Proceedings of the 2007 American Control Conference
Marriott Marquis Hotel at Times Square
New York City, USA, July 11-13, 2007

An LMI Design of Tracking Controllers for Nonholonomic Chained-Form System

Yu-Ping Tian and Ke-Cai Cao

***Abstract*— Global $\mathcal{K}$-exponential controllers are constructed for the tracking control problem of nonholonomic systems in chained-form whose reference targets are allowed to converge to a point exponentially. By using a novel transformation and the cascade-design method, the tracking control problem is converted into stabilization problem of two simple subsystems. Then an LMI design approach is developed for stabilizing subsystems. The assumption on the reference signal is much more relaxed than those given in the previous papers.**

I. INTRODUCTION

Since the chained-form system was first introduced by [14], it has been extensively studied by many researchers as a benchmark example in the area of control of nonholonomic systems (see, e.g., [12] and references therein). Motivated by a famous theorem of Brockett [3], which implies that there is no smooth or even continuous time-invariant state feedback law to asymptotically stabilize such a nonholonomic system, most efforts have been devoted to the stabilization problem (see e.g., [2] [10] [17] [19]), which still remains to be a very interesting topic today.

However, recent years have also seen increasing interests in the tracking control problem of the chained-form system [8][21][11][13]. A common assumption in most of the published papers regarding the tracking problem is that the reference signal satisfies some conditions such as persistent excitation (PE) or not converging to zero. And the fulfillment of these conditions implies that the desired reference target is “a moving target” which never stops at any point. In the case of tracking control of mobile robots, this assumption implies that the line velocity or angular velocity of the target must not converge to zero [7]. The restriction makes it impossible to treat the tracking problem and the stabilization problem simultaneously. Concerning the tracking for an n-dimensional chained-form system, [8] requires the first input must not converge to zero while [12] and [13] impose a PE condition for the same input, which means that tracking of a stabilized system is impossible. In other studies such as [1], the PE condition is also imposed on the first input of the second-order chained-form system.

Inspired by Samson’s time-varying stabilizer [17], [11] developed a time-varying “universal” controller to achieve both stabilization and tracking of mobile robots. However, as pointed in [11], there is no straight way to extend their method to an n-dimensional chained-from system. Universal controllers for mobile robots are also presented in [5][6]. Since some PE signals are introduced into the virtual control in the design of these universal controllers, the convergence rate is thus not exponential.

The tracking problem becomes much more challenging when the PE condition or not-converging-to-zero condition does not work. The purpose of this study is to construct global $\mathcal{K}$-exponential trackers for the chained-form system when the input of the reference system does not satisfy such conditions. The assumption on the reference signal in this paper is also much more relaxed than those given in our recent work [4][20]. A coordinate transformation based on dilation is introduced to extract the controllable part from the reference system and then the cascade-design method is adopted to transform the stabilization problem of a complicated error system into stabilization problems of two simple subsystems, one of which contains parameter uncertainty. The LMI tool is then used in designing stabilizers for the system with parameter perturbation. The controllers presented in this paper can be applied not only to tracking an exponentially stabilized system but also to tracking some targets moving along lines or circular paths. The obtained results show that the PE-kind condition is not necessary in the global $\mathcal{K}$-exponential tracking problem of the chained-form system.

II. PROBLEM FORMULATION

After a suitable change of coordinates and state feedback, many nonholonomic systems can be transformed into the following chained form [14]

$$\begin{aligned} \dot{x}_1 &= u_1, \\ \dot{x}_2 &= u_2, \\ \dot{x}_3 &= x_2\, u_1, \\ &\vdots \\ \dot{x}_n &= x_{n-1}\, u_1, \end{aligned} \tag{1}$$

where $u = (u_1, u_2)^T$ is the input and $x = (x_1, \cdots, x_n)^T$ is the state. And a vector-valued reference signal $x_d(t) = (x_{1d}(t), \cdots, x_{nd}(t))^T$ is produced by a virtual system which has the same formula as system (1) with inputs u_{1d}, u_{2d}.

Define the tracking error as $e_i = x_i - x_{id}$ and it is easy

This work was supported by the National Natural Science Foundation of China (under grant 60425308) and 863 program of China (under grant 2006AA04Z263)

Yu-Ping Tian and Ke-Cai Cao are with the Department of Automatic Control, Southeast University, Nanjing, 210096, China (yptian@seu.edu.cn, caokecai@163.com)

to obtain the tracking error dynamics given by

$$\begin{aligned}\dot{e}_1 &= u_1 - u_{1d},\\ \dot{e}_2 &= u_2 - u_{2d},\\ \dot{e}_3 &= u_{1d}\, e_2 + x_2(u_1 - u_{1d}),\\ &\vdots\\ \dot{e}_n &= u_{1d}\, e_{(n-1)} + x_{n-1}(u_1 - u_{1d}).\end{aligned} \tag{2}$$

In this paper we study the global tracking problem under the following assumption.

Assumption 1: $u_{1d} = e^{-\lambda(t-t_0)}D(t)$, where λ is a nonnegative constant and there exist nonzero constants D and T ($|D| > T > 0$) such that the bounded continuous $D(t)$ satisfies

$$|D(t) - D| \leq T.$$

Some related work on the tracking control of the chained-form system should be mentioned. [12] and [13] solved the tracking problem for the chained-from system under the PE condition of u_{1d}, which means that u_{1d} is not allowed to converge to zero. [7][8] presented the tracking control law based on state feedback under the assumption that u_{1d} is not converging to zero. Assumption 1 allows the first input u_{1d} to converge to zero exponentially and has no restrictions on the second input u_{2d}. Thus this assumption covers many interesting tracking tasks.

For example, in the tracking control of unicycle mobile robots, the first velocity u_{1d} of system in chained form corresponds to the angular velocity ω_d of the system in Cartesian coordinates [14]. Then, Assumption 1 covers the following tracking tasks.

(1) When $\lambda > 0$, u_{1d} converges to zero exponentially and Assumption 1 includes two important cases of the tracking control problems.
   - movement converging to a set-point: In this case the tracking target is a system whose linear and angular velocities are approaching zero.
   - movement along a straight line: In this case the linear velocity of the target does not converge to zero and it moves, for example, along a straight line.

(2) When $\lambda = 0$, u_{1d} is a constant or converging to a nonzero constant and Assumption 1 implies that the target's trajectory is a circular path.

III. PRELIMINARIES

A. $\mathcal{K}$-Exponential Stability of Cascaded System

Definition 3.1: [12] Consider a nonlinear system $\dot{x} = f(t,x)$. The equilibrium $x = 0$ of this system is said to be globally $\mathcal{K}$-exponentially stable if there exist a class $\mathcal{K}$ function $\alpha(\cdot)$ and a positive constant γ such that for all $t \geq t_0 \geq 0$ and $x(t_0) \in \mathbb{R}^n$ we have

$$\|x(t)\| \leq \alpha(\|x(t_0)\|)\exp(-\gamma(t-t_0)).$$

Consider a time-varying system given by

$$\begin{aligned}\dot{z}_1 &= f_1(t,z_1) + g(t,z_1,z_2)z_2,\\ \dot{z}_2 &= f_2(t,z_2).\end{aligned} \tag{3}$$

We call system (3) a cascaded system because it can be viewed as the following system

$$\Sigma_1 : \quad \dot{z}_1 = f_1(t,z_1) \tag{4}$$

perturbed by the output of the system

$$\Sigma_2 : \quad \dot{z}_2 = f_2(t,z_2). \tag{5}$$

Lemma 3.1: [12][15] The cascaded time-varying system (3) is globally $\mathcal{K}$-exponentially stable if the following assumptions are satisfied:

(1) the subsystems (4) is globally uniformly exponentially stable (GUES);

(2) the function $g(t,z_1,z_2)$ satisfies the following condition for all $t \geq t_0$

$$\|g(t,z_1,z_2)\| \leq \theta_1(\|z_2\|) + \theta_2(\|z_2\|)\|z_1\|,$$

where $\theta_i(\cdot) : R^+ \to R^+$ $(i = 1,2)$ are continuous functions;

(3) the subsystem (5) is globally $\mathcal{K}$-exponentially stable.

B. Quadratic Stabilization of Uncertain Systems

Consider the following system

$$\dot{x} = (A + \Delta A(t))x, \tag{6}$$

where $A \in R^{n\times n}$,$\Delta A(t) \in R^{n\times n}$ and $\Delta A(t) = E\Sigma(t)F$ with $E, F \in R^{n\times n}$ and unknown $\Sigma(t)$ belongs to the following set:

$$\Omega = \{\Sigma(t) | \Sigma(t)^T\Sigma(t) \leq I, \forall t\}.$$

Lemma 3.2: [9][16] System (6) is quadratically stable if there exist a positive definite matrix P and scalar a such that for all t

$$x^T(t)[A^TP + PA]x(t) + 2x(t)\Delta A^T(t)Px(t) \leq -a||x(t)||^2$$

is satisfied.

As shown in [18], from Lemma 3.2 it can be further obtained that if a system is quadratically stable, then the system is exponentially stable for all $\Delta A(t)(\Sigma(t) \in \Omega)$, and for any initial condition $x(0)$ the following inequality holds

$$||x(t)||^2 \leq \frac{\sigma_{\max}(P)}{\sigma_{\min}(P)}||x(0)||^2\exp(-\frac{a}{\sigma_{\max}(P)}t), \forall t \geq 0,$$

where $\sigma_{\max}(P)$ and $\sigma_{\min}(P)$ means the maximum and minimum eigenvalues of matrix P, respectively.

Lemma 3.3: [9][16] System (6) is quadratically stable if and only if A is stable and $||F(sI - A)^{-1}E||_\infty < 1$.

IV. MAIN RESULTS

A. Cascaded System Based on Dilation

Introduce a coordinate transformation for system (2) as

$$\begin{aligned} y_1 &= e_1/\exp(-(n-2)\lambda(t-t_0)),\\ y_2 &= e_2,\\ y_3 &= e_3/\exp(-\lambda(t-t_0)),\\ &\vdots\\ y_i &= e_i/\exp(-(i-2)\lambda(t-t_0)),\\ &\vdots\\ y_n &= e_n/\exp(-(n-2)\lambda(t-t_0)). \end{aligned} \tag{7}$$

Then the transformed system is given by

$$\begin{bmatrix} \dot y_2\\ \dot y_3\\ \vdots\\ \dot y_{n-1}\\ \dot y_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0\\ D(t) & \lambda & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & (n-3)\lambda & 0\\ 0 & 0 & \cdots & D(t) & (n-2)\lambda \end{bmatrix} \begin{bmatrix} y_2\\ y_3\\ \vdots\\ y_{n-1}\\ y_n \end{bmatrix}$$

$$+\begin{bmatrix}1\\0\\ \vdots\\0\\0\end{bmatrix}(u_2-u_{2d}) + \begin{bmatrix} 0\\ x_2\frac{u_1-u_{1d}}{\exp(-\lambda(t-t_0))}\\ \vdots\\ x_{n-2}\frac{u_1-u_{1d}}{\exp(-(n-3)\lambda(t-t_0))}\\ x_{n-1}\frac{u_1-u_{1d}}{\exp(-(n-2)\lambda(t-t_0))} \end{bmatrix} \tag{8}$$

and

$$\dot y_1 = (n-2)\lambda y_1 + (u_1-u_{1d})\exp((n-2)\lambda(t-t_0)). \tag{9}$$

Denote $Y=[y_2, y_3, \cdots, y_n]^T$ and $\Sigma(t) = \frac{D(t)-D}{T}, v = u_2 - u_{2d}$. Then the transformed system (8) (9) can be viewed as the following system

$$\dot Y = (A + E\Sigma(t)F)Y + Bv \tag{10}$$

cascaded by the system (9) with the interconnection term

$$\begin{bmatrix} 0\\ x_2\frac{u_1-u_{1d}}{\exp(-\lambda(t-t_0))}\\ \vdots\\ x_{n-2}\frac{u_1-u_{1d}}{\exp(-(n-3)\lambda(t-t_0))}\\ x_{n-1}\frac{u_1-u_{1d}}{\exp(-(n-2)\lambda(t-t_0))} \end{bmatrix}, \tag{11}$$

where A, B, E, F are defined as

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0\\ D & \lambda & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & (n-3)\lambda & 0\\ 0 & 0 & \cdots & D & (n-2)\lambda \end{bmatrix}, B = \begin{bmatrix}1\\0\\ \vdots\\0\\0\end{bmatrix},$$

$$E = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0\\ 1 & 0 & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & 0\\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, F = \begin{bmatrix} T & 0 & \cdots & 0 & 0\\ 0 & T & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & T & 0\\ 0 & 0 & \cdots & 0 & T \end{bmatrix},$$

respectively.

B. Design of Controller u_1

Under the control law $u_1 = u_{1d} - \exp(-(n-2)\lambda(t-t_0))k_1 y_1$ with $k_1 > (n-2)\lambda$, the following closed-loop system of system (9)

$$\dot y_1 = (-k_1 + (n-2)\lambda)y_1 \tag{12}$$

is globally exponentially stable.

C. Design of Controller u_2

Now we show that there exists a state-feedback law $v = KY$ such that the closed-loop system

$$\dot Y = (A + BK + E\Sigma(t)F)Y \tag{13}$$

is quadratically stable.

By Lemma 3.3, system (13) is quadratically stable if and only if $A+BK$ is stable and $||F(sI-A-BK)^{-1}E||_\infty < 1$.

According to the H_∞ control theory [18], $A + BK$ is stable and $||F(sI - A - BK)^{-1}E||_\infty < 1$ if and only if there exists a positive definite matrix P such that

$$(A+BK)^T P + P(A+BK) + PE(PE)^T + F^T F < 0. \tag{14}$$

By Schur's Complement Lemma the inequality (14) is equivalent to the following inequality

$$\begin{bmatrix} (A+BK)^T P + P(A+BK) & PE & F^T\\ E^T P & -I & 0\\ F & 0 & -I \end{bmatrix} < 0. \tag{15}$$

And (15) can be equivalently written as

$$\begin{bmatrix} P^{-1}(A+BK)^T + (A+BK)P^{-1} & E & (FP^{-1})^T\\ E^T & -I & 0\\ FP^{-1} & 0 & -I \end{bmatrix} < 0$$

by pre multiplying and post multiplying $diag\{P^{-1}, I, I\}$. Letting $X = P^{-1}, W = KX$, the above inequality can also be rewritten as

$$\begin{bmatrix} (AX+BW)^T + (AX+BW) & E & (FX)^T\\ E^T & -I & 0\\ FX & 0 & -I \end{bmatrix} < 0. \tag{16}$$

If there exist $X^\star$ and $W^\star$ such that (16) is satisfied, we say the above LMI is solvable.

D. Main Theorem

Theorem 4.1: Assume that u_{1d} satisfies Assumption 1 and that $x_{2d}, x_{3d}, \cdots, x_{(n-1)d}$ are bounded. Then, the state feedback control

$$u_1 = u_{1d} - \exp(-(n-2)\lambda(t-t_0))k_1 y_1 \quad (17)$$
$$u_2 = u_{2d} + W^\star (X^\star)^{-1} Y \quad (18)$$

globally $\mathcal{K}$-exponentially stabilize the tracking error system (2), where $k_1 > (n-2)\lambda$ and $W^\star, X^\star$ are solutions of (16).

Proof: Since the system formed by (8)(9) can be regarded as a system (10) cascaded by (9). Now we check three conditions of Lemma 3.1 for the cascaded system.

- Since the quadratical stability implies the exponential stability, the GUES property of (10) is obtained from the controller designed in Section IV-C.
- Under the control law u_1, the interconnection term (11) of system (10) and (9) can be rewritten as

  $$g(t, Y, Z)Z,$$

  where $g(t, Y, Z)$ is defined as

  $$-k_1 e^{-(n-3)\lambda(t-t_0)} \begin{bmatrix} 0 \\ y_2 \\ \vdots \\ y_{n-1} \end{bmatrix} - k_1 \begin{bmatrix} 0 \\ x_{2d} e^{-(n-3)\lambda(t-t_0)} \\ \vdots \\ x_{(n-1)d} \end{bmatrix}$$

  and $Z = y_1$ and Y is defined in Section IV-A. Since $x_{2d}, \cdots, x_{(n-1)d}$ are bounded, the second condition is satisfied.
- The GUES property of system (9) follows immediately from the controller constructed in Section IV-B. Therefore, the third condition is also satisfied.

Therefore, by Lemma 3.1 we conclude that the cascaded system formed by (8) and (9) is globally $\mathcal{K}$-exponentially stable. According to the transformation (7) we know that under the control input (17) (18) the original chained-form tracking error system (2) is globally $\mathcal{K}$-exponentially stable. ■

V. APPLICATION

Now we apply the control law proposed in Theorem 4.1 to a mobile robot described by

$$\begin{aligned} \dot{x}_c &= v \cos\theta_c, \\ \dot{y}_c &= v \sin\theta_c, \\ \dot{\theta}_c &= \omega. \end{aligned} \quad (19)$$

And the reference trajectory is generated by a virtual mobile robot which shares the same formula as system (19) with reference input v_d and ω_d. In the following discussion we let $t_0 = 0$ for simplicity.

A. case 1: $\lambda = 0.5 > 0$

If we set $\omega_d = \exp(-0.5t)(1 + 0.5\sin(t))$, then it is easy to check that $u_{1d} = \omega_d = \exp(-0.5t)(1+0.5\sin(t))$, $u_{2d} = v_d - (x_d \sin(\theta_d) - y_d \cos(\theta_d))\omega_d$ in chained form and $\lambda = 0.5, T = 0.5, D = 1$ in Assumption 1.

Initial conditions of reference system in chained form and the error systems are selected as $[-2.9856\ 0.1570\ 1.3153]$ and $[-0.080\ 0.8415\ -0.2579]$ respectively. Using the LMI tool in Matlab, the following

$$X^\star = \begin{bmatrix} 5.2461 & -1.964 \\ -1.964 & 1.2180 \end{bmatrix}, W^\star = \begin{bmatrix} -7.7397 & -1.0523 \end{bmatrix}$$

are obtained. Then control gain $W^\star(X^\star)^{-1} = [-4.5362\ -8.1771]$ and set $k_1 = 3$ where $k_1 > (n-2)\lambda$ is satisfied.

This paper does not impose any restrictions on u_{2d}. In simulation the following two cases are considered:

- $v_d = \exp(-0.5t)$

  In this case the reference target asymptotically approaches a steady point. Fig. 1 shows the move path of the mobile robot on the plane in this case. And the tracking errors versus time are presented in Fig. 2.

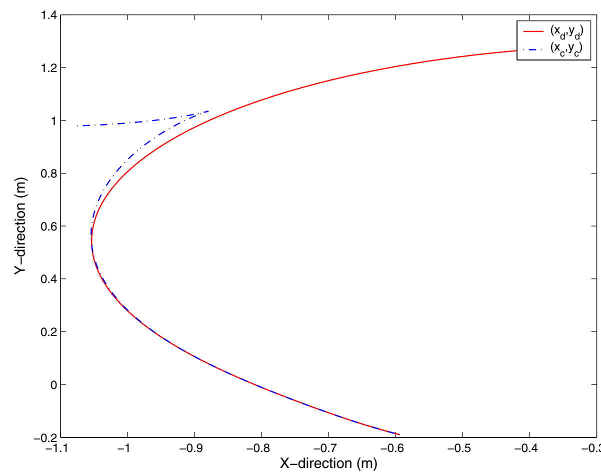

Fig. 1. Move path of mobile robots

- $v_d = \sin t$

  In this case the reference target is in a periodic movement along a line when time goes to infinite. Fig. 3 shows the move path of the mobile robot on the plane in this case. And the tracking errors versus time are presented in Fig. 4.

B. case 2: $\lambda = 0$

In this case initial conditions of reference system and error systems are the same as those in Section V-A. Two subcases that the target moves along the circular path with constant line velocity and line velocity approaching to zero are considered respectively.

- $v_d = 1, \omega_d = 1 + 0.05\sin(t)$

  In this case $D = 1$ and $T = 0.05$. Using the tool of LMI in Matlab it is easy to obtain that

  $$X^\star = \begin{bmatrix} 2.1474 & -1.059 \\ -1.059 & 2.1434 \end{bmatrix}, W^\star = \begin{bmatrix} -0.8172 & -2.1398 \end{bmatrix}$$

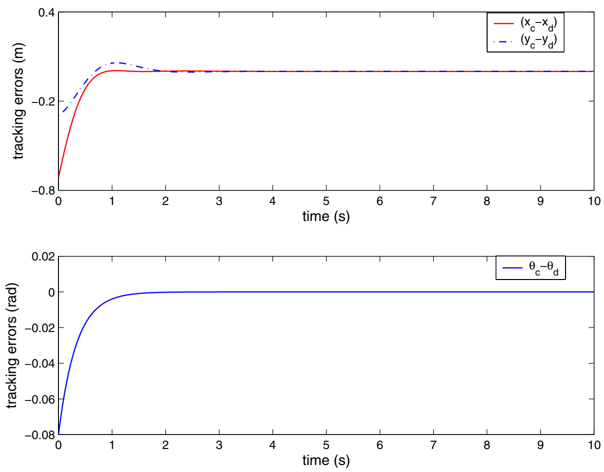

Fig. 2. Tracking errors

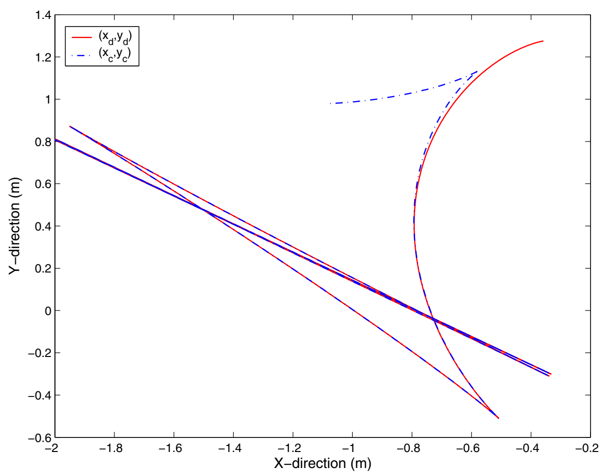

Fig. 3. Move path of mobile robots

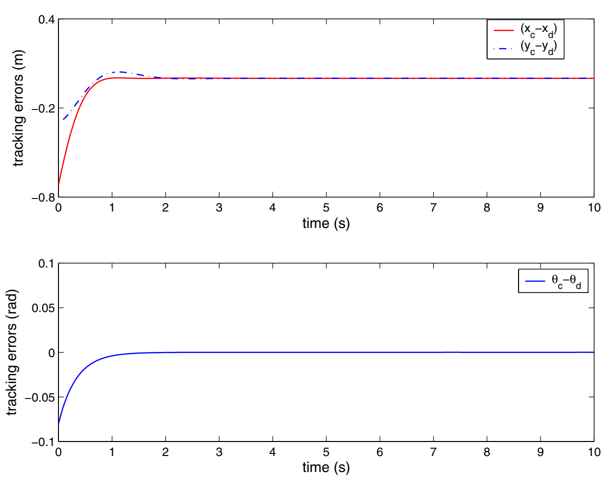

Fig. 4. Tracking errors

and control gain $W^\star(X^\star)^{-1} = \begin{bmatrix} -1.1543 & -1.5688 \end{bmatrix}$. Simulation results show that the reference target moves along a circular-like path with constant line velocity 1. Due to the perturbations the reference trajectory is not a circle. Fig. 5 shows the move path of the mobile robot on the plane. And the tracking errors versus time are presented in Fig. 6.

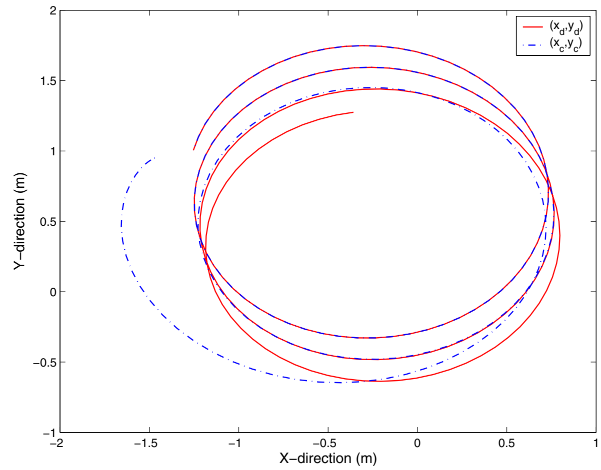

Fig. 5. Move path of mobile robots

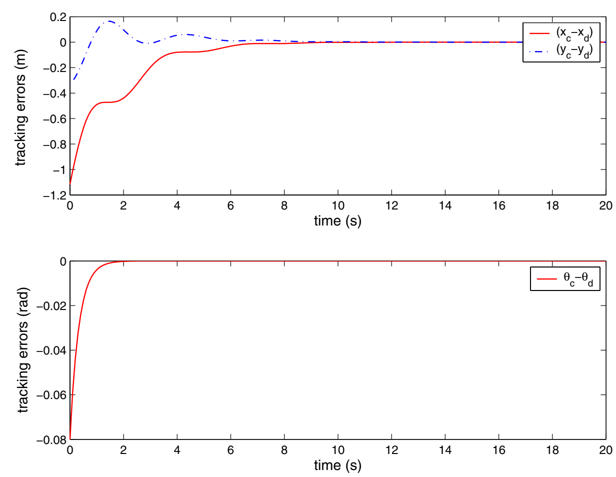

Fig. 6. Tracking errors

- $v_d = 1.5t - 1.5t/(t+10), \omega_d = 1 + 2t/(t+10) + 0.05\sin(t)$

  In this case $D = 3$ and $T = 0.05$. Using the tool of LMI in Matlab it is easy to obtain that

$$X^\star = \begin{bmatrix} 1.3917 & -0.2208 \\ -0.2208 & 1.3917 \end{bmatrix}, W^\star = \begin{bmatrix} -0.699 & -4.175 \end{bmatrix}$$

and then control gain $W^\star(X^\star)^{-1} = [-1.0037 \ -3.1592]$. Simulation results show that the reference target moves along a circular path with line velocity approaching to zero. Fig. 7 shows the move path of the mobile robot on the plane. And the tracking errors versus time are presented in Fig. 8.

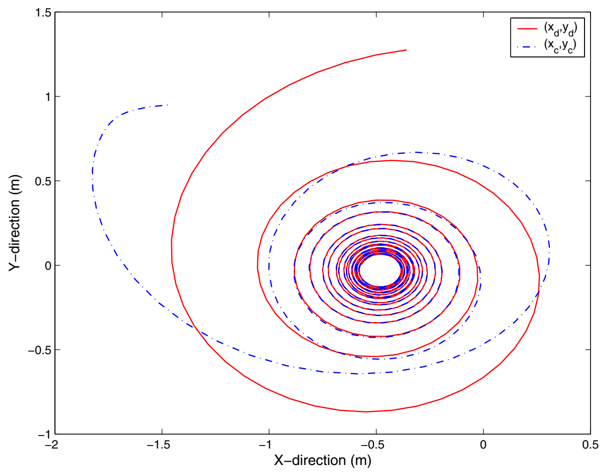

Fig. 7. Move path of mobile robots

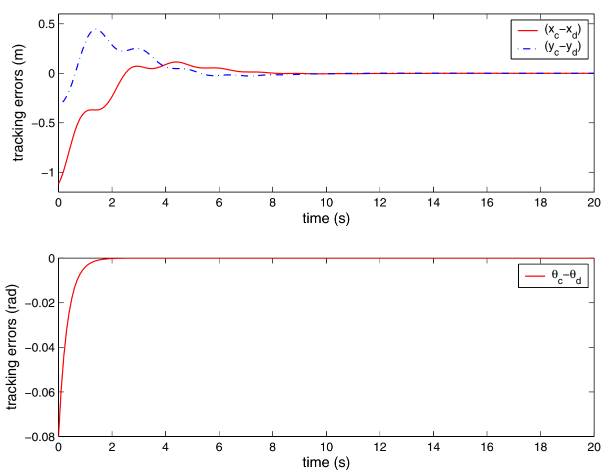

Fig. 8. Tracking errors

By Theorem 4.1, we can also design tracking controllers for the articulated vehicles which can be transformed into a 5-dimensional chained-form system. Here we omit the simulation due to the space limitation.

VI. CONCLUSIONS

Global $\mathcal{K}$-exponential tracking controllers are constructed for the tracking control problem of nonholonomic systems in chained form. A coordinate transformation based on dilation is introduced and then the cascade-design method is adopted to transform the tracking problem into two simple stabilizing problems. The results show that the popular condition of persistent excitation or not converging to zero is not necessary for the tracking control of nonholonomic systems in chained form.

REFERENCES

[1] N.P. Aneke, H. Nijimeijer, A.G.de Jager, "Tracking control of second-order chained form systems by cascade backstepping", *International Jounal of Robust and Nonlinear Control*, vol. 13, no. 2, 2003, pp. 95-115.

[2] A. Astolfi, "Discontinuous control of nonholonomic systems", *System & Control Letters*, vol. 27, no. 1, 1996, pp. 37-45.

[3] R.W. Brockett, "Asymptotic stability and feedback stabilization", *Differential Geometric Control Theory*, BirkhaK user, Boston, MA, USA; 1983, pp. 181-191.

[4] K.-C. Cao, Y.-P. Tian, "Tracking control of nonholonomic chained-form systems by output feedback", *IEEE Regional Inter-University Postgraduate Electrical and Electronics Engineering Conference*, Macau, 2006.

[5] K.D. Do, Z.-P. Jiang, J. Pan, "A global output-feedback controller for simultaneous tracking and stabilization of unicycle-type mobile robots", *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, 2004, pp. 589-594.

[6] K.D. Do, Z.-P. Jiang, J. Pan, "Simultaneous tracking and stabilization of mobile robots: an adaptive approach", *IEEE Transactions on Automatic Control*, vol. 49, no. 7, 2004, pp. 1147-1152.

[7] Z.-P. Jiang, H. Nijmeijer, "Tracking control of mobile robots: a case study in backstepping", *Automatica*, vol. 33, no. 7, 1997, pp. 1393-1399.

[8] Z.-P. Jiang, "A recursive technique for tracking control of nonholonomic systems in the chained form", *IEEE Transactions on Automatic Control*, vol. 44, no. 2, 1999, pp. 265-279.

[9] P.P. Khargonekar, I.R. Petersen, K. Zhou, "Robust stabilization of uncertain linear systems: quadratic stabilizability and H^∞ control theory", *IEEE Transactions on Automatic Control*, vol. 35, no. 3, 1990, pp. 356-361.

[10] I. Kolmanovsky, N.H. McClamroch, "Developments in nonholonomic control systems", *IEEE Control Systems Magazine*, vol. 15, no. 6, 1995, pp. 20-36.

[11] T.-C. Lee, K.-T. Song, C.-H. Lee, & C.-C. Teng, "Tracking control of unicycle modeled mobile robots using a saturation feedback controller", *IEEE Transactions Control Systems Technology*, vol. 9, no. 2, 2001, pp. 305-318.

[12] E. Lefeber, *Thesis: Tracking Control Of Nonlinear Mechanical Systems*, University of Twente, The Netherlands; 2000.

[13] E. Lefeber, A. Robertsson, H. Nijmeijer, "Linear controllers for exponential tracking of systems in chained-form", *International Journal of Robust Nonlinear Control*, vol. 10, no. 4, 2000, pp. 243-263.

[14] R.M. Murray, S.S. Sastry, "Noholonomic motion planning: steering using sinusoids", *IEEE Transactions on Automatic Control*, vol. 38, no. 5, 1993, pp. 700-716.

[15] E. Panteley, E. Lefeber, A. Loría and H. Nijmeijer, "Exponential tracking control of a mobile car using a cascaded approach", *Proceedings of the IFAC Workshop on Motion Control*, Grenoble, France, 1998, pp. 221-226.

[16] I.R. Petersen, D.C. McFarlane, "Optimal guaranteed cost control and filtering for uncertain linear systems", *IEEE Transactions on Automatic Control*, vol. 39, no. 9, 1994, pp. 1971-1977.

[17] C. Samson, "Control of chained system application to path following and time-varying point-stabilization of mobile robots", *IEEE Transactions on Automatic Control*, vol. 40, no, 1, 1995, pp. 64-77.

[18] T.-L. Shen, *Theory and Application of H_∞*, Press of Tinghua: Beijing (in Chinese), 1996.

[19] Y.-P. Tian, S. Li, "Exponential stabilization of nonholonomic dynamic systems by smooth time-varying control", *Automatica*, vol. 38, no. 7, 2002, pp. 1139-1146.

[20] Yu-Ping Tian, Ke-Cai Cao, "Time-varying linear controllers for exponential tracking of non-holonomic systems in chained form", *International Journal of Robust and Nonlinear Control*, in Press, 2006.

[21] G. Walsh, D. Tilbury, R. Murry and J.P. Launond, "Stabilization of trajectories for systems with nonholonomic constraints", *IEEE Transactions on Automatic Control*, vol. 39, 1994, pp. 216-222.