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Formation Control of Multiple Groups of Robots

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Abstract— **Formation of multiple groups of agents is an important issue to generate different geometric patterns and shapes. In this paper, a centroid based transformation (CBT), that captures the constraint relationship among agents, has been introduced for multiple groups of agents, and it decomposes the combined dynamics of double integrator systems into intra group shape dynamics, inter group shape dynamics, and the dynamics of the centroid. Upon application of the transformation, a modular architecture is formed, which gives the flexibility to design separate control laws for intra group, inter group shape variables, and the tracking control of the centroid. A new control law has been proposed such that the overall error dynamics becomes singularly perturbed system. Three time scale convergence of the error dynamics, has been achieved by selecting suitable controller gains, and also order of the system is significantly reduced after the convergence of the intra and inter group shape variables. Negative gradient of a potential based function has been appended to the controller to ensure collision avoidance among the agents. Simulation results has been provided to demonstrate the effectiveness of the proposed controller.**

I. INTRODUCTION

Though significant research has been carried out on single group of multi-agent system, less attempt has been made for the formation control of multiple groups of agents due to the inherent complexity in expressing the constraint relationship among the agents. The CBTs basically capture the constraint relationship among the agents. In this paper, we have introduced suitable CBTs for multiple groups of agents to get a modular architecture distinguishable in the form of **intra group shape variables**, **inter group shape variables** along with centroid. The main purpose of this work is to introduce a modular architecture, based on suitable CBT, for the formation control of multiple groups of agents. Based on this modular structure, a novel feedback control algorithm is proposed. The gains of controller is so selected that the closed loop dynamics become singularly perturbed system. Thus the model of the system gets significantly reduced and the asymptotic decoupling of the closed loop dynamics is achieved. Finally, the three time scale convergence analysis of the collectively coupled closed loop system is carried out in the singular perturbation framework. We've also used potential function, as given in [10], to avoid inter robot collision.

The concept of forming a specific pattern came into picture by observing the group behaviour or collective behaviour of living beings. [1] - [2]. These trials have gradually been replaced by a more concrete mathematical form of graph theory. The agents are being represented by nodes or vertices of the graph of desired formation and the edges of the given graph are being considered as the communication links. Leader-follower formation controller is the simplest of this kind of graphical formation approach [27]. Virtual leader is no real robot but very much required for some cases of formation control as shown in [5]. The problem associated with this approach is that with the failure of the leader, the entire structure fails to move. In [6], the authors have taken double integrator model to represent the network architecture by graphs. Proof of the stability of a network of agents, by using graph Laplacian matrix, can be found in [8] for switching networks. The problem with graph theory based formation control is that it doesn't offer flexibility of the rigid structure if change of structure is required in presence of obstacles or threat. Other novel graph theoretic results can be found in [28]-[29].

In virtual structure method as given in [15] - [17], the entire group of agents has been treated as a single rigid body and the structure is given a trajectory to follow. This method also carries the similar deficiency as in graph theory based architecture. Also this method is not suitable for the formation of large group of agents as the constraint relationship among agents becomes complicated. However, none of the above literatures addresses the problem associated with the decoupling of geometric formation control and trajectory tracking, which has become the prime focus of the later research on formation. Along with the mathematical realization of group behaviour, there is a formal introduction to the separation of formation controller and tracking controller, i.e., the group variables and shape variables has been separated [9]. A variation of this has been reported in [10], in which the control law guarantees that the agents should be inside a predefined geometric region and the centroid of the region will track the given trajectory. Another centroid based transformation approach to the this problem is shown in [7], where singular perturbation based controller has been designed so that formation is achieved faster than tacking. Literatures [11], [12] give an extensive exploitation of Jacobi transformation to separate formation control and tracking control. Some other approaches of formation control for single group of agents can be found in [24]-[27], where cluster space, distance based formation, slip dynamics of agents, and probability density function based approach has been demonstrated.

For collision avoidance between agents, potential force is generally used, which can be found in [5], [14], and [18].

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Most of the above literature consider single group of agents except [20], where a Leader Follower based multiple group formation architecture has been proposed on the basis of region based formation. In this paper, by means of a transformation matrix we introduce a modular architecture of the formation of multiple groups of agents. Three time scale has been viewed as a key ingredient of the modular architecture of the formation control of multiple groups of agents. This modular behaviour allows us to easily predict the behaviour of overall system of agents from the behaviour of the subgroups. We exploit singular perturbation based convergence analysis tools to prove the stability of the closed loop system.

The proposed technique is similar to deploying different parts of an army at different time at the time of war. This can be used in industrial application for locking a known object and carrying the same to destination. The main contributions of this paper are as follows

- Introduce CBTs for multiple groups of agents that separates intra group, inter group shape variables and centroid, and the general form of CBT for multiple group has been proposed.
- Apply the aforesaid transformations on the collective dynamics of double integrator agents to derive the modular architecture in the form of intra and inter group shape dynamics along with the dynamics of centroid,.
- To design separate formation controller for intra group shape dynamics, for the inter group shape dynamics, along with tracking control.
- Propose three time scale convergence analysis of error dynamics in the framework of singularly perturbed system.
- to design controller for the collision avoidance among the agents.

II. PROBLEM FORMULATION

Given a set of N agents with double integrator dynamics given by

$$\ddot{p}_i = u_i$$

whose positions are described by $p_i = [x_i, y_i]^T$, $i = 1, 2, \dots, N$ in the inertial coordinate frame, and u_i is the control input. Then, for a single group of agents, a linear transformation Φ, can be defined, that produces the following matrix equation

$$[z_1, z_2, \dots, z_{N-1}, z_c]^T = \Phi[p_1, p_2, \dots, p_N]^T \qquad (1)$$

where $z_i, i = 1, 2, \dots, (N-1)$ are the shape defining vectors or shape variables in transformed coordinate, and these vectors define the geometric shape of formation of swarms. Clearly, the transformation Φ generates shape variables along with the centroid for a single group of agents.

For multiple groups of agents, these shape variables can be categorized into two parts. The shape variables which represent the shape of each subgroup, are **intra group shape variables**. However, the variables which describe the interconnection between the groups, each group being considered as a single agent, concentrated onto the centroid of that group, are **inter group shape variables**. Suppose there are m subgroups and each subgroup contains ρ_i number of agents, where ($\sum_{i=1}^{m} \rho_i = N$, N being the total number of robots). Then the total number of intra group shape variables is $\sum_{i=1}^{m}(\rho_i - 1)$, and total number of inter group shape variables is $(m-1)$. The intra group shape variables for each subgroup is defined as

$$Z_1 = [z_{11}, z_{12}, \dots, z_{1(\rho_1-1)}]^T$$
$$Z_2 = [z_{21}, z_{22}, \dots, z_{2(\rho_2-1)}]^T$$
$$\vdots$$
$$Z_m = [z_{m1}, z_{m2}, \dots, z_{m(\rho_m-1)}]^T$$

Therefore, the intra groups shape vectors are defined in a compact form as

$$Z_s = [Z_1, Z_2, \cdots, Z_m]^T$$

. The inter group shape variables considering the centroid of each group as an agent, is defined as

$$Z_r = [z_{r1}, z_{r2}, \dots, z_{r(m-1)}]^T$$

The geometric center of mass, z_c is, defined by

$$z_c = \frac{1}{N}\sum_{i=1}^{N} p_i$$

Using the above definitions for multiple groups of robots, the intra group, inter group shape variable, and centroid can be written in compact form using a CBT Φ_M as

$$[Z_s, Z_r, z_c]^T = \Phi_M[p_1, p_2, \dots, p_N]^T$$

The detailed description of the matrix Φ and Φ_M is given in Section III and IV.

Define, the desired intra group shape variables Z_{sd}, the inter group shape variables Z_{rd}, and the desired trajectory of the centroid z_{cd}.

Let $Z = [Z_s, Z_r, z_c]^T$ and $X = [p_1, p_2, \dots, p_N]^T$ and $Z_d = [Z_{sd}, Z_{rd}, z_{cd}]^T$ and $X_d = [p_{1d}, p_{2d}, \dots, p_{Nd}]^T$. The following equation gives the transformation from Cartesian to the transformed coordinate.

$$Z = \Phi_M X; \;\; Z_d = \Phi_M X_d$$

Therefore, the convergence of $Z \to Z_d$ as $t \to \infty$ leads to the convergence of $X \to X_d$ as $t \to \infty$. However, our objective is that the intra group shape variables converge faster than the inter group shape variables, and the convergence of inter group shape variables will be faster than trajectory tracking of centroid. Using CBT, the resultant modular architecture of formation and tracking control law, is shown in Fig. 1. Based on this, the formation control problem has been divided into the following sub-problems.

Intra Group Formation Control: Given a reference constant Z_{sd}, determine a control law such that intra group shape variables $Z_s(t)$ converges to the desired value as

$$\lim_{t\to\infty} Z_s(t) \to Z_{sd}$$

Inter Group Formation Control: Given a reference constant Z_{rd} determine a control law such that inter group shape variable $Z_r(t)$ converges to the desired value as

$$\lim_{t\to\infty} Z_r(t) \to Z_{rd}$$

Trajectory Tracking: Given a reference time varying trajectory $z_{cd}(t)$ determine a control law such that the centroid $z_c(t)$ converges to the desired trajectory as

$$\lim_{t\to\infty} z_c(t) \to z_{cd}(t)$$

III. TRANSFORMATION MATRIX FOR MULTIPLE GROUPS OF AGENTS

This section mainly describes how to generate centroid based transformation matrices for multiple groups of agents. Fig. 3 shows three different groups of agents in triangular

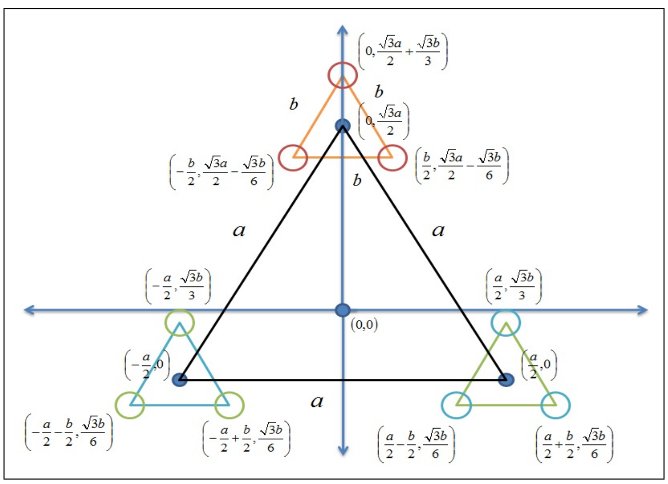

Fig. 1. Schematic Representation of Multiple Groups of Agents

formation and the centroid of each group when connected together, forms another bigger triangle. The goal is to generate intra group and inter group shape variables.

The importance of this modularity is that we don't need know what is the Jacobi or any other CBT for the entire group of agents (say, $N = 9$). Instead, we choose relatively small subgroups of agents (say, $N = 3$ or $N = 4$) and apply transformation. Then considering the centroid of each subgroup as a single agent, inter group relative vectors are generated. Let's illustrate with an example for the formation of agents described by Fig. 3. We've $N_g = 3$ groups of agents with $\rho_i = 3$ agents in each group. Therefore, we'll have 8 $(= 2 \times 3 + 2)$ shape variables along with the centroid of the groups. We choose to construct the transformation matrix for multiple groups of agents with the help of Jacobi transformation here in this example, although other transformations can be similarly used. We first write the shape variable and then give the transformation matrix for multiple groups.

$$\Phi_M \Rightarrow \begin{cases} Z_1 \Rightarrow \begin{cases} z_1 = \frac{1}{\sqrt{2}}(x_2 - x_1) \\ z_2 = x_3 - \frac{1}{2}(x_1 + x_2) \end{cases} \\ Z_2 \Rightarrow \begin{cases} z_3 = \frac{1}{\sqrt{2}}(x_4 - x_5) \\ z_4 = x_6 - \frac{1}{2}(x_4 + x_5) \end{cases} \\ Z_3 \Rightarrow \begin{cases} z_5 = \frac{1}{\sqrt{2}}(x_7 - x_8) \\ z_6 = x_9 - \frac{1}{2}(x_7 + x_8) \end{cases} \\ Z_r \Rightarrow \begin{cases} z_7 = \frac{1}{\sqrt{2}}(\mu_1 - \mu_2) \\ z_8 = \mu_3 - \frac{1}{2}(\mu_1 + \mu_2) \end{cases} \end{cases}$$

where,

$$\begin{cases} \mu_1 = \frac{1}{3}(x_1 + x_2 + x_3) \\ \mu_2 = \frac{1}{3}(x_4 + x_5 + x_6) \\ \mu_3 = \frac{1}{3}(x_7 + x_8 + x_9) \end{cases}$$

The transformation matrix, Φ_M, is given below.

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & 1 \\ -\frac{1}{3\sqrt{2}} & -\frac{1}{3\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} & 0 & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

A. General Form of The Transformation Matrix for Multiple Groups

Therefore, the general form of the transformation matrix for multiple groups of agents can be written as follows

$$\Phi_M^{'} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_m \\ \hline \Phi_r \\ \hline \Phi_c \end{bmatrix} \otimes I_2; \;\; \Phi_M = \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_m \\ \hline \Phi_r \\ \hline \Phi_c \end{bmatrix}$$

Where, $\Phi_1, \Phi_2, ..., \Phi_m$ are $(\rho_1 - 1 \times N), (\rho_2 - 1 \times N), ..., (\rho_m - 1 \times N)$ dimensional matrices and m is the total number of subgroups. The scalar $(\rho_i - 1),\ i = 1, 2, \cdots, m$ are the number of shape variables required to represent i^{th} subgroup. Φ_r is $(m - 1 \times N)$ and Φ_c is $(1 \times N)$. We write the transformation matrix in a more compact form given below.

$$\Phi_M^{'} = \begin{bmatrix} \mathbf{\Phi_m} \\ \hline \Phi_r \\ \hline \Phi_c \end{bmatrix} \otimes I_2; \Phi_M = \begin{bmatrix} \mathbf{\Phi_m} \\ \hline \Phi_r \\ \hline \Phi_c \end{bmatrix}$$

Suppose there are m groups of agents and in the i^{th} group, there are n_i number of agents. Then the dimension of the matrix $\mathbf{\Phi_m}$ is $((\rho_1 + \rho_2 + \cdots + \rho_m) - m) \times m$. Again $\mathbf{\Phi_m}$ can be written in block diagonal form as $\mathbf{\Phi_m} = diag\{\Phi_i\}$, where, Φ_i denotes the transformation matrix for the i_{th} group of agents containing ρ_i number of agents. The dimension of Φ_i is $(\rho_i - 1) \times \rho_i$.

B. Shape Variable Generation Algorithm for Multiple Groups of Agents

The following steps describe the generation algorithm for the shape variables of multiple groups of agents.
step 1: calculate intra group shape variables for each subgroup, i.e., $Z_1, \cdots, Z_m$.
step 2: calculate the centroid of each subgroup, i.e. $\mu_1, \cdots \mu_m$
step 3: calculate inter group shape variables for the overall group assuming each group as an agent.
step 4: calculate the overall centroid.
step 5: combine all the vectors to get the transformation matrix.

IV. FORMATION CONTROLLER DESIGN AND STABILITY ANALYSIS

A. Formation Dynamics

The entire formation of N double integrators can be viewed as a deformable body whose shape and movement can be described by vectors in transformed coordinate.
Let $Z = [Z_s, Z_r, z_c]^T$ and $X = [p_1, p_2, \ldots, p_N]^T$. The following equation gives the transformation from Cartesian to the transformed coordinate.

$$Z = \Phi_M X \tag{2}$$

Using (1), the overall translational dynamics of $i = 1, 2 \ldots, N$ agents in a compact form is given by

$$\ddot{X} = U \tag{3}$$

where

$$U = [u_1, u_2, \ldots, u_N]^T \tag{4}$$

Equation (3) can be written as

$$\ddot{Z} = F \tag{5}$$

where, $F = \Phi_M U$. With equation (5), we opt for designing a controller, by time scale division, such that, the convergence of the subgroup formation will be conducted first, then inter group formation, and finally trajectory tracking. For that we are required to use an important results of singular perturbation theory, given in the next subsection.

B. Stability of Singularly Perturbed Systems

The main result in the singular perturbation literature which will be used for the stability analysis of the proposed controller is given in this section. Extensive reviews of results on singular perturbation, has given in [13], [21], [22] and [23]. The following Lemma which is required to prove our results, is from [13], [7].
Lemma 1 Consider the singularly perturbed system

$$\begin{aligned} \dot{x} &= f(t, x, z, \epsilon) \\ \epsilon \dot{z} &= g(t, x, z, \epsilon) \end{aligned} \tag{6}$$

Assume that the following assumptions are satisfied for all $(t, x, \epsilon) \in [0, \infty) \times B_r \times [0, \epsilon_0]$:

1) $f(t, 0, 0, \epsilon) = 0, g(t, 0, 0, \epsilon) = 0.$
2) The equation $g(t, x, z, 0) = 0$ has an isolated solution $z = h(t, x)$ such that $h(t, 0) = 0$.
3) The functions f, g, h and their partial derivatives up to the second order are bounded for $y = z - h(t, x) \in B_\rho$, for some ρ.
4) The origin of the reduced system $\dot{x} = f(t, x, h(t, x), 0)$ is exponentially stable.
5) The origin of the boundary-layer system $\frac{dy}{d\tau} = g(t, x, y + h(t, x), 0)$ is exponentially stable, uniformly in (t, x).

Then, there exists ϵ^* such that for all ϵ, $0 < \epsilon < \epsilon^*$, the origin of (1) is exponentially stable.

C. Three time scale behaviour of multiple groups of agents

The collective dynamics of (3) can be separately written in the form of intra group shape dynamics (Z_s), as follows

$$\ddot{Z}_s = F_s \tag{7}$$

where, $Z_s = \mathbf{\Phi_m} X$; $F_s = \mathbf{\Phi_m} U$. The inter group shape dynamics (Z_r) is written as,

$$\ddot{Z}_r = F_r \tag{8}$$

where, $Z_r = \Phi_r X$; $F_r = \Phi_r U$. The dynamics of the centroid (z_c) is expressed as,

$$\ddot{z}_c = f_c \tag{9}$$

where, $f_c = \Phi_c U$.
We define the intra group shape error vector as $Z_{se} = Z_s - Z_{sd}$, the inter group shape error vector $Z_{re} = Z_r - Z_{rd}$, and the tracking error of centroid $z_{ce} = z_c - z_{cd}$, where Z_{sd}, Z_{rd}, and z_{cd} are the desired intra group, desired inter group shape variables, and desired trajectory of the centroid respectively. The controller is to be designed such that $Z_{se} \to 0$, $Z_{re} \to 0$, $z_{ce} \to 0$ as $t \to \infty$. To achieve the desired formation and tracking, the following controllers is proposed for (7)-(9).

$$\begin{aligned} F_s &= -K_{s1} Z_{se} - K_{s2} \dot{Z}_{se} - K'_{sr} \dot{Z}_{re} - K'_{sc} \dot{z}_{ce} + \ddot{Z}_{sd} \\ F_r &= -K_{r1} Z_{re} - K_{r2} \dot{Z}_{re} - K'_{rs} \dot{Z}_{se} - K'_{rc} \dot{z}_{ce} + \ddot{Z}_{rd} \\ f_c &= -k_{c1} z_{ce} - k_{c2} \dot{z}_{ce} - K'_{cs} \dot{Z}_{se} - K'_{cr} \dot{Z}_{re} + \ddot{z}_{cd} \end{aligned} \tag{10}$$

where, $K_{s1} = \frac{K_{fs1}}{(\epsilon_1 \epsilon_2)^2}$, $K_{fs1} = k_{fs1} I$, $K_{s2} = \frac{K_{fs1}}{\epsilon_1 \epsilon_2}$, $K_{fs2} = k_{fs2} I$, $K_{r1} = \frac{K_{fr1}}{\epsilon_1^2}$, $K_{fr1} = k_{fr1} I$, $K_{r2} = \frac{K_{fr2}}{\epsilon_1}$, $K_{fr2} = k_{r2} I$, and k_{fs1}, k_{fs2}, k_{fr1},k_{fr2}, k_{c1},k_{c2} are positive constants, I is an identity matrix of appropriate dimension. Using (10), the closed loop error dynamics is given as

$$\begin{aligned} \ddot{Z}_{se} &= -K_{s1} Z_{se} - K_{s2} \dot{Z}_{se} - K'_{sr} \dot{Z}_{re} - K'_{sc} \dot{z}_{ce} \\ \ddot{Z}_{re} &= -K_{r1} Z_{re} - K_{r2} \dot{Z}_{re} - K'_{rs} \dot{Z}_{se} - K'_{rc} \dot{z}_{ce} \\ \ddot{z}_{ce} &= -k_{c1} z_{ce} - k_{c2} \dot{z}_{ce} - K'_{cs} \dot{Z}_{se} - K'_{cr} \dot{Z}_{re} \end{aligned} \tag{11}$$

It is noted that two time scale analysis for the formation control of a single group is given [7]. Motivated by the results, three time scale convergence analysis of proposed control law (10) for multiple groups of robots, is stated in the form of the following theorem.

Theorem The controllers F_s, F_r, and f_c,as given in (10) locally exponentially stabilize system (11), for all $\epsilon_1 < \epsilon_1^*$ and $\epsilon_2 < \epsilon_2^*$ and for some small ϵ_1^* and ϵ_2^*. Moreover, the closed loop system is asymptotically decoupled. As a result, the intra group shape variable Z_s, inter group shape variables Z_r converge to their desired values. Also and the centroid z_c converge to the desired trajectory.
Proof: To write error dynamics define
Let,

$$E_c = \begin{bmatrix} z_{ce} \\ \dot{z}_{ce} \end{bmatrix}; E_r = \begin{bmatrix} \frac{1}{\epsilon_1} Z_{re} \\ \dot{Z}_{re} \end{bmatrix}; E_s = \begin{bmatrix} \frac{1}{\epsilon_1 \epsilon_2} Z_{se} \\ Z_{se} \end{bmatrix}$$

Define, $K_{r1} = \frac{K_{fr1}}{\epsilon_1^2}$, $K_{r2} = \frac{K_{fr2}}{\epsilon_1}$, $K_{s1} = \frac{K_{fs1}}{(\epsilon_1 \epsilon_2)^2}$ and $K_{s2} = \frac{K_{fs2}}{\epsilon_1 \epsilon_2}$. Hence, the error dynamics of (11) is written in the form of singularly perturbed system as follows

$$\dot{E}_c = \begin{bmatrix} 0 & 1 \\ -k_{c1} & -k_{c2} \end{bmatrix} E_c + \begin{bmatrix} 0 & 0 \\ 0 & -K'_{cs} \end{bmatrix} E_s + \begin{bmatrix} 0 & 0 \\ 0 & -K'_{cr} \end{bmatrix} E_r$$

$$\epsilon_1 \dot{E}_r = \begin{bmatrix} 0 & I \\ -K_{fr1} & -K_{fr2} \end{bmatrix} E_r +$$

$$\epsilon_1 \left(\begin{bmatrix} 0 & 0 \\ 0 & -K'_{rs} \end{bmatrix} E_s + \begin{bmatrix} 0 & 0 \\ 0 & -K'_{rc} \end{bmatrix} E_c \right)$$

$$\epsilon_1 \epsilon_2 \dot{E}_s = \begin{bmatrix} 0 & I \\ -K_{fs1} & -K_{fs2} \end{bmatrix} E_s +$$

$$\epsilon_1 \epsilon_2 \left(\begin{bmatrix} 0 & 0 \\ 0 & -K'_{sr} \end{bmatrix} E_r + \begin{bmatrix} 0 & 0 \\ 0 & -K'_{sc} \end{bmatrix} E_c \right)$$

For each equation Condition 1 and condition 3 of **Lemma** is satisfied. By setting $\epsilon_1 = \epsilon_2 = 0$, we've the slow manifolds $E_r = 0$, $E_s = 0$. And the boundary layer systems as follows

$$\frac{dE_r}{dt_1} = \begin{bmatrix} 0 & I \\ -K_{fr1} & -K_{fr2} \end{bmatrix} E_r \; ; \; t_1 = \frac{t}{\epsilon_1}$$

$$\frac{dE_s}{dt_2} = \begin{bmatrix} 0 & I \\ -K_{fs1} & -K_{fs2} \end{bmatrix} E_s \; ; \; t_2 = \frac{t}{\epsilon_1 \epsilon_2}$$

As the above boundary are all linear and time invariant, the exponential stability can easily be guaranteed by proper choice of the pair of positive definite gain matrices (K_{fr1}, K_{fr2}), (K_{fs1}, K_{fs2}) of each boundary layer equation above.
With proper choice of the slow gains k_{c1} and k_{c2}, the reduced system

$$\dot{E}_c = \begin{bmatrix} 0 & 1 \\ -k_{c1} & -k_{c2} \end{bmatrix} E_c$$

can also be shown to be exponentially stable. Hence, from *Lemma 1*, the overall system is exponentially stable for small values of ϵ_1, ϵ_2.■
Remark: As both the boundary layer systems and reduced system is linear, stability can easily be proved by calculating the eigenvalues. Stability of boundary layer systems (intra and inter group shape error dynamics) leads to the convergence to the desired intra and inter group shape variables. Whereas, the stability of the reduced system implies that tracking of the centroid to the desired trajectory, is also achieved. The order of the speed of convergence from high to low, is, intra group shape, inter group shape, tracking of centroid, respectively.

V. SIMULATION RESULTS

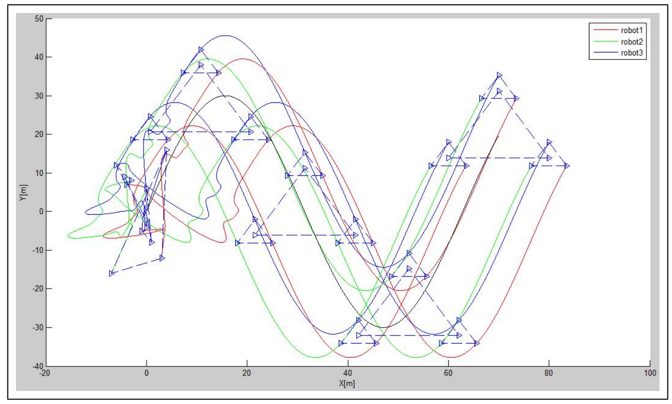

Fig. 2. Formation control using transformation Φ_M

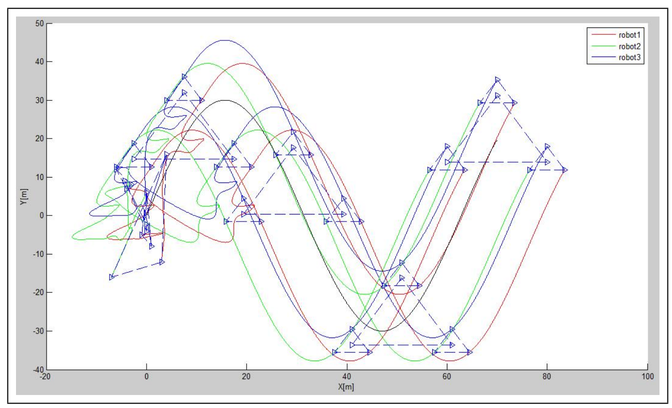

Fig. 3. Potential force based Formation control using transformation Φ_M

The controllers developed in section V-VI has been simulated on three groups of agents with three agents in each group. The controller gain parameters are chosen as $K_{fr1} = K_{fr2} = kI$, $K_{f11} = K_{f12} = kI$, where, $k = 1$, and $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$. The matrices K'_{sr}, K'_{sc}, K'_{rs}, K'_{rc}, K'_{cs}, K'_{cr} are chosen to be all 1s with appropriate dimension,so that the system becomes tightly coupled, although the degree of coupling is left as a choice for the user. All the figures in this section shows the trajectories of the agents moving in formation. The positions of the agents are marked by '$\triangleright$'. Potential force parameters are taken from [10]. The desired trajectory of the centroid of the formation is kept as $z_c = [t; 30sin(0.1t)]$. The rest of the desired vectors are given as $Z_{sd} = [(-4.9497, 0), (0, 6.0622), (-4.9497, 0), (0, 6.0622), (-4.9497, 0), (0, 6.0622)]^T$, $Z_{rd} = [(-14.1421, 0), (0, 17.3205)]^T$.

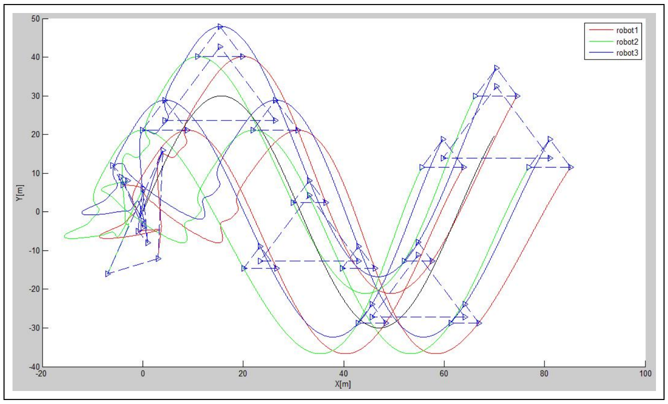

Fig. 4. Time Varying Formation control using transformation Φ_M

The initial values of position of 9 agents is respectively as follows $(x_1, y_1) = (-4, 0)$, $(x_2, y_2) =, (-3, 0)$, $(x_3, y_3) = (-2, 0)$, $(x_4, y_4) = (-1, 0)$, $(x_5, y_5) = (0, 0)$, $(x_6, y_6) = (1, 0)$, $(x_7, y_7) = (2, 0)$, $(x_8, y_8) = (3, 0)$, $(x_9, y_9) = (4, 0)$. We've shown formation with obstacle avoidance (the potential function being given in [7] or [10]) in Fig. 3 and also time varying formation in Fig. 4, where $a = 20 + 2sin(0.1t)$ and $b = 7 + 2sin(0.1t)$ are time varying distances among groups and among robots in a group respectively.

VI. CONCLUSION

In this paper, we propose a transformation for multiple groups of agents such that a modular architecture results, in the form of intra group, inter group shape variables, and centroid. Thus separate controller can be designed for each module of formation. The gains of the controller is so selected that the error dynamics of the overall system becomes singularly perturbed system. Three time scale behaviour of the system has been achieved. The convergence of intra group shape variables is faster than the inter group shape variables, and the convergence of inter group shape variables is faster than the tracking of the centroid. For collision avoidance, negative gradient of potential function has also been appended at the end of the designed controller, which guarantees the convergence of the proposed controller. Simulation results has also been given to show the performance of the proposed controllers.

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