Leader-follower and cascade system based formation control of nonholonomic intelligent vehicles

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Abstract: This paper presents a new framework for the formation control of nonholonomic intelligent vehicles combining the leader-follower and cascade system together. The formation model is obtained based on the leader-follower approach. The controllers are designed by using global fast terminal sliding mode control technique for the leader who tracks the reference trajectory and back-stepping method for the follower to keep the predetermined formation movement. Furthermore, the leader and follower of intelligent vehicles can be treated as every part of the cascade system, which relies on a kind of multi-platform information fusion technology to realize communication among vehicles. Simulation results show the correctness and effectiveness of the algorithm.

Key Words: Formation control, Nonholonomic intelligent vehicles, Leader-follower, Cascade system

1 INTRODUCTION

With the development of computer science and wireless communication technology, it has become possible for vehicles to coordinate and cooperation among vehicles, including multiple intelligent vehicles formation problem which is a typical problem in vehicles coordination. In recent years, multiple vehicles formation becomes a focal problem in academia. It improves the utilization rate of the actual traffic, eases traffic congestion effectively, enhances the smooth general characteristic of traffic and security, etc. At presence, many researchers have paid extensive attention on formation control research. There are several methodologies about vehicles formation control, including behavior-based approach, artificial potential field method, leader-follower approach, etc.

In the behavior-based approach [1-2], it is composed of a series of basic behaviors, including collision avoidance, obstacle avoidance, toward the goal, maintain formation and transformation formation, etc., which reach the overall behavior of vehicles through the design of vehicles basic behavior and local control rules. This approach is very easy to choose controller according to the current specific situation, but it is hard to guarantee the stability of the formation control.

By the artificial potential field method [3-4], it mainly carries on the analysis and control through the design of artificial potential field and potential field function to express the constraint relationship between the environment and formation of each vehicle. It uses the potential field method based on the principle of attracting the distant neighbors and rejecting too close neighbors in a certain range through adjusting formation by virtual leader in [3]. This method is easy to achieve real time control, but it is difficult to design the potential field function.

By the leader-follower approach [5-8], we can regard some vehicles as leaders and other vehicles as the followers in the group of multiple vehicles formation. In [5], vehicles achieve specified formation without collisions by the use of relative motion between the $l-\varphi$ control model [6] to design controllers, and to deal with uncertain parameters at the same time in this model. It depends on the cascade system theory and linear multi-agents collaborative control to design the nonholonomic constraints of mobile robot controller in [7]. What's more, we can enlighten the trajectory tracking of a single nonholonomic robot [8], we can convert formation control problem into tracking the leaders' position and direction of the followers.

As we all know, information-access among vehicles is a critical problem. First of all, every vehicle of the formation can act as a leader or a follower. They can get the information of neighbors by sensors, which do not need to consume a large amount of communication costs and time-delay. Secondly, we take an actual road model into consideration for an arbitrary practical road and its corresponding desired velocity. We can regard the leader vehicle as tracking the virtual leader and the followers maintain the spacing and velocity to track the leader, which can keep a complete formation behavior. We could analyze and design the cascade system by stitching together. In this paper, we use the cascade system technology to solve the problem of communication among vehicles and design the corresponding controller.

The rest of the paper is organized as follows. In section 2, the leader-follower formation model is briefly stated. The proposed nonholonomic intelligent vehicles formation control algorithm, its stability and convergence analysis are presented in section 3. We have a discussion of the communication problem in section 4. Section 5 presents the

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simulation and experimental studies. Finally, some concluding remarks are summarized in section 6.

2 FORMATION MODEL

Based on leader-follower vehicle formation control, there are two typical approaches including separation-bearing control method and separation-separation control method [6]. The purpose of separation-bearing formation control algorithm is to design a velocity control input such as

$$\lim_{t\to\infty}(\psi_{ijd}-\psi_{ij})=0\ ,\quad \lim_{t\to\infty}(l_{ijd}-l_{ij})=0 \tag{1}$$

where ψ_{ij} and l_{ij} are the follower's relative bearing and separation, ψ_{ijd} and l_{ijd} represent desired angles and distance respectively.

The leader-follower vehicles formation model shown in Figure.1 is a typical example of the $l-\varphi$ control model in a three-dimensional Cartesian coordinate system. Considering the two nonholonomic intelligent vehicles formation in Figure.1, three coordinate frame systems are defined: the global coordinate system $\{O,X,Y\}$, d_j is the distance between the vehicle center and the axis, $(x_i,y_i,\theta_i,v_i,\omega_i)$ and $(x_j,y_j,\theta_j,v_j,\omega_j)$ denote the Cartesian position, orientation, linear and angular velocity of the i th and j th respectively.

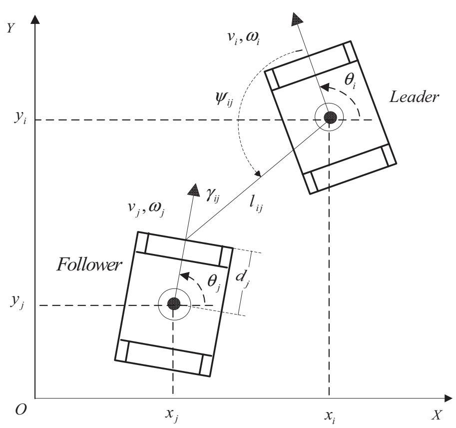

Fig 1. Leader-follower formation configuration

The actual position of follower j

$$\begin{aligned} x_j &= x_i - d_i\cos\theta_i + l_{ij}\cos(\psi_{ij}+\theta_i)\\ y_j &= y_i - d_i\sin\theta_i + l_{ij}\sin(\psi_{ij}+\theta_i)\end{aligned} \tag{2}$$

The desired position of follower j

$$\begin{aligned} x_{jr} &= x_i - d_i\cos\theta_i + l_{ijd}\cos(\psi_{ijd}+\theta_i)\\ y_{jr} &= y_i - d_i\sin\theta_i + l_{ijd}\sin(\psi_{ijd}+\theta_i)\\ \theta_{jr} &= \theta_i\end{aligned} \tag{3}$$

Then we can get the error system

$$e_j=\begin{bmatrix} e_{j1}\\ e_{j2}\\ e_{j3}\end{bmatrix}=\begin{bmatrix} l_{ijd}\cos(\psi_{ijd}+e_{j3})-l_{ij}\cos(\psi_{ij}+e_{j3})\\ l_{ijd}\sin(\psi_{ijd}+e_{j3})-l_{ij}\sin(\psi_{ij}+e_{j3})\\ \theta_i-\theta_j\end{bmatrix} \tag{4}$$

We transform the leader-follower vehicle formation problem into a tracking problem in order to remain at a fixed desired distance and a desired angle for vehicles formation. Therefore, our goal is to find a velocity control input for the followers and expect the error system $\lim_{t\to\infty} e_j = 0$.

In order to further analyze the error equations given in (4), let us suppose ψ_{ijd}, l_{ijd} are constant, and decompose the distance l_{ij} to the horizontal and vertical direction.

$$\begin{aligned} l_{ijx} &= x_i - x_j - d_j\cos\theta_j\\ l_{ijy} &= y_i - y_j - d_j\sin\theta_j\end{aligned} \tag{5}$$

Then we can get

$$\begin{aligned} \dot l_{ijx} &= v_i\cos\theta_i - v_j\cos\theta_j + d_j\omega_j\sin\theta_j\\ \dot l_{ijy} &= v_i\sin\theta_i - v_j\sin\theta_j - d_j\omega_j\cos\theta_j\end{aligned} \tag{6}$$

Noting that $l_{ij}^2 = l_{ijx}^2 + l_{ijy}^2$, $\psi_{ij} = \arctan(l_{ijy}/l_{ijx}) - \theta_i + \pi$. Then we could get

$$\begin{aligned} \dot l_{ij} &= v_j\cos\gamma_j - v_i\cos\psi_{ij} + d_j\omega_j\sin\gamma_j\\ \dot\psi_{ij} &= (v_i\sin\psi_{ij} - v_j\sin\gamma_j + d_j\omega_j\cos\gamma_j - l_{ij}\omega_i)/l_{ij}\end{aligned} \tag{7}$$

where $\gamma_j = \psi_{ij} + e_{j3}$. The changing rate of relative distance l_{ij} is caused by the translation component of v_i, v_j in the direction l_{ij} and the translation component of follower's angular velocity ω_j in the direction l_{ij}. The changing rate of relative angle ψ_{ij} is caused by the translation component of v_i, v_j in the perpendicular direction l_{ij} and the translation component of leader and follower's angular velocity ω_i, ω_j in the perpendicular direction l_{ij} .Using (4) and (7), the derivative of e_j becomes

$$\dot e_j=\begin{bmatrix} -v_j+v_i\cos e_{j3}+\omega_j e_{j2}-\omega_i l_{ijd}\sin(\psi_{ijd}+e_{j3})\\ -\omega_j e_{j1}+v_i\sin e_{j3}-d_j\omega_j+\omega_i l_{ijd}\cos(\psi_{ijd}+e_{j3})\\ \omega_i-\omega_j\end{bmatrix} \tag{8}$$

3 CONTROLLER DESIGN

We choose an intelligent vehicle as a leader which is responsible for the path planning and the rest of the vehicles as followers. The leader sends the next target to the follower by local communication and the followers adjust themselves to keep formation behavior.

3.1 Controller for the leader vehicle

The leader's trajectory tracking state has a great influence on the followers, so we should analyze the kinematics model of the leader and design the leader's controller. We develop the mathematical model for the leader vehicle, which can be described as follows

$$q_i=\begin{bmatrix} x_i\\ y_i\\ \theta_i\end{bmatrix}=\begin{bmatrix}\cos\theta_i & d_i\sin\theta_i\\ \sin\theta_i & -d_i\cos\theta_i\\ 0 & 1\end{bmatrix}\begin{bmatrix} v_i\\ \omega_i\end{bmatrix} \tag{9}$$

The tracking posture error can be expressed as

$$\dot x_r = v_r\cos\theta_r,\ \dot y_r = v_r\sin\theta_r,\ \dot\theta_r = \omega_r$$

$$e_i = T(q_r - q_i) = \begin{bmatrix} x_{ie} \\ y_{ie} \\ \theta_{ie} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_i \\ y_r - y_i \\ \theta_r - \theta_i \end{bmatrix} \tag{10}$$

where the desired vehicle posture $q_r = [x_r, y_r, \theta_r]^T$ and reference velocities $(v_r, \omega_r)^T$. Then we can get

$$\dot{e}_i = \begin{bmatrix} \dot{x}_{ie} \\ \dot{y}_{ie} \\ \dot{\theta}_{ie} \end{bmatrix} = \begin{bmatrix} \omega_i y_{ie} - v_i + v_r \cos\theta_{ie} \\ -\omega_i x_{ie} + v_r \sin\theta_{ie} \\ \omega_r - \omega_i \end{bmatrix} \tag{11}$$

In this paper, the control law [9] is adopted for the leader vehicle by the sliding mode control method. The sliding mode switch function is shown as

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} x_{ie} \\ \theta_{ie} + \arctan(v_r y_{ie}) \end{bmatrix} \tag{12}$$

We can design a sliding mode controller through this switching surface to ensure that $s_1 \to 0$, $s_2 \to 0$, furthermore, $\lim_{t\to\infty} \left\| (x_{ie}, y_{ie}, \theta_{ie})^T \right\| = 0$.

Compared with the constant velocity reaching law, exponential reaching law has better smoothness, just as the equation (13),

$$\dot{s} = -\varepsilon s^a \operatorname{sgn}(s) - k_1 s \tag{13}$$

where $\varepsilon > 0$, $k_1 > 0$ and $0 < a < 1$. In neighborhood of the sliding mode, the condition of sliding mode is $\dot{s}s < 0$, by formula (13)

$$\dot{s}s = -\varepsilon s^{1+a} \operatorname{sgn}(s) - k_1 s^2 \tag{14}$$

For $\varepsilon > 0$ and $k_1 > 0$, so $\dot{s}s < 0$ is established. It means that the exponential reaching law which is shown as formula (13) which meets the existence and reachability of sliding mode. Assume that the initial value $s(0) > 0$, then we can get

$$s^{1-a} = C \exp^{-(1-a)kt} - \frac{\varepsilon}{k_1} \tag{15}$$

Where $C = [s(0)]^{1-a} + \varepsilon / k_1$. The initial sliding mode $s(0)$ converges to $s = 0$ in limited time t .

$$t = \frac{1}{k_1(1-a)} \ln\left[1 + \frac{k_1 (s(0))^{1-a}}{\varepsilon}\right] \tag{16}$$

But this controller designed as shown above has not eliminated the problem of buffeting well. To solve this problem, the sign function can be substituted by the following continuous function

$$\operatorname{sgn}(s_i) = \frac{s_i}{|s_i| + \sigma} \qquad (i = 1, 2) \tag{17}$$

Sliding mode controller can be designed

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} y_{ie}\omega_i + v_r \cos\theta_{ie} + \varepsilon \dfrac{s_1^{1+a}}{|s_1| + \sigma} + k_2 s_1 \\ \dfrac{\omega_r + \dfrac{\partial\alpha}{\partial v_r}\dot{v}_r + \dfrac{\partial\alpha}{\partial y_{ie}}(v_r \sin\theta_{ie}) + \varepsilon \dfrac{s_2^{1+a}}{|s_2| + \sigma} + k_3 s_2}{1 + \dfrac{\partial\alpha}{\partial y_{ie}} x_{ie}} \end{bmatrix} \tag{18}$$

where ε , k_2 , k_3 are coefficient. Here, $\alpha = \arctan(v_r y_{ie})$, and then $\dfrac{\partial\alpha}{\partial v_r} = \dfrac{y_{ie}}{1 + (v_r y_{ie})^2}$, $\dfrac{\partial\alpha}{\partial y_{ie}} = \dfrac{v_r}{1 + (v_r y_{ie})^2}$.

First of all, we need to prove the switch function can converge to zero in finite time. Considering the following Lyapunov function for the formula (12),

$$V_s = \frac{1}{2} s^T s \tag{19}$$

Its derivation is derived as follows

$$\dot{V}_s = -k_2 x_{ie}^2 - k_2 x_{ie}^2 y_{ie}^2 - (\theta_{ie} + \arctan(v_r y_{ie}))^2 \le 0 \tag{20}$$

Therefore, the switching function can converge to zero in a finite time. When $s \to 0$, $\theta_e \to \alpha =: -\arctan(v_r y_e)$, and then the tracking posture error converges to zero.

3.2 Controller for the follower vehicle

According to the paper [10], the design of the control law for the follower vehicle is

$$\begin{bmatrix} v_j \\ \omega_j \end{bmatrix} = \begin{bmatrix} v_i \cos e_{j3} + k_4 e_{j1} \\ \omega_i + (v_i + k_v) k_5 e_{j2} + (v_i + k_v) k_6 \sin e_{j3} \end{bmatrix} + \begin{bmatrix} \gamma_{vj} \\ \gamma_{\omega j} \end{bmatrix} \tag{21}$$

where k_4 , k_5 , k_6 , k_v are positive real number. The number of k_v ensures that the system asymptotic stability.

$$\gamma_{vj} = -\omega_i l_{ijd} \sin(\psi_{ijd} + e_{j3}) \tag{22}$$

$$\gamma_{\omega j} = -\frac{|e_{j2}| (\omega_i (d_j + l_{ijd}) + k_6 (v_i + k_v) d_j + k_v)}{1/k_5 + |e_{j2}| d_j} \tag{23}$$

Considering the following Lyapunov function

$$V_j = \frac{1}{2}(e_{j1}^2 + e_{j2}^2) + \frac{1}{k_5}(1 - \cos e_{j3}) \tag{24}$$

where it is obvious that $V_j \ge 0$ and $V_j = 0$ if and only if $e_j = 0$. We can get the derivation of equation (24) as follows by formula (8), (21), (22) and (23).

$$
\begin{aligned}
\dot{V}_j &= e_{j1}(-v_j + v_i\cos e_{j3} + \omega_j e_{j2} - \omega_i l_{ijd}\sin(\psi_{ijd} + e_{j3})) + e_{j2}(-\omega_j e_{j1} \\
&\quad + v_i\sin e_{j3} - d_j\omega_j + \omega_i l_{ijd}\cos(\psi_{ijd} + e_{j3})) + \frac{1}{k_2}(\omega_i - \omega_j)\sin e_{j3} \\
&\le -k_4 e_{j1}^2 - k_5 d_j(v_i + k_v)e_{j2}^2 - \frac{k_6}{k_5}(v_i + k_v)\sin^2 e_{j3} \\
&\quad + \left|e_{j2}\right|(\omega_i(d_j + l_{ijd}) + k_v + k_6(v_i + k_v)d_j + (d_j + \frac{1}{k_5\left|e_{j2}\right|})\gamma_{\omega j}) \\
&\le -k_4 e_{j1}^2 - k_5 d_j(v_i + k_v)e_{j2}^2 - \frac{k_6}{k_5}(v_i + k_v)\sin^2 e_{j3} \le 0 \qquad (25)
\end{aligned}
$$

For all of the $v_i \ge 0$, we can get $\dot{V}_j \le 0$. With the velocity control input about the formula (21), (22) and (23), the dynamic error of the system achieves asymptotical stability about the formula (4) and (8).

4 COMMUNICATION PROBLEM

The designed controllers can achieve the purpose of the vehicle formation in certain environment. However，what make the controllers lose efficacy are that vehicles get the error messages and exist in the obstacles. At last, the formation system based on perceptual type sensor could do nothing. In order to solve this problem, a communication control method that bases on cascade system vehicle formation only needs to install vehicle communication equipment and related protocol, which is a kind of multi-platform information fusion technology. Vehicles formation control program flow chart for leader-follower approach is shown in Figure.2.

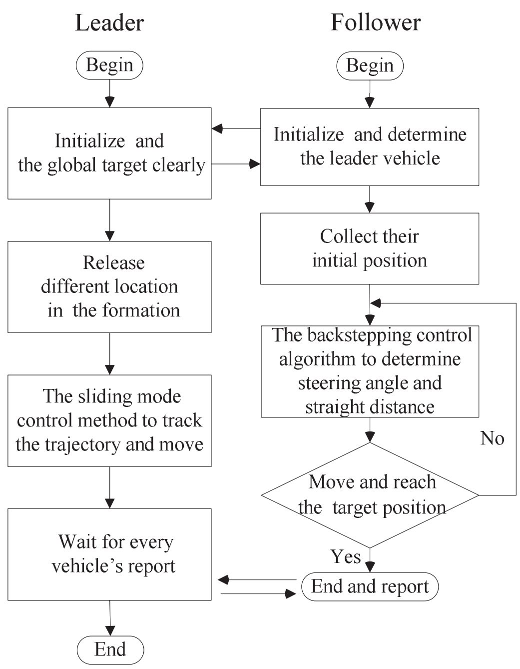

Fig 2. Formation control program flow chart

In this technology, the leader can obtain its own and followers'position and velocity by calculating the parameters such as relative distance and angle with follower,etc. Therefore, the leader sends the controller information for the followers to move and reach the target position through the information fusion processing. What's more, the follower adjusts its motion to arrive at the target position and send their own information to the leader.

The wireless communication system allows vehicles to send and receive information timely and quickly in order to adapt to a changing environment. The design of the controller depends largely on the communication between the vehicles.

5 SIMULATION RESULTS

In this section, we verify the correctness and effectiveness of the proposed control algorithm. Vehicle formation composes of two intelligent vehicles. The leader vehicle tracks virtual reference vehicle. Followers keep a certain distance and velocity track the leader vehicle, so we can regard it as three vehicles formation. The reference trajectory is a straight line, where $x_r(t) = t$, $y_r(t) = t$, $\theta_r(t) = \pi/3$ .The reference velocity are $\omega_r = 0$ and $v_r = 1m/s$. Simulation parameters are given as follows: $l_{12d} = 2m$, $\psi_{12d} = \pi$, $d = 0.45m$, $a = 0.1$, $\sigma = 0.01$, $k_2 = 6$, $k_3 = 9$, $k_4 = 10$, $k_5 = 30$, $k_6 = 5$, $k_v = 1$, $\varepsilon = 0.001$. The simulation results in 0-8s are shown in Figure 3-7.

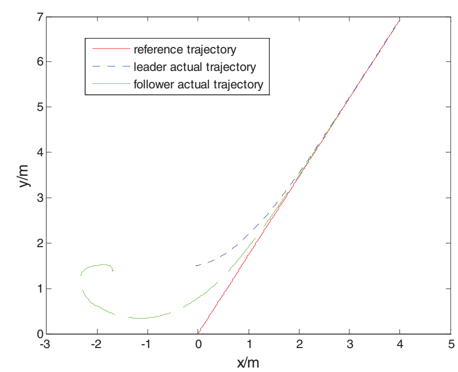

Fig 3. A straight line formation of 2 vehicles

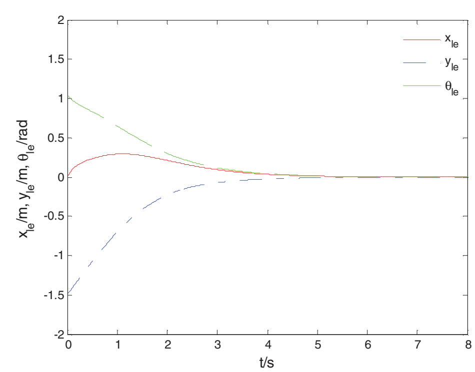

Fig 4. Posture errors of the leader

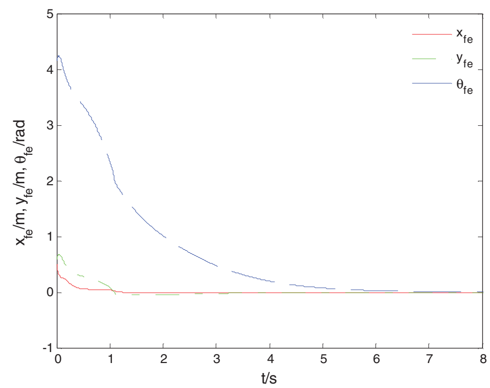

Fig 5. Posture errors of the follower

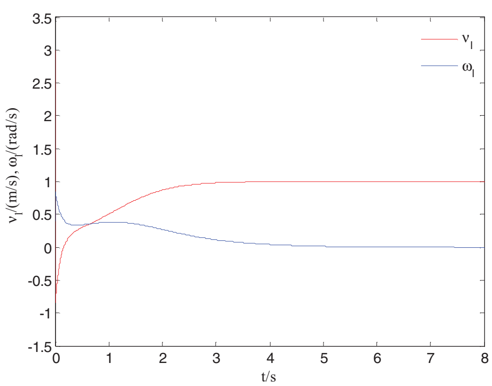

Fig 6. Velocity of the leader

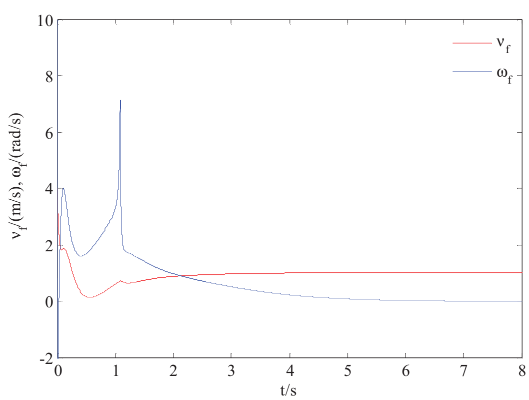

Fig 7. Velocity of the follower

From the simulation curves in Figure 3, the leader can fast track the desired reference trajectory about in 3s, but the follower needs to turn around and drive toward the start point of the reference trajectory to adjust its direction. At last, it can search for the direction of the reference trajectory and achieve the desired linear trajectory tracking.

As we can see from the simulation curves in Figure 4-5, the tracking posture errors convergence to zero quickly under the leader and follower's control strategy, and it has a good global stability. From Figure 6-7, the system comes into a stable state in 6s or so, and the leader and follower vehicles quick converge to the reference linear velocity and angular velocity. At the same time, it reaches the desired formation target.

6 CONCLUSIONS

In this paper, we build a new framework based on the leader-follower and cascade system. We combine them together to adopt the leader and follower's control law respectively by the multi-platform information fusion technology. This technology simplifies the communication difficulties to designing controllers. The stability of the system is rigorously proved by Lyapunov stability theory. The simulation is conducted based on Matlab/Simulink. Simulation results indicate the proposed framework can work well.

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