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Distributed trajectory tracking control for multiple nonholonomic mobile robots ⋆

Qingkai Yang *,**,*** **Hao Fang** *,** **Ming Cao** *** **Jie Chen** *,**

* *School of Automation, Beijing Institute of Technology, Beijing, 100081, China*
** *Key Laboratory of Intelligent Control and Decision of Complex Systems, Beijing, 100081, China*
*** *Faculty of Mathematics and Natural Sciences, University of Groningen, Groningen, 9747 AG, the Netherlands*

Abstract: In this paper, the distributed tracking problem for multiple nonholonomic mobile robots is investigated, in which the nonholonomic models are transformed into chained-form systems. By utilizing the dynamic oscillator strategy, the distributed controllers are constructed such that all the mobile robots' trajectories converge to the desired reference asymptotically. One advantage of the chained-form system solution that we propose is that it requires no other variable transformations, which could help reducing the system's complexity and broadening the proposed controller's practical applications. Simulations are presented to show the effectiveness of the proposed control algorithms.

Keywords: Multi-agent systems, Distributed control, Nonholonomic mobile robots, Tracking control

1. INTRODUCTION

With the features of robustness against failures, extendable in structures and reduced cost when separating a centralized task, multi-agent systems have received tremendous attentions in the past decade. Consequently, the distributed cooperative control problem for multi-agent systems has been intensively studied in many directions including consensus, trajectory tracking, formation control, containment, rendezvous, to name few.

As far as the system model is concerned, the majority of the research on distributed control starts from the simplest model, i.e., single integrator, see e.g. Jadbabaie et al. (2003); Fax and Murray (2004); Ren and Beard (2005). Due to the reason that the single integrator systems are sometimes too simplified to capture real agents' dynamics, efforts have been made to study double integrator systems, such as Hong et al. (2008); Lin and Jia (2009); Seyboth et al. (2013). Besides the simple linear systems, different topics on general linear dynamics have been discussed, for example, analytic synchronization for way-point model Cao et al. (2008), synchronization conditions being analyzed in Tuna (2009), observer-based consensus control in Li et al. (2010), and self-triggered control in Hu et al. (2015). Even though numerous control problems on linear systems have been extensively studied, the theories proposed so far cannot be directly put into practice in view of the nonlinearities and uncertainties existing in most of the mechanical systems. To narrow this gap, the cooperative control for Euler-Lagrange systems, which can represent a large class of mechanical systems, were considered in Mei et al. (2011); Nuno et al. (2011); Yang et al. (2014a,b). Moreover, efforts were also made to conduct distributed control for general nonlinear systems, such as Wang and Huang (2005) and Yu et al. (2010).

To be specific, due to the increasing need for nonholonomic systems in various applications, the results for distributed control of multiple nonholonomic mobile robots are reported more often in recent years. As shown in Brockett (1983), nonholonomic systems cannot be asymptotically stabilized by smooth state feedback control laws, which imposes difficulties for the control algorithm design. The tracking and stabilization problem for unicycle-type mobile robots were solved by employing the coordinate transformation and backstepping techniques in Do et al. (2004). For the geometric formation feasibility problem, Lin et al. (2005) presented the necessary and sufficient conditions for the existence of distributed controllers to stabilize the closed-loop system under the assumption that each nonholonomic mobile robot rotates freely. Using a special change of variable, distributed controllers were designed for multiple wheeled mobile robots to realize trajectory tracking under undirected graphs in Dong (2012). To deal with the nonholonomic constraints, Liu and Jiang (2013) made use of dynamic feedback linearization and small-gain methods to come up with a novel distributed controllers without global position measurements. Moreover, the adaptive distributed formation controllers were respectively developed for kinematics and dynamics using nonsmooth functions in Peng et al. (2016).

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E-mail addresses: fangh@bit.edu.cn(Hao Fang).

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In addition to directly analyzing the nonholonomic models, an equivalently chained-form system has been considered to implement the cooperative control tasks. Through variable transformations, dynamic distributed control laws have been proposed with the aid of a so called σ-process, which is also effective when it is subjected to communication delays Dong and Farrell (2008). Recently, a computationally simple controller for single wheeled mobile robot is developed based on Lyapunov's direct method and backstepping technique Wang et al. (2014). However, it should be noted that it is nontrivial to apply the results for a single agent to multi-agent systems. As an extension, control for high order chained-form systems was addressed in Cao et al. (2014), where cascading theory is used to overcome the difficulty when the group reference signal is not persistently exciting.

In this paper, we consider the distributed tracking problem for multiple nonholonomic mobile robots, where only a subset of the robots have access to the reference trajectory. The tracking errors can be guaranteed to asymptotically convergence to zero using our proposed distributed control laws, in which special components, serving as dynamic oscillator Dixon et al. (2000), are carefully designed with the aid of Lyapunov stability theory. Moreover, the control scheme is constructed under a general directed graph that contains a spanning tree, which has more potential applications.

The rest of the paper is organized as follows. Section 2 presents the nonholonomic models and the necessary preparations for the theoretical analysis. Distributed control laws and the rigorous theoretical proof are given in Section 3. Moreover, numeration simulations in Section 4 show that the proposed control algorithms are quite effective. Section 5 gives a short summary.

2. PROBLEM FORMULATION

Consider a group of n nonholonomic wheeled mobile robots, moving on a horizontal plane. The kinematics of robot i is described as

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \tag{1}$$

where $q_i = (x_i, y_i, \theta_i)$ are the position and orientation of robot i, and v_i and ω_i are the linear and angular velocities.

The neighboring relationships between the robots are described by a directed graph $\mathcal{G}$ with the vertex set $\mathcal{V} = \{1, 2, \cdots, n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. We use $A = (a_{ij})_{n\times n}$ to denote the adjacency matrix, where $a_{ij} > 0$ means there is an edge (j, i) between robot i and j, and robot i can obtain information from agent j, but not vice versa, and $a_{ij} = 0$ otherwise. The interaction relationships among the followers and the leader is denoted by matrix $B = \mathrm{diag}\{\mathrm{b}_1, \cdots, \mathrm{b}_\mathrm{n}\}$, where $b_i > 0$ if robot i is a neighbor of the leader, $b_i = 0$ otherwise. In this paper, the nonzero elements of A and B are chosen to be 1. The Laplacian matrix $L = (l_{ij})_{n\times n}$ is defined by $l_{ii} = \sum_{j\in\mathcal{N}_i} l_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$, where $\mathcal{N}_i$ denotes the set of neighbors of robot i. Let $D = \mathrm{diag}\{\mathrm{d}_1, \cdots, \mathrm{d}_\mathrm{n}\}$ represents the in-degree matrix, where $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, $i = 1, \cdots, n$.

To simplify the controller design, we introduce the following coordinate transformation and state feedback Murray and Sastry (1993):

$$\begin{bmatrix} z_{1i} \\ z_{2i} \\ z_{3i} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ \sin(\theta_i) & -\cos(\theta_i) & 0 \\ \cos(\theta_i) & \sin(\theta_i) & 0 \end{bmatrix}}_{\triangleq T_{1i}} \begin{bmatrix} x_i \\ y_i \\ \theta_i \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \underbrace{\begin{bmatrix} z_{2i} & 1 \\ 1 & 0 \end{bmatrix}}_{\triangleq T_{2i}} \begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix} \tag{3}$$

It can be seen that the matrices T_{1i} and T_{2i} are globally invertible, under which system (1) can be converted to the chained form as:

$$\begin{aligned} \dot{z}_{1i} &= u_{1i} \\ \dot{z}_{2i} &= u_{1i} z_{3i} \\ \dot{z}_{3i} &= u_{2i} \end{aligned} \tag{4}$$

Given a reference trajectory, $q_0(t) = (x_0(t), y_0(t), \theta_0(t))$, satisfying

$$\begin{aligned} \dot{x}_0 &= v_0 \cos(\theta_0) \\ \dot{y}_0 &= v_0 \sin(\theta_0) \\ \dot{\theta}_0 &= \omega_0 \end{aligned} \tag{5}$$

where all the states, i.e., q_0, v_0, ω_0, are available to parts of the n robots. Hereafter, we call the virtual agent following exactly the reference trajectory the leader and call the n robots represented by (1) followers. Since the two matrices T_{1i} and T_{2i} are globally nonsingular, under similar operation (2) and (3), we can equivalently obtain for the leader the transformed states $z_{l0}, l = 1, 2, 3$.

Control objective: Design control laws u_{1i} and u_{2i} for system i modeled by (4), such that the reference trajectory is tracked, namely,

$$\lim_{t\to\infty} (z_{li} - z_{l0}) = 0, \quad \forall\, l = 1, 2, 3,\ i = 1, \cdots, n \tag{6}$$

In order to achieve the objective, we need the following assumptions.

Assumption 1. The communication directed graph $\mathcal{G}$ has a spanning tree with the root node being that of the leader.

Assumption 2. The leader's inputs $u_{10}(t)$ and $u_{20}(t)$ are continuous. Moreover, there exist positive constants ϵ and T, such that for all $\tau \geq 0$,

$$\int_{\tau}^{\tau+T} [u_{10}(t)]^2 dt > \epsilon \tag{7}$$

Lemma 3. (Wang et al. (2014)) Let $V : R^+ \to R^+$ be continuously differentiable and $W : R^+ \to R^+$ uniformly continuous satisfying that, for each $t > 0$,

$$\dot{V}(t) \leq -W(t) + p_1(t)V(t) + p_2(t)\sqrt{V(t)} \tag{8}$$

with both $p_1(t)$ and $p_2(t)$ being non-negative and belonging to $\mathcal{L}_1$ space. Then, there exists a constant c, such that $W(t) \to 0$ and $V(t) \to c$ as $t \to \infty$.

Notations: In this paper $|x|$ and $\|x\|$ are used to denote the 1-norm and 2-norm of vector $x \in R^n$ respectively. When x is a scalar, $|x|$ denotes the absolute value of x. We use $\|X\|_1$ and $\|X\|$ to denote the corresponding induced 1-norm and 2-norm of square matrix X respectively.

3. MAIN RESULTS

3.1 Controller design

Define the tracking error for system i as

$$\tilde{z}_{li} = z_{li} - z_{l0}, \quad l = 1,2,3,\ i = 1,\cdots,n \tag{9}$$

and the local relative tracking error

$$e_{li} = \sum a_{ij}(z_{li} - z_{lj}) + b_i(z_{li} - z_{l0}), \quad l = 1,2,3 \tag{10}$$

$$e_{ri} = e_{2i} - z_{3i}e_{1i} \tag{11}$$

Eq. (10) can be written into the vector form as

$$e_l = (L+B)\tilde{z}_l = (L+B)z_l - B\mathbf{1}_n z_{l0} \tag{12}$$

$$e_r = e_2 - \mathrm{diag}(z_3)e_1 \tag{13}$$

where $e_l = [e_{l1},\cdots,e_{ln}]^T, z_l = [z_{l1},\cdots,z_{ln}]^T$, $\mathrm{diag}(z_3) = \mathrm{diag}\{z_{31},\cdots,z_{3n}\} \in R^{n\times n}$ and $\mathbf{1}_n$ is the column vector of all ones.

The control laws for agent i are given by

$$u_{1i} = \frac{1}{\sum\limits_{j\in\mathcal{N}_i} a_{ij} + b_i}\Big(- k_1 e_{1i} + b_i u_{10} + \sum_{j\in\mathcal{N}_i} a_{ij}u_{1j} + u_{2i}e_{ri}\Big) \tag{14}$$

$$u_{2i} = \frac{1}{\sum\limits_{j\in\mathcal{N}_i} a_{ij} + b_i}\Big(- k_3 e_{3i} + b_i u_{20} + \sum_{j\in\mathcal{N}_i} a_{ij}u_{2j} - k_2 e^{-\beta t}\frac{\sin(\alpha_i t)e_{ri}}{\sqrt{1+e_{1i}^2+e_{3i}^2}} + u'_{2i}\Big) \tag{15}$$

with

$$u'_{2i} = \frac{1}{\sum\limits_{j\in\mathcal{N}_i} a_{ij} + b_i}\Big(\sum_{j\in\mathcal{N}_i} a_{ji}u'_{2j} + \sum_{j\in\mathcal{N}_i} a_{ij}e_{ri}u_{1j} - \sum_{j\in\mathcal{N}_i} a_{ji}u_{1i}e_{rj} + b_i u_{10}e_{ri}\Big) \tag{16}$$

where k_j, $j = 1,2,3$, α_i and β are positive constants, satisfying $2k_3 > k_2$. The vector form of the control inputs can be written as

$$u_1 = (D+B)^{-1}\left(-k_1 e_1 + B\mathbf{1}_n u_{10} + Au_1 + \mathrm{diag}(u_2)e_r\right) \tag{17}$$

$$u_2 = (D+B)^{-1}\left(-k_3 e_3 + B\mathbf{1}_n u_{20} + Au_2 - k_2 u_{2p} + u'_2\right) \tag{18}$$

with

$$u'_2 = (D+B)^{-1}A^T u'_2 + (D+B)^{-1}\Big[\mathrm{diag}(\mathrm{e_r})(Au_1) - \mathrm{diag}(\mathrm{u_1})A^T e_r + u_{10}Be_r\Big] \tag{19}$$

where $u_\iota = [u_{\iota 1},\cdots,u_{\iota n}]^T$, $\iota = 1,2$ and

$$u_{2p} = e^{-\beta t}\left[\frac{\sin(\alpha_1 t)e_{r1}}{\sqrt{1+e_{11}^2+e_{31}^2}},\cdots,\frac{\sin(\alpha_n t)e_{rn}}{\sqrt{1+e_{1n}^2+e_{3n}^2}}\right]^T$$

Motivated by Khoo et al. (2009) , equivalently, (17) can be rewritten as

$$[I-(D+B)^{-1}A)]u_1 = (D+B)^{-1}\left(-k_1 e_1 + B\mathbf{1}_n u_{10} + \mathrm{diag}(u_2)e_r\right) \tag{20}$$

Substituting $I = (D+B)(D+B)^{-1}$ into (20), we get

$$(D+B)^{-1}(D+B-A)u_1 = (D+B)^{-1}\Big(- k_1 e_1 + B\mathbf{1}_n u_{10} + \mathrm{diag}(u_2)e_r\Big) \tag{21}$$

Noting that $D - A = L$, it follows

$$(L+B)u_1 = -k_1 e_1 + B\mathbf{1}_n u_{10} + \mathrm{diag(u_2)e_r} \tag{22}$$

Analogously, for the control input u_2, we have

$$(L+B)u_2 = -k_3 e_3 + B\mathbf{1}_n u_{20} - k_2 u_{2p} + u'_2 \tag{23}$$

By virtue of (19), it follows that

$$[I-(D+B)^{-1}A^T]u'_2 = (D+B)^{-1}\big[\mathrm{diag(e_r)}(Au_1) - \mathrm{diag(u_1)}A^T e_r + u_{10}Be_r\big] \tag{24}$$

Taking the transpose of both sides of (24) yields

$$(u'_2)^T[I - A(D+B)^{-1}] = \big[- \mathrm{diag(e_r)}(Au_1) + \mathrm{diag(u_1)}A^T e_r - u_{10}Be_r\big]^T (D+B)^{-1} \tag{25}$$

Rewriting I as $(D+B)(D+B)^{-1}$ and post-multiplying $D+B$ on both sides of (25) follows that

$$(u'_2)^T(D+B-A) = \big[- \mathrm{diag(e_r)}(Au_1) + \mathrm{diag(u_1)}A^T e_r - u_{10}Be_r\big]^T \tag{26}$$

Considering the fact that $L = D - A$, we get

$$(L+B)^T u'_2 = -\mathrm{diag(e_r)}(Au_1) + \mathrm{diag(u_1)}A^T e_r - u_{10}Be_r \tag{27}$$

Remark 4. u_{1p} and u_{2p} serve as dynamic oscillator or persisting excitation

3.2 Stability analysis

Lemma 5. (Samson (1995)) If a given differentiable function $f(x): R^+ \to R$ converges to some limit value when $x \to \infty$, and if the derivative $(df/dx)(x)$ of this function is the sum of two terms, one being uniformly continuous and the other one tending to zero when $x \to \infty$, then $(df/dx)(x) \to 0$ when $x \to \infty$.

Theorem 6. Under the Assumptions 1, the mobile robots modeled by (4) can be driven to track the dynamic leader using the distributed control laws (14) and (15).

Proof. Consider the Lyapunov function candidate as

$$V = \frac{1}{2}\sum_{i=1}^n \left(e_{1i}^2 + e_{ri}^2 + e_{3i}^2\right) = \frac{1}{2}(e_1^T e_1 + e_r^T e_r + e_3^T e_3) \tag{28}$$

Differentiating both sides of (11) in conjunction with (10), we have

$$\dot{e}_{ri} = \sum_{j\in\mathcal{N}_i} a_{ij}\left(z_{3i} - z_{3j}\right)u_{1j} + b_i(z_{3i} - z_{30})u_{10} - u_{2i}e_{1i} \tag{29}$$

Note that

$$\sum_{j\in\mathcal{N}_i} a_{ij}(z_{3i} - z_{3j})u_{1j} = \sum_{j\in\mathcal{N}_i} a_{ij}(\tilde{z}_{3i} - \tilde{z}_{3j})u_{1j} = \sum_{j\in\mathcal{N}_i} a_{ij}\tilde{z}_{3i}u_{1j} - \sum_{j\in\mathcal{N}_i} a_{ij}\tilde{z}_{3j}u_{1j} \tag{30}$$

and

$$\sum_{i=1}^n e_{ri}u_{2i}e_{1i} = [\mathrm{diag}(u_2)e_1]^T e_r = e_1^T \mathrm{diag}(u_2)e_r \tag{31}$$

Then, The compact form of (29) is given as

$$\dot{e}_r = \mathrm{diag}(\tilde{z}_3)(Au_1) - A[\mathrm{diag}(u_1)\tilde{z}_3] + u_{10}B\tilde{z}_3 - \mathrm{diag}(u_2)e_1 \tag{32}$$

The derivative of (28) along (22), (23) and (32) satisfies

$$\begin{aligned}\dot{V} =& e_1^T\left[(L+B)u_1 - B\mathbf{1}_n u_{10}\right] + e_3^T\left[(L+B)u_2 - B\mathbf{1}_n u_{20}\right]\\ &+ e_r^T\Big[\mathrm{diag}(\tilde{z}_3)(Au_1) - A[\mathrm{diag}(u_1)\tilde{z}_3]\\ &+ u_{10}B\tilde{z}_3 - \mathrm{diag}(u_2)e_1\Big]\\ =& e_1^T(-k_1e_1 + \mathrm{diag}(u_2)e_r) + e_3^T(-k_3e_3 - k_2u_{2p} + u_2')\\ &+ e_r^T\Big[\mathrm{diag}(\tilde{z}_3)(Au_1) - A[\mathrm{diag}(u_1)\tilde{z}_3]\\ &+ u_{10}B\tilde{z}_3 - \mathrm{diag}(u_2)e_1\Big] \end{aligned} \tag{33}$$

In view of (27), we obtain

$$\begin{aligned} e_3^Tu_2' &= \tilde{z}_3^T(L+B)^Tu_2'\\ &= -\tilde{z}_3^T\mathrm{diag}(\mathrm{e_r})(Au_1) + \tilde{z}_3^T\mathrm{diag}(\mathrm{u_1})A^Te_r - u_{10}\tilde{z}_3^TBe_r \end{aligned} \tag{34}$$

Since

$$\begin{aligned} &\tilde{z}_3^T\mathrm{diag}(\mathrm{e_r})(Au_1) - \tilde{z}_3^T\mathrm{diag}(\mathrm{u_1})A^Te_r\\ =&e_r^T\mathrm{diag}(\tilde{z}_3)(Au_1) - e_r^TA[\mathrm{diag}(u_1)\tilde{z}_3] \end{aligned} \tag{35}$$

Eq. (33) can be simplified into the following form

$$\dot{V} = -k_1e_1^Te_1 - k_3e_3^Te_3 - k_2e_3^Tu_{2p} \tag{36}$$

By using the Holder's inequality, we obtain

$$\dot{V} \le -k_1e_1^Te_1 - k_3e_3^Te_3 + k_2e^{-\beta t}\|e_3\|\|u_{2p}\| \tag{37}$$

Then, due to $\|u_{2p}\| \le \|e_r\|$, using Young's inequality yields

$$\begin{aligned}\dot{V} \le& -k_1\|e_1\|^2 - k_3\|e_3\|^2 + \frac{k_2}{2}e^{-\beta t}(\|e_3\|^2 + \|e_r\|^2)\\ \le& -k_1\|e_1\|^2 - \left(k_3 - \frac{k_2}{2}\right)\|e_3\|^2 + \frac{k_2}{2}e^{-\beta t}\|e_r\|^2\\ \le& -k_1\|e_1\|^2 - \left(k_3 - \frac{k_2}{2}\right)\|e_3\|^2 + k_2e^{-\beta t}V(t) \end{aligned} \tag{38}$$

Noting that the parameters k_j, $j = 1,2,3$ are chosen such that $k_1 > 0$ and $k_3 - k_2/2 > 0$ in the proposed controllers (14)-(16) and $k_2e^{-\beta t} \in \mathcal{L}_1$. Then, by means of Lemma 3, we obtain

$$\begin{cases}\lim\limits_{t\to\infty}\|e_1(t)\| = 0\\ \lim\limits_{t\to\infty}\|e_3(t)\| = 0\end{cases} \quad \text{and} \quad \lim_{t\to\infty}\|e_r(t)\| = c \tag{39}$$

where c is a non-negative constant, which implies that $e_1(t) \to \mathbf{0}$ and $e_3(t) \to \mathbf{0}$, as $t \to \infty$, equivalently, both e_{1i} and e_{3i}, $i = 1,\cdots,n$ tend to zero, as $t \to \infty$. It follows from Khoo et al. (2009) that $z_{li} = z_{l0}$, $l = 1,3$. Taking the derivative of (12) along (22), we get

$$\dot{e}_1 = -k_1e_1 + \mathrm{diag}(u_2)e_r \tag{40}$$

Furthermore, under Assumption 2, it can be verified that the control input u_2 presented in (18) is continuous. Hence, from Lemma 5, we know

$$\lim_{t\to\infty}\dot{e}_1(t) = \mathbf{0} \quad \text{and} \quad \lim_{t\to\infty}\mathrm{diag}(u_2)e_r(t) = \mathbf{0} \tag{41}$$

which implies that $\lim_{t\to\infty}u_{1i} = u_{10}$. Similarly, it also follows that $\lim_{t\to\infty}\dot{e}_3(t) = \mathbf{0}$, and thus $\lim_{t\to\infty}u_{2i} = u_{20}$. Combining (23), (27) and the fact that $\lim_{t\to\infty}\mathrm{diag}(u_2)e_r(t) = \mathbf{0}$, we have

$$u_{10}\underbrace{\begin{bmatrix} f_1(e_{r1}, e_{rj}),\ {}_{j\in\mathcal{N}_1}\\ \vdots\\ f_n(e_{rn}, e_{rj}),\ {}_{j\in\mathcal{N}_n}\end{bmatrix}}_{\triangleq f(e_r)} - k_2e^{-\beta t}\begin{bmatrix}\dfrac{\sin(\alpha_1t)e_{r1}^2}{\sqrt{1+e_{11}^2+e_{31}^2}}\\ \vdots\\ \dfrac{\sin(\alpha_nt)e_{rn}^2}{\sqrt{1+e_{1n}^2+e_{3n}^2}}\end{bmatrix} = \mathbf{0},$$

$$\text{as} \quad t\to\infty \tag{42}$$

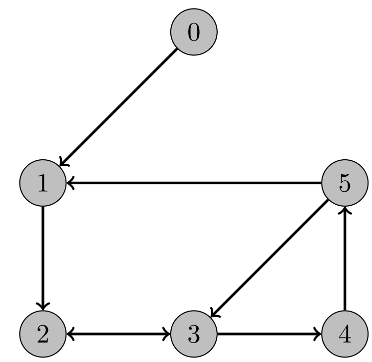

Fig. 1. Interaction topology

where $f_i(e_{ri}, e_{rj}),\ {}_{j\in\mathcal{N}_i}$, $i = 1,\cdots,n$, is a function of order two with respect to e_{ri} and e_{rj}, which is determined by the interaction relationships. Noting that $\lim_{t\to\infty}e^{-\beta t} = 0$, we naturally get $\lim_{t\to\infty}u_{10}f(e_r) = \mathbf{0}$ from (42). Consider that $A \ne A^T$ due to the asymmetric structure of the directed graph and u_{10} is persistently exciting from Assumption 2, therefore, $\lim_{t\to\infty}e_r = \mathbf{0}$. It follows from (13) that $\lim_{t\to\infty}e_2 = \mathbf{0}$, which implies $\lim_{t\to\infty}z_{2i} = z_{20}$. This completes the proof.

4. SIMULATIONS

Consider a group of 5 nonholonomic robots and the reference labeled by 0, whose interaction relationship is shown in Fig. 1, which is chosen to be the same as that in Liu and Jiang (2013). It can be seen that the reference is only available to robot 1, and there exists a spanning tree with 0 as the root.

The reference velocity v_0 and angular velocity w_0 are set as 5 and 1, respectively. Then, the kinematics of the reference trajectory is given by

$$\begin{aligned}\dot{x}_0 &= 5\cos(\theta_0)\\ \dot{y}_0 &= 5\sin(\theta_0)\\ \dot{\theta}_0 &= 1\end{aligned} \tag{43}$$

The initial values for $z_{\iota i} = 0.15*(i-3), i = 1,\cdots,5$. The control parameters k_1, k_2 and k_3 are set as 2, 6 and 20, respectively. In addition, for simplicity, α_i in (15) is chosen as the same value $\alpha = 0.1$, and β is taken as 0.01.

It can be seen from Fig. 2 and Fig. 3 that the tracking errors converge to zero using our proposed control algorithms.

5. CONCLUSION

We have studied the distributed tracking problem for multiple nonholonomic mobile robots under a directed graph. Based on the transformed chain-form system, distributed control algorithms have been proposed to solve the tracking problem by employing dynamic oscillator strategy. Moreover, the tracking errors for each robot have been shown to converge to zero asymptotically. Numerical simulation results also demonstrated that the nonholonomic robots can track the reference using proposed control laws.

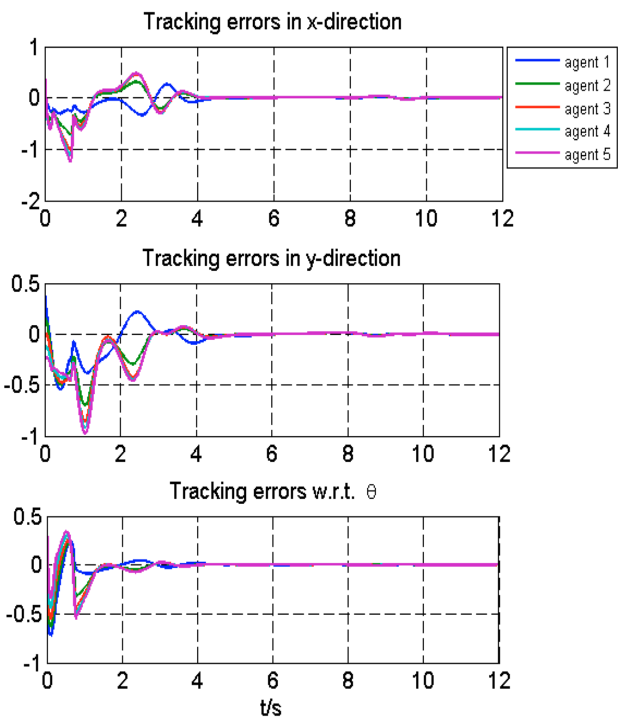

Fig. 2. Tracking errors of states x, y and θ

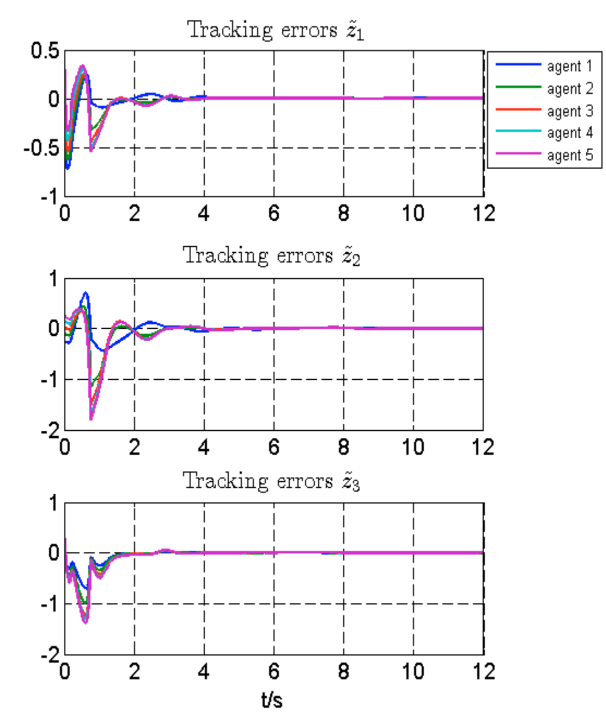

Fig. 3. Tracking errors of states z_1, z_2 and z_3

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