

Decentralized formation-tracking control of autonomous vehicles on straight paths

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Abstract—We present a simple leader follower tracking controller for autonomous vehicles following straight paths. The difficulty of this problem lies in the fact that the system is not controllable. We show that global tracking may be achieved with a controller which has a property of persistency of excitation, tailored for nonlinear systems. Roughly speaking the stabilization mechanism relies on exciting the system by an amount that is proportional to the tracking error. Then, the method is used to solve the problem of formation tracking of multiple vehicles interconnected on the basis of a spanning-tree topology *i.e.*, each vehicle communicates with one leader and one follower only.

I. INTRODUCTION

Over the last few years, formation control design of mobile robots is of great interest for researchers of several disciplines due to the necessity of developing complex tasks in an autonomous way. The control strategy relies on different objectives such as: obstacle-avoidance, target-seeking, leader-follower formations just to mention a few –see [1], [23] and [15]. Furthermore, the communication among the agents, which may either be all-to-all or fail among some agents, represents another challenge in the design of control strategies.

Several control approaches have been used to design formation-tracking controllers such as: backstepping [3], sliding mode [4], artificial neural network [9], feedback linearization [14], [6], *etc.* The seminal paper [14] shows how a feedback controller guarantees that a mobile robot follows a desired reference which is generated by “virtual” robot; the convergence proof is based on local stability results for time-varying systems.

In [11], the formation control problem in a leader-follower configuration without the need of knowing the leader’s velocity is studied. There are two leaders which govern the group’s motion. The stability analysis shows that the triangular formation is stable while the colinear one is not. In [5] the method of feedback linearization is used to design control laws to solve the problem of leader-follower for multiple robots under different geometries of formation. The motion of the leader robot is computed by minimizing a suitable cost function. In [10] the nonlinear formation control law for the coordination of a group of N mobile robots force the robots’ relative positions with respect to the center of the virtual

structure. Using the backstepping technique and Lyapunov’s direct method the control problem is solved for the follower robot. The proposed method guarantees asymptotic stability for the closed-loop error system dynamics. The authors of [8] use consensus-based controllers combined with a cascades-based approach to tracking control, resulting in a group of linearly coupled dynamical systems. Stability analysis relies on cascaded systems and nonlinear synchronization theory.

In this paper we present a simple controller for formation-tracking of mobile robots moving along straight lines. First, we study the problem of smooth tracking control of non-holonomic robots on straight-line paths in a leader-follower formation. Then, we extend this result to formation-tracking control of an arbitrary number of vehicles in a leader-follower configuration. We solve the problem by observing that a recursive implementation of the tracking leader-follower controller naturally leads to a spanning-tree communication topology. The rest of the paper is organized as follows. In Section II we establish the principal leader-follower tracking control problem and its extension to the case of formation-tracking control. Section III contains some illustrative simulation results and Section IV offers our concluding remarks.

II. MAIN RESULTS

A. Leader-follower tracking control

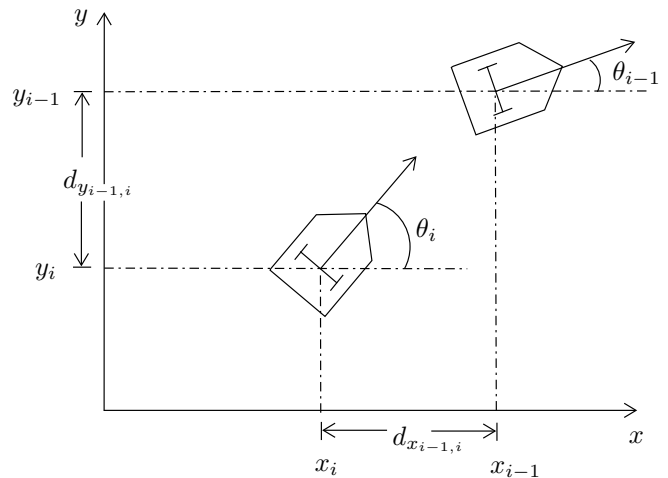


Fig. 1. Generic representation of a leader-follower configuration. For a swarm of n vehicles, any geometric topology may be easily defined by determining the position of each vehicle relative to its leader. This does not affect the kinematic model.

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After the seminal paper [14] the tracking control problem for mobile robots may be reformulated as that of controlling a robot in a leader-follower configuration as shown in Figure 1. Hence, the tracking control problem consists, for a mobile robot with kinematic model,

$$\begin{aligned}\dot{x}_1 &= v_1 \cos(\theta_1) \\ \dot{y}_1 &= v_1 \sin(\theta_1) \\ \dot{\theta}_1 &= w_1\end{aligned}$$

with forward velocity v_1 and angular velocity w_1 as control inputs, in following a fictitious vehicle

$$\dot{x}_0 = v_0 \cos(\theta_0) \quad (1a)$$

$$\dot{y}_0 = v_0 \sin(\theta_0) \quad (1b)$$

$$\dot{\theta}_0 = w_0.$$

That is, v_0 and w_0 are, respectively, forward and angular velocity references. From a control viewpoint, the goal is to steer to zero the differences between the Cartesian coordinates of the two robots, as well as orientation angles,

$$\begin{aligned}p_{1x} &= x_0 - x_1 - d_{xi-1,i} \\ p_{1y} &= y_0 - y_1 - d_{yi-1,i} \\ p_{1\theta} &= \theta_0 - \theta_1.\end{aligned}$$

Then, for the purpose of analysis we transform the error coordinates $[p_{1x}, p_{1y}, p_{1\theta}]$ of the leader robot from the global coordinate frame to local coordinates fixed on the robot that is,

$$\begin{bmatrix} e_{1x} \\ e_{1y} \\ e_{1\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} \\ p_{1y} \\ p_{1\theta} \end{bmatrix}. \quad (2)$$

In the new coordinates, the error dynamics between the virtual reference vehicle and the follower becomes

$$\dot{e}_{1x} = w_1 e_{1y} - v_1 + v_0 \cos e_{1\theta} \quad (3a)$$

$$\dot{e}_{1y} = -w_1 e_{1x} + v_0 \sin e_{1\theta} \quad (3b)$$

$$\dot{e}_{1\theta} = w_0 - w_1. \quad (3c)$$

The tracking control problem is transformed into that of stabilizing the origin for the error dynamics (3). This problem has been studied thoroughly; many contributions have been published using nonlinear control [13], cascades-based control [21], vision-based control [7] to mention a few. In [21] the control inputs

$$v_1 = v_0 + c_2 e_{1x}$$

$$w_1 = w_0 + c_1 e_{1\theta}$$

were proposed. As it was first observed for the first time in [21] and repeated in many subsequent references (e.g., [17], [11], [2]), this is of particular interest since it leads to the closed-loop system in cascaded form,

$$\begin{bmatrix} \dot{e}_{1x} \\ \dot{e}_{1y} \end{bmatrix} = \begin{bmatrix} -c_2 & w_0 \\ -w_0 & 0 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix} + d(t, e_\theta) + c_1 \begin{bmatrix} e_{1y} \\ -e_{1x} \end{bmatrix} e_{1\theta} \quad (4a)$$

$$\dot{e}_{1\theta} = -c_1 e_{1\theta} \quad (4b)$$

where we defined the interconnection term

$$d(t, e_\theta) := \begin{bmatrix} v_0(t)(\cos e_{1\theta} - 1) \\ v_0(t) \sin e_{1\theta} \end{bmatrix}. \quad (5)$$

Then, from adaptive control textbooks –see [20], [12], we know that if $e_{1\theta} = 0$ the origin of (4a) is uniformly exponentially stable if $c_2 > 0$ and w_1 is persistently exciting. In turn, the latter may be ensured provided that and the angular reference velocity possesses the same property that is, $w_0(s) := \psi(s)^2$ where

$$\int_t^{t+T} \psi(s)^2 ds \geq \mu, \quad \forall t \geq 0 \quad (6)$$

for some positive constants μ and T . A cascades argument establishes the stability result for the overall system.

The simplicity of these controllers comes at the price of ruling out the possibility of straight-path references, in which case $w_0 \equiv 0$. The persistency-of-excitation condition has been relaxed, for instance in [2], [16] where complex nonlinear time varying controls are designed to allow for reference velocity trajectories that converge to zero. It is worth to emphasize that [16] covers the case when also the forward velocity v_0 may converge to zero that is, tracking control towards a fixed point. In [2] the controller is designed so as to make the robot *converge* to the straight-line trajectory resulting in a path that makes it go back and forth on the path.

In this paper we solve the formation and tracking control problems on straight lines with fairly simple time-varying control laws inspired from [21]. Let $w_0 \equiv 0$ and let us introduce the control inputs

$$v_1 = v_0(t) + c_2 e_{1x}, \quad c_2 > 0 \quad (7a)$$

$$w_1 = h(t, e_{1y}) + c_1 e_{1\theta}, \quad c_1 > 0 \quad (7b)$$

where h is bounded, locally of linear order in e_{1y} , and continuously differentiable. Interestingly, the only difference between this controller and that of [21] is that instead of w_0 we use the term $h(t, e_{1y})$. Roughly speaking, it is required that h is persistently exciting in the case that the errors e_{1y} are different from zero; a precise definition is given farther below.

We show that the controller (7) stabilizes globally and uniformly the error dynamics. In order to understand the stabilization mechanism of the controller (7) it is convenient to examine the closed-loop equations, which result from using (7) in (3) to obtain

$$\dot{e}_{1x} = w_1 e_{1y} - c_2 e_{1x} + v_0 [\cos e_{1\theta} - 1] \quad (8a)$$

$$\dot{e}_{1y} = -w_1 e_{1x} + v_0 \sin e_{1\theta} \quad (8b)$$

$$\dot{e}_{1\theta} = -c_1 e_{1\theta} - h(t, e_{1y}). \quad (8c)$$

This system may be rewritten in compact form as

$$\begin{bmatrix} \dot{e}_{1x} \\ \dot{e}_{1y} \end{bmatrix} = \begin{bmatrix} -c_2 & w_1 \\ -w_1 & 0 \end{bmatrix} \begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix} + d(t, e_{1\theta}) \quad (9a)$$

$$\dot{e}_{1\theta} = -c_1 e_{1\theta} - h(t, e_{1y}) \quad (9b)$$

where we purposefully dropped the arguments of w_1 .

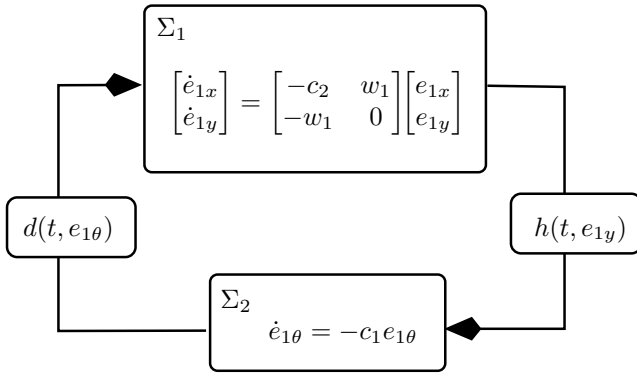


Fig. 2. “Small-gain” feedback representation of the closed-loop system with a persistently exciting controller

We are interested in establishing uniform global asymptotic stability of the origin of $(e_{1x}, e_{1y}, e_{1\theta}) = (0, 0, 0)$. To that end, we observe that the system (9) consists in the feedback interconnection of two systems as illustrated in Figure 2. Roughly speaking, after adaptive control systems theory, the system in the center upper block is uniformly asymptotically stable at the origin, provided that $c_2 > 0$ and w_1 is persistently exciting, globally Lipschitz and bounded. On the other hand, the origin of the system in the lower-center block is, clearly, exponentially stable if $c_1 > 0$. As a matter of fact, it may also be established that each of these subsystems is input to state stable. Moreover, the interconnection terms h and d are both uniformly bounded and satisfy $d(t, 0) \equiv 0$, $h(t, 0) \equiv 0$. Thus, the interconnected system (9) may be regarded as the feedback interconnection of two input to state stable (ISS) systems. Consequently, stability of the origin of (9) may be concluded invoking the small-gain theorem for ISS systems.

Since h and w_1 are functions of time and the state, we can no longer use the usual definition of persistency of excitation. Instead, we use a relaxed notion which was originally introduced in [19]; the following is a refined definition reported in [22].

Definition 1 (uδ-Persistency of excitation) Let $f(\cdot, \cdot)$ be such that the system $\dot{x} = f(t, x)$, with state $x = [x_1^\top \ x_2^\top]^\top$ and solution $x(t) = x(t, t_o, x_o)$ starting at $(t_o, x_o) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^n$ is forward complete.

The pair (ϕ, f) is called uniformly δ -persistently exciting (uδ-PE) with respect to x_1 if, for each r and $\delta > 0$, there exist constants $T(r, \delta)$ and $\mu(r, \delta) > 0$ such that, for all $(t_o, x_o) \in \mathbb{R}_{\geq 0} \times B_r$, all corresponding solutions satisfy¹

$$\left\{ \min_{s \in [t, t+T]} |x_1(s)| \geq \delta \right\} \Rightarrow \left\{ \int_t^{t+T} \phi(\tau, x(\tau, t_o, x_o)) \phi(\tau, x(\tau, t_o, x_o))^\top d\tau \geq \mu I \right\} \quad (10)$$

¹Notice that, in what the definition concerns, unicity of solutions is not required.

for all $t \geq t_o$.

In words, the pair (ϕ, f) is uδ-PE if the function $\phi(\cdot, x(\cdot))$ is PE in the usual sense of adaptive control, uniformly in initial conditions $(t_o, x_o) \in \mathbb{R}_{\geq 0} \times B_r$, whenever the trajectory $x(\cdot)$ is away from a δ -neighbourhood of the origin. For simplicity we may also say, with an abuse of terminology, that the function ϕ is uδ-PE in the understanding that the pair satisfies Definition 1. For instance, the function $\phi(t, x) := \psi(t) |x|$ is uδ-PE if ψ satisfies (6). In particular, the function $\phi(t) = \sin(t)\alpha(x)$ with α continuous, zero at zero, is uδ-PE.

There are several properties of uδ-PE functions which are useful in control design for nonholonomic systems; these are reported in [18]. One of them is that if w_1 is uδ-PE then there exists a function \tilde{w}_1 which depends only on time and which is persistently exciting in the usual sense that is,

$$\int_t^{t+T'} |\tilde{w}_1(\tau)|^2 d\tau \geq \mu' \quad \forall t \geq 0 \quad (11)$$

for some T' and $\mu' > 0$. Moreover, \tilde{w}_1 may be purposefully constructed to satisfy

$$\tilde{w}_1(t) := h(t, e_{1y}(t)) + c_1 e_{1\theta}(t) \quad \forall t : |e_{1y}(t)| \geq \delta. \quad (12)$$

Even though the function \tilde{w}_1 is parameterized by δ it is guaranteed that for any $\delta > 0$ there exists \tilde{w}_1 satisfying all of the above.

This property is useful because, for any δ and for all t such that $|e_{1y}(t)| \geq \delta$, the trajectories of Σ_1 in Figure 2 coincide with those of

$$\begin{bmatrix} \dot{e}_{1x} \\ \dot{e}_{1y} \end{bmatrix} = \underbrace{\begin{bmatrix} -c_2 & \tilde{w}_1(t) \\ -\tilde{w}_1(t) & 0 \end{bmatrix}}_{\tilde{A}_1(t)} \underbrace{\begin{bmatrix} e_{1x} \\ e_{1y} \end{bmatrix}}_{z_1} \quad (13)$$

which is linear. The clear advantage is that the behavior of the trajectories of (9a) with $d \equiv 0$ may be analyzed as those of a linear system, at least while the trajectories are away from the origin (strictly speaking away of any δ -neighbourhood). In particular, global exponential stability of the origin of (13) is easily concluded invoking classical results on adaptive control systems –see [12]. Consequently, one may use an intuitive contradiction argument to establish uniform global asymptotic stability of (9a) with $d \equiv 0$: assume that the origin is not attractive then, the trajectories (tend to) remain away of an arbitrary δ -neighbourhood of the origin². In that case, since they coincide with those generated by (13), which is exponentially stable, it follows that the trajectories of (9a) must converge to zero. The argument may be repeated for any arbitrarily small δ hence, the “exponential” rate of convergence diminishes but remains uniform in the initial conditions. Then, we have the following result.

Proposition 1 The origin of the system (9) is uniformly globally asymptotically stable if $c_1 > 0$, $c_2 > 0$, v_0 is bounded and w_1 is uδ-PE, bounded and locally Lipschitz in

²An “oscillating” behavior which would consist in the trajectories entering and exiting the δ -neighbourhood is excluded since the origin is stable.

e_{1y} uniformly in t . Moreover, $u\delta$ -PE of w_1 is also a necessary condition.

Next, we state under which conditions w_1 is $u\delta$ -PE. As a matter of fact, this has been established in the context of set-point stabilization, in [18] based on properties (for time-only dependent functions) well-known from adaptive control textbooks. Precisely, that the output of a low-pass filter driven by an input that is persistently exciting, is also persistently exciting. Therefore, w_1 which corresponds to a “filtered version” of \dot{h} , since it satisfies

$$\dot{w}_1 = -c_1 w_1 + \dot{h}(t, e_{1y}),$$

is $u\delta$ -PE if so is \dot{h} .

Proposition 2 Consider the system (3) in closed-loop with the controller (7). Let h be bounded, once continuously differentiable, such that $h(t, e_{1y})$ has a unique zero at $e_{1y} = 0$ for each fixed t ,

$$\sup_{t, e_{1y}} \left\{ |h(t, e_{1y})|, \left| \frac{\partial h(t, e_{1y})}{\partial e_{1y}} \right|, \left| \frac{\partial h(t, e_{1y})}{\partial t} \right| \right\} \leq c \quad (14)$$

for some positive constant c and assume that for any $\delta > 0$ there exist positive numbers μ and T such that

$$|e_{1y}| \geq \delta \implies \int_t^{t+T} |\dot{h}(\tau, e_{1y})| d\tau \geq \mu, \quad \forall t \geq 0. \quad (15)$$

Then, the origin of the closed-loop system is uniformly globally asymptotically stable.

Remark 1 The function h may be defined as a monotonic locally linear function of e_{1y} and smooth, persistently exciting in t ; for instance, $h(t, e_{1y}) = \psi(t) \text{sat}(e_{1y})$ where $\text{sat}(\cdot)$ is a saturation function and ψ is persistently exciting.

B. Leader-follower formation control

Now we extend the previous result to the case of formation-tracking control. Consider a group of n mobile robots with kinematic models,

$$\dot{x}_i = v_i \cos(\theta_i) \quad (16a)$$

$$\dot{y}_i = v_i \sin(\theta_i) \quad (16b)$$

$$\dot{\theta}_i = w_i, \quad i \in [1, n] \quad (16c)$$

where, for the i^{th} robot, x_i and y_i determine the position with respect to a globally-fixed frame, θ_i defines the heading angle –see Figure 1, and the linear and angular velocities are denoted by v_i and w_i respectively.

The control objective is to make the n robots take specific postures determined by the topology designer, and to make the swarm follow a path determined by a virtual reference vehicle labeled R_0 . Any physically feasible geometrical configuration may be achieved and one can choose any point in the Cartesian plane to follow the virtual reference vehicle.

We solve the problem using a spanning-tree communication topology and a recursive implementation of the tracking leader-follower controller (7). That is, the swarm has only one ‘leader’ robot tagged R_1 whose local controller uses

knowledge of the reference trajectory generated by the virtual leader R_0 . Therefore, in the communications graph, R_1 is the child of the root-node robot R_0 and the other robots are intermediate nodes labeled R_2 to R_{n-1} that is, R_i acts as leader for R_{i+1} and follows R_{i-1} . The last robot in the communication topology is denoted R_n and has no followers that is, it constitutes the tail node of the spanning tree.

The fictitious vehicle, which serves as reference to the swarm, describes the reference trajectory defined by (1); the desired linear and angular velocities v_0 and w_0 are communicated to the leader robot, R_1 , only. According to this communication topology, and following the setting for tracking control, the formation control problem reduces to that of stabilization of the error dynamics between any pair of leader-follower robots. For each $i \leq N$, this is

$$\dot{e}_{ix} = w_i e_{iy} - v_i + v_{i-1} \cos e_{i\theta} \quad (17a)$$

$$\dot{e}_{iy} = -w_i e_{ix} + v_{i-1} \sin e_{i\theta} \quad (17b)$$

$$\dot{e}_{i\theta} = w_{i-1} - w_i \quad (17c)$$

and for each $i \geq 1$ we define the control inputs v_i and w_i as

$$v_i = v_{i-1} + c_{2i} e_{ix} \quad (18a)$$

$$w_i = w_{i-1} + c_{1i} e_{i\theta} + h_i(t, e_{iy}) \quad (18b)$$

where h_i is once continuously differentiable, bounded and with bounded derivative. Then, the closed-loop equations yield

$$\begin{bmatrix} \dot{e}_{ix} \\ \dot{e}_{iy} \end{bmatrix} = \begin{bmatrix} -c_{2i} & w_i \\ -w_i & 0 \end{bmatrix} \begin{bmatrix} e_{ix} \\ e_{iy} \end{bmatrix} + \begin{bmatrix} v_{i-1}[1 - \cos e_{i\theta}] \\ v_{i-1} \sin e_{i\theta} \end{bmatrix} \quad (19a)$$

$$\dot{e}_{i\theta} = -c_{1i} e_{i\theta} + h_i(t, e_{iy}) \quad (19b)$$

which has the form of (9) and inherits similar properties; actually, similarly to Proposition 1 we have the following.

Proposition 3 The origin of the system (19) is uniformly globally asymptotically stable, for any $i \leq N$, if $c_{1i} > 0$, $c_{2i} > 0$, v_0 is bounded and w_i is $u\delta$ -PE, bounded and locally Lipschitz in e_{iy} uniformly in t . Moreover, $u\delta$ -PE of w_i is also a necessary condition.

Roughly speaking, the previous statement guarantees that the formation tracking control problem on straight paths may be simply solved by propagating the persistency-of-excitation effect from the leader robot, throughout the chain of systems interconnected in a spanning-tree topology. Technically, the result follows from the following observations: 1) the function h_i is, by assumption, continuous and bounded; 2) for (19a) with $e_{i\theta} = 0$, the origin is uniformly globally asymptotically stable provided that w_i is $u\delta$ -PE and 3) the interconnection term

$$d_i := \begin{bmatrix} v_{i-1}[1 - \cos e_{i\theta}] \\ v_{i-1} \sin e_{i\theta} \end{bmatrix}$$

is also bounded, along trajectories. To see the latter, consider first $i = 2$ then,

$$d_2 := \begin{bmatrix} v_1[1 - \cos e_{2\theta}] \\ v_1 \sin e_{2\theta} \end{bmatrix}$$

where $v_1 = v_0(t) + c_{21}e_{1x}$ is a function of t and e_{1x} . Hence, the function \tilde{d}_2 defined along trajectories as

$$\tilde{d}_2(t, e_{i\theta}) = \begin{bmatrix} v_1(t, e_{1x}(t))[1 - \cos e_{2\theta}] \\ v_1(t, e_{1x}(t)) \sin e_{2\theta} \end{bmatrix},$$

is also continuous and bounded if so is $v_1(t, e_{1x}(t))$. On the other hand, $e_{1x}(t)$ is part of the solution of (9) whose origin, after Proposition 1, is uniformly globally asymptotically stable. Therefore, $e_{1x}(t)$ is uniformly globally bounded and so is $v_1(t, e_{1x}(t))$. The statement of Proposition 3 for the case $i = 2$ follows hence, $v_2(t, \bar{e}_{2x}(t))$ where $\bar{e}_{2x} := [e_{1x} \ e_{2x}]^\top$, is uniformly bounded for any t . Using this and proceeding by induction, we conclude that the result of the lemma holds for any $i \geq 2$. We are ready to present our second main result.

Proposition 4 Consider the system (17) in closed loop with the controllers (18). Assume that, for each $i \leq N$, $h_i(t, e_{iy})$ has an isolated zero at $e_{iy} = 0$,

$$\sup_{t, e_{iy}} \left\{ |h_i(t, e_{iy})|, \left| \frac{\partial h_i(t, e_{iy})}{\partial e_{iy}} \right|, \left| \frac{\partial h_i(t, e_{iy})}{\partial t} \right| \right\} \leq c, \quad (20)$$

$\sum_{j=1}^i \dot{h}_j$ is $u\delta$ -persistently exciting and the control gains c_{1i} , c_{2i} are positive. Then, the origin of the closed-loop system is uniformly globally asymptotically stable.

Remark 2 In most cases, the condition that $\sum_{j=1}^i \dot{h}_j$ is $u\delta$ -persistently exciting for any $i \leq N$ holds if each \dot{h}_j is $u\delta$ -persistently exciting. For instance, it suffices to introduce N different harmonics:

$$h_j(t, e_{ey}) = \psi_j(\varpi_j t) \alpha(e_{iy})$$

where, for simplicity only, ψ_j is a periodic function of period $2\pi\varpi_j$ and α is a continuous odd function.

III. SIMULATION RESULTS

In order to illustrate our theoretical results we have performed some simulation tests under SIMULINKTM of MATLABTM.

We consider a group of five mobile robots. In a first stage of the simulation, the desired formation shape of the mobile robots consists in a linear parallel formation following a straight line trajectory. See Figure 3.

The initial conditions are set to $[x_1(0), y_1(0), \theta_1(0)]^\top = [0, -1, \pi/7]$, $[x_2(0), y_2(0), \theta_2(0)]^\top = [-0.5, 2, \pi/5]$, $[x_3(0), y_3(0), \theta_3(0)]^\top = [-1, -0.5, \pi/4]$, $[x_4(0), y_4(0), \theta_4(0)]^\top = [-1, 1, \pi/8]$ and $[x_5(0), y_5(0), \theta_5(0)]^\top = [1, 0.5, \pi/6]$.

The linear formation shape with a certain desired distance between the robots is obtained by defining $[d_{x1,2}, d_{y1,2}] = [0, 1]$ and $[d_{x2,3}, d_{y2,3}] = [0, -2]$ and $[d_{x3,4}, d_{y3,4}] = [0, 3]$ and $[d_{x4,5}, d_{y4,5}] = [0, -4]$. See Figure 3.

In order to obtain the reference trajectory of the leader robot, we set the reference linear velocity to $v_0(t) = 10\text{m/s}$, while the angular reference velocity is set to zero.

In order to show the flexibility of the formation and effectiveness of the proposed controller, after an arbitrary period

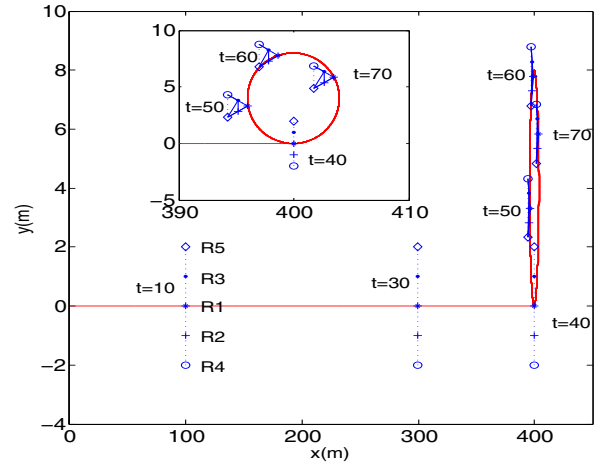


Fig. 3. Motion and relative positioning of the robots in straight line and triangular formation on the plane

of time, we allow the formation shape to change from linear to triangular and the desired trajectory from straight line to circular one respectively via $[d_{x1,2}, d_{y1,2}] = [\sqrt{3}/2, 0.5]$ and $[d_{x2,3}, d_{y2,3}] = [0, -1]$ and $[d_{x3,4}, d_{y3,4}] = [\sqrt{3}/2, -0.5]$ and $[d_{x4,5}, d_{y4,5}] = [0, 2]$ and the reference circular trajectory of the leader robot is obtained setting the linear and angular velocities to $[v_0(t), \omega_0(t)] = [12\text{ m/s}, 3\text{ rad/s}]$.

The control laws are given by

$$\begin{aligned} v_i &= v_{(i-1)} + c_{2i}e_{ix} \\ \omega_i &= \omega_{(i-1)} + c_{1i}e_{i\theta} + \varphi(t) \tanh(e_{iy}) \end{aligned}$$

with control gains $c_{1i} = 2$ and $c_{2i} = 5$. The function φ is generated as a square-pulse train signal of amplitude 0.5, period of four seconds and pulse width of 3.2 s. Note that this function is not smooth but it is persistently exciting hence, the term $\varphi(t) \tanh(e_{iy})$ induces enough excitation to stabilize the system in the y direction, as long as there is an error in this coordinate.

As previously explained the desired formation and trajectory change abruptly from aligned to triangular and from straight line to circular after 40s – see Figure 3.

The total simulation time is 70s. In Figure 3, we show the motion and relative positioning of the robots in either form. It is easy to see from the figure that the formation is established in less than 10s. In particular, each robot tracks its neighbor with its desired offset, while the leader tracking the reference trajectory with a satisfactory performance. In the same Figure, we show the change of the formation shape and the trajectory which occurs at $t = 40\text{s}$. As it is appreciated in the Figure, the triangular formation is also achieved after a short transient, inferior to 10s. The rapid response and excellent performance may be appreciated from the plots of the formation-tracking errors, depicted in Figures 4–6. The overshoots observed in the transient parts are due to the desired trajectory and the initial conditions at the moment when the reference trajectory changes abruptly. The figures show the short transients corresponding to each reference formation and trajectory.

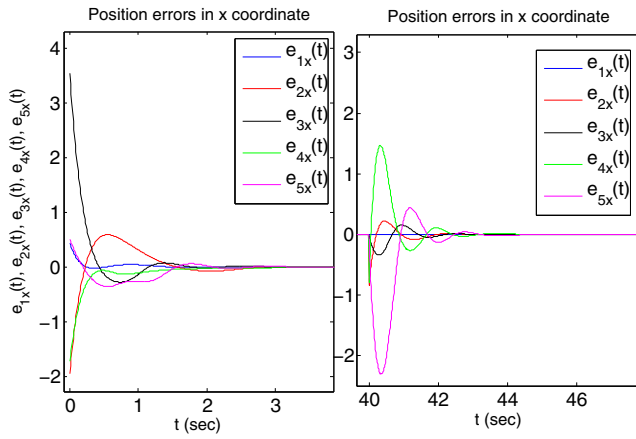


Fig. 4. Position errors in x coordinates with kinematic control algorithm. The reference trajectory changes from a straight-line to a circular path at $t = 40$ s. The first plot shows the transient corresponding to the case of a straight-line reference path and the second plot shows the transient corresponding to a circular reference.

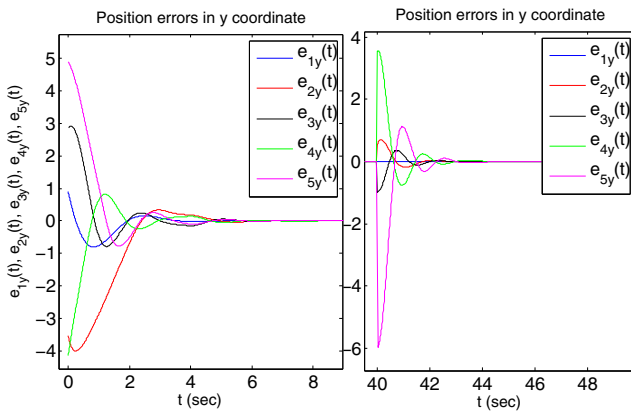


Fig. 5. Position errors in y coordinates with kinematic control algorithm. For the case of a straight-line reference trajectory (left) and a circular one (right).

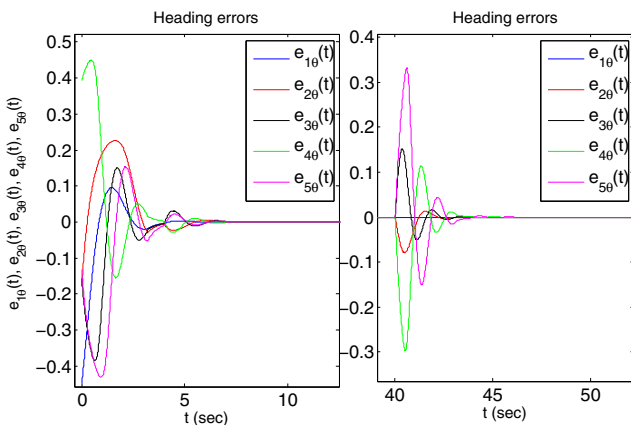


Fig. 6. Heading errors with kinematic control algorithm. For the case of a straight-line reference trajectory (left) and a circular one (right).

IV. CONCLUSION

We have presented an original a simple solution to the tracking control problem of autonomous vehicles in a leader

follower configuration. Our controllers are very simple to implement and ensure global tracking. Further research is oriented to establish formation tracking control in the case of switching communication topologies.

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