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Formation Control of Multiple Nonholonomic Mobile Robots Based on Cascade Design

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***Abstract*— Formation control of nonholonomic mobile robots is considered in this paper and decentralized linear controllers based on cascade design are proposed for a group of nonholonomic mobile robots. By using the theory of cascaded systems and cooperative control of linear agents, multiple nonholonomic robots' formation control problem is converted into multi-linear time-varying systems' stabilization problem. Linear decentralized cooperative controllers are brought out for the formation control problem when the reference targets satisfy the condition of persistent excitation which has relaxed the requirements on reference trajectory than those given in previous papers.**

I. INTRODUCTION

The last few years have seen growing researches in control and coordination of multiple mobile robots [1]-[4]. This stems from the fact that the challenging features of multiple mobile robots and cooperative tasks such as rescue mission, large object moving,troop hunting, formation control and satellites clustering are too difficult or impossible for a single robot [5]. Many control strategies have been proposed for the cooperative control of multiple mobile robots, e.g., behavior-based control [6], virtual structure based control [7], leader-follower based control [8][9], artificial based control [10] and graph theory based control [11].

This paper mainly focused on the formation control of multiple nonholonomic mobile robots. By formation control, we mean the control of positions and orientations of a group of mobile robots such that they track desired locations relative to reference points which can be virtual agents or another agents in the team [12][13][14]. Different to many previous researches on cooperative control of linear agents, this paper has discussed the formation control of nonholonomic mobile robots which is more challenging because of mobile robots' nonlinear dynamics and nonholonomic constraints on it. Recently there are a few papers considering the formation control problem of nonholonomic mobile robots. [15] only studied the cooperative control of positions of each robot in the group. By introducing a complex variable transformation, [13][14] considered the formation control of nonholonomic chained-form systems with the help of results obtained from cooperative control of linear agents. [16] studied the formation control problem using methods of leader-follower and dynamic feedback linearization. [17] considered the same problem using the nonlinear method of Backstepping. With the tool of passivity, [18] considered the agreement problem on velocity and the group coordination problem which encompass some previous results. The cooperative control laws presented in previous research is much complex [13][14] or has some singularity [16] that is difficult to realize in application.

Based on our previous researches on the tracking control of nonholonomic mobile robot [19][20], we construct cooperative control laws for a group of nonholonomic mobile robots with the help of cascade design method comes from nonautonomous systems [21]. To our knowledge, there are no papers introducing the cascade design method into the cooperative control of multiple nonholonomic mobile robots. Using the cascade design method, the complex problem of cooperative control of multiple nonholonomic mobile robots is simplified to a cooperative control problem of multi-linear time-varying systems.

Compare to previous results of formation and tracking problem on such systems [13]-[16], we only assume the reference trajectories' angular velocity to be a persistent exciting signal that may be zero at some time or does not converge to a nonzero constant. Theoretical proof and simulation results show that the formation and tracking problem of multiple nonholonomic mobile robots can be realized with linear decentralized time-varying control law which is much simpler than those obtained in previous papers.

The rest of the paper is organized as follows. Section 2 gives the system model and the problem statement. Some preliminary results are also included in this section. Our main results are presented in section 3. Section 4 presents simulation results of the proposed controller to formation and tracking control of three nonholonomic mobile robots. Conclusion is provided in Section 5.

II. PROBLEM FORMULATION

Consider m nonholonomic mobile robots indexed with $1, 2, \cdots, m$ which are moving on a plane. For simplicity, we assume that each member of the group has the same mechanical structure and each mobile robot can be described by the following equation in global coordinates:

$$\begin{aligned} \dot{x}_i &= u_i \cos\theta_i, \\ \dot{y}_i &= u_i \sin\theta_i, \\ \dot{\theta}_i &= \omega_i, \end{aligned} \tag{1}$$

where $i = 1, 2, \cdots, m$.

As done in [13], we also represent the m mobile robots as m vertices in $\mathcal{V}$ of a graph $G = (\mathcal{V}, E)$, where E denoting all

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edges in the graph G and one edge $(j,i) \in E$ means the state of robot j is available to robot i. For an undirected graph G with n robots the adjacency matrix $A = A(G) = (a_{ij})$ is the $n \times n$ where $a_{ij} = 1$ if there is one edge $(j,i) \in E$ otherwise $a_{ij} = 0$. Let N_i be a collection of neighbors of robot i. The desired geometric formation $\mathcal{F}$ is described by (h_{ix}, h_{iy}) for robot i where (h_{ix}, h_{iy}) is the position vectors of robots i in global coordinates. The desired trajectory $\mathcal{T}$ of the group of robots is described by:

$$\begin{aligned} \dot{x}_d &= u_d \cos\theta_d, \\ \dot{y}_d &= u_d \sin\theta_d, \\ \dot{\theta}_d &= \omega_d, \end{aligned} \tag{2}$$

where u_d, ω_d are known time varying functions. Our control problem is defined as follows.

Formation Control Problem: Design a controller for each follower based on its and its neighbors' states such that the group of robots comes into formation $\mathcal{F}$ and the group of robots moves along the desired trajectory $\mathcal{T}$, i.e., design control laws for systems (1)(2) such that

$$\lim_{t\to\infty} \left(\begin{bmatrix} x_l - x_j \\ y_l - y_j \end{bmatrix} - \begin{bmatrix} h_{lx} - h_{jx} \\ h_{ly} - h_{jy} \end{bmatrix} \right) = 0, \tag{3}$$

$$\lim_{t\to\infty} (\theta_l - \theta_j) = 0, \tag{4}$$

$$\lim_{t\to\infty} \left(\frac{1}{m} \sum_{i=1}^{m} x_i - x_d \right) = 0, \tag{5}$$

$$\lim_{t\to\infty} \left(\frac{1}{m} \sum_{i=1}^{m} y_i - y_d \right) = 0. \tag{6}$$

In order to solve the formation control problem, we make the following assumption on reference systems:

Assumption 1: v_d is bounded and ω_d is persistently exciting, i.e., there exist positive constants α_1, α_2 such that the following condition holds for all $t > 0$:

$$\alpha_1 I \le \int_t^{t+\delta} \omega_d(\tau)\omega_d^T(\tau) d\tau \le \alpha_2 I.$$

Now, we recall some lemmas that will be used in next Section.

Lemma 2.1: [21] Consider a linear time-varying system

$$\dot{x} = A(\phi(t))x + Bu \tag{7}$$

where $A(\phi)$ is continuous, $\phi : \mathbb{R} \to \mathbb{R}$ continuous. Assume that for all $s \neq 0$ the pair $(A(s), B)$ is observable. If $\phi(t)$ is bounded, Lipschitz and constants $\delta_c > 0$ and $\epsilon > 0$ exist such that

$$\forall t \ge 0, \exists s : t - \delta_c \le s \le t \text{ such that } |\phi(s)| \ge \epsilon,$$

then the system (7) is uniformly completely observable.

Consider a time-varying cascaded system $\dot{z} = f(t,z)$ that can be written as

$$\begin{aligned} \dot{z}_1 &= f_1(t, z_1) + g(t, z_1, z_2)z_2 \\ \dot{z}_2 &= f_2(t, z_2). \end{aligned} \tag{8}$$

Actually, system (8) can be regarded as the system

$$\Sigma_1 : \quad \dot{z}_1 = f_1(t, z_1) \tag{9}$$

perturbed by the output of the system

$$\Sigma_2 : \quad \dot{z}_2 = f_2(t, z_2). \tag{10}$$

Lemma 2.2: [21] The cascade time varying system (8) is globally $\mathcal{K}$-exponentially stable if the following conditions are satisfied:

1) The subsystem (9) is globally uniformly exponential stable;
2) The function $g(t, z_1, z_2)$ satisfies the following condition for all $t \ge t_0$:
$$g(t, z_1, z_2) \le \theta_1(\|z_2\|) + \theta_2(\|z_2\|)\|z_1\|$$
where $\theta_1 : R^+ \to R^+, \theta_2 : R^+ \to R^+$ are continuous functions;
3) The subsystem (10) is globally $\mathcal{K}$-exponentially stable.

III. CONTROLLERS DESIGN

Let $p_{ix} = x_i - x_d - h_{ix} + \frac{1}{m}\sum_{j=1}^{m} h_{jx}$, $p_{iy} = y_i - y_d - h_{iy} + \frac{1}{m}\sum_{j=1}^{m} h_{jy}$ and $p_{i\theta} = \theta_i - \theta_d$ where $i = 1, 2, \cdots, m$, then in order to solve the above formation control problem we only need to prove

$$\lim_{t\to\infty} \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{i\theta} \end{bmatrix} = 0.$$

For simplicity we convert the global coordinates representation to Cartesian coordinates fixed on each member of the group, the following global change of coordinates that proposed by [24] for tracking control of single nonholonomic robot system is introduced:

$$\begin{bmatrix} e_{ix} \\ e_{iy} \\ e_{i\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{ix} \\ p_{iy} \\ p_{i\theta} \end{bmatrix} \tag{11}$$

and in these new coordinates the error systems for robot i can be described as:

$$\begin{aligned} \dot{e}_{ix} &= \omega_i e_{iy} + u_i - u_d \cos e_{i\theta}, \\ \dot{e}_{iy} &= -\omega_i e_{ix} + u_d \sin e_{i\theta}, \\ \dot{e}_{i\theta} &= \omega_i - \omega_d, \end{aligned} \tag{12}$$

Subsystem (12) can be considered the following system (13)

$$\begin{aligned} \dot{e}_{ix} &= \omega_d e_{iy} + u_i - u_d, \\ \dot{e}_{iy} &= -\omega_d e_{ix}, \end{aligned} \tag{13}$$

cascaded by

$$\dot{e}_{i\theta} = \omega_L - \omega_F. \tag{14}$$

and the cascaded term Ξ_i for robot i is defined as follows:

$$\begin{bmatrix} (\omega_i - \omega_d)e_{iy} + u_d(1 - \cos e_{i\theta}) \\ -(\omega_i - \omega_d)e_{ix} + u_d \sin e_{i\theta} \end{bmatrix}. \tag{15}$$

Now the above formation control problem composed of m nonholonomic mobile robots (1) is reduced to m cascaded-linear-time-varying systems' stabilization problem.

And in the following, we will construct linear time-varying control laws for the m subsystems composed of (13)(14) where $i = 1$ to m.

A. Design of ω_F

Lemma 3.1: For system (14), control laws

$$\omega_i = \omega_d - k_1 e_{i\theta} - \sum_{j\in\mathcal{N}_i} a_{ij}(e_{i\theta} - e_{j\theta}),\ (1 \le i \le m) \quad (16)$$

make the subsystems composed of (14) is globally exponential stable, where $k_1 > 0, i = 1, 2, \cdots, m$.

Proof: The closed-loop system of e_θ can be described as

$$\dot{e}_\theta = -k_1 e_\theta - L_{\mathcal{G}} e_\theta \quad (17)$$

where $L_{\mathcal{G}}$ is the weighted Laplacian matrix [11], $e_\theta = [e_{1\theta}, e_{2\theta}, \cdots, e_{m\theta}]$.

Since the sum of each row of $L_{\mathcal{G}}$ is zero, $-k_1 I - L_{\mathcal{G}}$ is a strict diagonal dominant matrix with negative elements in the diagonal. Therefore the eigenvalues of a Linear-Time-Invariant system are all negative and then the closed-loop system (16) is globally exponentially stable. ■

B. Design of v_F

Lemma 3.2: For system (13), control laws

$$u_i = u_d - k_2 e_{ix} + k_3 \omega_d(t) e_{iy} - \sum_{j\in\mathcal{N}_i} a_{ij}(e_{ix} - e_{jx}), \quad (18)$$

make the subsystems composed of (13) is globally uniformly exponentially stable, where $i = 1, 2, \cdots, m, k_3 > -1$, and $k_2 > 0$ to be assigned in the proof.

Proof: Under the control law (18), the closed-loop system is

$$\begin{bmatrix}\dot{e}_{ix}\\ \dot{e}_{iy}\end{bmatrix} = \begin{bmatrix}-k_2 & (k_3+1)\omega_d\\ -\omega_d & 0\end{bmatrix}\begin{bmatrix}e_{ix}\\ e_{iy}\end{bmatrix} - \begin{bmatrix}\sum_{j\in\mathcal{N}_i} a_{ij}(e_{ix} - e_{jx})\\ 0\end{bmatrix} \quad (19)$$

Consider the Lyapunov function candidate

$$V = \sum_{i=1}^{m} k_2 e_{ix}^2 + \sum_{i=1}^{m} k_2(k_3+1) e_{iy}^2 \quad (20)$$

Differentiating (20) along solutions of the m subsystems (19) results in

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{m} -2k_2^2 e_{ix}^2 - \sum_{i=1}^{m} 2k_2 e_{ix}[\sum_{j\in\mathcal{N}_i} a_{ij}(e_{ix} - e_{jx})]\\ &= -\sum_{i=1}^{m} 2k_2(k_2 + \sum_{j\in\mathcal{N}_i} a_{ij}) e_{ix}^2 + \sum_{i=1}^{m} 2k_2 e_{ix}(\sum_{j\in\mathcal{N}_i} a_{ij} e_{jx})\\ &= -k_2[\sum_{i=1}^{m} 2(k_2 + \sum_{j\in\mathcal{N}_i} a_{ij}) e_{ix}^2 - \sum_{i=1}^{m} 2e_{ix}(\sum_{j\in\mathcal{N}_i} a_{ij} e_{jx})]\\ &\le -k_2[\sum_{i=1}^{m} 2(k_2 + \sum_{j\in\mathcal{N}_i} a_{ij}) e_{ix}^2 - \sum_{i=1}^{m}((m-1)e_{ix}^2 + \sum_{j\in\mathcal{N}_i} e_{jx}^2)]\\ &\le -\sum_{i=1}^{m} l_i e_{ix}^2 \end{aligned} \quad (21)$$

The last step of equality (21) can be rendered with appropriate positive constant l_i and then $\dot{V}$ is negative semi-definite.

Since $w_d(t)$ satisfies the persistent excitation condition, it follows immediately from Assumption 1 and Lemma 2.1 that the m subsystem (19) is uniformly completely observable. From Khalil [22] and Lefeber [21] it is easy to know that the m subsystem (19) is globally uniformly exponentially stable (GUES) under the control law of (18). ■

C. Main Theorem

Theorem 3.1: For system (1)(2), under Assumption 1, control laws (16) and (18) make the formation control problem is solvable, i.e. formula (3)-(6) are satisfied, where the control parameters are chosen as done in Lemmas 3.1-3.2.

Proof: The following proof mainly based on cascade design theory of nonautonomous systems (Lemma 2.2).

A. From Lemma 3.1, under the control law (16) the m subsystems composed of (14) is globally exponentially stable (GES).

B. From Lemma 3.2, under the control law (18) the m subsystems composed of (13) is globally uniformly exponentially stable (GUES).

C. From the previous results on tracking control of nonholonomic mobile robots [21], it is easy to obtain that the cascaded term of (15) between system (13) and system (14) satisfies the linear growth condition (the third condition of Lemma 2.2). Then it is easy to obtain that for all the members of the group the linear growth condition is also satisfied.

Therefore, by Lemma 2.2 we conclude that the m cascaded systems (12) formed by (13) and (14) is globally $\mathcal{K}$-exponentially stable under linear decentralized control laws (16) and (18). Thus the formation control problem for system (1) and (2) is solvable. ■

IV. SIMULATION RESULTS

In this section, we illustrate our control methodology with the help of three nonholonomic mobile robots.

Let $m = 3$ in system (1) and assume the desired formation is defined by $(h_{1x}, h_{1y}) = (0, 0)$, $(h_{2x}, h_{2y}) = (2, 1)$, $(h_{3x}, h_{3y}) = (2, -1)$ (See Fig. 1). The desired trajectory is generated by system (2) with $\omega_d(t) = -1rad/s$, $v_d(t) = 10m/s$. Then desired trajectory satisfy the assumption 1 and for simplicity the communication topology $\mathcal{G}$ is assumed to be fixed and strongly connected.

Under the cooperative controllers obtained in Theorem 3.1 the simulation results are shown in Fig. 2 to Fig. 5. Fig. 2 to Fig. 4 are the tracking errors respect to time of each robot in the group. Fig. 5 shows the paths of the centroid of the three robots and the geometric patterns of the three robots every 10 seconds.

Simulation results show that the three robots come into the desired formation and the centroid of the group of robots move along the desired trajectory.

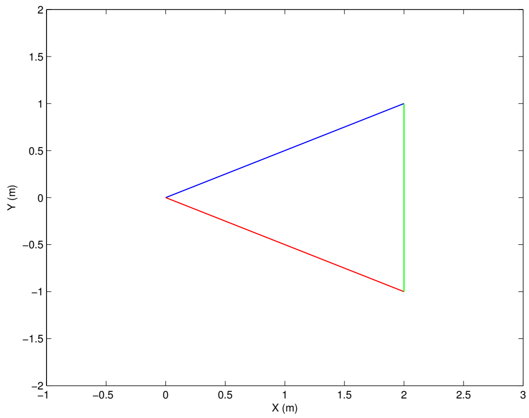

Fig. 1. Desired formation

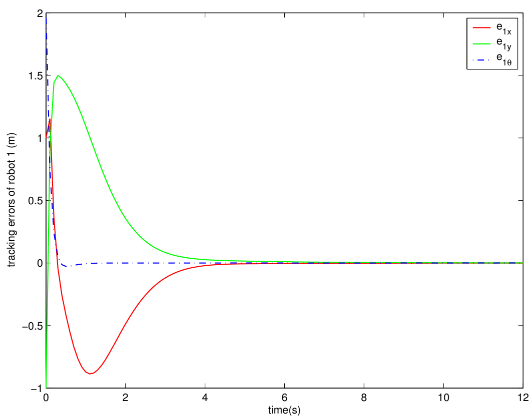

Fig. 2. Tracking errors of robot 1

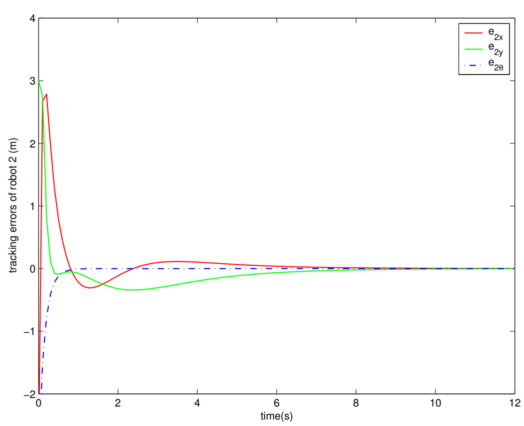

Fig. 3. Tracking errors of robot 2

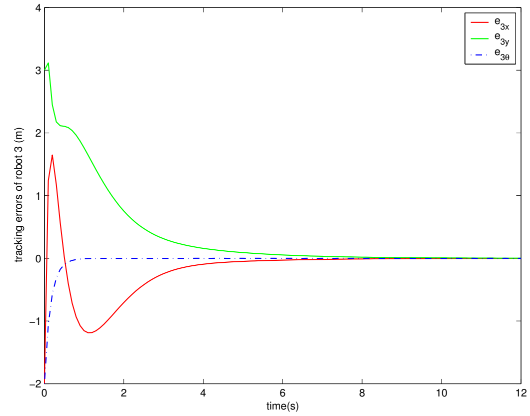

Fig. 4. Tracking errors of robot 3

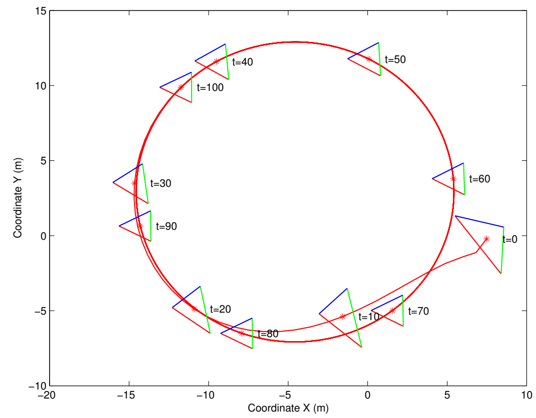

Fig. 5. paths of the centroid of the three robots and the geometric patterns every 10 seconds

V. CONCLUSIONS AND FUTURE WORKS

A. *Conclusions*

Linear decentralized controllers are constructed for the formation and tracking control problem of multiple non-holonomic mobile robots. By combining the cooperative control of linear agents and cascade design method of nonautonomous systems, the complex problem of formation and tracking control is simplified to a multi-linear time-varying systems' stabilization problem. Not converging to zero or does equal to zero all the time on reference signal has been replaced with the more relaxed condition of persistent excitation. Simulation results using Matlab validate the theoretic results.

B. *Future Works*

For simplicity the communication topology is assumed to be fixed and connected, the next step is to consider this problem when communication topology is switching or time-varying. Since communication delay is inevitable in cooperative control, future works maybe extend to consider the delay effects in the formation control of multiple nonholonomic mobile robots.

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REFERENCES

[1] Ali Jadbabaie, Jie Lin and A. Stephen Morse, Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbor Rules, *IEEE Transactions on Automatic Control*, vol. 48, no. 6, 2003,988-1001.

[2] Reza Olfati-Saber and Richard M. Murray, Consensus Problems in Networks of Agents With Switching Topology and Time-Delays, *IEEE Transactions on Automatic Control*, vol. 49, no. 9, 2004,1520-1533.

[3] Wei Ren and Randal W. Beard, Consensus Seeking in Multiagent Systems Under Dynamically Changing Interaction Topologies, *IEEE Transactions on Automatic Control*, vol. 50, no. 5, 2005,655-661.

[4] John T. Feddema, Chris Lewis and David A. Schoenwald, Decentralized Control of Cooperative Robotic Vehicles: Theory and Application, *IEEE Transactions on Robotics And Automation*, vol. 18, no. 5, 2002,852-864.

[5] Y.UNY Cao, Alex S. Fukunaga, Andrew B. Kahng, Cooperative mobile robotics: antecedents and directions, *Autonomous Robotics*, vol 4,1997,1-23.

[6] Tucker Balch and Ronald C. Arkin, Behavior-Based Formation Control for Multirobot Teams, *IEEE Transactions on Robotics And Automation*, vol. 14, no. 6, 1998,926-939.

[7] M. Anthony Lewis and Kar-Han Tan, High precision formation control of mobile robots using virtual structures, *Autonomous Robots*, vol. 4, 1997,387-403.

[8] Luca Consolini, Fabio Morbidi, Domenico Prattichizzo, Mario Tosques, "A Geometric Characterization of Leader-Follower Formation Control", *IEEE International Conference on Robotics and Automation*, Roma, Italy, 2007,2397-2402.

[9] H. G. Tanner, G. J. Pappas, and V. Kumar, Leader-to-formation stability, *IEEE Transactions on Automatic Control*, vol. 20, no. 3, 2004, 443-455.

[10] N. E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials and coordinated control of groups", *Proceeding of IEEE Conference on Decision Control*, 2001,2968-2973.

[11] J. Alexander Fax, and Richard M. Murray, Information Flow and Cooperative Control of Vehicle Formations, *IEEE Transactions on Automatic Control*, vol. 49, no. 9, 2004,1465-1476.

[12] K. D. Do, "Formation tracking control of unicycle-type mobile robots", *IEEE International Conference on Robotics and Automation*, Roma, Italy, 2007,2391-2396.

[13] W.J. Dong, Yi Guo, and J.A. Farrell, "Formation Control of Nonholonomic Mobile Robots", *Proceedings of the 2006 American Control Conference* Minneapolis, Minnesota, USA, 2006,5602-5607.

[14] Wenjie Dong and Jay A. Farrell, Cooperative Control of Multiple Nonholonomic Mobile Agents, *IEEE Transactions on Automatic Control*, vol. 53, no. 6, 2008,1434-1448.

[15] Zhiyun Lin, Bruce Francis and Manfredi Maggiore, Necessary and Sufficient Graphical Conditions for Formation Control of Unicycles, *IEEE Transactions on Automatic Control*, vol. 50, no. 1, 2005,121-127.

[16] Ghulam Mubashar Hassan, Khawaja M Yahya and Ihsan-ul-Haq, "Leader-Follower Approach using Full-State Linearization via Dynamic Feedback", *International Conference on Emerging Technologies*, Peshawar, Pakistan, 2006,13-14.

[17] Xiaohai Li, Jizong Xiao, Zijun Cai, "Backstepping Based Multiple Mobile Robots Formation Control", *Intelligent Robots and Systems*, 2005, 2-6: 887-892.

[18] Murat Arcak, Passivity as a Design Tool for Group Coordination, *IEEE Transactions on Automatic Control*, vol. 52, no. 8, 2007,1380-1390.

[19] K.-C. Cao and Y.-P. Tian, A time-varying cascaded design for trajectory tracking control of non-holonomic systems, *International Journal of Control*, vol. 80, no. 3, 2007, 416-429.

[20] Y.-P. Tian, K.-C. Cao. Time-varying linear controllers for exponential tracking of non-holonomic systems in chained form. *International Journal of Robust and Nonlinear Control*, vol. 17, no.7, 2007,631−647.

[21] E. Lefeber, *Thesis: Tracking Control of Nonlinear Mechanical Systems*, University of Twente, The Netherlands; 2000.

[22] H.K. Khalil, *Nonlinear Systems*, Prentice Hall, Upper Saddle River, NJ, 2nd edition, 1996.

[23] Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "A connection between formation control and flocking behavior in nonholonomic multiagent systems", *Proceedings of the 2006 IEEE International Conference on Robotics and Automation*, Orlando, Florida, 2006,940-945.

[24] Kanayama Y., Kimura Y., Miyazaki F. and Noguchi T., "A stable tracking control scheme for an autonomous mobile robot", *Proceedings of IEEE Internatioinal Conference on Robotics and Automation*, 1990, 384-389.