Modeling and Adaptive Path Control of a Differential Drive Mobile Robot

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Abstract: - This paper proposes an adaptive dynamic-based path control for a differential drive mobile robot. The control law involves error coordinates expressed in a coordinate frame which are partially linked to the robot. The construction of the controller is based on a two-layer structure using the principle of decomposition of the control problem. Assuming that the robot parameters are unknown but constant, for the inner (velocity) control loop, an adaptive control law is designed applying estimation-based approach. The design procedure and stability analysis of the closed-loop system is performed using Lyapunov stability theory. Simulation results illustrate the effectiveness of the proposed controller.

Key-Words: - Differential drive mobile robot, path following, adaptive control

1 Introduction

During the last two decades, the wheeled mobile robots have been increasingly presented in industrial and service robotics. At control level, important results have been established concerning specific control tasks as point stabilization (the parking problem), trajectory tracking and path following. Beyond the relevance in applications, stabilizing a mobile robot at given posture leads to specific control problems. It is known [1] that feedback point stabilization of nonholonomic systems like wheeled mobile robots can not be achieved via smooth time-invariant control law due to limitations imposed by Brockett's necessary condition [2] for feedback stabilization of such a system. Furthermore, the linearization of a nonholonomic system about any equilibrium point is uncontrollable and consequently, linear analysis and design techniques cannot be applied [3]. For trajectory tracking and path following tasks, standard linear [4] and nonlinear approaches are effective (feedback linearization [5], Lyapunov-based techniques [6, 7, 8, 9].

The most common way to build a mobile robot is to use two-wheel drive with differential steering and a free balancing wheel (castor). Controlling the two motors independently, such robots have good maneuvering and work well indoors on flat surfaces. Many commercial platforms based on this locomotion scheme exist, such as the mobile robot Pioneer 3-DX [10] from ROBOSOFT.

In this paper, we present an adaptive dynamic-based feedback path following controller for a differential drive mobile robot and in particular, with application to mobile robot Pioneer 3-DX, (Fig.1). A dynamic model of the robot using the Boltzmann-Hamel formalism in quasi-coordinates is derived. The construction of the controller is based on a two-layer structure using the principle of decomposition of the control problem: a kinematic path error control loop and velocity error control loop. Assuming that the robot parameters are unknown but constant, for the inner (velocity) control loop, an adaptive control law is designed applying estimation-based approach. The design procedure and stability analysis is performed via Lyapunov techniques.

Fig. 1 The mobile robot Pioneer 3-DX

The paper is organized as follows: In Section 2, the dynamic model of the robot is presented. In Section 3, the path following problem in error coordinates expressed in a moving reference frame partially linked to the robot is stated. In Section 4, the design of the proposed controller and stability analysis is given.

Simulation results are presented in Section 5. Section 6 contains some conclusions and future work.

2 Dynamic Modeling

A plan view of the mobile robot Pioneer 3-DX considered in this paper, is shown in Fig. 2. The kinematic scheme of the robot consists of platform with two driving wheels mounted on the same axis with independent actuators and one free wheel (castor). The mobile robot is steered by changing the relative angular velocities of the driving wheels. The wheels of the robot are assumed to roll without lateral sliding. Point P located at the centre of the wheel axle is used as reference point of the system.
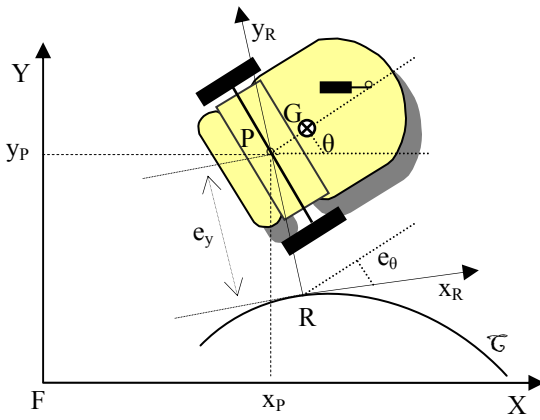

Fig. 2 A plan view of the differential drive mobile robot

The coordinates of point P with respect to an inertial frame Fxy, are denoted by (x_P, y_P). The angle θ is the orientation angle of the robot with respect to Fxy. The rotational angles of the wheels with respect to their proper axes are denoted by φ_i, $(i = 1, 2)$. The base of the robot (the distance between the driving wheels) is denoted by b. The wheel radii are denoted by r. The centre of mass G of the robot is on the longitudinal axis a c distance from point P, (Fig. 2). Using the coordinates of the reference point P, the configuration of the system is described by five generalized coordinates

$$q = [x_P, y_P, \theta, \varphi_1, \varphi_2]^T. \quad (1)$$

The system is characterized by the following nonholonomic constraints on the generalized velocities

$$A\dot{q} = 0 \quad (2)$$

where A is a $3x5$ matrix as follows

$$A = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 & 0 \\ \cos\theta & \sin\theta & -\dfrac{b}{2} & -r & 0 \\ \cos\theta & \sin\theta & \dfrac{b}{2} & 0 & -r \end{bmatrix}. \quad (3)$$

The mobile robot has two degrees of freedom (DOFs) in the plane. We introduce the following quasi-velocities

$$\eta = \begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \dot{\varphi}_1 & \dot{\varphi}_2 \end{bmatrix}^T. \quad (4)$$

The first three quasi-velocities are equal to zero with accordance to the nonholonomic constraints, and indicate that the wheels roll in the plan without slipping and lateral sliding. The last two quasi-velocities are equal to the general velocities $\dot{\varphi}_1$ and $\dot{\varphi}_2$.

The wheel angular velocities are related to the robot linear and angular velocities according to the following invertible relationship

$$\begin{bmatrix} v_{Px} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dfrac{r}{2} & \dfrac{r}{2} \\ \dfrac{r}{b} & -\dfrac{r}{b} \end{bmatrix} \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} \quad (5)$$

where v_{Px} is the linear velocity of reference point P and $\dot{\theta} = \omega$ is the angular velocity of the robot.

In order to derive a dynamic model of the robot, we use the Boltzmann-Hamel method [11], which is suitable for systems with nonholonomic constraints. The Boltzman-Hamel formulation of the system dynamic equations is given by

$$\frac{d}{dt}\frac{\partial T}{\partial \eta_s} - \frac{\partial T}{\partial \pi_s} + \sum_{r=1}^{n}\sum_{t=m+1}^{n} \gamma_{ts}^{r} \frac{\partial T}{\partial \eta_r} \eta_t = \Gamma_s \quad (6)$$
$$s = m+1,.....,n$$

where
- π_S are the so-called quasi-coordinates, $(d\pi_S/dt = \eta_S)$;
- $T(\eta)$ is the kinetic energy of the system expressed in terms of quasi-velocities η;
- γ_{ts}^{r} are the three-index coefficients;
- Γ_S are generalized forces associated with the quasi-coordinates.

In our case, $n = 5$ is the number of quasi-coordinates, $m = 3$ is the number of nonholonomic constraints. We note that the number of equations in (6) is equal to the

number of DOFs of the system. We form the following 5×5 matrices $\alpha(q)$ and $\beta(q)$

$$\alpha(q) := (a_{ij}(q)) = \begin{bmatrix} -\sin\theta & \cos\theta & 0 & 0 & 0 \\ \cos\theta & \sin\theta & -\dfrac{b}{2} & -r & 0 \\ \cos\theta & \sin\theta & \dfrac{b}{2} & 0 & -r \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$\beta(q) := (b_{ij}(q)) = \alpha^{-1}(q). \quad (8)$$

Using the elements of matrices (7) and (8), the three-index coefficients γ_{ts}^r and the expressions for the derivatives of the kinetic energy with respect to the quasi-coordinates can be determined by using the following expressions

$$\gamma_{ts}^r = \sum_{k=1}^n \sum_{l=1}^n a_{rl} \left(\frac{\partial b_{lt}}{\partial q_k} b_{ks} - \frac{\partial b_{ls}}{\partial q_k} b_{kt} \right) \quad (9)$$

$$\frac{\partial T}{\partial \pi_s} = \sum_{r=1}^n b_{rs} \frac{\partial T}{\partial q_r} \quad (10)$$

for $t = m+1, \ldots\ldots n; \ r = 1, \ldots\ldots n; \ s = m+1, \ldots n$.

If the inertia of the wheels with respect to their proper axes is ignored, the kinetic energy of the system expressed in terms of the quasi-velocities η is obtained in the form

$$T = \frac{1}{2} m \left[\frac{r^2}{4} (\eta_4 + \eta_5)^2 + \eta_1^2 \right] + Mc\eta_1 \frac{r}{b} (\eta_4 - \eta_5)$$
$$+ \frac{1}{2} J \frac{r^2}{b} (\eta_4 - \eta_5) \quad (11)$$

where m is the total (body and wheels) mass of the robot, and J is the total moment of inertia of the robot with respect to a vertical axis Pz_P.

Using (6)-(11), the dynamic equations of the vehicle are obtained in the following matrix form

$$M\ddot{\varphi} + H(\varphi)\dot{\varphi} = G \quad (12)$$

where

$$M = \begin{bmatrix} \dfrac{mr^2}{4} + \dfrac{r^2 J}{b^2} & \dfrac{mr^2}{4} - \dfrac{r^2 J}{b^2} \\ \dfrac{mr^2}{4} - \dfrac{r^2 J}{b^2} & \dfrac{mr^2}{4} + \dfrac{r^2 J}{b^2} \end{bmatrix} \quad (13)$$

$$H(\varphi) = \begin{bmatrix} 0 & -\dfrac{mcr}{b^2} (\dot{\varphi}_1 - \dot{\varphi}_2) \\ \dfrac{mcr}{b^2} (\dot{\varphi}_1 - \dot{\varphi}_2) & 0 \end{bmatrix} \quad (14)$$

$$\varphi = [\varphi_1 \quad \varphi_2]^T \quad (15)$$

$$\Gamma = \tau = [\tau_1 \quad \tau_2]^T \quad (16)$$

and τ_i, $(i = 1, 2)$, are the driving torques of the wheels.

3 Problem Formulation

The path following geometry used in this paper is represented in Fig. 2. Consider a differential-drive articulated mobile robot moving on a flat surface. It is assumed that the path $\mathcal{C}$ is a smooth planar curve. A moving reference coordinate frame $Rx_R y_R$ is defined such that the x_R axis is tangent to the path and oriented in the direction of motion to follow, and the y_R axis passes through the reference point P of the robot. It is supposed that the distance between points P and R is smaller than the reference curvature radius at point R and in that way, ensuring that the reference path is uniquely defined. Let $[x_P, y_P, \theta]^T$ and $[x_r, y_r, \theta_r]^T$ be the real and reference posture coordinates of the robot with respect to an inertial frame Fxy. The error posture $[e_x, e_y, e_\theta]^T$, i.e., the position and orientation of the robot with respect to the moving reference frame $Rx_r y_r$ is given by [12]

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_P - x_r \\ y_P - y_r \\ \theta - \theta_r \end{bmatrix}. \quad (17)$$

Differentiating (17), taking into account the nonholonomic constraints $v_{Ry} = v_{Py} = 0$, (v_{Ry} and v_{Py} are the projections of the velocities of points R and P on the Ry_R and Px_P axes, respectively), and using the fact that $e_x(t) \equiv \dot{e}_x \equiv 0$, after some work, we obtain a kinematic model of the system for path following applications in the form

$$\dot{e}_y = v_{Px}(t) \sin e_\theta$$
$$\dot{e}_\theta = e_\omega \tag{18}$$

where

$$e_\omega = \omega - \omega_R. \tag{19}$$

The reference angular velocity ω_r of the frame Rx_Ry_R can be expressed as a function of the velocity v_{Px} of the mobile robot as follows

$$\omega_r = \frac{c_r v_{Px} \cos e_\theta}{1 - c_r e_y}, \tag{20}$$

$c_r = 1/\rho_r$ is the curvature of the reference path at point R.

We assume that v_{Px} together with its derivative are bounded and also, that the following inequalities hold: $0 < v_{Pxmin} \le |v_{Px}(t)| \le v_{Pxmax}$, where v_{Pxmin} and v_{Pxmax} are positive constants. In this case, using the parameterization (e_y, e_θ) and given a path $\mathcal{C}$, the path following problem consists of finding a feedback control law for the system (12) and (18) such that the state vector $[\, e_y, e_\theta \,]^T$ tends to $[0, 0]^T$, as $t \to \infty$.

4 Feedback Control Design

In this Section, we present an adaptive dynamic-based path following controller for the mobile robot given by equations (12) and (18). The control objective is to regulate the state vector $[e_y, e_\theta]^T$ to zero. The control scheme is developed as a two-layer structure based on the principle of decomposition of the control problem: a kinematic path error control loop and velocity error control loop.

First, we consider the outer (kinematic-based) path following controller using the equations (18) for control design. Assuming that the orientation error is small, we linearize these equations, by setting $\sin e_\theta \approx e_\theta$. For the linearized system (18) we consider the Lyapunov function

$$V_1 = \frac{1}{2}e_y^2 + \frac{1}{2}e_\theta^2. \tag{21}$$

Applying the following feedback control with control input ω

$$\omega = -e_y v_{Px} + \omega_R - k_1 e_\theta, \quad k_1 = cte > 0 \tag{22}$$

the derivative of V_1 along the solutions of (18) becomes

$$V_1 = -k_1 e_\theta^2 \le 0. \tag{23}$$

Using (18) and (22), the closed-loop kinematic system can be written in the form

$$\dot{e}_y = v_{Px} e_\theta$$
$$\dot{e}_\theta = -v_{Px} e_y - k_1 e_\theta \tag{24}$$

The proposed control law (22) exponentially stabilizes the linearized system (18). Indeed, from (23) it follows that the positive-definite radially unbounded function (21) is non-increasing ($V_1(t) \le V_1(0)$) and this in turn implies that $e_y(t)$ and $e_\theta(t)$ are bounded. The function (21) is lower bounded and from (23) it follows that $\dot{V}_1$ is negative semi-definite. $\dot{V}_1$ is also uniformly continuous, since its derivative is bounded. Application of Barbalat's Lemma [13] indicates that $e_\theta(t) \to 0$ as $t \to \infty$. From the second equation of (24), it follows that $\dot{e}_\theta(t)$ is uniformly continuous, since its derivative is bounded. Applying again the Barabalat's Lemma, it follows that $\dot{e}_\theta(t) \to 0$ as $t \to \infty$. From the first equation of (24), it follows that $\dot{e}(t) \to 0$ as $t \to \infty$ since $v_{Px} - / \to 0$ as $t \to \infty$. From the second equation of (24), it follows that $e_y(t) \to 0$ as $t \to \infty$, since $e_\theta(t)$ and $\dot{e}_\theta(t)$ converge to zero as $t \to \infty$. From (22), since e_y, e_θ, v_{Px} and ω_R are bounded, it follows that the control $\omega(t)$ is also bounded.

For the design of the control law concerning the inner (velocity) error control loop, we first linearize equations (12) by ignoring the coupling effects described by the Coriolis/centripetal matrix H. Since the inertia matrix M is invertible, using (5) we obtain a dynamic model of the robot in terms of $[\dot{v}_{Px} \quad \dot{\omega}]^T$ in the following matrix form

$$\begin{bmatrix} \dot{v}_{Px} \\ \omega \end{bmatrix} = \begin{bmatrix} \dfrac{1}{m} & 0 \\ 0 & \dfrac{b}{2J} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{25}$$

where

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}. \tag{26}$$

The parameters m and J in (25) are constant but *unknown*. The model (25) consists of two decoupled

subsystems: the first one for the mobile robot linear velocity v_{Px} and the second – for the robot angular velocity ω. Since in this paper, we concern with the path following control problem, we use only the second equation of (25). We assume that the inner control loop for the mobile robot linear velocity v_{Px} has been already designed and the robot velocity is known bounded function of time. We also assume that the desired angular velocity $\omega(t) = \omega^d(t)$ of the robot (22) obtained from the design of the outer control loop is available for feedback control design. For simplicity in the derivation of the control law, we introduce the following notation

$$\rho = \frac{1}{a} \qquad (27)$$

where

$$a = \frac{b}{2J} = cte > 0 \qquad (28)$$

is an unknown parameter in the second equation of (25). We form the following Lyapunov function

$$V_2 = \frac{1}{2}e_\omega^2 + \frac{a}{2\gamma_\rho}\tilde{\rho}^2 \qquad (29)$$

where

$$\tilde{\rho} = \rho - \hat{\rho}. \qquad (30)$$

The derivative of V_2 is obtained in the following form

$$\dot{V}_2 = e_\omega(au_2 - \dot{\omega}^d) - \frac{a}{\gamma_\rho}\tilde{\rho}\dot{\hat{\rho}} \qquad (31)$$

Using (31), we choose the control for u_2 in the form

$$u_2 = \hat{\rho}(-k_2e_\omega + \dot{\omega}^d), \qquad k_2 = cte > 0. \qquad (32)$$

The estimate $\hat{\rho}$ in (32) is obtained from the following update law

$$\dot{\hat{\rho}} = -\gamma_\rho e_\omega(-k_2e_\omega + \dot{\omega}^d), \qquad (33)$$

Substituting (32) and (33) in (31), the derivative of V_2 results in

$$\dot{V}_2 = -k_2e_\omega^2 \leq 0. \qquad (34)$$

The closed-loop system is obtained as

$$\dot{e}_\omega = -k_2e_\omega - a\frac{\tilde{\rho}}{\hat{\rho}}u_2$$
$$\dot{\tilde{\rho}} = \gamma_\rho e_\omega(-k_2e_\omega + \dot{\omega}^d) \qquad (34)$$

where u_2 is given by (32).

The stability property of (34) follows from (29) and (34). Since V_2 is positive definite and $\dot{V}_2$ is negative semi-definite, the Lassale-Yoshizawa Theorem [14] guarantees that e_ω and $\tilde{\rho}$ are uniformly bounded and also that $e_\omega(t) \to 0$ as $t \to \infty$. From (32), since $\dot{\omega}^d(t)$ is bounded, it follows that the control $u_2(t)$ is also bounded.

5 Simulation Results

Simulation results were performed to illustrate the effectiveness of the proposed controller. The dynamic-based adaptive control law designed in Section 4 was implemented in MATLAB. A circular reference path with radius $R = 2m$ was chosen for the simulations. The velocity of the mobile robot was $v_{Px}(t) = 1m/s$. The robot base was $b = 0.5m$. The control gains were: $k_1 = 2$, $k_2 = 2$ and $\gamma_\rho = 2$. Initial conditions were chosen to be: $e_y(0) = 0.5m$, $e_\theta(0) = 0.2rad$, $e_\omega(0) = 0.1\ rad/s$. To test effectiveness of the proposed adaptive controller, it was assumed that the physical (true) value of the parameter $a = 5$ is different from the initial estimate $\hat{a}(0) = 1/\hat{\rho}(0) = 1$.

Evolution in time of the state coordinates $[\ e_y,\ e_\theta,\ e_\omega]^T$ of the closed-loop is depicted in Fig. 3.
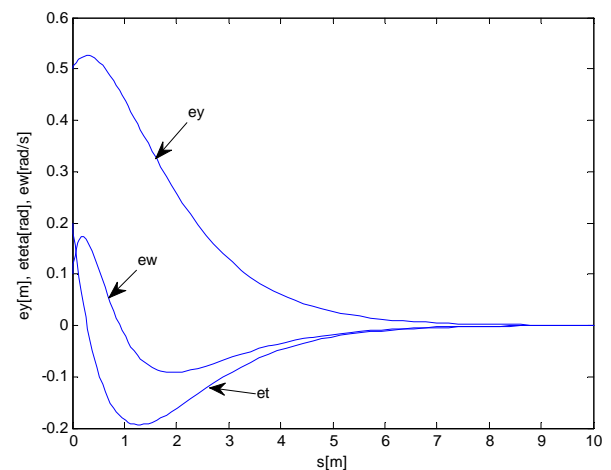

Fig. 3. Evolution in time of the state coordinates of the closed-loop system $[\ e_y,\ e_\theta,\ e_\omega\ ]^T$

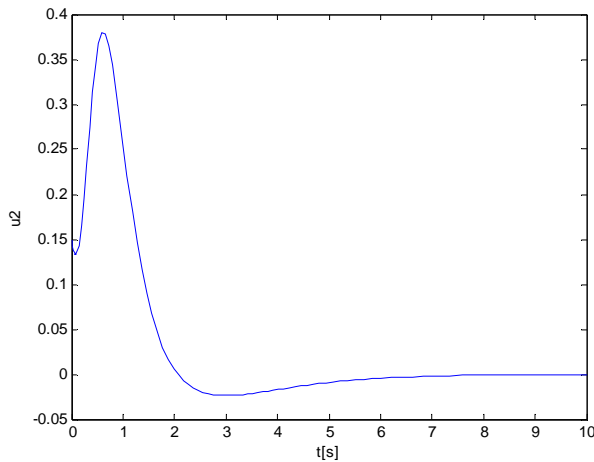Evolution in time of the control input u_2 is presented in Fig. 4.

Fig. 4. Evolution in time of the control input u_2

6 Conclusion

In this paper, an adaptive dynamic-based path controller for a differential drive mobile robot has been presented. A dynamic model of the robot using the Boltzmann-Hamel formalism in quasi-coordinates was derived. The construction of the controller is based on a two-layer structure using the principle of decomposition of the control problem: a kinematic path error control loop and velocity error control loop. Assuming that the robot parameters are unknown but constant, for the inner (velocity) control loop, an adaptive control law was designed applying estimation-based approach. Our future work will focus on the implementation of the proposed controller on the experimental mobile robot Pioneer 3-DX.

Acknowledgements

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