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Robust Distance-Angle Leader-Follower Formation Control of Non-Holonomic Mobile Robots

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***Abstract*— In this paper, we study formation control problem for a multi-robot system. Based on a leader-follower architecture, each follower keeps a desired relative position in the coordinate frame of a leader which determines the formation trajectory. It is supposed that each robot is exposed to an external disturbance. Hence, a nonlinear robust tracking controller is proposed to achieve a desired formation. Simulation results confirm the accuracy of the proposed formation control strategy.**

***Index Terms*— Nonholonomic mobile robots, formation control, leader-follower architecture, robust control, sliding mode control.**

I. Introduction

The problem of autonomous mobile robots is an interesting field of research in the area of robotics. This interest is motivated by the ability of this vehicles to operate remotely in hazardous and unstructured environments, especially when high degree of autonomy is required [1]–[3].

Recently, coordination of mobile robots in groups is a solution to increase robustness, exibility, and reliability of robots in doing missions. One of most important problems in the area of mobile robotics is formation control which can be defined as a collective behavior when mobile robots keep relative positions or distances from each other. Optimization in energy consumption, communication network, and robots field of view are the main advantages of formation keeping in multi-robot systems, and has a lot of civil applications such as rescue missions, surveillance, coverage missions, etc [4]–[6].

There are three main architectures for formation control problem namely: leader-follower, virtual, and behavioral each of whom has its own advantages and disadvantages [7], [8].

In the *leader-follower* architecture which this paper is devoted to, a robot is considered as a leader tracking a desired trajectory. Then, other robots follow it as followers while keeping a desired relative position or distance. Simplicity in analysis and scalability is the main advantages of this architecture. However, the dependency of the formation on a single robot is a drawback of this architecture [9]–[11].

In the *virtual* architecture, the formation is considered as a single rigid body and the desired trajectory is defined for this rigid body by defining a virtual leader for all the robots, and hence the formation behavior can be predictable. In this condition, high precision formation keeping can be realized. However, the virtual leader cannot make decision autonomously which is the main issue in the virtual structure [12]–[15]. Although in some missions, the virtual leader can be determined via robots interaction, autonomously [8].

The main idea of the *behavioral* architecture is devoting different behaviors such as collision avoidance, formation keeping, and target seeking for each single robot. In this condition, the behavior of each robot is the weighted linear combination of all the behaviors. However, complex mathematical formalization and stability analysis of the multi-robot system is the main drawback of this architecture [8], [16]–[18].

In general, two classes of strategies, namely, distance-angle and distance-distance, are introduced in the literature for maintaining formations based on the leader-follower architecture [19], [20]:

- Distance-angle: In this class of strategies, each follower should keep a relative distance and angle in the coordinate frame of the leader. In this condition, each follower can keep a fixed point in the leader coordinate frame.
- Distance-distance: Based on limitations in sensory systems of the mobile robots, it may be just possible to control relative distances. In this condition, it is necessary for each follower to follow more than one leading robot to achieve a specific formation. However, high communication cost is required.

In this paper, an novel distance-angle leader-follower structure for a multi-robot system is studied. By using a decentralized controller, each robot keeps a desired distance and angle in the leader coordinate frame. Therefore, based on the proposed architecture, each follower keeps a desired formation in the coordinate frame of the leader tracking a desired trajectory. In this condition, by using sliding mode controllers, formation keeping is guaranteed in the presence of bounded external disturbances.

The rest of the paper is organized as follows: In Section II, the dynamical equations of the mobile robots are introduced. In Section III, the formation architecture and the control strategy are presented. A simulation example is provided in Section IV. Finally, conclusions are given in Section V.

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II. Mobile Robots Dynamical Model

We consider a team of nonholonomic mobile robots containing a leader labeled by ℓ and N followers labeled by 1 to N which each robot dynamics are described in the global coordinate frame as follows:

$$\begin{aligned} \dot{x}_i &= V_i \cos(\theta_i), \\ \dot{y}_i &= V_i \sin(\theta_i), \\ \dot{V}_i &= \lambda_v (V_{ci} - V_i), \\ \dot{\theta}_i &= \lambda_\theta (\theta_{ci} - \theta_i) \end{aligned} \tag{1}$$

where $i \in \{\ell, 1, 2, \cdots, N\}$, (x_i, y_i) is the position of the ith robot, V_i is the speed, θ_i is heading angle, and λ_v and λ_θ are coefficients related to the robot structure. Moreover, V_{ci} and θ_{ci} are the speed and heading angle commands, respectively.

The objective is to find proper control commands V_{ci} and θ_{ci} such that the followers keep a desired formation with respect to the leader in presence of external disturbance. The following section introduces a control strategy to achieve this goal.

III. Formation Control Strategy

In this section, we design a robust formation control strategy such that each follower keeps a desired formation in the coordinate frame of the leader. Inspired by the virtual structure introduced in [21] and [22], we design a leader-follower structure; then, the robust formation control strategy is introduced.

A. Formation Control Architecture

To achieve a formation, each follower should maintain a desired distance and angle with respect to the leader coordinate frame. Assuming that the followers receives the state information of the leader, considering the architecture depicted in Fig. 1, the desired position of the ith follower is as follows:

$$\begin{bmatrix} x_{id} \\ y_{id} \end{bmatrix} = \begin{bmatrix} x_\ell \\ y_\ell \end{bmatrix} + \begin{bmatrix} \cos(\theta_\ell) & -\sin(\theta_\ell) \\ \sin(\theta_\ell) & \cos(\theta_\ell) \end{bmatrix} \begin{bmatrix} x_{id}^\ell \\ y_{id}^\ell \end{bmatrix}$$

where (x_{id}, y_{id}) denotes the follower desired position in the global coordinate frame, and $(x_{id}^\ell, y_{id}^\ell)$ indicates the follower desired position in the leader coordinate frame which is determined by a desired formation distance $d_{\ell i}$ and a desired formation angle $\theta_{\ell i}$ with respect to the leader as follows:

$$\begin{aligned} x_{id}^\ell &= -d_{\ell i} \cos(\theta_{\ell i}), \\ y_{id}^\ell &= -d_{\ell i} \sin(\theta_{\ell i}). \end{aligned} \tag{2}$$

Therefore, the objective is to design the control commands V_{ci} and θ_{ci} for the ith follower in order to follow the desired position defined in (2).

B. Formation Control Strategy

A popular technique for robust tracking control of nonlinear systems is sliding-mode. The base of this technique is to lead a system tracking error toward a stable surface in order

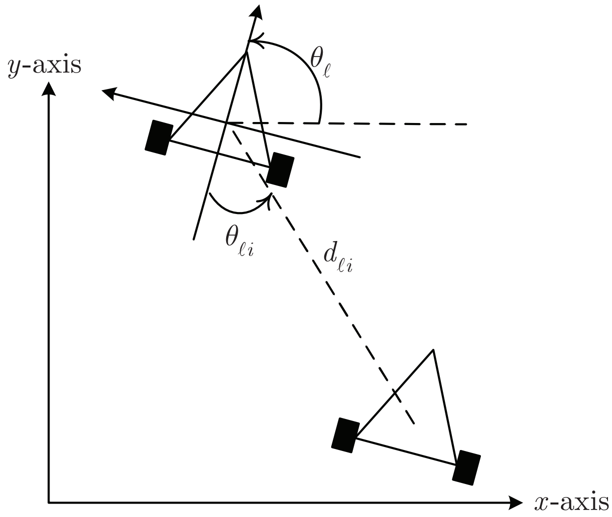

Fig. 1: The leader-follower formation architecture.

to make the error zero. Considering (1), at first let us restate the ith follower dynamical equation as follows:

$$\begin{aligned} \ddot{x}_i &= \lambda_v (V_{ci} - V_i) \cos\theta_i - V_i \lambda_\theta (\theta_{ci} - \theta_i) \sin\theta_i, \\ \ddot{y}_i &= \lambda_v (V_{ci} - V_i) \sin\theta_i + V_i \lambda_\theta (\theta_{ci} - \theta_i) \cos\theta_i \end{aligned}$$

where if we define $p_i = [x_i \ y_i]^T$ and $u_i = [V_{ci} \ \theta_{ci}]^T$, it can be stated as follows:

$$\ddot{p}_i = f_i + G_i u_i \tag{3}$$

where

$$\begin{aligned} f_i &= \begin{bmatrix} f_{xi} \\ f_{yi} \end{bmatrix} = \begin{bmatrix} -\lambda_v V_i \cos(\theta_i) + \lambda_\theta V_i \theta_i \sin(\theta_i) \\ -\lambda_v V_i \sin(\theta_i) - \lambda_\theta V_i \theta_i \cos(\theta_i) \end{bmatrix}, \\ G_i &= \begin{bmatrix} \lambda_v \cos(\theta_i) & -V_i \lambda_\theta \sin(\theta_i) \\ \lambda_v \sin(\theta_i) & V_i \lambda_\theta \cos(\theta_i) \end{bmatrix}. \end{aligned}$$

In this condition, we consider the external disturbance as additive terms affecting the followers dynamics as follows:

$$\begin{aligned} \dot{V}_i &= \lambda_v (V_{ci} - V_i) + w_{vi}, \\ \dot{\theta}_i &= \lambda_\theta (\theta_{ci} - \theta_i) + w_{\theta i} \end{aligned}$$

where w_{vi} and $w_{\theta i}$ are bounded external disturbances. In this condition, (3) can be restated as follows:

$$\ddot{p}_i = f_i + G_i u_i + w \tag{4}$$

where

$$w = \begin{bmatrix} w_{vi} \cos(\theta_i) - w_{\theta i} \sin(\theta_i), \\ w_{vi} \sin(\theta_i) + w_{\theta i} \cos(\theta_i). \end{bmatrix}$$

The objective is to design sliding mode controllers for control commands V_{ci} and θ_{ci} to guarantee the convergence of the following tracking errors zero:

$$\begin{aligned} e_{xi} &= x_i - x_{id}, \\ e_{yi} &= y_i - y_{id}. \end{aligned}$$

To design the sliding mode controllers, we consider the following sliding surfaces:

$$\begin{aligned} s_{xi} &= \dot{e}_{xi} + \lambda_{ix} e_{xi}, \\ s_{yi} &= \dot{e}_{yi} + \lambda_{iy} e_{yi} \end{aligned}$$

where $\lambda_{xi}, \lambda_{yi} > 0$. In this condition, the tracking errors go to zero as s_{xi} and s_{yi} go to zero. Hence, to guarantee the convergence of the sliding surfaces to zero, the following inequalities should be satisfied:

$$\begin{aligned} s_{xi}\dot{s}_{xi} &< \eta_{xi}|s_{xi}|, \\ s_{yi}\dot{s}_{yi} &< \eta_{yi}|s_{yi}| \end{aligned} \tag{5}$$

where $\eta_{xi}, \eta_{yi} > 0$. To achieve this goal, we define a new control input $\acute{u}_i = [\acute{u}_{xi}\ \acute{u}_{yi}] = G_i u_i$. Hence, considering (4), one can say:

$$\ddot{p}_i = f_i + \acute{u}_i + w. \tag{6}$$

In this condition, the following control law guarantees the convergence of the sliding surfaces to zero:

$$\begin{aligned} \acute{u}_{xi} &= -f_{xi} - \lambda_{xi} - k_{xi}\mathrm{sgn}(e_{xi}), \\ \acute{u}_{yi} &= -f_{yi} - \lambda_{yi} - k_{yi}\mathrm{sgn}(e_{yi}) \end{aligned}$$

where to satisfy (5), the following inequality should be satisfied [23]:

$$\begin{aligned} k_{xi} &\geq (\eta_{xi} + F_{xi}), \\ k_{yi} &\geq (\eta_{yi} + F_{yi}) \end{aligned}$$

where F_{xi} are F_{yi} should satisfy the following inequality:

$$\begin{aligned} |f_{xi} + w_{vi}\cos(\theta_i) - w_{\theta i}\sin(\theta_i)| &\leq F_{xi}, \\ |f_{yi} + w_{vi}\sin(\theta_i) + w_{\theta i}\cos(\theta_i)| &\leq F_{yi}. \end{aligned}$$

Remark 1: It should be noted that to obtain the ith mobile robot control input u_i from $\acute{u}$, we should have $\det(G)_i \neq 0$. On the other hand,

$$\det(G_i) = \lambda_v \lambda_\theta V_i.$$

Hence, the proposed control strategy should be employed when $V_i \neq 0$. However, to cope with this issue, we can choose $\det(G_i) = \epsilon$ when $\det(G_i) \leq \epsilon$.

Remark 2: Using $\mathrm{sgn}(.)$ function provides chattering in the the control commands which may not be practical. Therefore, to tackle this issue, $\tanh(.)$ function can be employed instead which is a smooth approximation of $\mathrm{sgn}(.)$ function.

In the following section, the performance of the proposed formation control strategy is investigated via an example.

IV. Simulation Results

In this section, we provide a simulation example to verify the validity of the proposed leader-follower formation control strategy. We have considered a team of mobile robots containing a leader and three followers to achieve a square formation with edge of $\sqrt{2}$ as depicted in Fig. 2. The robots initial positions are assumed to be $(4, 1, 0.5, \frac{\pi}{4})$, $(2, 0, 0.6, 0.1)$, $(4, -3, 0.1, -0.2)$, and $(5, -2, 0.3, -0.3)$, respectively. Moreover, it is supposed that each robot is exposed to an unknown disturbance with known bound as follows:

$$\begin{aligned} \dot{V}_i &= \lambda_v(V_{ci} - V_i) + \sin(t), \\ \dot{\theta}_i &= \lambda_\theta(\theta_{ci} - \theta_i) + \sin(t). \end{aligned}$$

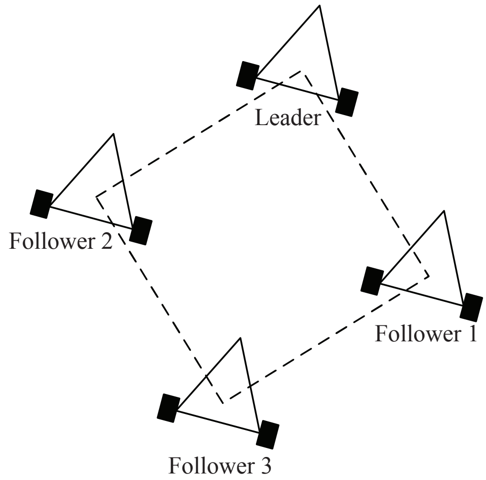

Fig. 2: The square formation of the four mobile robots.

Let us choose $\lambda_{xi} = \lambda_{yi} = 5, i \in \{\ell, 1, 2, 3\}$, and it is assumed that $\lambda_v = \lambda_\theta = 10$, and we have used $\tanh(5x)$ instead of $\mathrm{sgn}(x)$ function to reduce chattering in the control commands. Without loss of generality, the leader desired trajectory is defined as follows:

$$\begin{aligned} x_\ell &= -5\sin(\frac{t}{5}), \\ y_\ell &= \frac{t}{5}. \end{aligned}$$

Furthermore, to achieve a square formation, the formation parameters $\theta_{\ell i}$ and $d_{\ell i}$, $i \in \{1, 2, 3\}$, are designed as follows:

$$\begin{aligned} &\theta_{\ell 1} = \sqrt{2}, d_{\ell 1} = \frac{\pi}{4}, \\ &\theta_{\ell 2} = \sqrt{2}, d_{\ell 2} = -\frac{\pi}{4}, \\ &\theta_{\ell 3} = 2, d_{\ell 3} = 0. \end{aligned}$$

In this condition, the trajectories of the mobile robots are depicted in Fig. 3.

Moreover, the convergence of the formation distances and the formation angles to the desired values $d_{\ell i}$ and $\theta_{\ell i}$, $i \in \{1, 2, 3\}$, are depicted in Fig. 4 and Fig. 5, respectively, confirming that the square formation is achieved. Furthermore, the speed and heading angle commands are depicted in Fig. 6 and Fig. 7, respectively.

V. Conclusions

In this paper, decentralized formation control of mobile robots in a distance-angle leader-follower architecture was studied. Each follower kept a desired distance and angle in the coordinate frame of a leader to keep a desired formation. By using sliding-mode controllers, it was shown that the formation can be achieved in the presence of bounded external disturbances. Simulation results for a team of four mobile robots to achieve a square formation confirmed the accuracy of the proposed control strategy.

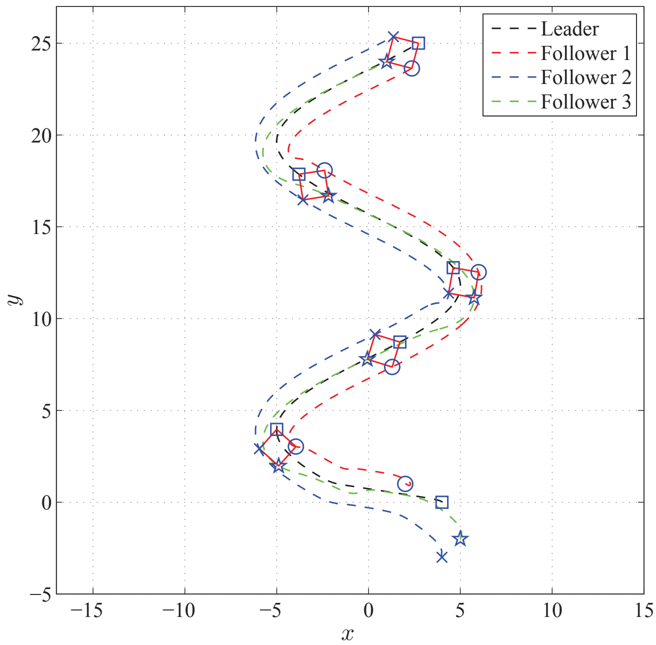

Fig. 3: The square formation of the robots.

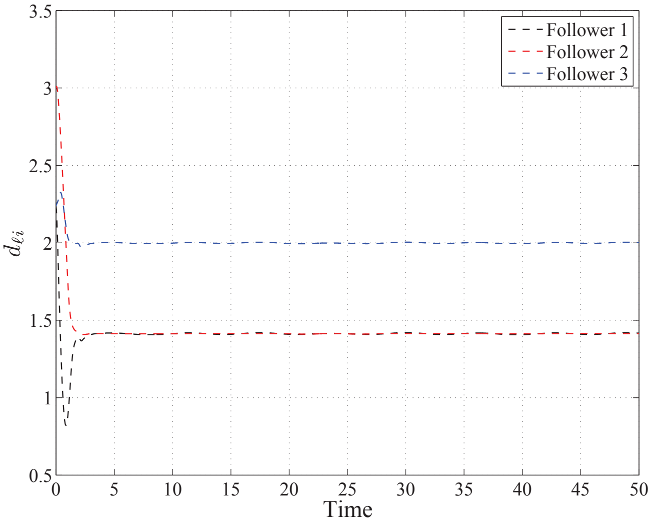

Fig. 4: The convergence of the formation distances to the desired values $d_{\ell i}, i \in \{1, 2, 3\}$.

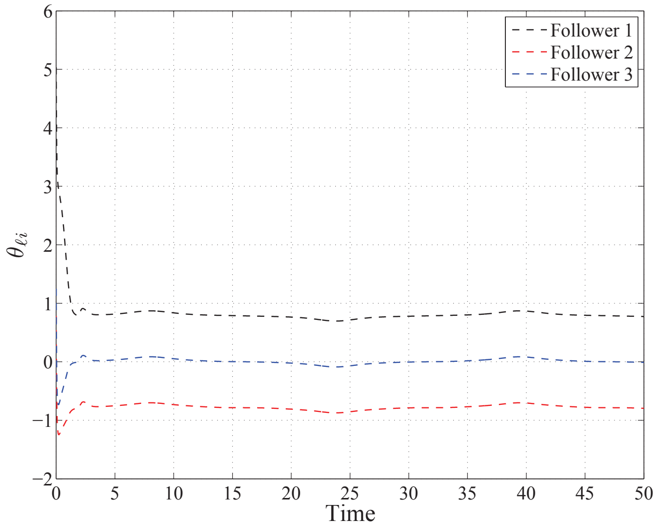

Fig. 5: The convergence of the formation angles to the desired values $\theta_{\ell i}, i \in \{1, 2, 3\}$.

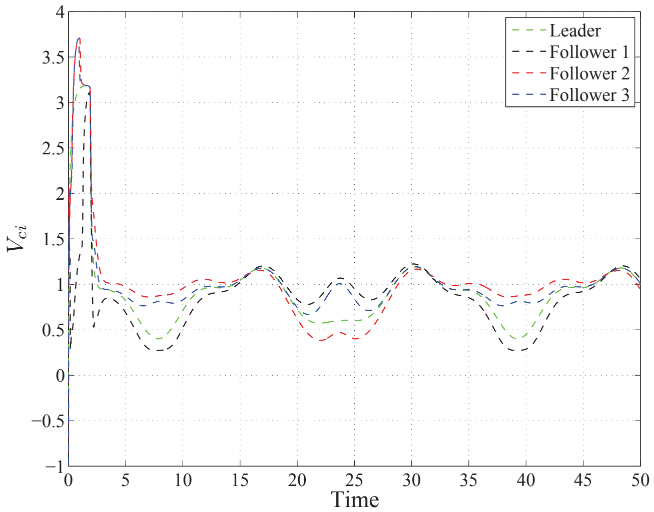

Fig. 6: The speed commands.

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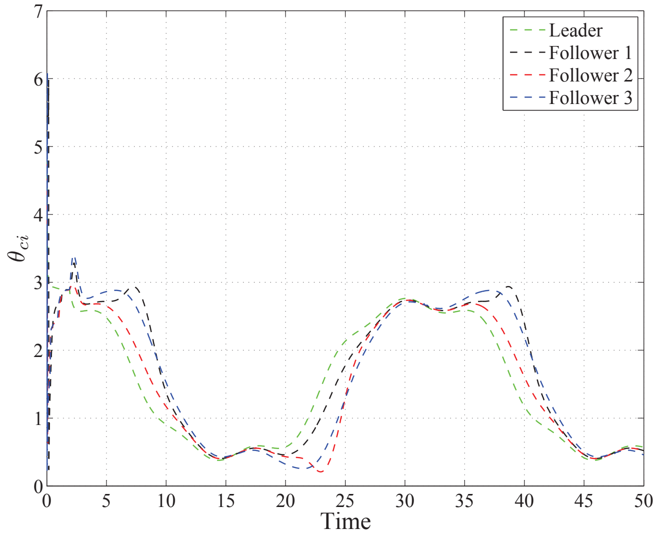

Fig. 7: The heading angle commands.

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