Decentralized Overlapping Control of a Platoon of Vehicles

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***Abstract*—In this paper a novel methodology is proposed for longitudinal control design of platoons of automotive vehicles within intelligent vehicle/highway systems (IVHSs). The proposed decentralized overlapping control law is obtained by using the inclusion principle, i.e., by decomposing the original system model by an appropriate input/state expansion, and by applying the linear quadratic (LQ) optimization to the locally extracted subsystems. The local quadratic criteria directly reflect the desired system performance. Optimization is carried out by using a sequential algorithm adapted to the lower block triangular (LBT) structure of the closed-loop system model. Contraction to the original space provides a decentralized platoon controller which preserves the asymptotic stability and the steady-state behavior of the controller obtained in the expanded space. Conditions for eliminating the "slinky effect" and obtaining the strict string stability are defined; it is shown that the corresponding constraints on the controller parameters are not too restrictive. A new dynamic platoon controller structure, consisting of a reduced order observer and a static feedback map, is obtained by applying the inclusion principle to the decentralized overlapping platoon control design in the case when the information from the preceding vehicle is missing. Numerous simulation results show that the proposed methodology provides a reliable tool for a systematic and efficient design of platoon controllers within IVHS.**

***Index Terms*—Decentralized overlapping control, inclusion principle, LQ optimal control, platoon of vehicles, string stability.**

I. Introduction

The problem of design of automated highway systems (AHSs) has recently attracted a considerable attention among researchers (e.g., [3], [14], [24]). It has been shown that control, communication and computing technologies can be effectively combined into an intelligent vehicle/highway system (IVHS) that can significantly increase safety and highway capacity. The IVHS architecture, proposed in [3], [14], [24], is based on the notion of platoons, groups of vehicles following the leading vehicle with small intraplatoon separation. Control of platoons of vehicles has been studied from different viewpoints [2], [13], [15], [22], [23]. It has been shown that an efficient decentralized platoon control law can be formulated when each individual vehicle is supplied with data representing its own velocity and acceleration, distance to the preceding vehicle, velocity and acceleration of the preceding vehicle, as well as the distance, velocity, and acceleration references [15] (different information structures leading to the so-called *spacing* and *headway* control strategies are discussed in [21] and [22]). However, in spite of the fact that successful technical solutions have been reported, tuning of the local controller parameters has been based mostly on intuitive arguments related to relative stability. To the authors knowledge, strategies taking systematically into account the desired system performance, optimality in any predefined sense, structural uncertainties, and possibilities to introduce dynamics into the regulator have not yet been reported (e.g., [2], [14], [15], [22]). Notice that there have been attempts to optimize strings of moving vehicles with respect to the quadratic criterion (e.g., [10] and [16]); however, the resulting control strategies are not in accordance with the information available in the individual vehicles in platoons within IVHS.

In this paper, the inclusion principle, which has been found to provide a good theoretical basis for practical solutions in the domain of decentralized control of large-scale systems [16], [18], [19], is applied to the design of decentralized overlapping longitudinal control of a platoon of vehicles, in which the local regulators, implemented within each vehicle, are designed starting from local quadratic performance indexes.

The first part of the paper contains the results related to platoon modeling, formulated in accordance with [2], [3], [14], [15], [22], and [24], together with a definition of general platoon control objectives within IVHS (e.g., [22]). In the following section (Section III), a novel platoon control strategy is presented *in extenso* from various design aspects. First, a linearized state model for a string of moving vehicles is derived on the basis of the results presented in [2], [10], [15], [16], and [22]. This state model is then expanded and decomposed into *overlapping subsystems* by using the methodology of the inclusion principle [16], [8], [19] (the original model is asssumed to represent a restriction of the expanded one). The subsystems are defined in such a way that their state vectors are composed of the measurements assumed to be available in each vehicle. Local control laws for the extracted subsystems are obtained by optimization with respect to local quadratic performance indexes, which give a measure of tracking accuracy of reference signals (distance, velocity, acceleration), as well as of velocities and accelerations of the preceding vehicles. Having in mind the desired control structure resulting from the assumed information structure within platoons, local LQ optimization of the subsystems is not done directly, but by applying a sequential optimization algorithm adapted to the lower block triangular (LBT) structure (see [12], [16], and [17]). Freedom in choosing weighting matrices in the local criteria allows a systematic generation of a

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variety of diverse, qualitatively different control laws. The resulting state feedback, ensuring both LQ suboptimality and the desired steady-state regime, is finally contracted from the expanded space to the original system space, according to the methodology of the inclusion principle [5], [7], [8], [16], [18], [19]. The contracted closed-loop system preserves in this case the main properties obtained by the design done in the expanded space (e.g., stability, suboptimality, and the transient regime). Starting from the general platoon objectives, a special care is taken of the "slinky effect" and the string stability (see, e.g., [4], [11], [14], and [22]). It is demonstrated through a rigorous analysis within a separate section how the corresponding constraints influence the final choice of the platoon controller parameters. The form of the extracted subsystems, together with the methodology of the inclusion principle applied to observers and dynamic controllers, allows also designing dynamic platoon controllers, composed of the observers and the corresponding suboptimal feedback maps. Such controllers are applicable in the situations when the information about the velocity and acceleration of the preceding vehicle is not available. The corresponding algorithm is described in Section III-F. The simulation part of the work encompasses characteristic examples illustrating properties of the proposed methodology for platoon control. It includes an analysis of the effect of disturbances of various kinds. Numerous curves presented in Section IV demonstrate that the methodology offers a systematic and reliable way of getting performance in accordance with *a priori* expressed requirements, superior to those obtainable by the existing approaches. Three Appendixes are added to the main body of the paper. In Appendix I, an outline of the main facts is presented which are related to the inclusion principle and is important for understanding the main ideas of the proposed approach. Appendix II contains a derivation of the applied sequential LQ optimization algorithm, while in Appendix III proofs of the theorems dealing with the string stability of the obtained platoon controller are given.

II. Model Description and Problem Formulation

It will be assumed in this paper that ith automotive vehicle in a close formation platoon consisting of n vehicles can be satisfactorily represented by the following nonlinear third-order model (see, e.g., [2] and [22]):

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= f_i(v_i, a_i) + g_i(v_i)\eta_i \end{aligned} \tag{1}$$

where $d_i = x_{i-1} - x_i$ is the distance between two consecutive vehicles, x_{i-1} and x_i being their positions, v_i and a_i are the velocity and acceleration, respectively, while η_i is the engine input. Functions $f_i(\cdot,\cdot)$ and $g_i(\cdot)$ are given by

$$\begin{aligned} f_i(v_i, a_i) &= -\frac{2K_{di}}{m_i} v_i a_i - \frac{1}{\tau_i(v_i)} \left[a_i + \frac{K_{di}}{m_i} v_i^2 + \frac{d_{mi}}{m_i} \right] \\ g_i(v_i) &= \frac{1}{m_i \tau_i(v_i)} \end{aligned} \tag{2}$$

where m_i represents the vehicle mass, τ_i the time-constant of its engine, K_{di} the aerodynamic drag coefficient and d_{mi} the mechanical drag [2]. The model (1) becomes close to the vehicle model considered in [22] when τ_i is small enough. Assuming that the parameters in (2) are *a priori* known, we shall adopt the following control law structure:

$$\eta_i = m_i u_i + K_{di} v_i^2 + d_{mi} + 2\tau_i K_{di} v_i a_i \tag{3}$$

where u_i is the input signal chosen to make the closed-loop system satisfy certain performance criteria. Obviously, this control law achieves feedback linearization, since, after introducing (3), the third equation in (1) becomes

$$\dot{a}_i = -\tau_i^{-1} a_i + \tau_i^{-1} u_i \tag{4}$$

assuming that τ_i is constant. The resulting linearized vehicle model has been considered in [15] as a basis for a general analysis of the platoon control strategies. Notice that the control law structure $\eta_i = (-f_i + u_i)/g_i$, proposed in [2], provides a different linearized model, since we have then $\dot{a}_i = u_i$. The adopted approach, leaving the time-constant of the engine as a parameter in the linearized equation, has been found to be more convenient, especially from the point of view of the proposed control design methodology.

The existing approaches to the platoon control design start from the following main platoon objectives derived from specific requirements within IVHS (e.g., [22]).

1) The entire closed-loop system should be asymptotically stable.
2) The steady-state spacing error e_i, defined as the deviation of d_i from the desired value d_r, that is, $e_i = d_i - d_r$, should be equal to zero for all the vehicles in the platoon.
3) The transient errors should not amplify with vehicle index due to any manoeuver of the leading vehicle. If $H(s)$ is the transfer function relating the spacing errors $E_i(s)$ and $E_{i-1}(s)$, i.e., $H(s) = E_i(s)/E_{i-1}(s)$, then $H(s)$ should be string stable in the strong sense, i.e., $h(t)$, the corresponding impulse response, should satisfy $\|h\|_1 \le 1$, having in mind that $\|e_i(t)\|_\infty \le \|h\|_1 \|e_{i-1}(t)\|_\infty$, (if $f(t)$ is any function, then $\|f\|_1 = \int_0^\infty |f(t)|\ dt$ and $\|f\|_\infty = \sup_{t\ge 0} |f(t)|$) [22].
4) The so-called "slinky effect" should be avoided, i.e., $\|H(j\omega)\| \le 1$, $\forall\omega$.

Permissible platoon control strategies should essentially be of decentralized type, having in mind the information structure implied by the availability of external measurements in individual vehicles. We shall assume, according to [15], that the local control input u_i depends on the local vehicle state variables $\{d_i, v_i, a_i\}$, as well as on the information about the velocity and acceleration of the preceding vehicle $\{v_{i-1}, a_{i-1}\}$ (which is assumed to be transmitted by appropriate communication channels). It will also be assumed that each vehicle is supplied with the information about the spacing, velocity, and acceleration reference commands $\{d_r, v_r, a_r\}$ (the so-called spacing control; see, e.g., [21] and [22]). The discussion given below will also encompass practically important cases in which some of the above data are not available.

III. Decentralized Overlapping Platoon Control

A. Decomposition to Subsystems

Based on (1) and (4), the following state model $\mathbf{S}$ of the entire platoon can be formulated (assuming that all the vehicles have identical models):

$$\mathbf{S}\colon \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \cdots \\ \dot{X}_n \end{bmatrix} = \begin{bmatrix} A_v & 0 & \cdots & & 0 \\ A_d & A_v & 0 & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots & \\ 0 & \cdots & 0 & A_d & A_v \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdots \\ X_n \end{bmatrix} + \begin{bmatrix} B_v & 0 & \cdots & 0 \\ 0 & B_v & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & B_v \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ u_n \end{bmatrix} \tag{5}$$

where $X_i^T = [d_i \;\; v_i \;\; a_i]$ ($x_0 = 0$ in d_1), and

$$A_v = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau^{-1} \end{bmatrix}; \quad A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$
$$B_v = \begin{bmatrix} 0 \\ 0 \\ \tau^{-1} \end{bmatrix}.$$

Notice that both the state and the input matrices are composed of identical diagonal blocks A_v and B_v, and that the interconnections between vehicles are modeled by the lower subdiagonal blocks A_d. The structure of the above model indicates that it would be possible to consider the platoon as being composed of a sequence of overlapping *subsystems*, defined by the following state models:

$$\mathbf{S_i}\colon \; \dot{\xi}_i = A_i\xi_i + B_i\zeta_i = \begin{bmatrix} A_L & 0 \\ \overline{A}_d & A_v \end{bmatrix} \xi_i + \begin{bmatrix} B_L & 0 \\ 0 & B_v \end{bmatrix} \zeta_i \tag{6}$$

where $\xi_i^T = [v_{i-1} \;\; a_{i-1} \;\; d_i \;\; v_i \;\; a_i]$ is the state vector of ith subsystem, $\zeta_i^T = [u_{i-1} \;\; u_i]$ represents its control vector and

$$A_L = \begin{bmatrix} 0 & 1 \\ 0 & -\tau^{-1} \end{bmatrix}; \quad \overline{A}_d^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad B_L = \begin{bmatrix} 0 \\ \tau^{-1} \end{bmatrix}.$$

The overlapping parts of $\mathbf{S_i}$ with $\mathbf{S_{i-1}}$ are, obviously, the block A_L in A_i and the block B_L in B_i, related to $\xi_{i,1}^T = [v_{i-1} \;\; a_{i-1}]$ and u_{i-1}, respectively, that is, the system $\mathbf{S}$ is both state and input overlapping [7], [16], [18], [19], [20]. The subsystems $\mathbf{S_i}$ can hardly be given any precise physical meaning; notice, however, that their state vectors contain exactly the measurements supposed to be available in each vehicle [15]. Their control vectors ζ_i contain two components: u_i, which represents the real control signal in ith vehicle, and u_{i-1}, which can be considered to represent, together with the corresponding part of the subsystem dynamics, the preceding part of the platoon, as seen by ith vehicle (for the second vehicle in the platoon this is exactly the leading vehicle dynamics).

The methodology of the inclusion principle offers a mathematically consistent way of dealing with the problem of decomposition of complex systems with overlapping structure. Extraction of the subsystems $\mathbf{S_i}$ from the platoon model can be achieved by applying an input-state expansion which transforms the original system model $\mathbf{S}$ into an expanded model $\tilde{\mathbf{S}}$ where the overlaping subsystems of $\mathbf{S}$ appear as disjoint. An outline of the main results related to the inclusion principle, which enables following the formal aspects of the platoon control design methodology proposed below, is presented in Appendix I (see also [7], [8], [16], [18], and [19] for detailed treatments of various aspects of the inclusion principle). Using the notation from Appendix I, the main point is to find such pairs of matrices (U, V) (for state contraction/expansion) and (Q, R) (for input contraction/expansion) which ensure both satisfactory decoupling of subsystems in the expanded system $\tilde{\mathbf{S}}$ and convenient contraction to $\mathbf{S}$ (after designing a control strategy in the expanded space, see, e.g., [8]). Adopting that $\mathbf{S}$ represents a restriction of $\tilde{\mathbf{S}}$ (see Definition A2 in Appendix I and [18] and [19]), we shall use the following transformation pair for the state contraction/expansion:

$$V^T = \begin{bmatrix} I & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & I & I & 0 & & & \\ 0 & 0 & 0 & I & & & \\ & & & & & \cdots & \\ & & & & & & I \end{bmatrix};$$
$$U = \begin{bmatrix} I & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & \frac{1}{2}I & \frac{1}{2}I & 0 & & & \\ 0 & 0 & 0 & I & & & \\ & & & & & \cdots & \\ & & & & & & I \end{bmatrix}$$

matrices Q and R for the input contraction/expansion are analogous to U and V, respectively, with appropriate dimensions. The resulting model in the expanded space, obtained after applying the corresponding formalisms outlined in Appendix I in relation with the conditions for restriction (Theorem A1), is

$$\tilde{\mathbf{S}}\colon \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \vdots \\ \dot{\xi}_n \end{bmatrix} = \begin{bmatrix} A_1 & 0 & \ldots & 0 \\ 0 & A_2 & \ldots & \\ & & \cdots & \\ 0 & & & A_n \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix} + \begin{bmatrix} B_1 & 0 & \ldots & 0 \\ 0 & B_2 & & \\ & & \cdots & \\ 0 & & & B_n \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_n \end{bmatrix}. \tag{7}$$

The choice of the inclusion type requires some additional comments. The adopted information structure enabling state feedback indicates, in general, the restriction as a convenient inclusion type, having in mind direct contractibility of state feedback gains designed in the expanded space (see, e.g., [8], [16], [18], and [19]). The chosen restriction type results from the arguments related to the input expansion/contraction and the desired model structure in the expanded space. An alternative to this choice is restriction (type c) [18], [19] (introduced and discussed in [5] under the name of *extension*), when we have, instead of $\tilde{B}R = VB$ in (33), the stronger relation $\tilde{B} = VBQ$, but also a possibility to contract any state feedback gain $\tilde{K}$ to the original space by $K = Q\tilde{K}V$ (see Theorem A2 and the corresponding comments). In this case, however, the resulting input matrix in the expanded platoon state model $\tilde{\mathbf{S}}$ does not have the appealing block-diagonal structure from (7), which immediately allows the application of the sequential design methodology proposed below. For example, for $n = 2$ this matrix would have the form

$$\begin{bmatrix} B'_L & 0 & 0 & 0 \\ 0 & \frac{1}{2}B_L & \frac{1}{2}B_L & 0 \\ 0 & \frac{1}{2}B_L & \frac{1}{2}B_L & 0 \\ 0 & 0 & 0 & B_v \end{bmatrix}$$

where $B'_L = [B_L^T \;\; 0]^T$. Notice that the structure in ((7) is obtained by an adequate choice of N_c in (34), satisfying (35) (see also [7], [8], and [16]).

After expansion, the subsystems in $\tilde{\mathbf{S}}$ represented by (7) appear as disjoint. Our immediate aim is then to design the state feedback controller $\tilde{\mathbf{F}}$ for $\tilde{\mathbf{S}}$ by local design of state feedback controllers $\mathbf{F_i}$: $\zeta_i = K\xi_i$ for the subsystems $\mathbf{S_i}$ (the gain K are assumed to be the same for all the subsystems for simplicity). We shall apply a methodology based on LQ optimization with respect to the local performance indexes attached to the subsystems. Connections between local and global performance indices in the context of the inclusion of performance indexes for $\tilde{\mathbf{S}}$ (i.e., for $\mathbf{S_i}$) and $\mathbf{S}$ are discussed from the general standpoint in [6], [16], and [17]. However, in the context of the desired performance and the assumed information structure in $\mathbf{S}$, we shall define the local performance indexes directly, starting from the basic platoon control requirements presented in Section II.

As the vehicle dynamics represents formally a part of each subsystem, we shall describe the details of the proposed control strategy sequentially, starting from the leading vehicle.

B. Leading Vehicle Control Law

The leading vehicle is supplied with the reference command and uses its own state vector for control design. Formally speaking, according to (1) and (4), it is represented by the following model:

$$\dot{X}_L = A_L X_L + B_L u_1 \tag{8}$$

where $X_L^T = [v_1 \;\; a_1]$. The optimal state feedback control law will be found from the condition for the minimum of the performance index

$$J_L = \int_{t_0}^{\infty} [(X_L - X_{1r})^T Q_L (X_L - X_{1r}) + R_L u_1^2] \, dt \tag{9}$$

where $X_{1r}^T = [v_r \;\; a_r]$ is a time-varying reference supplied to the first vehicle, which is known entirely in advance, and $Q_L \geq 0$ and $R_L > 0$ are the corresponding weighting matrices. According to [1], the resulting LQ optimal tracking control law is given by

$$\begin{aligned} u_1 &= -K_1 X_L - M_1 X_{1r} \\ K_1 &= R_L^{-1} B_L^T P_L \\ M_1 &= R_L^{-1} B_L^T (A_L - B_L K_1)^{-T} Q_L \\ P_L A_L + A_L^T P_L &- P_L B_L R_L^{-1} B_L^T P_L + Q_L = 0. \end{aligned} \tag{10}$$

The obtained control law is, in general, only suboptimal, since the feedforward matrix is constant [1]. It is not, however, far from the optimum, having in mind characteristic forms of the reference signals within platoons. *A priori* choice of the criterion weights provides different tracking properties.

Notice that the steady-state error with respect to a constant velocity reference always reduces to zero, i.e., $\overline{v}_1 = \lim_{t\to\infty} v_1(t) = v_r$ and $\overline{a}_1 = \lim_{t\to\infty} a_1(t) = 0$, having in mind that A_L is singular [1]. Namely, if $Q_L = \text{diag}\{q_{11}, q_{22}\}$, the only nonzero term in the two-dimensional vector M_1 in (10) is $M_1^1 = -R_L^{-1}(K_1^1)^{-1} q_{11}$ (for a vector B, B^i denotes its ith component). The condition $\dot{a}_1 = 0$ implies $-\tau^{-1}K_1^1 - \tau^{-1}M_1^1 = 0$ (since $\overline{a}_1 = 0$) and, having in mind that $K_1^1 = R_L^{-1}\tau^{-1}P_L^{12}$, we obtain finally $R_L^{-1}\tau^{-2}(P_L^{12})^2 = q_{11}$ (for a matrix A, A^{ij} denotes its (i,j)th element). The last equation coincides, however, with the scalar equation corresponding to the (1,1)th element in the matrix Riccati equation for P_L in (10).

C. Subsystem State Feedback

Control design for the subsystems defined by (6) can, in general, be based on the LQ optimal tracking methodology, providing the entire input vector $\zeta_i^T = [u_{i-1} \;\; u_i]$ as a function of the subsystem state vector ξ_i. However, such a solution is not in accordance with the assumed platoon controller structure, since the control signal u_{i-1} in the $(i-1)$st vehicle would become in such a way a function of d_i, v_i and a_i, i.e., of the state vector of the following, ith vehicle. Having this in mind, we shall propose in this section a different approach, based on sequential optimization, following the idea exposed in [12], [17] in relation with the systems possessing the general lower block triangular (LBT) structure. Namely, as the subsystem model (6) possesses the LBT structure with two main diagonal blocks, we shall attach to it two criteria

$$J_1 = \int_{t_0}^{\infty} [(\xi_{i,1} - X_{1r})^T Q'_L (\xi_{i,1} - X_{1r}) + R'_L u_{i-1}^2] \, dt \tag{11}$$

$$J_2 = \int_{t_0}^{\infty} [(\xi_i - X_{2r})^T Q (\xi_i - X_{2r}) + R u_i^2] \, dt \tag{12}$$

where $\xi_{i,1}^T = [\xi_i^1 \;\; \xi_i^2] = [v_{i-1} \;\; a_{i-1}]$, $X_{2r}^T = [v_r \;\; a_r \;\; d_r \;\; v_r \;\; a_r]$ is the complete set of reference commands corresponding to all the components of ξ_i, while $Q'_L \geq 0$, $R'_L > 0$, $Q \geq 0$ and $R > 0$ are appropriately defined weighting matrices.

The sequential design procedure starts from the top of the hierarchy by finding the optimal input u_{i-1} minimizing the criterion J_1 for the model $\dot{\xi}_{i,1} = A_L \xi_{i,1} + B_L u_{i-1}$ extracted from (6) (see general aspects of treating LBT structures in [12], [16], and [17]). The solution to this optimization problem coincides with the control law for the leading vehicle defined by (10) with $Q'_L = Q_L$ and $R'_L = R_L$ [the leading vehicle model represents formally a part of (6)], i.e.,

$$u_{i-1} = -K_1 \xi_{i,1} - M_1 X_{1r} \tag{13}$$

where K_1 and M_1 are defined by (10). After implementing (13) in (6), one obtains

$$\dot{\xi}_i = \begin{bmatrix} A_L - B_L K_1 & 0 \\ \overline{A}_d & A_v \end{bmatrix} \xi_i + \begin{bmatrix} 0 \\ B_v \end{bmatrix} u_i + \begin{bmatrix} -B_L M_1 \\ 0 \end{bmatrix} X_{1r}. \tag{14}$$

Now, following the idea of the sequential approach given in [12], the second step of the design consists of finding u_i in (14) by minimizing J_2. The state weighting matrix in J_2 is assumed here to have the following specific form, coming out basically from the general regulator structure adopted in [15]:

$$Q = \begin{bmatrix} p_1 & 0 & 0 & -p_1 & 0 \\ 0 & p_2 & 0 & 0 & -p_2 \\ 0 & 0 & q_{33} & 0 & 0 \\ -p_1 & 0 & 0 & q_{44}+p_1 & 0 \\ 0 & -p_2 & 0 & 0 & q_{55}+p_2 \end{bmatrix}. \tag{15}$$

In (15), q_{33} determines the relative importance of the spacing reference tracking, p_1 and p_2 influence tracking of the velocity and acceleration of the preceding vehicle, respectively, while q_{44} and q_{55} correspond to the velocity and acceleration reference tracking, respectively (having in mind that we have, in fact, $J_2 = \int_{t_0}^{\infty} [q_{33}(d_i - d_r)^2 + q_{44}(v_i - v_r)^2 + q_{55}(a_i - a_r)^2 + p_1(v_{i-1} - v_i)^2 + p_2(a_{i-1} - a_i)^2 + Ru_i^2]\ dt$). The problem posed belongs to the class of LQ optimal tracking problems with *a priori* known disturbances [1], having in mind that the last term in (14) [depending on X_{1r}, and introduced by u_{i-1} in (13)] becomes in the second step of the procedure a known disturbance input. An approximately optimal solution (in the sense that all the controller gains are assumed to be constant), is given by

$$\begin{aligned}
u_i &= -K_2\xi_i - M_2X_{2r} - M_3X_{1r}\\
K_2 &= R^{-1}B_v^T[P_{21} \vdots P_{22}]\\
M_2 &= R^{-1}B_v^TA_{22}^{-T}[Q_{21} \vdots Q_{22}]\\
M_3 &= -R^{-1}B_v^TA_{22}^{-T}P_{21}B_LM_1\\
&P_{22}A_v + A_v^TP_{22} - P_{22}B_vR^{-1}B_v^TP_{22} + Q_{22} = 0\\
&P_{21}A_{11} + A_{22}^TP_{21} = -Q_{21} - P_{22}\overline{A}_d\\
A_{11} &= A_L - B_LK_1; \quad A_{22} = A_v - B_vR_i^{-1}B_v^TP_{22}\\
Q_{22} &= \begin{bmatrix} q_{33} & 0 & 0 \\ 0 & q_{44}+p_1 & 0 \\ 0 & 0 & q_{55}+p_2 \end{bmatrix}\\
Q_{21} &= \begin{bmatrix} 0 & 0 \\ -p_1 & 0 \\ 0 & -p_2 \end{bmatrix}
\end{aligned} \tag{16}$$

(see Appendix II for derivation). Consequently, M_2 represents the feedforward gain for the complete reference X_{2r}, resulting directly from the supposed form of the criterion J_2, while the aim of the feedforward block M_3 is to compensate the effects of the disturbance term in (14). In (16), the first step is to calculate the matrix P_{22} by solving the given Riccati equation connected with the model $\dot{\xi}_{i,2} = A_v\xi_{i,2} + B_vu_i$, where $\xi_{i,2}^T = [\xi_i^3\ \xi_i^4\ \xi_i^5]$ (the second vehicle within the extracted subsystem $\mathbf{S_i}$). Once P_{22} is calculated, P_{21} is obtained by solving the corresponding Lyapunov equation in (16). Notice that Q_{22} (as a part of Q) influences the reference tracking directly, while Q_{21} determines the relative importance of tracking the preceding vehicle.

The resulting overall subsystem control law possesses the required information structure. The state feedback gain $K = [[K_1\ 0\ 0\ 0]^T \vdots K_2^T]^T$ has the LBT form, exactly in accordance with the information supposed to be locally available [15]. The subsystem model becomes, after introducing both u_{i-1} and u_i defined by (13) and (16)

$$\dot{\xi}_i = \overline{A}_i\xi_i + \overline{M}_i'X_{2r} = \overline{A}_i\xi_i + \overline{M}_iX_r \tag{17}$$

where

$$\begin{aligned}
\overline{A}_i &= \begin{bmatrix} A_L - B_LK_1 & 0 \\ \overline{A}_d - B_vK_{2,1} & A_v - B_vK_{2,2} \end{bmatrix}\\
\overline{M}_i' &= \begin{bmatrix} -B_LM_1 & 0 \\ -B_vM_3 & -R^{-1}B_v^TA_{22}^{-T}Q_{22} \end{bmatrix}\\
\overline{M}_i &= \begin{bmatrix} 0 \ \vdots -B_LM_1 \\ -R^{-1}B_v^TA_{22}^{-T}Q_{22} + [0 \vdots -B_vM_3] \end{bmatrix}.
\end{aligned}$$

$K_{2,1} = [K_2^1, K_2^2]$, $K_{2,2} = [K_2^3, K_2^4, K_2^5]$, and $X_r^T = [d_r\ \ v_r\ \ a_r]$. Notice that the resulting feedforward gain matrix $\overline{M}_i$ for the whole subsystem, which is obtained directly from M_1, M_2 and M_3, multiplies esentially the basic reference vector X_r. More specifically, in the five-dimensional vector M_2, the only nonzero term is $M_2^3 = -R^{-1}q_{33}(K_2^3)^{-1}$, while in the two-dimensional vector M_3 the only nonzero term is $M_3^1 = R^{-1}P_{21}^{12}(K_2^3)^{-1}(R_L')^{-1}\tau^{-1}(K_1^1)^{-1}$. Therefore, the matrix $\overline{M}_i$ has only three nonzero elements: $\overline{M}_i^{22} = \tau^{-1}M_1^1$, $\overline{M}_i^{51} = \tau^{-1}M_2^3$ and $\overline{M}_i^{52} = \tau^{-1}M_3^1$.

Having in mind the structure of (17), it is possible to show that for constant references the steady-state error is equal to zero, that is $\overline{\xi}_i = \lim_{t\to\infty}\xi_i(t) = X_{2r}$. One easily obtains, on the basis of the first four scalar equations in $\dot{\xi}_i = 0$ and using the results from the previous section, that $\overline{\xi}_i^1 = \overline{\xi}_i^4 = v_r$ and $\overline{\xi}_i^2 = \overline{\xi}_i^5 = 0$. The fifth scalar equation in $\dot{\xi}_i = 0$ gives

$$(K_2^1 + K_2^4)v_r + K_2^3\overline{\xi}_i^3 + M_2^3d_r + M_3^1v_r = 0. \tag{18}$$

However

$$\begin{aligned}
&K_2^1 + K_2^4 + M_3^1\\
&\quad = R^{-1}\tau^{-1}(P_{21}^{31} + P_{22}^{32} + P_{22}^{12}(K_2^3)^{-1}(K_1^1)^{-1})
\end{aligned} \tag{19}$$

having in mind the definition of K_2 and properties of K_1, given in the previous section. Starting from (16), one obtains the (1, 1)-element of the matrix Lyapunov equation and the (2, 1)-element of the matrix Riccati equation in the form

$$\begin{aligned}
P_{21}^{12}\tau^{-1}K_1^1 + P_{21}^{31}\tau^{-1}K_2^3 &= P_{22}^{11};\\
P_{22}^{11} + \tau^{-2}R^{-1}P_{22}^{13} &= 0
\end{aligned} \tag{20}$$

respectively. Introducing (19) into (18) we get $K_2^1 + K_2^4 + M_3^1 = 0$. Therefore, $\overline{\xi}_i^3 = d_r$, having in mind that the relation $\tau^{-2}R^{-1}(P_{22}^{13})^2 = q_{33}$, resulting from the (1, 1)-element of the matrix Riccati equation in (16), implies $(K_2^3)^2 = R^{-1}q_{33}$.

D. Platoon State Feedback

According to the main idea of the decentralized overlapping control design methodology based on the inclusion principle (see, e.g., [5], [8], [16], and [19]), the overall regulator $\tilde{\mathbf{F}}$ for the platoon model in the expanded space $\tilde{\mathbf{S}}$, composed of the local regulators $\mathbf{F}_i$ for the extracted subsystems $\mathbf{S_i}$, is to be finally contracted to the original space of the system $\mathbf{S}$ for implementation. Therefore, within the framework of the adopted information structure, the state feedback control law for the entire platoon has to be obtained from the set of the local subsystem feedback gains by applying the corresponding transformations derived on the basis of the chosen inclusion type and the selected pairs of matrices (U, V) and (Q, R). The main relevant theoretical results are formulated in Theorem A2 in Appendix I (for more details see [7], [17], and [19]). More precisely, as the overall state feedback gain in the expanded space has the block-diagonal form $\tilde{K} = \text{diag}\{K, \cdots, K\}$, where $K = [[K_1\ 0\ 0\ 0]^T \vdots K_2^T]^T$ [K_1 and K_2 are defined in (10) and (16)], the resulting state feedback gain K_P for the real platoon in the original space cannot be obtained directly by $K_P = Q\tilde{K}V$, having in mind that the condition $RK_P = \tilde{K}V$ (corresponding to the chosen inclusion type) cannot be satisfied for block-diagonal matrices $\tilde{K}$ ($\tilde{K}V = RQ\tilde{K}V$ does not hold in this case). Following the general methodology described in [8], the platoon state feedback controller will now be defined by $K_P = Q\tilde{K}_M V$, where $\tilde{K}_M$ is a modified version of $\tilde{K}$, satisfying the inclusion condition $\tilde{K}_M V = RQ\tilde{K}_M V$. As an illustration, consider the case of two subsystems. Then, we have, for example, by using the methodology of [8]

$$\tilde{K} = \begin{bmatrix} K_1' & 0 & 0 & 0 \\ K_{2,1}' & K_{2,2}' & 0 & 0 \\ 0 & 0 & K_1' & 0 \\ 0 & 0 & K_{2,1}' & K_{2,2}' \end{bmatrix}$$

$$\tilde{K}_M = \begin{bmatrix} K_1' & 0 & 0 & 0 \\ K_{2,1}' & K_{2,2}' & K_1' & 0 \\ K_{2,1}' & K_{2,2}' & K_1' & 0 \\ 0 & 0 & K_{2,1}' & K_{2,2}' \end{bmatrix}$$

$$K_P = \begin{bmatrix} K_1' & 0 & 0 \\ K_{2,1}' & \frac{1}{2}(K_1' + K_{2,2}') & 0 \\ 0 & K_{2,1}' & K_{2,2}' \end{bmatrix}$$

where $K_1' = [0\ K_1^1\ K_1^2]$, $K_{2,1}' = [K_2^1\ K_2^2\ K_2^3]$ and $K_{2,2}' = [K_2^4\ K_2^5]$.

Consequently, one obtains that the control input in each vehicle (excluding the leading one) is given by

$$u_i = -K_M \xi_i - M_M X_r \tag{21}$$

where

$$K_M = \left[K_2^1 \quad K_2^2 \quad K_2^3 \quad \frac{K_2^4 + K_1^1}{2} \quad \frac{K_2^5 + K_1^2}{2} \right].$$

The inclusion conditions satisfied for the overall platoon feedback gain in the original space ensure that the closed-loop system in the expanded space includes the closed-loop system in the original space, implying suboptimality and, consequently, stability in the original space [16].

The feedforward gain M_M in (21) could analogously be obtained by contracting the input matrix in the expanded space resulting from (17). However, having in mind that the feedforward block is out of the feedback loop, and that one of the main requirements is to obtain the zero steady-state error in the contracted space, we shall define M_M in (21) on the basis of the corresponding $\overline{M}_i$ in (17) simply by $M_M = \overline{M}_i^{\Delta} + \Delta M_M$, where $\overline{M}_i^{\Delta}$ is a 3 × 3 matrix composed of the last three rows of $\overline{M}_i$, and ΔM_M is aimed at reducing the steady-state error to zero. From (20) and the corresponding analysis, one easily obtains that the only nonzero element in ΔM_M is $\Delta M_M^{3,2} = 0.5(K_2^1 - K_2^4)$ (having in mind the form of K_M and the fact that $K_2^1 + K_2^4 + M_3^1 = 0$).

Consequently, after applying (21) for $i = 1, \ldots n$, the overall platoon tracks the command reference in a suboptimal way in the LQ sense, preserving the predefined information structure and ensuring the correct steady-state regime.

It is easy to see from the resulting closed-loop model and the corresponding parameter values that, finally, the proposed platoon controller, possesses the commonly adopted structure, i.e., for $i \geq 2$, we have in (4)

$$\begin{aligned} u_i = k_v(v_{i-1} - v_i) + k_a(a_{i-1} - a_i) + c_d(d_i - d_r) \\ + c_v(v_r - v_i) + c_a(a_r - a_i) \end{aligned} \tag{22}$$

where $k_v = -K_M^1, k_a = -K_M^2, c_d = -K_M^3, c_v = K_M^1 + K_M^4$ and $c_a = K_M^2 + K_M^5$. For $i = 1$, that is, for the leading vehicle, we have

$$u_1 = c_v'(v_r - v_1) + c_a'(a_r - a_i) \tag{23}$$

where $c_v' = K_1^1$ and $c_a' = K_1^2$.

E. String Stability

The above considerations have been focused primarily on getting near optimal local performance and the desired steady-state regime. According to the cited platoon objectives, some important issues, such as those related to the string stability [13], [22], [23], require an additional care. We shall present here an analysis based on the obtained platoon controller structure, and derive an additional set of constraints aimed at satisfying platoon objectives 3 and 4 from Section II.

Starting from (1), (22), and (23), one obtains

$$G_1(s) = \frac{V_1(s)}{V_r(s)} = \tau^{-1} \frac{c_a's + c_v'}{s^2 + \tau^{-1}[(1 + c_a')s + c_v']} \tag{24}$$

$$\begin{aligned} P(s)E_i(s) &= Q(s)V_{i-1}(s) - S(s)V_r(s) \\ P(s)E_i(s) &= T(s)E_{i-1}(s); \qquad (i \geq 2) \\ P(s) &= s^3 + \tau^{-1}[(1 + k_a + c_a)s^2 + (k_v + c_v)s + c_d] \\ Q(s) &= s^2 + \tau^{-1}[(1 + c_a)s + c_v] \\ S(s) &= \tau^{-1}(c_a s + c_v) \\ T(s) &= \tau^{-1}[k_a s^2 + k_v s + c_d]. \end{aligned} \tag{25}$$

In the above equations, $V_i(s)$, $V_r(s)$ and $E_i(s)$ stand for the Laplace transforms of $v_i(t)$, $v_r(t)$ and $e_i(t) = d_i(t) - d_r$, respectively. The transfer function $G_1(s)$ describes the performance of the leading vehicle, while the following two relations show how the preceding vehicle speed influences the spacing error, and how the spacing errors are amplified or attenuated in the platoon. Let $H(s) = (T(s)/P(s))$ and $G(s) = (Q(s)/P(s))$. The following theorem deals with the platoon objective 4.

Theorem 1: $|H(j\omega)| \leq 1$, $\forall\omega$, if

$$\begin{aligned} &[c_v(2k_v + c_v) \geq 2k_d(1 + c_a)] \\ &\quad \wedge [\tau^{-1}(1 + k_a + c_a)^2 - 2(k_v + c_v) - \tau^{-1}k_a^2 \geq 0]. \quad \square \end{aligned} \tag{26}$$

The proof is presented in Appendix III. Even a cursory observation indicates that (26) does not impose severe constraints on the parameters obtained by the above described methodology (see the simulation results). Notice that the first inequality alone implies the existence of such an $\omega_1 > 0$ ensuring $|H(j\omega)| \leq 1$, $\forall \omega \in [0, \omega_1]$; this is practically relevant, having in mind the relative importance of the low-frequency dynamics [4].

Starting from the platoon objective 3, one could pose, in general, the problem of minimizing $||h||_1$ and $||g||_1$ with respect to the admissible set of parameters, where $h(t)$ and $g(t)$ are the impulse responses corresponding to $H(s)$ and $G(s)$, respectively. Instead of pursuing strictly this line of thought, we shall concentrate our attention only on the corresponding constraints in the controller parameter space. Define the measure of string stability for a given set of parameters as $\gamma = ||h||_1$; obviously, it is required to obtain $\gamma \leq 1$.

Theorem 2: Let $P(s) = (s+\beta_1)(s+\beta_2)(s+\beta_3)$, where β_i $(i = 1, 2, 3)$ are real numbers satisfying $\beta_3 > \beta_2 > \beta_1 > 0$,

$$A = \frac{T(-\beta_2)}{T(-\beta_1)} \frac{\beta_3 - \beta_1}{\beta_3 - \beta_2} \quad \text{and} \quad B = \frac{T(-\beta_3)}{T(-\beta_1)} \frac{\beta_2 - \beta_1}{\beta_3 - \beta_2}.$$

Then, the condition $C1 \vee C2 \vee C3 \vee C4$ ensures $\gamma = 1$, where

$$\begin{aligned} C1 =& (\text{sgn}\, A = 1) \wedge (\text{sgn}\, B = 1) \\ &\wedge \left(B \frac{\beta_3 - \beta_2}{\beta_2 - \beta_1}\right)^{\beta_2 - \beta_1 / \beta_3 - \beta_1} \\ &\geq A \frac{\beta_3 - \beta_2}{\beta_3 - \beta_1} \\ C2 =& (\text{sgn}\, A = -1) \wedge (\text{sgn}\, B = 1) \\ C3 =& (\text{sgn}\, A = -1) \wedge (\text{sgn}\, B = -1) \wedge B \geq A - 1 \\ C4 =& (\text{sgn}\, A = 1) \wedge (\text{sgn}\, B = -1) \wedge A - B \leq 1. \quad \square \end{aligned}$$

Theorem 3: Let $P(s) = (s+\delta)[(s+\alpha)^2 + \beta^2]$, where α, β and δ are real numbers satisfying $\alpha > \delta > 0$. Then, $\gamma = 1$ if

$$\frac{|[(\alpha - \delta) + j\beta] T(\alpha + j\beta)|}{\beta |T(-\delta)|} \leq 1. \quad \square \tag{27}$$

Derivation of the Proofs of Theorems 2 and 3 is given in Appendix III. Without going too far into details, it is important to emphasize that the string stability requirements do not impose serious constraints. Namely, as $P(s) = s^3 + \tau^{-1}(1 + c_a)s^2 + \tau^{-1}c_v s + T(s)$, it follows that $c_v > 0$ is the parameter which plays the crucial role in achieving the string stability. Having in mind that all the parameters in $P(s)$ are positive ($P(s)$ is Hurwitz by assumption), one obtains, for c_v large enough, either case C1 from Theorem 2 or the conditions of Theorem 3 (this is obvious when $T(s) > 0$). The value of this regulator parameter is directly influenced by the choice of the parameter q_{44} in the weighting matrix Q in the criterion J_2 ((11) and (15)). On the other hand, the decrease of the time-constant τ makes β_3 (or δ) larger, i.e., for τ small enough condition C1 becomes satisfied (Theorem 3 is not applicable in this case). For $\tau \to 0$, one obtains a second-order model for the vehicle dynamics and a possibility to directly compare the above results to those presented in [22]. Notice here only that $c_v > 0$ makes it unnecessary to assume the existance of the measurements of the distance to the leading vehicle (see [22]). Moreover, it appears much more logical and better adapted to the conditions of Theorems 2 and 3 to assume the existance of the whole term $k_a(a_{i-1} - a_i)$ in (22), rather than $k_a a_{i-1}$ alone (as in [22]). The above conclusions related to string stability hold also when the reference signals are absent; connections with different variations of the headway control strategy are discussed in [21].

F. Dynamic Platoon Controller with Decentralized Observers

The above discussion has been related to the (static) state feedback controllers, having in mind the adopted information structure. It is, however, important for practice to analyze also situations in which the information about v_{i-1} and a_{i-1} is not available in ith vehicle. The inclusion principle provides a design methodology for this case based on the adopted decomposition to subsystems (see Appendix I, Definitions A3 and A4, Theorems A3 and A4). The corresponding additional tools at the local level are, obviously, state observers, which, according to the separation principle, should provide the estimates of the subsystem states to the LQ suboptimal state controllers already designed. The obtained subsystem dynamic controllers have to be contracted to the original space for implementation in such a way that the contracted vehicle controllers satisfy, at the same time, conditions for inclusion and observer decentralization.

Consider the subsystem model (6), and assume that the available measurements in the ith vehicle are, in general, $y_i = C\xi_i$, where C is an appropriately defined matrix; we shall consider the case when $C = [0 \vdots I_3]$, i.e., when the subsystem states d_i, v_i and a_i can be locally measured, but both v_{i-1} and a_{i-1} are not available. An estimate $\hat{\xi}_{i,1}$ of $\xi_{i,1} = [v_{i-1} \;\; a_{i-1}]^T$ can be obtained by using the standard procedure for constructing reduced order state estimators in the case of perfect measurements of a part of the state vector [9]. One obtains directly the following structure:

$$\begin{aligned} \dot{\hat{\xi}}_{i,1} =& A_L \hat{\xi}_{i,1} + B_L u_{i-1} \\ &+ L[\dot{\xi}_{i,2} - \overline{A}_d \hat{\xi}_{i,1} - A_v \xi_{i,2} - B_v u_i] \end{aligned} \tag{28}$$

where L is the observer gain (which can be defined, for example, in accordance with the Kalman filtering methodology [9]). We shall assume that the control law for $\mathbf{S_i}$ is now, according to the separation principle

$$\begin{aligned} u_{i-1} =& -K_1 \hat{\xi}_{i,1} - M_1 X_{1r} \\ u_i =& -K_{2,1} \hat{\xi}_{i,1} - K_{2,2} \xi_{i,2} - M_2 X_{2r} - M_3 X_{1r} \end{aligned} \tag{29}$$

where the corresponding matrices are defined in (10) and (16), so that one obtains, after replacing (29) into (28), that

$$\begin{aligned} \dot{\hat{\xi}}_{i,1} =& A_{11} \hat{\xi}_{i,1} + M_{i,1} X_r \\ &+ L[\dot{\xi}_{i,2} - (A_{22} - B_v K_{2,2}) \xi_{i,2} \\ &- (\overline{A}_d - B_v K_{2,1}) \hat{\xi}_{i,1} - M_{i,2} X_r] \end{aligned} \tag{30}$$

where $M_{i,1} = [0 \vdots -B_L M_1]$ and $M_{i,2} = -R^{-1} B_v^T A_{22}^{-T} Q_{22} + [0 \vdots -B_v M_3]$. In order to avoid differentiation of the measurements, introduce, in the standard way, a new variable q by $\hat{\xi}_{i,1} = q + L\xi_{i,2}$. Then, we have, after some calculations

$$\begin{aligned} \dot{q} =& [A_{11} - L(\overline{A}_d - B_v K_{2,1})] q + [A_{11} L - L(A_{22} - B_v K_{2,2}) \\ &- L(\overline{A}_d - B_v K_{2,1}) L] \xi_{i,2} + (M_{i,1} - L M_{i,2}) X_r. \end{aligned} \tag{31}$$

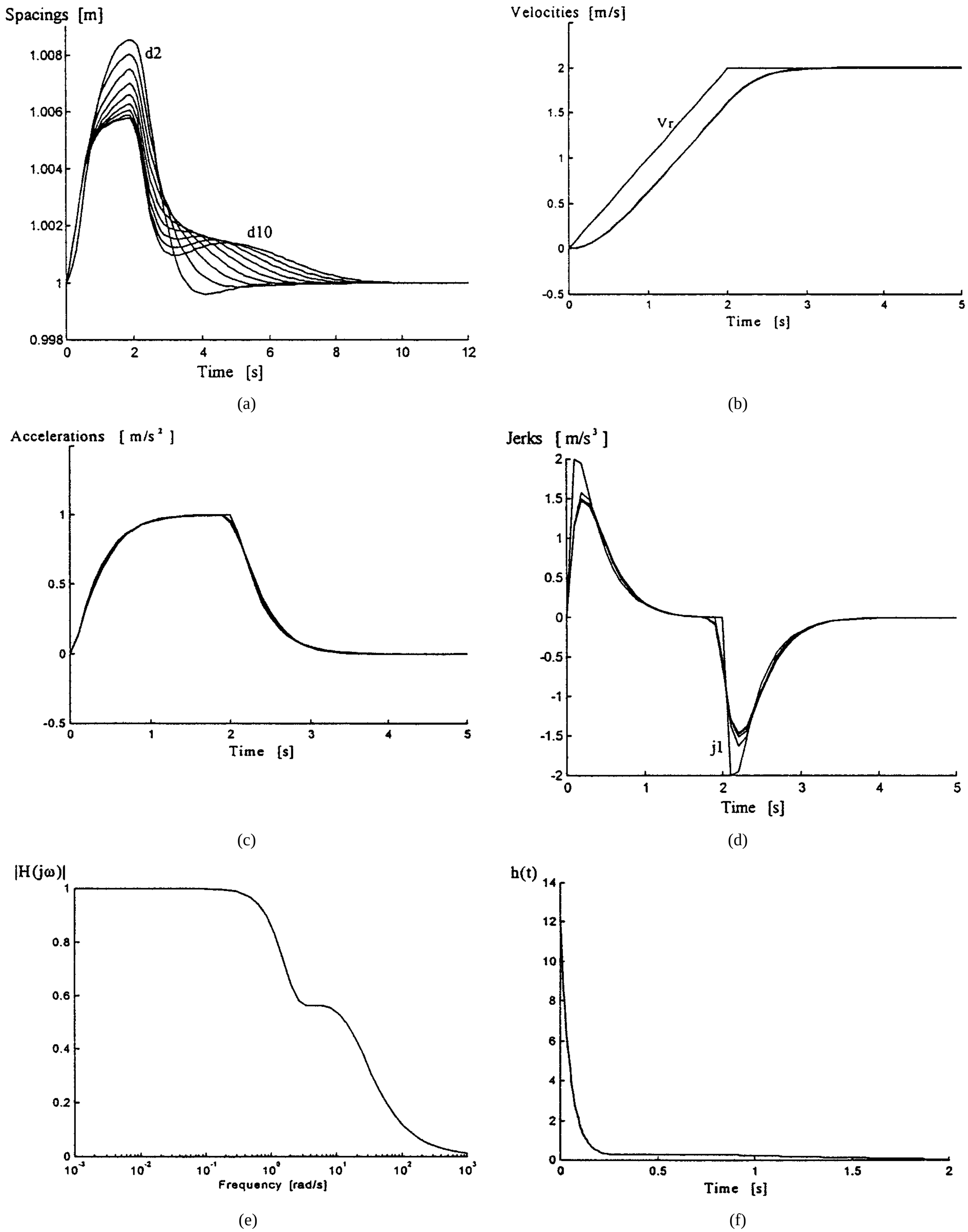


Fig. 1. Ten-vehicle platoon with $\tau = 0.1$s (speed increase): responses and spacing propagation characteristics. (a) Spacings; (b) Velocities; (c) Accelerations; (d) Jerks; (e) Frequency response; (f) Impulse response.

Obviously, the control law (29) can easily be reformulated as a function of q.

Using the notation from (37) and (38), the dynamic controller for the subsystem $\mathbf{S_i}$ defines a mapping from $y_i = \xi_{i,2}$ to the vector $[u_{i-1}\ \ u_i]$, defined as $[u_{i-1}\ \ u_i]^T = K\tilde{w}_i$, where $\tilde{w}_i = [\hat{\xi}_{i,1}^T, \xi_{i,2}^T]^T$. The overall controller in the expanded space consists of a set of n decoupled dynamic mappings of this form. Contraction to the original space should be done according to the corresponding theorems from Appendix I. It is to be emphasized that contraction of observers can be done separately from the contraction of the feedback gains. In order to preserve the decentralized platoon controller structure, we shall adopt that $D = E = I$ (following the idea presented in [5]), that is that the observer is invariant with respect to contraction. Contraction af-

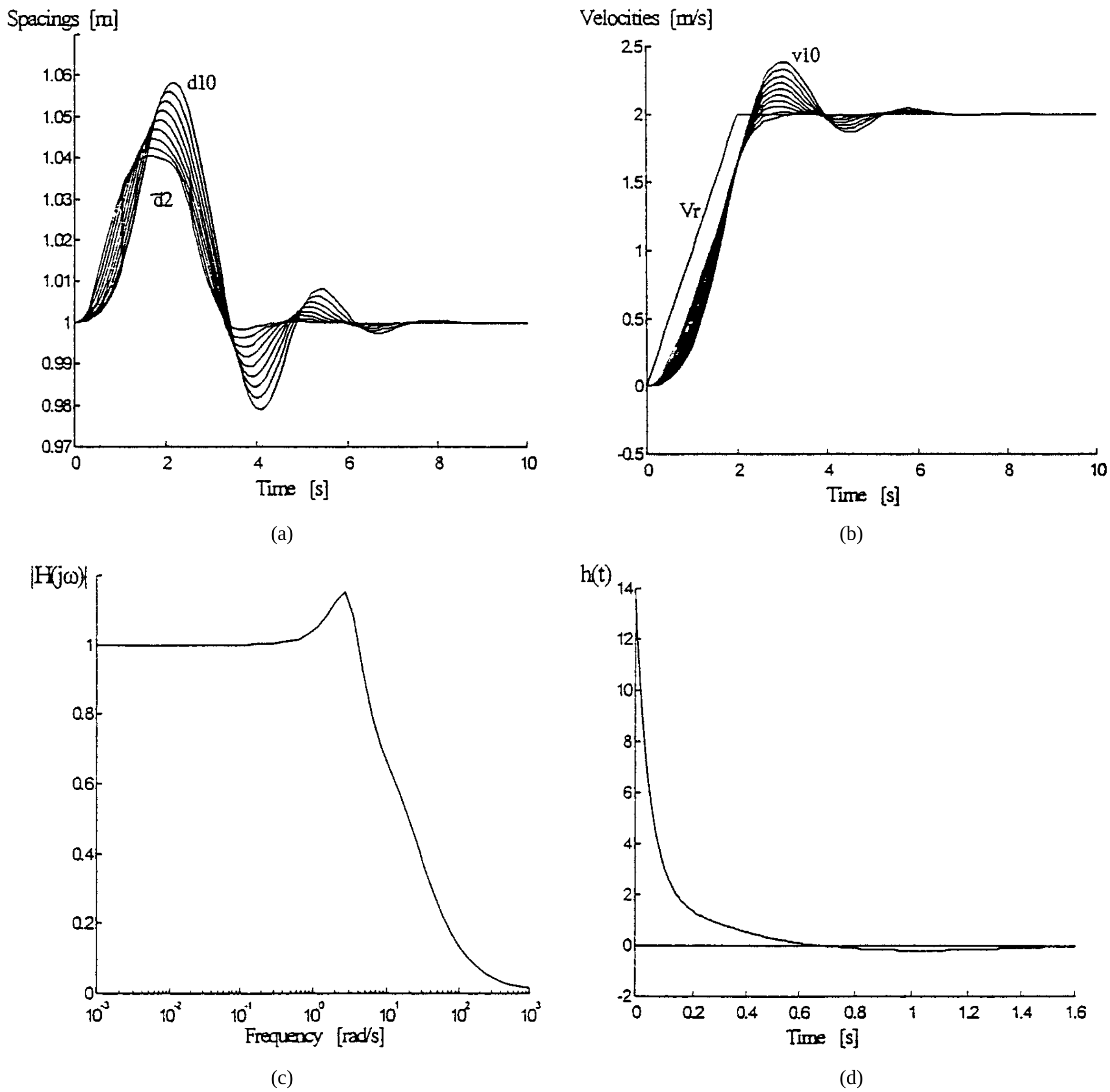


Fig. 2. Ten-vehicle platoon with $\tau = 0.1$s (speed increase): $\exists\omega : |H(j\omega)| > 1; h(t)$ changes sign. (a) Spacings; (b) velocities; (c) frequency response; (d) impulse response.

fects then only the feedback gain matrix, but exactly in the same way as in the case of the state feedback design. Therefore, according to (21), the local control input to the ith vehicle becomes finally given by $u_i = -K_M\tilde{w}_i - M_MX_r = -[K_M^1 K_M^2]\hat{\xi}_{i,1} - [K_M^3 K_M^4 K_M^5]\xi_{i,2} - M_MX_r$. Notice that, owing to the form of the extracted subsystem model (6), the proposed observer utilizes the part of the subsystem dynamics related to $\xi_{i,1}$, as well as the corresponding references, as an approximate model for the preceding part of the platoon.

A completely analogous algorithm can be constructed in the realistic case when only the measurement of a_{i-1} is missing (having in mind that v_{i-1} might be available together with d_i).

IV. Simulation Results

Numerous simulations have been undertaken in order to clarify the important aspects of the proposed methodology. In this section we shall present some characteristic results.

Time histories of a ten-vehicle platoon, together with its spacing propagation chacteristics expressed through $|H(j\omega)|$ and $h(t)$, are presented in Fig. 1. The platoon model with $\tau = 0.1$ has been used, while the design parameters have been: $Q_L = \text{diag}\{200, 10\}$, $R_L = 10$, $p_1 = 100$, $p_2 = 50$, $q_{33} = 500$, $q_{44} = 300$, $q_{55} = 10$, $R = 10$, giving the following state feedback vectors: $K_1 = [4.4721 \;\; 0.7103]$, $K_2 = [-4.0615 \;\; -1.2373 \;\; -7.0711 \;\; 6.7632 \;\; 1.3348]$ and the following feedforward parameters: $\overline{M}_i^{22} = 44.7214$, $\overline{M}_i^{51} = -70.7107$ and $\overline{M}_i^{52} = 27.3345$. The presented responses show an obvious advantage over those given in [15]; it is interesting to notice that the poles of the transfer function $H(s)$ are in this case: $s_1 = -3.2492$, $s_{2,3} = -1.589 \pm j0.9893$ (in [15] the regulator was designed by using the pole placement methodology, the desired poles being $s_{1,2} = -3$, $s_3 = -4$). An increase of the parameter R in J_2 leads to faster responses and higher tracking accuracy; however, the feedback gains become in this case higher, leading to higher jerks, and the system is less robust with respect to the measurement noise.

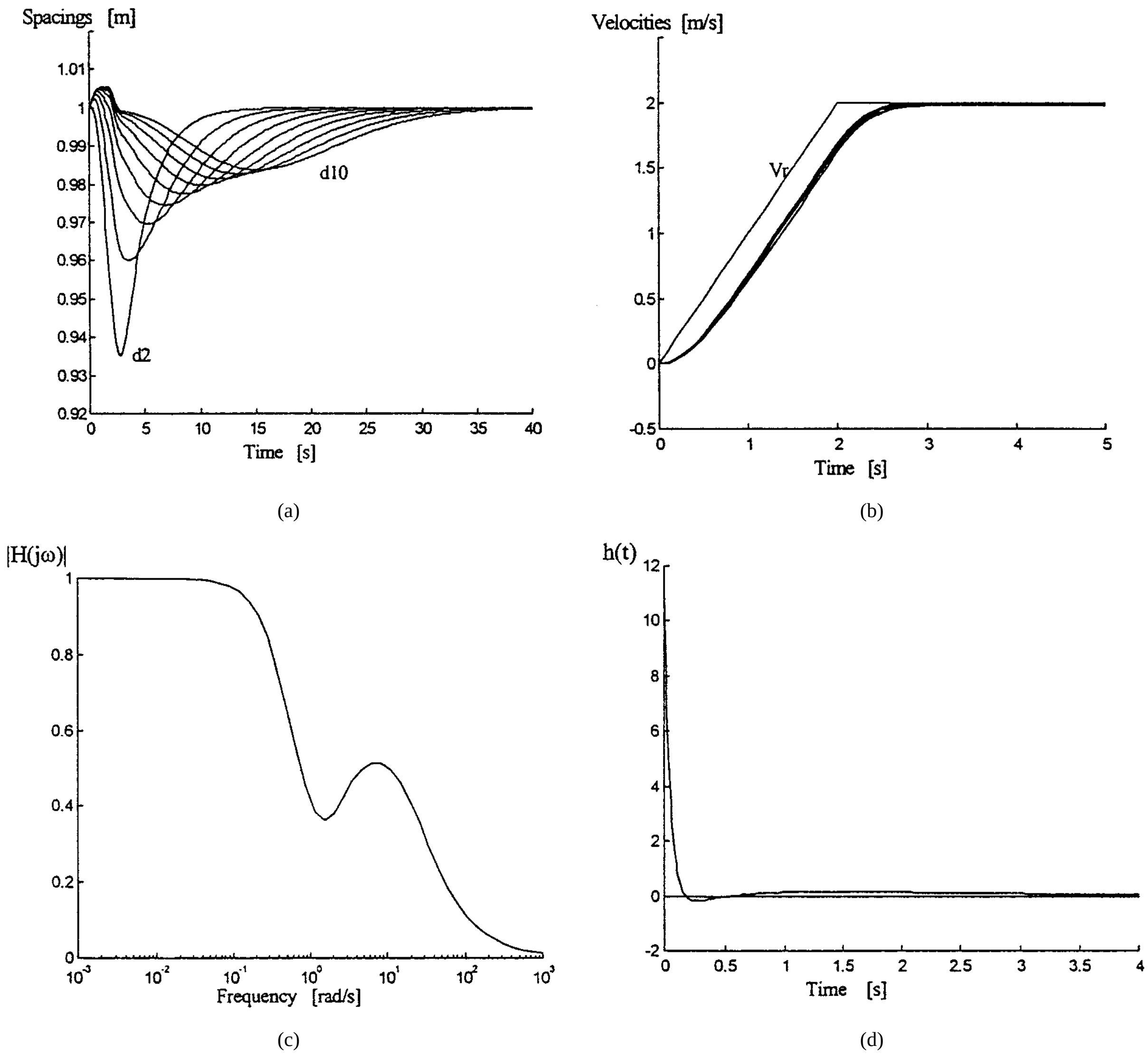


Fig. 3. Ten-vehicle platoon with $\tau = 0.1$s (speed increase): $vertH(j\omega)| < 1$; $h(t)$ changes sign. (a) Spacings; (b) velocities; (c) frequency response; (d) impulse response.

Obviously, both $|H(j\omega)|$ and $h(t)$ in Fig. 1 satisfy the platoon objectives 3 and 4 from Section II; it has been found by extensive simulation that this can be achieved for large ranges of the parameter values in both J_1 and J_2. An increase of q_{55} leads, in general, to the increase of the absolute values of the real system poles; however, when the pair of complex conjugate poles becomes dominant, overshoots of spacing and velocity become significant. On the other hand, an increase of q_{44} decreases the absolute value of the dominant real pole, but, as explained in the previous section, improves the performance from the point of view of string stability. The parameter q_{33} influences directly the spacing error; its value should be, according to the simulations, higher than the values of q_{44} and q_{55}. However, when q_{33} is very large, tracking accuracy may deteriorate and the system may become string unstable. Fig. 2 illustrates the situation when, in the above set of parameters, q_{33} is changed to 5000; the "slinky effect" is obvious from Fig. 2(c). Tracking accuracy is worse than in Fig. 1, and the maximal absolute value of the spacing error increases with the vehicle index. Low values of q_{33} are also not advisable; Fig. 3 corresponds to the case when $q_{33} = 50$. The "slinky effect" is not present, but the system is string unstable. The maximal absolute spacing error does not increase with the vehicle index, but is much higher than in Fig. 1.

Fig. 4 represents an illustration of the tracking performance obtainable in the case of higher engine time constants; it has been assumed that $\tau = 0.5$ (see [15]). The following values of the parameters in J_1 and J_2 have been adopted: $Q_L = \text{diag}\{200, 1\}$, $R_L = 0.1$, $p_1 = 50$, $p_2 = 1$, $q_{33} = 500$, $q_{44} = 100$, $q_{55} = 1$, $R = 0.1$, leading to the following feedback and feedforward parameters: $K_1 = [44.7214\ 6.4647]$, $K_2 = [-17.0102\ \ -1.6804\ \ -70.7107\ \ 48.3832\ \ 7.0057]$, $\overline{M}_i^{22} = 89.447$, $\overline{M}_i^{51} = -141.7214$, $\overline{M}_i^{52} = 62.7459$. The responses are very fast and accurate, in spite of a complex velocity reference, and better than those in [15].

Capabilities of the dynamic controller including an observer, proposed in Section III-F are illustrated by the responses presented in Fig. 5. The reduced order observer described in Section III-F has been used for esimating both v_{i-1} and a_{i-1}; it has been designed by using the

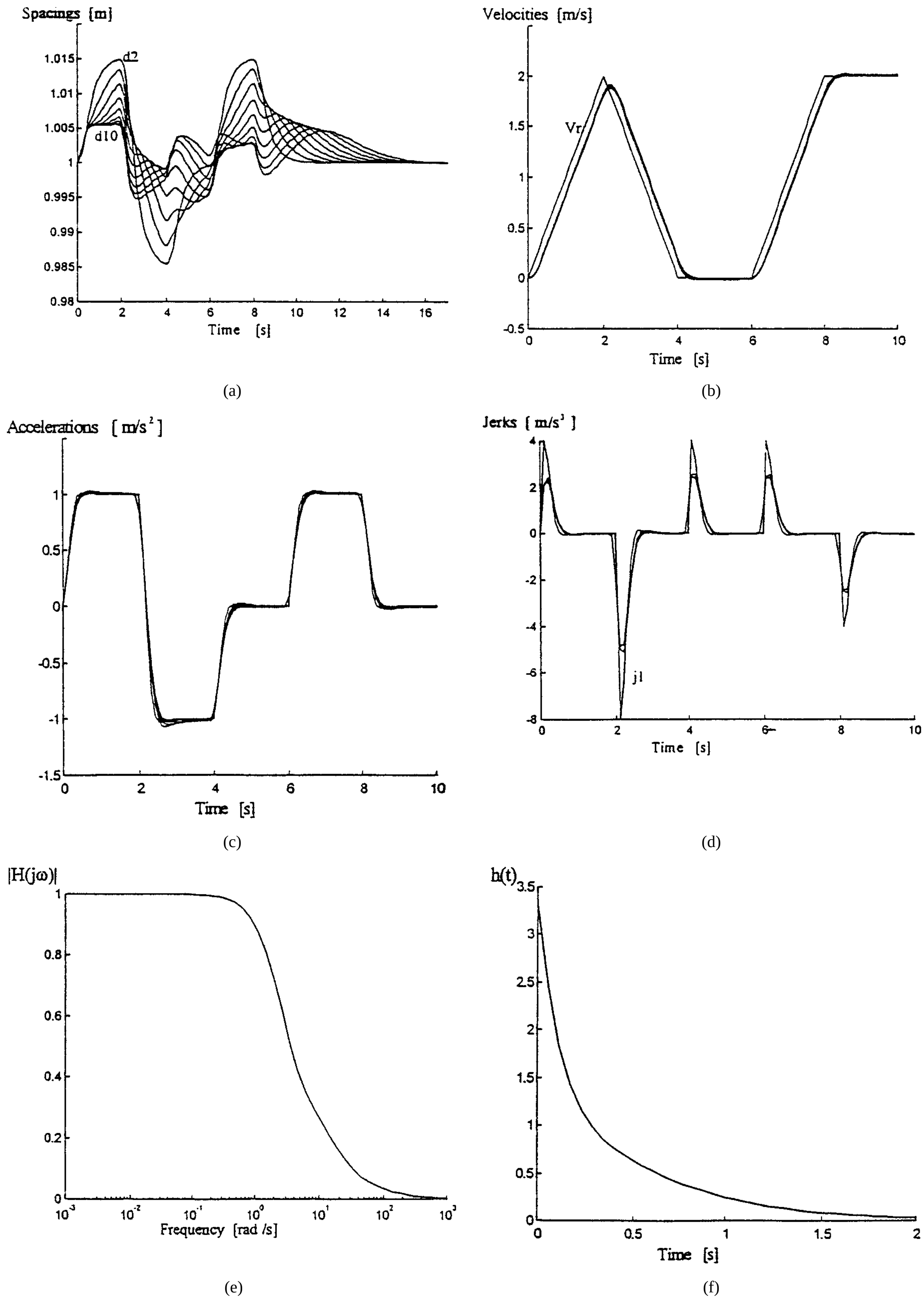
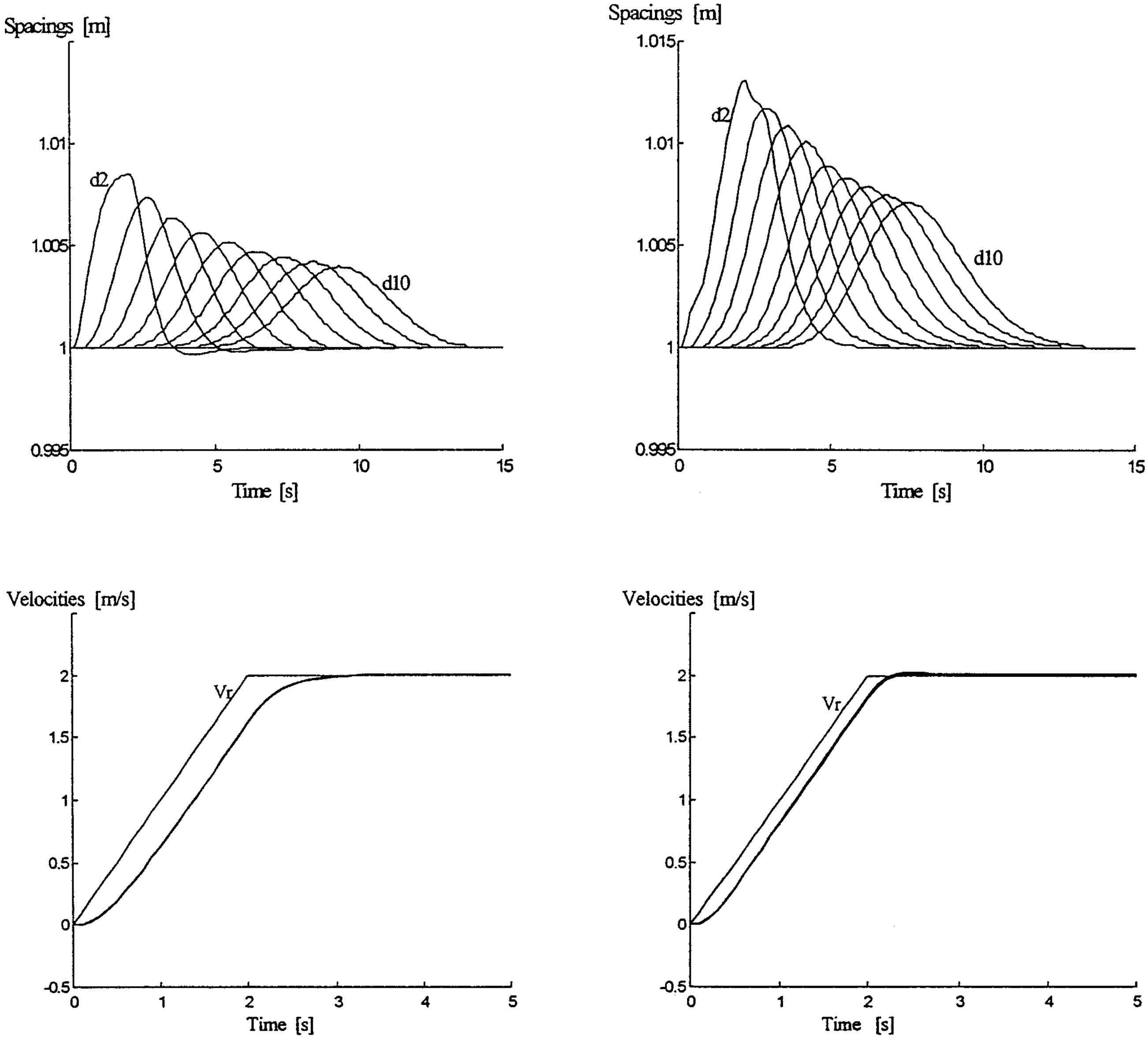


Fig. 4. Ten-vehicle platoon with $\tau = 0.5$s (complex velocity reference): responses and spacing propagation characteristics. (a) Spacings; (b) velocities; (c) accelerations; (d) jerks; (e) frequency response; (f) impulse response.

Kalman filtering methodology. Time histories in Fig. 5(a) are obtained by using the same feedback and feedforward gains as in Fig. 1, and those in Fig. 5(b) by using the gains corresponding to Fig. 4. Tracking accuracy is still very high; in general, the lack of measurements does not lead to significant performance deterioration owing to the chosen

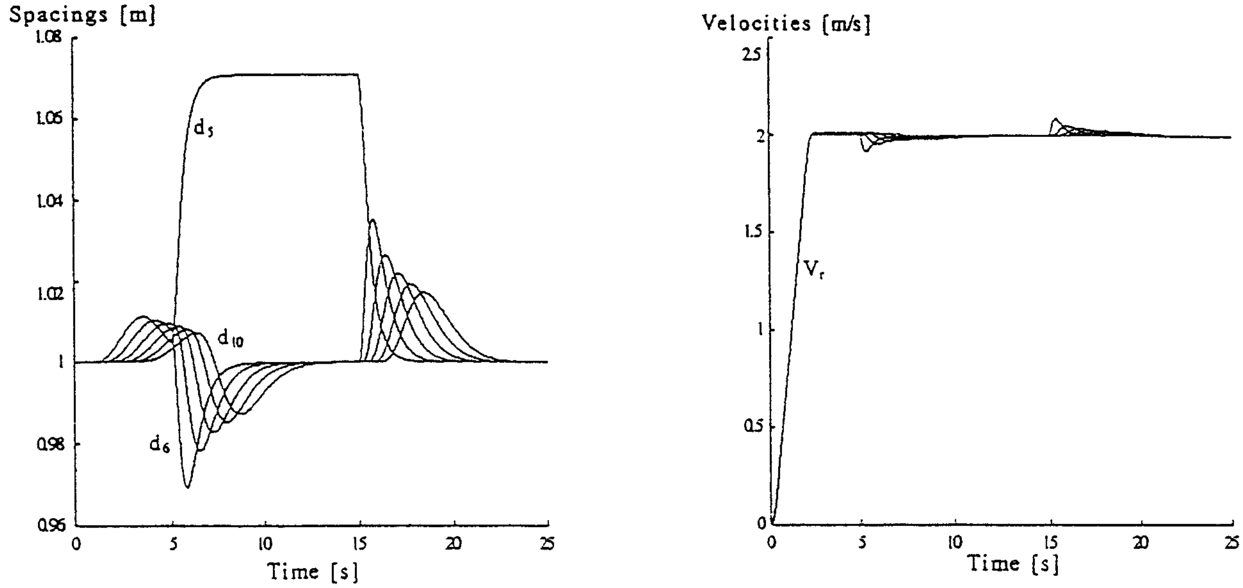


(a) (b)

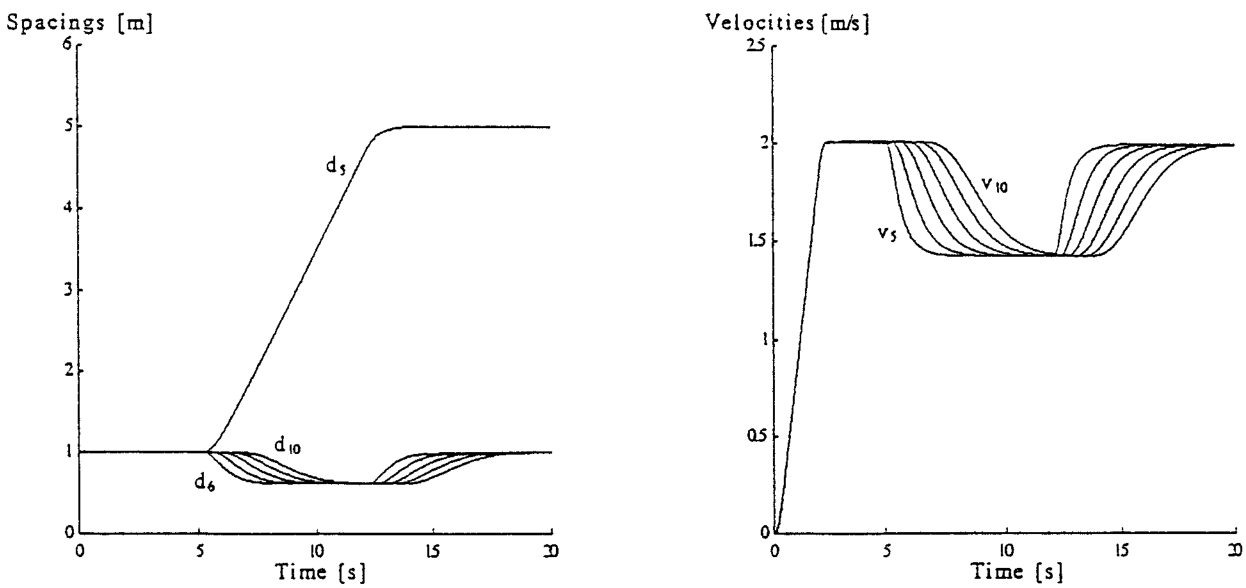
Fig. 5. Ten-vehicle platoon (speed increase): dynamic regulator with observer. (a) $\tau = 0.1$s, (b) $\tau = 0.5$s.

subsystem structure and the efficiency of the corresponding observer.

Time-histories presented in Fig. 6 are related to the important problem of the sensitivity of the proposed control system to the influence of disturbances. Various types of disturbances typical for real operation conditions have been taken into consideration. Fig. 6(a) corresponds to a ten-vehicle platoon with $\tau = 0.5$ s (the same as in Fig. 4), in which a considerable constant breaking force of 0.5 g is applied to the fifth vehicle in the interval [5 s, 15 s]. The dynamic controller described in Section III-F is applied. The influence to the velocity tracking is hardly visible; only the distance d_5 becomes slightly biased during breaking. The time-histories in Fig. 6(b) have been obtained by assuming that the fifth vehicle does not receive the information about the reference velocity; bias in d_5 is now more pronounced, but the velocity tracking is still satisfactory. This situation is closely related to the headway control strategy [22] and to the problem of autonomous vehicle control. It has been found in [21] that the proposed methodology can be successfully applied in the case of headway control. Fig. 6(c) corresponds to the situation when d_r for the fifth vehicle is time varying, changing from its initial value 1 to its final value 5 linearly between 6 s and 12 s (a vehicle intending to leave its lane). The results are extremely good. Fig. 6(d) represents an illustration of the capabilities of the proposed methodology to cope with noisy and missing measurements. A decentralized Kalman filter is applied to the estimation of d_i, v_i, a_i, v_{i-1} and a_{i-1} using noisy measurements of d_i, v_i and a_i, where the noise is assumed to be white and zero-mean with standard deviations 2 cm, 5 cm/s and 0.01 g, respectively. In the time-histories the curves with the indication "static feedback" correspond to the the state feedback described in Section III-C and III-D, directly applied to the noisy measurements of all the state variables (including v_{i-1} and a_{i-1}). The remaining curves, as well as all the curves repre-



(a)



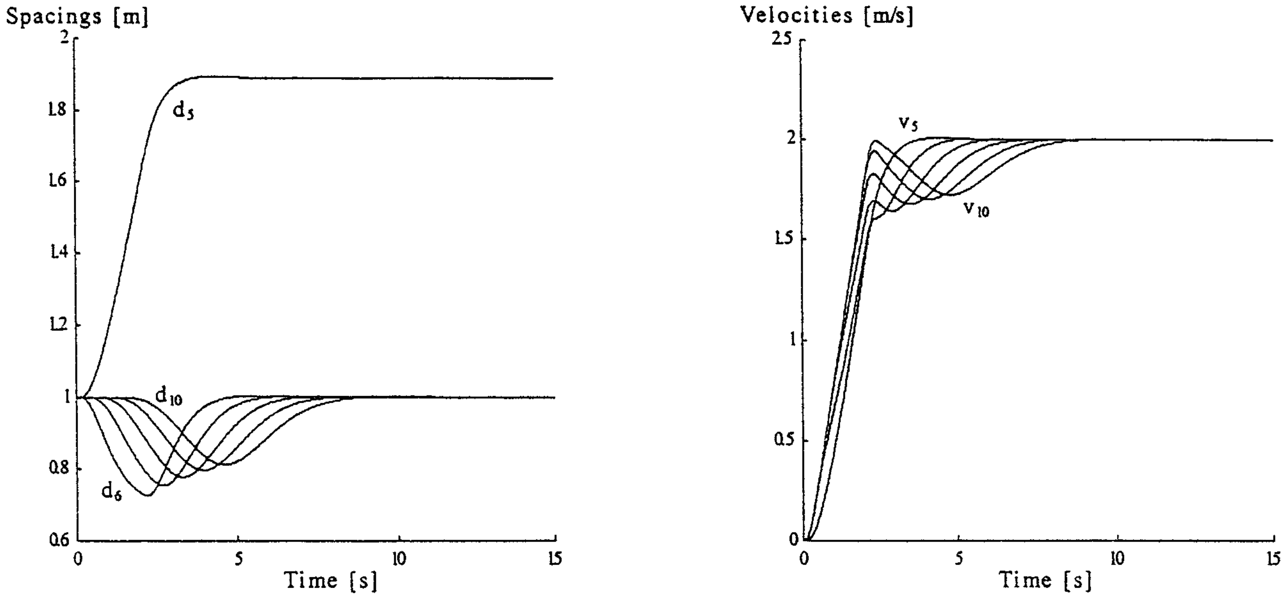
(b)

Fig. 6. Ten-vehicle platoon with $\tau = 0.5$ s: Effects of disturbances. (a) Breaking force. (b) Change of distance reference.

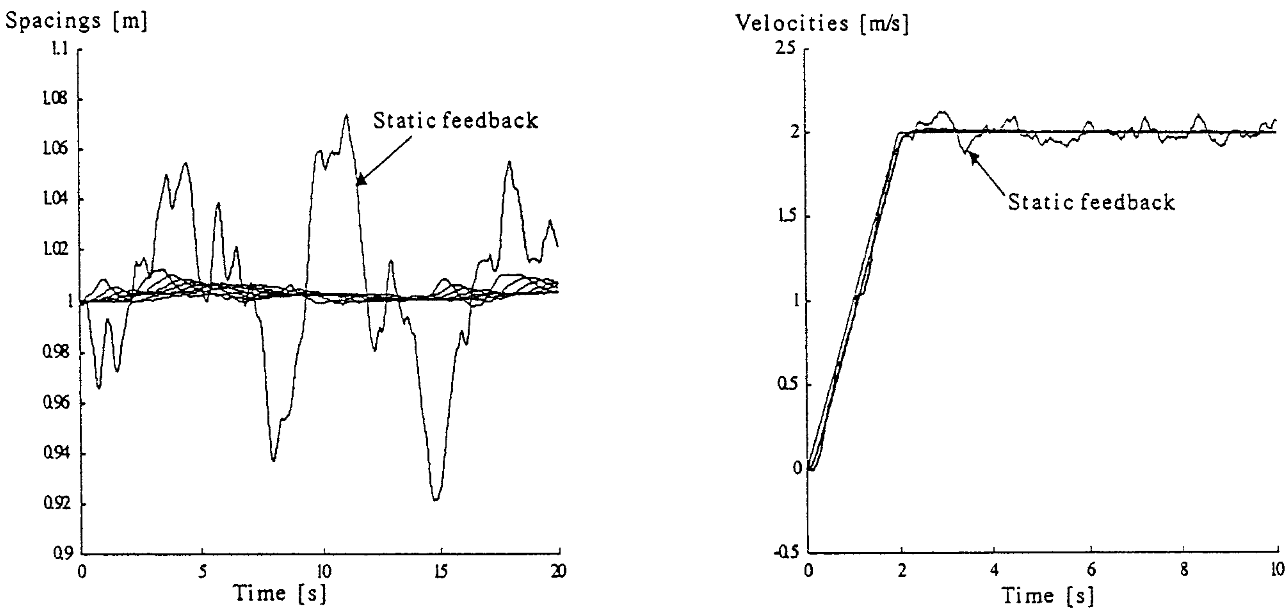
senting the distance time-histories, are obtained by applying the LQG dynamic regulator incorporating the above Kalman filter. The efficiency of dynamic regulators is obvious (platoon control in the stochastic case represents an interesting and important issue for research, see [20]).

V. Conclusion

In this paper a new methodology for control design of platoons of vehicles within IVHS is proposed. The presented decentralized overlapping control strategy is based on the application of the inclusion principle and local sequential LQ optimization of tracking both the given reference signals and the state of preceding vehicle. *A priori* choice of the weights in the local quadratic criteria allows obtaining diverse desired overall system characteristics. The obtained LQ suboptimal platoon controller is rigorously analyzed from the point of view of the general platoon objectives (asymptotic stability, steady-state regime, "slinky effect," string stability). It is shown that solutions satisfying all the given requirements can be obtained for a large variety of performance criterion forms. Moreover, the adopted platoon decomposition strategy is proven to be convenient for designing, again on the basis of the concepts of the inclusion principle, decentralized dynamic platoon controllers containing state observers (in the case when the information from the preceding vehicles is missing). The experimental part of the paper shows that the proposed platoon control design methodology represents a reliable tool for getting, in a systematic way, high platoon performance for a broad range of operating conditions and desired system performances.



(c)



(d)

Fig. 6. (*Continued*.) Ten-vehicle platoon with $\tau = 0.5$ s: Effect of disturbances. (c) Loss of velocity reference. (d) Stochastic disturbances.

The proposed methodology can be directly applied to the headway platoon control strategy [21]. An extension to the stochastic case, when the local LQG optimization is applied to the subsystems containing both measurement noise and stochastic input disturbances, leads to dynamic regulator structures (see [5] and [18]). Decentralized estimation is done by Kalman filters, using sets of noisy measurements [9]. Notice that, again, stability and suboptimality in the expanded space imply stability and suboptimality in the original space ([17], [18], [20]). An analysis of structural stability aspects of platoon control is also feasible owing to the fact that the inclusion principle has been taken as a basis for control design.

APPENDIX I
INCLUSION PRINCIPLE

In this Appendix we shall concisely present those definitions and theorems related to the inclusion principle that are directly utilized within the framework of the proposed control strategy.

Consider a pair $(\mathbf{S}, \tilde{\mathbf{S}})$ of linear time-invariant continuous-time dynamic systems represented by

$$\begin{aligned} &\mathbf{S}\colon\ \dot{x}(t) = Ax(t) + Bu(t); \qquad y(t) = Cx(t) \\ &\tilde{\mathbf{S}}\colon\ \dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t); \qquad \tilde{y}(t) = \tilde{C}\tilde{x}(t). \end{aligned} \tag{32}$$

where $x(t_0) = x_0$ and $\tilde{x}(t_0) = \tilde{x}_0$. In the above equations, $x(t) \in R^n$ and $\tilde{x}(t) \in R^{\tilde{n}}$ are the states, $u(t) \in R^p$ and $\tilde{u}(t) \in$

$R^{\tilde{p}}$ the inputs, and $y(t) \in R^q$ and $\tilde{y}(t) \in R^{\tilde{q}}$ the outputs of $\mathbf{S}$ and $\tilde{\mathbf{S}}$, respectively. It is assumed that $n \le \tilde{n}$, $p \le \tilde{p}$ and $q \le \tilde{q}$.

Definition A1: The system $\tilde{\mathbf{S}}$ includes the system $\mathbf{S}$ if there exists a quadruplet of full rank matrices $\{U, V, R, S\}$ satisfying $UV = I_n$, such that for any x_0 and $u(t)$ in $\mathbf{S}$ the conditions $\tilde{x}_0 = Vx_0$ and $\tilde{u}(t) = Ru(t)$ imply $x(t) = U\tilde{x}(t)$ and $y(t) = S\tilde{y}(t)$ [16]. □

A particular attention has been paid to restriction and aggregation, two special cases of inclusion; conditions covering different combinations of state, input and output contractions/expansions are presented in [18]. We shall focus our attention to one particular case of restriction [8].

Theorem A1: The system $\mathbf{S}$ is a restriction of $\tilde{\mathbf{S}}$ if there exist full rank matrices $\{V, R, T\}$ such that

$$\tilde{A}V = VA; \quad \tilde{B}R = VB; \quad \tilde{C}V = TC. \quad \square \tag{33}$$

If the matrices A, B, and C are specified, matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ can be, in general, expressed as

$$\begin{aligned} &\tilde{A} = VAU + M_c; \quad \tilde{B} = VBQ + N_c; \\ &\tilde{C} = TCU + L_c, \end{aligned} \tag{34}$$

where M_c, N_c and L_c are complementary matrices [16], [7], [8]. In the above case of restriction, these matrices have to satisfy

$$M_cV = 0; \quad N_cR = 0; \quad L_cV = 0. \tag{35}$$

Suppose that static state feedback control laws $\mathbf{F}$ and $\tilde{\mathbf{F}}$ are introduced

$$\mathbf{F}\text{: } u(t) = Kx(t) + v(t); \quad \tilde{\mathbf{F}}\text{: } \tilde{u}(t) = \tilde{K}\tilde{x}(t) + \tilde{v}(t) \tag{36}$$

where $v(t)$ and $\tilde{v}(t)$ are reference signals.

Definition A2: The controller $\tilde{\mathbf{F}}$ for $\tilde{\mathbf{S}}$ is contractible to the controller $\mathbf{F}$ for $\mathbf{S}$ if the closed-loop system $(\tilde{\mathbf{S}}, \tilde{\mathbf{F}})$ includes the closed-loop system $(\mathbf{S}, \mathbf{F})$ in the sense of Definition A1. □

Theorem A2: The state feedback controller $\tilde{\mathbf{F}}$ is contractible to the state feedback controller $\mathbf{F}$ if $\mathbf{S}$ is a restriction of $\tilde{\mathbf{S}}$, and the condition $RK = \tilde{K}V$ is satisfied. □

In the case of dynamic feedback, we have, in general, concatenations of observers and static linear feedback mappings [5], [8], [16], [18], [19]:

$$\begin{aligned} &\mathbf{F}\text{: } u(t) = Kw(t) + v(t); \\ &\dot{w}(t) = Fw(t) + Gu(t) + Ly(t); \qquad (w(0) = w_0) \end{aligned} \tag{37}$$

$$\begin{aligned} &\tilde{\mathbf{F}}\text{: } \tilde{u}(t) = \tilde{K}\tilde{w}(t) + \tilde{v}(t); \\ &\dot{\tilde{w}}(t) = \tilde{F}\tilde{w}(t) + \tilde{G}\tilde{u}(t) + \tilde{L}\tilde{y}(t); \qquad (\tilde{w}(0) = \tilde{w}_0) \end{aligned} \tag{38}$$

where $w(t) \in R^s$ and $\tilde{w}(t) \in R^{\tilde{s}}$ are the observer outputs $(s \le \tilde{s})$.

Definition A3: The observer in $\tilde{\mathbf{F}}$ (38) includes the observer in $\mathbf{F}$ (37) if there exist full rank matrices (U, V, R, D, E), satisfying $UV = I_n$ and $DE = I_s$, such that, for any given $X(0) = [x_0^T, w_0^T]^T$ and $u(t)$, the conditions $\tilde{X}_0 = V^*X_0$ and $\tilde{u}(t) = Ru(t)$ imply $X(t) = U^*\tilde{X}(t)$ $(\forall t \ge t_0)$, where $X(t) = [x(t)^T\ w(t)^T]^T$, $U^* = \text{diag}\{U, D\}$ and $V^* = \text{diag}\{V, E\}$ [19], [18]. □

Definition A4: The dynamic controller $\tilde{\mathbf{F}}$ includes the dynamic controller $\mathbf{F}$ if the observer in $\tilde{\mathbf{F}}$ includes the observer in $\mathbf{F}$ in the sense of Definition A3, and $\tilde{u}(t) = Ru(t)$ [8], [18], [5]. Then the closed-loop system consisting of the pair $(\tilde{\mathbf{S}}, \tilde{\mathbf{F}})$ includes the closed-loop system consisting of the pair $(\mathbf{S}, \mathbf{F})$. □

Theorem A3: The observer in $\mathbf{F}$ is a restriction of $\tilde{\mathbf{F}}$ if the conditions of Theorem A1 hold, together with $\tilde{F}E = EF$, $\tilde{G}R = EG$ and $\tilde{L}T = EL$ [19], [18]. □

Theorem A4: The dynamic controller $\tilde{\mathbf{F}}$ includes the dynamic controller $\mathbf{F}$ if the observer in $\tilde{\mathbf{F}}$ includes the observer in $\mathbf{F}$ in the sense of Theorem A3 and $\tilde{K}E = RK$ [19], [18]. □

Definition A2 gives, together with Theorem A2, the inclusion conditions for the static state feedback controllers, while Definitions A3 and A4, together with Theorems A3 and A4, present the inclusion conditions for the dynamic controllers. Both cases are treated within the main body of the paper. For example, if a static state feedback gain $\tilde{K}$ is defined for the expanded system $\tilde{\mathbf{S}}$, then the corresponding gain K in the space of $\mathbf{S}$ (satisfying the inclusion conditions), can be obtained by $K = Q\tilde{K}V$ if $\tilde{K}V = RQ\tilde{K}V$, where $QR = I_p$ (Theorem A2). Also, in the case of dynamic controllers, the observer gain L in $\mathbf{F}$ can be obtained from a predefined observer gain $\tilde{L}$ in $\tilde{\mathbf{F}}$ by $L = D\tilde{L}T$ if $\tilde{L}T = ED\tilde{L}T$ (Theorem A4; see also [19], [18], [5]). It is important to notice that the inclusion conditions guarantee that all the motions of $\mathbf{S}$ are immersed into the motions of $\tilde{\mathbf{S}}$ [16], and that, consequently, the basic properties of $\tilde{\mathbf{S}}$ are preserved in $\mathbf{S}$ (e.g. asymptotic stability).

Appendix II
Sequential LQ Optimization Algorithm

The posed problem of the sequential LQ optimization of tracking in the presence of a known disturbance represents a generalization of the regulation problem considered in [12], [17]. Pursuing the main line of thought, together with all necessary modifications related mainly to details, one obtains that the LQ optimal control input u_i at the second stage of the procedure (resulting from J_2 in (17)) is defined by

$$\begin{aligned} &u_i = -K_2\xi_i - M_2X_{2r} - M_3X_{1r} \\ &K_2 = R^{-1}B_i^TP; \quad M_2 = R^{-1}B_i^T(A_i - B_iK_2)^{-T}Q \\ &M_3 = R^{-1}B_i^T(A_i - B_iK_2)^{-T}PB_M \\ &PA_i + A_i^TP - PB_iR^{-1}B_i^TP + Q = 0 \end{aligned} \tag{39}$$

where

$$A_i = \begin{bmatrix} A_L - B_LK_1 & 0 \\ \overline{A}_d & A_v \end{bmatrix}; \qquad B_i^T = [0 \quad B_v];$$
$$B_M^T = [-M_1 \quad 0].$$

One obtains (21) after introducing $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ and performing all the matrix operations in the algebraic Riccati equation in (39). Notice that the Riccati equation for P is replaced in such a way by a Riccati equation for P_{22} and a Lyapunov equation for P_{21} in (16). The last equation depends on the important terms out of the main diagonal in Q, which take care of the preceding vehicle tracking (compare with [12], where only the diagonal form is considered).

APPENDIX III
THEOREM PROOFS

Proof of Theorem 1: The condition $|H(j\omega)| \le 1$ implies

$$\begin{aligned}\omega^4 &+ [\tau^{-2}(1+k_a+c_a)^2 - 2\tau^{-1}(k_v+c_v) - \tau^{-2}k_a^2]\omega^2 \\ &+ \tau^{-2}(k_v+c_v)^2 - 2\tau^{-2}c_d(1+k_a+c_a) \\ &- \tau^{-2}k_v^2 + 2\tau^{-2}c_d k_a \ge 0. \end{aligned} \tag{40}$$

The result of Theorem 1 follows directly as a sufficient condition for positiveness of the coefficients in (40). □

Proof of Theorem 2: Taking the inverse Laplace transform of $H(s)$, one obtains

$$\begin{aligned} h(t) = &\frac{T(-\beta_1)}{(\beta_2-\beta_1)(\beta_3-\beta_1)} e^{-\beta_2 t} \\ &\cdot \left[e^{(\beta_2-\beta_1)t} - \frac{T(-\beta_2)}{T(-\beta_1)} \frac{\beta_3-\beta_1}{\beta_3-\beta_2} \right. \\ &\left. - \frac{T(-\beta_3)}{T(-\beta_1)} \frac{\beta_2-\beta_1}{\beta_2-\beta_3} e^{(\beta_2-\beta_3)t} \right]. \end{aligned} \tag{41}$$

One has to derive conditions ensuring $h(t) \ge 0$ or $h(t) \le 0$, since then $||h||_1 = |H(0)|$, having also in mind that, in general, $||h||_1 \ge |H(0)|$ [22]. For sgn A = 1) ∩ (sgn B = 1), $h(t)$ does not change sign if

$$A \le Be^{[\beta_2-\beta_3)t} + e^{(\beta_2-\beta_1)t}. \tag{42}$$

The function of the right-hand side in (42) is minimum for $t_m = (1/\beta_3-\beta_1)\log(B(\beta_3-\beta_2/\beta_2-\beta_1))$. Replacing back t_m into (42) one gets C1. Conditions C2, C3, and C4 can be obtained in an analogous way; notice that in the case of C3 and C4 we have $t_m = 0$ for the corresponding functions. □

Proof of Theorem 3: In the case of a pair of complex conjugate roots, we have

$$\begin{aligned} h(t) = &\frac{T(-\alpha)}{|(\alpha-\delta)+j\beta|^2} e^{-\alpha t} \\ &\cdot \left[e^{(\alpha-\delta)t} + \frac{|[(\alpha-\delta)+j\beta]T(\alpha+j\beta)|}{\beta T(-\delta)} \sin(\beta t + \psi) \right] \end{aligned} \tag{43}$$

where

$$\psi = \arctan \frac{\mathrm{Im}\{T(\alpha+j\beta)\}}{\mathrm{Re}\{T(\alpha+j\beta)\}} - \arctan \frac{\beta}{\delta-\alpha}.$$

The result of Theorem 2 follows directly from (43) as a conservative condition ensuring that $h(t)$ does not change sign. Obviously, $h(t)$ changes sign for $\alpha < \delta$. □

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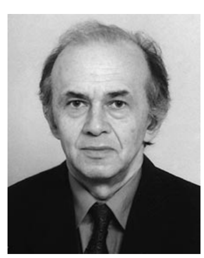

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