

Adaptive Tracking Control of a Class of Mechanical Systems

Ying Jin, and Jun Fu, *Senior Member, IEEE*

Abstract—This paper proposes a combined method for motion/force tracking control of a class of Lagrange mechanical systems by using cascaded methods and backstepping techniques. This paper first transforms error dynamics between the kinematic system and the desired trajectories into a cascaded system consisting of two subsystems and an interconnection function, then, designs virtual controllers for the kinematic systems under the framework of cascaded methods, last develops the tracking controller for the overall mechanical systems using adaptive backstepping techniques. Simulation studies are given to verify the effectiveness of the proposed method.

I. INTRODUCTION

NONHOLONOMIC systems abounding in robotics, as an important class of mechanical systems, range from unicycle- and car-like vehicles to more original systems such as rolling spheres, snake-like robots and roller racers [1]-[3]. In the last three decades, studies on the mechanical systems have attracted much attention from the control community [1]-[5], because no time-invariant smooth state feedback control law can stabilize the nonholonomic systems with restricted mobility to a desired configuration, as illustrated by Brockett's theorem [6]. Thus many publications have focused on the stabilization problem (see [9], [11] and references therein).

In practice, tracking a reference trajectory, as a more interesting issue, has received relatively less attention in the literature. According to the nonholonomic systems presenting at a kinematic or dynamic level, the tracking problem is usually classified into kinematic tracking or dynamic tracking problems. Thus the kinematic and dynamic tracking problems yield kinematic control such as driving speed and physical controls such as driving torques, respectively [7], [8] and [10]. Like the stabilization case, most of the work on the tracking problem in the literature focuses at the kinematic level [12]. Recognizing the importance of addressing the tracking control problem at the dynamic level, several works dealing with this problem have been reported in [10] and [15]. References [15] and [16] propose adaptive controllers for the nonholonomic dynamic systems with unknown inertia parameters. [10] and [17] develop robust adaptive controllers for the nonholonomic dynamic systems with some uncertainties in its dynamics or its constraints. [18] presents a sliding-mode-based robust controller for the nonholonomic

dynamic systems with disturbances. As pointed out in [19], the above-mentioned results suffer from an overly complicated structure stemming from a number of adaptive adjusting terms needed to construct the controllers against considered uncertainties. Although the high gain adaptive feedback controller is rather simple, in practice it may lead to instability or high noise or both [20]. On the other hand, control of the forces of the contact interactions is at least as important as the position control. Since physically the constraints provide the necessary reactions, no matter what the proposed control algorithms may be, it is not practical unless the position and the force of interaction are controlled in a simultaneous way. And yet, the literature on controlling the two objectives simultaneously is sparse [21, 22].

At a dynamic level, although each of the above-mentioned methods has different characteristics, in practice these control algorithms may be insufficient in solving the problems in terms of computation, once they are designed and implemented [23]. Realizing the advantages and disadvantages of the methods in the literature on motion and force tracking control of the nonholonomic systems and the practically computational insufficiencies in implementation, this paper proposes a new control strategy for motion and force tracking control of the nonholonomic dynamic systems by combining cascaded methods and backstepping techniques. The main feature of this paper lies in: 1) under the framework of cascaded methods, virtual controllers for the subsystems are designed to stabilize error dynamics between the kinematic system and the desired path with the aid of H-infinity control theory and Linear Matrix Inequalities (LMI); 2) the adaptive tracking controller for the overall dynamic system is designed by combining adaptive backstepping techniques and cascaded methods.

This paper is organized as follows. In Section II, a model of the mechanical systems and a control objective are given. Section III provides the main results, which consists of derivation of cascaded systems, virtual controller design, and adaptive control design. A case study on a mobile wheeled robot is carried out in Section IV. In Section V, a brief discussion is given for an extension from the considered mechanical system into an electrically driven nonholonomic dynamic systems. Some concluding remarks are used to sum up the paper in the last section.

II. MODEL DESCRIPTION OF MECHANICAL SYSTEM AND CONTROL OBJECTIVE

A. Model Description

Consider the following mechanical systems [10]

Ying Jin is with the School of Information Science and Engineering, Northeastern University, Shenyang, China, 110189, also with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, Canada, H3H 1M8. (Email: jymontreal@gmail.com).

Jun Fu is with the Department of Mechanical Engineering, Massachusetts Institute of Technology (MIT), Cambridge, MA, USA, 02139.

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = J^T(q)\lambda + B(q)\tau \quad (1)$$

$$J(q)\dot{q} = 0 \quad (2)$$

where q and τ denote the n vector of generalized coordinates and the r vector of generalized control input force, respectively; $\lambda \in R^m$ is the associated Lagrangian multiplier expressing the contract force; $D(q)$, $C(q, \dot{q})\dot{q}$ and $G(q)$ are $(n \times n)$ symmetric, bounded, positive definite inertia matrix, the n vector of centripetal and Coriolis torques, and the n vector of gravitational torques, respectively; $B(q)$ and $J(q)$ are $(n \times r)$ assumed known input transformation and $(m \times n)$ constraint matrix, respectively. Here assumptions are needed that Equation (2) is completely nonholonomic for all q and t , and $B(q)$ is a full-rank matrix and r is not less than $n - m$.

Before the control objective is stated, two simplifying properties [10] are presented about the mechanical systems, which will be used in the later design.

Property 1: A proper definition of $C(q, \dot{q})$ makes $\dot{D} - 2C$ skew symmetric.

Property 2: There exists an inertial parameter p vector β_p which depends on mechanical parameters, such as masses and inertia moments, such that

$$D(q) + C(q, \dot{q})v + G(q) = \Phi(q, \dot{q}, v, \dot{v}) \beta_p \quad (3)$$

where β_p is the vector of inertia parameters, and Φ is an $(n \times l)$ matrix of known function of q, \dot{q}, v, \dot{v} .

B. Control Objective

Given a desired contact force λ_d and desired trajectories q_d and \dot{q}_d which are assumed to be bounded and satisfy constraint (2), the control objective is to determine a control law for τ such that λ , q and \dot{q} asymptotically converge to λ_d , q_d and \dot{q}_d , respectively.

Before applying the strategy to be designed, the following transformation [10] on the system structure is needed.

Let v be a vector of independent generalized velocities and define $R(q)$ such that it maps vector v into a vector of feasible generalized velocities \dot{q} that satisfies constraint (2), namely

$$\dot{q} = R(q)v \quad (4)$$

Combining (2) and (4) yields

$$R^T(q)J^T(q) = 0 \quad (5)$$

Differentiation of (4) gives

$$\ddot{q} = R\dot{v} + \dot{R}v \quad (6)$$

Thus dynamic system (1) can be transformed into

$$\dot{q} = R(q)v \quad (7)$$

$$D(q)R(q)\dot{v} + C_1(q, \dot{q})v + G(q) = B(q)\tau + J^T(q)\lambda \quad (8)$$

where $C_1(q, \dot{q}) = D(q)\dot{R}(q) + C(q, \dot{q})R(q)$.

Remark 1: Please note that in systems (7) and (8) v will be viewed as a virtual controller in terms of backstepping technique, which will be illustrated in the following section to reach the control objective.

III. MAIN RESULTS

This section consists of three parts. The first part is the derivation of cascaded method for kinematic subsystem (7).

The second part is the design of a virtual controller with the aid of cascaded method, H-infinity control theory and Linear Matrix Inequalities (LMI). The third part is the adaptive control design for the overall dynamic systems (7) and (8) with adaptive backstepping techniques, and the stability analysis via Lyapunov techniques.

A. Derivation of Cascaded Method

According to [11], there exist a coordinate transformation, $x = \Psi(q)$, and a state feedback, $v = \Omega_1(q)u$, such that subsystem (7) with two independent generalized velocities can be transformed to the chained form

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_2 u_1 \\ &\vdots \\ \dot{x}_n &= x_{n-1} u_1 \end{aligned} \quad (9)$$

where $u = (u_1, u_2)^T$ is the input and x is the state.

Remark 2: Under the coordinate transformation [26, 10], dynamic model (8) is accordingly converted into

$$D_2(x)R_2(x)\dot{u} + C_2(x, \dot{x})u + G_2(x) = B_2(x)\tau + J_2^T(x)\lambda \quad (10)$$

where

$$D_2(x) = D(q)|_{q=\Psi^{-1}(x)},$$

$$R_2(x) = R(q)\Omega_1(q)|_{q=\Psi^{-1}(x)}$$

$$C_2(x, \dot{x}) = [D(q)R(q)\dot{\Omega}_1(q) + C_1(q, \dot{q})\Omega_1(q)]|_{q=\Psi^{-1}(x)}$$

$$G_2(x) = G(q)|_{q=\Psi^{-1}(x)}$$

$$J_2(x) = J(q)|_{q=\Psi^{-1}(x)}$$

$$B_2(x) = B(q)|_{q=\Psi^{-1}(x)}$$

The tracking problem for the chained-form system (9) is usually formulated as follows. To design an appropriate controller so that the state trajectory of system (2) can follow a vector-valued reference signal

$$x_d(t) = (x_{1d}(t), \dots, x_{nd}(t))^T$$

which is generated by a system of the same form as system (2)

with $u = (u_{1d}, u_{2d})^T$.

Assumption 1: Assume $u_{1d} = d(t)$ and there exist nonzero T and D such that $|D| > T > 0$ and the bounded continuous $d(t)$ satisfies $|d(t) - D| < T$.

Define the tracking errors as $y_i = x_i - x_{id}$, $i = 1, \dots, n$, and it is easy to obtain the tracking error dynamics described by

$$\begin{aligned} \dot{y}_1 &= u_1 - u_{1d} \\ \dot{y}_2 &= u_2 - u_{2d} \\ \dot{y}_3 &= u_{1d}y_2 + x_2(u_1 - u_{1d}) \\ &\vdots \\ \dot{y}_n &= u_{1d}y_{n-1} + x_{n-1}(u_1 - u_{1d}) \end{aligned} \quad (11)$$

System (11) can be arranged into

$$\begin{bmatrix} \dot{y}_2 \\ \dot{y}_3 \\ \vdots \\ \dot{y}_{n-1} \\ \dot{y}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ d(t) & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & d(t) & 0 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \\ \vdots \\ y_{n-1} \\ y_n \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} (u_2 - u_{2d}) + \begin{bmatrix} 0 \\ x_2 \\ \vdots \\ x_{n-2} \\ x_{n-1} \end{bmatrix} (u_1 - u_{1d}) \quad (12)$$

$$\dot{y}_1 = (u_1 - u_{1d}) \quad (13)$$

Denote

$$Y = [y_2, y_3, \dots, y_n]^T$$

and

$$\Sigma(t) = \frac{d(t) - D}{T}$$

System (12) can be rewritten as

$$\dot{Y} = (A + E\Sigma(t)F)Y + B(u_2 - u_{2d}) \quad (14)$$

where

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ D & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & D & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} T & 0 & \cdots & 0 & 0 \\ 0 & T & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & T & 0 \\ 0 & 0 & \cdots & 0 & T \end{bmatrix}.$$

interconnected with

$$f(x, (u_1 - u_{1d})) = (0, x_2, \dots, x_{n-2}, x_{n-1})^T (u_1 - u_{1d}).$$

So the error dynamics (11) can be treated as (14) cascaded by (13) with the interconnection term $f(t, x, (u_1 - u_{1d}))$. Thus the cascaded method [24, 25] can be applied to the system (12) and (13). The detailed design is given in the next subsection.

B. Stabilization of the Cascaded System

In this subsection, first the two sub-controllers are designed for the two subsystems of the cascaded system, and then the cascaded method is applied to the cascaded system for stabilization [25].

For the subsystem (13), it is easy to design

$$u_1 = u_{1d} - k_1 y_1, k_1 > 0 \quad (15)$$

to guarantee globally exponential stability of the corresponding closed-loop system.

For the subsystem (14), a controller can be designed as

$$u_2 = u_{2d} + KY \quad (16)$$

to ensure the stabilization of the following closed-loop subsystem

$$\dot{Y} = (A + BK + E\Sigma(t)F)Y \quad (17)$$

provided that K satisfies some norm conditions, which will be determined with the cascaded method in [25], and $\Sigma(t)$ belongs to the following set (see Assumption 1):

$$\Omega = \{\Sigma(t) \mid \Sigma(t)^T \Sigma(t) \leq I, \forall t\}.$$

As can be seen, system (17) is a linear system with a structural uncertainty. Therefore, (17) is quadratically stable if and only if $A + BK$ is stable and $\|F(sI - A - BK)^{-1}E\|_\infty < 1$

, which is equivalent to the existence of a positive definite matrix P [28] such that

$$(A + BK)^T P + P(A + BK) + PE(PE)^T + F^T F < 0 \quad (18)$$

Thus according to [23], by Schur's Complement Lemma the inequality (18) is converted into

$$\begin{bmatrix} P^{-1}(A + BK)^T + (A + BK)P^{-1} & E & (FP^{-1})^T \\ E^T & -I & 0 \\ FP^{-1} & 0 & -I \end{bmatrix} < 0 \quad (19)$$

In the end, LMI technique can be used to solve inequality (19) to determine K .

Thus according to the cascaded method [24, 25], the kinematic system in chained form is stabilized with (15) and (16), because the two sub-controllers globally stabilize the two subsystems and the interconnection term $f(t, x, (u_1 - u_{1d}))$ satisfies the linear growth norm condition [24, 25] as long as $k_1 > 0$. Please note that (15) and (16) will be treated as designed virtual controllers in the backstepping technique to be developed in the next section.

C. Adaptive Control of Dynamic Systems

In this section, by using backstepping techniques we will design a real controller for (10) and (11).

To develop the control law, we need the following assumptions.

Assumption 2: The trajectories y_d , their first time derivative and up to $(n-1)th$ time derivative inclusive are bounded.

Assumption 3: The determinate of $R_2^T R_2$ is not equal to zero for all $y \in R^n$ and $t \in R$.

Exploiting the structure of (10) obtains the following three useful properties under the above assumptions, which can be found in [10, 17].

Property 3: $D_3 = R_2^T D_2(q) R_2$ is symmetric and positive definite.

Property 4: If C is satisfied Property 2, $\dot{D}_3 - 2R_2^T C_2$ is skew-symmetric.

Property 5: In relation to the same suitable selected set of

inertia parameters in Property 1, the dynamics

$$D_2(x)R_2(x)\dot{u} + C_2(x, \dot{x})u + G_2(x) = \Phi_1(y, \dot{y}, u, \dot{u})\beta_p \quad (20)$$

where Φ_1 is a $(n \times l)$ regressor matrix and β_p is the l vector of inertia parameters, which will be estimated by the adaptive estimation design in this section.

Defining

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} u_1 - u_1^* \\ u_2 - u_2^* \end{bmatrix} \quad (21)$$

we have ξ - dynamics as follows

$$D_2(x)R_2(x)\dot{\xi} + C_2(x, \dot{x})\xi = \Phi_1(y, \dot{y}, u, \dot{u})\beta_p - B_2(x)\tau - J_2^T \lambda \quad (22)$$

and constructing a Lyapunov function candidate for

$$V = \frac{1}{2}y_1^2 + \frac{1}{2}Y^T P Y + \frac{1}{2}\xi^T D_3 \xi + \frac{1}{2}\tilde{\beta}_p^T \Gamma \tilde{\beta}_p \quad (23)$$

where P and D_3 are defined in the previous section, and

$$\tilde{\beta}_p = \beta_p - \hat{\beta}_p.$$

Differentiating (23) along (11) and (22) and yields

$$\begin{aligned} \dot{V} \leq & -k_1 y_1^2 + \frac{1}{2}Y^T [(A+BK)^T P + P(A+BK) + PE(PE)^T \\ & + F^T F] Y + N^T \xi + \frac{1}{2}\xi^T D_3 \xi + \frac{1}{2}\xi^T \dot{D}_3 \xi \\ & + \frac{1}{2}\xi^T D_3 \dot{\xi} + \tilde{\beta}_p^T \Gamma \dot{\tilde{\beta}}_p \end{aligned} \quad (24)$$

where $N := (Y^T P B, y_1)^T$ and the term $N^T \xi$ stems from differentiating augmented Lyapunov function candidate and using the previous Lyapunov function of the previous subsystem in a standard backstepping procedure [26].

Substituting (13), (15) and (18) into (24) gives

$$\begin{aligned} \dot{V} \leq & N^T \xi + \frac{1}{2}\xi^T D_3 \xi + \frac{1}{2}\xi^T \dot{D}_3 \xi + \frac{1}{2}\xi^T D_3 \dot{\xi} + \tilde{\beta}_p^T \Gamma \dot{\tilde{\beta}}_p \\ = & N^T \xi + \frac{1}{2}[(D_2 R_2)^{-1}(\Phi_1(\tilde{\beta}_p + \hat{\beta}_p) - B_2 \tau - C_2 \xi - J_2^T \\ & \cdot \lambda)]^T D_3 \xi + \frac{1}{2}\xi^T \dot{D}_3 \xi + \frac{1}{2}\xi^T D_3 [(D_2 R_2)^{-1}(\Phi_1(\tilde{\beta}_p \\ & + \hat{\beta}_p) - B_2 \tau - C_2 \xi - J_2^T \lambda)] - \tilde{\beta}_p^T \Gamma \dot{\tilde{\beta}}_p \end{aligned} \quad (25)$$

From Property 3, (25) is rearranged into

$$\begin{aligned} \dot{V} \leq & \xi^T \left(\frac{1}{2} \dot{D}_3 - R^T C_2 \right) \xi + (\xi^T R_2^T \Phi_1 - \dot{\tilde{\beta}}_p^T \Gamma) \tilde{\beta}_p \\ & + \xi^T R_2^T (R_2^{-1} N + \Phi_1 \hat{\beta}_p - B_2 \tau - J_2^T \lambda) \end{aligned} \quad (26)$$

Thus from Property 4, we can design the real control as

$$\begin{aligned} B_2 \tau &= R_2^{-1} N + \Phi_1 \hat{\beta}_p - \gamma R_2 \xi - J_2^T \lambda_c \\ \dot{\hat{\beta}}_p &= -\Gamma^{-1} \Phi^T R_2 \xi \end{aligned} \quad (27)$$

where γ is a positive definite matrix, which can be adjusted for practical requirements on convergence rate, and the force term λ_c is defined as $\lambda_d - K_\lambda(\lambda - \lambda_d)$, and K_λ therein is a constant matrix of force control feedback gains.

From (26) and (27), we can conclude that the time derivative of the Lyapunov function candidate V (23) is less

than or equal to zero, and in turn, that $y_i, (i=1, \dots, n)$ and ξ are bounded, and V will converge to its limit value as time goes to infinity. By $y_i = x_i - x_{id}, i=1, \dots, n$ and Assumption 2, $x_i, (i=1, \dots, n)$ are bounded. From the dynamic equations (11) and (22), \dot{y} and $\dot{\xi}$ are bounded, thus they are uniformly bounded. Noticing the boundedness of \ddot{V} and \dot{V} being uniformly continuous, we have that \dot{V} approaches zero, thus $y_i, (i=1, \dots, n)$ and ξ approach zero. Thus $x \rightarrow x_d$ and $\dot{x} \rightarrow \dot{x}_d$ as $t \rightarrow +\infty$. In addition, since

$$J^T(\lambda_c - \lambda) = -D_2 R_2 \dot{\xi} - \Phi_1 \tilde{\beta}_p - (\gamma R_2 + C_2)\xi + R_2^{-1} N \quad (28)$$

and the right hand side of (28) is bounded, noticing that $\lambda_c = \lambda_d - K_\lambda(\lambda - \lambda_d)$, thus we have

$$J^T(\lambda_d - \lambda) = (-D_2 R_2 \dot{\xi} - \Phi_1 \tilde{\beta}_p - (\gamma R_2 + C_2)\xi + R_2^{-1} N)(I - K_\lambda)^{-1}$$

which indicates that $\lambda - \lambda_d$ is bounded, and proportional to $(I - K_\lambda)^{-1}$ in terms of norm metric, therefore can be tuned by changing the gain matrix K_λ .

To this end, the main result is stated in the following theorem.

Theorem: Considering the system (9) and (10), the adaptive controller (27) ensures that all signals of the closed-loop system are bounded, that y asymptotically tracks the reference signal y_d , and that $\lambda - \lambda_d$ is proportional to $(I - K_\lambda)^{-1}$ in terms of norm metric, provided Assumptions 1-3 are satisfied.

Proof: It is obvious from the controller design and Lyapunov-based stability analysis above.

IV. APPLICATION TO A MOBILE WHEELED ROBOT

In order to investigate the effectiveness of our proposed controller, a simplified mobile wheeled robot model [10] is studied below.

A. System Model

The robot will move on a horizontal plane, constituted by a rigid trolley equipped with non-deformable wheels. The dynamic model is described by

$$\begin{aligned} m\ddot{x} &= \lambda \cos \theta - \frac{1}{P}(\tau_1 + \tau_2) \sin \theta \\ m\ddot{y} &= \lambda \sin \theta - \frac{1}{P}(\tau_1 + \tau_2) \cos \theta \\ I_0 \ddot{\theta} &= \frac{L}{P}(\tau_1 - \tau_2) \end{aligned} \quad (29)$$

where meanings of all variables and the values of all parameters can be found in [17, 10].

The nonholonomic constraint is written as

$$\dot{x} \cos \theta + \dot{y} \sin \theta = 0$$

The above kinematic system is transferred into the chained form

$$\begin{aligned}\dot{y}_1 &= u_1 \\ \dot{y}_2 &= u_2 \\ \dot{y}_3 &= y_2 u_1\end{aligned}\quad (30)$$

The corresponding dynamic model can be converted into equation (33) from [4] multiplied by R_2^T .

$$\begin{bmatrix} my_3^2 + I_0 & my_3 \\ my_3 & m \end{bmatrix} \ddot{u} + \begin{bmatrix} my_3 \dot{y}_3 & 0 \\ m \dot{y}_3 & 0 \end{bmatrix} \dot{u} = R_2^T B_2 \tau$$

where

$$R_2 = \begin{bmatrix} -y_3 \sin y_1 & y_3 \cos y_1 & 1 \\ -\sin y_3 & \cos y_1 & 0 \end{bmatrix}$$

B. Simulation Results

The control objective is to have the robot follow $q_d = [2 \cos t, 2 \sin t, t]^T$ and the contact force λ_d follow λ , with the initial conditions $q(0) = [3, 0, 0.5]^T$, $\dot{q}(0) = [0, 0, 0]^T$ and $\lambda_d = 10$.

Other data are set as follows:

$\bar{K} = \Gamma = \text{diag}(0.36, 0.36, 0.36)$, $[u_1(0), u_2(0)] = [[0; 0]$, $k_1 = 0.1$, $[m(0), I_0(0)] = [0.767; 0.475]$, $m = 0.5$, $K_e = \text{diag}(5, 5, 5)$, $[y_1(0), y_2(0), y_3(0)] = [0.5; -3 \sin(0.5); 3 \cos(0.5)]$, $I_0 = 0.5$.

Using the LMI solver embedded in MATLAB, we can obtain $K = [-1.1388 \ -1.5583]$.

The simulation is carried out for the mechanical system (9)-(10). Fig. 2-10 show the tracking error, time histories of y_1, y_2, y_3 and adaptive parameters, and geometric trajectory of x via y , and the desired geometric trajectory of $x - y - \theta$ and the trajectories tracking response, and force tracking response, respectively. From the simulation results, the performances of the proposed method are demonstrated.

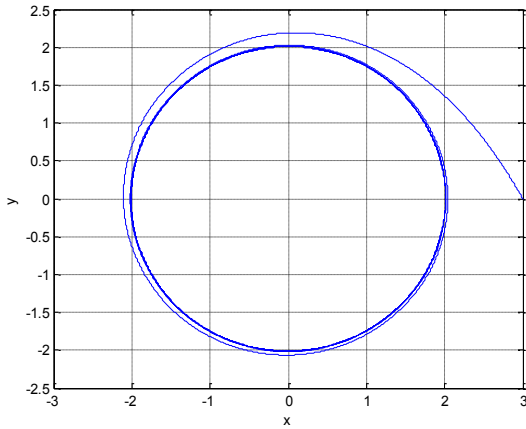


Fig. 8 Geometric trajectory of x via y

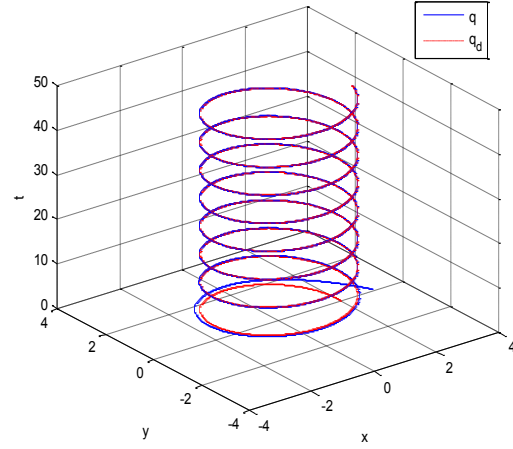


Fig. 9 The tracking responses of q_d and q

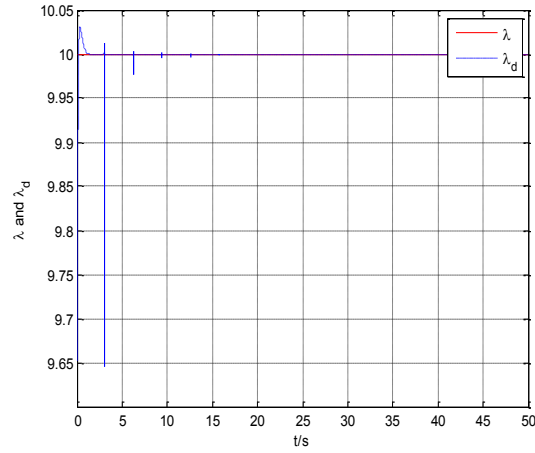


Fig. 10 The response of force tracking of λ_d and λ

V. CONCLUSION

This paper proposes a new control strategy for motion and force tracking controls of the nonholonomic dynamic systems by combining cascaded methods and backstepping techniques. For the kinematic level, first H-infinity control theory of linear systems and Linear Matrix Inequalities (LMI) technique are used to stabilize one subsystem of the kinematic dynamics, and a simple state feedback controller is given for the other subsystem, and then cascaded methods are designed to ensure the stabilization of the kinematic dynamics. For the overall mechanical system, an adaptive backstepping control law is proposed to achieve the tracking objective and guarantee the stability of all signals of the closed-loop systems. Simulation studies are carried out to verify the effectiveness of the proposed method.

ACKNOWLEDGMENT

The research in this paper is supported by the Natural Sciences and Engineering Research Council of Canada, the National Natural Science Foundation of China (61004009), the Fundamental Research Funds for the Central Universities (N100708001, N100408004), the Specialized Research Fund for the Doctoral Program of Higher Education of China under Grant 20100042120033, and by SRF for Returned Overseas Chinese Scholars of State Education Ministry.

REFERENCES

- [1] P. Morin and C. Samson, "Control of Nonholonomic Mobile Robots Based on the Transverse Function Approach", *IEEE Transactions on Robotics*, 25(5), pp. 1058-1073, 2009.
- [2] D. Lee, "Passive Decomposition and Control of Nonholonomic Mechanical Systems", *IEEE Transactions on Robotics*, 26(6), pp. 978-992, 2010.
- [3] B. S. Park, S. J. Yoo, J. B. Park, and Y. H. Choi, "A Simple Adaptive Control Approach for Trajectory Tracking of Electrically Driven Nonholonomic Mobile Robots", *IEEE Transactions on Control Systems Technology*, 18(5), pp. 1199-1206, 2010.
- [4] S.S. Ge, Z. Wang, and T. H. Lee, "Adaptive Stabilization of Uncertain Nonholonomic Systems by State and Output Feedback", *Automatica*, vol. 39, pp. 1451-1460, 2003.
- [5] A. Astolfi, "Discontinuous control of nonholonomic systems", *Systems and Control Letters*, vol. 27, pp. 37-45, 1996.
- [6] R. W. Brockett, "Asymptotic stability and feedback stabilization", In R. W. Brockett, R. S. Millman, and H. J. Sussmann (Eds.), *Differential geometric control theory*, Basel: Birkhauser, 1983.
- [7] T. Fukao, H. Nakawa, and N. Adachi, "Adaptive tracking control of a nonholonomic mobile robot", *IEEE Transactions on Robotics and Automation*, vol. 16, pp. 609-615, 2008.
- [8] Y. Hong, J. Wang, and Z. Xi, "Stabilization of Uncertain Chained Form Systems Within Finite Settling Time", *IEEE Transactions on Automatic Control*, 50(9), pp. 1379-1385, 2005.
- [9] V. Sankaranarayanan and A. D. Mahindrakar, "Control of a Class of Underactuated Mechanical Systems Using Sliding Modes", *IEEE Transactions on Robotics*, 25(2), pp. 459-467, 2009.
- [10] M. Oya, C.-Y. Su, and R. Katoh, "Robust adaptive motion/force tracking control of uncertain nonholonomic mechanical systems", *IEEE Transactions on Robotics and Automation*, Vol. 19, pp. 75-181, 2003.
- [11] I. Kolmanovsky and N. H. McClamroch, "Developments in nonholonomic control problems", *IEEE Control Systems Magazine*, 15, pp. 20-36, 1995.
- [12] Z.-P. Jiang and H. Nijmeijer, "A recursive technique for tracking control of nonholonomic systems in chained form", *IEEE Transactions on Automatic Control*, vol. 42, pp. 265-279, 1999.
- [13] W. Dong, W. L. Xu, and W. Huo, "Trajectory tracking control of dynamic nonholonomic systems with unknown dynamics", *Int. J. Robust Nonlinear Control*, vol. 9, pp. 905-922, 1999.
- [14] J.-M. Yang and J.-H. Kim, "Sliding-mode control for trajectory tracking of nonholonomic wheeled mobile robots", *IEEE Trans. Robot. Automat.*, vol. 15, pp. 578-587, 1999.
- [15] W. Dong and W. Huo, "Adaptive stabilization of uncertain dynamic non-holonomic systems", *International Journal of Control*, 72(18), pp. 1689-1700, 1999.
- [16] W. Dong, "On trajectory and force tracking control of constrained mobile manipulators with parameter uncertainty", *Automatica*, 38(9), pp. 1475-1484, 2002.
- [17] Z. Li, S. S. Ge, M. Adams, W. S. Wijesomab, "Robust adaptive control of uncertain force/motion constrained nonholonomic mobile manipulators", *Automatica*, 44, pp. 776 - 784, 2008.
- [18] M. Mauder, "Robust tracking control of nonholonomic dynamic systems with application to the bi-steerable mobile robot", *Automatica*, 44(10), pp. 2588-2592, 2008.
- [19] R. Michino and I. Mizumoto, "High Gain Adaptive Control of Uncertain nonholonomic Dynamic Systems", *Proceedings of International Conference on Modelling, Identification and Control*, pp. 332-337, Okayama, Japan, July 17-19, 2010.
- [20] J. Maples and J. Becker, "Experiments in force control of robotic manipulators", *IEEE International Conference on Robotics and Automation*, PP. 695-702, 1986.
- [21] C.-Y. Su and Y. Stepanenko, "Robot motion/force control of mechanical systems with classical nonholonomic constraints", *IEEE Trans. Automatic Control*, 1994, 39, pp. 609-614.
- [22] N. Sarkar, X. Yun, and V. Kumar, "Control of contact interactions with acatastatic nonholonomic constraints", *Int. J. Robot. Res.*, vol. 16, pp. 357-374, 1997.
- [23] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*, 15, SIAM, 1994.
- [24] E. Lefeber, A. Robertsson, and H. Nijmeijer, "Linear controllers for exponential tracking of systems in chained-form", *International Journal of Robust Nonlinear Control*, 10(4), 2000, 243-263.
- [25] Y. P. Tian and K. C. Cao, "An LMI Design of Tracking Controllers for Nonholonomic Chained-Form System", *Proceedings of the American Control Conference*, pp. 4512-4517, New York, 2007.
- [26] P. V. Kokotovic, "The joy of feedback: nonlinear and adaptive", *IEEE Control Systems Magazine*, 12 (3), pp. 7-17, 1992.
- [27] G. Walsh and L. G. Bushnell, "Stabilization of multiple input chained form control systems," *Systems and Control Letters*, vol. 25, pp. 227-234, 1995.
- [28] T.-L. Shen, *Theory and Application of H-infinity*, Press of Tsinghua: Beijing (in Chinese), 1996.