A HYBRID CONTROL STRATEGY FOR MULTIPLE MOBILE ROBOTS WITH NONHOLONOMIC CONSTRAINTS

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ABSTRACT

This paper explains a hybrid control strategy developed to coordinate multiple autonomous mobile robots with nonholonomic constraints. The robots are required to navigate in an obstacles populated environment with a predetermined geometric formation. The nonlinear robot dynamics are fully state feedback linearized in order to yield linear controllable systems while the dynamics of the environment are handled by an event based hybrid automata. The chattering effect of the switched system is tackled by incorporating sliding dynamics in the automata. The proposed system is implemented through simulation and the results are shown to verify its operation.

Index Terms— Mobile robots, Multi-robot formation, Hybrid automata, Nonlinear control, Motion control, MIMO

1. INTRODUCTION

Control and coordination of multiple mobile robots has been considered a prime research area due to its value in many fields. Intelligent highway, cooperative classification, air traffic control, military applications are some of the areas which can utilize multi robot systems [6]. Coordinating and controlling the multi robot systems in the face of dynamic environments, uncertainty of measurements, traffic delays, low bandwidth, and hardware constraints pose extra challenges to the problem. Multi robot systems perform a single task or a sequence of tasks cooperatively for ex: formation control, collective box pushing, map exploration or collective SLAM. Coordination strategies can be centralized, decentralized or it can be a combination of both [5]. Much research has been done in the centralized control schema where there is a central node or a set of central nodes which are called leaders and a set of other robots called followers [1]. The leader robots are autonomous, such that they make their own decisions while the followers depend on the instructions given by the respective leader. Decentralized control for coordination includes consensus control strategies [9]. Mesh and string stability control are the other decentralized control strategies. There is also growing research for hybrid control strategies which can combine the merits of both mainstream strategies. This paper proposes a leader based coordination control platform for nonholonomic mobile robots which can handle formation control, obstacle avoidance and path planning. The nonholonomic robot system is explicitly nonlinear. It belongs to a special class of Euclidean *SE(2)* dynamic systems. And according to Brockett [8] the nonholonomic robot system can not be controlled by smooth linear time invariant controls laws. However the rank of the accessibility matrix of the unicycle robot system being equal to the number of states of the system proves it as controllable [1]. Thus with the use of input output feedback linearization we derive a set of controller subsystems to handle formation control, obstacle avoidance and goal navigation of the nonholonomic robots. The continuous dynamics of the developed subsystems have to be switched between or run parallel when the external environment changes i.e. obstacle is encountered. Running the continuous dynamics of different subsystems parallel will be mechanically impossible to combine. Hence the idea is to building a hybrid automaton which combines the continuous dynamics of subsystems with discrete switching between the subsystems [2], [7] triggered by events of the external environment or the robot itself.

A switched system may exhibit chattering known as zeno effect [2]. Zeno effect in theory makes infinite transitions in finite time between the states of the system and is responsible for chattering and system instability. These switched systems form a special class of hybrid automaton called zeno hybrid automaton since they exhibit a sliding property in the sense of Fillipov [2] along the switching surface. The idea is to use this sliding property of the switched system to define another intermediate state in between the switching states to incorporate sliding dynamics. Hence at the switching boundary we can make the system to slide along the boundary instead of switching between different states. This minimizes the chattering effect. Two separate hybrid schemes are developed, one for the leader robot and the other for the followers. The leader robot navigates along goal-points while avoiding obstacles and the followers keep a predetermined geometric formation with the respective leader while also avoiding obstacles.

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2. NONLINEAR CONTROL

Nonlinear feedback control is used to full state linearize the nonlinear dynamics. Differential geometrical approach is used to feedback linearize the system. Consider a general single-input-single-output (SISO) nonlinear system given by the following.

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x)$$

It can possibly be linearized by the combination of a change of coordinates and a state feedback [3], [4]. The relative degree of the nonlinear system is the number of times the output y has to be differentiated before the control u appears. If the relative degree is equal to the number of states of the system, then the system is turned in to Brunowsky form, which is linear and controllable. If the relative degree is less than the number of states of the system then there is internal dynamics present. Internal dynamics are what the control doesn't reach. Sometimes they make the system unstable. So the internal dynamics are analyzed in the sense of zero dynamics to make things simpler [3]. The multi-input-multi-output MIMO formulation is an extension from the SISO geometric feedback linearization. Since this system is MIMO, geometric MIMO feedback linearization is used.

3. ROBOT CONTROL DYNAMICS FORMULATION

Voronoi decomposition is performed on a given map and A* algorithm is utilized to find the minimum cost path from a starting point to a goal location and the sub goals are also recorded. The leader robot is navigated to the end goal via the sub goals. Every time the leader robot approaches a sub goal the robot is turned to the next sub goal at a distance d from the sub goal which is equivalent to the maximum radius of the robot.

3.1. Leader Goal Navigation Formulation

Unicycle dynamics can be stated as,

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

v and ω are the linear and rotational velocities of the robot defined in the Eucledean *SE (2)* coordinate system respectively. It is seen that a relative degree can not be defined for the unicycle robot system above. In such situations one can resort to dynamic feedback linearization or changing the measurement output at an offset from the current measurement coordinates. Former introduce unstable singularities to the system. Hence by the latter it is found,

$$\begin{pmatrix} X_{new} \\ Y_{new} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} X \\ Y \end{pmatrix}$$

a and b are the offsets in X and Y robot coordinates from it's origin. For simplicity we make b = 0 and proceed with a, a small offset from the origin of the robot coordinates towards X direction. Hence through some manipulations,

$$\begin{pmatrix} V \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta & -a\sin\theta \\ \sin\theta & a\cos\theta \end{pmatrix}^{-1} \begin{pmatrix} c_1(x_d - x) \\ c_2(y_d - y) \end{pmatrix}$$

C_1, C_2 are constants. X_d, Y_d are sub goal locations while x & y are the new measured coordinates. The internal dynamics of the system can be proved to be stable.

3.2. Formation Control for Followers

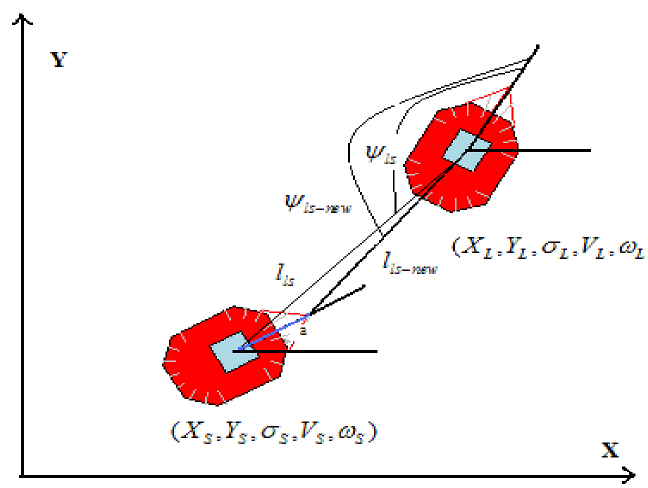

Fig 1: Two robot formation Control

The follower's objective is to follow the lead robot with a desired relative distance and a bearing. It is seen that for the formation control system in Fig.1 a relative degree can not again be defined. Hence in the follower robot, we make the observation from a small shift *'a'* in X direction of the follower robot coordinate frame. The formulated dynamics for the follower robot is,

$$\begin{pmatrix} \dot{l}_{ls-n} \\ \dot{\psi}_{ls-n} \\ \dot{\theta}_{ls} \end{pmatrix} = \begin{pmatrix} \cos\gamma & a\sin\gamma \\ \frac{-\sin\gamma}{l_{ls-new}} & \frac{a\cos\gamma}{l_{ls-new}} \\ 0 & -1 \end{pmatrix} \begin{pmatrix} V_s \\ \omega_s \end{pmatrix} + \begin{pmatrix} \cos(\psi_{ls-n}) & 0 \\ \frac{-\sin(\psi_{ls-n})}{l_{ls-new}} & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} V_L \\ \omega_L \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} l_{ls-new} \\ \psi_{ls-new} \\ \theta_{ls} \end{pmatrix}$$

Where $\gamma = \theta_{ls} + \psi_{ls-new}$, $\theta_{ls} = \theta_L - \theta_S$, $\mathrm{X}_{ls-n} = X_{ls-new}$

The system in Fig.1 represents a MIMO system with an exogenous input. The lead robot dynamics are the exogenous input to the follower robot system. Also the robot inputs are developed to decouple the exogenous inputs from the outputs, which in this case happen to be the relative distance and bearing [3]. Since the given system adheres to some differential geometrical laws developed for feedback linearized systems see [3] it is possible to decouple the outputs from the inputs here. The formation controller above looks like,

$$\mathbf{y}^r = \mathbf{J}(x)\mathbf{u} + \mathbf{K}(x)\mathbf{w}$$

Where $\mathbf{y}^r = \begin{pmatrix} \dot{l}_{ls-n} & \dot{\psi}_{ls-n} & \dot{\theta}_{ls} \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} v_s & \omega_s \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} v_L & \omega_L \end{pmatrix}$ while $\mathbf{J}(x)$ and $\mathbf{K}(x)$ are as above and r is the degree of differentiation. In order to keep a desired relative distance and a bearing with the leader, the proposed control laws can be stated as,

$$\mathbf{u} = \mathbf{J}(x)^{-1}\left(\mathbf{y}^r - \mathbf{c}_{r-1}\left(\mathbf{y}^{r-1} - \mathbf{y}_{\mathbf{d}}^{r-1}\right)\cdots\mathbf{c}_1\left(\mathbf{y} - \mathbf{y}_{\mathbf{d}}\right) - \mathbf{K}(x)\mathbf{w}\right)$$

The above closed loop system's internal dynamics are again proved stable. Thus the whole system is stable. $c_{r-1}\ldots c_1$ are positive constants of the system and y_d are the desired values.

As $t \to \infty$

$$l_{ls}^d - l_{ls} \to 0,\ \psi_{ls}^d - \psi_{ls} \to 0 \text{ and } |\theta_{ls}| \le \delta \text{ for small } \delta \ge 0$$

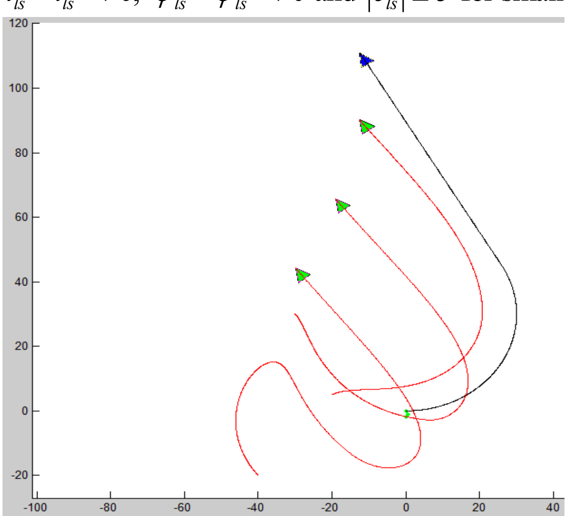

Fig 2: A leader (blue) and three followers formation Control

Also the formation can be formulated through two designated leader robots. The formation dynamics for such a system is given can be described by,

$$\begin{pmatrix} \dot{l}_{13} \\ \dot{l}_{23} \\ \dot{\theta}_{123} \end{pmatrix} = \begin{pmatrix} \cos(\gamma_{13}) & a\sin(\gamma_{13}) \\ \cos(\gamma_{23}) & a\sin(\gamma_{23}) \\ 0 & -1 \end{pmatrix} \begin{pmatrix} V_3 \\ \omega_3 \end{pmatrix} + \begin{pmatrix} -\cos(\psi_{13}) & 0 \\ 0 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} V_1 \\ \omega_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\cos(\psi_{23}) & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} V_2 \\ \omega_2 \end{pmatrix}$$

and

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} l_{13} \\ l_{23} \\ \theta_{123} \end{pmatrix}$$

Subscript 1 & 2 are for the lead robots and subscript 3 is for the follower. Other notations are similar to first system. The follower robot can be made to drive to a desired pose by incorporating a closed loop system equivalent to the first formation control closed loop system above. The internal dynamics of the systems are again proved to be stable. And the whole system is,

As $t \to \infty$

$$l_{13}^d - l_{13} \to 0,\ l_{23}^d - l_{23} \to 0 \text{ and } |\theta_{123}| \le \delta \text{ for small } \delta \ge 0$$

3.3. Obstacle Avoidance Control

Obstacle avoidance for leader robot and the follower robots are similar; the difference being the former must navigate to a *static* sub-goal position while the latter must keep a desired relative distance to the former while avoiding obstacles. Here the followers are only limited to keeping a desired relative distance because keeping a relative bearing is impossible when avoiding obstacles. In obstacle avoidance all robots are required to keep a safer distance from the obstacle 'd_{obs}' which in this case taken as twice the maximum turning radius of the robot '$2R$'. If the obstacle dynamics are known one can easily incorporate them in the dynamic system. But the following derivation is carried out with the assumption that the obstacles are static.

$$\begin{pmatrix} V_s \\ \omega_s \end{pmatrix} = \begin{pmatrix} \cos\gamma & a\sin\gamma \\ \cos\phi & a\sin\phi \end{pmatrix} \begin{pmatrix} c_1(l_d - l) + \cos\psi * V_L \\ c_2(2R - d_{obs}) \end{pmatrix}$$

Where $\phi = \alpha_{\text{obs}} - \theta_s$, c_1, c_2 positive constants,

$-\pi \le \alpha_{\text{obs}} \le \pi$ and $\alpha_{obs} = \text{atan2}(y_{obs} - y_s, x_{obs} - x_s)$

(x_{obs}, y_{obs}) is the location of the obstacle. Subscript S stands for the follower robot while L stands for the leader and other notations are similar to the earlier cases. For the followers the above dynamics can directly be applied but for the leader V_L is zero while γ is chosen by assuming the sub goal has a heading to the next sub goal or in the case of last goal an arbitrary heading. Also l_d is the desired distance from the follower to the leader in the leader follower case, and zero for leader only goal navigation with obstacle avoidance.

4. HYBRID AUTOMATON

It is observed that in the face of dynamic changes in the environment the developed continuous dynamic subsystems must be switched between. So a hybrid automaton is developed, which has discrete states and these occupy the continuous dynamics. Depending on external or deliberate events sensed the switching is triggered between the states. Such a switched system will probably have chattering and can be tackled by a continuous dynamic subsystem which incorporates sliding dynamics. If the relative angle between the robot and obstacle is α_{obs} and the robot orientation is θ, linear & angular velocities set are,

If $(\alpha_{obs} + \pi/2) \le \theta \le (\alpha_{obs} + \pi),\ -\pi \le \alpha_{obs} \le \pi$

$V = C,\ \omega = -|\theta - (\alpha_{obs} + \pi/2)|$ and C-constant

of the robot and if,

$(\alpha_{obs} - \pi) \le \theta \le (\alpha_{obs} - \pi/2),\ -\pi \le \alpha_{obs} \le \pi$

$V = C,\ \omega = |\theta - (\alpha_{obs} - \pi/2)|$ and C-constant

And if α_{obs} does not belong to the above regions there is no transition to obstacle avoidance in this hybrid automaton. Derived dynamics above provides a continuous flow along the surface. The hybrid automatons for leader and followers are shown in Fig.3.

5. CONCLUSION

The system is simulated in an obstacle populated environment. It is identified that when the leader moves without making hard turns, the formation controllers guarantee fast convergence. In the two-leader controller, if the two leaders don't run in an accurate formation the followers become quite unstable. Also the followers depend heavily on accurate measurements of leader's pose and velocity, 'which is communicated here and also on its own sensed information like odometry. Obtaining such accurate measurements becomes really challenging in a real world robot application. It is also observed that the hybrid automata handle the discrete switching well enough such that the zeno effect is well tackled for. Hence the system chattering is reduced. For both leader and the follower the static obstacles avoidance controllers avoid the obstacles smoothly, while keeping the other objective fulfilled, goal navigation for the leader and leader following for the followers. Future enhancements for the system will probably include dealing with uncertainty of measurements.

6. ACKNOWLEDGEMENT

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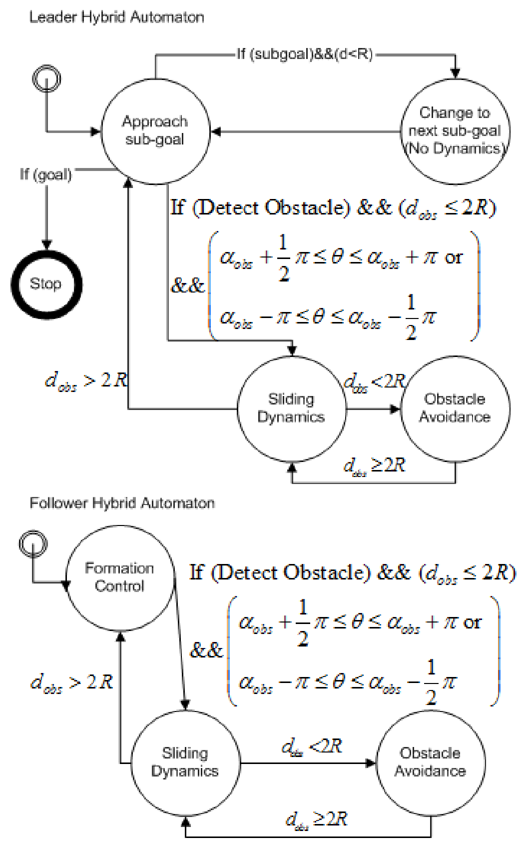

Fig 3: Hybrid Automaton for both leader and follower

7. REFERENCES

[1] A. Das, James P. Ostrowski, V. Kumar, "Modeling and control of nonholonomic mobile robots." *IEEE Transaction on Robotics and Automation*, vol.17, pp. 905-908, 2001.

[2] Magnus Egerstedt, Xiaoming Hu, "A hybrid control approach to coordination for mobile robots," *Automatica*, 2002.

[3] Alberto Isidori, *Nonlinear Control Systems*, Springer, 1989.

[4] Hassan K. Khalil, *Nonlinear Systems*, Prentice Hall, 1996.

[5] Aveek K. Das, Rafael Fierro, Vijay Kumar, James P. Ostrowski, John Spletzer and Camilo J. Taylor, " A Vision-Based formation Control Framework," *IEEE Transaction on Robotics & Automation*, vol. 18, pp. 813-825, 2002.

[6] T. Balch and R. Arkin, "behavior based formation control for multi robotics teams," *IEEE Transactions on Robotics and Automation*, vol. 14, pp. 926-934, December 1998.

[7] Jana Kosecka, Ruzena Bajcsy, "Discrete Event Systems for Mobile Agents," *Internal report- Grasp laboratory*, University of Pennsylvania, USA, Oct 1993.

[8] S. L. de Olivera Kothare and M. Morari, "Contractive Model Predictive Control for Constrained Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 45(6), pp. 1053-1071, 2000.

[9] Wei Ren, "Consensus based formation control strategies for multi-vehicle systems," *American Control Conference*, Minnesota, USA, June 2006.