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Simultaneous Stabilization and Tracking of Wheeled Mobile Robots via Chained Form*

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***Abstract*—A time-varying controller is proposed to simultaneously solve the stabilization and the tracking problem of wheeled mobile robots via their equivalent chained form. The control is designed based on Lyapunov's direct method and the backstepping technique. The outstanding feature of the proposed controller is computationally simple due to its full use of the existing results on stabilization and tracking control for the equivalent chained form systems of wheeled mobile robots. Simulation results for a wheeled mobile robot are provided to illustrate the effectiveness of the proposed controller.**

I. INTRODUCTION

In the past decade, the control of mechanical systems with nonholonomic constraints has attracted considerable attention in the control community. Mobile robots with two independent drive wheels, or their feedback equivalent chained form systems, have served as benchmark mechanical example with substantial engineering interest. The challenge of controlling such systems comes from the fact that the motion of a wheeled mobile robot in a plane possesses three degrees of freedom (DOF); while there are only two available control inputs under the nonholonomic constraint. By applying Brockett's theorem [1], it is shown that a nonholonomic system cannot be asymptotically stabilized by smooth or even continuous, pure-state feedback. Consequently, a good number of novel ideas and feedback design strategies have been proposed for the stabilization problem, including: discontinuous time-invariant feedback [2]– [4], smooth time-varying feedback [5]– [9], and hybrid feedback [10], [11]. See the survey paper [12] for more details. Specifically, the first time-varying control method was proposed by Samson in [5] for the stabilization of cart. This approach was further developed for the stabilization of a car-like mobile robot with a steering wheel [6]. The essential idea of Samson's result is using a heat function to provide "persistent excitation" and a feedback term to introduce "dissipation".

In parallel, the tracking control problem of nonholonomic systems has also received a great deal of attention because of its practical importance. In the case of mobile robot, solutions to trajectory tracking were introduced in [13]– [16], and [17]–[22] to cover a broader class of systems in the chained form. To ensure asymptotic tracking, the controllers in these references require that the reference velocities satisfy some kind of persistent excitation (PE) condition. For example, in [13], the reference linear velocity must nonzero; in [14], the reference angular velocity is assumed to satisfy the PE condition; and in [15] and [16], the reference linear velocity or angular velocity must not converge to zero. Roughly speaking, the fulfillment of a PE condition implies that the desired reference trajectory is "a moving trajectory", instead of fixed set-point. These assumptions make it impossible for a single controller to solve both the stabilization problem and the tracking problem.

The stabilization and tracking problems of nonholonomic systems are studied separately. However, in practical applications, it is preferable to solve the stabilization problem and the tracking problem simultaneously using a single controller. This problem was solved in [23] for the first time by introducing a time-varying velocity feedback controller to achieve both stabilization and tracking of unicycle-modeled mobile robots at the kinematics level. Recently, Do et al. [24], [25] focused on the dynamic model using the backstepping technique, where the torque and force were taken as the control inputs. However, their controller is quite complicated and computationally demanding, and the control gains must be selected very carefully to satisfy some constraints.

Inspired by Samson's time-varying feedback method, this paper presents a novel controller at the torque level to simultaneously solve the stabilization and the tracking problem of mobile robot. The outstanding feature of our controller is computationally simple due to its fully use of the existing results on stabilization [6] and tracking control [20], [21] for the equivalent chained form systems of wheeled mobile robots. The controller is developed based on backstepping technique and Lyapunov's direct method, and guarantees the global asymptotic convergence of the regulation and tracking error to the origin. Simulation results for a wheeled mobile robot are presented to illustrate the effectiveness of the proposed controller.

II. PROBLEM FORMULATION

As shown in Fig. 1, the configuration of the differential-drive wheeled mobile robot (WMR) can be described by $q = [x, y, \theta]^T$, where (x, y) are the coordinates of the midpoint Q between the two driving wheels, and θ is the orientation angle of the mobile robot. The mobile base satisfies the "pure rolling without side slipping" condition, which is described by the following nonholonomic constraint:

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \tag{1}$$

Define the constraint matrix $J(q)$ as

$$J(q) = [\sin\theta, -\cos\theta, 0] \tag{2}$$

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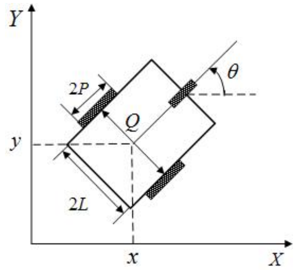


Fig. 1. Wheeled mobile robot configuration.

The constraint (1) can be rewritten as

$$J(q)\dot{q} = 0 \tag{3}$$

According to the Euler-Lagrangian formulation, the dynamics of the mobile robot can be described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J^T(q)\lambda + B(q)\tau \tag{4}$$

where $M(q) \in R^{3\times 3}$ is the symmetric, positive definite inertia matrix, $C(q,\dot{q}) \in R^{3\times 3}$ is the centripetal and coriolis matrix, $G(q) \in R^3$ is the gravitational vector, $J(q) \in R^{1\times 3}$ is the constrained matrix, $\lambda \in R$ is the associated Lagrange multiplier, $\tau \in R^2$ is the vector of control inputs force, $B(q) \in R^{3\times 2}$ is the input transformation matrix.

Let $S(q) \in R^{3\times 2}$ be a full rank matrix defined by

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

It is easy to verify that $S^T(q)J^T(q) = 0$, then there exists an auxiliary vector of independent generalized velocities $v \in R^2$ that make the whole system (3) and (4) be transformed into a more appropriate representation for control purposes:

$$\dot{q} = S(q)v \tag{6}$$

$$M_1(q)\dot{v} + C_1(q,\dot{q})v + G_1(q) = B_1(q)\tau \tag{7}$$

where $M_1(q) = S^T M(q) S, C_1(q,\dot{q}) = S^T(M(q)\dot{S} + C(q,\dot{q})S), G_1(q) = S^T G(q), B_1(q) = S^T B(q)$.

For ease of controller design in this article, the existing results for the control of nonholonomic canonical forms in the literature are exploited [26]. Considering the coordinate transformation$z = T_1(q)$ and state feedback $v = T_2(q)u$ which are defined as

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{9}$$

Using the above coordinate transformation, the kinematic model of WMR given by Eq. (6) can be converted to the chained form

$$\dot{z}_1 = u_1, \ \dot{z}_2 = u_1 z_3, \ \dot{z}_3 = u_2 \tag{10}$$

where $z = [z_1, z_2, z_3]^T \in R^3$ is the state, $u = [u_1, u_2]^T \in R^2$ is the generalized velocity vector. Based on the above transformation, the dynamic model is converted as

$$M_2(z)\dot{u} + C_2(z,\dot{z})u + G_2(z) = B_2(z)\tau \tag{11}$$

where $M_2(z) = T_2^T(q)M_1(q)T_2(q)|_{q=T_1^{-1}(z)}, C_2(z,\dot{z}) = T_2^T(q)(M_1(q)\dot{T}_2(q) + C_1(q,\dot{q})T_2(q))|_{q=T_1^{-1}(z)}, G_2(z) = T_2^T(q)G_1(q)|_{q=T_1^{-1}(z)}, B_2(z) = T_2^T(q)B_1(q)|_{q=T_1^{-1}(z)}$. The following properties of the model (11) can be easily proved:
Property 1: Matrix $M_2(z)$ is symmetric positive definite; $M_2(z), C_2(z,\dot{z})$ and $G_2(z)$ are bounded.
Property 2: $\dot{M}_2 - 2C_2$ is a skew-symmetric matrix.

We assume that the reference trajectory is generated by the virtual robot:

$$\begin{aligned} \dot{x}_d &= v_{1d}\cos(\theta_d) \\ \dot{y}_d &= v_{1d}\sin(\theta_d) \\ \dot{\theta}_d &= v_{2d} \end{aligned} \tag{12}$$

where $q_d = [x_d, y_d, \theta_d]^T$ are the position and orientation of the virtual robot. $v_d = [v_{1d}, v_{2d}]^T$ are the velocities of the virtual robot. Applying the coordinate transformation $z_d = T_1(q_d)$ and the state feedback $v_d = T_2(q_d)u_d$, so that z_d and u_d satisfy the equation(10). It is assumed that $u_d = [u_{1d}, u_{2d}]^T$ satisfy the following assumption:
Assumption 1: The reference signals $u_{1d}, u_{2d}, \dot{u}_{1d}$ and $\dot{u}_{2d}$ are bounded. In addition, one of the following conditions holds:

C 1) There exist T, $\mu_1 > 0$ such that

$$\int_t^{t+T} (u_{1d}^2(s) + u_{2d}^2(s))\mathrm{d}s \geq \mu_1, \ \forall t \geq 0 \tag{13}$$

C 2) There exists $\mu_2 > 0$ such that

$$\int_0^{\infty} (|u_{1d}(s)| + |u_{2d}(s)|)\mathrm{d}s \leq \mu_2 \tag{14}$$

In this paper, our purpose is to design a (single) continuous feedback control law τ for the system (10)-(11) that simultaneously solves stabilization and tracking for a desirable reference trajectory $z_d(t)$, in particular,

$$\lim_{t\to\infty} (z(t) - z_d(t)) = 0 \tag{15}$$

Remark 1: It should be noted that asymptotic convergence of the regulation and tracking error to the origin is achieved in pur paper. Based on dynamic oscillators [27] and transverse functions [28], unified frameworks were provided for both the tracking and the regulation problem, however, only uniformly ultimately bounded rather than asymptotic convergence of the tracking error was achieved.

III. Control Design

In this section, the controller at torque level is designed using Lyapunov-based approach and the backstepping technique. The structure of the control system is illustrated in Fig. 2.

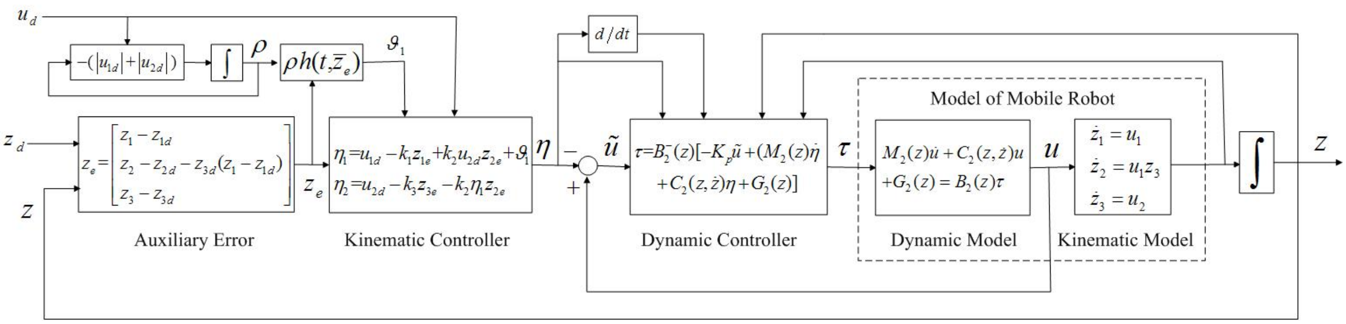


Fig. 2. Structure of the proposed control system.

A. Kinematic Control

As a precursor to the full dynamic system study, let us first neglect the dynamics of system (10)-(11) and consider the kinematic tracking problem only. Now u is assumed to be the virtual control, the object is to design the control input u such that (15) holds. To facilitate controller design, an appropriate tracking error is introduced as follows:

$$z_e=\begin{bmatrix} z_1-z_{1d} \\ z_2-z_{2d}-z_{3d}(z_1-z_{1d}) \\ z_3-z_{3d} \end{bmatrix} \tag{16}$$

Then, the following error model can be attained:

$$\begin{aligned} \dot{z}_{1e} &= u_1-u_{1d} \\ \dot{z}_{2e} &= u_1z_{3e}-u_{2d}z_{1e} \\ \dot{z}_{3e} &= u_2-u_{2d} \end{aligned} \tag{17}$$

Considering the following Lyapunov function candidate

$$V_1=\frac{1}{2}(z_{1e}^2+k_2z_{2e}^2+z_{3e}^2) \tag{18}$$

where k_2 is a positive constant. Along the trajectories of (17), the time derivative of is given as

$$\dot{V}_1=z_{1e}(u_1-u_{1d}-k_2u_{2d}z_{2e})+z_{3e}(u_2-u_{2d}+k_2u_1z_{2e}) \tag{19}$$

Choosing the control u as

$$\begin{aligned} u_1 &= u_{1d}-k_1z_{1e}+k_2u_{2d}z_{2e}+\vartheta_1 \\ u_2 &= u_{2d}-k_3z_{3e}+k_2u_1z_{2e} \end{aligned} \tag{20}$$

where k_1, k_3 are positive constants. Then substituting the expression of u into (19), we have

$$\dot{V}_1=-k_1z_{1e}^2-k_3z_{3e}^2+\vartheta_1z_{1e} \tag{21}$$

Moreover, the time-varying signal ϑ_1 is defined by

$$\vartheta_1=\rho(t)h(t,\bar{z}_e) \tag{22}$$

with

$$\dot{\rho}=-(|u_{1d}(t)|+|u_{2d}(t)|)\rho,\ \rho(0)=1 \tag{23}$$

and $\bar{z}_e=[z_{2e},z_{3e}]^T$, $h(t,\bar{z}_e)$ is a function of class C^{p+1}, with all successive partial derivatives uniformly bounded with respect to t, and is required to have the following two properties:

P 1) $h(t,\bar{z}_e)$ is uniformly bounded with respect to t and $\bar{z}_e$, i.e., there exists a constant $h_0>0$, such that

$$|h(t,\bar{z}_e)|\leq h_0,\ \forall t>0, \bar{z}_e\in R^2 \tag{24}$$

P 2) $h(t,0)=0$, and there is a time-diverging sequence $\{t_i\}_{i\in N}$, and a positive continuous function $\alpha(\cdot)$, such that

$$\|\bar{z}_e\|\geq l>0 \Longrightarrow \sum_{j=1}^{j=p}\left(\frac{\partial^j h}{\partial t^j}(t_i,\bar{z}_e)\right)^2\geq\alpha(l)>0,\ \forall i \tag{25}$$

where N denotes the set of natural numbers.

Remark 2: The conditions imposed by P 1 and P 2 upon $h(t,\bar{z}_e)$ are not severe and can be easily met. For example, the following three functions all satisfy the properties

$$\begin{aligned} h(t,\bar{z}_e) &= h_0\tanh(a\|\bar{z}_e\|^b)\sin(ct) \\ h(t,\bar{z}_e) &= \frac{2h_0\|\bar{z}_e\|^b}{1+\|\bar{z}_e\|^{2b}}\sin(ct) \\ h(t,\bar{z}_e) &= \frac{2h_0}{\pi}\arctan(a\|\bar{z}_e\|^b)\sin(ct) \end{aligned}$$

with $a\neq 0, b>0, c\neq 0$.

Based on the above analysis, the original system (17) is transformed into

$$\begin{aligned} \dot{z}_{1e} &= -k_1z_{1e}+k_2u_{2d}z_{2e}+\vartheta_1 \\ \dot{z}_{2e} &= u_1z_{3e}-u_{2d}z_{1e} \\ \dot{z}_{3e} &= -k_2u_1z_{2e}-k_3z_{3e} \end{aligned} \tag{26}$$

Before giving the stability analysis of closed-loop system (26), we first present a technical lemma.

Lemma 1 *Let $V: R^+\to R^+$ be continuously differentiable and $W: R^+\to R^+$ uniformly continuous satisfying that, for each $t\geq 0$,*

$$\dot{V}(t)\leq -W(t)+p_1(t)V(t)+p_2(t)\sqrt{V(t)} \tag{27}$$

with both $p_1(t)$ and $p_2(t)$ are non-negative and belong to L_1-space. Then, there exists a constant c, such that $W(t)\to 0$ and $V(t)\to c$ as $t\to\infty$.

Proof: First, we prove that $V(t)$ is bounded. According to (27), we have

$$\dot{V}(t)\leq p_1(t)V(t)+p_2(t)\sqrt{V(t)} \tag{28}$$

which implies the following inequality when $V \neq 0$:

$$\frac{\mathrm{d}(\sqrt{V(t)})}{\mathrm{d}t} \leq \frac{p_1(t)}{2}\sqrt{V(t)} + \frac{p_2(t)}{2} \quad (29)$$

When $V = 0$, it can be verified that the upper right-hand derivative $D^+(\sqrt{V(t)}) \leq p_2(t)/2$, Hence, $D^+(\sqrt{V(t)})$ satisfies (29) for all values of V. By the comparison lemma, $\sqrt{V(t)}$ satisfies the inequality

$$\begin{aligned} \sqrt{V(t)} \quad \leq \quad & \left(\sqrt{V(0)} + \int_0^t \exp(-\int_0^s \frac{p_1(\tau)}{2}\mathrm{d}\tau)\frac{p_2(s)}{2}\mathrm{d}s\right) \\ & \times \exp\left(\int_0^t \frac{p_1(s)}{2}\mathrm{d}s\right) \end{aligned} \quad (30)$$

Since $p_1(t)$ and $p_2(t)$ are non-negative, and $\exp(-\int_0^s \frac{p_1(\tau)}{2}\mathrm{d}\tau) \leq 1, \forall s \geq 0$, equation (30) implies that

$$\sqrt{V(t)} \leq \left(\sqrt{V(0)} + \int_0^t \frac{p_2(s)}{2}\mathrm{d}s\right)\exp\left(\int_0^t \frac{p_1(s)}{2}\mathrm{d}s\right) \quad (31)$$

Since both $p_1(t)$ and $p_2(t)$ belong to L_1 space, $V(t)$ is bounded. That is, for any $r > 0$ and for any initial condition $\sqrt{V(0)} \leq r$, there exists a positive constant δ such that

$$\sqrt{V(t)} \leq \delta, \forall t \geq 0. \quad (32)$$

Then from (27), we have

$$\dot{V}(t) \leq -W(t) + \delta^2 p_1(t) + \delta p_2(t) \quad (33)$$

which implies

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(V(t) - \delta^2\int_0^t p_1(s)\mathrm{d}s - \delta\int_0^t p_2(s)\mathrm{d}s\right) \leq 0 \quad (34)$$

It follows that $(V(t) - \delta^2\int_0^t p_1(s)\mathrm{d}s - \delta\int_0^t p_2(s)\mathrm{d}s)$ is non-increasing. Since $V(t)$ is bounded from below by zero, $V(t)$ tends to a finite nonnegative constant.

On the other hand, it follows from (27) that

$$\begin{aligned} V(t) + \int_0^t W(s)\mathrm{d}s \leq V(0) + \delta^2\int_0^t p_1(s)\mathrm{d}s \\ + \delta\int_0^t p_2(s)\mathrm{d}s < \infty,\ \forall\sqrt{V(0)} \leq r, \forall t \geq 0. \end{aligned} \quad (35)$$

The above inequality implies that $W(t)$ belongs to L_1-space. Thus, by Barbalat's lemma, $W(t)$ asymptotically converges to zero. This completes the proof. ■

We now use Lemma 1 to investigate the stability property of closed-loop system (26), the result is stated in the following theorem:

Theorem 1 *Consider the kinematic simultaneous stabilization and tracking problem of subsystem (10). Under Asumption 1, the closed-loop system (26) is globally asymptotically stable. Thus, the time-varying control law (20) makes (15) holds.*

Proof: Solving the differential equation (23), we have

$$\rho(t) = \exp\left(-\int_0^t (|u_{1d}(s)| + |u_{2d}(s)|)\mathrm{d}s\right) \quad (36)$$

which means $0 \leq \rho(t) \leq 1$. If C 1 holds, then based on the stability results on linear time-varying systems, $\rho(t)$ is exponentially convergent, and $\rho(t) \in L_1$. If C 2 holds, then $0 < \exp(-\mu_2) < \rho(t) \leq 1$.

We first consider the case of C 1. Since $|h(t, \bar{z}_e)| \leq h_0$, and $|z_{1e}| \leq \sqrt{2V_1}$, from (21), we have

$$\begin{aligned} \dot{V}_1 \quad & \leq \quad -k_1 z_{1e}^2 - k_3 z_{3e}^2 + \rho(t)|h(t, \bar{z}_e)||z_{1e}| \\ & \leq \quad -k_1 z_{1e}^2 - k_3 z_{3e}^2 + h_0\rho(t)\sqrt{2V_1} \end{aligned} \quad (37)$$

Because C 1 holds, $\rho(t) \in L_1$. By means of Lemma 1, we have z_{1e}, z_{3e} tend to zero, and z_{2e} tends to a constant as $t \to \infty$. Since $\lim_{t\to\infty} z_{1e}(t) = 0$, applying the extended version of Barbalat's lemma [6] to the first equation of (26), yields

$$\lim_{t\to\infty} u_{2d}(t)z_{2e}(t) = 0 \quad (38)$$

Similarly, because $\lim_{t\to\infty} z_{3e}(t) = 0$, applying the extended version of Barbalat's lemma to the last equation of (26), yields

$$\lim_{t\to\infty} u_{1d}(t)z_{2e}(t) = 0 \quad (39)$$

Combining (38) and (39), we have

$$\lim_{t\to\infty}(u_{1d}^2(t) + u_{2d}^2(t))z_{2e}(t) = 0 \quad (40)$$

Following (40), it can be easily proved that $\lim_{t\to\infty} z_{2e}(t) = 0$ by contradiction. Thus, $\|z_e\|$ is asymptotically convergent.

We now move to the case of C2. We first show that z_{1e} is bounded. From (21), we have

$$\begin{aligned} \dot{V}_1 \quad & \leq \quad -k_1 z_{1e}^2 - k_3 z_{3e}^2 + \rho(t)|h(t, \bar{z}_e)||z_{1e}| \\ & \leq \quad -k_1 z_{1e}^2 + h_0|z_{1e}| \end{aligned} \quad (41)$$

It follows that if $|z_{1e}| \geq h_0/k_1$, then $\dot{V}_1 \leq 0$, thus z_{1e} is bounded.

Then we will shown that z_{2e} and z_{3e} tend to zero as $t \to \infty$. For the last two equations of (26), we define the Lyapunov function as

$$V_2 = \frac{1}{2}(k_2 z_{2e}^2 + z_{3e}^2) \quad (42)$$

the time derivative of V_2 is given as

$$\dot{V}_2 = -k_3 z_{3e}^2 - k_2 u_{2d} z_{1e} z_{2e} \quad (43)$$

Since $|z_{2e}| \leq \sqrt{2V_2/k_2}$, we have

$$\dot{V}_2 \leq -k_3 z_{3e}^2 - k_2|u_{2d}||z_{1e}|\sqrt{2k_2V_2} \quad (44)$$

Because z_{1e} is bounded and $u_{2d} \in L_1$, $u_{2d}z_{1e} \in L_1$. By means of Lemma 1, we have z_{3e} tends to zero, and z_{2e} tends to a constant as $t \to \infty$. Since $\lim_{t\to\infty} z_{3e}(t) = 0$, applying the extended version of Barbalat's lemma to the last equation of (26), yields

$$\lim_{t\to\infty} u_1(t)z_{2e}(t) = 0 \quad (45)$$

We now proceed by contradiction. Assume that $u_1(t)$ does not tend to zero. Considering that $z_{2e}(t)$ tends to a constant together with equation (45), then $z_{2e}(t)$ tends to zero. By uniformly continuity and since $\vartheta_1(t, 0) = 0$, $\vartheta_1(t, \bar{z}_e)$ also tends to zero. Considering that z_{2e} is bounded and $u_{2d} \in L_1$, then $u_{2d}z_{2e} \in L_1$, and the first equation of (26):

$$\dot{z}_{1e} = -k_1 z_{1e} + k_2 u_{2d} z_{2e} + \vartheta_1 \tag{46}$$

can be viewed as a stable linear system subjected to an additive perturbation which asymptotically vanishes. As a consequence, z_{1e} tends to zero. As z_{1e} and ϑ_1 tend to zero, u_{1d} and $u_{2d}z_{2e}$ belong to L_1, this in turn implies that $u_1(t)$ tends to zero, yielding a contradiction. Therefore, $u_1(t)$ must asymptotically tend to zero.

Differentiating the expression of u_1 with respect to time and using the convergence of $u_{1d}, u_{2d}, \dot{u}_{1d}, \dot{u}_{2d}$ and $\|\dot{\bar{z}}_e\|$ to zero, we get

$$\begin{aligned} \dot{u}_1(t) &= \frac{\partial \vartheta_1}{\partial t}(t, \bar{z}_e) + o(t) \\ &= \rho(t)\frac{\partial h}{\partial t}(t, \bar{z}_e) + \frac{\partial \rho}{\partial t} h(t, \bar{z}_e) + o(t) \\ &= \rho(t)\frac{\partial h}{\partial t}(t, \bar{z}_e) + o'(t) \end{aligned} \tag{47}$$

where $\lim_{t\to\infty} o(t) = 0$, and

$$o'(t) = -(|u_{1d}(t)| + |u_{2d}(t)|)\vartheta_1(t, \bar{z}_e) + o(t).$$

Because $u_{1d}(t)$ and $u_{2d}(t)$ belong to L_1, $\vartheta_1(t, \bar{z}_e)$ is uniformly bounded, we conclude that $o'(t)$ tends to zero. Since $(\partial h/\partial t)(t, \bar{z}_e)$ is uniformly continuous and $0 < \exp(-\mu_2) < \rho(t) \le 1$, $(\partial h/\partial t)(t, \bar{z}_e)$ tends to zero by application of the extended version of Barbalat's lemma.

By repeating the same procedure as many times as necessary, we show that $(\partial^j h/\partial t^j)(t, \bar{z}_e)$ tend to zero, $(1 \le j \le p)$. Therefore

$$\lim_{t\to\infty} \sum_{j=1}^{j=p} \left(\frac{\partial^j h}{\partial t^j}(t, \bar{z}_e)\right)^2 = 0 \tag{48}$$

Assume now that $\|\bar{z}_e(t)\|$ remains larger than some positive number l. The previous convergence result is then not compatible with the P 2 property imposed on the function $h(t, \bar{z}_e)$. Therefore, $\|\bar{z}_e(t)\|$ asymptotically converges to zero. Then by uniformly continuity and $\vartheta_1(t, 0) = 0$, $\vartheta_1(t, \bar{z}_e)$ tends to zero.

In view of the expression of u_1, asymptotical convergence of $z_{1e}(t)$ to zero readily follows. Thus, $z_e(t)$ is asymptotically convergent independent of initial state. This completes the proof of Theorem 1. ■

B. Dynamic Control

In this part, the control law τ is designed based on the kinematic controller using the bacstepping approach, so that the problem of simultaneous stabilization and tracking can also be solved using mechanical torques.

To this end, we reformulate the kinematic controller given in (20) as a desired signal as follows

$$\begin{aligned} \eta_1 &= u_{1d} - k_1 z_{1e} + k_2 u_{2d} z_{2e} + \vartheta_1 \\ \eta_2 &= u_{2d} - k_3 z_{3e} - k_2 \eta_1 z_{2e} \end{aligned} \tag{49}$$

Denote $\tilde{u} = u - \eta$, the subsystem (11) can be rewritten as

$$\begin{aligned} M_2(z)\dot{\tilde{u}} + C_2(z, \dot{z})\tilde{u} &= B_2(z)\tau \\ &\quad - (M_2(z)\dot{\eta} + C_2(z, \dot{z})\eta + G_2(z)) \end{aligned} \tag{50}$$

If the control law is chosen as

$$\tau = B_2^{-1}(z)\left[-K_p\tilde{u} + (M_2(z)\dot{\eta} + C_2(z, \dot{z})\eta + G_2(z)\right] \tag{51}$$

where K_p is a positive definite matrix, it can be proved that $z_e(t)$ asymptotically converge to zero along a similar reasoning process in the proof of Theorem 1. The completed stability analysis is omitted due to limited space, however it will appear in our further work.

IV. SIMULATION RESULTS

A simplified model of wheeled mobile robot moving on a horizontal plane is used for simulation. The dynamic model can be expressed as

$$\begin{aligned} m\ddot{x} &= \frac{1}{P}(\tau_1 + \tau_2)\cos\theta + \lambda \sin\theta \\ m\ddot{y} &= \frac{1}{P}(\tau_1 + \tau_2)\sin\theta - \lambda \cos\theta \\ I_0\ddot{\theta} &= \frac{L}{P}(\tau_1 - \tau_2) \end{aligned} \tag{52}$$

where m is the mass of the mobile robot, I_0 being its inertia moment around the vertical axis at point Q, P is the radius of the wheels and $2L$ the length of axis of the front wheels, and τ_1 and τ_2 are the torques provided by motors.

The following four cases are simulated to illustrate the effectiveness of the proposed controller:

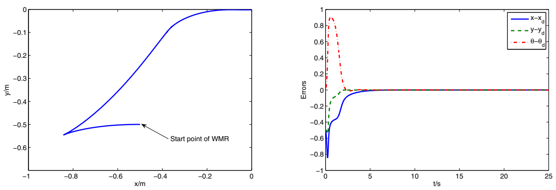
Case 1: Set-point stabilization: $v_{1d} = 0, v_{2d} = 0$;
Case 2: Approach to a set-point: $v_{1d} = e^{-0.1t}, v_{2d} = e^{-t}$;
Case 3: Tracking a line path: $v_{1d} = 1, v_{2d} = 0$;
Case 4: Tracking a circle: $v_{1d} = 1, v_{2d} = 1$.

The reference trajectory $q_d(t) = [x_d(t), y_d(t), \theta_d(t)]^T$ is generated by the desired velocities $v_{1d}(t)$ and $v_{2d}(t)$ with the initial conditions as $q_d(0) = [0, 0, 0]^T$.

In the simulation, the physical parameters of WMR are taken as $m = 12, I_0 = 5, L = 0.2, P = 0.1$. The initial positions and velocities of the wheeled robot are picked as $q(0) = [-0.5, -0.5, 0]^T$, $v(0) = [0, 0]^T$. The control parameters are set as $k_1 = 3, k_2 = 5, k_3 = 5$, $K_p = diag[10, 10]$. The nonlinear time-varying function $h(t, \bar{z}_e)$ is chosen as $h(t, \bar{z}_e) = 15\tanh(z_{2e}^2 + z_{3e}^2)\sin 2t$. Simulation results are shown in Figs. 3-6. It can be seen from the figures that the tracking errors all approach to zero and the mobile robot follows the desired path with perfect performance.

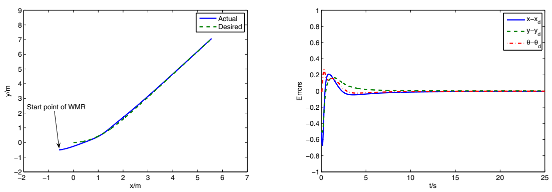
V. CONCLUSION

A single controller is developed to simultaneously solve the tracking and regulation problems of a mobile robot. The controller fully exploit the existing results on stabilization and tracking control for the equivalent chained form systems of wheeled mobile robots. Simulation results confirm the effectiveness of the proposed time-varying controller. Future works may to extend the proposed methodology to mobile robots with dynamic uncertainties.



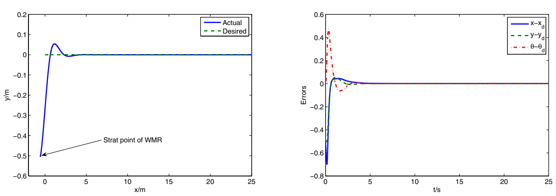
(a) Robot position in (x, y) plane (b) Tracking errors

Fig. 3. Simulation results of Case 1.



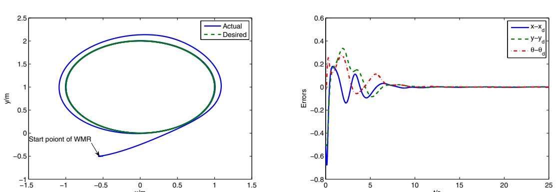
(a) Robot position in (x, y) plane (b) Tracking errors

Fig. 4. Simulation results of Case 2.



(a) Robot position in (x, y) plane (b) Tracking errors

Fig. 5. Simulation results of Case 3.



(a) Robot position in (x, y) plane (b) Tracking errors

Fig. 6. Simulation results of Case 4.

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