TRACKING CONTROL OF MULTIPLE MOBILE ROBOTS: A CASE STUDY OF INTER-ROBOT COLLISION-FREE PROBLEM

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ABSTRACT

Tracking control problem of multiple mobile robots is considered. Our system is composed of a reference and two follower robots of unicycle type, which have their own priority numbers. The purpose is to control the two followers so that the reference is tracked with arbitrary desired clearance and also to avoid an inter-robot collision, which can occur randomly during the control process subjected to communication range limitation. We introduce two switching controllers and appropriately using their convergence properties leading to the collision-free movement. Simulation results prove efficiency of our control techniques.

KeyWords: Multiple mobile robots, feedback, collision, tracking control, terminal attractor.

I. INTRODUCTION

Control of multiple mobile robots has been subject of a considerable research effort over the last few years. The rapid progress in this field has been the interplay of systems, theories and problems [1]. Three examples are introduced as the recent research trends, namely foraging, box-pushing and traffic control, foraging addresses the problem of learning and distributed algorithm; box-pushing focuses particularly on cooperative manipulation; and traffic control addresses the problem occurring when multiple robots move in a common environment that typically attempt to avoid collisions.

Despite the vast amount of papers published on the collision avoidance motivated by the case of traffic control, the majority has concentrated on centralized path and motion plannings, which mostly utilize feed-forward control in order to accomplish a goal, leading also to a computational time consumption, see for examples Arkin [2] and Lavalle [3], while less attention has been paid to control of the multiple mobile robots by a concept of feedback control. Desai [5] has studied the problem of control and formation where a notation of *l-l* control is originally presented, however, the issue of sensor capability is absent. *l-l* control gives a motivation of using relative dynamics in the system, which will be further extended in our paper.

In this paper, we focus on tasks in which the multiple mobile robots are required to follow a given trajectory in leader-follower formation while avoiding collision among themselves. In addition, because of the high-bandwidth of inter-robot communication, consideration of communication capability becomes necessary. Here, a limited range of communication is considered. The problem of control and coordination for multiple mobile robots is formulated using decentralized controllers. We assume that there are no obstacle in the work space, which simplifies the problem to avoiding inter-robot collision. We also propose a simple technique that helps in decision process of collision avoidance based on geometric analysis. This proposition corresponds to the issue of Latombe [4] stated that collision avoidance is an intrinsically geometric problem in configuration space.

The paper is organized as follows: Section 2 describes model of the wheeled mobile robot of a unicycle type and formulates the tracking problem for a group of three mobile robots without the presence of obstacle. Section 3 introduces a simple collision detection model, virtual robot tracking control (VR control) as a main controller and collision avoidance control (*l-l* control), which is the extended version of the original work of Desai [5]. Analysis of each controllers' properties and combination is also provided, which leads to the main issue. Section 4 contains simulation results that illustrate the performance of the proposed control strategies. The paper concludes with the summary of results and recommendations for further research.

II. PROBLEM FORMULATION

A unicycle-type mobile robot, of which kinematic model is defined by

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$$\dot{q}_i = B_i u_i = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} u_i \tag{1}$$

where $q_i = [x_i, y_i, \theta_i]^T$ and $u_i = [\upsilon_i, \omega_i]^T$ is considered. This model captures the characteristic of a class of restricted mobile robot which is what we need to investigate. The robot are assumed to satisfy pure rolling and non-slipping conditions, which lead to nonholonomic velocity constraints,

$$\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0, \tag{2}$$

$$\dot{x}_i \cos\theta_i + \dot{y}_i \sin\theta_i = \upsilon_i \tag{3}$$

2.1 Assumptions

We summarize here all the assumptions used in this paper.

(i) System is homogeneous, i.e., all robots are of the same model described in (1) and satisfy the velocity constraints, (2) and (3).
(ii) The workspace (*xy*-plane) is flat and contains no obstacle. This comforts us in finding solution focusing on only inter-robot collision problem.
(iii) The reference robot follows a smooth trajectory, and maintains positive velocity. We need this assumption in order to guarantee that our problem is tracking not stabilizing control, see [7].
(iv) Each follower robot is indexed by different priority number. Then, system becomes hierachy and we can manage the behavior of each robot during switching control as will be seen in algorithm in Section 3.
(v) Each robot can extract any necessary information from its communication equipment with negligible delay. In practical case, high-speed ceiling camera should be used to track the absolute position of each robot.

2.2 Problem statement

To keep our investigation on the way, the problem is preliminarily stated as follows.

Statement 1. Giving initial positions and orientations, $q_i(0)$ for the follower robot *i*, and the motion of reference robot, it is required that control law u_i gives a motion satisfying assumptions (i)-(v) such that, as $t \to \infty$,

1 formation is established,
2. no collision occurs among robot *i* and any robot *j*, and
4. a whole motion satisfies the limitation of communication range.

III. CONTROLLER DESIGN AND ANALYSIS

We arrive here with assumptions defined in the last section. We will first apply a VR tracking control approach[1] to the system to complete the requirement in item (1) of the problem while adding an ability of collision checking. The check is done in a sufficiently small sampling time. When a robot detects possibility to collide with another, it checks if its priority number is higher (small value) or not. If not, it has to change the control to *l-l* control[2] in order to avoid the collision and complete the requirement of item (2) of the problem.

It is easy to understand the algorithm above by the flowchart shown in Fig. 1.

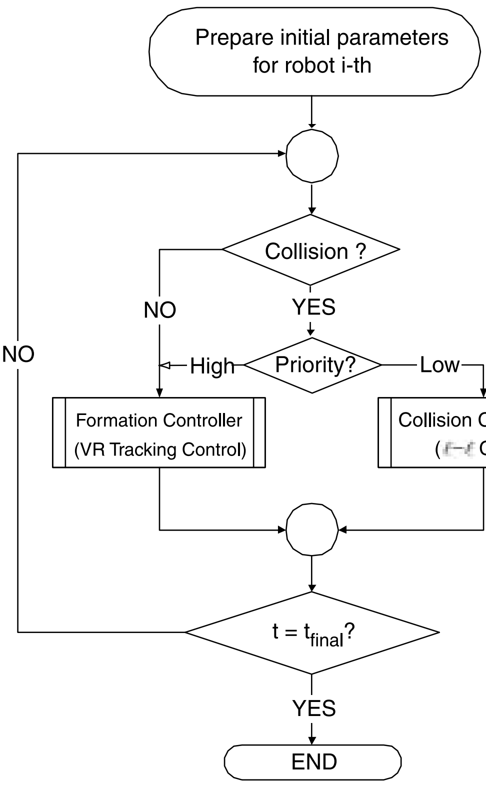

Fig. 1. Algorithm flowchart.

Referring to this algorithm, we need to design a simple collision detecting technique and introduce two feedback control approaches, VR tracking control and *l-l* control, that also fulfill the requirement in item (3) of the problem, which are the keys of the main issue.

3.1 Detecting collision

A simple technique used to detect the collision between any two robots can be realized by modeling both

[1]See details in Section 3.2.
[2]See details in Section 3.3.

robots to be covered with double circles centering at the control points, referring to Fig. 2. The solid ones cover the entire robots with radius D while the outer dotted circles with radius $D + d$ is designed for the required clearances between robots. For convenience, we define the distance between two robots in the following definition.

Definition 1. Let (x_i, y_i) and (x_j, y_j) denote the control points of robot i and robot j, respectively. Distance between robots is defined by

$$\rho_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{4}$$

Then, we define a function that can be used to check the collision between robots i and j as follows.

$$\begin{aligned} f_{ij} &= \rho_{ij}^2 - 4(D + d)^2 \\ f_{ij} &> 0 \rightarrow \text{safety} \\ &\leq 0 \rightarrow \text{collision} \end{aligned} \tag{5}$$

Here, f_{ij} mathematically represents a condition for item (2) of the problem statement. Note that the choice of d should be designed with appropriate value. A large d increases safety level but decreases the smoothness of the robot because it detects the possibility of collision more often, which yields more switching of control inputs.

3.2 Virtual robot tracking control

This approach offers a tracking control with arbitrary clearance between robots, which is, in fact, necessary for navigating the multiple mobile robots while maintaining the formation as we need. However, the collision that can occur during transient state of the control is not considered. We give the detail of this approach in this section.

To easily separate many robot configuration symbols, we will use the following definitions thereafter. $q_r = [x_r, y_r, \theta_r]^T$ refers to a reference robot, $q_i = [x_i, y_i, \theta_i]^T$, $i = 1, 2, \ldots$, refers to follower robot i and $q_{vi} = [x_{vi}, y_{vi}, \theta_{vi}]_T$ is for VR of follower robot i.

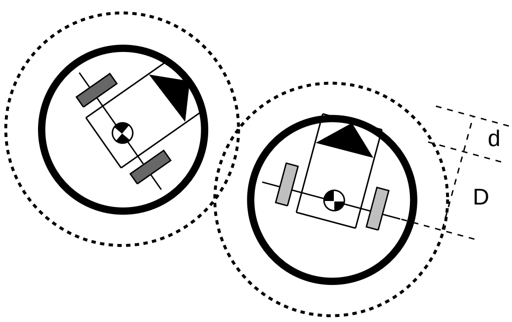

Fig. 2. Collision avoidance model.

In order to separate follower from reference robot avoiding collision, a concept of VR is defined with additional objective to force error parameters to zero when system approaches final time, as follows.

Definition 2. Virtual robot (VR): a hypothetical robot whose orientation is identical to that of its corresponding follower robot, but position is placed apart from it by the predefined $r - l$ clearances. The symbols l, r denote longitudinal clearance and clearance along rear wheel axis, respectively.

The relation between VR and the follower robot is written as follows.

$$\begin{aligned} x_{vi} &= x_i - r \sin \theta_i + l \cos \theta_i \\ y_{vi} &= y_i + r \cos \theta_i + l \sin \theta_i \\ \theta_{vi} &= \theta_i \end{aligned} \tag{6}$$

Note that the above equation is satisfied when the follower is on VR's right-hand side.

From (6) and assumptions (2), (3), we can derive the kinematic model of VR as follows.

$$\dot{q}_{vi} = \begin{bmatrix} \cos \theta_i & -r \cos \theta_i - l \sin \theta_i \\ \sin \theta_i & -r \sin \theta_i + l \cos \theta_i \\ 0 & 1 \end{bmatrix} u_i = \begin{bmatrix} \mathbf{B_{vi}} \\ \hline 0 \quad 1 \end{bmatrix} u_i \tag{7}$$

See proof in appendix A. Figure 3 describes the parameters of VR on xy-plane.

A standard technique of I/O linearization is used to generate a control law for each follower [9]. A property inherited in this controller is described by the following statement.

Property 1. VR tracking control gives solution that exponentially converges in an internal shape x_e and y_e.

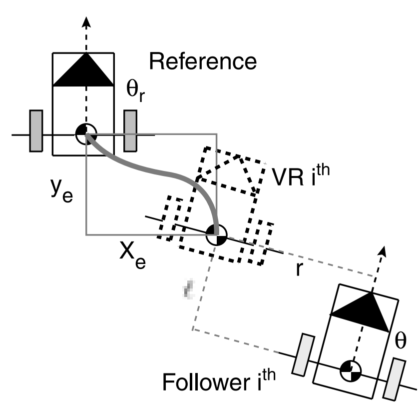

Fig. 3. VR tracking model.

Proof. An error model is formulated as

$$\dot{z}_{ei} = \dot{z}_{\upsilon i} - \dot{z}_r = \mathbf{B}_{\mathbf{vi}} u_i - \mathbf{b}_\mathbf{r} \upsilon_r \tag{8}$$

where $\mathbf{B}_{\mathbf{vi}}$ is the matrix defined in (7), υ_r denotes reference robot's translational velocity and u_i denotes follower robot's velocity vector, and

$$z_{\upsilon i} = \begin{bmatrix} x_{\upsilon i} \\ y_{\upsilon i} \end{bmatrix},\ z_r = \begin{bmatrix} x_r \\ y_r \end{bmatrix},\ \mathbf{b}_\mathbf{r} = \begin{bmatrix} \cos\theta_r \\ \sin\theta_r \end{bmatrix}$$

After applying I/O linearization, we obtain

$$u_i = \mathbf{B}_{\mathbf{vi}}^{-1}(\mathbf{b}_\mathbf{r}\upsilon_r - \lambda z_{ei}) \tag{9}$$

where λ denotes positive-constant diagonal matrix.

The solution of this controlled system, is

$$z_{ei} = e^{-\lambda t} z_{ei}(0) \tag{10}$$

which itself tells that the solution exponentially converges in an internal shape x_e and y_e. Moreover, solution is well-defined because parameter l is designed not to be zero, and hence, $\mathbf{B}_{\mathbf{vi}}^{-1}$ is non-singular. Curve in Fig. 3 illustrates this property.

In addition, zero dynamics given in term of θ_i must be proved to be stable in order to guarantee the use of control law in (9). The differential equation for θ_i is given by

$$\dot{\theta}_i = \left(\frac{\lambda_1 x_{ei} - \upsilon_r \cos\theta_r}{l}\right)\sin\theta_i + \left(\frac{-\lambda_2 y_{ei} - \upsilon_r \sin\theta_r}{l}\right)\cos\theta_i \tag{11}$$

or can be written in the form

$$\dot{\theta}_i = k_1 \sin(\theta_i + k_2) \tag{12}$$

where

$$k_1 = \frac{1}{l}\sqrt{(\lambda_1 x_e - \upsilon_r \cos\theta_r)^2 + (-\lambda_2 y_e + \upsilon_r \sin\theta_r)^2}$$

$$k_2 = \arctan \frac{-\lambda_2 y_e + \upsilon_r \sin\theta_r}{\lambda_1 x_e - \upsilon_r \cos\theta_r}$$

The trajectory, $\theta_i^e = -k_2 - \arcsin\frac{-w_r}{k_1}$ is shown to be exponentially stable by linearization [10] of (12) as follows.

$$\Delta\dot{\theta}_i = k_1 \cos(\theta_i + k_2)^{\theta_i = \theta_i^e} \Delta\theta_i \tag{13}$$

$$= -\sqrt{k_1^2 - w_r^2}\,\Delta\theta_i \tag{14}$$

Note that $\cos(\arcsin(a)) = \sqrt{1 - a^2}$ and Δ denotes a deviation operator. ■

Corollary 1. Property 1 ensures that applying (9) to follower robot will automatically avoid collision with reference robot if the condition below is satisfied,

$$\rho_{ri}(t_0) > 2\,(d + D) \tag{15}$$

where ρ_{ri} denotes distance between reference and follower robot i and t_0 is initial time.

Proof. Recall that (10) ensures exponential convergence of ρ_{ri}. Also, the predetermined $r - l$ clearance implies that $\rho_{ri} \to \sqrt{r^2 + l^2}$. Obviously, if robots do not start with collision, *i.e.* (15) is satisfied, the motions of follower robots are free of collision with reference robot. ■

Remark. Corollary 1 only guarantees collision-free motion between reference and one of follower robot. Hence, there is no guarantee of the motion between follower robots themselves.

3.3 *l-l* control

The aim of this control is to maintain the desired lengths, l_{13}^d and l_{23}^d of considered robot (Robot 3 in Fig. 4) from its two leaders. Referring to the main algorithm, at any collision occurrence, the lowest priority robot will be switched from VR tracking control to this control. *l-l* control is originally presented in Desai [5]; however, the convergence of states is not established within finite time and is aimed for maintaining formation. Here, we utilize *l-l* control as collision avoidance controller, which requires that formation must be established within finite time.

The kinematic equations for the Robot 3 is given as follows,

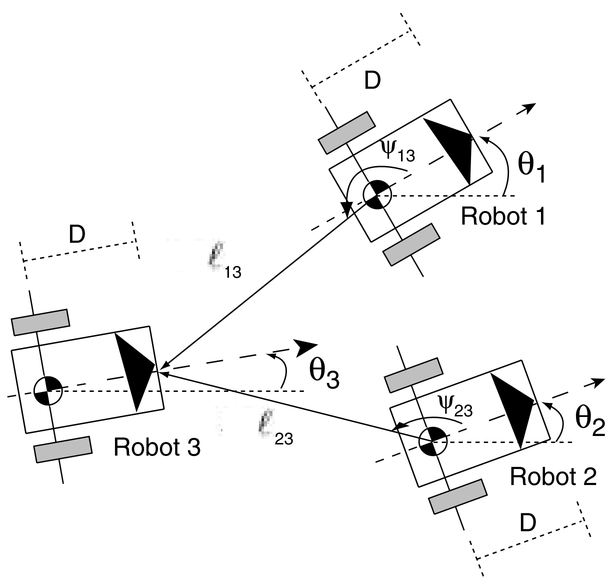

Fig. 4. Notation for *l-l* control model.

$$\begin{bmatrix} \dot{l}_{13} \\ \dot{l}_{23} \end{bmatrix} = \begin{bmatrix} \cos\gamma_1 & D\sin\gamma_1 \\ \cos\gamma_2 & D\sin\gamma_2 \end{bmatrix} \begin{bmatrix} \upsilon_3 \\ \omega_3 \end{bmatrix} - \begin{bmatrix} \upsilon_1\cos\psi_{13} \\ \upsilon_2\cos\psi_{23} \end{bmatrix}$$

$$= \mathbf{B}_{ll} u_3 - \upsilon_{ll} \tag{16}$$

and

$$\dot{\theta}_3 = \omega_3$$

where $\gamma_i = \theta_i + \psi_{i3} - \theta_3$, ($i = 1, 2$).

Remark. In (16), $\mathbf{B}_{ll}$ is singular only if $\gamma_2 - \gamma_1 = n\pi$, ($n = \ldots, -1, 0, 1, \ldots$) which determines the three robots lie on the same line connecting them. We will refer it to *singular line* later in the context.

For the purpose of collision avoidance, a property below is realized by I/O linearization.

Property 2. *l-l* control gives solution that monotonically converges in an internal shape l_{13} and l_{23} within finite time.

Proof. Required that the controlled variables l_{13} and l_{23} monotonically converge to l_{13}^d and l_{23}^d, respectively, within finite time T_r, we set

$$\begin{bmatrix} \dot{l}_{13} \\ \dot{l}_{23} \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} (l_{13}^d - l_{13})^{\frac{p}{q}} \\ (l_{23}^d - l_{23})^{\frac{p}{q}} \end{bmatrix} \tag{17}$$

$$= \alpha \cdot l_e \tag{18}$$

which is identical to a terminal attractor model where (p, q) can be, for example, set to (2, 3). See Appendix B for details.

By arbitrary setting reaching time T_r, we obtain a formula for finding α_1, α_2 as follows.

$$\alpha_1 = sign\,(l_{13}^d - l_{13}(0)) \left| l_{13}^d - l_{13}(0) \right|^{\frac{1}{3}} / (\frac{T_r}{3})$$

$$\alpha_2 = sign\,(l_{23}^d - l_{23}(0)) \left| l_{23}^d - l_{23}(0) \right|^{\frac{1}{3}} / (\frac{T_r}{3})$$

The $sign(\cdot)$ and $|\cdot|$ are used to avoid complex solutions. We can generate a feedback control law as follows.

$$u_3 = \mathbf{B}_{ll}^{-1}(\upsilon_{ll} + \alpha \cdot l_e), \quad 0 \le t \le T_r \tag{19}$$

The solution of this controlled system is

$$l_e = \left((l_{i3}^d - l_{i3}(0))^{\frac{1}{3}} - \frac{\alpha_i}{3} t \right)^3, \quad i = 1, 2, \tag{20}$$

which proves the properties. Note that the term in parenthesis is linear.

Again, zero dynamics in term of θ_3 can be proved stable, clearly when two leaders follows parallel straight lines with the same velocities or $\upsilon_1 = \upsilon_2 = \overline{\upsilon} > 0$, $\omega_1 = \omega_2 = 0$, $\theta_1(0) = \theta_2(0) = \theta_0$. From (16), (19), we have

$$\dot{\theta}_3 = \frac{\overline{\upsilon}\cos\psi_{13}\cos(\psi_{23} + \theta_0 - \theta_3)}{D\sin(\psi_{13} - \psi_{23})} - \frac{\overline{\upsilon}\cos\psi_{23}\cos(\psi_{13} + \theta_0 - \theta_3)}{D\sin(\psi_{13} - \psi_{23})}$$

$$= \frac{\overline{\upsilon}}{D}\sin(\theta_0 - \theta_3) \tag{21}$$

Lyapunov candidate, $V = 1 - \cos(\theta_0 - \theta_3)$ guarantees the stability of (21). In case of more complicate motions of two leaders, the stability is guaranteed by observing that ρ_{12} exponentially decreases according to Property 1. This means l_{13}, l_{23} are always determined or the only singularity $\gamma_1 - \gamma_2 = 0$ is not passed. ■

Remark. Note that (20) also gives an interesting convergent motion in an internal shape l_{13} and l_{23}. It matches with our required motion when collision is detected, i.e., when Robot 3 gets close to the two leaders, with the control (19), it tends to monotonically separate from the leaders avoiding collision.

3.4 Combination of VR and *l-l* control

We can formulate a control of the whole system using the combination of three techniques as stated in Section 3.1-3.3. The flow of control process is formerly presented in this section; however, conditions in which the control process can be performed properly are not analyzed. We summarize all limitations occurs from the combination as follows.

$$1)\ r^2 + l^2 > 4(D + d)^2 \tag{22}$$

$$2)\ l_{i3}^d > 2(D + d), \ i = 1, 2 \tag{23}$$

$$3)\ \rho_{12} < l_{13}^d + l_{23}^d \tag{24}$$

Condition (22) is extracted from the first requirement in item (1) of the problem statement and the design of collision detection in Section 3.1. Condition (23) comes from the restriction of the robots not to get closer than safety regions when *l-l* control is performed. Again, in *l-l* control phase, l_{13}^d and l_{23}^d must be designed in such a way that three robots can form a triangle formation at the end of control, which yields (24).

As stated in Section 3.3, when three robots lie on the same line connecting them, the control law u_3 in (19) is not determined. In addition, the requirement in item (3) of the problem states that communication range is limited, namely,

$\overline{R}$ as maximum range. These two facts enable us to define the *accessible area*, shaded ones in Fig. 5 and Fig. 6, where the low priority robot can move on during collision avoidance. Accessible area comes from the intersection between the circle $\overline{R}$ and the area devided by singular line in which the lowest priority robot exists. Hence, in *l*-*l* control phase, the design of l_{13}^d and l_{23}^d (equivalent to the design of P_x) is simply done by setting the values within accessible area.

Physically, P_x is the point where follower is supposed to be moved to at the end of *l*-*l* control. In fact, the design of P_x can be classified into two cases: desired target is outside accessible area, Fig. 5, and inside the area, Fig. 6. The two cases can be distinguished by the condition below. (For the sake of simplicity, we assume that reference robot moves along the positive *y*-direction.)

if $\theta_a < \theta_b$
Target is outside accessible area
else Target is inside accessible area

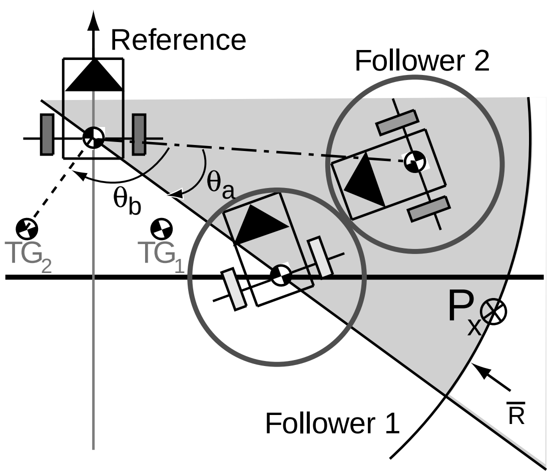

Fig. 5. Target TG_2 is outside accessible area : $\theta_a < \theta_b$.

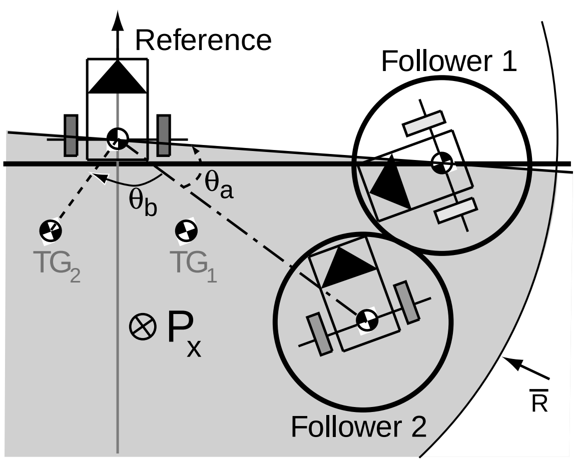

Fig.6. Target TG_2 is inside accessible area: $\theta_a > \theta_b$.

where θ_a, $\theta_b \in [0, 2\pi)$ are measured in clockwise fashion. θ_a is an angle measured from follower 2 to follower 1 centered at reference robot while θ_b is measured from follower 2 to its formation target TG_2.

Remark. In case like Fig. 5, it is strongly recommended to define P_x behind follower 1 to ensure that collision will hardly or not repeat after switching to VR tracking control.

3.5 Main issue

Finally, we summarize all results in this paper as a main issue in theorem below.

Theorem 1. A system of three mobile robots whose models are defined by (1), operating in an environment described by assumptions (i)-(v) and controlled by VR tracking control law: u_i in (9) as a formation controller, and *l*-*l* control law: u_3 in (19) as a collision controller, gives a motion of the form,

$$\rho_{ri} \leq \overline{R}, \; i = 1, 2 \tag{25}$$

while avoiding inter-robot collision and maintaining a predetermined formation in the steady state. $\overline{R}$ is positive scalar corresponding to maximum communication length of reference robot.

Proof. It can be simply checked that the distribution of the three robots is guaranteed to be inside a circle of radius $\overline{R}$ centered at reference robot by Property 1, Property 2 and design process in Section 3.4 along the motion if the two follower robots are initially in this circle. ■

IV. SIMULATION RESULTS

The aim of these simulations is to validate our tracking control method for a multiple mobile robot (three-robot case). Our goal is to make the three robots form a leader-wing motion. Two cases when target is initially located outside and inside accessible area are shown. We start from the former case where initial parameters are set as follows.

Common: $Tr = 1.5s$, $D = 2.25$, $d = 1$, $\overline{R} = 25$

Reference: $q_r(0) = [0, 0, \frac{\pi}{2}]^T$, $u_r = [5, 0]^T$

Follower 1: $q_1(0) = [6, -6, \frac{\pi}{2}]^T$, $(r, l) = (6, 6)$

Follower 2: $q_2(0) = [10, 2, \frac{3\pi}{4}]^T$, $(r, l) = (-6, 6)$

Assume also that reference robot has the highest priority and follower robot 2 has the lowest one.

From Fig. 8, we see that motion of follower robot 2 is kept within the maximum communication range $\overline{R}$ with a desired formation established in the steady state. See snapshot in Fig. 7 for an entire motion of system. According to Table 1, four of collision occurrences between the two followers are found. The first two occurences fall to the case when target is outside accessible area, while the successive ones fall to the case when target is inside the area. The number of collision can be decreased by modifying l_{13}^d, l_{23}^d and T_r in each occurrence, and here we simply use constant values.

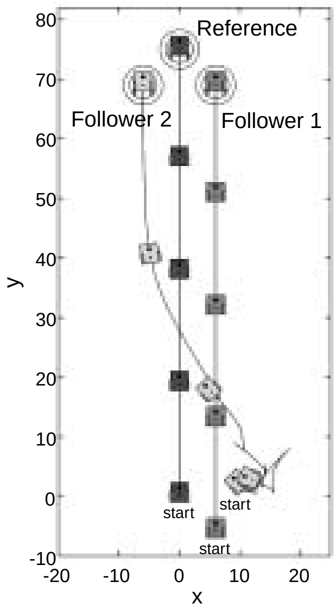

Fig. 7. Snapshot of motion (target is outside accessible area)

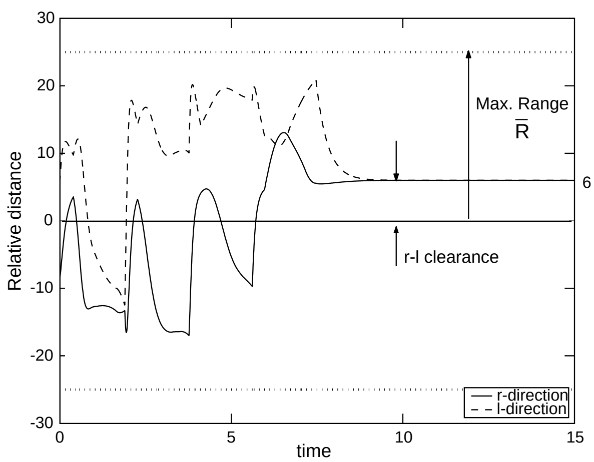

Fig. 8. Distance from reference to follower 2 in $r - l$ direction when target is outside.

Table 1. Collision information (Target outside).

No. Info.	t	l_{13}^d	l_{23}^d	case
1	0.3925	18.1325	10.0000	outside
2	2.2648	18.1325	10.0000	outside
3	4.1099	18.1325	10.0000	inside
4	5.9663	18.1325	10.0000	inside

On the other hand, the case when target is initially located inside accessible area is done by setting,

$$\text{Follower 1: } q1(0) = [8, -3, \frac{5\pi}{4}]^T$$

$$\text{Follower 2: } q2(0) = [8, -9, \frac{5\pi}{8}]^T$$

where other parameters are identical to the last example.

Again, the motion of follower robot 2 (Figs. 9, 10) is guaranteed to be inside communication range. The design of l_{13}^d, l_{23}^d for this case gives only one collision, referring to Table 2.

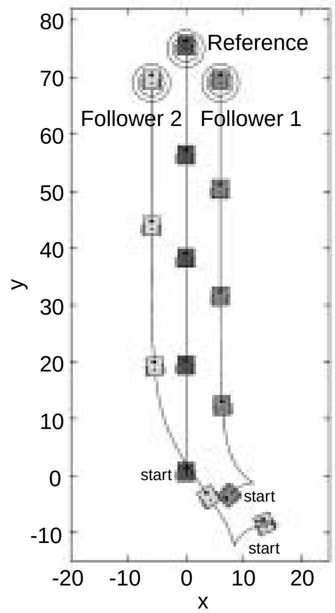

Fig. 9. Snapshot of motion (target is inside accessible area).

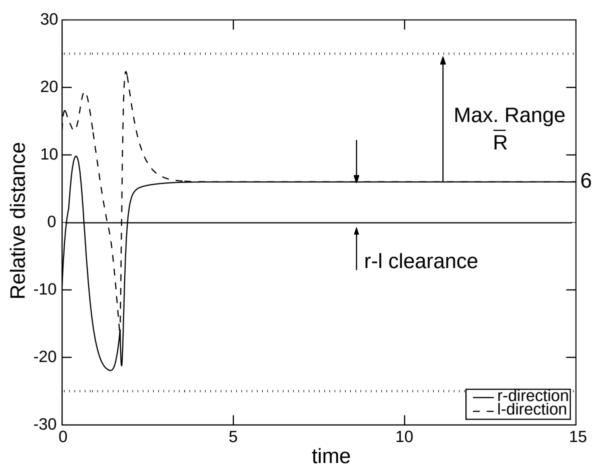

Fig. 10. Distance from reference to follower 2 in $r - l$ direction when target is inside.

Table 2. Collision information (Target inside).

No. Info.	t	l_{13}^{d}	l_{23}^{d}	case
1	0.1940	17.0515	13.8432	inside

V. CONCLUSION

In this paper, we have studied strategies for controlling multiple mobile robots (three-robot case) focused on a collision-free movement. The follower robots track the gait of reference robot using virtual robot tracking control. The extended *l-l* control technique used to avoid the collision of the three-tuple robots is applied during finite time interval in such a way that the lowest priority robot can move away from the others. After the collision-free condition is established, the controller is switched back to the virtual robot tracking control and maintain the desired formation and so on. The convergences and stabilities of both controllers are throughly studied, leading to the main issue in Theorem 1. The problem arises from the non-robustness of the terminal attractor function used in *l-l* control, and hence before we move to real experiment, the function should be substituted by the more robust one. We are currently working with this issue.

We also plan to enlarge our framework to the problems of obstacle avoidance and formation changes, of which we believe the successful strategies are strongly based on the geometric calculation as stated in Latombe [4]. According to Theorem 1, the extension of our work to the system of more than three robots is possible particularly when we have a number of groups of three-tuple robots to be controlled. These topics are the subject of our future work.

APPENDIX

A. Proof of VR kinematic (7)

Proof. Using real robot kinematic equation (1) and time derivative of a right-hand model (6), we obtain

$$\dot{x}_{\upsilon} = \dot{x} - r\cos\theta\dot{\theta} - l\sin\theta\dot{\theta}$$

$$= \upsilon\cos\theta + \omega(-r\cos\theta - l sin\theta) \quad \text{(A1)}$$

$$\dot{y}_{\upsilon} = \dot{y} - r\sin\theta\dot{\theta} - l\cos\theta\dot{\theta}$$

$$= \upsilon\sin\theta + \omega(-r\sin\theta + l cos\theta) \quad \text{(A2)}$$

$$\dot{\theta}_{\upsilon} = \dot{\theta}$$

$$= \omega \quad \text{(A3)}$$

Rewrite the above equations in driftless matrix form yields (7). ■

B. Terminal attractor

To establish the convergence of z_{ei} in arbitrary finite time, T_r, consider the following lemma.

Lemma 1. The origin of a differential equation

$$\dot{z} = -\lambda z^{\frac{p}{q}},\ \lambda > 0 \quad \text{(A4)}$$

is a terminal attractor with a finite reaching time

$$T_r = \frac{z(0)^{1-\frac{p}{q}}}{\lambda(1-\frac{p}{q})},\ \forall\frac{p}{q} \in (0, 1) \quad \text{(A5)}$$

where $p, q \in I^{+}$, and I^{+} = {positive integer}

Proof. Direct integration of (A4) yields

$$z(t)^{1-\frac{p}{q}} = z(0)^{1-\frac{p}{q}} - (1-\frac{p}{q})\lambda t \quad \text{(A6)}$$

Requiring $z(T_r) = 0$ implies formula (A5). ■

Note that T_r can be chosen arbitrary from the choice of λ. We should take care of the choice p, q, which is suggested in proposition below.

Proposition 1. The choice of p and q in (A4) should be selected such that $(p, q) \in \Omega$.

$$\Omega = \{(p, q)\,|\, p < q,\ q\ \ is\ \ odd\ \} \quad \text{(A7)}$$

According to (A4) and (A5), $\frac{p}{q}$ and $1-\frac{p}{q}$ must be designed to be odd-root in order to avoid $2i$-root of negative values. In addition with the assumption that $\frac{p}{q}$ <1, Ω is established.

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