Leader-following formation control of multiple mobile vehicles

J. Shao, G. Xie and L. Wang

Abstract: A framework for controlling groups of autonomous mobile vehicles to achieve predetermined formations based on a leader-following approach is presented. A three-level hybrid control architecture is proposed to implement both centralised and decentralised cooperative control. Under such architecture, the global-level formation control problem of n vehicles is decomposed into decentralised control problems between $n - 1$ pairs of follower and their designated leader. In the leader–follower control level, two basic controllers are proposed to make the following robot keep a relative position with respect to the leader and avoid collisions in the presence of obstacles. Then, graph theory is introduced to formalise specified formation patterns in a simple but effective way, and two types of switching between these formations are also proposed. Numerical simulations and physical robot experiments show the effectiveness of our approach.

1 Introduction

In recent years, there has been a tremendous interest in the coordinated control of multiple autonomous mobile vehicles. This is because there are many potential advantages of such systems over a single robot, including greater flexibility, adaptability to unknown environments and robustness. Among all the topics of study in this field, the formation control has attracted considerable attention from many researchers. Formation control is defined as the coordination of a group of robots that enter into and maintain a formation within specified geometrical shapes, such as a wedge or a chain. Potential application areas of formation control include many cooperative tasks such as exploring, surveillance, search and rescue, transporting large objects and control of arrays of satellites.

The formation control issues have been extensively investigated in the literature [1–9]. In Balch and Arkin [1], a behaviour-based approach is proposed. Reactive formation behaviours for four formations are integrated with navigational behaviours, implementing formation keeping and obstacle avoidance. Three formation reference types are also presented. Fredslund and Mataric [2] show how a group of distributed vehicles achieve global-level formations using only local sensing and minimal communication. Both the approaches are behavioural, reactive and seem to be reliable through experiments; however, they did not analyse the dynamics of robot vehicles and so the performance cannot be guaranteed mathematically. Das *et al.* [3] formulate the formation control problem as a hybrid control problem and propose a suite of control algorithms and a switching paradigm that allows the vehicles to maintain a prescribed formation and change formation in the presence of obstacles. In Lawton *et al.* [8], the authors propose a behavioural method of decomposing complex formation manoeuvres into a sequence of manoeuvres between formation patterns. Three control strategies are considered. Although Das *et al.* [3] and Lawton *et al.* [8] take full consideration for dynamics of vehicles, they recur to an off-the-axis point as offset and use feedback linearisation, which may cause some ill-conditioned control action. Another interesting approach named ‘virtual structures’ is presented in Ögren *et al.* [9]. On the basis of the assumption that vehicles have ‘control Lyapunov functions’, a stable coordination strategy is developed and formation maintenance, task completion time and formation velocity are also proved. Among all the approaches to formation control reported in the literature, the leader-following method has been adopted by many researchers [2–6]. In this method, each robot takes another neighbouring robot as a reference point to determine its motion. The referenced robot is called a leader and the robot following it is called a follower. Thus, there are many pairs of leaders and followers, and complex formations can be achieved by controlling relative positions of these pairs of vehicles, respectively. This approach is characterised by simplicity, reliability and no need for global knowledge and computation.

In this paper, we develop a new framework based on the leader-following approach to investigate formation control problems in teams of multiple mobile vehicles. Using distributed control and communication protocols, the global-level formation tasks are converted into local tracking problems in which the following vehicles track their leaders with the desired separations and desired relative bearings. The control algorithms use only relative information and no global positioning system is needed. On the basis of the formations of a two-robot system, more complex general configurations (n vehicles) are considered and formalised using tree-graph theory.

When compared with previous related work, our contributions are two-fold. First, we develop a hierarchical top-down control architecture that combines the advantages of the leader-following approach, distributed control and graph theory. Our second contribution is in the controller design. On the basis of the dynamic model of vehicle,

doi:10.1049/iet-cta:20050371

Paper first received 28th September 2005 and in revised form 7th March 2006
The authors are with the Center for Systems and Control, Department of Mechanics and Engineering Science, Peking University, Beijing 100871, People’s Republic of China

E-mail: longwang@mech.pku.edu.cn

we make full use of the separation in formation pattern and realise feedback linearisation more effectively. Additionally, a virtual robot is introduced and the leader-following formation is achieved by a tracking controller.

2 Hybrid control architecture for leader–follower formations

Before describing the control architecture, a brief discussion about the system is presented. A team of n non-holonomic vehicles is studied and each is equipped with sonar, a laser, a pan-tilt-zoom camera and a radio device. The vision sensor has a limited field of view. The vehicles are labelled and each has a unique identification number (ID). They can identify each other by their IDs, just as in the work of Fredslund and Mataric [2] and Das *et al.* [3]. The objective of formation control is to ensure that these vehicles move through unknown environments and reach their destination while maintaining desired geometrical shapes. To achieve this goal, it is important to describe the relations as well as organisation between these vehicles.

We will model the formation control in a top-down approach based on the leader-following model. A three-level hybrid architecture is developed as a solution. In the first level, a vehicle is identified as the lead vehicle that is responsible for planning a proper path for the whole team to reach their destination. As the coordinator of the system, the lead vehicle regularly exchanges information with other members, receiving their state information and broadcasting formation patterns to them. Any vehicle can take on this role. When the lead vehicle fails during the motion, other vehicles will prompt a new lead vehicle according to the role assignment mechanism. The dynamic leadership-changing mechanism provides the system with more flexibility and fault tolerance. Fig. 1 depicts a diagram of the first level.

In the second level, we focus on a leader–follower pair. Each vehicle within the formation, as already mentioned above, should determine its motion relative to one of the others (its leader) except the lead vehicle that does not follow any vehicle. Thus, there are many pairs of leaders and followers within the formation and the team of n vehicles can be decomposed into $n-1$ decentralised sub-systems of two vehicles. In each subsystem, the following vehicle tries to maintain a desired distance and desired angle relative to its leader. When all the vehicles are in the expected positions, the desired formations are established. We formulate the relation in each pair as a typical feedback control that is described in Fig. 2.

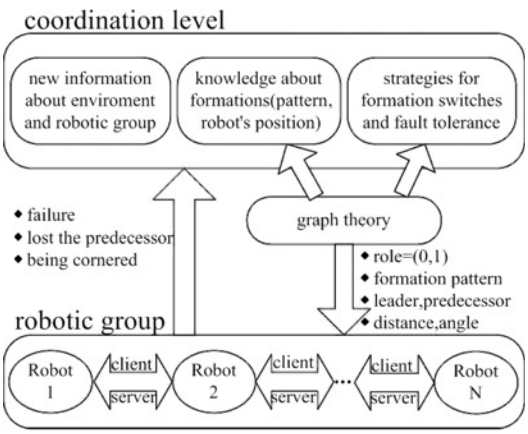

Fig. 1 *Coordination level*

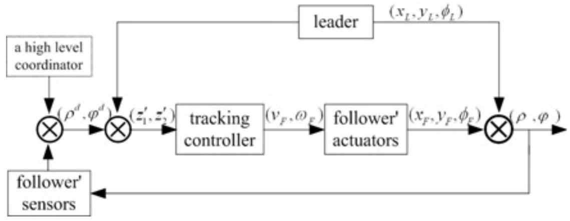

Fig. 2 *Leader–follower control level*

The third level is the entity-control level, in which two modes represent two different roles that the vehicle can take on. If the role of lead vehicle is assigned to it, the vehicle will take over the leadership of the group, make coordination between other vehicles and 'drag' the whole formation to their destination. Otherwise, the vehicle will take the follower mode. In this mode, two controllers can be chosen to implement formation keeping and obstacle avoidance. The control laws of the follower mode are explained in the next section. Fig. 3 illustrates the framework of individual vehicle architecture consisting of two modes.

3 Controller design

In this section, details of our controllers are presented in the follower mode: a tracking controller for establishing as well as maintaining formation and an obstacle avoidance controller for negotiating obstruction during the movement.

3.1 Tracking controller

Throughout this paper, we consider the mobile vehicles whose motion dynamics are determined by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} \cos\phi(t) & 0 \\ \sin\phi(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \tag{1}$$

where (x, y, ϕ) denotes the position and the orientation of the vehicle with respect to an inertial coordination frame and v and ω stand for the linear and angular velocities, respectively.

It is noted that (1) is a typical non-holonomic system. For this system, Brockett's feedback stabilisation theory [10] shows that a continuous time-invariant feedback control law, which stabilises the origin $x = y = \phi = 0$, does not exist. Other control strategies such as approximate inversion [11], discontinuous [12] or hybrid control [13] are used while not violating Brockett's theorem. In formation

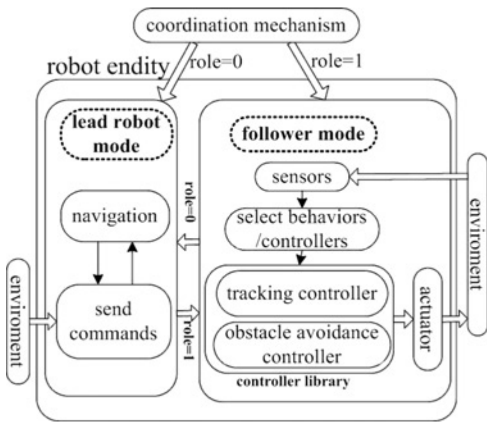

Fig. 3 *Entity control level*

control, the vehicles are, in fact, to be controlled to track some reference trajectories. As we know, in order to deal with the non-holonomic constraint, an off-the-axis point should be considered as a handling point that lies on the vehicle's axis of orientation, a distance d from the centre [3, 8]. Then, feedback linearisation can be utilised. However, in this approach, if the offset d is large, it will cause a large error to the formation, whereas if the offset is a smaller value [3], it may lead to large and ill-conditioned control action when using feedback linearisation.

In fact, when vehicles are required to establish certain formations, they will inevitably keep separation between each other; we can take full advantage of this separation to deal with the offset problem. Fig. 4 shows a simple triangular formation, in which R_l denotes the leader, and R_f is one of the followers. In order to keep such a formation, R_f needs to maintain a desired distance ρ^d and desired angle φ^d with respect to its leader R_l.

Then, let us focus on the configuration of this simple leader–follower pair (R_f, R_l), as shown in Fig. 5. For the follower R_f, we consider an off-the-axis point h as the handling point, which locates a distance L from the centre of R_f, where $L = \rho^d \cos \varphi^d$.

On the basis of (1), h is defined by

$$\begin{aligned} x_h &= x_f + L\cos\phi_f \\ y_h &= y_f + L\sin\phi_f \\ \phi_h &= \phi_f \end{aligned}$$

After derivation, we obtain

$$\begin{aligned} \dot{x}_h &= v_f\cos\phi_f - \omega_f L\sin\phi_f \\ \dot{y}_h &= v_f\sin\phi_f + \omega_f L\cos\phi_f \\ \dot{\phi}_h &= \omega_f \end{aligned} \tag{2}$$

Then, we assume that there exists a virtual leader named R_v, as shown in Fig. 5, which is located at the line perpendicular to the orientation of R_l, with a separation L' from the centre of R_l, where $L' = \rho^d \sin \varphi^d$. Then, if we want the follower R_f to keep a relative pose (ρ^d, φ^d) to its leader R_l, what we should do is to control the handling point h to track the virtual vehicle R_v.

Using the leader as a reference, the position of R_v can be obtained

$$\begin{aligned} x_v &= x_l + L'\cos\left(\phi_l - \frac{\pi}{2}\right) \\ y_v &= y_l + L'\sin\left(\phi_l - \frac{\pi}{2}\right) \\ \phi_v &= \phi_l \end{aligned} \tag{3}$$

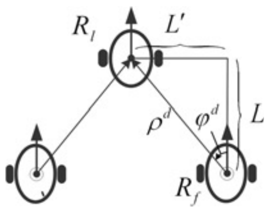

Fig. 4 *Simple triangle formation of three vehicles*

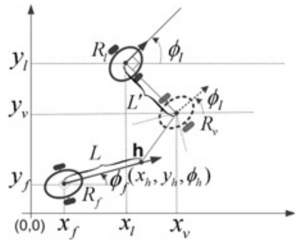

Fig. 5 *Illustration of tracking controller, using h as an offset and introducing a virtual vehicle*

After derivation, we obtain the dynamics of R_v

$$\begin{aligned} \dot{x}_v &= v_l\cos\phi_l - L'\sin\left(\phi_l - \frac{\pi}{2}\right)\omega_l \\ \dot{y}_v &= v_l\sin\phi_l + L'\cos\left(\phi_l - \frac{\pi}{2}\right)\omega_l \\ \dot{\phi}_v &= \omega_l \end{aligned} \tag{4}$$

Thus, the tracking error between R_v and h can be obtained as

$$\begin{aligned} \dot{\bar{x}} &= v_l\cos\phi_l - v_f\cos\phi_f - L'\sin\left(\phi_l - \frac{\pi}{2}\right)\omega_l \\ &\quad + L\omega_f\sin\phi_f \\ \dot{\bar{y}} &= v_l\sin\phi_l - v_f\sin\phi_f + L'\cos\left(\phi_l - \frac{\pi}{2}\right)\omega_l \\ &\quad - L\omega_f\cos\phi_f \\ \dot{\bar{\phi}} &= \omega_l - \omega_f \end{aligned} \tag{5}$$

where $\bar{x} = x_v - x_h$, $\bar{y} = y_v - y_h$, $\bar{\phi} = \phi_v - \phi_h$.

By the following coordinates transformation

$$\begin{aligned} z_1 &= \bar{x}\cos\phi_f + \bar{y}\sin\phi_f \\ z_2 &= \bar{x}\sin\phi_f - \bar{y}\cos\phi_f \end{aligned} \tag{6}$$

we can rewrite the error system in the new coordinates as (Fig. 6)

$$\begin{aligned} \dot{z}_1 &= \dot{\bar{x}}\cos\phi_f - \bar{x}\sin\phi_f\cdot\omega_f + \dot{\bar{y}}\sin\phi_f + \bar{y}\cos\phi_f\cdot\omega_f \\ &= -v_f + v_l\cos\bar{\phi} - z_2\omega_f - L'\cdot\omega_l\sin\left(\bar{\phi} - \frac{\pi}{2}\right) \\ \dot{z}_2 &= \dot{\bar{x}}\sin\phi_f + \bar{x}\cos\phi_f\cdot\omega_f - \dot{\bar{y}}\cos\phi_f + \bar{y}\sin\phi_f\cdot\omega_f \\ &= -v_l\sin\bar{\phi} + z_1\omega_f + L\omega_f - L'\cdot\omega_l\cos\left(\bar{\phi} - \frac{\pi}{2}\right) \\ \dot{\bar{\phi}} &= -\omega_f + \omega_l \end{aligned} \tag{7}$$

where (z_1, z_2) stands for position errors between R_v and h in the new coordinates, and (v_l, ω_l) are the leader's linear

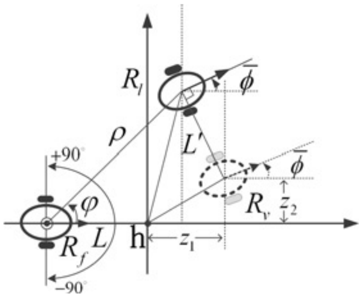

Fig. 6 *Error system in new coordinates*

velocity and angular velocity, which can be treated as exogenous inputs.

The next step is to find a control law (v_f, ω_f) to minimise the tracking error in (7). We choose the following inputs

$$\begin{aligned} v_f &= v_l \cos\bar{\phi} + k_1 z_1 + \rho^d \omega_l \sin(\varphi^d + \bar{\phi}) \\ &\quad - L' \cdot \omega_l \sin\left(\bar{\phi} - \frac{\pi}{2}\right) \\ \omega_f &= \frac{1}{L}\left[v_l \sin\bar{\phi} - k_2 z_2 + L' \cdot \omega_l \cos\left(\bar{\phi} - \frac{\pi}{2}\right)\right] \end{aligned} \tag{8}$$

where k_1, k_2 are the selected positive gains, and v_l, ω_l are treated as exogenous inputs that can be obtained by the follower through communicating with the leader. Now, the problem is that one cannot use distance sensors to measure (z_1, z_2) in the real system, as R_v is a virtual vehicle. In fact, from Fig. 6, we can easily obtain the following equations

$$\begin{aligned} z_1 &= \rho\cos\varphi + L'\cos\left(\bar{\phi} - \frac{\pi}{2}\right) - L \\ z_2 &= \rho\sin\varphi + L'\sin\left(\bar{\phi} - \frac{\pi}{2}\right) \end{aligned} \tag{9}$$

where ρ and φ are the distance and angle between R_f and R_v, which are measurable.

Inserting (9) into (8), we end up with

$$\begin{aligned} v_f &= v_l \cos\bar{\phi} + k_1\left(\rho\cos\varphi + L'\cos\left(\bar{\phi} - \frac{\pi}{2}\right) - L\right) \\ &\quad + \rho^d\omega_l \sin(\varphi^d + \bar{\phi}) - L' \cdot \omega_l \sin\left(\bar{\phi} - \frac{\pi}{2}\right) \\ \omega_f &= \frac{1}{L}\left[v_l \sin\bar{\phi} - k_2\left(\rho\sin\varphi + L'\sin\left(\bar{\phi} - \frac{\pi}{2}\right)\right)\right. \\ &\quad \left. + L' \cdot \omega_l \cos\left(\bar{\phi} - \frac{\pi}{2}\right)\right] \end{aligned} \tag{10}$$

Remark 1: For applications of the proposed controller above, one can easily find that only relative distance and angle between the following vehicle and its leader are of interest; therefore the control algorithm is independent of the global coordinate system.

Remark 2: In our algorithm, we adopt the separation between the follower and its leader, that is, $L = \rho^d \cos\varphi^d$ as the offset, instead of using a fixed value as in Das *et al.* [3] and Lawton *et al.* [8]. Because $-\pi/2 < \varphi^d < \pi/2$, this offset will not equal to zero. Of course, when φ^d approaches $\pm\pi/2$, that is, the vehicles make a line-like formation, the offset may approach zero. In this special circumstance, we should make a fixed offset in order to avoid ill-conditioned control action. In practice, we use the following constraints to solve this problem: if ($\varphi^d < \pi/2$), then $\varphi^d = 89\pi/180$.

In the following, we will discuss the stability properties of the tracking controller under some reasonable assumptions on the motion of the leader and some normal operating conditions.

Assumption 1: 1. The leader's translational velocity is lower bounded and rotational velocity bounded, that is, $v_l > 0$, $\|\omega_l\| < K$ and 2. The initial orientation error $\bar{\phi}(0)$ is bounded away from $\pm\pi$.

Theorem 1: For the error system described in (6), under (1) of Assumption 1, if the tracking controller (8) is applied to the follower, the position errors z_1, z_2 will converge asymptotically to zero and the orientation error $\bar{\phi}$ will be bounded.

Proof: Inserting (8) into (6), we have

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 - z_2\omega_f \\ \dot{z}_2 &= z_1\omega_f - k_2 z_2 \\ \dot{\bar{\phi}} &= -\frac{1}{L}\left[v_l \sin\bar{\phi} - k_2 z_2 + L' \cdot \omega_l \cos\left(\bar{\phi} - \frac{\pi}{2}\right)\right] + \omega_l \end{aligned} \tag{11}$$

For the position error system

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 - z_2\omega_f \\ \dot{z}_2 &= z_1\omega_f - k_2 z_2 \end{aligned} \tag{12}$$

we consider the following Lyapunov function candidate

$$V = \frac{1}{2}(z_1^2 + z_2^2) \tag{13}$$

where $V \geq 0$, and $V = 0$ only when $z_1 = z_2 = 0$. Furthermore, by using (10), we obtain

$$\dot{V} = z_1\dot{z}_1 + z_2\dot{z}_2 = -k_1 z_1^2 - k_2 z_2^2 \tag{14}$$

Clearly, $\dot{V} \leq 0$ and the equality holds if and only if $z_1 = z_2 = 0$. Thus, the position error system is asymptotically stable.

Next, we will prove that the orientation error $\bar{\phi}$ is bounded. We have

$$\begin{aligned} \dot{\bar{\phi}} &= -\frac{1}{L}v_l \sin\bar{\phi} + \frac{1}{L}k_2 z_2 - \frac{L'}{L}\omega_l \cos\left(\bar{\phi} - \frac{\pi}{2}\right) + \omega_l \\ &= -\frac{1}{L}v_l \sin\bar{\phi} + \zeta(z_2, \bar{\phi}) \end{aligned} \tag{15}$$

where $\zeta(z_2, \bar{\phi}) = (1/L)k_2 z_2 - (L'/L)\omega_l \cos(\bar{\phi} - \pi/2) + \omega_l$.

Consider the nominal system

$$\dot{\bar{\phi}} = -\frac{1}{L}v_l \sin\bar{\phi} \tag{16}$$

which is asymptotically stable because $\|\bar{\phi}(0)\| < \pi$ and $v_l > 0$. Furthermore, as $(1/L)k_2 z_2 \to 0$ as $t \to \infty$, $\|\omega_l\| < K$ and thus $\zeta(z_2, \bar{\phi})$ is bounded, using stability theory of perturbed systems, we can obtain that $\bar{\phi}$ is bounded. Until now, we proved that using the proposed tracking controller the leader–follower model is stable, that is, we can regulate two outputs of the vehicles' dynamics (x, y) with a bounded orientation error $\bar{\phi}$. □

Next, we describe how the tracking controller is utilised to implement obstacle avoidance. Let l^d be a security distance to obstacles. If the follower detects an obstacle within the security range, as shown in Fig. 7, it will drive away from the obstruction by turning a small angle. After incorporating this angle with the requirements for formation maintenance, a new desired angle φ'^d for the controller will be calculated using the following formula

$$\varphi'^d = \varphi^d - \frac{\varphi^d - \gamma}{\|\varphi^d - \gamma\|} \arccos\sqrt{1 - \frac{(l^d - l)^2}{2(\rho^d)^2}} \tag{17}$$

where l is the distance between the centre point of the follower's body and the closest point on the obstacle, and γ

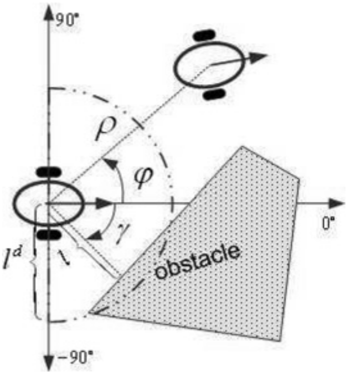

Fig. 7 *Follower negotiates an obstacle*

denotes the relative orientation of the nearest point with respect to the heading direction of the following vehicle. By using such strategy, the follower will avoid collision with the nearest obstacle and simultaneously maintain the desired formation to some extent.

Remark 3: As it is observed, there are limitations in this collision-avoiding strategy. If there are so many obstacles in the environment, the follower may fail to avoid all collisions. However, this strategy can integrate obstacle-avoiding control into formation-keeping control very well and so there is no need to design another special controller for avoiding collision and switching strategy between these controllers.

4 Extension to *n*-vehicle formations

In Section 3.1, we addressed how a two-vehicle pair establishes and keeps desired formations by using two simple controllers. Here, we show that our approach can be easily extended to more general complex formations of n vehicles based on graph theory.

First, let us review some definitions in graph theory [14].

1. Directed graph: a directed graph G is a triplet (V, E, f), where V is a set of vertices or nodes, E is a set of edges and f is a mapping from the edges to the ordered set of vertex pairs: $E \rightarrow V \times V$. The outdegree of a node $v \in V$ denotes the number of edges that have v as an initial vertex, whereas the indegree represents the number of edges that take v as a terminal vertex.
2. Tree (directed): a tree is a directed graph that has no cycles. There is only one node that has indegree 0 and is called root, other nodes of indegree 1 are called leaves.
3. Adjacency matrix: the adjacency matrix $\boldsymbol{A} = (a_{ij})_{n\times n}$ of G is defined by

$$a_{ij} = \begin{cases} 1, & \text{if } v_i v_j \in E \\ 0, & \text{otherwise} \end{cases}$$

Using the above definitions, we can establish corresponding relations between a tree graph to a leader-following formation of vehicles, that is, nodes of tree denote the distributed vehicles, the root stands for the unique lead vehicle, leaves are the other vehicle members and the directed edges represent connections of leader–follower pairs. Specifically, we use an example to illustrate it in detail. Fig. 8*a* shows a typical tree structure of six vehicles establishing a wedge-like formation, where nodes of the tree denote the vehicles and the directed edges indicate the relations in leader–follower pairs. For example, $R_1 \rightarrow R_3$ means R_1 is R_3's leader. The adjacency matrix $\boldsymbol{M}_i$ of this tree is

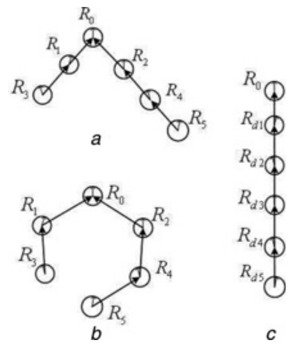

Fig. 8 *Tree structure of six vehicles*

$$\begin{array}{c} \\ R_0 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{array}
\begin{array}{c} \begin{array}{cccccc} R_0 & R_1 & R_2 & R_3 & R_4 & R_5 \end{array} \\
\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

which indicates the interconnection of this wedge-like formation, where each row denotes a follower and each column represents a leader candidate for it. If row i has a non-zero entry in column j, then vehicle i follows vehicle j. As the lead vehicle R_0 does not follow any vehicle, there is no non-zero entry in the first row.

On the basis of the above discussion, we notice that a team of n vehicles can be decomposed into $n-1$ pairs of leader and follower through decomposing a n-node tree into $n-1$ edges. For every edge, that is, every leader–follower pair, a tracking controller designed in Section 3 is responsible for regulating the relative attitudes. As long as the assumptions of Theorem 1 are satisfied in every leader–follower pair, the whole chain of leader-following formation is stable. This construction is familiar to the cyclic pursuit framework [15–17]. The difference is, in cyclic pursuit, one leader has only one follower and vice versa, whereas in our algorithm, one leader may have multiple followers.

In the tracking control of each pair, there are two critical parameters ρ and φ that determine the geometric shape of the two-vehicle subsystem. On the basis of such consideration, we propose another matrix $\boldsymbol{M}_{\mathrm{p}}$, namely parameter matrix, to describe the formation shape. We still use the six vehicles as an example. Assume that the triangle formation is equilateral and the sides are 2 units in length. Then, its parameter matrix $\boldsymbol{M}_{\mathrm{p}}$ can be denoted as

$$\begin{array}{c} \\ R_0 \\ R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{array}
\begin{array}{c} \begin{array}{cccc} D & A & \bar{\phi} & P \end{array} \\
\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & -\dfrac{\pi}{6} & \bar{\phi}_1 & 1 \\ 1 & \dfrac{\pi}{6} & \bar{\phi}_2 & 1 \\ 1 & -\dfrac{\pi}{6} & \bar{\phi}_3 & 1 \\ 1 & \dfrac{\pi}{6} & \bar{\phi}_4 & 1 \\ 1 & \dfrac{\pi}{6} & \bar{\phi}_5 & 1 \end{bmatrix} \end{array}
\qquad \begin{array}{l} D = \text{distance} \\ A = \text{angle} \\ \bar{\phi} = \text{orientation error} \\ P = \text{presence} \end{array}$$

where each row defines the pose of the follower with respect to its leader and P describes the leader's presence in the field of view of the follower. Especially, $P = 0$ means that the follower loses its leader and the connection between them breaks off.

By using these two matrices, we can easily characterise a formation both in structure and in geometric shape. Naturally, two types of switches between different formations can be defined. A switch in which only parameter matrix need to be changed is called a Type A switch and a switch in which both parameter and adjacency matrices change is called a Type B switch. We still use the above six vehicles as an example. Assume that they have established a formation as in Fig. 8*a*, whereas the desired formation is a hexagon, as shown in Fig. 8*b*.

In this case, the adjacency matrix stays the same and only the parameter matrix needs to be changed to the desired one. The type B switch is more complex, for example, if the desired formation is a column as in Fig. 8*c*. In order to implement such switching, both the adjacency and the parameter matrices should be changed. The desired matrices should be

$$\begin{array}{c} \\ R_0 \\ R_{d1} \\ R_{d2} \\ R_{d3} \\ R_{d4} \\ R_{d5} \end{array} \begin{array}{c} \begin{array}{cccccc} R_0 & R_{d1} & R_{d2} & R_{d3} & R_{d4} & R_{d5} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

and

$$\begin{array}{c} \\ R_0 \\ R_{d1} \\ R_{d2} \\ R_{d3} \\ R_{d4} \\ R_{d5} \end{array} \begin{array}{c} \begin{array}{cccc} D & A & \bar{\phi} & P \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ \rho_{d1}^{d} & \varphi_{d1}^{d} & 0 & 1 \\ \rho_{d2}^{d} & \varphi_{d2}^{d} & 0 & 1 \\ \rho_{d3}^{d} & \varphi_{d3}^{d} & 0 & 1 \\ \rho_{d4}^{d} & \varphi_{d4}^{d} & 0 & 1 \\ \rho_{d5}^{d} & \varphi_{d5}^{d} & 0 & 1 \end{bmatrix} \end{array}$$

Note that in this kind of switching, the leader–follower relations have totally changed and should be re-assigned. Thus, each vehicle (except the lead vehicle) is required to find and follow a new leader. We assume that this re-assignment is performed by a high-level coordinator that is not considered here. Here, we also assume that there is no failure among the group of vehicles. In fact, under this structure, if one leader malfunctions, we can select one of its followers as a successor and the whole formation can be maintained to some extent.

5 Simulations and experimental results

In this section, we provide extensive experiments, both simulation and with real vehicles, to illustrate the effectiveness of our approach.

5.1 Simulations

The simulation experiments were done using a Player/Stage platform [18–20]. We conducted a series of experiments to demonstrate the stability, robustness and agility of the proposed algorithms. The first set of experiments

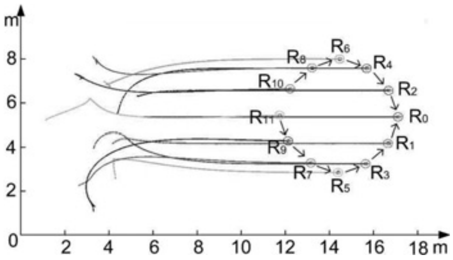

Fig. 9 *Twelve vehicles establish a circle*

were conducted to demonstrate the stability of the tracking controller. Fig. 9 shows that a team of 12 vehicles started from initial points with arbitrary attitudes and aimed to establish a circle formation. The expected diameter was 5.5 m. During moving, R_0 was selected as the lead vehicle and moved at a constant speed 1 m/s. The 11 leader–follower pair connections are indicated using arrows. It was noted that the formation was performed and maintained after about 5 s.

In Fig. 10, six vehicles performed a triangular formation (the side is 1.5 m) and the lead vehicle R_0 had a fixed translation velocity (0.5 m/s) and rotational velocity (0.03 rad/s). The trajectories in these figures illustrated that the tracking behaviours were quite smooth in all of the leader–follower pairs. The two matrices for this formation at 10 s are

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{3}{4} & -\frac{\pi}{6} & 0.001 & 1 \\ \frac{3}{4} & \frac{\pi}{6} & 0.003 & 1 \\ \frac{3}{4} & -\frac{\pi}{6} & 0.001 & 1 \\ \frac{3}{4} & \frac{\pi}{6} & 0.002 & 1 \\ \frac{3}{4} & \frac{\pi}{6} & 0.004 & 1 \end{bmatrix}$$

Fig. 11 shows the experimental results of a five-vehicle team exploring in a cave-like scenario. Initially, these vehicles established a wedge-shape formation and drove forward. The lead vehicle navigated and planned the heading direction for the whole group with the followers tracking their designated leaders, respectively. This experiment demonstrated that the vehicles could avoid collisions with obstacles and simultaneously maintain the desired geometric formation to some extent. It should be mentioned that if the available passage is very narrow, the tracking controller may fail to avoid all obstacles, and under this circumstance, transformation should be adopted as a solution.

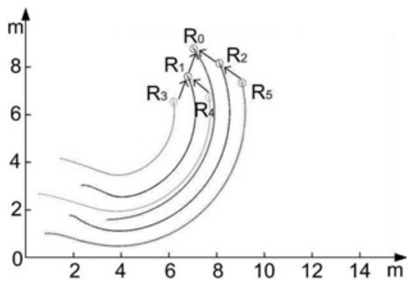

Fig. 10 *Leader has a rotational velocity*

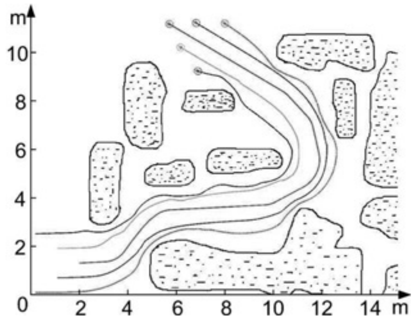

Fig. 11 *Snapshot of five vehicles in a wedge formation exploring in a cave-like environment*

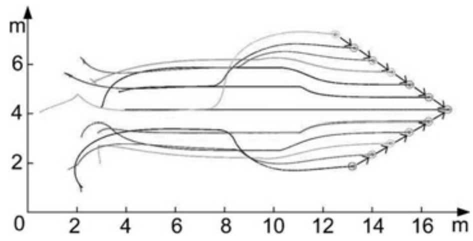

Fig. 12 *Switch from a circle to a wedge-like formation*

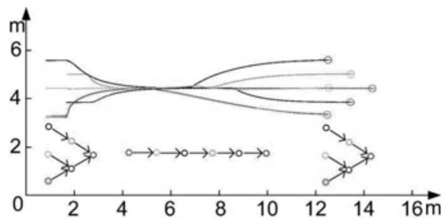

Fig. 13 *Five vehicles switch from a wedge to a column and then to a wedge*

Figs. 12 and 13 depict the switching results between different geometric formations. In Fig. 12, a group of 12 vehicles was required to establish a circle and then switch to a wedge-like formation. In this switching, each vehicle followed the same leader and only the relative distance and angle were changed (Type A switch). Fig. 13 shows trajectories of another simulation run where five vehicles switched from a wedge to a column and back again to a wedge. These two types of switches are both categorised as Type B. The change of leader–follower connections is illustrated using arrows.

5.2 Physical vehicles experiments

In physical experiments, we used three Pioneer 3DX vehicles each equipped with laser scanner, sonars and a pan-tilt-zoom camera as shown in Fig. 14*a*. We established a column with an individual colour on the top of every vehicle as an ID indicator. Below the ID column was an octahedron crown with a different colour. This crown was used to provide orientation information that was necessary for the tracking controller. The camera identified its leader and obtained the orientation deflection between its leader and itself. The laser detected the distance between the vehicle and its leader or surrounding obstacles. Fig. 14*b* illustrates the orientation measurement. Because the crown was octahedral and every side had an individual colour, when other vehicles viewed it from different angles, they could identify the orientation by calculating the

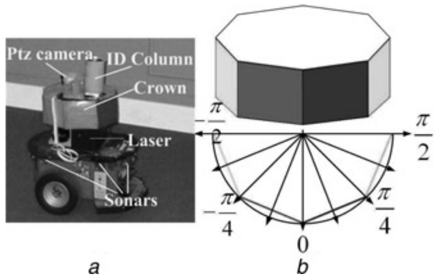

Fig. 14 *Pioneer 3DX vehicle and orientation detection*

a Pioneer vehicle with laser, camera, sonars, ID column and colourful crown
b Orientation detection using the colourful crown

proportion of areas of different sides, that is, different colours. Combining this orientation information and the pan angle of its camera, every follower vehicle could get φ and $\bar{\phi}$, which were necessary to track its leader vehicle.

One set of experiments was conducted to examine the stability and switching ability. The three vehicles, starting from initial random attitudes, aimed to converge to a triangle formation. Initially, R_0 was designated as the lead vehicle with R_1 and R_2 following R_0. They first performed a triangle formation. Then, after maintaining triangle formation for several seconds, the three vehicles were ordered to switch to a column formation. Note that this kind of transformation belongs to a Type B switch and that the leader–follower connections will change. R_2 broke the relation with R_0 and took R_1 as the new leader. As the view field of the pan-tilt-zoom camera was limited, R_2 first turned to search for its new leader R_1. After finding R_1, R_2 followed its new leader through the tracking controller. Fig. 15 shows the scenarios of the experiment.

In the following set of experiments, we demonstrated the obstacle avoidance ability of the vehicles. First, the three vehicles established a triangle formation and moved forwards. An obstacle was placed in the path of the formation and blocked R_2. When R_2 detected the obstacle within its security range, it regulated its advancing direction and avoided collision successfully. Fig. 16 shows how R_2 negotiates the obstacle.

For a full report of the results, please visit our website http://www.mech.pku.edu.cn/~wanglong/mrf.html to view more results. They have not been presented here due to space limitations.

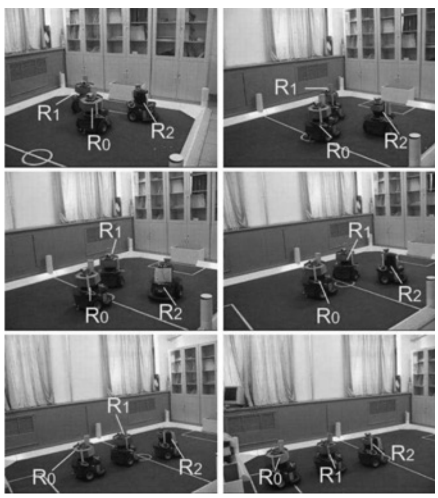

Fig. 15 *Three Pioneer vehicles switch from a triangle to a column (order is left to right and top to bottom)*

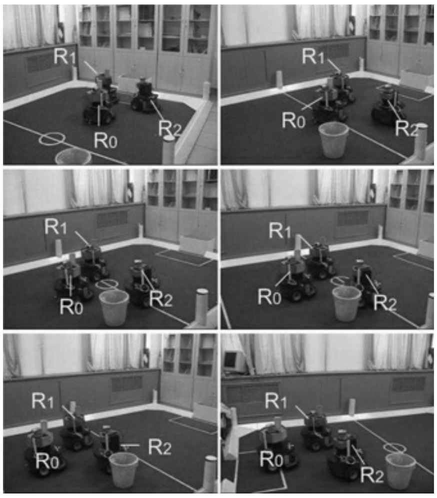

Fig. 16 *Follower avoids an obstacle while keeping to a triangular formation (order is left to right and top to bottom)*

6 Conclusions and future work

In this paper, we proposed a new method to investigate formation control of multiple mobile vehicles based on the leader–follower approach. A reactive tracking controller was proposed to make each following vehicle maintain a desired position to its leader. After introducing such controllers, the global-level formation behaviour of n vehicles was achieved through decentralised tracking control in a chain of $n - 1$ leaders and followers. Simulation and experimental results demonstrated the effectiveness of our approach.

7 Acknowledgments

The authors are grateful to Prof. Mario Garcia-Sanz and the anonymous reviewers for their valuable suggestions that have improved the presentation of this paper. This work was supported by NSFC (10372002 and 60528007) and National 973 Program (2002CB312200).

8 References

1 Balch, T., and Arkin, R.C.: 'Behavior-based formation control for multirobot team', *IEEE Trans. Robot. Autom.*, 1998, **14**, (6), pp. 926–939

2 Fredslund, J., and Mataric, M.J.: 'A general algorithm for robot formations using local sensing and minimal communication', *IEEE Trans. Robot. Autom.*, 2002, **18**, (5), pp. 837–846

3 Das, A.K., Fierro, R., Kumar, V., Ostrowski, J.P., Spletzer, J., and Taylor, C.J.: 'A vision-based formation control framework', *IEEE Trans. Robot. Autom.*, 2002, **18**, (5), pp. 813–825

4 Bicho, E., and Monteiro, S.: 'Formation control for multiple mobile robots: a non-linear attractor dynamics approach'. Proc. IEEE Int. Conf. on Intelligent Robots and Systems, Las Vegas, Nevada, October 2003, pp. 2016–2022

5 Chio, T., and Tarn, T.: 'Rules and control strategies of multi-robot team moving in hierarchical formation'. Proc. IEEE Int. Conf. on Robotics and Automation, Taiwan, September 2003, pp. 2701–2706

6 Chen, X., Serrani, A., and Ozboy, H.: 'Control of leader-follower formations of terrestrial UAVs'. Proc. IEEE Conf. on Decision and Control, Maui, Hawaii, USA, December 2003, pp. 498–503

7 Michaud, F., Letourneau, D., Guilbert, M., and Valin, J.: 'Dynamic robot formations using directional visual perception'. Proc. IEEE Int. Conf. on Intelligent Robots and Systems, Switzerland, October 2002, Sponsored by EPFL, pp. 2740–2745

8 Lawton, J., Beard, R., and Young, B.: 'A decentralized approach to formation maneuvers', *IEEE Trans. Robot. Autom.*, 2003, **19**, (6), pp. 933–941

9 Ögren, P., Egerstedt, M., and Hu, X.: 'A control Lyapunov function approach to multi-agent coordination', *IEEE Trans. Robot. Autom.*, 2002, **18**, (5), pp. 847–851

10 Brockett, R.W.: 'Asymptotic stability and feedback stabilization' in Brockett, R.W., Millman, R.S., and Sussmann, H.J. (eds): 'Differential geometric control theory' (Birkhauser, Boston, 1983), pp. 181–191

11 Brockett, R.W.: 'Characteristic phenomena and model problems in nonlinear control'. Proc. IFAC Congress, 1996, pp. 135–140

12 Astolfi, A.: 'Exponential stabilization of a wheeled mobile robot via discontinous control'. Nonlinear Control System Design Symposium, 1995, pp. 741–746

13 Kolmanovsky, I., and McClamroch, N.H.: 'Hybrid feedback laws for a class of cascade nonlinear control systems', *IEEE Trans. Autom. Control*, 1996, **41**, (9), pp. 1271–1282

14 Diestel, R.: 'Graph theory' (Springer-Verlag, New York, 1997)

15 Marshall, J.A., Broucke, M.E., and Francis, B.A.: 'Formation of vehicles in cyclic pursuit', *IEEE Trans. Autom. Control*, 2004, **49**, (11), pp. 1963–1974

16 Richardson, T.: 'Stable polygons of cyclic pursuit', *Ann. Math. Artif. Intell.*, 2001, **31**, pp. 147–172

17 Sinha, A., and Ghose, D.: 'Generalization of the cyclic pursuit problem'. Proc. Amer. Control Conf., Portland, OR, USA, 2005, pp. 4997–5002

18 Gerkey, B.P., Vaughan, R.T., and Howard, A.: 'The Player/Stage project: tools for multi-robot and distributed sensor systems'. Proc. Int. Conf. on Advanced Robotics, Coimbra, Portugal, 2003, pp. 317–323

19 Gerkey, B.P., and Matarić, M.: 'Most valuable player: a robot device server for distributed control'. Proc. IEEE Conf. on Intelligent Robots and Systems, 2001, pp. 1226–1231

20 Vaughan, R.T., Howard, A., and Gerkey, B.P.: 'Player/Stage user manuals, Player/Stage project', http://playerstage.sourceforge.net, 2004