Robust Leader-following Formation Control of Multiple Mobile Robots using Lyapunov Redesign

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***Abstract*-This paper proposes a novel formation control strategy for multiple nonholonomic mobile robots using Lyapunov redesign. Leader-follower $(l-\varphi)$, and leader-leader $(l-l)$ formation control has been formulated based on the first-order kinematics model of robots. The effect of the absolute velocity of leader robot, which is difficult to accurately measure, treated as model uncertainty of the system. Using Lyapunov redesign technique, a formation controller is designed to stabilize the overall system and makes the formation control system robust against the unmeasured velocity of the leader robot. Our approach is implemented through simulation and the results are shown to verify its operation**.

Keywords: Leader-follower formation, Leader-leader formation, Lyapunov redesign technique.

I. INTRODUCTION

In recent years, there has been a tremendous interest in the coordinated control of multiple autonomous mobile vehicles. There are several advantages in using a team of robots in tasks such as search and rescue operations, mapping unknown or hazardous environments, and security. Various control methods have been proposed and applied to the formation design of robotic networks, such as behavior-based approach [7, 8], virtual structure approach [9-12], and the leader-follower approach [1-3]. Each of them has some advantages and weaknesses. The leader-follower formation control of mobile robots, one of the main approaches in this field, has been studied by many researchers. In a robot formation with leader-follower configuration, one or more robots are selected as leaders, which are responsible for guiding the formation, and the rest of the robots are controlled to follow the leaders. The control objective is to make the follower robots track the leaders with some prescribed offsets.

Desai *et al.*[1-3] presented a feedback linearization control method for the formation of nonholonomic mobile robots using the leader-follower approach. In [5], a robust method is presented to keep the follower in formation with the leader, and absolute acceleration of the leader robot is treated as model uncertainty of the system. Based on the second order kinematics model of formation, robust $(l-\varphi)$ formation using a sliding mode controller is considered. Dierks *et. al.* in [13], control a differentially steered robot by backstepping kinematics into dynamics. Li *et. al.* [6] presented a kinematics model for the leader following based formation control of tricycle mobile robots and a back stepping based stabilizing controller is derived under the conditions of perfect velocity tracking and no disturbances. Desai *et al.* in [2], used the kinematics model and graph theory to design a controller for multiple mobile robot formations, however there is no uncertainty was considered in the system. In [1] and [3], the $(l-l)$ and $(l-\varphi)$ formation control methodologies have been considered however the follower robot must know the absolute velocity of the leaders.

In this paper, a new formation methodology is presented which is based on the first-order kinematics model of robots. The key point of our algorithm is that the system is robust against absolute velocity of leader(s). First, the kinematics model for robots formation is formulated based on the relative motion states between the robots and the local motion of the follower robot. In Section 2, leader-follower formation and leader-leader formation is formulated based on kinematic equation of robots in formation. In Section 3, a formation controller is provided with a same design for both leader-follower and leader-leader formation. The proposed controller generates the commanded velocity for the follower robot and makes the formation control system robust to the effect of unmeasured velocity of the leader robots. Simulation results in section 4 are used to validate the proposed methodologies. Finally, conclusions are drawn in Section 5.

II. PROBLEM FORMULATION

A. Leader-follower formation

In this section, the kinematics model of the leader-follower robots in formation $(l-\varphi)$ is given. The leader-follower setup considered in this paper is presented in Fig. 1. The aim of the formation control is to make the follower robot R_2 track the leader robot R_1 with the desired separation l_{12}^d and the desired relative bearing φ_{12}^d between the robots. And a relative motion sensor is mounted at point c on the follower robot R_2.

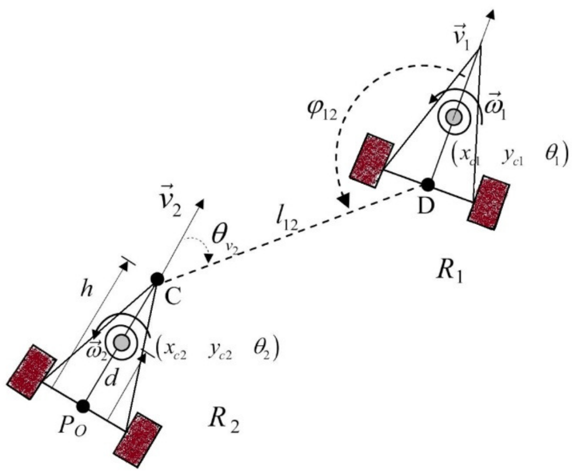


Fig. 1 Two robots in leader-follower formation

The kinematic equations of the robots are given by the following equations:

$$\begin{cases} \begin{pmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{pmatrix} = \begin{pmatrix} cos\theta_i & -dsin\theta_i \\ sin\theta_i & dcos\theta_i \end{pmatrix} \begin{pmatrix} v_i \\ \omega_i \end{pmatrix} \\ \dot{\theta}_i = \omega_i \end{cases} \quad (1)$$

Where $(x_{ci} \quad y_{ci})$ are the coordinates of the center of mass P_c in the world coordinates system, and θ_i is the heading angle of the robot. As shown in the Fig. 1, $\vec{v}_i$ and $\vec{\omega}_i$ are the linear and angular velocities of the robot R_i . P_0 is the intersection of the axis of symmetry with the driving wheel axis; d is the distance from the center of mass to point P_0 and h is the distance from the reference point C to point P_0 .

In [1-3], the kinematics of the leader and follower robots in formation is described by the following equations:

$$\dot{l}_{12} = v_2 cos\gamma - v_1 cos\psi_{12} + d\omega_2 sin\gamma$$

$$\dot{\psi}_{12} = \frac{1}{l_{12}}\{v_1 sin\psi_{12} - v_2 sin\gamma + d\omega_2 cos\gamma - l_{12}\omega_1\}$$

$$\dot{\theta}_{12} = \omega_1 - \omega_2 \quad (2)$$

Where $\theta_{12} = \theta_1 - \theta_2$ and $\gamma = \theta_{12} + \varphi_{12}$.

Let set $\theta_{v2} = \pi - \varphi_{12} - \theta_{12}$ that is the relative bearing between velocity v_2 and line l_{12} and then rewrite (2) in the following form:

$$M(l_{12}, \theta_{v2}) \begin{pmatrix} \dot{l}_{12} \\ \dot{\varphi}_{12} \end{pmatrix} - N(l_{12}, \theta_{12}, \theta_{v2}) \begin{pmatrix} v_1 \\ \omega_1 \end{pmatrix} = \begin{pmatrix} v_2 \\ \omega_2 \end{pmatrix} \quad (3)$$

Where the matrices M and N are defined as:

$$M = \begin{pmatrix} -cos\theta_{v2} & -l_{12} sin\theta_{v2} \\ (sin\theta_{v2})/h & -(l_{12} cos\theta_{v2})/h \end{pmatrix}$$

$$N = \begin{pmatrix} -cos\theta_{12} & l_{12} sin\theta_{v2} \\ -(sin\theta_{12})/h & (l_{12} cos\theta_{v2})/h \end{pmatrix}$$

Defining the state variables as $q = (l_{12} \quad \varphi_{12})^T$ and $\dot{q} = (\dot{l}_{12} \quad \dot{\varphi}_{12})^T$ and the input of the robot formation system as $u = (v_2 \ \omega_2)^T$ we can rewrite model (3) in the following form:

$$\dot{q} = G \times (u + \delta) \quad (4)$$

Where matrix G and vector δ are defined as:

$$G = M^{-1} = \begin{pmatrix} -cos\theta_{v2} & hsin\theta_{v2} \\ -sin\theta_{v2}/l_{12} & -hcos\theta_{v2}/l_{12} \end{pmatrix}$$

$$\delta = N(l_{12}, \theta_{12}, \theta_{v2}) \begin{pmatrix} v_1 \\ \omega_1 \end{pmatrix}$$

Remark 1. The last term δ in (4) represents the effect of the absolute velocity of the leader robot, which is difficult to accurately measure or estimate due to the limitation of the motion sensors and treated as model uncertainty of the system.

Position and velocity errors are defined as:

$$\tilde{q} = q - q^d$$
$$\dot{\tilde{q}} = \dot{q} - \dot{q}^d$$

We can rewrite model (4) in the following form:

$$\dot{x} = f + G \times (u + \delta) \quad (5)$$

Where $x = \tilde{q}$ and $f = -\dot{q}^d$.

B. leader-leader formation

In Fig. 2 we show a system with three nonholonomic mobile robots. In the leader-leader formation *(l-l)* control, the aim is to maintain the desired lengths, l_{13}^d and l_{23}^d of the third robot from its two leaders. In particular this requires that the two lead robots never separate by a distance greater than $l_{13}^d + l_{23}^d$.

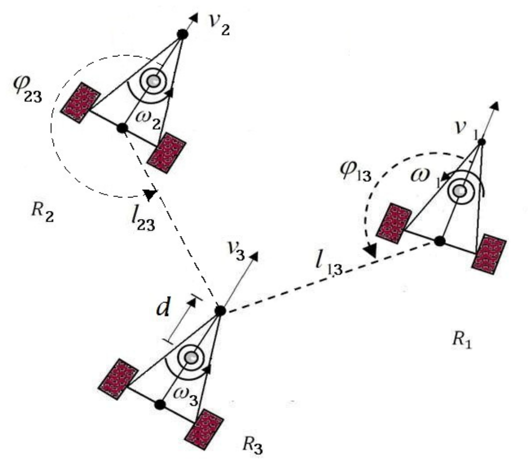


Fig. 2 Three robots in leader-leader formation

The kinematic equations of *l-l* formation is given by [1-3]:

$$\dot{l}_{13} = v_3 cos\gamma_1 - v_1 cos\varphi_{13} + d\omega_3 sin\gamma_1$$

$$\dot{l}_{23} = v_3 cos\gamma_2 - v_2 cos\varphi_{23} + d\omega_3 sin\gamma_2$$

$$\dot{\theta}_3 = \omega_3 \qquad (6)$$

Where, $\gamma_i = \theta_i + \varphi_{i3} - \theta_3 \quad (i = 1,2)$.

Now, (6) can be expressed as:

$$\begin{bmatrix} \dot{l}_{13} \\ \dot{l}_{23} \end{bmatrix} = \begin{bmatrix} cos\gamma_1 & dsin\gamma_1 \\ cos\gamma_2 & dsin\gamma_2 \end{bmatrix} \begin{bmatrix} v_3 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -cos\varphi_{23} & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} -cos\varphi_{13} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ \omega_1 \end{bmatrix} \qquad (7)$$

Remark 2. What is important in leader-leader formation is the distance between the follower robot with leader robots and as can be seen from (7), the impact of angular velocity of the leaders is not considered.

According to Remark 2, equation (7) can be simplified as below:

$$\begin{bmatrix} \dot{l}_{13} \\ \dot{l}_{23} \end{bmatrix} = \begin{bmatrix} cos\gamma_1 & dsin\gamma_1 \\ cos\gamma_2 & dsin\gamma_2 \end{bmatrix} \begin{bmatrix} v_3 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} -cos\varphi_{13} & 0 \\ 0 & -cos\varphi_{23} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad (8)$$

Solving equation (8) according to linear and angular velocity of follower robot yields

$$\begin{bmatrix} v_3 \\ \omega_3 \end{bmatrix} = M \begin{bmatrix} \dot{l}_{13} \\ \dot{l}_{23} \end{bmatrix} - N \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad (9)$$

Where the matrices M and N are defined as *:*

$$M = \frac{1}{dsin(\gamma_2 - \gamma_1)} \begin{bmatrix} dsin(\gamma_2) & -dsin(\gamma_1) \\ -cos\,(\gamma_2) & cos\,(\gamma_1) \end{bmatrix}$$

$$N = \frac{1}{dsin(\gamma_2 - \gamma_1)} \begin{bmatrix} -dcos(\varphi_{13}) sin\,(\gamma_2) & dsin(\gamma_1) cos\,(\varphi_{23}) \\ cos\,(\gamma_2) cos\,(\varphi_{13}) & -cos\,(\gamma_1) cos\,(\varphi_{23}) \end{bmatrix}$$

Defining the state variables as $q = (l_{13} \;\; l_{23})^T$ and $\dot{q} = (\dot{l}_{13} \;\; \dot{l}_{23})^T$ and the input of the robot formation system as $u = (v_3 \;\; \omega_3)^T$ we can rewrite model (9) in the following form:

$$\dot{q} = G \times (u + \delta) \qquad (10)$$

Where matrix G and vector δ are defined as:

$$G = M^{-1} = \begin{bmatrix} cos\gamma_1 & dsin\gamma_1 \\ cos\gamma_2 & dsin\gamma_2 \end{bmatrix}$$

$$\delta = N \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The last term δ in (10) represents the effect of the absolute linear velocity of the leader robots, which is treated as model uncertainty of the system.

Position and velocity errors are defined as:

$$\tilde{q} = q - q^d$$
$$\dot{\tilde{q}} = \dot{q} - \dot{q}^d$$

Similar to (5) we can rewrite model (10) in the following form:

$$\dot{x} = f + G \times (u + \delta) \qquad (11)$$

where $x = \tilde{q}$ and $f = -\dot{q}^d$.

III. THE PROPOSED ROBUST FORMATION CONTROL SCHEME

In this section, a robust controller is designed to stabilize the system in the presence of modeling uncertainties.

Suppose we have succeeded to design a feedback control law u_0 such that the origin of the nominal closed-loop system

$$\dot{x} = f + Gu_0 \quad (12)$$

is uniformly asymptotically stable. Suppose further that we know a Lyapunov function for (12); that is, we have a continuously differentiable function $V(t,x)$ that satisfies the inequalities

$$\alpha_1(\|x\|) \le V(t,x) \le \alpha_2(\|x\|) \qquad (13)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}[f + Gu_0] \le -\alpha_3\|x\| \qquad (14)$$

Where α_1 , α_2 and α_3 are class $\mathcal{K}$ functions. Assume that, with $u = u_0 + u_1$ the uncertain term δ satisfies the inequality

$$\|\delta(t,x)\| \le \rho(t,x) + k\|u_1\| \qquad 0 \le k < 1 \quad (15)$$

Where $\rho: [0,\infty) \times R^2 \to R$ is a nonnegative continuous function. The function ρ is a measure of the size of the uncertainty. It is important to emphasize that we will not require ρ to be small. We will only require it to be known. Our goal in this section is to show that with the knowledge of the Lyapunov function V, the function ρ, and the constant k in (15), we can design an additional feedback control u_1 such that the overall control $u = u_0 + u_1$ stabilizes the actual system (5) or (11) in the presence of the uncertainty. The design of v is called *Lyapunov redesign [14].*

Consider now the system (5) and (11) and apply the control $u = u_0 + u_1$ .The closed-loop system

$$\dot{x} = f + G(t,x)u_0 + G(t,x)[u_1 + \delta(t,x)] \qquad (16)$$

is a perturbation of the nominal closed-loop system (12). The stability of the resultant closed-loop (16) can be proved using the *Lyapunov redesign*.

Theorem 1. *Consider the systems* (5) and (11). *Let* $D \epsilon R^2$ *be a domain that contains the origin. And* u_0 *be a stabilizing feedback control law for the nominal system* (12) *with a Lyapunov function V(t, x) that satisfies (13) and* (14) *in 2-norm for all* $t \ge 0$*and all*$x \,\epsilon\, D$, *with some* $\mathcal{K}$ *class functions* α_1 , $\alpha_2$and α_3. *Suppose the uncertain term* δ *satisfies (15) in 2-norm for all* $t \ge 0$ *and al l*$x \,\epsilon\, D$. closed-loop system (16) under control law $u = u_0 + u_1$ is asymptotically stable at the origin with the u_1 selected as

$$u_1 = \frac{-\eta(t,x)}{1-k} \cdot \frac{w}{\|w\|_2} \qquad (17)$$

Where $w = G^T \left[\frac{\partial V}{\partial x}\right]^T$ and $\eta(t,x) \ge \rho(t,x)$ for all $(t,x) \epsilon [0,\infty] \times D$ and $0 \le k < 1$.

Proof. Let us calculate the derivative of $V(t,x)$ along the trajectories of (16). For convenience we will not write the argument of the various functions. We have

$$\dot{V} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}(f + Gu_0) + \frac{\partial V}{\partial x}G(u_1 + \delta)$$
$$\le -\alpha_3\|x\| + \frac{\partial V}{\partial x}G(u_1 + \delta)$$

Set $w^T = [\frac{\partial V}{\partial x}]G$ and rewrite the last inequality as

$$\dot{V} \le -\alpha_3\|x\| + w^T u_1 + w^T \delta$$

It is possible to choose u_1 to cancel the (destabilizing) effect of δ on $\dot{V}$.

Suppose inequality (15) is satisfied with$\| \,.\, \|_2$; that is,

$$\|\delta(t,x)\|_2 \le \rho(t,x) + k\|u_1\|_2 \qquad 0 \le k < 1$$

We have

$$w^T u_1 + w^T \delta \le w^T u_1 + \|w\|_2 \|\delta\|_2$$
$$\le w^T u_1 + \|w\|_2[\rho(t,x) + k\|u_1\|_2]$$

Choosing u_1 in (17) yields

$$w^T u_1 + w^T \delta \le -\frac{\eta}{1-k}\|w\|_2 + \rho\|w\|_2 + \frac{k\eta}{1-k}\|w\|_2$$
$$= -\eta\left(\frac{1}{1-k} - \frac{k}{1-k}\right)\|w\|_2 + \rho\|w\|_2$$
$$\le -\rho\|w\|_2 + \rho\|w\|_2 = 0$$

Hence, with the control (17), the derivative of $V(t,x)$ along the trajectories of the closed-loop system (16) is negative definite. Thus, Theorem 1 is proved. □

Converging the state of system to origin means $x = \tilde{q} \to 0$ thus $q(t) \to q^d(t)$ and so the follower robot tracks the leader(s) with the desired configuration.

Remark 3. The control law given by (17) is discontinuous functions of the state *x*. This discontinuity causes some theoretical as well as practical problems. Practically, the implementation of such discontinuous controllers is characterized by the phenomenon of *chattering*. We can approximate the discontinuous control law (17) by a continuous one and consider the below feedback control law:

$$u_1 = \begin{cases} -\frac{\eta(t,x)}{1-k} \cdot \frac{w}{\|w\|_2} & \eta(t,x)\|w\|_2 \geq \varepsilon \\ -\frac{\eta^2(t,x)}{1-k} \cdot \frac{w}{\varepsilon} & \eta(t,x)\|w\|_2 < \varepsilon \end{cases} \quad (18)$$

IV. SIMULATION RESULTS

We will now illustrate the effectiveness of the proposed control scheme. The proposed formation control algorithm is verified by the simulations carried out using MATLAB. In the first scenario, leader-follower formation is used. The desired formation is defined as:

$$q_{des} = [l_{12}^d \ , \ \varphi_{12}^d]^T = [200 \ , \ \frac{2\pi}{3}]^T$$

Where h is set at $h = 100\ mm$. The Lyapunov function can be written as

$$V = \frac{1}{2} x^T Q x \qquad (19)$$

Where Q is a symmetric positive definite matrix. The nominal control has been defined as

$$u_0 = G^{-1}[-\lambda x - f] \ \ , \ \ \lambda > 0 \qquad (20)$$

The nominal control u_0 in (20) leads to a linear closed-loop equation of the nominal system

$$\dot{x} = -\lambda x$$

The control parameters are selected as:

$$\alpha_1(r) = \frac{1}{3} r^2 \ , \alpha_2(r) = r^2 \ , \alpha_3(r) = 5r^2 , Q = I_{2\times 2}$$

$$\lambda = 10 \ , \rho(t,x) = 20 \ , \eta(t,x) = 40 \ , k = 0.5$$

Constituting control parameters in (17) yields:

$$u_1 = -80 \frac{G^T x}{\|G^T x\|_2} \qquad (21)$$

Then, the overall control is given by

$$u = u_0 + u_1 = G^{-1}[-\lambda x - f] - 80 \frac{G^T x}{\|G^T x\|_2} \qquad (22)$$

Under controller (22) Distance and relative bearing errors are shown in Fig. 3. State $x = \tilde{q} = q - q^d$ converges to zero and Both the relative distance and the relative bearing converge to their desired values and the follower robot track The leader with desired configuration.

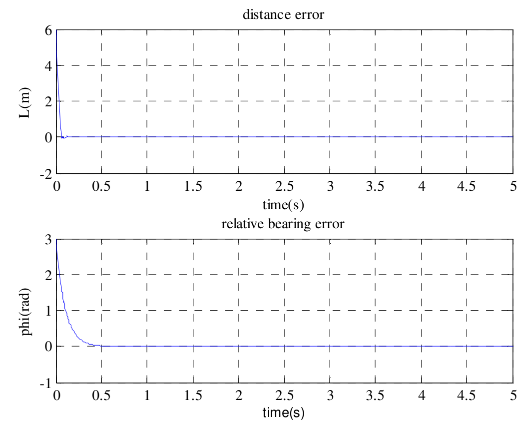
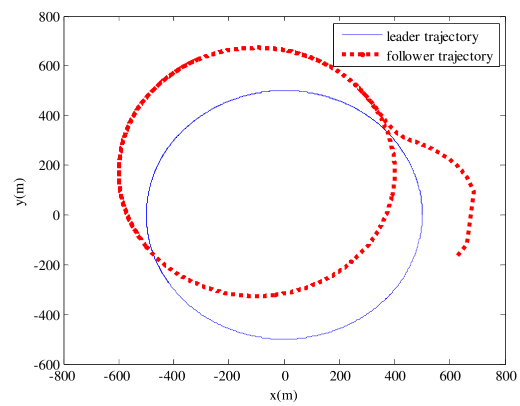
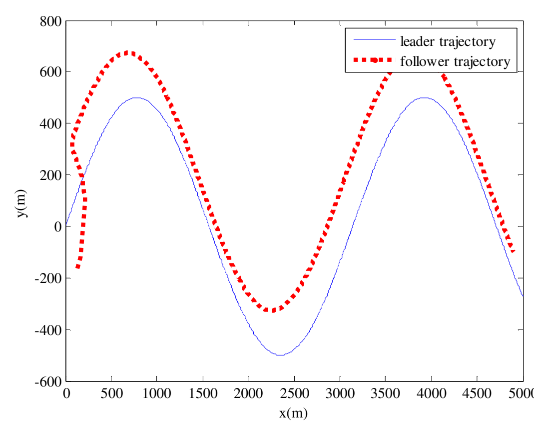


Fig. 3 The distance and relative bearing errors in leader follower formation

The trajectories of the two robots are depicted in Fig. 4. The leader robot has a circular path in Fig. 4.a whereas in Fig. 4.b has a cosine trajectory.



(a)



(b)

Fig. 4 Trajectory of leader-follower formation

In the second experiment, the leader-obstacle formation is used. The desired formation is defined as:

$$q_{des} = [l_{13}^d \;,\; l_{23}^d]^T = [200 \;,200]^T$$

Control parameters are similar to the first simulation. Distance errors are shown in Fig.5

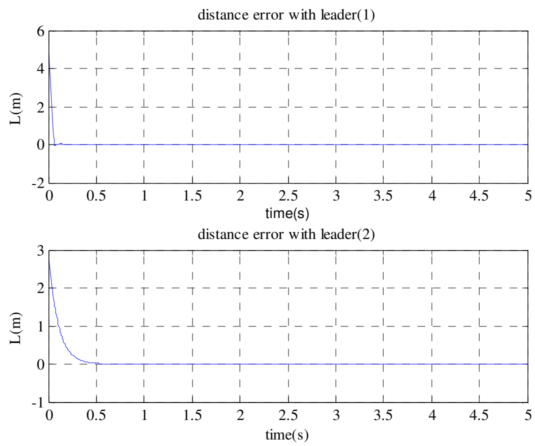


Fig. 5 The distance errors in leader-leader formation

V. CONCLUSIONS

In this paper, a new formation methodology has been presented which is based on the first order kinematics model of robots. . Using Lyapunov redesign technique, a robust controller has been proposed to control the leader-follower and leader-leader formations. The proposed controller makes the closed-loop formation system robust to the uncertainty associated with the absolute velocity of leader(s). Simulation results demonstrated the effectiveness of the proposed method.

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