems with monotone and slope-restricted nonlinearities," *SIAM J. Control*, vol. 6, no. 1, pp. 89–108, 1968.

Sean C. Warnick received the B.S.E. degree (*summa cum laude*) in 1993 from Arizona State University, Tempe, and the S.M. degree in 1995 from the Massachusetts Institute of Technology, Cambridge. He is currently working toward the Ph.D. degree at the Massachusetts Institute of Technology.

His research interests have focused on the robustness and complexity of multivariable feedback designs for various nonlinear systems. Applications have included IVHS, spectroscopic ellipsometry as a sensor technology, and the real-time control of metalorganic chemical vapor deposition (MOCVD) and related processes. He has held consulting positions with various companies including Spire Corporation, Textron Specialty Materials, and Neurodyne, Inc. He currently sits on the Board of Directors of Crescent Technologies, Inc.

Mr. Warnick was named the Outstanding Graduate of 1993 and received the Fredrick C. Hennie III Award for excellence in teaching in 1997.

Armando A. Rodriguez received the Ph.D. degree from the Massachusetts Institute of Technology, Cambridge.

He joined the Electrical Engineering Department, Arizona State University, Tempe. His main areas of research include control of distributed parameter systems, approximation theory, robust control of flexible autonomous machines operating in uncertain environments (FAME), and development of interactive multivariable control system design tools. Application areas include robotics, aerospace, IVHS, and semiconductor manufacturing. He has more than 90 publications which span the above areas. He has worked and/or consulted for various companies including AT&T, Bell Laboratories, Boeing, Englin Air Force Base, Honeywell, and IBM.

Dr. Rodriguez is an AT&T Bell Laboratories Fellow and a Boeing A.D. Welliver Fellow. He is the recipient of various teaching excellence awards and IEEE advising awards. In 1998, he received a White House Presidential Excellence Mentoring Award. The award was presented in a White House ceremony with President Bill Clinton.