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A Time-varying Cascaded Design for Trajectory Tracking Control of Nonholonomic Systems

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Abstract: This paper deals with the tracking control problem for a general class of nonholonomic dynamic systems with reference signals that may exponentially decay. By introducing a time-varying coordinate transformation and using the cascade-design approach, smooth time-varying controllers are constructed, which render the tracking error dynamics globally $\mathcal{K}$-exponentially stable. Obtained result shows that the popular condition of persistent-excitation or not converging to zero imposed on the reference signals is not necessary even for the globally $\mathcal{K}$-exponential tracking of nonholonomic systems. Application to the tracking control of an underactuacted surface vessel validates the effectiveness of the proposed method.

Key Words: Trajectory Tacking, Nonholonomic Systems, Cascaded System, Persistent Excitation, Exponential Stability.

1 INTRODUCTION

Over the last decade, considerable efforts have been devoted to the control of nonlinear mechanical systems with nonholonomic constraints. This problem is known to be rather difficult, largely due to the impossibility of asymptotically stabilizing nonholonomic systems via smooth time-invariant state feedback, a well-recognized fact uncovered by Brockett's necessary condition for asymptotic feedback stabilization [2]. Extensive research has been conducted in order to find alternative control methods such as discontinuous feedback, time-varying feedback and hybrid control laws (see e.g., [1, 16, 18, 21, 6]).

In contrast to the stabilization problem, the tracking problem for nonholonomic systems has received less attention. From a practical point of view, however, the tracking problem—sometimes called the stabilization of trajectories—is more important and difficult than the stabilization problem. This problem becomes very challenging when the reference signal converges to zero because it not only requires the controlled system rests upon the stop point in the end but also requires the controlled system tracks the reference system's trajectory before it stops (see e.g., [22, 10]). After many attempts to design local tracking controllers for some special classes of nonholonomic systems by using local linearization, input-output linearization or differential flatness methods (see [4] for an overview), the first global tracking control law was presented in [15] for a nonholonomic wheeled cart, to our knowledge. By means of backstepping Jiang & Nijmeijer [4] derived semi-global tracking controllers for general chained-form systems and achieved global tracking results for some special cases. Using cascade-design method, Lefeber, et al. [9, 8] proposed global $\mathcal{K}$-exponential tracking controllers for chained-form systems and underactuated ships. However, in all of the above mentioned studies there are some constraints on the target's movement. For examples, in [4] and [16] it was assumed that the linear velocity or angular velocity of the target (or the first reference control signal of the chained-form system) must not converge to zero; and in [9] and [8] it was assumed that the first reference control signal must satisfy the persistent excitation condition, which also implies that the signal must not converge to zero.

Recently, motivated by the time-varying control method of Samson [16], Lee, et al. [10] and Do, et al. [3] proposed time-varying controllers to achieve both stabilization and tracking for unicycle mobile robots and underactuated ships, respectively. The main idea of [10] and [3] is introducing a time-varying persistently exciting signal in the virtual control of backsteepping. This, however, implies that the rate of convergence of the tracking error dynamics cannot be exponential. Moreover, the results of [10] and [3] are restricted to mobile robots and underactuated ships only, and there is no easy way to extend the similar results to a general class of nonholonomic systems.

The purpose of this paper is to design tracking controllers for a general class of nonholonomic systems whose reference system is allowed to converge exponentially. Motivation of our study mainly comes from the results on the stabilization problem for nonholonomic systems given by Astolfi [1], Laiou & Astolfi [7] and Tian & Li [19]. Astolfi [1] first constructed a discontinuous control law to stabilize the nonholonomic chained-form system by transforming a

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smooth system into a discontinuous one via the so-called δ-process. The existence of smooth time-varying control laws which exponentially stabilize nonholonomic systems was pointed out by Laiou & Astolfi [7] and Tian & Li [19] independently. Both [7] and [19] introduced an assistant state to complete a coordinate transformation, while the former used a state scaling transformation and the latter used a transformation based on the minimal dilation degree. The idea of the minimal dilation degree can also be extended to the tracking control problem for nonholonomic systems when the target's first velocity is composed of "slow" and "fast" modes. Different from the above mentioned papers, the focus of this paper is on the tracking control of a general class of nonholonomic systems and the transformation based on the minimal dilation degree is introduced for the reference target where no assistant states are involved.

2 STATEMENT OF PROBLEM AND PRELIMINARIES

The nonholonomic systems to be studied is described as

$$\begin{aligned} \dot{x}_1^{(r_1)} &= u_1, \qquad (1)\\ \dot{x}_i^{(r_i)} &= A_{i1}(Y_1,u_1)u_2 + \sum_{j=2}^{n} A_{ij}(Y_1,u_1)Y_j, \end{aligned}$$

where $i = 2,\cdots,n$ and r_i denotes the order of the time-differentiation on the variable x_i, $Y_i = [x_i^{(0)}, x_i^{(1)}, \cdots, x_i^{(r_i-1)}]^T$ is the extended state vector, $A_{ij} \in R^{1\times r_j}, i = 2,3,\cdots,n, j = 2,\cdots,n$, and $A_{i1} \in R$ are multivariable polynomial functions of $x_1, x_1^{(1)}, \cdots, x_1^{(r_1-1)}$ and u_1, i.e.,

$$A_{ij}(Y_1,u_1) = \sum_{\substack{i_0+i_1+\\ \cdots+i_{r_1}=1}}^{N} \bar{A}^{ij}_{i_0 i_1 \cdots i_{r_1}} (x_0^{(0)})^{i_0} \cdots (x_1^{(r_1-1)})^{i_{r_1-1}} u_1^{i_{r_1}}$$

where $i_0, i_1, \cdots, i_{r_1}$ are nonnegative integers. Eq. (1) is a dynamic model for nonholonomic systems, which was first proposed in [21]. As shown in [20, 21], this model is powerful enough to describe many typical nonholonomic systems such as extended chained systems, extended power systems, underactuated surface vessel systems, etc.
Assume that the reference trajectory is generated by the following equations:

$$\begin{aligned} \dot{x}_{1d}^{(r_1)} &= u_{1d}, \qquad (2)\\ \dot{x}_{id}^{(r_i)} &= A_{i1}(Y_{1d},u_{1d})u_{2d} + \sum_{j=2}^{n} A_{ij}(Y_{1d},u_{1d})Y_{jd} \end{aligned}$$

We make the following assumptions for the reference system.

Assumption 1: If $A_{ij}(Y_{1d}, u_{1d}), i = 2,\cdots,n,\ j = 1,\cdots,n$, are not identically to zero, then there exist nonnegative integers λ, p_{ij} and nonzero constant row vectors D_{ij} such that

$$\lim_{t\to\infty} \frac{A_{ij}(Y_{1d},u_{1d})}{\exp(-\lambda p_{ij}(t-t_0))} = D_{ij},$$

$$\int_0^{\infty} \| \frac{A_{ij}(Y_{1d},u_{1d})}{\exp(-\lambda p_{ij}(t-t_0))} - D_{ij} \| dt < +\infty;$$

If $A_{ij}(Y_{1d}, u_{1d}) \equiv 0$, it is assumed that $p_{ij} = \infty$.

Assumption 2: All the states and inputs of the reference system are bounded signals.

Under Assumption 1, we suppose $\Pi = [p_1,\ p_2,\ \cdots,\ p_n]^T$ is the minimal dilation degree for system (2). For the definition of minimal dilation degree and the method to obtain the minimal dilation degree, we refer to [21].
Now, we define the tracking errors as $e_i^{(r_i)} = x_i^{(r_i)} - x_{id}^{(r_i)}$. Then, from (1) and (2) it follows that

$$\dot{e}_1^{(r_1)} = u_1 - u_{1d} \qquad (3)$$

$$\begin{aligned} \dot{e}_i^{(r_i)} &= A_{i1}(Y_{1d},u_{1d})(u_2-u_{2d}) + [A_{i1}(Y_1,u_1) - \\ &\quad A_{i1}(Y_{1d},u_{1d})]u_2 + \sum_{j=2}^{n} A_{ij}(Y_{1d},u_{1d})[Y_j - Y_{jd}] \\ &\quad + \sum_{j=2}^{n}[A_{ij}(Y_1,u_1) - A_{ij}(Y_{1d},u_{1d})]Y_j. \qquad (4) \end{aligned}$$

Next, we first recall some lemmas that will be needed in the proof of our main result.

Lemma 2.1 [21] *Consider a linear time-varying control system*

$$\dot{x} = (A_0 + A_1(t))x + (B_0 + B_1(t))u$$

with $x \in R^n, u \in R^m$. Suppose the system satisfy the following properties

(1) $\lim_{t\to\infty} A_1(t) = 0, \quad \int_0^{\infty} \|A_1(t)\|dt < \infty$;

(2) $\lim_{t\to\infty} B_1(t) = 0, \quad \int_0^{\infty} \|B_1(t)\|dt < \infty$;

(3) (A_0, B_0) *is a stabilizable pair.*

Then there exist a state feedback $u = -Kx$ which makes the closed-loop system uniformly exponentially stable, where K is gain matrix such that $A_0 - B_0K$ is Hurwitz.

Now, we consider a time-varying cascaded system given by

$$\begin{aligned} \dot{z}_1 &= f_1(t,z_1), \qquad (5)\\ \dot{z}_2 &= f_2(t,z_2) + g(t,z_1,z_2)z_1. \end{aligned}$$

Note that this system can be viewed as the following system

$$\Sigma_2 : \quad \dot{z}_2 = f_2(t,z_2) \qquad (6)$$

perturbed by the output of the system

$$\Sigma_1 : \quad \dot{z}_1 = f_1(t,z_1). \qquad (7)$$

Lemma 2.2 [8] *The cascaded time varying system* (5) *is global $\mathcal{K}$-exponentially stability if the following assumptions are satisfied*

(1) *the subsystems* (6) *is globally uniformly exponentially stable (GUES);*

(2) *the function $g(t, z_1, z_2)$ satisfies the following condition for all $t \geq t_0$*

$$\|g(t, z_1, z_2)\| \leq \theta_1(\|z_1\|) + \theta_2(\|z_1\|)\|z_2\|,$$

where $\theta_1(\cdot), \theta_2(\cdot)$ are continuous functions;

(3) *the subsystem* (7) *is globally $\mathcal{K}$-exponentially stable.*

3 CONTROLLER DESIGN

In this section we present a time-varying cascade design procedure for the tracking control of the nonholonomic system (1)(2).

3.1 Control Law for u_1

Introduce a time-varying coordinate transformation for system (3) as follows

$$Z_1 = E_1 / \exp(-\lambda p^\star (t - t_0)), \tag{8}$$

where $Z_1 = [Z_1^{(0)}, \cdots, Z_1^{(r_1-1)}], E_1 = [e_1^{(0)}, \cdots, e_1^{(r_1-1)}]$, and $p^\star = \max\{p_{ij} + p_j\}, i = 2, 3, \cdots, n;\ j = 1, 2, \cdots, n$.
Here and throughout the paper, t_0 represents the initial time from which the control action starts. Under control law $u_1 = u_{1d} - \exp(-\lambda p^\star(t - t_0))K_1[z_1^{(0)}, \cdots, z_1^{(r_1-1)}]$, the formula (3) can also be rewritten as

$$\begin{bmatrix} \dot{z}_1^{(0)} \\ \dot{z}_1^{(1)} \\ \vdots \\ \dot{z}_1^{(r_1-1)} \end{bmatrix} = \begin{bmatrix} \lambda p^\star & 1 & \cdots & 0 \\ 0 & \lambda p^\star & \ddots & 0 \\ \vdots & & \ddots & 1 \\ k_{10} & k_{11} & \cdots & k_{1(r_1-1)} + \lambda p^\star \end{bmatrix} \begin{bmatrix} z_1^{(0)} \\ z_1^{(1)} \\ \vdots \\ z_1^{(r_1-1)} \end{bmatrix}, \tag{9}$$

from which we can choose an appropriate K_1 such that the resulted closed-loop system is GUES.

3.2 Control Law for u_2

Introduce the following transformation for system (4)

$$Z_i = E_i / \exp(-\lambda p_i (t - t_0)),\ i = 2, \cdots, n \tag{10}$$

where $Z_i = [z_i^{(0)}, \cdots, z_i^{(r_1-1)}], E_i = [e_i^{(0)}, \cdots, e_i^{(r_1-1)}]$.
Then the system (4) is transformed into

$$\begin{aligned} \dot{z}_i^{(0)} &= \lambda p_i z_i^{(0)} + z_i^{(1)}, \\ \dot{z}_i^{(1)} &= \lambda p_i z_i^{(1)} + z_i^{(2)}, \\ &\vdots \\ \dot{z}_i^{(r_i-1)} &= \lambda p_i z_i^{(r_i-1)} + \tfrac{e_i^{(r_i)}}{\exp(-\lambda p_i (t-t_0))}. \end{aligned} \tag{11}$$

The last equation in system (11) can also be written as

$$\begin{aligned} &\frac{d}{dt}\Big(\frac{e_i^{(r_i-1)}}{\exp(-\lambda p_i(t-t_0))}\Big) = \lambda p_i \frac{e_i^{(r_i-1)}}{\exp(-\lambda p_i(t-t_0))} \\ &\quad + \left\{ \left[F_{i1} + \frac{A_{i1}(Y_{1d}, u_{1d})}{\exp(-\lambda p_i(t-t_0))} - F_{i1} \right] (u_2 - u_{2d}) \right\} \\ &+ \left\{ \sum_{j=2}^{n} \left\{ F_{ij} + e^{-\lambda(p_j + p_{ij} - p_i)(t-t_0)} \frac{A_{ij}(Y_{1d}, u_{1d})}{e^{-\lambda p_{ij}(t-t_0)}} - F_{ij} \right\} \frac{Y_j - Y_{jd}}{e^{-\lambda p_j(t-t_0)}} \right\} \\ &+ \sum_{j=2}^{n} e^{-\lambda(p_j + p_{ij} - p_i)(t-t_0)} \frac{A_{ij}(Y_1, u_1) - A_{ij}(Y_{1d}, u_{1d})}{e^{-\lambda p_{ij}(t-t_0)}} \frac{Y_j}{e^{-\lambda p_j(t-t_0)}} \\ &\quad + \frac{A_{i1}(Y_1, u_1) - A_{i1}(Y_{1d}, u_{1d})}{\exp(-\lambda p_i(t-t_0))} u_2. \end{aligned} \tag{12}$$

For simplicity, we denote

$$W_1 = Z_1, W_2 = [Z_2, \cdots, Z_n]^T, \tag{13}$$

and

$$H_{ij}(t) = e^{-\lambda(p_j + p_{ij} - p_i)(t-t_0)} \frac{A_{ij}(Y_{1d}, u_{1d})}{e^{-\lambda p_{ij}(t-t_0)}} - F_{ij},$$

where $i = 2, \cdots, n;\ j = 1, \cdots, n$.
Then, system (11) can also be rewritten as

$$\begin{aligned} \dot{W}_2 =& (A_0(t) + A_1(t))W_2 + (B_0 + B_1(t))(u_2 - u_{2d}) \\ &+ \bar{g}(t, W_1, W_2), \end{aligned} \tag{14}$$

where $A_0, A_1(t)$ are defined as

$$\begin{bmatrix} \lambda p_2 & 1 & \cdots & 0 & & & & \\ & \ddots & \ddots & & & & & \\ 0 & \cdots & \lambda p_2 & 1 & & & & \\ f_{221} & \cdots & f_{22(r_2-1)} & f_{22r_2} + \lambda p_2 & \cdots & f_{2n1} & \cdots & f_{2n(r_n)} \\ & & & & \ddots & & & \\ & & & & & \lambda p_n & \ddots & 0 \\ & & & & & & \ddots & \\ & & & & & & & 1 \\ f_{n21} & \cdots & f_{n2(r_2-1)} & f_{n2r_2} & \cdots & f_{nn1} & \cdots & f_{nn(r_n)} + \lambda p_n \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & & & & \\ & \ddots & \ddots & & & & & \\ 0 & \cdots & 0 & 0 & & & & \\ h_{221}(t) & \cdots & \cdots & f_{22r_2}(t) & \cdots & h_{2n1}(t) & \cdots & h_{2n(r_n)}(t) \\ & & & & \ddots & & & \\ & & & & & 0 & \ddots & 0 \\ & & & & & & \ddots & \\ & & & & & & & 0 \\ h_{n21}(t) & \cdots & \cdots & f_{n2r_2}(t) & \cdots & h_{nn1}(t) & \cdots & h_{nn(r_n)}(t) \end{bmatrix},$$

and

$$B_0 = \begin{bmatrix} 0 \cdots F_{21} \cdots \cdots 0 \cdots F_{n1} \end{bmatrix}^T,$$

$$B_1(t) = \begin{bmatrix}0 \cdots H_{21}(t) \cdots \cdots 0 \cdots H_{n1}(t)\end{bmatrix}^T,$$

and the last term in the right of (14) is

$$\begin{bmatrix}0 \ \cdots 0 \ \bar{g}_2(t, W_1, W_2) \cdots 0 \cdots 0 \ \bar{g}_n(t, W_1, W_2))\end{bmatrix}^T,$$

where $\bar{g}_i(t, W_1, W_2)$ is

$$\sum_{j=2}^{n} e^{-\lambda(p_j+p_{ij}-p_i)(t-t_0)} \frac{A_{ij}(Y_1,u_1)-A_{ij}(Y_{1d},u_{1d})}{\exp(-\lambda p_{ij}(t-t_0))} \frac{Y_j}{e^{-\lambda p_j(t-t_0)}}$$
$$+\frac{A_{i1}(Y_1,u_1) - A_{i1}(Y_{1d},u_{1d})}{\exp(-\lambda p_i(t-t_0))} u_2 \quad (i = 2, \cdots, n). \quad (15)$$

Under Assumption 1 and definition of $H_{ij}(t)$, system

$$\dot{W}_2 = (A_0(t)+A_1(t))W_2+(B_0+B_1(t))(u_2-u_{2d}) \quad (16)$$

can be globally uniformly exponentially stabilized by the feedback $u_2 = u_{2d} - K_2W_2$ if (A_0, B_0) is a stabilizable pair and K_2 is a gain vector such that $A_0-B_0K_2$ is Hurwitz by using Lemma 2.1.

3.3 Main Results

Theorem 3.1 *If* (A_0, B_0) *is stabilizable, then, under Assumptions 1 and 2, the controller*

$$u_1 = u_{1d} - K_1[e_1^{(0)} \cdots e_1^{(r_1-1)}]^T, \quad (17)$$
$$u_2 = u_{2d} - K_2[\frac{e_2^{(0)}}{e^{-\lambda p_2(t-t_0)}} \cdots \frac{e_2^{(r_2-1)}}{e^{-\lambda p_2(t-t_0)}} \cdots \frac{e_n^{(0)}}{e^{-\lambda p_n(t-t_0)}} \cdots \frac{e_n^{(r_n-1)}}{e^{-\lambda p_n(t-t_0)}}]^T \quad (18)$$

renders the closed-loop system formed by (3)(4) *and* (17) (18) *globally* $\mathcal{K}$*-exponentially stable.*

Due to the space limitation, the proof of the theorem is omitted here.

4 TRACKING CONTROL OF AN UNDERACTUATED SURFACE VESSEL

4.1 System Model and Controller Design

Stabilization and tracking control of an underactuated surface vessel (USV) have received considerable attention from the control community. The exponential stabilization problem for this system has been studied by Pettersen & Egeland [13], Reyhanoglu [14] and Tian & Li [21]. A global tracking controller for the USV has been derived by using the cascaded approach [9, 8]. A universal controller for both stabilization and tracking of the USV has also been presented in [3] by introducing a persistent signal in the backsteppping design.

In this section we apply the time-varying cascade design method for the tracking control of the USV, which is modeled by

$$\begin{aligned}
\dot{x}_1 &= x_5 \\
\dot{x}_2 &= x_6 + x_3x_5 \\
\dot{x}_3 &= x_4 - x_2x_5 \\
\dot{x}_4 &= -\alpha x_4 - \beta x_5x_6 \\
\dot{x}_5 &= v_1 \\
\dot{x}_6 &= v_2
\end{aligned} \quad (19)$$

where $\alpha, \beta > 0$ are some constants. First we take a transformation for control variables as $u_1 = v_1, u_2 = v_2+x_3^{(1)}x_1^{(1)}+x_3v_1$, then represent x_4, x_5, x_6 by x_1, x_2, x_3 and $\dot{x}_1, \dot{x}_2, \dot{x}_3$. The model (19) can be transformed into the canonical form (1) with $A_{21}(Y_1, u_1) = 1, A_{22}(Y_1, u_1) = A_{23}(Y_1, u_1) = A_{31}(Y_1, u_1) = 0, A_{32}(Y_1, u_1) = [-u_1 - \alpha x_1^{(1)}, -(\beta+1)x_1^{(1)}], A_{33}(Y_1, u_1) = [\beta(x_1^{(1)})^2, -\alpha]$. The error system for the USV can be easily obtained from A_{ij} which is not given here due to the space limitation. As the main purpose of this paper is to study the tracking control for an exponentially converging reference, we assume that the reference system for the USV is being stabilized. The state of the reference system $x_{1d}^{(1)}$ is assumed to satisfy

$$x_{1d}^{(1)} = \exp(-\lambda(t-t_0))g(t),$$

where λ is a positive constant, $g^{(1)}(t)$ is uniformly continuous over $[0, \infty)$, and

$$\lim_{t\to\infty} g(t) = G \quad (G \neq 0).$$

After simple computation, it follows that $u_{1d} = x_{1d}^{(2)} = \exp(-\lambda(t-t_0))(-\lambda g(t) + g^{(1)}(t))$. And $g^{(1)}(t) \to 0$ as $t \to 0$ can be obtained under the condition $\lim_{t\to\infty} g(t) = G$ and $g^{(1)}(t)$'s uniform continuity over $[0, \infty)$ by using the well-known Barbarlat lemma [5]. Therefore, the expression of u_{1d} can be simply written as

$$u_{1d} = \exp(-\lambda(t-t_0))g_1(t)$$

where $g_1(t) = -\lambda g(t) + g^{(1)}(t)$ satisfies

$$\lim_{t\to\infty} g_1(t) = -\lambda G.$$

From Proposition 2 of [21], the existence of p_{ij} defined in Assumption 1 is guaranteed and p_{ij} can be obtained as $p_{21} = 0, p_{22} = p_{33} = p_{31} = \infty, p_{32} = 1, p_{33} = 0$. Using the method of computing the minimal dilation degree given by Proposition 3 of [21], we get the minimal dilation degree as $p_2 = 0, p_3 = 1$. And simple computation shows that the parameter $p^\star$ defined in section 3.1 is $p^\star = 1$. Consequently, the coordinate transformation for the error system is as follows

$$W = \left[\frac{e_1^{(0)}}{e^{-\lambda(t-t_0)}} \ \frac{e_1^{(1)}}{e^{-\lambda(t-t_0)}} \ e_2^{(0)} \ e_2^{(1)} \ \frac{e_3^{(0)}}{e^{-\lambda(t-t_0)}} \ \frac{e_3^{(1)}}{e^{-\lambda(t-t_0)}}\right],$$

which was obtained from the value of parameters $p^\star, p_1, p_2$.

For simplicity of notation, let

$$\sigma_1(t) = e^{\lambda(t-t_0)}(-u_{1d} - \alpha x_{1d}^{(1)})$$
$$\sigma_2(t) = -(\beta+1)e^{\lambda(t-t_0)}x_{1d}^{(1)}$$
$$\sigma_3(t) = \beta(x_{1d}^{(1)})^2$$
$$\sigma_4(t) = \lambda - \alpha$$

then the error model of USV can be regarded as the following system

$$\begin{bmatrix}\dot\omega_3\\ \dot\omega_4\\ \dot\omega_5\\ \dot\omega_6\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \lambda & 1\\ \sigma_1(t) & \sigma_2(t) & \sigma_3(t) & \sigma_4(t)\end{bmatrix}\begin{bmatrix}\omega_3\\ \omega_4\\ \omega_5\\ \omega_6\end{bmatrix} + \begin{bmatrix}0\\1\\0\\0\end{bmatrix}(u_2 - u_{2d})$$

cascaded by

$$\dot\omega_1 = \lambda\omega_1 + \omega_2$$
$$\dot\omega_2 = \lambda\omega_2 + e^{\lambda(t-t_0)}(u - u_{1d}). \tag{20}$$

And the interconnection term is

$$\begin{bmatrix}0\\0\\0\\g(t,z)\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ g_1(t,z) & g_2(t,z)\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix}, \tag{21}$$

where

$$g_1(t,W) = k_1 w_3 + k_1 x_{2d},$$
$$g_2(t,W) = k_2 w_3 - \alpha w_3 - (\beta+1)w_4 + k_2 x_{2d}$$
$$-\alpha x_{2d} - (\beta+1)x_{2d}^{(1)} + 2\beta e^{-\lambda(t-t_0)}x_{1d}^{(1)}w_5$$
$$+2\beta x_{1d}^{(1)}x_{3d} + \beta e^{-2\lambda(t-t_0)}w_2 w_5 + \beta e^{-\lambda(t-t_0)}w_2 x_{3d}$$

Under the assumption $\lambda \neq \alpha$ which ensures the pair (A_0, B_0) given by

$$A_0 = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \lambda & 1\\ (\lambda-\alpha)G & -(\beta+1)G & 0 & \lambda-\alpha\end{bmatrix} B_0 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}.$$

is stabilizable, the controller given in theorem 3.1 renders the tracking error system globally uniformly $\mathcal{K}$-exponentially stable, where K_1 is a stabilizing gain of (20) and K_2 is a stabilizing gain of (A_0, B_0), i.e., $A_0 - B_0K_2$ is Hurwitz.

4.2 Simulation Results

Case A:converging to a point In simulation the reference target's linear velocity and angular velocity are $\exp(-t)$ and $-5\exp(-t) + 6\exp(-2t)$, respectively. The parameters and the initial conditions are selected as $\alpha = 0.4, \beta = 0.8$, $(x_{1d}(0), x_{2d}(0), x_{3d}(0), x_{4d}(0), x_{5d}(0), x_{6d}(0)) = (2, 1, -1, 1, 1, 1)$, $(x_1(0), x_2(0), x_3(0), x_4(0), x_5(0), x_6(0)) = (2.1, 1.2, -0.7, 1.2, 1.2, 0.7)$, respectively. The gain vectors are $K_1 = [-12\ -7], K_2 = [-9.4965\ -4.1850\ -2.0337\ -0.2925]$. Figure 1 shows the moving path of the USV. The tracking errors are presented in Figure 2.

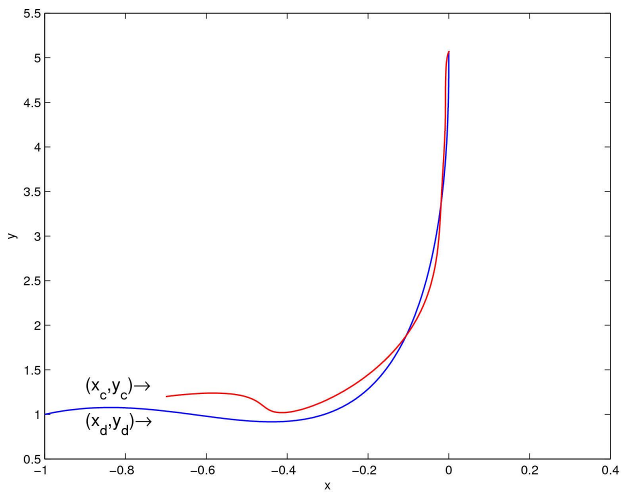

Figure 1: Case A: moving path of the USV.

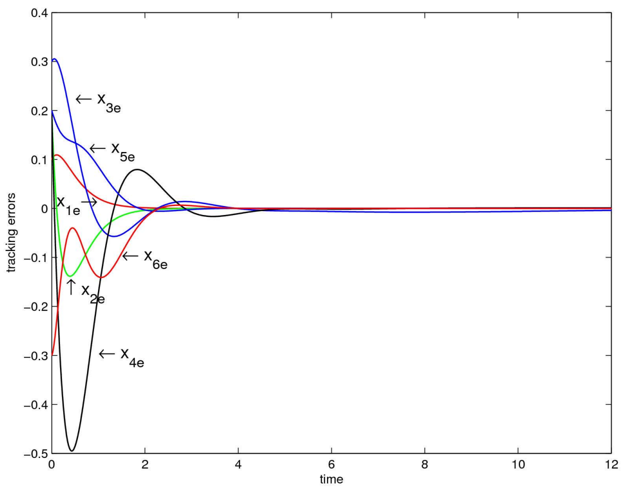

Figure 2: Case A: tracking errors.

Case B:converging to a straight line In this case the reference target's linear velocity and angular velocity are $\sin(t)$ and $-5\exp(-t)\ +6\exp(-2t)$, respectively. The initial conditions are chosen as $(x_{1d}(0), x_{2d}(0), x_{3d}(0), x_{4d}(0), x_{5d}(0), x_{6d}(0)) = (2, 1, -1, 0, 1, 1), (x_1(0), x_2(0), x_3(0), x_4(0), x_5(0), x_6(0)) = (2.1, 1.2, -0.7, 0.2, 1.3, 0.7)$. The gain vectors are $K_1 = [-20\ -9], K_2 = [-6.2192\ -4.5929\ -1.5655\ -0.4327]$. Figure 3 shows the moving path of the USV. The tracking errors are presented in Figure 4.

5 CONCLUSION

This paper concentrates on designing tracking controllers for a general class of nonholonomic dynamic systems, whose reference systems are allowed to converge exponentially. By introducing a time-varying coordinate transformation based on the minimal dilation degree and using the cascade-design method, a global $\mathcal{K}$-exponential controller is derived under some assumptions on the reference system. The main result shows that the popular condition of

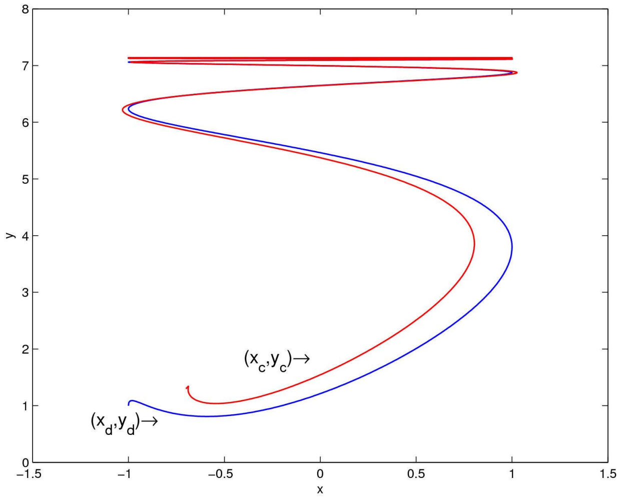

Figure 3: Case B: moving path of the USV.

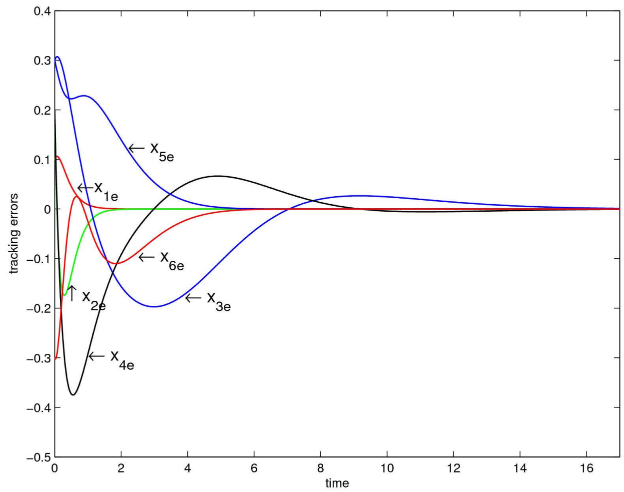

Figure 4: Case B: tracking errors.

persistent excitation or not converging to zero that imposed on the reference target is not necessary.

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