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Control Design for Flexible Hierarchical Formation of Multiple Robots*

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***Abstract*— In this paper, we consider flexible formation and related control design of multi-robot systems. A hierarchical structure is provided for flexible formation of unicycle robots. With the virtual hierarchy defined in light of interconnection graphs, the formation and related parameters can be changed in different levels of the whole structure. The formation control design is given from a leader-following viewpoint. Stability analysis is also provided after that.**

***Index Terms*— Multi-robot systems, flexible formation, hierarchical product graph, nonholonomic control.**

I. INTRODUCTION

In recent years, multi-agent coordination has been drawn much attention [1], [2], [15]. As a main application background, multi-robot systems have been widely studied and been investigated from different perspectives. For example, Balch and Arkin proposed a behavior-based approach to robots' coordination in [4]; Fax and Murray reported some results in distributed coordination control for linear vehicle models in [1].

Formation control is an important problem in multi-robot coordination, and many coordination problem can be somehow converted into a formation problem. Formation problem and its stability has been investigated using different approaches including leader-following, potential functions, and virtual structure ([5], [12], [13]). Among these methods, virtual structure approach first introduced by [6] takes the whole formation as a rigid body. One of the main advantages of virtual structure approach is its simplicity of the description of the coordinated behavior for the robots. Moreover, the formation-keeping can be well maintained during maneuvering since the whole formation can maintain a prescribed geometric relationship.

However, many practical problems get involved when the formation shape is time-varying or frequently changed, and many formation methods may not work well. Efforts were taken to overcome the difficulties, for example, in [12], where control with affine transformations in robot formation was investigated, but the result only considers circular formation or formation in a closed curve.

In our paper, we define a new hierarchical formation based on pseudo-rigid transformation, which can endure formation shape changing during maneuvers. Due to its hierarchy, the formation task can be separated into several levels, which facilitates the control of multi-agent coordination. The main problem is to investigate its control problem during movements. Actually, hierarchical formation has been investigated by several researchers. A hierarchical formation was introduced, and several control rules were given only for the linear integrators in [5], while another hierarchical formation for linear systems and related stability was investigated in [17]. However, the existing hierarchical formation methods cannot be directly applied to our problem for at least two reasons: the considered system of a group of nonholonomic robots is nonlinear and the proposed control is given for time-varying flexible formation.

The rest of the paper is organized as follows. First, we give some preliminaries in section 2. Next, we give the problem formulation and analysis of formation structure in section 3. Then, the main results with design procedures are presented in section 4, where we directly investigate this problem from the leader-following viewpoint by assigning a virtual leader for each robot to guarantee the formation stability. Finally, concluding remarks are given in section 5.

II. FORMATION CHANGE

In our paper, we discuss how to describe the formation change for a group of unicycle robots.

At first, we consider robot model in kinematics, which is usually taken as follows:

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{1}$$

where v, ω are velocities and taken as control inputs. This type of mobile robots is subject to nonholomonic constraints as: $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$.

Next, we consider the description of formation in order for the formation control of robots. As we mentioned, virtual structure approach is an important method. This method take the whole formation as a rigid body. The main advantages of virtual structure is simplicity to prescribe the coordinate behavior for the group, and formation-keeping can be well

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maintained during maneuvering; that is, the whole formation can maintain a prescribe geometric relationship. However, this method is not suitable when formation shape is time-varying or frequently changed, which will be taken into consideration in this paper. Therefore, we define a new virtual structure which apparently a flexible formation and permits some formation transformations. For simplicity, we only consider this problem on a plane. The extension to high dimensional space is easy in concepts but tedious in procedures, which will not be discussed here.

To describe variable formation, we introduce pseudo-rigid transformation since rigid-body transformation cannot change the formation. A pseudo-rigid body was introduced in mechanism to describe the deformation and motion of a moving body [10]. To be specific, we denote the position of a point in the body as $\xi(t) = [x(t), y(t)]^T$, and position of the reference center is $\xi^C(t) = [x_0(t), y_0(t)]^T$. Then we consider the transformation for pseudo-rigid bodies (referring to see [11]):

$$\xi(t) - \xi^C(t) = F(t)(\xi(t_0) - \xi^C(t_0)),$$

or equivalently,

$$\xi(t) = \xi^C(t) + F(t)(\xi(t_0) - \xi^C(t_0)) = \xi^C(t) + F(t)\xi^H,$$

Here F is the pseudo-rigid transformation matrix and $\xi^H \triangleq [h_x, h_y]'$ is the relative vector from the reference center to this point at time t_0. For simplicity, F is also called a flexible formation matrix.

With the Cauchy-Stokes Deposition on F, we obtain

$$F = R \cdot U = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \alpha & \gamma \\ \gamma & \beta \end{bmatrix} \tag{2}$$

where $R \in SO(2)$ depending on ϕ represents the body rotation on a plane and U is a symmetric positive-definite matrix to characterize the stretching deformation. In fact, α, β describe the stretching of the formation in two orthogonal directions, and γ does the shearing change of this formation.

The change in formation via $F(t)$ is called pseudo-rigid transformation. As we know, once the parameters $\alpha, \beta, \gamma, \phi$ are given, F is fixed.

Remark 1: In [12], the authors considered the parameters $c_x, c_y, \alpha, s_x, s_y$ corresponding to our parameters $h_x, h_y, \phi, \alpha, \beta$, respectively. Clearly, the transformation discussed in [12] is a special case of pseudo-rigid transformation discussed here by taking the shearing value $\gamma = 0$. However, it is worth to mention that if we only consider two points, and take one of them as the reference center in the above discussion, then we can get a transformation matrix with $\gamma = 0$ in this simple case.

With the transformation matrix $F(t)$, we can actually describe the flexible formation change in the way of pseudo-rigid body. In the following sections, we apply this concept and property to the formation problem of multi-robot system, which is apparently very important and useful in some complicated circumstance for multi-robot coordination.

III. Flexible hierarchical formation

In the last section, we introduced a transformation matrix $F(t)$ to describe the time-varying flexible formation. In fact, based on the transformation matrix, we can further define flexible hierarchical formation. In other words, we discuss the formation changes in several levels, where the change at each level is assigned with a pseudo-rigid transformation matrix. Therefore, all the control design will be in the same form though the parameters related to the pseudo-rigid transformation may be different.

In the hierarchical formation, robots have different priorities and play different roles. There are the leader, the subleaders; and followers, for example. To be clear, we give detailed analysis in the following subsections.

A. Hierarchical Topology

Here we depict the hierarchical communicative topology using interconnection graphs.

First, we introduce a simple definition [17]: For graphs $G_1, G_2, \cdots, G_n$, we call graph G a n-level hierarchical product graph if G is constructed through replacing all nodes of G_{i-1} with the identical node of G_i, and attaching G_i to this modified G_{i-1} $(1 < i \leq n)$. We denote it as $G = G_1 \otimes \cdots \otimes G_{n-1} \otimes G_n$.

In real multi-robot system we often employ one-way communication for simplicity and resource limitation. Here we use directed graphs to describe its communication topology of multi-robot system. In order to assure the connectivity in communication, we give a modified definition to focus our attention to a class of graphs with a directed spanning tree. To be strict, for directed graphs $G_1, G_2, \cdots, G_n$ with spanning trees, we call graph $G = G_1 \otimes \cdots \otimes G_{n-1} \otimes G_n$ a n-level directed hierarchical product graph if G is constructed through replacing all nodes of G_{i-1} except its root with the root of G_i, and attaching G_i to this modified G_{i-1} $(1 < i \leq n)$.

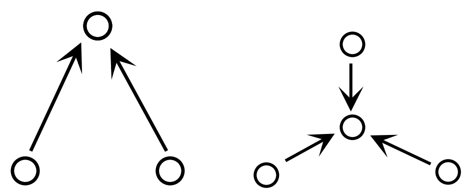
The root of the whole graph is the leader of the robot teams, and nodes connected with the leader are the virtual subleaders to be tracked.

For simplicity, we consider a 2-level hierarchical graph $S = S_1 \otimes S_2$, where S_1, S_2 are both star-like graph with its root in the center (see Figure. 1, where S_1 and S_2 are both star-like with respective 3 and 4 nodes).

Two hierarchical product graphs of these two graphs are exhibited in Figure. 2. Note that this hierarchical product is not commutative, similar to Kronecker product.

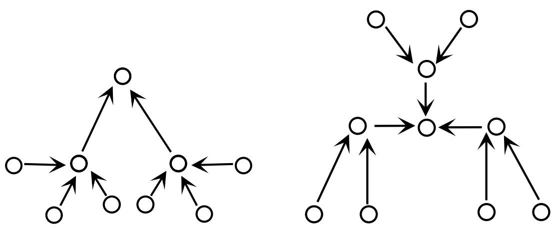
These results can be easily expanded into more levels hierarchical formation. Actually, it is equal to considering general directed graphs in the sense of information flow.

Still consider a 2-level case. Assume S_1 with $m+1$ nodes and S_2 with $n+1$. Then $S = S_1 \otimes S_2$ has $m(n+1)$ nodes.



(a) S_1 (b) S_2

Fig. 1. Graph S_1 and S_2.



(a) $S = S_1 \otimes S_2$ (b) $S = S_2 \otimes S_1$

Fig. 2. The two hierarchical product graph of S_1 and S_2.

We use two dimensional labels (i, j) for all robots in the following ways: if a robot is in level 1, then $i = 0$, j is the number label for this robot in level 1 and $j = 0$ if it is the root of S_1; if the robot is in level 2, then i is its root's second label, j is its label in graph S_2.

B. Hierarchy in Formation

Based on the hierarchical topology, we define a hierarchical virtual structure for formation change and related control of the multi-robot system. In our proposed hierarchical formation, the robots (or nodes) at the same level follow the same command determined by the flexible formation matrix.

From the flexible formation matrix, the desired position of a robot can be obtained based on the information of its leader (its root in the graph, or reference center in the body).

In fact, with its initial relative position vector to the reference center in a pseudo-rigid body, we can obtain its position $\xi(t)$ via pseudo-rigid transformation matrix $F(t)$.

As we discussed, the root of the interconnection graph is the leader for a multi-robot system. In the first level, all the nodes are directly connected to the leader. Based on the flexible formation matrix assigned for the first level, we can define the formation change of the first level. In this case, the reference center of the first level is put at the position of the leader. Then we find the desired positions (as virtual leaders) of the first-level nodes based on the formation matrix. For example, for a group of first-level robots labeled as $(0, k)$ for $k = 1, 2, \cdots, l$, the node (say node O) with label (0,0) as the leader provides the reference center. The relative position vectors with respect to O can determine the desired formation positions of the robots. To be strict, we denote the desired relative position vector for robot k as ξ_k^H based on $\xi_k^H = F_1(t)\xi_{k0}^H$, with the transformation matrix F_1 for level-one time-varying formation matrix. Therefore, the reference trajectory of the desired virtual leader for robot $(0, k)$ is given via pseudo-rigid transformation matrix as

$$\xi_k(t) = \xi^C(t) + \xi_k^H = \xi^C(t) + F_1(t)\xi_{k0}^H,$$

where $\xi_k(t)$ is desired position for robot $(0, k)$ (viewed as its virtual leader position), $\xi^C(t)$ is the position of the leader O, ξ_{k0}^H are initial or normalized relative position vectors for the initial virtual structure of the formation.

Remark 2: If we let $\xi_k^H = R(\cos(\theta_k), \sin(\theta_k))', (k = 1, \cdots, n)$, where $\theta_k = \frac{2k\pi}{n}$, with proper transformation matrix selected, then the circular formation in [12] and its deformed formation enduring affine transformations can be perfectly represented, too.

Then we can extend the above idea to other levels in the hierarchy in formation. To illustrate this problem explicitly, we still take 2 level ones as an example:

- The root of the interconnection defines the leader of the whole group of robots; then nodes connected to the leader form the first-level node set. Each first-level node has the nodes to be connected to it, which forms its second-level nodes. In this way, the whole group of robots are divided into several subgroups: the root of the graph is taken by the leader; the first-level nodes become the subleaders of the second-level of nodes in the graph.
- To achieve the hierarchical formation, once the leader position is fixed, the virtual leaders for first-level nodes are obtained with the given first-level transformation matrix $F_1(t)$. Then the formation control is given to allow the first-level node to follow its corresponding virtual leader.
- The nodes at the same level follow the same formation command determined by the transformation matrix and the jth-level formation transformation matrix is denoted as $F_j(t)$ for convenience. Note that each first-level node has its own subgroup, and it is the root of its subgraph, which is regarded the leader of the subgroup (or subleader of the whole group). These subleaders form a new virtual structure with their own as the reference center. Namely, their positions become references for the second-level nodes to calculate their desired positions (or the corresponding virtual-leader positions). Similarly, the second-level nodes will follow their virtual leaders.

Obviously, if all $F_i(i \neq 0)$ are identical, then the formation keep time-invariant.

By the proposed hierarchical formation formulation, the formation changes are decoupled in different levels, which reduces the information of formation changes and facilitates the control calculation in the design. In the hierarchical

Multi-robot control scheme	
Step 1:	Virtual leaders map $\mathcal{T}(\cdot)$ based on transformation matrix F at the given level.
Step 2:	Control and coordination for robots in different level with the control generated by $\mathcal{C}(\cdot)$
Step 3:	Feedback from the environment, and return to Step 1 for regenerating a trajectory.

TABLE I

OUTLINE OF OUR CONTROL SCHEME

formation problems, the formation of subgroups of robots takes a uniform shape, which may reduce the computational complexity, and we need not change the whole formation by decoupling the formation changes, which may save the energy or cost. In fact, the conventional virtual structure method requires to change all the parameters of the formation robots, which is much complicated and time-consuming. In a contrast, in this hierarchical formation, only a change of the parameters in the corresponding level's transformation matrix is also quite effective to drive the whole robot team to form a new shape.

IV. HIERARCHICAL FORMATION CONTROL

In this section, we are ready for the formation control design to achieve the hierarchical formation described in the preceding sections. Note that all the formation matrices are time-varying and pseudo-rigid transformation matrices.

To clarify the formation control design, we give an outline of our control procedure as shown in Table.I.

Details will be specified in the following subsections.

A. Reference Calculation

As we know, motion planning for the formation should be given first based on the environment information or coordination tasks, and there are few works for the flexible formation planning (see [11]). Here we skip this part and assume that the desired formation has been planned and the formation command has been sent to the robots with the transformation matrix F (or equivalent information of parameters $\phi, \alpha, \beta, \gamma$) at the given level.

For simplicity, our discussion is only for the first level, which can be easily extended to other levels. In this case, we will determine the virtual leader map $\mathcal{T}(\cdot)$.

Without of loss of generality, we assume the state of the leader of this level is described by $x_0, y_0, \theta_0, v_0, \omega_0$ and the formation command is expressed as the pseudo-rigid transformation matrix F or $\phi, \alpha, \beta, \gamma$. Then we will determine the state of the virtual leader for the formation of each robot, which is a mapping $\mathcal{T}(\cdot)$:

$$(x, y, \theta, v, \omega) = \mathcal{T}(F, x_0, y_0, \theta_0, v_0, \omega_0)$$

defined by

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + F(\phi(t), \alpha(t), \beta(t), \gamma(t)) \begin{bmatrix} h_x \\ h_y \end{bmatrix} \\ \theta &= \arctan(\dot{y}/\dot{x}) \\ v &= \sqrt{\dot{x}^2 + \dot{y}^2} \\ \omega &= \frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^2 + \dot{y}^2} \end{aligned} \tag{3}$$

where $(h_x, h_y)'$ is the initial offset of the considered robot.

If we send the position of robot $(0, 0)$ and F_1, we can generate the virtual leader for robot $(0, i)$, denoted as

$$(x, y, \theta, v, \omega)_{(0,i)} = \mathcal{T}(F_1, (x, y, \theta, v, \omega)_{(0,0)}).$$

Then we take the first-level robot, robot $(0, i)$ as the reference center of its subgroup. With the help of matrix F_2, the virtual leaders or the corresponding reference trajectories can be obtained for second level robots in the group of robot $(0, i)$.

$$\begin{aligned} (x, y, \theta, v, \omega)_{(i,j)} &= \mathcal{T}(F_2, (x, y, \theta, v, \omega)_{(0,i)}) \\ &= \mathcal{T}(F_2, \mathcal{T}(F_1, (x, y, \theta, v, \omega)_{(0,0)})) \end{aligned}$$

Repeating the procedure, we will get all the virtual leaders for the corresponding robots at different levels, where the virtual leader (i, j) is generated as the reference trajectory for robot (i, j). In this way, formation tracking is decoupled into several one-to-one tracking subproblems, which we will investigate in the following subsection.

B. Tracking a virtual leader

With the hierarchical decoupling and virtual-leader assignment, the formation control can be designed from the viewpoint of leader following or formation tracking. In other words, the feedback control is designed to guide all robots in formation to follow the reference trajectories of their virtual leaders.

Denote the reference trajectory as $(\bar{x}, \bar{y}, \bar{\theta})$, with its velocities are $\bar{v}$ and $\bar{\omega}$. Following [3], the tracking error in posture with respect to a local coordinate frame is given by

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x} - x \\ \bar{y} - y \\ \bar{\theta} - \theta \end{bmatrix} \tag{4}$$

then the error dynamics are

$$\begin{aligned} \dot{x}_e &= \omega y_e - v + \bar{v}\cos\theta_e \\ \dot{y}_e &= -\omega x_e + \bar{v}\sin\theta_e \\ \dot{\theta}_e &= \bar{\omega} - \omega \end{aligned} \tag{5}$$

Our goal is to propose appropriate feedback control laws

$$\begin{aligned} v &= v(x_e, y_e, \theta_e, \bar{v}, \bar{\omega}) \\ \omega &= \omega(x_e, y_e, \theta_e, \bar{v}, \bar{\omega}) \end{aligned} \tag{6}$$

such that, for initial errors $(x_e(0), y_e(0), \theta_e(0))$, the closed-loop trajectories of (5) and (6) are uniformly bounded and converge to zero.

To make the formation error converge into zero, we can take v as a control input to cancel the nonlinear part $\bar{v}\cos\theta$ to stabilize x_e. Noticing that y_e cannot be directly controlled, we seek to select appropriate ω to alleviate the effect of $y_e\omega$ and $x_e\omega$.

First, we stabilize x_e, and give

$$v = K_x x_e + \bar{v}\cos\theta_e \triangleq \bar{u}_v, \qquad (7)$$

the close-loop dynamics of x_e is rewritten as

$$\dot{x}_e = -K_x x_e + \omega y_e. \qquad (8)$$

From (8), we can conclude that, if y_e is zero, then x_e can exponentially converge to zero. To determine the control input ω, we give a Lyaponuv function as

$$V(t) = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1-\cos\theta_e}{K_y} \qquad (9)$$

with K_y as a positive constant.

The time derivative of (9) along the solution of (5) and (6) is then

$$\begin{aligned}\dot{V}(t) &= x_e\dot{x}_e + y_e\dot{y}_e + \frac{1}{K_y}\sin\theta_e\dot{\theta}_e \\ &= -K_x x_e^2 + \bar{v}y_e\sin\theta_e + \frac{1}{K_y}\sin\theta_e(\bar{\omega}-\omega) \\ &= -K_x x_e^2 + \sin\theta_e[\bar{v}y_e + \frac{1}{K_y}(\bar{\omega}-\omega)]\end{aligned}$$

To make the derivative of the Lyapunov function V negative, we choose

$$\omega = \bar{\omega} + K_y\bar{v}y_e + K_\theta K_y\sin\theta_e \triangleq \bar{u}_\omega \qquad (10)$$

where K_θ is a selected positive constant. After some manipulation, we obtain

$$\dot{V}(t) = -K_x x_e^2 - K_e\sin^2\theta_e \leq 0 \qquad (11)$$

We give our first result for formation tracking problem:

Theorem 1: Under the control inputs $v = \bar{u}_v$ and $\omega = \bar{u}_w$ given in (7) and (10), the equilibrium $e = (x_e, y_e, \theta_e)'$ of system (5) is stable. Furthermore, if $\bar{v}, \bar{\omega}$ are bounded and uniformly continuous in t on $[t_0, +\infty)$, the tracking error asymptotically converges to zero.

Proof With v and ω given in (7) and (10), $V(t)$ is positive definite and $\dot{V}(t)$ is negative semi-definite around the origin. Therefore, $V(t)$ is a Lyaponuv function, which indicates the local stability.

Clearly, from (9), $V(t) = 0$ if and only if $X_e \triangleq (x_e, \theta_e)' = 0$ and $y_e = 0$. Since (5) is local Lipschitz in X_e and uniformly in time t and $\bar{v}, \bar{\omega}$ are bounded and uniformly continuous in t on $[t_0, +\infty)$, the state vector $(X_e, y_e)'$ is bounded and consequently its derivative is bounded. Therefore, $||V(t)|| \leq +\infty$ which show that $V(t)$ is uniformly continuous.

According to Barbalat Lemma, we conclude that $\lim\limits_{t\to\infty}\dot{V} = 0$, that is $-K_x x_e^2 - K_e\sin^2\theta_e = 0$, and this implies that $x_e = 0$ and $\theta_e = 0$, then it is straight forward to obtain $\lim\limits_{t\to\infty} y_e = 0$. This completes the proof. ∎

Here it is not hard to obtain a feedback control in the form of $\mathcal{C}(\cdot)$ as

$$(v, \omega) = \mathcal{C}(\bar{x}, \bar{y}, \bar{\theta}, \bar{v}, \bar{\omega})$$

is defined by:

$$\begin{aligned} v &= K_x x_e + \bar{v}\cos\theta_e \qquad (12) \\ \omega &= \bar{\omega} + K_y\bar{v}y_e + K_\theta K_y\sin\theta_e \qquad (13)\end{aligned}$$

Remark 3: When $\bar{v} > 0$, the local stability is solved in [3], while in Theorem 1, we obtain the local stability even under the condition when $\bar{v} = 0$, which is not mentioned in [3].

We usually assume that the formation is achieved initially in the study of formation change, and the matrix F_i changes very slowly. Therefore, the tracking error is not very large. The following result demonstrates the exponential stability when the tracking error is not large.

Theorem 2: Under the conditions given in Theorem 1, if $\dot{\bar{v}}$ and $\dot{\bar{\omega}}$ are continuous and bounded, at least one of $\bar{v}$ and $\bar{\omega}$ does not converge to zero as t tends to ∞, then the zero solution is locally exponentially stable.

Proof It is easy to see that the local exponential stability cannot be obtained from the former Lyapunov function given in 1. To solve the problem, we first linearize the tracking nonlinear systems around the origin as follows:

$$\dot{e} = Ae + f_{h.o.t}(\cdot) \qquad (14)$$

where $e = (x_e, y_e, \theta_e)'$, $f_{h.o.t}$ is the high order terms, and

$$A = \begin{bmatrix} -K_x & \bar{\omega} & 0 \\ -\bar{\omega} & 0 & \bar{v} \\ 0 & -K_y\bar{v} & -K_\theta K_y \end{bmatrix}$$

It is easy to verify the terms (denoted as $f(\cdot)$) on the right hand side of (5) with the control inputs are continuous differentiable and the Jacobian matrix $[\partial f/\partial e]$ is bounded and satisfy the Lipschitz condition, uniformly in t.

Combined with the characterization equation of A which is

$$a_3s^3 + a_2s^2 + a_1s + a_0 = 0$$

where

$$\begin{cases} a_0 = 1 \\ a_1 = K_\theta K_y + K_x \\ a_2 = \bar{\omega}^2 + K_y\bar{v}^2 + K_xK_yK_\theta \\ a_3 = K_xK_y\bar{v}^2 + K_\theta K_y\bar{\omega}^2 \end{cases}$$

Since all coefficients are positive and $a_1a_2 - a_0a_3 > 0$, the real parts of all roots are negative by Routh-Hurtwiz Criterion. Thus, our conclusion are obtained. ∎

C. Hierarchical Formation Stability

With the map $\mathcal{T}$ and $\mathcal{C}$ determined, we return to the control of the hierarchical formation of the whole system. For the 2-level flexible hierarchical formation defined above, if the root of whole structure, i.e. the root of graph in level 1, robot (0,1) can get the exact poses of a reference target, and there are no disturbances in communication, that is, robot in level 2 can get all exact information including poses and velocities of its leader in level 1. In fact, we can obtain the following formation tracking result.

Theorem 3: With control inputs for each robot as (7) and (10), the formation is stable. Furthermore, if $\bar{v}$, $\bar{\omega}$, and the transformation matrix are all bounded and uniformly continuous, then the formation error asymptotically converges to zero.

Proof As we discussed, the formation tracking problem can be decoupled into $m(n+1)$ trajectory following problems, then sum up all control inputs for each robot, we obtain the control inputs for this problem, if we take a Lyapunov function $V(t) = \sum_{i,j} V_{(i,j)}(t)$, the proof of the stability and convergence is analogous to one-to-one tracking case, and omitted here. ∎

These results were obtained by directly investigating the nonholomonic robot's kinematics, unlike linearization method used in many papers such as [14], where the kinematics of robots was somehow converted into double-integrator.

Example 1: We consider the graph $S_1 \otimes S_2$ and the transformation matrices are

$$F_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad F_2 = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}$$

Then the controller will be:

(i) for robots in level 1,

$$\begin{aligned} v_{(0,i)} &= K_{x_{(0,i)}}[\cos\theta_{(0,i)}(x_{(0,0)} + h_{x_{0,i}} - x_{(0,i)}) \\ &+ \sin\theta_{(0,i)}(y_{(0,0)} + 2h_{y_{0,i}} - y_{(0,i)})] \\ &+ v_{(0,0)}\cos(\theta_{(0,0)} - \theta_{(0,i)}), \\ \omega_{(0,i)} &= \omega_{(0,0)} + K_{\theta_{(0,i)}} K_{y_{(0,i)}} \sin(\theta_{(0,0)} - \theta_{(0,i)}) \\ &+ K_{y_{(0,i)}} v_{(0,0)}[\cos\theta_{(0,i)}(y_{(0,0)} + 2h_{y_{0,i}} - y_{(0,i)}) \\ &- \sin\theta_{(0,i)}(x_{(0,0)} + h_{x_{0,i}} - x_{(0,i)})] \end{aligned}$$

(ii) for robots in level 2,

$$\begin{aligned} v_{(i,j)} &= K_{x_{(i,j)}}[\cos\theta_{(i,j)}(x_{(0,i)} + \sqrt{3}h_{x_{(i,j)}}/2 + h_{y_{i,j}}/2 \\ &- x_{(i,j)}) + \sin\theta_{(i,j)}(y_{(0,i)} - h_{x_{i,j}})/2 + \sqrt{3}h_{y_{i,j}}/2 \\ &- y_{(i,j)})] + v_{(0,0)}\cos(\theta_{(0,0)} - \theta_{(i,j)}), \\ \omega_{(i,j)} &= \omega_{(0,0)} + K_{\theta_{(i,j)}} K_{y_{(i,j)}} \sin(\theta_{(0,0)} - \theta_{(i,j)}) \\ &+ K_{y_{(i,j)}} v_{(0,0)}[\cos\theta_{(i,j)}(y_{(0,i)} - h_{x_{i,j}}/2 \\ &+ \sqrt{3}h_{y_{i,j}}/2 - y_{(i,j)}) - \sin\theta_{(i,j)}(x_{(0,i)} + \sqrt{3}h_{x_{i,j}}/2 \\ &+ \frac{1}{2}h_{y_{i,j}} - x_{(i,j)})] \end{aligned}$$

where $(x_{(0,0)}, y_{(0,0)}, \theta_{(0,0)}, v_{(0,0)}, \omega_{(0,0)})$ is the formation's reference center and $h_{x_{i,j}}, h_{y_{i,j}}$ are parameters in initial formations with $K_{(\cdot)}$ as positive gains from Theorem 1.

V. Conclusions

In this paper, a formation structure of a team of nonholomonic robots was given for the effective formation changes, and its tracking problem was investigated. We first introduce a special kind of matrix called pseudo-rigid transformation matrix. With these matrices and hierarchical product graph defined later, flexible hierarchical formation was constructed attached with some illustrated examples. To control this formation to track a reference trajectory, a design procedure was given, along with related stability analysis.

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