Proceedings of the 2006 American Control Conference
Minneapolis, Minnesota, USA, June 14-16, 2006

Decentralized formation control and obstacle avoidance for multiple robots with nonholonomic constraints

Yi Liang and Ho-Hoon Lee

Abstract— **This paper proposes a formation control scheme for a group of mobile robots based on a multi-objective potential force. The angle of the potential force, with respect to the global coordinate system, is used to generate trajectories for the navigation of a group of nonholonomic mobile robots. A smooth and continuous control law, based on translational force input and rotational torque input, is designed to reduce the global orientation error asymptotically to zero while maintaining proper formation for a target configuration. Lyapunov stability theorem is applied to construct the proposed control with smooth continuous feedback, in which stability is proved for the control of multiple mobile robots. The formation regulation, flocking, and obstacle avoidance of the proposed control are validated through numerical simulation.**

I. INTRODUCTION

For the last ten years, extensive research has been performed for the coordinated motion control of autonomous aggregations. Coordination of the collective motions has various engineering applications including massive network sensing in an uncertain environment, gradient climbing or descending, cooperative transportation of a large payload, formation flying of unmanned aerial vehicles [13], and multi-agent gaming such as robot soccer and robot rescue.

Coordinated behavior can be widely observed in the nature ranging from chemical crystal lattice to animal flocks. Reynolds [14] made a computer simulation model of coordinated animal motion such as bird flocks and fish schools. Toner et al. [18] formulated a continuum model for flocking dynamics and obtained some results for a 2-dimensional model describing the 2D motion of land flocks. In their later work [19], they presented a quantitative continuum theory of flocking. In [5], Vicsek et al. simulated the convergence property of a group of particles using the nearest neighbor rule and its mathematical stability analysis was presented in [6].

A vast number of decentralized control strategies have been designed in the last ten years to stabilize a group of mobile robots in achieving a global objective such as a tight formation with fixed pair-wise inter-vehicle distances. However, most of them are based on point-mass dynamics [1-4]. The flocking control method in [3] assumed point-mass idealization and structural stabilization for robots within the system. In reality, most of the mobile robots are subject to nonholonomic constraints. It has been shown in [7] that there is no smooth time-invariant feedback control law that can locally stabilize a nonholonomic system about an arbitrary point.

Y. Liang is with Mechanical Engineering, Tulane University, New Orleans, Louisiana 70118, yliang@tulane.edu
H. Lee is with the Faculty of Mechanical Engineering, Tulane University, New Orleans, Louisiana 70118, hhlee@tulane.edu

Both discontinuous [8-10] and time-varying type [11, 12] control laws have been applied to stabilize the mass center of a nonholonomic mobile robot. However, time-varying control laws suffer from slow convergence. Discontinuous control laws guarantee fast convergence, but complicate the stability analysis with Lyapunov stability theory, because they involve differential inclusion and nonsmooth Lyapunov candidate functions. Pomet [15] proposed a stabilizing control of an off-axis point along the heading direction of a nonholonomic mobile robot. Pathak et al. [16] controlled a nonholonomic robot in a known environment by combining ring-shaped potential fields. Brock et al. [17] applied a bubble expansion method to create a local-minimum-free path-following controller.

Cooperative and coordinated control of multiple mobile robots with nonholonomic constraints has been addressed by a few researchers. Lawton [20] developed a feedback linearization technique for the dynamic model of nonholonomic mobile robots by allowing an off-axis point along the robot's heading direction to be controlled as a point-mass. Loizou et al. [21] used a dipolar potential field and a kinematic controller to globally stabilize a group of nonholonomic mobile robots from an initial configuration to a final configuration. However, these methods have focused on kinematic models of the electromechanical systems, leading to a direct control of linear and angular velocity, while paying less attention to the control of nonholonomic systems where forces and torques are the true inputs.

The objective of this research is to develop a decentralized control architecture for a group of nonholonomic mobile robots. In this study, a decoupled controller, consisting of a force controller and a torque controller, is designed to achieve a smooth control of translational and rotational motion of a group of mobile robots subject to nonholonomic constraints while maintaining prescribed formation and avoiding inter-vehicle and obstacle collisions.

The remainder of the paper is organized as follows. In Section 2, the kinematics and dynamics of a wheeled robot are described. Section 3 describes an interactive potential energy function. In Section 4, a regulating control law stabilizing a flock of nonholonomic robots in an unknown environment with obstacles is derived and its stability is proven. Simulation results are given in Section 5. Finally, in Section 6, this study is discussed and concluded.

1-4244-0210-7/06/$20.00 ©2006 IEEE

II. Kinematics and Dynamics

In this section, the kinematics and dynamics of mobile robots are described. In Fig. 1, a differentially-driven mobile robot is characterized by two driving wheels mounted symmetrically along an axis, which passes through the mass center of the robot. Two additional castor wheels are installed along an axis which is perpendicular to the driving axis.

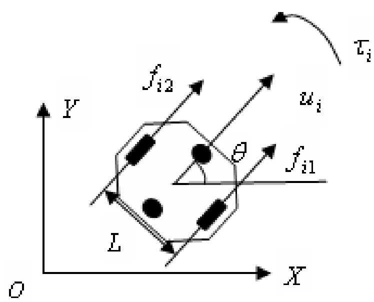

Fig. 1. A wheeled mobile robot of unicycle type

In this study, the following assumptions are presumed: (1) the driving wheels allow only pure rolling without slipping, (2) the effects of the castor wheels are ignored, (3) the masses and the mass moment of inertia of the driving wheels are negligible, compared to the mass of the mobile robot, and (4) the mass center of the mobile robot is aligned with the geometric center. The kinematic and dynamic equations of motion of the i^{th} individual robot are given as

$$\begin{aligned} \dot{x}_i &= v_i \cos\theta_i \\ \dot{y}_i &= v_i \sin\theta_i \\ \dot{\theta}_i &= \omega_i \\ \dot{v}_i &= u_i/m_i \\ \dot{\omega}_i &= \tau_i/J_i \end{aligned} \tag{1}$$

where the position of the mass center of the robot is given by $\mathbf{r}_i = (x_i, y_i)$; θ_i is the heading direction of the robot in the global coordinates; v_i is the translational velocity of the robot in the direction of θ_i; ω_i is the angular velocity of the robot about the vertical axis passing through the mass center; u_i is the applied translational force; τ_i is the applied rotational torque; m_i and J_i are the mass and the mass moment of inertia of the robot, respectively.

As long as the control inputs u_i and τ_i are specified for the robot, the input torque to each driving wheel can be readily computed as

$$f_{i1} = (u_i + \tau_i/L)/2 \tag{2}$$

$$f_{i2} = (u_i - \tau_i/L)/2 \tag{3}$$

where L is the distance between the two driving wheels.

III. Interactive Potential Energy

A. Graph theory

This section presents a structural model for a group of mobile robots navigating in formation, where each robot is described by the kinematic and dynamic equations shown in Eq. (1). It is assumed that the group of robots has certain interconnection topology and the state information is exchanged among the robots by sensing or communicating. For simplicity, it is also assumed that the mass and the mass moment of inertia of the robot are unity. A group consists of n autonomous robots labeled 1 through n. The i^{th} robot is denoted with the position vector $\mathbf{r}_i = (x_i, y_i)$. The internal topology of the robot can be defined as a graph. Then the relative positions of the robots can be studied by applying the graph theory.

The set of robots in the detecting range of a robot is called the neighborhood of the robot. Each robot in the group collects the same amount of information from the field. Each edge, which connects two robots, is undirected. The topology of the group is rigid and unambiguous to guarantee that the graph is not foldable.

Definition 1 (Neighborhood): The j^{th} robot is said to be a neighbor of the i^{th} robot at time t if and only if the j^{th} robot is a component of $N_i(\mathbf{r}_i, rad_i)$ at time t with $N_i(\mathbf{r}_i, rad_i)$ defined as

$$N_i(\mathbf{r}_i, rad_i) = \{\mathbf{r}_j \in R^2 : \|\mathbf{r}_j - \mathbf{r}_i\| \le rad_i\} \tag{4}$$

where $N_i(\mathbf{r}_i, rad_i)$ is called the neighborhood set of the i^{th} robot; rad_i is the sensing radius of the i^{th} robot [22]. If the graph is undirected, then the following property holds

$$\mathbf{r}_j \in N_i(\mathbf{r}_i, rad_i) \leftrightarrow \mathbf{r}_i \in N_j(\mathbf{r}_j, rad_j) \tag{5}$$

B. Interactive Potential Energy

An artificial potential function is derived to control the interconnecting distance between a pair of mobile robots, and to steer them towards their desired distances without collision. In this study, it is assumed that the potential energy only takes effects when the robots are in the neighborhood of each other. The formation distance error q_{ij}, between the i^{th} and the j^{th} robot, is defined as

$$q_{ij} = \|\mathbf{r}_j - \mathbf{r}_i\| - d_{ij} \tag{6}$$

where d_{ij} is the desired distance between the two robots.

Using the formation distance error, the following hyperbolic function is chosen to represent the potential energy between the i^{th} and j^{th} robots:

$$V_{ij}(\mathbf{r}_i, \mathbf{r}_j) = K_{ij} \ln(\cosh(q_{ij})) \tag{7}$$

where K_{ij} is a gain to regulate the magnitude of the potential energy, henceforth to affect the magnitude of the interactive force generated from the potential energy. K_{ij} can be selected such that the control force goes saturated as the power of the driving motor approaches its maximum. Typically, it can be set as $K_{ij} = 4\tau_{max}/L$, where τ_{max} is the maximum output torque of the driving motor. K_{ij} can be smaller when the driving power is sufficient, while it can be bigger if fast response is preferred.

Correspondingly, the total structural potential energy from all neighbors around the i^{th} robot is computed from V_{ij}:

$$U_i(\mathbf{r}_i) = \sum_{j \neq i} V_{ij}(\mathbf{r}_i, \mathbf{r}_j) \tag{8}$$

The interactive structural force is the gradient of the potential energy:

$$f_{ij}(\mathbf{r}_i, \mathbf{r}_j) = \nabla V_{ij} = K_{ij} \tanh(q_{ij}) \frac{\mathbf{r}_j - \mathbf{r}_i}{\|\mathbf{r}_j - \mathbf{r}_i\|} \quad (9)$$

where repulsive forces are negative while attractive forces are positive.

The potential energy in Eq. (7) and the potential force in Eq. (9) are shown in Fig. 2. It is shown that the potential force is attractive when the distance between the two robots is greater than the desired distance, while it is repulsive when the distance is less than the desired distance.

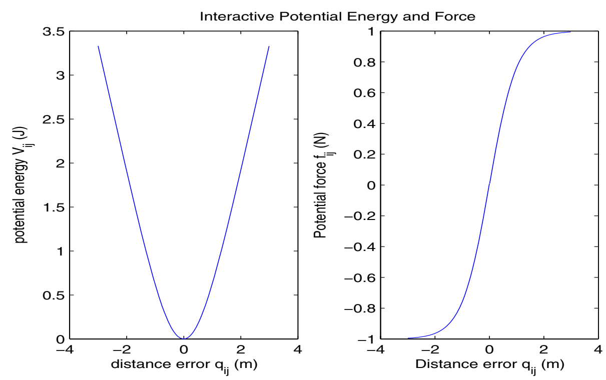

Fig. 2. A wheeled mobile robot of unicycle type

The total structural force acting on the i^{th} robot, due to the potential force contributions from all of its neighboring robots, is given as

$$F_i(\mathbf{r}_i) = \sum_{j \neq i} f_{ij}(\mathbf{r}_i, \mathbf{r}_j) \quad (10)$$

IV. Formation Control with Obstacle Avoidance

A. Regulation control to a target configuration

An important feature of a formation control system is that the robots within the system reach their target configuration. Multiple mobile robots in formation are able to approach a targeted goal position using a regulating control with an artificial potential field that attracts the robots to their final target positions. In order to ensure the stability of the system and to maintain the formation, the targeting potential functions should be incorporated within a structural potential function, which guarantees structural stability of the formation.

The destination target can be defined as a virtual robot, for example, as a vertex of the formation topology. Each real robot has a specified distance relationship with this virtual robot, as defined in Eq. (6). As long as all of the required distances are satisfied, a formation is accomplished and the target position is reached.

In this study, the destination position is defined as the target for a specified robot in the group. An artificial potential function is built between the leader robot and its destination. This targeting potential function is inserted into the interactive potentials from all neighbors of the leader. While the leader is being attracted to the target, the internal potentials attract all of the other robots to the leader. Therefore, the group is attracted to the destination while maintaining the formation. The targeting potential gain K_{i0} is kept small, compared to the other structural potential gains K_{ij}, to keep the strong regulation forces from overwhelming the other structural forces acting on the robots.

Control of nonholonomic mobile robots in formation is based on the incorporation of the potential forces. The synthesized force for the i^{th} robot, F_i, is given by

$$F_i = \nabla_{\mathbf{r}_i} U_i \quad (11)$$

where $\nabla_{\mathbf{r}_i} U_i$ includes the potential energy from the destination target. Fig. 3 demonstrates the distribution of the potential forces summed to determine the synthesized force.

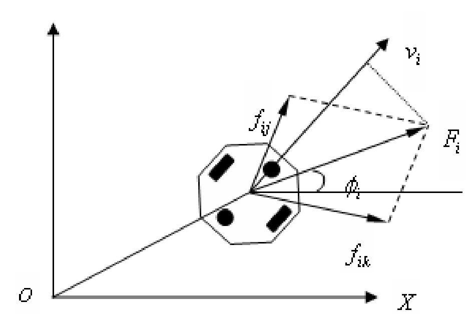

Fig. 3. Illustration of synthesized force and angle

The objective of the control is to regulate a group of nonholonomic mobile robots. ϕ_i, the angle of the synthesized force, is used as the desired orientation for the heading direction of the robot. The magnitude of the synthesized force projected in the heading direction of the robot is used to control the translational acceleration of the robot.

In order to traverse an unknown environment, the control law attempts to reduce the orientation error, $e_i = \theta_i - \phi_i$, asymptotically to zero while maintaining the proper structural distances in formation and approaching the target. Because the angle of the synthesized force acts as a trajectory, the orientation error evolves through time as

$$\begin{aligned} e_i &= \theta_i - \phi_i \\ \dot{e}_i &= \dot{\theta}_i - \dot{\phi}_i \\ \ddot{e}_i &= \ddot{\theta}_i - \ddot{\phi}_i \end{aligned} \quad (12)$$

Theorem 1: Consider a group of mobile robots, the motion of which is described by Eq. (1). Suppose that the synthesized force defined by the structural and targeting potentials is applied to each robot, and that the angle of the synthesized force in the global coordinates is defined as the desired orientation of each robot. Then, the following decentralized control law stabilizes the formation of the group of robots while they are approaching the target configuration:

$$u_i = -f_{ix}\cos\theta_i - f_{iy}\sin\theta_i - k_{vi}v_i \quad (13)$$

$$\tau_i = -k_{pi}(\theta_i - \phi_i) - k_{di}(\dot{\theta}_i - \dot{\phi}_i) + \ddot{\phi}_i \quad (14)$$

where k_{pi}, k_{vi}, and k_{di} are positive gains; f_{ix} and f_{iy} are the components of the synthesized force in the x and y directions, respectively.

Proof: Consider the following Lyapunov candidate:

$$W = \frac{1}{2}\sum_{i=1}^{N} U_i + \frac{1}{2}\sum_{i=1}^{N} k_{pi}e_i^2 + \frac{1}{2}\sum_{i=1}^{N} \dot{e}_i^2 + \frac{1}{2}\sum_{i=1}^{N} k_{vi}v_i^2 \quad (15)$$

It should be noted that the candidate satisfies the prerequisites of a Laypunov function in that (1) $W(q_{ij}, e_i, \dot{e}_i, v_i) = 0$ if and only if $(q_{ij}, e_i, \dot{e}_i, v_i) = 0$, and (2) $W(q_{ij}, e_i, \dot{e}_i, v_i)$ is a positive, continuously differentiable function with $W(q_{ij}, e_i, \dot{e}_i, v_i) > 0$, $\forall (q_{ij}, e_i, \dot{e}_i, v_i) \neq 0$.

The time derivative of W is computed as

$$\dot{W} = \sum_{i=1}^{N}((\nabla_{r_i}U_i)^T\dot{\mathbf{r}}_i + k_{pi}e_i\dot{e}_i + \dot{e}_i\ddot{e}_i + k_{vi}v_i\dot{v}_i) \quad (16)$$

where

$$\begin{aligned}(\nabla_{r_i}U_i)^T\dot{\mathbf{r}}_i &= ((\nabla_{r_i}U_i)_x, (\nabla_{r_i}U_i)_y)\begin{pmatrix} v_i\cos\theta_i \\ v_i\sin\theta_i \end{pmatrix} \\ &= (f_{ix}, f_{iy})\begin{pmatrix} v_i\cos\theta_i \\ v_i\sin\theta_i \end{pmatrix} \end{aligned} \quad (17)$$

Inserting the control law and the kinematics (1) into $\dot{W}$:

$$\begin{aligned}\dot{W} &= \sum_{i=1}^{N}[(f_{ix}, f_{iy})\begin{pmatrix} v_i\cos\theta_i \\ v_i\sin\theta_i \end{pmatrix} \\ &\quad + k_{pi}e_i\dot{e}_i + \dot{e}_i(-k_{pi}e_i - k_{di}\dot{e}_i) \\ &\quad + v_i(-f_{ix}\cos\theta_i - f_{iy}\sin\theta_i - k_{vi}v_i)] \\ &= \sum_{i=1}^{N}(-k_{di}\dot{e}_i^2 - k_{vi}v_i^2) \end{aligned} \quad (18)$$

Remark 1: LaSalle's invariance principle states that the solution of the system will converge to the largest invariant set contained in $S = \{(q_{ij}, e_i, \dot{e}_i, v_i) : \dot{W}(t) = 0\}$. For the condition $\dot{W}(t) = 0$ to be satisfied, $\dot{e}_i \to 0$ and $v_i \to 0$ as $t \to \infty$. By the definition of the candidate function $W(t) \geq 0$ and the result that $\dot{W}(t) \leq 0$, it is implied that $(q_{ij}, e_i, \dot{e}_i, v_i)$ is bounded such that $(q_{ij}, e_i, \dot{e}_i, v_i) \in L^\infty$ and $(\dot{e}_i, v_i) \in L^2$. $\phi_i \in [-\pi, \pi]$ is bounded. As a result, the boundedness of $(\ddot{e}_i, \dot{v}_i)$ follows from Eqs. (1) and (14) such that $(\ddot{e}_i, \dot{v}_i) \in L^\infty$. Then, as a consequence of Barbalat's Lemma, $(\dot{e}_i, v_i) \to 0$ asymptotically as $t \to \infty$, which implies $(\ddot{e}_i, \dot{v}_i) \to 0$ asymptotically as $t \to \infty$ since $(\ddot{e}_i, \dot{v}_i) \in L^\infty$. Finally, by the definition of the orientation error in Eq. (12), the following conclusion holds: $\dot{\theta}_i \to \dot{\phi}_i$, and $\ddot{\theta}_i \to \ddot{\phi}_i$. Therefore $\theta_i \to \phi_i$ from the control law (14). Moreover, $(v_i, \dot{v}_i) \to 0$ means

$$(\nabla_{\mathbf{r}_i}U_i)_x\cos\theta_i + (\nabla_{\mathbf{r}_i}U_i)_y\sin\theta_i \to 0 \quad (19)$$

but this is not sufficient to prove that $\nabla_{\mathbf{r}_i}U_i \to 0$ as $t \to \infty$, which is discussed in Remark 2.

Remark 2: Since $\theta_i \to \phi_i$, it can be readily derived from Eq. (19) that

$$\frac{\sin(\phi_i)}{\cos(\phi_i)} \to \frac{-(\nabla_{\mathbf{r}_i}U_i)_x}{(\nabla_{\mathbf{r}_i}U_i)_y} \quad (20)$$

According to the definition of the synthesized force, the definition of the desired orientation is

$$\tan(\phi_i) = \frac{(\nabla_{\mathbf{r}_i}U_i)_y}{(\nabla_{\mathbf{r}_i}U_i)_x} \quad (21)$$

Then, from Eq. (20) and Eq. (21)

$$\frac{-(\nabla_{\mathbf{r}_i}U_i)_x}{(\nabla_{\mathbf{r}_i}U_i)_y} \to \frac{(\nabla_{\mathbf{r}_i}U_i)_y}{(\nabla_{\mathbf{r}_i}U_i)_x} \quad (22)$$

that is,

$$(\nabla_{\mathbf{r}_i}U_i)_y^2 + (\nabla_{\mathbf{r}_i}U_i)_x^2 \to 0 \quad (23)$$

Since $(\nabla_{\mathbf{r}_i}U_i)_x$ and $(\nabla_{\mathbf{r}_i}U_i)_y$ are the two components of the vector $\nabla_{\mathbf{r}_i}U_i$, it can be readily concluded that $\nabla_{\mathbf{r}_i}U_i \to 0$ as $t \to \infty$. This means that the structural potential approaches a minimum point as time approaches infinity. Therefore, the inter-vehicle collision-free property is guaranteed and the robots approach the desired formation.

Remark 3: Since the decoupled translational control and rotational control are both PD controls, the system is very robust. The control gains k_{pi}, k_{di} and k_{vi} are mainly determined by the mass and the mass moment of inertia of the mobile robots.

B. Obstacle avoidance

Obstacle avoidance is an essential part within a control system to ensure that multiple mobile robots cooperatively traverse unknown environments. For efficient navigation, the robots must be capable of locally sensing obstacles along their paths and of taking proper actions to maintain the formation, while avoiding collisions.

When a robot is approaching an obstacle having a convex surface, e.g., a regular round shape, the robot projects itself onto the surface of the obstacle. Therefore, the geometric and kinematic relationships between the robot and the obstacle can be described by Fig. 4, where $\mathbf{q}_i(x_r, y_r)$ is the position of the robot, $\mathbf{q}_{obs}(x_{obs}, y_{obs})$ represents the projection point of the robot onto the surface of the obstacle, and $\mathbf{O}_k(x_k, y_k)$ is the center of the obstacle, which can be neglected for a real time measurement system.

From Fig. 4, it can be readily derived that

$$\mathbf{q}_{obs} = \frac{r_{obs}}{\|\mathbf{q}_i - \mathbf{O}_k\|}\mathbf{q}_i + (1 - \frac{r_{obs}}{\|\mathbf{q}_i - \mathbf{O}_k\|}\mathbf{q}_i)\mathbf{O}_k \quad (24)$$

where r_{obs} is the radius of the obstacle. Furthermore, it can be derived that the projection point is moving with a velocity

$$v_{obs} = \frac{v_i r_{obs}\sin\alpha_i}{\|\mathbf{q}_i - \mathbf{q}_{obs}\|} \quad (25)$$

where v_i is the velocity of the robot; α_i is the angle measured from the orientation of the i^{th} robot to the straight line which connects the robot to the obstacle.

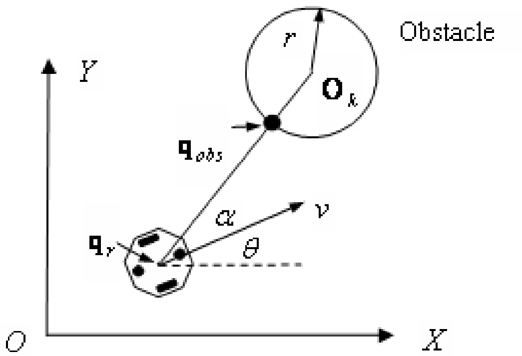

Fig. 4. Relationship of a robot and a obstacle

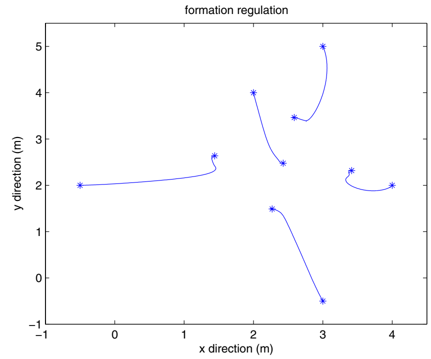

Fig. 5. Regulating a group to form a formation

Since the projection point has individual position and velocity, it can be considered an independent virtual robot. If the potential between the real robot and the virtual robot is synthesized into the total potentials of the real robot, the real robot is capable of steering from the virtual robot. The steering action takes place as long as the virtual robot enters the view of the real one; therefore, the robot is safe from the obstacle. Since all robots in the group have identical controller, the group is able of avoiding obstacles in the environment when the controller in Theorem 1 is applied.

V. Simulation

In the simulation, the mobile robots and the obstacle were idealized as circular objects with radii equivalent to their dimension. The proposed control law described in Theorem 1 was applied for formation regulation, target reaching and obstacle avoidance, in which all of the robots started from a non-formation configuration. The fixed sampling rate of 5 ms was applied to model the perceived sampling time for future real-time application.

A. Formation regulation of multiple mobile robots

In this simulation, a group of mobile robots was attracted to the desired formation using the regulation control law derived in Theorem 1. In the simulation, all of the robots started from arbitrary positions, but eventually reached a star formation as defined by the following distance matrix:

$$(\mathbf{d}_{ij}) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1.414 & 2 & 1.414 \\ 1 & 1.414 & 0 & 1.414 & 2 \\ 1 & 2 & 1.414 & 0 & 1.414 \\ 1 & 1.414 & 2 & 1.414 & 0 \end{pmatrix}$$

B. Formation flocking of multiple mobile robots

The objective of this simulation is to allow a group of robots in a formation to travel in an unopposed environment to a given target position. It can be readily read from Fig. 6 that the robots started from arbitrary positions, and then formed a formation in a couple of seconds, after that the group of robots maintained the formation while traveling to the target. Fig. 7 and Fig. 8 illustrate the velocities and orientations of the robots in formation, respectively. In the figures, all of the robot kept the same velocity and orientation while approaching the target. In this simulation, the target position was defined as the target of the 1^{st} robot. When the 1^{st} robot was attracted to the target position, all of the others were attracted by the 1^{st} robot; therefore the whole formation reached the target while formation was maintained.

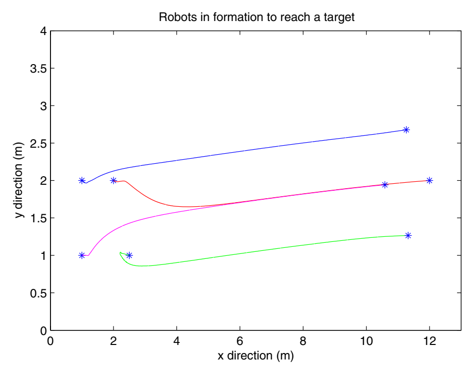

Fig. 6. A group of robot form a formation while reaching the target

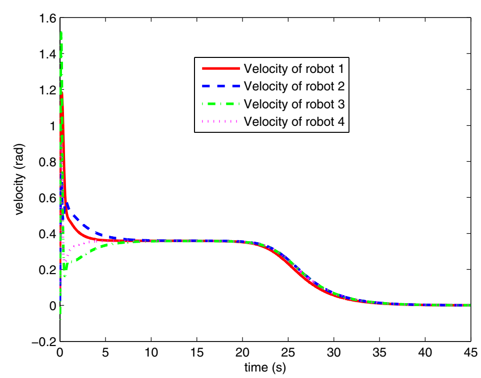

Fig. 7. Velocities of robots in a formation while reaching the target

C. Formation flocking with obstacle avoidance

In this simulation, a formation of nonholonomic mobile robots traveled in an unknown environment with a convex

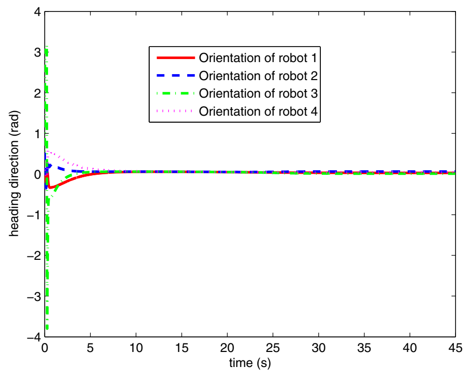

Fig. 8. Orientations of robots in a formation while reaching the target

obstacle. Fig. 9 shows the trajectories of the robots from the initial to the final configuration. The robots were able to travel in a space confined by an obstacle while maintaining the formation.

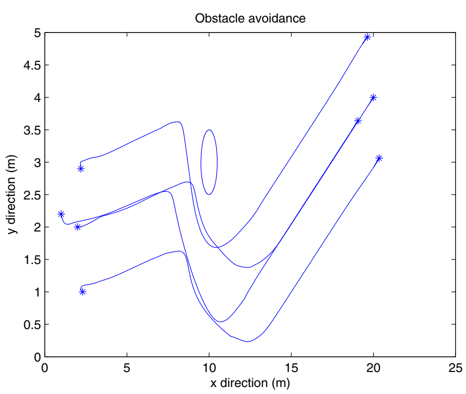

Fig. 9. Obstacle avoidance

VI. Conclusions

This study has presented the initial framework for the development of a decentralized control architecture for a group of autonomous nonholonomic mobile robots. In this study, Lyapunov stability theorem was used to construct a convergent control law with smooth continuous feedback.

With the global angle of the synthesized potential force used as a desired trajectory, the proposed technique allows the generation of admissible minimum-energy trajectories through unknown environments without inter-vehicle and obstacle collision. The proposed algorithm has been validated with numerical simulations.

As a future research, the proposed control will be implemented for the navigation of a group of mobile robots, which have already been built for this research in the Department of Mechanical Engineering at Tulane University.

References

[1] E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials, and coordinated control of groups," *Proc. of the 40th IEEE Conference on Decision and Control*, 2001, pp. 2968-2973.

[2] R. O. Saber and R. M. Murray, "Distributed Structure Stabilization and Tracking for Formations of Dynamic Multi-Agents," *Technical Report CDC02-REG0736*, California Institute of Technology, CA, 2002.

[3] R. O. Saber, "Flocking with Obstacle Avoidance," *Technical Report CIT-CDS 03-006*, California Institute of Technology, CA, 2003.

[4] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Stable Flocking of Mobile Agents, Part I: Fixed Topology," *The 42nd IEEE Conference on Decision and Control*, Maui Hawaii, 2003, pp. 2010-2015.

[5] T. Vicsek, A. Czirok, E. Ben Jacob, I. Cohen, and O. Schochet, "Novel type of phase transitions in a system of self-driven particles," *Physical Review Letters*, vol. 75, 1995, pp. 1226-1229.

[6] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile agents using nearest neighbor rules," The IEEE Trans. on Automatic Control, vol. 48, 2003, pp. 988-1001.

[7] R. W. Brockett, "Asymptotically stability and feedback stabilization," *Differential Geometry Control Theory*, Birkhauser, Boston, USA, 1983, pp. 181-191.

[8] C. C. Wit and O. Sordanlen, "Exponential stabilization of mobile robot with nonholonomic constraints," *IEEE Transaction on Automatic Control*, vol. 37, 1992, pp. 1791-1797.

[9] A. Astolfi, "Discontinuous control of nonholonomic systems," *Systems and Control Letters*, vol. 27, 1996, pp. 37-45

[10] H. G. Tanner and K. J. Kyriakopoulos, "Discontinuous backstepping for stabilization of nonholonomic mobile robots," *Proc. of the 2002 IEEE International Conference on Robotics and Automation (ICRA)*, 2002, pp. 3948-3953.

[11] R. Murray and S. Sastry, "Nonholonomic motion planning-steering using sinusoids," *IEEE Transaction on Automatic Control*, vol. 38, 1993, pp. 700-716.

[12] W. E. Dixon, Z. P. Jiang, and D. M. Dawson, "Global exponential setpoint control wheeled mobie robot: a Lyapunov approach," *Automatica*, vol. 36, 2000, pp. 1741-1746.

[13] E. W. Justh and P. S. Krishnaprasad, "A Simple Control Law for UAV Formation Flying," *Technical Report TR 2002-38*, Institute for Sytem Research, University of Maryland, MD, USA, 2002.

[14] C. Reynolds, "Flocks, birds, and schools: a distributed behavioral model," *Computer Graphics*, vol. 21, 1987, pp. 25-34

[15] J. B. Pomet, "A hybrid strategy for the feedback stabilization of nonholonomic mobile robots," *Proc. of the 1992 IEEE International Conference on Robotics and Automation (ICRA)*, 1992, pp. 129-134.

[16] K. Pathak and S. K. Agrawal, "Planning and control of a nonholonomic unicycle using ring shaped local potential fields," *American Control Conference*, 2004, pp. 2368-2373.

[17] O. Brock and L. E. Kavraki, "Decomposition-based motion planning: a framework for real-time motion planning in high-dimensional configuration spaces," *Proc. of the 2001 IEEE International Conference on Robotics and Automation (ICRA)*, 2001, pp. 1469-1474.

[18] J. Toner and Y. Tu, "Long-range order in a two-dimensional dynamical XY model: how birds fly together," *Physical Review Letters*, vol. 75, 1995, pp. 4326-4329

[19] J. Toner and Y. Tu, "Flocks, herds, and schools: a quantitative theory of flocking," *Physical Review E*, vol. 80, 1998, pp. 4828-4858.

[20] J. R. Lawton, R. W. Beard, and B. J. Young, "A decentralized approach to formation maneuvers," *IEEE Transactions on Robotics and Automation*, vol 19, 2003, pp. 933-941.

[21] S. G. Loizou, D. V. Dimarogonas, and K. J. Kyriakopoulos, "Decentralized feedback stabilization and collision avoidance of multiple nonholonomic agents," *Technical Report No. 01-14*, National Technical University of Athens, 2004.

[22] Y. Liang, H-H. Lee, "Avoidance of Multiple Obstacles for a Mobile Robot with Nonholonomic Constraints," *IMECE2005*, paper number: IMECE2005-81744 2004.