Multirate Sampled Data Control of Nonholonomic Systems in Time-State Control Form Based on Periodic Switching

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Multirate Sampled Data Control of Nonholonomic Systems in Time-State Control Form Based on Periodic Switching†

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This paper proposes a new feedback control system based on a multirate sampled data control and a periodic switching for a class of nonholonomic systems in time-state control form. A coordinate transformation is introduced to transform the discretized nonholonomic system with a zero order hold and a sampler into a linear time-invariant discrete-time system. Moreover, a simple necessary and sufficient condition is derived to assure the controllability of the transformed linear system. The contributions are as follows. First, the problem of finding a controller to stabilize the nonholonomic systems is reduced to the well-known pole assignment problem. As a result, a simple and explicit design method of the stabilizing controllers is obtained. Secondly, a quadratic regulator problem is posed and is solved of finding a controller to minimize a cost functional consisting of the state and the control input. This minimization problem is reduced to a linear quadratic regulator (LQR) problem, and a simple design method of the optimal feedback gain is presented. Finally, a simulation for a two-wheeled vehicle demonstrates the effectiveness of the proposed method.

Key Words: nonholonomic systems, chained form, time-state control form, multirate sampled data control, optimal regulator

1. Introduction

This paper addresses a feedback stabilization problem for a class of nonholonomic systems described in chained form. A chained form is a canonical form introduced by Murray and Sastry[12] and it is widely accepted that many mechanical systems with nonholonomic constraints can be locally, or globally, converted to the chained form. For example, such mechanical systems include cars with several trailers and space robot and so on. A nonholonomic system described in the chained form is called chained system. The feedback stabilization problem for the chained systems has been studied by many researches. The major obstruction to the stabilization problem was the fact that there exists no continuous time-invariant state feedback controller to stabilize the chained systems[2]. During the last few years, several novel controller designs have been proposed to stabilize the chained systems[1),3),5)~8),11)~18)], for example, the methods based on discontinuous feedback control, on time-varying feedback control, on multirate digital control and on time-state control form, and so on. For detail, see 10).

Monaco and Cyrot[11] have proposed a control strategy based on a multirate digital control for the chained systems. Nam and Mita[13] have extended this control strategy to high degree chained systems and have presented a design method of the stabilizing controllers. The objective of the control low has been focused mainly on a deadbeat regulation that settles the state in the chained systems to perfectly zero in a finite time. On the other hand, Kiyota and Sampei[8], Fujimoto et al.[3] and Hoshi and Sampei[5] have considered feedback stabilization problems for the chained systems via time-state control form introduced by Sampei[16] and have proposed control systems based on logic-based switching of controllers. In 8) and 5), some switching conditions to assure the stability of the feedback systems have been derived by using Lyaponov functions. In 3), a periodic switching low has been proposed and a simple method of determining whether the feedback system under the given switching low is stable or not has been shown. Moreover, for some limited cases, a design method of the stabilizing controllers has been presented.

This paper deals with a synthesis problem of the stabilizing controllers for the chained systems via the time-state

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control form and proposes a new multirate sampled data control system based on the periodic switching control low as 3). The key idea is the use of a coordinate transformation to transform the chained system discretized with a zero order hold and a sampler into a linear time-invariant discrete-time system. Moreover a simple necessary and sufficient condition is derived to assure the controllability of the transformed linear discrete-time system. As a result, the following contributions are obtained. First, the problem of finding a controller to stabilize the chained systems is reduced to the well-known pole placement problem. Accordingly, a simple design method of the stabilizing controllers is obtained in an explicit form. Secondly, in order to improve the control performance, a quadratic regulator problem of finding a controller to minimize a cost functional consisting of the state and the control input is posed and is solved. This minimization problem is reduced to a linear quadratic regulator (LQR) problem, and a simple design method of the optimal feedback gain is presented. Finally, a simulation for a two-wheeled vehicle demonstrates the effectiveness of the proposed method.

2. Chained form and Time-state control form

The class of nonholonomic systems to be studied in this paper is described in the following chained form.

$$\frac{d}{dt}\boldsymbol{x}_e=\boldsymbol{g}_1(\boldsymbol{x}_e)u_1+\boldsymbol{g}_2u_2, \tag{2.1}$$

where $\boldsymbol{x}_e=[x_0, \cdots, x_n]^T\in\boldsymbol{R}^{n+1}$ is the state and $u_i\in\boldsymbol{R}$, $i=1$, 2 are two control inputs and

$$\boldsymbol{g}_1(\boldsymbol{x}_e)=[1, 0, x_1, \cdots x_{n-1}]^T\in\boldsymbol{R}^{n+1}$$
$$\boldsymbol{g}_2=[0, 1, 0 \cdots 0]^T\in\boldsymbol{R}^{n+1}.$$

Consider the following control input transformation.

$$\mu=u_1,\ u=u_2/u_1. \tag{2.2}$$

Then the chained system of eq. (2.1) can be represented in the following form called time-state control form[16];

$$\frac{d}{dt}x_0=\mu \tag{2.3}$$

and

$$\frac{d}{dx_0}\boldsymbol{x}=A_1\boldsymbol{x}+\boldsymbol{b}_1u, \tag{2.4}$$

where

$$\boldsymbol{x}=\begin{bmatrix} x_n \\ \vdots \\ x_2 \\ x_1 \end{bmatrix}\in\boldsymbol{R}^n,\ A_1=\begin{bmatrix} 0 & 1 & & 0 \\ \vdots & \ddots & \ddots & \\ \vdots & & \ddots & 1 \\ 0 & \cdots & \cdots & 0 \end{bmatrix}\in\boldsymbol{R}^{n\times n},$$

$$\boldsymbol{b}_1=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}\in\boldsymbol{R}^n. \tag{2.5}$$

This form consists of two state equations. Equations (2.3) and (2.4) are called the time control part and the state control part, respectively. Note that this state control part consists of a controllable linear state equation on the state x_0 instead of the actual time t.

3. Problem formulation

The stability of the chained system is defined as follows.

Definition[3]

The chained system of eq. (2.1) is said to be stable, for each $\varepsilon>0$ and $\boldsymbol{x}_e(0)\in\boldsymbol{R}^{n+1}$, if there exists a finite time $t_f(\varepsilon, \boldsymbol{x}_e(0))\geq 0$ such that

$$|x_0(t)|+\|\boldsymbol{x}(t)\|<\varepsilon,\ \forall t>t_f(\varepsilon, \boldsymbol{x}_e(0)), \tag{3.1}$$

where $\|\ \|$ denotes the Euclidean norm.

Note that the state $x_0(t)$ in the time control part of eq. (2.3) depends on only $\mu(t)$, and is independent of the remaining state $\boldsymbol{x}(t)$ and another control input $u(t)$. In this paper, the following control strategy based on a periodic switching of $\mu(t)$ similar to 3) is proposed to stabilize the chained system of eq. (2.1).

Control strategy

(Step 1) Set $d_1, d_2\in\boldsymbol{R}$ such that $d_1\leq 0\leq d_2$ and $d_1\neq d_2$. If $x_0(0)\notin[d_1, d_2]$, then achieve that $x_0(t_0)\in[d_1, d_2]$ by an appropriate control input $\mu(t)$, $0\leq t\leq t_0$.

(Step 2) Switch periodically the sign of the control input $\mu(t)$ such that $x_0(t)$ goes forward and backward between d_1 and d_2. In other words, when x_0 arrives at d_1, switch the sign of $\mu(t)$ to positive, and when x_0 arrives at d_2, switch the sign to negative. Under such a periodic switching, by using another control input $u(t)$, $t_0\leq t\leq t_f$, achieve that

$$|x_0(t_f)|+\|\boldsymbol{x}(t_f)\|<\varepsilon. \tag{3.2}$$

(Step 3) Set $u(t)=\mu(t)=0$, $t>t_f$. Then the procedure to stabilize the chained system is completed.

It is straightforward to determine the control input $\mu(t)$, $0\leq t\leq t_0$ satisfying Step 1 from eq. (2.3). In this paper, we will focus on Step 2. Without loss of generality, let $t_0=0$ and $x_0(0)=d_1$. For sake of simplicity, the following switching law on $\mu(t)$ is utilized to satisfy Step 2 as shown in **Fig. 1**.

$$\mu(t)=\begin{cases} \mu>0 & (2iT\leq t<(2i+1)T) \\ -\mu<0 & ((2i+1)T\leq t<(2i+2)T) \end{cases} \tag{3.3}$$

for $i=0, 1, 2, \cdots$, where

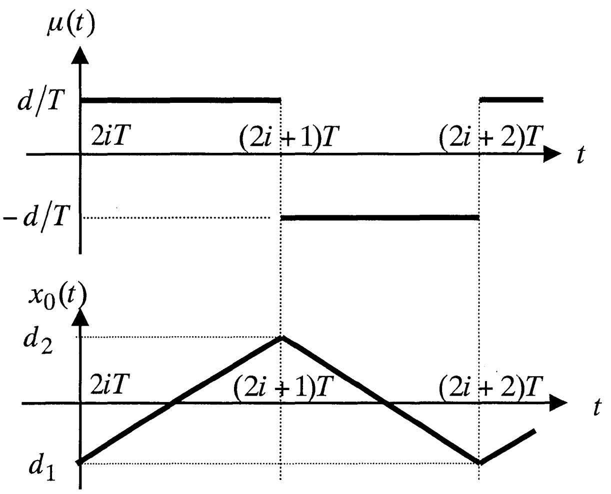

Fig. 1 Control input $\mu(t)$ and state $x_0(t)$

$$\mu=\frac{d}{T},\ d=d_2-d_1. \tag{3.4}$$

Then the response of $x_0(t)$ is given by

$$x_0(t)=\begin{cases} d_1+(t-2iT)\mu \\ \quad (2iT\leq t<(2i+1)T) \\ d_2-(t-(2i+1)T)\mu \\ \quad ((2i+1)T\leq t<(2i+2)T) \end{cases} \tag{3.5}$$

for $i=0, 1, 2 \cdots$. Let

$$\tau=\mu t. \tag{3.6}$$

Then the system of eq.(2.4) is represented in a piecewise linear system as follows:

$$\frac{d}{d\tau}\boldsymbol{x}=\begin{cases} A_1\boldsymbol{x}+\boldsymbol{b}_1u & (2iT\leq t<(2i+1)T) \\ A_2\boldsymbol{x}+\boldsymbol{b}_2u & ((2i+1)T\leq t<(2i+2)T) \end{cases} \tag{3.7}$$

for $i=0, 1, 2 \cdots$, where

$$A_2=-A_1,\ \boldsymbol{b}_2=-\boldsymbol{b}_1. \tag{3.8}$$

The main objective of the remaining part in this paper is to present the control input $u(t)$ so as to achieve Step 2. **Fig. 2** shows the multirate sampled data feedback control system proposed in this paper. The sampling period is set to the switching one of $\mu(t)$, i.e., $T\in R$ in eq.(3.3). S_T is a sampler with the sampling period T, and the state in the state control part of eq.(2.4) is sampled at the sampling instants. $H_{T/m}$ is a zero order hold with the segmented sampling period T/m, where this m is a natural number to be determined. As mentioned later, it is interesting that the controllability of the chained system discretized with the zero order hold and the sampler depends on this natural number. The section 4 will give how to determine this natural number to assure the controllability. In the controller, the control input sequences, $u_j[i]$, at $t=iT+(j-1)T/m\in\boldsymbol{R}$, i.e., at the segmented sampling instants, are determined from the sampled state, $\boldsymbol{x}(iT)$, at $t=iT\in\boldsymbol{R}$, i.e., at the sampling instants. Then the control input

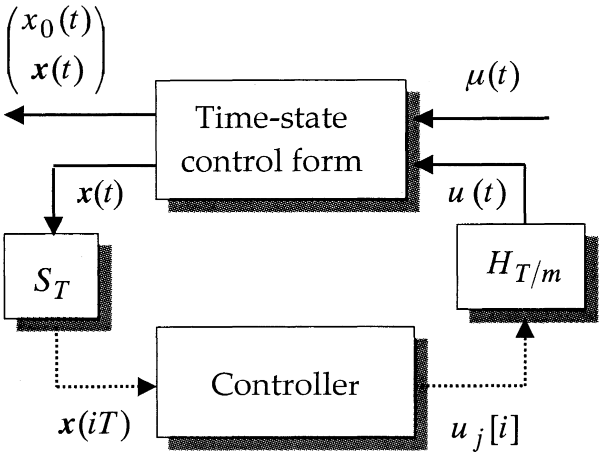

Fig. 2 Multirate sampled data feedback control system for time-state control form

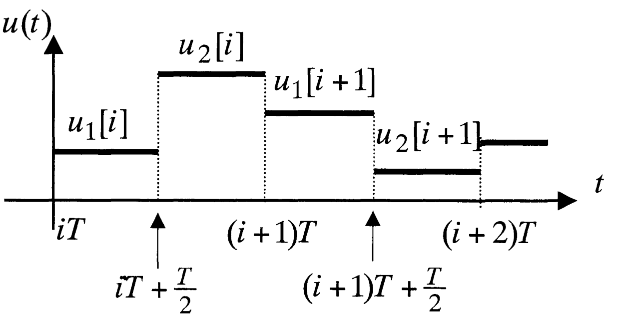

Fig. 3 Control input $u(t)$ for the case of $m=2$

$u(t)$ is given by

$$u(t)=u_j[i]\quad (iT+(j-1)T/m\leq t<iT+jT/m) \tag{3.9}$$

for $j=1\cdots m$, $i=0, 1, 2\cdots$. **Fig. 3** shows the response of the control input $u(t)$ in the proposed multirate sampled data control system for the case of $m=2$.

In Section 4, the problem of finding a controller to stabilize the chained system is solved under the proposed multirate sampled data control system. In Section 5, the following optimal regulator problem is solved in order to improve the responses of the state $\boldsymbol{x}(t)$ and the control input $u(t)$.

Optimal Regulator Problem

Find the optimal control input $u(t)$ of eq.(3.9) not only to stabilize the chained system but also to minimize the following cost functional;

$$J=\int_0^\infty \boldsymbol{x}(t)^T Q_c\boldsymbol{x}(t)+r_c u(t)^2 dt, \tag{3.10}$$

where $Q_c\in\boldsymbol{R}^{n\times n}$ and $r_c\in\boldsymbol{R}$ are given such that

$$Q_c=\mathrm{diag}(q_i,\ i=1\cdots n)>0,\ r_c>0. \tag{3.11}$$

Remark: Note that t in eq.(3.10) means actual time, which is not x_0 nor τ. It is interesting that, from eqs.(2.2) and (3.3), the cost functional of eq.(3.10) can be represented in the following form.

$$J=\int_0^\infty \boldsymbol{x}(t)^T Q_c \boldsymbol{x}(t)+r_c' u_2(t)^2 dt, \tag{3.12}$$

where

$$r_c'=\frac{r_c}{\mu^2}>0. \tag{3.13}$$

From the viewpoint of the chained form of eq.(2.1), the optimal regulator problem mentioned above means the optimal one that minimizes the cost functional of eq.(3.12) consisting of the state $\boldsymbol{x}(t)$ and the control input $u_2(t)$ under a given $u_1(t)$ of eq.(3.3). Accordingly, this problem provides the optimal transient responses of the state $\boldsymbol{x}(t)$ and the control input $u_2(t)$ for the chained system of eq.(2.1)

4. Design of stabilizing controllers

The purpose of this section is to present controllers to stabilize the chained system. From eqs.(3.6) and (3.7), under the proposed multirate sampled data control system, the response of the state of the chained system can be expressed as follows.

$$\boldsymbol{x}(t)=\begin{cases} e^{A_1(\mu t-id)}\boldsymbol{x}(iT)+H(t,i,j)\boldsymbol{u}[i] \\ (i=0,2,4\cdots 2l,\cdots) \\ e^{A_2(\mu t-id)}\boldsymbol{x}(iT)+\hat{H}(t,i,j)\boldsymbol{u}[i] \\ (i=1,3,5\cdots 2l+1,\cdots) \end{cases}, \tag{4.1}$$

$$(iT+(j-1)T/m\le t\le iT+jT/m),\ j=1\cdots m,\ i=0,1\cdots,$$

where

$$\begin{aligned} &\boldsymbol{u}[i]=[u_j[i],\ j=1\cdots m]^T\in\boldsymbol{R}^m \\ &H(t,i,j)=[\boldsymbol{h}(^{\mu t-id}_{\mu t-id-d/m}),\cdots,\boldsymbol{h}(^{\mu t-id-(j-1)d/m}_{0}),0\cdots 0] \\ &\qquad\in\boldsymbol{R}^{n\times m} \\ &\boldsymbol{h}(^{t_1}_{t_2})=\int_{t_2}^{t_1} e^{A_1\sigma}\boldsymbol{b}_1 d\sigma\in\boldsymbol{R}^n \\ &\hat{H}(t,i,j)=[\hat{\boldsymbol{h}}(^{\mu t-id}_{\mu t-id-d/m}),\cdots,\hat{\boldsymbol{h}}(^{\mu t-id-(j-1)d/m}_{0}),0\cdots 0] \\ &\qquad\in\boldsymbol{R}^{n\times m} \\ &\hat{\boldsymbol{h}}(^{t_1}_{t_2})=\int_{t_2}^{t_1} e^{A_2\sigma}\boldsymbol{b}_2 d\sigma\in\boldsymbol{R}^n. \end{aligned} \tag{4.2}$$

It is easy to check that the relationship between $(A_1, \boldsymbol{b}_1)$ and $(A_2, \boldsymbol{b}_2)$ of eq.(3.8) is given as follows.

$$A_2=E_nA_1E_n,\ \ \boldsymbol{b}_2=(-1)^n E_n\boldsymbol{b}_1, \tag{4.3}$$

where

$$E_n=\mathrm{diag}((-1)^{i-1},\ i=1\cdots n)\in\boldsymbol{R}^{n\times n}. \tag{4.4}$$

Using this relationship, the response of the state $\boldsymbol{x}(t)$ can be expressed in a single and simple form as follows.

$$\begin{aligned} &\boldsymbol{x}(t)=E_n^i e^{A_1(\mu t-id)}E_n^i\boldsymbol{x}(iT)+E_n^iH(t,i,j)(-1)^{ni}\boldsymbol{u}[i], \\ &(iT+(j-1)T/m\le t\le iT+jT/m),\ j=1\cdots m,\ i=0,1\cdots, \end{aligned} \tag{4.5}$$

Especially, at the switching instants, we obtain a linear time-varying discrete-time state equation as follows;

$$\begin{aligned} &\boldsymbol{x}((i+1)T)=A(i)\boldsymbol{x}(iT)+B(i)\boldsymbol{u}[i], \\ &A(i)=E_n^i e^{A_1 d}E_n^i,\ B(i)=E_n^i[\boldsymbol{h}(^{d}_{d-d/m}),\cdots,\boldsymbol{h}(^{d/m}_{0})](-1)^{ni}. \end{aligned} \tag{4.6}$$

We introduce a coordinate transformation as follows;

$$\bar{\boldsymbol{x}}(iT)=E_n^i\boldsymbol{x}(iT),\ \ \bar{\boldsymbol{u}}[i]=(-1)^{ni}\boldsymbol{u}[i]\in\boldsymbol{R}^m. \tag{4.7}$$

This transforms time-varying discrete-time state equation of eq.(4.6) into a time-invariant one as shown in the following lemma, which plays an important role on designing the stabilizing controllers in this paper.

Lemma 4.1

Under the proposed multirate sampled data control system, the state sequences $\boldsymbol{x}(iT)$ of the chained system at the switching instants can be expressed in the following linear, time-invariant, discrete-time state equation with m inputs;

$$\bar{\boldsymbol{x}}((i+1)T)=\bar{A}_n\bar{\boldsymbol{x}}(iT)+\bar{B}_{n,m}\bar{\boldsymbol{u}}[i] \tag{4.8}$$

and the following coordinate transformation;

$$\boldsymbol{x}(iT)=E_n^i\bar{\boldsymbol{x}}(iT), \tag{4.9}$$

where $\bar{u}$ are given by eq.(4.7), and

$$\begin{aligned} &\bar{A}_n=E_ne^{A_1d}\in\boldsymbol{R}^{n\times n} \\ &\bar{B}_{n,m}=E_n[\boldsymbol{h}(^{d}_{d-d/m}),\cdots,\boldsymbol{h}(^{d/m}_{0})]\in\boldsymbol{R}^{n\times m}. \end{aligned} \tag{4.10}$$

Proof: The proof is straightforward from eqs.(4.6) and (4.7), and it is omitted. ∎

The following theorem gives how to determine the natural number, m of eq.(3.9), to assure the controllability of the transformed linear discrete-time system in an explicit form.

Theorem 4.2

The discrete-time state equation $(\bar{A}_n, \bar{B}_{n,m})$ of eq.(4.8) is controllable if and only if

$$m\ge n. \tag{4.11}$$

Proof: To begin with, let us show the matrices $\bar{A}_n$ and $\bar{B}_{n,m}$ of eq.(4.10) in an explicit form. From eq.(2.5), a simple calculation of eq.(4.10) can provide the following equations.

$$\begin{aligned} &\bar{A}_n=\begin{bmatrix} 1 & \frac{d}{1!} & \frac{d^2}{2!} & \cdots & \frac{d^{n-1}}{(n-1)!} \\ 0 & -1 & -\frac{d}{1!} & \cdots & -\frac{d^{n-2}}{(n-2)!} \\ \vdots & \ddots & (-1)^2 & & \vdots \\ \vdots & & \ddots & \ddots & \frac{(-1)^{n-2}d}{1!} \\ 0 & \cdots & \cdots & 0 & (-1)^{n-1} \end{bmatrix}\in\boldsymbol{R}^{n\times n}, \\ &\bar{B}_{n,m}=[\bar{\boldsymbol{b}}_{n,m}(l),\ l=1,\cdots,m]\in\boldsymbol{R}^{n\times m}, \\ &\bar{\boldsymbol{b}}_{n,m}(l)=\begin{bmatrix} \bar{b}_{n,m}(1,l) \\ \vdots \\ \bar{b}_{n,m}(n,l) \end{bmatrix}\in\boldsymbol{R}^n, \\ &\bar{b}_{n,m}(j,l)=\frac{(-1)^{j-1}}{(n-j+1)!}\left[\left\{\left(1-\frac{l-1}{m}\right)d\right\}^{n-j+1}\right. \\ &\qquad\left.-\left\{\left(1-\frac{l}{m}\right)d\right\}^{n-j+1}\right]. \end{aligned} \tag{4.12}$$

Note that $\bar{B}_{n,m}$ can be expressed in the following form.

$$\bar{B}_{n,m}=X_1 V_{n,m} X_2^{-1}, \tag{4.13}$$

where

$$X_1=\mathrm{diag}\left\{\frac{(-1)^{j-1}}{(n-j+1)!}, j=1,\cdots,n\right\}\in \boldsymbol{R}^{n\times n}$$

$$X_2=\begin{bmatrix}1 & & 0\\ \vdots & \ddots & \\ 1 & \cdots & 1\end{bmatrix}\in \boldsymbol{R}^{m\times m} \tag{4.14}$$

$$V_{n,m}=\begin{bmatrix}(m\nu)^n & \cdots & (2\nu)^n & \nu^n\\ \vdots & & \vdots & \vdots\\ (m\nu)^2 & \cdots & (2\nu)^2 & \nu^2\\ m\nu & \cdots & 2\nu & \nu\end{bmatrix}\in \boldsymbol{R}^{n\times m},\ \nu=\frac{d}{m}.$$

(Proof of the part of if)

Suppose that $m\geq n$. Since $V_{n,m}$ is a Vandamonde matrix and both X_1 and X_2 are nonsingular matrix, it follows that

$$\mathrm{rank}\,\bar{B}_{n,m}=n, \tag{4.15}$$

Accordingly,

$$\mathrm{rank}(\bar{A}_n-zI_n,\ \bar{B}_{n,m})=n\quad \forall z\in \boldsymbol{C}. \tag{4.16}$$

This means that the discrete-time state equation of $(\bar{A}_n, \bar{B}_{nm})$ is controllable.

(Proof of the part of only if)

We will show that if $m=n-1$, then the discrete-time state equation of $(\bar{A}_n, \bar{B}_{nm})$ is not controllable.

Suppose that $m=n-1$. Note that, from eq. (4.12), $\bar{A}_n$ and $\bar{B}_{n,n-1}$ can be decomposed as follows.

$$\bar{A}_n=\begin{bmatrix}1 & \bar{\alpha}_n\\ 0 & -\bar{A}_{n-1}\end{bmatrix},$$

$$\bar{\alpha}_n=\left[\frac{1}{1!}d\ \ \frac{1}{2!}d^2\ \cdots\ \frac{1}{(n-1)!}d^{n-1}\right]\in \boldsymbol{R}^{n-1},$$

$$\bar{B}_{n,n-1}=\begin{bmatrix}\bar{\beta}_n\\ -\bar{B}_{n-1,n-1}\end{bmatrix}, \tag{4.17}$$

$$\bar{\beta}_n=[\,\bar{b}_{n,n-1}(1,1)\ \cdots\ \bar{b}_{n,n-1}(1,n-1)]\in \boldsymbol{R}^{n-1}.$$

From eq. (4.13), $\bar{B}_{n-1,n-1}\in \boldsymbol{R}^{(n-1)\times(n-1)}$ is nonsingular. From eq. (A. 1) of Lemma A. 1 in Appendix A, it follows that

$$\begin{aligned}&[\bar{A}_n-zI_n,\ \bar{B}_{n,n-1}]\\ &=\begin{bmatrix}1 & 0\\ 0 & -\bar{B}_{n-1,n-1}\end{bmatrix}\begin{bmatrix}1-z & \bar{\alpha}_n\bar{B}_{n-1,n-1} & \bar{\beta}_n\\ 0 & J_{n-1}+zI_{n-1} & I_{n-1}\end{bmatrix}\\ &\cdot\begin{bmatrix}1 & 0 & 0\\ 0 & \bar{B}_{n-1,n-1}^{-1} & 0\\ 0 & 0 & I_{n-1}\end{bmatrix},\end{aligned} \tag{4.18}$$

where

$$J_{n-1}=(-1)^{n-2}\begin{bmatrix}0 & & 1\\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0\end{bmatrix}\in \boldsymbol{R}^{(n-1)\times(n-1)}. \tag{4.19}$$

Accordingly,

$$\mathrm{rank}[\bar{A}_n-zI_n,\ \bar{B}_{n,n-1}]_{z=1}=\mathrm{rank}\begin{bmatrix}0 & \bar{\alpha}_n\bar{B}_{n-1,n-1} & \bar{\beta}_n\\ 0 & J_{n-1}+I_{n-1} & I_{n-1}\end{bmatrix}. \tag{4.20}$$

Since

$$\bar{\alpha}_n\bar{B}_{n-1,n-1}=\bar{\beta}_n(J_{n-1}+I_{n-1}), \tag{4.21}$$

from eq. (A. 2) of Lemma A. 1, we have the following equation,

$$\begin{aligned}&\mathrm{rank}[\bar{A}_n-zI_n,\ \bar{B}_{n,n-1}]_{z=1}\\ &=\mathrm{rank}\begin{bmatrix}0 & 0 & \bar{\beta}_n\\ 0 & 0 & I_{n-1}\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & I_{n-1} & 0\\ 0 & J_{n-1}+I_{n-1} & I_{n-1}\end{bmatrix}\\ &=n-1.\end{aligned} \tag{4.22}$$

This means that the discrete-time state equation of $(\bar{A}_n, \bar{B}_{n,n-1})$ is not controllable.

The proof for the case of $m<n-1$ is similar, and it is omitted. The proof is completed. ∎

Consider the following state feedback controller in the proposed multirate sampled data control system.

$$u_j[i]=\boldsymbol{k}_j[i]\boldsymbol{x}(iT),\ j=1\cdots m,\ i=0,1,\cdots, \tag{4.23}$$

where $\boldsymbol{k}_j[i]\in \boldsymbol{R}^n$, $j=1\cdots m$ are state feedback gains to be designed. By using the coordinate transformation of eq. (4.7), it follows that

$$\bar{\boldsymbol{u}}[i]=(-1)^{ni}\begin{bmatrix}\boldsymbol{k}_1[i]\\ \vdots\\ \boldsymbol{k}_m[i]\end{bmatrix}E_n^i\bar{\boldsymbol{x}}(iT), \tag{4.24}$$

Therefore, from eq. (4.8), the state sequences $\bar{\boldsymbol{x}}(iT)$ of the chained system at the switching instants can be expressed as

$$\bar{\boldsymbol{x}}((i+1)T)=(\bar{A}_n+\bar{B}_{n,m}K[i])\bar{\boldsymbol{x}}(iT), \tag{4.25}$$

where

$$K[i]=(-1)^{ni}\begin{bmatrix}\boldsymbol{k}_1[i]\\ \vdots\\ \boldsymbol{k}_m[i]\end{bmatrix}E_n^i\in \boldsymbol{R}^{m\times n}. \tag{4.26}$$

Consequently, the feedback stabilization problem for the chained system is reduced to the problem of finding $K[i]\in \boldsymbol{R}^{m\times n}$ to stabilize the linear discrete-time state equation of eq. (4.25). Many conventional linear control theories can provide the solutions of this reduced controller synthesis problem. The following corollary presents a simple and explicit design method of the stabilizing controller.

Corollary 4. 3

For $m\geq n$, the state feedback gains $\boldsymbol{k}_j[i]\in \boldsymbol{R}^n$, $j=1\cdots m$ of

$$\begin{bmatrix}\boldsymbol{k}_1[i]\\ \vdots\\ \boldsymbol{k}_m[i]\end{bmatrix}=(-1)^{ni}KE_n^i\in \boldsymbol{R}^{m\times n}, \tag{4.27}$$

stabilize the chained system under the proposed multirate sampled data control system, where

$$K=\bar{B}_{n,m}^{+}(\bar{A}_s-\bar{A}_n)+\bar{B}_{n,m}^{-}F\in \boldsymbol{R}^{m\times n}, \tag{4.28}$$

$\bar{B}_{n,m}^{+}$, $\bar{B}_{n,m}^{-}\in \boldsymbol{R}^{m\times n}$ are matrices such that

$$\bar{B}_{n,m}\bar{B}_{n,m}^{+}=I_n \tag{4.29}$$

and

$$\bar{B}_{n,m}\bar{B}_{n,m}^{-}=\boldsymbol{0}_{n\times n}, \tag{4.30}$$

respectively. $\bar{A}_s\in \boldsymbol{R}^{n\times n}$ is a matrix such that the absolutes of all eigenvalues are less than one, i.e.,

$$|\lambda_j(\bar{A}_s)|<1,\ j=1,\cdots,n, \tag{4.31}$$

and $F\in\boldsymbol{R}^{n\times n}$ is arbitrary.

Proof: From eq.(4.25), $K[i]\in\boldsymbol{R}^{m\times n}$ satisfying

$$\bar{A}_n+\bar{B}_{n,m}K[i]=\bar{A}_s, \tag{4.32}$$

for an $\bar{A}_s\in\boldsymbol{R}^{n\times n}$ of eq.(4.31) provides that

$$\lim_{i\to\infty}\|\bar{\boldsymbol{x}}(iT)\|=0. \tag{4.33}$$

Then, from eq.(4.7), it follows that

$$\lim_{i\to\infty}\|\boldsymbol{x}(iT)\|=0. \tag{4.34}$$

Moreover, from eq.(4.5), since

$$\begin{aligned}&\boldsymbol{x}(t)=E_n^i(e^{A_1(\mu t-id)}+H(t,i,j)K[i])\bar{\boldsymbol{x}}(iT)\\&(iT+(j-1)T/m\leq t\leq iT+jT/m),\end{aligned} \tag{4.35}$$

the response of $\boldsymbol{x}(t)$ between the switching periods also converges to zero. Consequently, a $K[i]\in\boldsymbol{R}^{m\times n}$ satisfying eq.(4.32) stabilizes the chained system. Since $\bar{B}_{n,m}$ is row full rank from eq.(4.15), there exist $\bar{B}^{+}_{n,m}, \bar{B}^{-}_{n,m}\in\boldsymbol{R}^{m\times n}$ satisfying eqs.(4.29) and (4.30), respectively. All $K[i]\in\boldsymbol{R}^{m\times n}$ satisfying eq.(4.32) is represented by $K\in\boldsymbol{R}^{m\times n}$ of eq.(4.28). Then, from eq.(4.26), the state feedback gain, $\boldsymbol{k}_j[i]\in\boldsymbol{R}^n$, is obtained by eq.(4.27). The proof is completed. ∎

Remark: Note that the eigenvalues of $\bar{A}_s\in\boldsymbol{R}^{n\times n}$ in eq.(4.28) is equal to the poles of the discrete-time state equation of eq.(4.25) on the state sequence $\bar{\boldsymbol{x}}(iT)$ at the switching instants. From eq.(4.7),

$$\|\bar{\boldsymbol{x}}(iT)\|=\|\boldsymbol{x}(iT)\|. \tag{4.36}$$

Consequently, by assignment of the eigenvalues, the convergent rate of $\|\boldsymbol{x}(iT)\|$ of the state in the chained system at the switching instants can be adjusted arbitrarily.

5. Design of optimal regulator

The purpose of this section is to present the solution of the optimal regulator problem mentioned above.

Lemma 5.1

The cost functional of eq.(3.10) is represented in the following form consisting of $\bar{\boldsymbol{x}}(iT)$ and $\bar{\boldsymbol{u}}[i]$.

$$J=\sum_{i=0}^{\infty}\bar{\boldsymbol{x}}(iT)^T\bar{Q}\bar{\boldsymbol{x}}(iT)+2\bar{\boldsymbol{x}}(iT)^T\bar{S}\bar{\boldsymbol{u}}[i]+\bar{\boldsymbol{u}}[i]^T\bar{\boldsymbol{R}}\bar{\boldsymbol{u}}[i], \tag{5.1}$$

where $\bar{Q}\in\boldsymbol{R}^{n\times n}$, $\bar{S}\in\boldsymbol{R}^{n\times m}$ and $\bar{R}\in\boldsymbol{R}^{m\times m}$ are matrices with real entries and are given by.

$$\begin{aligned}\bar{Q}&=\int_0^T(e^{A_1\mu\sigma})^TQ_c\,e^{A_1\mu\sigma}d\sigma\in\boldsymbol{R}^{n\times n}\\ \bar{S}&=\sum_{j=1}^{m}\int_{(j-1)\frac{T}{m}}^{j\frac{T}{m}}(e^{A_1\mu\sigma})^TQ_cH(\sigma,0,j)d\sigma\in\boldsymbol{R}^{n\times m}\\ \bar{R}&=\sum_{j=1}^{m}\int_{(j-1)\frac{T}{m}}^{j\frac{T}{m}}(H(\sigma,0,j))^TQ_cH(\sigma,0,j)d\sigma\\&\quad+\mathrm{diag}\left\{\frac{Tr_c}{m},\cdots,\frac{Tr_c}{m}\right\}\in\boldsymbol{R}^{m\times m}.\end{aligned} \tag{5.2}$$

Proof: Note that

$$(E_n^i)^TQ_cE_n^i=Q_c. \tag{5.3}$$

By substituting eq.(4.5) into the cost functional of eq.(3.10), the proof is straightforward. ∎

Remark: From Lemmas 4.1 and 5.1, the optimal regulator problem mentioned above is reduced to the problem of finding $\bar{\boldsymbol{u}}[i]\in\boldsymbol{R}^m$ that minimizes the cost functional of eq.(5.1) under the linear, time-invariant, discrete-time state equation of eq.(4.8). Note that this reduced problem is the standard *discrete-time LQR problem*, since $\bar{Q}\in\boldsymbol{R}^{n\times n}$, $\bar{S}\in\boldsymbol{R}^{n\times m}$ and $\bar{R}\in\boldsymbol{R}^{m\times m}$ are constant matrices. The following theorem gives the solution of this problem.

Theorem 5.2

For $m\geq n$, the optimal control input $u(t)$ of eq.(3.9) that minimizes the cost functional of eq.(3.10) is obtained in the state feedback controller form as follows,

$$u_j[i]=\boldsymbol{k}_j[i]\boldsymbol{x}(iT),\ j=1\cdots m,\ i=0,1,\cdots, \tag{5.4}$$

where $\boldsymbol{k}_j[i]\in\boldsymbol{R}^n$, $j=1\cdots m$ are given by

$$\begin{bmatrix}\boldsymbol{k}_1[i]\\ \vdots\\ \boldsymbol{k}_m[i]\end{bmatrix}=-(-1)^{ni}\bar{K}_{opt}E_n^i\in\boldsymbol{R}^{m\times n} \tag{5.5}$$

$$\bar{K}_{opt}=(\bar{R}+\bar{B}_{n,m}{}^TP\bar{B}_{n,m})^{-1}(\bar{B}_{n,m}{}^TP\bar{A}_n+\bar{S}^T)\in\boldsymbol{R}^{m\times n}, \tag{5.6}$$

$P\in\boldsymbol{R}^{n\times n}$ is unique positive definite matrix satisfying the following Riccati equation,

$$P=\hat{A}_n^TP\hat{A}_n+\hat{Q}-\hat{A}_n^TP\bar{B}_{n,m}(\bar{R}+\bar{B}_{n,m}{}^TP\bar{B}_{n,m})^{-1}\bar{B}_{n,m}{}^TP\hat{A}_n \tag{5.7}$$

and

$$\hat{A}_n=\bar{A}_n-\bar{B}_{n,m}\bar{R}^{-1}\bar{S}^T \tag{5.8}$$

$$\hat{Q}=\bar{Q}-\bar{S}\bar{R}^{-1}\bar{S}^T. \tag{5.9}$$

Moreover, the state feedback controller of eq.(5.4) stabilizes the chained system.

Proof: Note that $(\hat{A}_n,\bar{B}_{n,m})$ is controllable from Theorem 4.2 and $(\hat{Q}^{1/2},\hat{A}_n)$ is observable from eq.(3.11) and Appendix B. From 4) and 9), $\bar{\boldsymbol{u}}[i]\in\boldsymbol{R}^m$ that minimizes the cost functional of eq.(5.1) under the linear, time-invariant, discrete-time state equation of eq.(4.8) is obtained by

$$\bar{\boldsymbol{u}}[i]=-\bar{K}_{opt}\,\bar{\boldsymbol{x}}(iT), \tag{5.10}$$

where $\bar{K}_{opt}\in\boldsymbol{R}^{m\times n}$ is given by eq.(5.6). By using a coordinate transformation of eq.(4.7), eq.(5.10) is expressed by

$$\boldsymbol{u}[i]=-(-1)^{ni}\bar{K}_{opt}E_n^i\boldsymbol{x}(iT) \tag{5.11}$$

From eq.(5.5), we obtain eq.(5.4). The proof is completed. ∎

6. Numerical example

In this section, we will show simulation results where the proposed control strategy is applied to the control problem for a 2-wheeled vehicle system as shown in **Fig. 4**. As usual, we assume that the wheels of this vehicle are all-

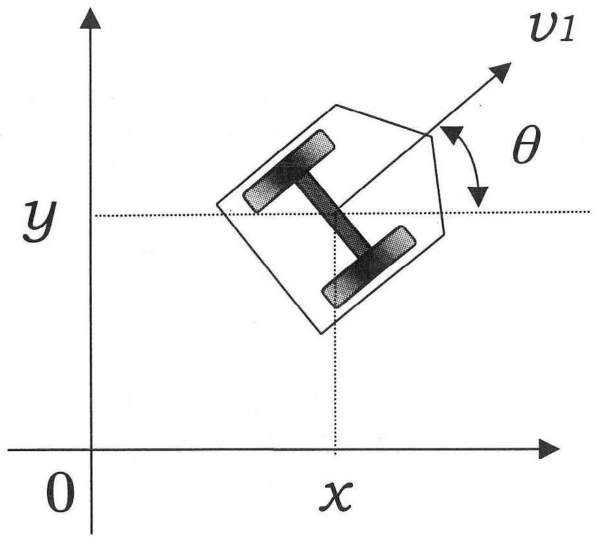

Fig. 4 Coordinates of 2-wheeled vehicle

owed to roll and spin but not slip. Under this assumption, the kinematic motion of the vehicle is described as follows[10]:

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}\nu_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\nu_2. \tag{6.1}$$

(x, y) denotes the position of the center of the vehicle and θ is the orientation to x-axis. ν_1 and ν_2 denote the forward velocity input and the angular velocity one, i.e., $\nu_2=\dot{\theta}$, respectively. Under the constraint of $\theta\neq 90°$, by using the input transformations given by

$$\begin{aligned} u_1 &= \mu = \nu_1\cos\theta \\ u_2 &= \frac{\nu_2}{\nu_1(\cos\theta)^3}, \end{aligned} \tag{6.2}$$

the system of eq. (6.1) is transformed into the chained form of eq. (2.1) for $n=2$, where

$$\begin{aligned} x_0 &= x \\ x_1 &= \tan\theta \\ x_2 &= y. \end{aligned} \tag{6.3}$$

The switching conditions on the control input of μ are given by $(d_1, d_2)=(-1, 1)$ and $T=2$. The optimal controllers of eq. (5.4) are designed for three cases of $(m, r_c)=(2, 0.01)$, $(6, 0.01)$ and $(2, 1)$ under $Q_c=\mathrm{diag}(1, 1)$. In all simulations, the initial states of the vehicle are given by $[x(0), y(0), \theta(0)]=[-1, 3, 45°]$.

First, in order to investigate the relationship between m, i.e., the number of the segmented period, and the control performance, we compare two cases of $(m, r_c)=(2, 0.01)$ and $(6, 0.01)$. **Fig. 5** and **6** show the trajectories and the control inputs, respectively. **Fig. 7** and 8 are the responses of the states in the chained form. These figures demonstrate that the proposed control strategy can provide excellent regulations on the states and the control input u_2 to zero, and moreover, as increasing m, the convergence rate of the state is made faster.

Secondly, in order to investigate the relationship between the weighting r_c of the cost functional and the control performance, we compare two cases of $(m, r_c)=(2,$

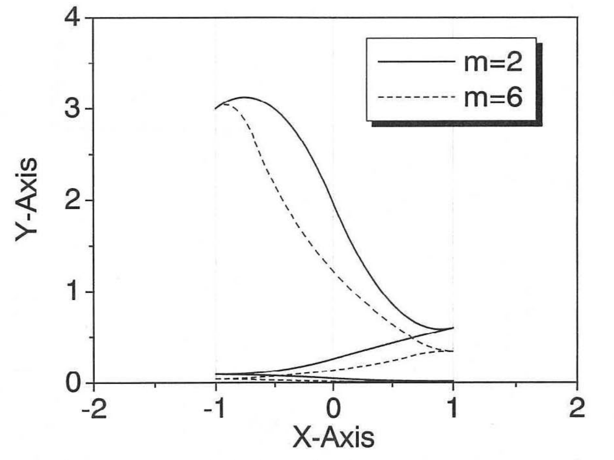

Fig. 5 Trajectories of 2-wheeled vehicle for the cases of $(m, r_c)=(2, 0.01)$ and $(6, 0.01)$

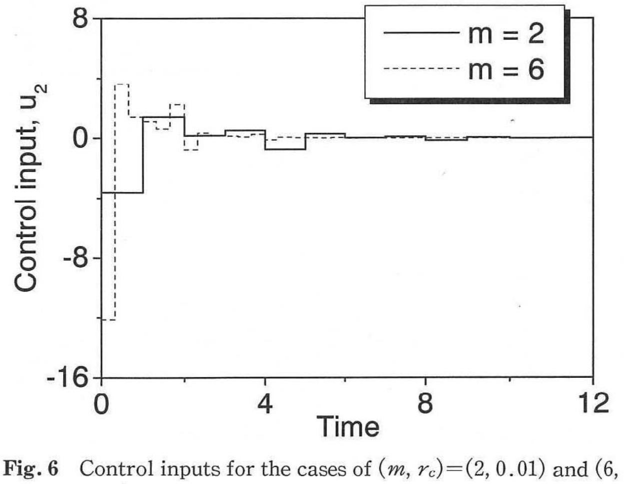

Fig. 6 Control inputs for the cases of $(m, r_c)=(2, 0.01)$ and $(6, 0.01)$

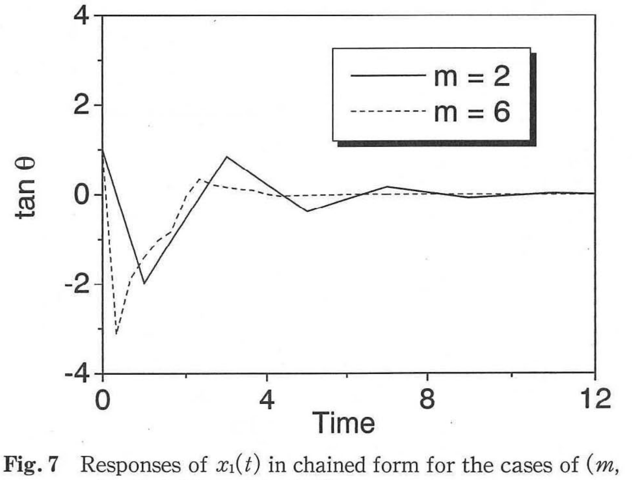

Fig. 7 Responses of $x_1(t)$ in chained form for the cases of $(m, r_c)=(2, 0.01)$ and $(6, 0.01)$

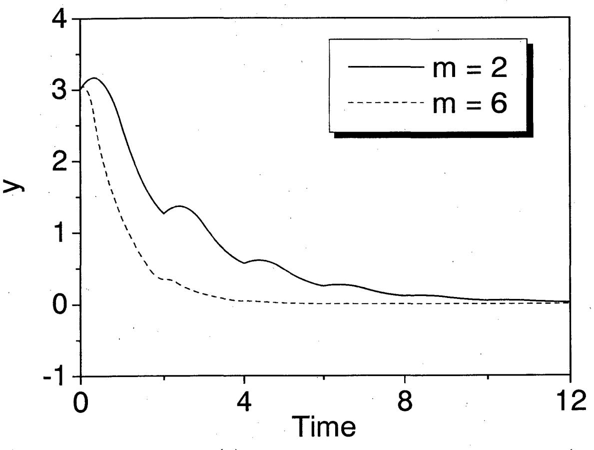

Fig. 8 Responses of $x_2(t)$ in chained form for the cases of $(m, r_c)=(2, 0.01)$ and $(6, 0.01)$

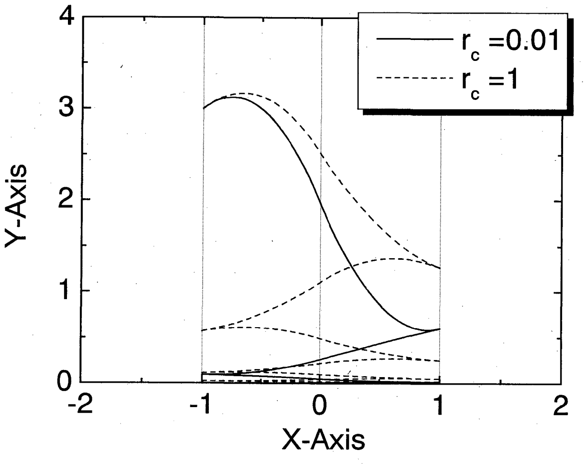

Fig. 9 Trajectories of 2-wheeled vehicle for the cases of $(m, r_c)=(2, 1)$ and $(2, 0.01)$

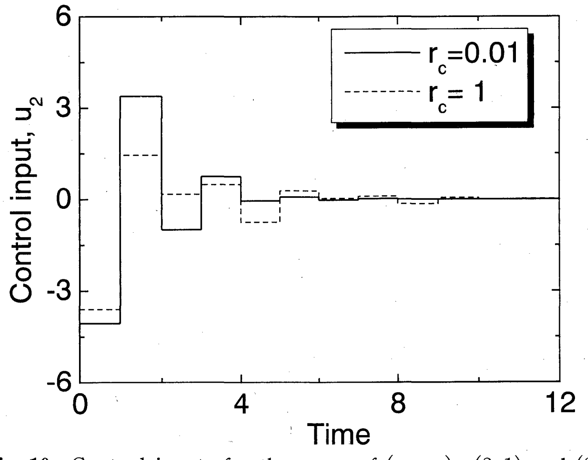

Fig. 10 Control inputs for the cases of $(m, r_c)=(2, 1)$ and $(2, 0.01)$

0.01) and (2, 1). **Fig. 9** and **10** show the trajectories and the control inputs, respectively. These figures demonstrate that as increasing the r_c, the control input u_2 is made smaller, although the convergence rate of the state is made slower.

7. Conclusion

This paper has proposed a new feedback control system based on a multirate sampled data control and a periodic switching for a class of nonholonomic systems in the time-state control form. A simple design method of the optimal regulator has been presented not only to stabilize the system but also to minimize the cost functional of eq. (3.10) consisting of the state and the control input.

The future directions of our research are to make clear the robustness of the proposed multirate sampled data control system and to extend the control strategy to tracking problem. The authors would like to thank reviewers for their valuable comments.

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Appendix A

In this appendix, two equations will be shown used in the proof of Theorem 4.2.

Lemma A. 1

$$\bar{B}_{n,n}^{-1}\bar{A}_n\bar{B}_{n,n}=J_n \tag{A. 1}$$

and

$$\bar{\alpha}_n\bar{B}_{n-1,n-1}=\bar{\beta}_n(J_{n-1}+I_{n-1}), \tag{A. 2}$$

where

$$J_n=(-1)^{n-1}\begin{bmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{bmatrix}\in \boldsymbol{R}^{n\times n}. \tag{A. 3}$$

Proof: For any $p, q\in \boldsymbol{R}$, we have

$$(p-q)^h=\sum_{j=1}^{h}\frac{h!}{j!(h-j)!}p^{h-j}(-q)^j. \tag{A. 4}$$

First, let

$$[\xi(j, l), j, l=1\cdots n]=\bar{A}_n\bar{B}_{n,n}\in \boldsymbol{R}^{n\times n}. \tag{A. 5}$$

By using two equations of eq. (A. 4) substituting

$$(p, q, h)=((1-(l-1)/n)d, d, n-j+1) \tag{A. 6}$$

and

$$(p, q, h)=((1-l/n)d, d, n-j+1), \tag{A. 7}$$

respectively, we obtain the following equation;

$$\begin{aligned}\xi(j, l)=&\sum_{i=0}^{n-j}\frac{(-1)^i}{i!((n-j+1)-i)!}\\ &\cdot\left\{\left(1-\frac{l-1}{n}\right)d\right\}^{(n-j+1)-i}d^i\\ &-\sum_{i=0}^{n-j}\frac{(-1)^i}{i!((n-j+1)-i)!}\left\{\left(1-\frac{l}{n}\right)d\right\}^{(n-j+1)-i}d^i\\ =&\left[\frac{1}{(n-j+1)!}\left\{\left(1-\frac{l-1}{n}\right)d-d\right\}^{n-j+1}\right.\\ &\left.-(-d)^{n-j+1}\right]\\ &-\left[\frac{1}{(n-j+1)!}\left\{\left(1-\frac{l}{n}\right)d-d\right\}^{n-j+1}\right.\\ &\left.-(-d)^{n-j+1}\right]\\ =&(-1)^{n-1}\frac{(-1)^{j-1}}{(n-j+1)!}\left[\left\{\frac{l}{n}d\right\}^{n-j+1}\right.\\ &\left.-\left\{\left(\frac{l-1}{n}\right)d\right\}^{n-j+1}\right]\\ =&(-1)^{n-1}\bar{b}_{n,n}(j, n-l+1).\end{aligned} \tag{A. 8}$$

Therefore

$$\bar{A}_n\bar{b}_{n,n}(l)=(-1)^{n-1}\bar{b}_{n,n}(n-l+1). \tag{A. 9}$$

This can be expressed by

$$\bar{A}_n\bar{B}_{n,n}=\bar{B}_{n,n}J_n. \tag{A. 10}$$

Accordingly, we obtain eq. (A. 1).

Secondly, by using two equations of eq. (A. 4) substituting

$$(p, q, h)=((1-(l-1)/(n-1))d, d, n) \tag{A. 11}$$

and

$$(p, q, h)=((1-l/(n-1))d, d, n), \tag{A. 12}$$

respectively, we obtain the following equation;

$$\begin{aligned}\bar{\alpha}_n\bar{\boldsymbol{b}}_{n-1,n-1}(l)=&\sum_{j=1}^{n-1}\frac{(-1)^{j-1}}{j!(n-j)!}\left\{\left(1-\frac{l-1}{n-1}\right)d\right\}^{n-j}d^j\\ &-\sum_{j=1}^{n-1}\frac{(-1)^{j-1}}{j!(n-j)!}\left\{\left(1-\frac{l}{n-1}\right)d\right\}^{n-j}d^j\\ =&\frac{(-1)^{n-2}}{n!}\left[\left\{\frac{l}{n-1}d\right\}^n-\left\{\frac{l-1}{n-1}d\right\}^n\right]\\ &+\frac{1}{n!}\left[\left\{\left(1-\frac{l-1}{n-1}\right)d\right\}^n\right.\\ &\left.-\left\{\left(1-\frac{l}{n-1}\right)d\right\}^n\right]\\ =&(-1)^{n-2}\bar{b}_{n-1,n-1}(1, n-l)+\bar{b}_{n-1,n-1}(1, l).\end{aligned} \tag{A. 13}$$

This can be expressed by

$$\begin{aligned}\bar{\alpha}_n\bar{B}_{n-1,n-1}=&[(-1)^{n-2}\bar{b}_{n-1,n-1}(1, n-l)\\ &+\bar{b}_{n-1,n-1}(1, l), l=1\cdots, n-1]\\ =&\bar{\beta}_n(J_{n-1}+I_{n-1}).\end{aligned} \tag{A. 14}$$

Consequently, we obtain eq. (A. 2). The proof is completed. ∎

Appendix B

In this appendix, it will be proven that $(\hat{Q}^{1/2}, \hat{A}_n)$ is observable.

Lemma B. 1

$(\hat{Q}^{1/2}, \hat{A}_n)$ is observable.

Proof: Let

$$W=\begin{bmatrix}\bar{Q} & \bar{S}\\ \bar{S}^T & \bar{R}\end{bmatrix}. \tag{B. 1}$$

Since $\bar{R}\in \boldsymbol{R}^{m\times m}$ of eq. (5.2) is positive definite, if the matrix W is positive definite, then the matrix $\hat{Q}$ is so, and then $(\hat{Q}^{1/2}, \hat{A}_n)$ is observable. In the remaining part in this proof, we will show that the matrix W is positive definite.

Suppose that the matrix W is not positive definite. Note that, from eq. (5.2), the matrix W is represented as follows

$$W=\sum_{j=1}^{m}Q_j, \tag{B. 2}$$

where

$$Q_j=\int_{(j-1)\frac{T}{m}}^{j\frac{T}{m}}\begin{bmatrix} e^{A_1\mu\sigma} & H(\sigma,0,j) \\ \boldsymbol{0} & \boldsymbol{e}_j \end{bmatrix}^T \begin{bmatrix} Q_c & 0 \\ 0 & r_c \end{bmatrix}$$

$$\begin{bmatrix} e^{A_1\mu\sigma} & H(\sigma,0,j) \\ \boldsymbol{0} & \boldsymbol{e}_j \end{bmatrix} d\sigma \in \boldsymbol{R}^{(m+n)\times(m+n)}. \qquad \text{(B. 3)}$$

$$\boldsymbol{e}_j=[\underbrace{0\ \cdots\ 0}_{j-1}\ 1\ 0\ \cdots\ 0]\in \boldsymbol{R}^m$$

Since $Q_j \geq 0$, $j=1\cdots m$, there exists a nonzero vector $w\in \boldsymbol{R}^{(m+n)}$ such that $w^T Q_j w=0$, $j=1\cdots m$, which means that

$$\begin{bmatrix} e^{A_1\mu\sigma} & H(\sigma,0,j) \\ \boldsymbol{0} & \boldsymbol{e}_j \end{bmatrix} w=0,\ \forall\sigma\in\left[(j-1)\frac{T}{m},\ j\frac{T}{m}\right],$$

$$j=1\cdots m. \qquad \text{(B. 4)}$$

from eq. (3.11). Let $w^T=[w_1^T, w_2^T]$, where $w_1\in \boldsymbol{R}^n$ and $w_2\in \boldsymbol{R}^m$. From eq. (B. 4), we have $\boldsymbol{e}_j w_2=0$, $j=1\cdots m$, which means that $w_2=0$. Then, it follows that $w_1=0$, which is the contradiction with $w\neq 0$. Accordingly, the matrix W is positive definite. The proof is completed. ∎

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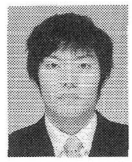

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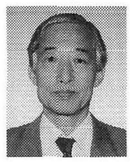

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