Proceedings of the 2014 IEEE
International Conference on Robotics and Biomimetics
December 5-10, 2014, Bali, Indonesia

Formation Control of Autonomous Robots to Track a Moving Target in an Unknown Environment

Anh Duc Dang and Joachim Horn

Abstract—**This paper presents a novel approach to path planning for the formation of a swarm of autonomous robots to track a moving target in an unknown environment. This approach is based on the traditional potential fields combined with the rotational vector field. The potential field is used to repel the robots away from obstacles while the rotational vector field is added to drive robots to avoid obstacles in the direction of the target's trajectory. Under the effect of the blended vector field, autonomous robots can easily escape obstacles in order to quickly reach the target. In this approach, the neighboring robots in a swarm will be linked to each other by the attractive and the repulsive vector field between them in order to generate a stable formation. Moreover, information about obstacles, which each member robot detects from the environment, will be sent to all other member robots in the swarm. Hence, the formation of the swarm is maintained while tracking a moving target. The effectiveness of this proposed approach has been verified in simulations.**

Keywords—**Formation control, swarm intelligence, obstacle avoidance, vector fields, multi-agent systems**

I. Introduction

Formation control of multi-robot systems has been one of the interesting research topics in the control community all over the world in recent years. Its potential applications in many areas, such as search and rescue missions, and forest fire detection and surveillance, is the motivation for this attraction.

Obstacle avoidance is one of the important problems in path planning for autonomous robots to reach the target. Furthermore, when the target moves in an unknown environment, the direction for the robots to avoid obstacles has a great influence to finding a fastest way towards the target. The artificial potential field, shown in [1]-[4], is known as a positive method in order to solve this problem. In recent years, the potential field method has been widely studied and been applied powerfully to formation control of a swarm of multi-agents to reach the position of the target in a simple environment, see [6]-[9]. The obstacle avoidance of a swarm is successful but, while avoiding obstacles, the formation of the swarm is broken. Although the artificial potential field method is advantageous for its ease of use in path planning for autonomous robots, this method is constrained in several cases due to local minima. For example, when the attractive force of the target and the repulsive force of the obstacles are equal and collinear but in opposite directions, the total force on the robot is zero and the robot's motion is stopped. Moreover, in a complex environment with the U-shaped obstacles or connected walls, the application of the potential field method to path planning for a swarm of the autonomous robots is very difficult. Robots can be trapped in these obstacles before reaching the target, see [5].

In this paper, we propose a novel approach to path planning for the formation of a swarm of autonomous robots to track a moving target in an unknown environment. In this approach, an attractive vector field is built around the target to drive all member robots towards the target position. When each robot in the swarm detects the obstacle in the environment, the repulsive and rotational force field will be used to drive robot to avoid this obstacle. The repulsive vector field, which is stronger when the robot is closer to the obstacle, will repel the robots away from the obstacle to avoid collision. The rotational vector field is added to drive the robots to avoid obstacles in the direction of the target's trajectory. Under the effect of this blended vector field, the robots can easily escape the obstacles to quickly catch the target. However, in order that robots can quickly exit the complex obstacles, the computed rotational force must be larger than the sum of the repulsive force of obstacles and the attractive force of the target. The target's direction is determined based on the relative position between the current position and the future position of the target with the preselected time-step Δt. In addition, in order to avoid collisions and maintain the stability in the formation of a swarm, the neighboring robots will be connected to each other by the attractive and repulsive vector field between them. Information about obstacles in the environment will be sent to all other member robots in the swarm. Therefore, the velocity of the robots in a formation while avoiding obstacle is same.

The rest of this paper is organized as follows: The problem statement is given in the next section. Section III presents the formation control algorithm. In section IV, the obstacle avoiding direction control is presented. Simulation results are presented in section V. Finally, section VI concludes this paper and proposes new research directions.

II. Problem Statement

In this section we consider a swarm of N robots (N≥2) that moves in a two-dimensional Euclidean space $\{R^2\}$ with M obstacles in the environment. Each robot's motion, which

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978-1-4799-7397-2/14/$31.00

is assumed as a moving point in the space, is described by the dynamic model as:

$$\begin{cases} \dot{p}_i = v_i \\ m_i \dot{v}_i = u_i \end{cases} \quad i = 1, \dots N. \tag{1}$$

Here $(p_i, v_i, u_i) \in \{R^2\}$ and m_i are the position vector, the velocity vector, the control input and the mass of the robot *i*, respectively.

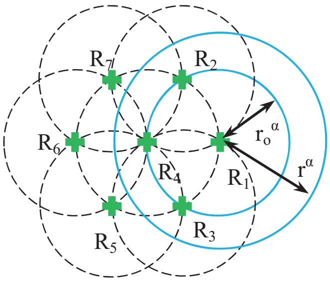

Fig.1 Configuration of a formation of seven member-robots.

In the formation of a desired swarm, the neighboring robots have to link with each other to generate the constant distances among them (example in Fig.1). Let N_i^α be the set of the robots in the neighborhood of robot *i* at time (t), such that:

$$N_i^\alpha(t) = \left\{ \forall j : d_i^j \le r^\alpha, j \in \{1,..N\}, j \ne i \right\}, \tag{2}$$

where $r^\alpha > 0$, and $d_i^j = \left\| p_i - p_j \right\|$ are the interaction range (radius of neighborhood circle, shown in Fig.1), and the Euclidean distance, respectively. For example, in Fig.1, the robot R_1 has three neighbors: R_2, R_3, R_4.

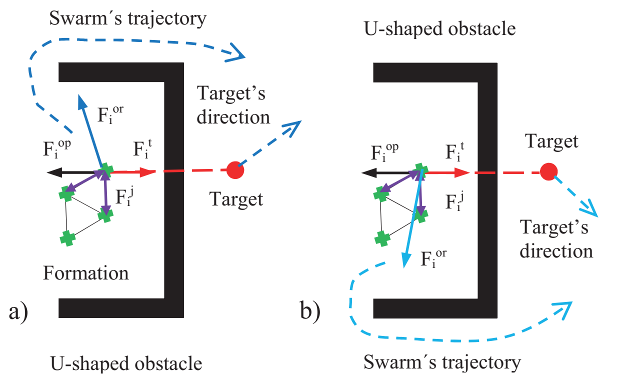

Fig.2 The description of the obstacle avoidance of a swarm of four robots to track a moving target: clockwise (a) and counter-clockwise b).

The formation control for a swarm of autonomous robots to track a moving target in an unknown environment is shown in Fig.2. This swarm must overcome the U-shaped obstacle in order to reach the target. In an unknown environment, it is very difficult to determine the optimal direction for the robots to easily escape obstacles and simultaneously reach the target quickly. The best way to solve this problem is to control the robots to avoid obstacles in the direction of the target's trajectory. Moreover, while these robots avoid obstacles, the stability and robustness of their formation must be maintained. Therefore, in order to execute this idea, each robot will be controlled by a total force that consists of the attractive force F_i^t of the target, the repulsive force F_i^{op} of the obstacle, the rotational force F_i^{or} around the obstacle, the connecting force between this robot with its neighbors F_i^j and the obstacle avoidance forces from other member robots send to this robot.

III. Formation Control Algorithm

This section presents the formation control algorithm for a swarm of N robots, which passes through M obstacles to track a moving target. As stated in section II, the final aim of the member robots in a swarm is to escape obstacles in the environment to reach the target while staying together. Accordingly, the control algorithm for each member robot *i* is given as follows:

$$u_i = f_i^t + f_i^o + f_i^j. \tag{3}$$

A. Target-tracking control

In order to control robot *i* as it moves towards the target position, the first component f_i^t in (3) is proposed as:

$$f_i^t = F_i^t(p_i) - k_{iv}^t(v_i - v_t). \tag{4}$$

In this equation, the relative velocity vector ($v_i - v_t$) between the robot *i* and the target is added as a damping term with the damping scaling factor k_{iv}^t. Under the effect of the attractive force $F_i^t(p_i)$ from the target, the robot *i* will always move towards the target position until it reaches this target. This attractive force is designed as follows:

$$F_i^t(p_i) = \begin{cases} -k_{ip}^t(p_i - p_t)/r^\tau, & if \ d_i^t < r^\tau \\ -k_{ip}^t(p_i - p_t)/d_i^t, & otherwise. \end{cases} \tag{5}$$

Here $r^\tau > 0$ is the range around the target, at which the robot's speed is reduced before reaching the target, and $(p_i - p_t)$ is the relative position vector between the robot *i* and the target. The magnitude of this force is depended on the control factor k_{ip}^t and the distance $d_i^t = \left\| p_i - p_t \right\|$ between robot *i* and the target.

B. Obstacle-avoiding control

The second component f_i^o of (3) is used to control the obstacle avoidance for each member robot of the swarm. This component is projected for each robot *i* as follows:

$$f_i^o = \sum_{k=1}^{N} \sum_{o=1}^{M} \left(F_k^{op}(\mathrm{p}_k) + F_k^{or}(\mathrm{p}_k) + k_{kv}^o c_k^o (v_k - v_o) \right), \tag{6}$$

where the relative velocity vector ($v_k - v_o$) between the robot *k* and its neighbor-obstacle *(o)* is used as a damping term with the damping scaling factor k_{kv}^p. Let $N_k^\beta(t)$ be the set of

β neighboring obstacles of the robot k at time (t). The neighbor-obstacle *(o)* of the robot k ($o \in N_k^\beta$), which the robot k must avoid, is defined similar to (2) as:

$$N_k^\beta(t) = \left\{ \forall o : d_k^o \le r^\beta, o \in \{1,..M\}, o \ne j \right\}. \quad (7)$$

Here $r^\beta > 0$ and $d_k^o = \|p_k - p_o\|$ are the obstacle detection range and the Euclidean distance, respectively. The scalar c_k^o, which is used to determine if an obstacle *(o)* is a neighboring obstacle of robot k, is defined as:

$$c_k^o = \begin{cases} 1 & if \;\; o \in N_k^\beta(t) \\ 0 & if \;\; o \notin N_k^\beta(\mathrm{t}). \end{cases} \quad (8)$$

The repulsive force field $F_k^{op}(p_k)$ is created around the neighboring obstacle (o) to repel the robot k away from this obstacle, see Fig.3. This force is designed as:

$$F_k^{op}(p_k) = c_k^o \left(\left(\frac{1}{d_k^o} - \frac{1}{r^\beta} \right) \frac{k_{kp}^{op}}{(d_k^o)^2} - k_{kp}^{o\delta} \left(d_k^o - r^\beta \right) \right) \mathrm{n}_k^o . \quad (9)$$

The positive constants $k_{kp}^{op}, k_{kp}^{o\delta}$ are used to control the fast obstacle avoidance, n_k^o is a unit vector from the obstacle to robot k.

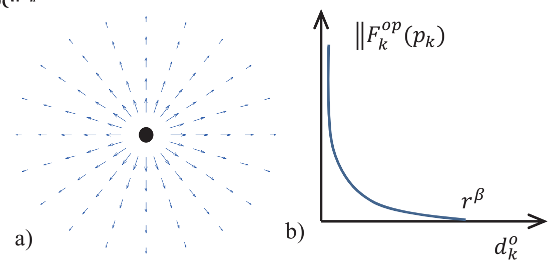

Fig.3 The repulsive force field around a neighboring obstacle of the robot k (a) and its amplitude (b).

The rotational force field $F_k^{or}(p_k)$ is also generated around the obstacle (o) to help the repulsive force to quickly drive robot k to escape this obstacle. The rotational direction of this force field can be clockwise rotation (see Fig.4a) or counter-clockwise rotation (see Fig.4b). This rotational force is built as follows:

$$F_k^{or}(p_k) = w_k^o c_k^o n_k^{or} . \quad (10)$$

Here, the unit vector n_k^{or} is given as:

$$n_k^{or} = \frac{c_k^{or}}{d_k^o} \left((\mathrm{y}_k - \mathrm{y}_o) \quad -(\mathrm{x}_k - \mathrm{x}_o) \right)^T . \quad (11)$$

In the equation (11) the scalar c_k^{or} is used to define the rotational direction for this force. This equation shows that the rotational force is clockwise if $c_k^{or} = 1$ and counter-clockwise if $c_k^{or} = -1$. The direction of this rotational force depends on the moving direction of the target, which will be discussed in section IV. The positive gain factor w_k^o is used as a control element to control robots to quickly escape obstacles. However, their velocity does not overcome the limited velocity. This control element is designed such that the total force on robots always has the direction in the selected rotational direction. Hence, this control element w_k^o is built as follows:

$$w_k^o = (1+c)\left(\left\| F_k^{top}(\mathrm{p}_k) \right\| \right) . \quad (12)$$

Here, the force $F_k^{top}(\mathrm{p}_k)$, which is the sum of the attractive force of the target $F_k^t(p_k)$ and the repulsive force of obstacle $F_k^{op}(p_k)$ on the robot k, is describes as:

$$F_k^{top}(p_k) = F_k^t(p_k) + F_k^{op}(p_k) . \quad (13)$$

The constant c, which depends on the angle α between the vector $F_k^{top}(p_k)$ and the unit vector n_k^{or}, is described as:

$$c = \begin{cases} c_1 \; , & if \;\; 0 \le \alpha < \pi/2 \\ c_2 \; , & otherwise, \end{cases} \quad (14)$$

where two constants c_1 and c_2 can be chosen as follows $-1 < c_1$, $0 < c_2$ and $c_1 < c_2$.

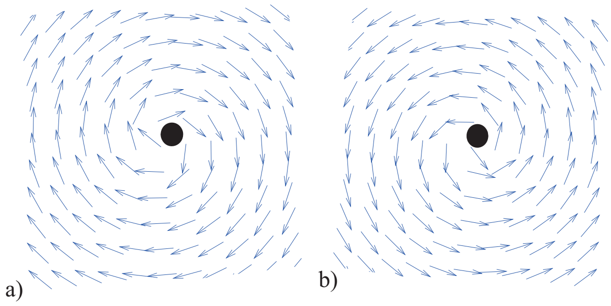

Fig.4 The clockwise rotational force field (a) and the counter-clockwise rotational force field (b)

The equation (6) shows that each member robot will obtain the environmental information from other members in the swarm. While avoiding obstacles, all member robots will be driven by the same force. Therefore, the formation of these robots is always maintained.

C. Swarm-connection control

The third component f_i^j of (3) is used to control the robust connection between neighboring robots in the formation. As shown in [9], this connection is controlled based on the combination of the attractive vector field and the repulsive vector field between the neighboring robots. Furthermore, in order to obtain the quick stability at the balance point, at which the distance between the neighboring robots is constant, the relative velocity vector ($v_i - v_j$) between them is added as a damping term with the scaling factor k_{iv}^{j}. This control component is implemented as follows:

$$f_i^j = \sum_{j=1, j\neq i}^{N} \left(F_i^j(p_i) - k_{iv}^j c_i^j (v_i - v_j) \right). \tag{15}$$

In this equation, the scalar c_i^j is used to determine if the robot j is a neighbor of robot i. It is defined as:

$$c_i^j = \begin{cases} 1 & \text{if } j \in N_i^\alpha(t) \\ 0 & \text{if } j \notin N_i^\alpha(\mathrm{t}). \end{cases} \tag{16}$$

To create the attractive/repulsive force field $F_i^j(p_i)$ between the robot i and its neighbor j, a respective potential function is proposed as:

$$V_i^j(\mathrm{p}_i) = \frac{c_i^j}{2}\left(\left(\frac{k_{ip}^{1j}}{d_i^j} - k_d \right)^2 + k_{ip}^{2j}\left(d_i^j - r_1^\alpha\right)^2 \right). \tag{17}$$

Taking the negative gradient (see [1]-[4]) of this potential function at p_i, we obtain the attractive/repulsive force, which is described in Fig.5, as follows:

$$F_i^j(\mathrm{p}_i) = c_i^j \left(\left(\frac{k_{ip}^{1j}}{d_i^j} + k_d \right) \frac{k_{ip}^{1j}}{(d_i^j)^2} - k_{ip}^{2j}\left(d_i^j - r_1^\alpha\right) \right) n_i^j \tag{18}$$

where $n_i^j = \left(p_i - p_j\right) / \left\| p_i - p_j \right\|$ is a unit vector along the line connecting p_i *to* p_j and d_i^j is the Euclidean distance shown in equation (2). The positive constants ($k_{ip}^{1j}, k_{ip}^{2j},$) are used to regulate the fast collision avoidance, and the stability in the set of the α neighbors of the robot i. The distance r_1^α is a minimum desired distance at which the attractive/repulsive forces balance. The positive factor k_d is used as a control element to control the balance position between the attraction and the repulsion.

By equating $\left(\left(k_{ip}^{1j}/d_i^j + k_d\right)k_{ip}^{1j}/(d_i^j)^2 - k_{ip}^{2j}\left(d_i^j - r_1^\alpha\right)\right) = 0$, one can find a value $d_i^j = r_0^\alpha$ at which the sum of the attractive force and the repulsive is zero. In other words, if there is a given value $r_0^\alpha \geq r_1^\alpha$ and the line $-k_{ip}^{2j}\left(d_i^j - r_1^\alpha\right)$ is not changed, then the control element k_d is determined as a function of the r_0^α. This function is described as

$$k_d = k_{ip}^{2j}\left(r_0^\alpha - r_1^\alpha\right)\left(r_0^\alpha\right)^2 / k_{ip}^{1j} - k_{ip}^{1j} / r_0^\alpha . \tag{19}$$

The interacting ranges ($r_0^\alpha, r^\alpha > 0$, shown in Fig.1) describe the influence of the force $F_i^j(\mathrm{p}_i)$ on the robot i. When $0 < d_i^j < r_0^\alpha$, then robots i and j repel each other to avoid the collisions between them. Otherwise, when $r_0^\alpha < d_i^j \leq r^\alpha$, they attract each other to achieve the equilibrium position $(d_i^j = r_0^\alpha)$ in the set of α neighbors of robot i. In case $d_i^j > r^\alpha$ there is no interaction between these members.

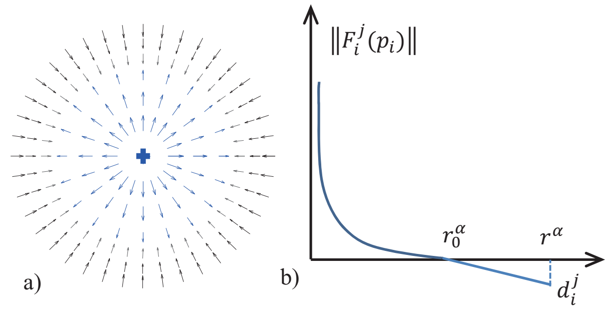

Fig.5 The attractive/repulsive force field (a) and the amplitude of the force of robot j acts on robot i (b).

IV. Obstacle Avoiding Direction

This section presents the algorithm to determine the obstacle-avoidance direction for robots according to the direction of the target's trajectory. This algorithm is built based on the geometry, as depicted in Fig.6.

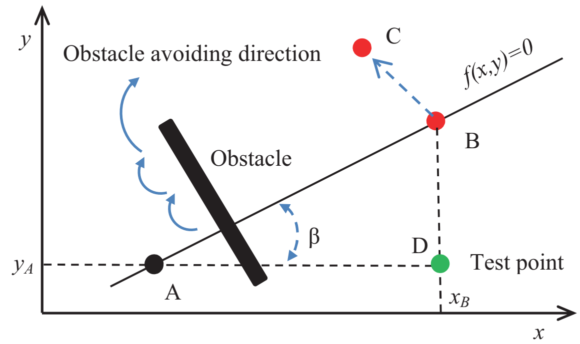

Fig.6 The description of the obstacle avoiding direction of a swarm according to the clockwise direction in case $0 < \beta < \pi / 2$.

In Fig.6, points B and C are the position of the target at time (t) and at time (t+Δt), respectively. The positive constant Δt is a preselected time-step used to determine the relative position between B and C. The point A, which is the center of the swarm at time (t), is calculated as follows:

$$A = \frac{1}{N}\sum_{i=1}^{N} p_i(\mathrm{t}). \tag{20}$$

The line *f(x,y)=0* through points A and B (see in [10], [11]) is used as the basis to determine the moving direction of the target. Here *f(x,y)* is described as follows:

$$f(x,y) = \frac{x - x_A}{x_B - x_A} - \frac{y - y_A}{y_B - y_A}. \tag{21}$$

The angle β between the vector BA and the unit vector along the coordinate x-axis is added to determine the direction of the relative position vector between B and A. As shown in [10], the constituted line from two points A and B *f(x,y)=0* will split the coordinate plane *xoy* into two halfplanes. One side of this boundary line consists of all points that satisfy the inequality $f(x,y)<0$. Otherwise, all points on the opposite side satisfy the inequality $f(x,y)>0$. In order to know which side of the boundary line $f(x,y)=0$ the target is moving towards, we choose a test point $D=(x_B, y_A)^T$. This test point is used for comparison against the position of the target (point C) at time ($t+\Delta t$), see Fig.6. If point C lies on the half-plane containing the test point D then $f(C)f(D)>0$. In contrast, if C and D lie on the different sides of the boundary line $f(x,y)=0$ then $f(C)f(D)<0$. In Fig.6, the rotational direction c_k^{or} of the rotational force, which is presented in section III, is depicted in the case $0<\beta<\pi/2$. This rotational direction is determined by the moving direction of the target. It is described as follows:

$$c_k^{or} = \begin{cases} 1 & \text{if } f(C)f(D)<0 \\ -1 & \text{Otherwise}. \end{cases} \tag{22}$$

However, in practice the angle β can span any of the quadrants of the coordinate system xoy. Moreover, if the target does not move or the test point (D) sits on the boundary line $f(x,y)=0$, then $f(C)f(D)=0$. In these situations, the scalar c_k^{or} can be chosen arbitrary as $c_k^{or}=1$ or $c_k^{or}=-1$ (in this paper $c_k^{or}=1$ is chosen). In summary, the obstacle avoiding direction for the robots is proposed in table I.

TABLE I.
DETERMINATION ROTATIONAL DIRECTION

β	$f(C)f(D)$	c_k^{or}
$0 \le \beta < \pi/2$	$f(C)f(D) \le 0$	1
	$f(C)f(D) > 0$	-1
$\pi/2 \le \beta < \pi$	$f(C)f(D) \ge 0$	1
	$f(C)f(D) < 0$	-1
$\pi \le \beta < 3\pi/2$	$f(C)f(D) \le 0$	1
	$f(C)f(D) > 0$	-1
$3\pi/2 \le \beta < 2\pi$	$f(C)f(D) \ge 0$	1
	$f(C)f(D) < 0$	-1

V. SIMULATION RESULTS

This section presents the simulation results of the formation control of robots to track a moving target in an unknown environment.

TABLE II
PARAMETER VALUES

Parameter	Definition	Value
r_1^{α}	Minimum desired distance for neighbors	10
r_0^{α}	Desired distance for neighbors	20
r^{β}	Obstacle detecting range	30
r^{τ}	Distance of approach to target position	50
c_1	Constant	0,7
c_2	Constant	1,8
k_{iv}^{j}, k_{iv}^{t}	Damping factors	1,2
k_{ip}^{1j}, k_{ip}^{2j}	Factors for fast links	80
k_{ip}^{op}, $k_{ip}^{o\delta}$	Constants for fast obstacle avoidance	100
k_{ip}^{t}	Constant for fast approach to target	2,8

For the simulations, the general parameters for the simulations are listed in table II. Firstly, we test the path planning algorithm for the formation of a swarm of four robots. This swarm must avoid obstacles, which are U-shaped, to track a moving target. For these simulations, the initial positions of the robots are chosen as follows: $p_1=(20,180)^T$, $p_2=(30,230)^T$, $p_3=(40,170)^T$, $p_4=(10,210)^T$.

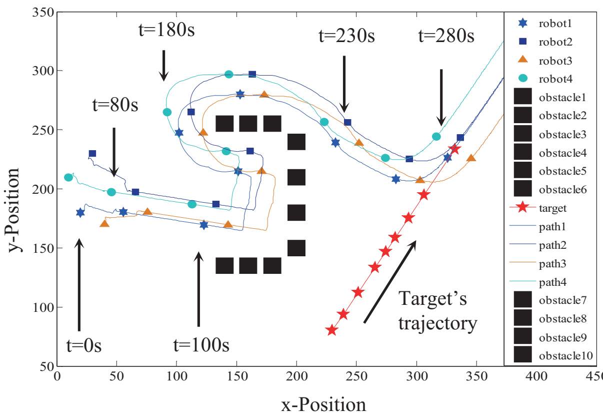

Fig.7 U-shaped obstacle avoidance of a swarm to track a moving target p_{t1}.

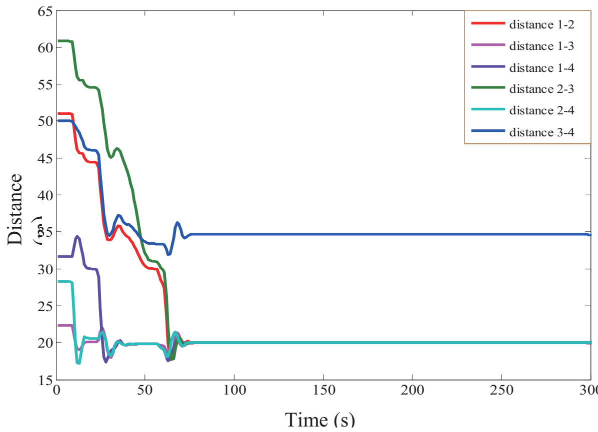

Fig.8 The distance between robots in swarm at time t.

The first case, in which the target moves along the trajectory $p_{t1}=(0.6t+230, 0.9t+80)^T$, is depicted in Fig.7 and Fig.8. The results of the simulations in Fig.7 and Fig.8 show that the formation of a swarm of four robots is maintained while the swarm tracks a moving target. At initial time, all robots move freely, but after a time of circa 80s they are linked to each other in order to generate a desired formation. Then, this formation moves towards the target position by the attractive force field from the target until it meets the obstacles. When the swarm detects the obstacle it changes its moving direction to avoid collision with this obstacle and searches the new path towards the target. Fig.7 shows that the obstacle avoidance of the swarm according the moving direction of the target is successful. The robots can easily escape the U-shaped obstacle without breaking the formation. The distance between the neighboring robots in the swarm is kept constant, see Fig.8. After the robots overcome the obstacle, they continue to chase the target until it is reached at time t=280s, see Fig.7

The second case, in which the target moves along the trajectory $p_{t2} = (0.6t + 230, -0.9t + 320)^T$, is depicted in Fig.9. The simulation result in this case shows that the intelligence of a swarm while pursuing a moving target. The robots in the swarm move according to the movement direction of the target in order to quickly catch the target.

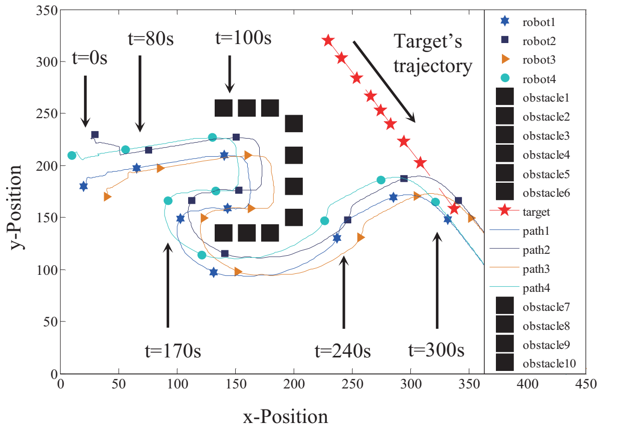

Fig.9 U-shaped obstacle avoidance of a swarm to track a moving target p_{t2}.

Secondly, the path planning algorithm for the formation of a swarm of four robots in an environment with the connected walls will be tested. The target's trajectory is $p_{t3} = (-0.4t + 150, -0.9t + 250)^T$. For this simulation, the initial positions of the robots are chosen as follows: $p_1 = (270,320)^T$, $p_2 = (300,300)^T$, $p_3 = (310,340)^T$, $p_4 = (330,310)^T$,

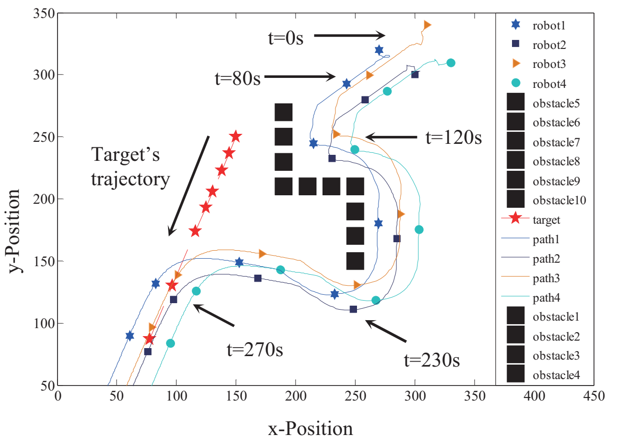

Fig.10 Wall-shaped obstacle avoidance of a swarm to track a moving target.

The simulation result depicted in Fig.10 shows that the swarm of four robots successfully escapes the wall-shaped obstacle. The robots can quickly exit this wall-shaped obstacle in order to move towards the target. The directional movement of these robots is driven towards the right of the wall-shaped obstacle (clockwise direction) at time t=120s. The change in direction helps the robots avoid collision with the obstacles and find the fastest way to chase the target. Moreover, the formation of the robots is not broken.

VI. CONCLUSION

This paper has presented an approach to path planning for the formation of a swarm of autonomous robots to track a moving target in an unknown environment based on the combination of the traditional potential fields and the rotational vector field. The rotational force field is added to help the robots to quickly escape obstacles. The movement direction for the robots to avoid obstacles is designed to be in the moving direction of the target, such that the robots can easily escape the obstacles and find the fastest path towards the target. The results of the simulations have shown that, under the effect of the blended force field, a swarm of autonomous robots can easily find a path to track a moving target in an unknown environment. Moreover, the robots in a swarm are connected to each other and they obtain information about the obstacles of the environment from other member robots. Thus, the formation of the swarm is maintained while tracking a moving target. Formation control of multi robots to avoid moving obstacles is an interesting topic for our future research.

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