49th IEEE Conference on Decision and Control
December 15-17, 2010
Hilton Atlanta Hotel, Atlanta, GA, USA

Trajectory tracking for non-holonomic cars: A linear approach to controlled leader-follower formation

H. Sira-Ramírez and R. Castro-Linares

Abstract— **This article describes the design of a linear robust dynamic output feedback control scheme for output reference trajectory tracking tasks in a leader-follower non-holonomic car formation problem using the cars' kinematic models. A simplification is proposed on the follower's exact open loop position tracking error dynamics, obtained by flatness considerations, resulting in a system described by an additively disturbed set of two, second order, integrators with non-linear velocity dependent control input matrix gain. The unknown disturbances are modeled as absolutely bounded, additive, unknown time signals which may be locally approximated by arbitrary elements of, a, fixed, sufficiently high degree family of Taylor polynomials. Linear Luenberger observers may be readily designed, which include the, self updating, internal model of the unknown disturbance input vector components as generic time-polynomial models. The proposed Generalized Proportional Integral (GPI) observers, which are the dual counterpart of GPI controllers ([11]), achieve a, simultaneous, disturbance estimation and tracking error phase variables estimation. This, on-line, gathered information is used to advantage on the follower's linear output feedback controller thus allowing for a simple, yet efficient, disturbance and control input gain cancelation effort. The results are applied to control the fixed time delayed trajectory tracking of the leader path on the part of the follower. Simulations are presented which illustrate the robustness of the proposed approach.**

I. INTRODUCTION

Research on multi-robot cooperation and coordination has recently increased due to its potential applications in many fields. These applications require multiple robots to move on a specific geometric path in order to accomplish complex tasks; such as transportation of objects [15], [24]. Areas of interest include: space exploration, in the context of robot assisted construction of space stations [2], security surveillance [8], hazardous cleaning labors [23] and exploration of unknown, or dangerous, environments [3], [13]. One of the central problems in mobile robot cooperation tasks is that of robot formation.

Three main approaches have been developed for the study of multi-vehicle formations. In the behavior-based approach [1], [17], a desired behavior is assigned to each robot, in accordance to a system's excitation, thus leading to individual controls that depend on all other robot's behaviors. In the virtual structure approach [7], [18], each robot is treated as an element of a rigid virtual structure, each of them having a local control that makes possible the structure to maintain a given arrangement. Finally, in the leader-follower approach [4], [6], the objective is to maintain a desired distance, and relative bearing, of the follower(s) respect to the leader(s). Other approaches are based on generalized coordinates [21], graph theory [14], synchronization [22], and artificial potentials [19]. In this work, we are specifically interested in the problem of mobile robot formations in accordance with a "leader-follower" synchronization task. We emphasize that many of the controllers proposed in the literature, for leader-follower formations, lack of robustness to disturbances and model uncertainties. In addition, it is well known that in practical situations one has to deal with uncertainty, nonlinearities and external disturbances acting on the robots [5], [6]

In this article, we present a decentralized formation controller for a team of two autonomous mobile robots with non-holonomic constraints. Our controller design approach is focused on tasks in which the leader robot is required to follow a given path on the plane, while the follower robot is forced to asymptotically track the leaders' path with a fixed time delay, thus keeping a convenient separation with respect to the leader. To achieve this decentralized cooperation target, the differentially flat properties of the robots are used to obtain a simplified reference trajectory tracking error dynamics, for the follower, that is independent of all the leader's retarded dynamics, as well as of his applied control input actions; only the leader's position measurement is available to the follower's controller, including a fixed delay. The rest of the leader's influence on the multivariable trajectory tracking error dynamics is reduced to that of an unknown but bounded, yet trivially observable, perturbation, or disturbance, input. Such an unknown disturbance signal can be rather closely, on-line, estimated via a linear Luenberger observer and subsequently eliminated by the local follower's controller actions via an on-line cancelation effort. The idea of regarding state-dependent perturbations inputs, or disturbances, as unknown but bounded time signals that need to be directly overcome, or, else, estimated and then canceled, is extensively used in sliding mode control theory. It has also been advocated, as *active disturbance rejection* by the late Prof. J. Han in a number of academic and successful industrial applications [12]. The idea is central iin *intelligent PID control*, recently introduced by Fliess and Join [9]. the Very interesting extensions, of the same idea, have been advocated by Prof. C.D. Johnson, since the nineteen seventies, under the name of *disturbance accommodation* (See, for example, [20]).

In section II we present some generalities about state-

H. Sira-Ramírez and R. Castro Linares are with the Department of Electrical Engineering, CINVESTAV-IPN, Av. IPN No. 2508, Col. San Pedro Zacatenco, 07360, México City, México, Ph.: +52-55-50613800, Fax: +52-55-50613866. hsira@cinvestav.mx, rcastro@cinvestav.mx

dependent disturbance estimation-disturbance elimination linear output feedback strategy. Section III applies the obtained results to the trajectory tracking problem in a two non-holonomic, leader-follower, formation car problem and presents the linear controller design approach. Illustrative simulations are presented in section IV. Section V presents some conclusions and suggestions for future work.

II. GENERALITIES ON LINEAR CONTROL OF NONLINEAR SYSTEMS

Consider a smooth, square, multivariable, nonlinear system described by the following set of input-output nonlinear differential equations:

$$\begin{aligned} y_k^{(n_k)} &= \psi_k(y, \dot{y}, ..., y^{(\underline{n-1})}) + \sum_{j=1}^{m} \mu_{kj}(y, \dot{y}, ..., y^{(\underline{n-1})}) u_j \\ & \quad k = 1, 2, ..., m \end{aligned} \tag{1}$$

where the underlined integer: $\underline{n}$, is a multi-index of integer components, n_i, $i = 1, 2, ..., m$, such that $\sum_i^m n_i = n$. The symbol $\underline{n-1}$ has the corresponding interpretation. We summarize (1) into a single vector differential equation, as follows:

$$y^{(\underline{n})} = \psi(y, \dot{y}, ..., y^{(\underline{n-1})}) + \mu(y, \dot{y}, ..., y^{(\underline{n-1})}) u \tag{2}$$

where y is the vector of outputs, which, for simplicity, we assume is constituted by the set of *flat*, or linearizing, outputs $\{y_1, ..., y_m\}$ (See Fliess *et al.* [10]). The matrix function, $\mu(\cdot)$ is, clearly, a square $m \times m$ matrix, that we assume to be locally invertible. Some of the components of the control input vector, u, may already be finitely *extended inputs* in relation to some original set of m inputs. This means that some components of u represent finite time-differentiations of original u's with the highest time derivative acting as an actual control input. The lower time derivatives of such control inputs are relegated to conform measured states that can, in turn, be differentially parameterized by the flat outputs in y. Flatness implies that there is no zero-dynamics associated with the set of special outputs y.

Assume that the analytic expression of the matrix $\mu(y, \dot{y}, ..., y^{(\underline{n-1})})$ is known, although all of its arguments, except for the components of y, are not assumed to be measured. We consider then the following simplified linear time-varying system [1]

$$y^{(\underline{n})} = \hat{\mu}(t) u + \xi(t) \tag{3}$$

where $\xi(t)$ comprises, into a lumped vector of time signals, the state-dependent, smooth, vector of system nonlinearities. The input vector $\xi(t)$ may also include unknown but bounded time signals, denoted below by $\zeta(t)$, acting as external disturbance inputs affecting the system behavior,

$$\xi(t) = \psi(y(t), \dot{y}(t), ..., y^{(n-1)}(t)) + \zeta(t) \tag{4}$$

[1]Linear non-phenomenological models of nonlinear systems have proven to be effective in control theory. Recently, non-phenomenological models of nonlinearities have been exploited in a novel algebraic approach to perturbation estimation in [9]. The idea has also been elegantly exploited in a recent article, [12], by the late Professor Jingqing Han.

we refer to $\xi(t)$ as the *perturbation input* reflecting our lack of knowledge, or lack of measurements, of the system nonlinearities. The estimated control gain, $\hat{\mu}(t)$, is computed as $\mu(y, \widehat{\dot{y}}, ..., \widehat{y^{(\underline{n-1})}})$, where $\widehat{\dot{y}}$,...,$\widehat{y^{(\underline{n-1})}}$, are estimated values of the relevant time derivatives of the components of the measured output vector y. We formulate our problem as follows:

Based only on the set of measured flat outputs, y, and regardless of the unknown but bounded, smooth, disturbance input vector $\xi(t)$, it is desired to have the vector of flat outputs, y, asymptotically track a given, desired, vector of reference trajectories, $y^*(t)$, within an arbitrarily small bounding disk for the set of tracking error phase variables: $e_y = y - y^*(t)$, $\dot{e}_y = \dot{y} - \dot{y}^*(t)$,...,$e_y^{(\underline{n-1})}(t) = y^{(\underline{n-1})} - y^{*(\underline{n-1})}(t)$.

We consider the following controller with auxiliary control input vector v, given by

$$\begin{aligned} u &= [\hat{\mu}(t)]^{-1} v = \left[\mu^{-1}(y, \hat{\dot{y}}(t), ..., \widehat{y^{(\underline{n-1})}(t)})\right] v \\ v &= \left[-\hat{\xi}(t) - [y^*(t)]^{(n)} - \sum_{j=1}^{n-1} K_j \left(\widehat{y^{(j)}} - [y^*(t)]^{(j)} \right) \right] \end{aligned} \tag{5}$$

with K_j being controller design parameter matrices, preferably chosen as diagonal matrices, which render an asymptotically exponentially set of closed loop tracking error trajectories smoothly converging towards a small vicinity of the origin of the tracking error phase space of each flat output y_i. In other words, these design parameter matrices are chosen so that the closed loop tracking error vector dynamics, $e = y - y^*(t)$, satisfies:

$$e^{(\underline{n})} + K_{n-1} e^{(\underline{n-1})} + \cdots + K_1 \dot{e} + K_0 e = \phi(t) \tag{6}$$

with the vector, $\phi(t)$, being a tracking error phase variable-dependent and, possibly, control input dependent, vector of time signals with unknown but bounded components that act as perturbation inputs into the multi-variable closed loop tracking error dynamics. The appropriate choice of the K_j matrices, so that the roots of the, m, closed loop characteristic polynomials are located sufficiently far into the left half of the complex plane, render the tracking error trajectory components to converge, rather independently of $\phi(t)$, towards a small as desired disk in the tracking error phase space for each one of the controlled flat outputs.

The quantities, $\hat{\xi}(t)$ and $\widehat{y^{(i)}}$, $i = 1, 2, ...$, in the proposed controller (5) are, respectively, the estimates of the, unknown but bounded, perturbation input, $\xi(t)$, and of the phase variables associated with each one of the flat outputs. These on-line estimates are generated as follows:

For each flat output, y_i, $i = 1, 2, ..., m$, devise the following linear Luenberger observer of the Generalized Proportional Integral (GPI) type,

$$
\begin{aligned}
\dot{\eta}_{i,1} &= \eta_{i,2} + \lambda^i_{n_i+p_i-1}(y_i - \eta_{i,1}) \\
\dot{\eta}_{i,2} &= \eta_{i,3} + \lambda^i_{n_i+p_i-2}(y_i - \eta_{i,1}) \\
&\vdots \\
\dot{\eta}_{i,n_i} &= \sum_{j=1}^{m} \mu_{ij}(y, \widehat{\dot{y}}, ..., \widehat{y^{(n-1)}})u_j + z_{i,1} + \lambda^i_{p_i}(y_i - \eta_{i,1}) \\
\dot{z}_{i,1} &= z_{i,2} + \lambda^i_{p_i-1}(y_i - \eta_{i,1}) \\
\dot{z}_{i,2} &= z_{i,3} + \lambda^i_{p_i-2}(y_i - \eta_{i,1}) \\
&\vdots \\
\dot{z}_{i,p_i} &= \lambda^i_0(y_i - \eta_{i,1}) \qquad (7)
\end{aligned}
$$

with

$$
\begin{aligned}
\hat{\xi}_i(t) &= z_{i,1}, \ \ \widehat{y_i^{(q_i)}} = \eta_{i,q_i+1}, \\
q_i &= 1, ..., n_i - 1, \ i = 0, 1, 2, ..., m, \qquad (8)
\end{aligned}
$$

and p_i is a finite integer denoting the fixed order of the time polynomial, adopted as an internal model for the observer dynamics, modeling the unknown but bounded perturbation input component $\xi_i(t)$. The reconstruction error $\epsilon_i = y_i - \eta_{i,1}$ is seen to satisfy the following predominantly linear dynamics

$$
\epsilon_i^{(n_i+p_i)} + \lambda^i_{n_i+p_i-1}\epsilon_i^{(n_i+p_i-1)} + \cdots + \lambda^i_1\dot{\epsilon}_i + \lambda^i_0\epsilon_i = \theta_i(t) \quad (9)
$$

where $\theta_i(t)$ is a bounded time signal containing the, self-updating, residuals of the time polynomial approximation of the perturbation input component $\xi_i(t)$; it also contains the temporary mismatches between the row components of the control input gain matrix, μ, and its estimated values. The components of $\theta_i(t)$ are, therefore, state dependent and input dependent bounded perturbation inputs to the linear reconstruction error dynamics whose design coefficients λ^i_j, $j = 0, ..., n_i + p_i - 1$ are chosen sufficiently large so that the predominantly linear dynamics converges to a small as desired vicinity of the origin of the reconstruction error space and its associated phase variables.

III. LINEAR SYNCHRONIZATION OF NON-HOLONOMIC CARS

Consider the problem of synchronizing two non-holonomic cars (see Figure 1), a leader and a follower, evolving on a plane of coordinates x, y, in which it is desired that the follower asymptotically tracks exactly the same path of the leader with a fixed time delay, T. The follower has no information, whatsoever, about the leader control actions, his state variables, nor his system model. The follower performs only accurate measurements of the leader position coordinates.

A. Formulation of the problem and main assumptions

We state, respectively, the leader and follower car kinematic models as follows:

$$
\begin{aligned}
\dot{x}_L &= v_L\cos(\theta_L) & \dot{x}_F &= v_F\cos(\theta_F) \\
\dot{y}_L &= v_L\sin(\theta_L) & \dot{y}_F &= v_F\sin(\theta_F) \\
\dot{\theta}_L &= w_L & \dot{\theta}_F &= w_F
\end{aligned}
$$

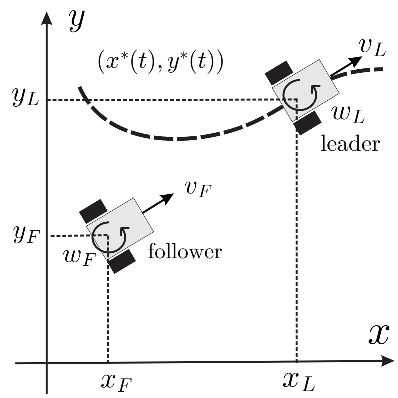

Fig. 1. Leader follower car arrangement

The variables (x_k, y_k), $k = L, F$ represent, respectively, the leader and the follower position coordinates in the plane. The variables θ_L and θ_F are the instantaneous orientations of both vehicles with respect to the positive x coordinate axis. The control variables v_k, w_k, $k = L, F$ are the control inputs, representing, respectively, the forward velocity and the velocity of change of orientation.

B. The leader's tracking problem

We assume that it is desired that the leader tracks, independently of the follower, a trajectory specified by the position reference signals: $x^*(t), y^*(t)$.

The leader's reference trajectory tracking problem may be stated as follows:

Given the leader's non-holonomic car kinematic model

$$
\dot{x}_L = v_L\cos(\theta_L), \quad \dot{y}_L = v_L\sin(\theta_L), \quad \dot{\theta}_L = w_L \qquad (10)
$$

devise a, possibly dynamic, linear output feedback controller, for the control input variables: v_L, w_L, so that the leader's position output variables, x_L, y_L, asymptotically track a given reference trajectory, $x^*(t)$, $y^*(t)$, with a certain level of accuracy.

The leader's kinematic car model is *differentially flat* since all system variables may be differentially parameterized by the output variables x_L, y_L and a finite number of their time derivatives. Indeed,

$$
\theta_L = \tan^{-1}\left(\frac{\dot{y}_L}{\dot{x}_L}\right), w_L = \frac{\ddot{y}_L\dot{x}_L - \ddot{x}_L\dot{y}_L}{\dot{x}_L^2 + \dot{y}_L^2}, v_L = \sqrt{\dot{x}_L^2 + \dot{y}_L^2} \qquad (11)
$$

From this differential parametrization, it is immediate to see that the control input variable, v_L, requires a first order *extension* in order to have a well defined relative degree and achieve invertibility of the matrix defining the input to output's highest time derivatives relation. The input-output model with $\dot{v}_L$ as a new auxiliary control input variables yields,

$$
\begin{bmatrix} \ddot{x}_L \\ \ddot{y}_L \end{bmatrix} = \begin{bmatrix} \frac{\dot{x}_L}{\sqrt{\dot{x}_L^2+\dot{y}_L^2}} & -\dot{y}_L \\ \frac{\dot{y}_L}{\sqrt{\dot{x}_L^2+\dot{y}_L^2}} & \dot{x}_L \end{bmatrix} \begin{bmatrix} \dot{v}_L \\ w_L \end{bmatrix} \qquad (12)
$$

We have the following result.

Proposition. : The following output tracking error based dynamical feedback controller

$$\begin{bmatrix} \dot{v}_L \\ w_L \end{bmatrix} = \begin{bmatrix} \frac{\hat{x}_L}{\sqrt{\hat{x}_L^2+\hat{y}_L^2}} & \frac{\hat{y}_L}{\sqrt{\hat{x}_L^2+\hat{y}_L^2}} \\ -\frac{\hat{y}_L}{\hat{x}_L^2+\hat{y}_L^2} & \frac{\hat{x}_L}{\hat{x}_L^2+\hat{y}_L^2} \end{bmatrix} \times$$
$$\begin{bmatrix} \ddot{x}_L^*(t) - k_1^{x,L}(\hat{x}_{L,sm} - \dot{x}_L^*(t)) - k_0^{x,L}(x_L - x_L^*(t)) \\ \ddot{y}_L^*(t) - k_1^{y,L}(\hat{y}_{L,sm} - \dot{y}_L^*(t)) - k_0^{y,L}(y_L - y_L^*(t)) \end{bmatrix} \quad (13)$$

where $\hat{x}_{L,sm}$, $\hat{y}_{L,sm}$, are "smoothed" versions, respectively, of the variables $\hat{x}_L = \hat{x}_{2L}$, $\hat{y}_L = \hat{y}_{2L}$, generated by the following linear GPI observers:

$$\begin{aligned} \frac{d}{dt}\hat{x}_{1L} &= \hat{x}_{2L} + \lambda_4^x(x_L - \hat{x}_{1L}) \\ \frac{d}{dt}\hat{x}_{2L} &= \left(\frac{\hat{x}_{2L}}{\sqrt{\hat{x}_{2L}^2 + \hat{y}_{2L}^2}}\right)\dot{v}_L - \hat{y}_{2L}w_L + z_{1xL} \\ &\qquad + \lambda_3^x(x_L - \hat{x}_{1L}) \\ \dot{z}_{1xL} &= z_{2xL} + \lambda_2^x(x_L - \hat{x}_{1L}) \\ \dot{z}_{2xL} &= z_{3xL} + \lambda_1^x(x_L - \hat{x}_{1L}) \\ \dot{z}_{3xL} &= \lambda_0^x(x_L - \hat{x}_{1L}) \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d}{dt}\hat{y}_{1L} &= \hat{y}_{2L} + \lambda_4^y(y_L - \hat{y}_{1L}) \\ \frac{d}{dt}\hat{y}_{2L} &= \left(\frac{\hat{y}_{2L}}{\sqrt{\hat{x}_{2L}^2 + \hat{y}_{2L}^2}}\right)\dot{v}_L + \hat{x}_{2L}w_L + z_{1yL} \\ &\qquad + \lambda_3^y(y_L - \hat{y}_{1L}) \\ \dot{z}_{1yL} &= z_{2yL} + \lambda_2^y(y_L - \hat{y}_{1L}) \\ \dot{z}_{2yL} &= z_{3yL} + \lambda_1^y(y_L - \hat{y}_{1L}) \\ \dot{z}_{3yL} &= \lambda_0^y(y_L - \hat{y}_{1L}) \end{aligned} \quad (15)$$

asymptotically exponentially drives the leader's trajectory tracking error to a small as desired vicinity of the origin of tracking errors phase space coordinates.

The follower is assumed, as already mentioned, to accurately measure the leader position, $(x_L(t), y_L(t))$, at each instant of time and he is to device his feedback control scheme so as to track the known set of delayed signals $x_L(t-T), y_L(t-T)$. The follower will asymptotically track the leaders path, with a fixed time delay, T.

We concentrate now on the follower's trajectory tracking problem. Note that the follower's tracking problem involves the leader's car model. We state the problem as follows:

Given the follower's car model

$$\dot{x}_F = v_F\cos(\theta_F), \quad \dot{y}_F = v_F\sin(\theta_F), \quad \dot{\theta}_F = w_F \quad (16)$$

and given a measured reference trajectory: $(x_L(t-T), y_L(t-T))$ where T is a fixed, known, time delay, it is desired that feedback control actions be specified for the control inputs v_F, w_F based, solely, on the tracking error information $e_x(t) = x_F - x_L(t-T)$, $e_y(t) = y_F - y_L(t-T)$, so that the tracking errors $e_x(t)$ and $e_y(t)$ asymptotically converge to a small vicinity of the origin. It is therefore specifically assumed that the follower does not have information about the leader's car dynamic model, nor of his control actions.

The follower's car model is also *differentially flat*. Indeed, all system variables are differentially parameterizable in terms of the car position coordinates x_F, y_F.

$$\theta_F = \tan^{-1}\left(\frac{\dot{y}_F}{\dot{x}_F}\right), w_F = \frac{\ddot{y}_F\dot{x}_F - \ddot{x}_F\dot{y}_F}{\dot{x}_F^2 + \dot{y}_F^2}, v_F = \sqrt{\dot{x}_F^2 + \dot{y}_F^2} \quad (17)$$

An invertible input-output relation is then readily obtained via a first order extension of the control input v_F, thus letting the overall system to have a well defined relative degree. One obtains the following control input parametrization:

$$\begin{bmatrix} \dot{v}_F \\ w_F \end{bmatrix} = \begin{bmatrix} \frac{\dot{x}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & \frac{\dot{y}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} \\ -\frac{\dot{y}_F}{\dot{x}_F^2+\dot{y}_F^2} & \frac{\dot{x}_F}{\dot{x}_F^2+\dot{y}_F^2} \end{bmatrix} \begin{bmatrix} \ddot{x}_F \\ \ddot{y}_F \end{bmatrix} \quad (18)$$

in other words,

$$\begin{bmatrix} \ddot{x}_F \\ \ddot{y}_F \end{bmatrix} = \begin{bmatrix} \frac{\dot{x}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & -\dot{y}_F \\ \frac{\dot{y}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & \dot{x}_F \end{bmatrix} \begin{bmatrix} \dot{v}_F \\ w_F \end{bmatrix} \quad (19)$$

Let $e_{xF} = x_F(t) - x_L(t-T)$, $e_{yF} = y_F(t) - y_L(t-T)$. The open loop tracking error dynamics is readily obtained by subtracting the delayed version of (12) from (19). One obtains

$$\begin{bmatrix} \ddot{e}_{xF} \\ \ddot{e}_{yF} \end{bmatrix} = \begin{bmatrix} \frac{\dot{x}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & -\dot{y}_F \\ \frac{\dot{y}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & \dot{x}_F \end{bmatrix} \begin{bmatrix} \dot{v}_F \\ w_F \end{bmatrix}$$
$$- \begin{bmatrix} \frac{\dot{x}_L(t-T)}{\sqrt{\dot{x}_L^2(t-T)+\dot{y}_L^2(t-T)}} & -\dot{y}_L(t-T) \\ \frac{\dot{y}_L(t-T)}{\sqrt{\dot{x}_L^2(t-T)+\dot{y}_L^2(t-T)}} & \dot{x}_L(t-T) \end{bmatrix} \begin{bmatrix} \dot{v}_L(t-T) \\ w_L(t-T) \end{bmatrix} \quad (20)$$

Clearly, the tracking error dynamics, for the follower, depends on the leader's control actions $(\dot{v}_L(t-T), w_L(t-T))$. Moreover, the time derivatives of the displacement variables of the leader, $\dot{x}_L, \dot{y}_L$, explicitly influence the follower's tracking error dynamics. According to the previous assumptions; this information is not available to the follower agent. A control strategy, that is largely independent of the leader's state variables and control inputs, hence robust, is based on considering the unmeasured additive signals in the right hand side of (20) as disturbance inputs. These unknown inputs need to be rejected, after they are properly identified, or estimated. We thus propose the following simplified tracking error dynamics model for the follower's tracking problem:

$$\begin{bmatrix} \ddot{e}_{xF} \\ \ddot{e}_{yF} \end{bmatrix} = \begin{bmatrix} \frac{\dot{x}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & -\dot{y}_F \\ \frac{\dot{y}_F}{\sqrt{\dot{x}_F^2+\dot{y}_F^2}} & \dot{x}_F \end{bmatrix} \begin{bmatrix} \dot{v}_F \\ w_F \end{bmatrix} + \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} \quad (21)$$

The proposed simplified model, purposely, neglects all the leader retarded dynamics and considers its influence as a trivially observable perturbation that can be eliminated from the local follower controller actions via a suitable on-line estimation. We state our main result.

Proposition. : The following tracking error based feedback controller with a suitable cancelation action

$$\begin{bmatrix} \dot{v}_F \\ w_F \end{bmatrix} = \begin{bmatrix} \frac{\hat{\dot{x}}_F}{\sqrt{\hat{\dot{x}}_F^2+\hat{\dot{y}}_F^2}} & \frac{\hat{\dot{y}}_F}{\sqrt{\hat{\dot{x}}_F^2+\hat{\dot{y}}_F^2}} \\ -\frac{\hat{\dot{y}}_F}{\hat{\dot{x}}_F^2+\hat{\dot{y}}_F^2} & \frac{\hat{\dot{x}}_F}{\hat{\dot{x}}_F^2+\hat{\dot{y}}_F^2} \end{bmatrix} \times \begin{bmatrix} -z_{1xs} - k_1^x \hat{e}_{2xs} - k_0^x e_x \\ -z_{1ys} - k_1^y \hat{e}_{2ys} - k_0^y e_y \end{bmatrix} \tag{22}$$

where $\hat{e}_{2xs}$, $\hat{e}_{2ys}$, z_{1xs} and z_{1ys} are "smoothed" versions, respectively, of the variables $\hat{e}_{2x}$, $\hat{e}_{2y}$, z_{1x} and z_{1y} and

$$\hat{\dot{x}}_F = \hat{e}_{2xs} + \dot{x}_L^*(t-T),\ \hat{\dot{y}}_F = \hat{e}_{2ys} + \dot{y}_L^*(t-T), \tag{23}$$

generated by the following linear GPI observers:

$$\begin{aligned}
\frac{d}{dt}\hat{e}_{1x} &= \hat{e}_{2x} + \lambda_4^x(e_{1x} - \hat{e}_{1x}) \\
\frac{d}{dt}\hat{e}_{2x} &= \left(\frac{\hat{\dot{x}}_F}{\sqrt{\hat{\dot{x}}_F^2 + \hat{\dot{y}}_F^2}}\right)\dot{v}_F - \left(\hat{\dot{y}}_F\right) w_F + z_{1x} \\
&\qquad + \lambda_3^x(e_{1x} - \hat{e}_{1x}) \\
\dot{z}_{1x} &= z_{2x} + \lambda_2^x(e_{1x} - \hat{e}_{1x}) \\
\dot{z}_{2x} &= z_{3x} + \lambda_1^x(e_{1x} - \hat{e}_{1x}) \\
\dot{z}_{3x} &= \lambda_0^x(e_{1x} - \hat{e}_{1x})
\end{aligned} \tag{24}$$

$$\begin{aligned}
\frac{d}{dt}\hat{e}_{1y} &= \hat{e}_{2y} + \lambda_4^y(e_{1y} - \hat{e}_{1y}) \\
\frac{d}{dt}\hat{e}_{2y} &= \left(\frac{\hat{\dot{y}}_F}{\sqrt{\hat{\dot{x}}_F^2 + \hat{\dot{y}}_F^2}}\right)\dot{v}_F + \left(\hat{\dot{x}}_F\right) w_F + z_{1y} \\
&\qquad + \lambda_3^y(e_{1y} - \hat{e}_{1y}) \\
\dot{z}_{1y} &= z_{2y} + \lambda_2^y(e_{1y} - \hat{e}_{1y}) \\
\dot{z}_{2y} &= z_{3y} + \lambda_1^y(e_{1y} - \hat{e}_{1y}) \\
\dot{z}_{3y} &= \lambda_0^y(e_{1y} - \hat{e}_{1y})
\end{aligned} \tag{25}$$

asymptotically exponentially drives the follower's trajectory tracking error to a small as desired vicinity of the origin of tracking errors phase space coordinates.

The reconstruction errors of the tracking error dynamics $\tilde{e}_x = (e_{1x} - \hat{e}_{1x})$ and $\tilde{e}_y = (e_{1y} - \hat{e}_{1y})$, respectively satisfy the perturbed linear dynamics:

$$\begin{aligned}
\tilde{e}_x^{(5)} + \lambda_4^x \tilde{e}_x^{(4)} + \cdots + \lambda_0^x \tilde{e}_x &= \xi_1^{(3)}(t) \\
\tilde{e}_y^{(5)} + \lambda_4^y \tilde{e}_y^{(4)} + \cdots + \lambda_0^y \tilde{e}_y &= \xi_2^{(3)}(t)
\end{aligned} \tag{26}$$

So that, if the λ's are chosen in such a manner that the polynomials in the frequency domain variables, s:

$$\begin{aligned}
p_x(s) &= s^5 + \lambda_4^x s^4 + \cdots + \lambda_1^x s + \lambda_0^x \\
p_y(s) &= s^5 + \lambda_4^y s^4 + \cdots + \lambda_1^y s + \lambda_0^y
\end{aligned} \tag{27}$$

exhibit their roots deep into the left half of the complex plane, then the reconstruction errors of the tracking errors asymptotically converge towards a small disk in the reconstruction error phase space. This is a well known bounded input bounded output result for linear systems (see [16]).

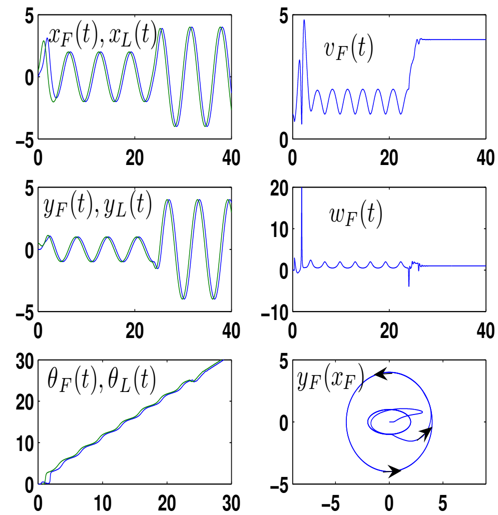

Fig. 2. Performance of GPI observer-based controller for leader-follower car formation problem.

As usual, the smoothing of the observer variables $\hat{e}_{2x}$, $\hat{e}_{2y}$, z_{1x} and z_{1y} is carried out by means of a suitable *clutch* avoiding possible large peaks in their high gain induced responses.

We set the following expressions for the clutches that avoid large initial peaks in the controlled response of the cars,

$$\hat{e}_{2js} = \begin{cases} \hat{e}_{2j}\sin^8(\frac{\pi t}{2\epsilon}) & \text{for } t < \epsilon \\ \hat{e}_{2j} & \text{for } t \geq \epsilon \end{cases} \quad j = x, y \tag{28}$$

and

$$z_{1js} = \begin{cases} z_{1j}\sin^8(\frac{\pi t}{2\epsilon}) & \text{for } t < \epsilon \\ z_{1j} & \text{for } t \geq \epsilon \end{cases} \quad j = x, y \tag{29}$$

IV. SIMULATION RESULTS

It is desired to have the leader follow an elliptical trajectory centered at the origin of the x,y plane with semi-axes given by: $a = 2$ and $b = 1$. The follower is required to track the leader trajectories with a time delay of $T = 0.5$ [s]. At certain point in time, it is also desired that the leader smoothly changes its trajectory in the plane to that of a circle of radius 4, also centered at the origin. The follower must follow suit. The GPI observers were designed with the help of the following dominating Hurwitz characteristic polynomial: $(s^2 + 2\zeta_o\omega_{no}s + \omega_{no}^2)(s + p_o)$ with $\zeta = 1$, $\omega_n = 20$, $p = 20$. The feedback controller gains were set according to the polynomial $(s^2 + 2\zeta_c\omega_{nc}s + \omega_{nc}^2)$ with $\zeta_c = 1$, $\omega_{nc} = 2$. ϵ was set to 2.5 [s].

The simulations, shown in Figure 2, depict the performance of the leader and of the follower in the formulated synchronization problem.

Figure 3, depicts the performance, in the (x, y) plane, of the proposed controller in the trajectory tracking problem for the leader and the follower

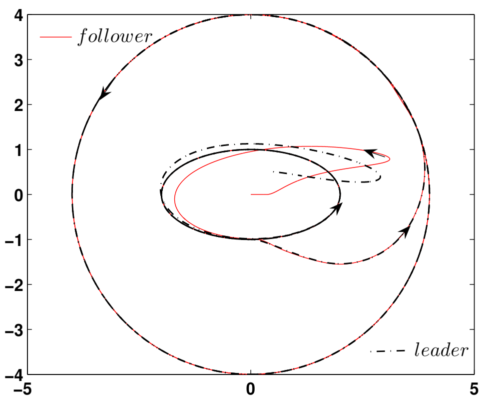

Fig. 3. Performance, in the (x, y) plane, of the GPI observer based controller for leader-follower car formation problem.

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

In this article, we have presented a linear output dynamic feedback control scheme, of decentralized nature, for the efficient formation of a leader-follower trajectory tracking task in non-holonomic mobile robots. The proposed linear controller design approach solves the follower reference path tracking problem with a fixed delay, thus allowing for a safe separation between the controlled cars. The differential flatness property of non-holonomic robots is the key consideration to obtain a simplified reference trajectory tracking error dynamics, for the follower, which is independent of the leader's dynamics and of his applied control inputs. The follower only needs the leader's delayed position measurement. The rest of the leader's influence on the multivariable trajectory tracking error dynamics is reduced to that of an unknown but bounded, yet trivially observable, perturbation, or disturbance, input. Such an unknown disturbance signal can be rather closely, on-line, estimated via a linear Luenberger observer and subsequently eliminated by the local follower's controller actions via an on-line cancelation effort.

B. Future work

The encouraging simulation results, obtained thus far, motivate us to try the proposed linear dynamic output feedback control scheme on an actual laboratory experimental setup, using non-holonomic car prototypes. A possible extension of this work contemplates the inclusion of retarded control input actions, on the part of the follower, due to communication delays and on-board data processing tasks resulting in important time delays that affect the on-line decisions. We hope to report results of this kind in a near future.

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