Formation Control of Autonomous Robots Following Desired Formation During Tracking a Moving Target

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Abstract-In this paper, we propose a novel method for control the formation of the autonomous robots following to the desired formations during tracking a moving target under the influence of the dynamic environment. The V-shape formation is used to track a moving target when the distance from this formation to the target is longer than the target approaching radius. Furthermore, when the leader moves in the target approaching range, the circling shape formation is used to encircle the target. The motion of the robots to the optimal positions in the desired formations are controlled by the artificial force fields, which consist of local and global potential fields around the virtual nodes in the desired formations. Using the global attractive force field around the target, the formation of robots is always driven towards the target position. Moreover, using the repulsive/rotational vector fields in the obstacle avoiding controller, robots can easily escape the obstacle without collisions. The success of the proposed method is verified in simulations.

Keywords: Formation control, swarm intelligence, collision avoidance, artificial vector fields

I. INTRODUCTION

In recent years, multi-agent systems have widely been researched in many areas, such as physic, chemic, biology, cybernetic, and automatic control over the world. Formation control is one of the necessary and important problems in the research field on the multi-agent systems. The formation control of autonomous roots, such as the formation of unmanned air vehicles see [10], autonomous underwater vehicles see [9], mobile sensor networks see [1]-[8], etc., and its potential applications, such as search and rescue missions, forest fire detection and surveillance, etc., have attracted a lot of attention from researchers worldwide.

Formation control of autonomous robots is desired based on the formations in the nature, for example schools of fishes, flocks of bees, swarm of ants, etc., and guaranteed that the members in the formation have to move together under the velocity matching without collisions among them. In order to solve these problems there are many approaches, such as the random connections control between neighboring members in the formation as an α -lattice configuration see [4]-[8], the motion of all robots is driven by the a given dynamic framework see [11]-[17]. In the first approach, the neighboring robots are linked to each other by the attractive/repulsive force fields between them to create the robust formation without collisions. These force fields are



Fig.1 The V-shape flying formation of birds. (Source: http://www.grahamowengallery.com/photography/geese.html)

desired based on the artificial potential field method see [1]-[6]. In contrast, using the second control approach, the robots are driven inside a given dynamic framework to move towards the target. The published results in this approach have shown that robots are able to adjust the formation by rotating and scaling during movement together.

This paper considers a novel approach for the formation control of autonomous robots following desired shapes under the effect of the artificial vector fields. In this paper, the V-shape and circle-shape formation are used as the desired formations. The V-shape formation is used to track the moving target in the global potential field from the target. As analysed in literatures [15]-[17], the V-shape formation of birds (for example the formation flight of the Canada geese during migration, see Fig 1) has a lot of advantages, such as energy savings during flight. The research results in these literatures indicated that formation members realized up to 51% in energy savings over solo flight. Moreover, by flying in V-shape formation the formation members can easily communicate with each other, etc. Furthermore, the circle-shape formation is used to encircle the target when the potential field around the target is local.

In this approach, the desired formation with the constant distances between virtual nodes is firstly generated based on the relative position between the leader of the formation and the target. The leader is chosen from a member robot that has the closest distance to target. This leader plays important role to create and lead the desired formation towards the target in a stable direction. Hence, in many undesired cases,

such as the leader is broken or trapped in the obstacles (for example U-shape obstacle), one new leader is replaced in order to continue to lead the swarm towards the target. Under the effect of the artificial force fields, which are the global attractive force field to the free virtual nodes, the local attractive force field from the active virtual nodes, and the local repulsive force field around each robot, all free robots will automatically find their optimal position at virtual nodes in desired formation without collisions. In addition, the global attractive potential field from the target combined with the orientation controller will easily drive the formation of the robots towards the target position in a desired direction.

The remaining sections of this paper are organized as follows: The problem formulation is presented in the section II. Section III gives the method in order to build the desired formations. In section IV, the formation control algorithms are presented. Simulation results are discussed in section V. Finally, section VI concludes this paper and proposes the future research topic.

II. PROBLEM FORMULATION

In this section, we consider a swarm of N robots that has the mission to track and encircle a moving target in a two-dimensional Euclidean space. The direction of motion of this formation must be kept stable during movement. Let $p_i = (x_i, y_i)^T, v_i = (v_{ix}, v_{iy})^T$ and u_i be the position, velocity vector and control input of the robot i, respectively. The dynamic model of each robot ith is described as follows:

$$\dot{p}_i = v_i
\dot{v}_i = u_i \qquad i = 1,..,N.$$
(1)

Definition 1. The V-shape formation of the autonomous robots is a formation that is linked by two linear formations that own a leader together and deviate a desired formation angle ∂_d . In the linear formations, the distance between the neighboring members is constant.

Definition 2. Robot p_i (i=1,2,...,N) is an active robot if the distance from it to the virtual node q_j (j=1,2,...,N) of the desired formation is smaller than the radius of the active circle around each virtual node ($d_i^j < r_a$, $r_a = d/2$), see Fig.2. In other words, robot i is active if it lies inside of one active circle j. Otherwise, it is a free robot.

Definition 3. A virtual node q_j (j=1,2,...,N; $q_j=(x_j, y_j)^T)$ of the desired formation is active if there is a robot i (i=1,2,...,N) in the active circle of this virtual node. In the case where there is no robot in the active circle j, then the virtual node j is free.

Definition 4. Optimal position for robot i in the desired formation is a virtual node j at which $\lim_{t\to\infty}(p_i(t)-q_j(t))=0$ and

the virtual node (*j-1*) is also active.

Remark 1. Consider a V-shape desired formation including N virtual nodes that are equally spaced with the desired distance $d = \|q_{j-1} - q_j\|$, and a constant formation angle ∂_d as

shown in Fig.2. Robots have to find the optimal position in this desired formation. Firstly, each free robot i will pursue the closest free node in order to become an active robot at this virtual node. Secondly, if the position of the active robot at the active node j is not optimal (for example robot p_k $(k=1,2,...,N, k\neq l)$, then this active robot will automatically move into virtual nodes (j-l) until it achieves an optimal position in the desired formation. The formation angle ∂_d can be chosen from ∂_{dmin} to π . Here, the formation must guarantee that there aren't collisions between the members in the formation. In other words, it depends on the repulsive radius around each robot. Hence, this formation angle is computed as follows: $\partial_{dmin} = 2 \arcsin(r_r/2d)$.

Remark 2. The motion of the formation depends on the relative position between the leader and the target. At initial time, one robot, which is closest to the target, is chosen as the leader, and it is saved in order to lead its formation towards the target. During movement, if this leader meets any risk, such as it is broken or hindered by the environment, a new leader is selected. This new leader will reorganize the formation and continue to lead the new formation towards the target.

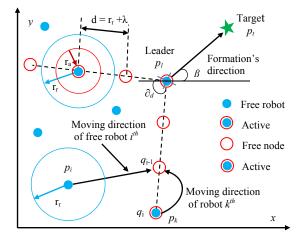


Fig.2 Formation control following the V-shape desired structure.

III. DESIRED FORMATIONS

In this section, we present the method in order to build the desired formations based on the relative position between the target and the leader combined with the coordinate system rotation and translation. The V-shape formation is used to track the target when the distance between the leader and the target is larger than the target reach radius $d_i^t = \|p_i - p_i\| > r^t$. On the other hand, the circling formation is used to encircle the target when $d_i^t = \|p_i - p_i\| \le r^t$.

A. V-shape desired formation

The V-shape desired formation is designed as Fig.3. Assume that the leader's position is determined at $q_l = (x_l, y_l)^T$. Now, in order to build the V-shape desired formation, firstly, we design the right side of the V-shape

formation based on the desired formation angle ∂_d and the relative position between the leader and the target. As depicted in Fig.3, the coordinates of the base node q_μ on the coordinate system x'y' $(q'_\mu = (x'_\mu, y'_\mu)^T)$ are determined as follows:

$$\begin{pmatrix} x'_{\mu} \\ y'_{\mu} \end{pmatrix} = \|q_{\mu} - p_{l}\| \begin{pmatrix} \cos \delta_{d} \\ \sin \delta_{d} \end{pmatrix}.$$
 (2)

In equation (2), the angle δ_d is equal to $\partial_d/2$. By rotating and translating equation (2) according to coordinate systems x''y'' and xy, see [18], we obtain the position of the desired node q_u on the coordinate system xy as follows:

$$q_{\mu} = p_l + Rq_{\mu}^{'}. \tag{3}$$

In equation (3), the rotation matrix R depends on the rotation angle theta θ . If the angle theta θ rotates clockwise as described in Fig.3, then this rotation matrix is determined as:

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} , \tag{4}$$

and in contrast, this matrix is given as follows:

$$R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \tag{5}$$

From the base node q_{μ} and the leader we obtain a unit vector along the line connecting from q_{μ} to p_l , this unit vector is $n_{\mu l} = (p_l - p_{\mu}) / \|p_l - p_{\mu}\|$. Now, a virtual node q_j (j=1,2...,N; $q_j=(x_j, y_j)^T$; $d_{jl}=jd$) is determined by the unit vector $n_{\mu l}$ as follows:

$$(q_i - p_l) = jdn_{ul} . (6)$$

The equation (6) can be rewritten as follows:

$$\begin{pmatrix} x_j \\ y_j \end{pmatrix} = (I+j) \begin{pmatrix} x_l \\ y_l \end{pmatrix} - j \begin{pmatrix} x_\mu \\ y_\mu \end{pmatrix} .$$
 (7)

The equation (7) shows that when j changes, we get a formation of the virtual nodes that lie on the line through p_l and q_u , and are equally spaced.

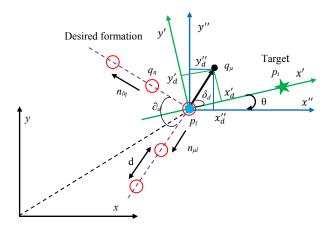


Fig.3 The description of the method to build the V-shape desired formation.

Similarly, the virtual nodes q_j on the left side of the V-shape formation is also designed as follows:

$$q_{i} = (1 - j)p_{i} + jq_{n}$$
 (8)

In this equation, q_{η} is the position of the base node on the line deviates with the line through the leader and the target an angle $(\pi$ - $\delta_d)$, see Fig.3. This node is determined similar to equation (3).

Finally, the algorithm to generate the desired V-shape formation of the N-I members (j=1,2..,N-I) is proposed as equation (9). Here, the positive factor ξ is described as $\xi = j$ -floor(N/2). Using this algorithm, the virtual nodes will be evenly distributed to both sides of the leader as depicted in Fig.3.

$$q_{j} = \begin{cases} (l+j)p_{l} - jq_{\mu}, & \text{if } j \leq N/2\\ (l-\xi)p_{l} + \xi q_{d}, & \text{Otherwise} \end{cases}, \tag{9}$$

B. Circling desired formation

The circling desired formation is designed based on the relative position between the target and the leader, see Fig.4. The position of the virtual nodes q_m (m=1,2...,N-1) on the circle with the center at the target's position p_t and the radius $d_t^t = ||p_t - p_t||$ is computed as follows:

$$q_m = p_t + Rq_m'. ag{10}$$

In this equation, the rotation matrix R is determined similar to equations (4) and (5). The position of the virtual nodes on the coordinate system x'y' $(q'_m = (x'_m, y'_m)^T)$ is computed as:

$$\begin{pmatrix} x'_{m} \\ y'_{m} \end{pmatrix} = d_{l}^{t} \begin{pmatrix} \cos(2k\pi/N) \\ \sin(2k\pi/N) \end{pmatrix}, \quad m = 1, 2..., N - 1.$$
 (11)

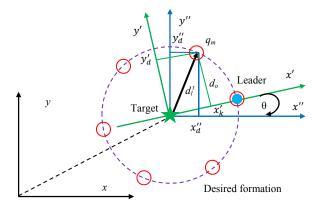


Fig.4 The description of the method to build the circling desired formation.

IV. CONTROL ALGORITHMS

This section presents the algorithms for control wherein all robots will automatically find the optimal position in the desired formation. While tracking the moving target, the stability of the formation must be maintained, and guarantee that there are no collisions between members. Furthermore, after avoiding obstacle, robots have to find their swarm and continue with their formation to track the target.

A. Attractive forces from virtual nodes

Around the virtual nodes of the desired formation, the attractive force fields are created to drive the free robots towards the optimal positions in the desired formation. These attractive forces are built as follows:

Algorithm 1: Reach the optimal position at the virtual nodes in the desired formation

Consider: a robot p_i and virtual nodes q_j (i,j=1,...,N, $i\neq l$). Determine the shortest distance from p_i to all the virtual nodes q_i and the scalar factor \mathbf{c}_i^j as follows:

In algorithm $1, k_{i1}^j, k_{i2}^j, k_{i3}^j, k_{iv}^j$, $(p_i - q_j), (v_i - v_j)$ and d_i^j are the positive gain factors, the relative position vector, the relative velocity vector and the Euclidean distance.

B. Collision avoidance between robots

In order to avoid collision between robots i and k $(i,k=1,2,...,N;i\neq k,i\neq 1)$ during movement, the local repulsive force field is created around each robot within the repulsive radius r_r $(r_r = d - \lambda, \lambda)$ is positive constant), see Fig.2. This vector field is given as follows:

$$F_i^k(p_i) = \left(\left(\frac{1}{d_i^k} - \frac{1}{r_r} \right) \frac{k_{i1}^k}{(d_i^k)^2} - k_{i2}^k \left(d_i^k - r_r \right) \right) c_i^k n_i^k.$$
 (12)

Here k_{li} , k_{2i} , $n_i^k = (p_i - p_k)/\|p_i - p_k\|$, and $d_i^k = \|p_i - p_k\|$ are the positive factors, the unit vector along line from robot k to robot i, and the Euclidean distance between k and i, respectively. The scalar c_i^k is defined as:

$$c_i^k = \begin{cases} 1 & \text{if } d_i^k \le r_r \\ 0 & \text{otherwise.} \end{cases}$$
 (13)

The control algorithm for collision avoidance is built based on the repulsive vector field combined with the relative velocity vector $k_{iv}^k(v_i - v_k)$ between k and i as follows:

$$f_i^k(p_i) = \sum_{k=1, k \neq i}^{N} \left(F_i^k(p_i) - k_{iv}^k c_i^k(v_i - v_k) \right). \tag{14}$$

C. Obstacle-avoiding control algorithm

Similar to the equation (14), the obstacle-avoiding control algorithm for robot i (i=1,2,...,N) is also designed as follows:

$$f_i^o(p_i) = \sum_{o=1 \text{ ord}_k}^M \left(f_i^{op}(p_i) + f_i^{or}(p_i) + k_{iv}^o c_i^o(v_i - v_o) \right). \tag{15}$$

In this equation, $(v_i - v_o)$, and k_{iv}^o are the relative velocity between the robot i^{th} and the obstacle (o), and the positive constant, respectively. The scalar c_i^k is defined as:

$$c_i^o = \begin{cases} 1 & \text{if } d_i^o \le r^\beta \\ 0 & \text{otherwise.} \end{cases}$$
 (16)

Here $d_k^o = ||p_k - p_o||$ and $r^\beta > 0$ are the Euclidean distance and an obstacle detecting range, respectively. The force field $f_i^{op}(p_i)$ is used to repel the robot away from the detected obstacle $(c_i^o = 1)$. Hence, this vector field is given as follows:

$$f_{i}^{op}(p_{i}) = c_{i}^{o} \left(\left(\frac{1}{d_{i}^{o}} - \frac{1}{r^{\beta}} \right) \frac{k_{i1}^{o}}{(d_{i}^{o})^{2}} - k_{i2}^{o} \left(d_{i}^{o} - r^{\beta} \right) \right) \mathbf{n}_{i}^{o}. (17)$$

In this equation, k_{i1}^o , k_{i2}^o , $n_i^o = (p_i - p_o) / \|p_i - p_o\|$ are the positive factors, and the unit vector from the obstacle $p_o = (x_o, y_o)^T$ to the robot *i*. The rotational force field $f_i^{or}(p_i)$ is added to combine with the repulsive force field to drive the robot to quickly escape the obstacle. This rotational force is built as follows:

$$f_i^{or}(p_i) = w_i^o n_i^{or} . (18)$$

Here, the unit vector n_i^{or} is described as follows:

$$n_i^{or} = \left(\frac{(y_i - y_o)}{d_i^o} - \frac{-(x_i - x_o)}{d_i^o}\right)^T.$$
 (19)

The element w_i^o is used to control robot to quickly escape the obstacle, but its velocity does not overcome the limited velocity.

D. Target tracking control algorithm

Firstly, one robot in a swarm is selected as the leader in order to generate the desired formation. Then, this leader is saved to lead its formation to track a moving target. If the leader meets risks, such as it is broken or trapped in obstacles, it must transfer its leadership to one other.

The target tracking controller, which is designed based on the relative position between the leader and the target, has to guarantee that the formation's motion is always driven towards the target. This control law is designed as follows:

$$f_l^t(p_l) = F_l^t(p_l)n_l^t - k_{lv}^t(v_l - v_t).$$
 (20)

In this equation, $n_l^t = (p_l - p_t) / \|p_l - p_t\|$ is the unit vector along the line connection from the target to the leader, and the component $-k_{lv}^t(v_l - v_t)$ is used as a damping term. The value of the attractive force $F_l^t(p_l)$ is computed as follows:

$$F_{l}^{t}(p_{l}) = \begin{cases} -k_{l1}^{t}, & \text{if } d_{l}^{t} > r^{t} \\ \left(\frac{1}{d_{l}^{t}} - \frac{1}{r^{\tau}}\right) \frac{k_{l2}^{t}}{(d_{l}^{t})^{2}} - \frac{k_{l1}^{t}(d_{l}^{t} - r^{\tau})}{(r^{t} - r^{\tau})}, & \text{otherwise.} \end{cases}$$
(21)

Here, k_{11}^t, k_{12}^t, r^t and r^r are the positive factors, and the radius to reach towards the target, respectively. The leader is selected as follows:

Algorithm 2: Leader selection

Update data: The actual position of robots p_i (i=1,...,N, $i\neq l$), obstacle's information, the target's position p_l , the actual position of the leader ($p_{\varepsilon}=p_l$).

if time t=0 (at initial time) then

Compute the shortest distance from the robot p_i to the target p_t in order to determine the leader as follows:

$$d_{imin1}^{t} = \min\{\|p_i - p_t\|, i = 1,..,N\}; p_l = p_{imin1}$$

else

if the actual leader meets obstacle or is broken then
 Leadership is transferred to other that is free and has the closest distance to the target.

$$d_{imin2}^{t} = \min\{\|p_i - p_i\|, i = 1,., N, i \neq \xi, free\}; p_i = p_{imin2}$$

else

Maintain the leadership of the actual leader.

 $p_l = p_s$

end

end

V. SIMULATION RESULTS

In this section, we present the results of the simulations of the above proposed control algorithms. For these simulations, we assume that the initial velocities of the robots and target are set to zero, and obstacles of the environment are stationary. The position of the Obstacles and the random initial position of the robots are depicted in Fig.5 and Fig.6. The target moves in a sine wave trajectory $p_t = (0.9t + 640, 160\sin(0.01t) + 250)^T$. The general parameters of the simulations are listed in table I.

Firstly, we test the algorithms to generate the desired formations (V-shape formation (9), and circling formation (10)), the algorithm to control the robots towards the virtual nodes in the desired formation (algorithm 1). Moreover, the stability of a swarm following the desired formations under the influent of the dynamic environment is also tested.

TABLE I: PARAMETER VALUES

Parameter	Definition	Value
d	Desired distance between robots	60
λ	Positive constant	0.5
N	Number of robots	9
∂_d	Formation angle	$\pi/3$
r^{β}	Obstacle detecting range	20
r^t	Target reaching radius	100
r^{τ}	Active radius around the target	60
k_{l1}^{t}, k_{l2}^{t}	Factors for approach to target	9, 0.6
$k_{i1}^{\ k}, k_{i2}^{\ k}$	Positive factors for fast repulsion	80, 12
k_{i1}^{o} , k_{i2}^{o}	Constants for fast obstacle avoidance	90, 15
$k_{i1}^{j}, k_{i2}^{j}, k_{i3}^{j}$	Positive constants	3, 4, 9
$k_{iv}^{j}, k_{iv}^{t}, k_{iv}^{k},$	Damping factors	1.4

The results of the simulations in Fig.5 show that the desired formations can easily be created. Robots, which have the random initial positions, have achieved the optimal positions in this desired formation while reaching the target without collisions. The position permutations between the members in a swarm are happed, but they do not influence on the structure of the formation during the target tracking. At initial time, one robot is chosen as the leader of the swarm, and then it is saved in order to drive its formation towards the target in a V-shape formation. At time t=90s, the formation of a swarm is made based on the V-shape desired structure, and it is kept until the square robot detects the obstacle O_I . While avoiding the obstacle O_I , the virtual node, which was being owned by the square robot, became a free node, and it attracted the triangular robot to become the active node at time t=160s. After escaping the obstacle, the square robot quickly reached the remaining free node of the desired formation, see Fig.5 at time t=250s. Similarly, the rhombus robot is permuted with other robot in the swarm while avoiding the obstacle O_2 . At time t=250s, the V-shape formation changed to the circling formation in order to encircle the target. Then, this circling formation is kept around the target at the active radius r^{τ} .

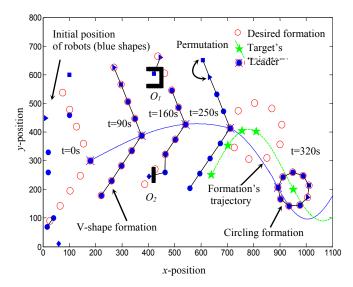


Fig.5 Simulate the motion of a swarm of the autonomous robots following the desired formations under the influence of the dynamic environment while tracking a moving target.

Secondly, we test the intelligence of a swarm when the leader is trapped in the complex obstacle (for example Ushape obstacle, see Fig.6). Using the algorithm 1 and control law (20), the actual leader has to transfer its leadership for other member in the swarm. The simulation results in Fig.8 show that, at time t=0s, the square robot is chosen as the leader and its leadership is kept until it is trapped in the Ushape obstacle at time t=180s. While avoiding obstacle, the square leader transferred its leadership to the triangular robot, which is not hindered and has the closest distance to the target. Then, the square leader became a free robot. It automatically found a way to escape this obstacle in order to continue to follow its formation. After receiving the leadership, the triangular robot reorganized a new formation, and continued to lead this formation in the target tracking. Fig.6 shows that, the position change between the square leader and the triangular leader does not influence on the desired structure of the formation.

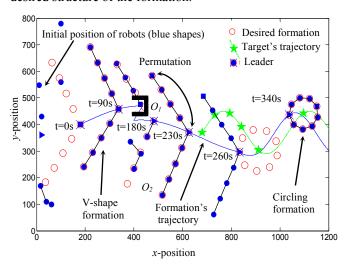


Fig.6 Simulate the leader permutation of a V-shape formation while tracking a moving target in a dynamic environment.

VI. CONCLUSION

This paper has proposed a novel approach to formation control of autonomous robots following the desired formations (the V-shape and circling structure) to track a moving target in a dynamic environment. The desired formations are built based on the relative position between the target and the leader of the swarm. The trajectory of the robots is driven by the artificial force fields from the virtual attractive nodes of the desired formation. In this approach, the leader plays an important role while tracking the target. The mission of the leader is to generate the desired formations, and lead its swarm towards the target. Moreover, using the leader select algorithm, the formation of a swarm can easily pass the obstacles of the environment in order to track the moving target. Furthermore, the repulsive force fields between robots are used to guarantee that there are no collisions in the swarm during movement. The results of the simulations have shown that using the proposed control algorithms, the member robots have quickly achieved the

optimal positions in the desired formation. In some cases, such as the obstacle avoidance, the position of some robots in the formation can be permuted, but the structure and the motion direction of the formation are kept. The development and application of this proposed approach for formation control of the autonomous robots in 3D space will be our future research.

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