Potential Field Based Formation Control in Trajectory Tracking and Obstacle Avoidance Tasks

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***Abstract*—An approach using potential functions applied to formation control (including aggregation, collision-free goal following behavior, and trajectory tracking one) is proposed. The formation control is considered as a special form of agent aggregation, where the final aggregated form has to constitute a particular previously determined geometrical configuration that is defined by a set of desired inter-agent distance values. This is achieved by defining an appropriate potential function, which reaches global minimum at the desired shape of formation. The innovation in this paper is the modification of the potential function by adding new parts that take into consideration: (1) the desired orientation of the formation as a whole; (2) obstacle avoidance behavior; and (3) goal following one. The control strategy is based on forcing the motion of the agents along the negative gradient of the potential field and taking into account the agents' non-holonomic kinematics. The validation of the proposed formation control is confirmed by simulations in MATLAB environment.**

Keyword: formation control; non-holonomic robots; potential field; trajectory tracking control; obstacle avoidance

I. INTRODUCTION

The increased interest in control of multi-robot systems is provoked by the abilities of these systems to work in the environments that are difficult for accessing or dangerous for the human beings (e.g. mobile sensor networks for topography or monitoring of the environment; underwater or space explorations; mastering new planets; etc.). In these applications the desired goal is a system, composed of a majority of comparatively simple cooperatively functioning robots because of its size, cost, flexibility and fault tolerance.

The approaches to robot formation control fall in general under three basic categories [1, 2]: "leader - follower" control [3, 4], behavioral control [5, 6] and virtual structures [7, 8]. Some of the more popular techniques (methods) used in these approaches are: potential field [7], graphs [9], sliding mode [7], etc. The "virtual structure" approach considers the whole formation as a rigid body. A "virtual leader" is usually synthesized [9, 10]. This approach is realized in the works of Gazi et al. [7, 8] as a combination of the potential field method and the sliding mode control.

The idea behind using potential fields is to contribute the design of formation control laws that are relatively simple at the individual level but lead to emergent intelligent behavior at the group level. This idea is inspired by observations of biologists over the social behavior of some animals (social insects, schools of fish, flocks of birds, etc.). Biological agents usually obey simple rules, possess limited local sensory information, which includes knowledge about their neighbors, and they have no information about the intelligent behavior of the group as a whole. For example, schooling in fish is a consequence of the tendency of the fish to avoid others that are close, to align its body with those at intermediate distances, and to move towards others that are far away [11]. These local traffic rules could be encoded by means of local artificial potentials that define interactions between neighboring agents. Each of these potentials is a function created to satisfy a definite criterion, which contributes to emergence of the desired group behavior.

In [7, 8] the design procedure is based on a potential function which is selected so that the corresponding potential field is attractive for agent pairs with large inter-agent distances (in order to result in aggregation) and repulsive for short inter-agent distances (in order to avoid collisions between the agents). The motion of the agents is along the negative gradient of the potential function. Due to the fact that potential functions may have many local minima, convergence to the desired formation may not be guaranteed. Hence, the main drawback of the approach is the "fixture in a local minimum of the potential function". Besides, the orientation of the obtained formation is arbitrary (not a priori determined).

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The problems, discussed in the most publications are related to formation control in homogeneous environments, including aggregation, trajectory tracking, etc. Less attention has been paid to formation maneuvering in the presence of obstacles. In [12, 13] a potential field based control has been applied to solve obstacle avoidance problems. To overcome the local minima problem the authors in [13] have proposed the use of simple goal-directed fields (that are not specifically designed to avoid obstacles or neighboring robots) in a combination with the simulated dynamics of a visco-elastic collision in the vicinity of obstacles.

The purpose of the paper is to propose a new potential function that takes into consideration both the distance between every two agents and the desired orientation of the formation as a whole, and then to adapt this function to a collision-free goal following problem. The proposed gradient-based formation creation algorithm is combined with trajectory tracking control of a non-holonomic autonomous robot and the result is validated in MATLAB environment.

II. POTENTIAL FUNCTIONS

A. Potential Function Based on Inter-Agent Distances

A previously defined geometrical shape of formation can be described by a set of desired inter-agent distances δ_{ij}. In Gazi et al. [7, 8] it has been shown for a certain class of potential functions $J(\mathbf{p})$ that if the agents move in the space $\mathbf{R}^n$ based on:

$$\dot{\mathbf{p}}_i = -\nabla_{p_i} J(\mathbf{p}), \qquad (1)$$

where $J: \mathbf{R}^{nN} \to \mathbf{R}$ is the potential function, $\mathbf{p} = [\mathbf{p}_1^{\mathrm{T}}, ..., \mathbf{p}_N^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{nN}$ is the lumped vector of the positions $\mathbf{p}_i \in \mathbf{R}^n$ of the agents $(i = 1, ..., N)$, then aggregation in the desired formation will be achieved. The potential functions considered in [7, 8] satisfy

$$\nabla_{p_i} J(p) = \sum_{j=1, j\neq i}^{N} \mathbf{g}(\mathbf{p}_i - \mathbf{p}_j), \ i = 1, ..., N, \qquad (2)$$

where $\mathbf{g}: \mathbf{R}^n \to \mathbf{R}^n$ are odd functions, that represent the attraction and repulsion relationships between the individuals. Moreover, it has been assumed that for any $\overline{\mathbf{p}} \in \mathbf{R}^n$ (where $\overline{\mathbf{p}} = \mathbf{p}_i - \mathbf{p}_j$), $\mathbf{g}(\overline{\mathbf{p}})$ satisfies

$$\mathbf{g}(\overline{\mathbf{p}}) = -\overline{\mathbf{p}}[\mathbf{g}_a(\|\overline{\mathbf{p}}\|) - \mathbf{g}_r(\|\overline{\mathbf{p}}\|)], \qquad (3)$$

where $\mathbf{g}_a(\|\overline{\mathbf{p}}\|)$ represents the attractive part which dominates on large distances and $\mathbf{g}_r(\|\overline{\mathbf{p}}\|)$ represents the repulsive part which dominates on short distances. One potential function which satisfies these assumptions is [14, 7]:

$$J_\delta(\mathbf{p}) = \frac{1}{2}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{1}{2}(\|\mathbf{p}_i - \mathbf{p}_j\|^2 - \delta_{ij}^{\ 2})^2, \qquad (4)$$

for which (2) (under: $J \leftarrow J_\delta$) is satisfied with

$$\mathbf{g}_\delta(\mathbf{p}_i - \mathbf{p}_j) = (\mathbf{p}_i - \mathbf{p}_j)(\|\mathbf{p}_i - \mathbf{p}_j\|^2 - \delta_{ij}^{\ 2}), \qquad (5)$$

where δ_{ij} are the desired inter-agent distances of the formation. The procedure is not limited to that potential function only and could be used with other potential functions. In the particular case (this paper) $\mathbf{p} = [\mathbf{p}_1^{\mathrm{T}}, ..., \mathbf{p}_N^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{2N}$ and $\mathbf{p}_i = [x_i, y_i]^{\mathrm{T}} \in \mathbf{R}^2$ for $i = 1, ..., N$.

B. Potential Function Related to the Formation Orientation

Let agent 1 plays the role of a leader and all the other agents have to draw up in formation following the leader. Consider the following potential function [15]:

$$J_\alpha(\mathbf{p}) = \frac{1}{2}\sum_{i=2}^{N}(\theta_{\text{form, ref}} + \alpha_{i1} - \theta_{i1})^2, \qquad (6)$$

where α_{i1} is the desired angle at which the agent i sees the agent 1 with respect to the mobile basis $\{i, x_m, y_m\}$, and θ_{i1} - the actual angle, with respect to an inertial Cartesian frame (immovable). Fig.1 and tabl.1 illustrate a particular case of a triangular formation consisting of $N = 6$ agents.

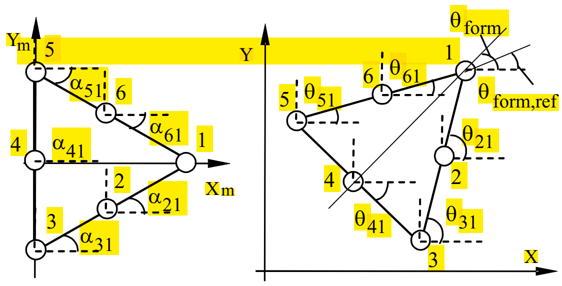

Figure 1. Angles between the leader and the followers

TABLE I. ANGLES AT WHICH THE FOLLOWERS SEE THE LEADER

α_{i1} Robot #	α_{21} 2	α_{31} 3	α_{41} 4	α_{51} 5	α_{61} 6
1	$\pi/6$	$\pi/6$	0	$-\pi/6$	$-\pi/6$

The actual angles θ_{i1} $(i = 2, ..., N)$ can be calculated using positions of the agent i and agent 1:

$$\theta_{i1} = \arctan\frac{(y_1 - y_i)}{(x_1 - x_i)}, \bmod 360^0, \ i = 2, ..., N. \qquad (7)$$

The derivatives of the potential function with respect to x_i and y_i ($i = 2, ..., N$) are:

$$\frac{\partial J_\alpha}{\partial x_i} = -\frac{\theta_{form,ref} + \alpha_{i1} - \arctan\frac{(y_1 - y_i)}{(x_1 - x_i)}}{(x_1 - x_i)^2 + (y_1 - y_i)^2}(y_1 - y_i);$$

$$\frac{\partial J_\alpha}{\partial y_i} = \frac{\theta_{form,ref} + \alpha_{i1} - \arctan\frac{(y_1 - y_i)}{(x_1 - x_i)}}{(x_1 - x_i)^2 + (y_1 - y_i)^2}(x_1 - x_i). \quad (8)$$

C. Potential Functions for Obstacle Avoidance and Goal Following Behaviors

The proposed obstacle-oriented potential function has the following form:

$$J_o(\mathbf{p}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{o=1}^{n_{o,i}}\frac{1}{2}(\|\mathbf{p}_i - \mathbf{p}_o\|^2 - \delta_{io}{}^2)^2, \quad (9)$$

where $n_{o,i}$ is the number of obstacles detected in the vicinity of the agent i, δ_{io} is the range of obstacle detection of the agent i ($i = 1,...,N$) and also the desired distance between the agent i and the obstacle o, and $\mathbf{p}_o$ is the position of the obstacle o (the coordinates of the obstacle's point, which is nearest to the i-th agent). The idea behind this potential function is rather to derive wall following behavior than the obstacle avoidance one, because the former turns out to be more successful in the presence of big and complex obstacles (Π-shaped obstacles).

Each term of the gradient of (9) has the same form as (5):

$$\mathbf{g}_o(\mathbf{p}_i - \mathbf{p}_o) = (\mathbf{p}_i - \mathbf{p}_o)(\|\mathbf{p}_i - \mathbf{p}_o\|^2 - \delta_{io}{}^2). \quad (10)$$

The used goal-oriented potential function is:

$$J_g(\mathbf{p}) = \frac{1}{4}(\|\mathbf{p}_l - \mathbf{p}_g\|^2 - \delta_{lg}^2)^2, \quad (11)$$

where $\delta_{lg} = 0$, $\mathbf{p}_l$ is the position of the robot that plays the role of a formation's leader (according to tabl.1 $l = 1$), and $\mathbf{p}_g$ is the position of the goal, which could be a static or moving goal. The gradient of (11) is:

$$\nabla_{p_l} J_g(p) = \mathbf{g}_l(\mathbf{p}_l - \mathbf{p}_g) = (\mathbf{p}_l - \mathbf{p}_g)\|\mathbf{p}_l - \mathbf{p}_g\|^2. \quad (12)$$

D. Total Potential Function

The total potential function has to satisfy multiple criteria directed to: (1) creation of a formation with a specific shape; (2) maintenance of a previously determined orientation of a whole formation; (3) obstacle avoidance behavior; and (4) goal following behavior. Actually the potential functions of the leader and the followers have to be different because their basic behavior is different. The task of the followers is to follow the leader, to maintain the specific formation (shape, or shape and orientation), and to avoid collisions with obstacles. Hence, the total potential function $J_{\mathrm{F}}(\mathbf{p})$ of the followers can be obtained by the weighted sum of the potential functions (4), (6), and (9):

$$J_{\mathrm{F}}(\mathbf{p}) = k_\delta J_\delta(\mathbf{p}) + k_\alpha J_\alpha(\mathbf{p}) + k_o^{\mathrm{F}} J_o(\mathbf{p}), \quad (13)$$

where k_δ, k_α, and k_o^{F} are positive weight coefficients.

The leader has to perform a collision-free goal following behavior and/or trajectory tracking one. Usually, the leader does not receive any information feedback from the followers. Thus, its total potential function $J_{\mathrm{L}}(\mathbf{p})$ is the weighted sum of the potential functions (9) and (11):

$$J_{\mathrm{L}}(\mathbf{p}) = k_o^{\mathrm{L}} J_o(\mathbf{p}) + k_g J_g(\mathbf{p}), \quad (14)$$

where k_o^{L} and k_g are positive weight coefficients. The desired formation will be achieved if the agents move in the work space following the negative valued gradient of the correspondent potential function $J_{\mathrm{L}}(\mathbf{p})$ and $J_{\mathrm{F}}(\mathbf{p})$ as it is shown in (1).

III. Formation Trajectory Tracking Control

A. Trajectory Tracking Control of a Non-holonomic Mobile Robot

The position of the robot in an inertial Cartesian frame $\{\mathrm{O,X,Y}\}$ (Fig.2) is completely specified by the *posture* $\mathbf{q} = [x, y, \theta]^{\mathrm{T}}$ where (x, y) and θ are the coordinates of the reference point C, and the orientation of the mobile basis $\{\mathrm{C}, \mathrm{X}_C, \mathrm{Y}_C\}$ with respect to the inertial basis, respectively. The motion of the mobile robot is controlled by its *linear velocity* υ and *angular velocity* ω, which are also functions of time. The kinematics of the vehicle is defined by Jacobian matrix $\mathbf{J}$, which transforms velocities $\mathbf{v} = [\upsilon,\ \omega]^{\mathrm{T}}$ expressed in mobile basis into velocities $\dot{\mathbf{q}}$ expressed in Cartesian one [16]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \dot{\mathbf{q}} = \mathbf{Jv} = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix}. \quad (15)$$

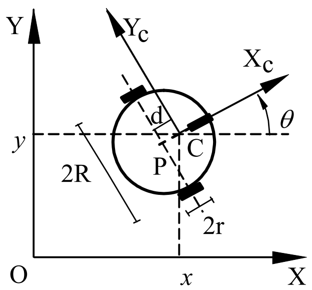

Figure 2. Autonomous mobile robot

The block-diagram for the mobile robot tracking control system is represented in Fig.3. The trajectory tracking problem is posed as in Kanayama et al. [17] and Fierro and Lewis [16]. The tracking error posture

$\mathbf{e}(t)=[e_1(t),e_2(t),e_3(t)]^T$ is expressed on the basis of a mobile frame linked to the mobile robot [16]:

$$\mathbf{e}=\mathbf{T}_e(\mathbf{q}_r-\mathbf{q}),$$
$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_r-x \\ y_r-y \\ \theta_r-\theta \end{bmatrix}, \quad (16)$$

where $\mathbf{q}_r(t)=[x_r(t),y_r(t),\theta_r(t)]^{\mathrm{T}}$ is the reference posture of the robot. The auxiliary velocity control input (target velocity vector $\mathbf{v}_t(t)$) that achieves tracking for (16) is given by [17]:

$$\mathbf{v}_t=f(\mathbf{e},\mathbf{v}_r)=\begin{bmatrix} \upsilon_r\cos e_3+k_1e_1 \\ \omega_r+k_2\upsilon_r e_2+k_3\upsilon_r\sin e_3 \end{bmatrix}, \quad (17)$$

where k_1 , k_2 and k_3 are positive constants, and $\upsilon_r>0$. Block $\mathbf{T}_v$ transforms target velocities $\mathbf{v}_t$ into real velocities $\mathbf{v}$ of the vehicle. In simulations for the sake of simplicity it is assumed that $\mathbf{v}=\mathbf{v}_t$ ($\mathbf{T}_v$ is the identity transformation), called "perfect velocity tracking" [17, 16].

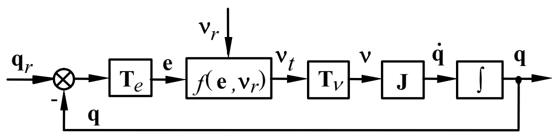

Figure 3. Tracking control structure

B. Robots' Reference Trajectories Generation Using Artificial Potential Functions

The formation control is designed via forcing the motion of each individual agent along the negative gradient of the potential function $J(\mathbf{p})$ ($J_{\mathrm{L}}(\mathbf{p})$ and $J_{\mathrm{F}}(\mathbf{p})$), i.e. forcing each agent to obey equation (1) where J is as defined in (13) and (14). This will be achieved by the use of the tracking controller (17) where reference posture $\mathbf{q}_r$ and reference velocities $\mathbf{v}_r$ will be determined by the potential field's gradient vector. Consider a reference cart that will produce these reference values:

$$\dot{x}_r=\upsilon_r\cos\theta_r,\ \dot{y}_r=\upsilon_r\sin\theta_r,\ \dot{\theta}_r=\omega_r, \quad (18)$$
$$\mathbf{q}_r=[x_r,y_r,\theta_r]^T,\ \mathbf{v}_r=(\upsilon_r,\ \omega_r)^{\mathrm{T}},$$

with $\upsilon_r>0$ for all t. Let the smooth velocity control input $\mathbf{v}_t=f(\mathbf{e},\mathbf{v}_r)$ (17) is such that $\lim\limits_{t\to\infty}(\mathbf{q}_r-\mathbf{q})=0$, and let the gradient of the potential at $\mathbf{p}_i$ satisfies (1) and (18), i.e.

$$-\nabla_{p_i}J(\mathbf{p})=\begin{bmatrix} -\partial J(\mathbf{p})/\partial x_i \\ -\partial J(\mathbf{p})/\partial y_i \end{bmatrix}=\begin{bmatrix} \upsilon_{r,i}\cos\theta_{r,i} \\ \upsilon_{r,i}\sin\theta_{r,i} \end{bmatrix}. \quad (19)$$

In other words, the reference direction of motion can be determined by the direction of the negative gradient, and the reference linear velocity - by its magnitude:

$$\theta_{r,i}=\arctan\left(\frac{\partial J(\mathbf{p})/\partial y_i}{\partial J(\mathbf{p})/\partial x_i}\right),\ \mathrm{mod}\,360^0$$
$$\upsilon_{r,i}=\|\nabla_{p_i}J(\mathbf{p})\|. \quad (20)$$

The magnitude of the gradient could be very big or very small and thus the value of the linear velocity can make difficult controlling the physical vehicles. Because of this reason the linear velocity is proposed to be normalized so that it is limited between zero and $\upsilon_{\max}$:

$$\upsilon_{r,i}^{\mathrm{norm}}=\frac{\upsilon_{r,i}}{\max\limits_i\,[\upsilon_{r,1}..\upsilon_{r,i}..\upsilon_{r,N}]}\upsilon_{\max},\ i=1,\ldots,N. \quad (21)$$

In the time-discrete domain $x_{r,i}$ and $y_{r,i}$ components of the reference postures of the agents can be calculated as follows:

$$\begin{aligned} x_{r,i}(k)&=x_{r,i}(k-1)+\upsilon_{r,i}^{\mathrm{norm}}(k)\cos\theta_{r,i}(k)\Delta t \\ y_{r,i}(k)&=y_{r,i}(k-1)+\upsilon_{r,i}^{\mathrm{norm}}(k)\sin\theta_{r,i}(k)\Delta t \end{aligned}. \quad (22)$$

The leader is controlled independently from the other robots. It tracks a preliminarily determined reference trajectory and avoids collisions with randomly appearing obstacles. For this purpose the trajectory points (time-indexed path) are considered as moving goals creating artificial potentials (11), which can be combined with obstacle oriented potentials (9) to obtain the leader's total potential function (14).

IV. Simulation Results and Discussions

To illustrate the performance of the proposed formation control using artificial potential functions and gradient descent techniques it was simulated in MATLAB environment and tested on several examples. The kinematics model of a mobile robot with two driving wheels and a front free wheel was used (the agent). For simulations the parameters of trajectory tracking controller (Fig.3) were chosen to correspond to a critical damping case [17] ($k_1=10$, $k_2=64$ and $k_3=16$), reference linear velocities for all agents were calculated as shown in (21), and angular velocity was chosen to be $\omega_{\mathrm{r}}=0$ rad/s. The maximum reference linear velocity of the robots was $\upsilon_{\max}=0.6$ m/s. The sampling time was set to $\Delta t=0.01$ s. The formation shape was chosen to be a 6-agent-based triangle with the following inter-agent distance matrix:

$$\mathbf{D}=\begin{bmatrix} 0 & 1 & 2 & \sqrt{3} & 2 & 1 \\ 1 & 0 & 1 & 1 & \sqrt{3} & 1 \\ 2 & 1 & 0 & 1 & 2 & \sqrt{3} \\ \sqrt{3} & 1 & 1 & 0 & 1 & 1 \\ 2 & \sqrt{3} & 2 & 1 & 0 & 1 \\ 1 & 1 & \sqrt{3} & 1 & 1 & 0 \end{bmatrix}.$$

In the beginning the movement of a formation of material points along the negative gradient of the potential function $J_\delta(\mathbf{p})$ (4) was investigated. The simulation results are shown in Fig.4, where the case (a) treats uniform agents, while in the case (b) the position of the agent 1 is fixed and the agent is maintained immovable as long as the formation was created. In the first case the position and orientation of the created formation were not previously determined, while in the second case the position of the formation was given a priori by fixing the position of the agent 1 (that will be considered as a leader in the next simulations). The measuring units on the two axes of the agent's rectangular work area are meters. The symbol 'o' filled with black color denotes the agent 1 (the leader). The experiments were repeated using a combination of the two potential functions (4) and (6): $k_\delta J_\delta(\mathbf{p}) + k_\alpha J_\alpha(\mathbf{p})$, where the weight coefficients were set to $k_\delta = 1$ and $k_\alpha = 20$. For various set-points of formation orientation the results are shown in Fig.5. Since in these experiments the position of the agent 1 was not maintained fixed, the formation was created in a position unknown a priori, but with a given orientation $\theta_{\text{form, ref}}$.

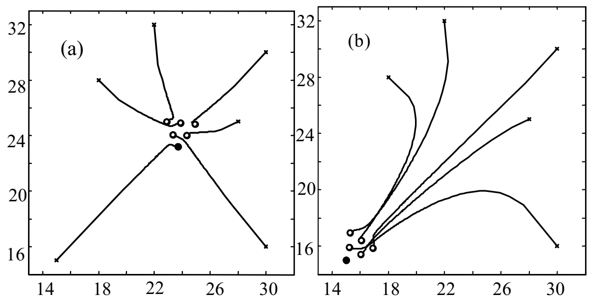

Figure 4. Creation of a formation of material points using an inter-agent distances based potential function J_δ: (a) uniform agents, and (b) immovable agent 1.

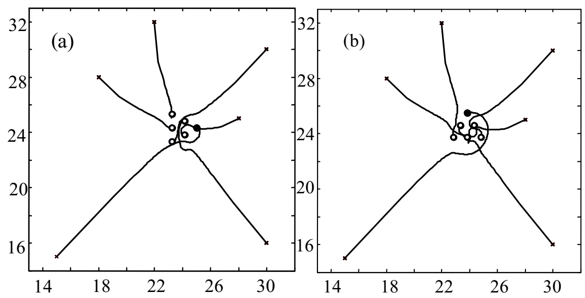

Figure 5. Creation of a formation of material points using the potential function $k_\delta J_\delta(\mathbf{p}) + k_\alpha J_\alpha(\mathbf{p})$, where $k_\delta = 1$ and $k_\alpha = 20$: (a) $\theta_{\text{form, ref}} = 0$; (b) $\theta_{\text{form, ref}} = \pi/2$ rad.

The trajectory tracking behavior of the formation as a whole was simulated in the presence of agents' non-holonomic kinematics. The set-points of the agents' controllers (17) were calculated using the negative gradient of the total potential functions (13) and (14). At the first time the obstacle avoidance behavior was rejected by setting the weight coefficients $k_o^{\text{L}} = k_o^{\text{F}} = 0$. The results are shown in Fig.6, where the agents are gathered together in a formation, which tracks a reference trajectory marked with a dash line. The case shown in Fig.6a does not take into account controlling the formation's orientation, i.e., the coefficient $k_\alpha = 0$ (the rest coefficients were $k_\delta = 10$, and $k_g = 100$). Fig.6b illustrates an experiment where the formation's orientation was maintained constant all the time - $\theta_{\text{form, ref}} = 3\pi/4$ rad ($k_\alpha = 100$).

In the end the trajectory tracking behavior of the formation was combined with obstacle avoidance one and the results are shown in Fig. 7. The parameter δ_{io} in (9) and (10) was chosen to be $\delta_{io} = 2.2$ m. In the case (a) the formation as a whole does not maintain any reference orientation ($k_\alpha = 0$), and in the case (b) the formation's orientation was constant all the time - $\theta_{\text{form, ref}} = -\pi/2$ rad ($k_\alpha = 100$). The weight coefficients for the obstacle-oriented part are different in the total potential function of the leader and the followers - $k_o^{\text{L}} = 100$ and $k_o^{\text{F}} = 3$, respectively. The implemented experiments confirmed the efficiency of the formation control using the proposed total potential function that manipulates the reference orientation of the formation and preserves it from collisions with obstacles.

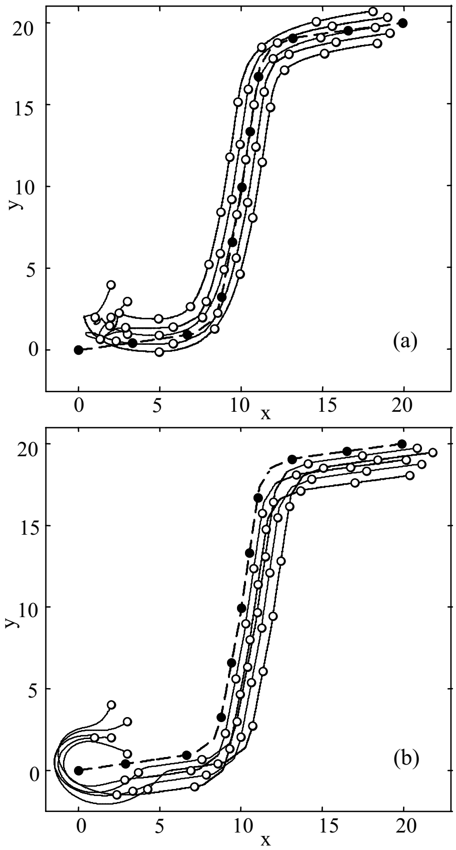

Figure 6. Tracking control of a formation of non-holonomic mobile agents using the potential functions (13) and (14) with coefficients $k_o^{\text{L}} = k_o^{\text{F}} = 0$, $k_\delta = 10$, $k_g = 100$, and: (a) $k_\alpha = 0$; and (b) $k_\alpha = 100$, and $\theta_{\text{form, ref}} = 3\pi/4$ rad.

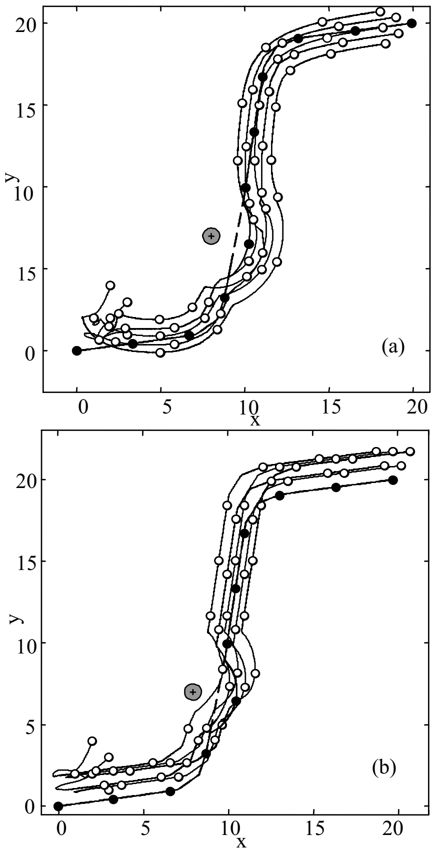

Figure 7. Tracking control combined with obstacle avoidance behavior of a formation of non-holonomic mobile agents using the potential functions (13) and (14) with coefficients $k_o^{\mathrm{L}} = 100$ and $k_o^{\mathrm{F}} = 3$ ("L" – for the leader, and "F" – for the followers), $k_\delta = 10$, $k_g = 100$, and: (a) $k_\alpha = 0$; (b) $k_\alpha = 100$, and $\theta_{\mathrm{form,\,ref}} = -\pi/2$ rad

V. Conclusion

An approach for formation control of a group of non-holonomic agents is presented. It combines a few types of potential functions using parts, related to: (1) inter-agent distances; (2) the formation orientation; (3) the obstacle avoidance behavior; and (4) the goal following one. The first of these parts is well known in the literature, while the second one together with its combinations with obstacle avoidance part and the other parts are the innovations proposed in the paper. They let the formation maintain and manipulate its own orientation during the motion, which is a trajectory tracking collision-free motion. The part related to formation's orientation causes the local minima problem. The performance of the formation navigation depends on the choice of the weight parameters in the potential functions. Future work will extend the control law taking into consideration the robot dynamics, as well as including the investigation of the stability of the formation control, and further verification of that control on physical mobile robots.

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