

Swarm Tracking Using Artificial Potentials and Sliding Mode Control

Jingyi Yao, Raúl Ordóñez, and Veysel Gazi

Abstract—In this paper, we present a stable and decentralized control strategy for multi-agent systems (swarms) to capture a moving target in a specific formation. The coordination framework uses artificial potentials to take care of both tracking and formation tasks. First, a basic controller is designed based on a kinematic model. After that, sliding mode control technique is used to force the agents with general vehicle dynamics to obey the required motion. Finally, specific potential functions are discussed and corresponding simulation results are given.

I. INTRODUCTION

In recent years it has become popular to study the biological world and apply the derived principles to the design of engineering. The topic of distributed coordination and control of multiple autonomous agents has gained lots of attention [1], [2], [3]. Cooperative agents can often be used to perform tasks that are too difficult for a single one to perform. Instead of the traditional trajectory tracking problem, people began to study coordinated tracking [4], [5]. Tasks can be performed more efficiently by controlling the group to move in formation. Possible applications could range from autonomous robot assembly to UAVs scout and counterwork. Compared to individuals, swarms, flocks, and schools can have remarkable group-level characteristics, which may allow them to perform complicated task efficiently.

The coordinated tracking problem is to find a coordinated control scheme for a group of agents to make them achieve and maintain some given geometrical formation, at the same time, the agents viewed as a group have to track a target or a trajectory (e.g. which may be required in order to execute a given task). Thus, there is a trade-off between maintaining formation and arriving at the final goal. Possible approaches for formation control include leader-following and the virtual structure approaches [6], [7]. In both approaches, one agent, which could be real or virtual, is designated as leader to perform the tracking task without considering the followers, and the remaining agents only have to stay from the leader within a desired offset, without considering the target. However, these approaches are centralized and therefore not

robust - if the leader fails, the task would also fail. In that case, decentralized formation control is preferred as discussed in [8], where feedback control laws are used to keep the formation during tracking process.

In this article, we consider a control strategy based on artificial potential functions and sliding mode control. Artificial potential functions have been first widely used for robot navigation and control including multi-agent coordination [6], [9], [10], [11]. The potential function is created to encode the interaction rule for the group. In order to perform the task in a decentralized way, in this paper we consider a potential function composed of two parts. The inter-connection part makes the agent constrained by its neighbor to maintain a group structure. This part includes functions of the relative distance between each pair of neighbors. In addition, pursuer-target potential function is introduced to direct the group behavior, which is to catch up with the target. The specific form of potential function is defined according to the desired geometric formation. The choice of a potential function is important because different potentials might result in different performance even with the same control algorithm. In particular, existence of multiple local minima in the potential function leads to guaranteeing only local convergence to the desired formation. Nevertheless, we show that by appropriate choice of the potential function one can always guarantee that eventually the target will be surrounded or “enclosed” by the tracking agents.

The sliding mode control technique has the important properties of suppressing disturbances and model uncertainties. It becomes attractive for two main advantages: (i) the dynamic behavior of the system may be tailored by the particular choice of switching function, and (ii) the closed-loop response becomes totally insensitive to a particular class of uncertainties. This idea has already been successfully used to implement engineering aggregating swarm [12] and tracking moving targets [13]. In this article we extend the work to the case of multiple agents tracking or capturing/enclosing a moving target (possibly) in a formation.

The paper is organized as follows: In Section 2, we define a general potential function and develop the control algorithm for the “kinematic” model. In Section 3, we consider a general fully actuated dynamic model of the agents and derive a new controller based on the sliding mode control method. In Section 4, specific potential functions are applied to the problem and results are presented and discussed. Finally, conclusions are made in Section 5.

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II. BASIC “KINEMATIC MODEL”

Consider a multi-agent system (i.e., a swarm) consisting of N individuals in an n -dimensional Euclidean space. To begin with, we model the individuals as points and ignore their dimensions. Moreover, we assume synchronous motion and no time delays. Let $x_i, x_t \in \mathbb{R}^n$ denote the position vector of individual i and the target respectively. Also, temporarily assume that the motion dynamics for the agents are given by

$$\dot{x}_i = u_i \quad (1)$$

which we call the “kinematic model.” Later we will show how the derivations in this section can be extended to the case in which the agents have fully actuated dynamics. Our objective is to make the entire group aggregate around the target and move together with it possibly in a specific formation regardless of the target’s movement. This is a simple case of the coordinated tracking problem.

We can design a controller in a simpler form if \dot{x}_t is available [13]. However, assuming that \dot{x}_t is known is a strong assumption since the current velocity of the target is unknown in most circumstances. It is more realistic to assume that $\|\dot{x}_t\| \leq \gamma_t$ for some known $\gamma_t > 0$ since any realistic agent has a bounded velocity. We also assume that the relative position of the target x_t is known and each agent knows the exact relative positions of all the other individuals.

With the assumptions above, we choose the control laws such that each agent is moving based on the equation

$$\dot{x}_i = u_i = -\alpha \nabla_{x_i} J(x, x_t) - \beta \text{sign}(\nabla_{x_i} J(x, x_t)) \quad (2)$$

for all $i = 1, \dots, N$ where $\alpha > 0$ and $\beta \geq \gamma_t$ are positive constants, $\text{sign}(\cdot)$ is the signum function operated elementwise for a vector $y \in \mathbb{R}^n$, and $J : \mathbb{R}^{n \times N} \times \mathbb{R}^n \rightarrow \mathbb{R}$ is a potential function (to be defined below). It is specified by the multi-agent system designer based on the desired structure and/or behavior of the swarm. Such potential functions are being used for swarm aggregations, formation control, and multi-agent coordination and so on [12], [14]. Note also that in the above equation we implicitly defined $x^\top = [x_1^\top, \dots, x_N^\top] \in \mathbb{R}^{n \times N}$.

In order to satisfy both the tracking and formation control specifications we consider potential functions $J(x, x_t)$ which are composed of two parts - the inter-agent interactions (or formation control) part and the agent-target interaction (or tracking) part. In particular, we consider potential functions of the form

$$J(x, x_t) = K_T \sum_{i=1}^N J_{it}(\|x_i - x_t\|) + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N J_{ij}(\|x_i - x_j\|) \quad (3)$$

where $J_{it}(\|x_i - x_t\|)$ is the potential between agent and target, while $J_{ij}(\|x_i - x_j\|)$ is the potential between agents in the group. With such a form, each agent takes care of the tracking task by itself and keeps certain distances between itself and its neighbors. For the tracking part, it is required that $J_{it}(\|x_i - x_t\|)$ has a unique minimum at a particular distance from the target so that the individuals catch up with the target and encircle/enclose it. As for the formation part,

$J_{ij}(\|x_i - x_j\|)$ is required also to have a unique minimum at the desired distance between the agent based on the formation expected to be achieved. Note that $J_{ij}(\|x_i - x_j\|)$ can be different for different pairs. (The same could be the case for $J_{it}(\|x_i - x_t\|)$ as well although we used the same $J_{it}(\|x_i - x_t\|)$ for all i in this paper.) The coefficients K_T and K_F weigh the relative importance of tracking versus formation keeping.

Assume $J_{it}(\|x_i - x_t\|)$ satisfies:

(A) There exist corresponding function $h^{it} : \mathbb{R}^+ \rightarrow \mathbb{R}$ such that

$$\nabla_y J_{it}(\|y\|) = y h^{it}(\|y\|) \quad (4)$$

(B) There exist unique distances δ_{it} at which we have $h^{it}(\|y\|) = 0$.

Assume $J_{ij}(\|x_i - x_j\|)$ satisfies:

(A) The potentials $J_{ij}(\|x_i - x_j\|)$ are symmetric and satisfy

$$\nabla_{x_i} J_{ij}(\|x_i - x_j\|) = -\nabla_{x_j} J_{ij}(\|x_i - x_j\|) \quad (5)$$

(B) There exist corresponding function $g^{ij} : \mathbb{R}^+ \rightarrow \mathbb{R}$ such that

$$\nabla_y J_{ij}(\|y\|) = y g^{ij}(\|y\|) \quad (6)$$

(C) There exist unique distances δ_{ij} at which we have $g^{ij}(\|y\|) = 0$.

Then,

$$\begin{aligned} \nabla_{x_i} J(x, x_t) &= K_T (x_i - x_t) h^{it}(\|x_i - x_t\|) \\ &\quad + K_F \sum_{j=1, j \neq i}^N (x_i - x_j) g^{ij}(\|x_i - x_j\|) \quad (7) \\ \nabla_{x_t} J(x, x_t) &= -K_T \sum_{i=1}^N (x_i - x_t) h^{it}(\|x_i - x_t\|) \quad (8) \end{aligned}$$

By observing the equalities in (7) and (8), the equality in (8) can be rewritten as

$$\begin{aligned} \nabla_{x_t} J(x, x_t) &= -\sum_{i=1}^N \nabla_{x_i} J(x, x_t) \\ &\quad + K_F \sum_{i=1}^N \sum_{j=1, j \neq i}^N (x_i - x_j) g^{ij}(\|x_i - x_j\|) \quad (9) \end{aligned}$$

Moreover, since we have

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N (x_i - x_j) g^{ij}(\|x_i - x_j\|) = 0 \quad (10)$$

which follows from (5), we obtain

$$\nabla_{x_t} J(x, x_t) = -\sum_{i=1}^N \nabla_{x_i} J(x, x_t) \quad (11)$$

Definition (Potential function): $J : \mathbb{R}^{n \times N} \times \mathbb{R}^n \rightarrow \mathbb{R}$ is a differentiable, nonnegative, decrescent, radially unbounded function of the distance $\|x_i - x_t\|$ between agent i and target, and the distance $\|x_i - x_j\|$ between agent i and j , such that

- J has at least one isolated minimum.
- $\nabla_{x_t} J(x, x_t) = -\sum_{i=1}^N \nabla_{x_i} J(x, x_t)$.

The time derivative of function J is given by

$$\dot{J} = \sum_{i=1}^N [\nabla_{x_i} J(x, x_t)]^\top \dot{x}_i + [\nabla_{x_t} J(x, x_t)]^\top \dot{x}_t \quad (12)$$

Substituting the agent dynamics in (2) and the condition in (11) in the \dot{J} equation, one obtains

$$\begin{aligned} \dot{J} &= - \sum_{i=1}^N [\nabla_{x_i} J(x, x_t)]^\top [\alpha \nabla_{x_i} J(x, x_t) + \beta \text{sign}(\nabla_{x_i} J(x, x_t))] \\ &\quad - \sum_{i=1}^N [\nabla_{x_t} J(x, x_t)]^\top \dot{x}_t \\ &\leq -\alpha \sum_{i=1}^N \|\nabla_{x_i} J(x, x_t)\|^2 - \beta \sum_{i=1}^N \|\nabla_{x_i} J(x, x_t)\| + \gamma \sum_{i=1}^N \|\nabla_{x_t} J(x, x_t)\| \end{aligned} \quad (13)$$

Since we have $\beta \geq \gamma$, the time derivative of J is bounded by

$$\dot{J} \leq -\alpha \sum_{i=1}^N \|\nabla_{x_i} J(x, x_t)\|^2 \quad (14)$$

This equation implies that as time tends to infinity, we have $\dot{J} \rightarrow 0$, which indicates J converges to a constant value implying that the system converges to a constant configuration corresponding to a minimum of J . This is because from LaSalle Yoshizawa theorem, (14) implies that $\lim_{t \rightarrow \infty} \sum_{i=1}^N \|\nabla_{x_i} J(x, x_t)\|^2 = 0$, implying $\|\nabla_{x_i} J(x, x_t)\| \rightarrow 0$ for all i . Also from (11) we see that we also have $\|\nabla_{x_t} J(x, x_t)\| \rightarrow 0$. In other words, as $t \rightarrow \infty$ we have $(x, x_t) \rightarrow \Omega \subset \{(x, x_t) | \dot{J} = 0\}$ where

$$\Omega = \{(x, x_t) | \nabla_{x_i} J(x, x_t) = 0, \nabla_{x_t} J(x, x_t) = 0, i = 1 \dots N\}$$

If the potential function we choose has one unique minimum, we will achieve our goals. Unfortunately, most potential functions may have multi-minima. If the distance between any pair of objects including agents and target ($\|x_i - x_t\|$ and $\|x_i - x_j\|$) is the desired one (δ_{it} and δ_{ij}), the potential function J reaches its global minimum, which corresponds to our tracking and formation objectives. However, there are other feasible configurations different from the desired one, at which the potential function J achieves a local minimum. Therefore, in general, unless the initial configuration of the agents is “close enough” to the global minimum, it might be the case that J converges to such a local minimum resulting in a different configuration.

Then, from (8) we have

$$-K_T \sum_{i=1}^N (x_i - x_t) h^{it}(\|x_i - x_t\|) = 0$$

Rearranging this equation, we obtain

$$\sum_{i=1}^N x_i h^{it}(\|x_i - x_t\|) = x_t \sum_{i=1}^N h^{it}(\|x_i - x_t\|) \quad (15)$$

which is guaranteed to be achieved as $t \rightarrow \infty$. This is an important observation because it provides a relation of the position of the target to the position of the agents at equilibrium and allows the designer to appropriately choose the $h^{it}(\|x_i - x_t\|)$ (the tracking part of the potential function).

First of all note that at the desired formation, since $\|x_i - x_t\| = \delta_{it}$ for all i , we have $h^{it}(\|x_i - x_t\|) = 0$ for all i and (15) is satisfied. Second issue to note is that if $h^{it}(\|x_i - x_t\|)$ are chosen such that $\sum_{i=1}^N h^{it}(\|x_i - x_t\|) = 0$ is feasible (excluding the case at the desired formation), then the position of the target x_t cannot be specified (meaning that it could be anywhere in the state space). To avoid this situation one can choose $h^{it}(\|y\|) > 0$ for all y except $\|y\| = \delta_{it}$. Then, assuming that $\sum_{i=1}^N h^{it}(\|x_i - x_t\|) \neq 0$ we obtain

$$x_t = \frac{\sum_{i=1}^N x_i h^{it}(\|x_i - x_t\|)}{\sum_{i=1}^N h^{it}(\|x_i - x_t\|)}$$

Defining

$$\eta_i = \frac{h^{it}(\|x_i - x_t\|)}{\sum_{i=1}^N h^{it}(\|x_i - x_t\|)}, i = 1, \dots, N,$$

we obtain

$$x_t = \sum_{i=1}^N \eta_i x_i$$

With the choice of $h^{it}(\|y\|) \geq 0$ or for all y we see that $0 \leq \eta_i \leq 1$ for all i and $\sum_{i=1}^N \eta_i = 1$ implying that as $t \rightarrow \infty$ we will have $x_t \rightarrow \text{conv}\{x_1, x_2, \dots, x_N\}$, where $\text{conv}\{x_1, x_2, \dots, x_N\}$ is the convex hull of the positions of the agents. In other words, by choosing $h^{it}(\|y\|)$ as above one can guarantee that as $t \rightarrow \infty$ the agents will “surround” or “enclose” the target, which is an important result.

Despite the beauty of the above results there is one shortcoming, which is that the model in (1) does not represent the dynamics of realistic agents. As was discussed in [14] it is possible to argue that in some cases it can be viewed as an approximation of the point mass dynamics described by

$$\begin{aligned} \dot{x}_i &= v_i \\ m_i \dot{v}_i &= -k_v v_i + u_i \end{aligned} \quad (16)$$

where $-k_v v_i$ is a velocity damping term, and u_i is the control acting on agent i . This is possible for the cases in which m_i is very small ($m_i \approx 0$) and the viscosity of the environment is high (as is the case for some bacteria) which allows one to ignore m_i . Then setting $m_i = 0$ and taking $k_v = 1$ one can see that (16) reduces to (1).

Although here we are not concerned with the motion of bacteria and (1) does not correspond to the dynamics of realistic agents, the results derived are still of interest since they serve as proof of concept for the behavior considered. Although they do not specify how that desired behavior could be achieved in engineering applications with given agent dynamics, they can serve as guidelines for designing such applications. In the next section, we discuss a control algorithm based on sliding mode control theory which could be applied for agents with general fully actuated dynamics (such as omni-directional robots).

III. SLIDING MODE CONTROL FOR AGENTS WITH VEHICLE DYNAMICS

In this section, we consider all the agents in the system have the same dynamics, which could be described by the

equation

$$M(x_i)\ddot{x}_i + f_i(x_i, \dot{x}_i) = u_i \quad (17)$$

where $x_i \in \mathbb{R}^n$ is the position of the agent, $M(x_i) \in \mathbb{R}^{n \times n}$ is the mass or inertia matrix, and $u_i \in \mathbb{R}^n$ represents the control inputs. The function $f_i(x_i, \dot{x}_i) \in \mathbb{R}^n$ represents disturbances and other effects and we assume that

$$f_i(x_i, \dot{x}_i) = f_i^k(x_i, \dot{x}_i) + f_i^u(x_i, \dot{x}_i)$$

where $f_i^k(\cdot, \cdot)$ represents the known part and $f_i^u(\cdot, \cdot)$ represents the unknown part. For the unknown part, we assume that $\|f_i^u(x_i, \dot{x}_i)\| \leq \bar{f}_i$, where $\bar{f}_i < \infty$ is a known constant. Moreover, it is assumed that for all x_i the matrix $M(x_i)$ satisfies

$$\underline{M}\|y\|^2 \leq y^T M(x_i)y \leq \bar{M}\|y\|^2$$

where \underline{M} and \bar{M} are known and $y \in \mathbb{R}^n$ is arbitrary.

Given the dynamics above, we would like to choose the control input u_i to enforce the velocity of the agent to satisfy (2). Then, the discussion in the preceding section guarantees that the group catches up with the target, or at least encloses it, and forms the desired shape. Here, we will use the sliding mode control theory, following a procedure similar to that in [13].

Define the n -dimensional sliding manifold for agent i as

$$s_i = \dot{x}_i + \alpha \nabla_{x_i} J(x, x_t) + \beta \text{sign}(\nabla_{x_i} J(x, x_t)) \quad (18)$$

Now, let's design the control input u_i to enforce the occurrence of sliding mode. If $s_i^T \dot{s}_i < 0$, the sliding manifold is asymptotically reached. Differentiating the sliding manifold equation we obtain

$$\dot{s}_i = \ddot{x}_i + \frac{d}{dt}[\alpha \nabla_{x_i} J(x, x_t)] + \frac{d}{dt}[\beta \text{sign}(\nabla_{x_i} J(x, x_t))] \quad (19)$$

Let us assume for now that $\|\frac{d}{dt}[\beta \text{sign}(\nabla_{x_i} J(x, x_t))]\| \leq \bar{J}_s$ and $\|\frac{d}{dt}[\alpha \nabla_{x_i} J(x, x_t)]\| \leq \bar{J}$ where \bar{J}_s and \bar{J} are known positive constants. Since the potential function is to be chosen by the designer, he or she should make sure that $\|\frac{d}{dt}[\alpha \nabla_{x_i} J(x, x_t)]\| \leq \bar{J}$ is satisfied for some \bar{J} . In the next section we will consider potential functions which satisfy these bounds. Note also that $\|\frac{d}{dt}[\beta \text{sign}(\nabla_{x_i} J(x, x_t))]\|$ is unbounded at the instances at which $\text{sign}(\nabla_{x_i} J(x, x_t))$ changes sign. This problem will be solved by using a low pass filter and will be discussed below.

By choosing

$$u_i = -u_0 \text{sign}(s_i) + f_i^k(x_i, \dot{x}_i) \quad (20)$$

and

$$u_0 > \bar{M}(\frac{1}{\underline{M}}\bar{f}_i + \bar{J} + \bar{J}_s + \varepsilon) \quad (21)$$

we can guarantee that $s_i^T \dot{s}_i < -\varepsilon \|s_i\|$ for any $\varepsilon > 0$, which implies that the manifold is reached in finite time.

Once the sliding manifold is reached, the system remains on that manifold for all time. Then, the results discussed in the preceding section are recovered despite the model uncertainties in (17). This result is achieved thanks to the robustness properties of the sliding mode control method.

In order to derive the above result we assumed that $\|\frac{d}{dt}[\beta \text{sign}(\nabla_{x_i} J(x, x_t))]\| \leq \bar{J}_s$, which is not the case. This problem could be solved by passing the switching signal through a low pass filter [13]. Define

$$\mu \dot{z} = -z + \beta \text{sign}(\nabla_{x_i} J(x, x_t)) \quad (22)$$

where μ is a small positive constant. With proper choice of the parameter μ and using an approach similar to one of sliding mode observers based on the equivalent control method [15] we have

$$z \approx [\beta \text{sign}(\nabla_{x_i} J(x, x_t))]_{eq} \quad (23)$$

where we used the subscript *eq* to denote the *equivalent* (effective or average) value of the discontinuous signal. Therefore, although $\beta \text{sign}(\nabla_{x_i} J(x, x_t))$ is not differentiable, its approximation z is differentiable and can be used in the definition of the sliding manifold.

Moreover, we have

$$\left\| \frac{d}{dt} [\beta \text{sign}(\nabla_{x_i} J(x, x_t))]_{eq} \right\| = \|\dot{z}\| \leq \frac{2\beta}{\mu} \triangleq \bar{J}_s \quad (24)$$

Therefore, the sliding manifold can be redefined as

$$s_{i,new} = \dot{x}_i + \alpha \nabla_{x_i} J(x, x_t) + z. \quad (25)$$

Utilizing z in the sliding manifold makes the algorithm implementable. One issue to notice is that μ has to be chosen properly so that the low-pass filter is able to extract the actual "average" or equivalent value of its input.

IV. POTENTIAL FUNCTION CASES DISCUSSION

As mentioned above the algorithm may have different performance based on the chosen potential function $J(x, x_t)$. In this section, we illustrate how the choice of potential function influences the application by presenting several examples. We set $n = 2$ here, but the results should work for higher dimensions as well.

Consider the group consisting of agents with the same dynamics

$$M(x_i)\ddot{x}_i + f_i(x_i, \dot{x}_i) = u_i \quad (26)$$

and unity mass $M_i = 1$. Choose $\underline{M} = 0.5, \bar{M} = 1.5$ and let $f_i(x_i, \dot{x}_i) = \sin(0.2t)$ be the unknown uncertainty in the system.

Assume the target dynamics as

$$\begin{aligned} \dot{x}_{t1} &= 0.25 \\ \dot{x}_{t2} &= \sin(0.25t) \end{aligned}$$

We set the initial conditions of agents randomly within an area near the origin, and set the target initially at $[5, 5]$, which is located out of the agents.

Let $N = 3$, our task is to make the three agents form an equilateral triangle with the target in the mid-point. If we let $\delta_{ij} = \sqrt{3}$ as the distance between agents in the triangle, then the distance between the target and agent $\delta_{it} = 1$ can be deduced from geometry.

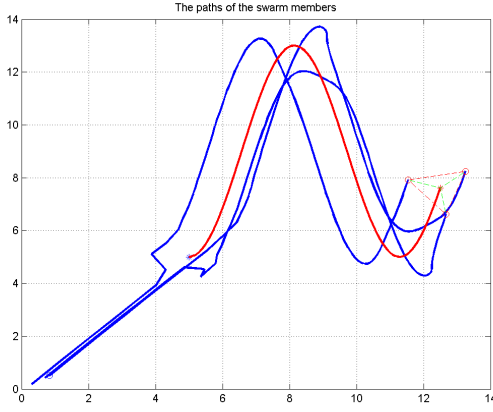


Fig. 1. The paths of the swarm members.

Case 1: For this case we chose the potential function as [16]

$$J(x, x_t) = K_T \sum_{i=1}^N \frac{1}{2} (\|x_i - x_t\|^2 - \delta_{it}^2)^2 + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left[\frac{1}{2} (\|x_i - x_j\|^2 - \delta_{ij}^2)^2 \right] \quad (27)$$

where δ_{it} and δ_{ij} as defined before. For this function, it can be shown that

$$\begin{aligned} \left\| \frac{d}{dt} [\alpha \nabla_{x_i} J(x, x_t)] \right\| &\leq K_T \alpha (K_T \alpha \|x_i(0) - x_t(0)\| + \\ &K_F \sum_{j=1, j \neq i}^N (a_{ij} - b_{ij}) \|x_i(0) - x_j(0)\|) + \\ &K_F K_T \alpha^2 \sum_{j=1, j \neq i}^N (\|x_i(0) - x_t(0)\| + \\ &\|x_j(0) - x_t(0)\|) + K_T \alpha \beta + K_T \alpha \gamma \\ &\triangleq \bar{J} \end{aligned} \quad (28)$$

It is obvious that $J(x, x_t) \geq 0$ is quadratic and has a unique global minimum at $J(x, x_t) = 0$ which occurs when $\|x_i - x_t\| = \delta_{it}$ for all i and $\|x_i - x_j\| = \delta_{ij}$ for all pairs (i, j) . Therefore, in light of the discussions in the preceding sections we would expect the formation control and the capture of the target to be exactly achieved (which is the case as seen below).

For the controller parameters, we use $\alpha = 0.01, \beta = 2.0, \bar{f}_i = 1$, and $\varepsilon = 1$. For the filter, we chose $\mu = 0.1$. With the proper choice of the agent initial conditions, we get \bar{J} and \bar{J}_s from (28) and (24), which produce the value of u_0 .

For the simulations below we let $\delta_{it} = 1$ as the desired distance between the target and agent, $\delta_{ij} = \sqrt{3}$ for all i and j as the desired distance between agents.

From Figure 1, we can see that after a short period, the group catches up with the target, and Figure 2 shows the target rightly stays at the mid-center of the triangle.

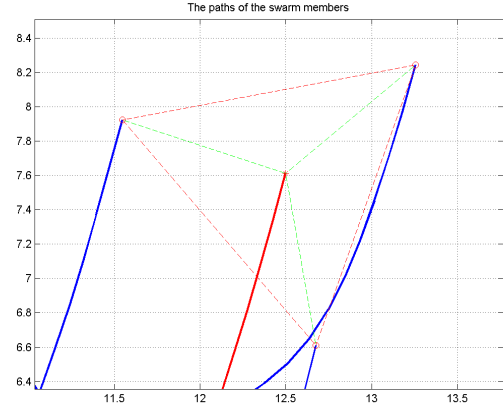


Fig. 2. Zoomed out final position of the swarm.

Case 2: For this case we chose the potential function as

$$J(x, x_t) = K_T \sum_{i=1}^N \frac{1}{2} (\|x_i - x_t\|^2 - \delta_{it}^2)^2 + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left[\frac{a_{ij}}{2} \|x_i - x_j\|^2 + \frac{b_{ij} c_{ij}}{2} \exp\left(-\frac{\|x_i - x_j\|^2}{c_{ij}}\right) \right] \quad (29)$$

where the parameters a_{ij}, b_{ij} and c_{ij} depend on the desired relative distances δ_{ij} of the individuals [12]. Here $\frac{a_{ij}}{2} \|x_i - x_j\|^2$ is the attraction part between agents and $\frac{b_{ij} c_{ij}}{2} \exp\left(-\frac{\|x_i - x_j\|^2}{c_{ij}}\right)$ is the repulsion part. The distance $\delta_{ij} = \sqrt{c_{ij} \ln(b_{ij}/a_{ij})}$ is the distance at which the attraction and repulsion balance [17], [18], which also defines the distance between the agents in the desired formation. We notice that J has a minimum at $J(x, x_t) = 0$ which occurs when $\|x_i - x_t\| = \delta_{it}$ for all i and $\|x_i - x_j\| = \delta_{ij}$ for all pairs (i, j) .

For the simulations we let $\delta_{it} = 1/3$ and $\delta_{ij} = \sqrt{3}/3$. We got the controller parameter with the same procedure as before. From Figures 3, we can see that after a short period, the group follows the target, but doesn't achieve the formation we want. Here, the system is locked at a local minimum, which corresponds to an undesired configuration.

Now, redefine the potential function as

$$J(x, x_t) = K_T \sum_{i=1}^N \frac{1}{3} [(\|x_i - x_t\|^2 - \delta_{it}^2)^3 + \delta_{it}^6] + K_F \sum_{i=1}^{N-1} \sum_{j=i+1}^N \left[\frac{a_{ij}}{2} \|x_i - x_j\|^2 + \frac{b_{ij} c_{ij}}{2} \exp\left(-\frac{\|x_i - x_j\|^2}{c_{ij}}\right) \right] \quad (30)$$

It satisfies the condition that $h^i(\|y\|) > 0$ for all y except $\|y\| = \delta_{it}$. Therefore, we would expect that the agents would at least enclose the target, regardless of initial conditions. From Figures 4, we can see that after a short period, the group encloses the target.

It is well known that, practically, potential functions may have many local minima. Therefore, usually when potential functions are used for coordinated tracking in a formation

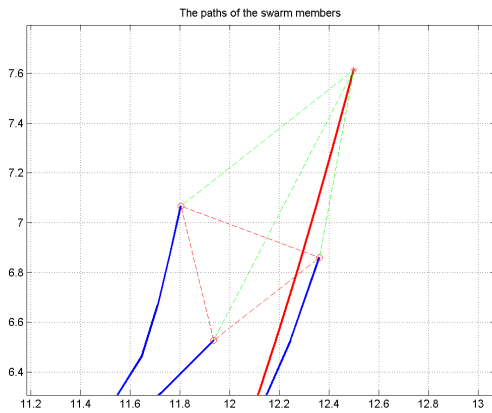


Fig. 3. Zoomed out final position of the swarm.

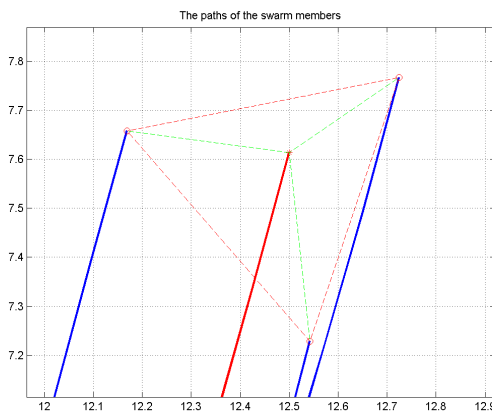


Fig. 4. Zoomed out final position of the swarm.

it is impossible to drive with global results, but only local results can be obtained. This problem has been observed in many other practical implementations [10]. In particular, for the formation control problem we are considering there may be several different families of minima. The first family is at the desired formation and the other families occur when the agents are at points at which the attraction and repulsion forces balance. The beauty of the sliding mode control method discussed here is that it is independent on the potential function chosen as long as the boundedness assumptions are satisfied. Moreover, the objectives are achieved despite the model uncertainties.

V. CONCLUSIONS

In this paper we extended the work of [13] to the case of multiple agents tracking a target in a formation. We presented a procedure to implement coordinated tracking problem. Here, the potential function for individual includes both the tracking term and the formation term, so that each agent will react if any given agent cannot keep up with the target. This method maintains a more robust formation than the leader-follower model. We obtained the stability proof

based on the potential function in a general form. Sliding mode control technique is used to apply the basic design result to the group with general vehicle agents. Results were obtained from two different potential functions. Future research may focus on formation reconfiguration (time varying or dynamic formations) as well as including disturbance and uncertainties as well as measurement errors.

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