Formation Control of Multiple Wheeled Mobile Robots via Leader-Follower Approach

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Abstract: This paper investigates the formation control of multiple wheeled mobile robots with differential drive. Based on the kinematics models of the leader-follower system in Cartesian coordinates, four control laws are derived by the straightforward input-output linearization method, and asymptotical stable for the formation control system is achieved by the proposed control laws. Simulation studies are included to demonstrate the effectiveness of the proposed method.

Key Words: Multiple wheeled mobile robots, differential drive, formation control, leader-follower, nonlinear system

1 Introduction

In recent years, the planning, coordination and competition of multiple distributed robots have drawn an extensive research attention in robotics and control and artificial intelligence community, because some complex tasks can completed by the multiple robots rather than a single robot [1-3]. And due to the mobile robot has batter distribution of time and spatial than the stationary robot lead to many advantages for various application areas, it develops an important direction of the robot community now. The applications include rescue mission, moving of a large object, troop hunting, formation control, cluster of satellites, distributed localization and mapping of unknown or partially known environments, perimeter security, reconnaissance, surveillance and exploration, and so on. For these reasons, control of multiple mobile robots moving in formation has drawn an extensive research attention in robotics and control community [3-17]. The results can also to unmanned air vehicles (UAVs), autonomous underwater vehicles (AUVs) and aircrafts, etc [18, 19].

For the formation control of multiple mobile robots, diverse strategies and methods have been proposed, and virtual structure, behavior based and leader-following are three most commonly used methods, each of them has its own characteristics and advantages [4]. It is core idea of the virtual structure method that entire formation is treated as a single virtual rigid structure and the desired motion is assigned to the virtual structure as a whole [8, 18]. As a direct result, the behavior of the robot team can be easily prescribed, and the centralization leads a single point of failure for the whole system which greatly limited it's application. The motion of each robot is divided into several desired behaviors by behavior based approach, and possible desired behaviors mainly include collision avoidance, obstacle avoidance, goal seeking, ect [9, 10]. The final action of each robot is derived by weighting the relative importance

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of each behavior, but difficulty to analyze mathematically is the primary limitation of behavior based method. In the leader-follower method [12–16], at least one of the robots is designated as the leader and the rest of the robots are followers. In the maneuvers, the follower robots need to position and maintain the desired relative position with respect to the leader. Consequently, it can be essentially viewed as a natural extension of the trajectory tracking problem, and facilitate the mathematical analysis.

Based on these control methods, some approaches commonly used in trajectory tracking control [20-23] are used to design the control law for the leader-follower based formation control of multiple wheeled mobile robots whit nonholonomic constraints. In the literatures [13-15], Desai et al. propose some well known models and control laws for multiple wheeled mobile robots in polar coordinate. Since the desired relative position of the follower with respect to the leader can be simply represented in polar coordinates, the formation control system model can be easily represented in polar coordinates. However, primary shortcoming of the models is that inherent singularity problem of control law design is inevitable because of the polar coordinate representation. To solve this disadvantage, Li in literature [16] develops a model in Cartesian coordinates to avoid the singular points in polar coordinates representation, and a control law is designed for fixed desired relative distance by input-output linearization. For control of multiple mobile robots, many control laws are designed by several control ways, such as backstepping [6, 7, 24], sliding mode [11, 17], switching control [5], neural network [25, 26] and so on.

According to the above state, however, the most of results are only discuss the fixed desired relative distance. In fact, the desired relative distance and the desired relative angle are usually changed during actual movement of the mobile robot formation. Thus, this paper considers the change of the desired relative distance and the desired relative angle in control law design, and it is used in this paper that kine-

matics models of the leader-follower configuration of multiple wheeled mobile robots with nonholonomic constraints in Cartesian coordinate due to the advantages stated above. Several control laws with certain particular characteristics for the followers are derived in a simple manner, and the application of control laws designed is strengthened.

The paper is organized as follows. The problem formulation of this paper and the kinematics models presented in Cartesian coordinates for the formation control of two non-holonomic mobile robots are introduced in Section 2. In Section 3, four simple control laws are derived from the models and the stability of the whole system is analyzed. In Section 4, we present some simulation results to demonstrate the the effectiveness of the proposed approach, and the conclusion and future work is included in Section 5.

2 Problem Formulation and System Model

Consider formation movement of N differential drive mobile tricycle robots with a leader, and movement trajectory of the leader has been accomplishment at motion planning. Therefore, the control law of robot need been designed for formation control.

Based on the problem formulation above, this formation control problem can be decompose into trajectory tracking problem of pairwise, which is a base of robot team. The configuration of the leader and follower robots is shown in Figure 1, and where X-Y is the ground coordinates, X_i-Y_i is the Cartesian coordinates fixed on the ith robot's body. Moreover, suppose that the robot moves forward along $+X_i$, as shown in Figure 1. (x_i,y_i) and (x_j,y_j) are global positions of the centers of driving axes of the ith and jth robots respectively, and let the ith robot (leader) leads the jth robot (follower).

The robot considered in this paper has two drive wheels and one steering wheel, each with diameter r. Given a point O_i centered between the two drive wheels, each wheel is a distance D from O_i , and the spinning speed of each wheel $\dot{\phi}_i^L$ and $\dot{\phi}_i^R$, in which superscript 'L' and 'R' stand for left and right wheel respectively; v_i, v_j are linear velocities of two robots respectively; θ_i, θ_j are their orientation angles. And l_{ij}, φ_{ij} are the jth robot's relative distance and angle with respect to the ith robot in $X_i - Y_i$ Cartesian coordinates. d is the length between the center of steering wheel and the center of the driving axis.

Suppose the build target formation and the movement of leader has been determined by motion planning, this means the follower's position will be uniquely determines by a given the leader's position and orientation. So (l_{ij}, φ_{ij}) can represent the formation between leader and follower, if l_{ij} and φ_{ij} can be control to the desired value, namely the relative position and relative angle of the followers need to control to the desired value, the desired formation will to be achieved. Such as the desired formation is $F^* = \{l_{ij}^*, \varphi_{ij}^*\}$, then the control task is to make $l_{ij} \to l_{ij}^*$ and $\varphi_{ij} \to \varphi_{ij}^*$.

Since the formation system model represented by l- φ polar coordinates will lead to the inevitably singular point in the control law as in [13–15], the formation system model

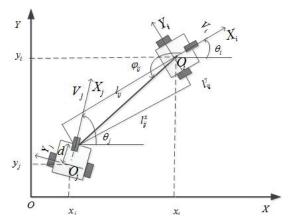


Figure 1: Sketch of leader-following configuration of two robots [16]

set up in Cartesian coordinates to eliminate the singularity point problem, and the relative distance between the leader and follower need to project into x,y directions in X_i-Y_i Cartesian coordinates. Consequently, the control task transform into $l^x_{ij} \to l^{\star x}_{ij}, l^y_{ij} \to l^{\star y}_{ij}$ and $\varphi_{ij} \to \varphi^{\star}_{ij}$, in which $l^x_{ij}, l^y_{ij}, l^{\star x}_{ij}$ and $l^{\star y}_{ij}$ are the follower's relative position and the desired value along x and y directions in X_i-Y_i Cartesian coordinates respectively.

The detail derived of following formula can be refer to [16]:

$$\begin{cases} l_{ij}^{x} = -(x_i - x_j - d\cos\theta_j)\cos\theta_i \\ -(y_i - y_j - d\sin\theta_j)\sin\theta_i \\ l_{ij}^{y} = -(x_i - x_j - d\cos\theta_j)\sin\theta_i \\ -(y_i - y_j - d\sin\theta_j)\cos\theta_i. \end{cases}$$
(1)

Define the error variables

$$\begin{cases} e_{ij}^{x} = l_{ij}^{xx} - l_{ij}^{x} \\ e_{ij}^{y} = l_{ij}^{xy} - l_{ij}^{y} \\ e_{ij}^{\theta} = \theta_{j} - \theta_{i}. \end{cases}$$
 (2)

where $l_{ij}^{\star x} = l_{ij}^{\star} \cos \varphi_{ij}^{\star}$ and $l_{ij}^{\star y} = l_{ij}^{\star} \sin \varphi_{ij}^{\star}$. Using the nonholonomic constraints of a mobile robot, the following error dynamic model can derived [16]

$$\begin{cases}
\dot{e}_{ij}^{x} = \dot{l}_{ij}^{\star} \cos \varphi_{ij}^{\star} + v_{i} - \left(l_{ij}^{\star} \sin \varphi_{ij}^{\star} - e_{ij}^{y}\right) \omega_{i} \\
-l_{ij}^{\star} \dot{\varphi}_{ij}^{\star} \sin \varphi_{ij}^{\star} - v_{j} \cos e_{ij}^{\theta} + \omega_{j} d \sin e_{ij}^{\theta} \\
\dot{e}_{ij}^{y} = \dot{l}_{ij}^{\star} \sin \varphi_{ij}^{\star} + \left(l_{ij}^{\star} \cos \varphi_{ij}^{\star} - e_{ij}^{\star}\right) \omega_{i} \\
+l_{ij}^{\star} \dot{\varphi}_{ij}^{\star} \cos \varphi_{ij}^{\star} - v_{j} \sin e_{ij}^{\theta} - \omega_{j} d \sin e_{ij}^{\theta} \\
\dot{e}_{ij}^{\theta} = \omega_{j} - \omega_{i}.
\end{cases} (3)$$

The l_{ij}^{\star} , φ_{ij}^{\star} , ω_i and v_i all have sufficiently smooth derivatives are assumed in the rest of this paper. Because of the saturation of the motors on the mobile robots, ω_i and v_i are bounded, and so are their first derivatives. For keeping formation, due to the movement of leader has been determined by motion planning, that is need to design the control law for ω_j and v_j make e_{ij}^x and e_{ij}^y asymptotically stable and e_{ij}^θ needs to make stable at least. However, the change of ω_j and v_j is finally implemented by change of wheel's movement state for differential derive robot. The relationship is introduced in following.

For v_j , it can assume one wheel spins while the other wheel contributes nothing and is stationary, and O_j is halfway between the two wheels, it will move instantaneously with half the speed for the left and right wheel respectively: $\dot{x}_j^L = \frac{r\dot{\phi}_j^L}{2}$ and $\dot{x}_j^R = \frac{r\dot{\phi}_j^R}{2}$. Then, these two contributions can simply be added to calculate in a differential drive robot:

$$v_j = \frac{r\dot{\phi}_j^L}{2} + \frac{r\dot{\phi}_j^R}{2}.\tag{4}$$

A similar analysis as above for ω_j , forward spin of this wheel results in *counterclockwise* and *clockwise* rotation at point O_j . Since the wheel is instantaneously moving along the arc of a circle of radius 2D, the rotation velocity of left and right wheel at O_j can be computed as $\omega_j^L = \frac{-r\dot{\phi}_j^L}{2D}$ and $\omega_j^R = \frac{r\dot{\phi}_j^R}{2D}$.

Consequently, in a differential drive robot, these two contributions can simply be added to calculate:

$$\omega_j = \frac{r\dot{\phi}_j^R}{2D} + \frac{-r\dot{\phi}_j^L}{2D}.\tag{5}$$

Therefore, the problem is finally converted to design control law for $\dot{\phi}^L_j$ and $\dot{\phi}^R_j$ to make e^x_{ij} and e^y_{ij} asymptotically stable, and e^g_{ij} stable at least.

3 Control Law Design

In this section, we discuss design of control laws for these cases based on the presented models. For the common formation control problem, the desired relative distance l_{ij} and relative angle φ_{ij} between the leader and follower robots could be constant or varying. Therefore, there are four cases as follows:

Case 1: Keep the configuration unchanged during the movement, this means l_{ij}^{\star} and φ_{ij}^{\star} are known and fixed, that is $\dot{l}_{ij}^{\star}=0$ and $\dot{\varphi}_{ij}^{\star}=0$. Then, the error dynamic model (3) can be rewritten as:

$$\begin{cases}
\dot{e}_{ij}^{x} = e_{ij}^{y} \omega_{i} - v_{j} \cos e_{ij}^{\theta} + \omega_{j} d \sin e_{ij}^{\theta} \\
-l_{ij}^{\star} \omega_{i} \sin \varphi_{ij}^{\star} + v_{i}, \\
\dot{e}_{ij}^{y} = -e_{ij}^{x} \omega_{i} - v_{j} \sin e_{ij}^{\theta} - \omega_{j} d \sin e_{ij}^{\theta} \\
+l_{ij}^{\star} \omega_{i} \cos \varphi_{ij}^{\star}, \\
\dot{e}_{ij}^{\theta} = \omega_{i} - \omega_{j}.
\end{cases} (6)$$

Case 2: The configuration changes during the movement, and the desired distance between the leader and follower robots l_{ij}^{\star} is a constant value, (i.e., $\dot{l}_{ij}^{\star}=0$), and the relative angle φ_{ij}^{\star} could be a changing variable. Then, the error dynamic model (3) can be rewritten as:

$$\begin{cases} \dot{e}_{ij}^{x} = & e_{ij}^{y} \omega_{i} - v_{j} \cos e_{ij}^{\theta} + \omega_{j} d \sin e_{ij}^{\theta} \\ & -l_{ij}^{*} \omega_{i} \sin \varphi_{ij}^{*} + v_{i} - l_{ij}^{*} \dot{\varphi}_{ij}^{*} \sin \varphi_{ij}^{*}, \\ \dot{e}_{ij}^{y} = & -e_{ij}^{x} \omega_{i} - v_{j} \sin e_{ij}^{\theta} - \omega_{j} d \sin e_{ij}^{\theta} \\ & + l_{ij}^{*} \dot{\varphi}_{ij}^{*} \cos \varphi_{ij}^{*} + l_{ij}^{*} \omega_{i} \cos \varphi_{ij}^{*}, \\ \dot{e}_{ij}^{\theta} = & \omega_{i} - \omega_{j}. \end{cases}$$
(7)

Case 3: The configuration changed during the movement, and the desired distance between the leader and follower

robots l_{ij}^{\star} could be a changing variable, and the relative angle φ_{ij}^{\star} is a constant value, (i.e., $\dot{\varphi}_{ij}^{\star}=0$). Then, the error dynamic model (3) can be rewritten as:

$$\begin{cases}
\dot{e}_{ij}^{x} = e_{ij}^{y}\omega_{i} - v_{j}\cos e_{ij}^{\theta} + v_{i} + \omega_{j}d\sin e_{ij}^{\theta} \\
+ \dot{l}_{ij}^{\star}\cos\varphi_{ij}^{\star} - l_{ij}^{\star}\omega_{i}\sin\varphi_{ij}^{\star}, \\
\dot{e}_{ij}^{y} = -e_{ij}^{x}\omega_{i} - v_{j}\sin e_{ij}^{\theta} - \omega_{j}d\sin e_{ij}^{\theta} \\
+ l_{ij}^{\star}\omega_{i}\cos\varphi_{ij}^{\star} + \dot{l}_{ij}^{\star}\sin\varphi_{ij}^{\star}, \\
\dot{e}_{ij}^{\theta} = \omega_{i} - \omega_{j}.
\end{cases} (8)$$

Case 4: The configuration changed during the movement, and the desired distance between the leader and follower robots l_{ij}^{\star} and the relative angle φ_{ij}^{\star} could be a changing variable. Then, the error dynamic model is (3):

$$\begin{cases}
\dot{e}_{ij}^{x} = e_{ij}^{y} \omega_{i} - l_{ij}^{*} \dot{\varphi}_{ij}^{*} \sin \varphi_{ij}^{*} - l_{ij}^{*} \omega_{i} \sin \varphi_{ij}^{*} \\
-v_{j} \cos e_{ij}^{\theta} + \omega_{j} d \sin e_{ij}^{\theta} + v_{i}, \\
\dot{e}_{ij}^{y} = -e_{ij}^{x} \omega_{i} + \dot{l}_{ij}^{*} \sin \varphi_{ij}^{*} + l_{ij}^{*} \dot{\varphi}_{ij}^{*} \cos \varphi_{ij}^{*} \\
-v_{j} \sin e_{ij}^{\theta} - \omega_{j} d \sin e_{ij}^{\theta} + l_{ij}^{*} \omega_{i} \cos \varphi_{ij}^{*}, \\
\dot{e}_{ij}^{\theta} = \omega_{i} - \omega_{j}.
\end{cases} \tag{9}$$

Firstly, we analysis Case 1. For simplicity denotation, we define

$$\begin{cases} f_1 = -l_{ij}^* \omega_i \sin \varphi_{ij}^* + v_i, \\ f_2 = l_{ij}^* \omega_i \cos \varphi_{ij}^*. \end{cases}$$
 (10)

Obviously, if the assumptions about l_{ij}^{\star} , φ_{ij}^{\star} , ω_i and v_i in Section 2 are satisfied, f_1 and f_2 are known, bounded, and sufficiently smooth functions.

Then error dynamic model (6) can be rewritten as:

$$\begin{cases} \dot{e}_{ij}^{x} = e_{ij}^{y}\omega_{i} - v_{j}\cos e_{ij}^{\theta} + \omega_{j}d\sin e_{ij}^{\theta} + f_{1}, \\ \dot{e}_{ij}^{y} = -e_{ij}^{x}\omega_{i} - v_{j}\sin e_{ij}^{\theta} - \omega_{j}d\sin e_{ij}^{\theta} + f_{2}, \\ \dot{e}_{ij}^{\theta} = \omega_{j} - \omega_{i}. \end{cases}$$
(11)

Let $z = \left[e^x_{ij}, e^y_{ij}\right]^T$, where superscript T stands for transpose, then the first and second equation of (11) can be expressed as a matrix as follows:

$$\dot{z} = AZ + BU + f,\tag{12}$$

where

$$A = \begin{bmatrix} 0 & \omega_i \\ \omega_i & 0 \end{bmatrix}, B = \begin{bmatrix} -\cos e_{ij}^{\theta} & d\sin e_{ij}^{\theta} \\ -\sin e_{ij}^{\theta} & -d\sin e_{ij}^{\theta} \end{bmatrix}.$$
(13)

$$U = [v_j, \omega_j]^T, \quad f = [f_1, f_2]^T.$$
 (14)

Since $det(B)=d\neq 0$, we can do input-output linearization on (12), and get

$$U = [v_j, \omega_j]^T = B^{-1}(-kz - Az - f),$$
 (15)

with $k = [k_1, k_2] > 0$.

As a result, the control law is derived as

$$v_{j} = (k_{1}e_{ij}^{x} + \omega_{j}e_{ij}^{y} + f_{1})\cos e_{ij}^{\theta} - (-k_{2}e_{ij}^{y} + \omega_{j}e_{ij}^{x} - f_{2})\sin \theta_{ij},$$
(16)
$$\omega_{j} = \frac{1}{d} \left[-(k_{1}e_{ij}^{x} + \omega_{j}e_{ij}^{y} + f_{1})\sin e_{ij}^{\theta} + (k_{2}e_{ij}^{y} - \omega_{j}e_{ij}^{x} + f_{2})\cos e_{ij}^{\theta} \right].$$
(17)

with k_1 and k_2 are positive adjustable design parameters. According to equation (3), (4), (16) and (17), we can get:

$$\dot{\phi}_{j}^{R} = \frac{2D\left(k_{1}e_{ij}^{x} + \omega_{j}e_{ij}^{y} + f_{1}\right)\left(\cos e_{ij}^{\theta} - \frac{\sin\theta_{ij}}{d}\right)}{rD + r} + \frac{2D\left(k_{2}e_{ij}^{Y_{i}} - \omega_{j}e_{ij}^{X} + f_{2}\right)\left(\sin e_{ij}^{\theta} + \frac{\cos e_{ij}^{\theta}}{d}\right)}{rD + r},$$

$$\dot{\phi}_{j}^{L} = \frac{2D\left(k_{1}e_{ij}^{x} + \omega_{j}e_{ij}^{y} + f_{1}\right)\left(\cos e_{ij}^{\theta} + \frac{1}{d}\sin\theta_{ij}\right)}{rD + r} + \frac{2D\left(k_{2}e_{ij}^{y} - \omega_{j}e_{ij}^{x} + f_{2}\right)\left(\sin e_{ij}^{\theta} - \frac{1}{d}\cos e_{ij}^{\theta}\right)}{rD + r}.$$

$$(19)$$

Theorem 1. Considering the formation control of two wheeled mobile robots shown in Figure 1, for any given bounded and sufficiently smooth leader's path, the formation control of Case 1 can be achieved asymptotically by the proposed control law consisted of (18) and (19), and the whole system (6) will also be stable.

Proof: Substitute (18) and (19) into error system (12), then

$$\begin{cases} \dot{e}_{ij}^x = -k_1 e_{ij}^x, \\ \dot{e}_{ij}^y = -k_2 e_{ij}^y. \end{cases}$$
 (20)

Namely,

$$\begin{cases} e_{ij}^x = e^{-k_1 t}, \\ e_{ij}^y = e^{-k_2 t}. \end{cases}$$
 (21)

Therefore, $e^x_{ij} \to 0$ and $e^y_{ij} \to 0$ as $t \to \infty$. Therefore, if only the equation (18) and (19) are satisfied, the e^x_{ij} and e^y_{ij} exponentially stable. The rest of the problem is only to test the stability of e^θ_{ij} .

From (17), we have

$$\dot{e}_{ij}^{\theta} = \omega_j - \omega_i = \frac{1}{d} \left[\left(-k_1 e_{ij}^x - \omega_j e_{ij}^y - f_1 \right) \sin e_{ij}^{\theta} + \left(k_2 e_{ij}^y - \omega_j e_{ij}^x + f_2 \right) \cos e_{ij}^{\theta} \right] - \omega_i.$$
(22)

It is easily to know $\|e_{ij}^x\| < \infty$, $\|e_{ij}^y\| < \infty$, and f_1, f_2 are also bounded either. Then, equation (17) can be rewritten as

$$\dot{e}_{ij}^{\theta} = -a\sin(e_{ij}^{\theta} + \gamma) - \omega_i. \tag{23}$$

with

$$a = \frac{1}{d} \left[\left(k_1 e_{ij}^x + \omega_j e_{ij}^y + f_1 \right)^2 + \left(k_2 e_{ij}^y - \omega_j e_{ij}^x + f_2 \right)^2 \right]^{1/2}, \tag{24}$$

$$\gamma = \arctan\left(\frac{k_2 e_{ij}^y - \omega_j e_{ij}^x + f_2}{k_1 e_{ij}^x + \omega_j e_{ij}^y + f_1}\right),\tag{25}$$

where ω_i can be considered as a disturbance term. With a bounded ω_i , it is straightforward to see it is stable by

doing linearization on (23) locally, which means that the zero dynamics of whole system (6) is stable. So, the whole system (6) is stable.

Due to the only difference of the four cases is different f in each case, similar analysis and proof are used for the Case 2, Case 3 and Case 4, the formation control theorems and control laws similar to theorem 1 and (18) and (19) with corresponding f respectively, and the stability of e_{ij}^{θ} can be similarly proved.

4 Simulation

To verify the effectiveness of presented model and control law as stated above, we simulate one team of 2 tricycle mobile robots as shown in Figure 1, in which $d=4\mathrm{cm}$, $r=1\mathrm{cm}$, $D=3\mathrm{cm}$ and the desired relative distance and angle between them are 30cm and 120° , and the initially $e_{ij}^x=10\mathrm{cm}$ and $e_{ij}^y=10\mathrm{cm}$, $e_{ij}^\theta=30^{\circ}$.

 $e_{ij}^x=10\mathrm{cm}$ and $e_{ij}^y=10\mathrm{cm}$, $e_{ij}^\theta=30^\circ$. Figures 2 and 3 show e_{ij}^x and e_{ij}^y are asymptotic convergence to zero respectively, where the leader undergoes uniform linear motion at speed $v_i=5\mathrm{cm/s}$ and $\omega_i=0$. Figures 4 shows the tracking trajectory of mobile tricycle robots. As we can see from the simulation results, the desired formation can achieve asymptotically by the proposed control law as stated above.

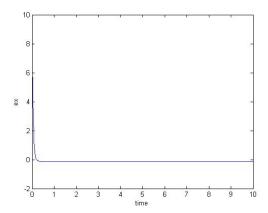


Figure 2: Convergence curve of e_{ij}^x .

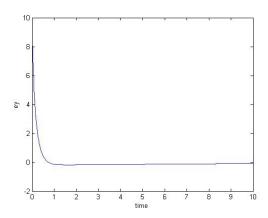


Figure 3: Convergence curve of e_{ij}^y .

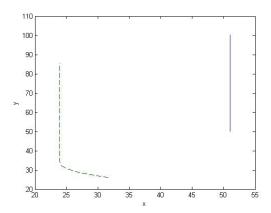


Figure 4: The tracking trajectory of mobile tricycle robots: leader: " – ", follower: "- -".

5 Conclusion

In this paper, the formation control of multiple wheeled mobile tricycle robots convert to a tracking problem with desired relative distance and relative angle between leader robot and follower robot. The kinematics models in Cartesian coordinates, which good for avoid the inevitable singularity problem in control law design, are used to to derive control laws. Based the models, the four cases of formation control have been consider, and four control laws have proposed corresponding to control tasks respectively, and asymptotical stable of the formation control system can achieve by the proposed control law. Simulation studies are included to demonstrate the effectiveness of the proposed method. The model uncertainties, noise and disturbances acting on the robot are not much considered in robot dynamics is the shortcoming of this paper. In following works, these factors will be take into account to be more close to the real situation.

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