Artificial Potential Based Control for a Large Scale Formation of Mobile Robots[1]

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Abstract

This paper presents control method based on a set of artificial potential functions. The kind of used artificial potential depends on particular objective: avoiding collisions between robots and keeping them in the ordered formation, executing task by the formation (e.g. moving formation into desired position), building formation and avoiding collisions with obstacles. In this paper we expand existing framework proposing attraction area potential function which allows to build formation and repulsion potential function which allows to avoid collisions with obstacles. Main advantage of presented approach is that it is easy scalable because control is based on general rule that determines behaviour of all robots. Presented solutions are illustrated with simulation results.

Keywords: cooperation of robots, formation of robots, mobile robots, building formation, artificial potentials.

1 Introduction

Multiple robot coordination became intensively investigated area of robotics in last few years. This intensive study caused remarkable growth of knowledge of distributed systems and interactions between their autonomous subsystems. There are two main reasons of great development in this field. First is that contemporary industry requires more and more flexible, efficient and cheaper solutions. Second reason is availability of low-cost components that are necessary to build distributed and easy-scalable multi-agent solutions.

Muli-robot systems has many advantages in comparison with old-style, centralized solutions, however, they issue new challenges for engineers and scientists. These challenges are not trivial and there still remain many questions without answers.

A large majority of multi-robot coordination methods developed in last years belong to one of specified classes of approaches: virtual structure approach [2], [4]- [8], behavioral approach [10]- [17] and leader follower scheme [1], [3] (often treated as a combination of first two approaches). Each of them has some advantages and weaknesses. There exist some solutions with characteristic features of more then one approach [9]. Detailed comparison of approaches to multi-robot coordination is presented in [18] and [20].

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In section two we describe artificial potential based control. I subsection 2.1 we present artificial potential function that avoid collisions between robots of the formation and keeps robots in order. In section 2.2 we introduce virtual leaders and present control method that utilize both previously described artificial potential functions. In section 2.3 we present attraction area artificial potential function. In section 2.4 we introduce artificial potential that avoid collisions with obstacles.

In section three we present simulation results for control methods presented in section two.

2 Artificial potential based control

In his section we describe artificial potential based control. In paper [21] this control method was used to control formation using virtual leaders. Attraction area and repulsion to the obstacles are expansion of concept proposed in that paper.

Presented control law assumes that all robots are fully actuated. Dynamics of single robot is given by equation:

$$\ddot{x}_i = u_i, \tag{1}$$

where x_i is position vector of i-th robot, $x_i \in \mathbb{R}^2$, u_i - control force vector on the i-th robot, $u_i \in \mathbb{R}^2$, $i = 1, 2, ..., N$, N - number of robots.

By using feedback linearization system dynamics is transformed into fully actuated double integrator equations of motion [22], [23].

In control method based on artificial potentials the role of particular robot in the formation is not explicitly specified by the high-level planner. There are no specially distinguished robots. User specifies only general rule that determines behaviour of all robots of the formation. This feature is an important advantage in the case of huge formation of robots (dozens and more robots).

Control based on artificial potentials assume that single robot is fully actuated. This is a very strong assumption. It requires robots to be in fact fully actuated or special technique must be used to transform underactuated robot into fully actuated (it can be done using feedback linearization). The strength of this is that formation control can be implemented separately form robots motion control and is not depended on architecture and properties of robots.

2.1 Interactions between robots

Physical rules of short-distance interactions between atoms inspired basic concepts of presented control method for formation of mobile robots. Artificial potentials between robots keep formation in order and avoid collisions between robots. Artificial potentials cause artificial forces between robots. Strength of that forces depend on distances between formation members. Artificial forces cause repulsion when robots are too close; when they are too far artificial forces cause attraction, but when distance exceed specified limit attraction forces are reduced to zero.

Let denote $l_{ij} = x_i - x_j$, $l_{ij} \in \mathbb{R}^2$ - distance between i-th robot and j-th robot.

Artificial potentials between robots are given by the following equation:

$$V_I = \begin{cases} \alpha_I(\ln(l_{ij}) + \frac{d_0}{l_{ij}}) & 0 < l_{ij} < d_1 \\ \alpha_I(\ln(d_1) + \frac{d_0}{d_1}) & l_{ij} \geq d_1. \end{cases} \tag{2}$$

Parameter d_0 designates equilibrium point between attraction and repulsion along line between robots. Parameter d_1 designates limit of distance between robots for which attraction force decay. The shapes of V_I and f_I depend on α_I parameter. It determines maximum value and slope of artificial force function along line between robots.

Artificial force f_I between robots caused by artificial potential V_I is given by following equation:

$$f_I = \begin{cases} \alpha_I(\frac{1}{l_{ij}} - \frac{d_0}{(l_{ij})^2}) & 0 < l_{ij} < d_1 \\ \alpha_I(\frac{1}{d_1} - \frac{d_0}{(d_1)^2}) & l_{ij} \geq d_1. \end{cases} \tag{3}$$

In Figure 2 we present graph of artificial force f_I.

2.2 Virtual leaders

Virtual leaders can be used to keep formation in prescribed shape or change the shape of the formation. Usually robots

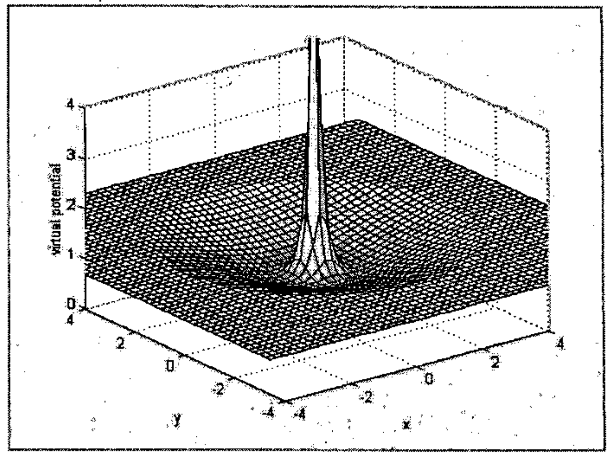

Figure 1: Artificial potential between robots and virtual leaders and between robots themselves

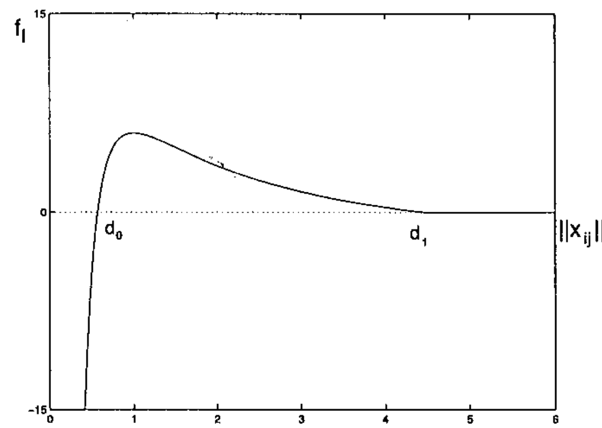

Figure 2: Artificial potentials between robots $f_I = f(||x_{ij}||)$

are surrounded by virtual leaders. Artificial potentials that surround virtual leaders are similar to ones that surround robots with an accuracy to the coefficients.

We introduce M virtual leaders. The position of k-th virtual leader is $b_k \in \mathbb{R}^2$, $k = 1, 2, ..., M$.

Let denote $h_{ik} = x_i - b_k$, $h_{ik} \in \mathbb{R}^2$ - distance between i-th robot and k-th virtual leader.

Artificial potentials between robots and virtual leaders are given by the following equation:

$$V_h = \begin{cases} \alpha_h(\ln(h_{ik}) + \frac{h_0}{h_{ik}}) & 0 < h_{ik} < h_1 \\ \alpha_h(\ln(h_1) + \frac{h_0}{h_1}) & h_{ik} \geq h_1. \end{cases} \tag{4}$$

Artificial force f_I between robots caused by artificial potential V_I is given by following equation:

$$f_h = \begin{cases} \alpha_h(\frac{1}{h_{ik}} - \frac{h_0}{(h_{ik})^2}) & 0 < h_{ik} < h_1 \\ \alpha_h(\frac{1}{h_1} - \frac{h_0}{(h_1)^2}) & h_{ik} \geq h_1. \end{cases} \quad (5)$$

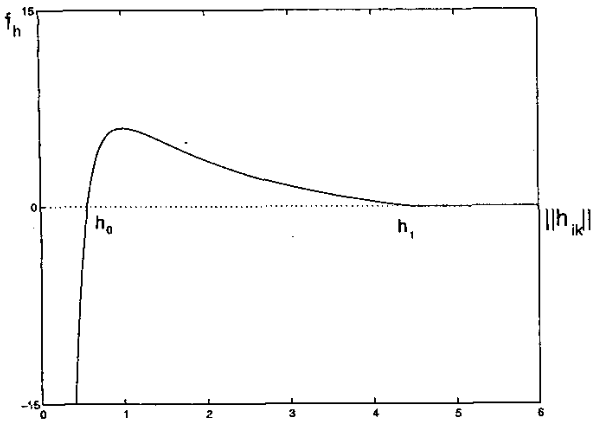

Figure 3: Artificial potentials between robots and virtual leaders $f_h = f(\|h_{ik}\|)$

Parameter h_0 designates equilibrium point between attraction and repulsion along line between robot and virtual leader. Parameter h_1 designates limit of distance between robot and virtual leader for which attraction force decay. The shapes of V_h and f_h depend on α_h parameter. It determines maximum value and slope of artificial force function along line between robot and virtual leader.

In Figure 3 graph of artificial force f_h is presented.

At this point we can present control equation that combine both previously presented artificial potential functions. The control applied to the i-th robot is as follows:

$$u_i = -\sum_{j=1, j\neq i}^{N} \nabla_{x_i} V_I(l_{ij}) - \sum_{k=1}^{M} \nabla_{x_i} V_h(h_{ik}) - K\dot{x}_i \quad (6)$$

$$= -\sum_{j=1, j\neq i}^{N} \frac{f_I(l_{ij})}{\|l_{ij}\|} l_{ij} - \sum_{k=1}^{M} \frac{f_h(h_{ik})}{\|h_{ik}\|} h_{ik} - K\dot{x}_i, \quad (7)$$

where $\nabla_{x_i} V_I(l_{ij})$ is the gradient of the potential between i-th and j-th robot, $\nabla_{x_i} V_h(h_{ik})$ is the gradient of the potential between i-th robot and k-th virtual leader, K is positive definite matrix of linear dumping coefficients, $\|l_{ij}\|$ is distance between i-th and j-th robot, $\|h_{ik}\|$ is distance between i-th robot and k-th virtual leader.

2.3 Attraction area

Artificial potential function that defines 'attraction area' can be used when robots are distributed in the environment and they are expected to build ordered formation in desired position. The shape of attraction area is a circle with radius r. The value of the artificial potential in attraction area is constant and grows outside the circle.

Attraction area artificial potential can be used before execution of complex task that require robots to be in ordered formation. In particular it can be useful before we use virtual leaders, because they assume that initially robots are ordered.

Artificial potential that acts on robots is given by the following equation:

$$V_a = \begin{cases} C & 0 < s_i < r \\ \alpha_a (s_i - r)^{\frac{3}{2}} + C & s_i \geq r \end{cases} \quad (8)$$

where α_a is the factor that determines slope of the artificial potential outside attraction area, s_i is distance between i-th robot and the center of the attraction area, C is constant that determines the level shift of the potential function.

Artificial potential function given by equation 8 is presented on the Figure 4.

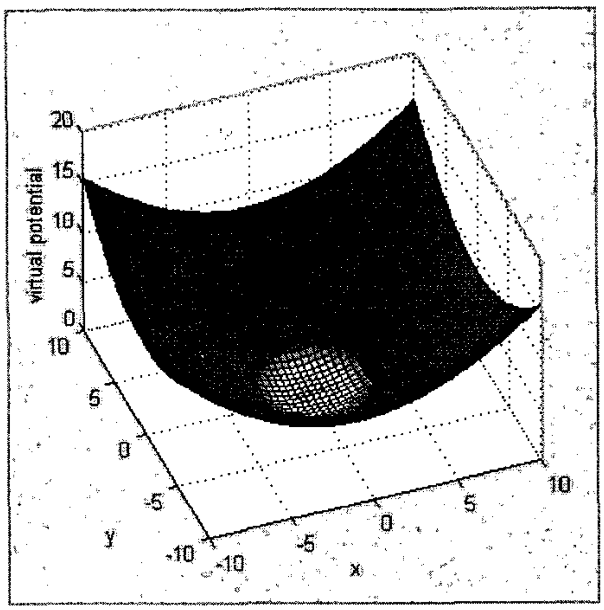

Figure 4: Artificial potential in attraction area and around it; potential is constant in the circle in the middle and grows outside when distance grows

Artificial potential given by 8 causes the following artificial force:

$$f_a = \begin{cases} 0 & 0 < s_i < r \\ \frac{3}{2}\alpha_a \sqrt{s_i - r} & s_i \geq r \end{cases} \quad (9)$$

In Figure 5 we present graph of artificial force f_a caused by artificial potential V_a.

The control for i-th robot that combines both attraction area

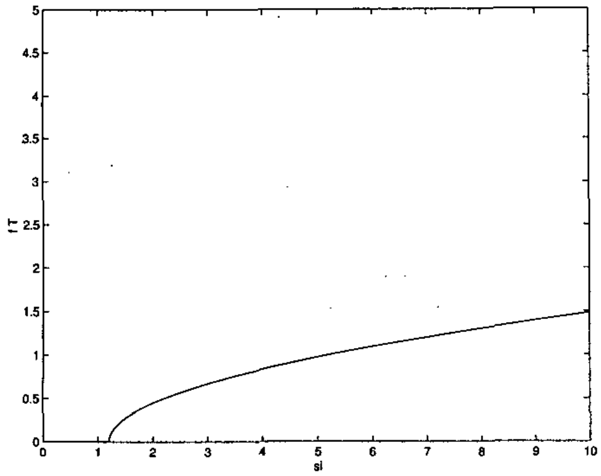

Figure 5: Artificial forces along distance to the center of attraction area $f_a = f(\|s_i\|)$

and interaction between robots artificial potentials is given by the following equation:

$$u_i = -\sum_{j=1, j\neq i}^{N} \nabla_{x_i} V_I(l_{ij}) - \nabla_{x_i} V_a(s_i) - K\dot{x}_i \quad (10)$$

where $\nabla_{x_i} V_a(s_i)$ is the gradient of the potential of attraction area that acts on the i-th robot.

2.4 Repulsion to the obstacles

To avoid collisions with the obstacles we introduce repulsion potentials that act close to obstacles and decay when distances to them grow.

Artificial potential that acts on robots when they are near to the obstacle is given by the following equation:

$$V_o = \begin{cases} not\ defined & 0 < g_i < r \\ \frac{\alpha_o}{g_i - r} + g_i \frac{\alpha_o}{(q-r)^2} & r \leq g_i < q \\ \alpha_o\left(\frac{2q-r}{(q-r)^2}\right) & g_i \geq q \end{cases} \quad (11)$$

where r is the radius of the forbidden area that surrounds obstacle, q is the radius of the area where repulsion acts (this repulsion decays outside), g_i is distance between the i-th robot and the center of the forbidden area and α_o determines slope of the artificial potential around forbidden area. The condition $q > r$ must be fulfilled.

The value of the artificial potential in the forbidden area is not defined. Considerable slope of the potential function near border of the forbidden area avoid robots to get inside.

Artificial potential function given by equation 11 is presented on the Figure 6.

Artificial potential causes the following artificial forces:

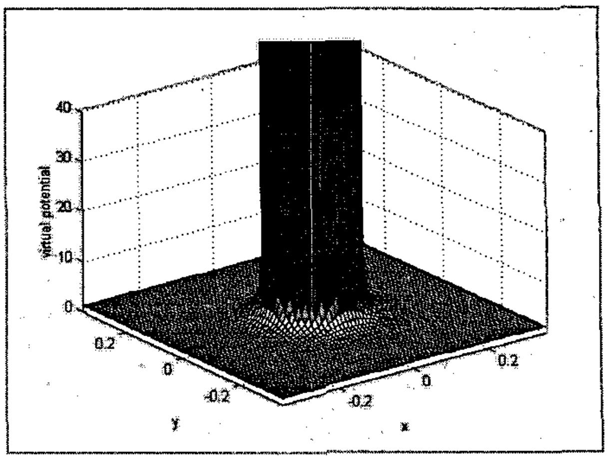

Figure 6: Artificial potential around obstacle; inside forbidden area artificial potential is not defined, close to the obstacle it has considerable value and quickly decay when distance to the obstacle grows

$$f_o = \begin{cases} not\ defined & 0 < g_i < r \\ -\frac{\alpha_o}{(g_i - r)^2} + \frac{\alpha_o}{(q-r)^2} & r \leq g_i < q \\ 0 & g_i \geq q \end{cases} \quad (12)$$

In Figure 7 we present graph of artificial force f_o caused by artificial potential V_o.

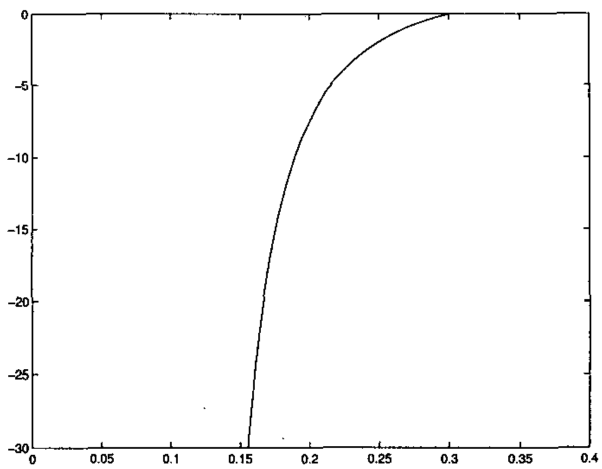

Figure 7: Artificial forces along distance to the center of forbidden area (around obstacle) $f_o = f(\|g_i\|)$

The control for i-th robot that combines repulsion to the obstacle, attraction area and interaction between robots artificial potentials is given by the following equation:

$$u_i = -\sum_{j=1, j\neq i}^{N} \nabla_{x_i} V_I(l_{ij}) - \nabla_{x_i} V_a(s_i) - \nabla_{x_i} V_o(g_i) - K\dot{x}_i \tag{13}$$

where $\nabla_{x_i} V_o(g_i)$ is the gradient of the potential of obstacle that acts on the i-th robot.

Control method presented in this subsection can be easy expanded to deal with multiple obstacles, however, some additional conditions must be taken into account to avoid locks that precludes proper execution of the task. Improper choice of forbidden areas around obstacles can lead to emergence of local minimums of artificial potential.

3 Simulation results

In this section we present simulation results for control of the formation of nineteen robots.

In Figure 8 formation of nineteen robots is surrounded by 19 virtual leaders. Initially the center of the formation is positioned at the origin of coordination frame. The motion of the virtual leaders cause motion of whole formation. Control law given by equation 7 was used in presented simulation.

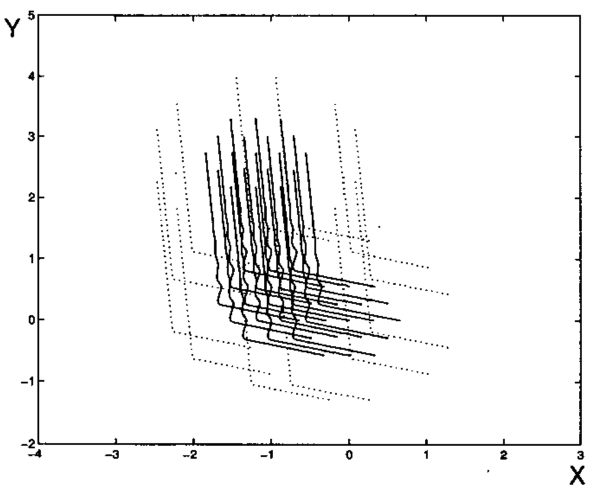

Figure 8: Formation of nineteen robots surrounded by twelve virtual leaders

In Figure 9 we present simulation results for nineteen robots that use combination of interaction between robots and attraction area artificial potentials to execute motion from the origin of the coordination frame to the area with the center in the position $(1.2, 1.2)$. Initially robots are ordered. The shape of the formation does not change during execution of the task. Formation is kept only with interaction between robots artificial potential.

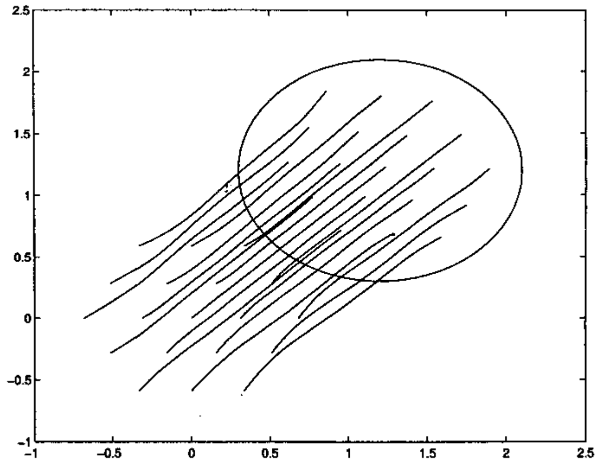

Figure 9: Formation of nineteen robots (initially ordered) with desired attraction area

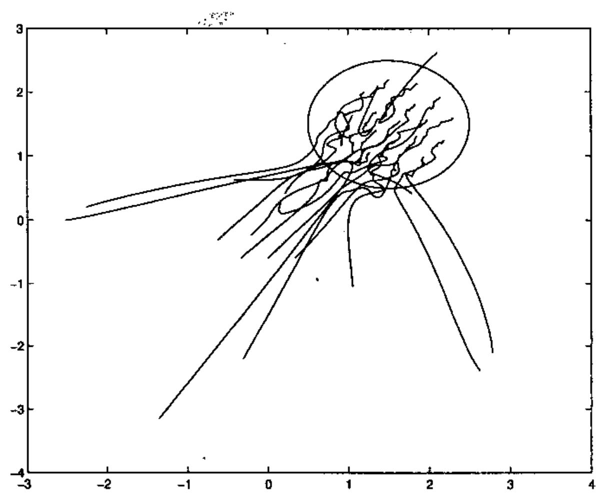

Figure 10: Nineteen robots (initially scattered) with desired attraction area

In Figure 10 we present simulation results for nineteen robots that initially are scattered in the environment. Used control combine interaction between robots and attraction area artificial potentials to execute task. Finally robots build ordered formation at the position $(1.5, 1.5)$.

In Figure 11 we present formation of nineteen robots that drives between two obstacles. Clearance between obstacle is narrower then the width of the formation. Initially formation is positioned at the origin of coordination frame.

In Figure 12 we present formation of nineteen robots that drives through the row of obstacles. Initially robots are positioned at the origin of coordination frame.

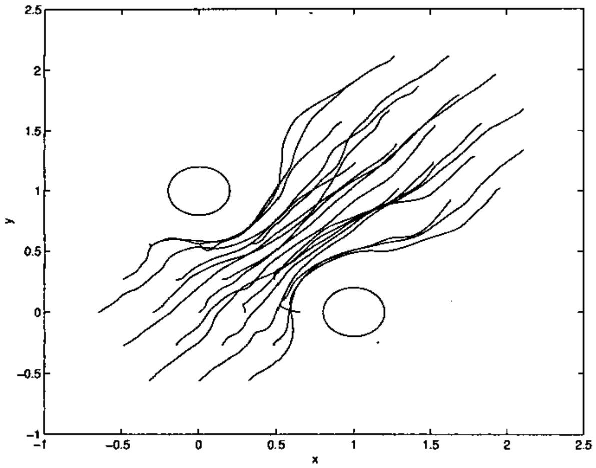

Figure 11: Formation of nineteen robots drives between two obstacles

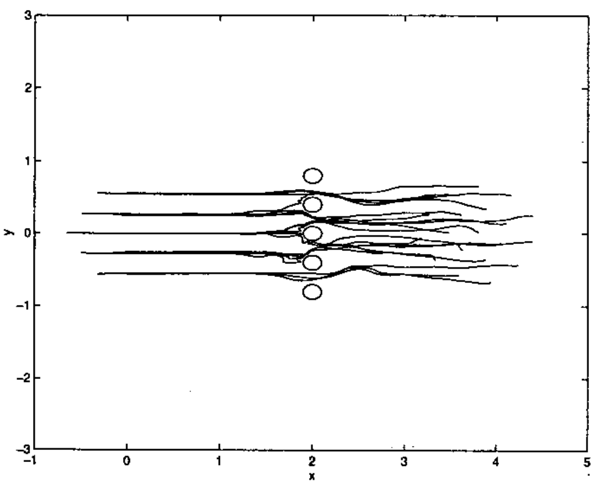

Figure 12: Formation of nineteen robots drives through the row of obstacles

4 Conclusion

Attraction area and repulsion to the obstacle artificial potential functions are extensions of control framework based on artificial potentials. Proposed approaches can be used to perform motion to the desired area, to build ordered formation in the desired area and to avoid collisions with the obstacles during task execution. Main advantage of presented approach is that control is entirely distributed. Second important advantage is that the method is easy scalable because behaviour of all robots of the formation is determined with one common rule.

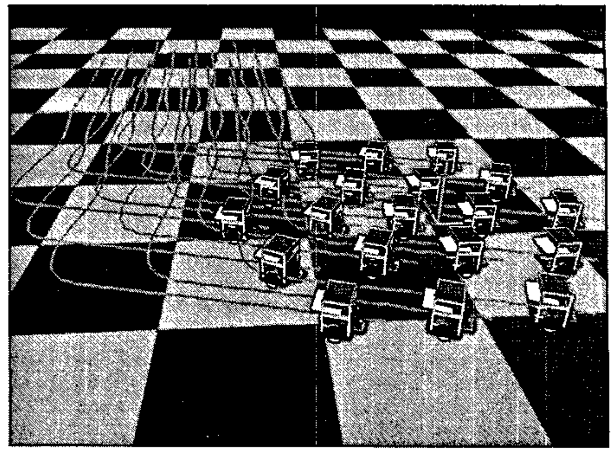

Figure 13: Screen-shot from the software visualization tool used to verify correctness of obtained simulation results (simulation for the same case as in Figure 8)

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