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Path Planning for a Formation of Autonomous Robots in an Unknown Environment Using Artificial Force Fields

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***Abstract*—A novel approach to path planning for the formation of a swarm of autonomous robots in an unknown environment is presented in this paper. This approach is designed based on the traditional potential fields combined with the rotational vector field. Under the action of this blended vector field, autonomous robots can easily avoid obstacles in order to reach the target position. Furthermore, in this approach, the neighboring robots in a swarm will be linked to each other by the attractive and the repulsive vector field between them in order to generate a stable formation. Information about obstacles, which each member robot detects from the environment, will be sent to all other member robots in the swarm. Hence, while these autonomous robots move towards the target position, the formation of them is maintained and there are no collisions between them. Simulation results and experiments have verified the success of this approach.**

***Keywords*—Swarm intelligence; obstacle avoidance; vector fields; mobile sensor network**

I. INTRODUCTION

In recent years, the mobile robot network [11]-[16] has been an interesting research topic in the control community all over the world. In many practical areas, such as rescue and search missions, and forest fire detection, etc., the applications of the autonomous robot network play an important role.

One of the simple and effective methods to control the trajectory of autonomous robots towards the target position is the artificial vector field, which considers the relative positions between the target, robots, and obstacles in the environment, see [1]-[9]. In this method, a repulsive vector field is generated around each obstacle to drive robots away from this obstacle and an attractive vector field is created around the target to attract these autonomous robots to the target position. The actual path of these robots is determined by the resultant of the presented vector fields. In recent years, this method has been widely researched and been applied powerfully to flocking control of multi-agents to follow the trajectory of the target in a simple environment, see [13]-[18]. The obstacle avoidance of a swarm is successful but while avoiding obstacles the formation of the swarm is broken. Although artificial potential field is known as a positive method to path planning for autonomous robots, in several cases of local minima this method is limited, such as, when the attractive force from the target and the repulsive force from the obstacles are equal and collinear but opposite direction, the total force on robot is zero and the robot's motion is stopped. Moreover, in a complex environment, such as, in that there are U-shaped obstacles or long walls, the application of the potential field method in order to control autonomous robots during tracking is very difficult. Robots can be trapped in these complex obstacles before reaching the target, see [2], [5]. This is an interesting topic that attracts the attention from researchers in recent years. Some research results around this topic are extended from potential field method, such as: Using a virtual obstacle presented in [7], using a virtual goal presented in [4], [5], [6]. However, these research results are achieved exclusively in a predetermined environment.

In this paper, a novel approach to path planning for formation of a swarm of multi-robots in an unknown environment is proposed. In this approach, an attractive vector field is built from the target to drive all member robots towards the target position. Around each obstacle two vector fields are built, which consist of a repulsive and a rotational vector field. The repulsive vector field, which is stronger when the robot is closer to obstacle, is used to drive robots to avoid the collision with this obstacle. In other words, this vector field always repels robots away from obstacles. The rotational vector field is added to solve the local minimum problems, which constraints the potential field method. Under the influence of this added rotational vector field, the robot always escapes the trapping point at which the attractive force of the target and the repulsive force of the obstacles are balanced. Especially, when robots are trapped in complex obstacle then the added rotational vector field will help them to find a new path to escape these obstacles. However, in order that robots can quickly exit the complex obstacles the rotational force must be computed larger than the sum of the attractive force and the repulsive force on these robots. In addition, in order to avoid collisions and maintain the stability in the formation of a swarm, the neighboring robots will be connected to each other by the attractive and the repulsive vector field between them. The obstacle information, which each member robot obtains from the environment, will be sent to all other members of the swarm.

The rest of this paper is organized as follows: The problem formulation is presented in the section II. In next Section, the background of the traditional potential field method is introduced. Section IV presents the path planning algorithm for the formation of a swarm of autonomous robots in an unknown environment. The simulation and experimental results for the path planning algorithm are shown in section V. Finally, the summary and outlook for this paper is presented in section IV.

I. PROBLEM FORMULATION

Consider a swarm of N robots ($N \geq 2$) in a two-dimensional Euclidean space $\{R^2\}$. Assume that each robot is a moving point in this space, and it has mathematical model as follows:

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = u_i \end{cases} \qquad i = 1, \ldots N. \tag{1}$$

In this equation, $p_i = (x_i, y_i)^T, v_i = (v_{ix}, v_{iy})^T$ and u_i are the position vector, velocity vector and control input of the robot i, respectively.

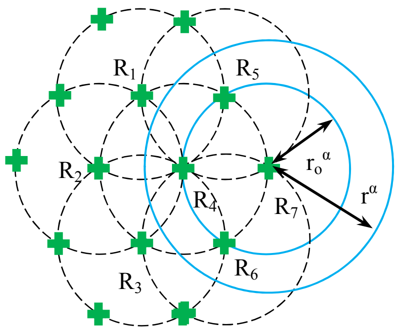

Fig.1 The description of the formation of a swarm.

In the formation of a desired swarm, the neighboring robots have to link each other in order to generate the constant distances among them (example in Fig.1). Let N_i^α be the set of the robots in the neighborhood of robot i at time (t), such that:

$$N_i^\alpha(t) = \left\{ \forall j : d_i^j \leq r^\alpha, j \in \{1,..N\}, j \neq i \right\}. \tag{2}$$

Here, $r^\alpha > 0$ is a radius of neighboring circle (shown in Fig.1) and $d_i^j = \left\| p_i - p_j \right\|$ is the Euclidean distance between robot i and robot j. For example, in Fig.1, the robots R_4, R_5, R_6 are the neighbors of robot R_7.

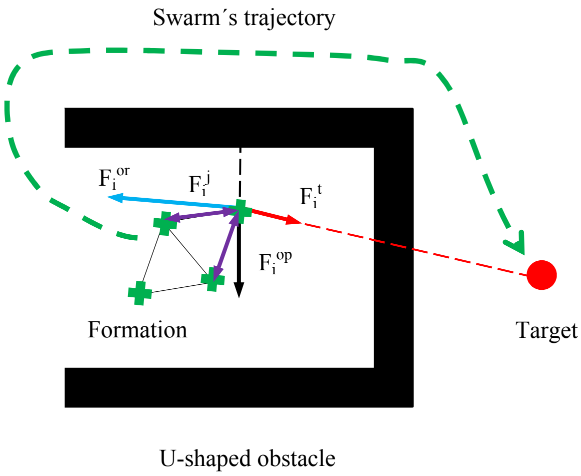

Fig.2 The description of path planning for a swarm of four robots in an environment with a U-shaped obstacle.

The path planning for a swarm of multi-robots is shown in Figure 2. This swarm must overcome the U-shaped obstacle in order to continue to move towards the target. Furthermore, while moving the swarm's formation must be maintained. Therefore, in order to perform this idea each robot will be controlled by a total force. This total force consists of the repulsive force F_i^{op}, rotational force F_i^{or} of obstacles, the attractive force F_i^t of the target, the connecting force between this robot with its neighbors F_i^j and the obstacle avoidance forces of other member robots send to this robot.

II. BACKGROUND

The artificial potential field is known in control technology as an effective method for robot's path planning. In this potential field, the attractive force field to the target is combined with the repulsive force fields around obstacles, see [1]-[6]. These vector fields are created by the negative gradient of the respective potential functions, see [2].

A. Attractive vector field

As presented in [2] and [3], the attractive vector field has a respective potential function as:

$$V = \frac{1}{2} k \, (p_i - p_t)^T (p_i - p_t) \, . \tag{3}$$

In equation (3), (p_i - p_t) is a relative position vector between robot and target, and k is a positive constant. Using the negative gradient of the potential function (3), the attractive force field, which is depicted in Fig.3a, is given as follows:

$$F = -\nabla V(p) = -k \, (p_i - p_t) \, . \tag{4}$$

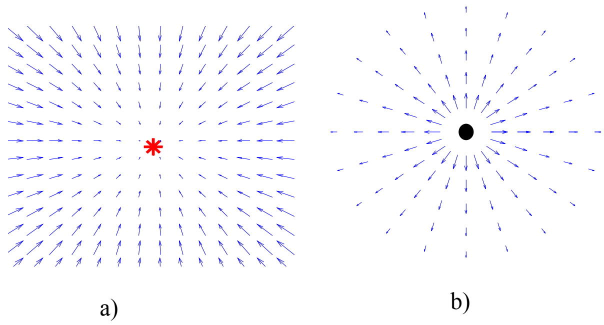

Fig.3 The description of attractive force field to the target (a) and the repulsive force field from the obstacle (b).

B. Repulsive vector field

The repulsive vector field in [2], which is used to drive robot away from obstacles while reaching the target position, is described as follows:

$$F^o = \begin{cases} k^o \left(\dfrac{1}{d} - \dfrac{1}{r^\beta} \right) \dfrac{1}{d^3} (p_i - p_o), \; 0 < d \leq r^\beta \\ 0 \qquad , \qquad otherwise. \end{cases} \tag{5}$$

Here, $(p_i, p_o) \in \{R^2\}$, d, r^β and k^o are the robot's position, the obstacle's position, the obstacle detection radius, the distance from robot to obstacle, and positive factor, respectively. This vector field is depicted in Fig.3b, and it also has a respective potential function as follows:

$$V^o = \begin{cases} \frac{1}{2}k^o\left(\frac{1}{d}-\frac{1}{r^\beta}\right)^2 & , \ 0<d\le r^\beta \\ 0 & , \ otherwise. \end{cases} \quad (6)$$

Finally, the control law for path planning of a robot towards the target in a dynamic environment, at which there are M obstacles, is given as follows:

$$u_i = F + \sum_{o=1}^{M} F^o. \quad (7)$$

III. Path Planning Algorithm

This section presents the formation control algorithm for a swarm of N robots, which passes through M obstacles to reach a target. The member robots have to escape obstacles in the environment to reach the target while staying together. Accordingly, the control algorithm for each robot i is proposed as follows:

$$u_i = f_i^t + f_i^o + f_i^j. \quad (8)$$

A. Target-reaching control algorithm

In order to control the each robot as it moves towards the target, the first component in equation (8) is built as follows:

$$f_i^t = F_i^t(p_i) - k_{iv}^t(v_i - v_t). \quad (9)$$

In this equation, the relative velocity vector ($v_i - v_t$) between the robot i and the target is used as a damping term and k_{iv}^t is the positive factor. The force field $F_i^t(p_i)$ is generated from the target in order to drive robots always move towards the target position until they reaches this target. This attractive force field is designed as follows:

$$F_i^t(p_i) = \begin{cases} -\frac{k_{ip}^t}{r^\tau}(p_i - p_t), & if \ d_i^t < r^\tau \\ -k_{ip}^t\frac{(p_i - p_t)}{\|p_i - p_t\|}, & otherwise. \end{cases} \quad (10)$$

Here, the positive constant r^τ is the range around the target, at which the robot's speed is reduced before reaching the target, and $(p_i - p_t)$ is the relative position vector between the robot i and the target. The magnitude of this force is depended on the control factor k_{ip}^t and the distance $d_i^t = \|p_i - p_t\|$ between robot i and the target.

B. Obstacle-avoiding control algorithm

The component f_i^o of equation (8) is used to control the obstacle avoidance for each member robot of the swarm. This component is projected for each robot i as:

$$f_i^o = \sum_{k=1}^{N}\sum_{o=1}^{M}\left(F_k^{op}(p_k) + F_k^{or}(p_k) + k_{kv}^o c_k^o (v_k - v_o)\right). \quad (11)$$

Here, v_k, v_o, and k_{kv}^o are the velocity of the robot k ($k=1,2...N$), the velocity of the obstacle *(o)* and the positive scaling factor, respectively. Similar to equation (2), the set of β neighboring obstacles of the robot k at time (t) is also defined as:

$$N_k^\beta(t) = \left\{\forall o : d_k^o \le r^\beta, o \in \{1,..M\}, o \ne j\right\}. \quad (12)$$

Here r^β and $d_k^o = \|p_k - p_o\|$ are the obstacle detection range and the Euclidean distance, respectively. The scalar c_k^o, which is used to determine if an obstacle *(o)* is a neighboring obstacle of robot k, is defined as:

$$c_k^o = \begin{cases} 1 & if \ o \in N_k^\beta(t) \\ 0 & if \ o \notin N_k^\beta(t). \end{cases} \quad (13)$$

The repulsive force field $F_k^{op}(p_k)$ around each obstacle (o) is used to repel the robot k away from this obstacle. Similar to equation (5) this force field is designed as:

$$F_k^{op}(p_k) = c_k^o\left(\left(\frac{1}{d_k^o} - \frac{1}{r^\beta}\right)\frac{k_{kp}^{op}}{(d_k^o)^2} - k_{kp}^{o\delta}\left(d_k^o - r^\beta\right)\right)n_k^o. \quad (14)$$

In equation (14), k_{kp}^{op}, $k_{kp}^{o\delta}$ are the positive constants, and $n_k^o = (p_k - p_o)/\|p_k - p_o\|$ is a unit vector, which has direction from the obstacle to robot k.

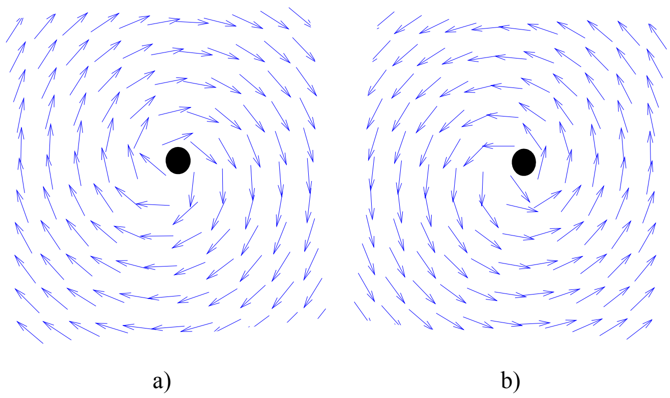

Fig.4 The clockwise rotational force field (a) and the counter-clockwise rotational force field (b).

The rotational force field $F_k^{or}(p_k)$ is also generated around the obstacle (o) to help the repulsive force to quickly drive robot k to escape this obstacle. The rotational direction of this force field can be clockwise rotation (see Fig.4a) or counter-clockwise rotation (see Fig.4b). This rotational force is built as follows:

$$F_k^{or}(p_k) = w_k^o c_k^o n_k^{or}. \quad (15)$$

Here, the unit vector n_k^{or} is given as:

$$n_k^{or} = c_k^{or}\left(\frac{(\mathrm{y}_k - \mathrm{y}_o)}{d_k^o} \quad \frac{-(\mathrm{x}_k - \mathrm{x}_o)}{d_k^o}\right)^T . \qquad (16)$$

In equation (16) the scalar c_k^{or} is used to define the rotational direction for this rotational force, it is described as

$$c_k^{or} = \begin{cases} 1\,, F_k^{or}(p_k) \textit{ is clockwise rotation} \\ -1,\ F_k^{or}(p_k) \textit{ is counter} - \textit{clockwise rotation}\ . \end{cases} \qquad (17)$$

The positive gain factor w_k^o is used as a control element to control robots to quickly escape obstacles. However, their velocity does not overcome the limited velocity. Therefore, this control element is designed such that the total force on the robots always has the direction in the selected rotational direction. Hence, this control element w_k^o is built as follows:

$$w_k^o = (1+c)\left(\left\|F_k^{top}(\mathrm{p}_k)\right\|\right). \qquad (18)$$

Here, the total force $F_k^{top}(\mathrm{p}_k)$ on the robot k is described as:

$$F_k^{top}(p_k) = F_k^t(p_k) + F_k^{op}(p_k)\,. \qquad (19)$$

The constant c, which depends on the angle α between the vector $F_k^{top}(p_k)$ and the unit vector n_k^{or}, is described as:

$$c = \begin{cases} c_1\ , & \textit{if } 0 \le \alpha < \pi/2 \\ c_2\ , & \textit{otherwise,} \end{cases} \qquad (20)$$

where two constants c_1 and c_2 can be chosen as follows $-1 < c_1$, $o < c_2$ and $c_1 < c_2$.

Equation (11) shows that each member robot will obtain the environmental information from other members in the swarm. While avoiding obstacles all member robots will be driven by the same force. Therefore, the formation of these robots is always maintained.

C. Swarm-connection control algorithm

As shown in [10], the connection between neighboring robots in the formation of a swarm is controlled by the repulsive/attractive forces between them. Furthermore, in order to obtain the quick stability at the balance point, at which the distance between the neighboring robots is kept constant, the relative velocity vector ($v_i - v_j$) between them is added with a positive control factor k_{iv}^j. Therefore, the control component f_i^j of equation (8) is implemented as follows:

$$f_i^j = \sum_{j=1, j\neq i}^{N}\left(F_i^j(p_i) - k_{iv}^j c_i^j (v_i - v_j)\right). \qquad (21)$$

In this equation, the scalar c_i^j is used to determine if the robot j is a neighbor of robot i at time (t). It is defined as:

$$c_i^j = \begin{cases} 1 & \textit{if } j \in N_i^\alpha(t) \\ 0 & \textit{if } j \notin N_i^\alpha(\mathrm{t}). \end{cases} \qquad (22)$$

To create the attractive/repulsive force field $F_i^j(p_i)$ between neighboring robots (robot i and its neighbor j), a respective potential function of this vector field is proposed as:

$$V_i^j(\mathrm{p}_i) = \frac{c_i^j}{2}\left(\left(\frac{k_{ip}^{1j}}{d_i^j} - k_d\right)^2 + k_{ip}^{2j}\left(d_i^j - r_0^\alpha\right)^2\right). \qquad (23)$$

Taking the negative gradient (see [1]-[3]) of the potential function (23) at p_i, we receive the repulsive/attractive force field as follows:

$$F_i^j(\mathrm{p}_i) = c_i^j\left(\left(\frac{k_{ip}^{1j}}{d_i^j} + k_d\right)\frac{k_{ip}^{1j}}{(d_i^j)^2} - k_{ip}^{2j}\left(d_i^j - r_0^\alpha\right)\right)n_i^j . \qquad (24)$$

This force field is depicted in Fig.5. In the equation (24), the unit vector $n_i^j = (p_i - p_j)/\|p_i - p_j\|$ has direction along the line connecting from p_j to p_i, d_i^j is the Euclidean distance shown in equation (2). The fast stability at the desired distance r_0^α (see Fig.1), at which the repulsive force is equal to the attractive force, is regulated by the positive constants ($k_{ip}^{1j}, k_{ip}^{2j},$). The positive factor k_d is used as a control element to control the balance position between the attraction and the repulsion.

By equating $\left(\left(k_{ip}^{1j}/d_i^j + k_d\right)k_{ip}^{1j}/(d_i^j)^2 - k_{ip}^{2j}\left(d_i^j - r_0^\alpha\right)\right) = 0$, one can find a value $d_i^j = r_1^\alpha$ at which the repulsive/attractive force is zero. In other words, if there is a given value $r_1^\alpha \ge r_0^\alpha$ and the line $-k_{ip}^{2j}\left(d_i^j - r_0^\alpha\right)$ is not changed, then the control element k_d is determined as a function of the r_1^α. This function is described as

$$k_d = \frac{k_{ip}^{2j}\left(r_1^\alpha - r_0^\alpha\right)\left(r_0^\alpha\right)^2}{k_{ip}^{1j}} - \frac{k_{ip}^{1j}}{r_1^\alpha}\ . \qquad (25)$$

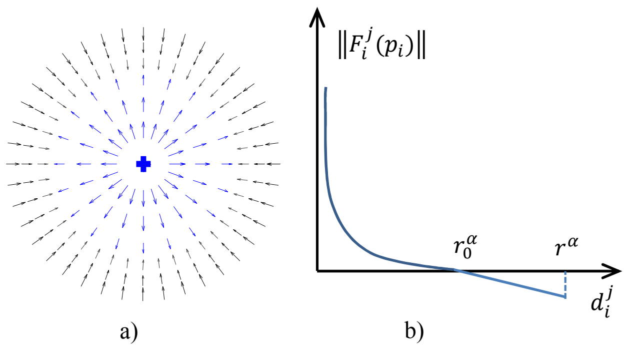

Fig.5 The repulsive/attractive force field (a) and its amplitude (b).

The effect of the repulsive/attractive force $F_i^j(\mathrm{p}_i)$ of the robot j (j is neighbor of i) on the robot i is depended on the relative position between robot j and robot i. Fig.5 shows that under the effect of the repulsive/attractive force the neighboring robots will quickly reach the equilibrium position $(d_i^j = r_0^\alpha)$, at which the force $F_i^j(\mathrm{p_i})$ is equal to zero.

IV. RESULTS

In this section, the path planning algorithm for a formation of a swarm of autonomous robots to reach a stationary target in an unknown environment will be tested in simulations. The general parameters, which are used to simulate, are listed in below table.

TABLE I. PARAMETER VALUES

Parameter	Definition	Value
r_1^α	Minimum desired distance for neighbors	10
r_0^α	Desired distance for neighbors	20
k_{iv}^j	Damping factor for approach to balance point	1.3
k_{ip}^{1j}	Scaling factors for fast connection between neighbors	50
k_{ip}^{2j}		1.2
r^β	Obstacle detecting range	30
k_{ip}^{op}	Constants for fast obstacle avoidance	70
$k_{ip}^{o\delta}$		1.4
r^τ	Distance of approach to target position	50
k_{ip}^t	Constant for fast approach to target position	2.8
k_{iv}^t	Damping factor for approach to target	0.9
c_1	Constant	0.7
c_2	Constant	1.8

Firstly, the path planning algorithm for the formation of four robots is tested. This swarm must avoid obstacles in the complex environment to reach the target. In this simulation case, we use the clockwise rotational force to control the robots to escape obstacles. The target is assumed as a stationary point with the position $p_t = (280,180)^T$, and the obstacles are linked as U-shaped. The initial positions of the obstacles and the robots for simulations are chosen as follows: $\mathrm{p_{o1}}=(200, 150)^\mathrm{T}$, $\mathrm{p_{o2}}=(200, 180)^\mathrm{T}$, $\mathrm{p_{o3}}=(200, 210)^\mathrm{T}$, $\mathrm{p_{o4}}=(200, 400)^\mathrm{T}$, $\mathrm{p_{o5}}=(180, 135)^\mathrm{T}$, $\mathrm{p_{o6}}=(160, 135)^\mathrm{T}$, $\mathrm{p_{o7}}=(140, 135)^\mathrm{T}$, $\mathrm{p_{o8}}=(180, 255)^\mathrm{T}$, $\mathrm{p_{o9}}=(180, 255)^\mathrm{T}$, $\mathrm{p_{o10}}=(160, 255)^\mathrm{T}$, $\mathrm{p_1}=(20, 180)^\mathrm{T}$, $\mathrm{p_2}=(30, 230)^\mathrm{T}$, $\mathrm{p_3}=(40, 170)^\mathrm{T}$, $\mathrm{p_4}=(10, 210)^\mathrm{T}$.

The simulation results in Fig.6 show that the preservation of the formation of four robots is successful while this swarm moves towards the target. All robots have random initial position, and between them there are no the connections to each other. At time t= 80s these free robots are begun to link to each other in order to generate a desired formation with the constant distances between them, see Fig.6 and Fig.7. Then, this formation is kept and while moving towards the target position until it meets the obstacles. When the swarm detects the obstacles it changes its motion direction to avoid the collision with these obstacles and searches the new path towards the target. The robots follow the clockwise rotational direction around the obstacles to find a path to deviate these obstacles. Fig.6 shows that the obstacle avoidance of the swarm is successful. The robots can easily escape the U-shaped obstacle without breaking the formation. The distance between the neighboring robots in the swarm is kept constant during the simulation, see Fig.7. After the robots overcome the obstacles they further move to the target and they reach this target position at t=270s, see Fig.6.

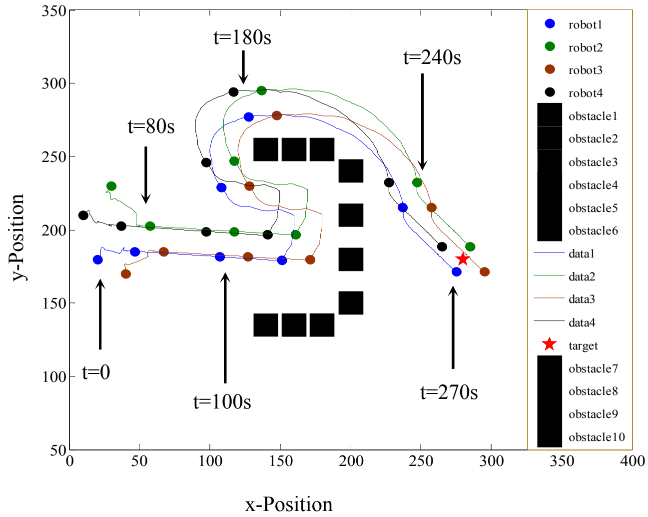

Fig.6 Path planning for a swarm of four robots using clockwise rotational vector field combined with the potential field.

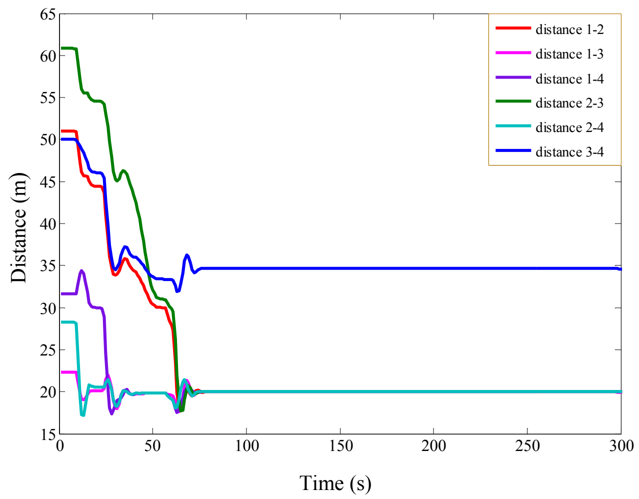

Fig.7 The stable distance between members in formation of four robots at time t in an environment with U-shaped obstacle.

Secondly, the path planning algorithm for formation of a swarm of four robots using the counter-clockwise rotational force field combined with the traditional potential field was tested. Similar to the first simulation case, the target position is also assumed as a stationary point in the space $p_t = (300,300)^T$. The obstacles are connected to generate a complex wall that hinders the robots go towards the target. The position of obstacles and robots are selected at initial time for this simulation case as follows: $\mathrm{p_{o1}}=(230, 240)^\mathrm{T}$, $\mathrm{p_{o2}}=(230, 270)^\mathrm{T}$, $\mathrm{p_{o3}}=(230, 300)^\mathrm{T}$, $\mathrm{p_{o4}}=(230, 330)^\mathrm{T}$, $\mathrm{p_{o5}}=(140, 240)^\mathrm{T}$, $\mathrm{p_{o6}}=(170, 240)^\mathrm{T}$, $\mathrm{p_{o7}}=(200, 240)^\mathrm{T}$, $\mathrm{p_{o8}}=(140, 150)^\mathrm{T}$, $\mathrm{p_{o9}}=(140, 180)^\mathrm{T}$, $\mathrm{p_{o10}}=(140, 210)^\mathrm{T}$, $\mathrm{p_1}=(10, 340)^\mathrm{T}$, $\mathrm{p_2}=(30, 290)^\mathrm{T}$, $\mathrm{p_3}=(40, 320)^\mathrm{T}$, $\mathrm{p_4}=(60, 360)^\mathrm{T}$.

The simulation result depicted in Fig.8 shows that the complex wall-shaped obstacle escape of a swarm of four robots is successfully achieved. The robots can quickly exit this wall-shaped obstacle in order to move towards the target. The motion direction of these robots is driven towards the left of this wall-shaped obstacle (counter-clockwise direction) at time t=120s, see Fig.8. This direction change helps the robots to avoid the collision with obstacles and they can find a path to move to the target position, without influencing the connection between robots of the swarm. The formation of the robots is not broken. In other words, the constant distance between the neighboring robots in the swarm is kept, as shown in Fig.8.

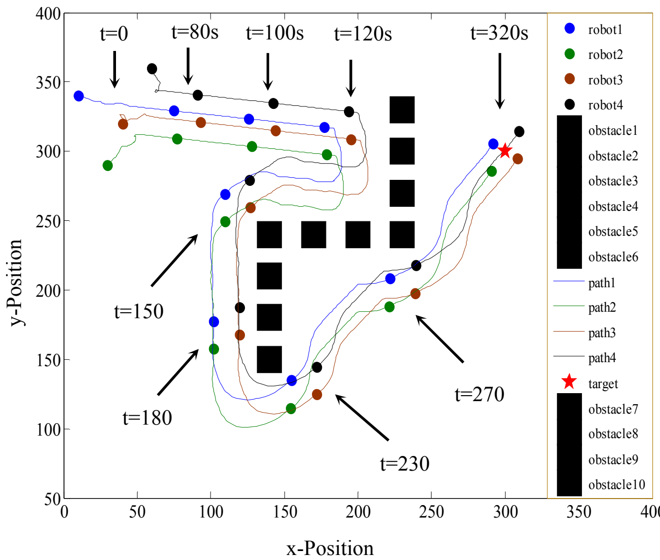

Fig.8 Path planning for a swarm of four robots using counter-clockwise rotational vector field combined with the potential field.

V. CONCLUSION

In this paper, we have proposed a novel approach to path planning for the formation of a swarm of autonomous robots to reach a target in an unknown environment. This approach is built based on the rotational vector field combined with the traditional potential fields. The rotational force field is added around obstacles to help robots to quickly escape obstacles. Especially, in case the robots are trapped in complex obstacles, such as U-shaped obstacles, this rotational vector field plays an important role in helping the robots to find a new path to escape this environment. The results of the simulations have shown that under the effect of this blended force field a swarm of robots can easily find a path to move towards the target in an unknown environment. Using the added rotational force field, the obstacle avoidance of this swarm is successfully achieved. Moreover, the robots in a swarm are connected to each other and they obtain information about the obstacles of the environment from other member robots. Thus, the formation of the swarm is maintained while reaching a target. The development of this approach to path planning of the formation of a swarm to track the moving targets in an unknown environment will be an interesting research direction for our in the future.

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