Proceedings of the 34th
Conference on Decision & Control
New Orleans, LA - December 1995

FP02 3:50

CONTROL OF A NONHOLONOMIC MOBILE ROBOT: BACKSTEPPING KINEMATICS INTO DYNAMICS

R. Fierro and F. L. Lewis
Automation and Robotics Research Institute. The University of Texas at Arlington
7300 Jack Newell Blvd. South. Fort Worth, Texas 76118-7115
email. rfierro@arri.uta.edu

Abstract

A dynamical extension that makes possible the integration of a kinematic controller and a torque controller for nonholonomic mobile robots is presented. A combined kinematic/torque control law is developed using backstepping and asymptotic stability is guaranteed by Lyapunov theory. Moreover, this control algorithm can be applied to the three basic nonholonomic navigation problems: tracking a reference trajectory, path following and stabilization about a desired posture. A general structure for controlling a mobile robot results that can accommodate different control techniques ranging from a conventional *computed-torque* controller, when all dynamics are known, to adaptive controllers.

1. Introduction

A mobile robot is suitable for a variety of applications in unstructured environments where a high degree of autonomy is required. This desired autonomous or *intelligent* behavior has motivated an intensive research in the last decade. Much research effort has been oriented to solving the problem of motion under nonholonomic constraints using the kinematic model of a mobile robot. On the other hand, less research has been devoted to the problem of integration of the nonholonomic kinematic controller and the dynamics of the mobile platform [9].

The navigation problem may be divided into three basic problems [4]: tracking a reference trajectory, following a path, and point stabilization. Some nonlinear feedback controllers have been proposed in the literature [6], [11] for solving these problems. The main idea behind these algorithms is to define velocity control inputs which stabilize the closed-loop system. All these controllers consider only the kinematic model (*e.g.*, steering system) of the mobile robot, and 'perfect velocity' tracking is assumed to generate the actual vehicle control inputs.

The dynamical extension proposed in this paper provides a rigorous method of taking into account the specific vehicle dynamics to convert a steering system command into control inputs for the actual vehicle. It can be considered as a *backstepping* control approach. First, feedback velocity control inputs are designed for the kinematic steering system to make the position error asymptotically stable. Second, a feedback velocity-following control law is designed such that the mobile robot's velocities converge asymptotically to the given velocity inputs. Finally, this second control signal is used by the computed-torque feedback controller to compute the required torques for the actual mobile robot. This control approach can be applied to a class of *smooth* kinematic system control velocity inputs. Therefore, the same design procedure works for all of the three basic navigation problems mentioned above.

This paper is organized as follows. In Section 2, we present the theoretical background of a nonholonomic mobile robot including an important *skew-symmetry* property. Although our method can be applied to path following and point stabilization, for illustration, the design algorithm for tracking a reference trajectory is presented in Section 3. Section 4 presents some simulation results. Finally, Section 5 gives some concluding remarks.

2. A Nonholonomic Mobile Robot

A mobile robot system having an n-dimensional configuration space $\mathbf{C}$ with generalized coordinates $(q_1,\ldots,q_n)$ and subject to m constraints can be described by

$$\mathbf{M}(q)\ddot{q}+\mathbf{V}_m(q,\dot{q})\dot{q}+\mathbf{F}(\dot{q})+\mathbf{G}(q)+\tau_d=\mathbf{B}(q)\tau-\mathbf{A}^T(q)\lambda, \quad (1)$$

where $\mathbf{M}(q) \in \Re^{n\times n}$ is a symmetric, positive definite inertia matrix, $\mathbf{V}_m(q,\dot{q}) \in \Re^{n\times n}$ is the centripetal and coriolis matrix, $\mathbf{F}(\dot{q}) \in \Re^n$ denotes the surface friction, $\mathbf{G}(q) \in \Re^n$ is the gravitational vector, τ_d denotes bounded unknown disturbances including unstructured unmodelled dynamics, $\mathbf{B}(q) \in \Re^{n\times r}$ is the input transformation matrix, $\tau \in \Re^r$ is the input vector, $\mathbf{A}(q) \in \Re^{m\times n}$ is the matrix associated with the constraints, and $\lambda \in \Re^m$ is the vector of constraint forces.

We consider that all kinematic equality constraints are independent of time, and can be expressed as

$$\mathbf{A}(q)\dot{q} = 0. \tag{2}$$

Let $\mathbf{S}(q)$ be a full rank matrix $(n\text{-}m)$ formed by a set of smooth and linearly independent vector fields spanning the null space of $\mathbf{A}(q)$, *i.e.*,

$$\mathbf{S}^T(q)\mathbf{A}^T(q) = 0. \tag{3}$$

The involutivity properties of the distribution Δ spanned by the vectors of $\mathbf{S}(q)$ are closed related to the nature of the constraints as is pointed out in [2]. According to (2) and (3), it is possible to find an auxiliary vector time function $\nu(t) \in \Re^{n-m}$ such that, for all t

$$\dot{q} = \mathbf{S}(q)\nu(t). \tag{4}$$

2.1 Kinematics and Dynamics of a Mobile Platform

The mobile robot shown in Fig. 1 is a typical example of a nonholonomic mechanical system. It consists of a vehicle with two driving wheels mounted on the same axis, and a front free wheel. The motion and orientation are achieved by independent actuators, *e.g.*, DC motors providing the necessary torques to the rear wheels. Another common configuration uses the front wheel for driving and steering. Basically both configurations share the same control and structural properties [14].

The position of the robot in an inertial Cartesian frame {O, X, Y} is completely specified by the vector $q = [x_c\ y_c\ \theta]^T$ where (x_c, y_c) and θ are the coordinates of the reference point **C**, and the orientation of the basis {C, Xc, Yc} with respect to the inertial basis respectively. Additionally, Fig. 1 shows the geometry of the mobile base that will be used to develop a mathematical model of the vehicle.

A well-known result states that linearization control techniques fail at point **P** [5], *i.e.*, the intersection of the wheel axis and the axis of symmetry. A common solution to this problem is to redefine a new reference point located at a certain distance d from **P**. We choose a fixed point **C**, and use this point as a reference point to develop the mathematical model.

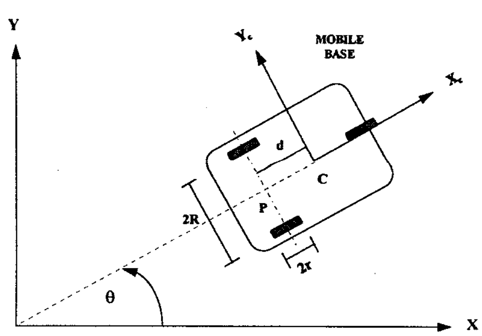


Fig. 1. A nonholonomic mobile platform

The nonholonomic constraint states that the robot can only move in the direction normal to the axis of the driving wheels, *i.e.*, the mobile base satisfies the conditions of *pure rolling and non slipping* [1], [13]

$$\dot{y}_c \cos\theta - \dot{x}_c \sin\theta - d\dot{\theta} = 0. \tag{5}$$

It is easy to verify that $\mathbf{S}(q)$ is given by

$$\mathbf{S}(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}. \tag{6}$$

Using the above expressions is possible to derive the forward kinematics of the mobile base. Forward kinematics is used to estimate positions and velocities in Cartesian space from a set of joint variables. The kinematic equations of motion (4) of **C** in terms of its linear velocity and angular velocity are

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \tag{7}$$

where $|v_1| \le V_{max}$ and $|v_2| \le W_{max}$. V_{max} and W_{max} are the maximum linear and angular velocities of the mobile robot. System (7) is called the *steering system* of the vehicle.

The Lagrange formalism is used to derived the dynamic equations of the mobile robot. In this case $\mathbf{G}(q) = 0$, because the trajectory of the mobile base is constrained to the horizontal plane, *i.e.*, since the system cannot change its vertical position, its potential energy, U, remains constant. The kinetic energy K is given by [8]

$$K = \frac{1}{2}\dot{q}^T \mathbf{M}(q)\dot{q}. \tag{8}$$

The dynamical equations of the mobile base in Fig. 1 can be expressed in the matrix form (1) where

$$\mathbf{M}(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix},$$

$$\mathbf{V}(q,\dot{q}) = \begin{bmatrix} md\dot{\theta}^2\cos\theta \\ md\dot{\theta}^2\sin\theta \\ 0 \end{bmatrix}, \quad \mathbf{B}(q) = \frac{1}{r}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix}, \tag{9}$$

$$\mathbf{G}(q) = \mathbf{0}, \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}, \quad \mathbf{A}^T(q) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ -d \end{bmatrix},$$

$$\lambda = -m(\dot{x}_c\cos\theta + \dot{y}_c\sin\theta)\dot{\theta}.$$

2.2 Structural Properties of a Mobile Platform

The mobile robot dynamics (1) have the following standard properties [8]:

Boundedness: $\mathbf{M}(q)$, the norm of the matrix $\mathbf{V_m}(q,\dot{q})$, and τ_d are bounded.

Skew-symmetry: The matrix $\dot{\mathbf{M}} - 2\mathbf{V_m}$ is skew symmetric. This property is particularly important in the stability analysis of the control system.

The system (1) is now transformed into a more appropriate representation for controls purposes. Differentiating equation (4), substituting this result in (1), and then multiplying by $\mathbf{S}^T$, we can eliminate the constraint matrix $\mathbf{A}^T(q)\lambda$. The complete equations of motion of the nonholonomic mobile platform are given by

$$\dot{q} = \mathbf{S}v, \tag{10}$$

$$\mathbf{S}^T\mathbf{M}\mathbf{S}\dot{v} + \mathbf{S}^T(\mathbf{M}\dot{\mathbf{S}} + \mathbf{V_m}\mathbf{S})v + \overline{\mathbf{F}} + \overline{\tau}_d = \mathbf{S}^T\mathbf{B}\tau, \tag{11}$$

where $v(t) \in \Re^{n-m}$ is a velocity vector. By appropriate definitions we can rewrite equation (11) as follows

$$\overline{\mathbf{M}}(q)\dot{v} + \overline{\mathbf{V}}_\mathbf{m}(q,\dot{q})v + \overline{\mathbf{F}}(v) + \overline{\tau}_d = \overline{\mathbf{B}}(q)\tau, \tag{12}$$

where $\overline{\mathbf{M}}(q) \in \Re^{r\times r}$ is a symmetric, positive definite inertia matrix, $\overline{\mathbf{V}}_\mathbf{m}(q,\dot{q}) \in \Re^{r\times r}$ is the centripetal and coriolis matrix, $\overline{\mathbf{F}}(v) \in \Re^r$ is the surface friction, $\overline{\tau}_d$ denotes bounded unknown disturbances including unstructured unmodelled dynamics, $\overline{\mathbf{B}}(q) \in \Re^{r\times r}$ is the input transformation matrix, and $\tau \in \Re^r$ is the input vector. If $r = n\text{-}m$, $\overline{\mathbf{B}}(q)$ is nonsingular. Equation (12) describes the behavior of the nonholonomic system in a new set of *local* coordinates, *i.e.*, $\mathbf{S}(q)$ is a Jacobian matrix which transforms velocities in mobile base coordinates v to velocities in Cartesian coordinates $\dot{q}$. Therefore, the properties of the original dynamics hold for the new set of coordinates [8]. These properties have not explicitly been displayed in the literature:

Boundedness: $\overline{\mathbf{M}}(q)$, the norm of the $\overline{\mathbf{V}}_\mathbf{m}(q,\dot{q})$, and $\overline{\tau}_d$ are bounded.

Skew-symmetry: The matrix $\dot{\overline{\mathbf{M}}} - 2\overline{\mathbf{V}}_\mathbf{m}$ is skew symmetric.

Proof: As $\dot{\mathbf{M}} - 2\mathbf{V_m}$ is skew-symmetric, it is straightforward to show that (13) is skew-symmetric also.

$$\dot{\overline{\mathbf{M}}} - 2\overline{\mathbf{V}}_\mathbf{m} = \dot{\mathbf{S}}^T\mathbf{M}\mathbf{S} - (\dot{\mathbf{S}}^T\mathbf{M}\mathbf{S})^T + \mathbf{S}^T(\dot{\mathbf{M}} - 2\mathbf{V_m})\mathbf{S}\,. \tag{13}$$

□

3. Control Design

The complete dynamics (10), (11) consist of the kinematic steering system (10) plus some extra dynamics (11). Standard approaches to nonholonomic controls design deal only with (10), ignoring the actual vehicle dynamics. In this paper we correct this omission.

Let u be an auxiliary input, then by applying the nonlinear feedback

$$\tau = f_\tau(q,\dot{q},v,u) = \overline{\mathbf{B}}^{-1}(q)[\overline{\mathbf{M}}(q)u + \overline{\mathbf{V}}_\mathbf{m}(q,\dot{q})v + \overline{\mathbf{F}}(v) + \overline{\tau}_d], \tag{14}$$

one can convert the dynamic control problem into the kinematic control problem

$$\dot{q} = \mathbf{S}(q)\mathrm{v}, \tag{15.a}$$

$$\dot{\mathrm{v}} = u. \tag{15.b}$$

Equation (15) represents a state-space description of the nonholonomic mobile robot and constitutes the basic framework for defining its nonlinear control properties [2], [3], [11], [13].

In performing the input transformation (14), it is assumed that all the dynamical quantities (*e.g.*, $\overline{\mathbf{M}}(q)$, $\overline{\mathbf{F}}(v)$, $\overline{\mathbf{V}}_\mathbf{m}(q,\dot{q})$) of the vehicle are exactly known. It is straightforward to incorporate standard adaptive or robust control techniques if this is not the case [8].

3.1 Backstepping Control Design

Many approaches exist to selecting a velocity control $v(t)$ for the steering system (10). In this section, we desire to convert such a prescribed control $v(t)$ into a torque control $\tau(t)$ for the actual physical cart. Therefore, our objective is to select $\tau(t)$ in (11) so that (10), (11) exhibits the desired behavior motivating the specific choice of the velocity $v(t)$. This allows the steering system commands $v(t)$ in the literature to be converted to torques $\tau(t)$ that take into account the mass, friction, etc. parameters of the actual cart.

Considering that each one of the basic navigation problems may be solved by using adequate smooth velocity control inputs. If the mobile robot system can track a class of velocity control inputs, then tracking, path following and stabilization about a desired posture may be solved under the same control structure.

The smooth steering velocity control, denoted by v_c, can be found by any technique in the literature. Using the algorithm to be derived and proved in Section 3.2, the three basic navigation problems are solved as follows:

Tracking: Given a reference cart

$$\dot{x}_r = \mathrm{v}_r\cos\theta_r,\quad \dot{y}_r = \mathrm{v}_r\sin\theta_r,\quad \dot{\theta}_r = \mathrm{w}_r, \qquad q_r = [x_r\ \ y_r\ \ \theta_r]^T,\quad v_r = [\mathrm{v}_r\ \ \mathrm{w}_r]^T, \tag{16}$$

with $\mathrm{v}_r > 0$ for all t, find a smooth velocity control input $v_c = f_c(e, v_r, K)$ such that $\lim_{t\to\infty}(q_r - q) = 0$.

Path Following: Given a path $\mathbf{P}$ in the plane and the mobile robot linear velocity $\upsilon(t)$, find a smooth velocity control input $v_c = f_c(e_\theta, \upsilon, b, K)$, where e_θ and $b(t)$ are the orientation error and the distance between a reference point in the mobile robot and the path $\mathbb{P}$

respectively, such that $\lim_{t\to\infty}(e_\theta)=0$ and $\lim_{t\to\infty}(b(t))=0$.

Point Stabilization: Given an arbitrary configuration q_r, find a smooth time-varying velocity control input $v_c = f_c(e, v_r, K, t)$ such that $\lim_{t\to\infty}(q_r - q) = 0$.

□

Then define an auxiliary feedback control law $u = \dot{v}_c + K_4(v_c - v)$ such that $v \to v_c$ as $t \to \infty$. Finally, compute the torque $\tau = f_\tau(q, \dot{q}, v, u)$ using (14).

3.2 Tracking a Reference Trajectory

Many workers have designed nonlinear feedbacks for the kinematic model (15.a) of a mobile robot that solve the three basic problems just mentioned, providing a steering system input v_c. Unfortunately, the means of selecting actual torque inputs $\tau(t)$ from steering inputs $v(t)$ has not been addressed.

A general structure for the tracking control system is presented in Fig. 2. In this figure, complete knowledge of the dynamics of the cart is assumed, so that (14) is used to compute $\tau(t)$ given $u(t)$. The contribution of this paper lies in deriving a suitable $u(t)$ and $\tau(t)$ from a specific $v_c(t)$ that controls the steering system (15.a). It is common in the literature to address the problem by assuming 'perfect velocity tracking', which may not hold in practice. A better alternative to this unrealistic assumption is the *integrator backstepping method* now developed.

To be specific, it is assumed that the solution to the steering system tracking problem in [6] is available. This is denoted as $v_c(t)$.

The tracking error vector is expressed in the basis of a frame linked to the mobile platform [4]

$$e=\mathbf{T}_e(q_r-q),\quad \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}, \tag{17}$$

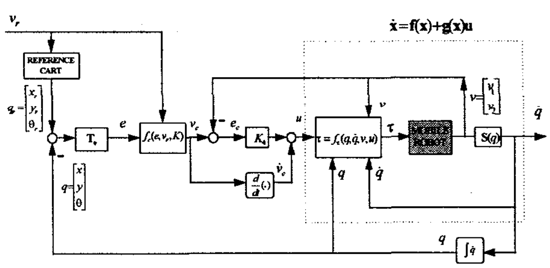


Fig. 2. Tracking Control Structure

and the derivative of the error is

$$\dot{e} = \begin{bmatrix} v_2 e_2 - v_1 + \mathrm{v}_r \cos e_3 \\ -v_2 e_1 + \mathrm{v}_r \sin e_3 \\ \mathrm{w}_r - v_2 \end{bmatrix}. \tag{18}$$

The auxiliary velocity control input that achieves tracking for (15.a) is given by

$$v_c = \begin{bmatrix} \mathrm{v}_r \cos e_3 + k_1 e_1 \\ \mathrm{w}_r + k_2 \mathrm{v}_r e_2 + k_3 \mathrm{v}_r \sin e_3 \end{bmatrix},\quad v_c = f_c(e,\ v_r,\ K). \tag{19}$$

The derivative of v_c becomes

$$\dot{v}_c = \begin{bmatrix} \dot{\mathrm{v}}_r \cos e_3 \\ \dot{\mathrm{w}}_r + k_2 \dot{\mathrm{v}}_r e_2 \end{bmatrix} + \begin{bmatrix} k_1 & 0 & -\mathrm{v}_r \sin e_3 \\ 0 & k_2 \mathrm{v}_r & k_3 \mathrm{v}_r \cos e_3 \end{bmatrix} \dot{e}, \tag{20}$$

and, assuming that the linear and angular reference velocities are constants, we obtain

$$\dot{v}_c = \begin{bmatrix} k_1 & 0 & -\mathrm{v}_r \sin e_3 \\ 0 & k_2 \mathrm{v}_r & k_3 \mathrm{v}_r \cos e_3 \end{bmatrix} \dot{e}. \tag{21}$$

Then the proposed nonlinear feedback acceleration control input is

$$u = \dot{v}_c + K_4(v_c - v), \tag{22}$$

where K_4 is a positive definite, diagonal matrix given by

$$K_4 = k_4 \mathbf{I}. \tag{23}$$

Note that equation (22) is also valid for the case when $\mathrm{v}_r(t)$ and $\mathrm{w}_r(t)$ are time-varying functions. It is common in the literature to assume simply that $u = \dot{v}_c$, called 'perfect velocity tracking', which cannot be assured to yield tracking for the actual cart.

<u>Theorem</u>: Given a nonholonomic system (10), (11) with n generalized coordinates q, m independent constraints, r actuators, let the following assumptions hold: **a.1** the number of actuators is equal to the number of degrees of freedom (*i.e.*, $r = n\text{-}m$). **a.2** The reference linear velocity is nonzero and bounded, $\mathrm{v}_r > 0$ for all t. The angular velocity w_r is bounded. **a.3** A smooth auxiliary velocity control input v_c is given by (19). **a.4** $K=[k_1\ k_2\ k_3]^T$ is a vector of positive constants. **a.5** k_4 is a sufficiently large positive constant.

Let the nonlinear feedback control $u \in \Re^{n-m}$ given by (22) be used and the vehicle input commands be given by (14). Then, the origin $e = 0$ is uniformly asymptotically stable, and the velocity vector of the mobile base satisfies $v \to v_c$ as $t \to \infty$.

Proof: Define an auxiliary velocity error

$$e_c = v - v_c = \begin{bmatrix} e_4 \\ e_5 \end{bmatrix} = \begin{bmatrix} v_1 - \mathrm{v}_r \cos e_3 - k_1 e_1 \\ v_2 - \mathrm{w}_r - k_2 \mathrm{v}_r e_2 - k_3 \mathrm{v}_r \sin e_3 \end{bmatrix}, \tag{24}$$

by using (22), we obtain

$$\dot{e}_c = -K_4 e_c, \tag{25}$$

under assumption **a.5** the auxiliary velocity vector converges exponentially to zero. ∴ the velocity vector of the mobile base satisfies $v \to v_c$ as $t \to \infty$.

Consider the following Lyapunov function candidate:

$$V = k_1(e_1^2 + e_2^2) + \frac{2k_1}{k_2}(1 - \cos e_3) + \frac{1}{2k_4}(e_4^2 + \frac{k_1}{k_2 k_3 \mathrm{v}_r} e_5^2), \tag{26}$$

where $V \geq 0$, and $V = 0$ only if $e = 0$ and $e_c = 0$. Furthermore, by using (18), (24) and (25)

$$\begin{aligned} \dot{V} = &-k_1^2 e_1^2 - \frac{k_1 k_3}{k_2} \mathrm{v}_r \sin^2 e_3 - (e_4 + k_1 e_1)^2 \\ &- \frac{k_1}{k_2 k_3 \mathrm{v}_r}(e_5 + k_3 \mathrm{v}_r \sin e_3)^2, \end{aligned} \tag{27}$$

clearly, $\dot{V} \leq 0$ and the entire error $\mathbf{e} = [e \;\; e_c]^T$ is bounded. Using equations (18), (24), (27), and assumption **a.3**, one deduces that $\|\mathbf{e}\|$ and $\|\dot{\mathbf{e}}\|$ are bounded, so that $\|\ddot{V}\| < \infty$, *i.e.*, $\dot{V}$ is uniformly continuous. Since $V(t)$ does not increase and converges to some constant value, by Barbalat's lemma, $\dot{V} \to 0$ as $t \to \infty$. Considering that $e_c = [e_4 \;\; e_5]^T \to 0$ as $t \to \infty$, then in the limit

$$0 = k_1 e_1^2 + \frac{k_3}{k_2} \mathrm{v}_r \sin^2 e_3 . \tag{28}$$

Equation (28) implies that $[e_1 \;\; e_3]^T \to 0$ as $t \to \infty$. Finally, using the definition of e_c, it is easy to show that $e_2 \to 0$ as $t \to \infty$. ∴ The equilibrium point $\mathbf{e} = \mathbf{0}$ is uniformly asymptotically stable.

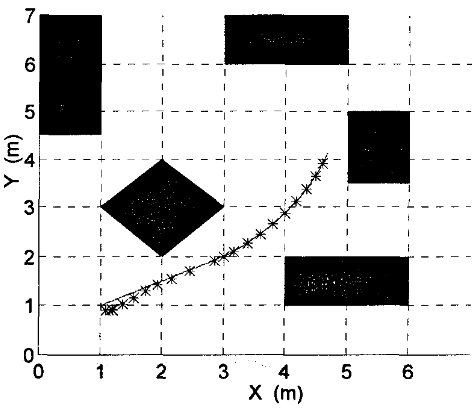
□

4. Simulation Results

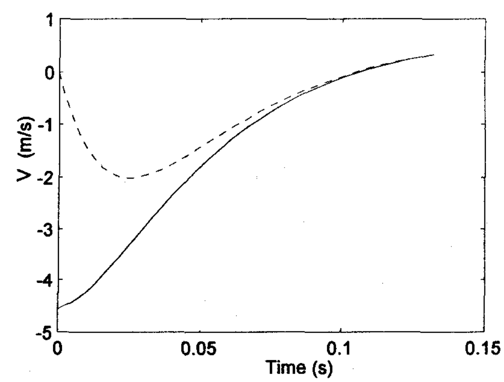
A simulation was implemented in MATLAB. The controller gains were chosen so that the closed-loop system exhibits a critical damping behavior. We consider a trajectory that consists of a straight line segment and an arc line segment as is shown in Fig. 3. This may represent the output of a typical path planner [7]. Clearly, the mobile platform is able to track the reference trajectory. Furthermore, the actual velocities of the cart converge to the control velocities. This is a key point in our backstepping approach, where the position of the vehicle is indirectly controlled by using velocity control inputs. These velocities are converted into torques for the driving wheels considering the parameters of the actual cart and the nonholonomic constraints.

Desired trajectory 1. $(1,1) \to (3,2)$, $\theta_r = 26°$. $\mathrm{v}_r = 0.5$ (m/s), $\mathrm{w}_r = 0$. Desired trajectory 2. $(3,2) \to (4.6,4.1)$, $\theta_r = \mathrm{w}_r * t$, $\mathrm{w}_r = 0.125$ (rad/s).

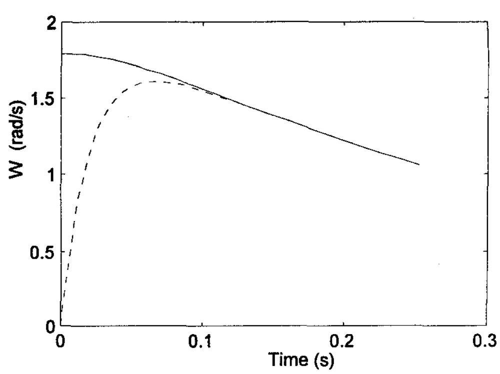
Desired (-) and Actual (*) Trajectories



Control (-) and Actual (..) Linear Velocities v (m/s),



Control (-) and Actual (..) Angular Velocities ω (rad/s)



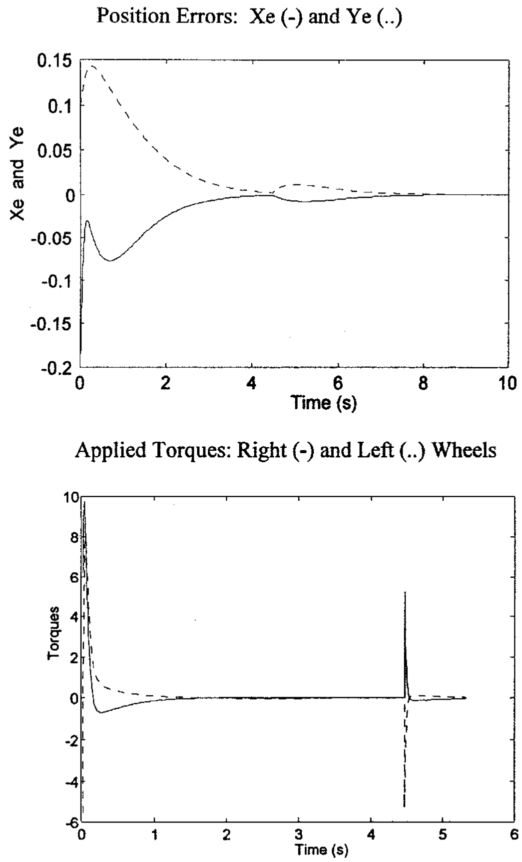


Fig. 3

5. Conclusions

A stable control algorithm capable of dealing with the three basic nonholonomic navigation problems, and that considers the complete dynamics of a mobile robot has been derived using backstepping. This feedback servo-control scheme is valid as long as the velocity control inputs are smooth and bounded, and the dynamics of the actual cart are complete known.

In fact, perfect knowledge of the mobile robot parameters is unattainable, *e.g., friction* is very difficult to model by conventional techniques. To confront this, robust-adaptive control approaches can be implemented in this general structure (Fig. 2) if the computed-torque block is properly modified.

References

[1] J. Barraquand and J-C. Latombe, "Nonholonomic Multibody Mobile Robots: Controllability and Motion Planning in the Presence of Obstacles," *Proc. IEEE International Conference on Robotics and Automation*, pp. 2328-2335, Sacramento, CA, April 1991.

[2] G. Campion, B. d'Andréa-Novel and G. Bastin, "Controllability and State Feedback Stabilizability of Nonholonomic Mechanical Systems," in *Lecture Notes in Control and Information Science*, C. Canudas de Wit ed., pp. 106-124, Springer-Verlag, 1991.

[3] G. Campion, B. d'Andréa-Novel and G. Bastin, "Modeling and State Feedback Control of Nonholonomic Mechanical Systems," *Proc. 30th IEEE Conference on Decision and Control*, pp. 1184-1189, Brighton, England, December 1991.

[4] C. Canudas de Wit, H. Khennouf, C. Samson and O. J. Sordalen, "Nonlinear Control Design for Mobile Robots," in *Recent Trends in Mobile Robots*, Y. F. Zheng ed., pp. 121-156, World Scientific, 1993.

[5] S. Jagannathan, F. L. Lewis and K. Liu, "Modeling, Control and Obstacle Avoidance of A Mobile Robot With an Onboard Manipulator," *Proc. IEEE International Symposium on Intelligent Control,* pp. 196-201, August 1993.

[6] Y. Kanayama, Y. Kimura, F. Miyazaki and T. Noguchi, "A Stable Tracking Control Method for an Autonomous Mobile Robot," *Proc. IEEE International Conference on Robotics and Automation*, vol. 1, pp. 384-389, May 1990.

[7] J-C. Latombe, *Robot Motion Planning*, Kluwer Academic Publishers, 1991.

[8] F. L. Lewis, C. T. Abdallah and D. M. Dawson, *Control of Robot Manipulators*, MacMillan, New York, 1993.

[9] C. Samson, "Velocity and Torque Feedback Control of a Nonholonomic Cart," in *Lecture Notes in Control and Information Science*, C. Canudas de Wit ed., pp. 125-151, Springer-Verlag, 1991.

[10] C. Samson, "Time-varying Feedback Stabilization of Car-like Wheeled Mobile Robots," *The International Journal of Robotics Research*, vol. 12, no. 1, February 1993.

[11] C. Samson and K. Ait-Abderrahim, "Feedback Control of a Nonholonomic Wheeled Cart in Cartesian Space," *Proc. IEEE International Conference on Robotics and Automation*, vol. 2, pp. 1136-1141, April 1991.

[12] J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, 1991.

[13] Y. Yamamoto and X. Yun, "Coordinating Locomotion and Manipulation of a Mobile Manipulator," in *Recent Trends in Mobile Robots*, Y. F. Zheng ed., pp. 157-181, World Scientific, 1993.

[14] Y. Zhao and S., BeMent, "Kinematics, Dynamics and Control of Wheeled Mobile Robots," *Proc. IEEE International Conference on Robotics and Automation*, pp. 91-96, Nice, France, May 1992.