Proceedings of the 2011 IEEE
International Conference on Mechatronics
April 13-15, 2011, Istanbul, Turkey

Leader-Follower Based Formation Control of Nonholonomic Robots Using the Virtual Vehicle Approach

Jawhar Ghommam*, Hasan Mehrjerdi† and Maarouf Saad†
*Research Unit on Intelligent Control and Optimization of Complex Systems
National School of Engineer of Sfax, ENIS, Tunisia
Email: jghommam@gmail.com
†Department of Electrical Engineering,École de technologie supérieure,
1100, rue Notre-Dame Ouest Montreal, Québec Canada.
Email: hasan.mehrjerdi.1@ens.etsmtl.ca, Maarouf.Saad@etsmtl.ca

***Abstract*— In this paper we investigate the leader follower motion coordination of multiple nonholonomic mobile robots. A combination of the virtual vehicle and trajectory tracking approach is used to derive the formation architecture. A virtual vehicle is steered in such away it stabilizes to a shifted reference position/heading defined by the leader, the velocity of the virtual vehicle is then provided for further use in designing control law for the follower independent from the measurement of leader's velocity. Position tracking control is then constructed for the follower to track the virtual vehicle using the backstepping and Lyapunov direct design technique. Simulations are provided to show the effectiveness of the proposed approach.**

I. Introduction

There are distinct advantages of using a group of homogeneous or heterogeneous mobile robots to accomplish a task, rather than a more elaborated single robot. When comparing the mission outcome of a group of multi mobile robots (MMR) operating in the same environment to that of a single robot, it is easy to see that the overall performance of the MMR group can improve task allocation, performance, the time duration required and the system effectiveness and safety to achieve the outcome [2].

The multi robot system includes different typologies of robotic systems such as: a team of cooperating autonomous mobile robots (grounded, aerial, underwater robots or autonomous marine surface vessels), and multiple industrial manipulators. Some applications that are applicable for multiple mobile robots are: surveillance [3], exploration [4] satellite clustering [5], underwater autonomous vehicles and a fleet of marines, [6], aerial vehicles and unmanned aerial vehicle (UAV) [7], cooperative robot reconnaissance [8], and manipulation cooperation. In such applications, multiple robots are required to travel autonomously between different locations, while avoiding collisions with static or dynamic obstacles and other robots.

Previous work

In the leader follower method, one of the robots is designated as the leader, with the rest robots as followers. The follower robots need to position themselves relative to the leader and to maintain a desired relative position with respect to the leader. This method is characterized by simplicity and reliability. In this method, there is no explicit feedback from the followers to the leader and that is the disadvantage of this method. Deasi et al. [1] develop a set of decentralized control laws that allows each robot to maintain a desired position within a formation and to enable changes in the shape of a team. Separation-separation and separation-bearing are two popular techniques in leader-follower formation control, and in this work, the latter will be considered, where the followers stay at specified separation and bearing from its designated leader. Huang et al. [9] present the control and localization of a heterogeneous group of mobile robots where inexpensive sensor-limited and computationally limited robots follow a leader robot in a desired formation over long distances. The proposed method is limited in that the leader needs to generally maintain line-of-sight contact with the followers. The paper [91] deals with leader-follower formations of nonholonomic mobile robots where robots' control inputs are forced to satisfy suitable constraints that restrict the set of leader possible paths and admissible positions of the follower with respect to the leader. In this algorithm follower position is not rigidly fixed with respect to the leader but varies in proper circle arcs centered in the leader reference frame. Chen et al. [10] present a receding-horizon leader follower control to yield a fast convergence rate of the formation tracking errors and to solve the formation problem of multiple nonholonomic mobile robots with a rapid error convergence rate. A separation-bearing-orientation scheme for two-robot formations and separation-separation-orientation scheme for three-robot formations is presented to maintain the

978-1-61284-985-0/11/$26.00 ©2011 IEEE

desired leader-follower relationship. Recently Defoort et al. [11] proposed a coordinated sliding mode controllers scheme based on leader-follower approach to asymptotically stabilize the vehicles to a time varying desired formation.

Main contribution

Inspired by the developments in the field, this paper tackles a problem in coordinated vehicle motion that departs slightly from mainstream work reported in [12]. Specifically, we consider the problem of coordinating a group of nonholonomic mobile robots to a global leader vehicle. The only information provided from the leader vehicle is its position/heading measurement. The approach that we propose here precludes the necessity to contract an observer to estimate the leader's velocity, it relies however on a filtering technique to generate pseudo filtered tracking error signals to eliminate the need for velocity measurements from the leader. A virtual vehicle is therefore designed on the basis of the filter technique to track the reference trajectory of the follower. Using the backstepping technique a control law input for the follower is designed such that the follower converges asymptotically to the trajectory of the virtual

II. Dynamical model and problem formulation

In this section, the motion and dynamic description of the car-like mobile robot is reviewed. The problem formulation for the leader-follower formation control is provided thereafter .

A. Mobile robot modeling

We consider a group of n mobile robots, of which each has the following dynamics

$$\begin{aligned} \dot{x}_i &= v_i \cos\phi_i \\ \dot{y}_i &= v_i \sin\phi_i \\ \dot{\phi}_i &= \omega_i \\ \overline{\mathbf{M}}_i \dot{\varpi}_i &= -\overline{\mathbf{C}}_i(\dot{\mathbf{p}}_i)\varpi_i - \overline{\mathbf{D}}_i \varpi_i + \overline{\mathbf{B}}_i F_i \end{aligned} \tag{1}$$

where **Assumption 1** The robot velocities of the followers $(\omega_{1i}, \omega_{2i})$ or (v_i, ω_i) are measurable and available for control design.

Remark 1: In this paper, we assume that the follower robots are with identical dynamics. Non-heterogenous robots can also be considered since the control law that will ensure formation, only require local information from onboard sensors and relative state information from the leader. The leader vehicle can be of any dynamical model and would not affect the follower's control input.

B. Leader-follower problem formulation

In this paper, we are primarily interested in leader-follower trailing control. In this scheme, one main leader is defined, that doesn't follow any other vehicle, while the desired position of the followers are defined relative to the state of the leader, such that the followers keep a predefined distance and orientation with respect to their leader. When the followers reach their expected positions, the group formation is therefore established. We formulate the leader-follower problem along the line of [11], [12] with a slight modification: The reference trajectory for the follower $\mathcal{R}_r$ is generated using a copy of the leader's trajectory $\mathcal{R}_L$ shifted a certain distance from the leader. Assume a virtual vehicle henceforth designated by $\mathcal{R}_V$, tries to cruise along this generated trajectory, then this virtual vehicle is separated a distance l_{ik} from the leader with a relative bearing angle ψ_{ik} with respect to the leader (see Fig. 1). We suppose that the only measurement available from the leader is the position/heading of the vehicle. We propose to design a controller for the virtual vehicle based on its velocity vector in such away, we ensure convergence of the virtual vehicle to the reference trajectory. Through the use of this intermediate controlled virtual vehicle, we implicitly reconstructed an estimate of the leader's velocity. Controlling the real physical follower to track the phantom vehicle would be easier using a complete information about its position/heading and velocity.

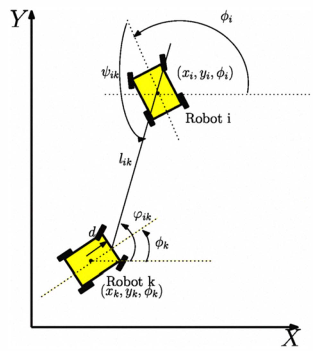

Fig. 1. Leader follower configuration.

Remark 2: The leader-follower scheme presented so far, seems to be compliant to only one follower. It still possible however to extend this scheme to any number of followers within the formation, only if we provide the followers with collision avoidance scheme. This will be detailed far away in this paper.

III. Virtual vehicle design

In this section, we will describe the relationship between the the formation's main leader and the cascades of several other pairs of leader-follower vehicles as well as their organization in the formation scheme.

A. Leader-follower scheme

A multilayer high level formation can be realized by considering the team of n vehicles as a set of $n-1$ decentralized subsystems of two vehicles and one main leader. As in [1], we model the interaction topology among the group of $n-1$ mobile robots as a directed graph, where the ith node represents the ith robot $\mathcal{R}_i$ and a directed edge from robot $\mathcal{R}_i$ to robot $\mathcal{R}_j$ denotes the flow of information exchange between robot $\mathcal{R}_i$ and $\mathcal{R}_j$. For the leader-follower formation scheme discussed in this paper,the formation graph can be viewed as a tree with the main leader of the formation $\mathcal{R}_L$ as a head of the tree. For the clarity of the development, we will consider a pair of leader-follower vehicles and derive a kinematic controller which will help the follower vehicle reaches its position with respect to the formation scheme.

Referring to Fig. **??** the virtual robot k will ideally follow the the reference vehicle placed a distance l_{lk} from the leader l. Let $l_{lk} \in \mathbb{R}_{>0}$ denotes the relative distance between the reference vehicle and the leader and is given as

$$l_{lk} = \sqrt{(x_l - x_k - d\cos\phi_k)^2 + (y_l - y_k - d\sin\phi_k)^2} \quad (2)$$

where d as shown in Fig. 1 is the distance between the center of gravity and the front of the robot. The relative bearing ψ_{lk} is defined as

$$\psi_{lk} = \pi + \varphi_{lk} - \phi_l \quad (3)$$

with $\varphi_{lk} = \arctan\Big[(y_l - y_k - d\sin\phi_k)/(x_l - x_k - d\cos\phi_k)\Big]$. Differentiating equations (2) and (3) with respect to time and let $e_{lk} = [l_{lk}, \psi_{lk}]^\top$, the kinematic equations of the two robots are given by []:

$$\dot{e}_{lk} = \mathbf{H}u_k + \mathbf{G}u_l \quad (4)$$

where $\mathbf{H} = \begin{pmatrix} \cos\gamma_{lk} & d\sin\gamma_{lk} \\ -\frac{\sin\gamma_{lk}}{l_{lk}} & \frac{d\cos\gamma_{lk}}{l_{lk}} \end{pmatrix}$, $\mathbf{G} = \begin{pmatrix} -\cos\psi_{lk} & 0 \\ -\frac{\sin\psi_{lk}}{l_{lk}} & -1 \end{pmatrix}$, $u_k = [v_k, \omega_k]^\top$ is the input for the virtual robot k and $u_l = [v_l, \omega_l]^\top$ is the input for the leader robot l. The angle γ_{lk} is defined as the sum of the bearing angle and the difference between the robot's heading, i.e, $\gamma_{lk} = \psi_{lk} + (\phi_l - \phi_k)$.

B. Control design for the virtual vehicle

The key idea behind designing a control law u_k for the new kinematic equations (4), is to stabilize the vector state e_{lk} to a desired output. A leader follower control law was proposed in [] based on input-output feedback linearization technique. However the controller derived in [] is based on the knowledge of the velocity measurement of the leader. In this paper, we consider the velocity u_k of the virtual vehicle as control input and design it in such away it depends only on the measurement of the leader's position and ensures that virtual vehicle tracks the leader's reference trajectory as shown in Fig. **??**. To design the control input for the virtual leader, we generate a pseudo filtered tracking error signal [], which will eliminate the need for velocity measurement to be included in the control design. To achieve the control objective we define the error variable $r_{elk} = \tilde{e}_{lk} + \Omega_l$ with $\tilde{e}_{lk} = e_{lk} - e_l^d$, e_l^d is the desired distance/bearing of the reference vehicle with respect to the leader, $\Omega_l = [\Omega_{1l}, \Omega_{2l}]^\top$ is the output of the filter given by

$$\dot{\Omega}_l = -\boldsymbol{\beta}\Omega_l - \mathbf{K}r_{elk}, \quad \Omega_l(0) = 0 \quad (5)$$

the gain matrices of the filter (5) are assumed to have the form $\boldsymbol{\beta} = \mathrm{diag}[\beta]$ and $\mathbf{K} = \mathrm{diag}[k]$ where β and k positive scalar constants. The filter (5), generates a velocity related signal from the error track. Let the virtual velocity control input for the virtual vehicle be designed as

$$u_k = \mathbf{H}^{-1}\Big[\dot{e}_l^d - \boldsymbol{\alpha}\tilde{e}_{lk} + (\boldsymbol{\beta} - \boldsymbol{\alpha})\Omega_l + \mathbf{K}\Pi\Big] \quad (6)$$

where $\boldsymbol{\alpha} = \mathrm{diag}[\alpha], \alpha > 0$ is feedback gain matrix and the last term in (6) is defined as

$$\Pi = \begin{bmatrix} \tanh(\delta\Omega_{1l}) \\ \tanh(\delta\Omega_{2l}) \end{bmatrix}$$

Taking the time derivative of r_{elk} and using the virtual control u_k we obtain

$$\dot{r}_{elk} = -(\boldsymbol{\alpha} + \mathbf{K})r_{elk} + \mathbf{K}\Pi + \mathbf{G}u_l \quad (7)$$

Clearly the solutions of equation (7) are bounded provided that the last term $\mathbf{G}u_l$ is bounded. We assume for practical reasons, that the velocity of the leader vehicle is bounded as:

$$\sup_t(u_l) = \bar{V}_L, \quad \bar{V}_L > 0 \quad (8)$$

The stability analysis is stated in the following theorem:

Theorem 1: Consider the filter dynamic described by (5) and (7). Let the bounds on the leader's velocity be defined by (8), with the control input for the virtual vehicle (6), then for any bounded initial conditions, all the closed loop solutions are UPAS.

Proof: Consider the positive definite Lyapunov function

$$\begin{aligned} V_{lk}(t) &= \frac{1}{2}r_{elk}^\top r_{elk} + \begin{bmatrix} \sqrt{\ln\cosh(\delta\Omega_{1l})} \\ \sqrt{\ln\cosh(\delta\Omega_{2l})} \end{bmatrix}^\top \Delta^{-1} \\ &\quad \times \begin{bmatrix} \sqrt{\ln\cosh(\delta\Omega_{1l})} \\ \sqrt{\ln\cosh(\delta\Omega_{2l})} \end{bmatrix} \end{aligned} \quad (9)$$

with $\Delta = \mathrm{diag}[\delta]$ is a positive gain matrix. Taking the derivative of (9) along the solutions of (7), yields

$$\begin{aligned} \dot{V}_{lk} &= -r_{elk}^\top(\boldsymbol{\alpha} + \mathbf{K})r_{elk} - \boldsymbol{\beta}\Omega_i^\top\Pi + r_{ei}^\top\mathbf{G}u_l \quad (10) \\ &\le -\Big(\min\{\alpha + k\} - \frac{1}{2} - \frac{3\bar{V}_L\|\mathbf{G}\|}{\|r_{elk}\|}\Big)\|r_{elk}\|^2 - \boldsymbol{\beta}\Omega_l^\top\Pi \end{aligned}$$

let ε be any given positive constant, we choose the gain

$$\min\{\alpha + k\} - \frac{1}{2} - \frac{3\bar{V}_L\|\mathbf{G}\|}{\|\varepsilon\|} > 0$$

we obtain for $\|r_{elk}\|^2 > \varepsilon^2 \Rightarrow \dot{V}_{lk} \le -\|r_{elk}\|^2 - \boldsymbol{\beta}\Omega_l^\top\Pi$. By using the fact that $x\tanh(x)$ is a positive definite function for all $x \in \mathbb{R}^n$, we conclude that $\dot{V}_{lk}$ is negative definite, however due to the linear dependency of $1/\varepsilon$ in $\dot{V}_{lk}$ then based on [12] we conclude that the solutions of the closed loop system are UPAS. This concludes the proof. ■

Remark 3: At this stage, all we did was to ensure that the virtual vehicle converges to a reference trajectory defined relative to the leader vehicle using measurement of the available signals like the position and heading. The virtual vehicle therefore becomes defined by its position/heading $(\mathbf{p}_k, \phi_k)$ and this time by the measurement of its velocity u_k which was implicitly estimated form the leader's velocity. These signals will be exploited later to position tracking of a real physical mobile robot to its virtual vehicle.

IV. Position tracking of the follower robot

As discussed earlier, the formation control objective is realized only if the follower physical robot reaches its desired position with respect to the leader. Since, the velocity of the leader is unavailable for design control, a virtual mobile robot is controlled such that it converges to the defined reference trajectory by eliminating the need for this velocity. The virtual leader now with known information for the design control will substitute the reference trajectory to be tracked by the follower. The kinematic of the virtual vehicle is given by

$$\dot{x}_k = v_k \cos\phi_k, \quad \dot{y}_k = v_k \sin\phi_k, \quad \dot{\phi}_k = \omega_k \tag{11}$$

The control objective for the position tracking under assumption 1 is to design a control input F_i for follower i such that it asymptotically tracks the virtual vehicle's position that we denote by $\mathbf{p}_k = (x_k, y_k)^\top$ and heading ϕ_k, that is

$$\lim_{t\to\infty} \|\mathbf{p}_i - \mathbf{p}_k\| = 0, \quad \lim_{t\to\infty} (\phi_i - \phi_k) = 0 \tag{12}$$

A close look at the dynamic system (1) shows that this system is of a strict feedback form [] to which, direct Lyapunov method and backstepping technique can be applied to design the control input F_i. The control design is two stage design strategies. In the first stage of the design, the kinematic equations of the mobile robot i are considered with v_i and ω_i are viewed to be an immediate controls to regulate the position of the actual robot at the virtual vehicle position. In the second stage, the dynamic equation of the follower robot is considered to design the control input F_i.

A. Stage 1: Kinematic design

The kinematic controller of the mobile robot is responsible for the task of position tracking of the follower robot. To derive the kinematic controller, two steps of the backstepping technique are used to fulfil the position tracking. In the first step, the heading ϕ_i and the linear velocity v_i of the mobile robot i are used as controls to preform the position tracking control problem. In the second step, the angular velocity ω_i is used as intermediate control to assure that the error between this actual angular velocity and its immediate value stabilize at the origin. Inspired by the work of [13], we define the following error variables

$$v_{ei} = v_i - \alpha_{vi}, \quad \phi_{ei} = \phi_i - \alpha_{\phi i} \tag{13}$$

where α_{vi} and $\alpha_{\phi i}$ are virtual controls to be determined later. The three first equations of (1) in terms of the new error variables re-write

$$\dot{\mathbf{p}}_i = \bar{u}_i + \Psi_{1i} + \Psi_{2i} \tag{14}$$

where $\bar{u}_i = \boldsymbol{\rho}(\alpha_{\phi i})\alpha_{vi}$, $\Psi_{2i} = \boldsymbol{\rho}(\phi_i)v_{ei}$, with $\boldsymbol{\rho}(\bullet) = [\cos(\bullet), \sin(\bullet)]^\top$ and

$$\Psi_{1i} = \begin{bmatrix} (\cos(\phi_{ei}) - 1)\cos(\alpha_{\phi i}) - \sin(\phi_{ei})\sin(\alpha_{\phi i}) \\ \sin(\phi_{ei})\cos(\alpha_{\phi i}) + (\cos(\phi_{ei}) - 1)\sin(\alpha_{\phi i}) \end{bmatrix}$$

From the definition of $\bar{u}_i$, it would be easy to determine the immediate control $\alpha_{\phi i}$ and α_{vi} from the knowledge of the expression of $\bar{u}_i$. The following Lyapunov function will allow the design of the virtual control $\bar{u}_i$ in order to fulfil the task of position tracking:

$$V_{1i} = \sqrt{1 + \|\mathbf{p}_i - \mathbf{p}_k\|^2} - 1 \tag{15}$$

which derivative along the solutions of (14) gives

$$\dot{V}_{1i} = \frac{(\mathbf{p}_i - \mathbf{p}_k)^\top}{\sqrt{1 + \|\mathbf{p}_i - \mathbf{p}_k\|^2}}(\bar{u}_i + \Psi_{1i} + \Psi_{2i} - \dot{\mathbf{p}}_k) \tag{16}$$

the velocity $\dot{\mathbf{p}}_k$ of the virtual robot is known for the design, we can design a bounded virtual control for $\bar{u}_i$ as follows

$$\bar{u}_i = -k_i\bar{V}_L \frac{(\mathbf{p}_i - \mathbf{p}_k)}{\sqrt{1 + \|\mathbf{p}_i - \mathbf{p}_k\|^2}} + \dot{\mathbf{p}}_k \tag{17}$$

where k_i is a positive constant chosen such that

$$k_i > \frac{v_k^{\min}}{2\bar{V}_L} \tag{18}$$

where $v_k^{\min}$ is the minimum velocity of the virtual vehicle such that $\|\dot{\mathbf{p}}_k\| > v_k^{\min}$. We need to solve for $\alpha_{\phi i}$ and α_{vi} from the definition of $\bar{u}_i$ and its expression in (17). The expressions for $\alpha_{\phi i}$ and α_{vi} are given in (19)-(20). Note that the expression in (19) is well defined since the term $-k_i\bar{V}_L(x_i - x_k)\cos\phi_k - k_i\bar{V}_L(y_i - y_k)\sin\phi_k + v_k\sqrt{1 + \|\mathbf{p}_i - \mathbf{p}_k\|^2} \neq 0$ since the condition (18) on the constant gain k_i is satisfied.

with the virtual control (17), the time derivative of the Lyapunov function (21) rewrites:

$$\dot{V}_{1i} = -k_i\bar{V}_l \frac{\|\mathbf{p}_i - \mathbf{p}_k\|^2}{1 + \|\mathbf{p}_i - \mathbf{p}_k\|^2} + \frac{(\mathbf{p}_i - \mathbf{p}_k)^\top}{\sqrt{1 + \|\mathbf{p}_i - \mathbf{p}_k\|^2}}(\Psi_{1i} + \Psi_{2i}) \tag{21}$$

Clearly if the second term of (21) was zero, we would conclude about the asymptotic stability of the error position tracking. The next steps consist in canceling those term in order to conclude

$$\alpha_{\phi i} = \phi_k + \arctan\Big(\frac{-k_i\bar{V}_L(x_i-x_k)\sin\phi_k + k_i\bar{V}_L(y_i-y_k)\cos\phi_k}{-k_i\bar{V}_L(x_i-x_k)\cos\phi_k - k_i\bar{V}_L(y_i-y_k)\sin\phi_k + v_k\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\Big) \quad (19)$$

$$\alpha_{vi} = -k_i\bar{V}_L\frac{x_i-x_k}{\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\cos\alpha_{\phi i} - k_i\bar{V}_L\frac{y_i-y_k}{\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\sin\alpha_{\phi i} + v_k\cos(\phi_k-\alpha_{\phi i}) \quad (20)$$

---

about the stability of the closed loop system. The final step in the first stage of the design consists in using the angular velocity ω_i as an immediate control to stabilize the error between the actual heading of the vehicle and its immediate value. Note that if the error position tracking is zero (i.e, $x_i - x_k = 0$ and $y_i - y_k = 0$) then from (19), the heading of the follower vehicle will ultimately converge to the heading of the virtual vehicle (i.e,ϕ_k).

We now consider ω_i as an immediate control to stabilize ϕ_{ei}, define then the following error variable

$$\omega_{ei} = \omega_i - \alpha_{\omega_i} \quad (22)$$

where α_{ω_i} is a virtual control of ω_i. To determine this virtual control, we propose a the following Lyapunov function candidate

$$V_{2i} = V_{1i} + \frac{1}{2}\phi_{ei}^2 \quad (23)$$

which derivative along the solutions of (21) and the third equation of (1) would allow to chose α_{ω_i} to cancel the first cross product term $\frac{(\mathbf{p}_i-\mathbf{p}_k)^\top}{\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\Psi_{1i}$ in (21) as follows:

$$\begin{aligned}\alpha_{\omega_i} &= -k_{\phi_{ei}}\phi_{ei} + \frac{\partial\alpha_{\phi i}}{\partial\phi_k}\omega_k + \frac{\partial\alpha_{\phi i}}{\partial\mathbf{p}_k}v_k\boldsymbol{\rho}(\phi_k) + \frac{\partial\alpha_{\phi i}}{\partial v_k}\dot{v}_k \\ &\quad - \frac{(\mathbf{p}_i-\mathbf{p}_k)^\top}{\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\frac{\Psi_{1i}}{\phi_{ei}} + \frac{\partial\alpha_{\phi i}}{\partial\mathbf{p}_i}(\bar{u}_i+\Psi_{1i}) \quad (24)\end{aligned}$$

with this choice the time derivative of the Lyapunov function (23) is given by

$$\begin{aligned}\dot{V}_{2i} &= -k_i\bar{V}_L\frac{\|\mathbf{p}_i-\mathbf{p}_k\|^2}{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2} - k_{\phi_{ei}}\phi_{ei}^2 \\ &\quad + \left[\frac{(\mathbf{p}_i-\mathbf{p}_k)^\top}{\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\boldsymbol{\rho}(\phi_i), \quad \phi_{ei}\right]\varpi_{ei} \quad (25)\end{aligned}$$

where $\varpi_{ei} = [v_{ei}, \omega_{ei}]^\top$. Note that the term $\frac{\Psi_{1i}}{\phi_{ei}}$ is well defined, since $\lim_{\phi_{ei}\to 0}\frac{\sin\phi_{ei}}{\phi_{ei}} = 1$ and $\lim_{\phi_{ei}\to 0}\frac{\cos\phi_{ei}-1}{\phi_{ei}} = 0$. Next step in the backstepping control concerns the last stage of the design which is the dynamic part of the follower robot. Obviously in the second stage, the last term in (25) will be canceled by choosing appropriately the control design F_i.

B. Stage 2: Dynamic design

In this stage, we design the real control input vector F_i for the follower robot. Before proceeding to the design of the control input, we re-write the dynamic of the robot in term of the error variable ϖ_{ei} as follows:

$$\overline{\mathbf{M}}_i\dot{\varpi}_{ei} = -\overline{\mathbf{C}}_i(\dot{\mathbf{p}}_i)\varpi_i - \overline{\mathbf{D}}_i\varpi_i - \overline{\mathbf{M}}[\dot{\alpha}_{vi}, \dot{\alpha}_{\phi i}]^\top + \overline{B}_iF_i \quad (26)$$

Note form (19)-(20), that α_{vi} and $\alpha_{\phi i}$ are function of the states of actual robot and the states of the virtual vehicle, their time derivatives are easily obtained. To determine the control input vector F_i, we propose the following Lyapunov function:

$$V_{3i} = V_{2i} + \varpi_{ei}^\top\overline{\mathbf{M}}_i\varpi_{ei} \quad (27)$$

Differentiating the Lyapunov function (27) along the solutions of (25) and (26) and choose for the control input F_i, the following

$$\begin{aligned}F_i &= \overline{B}_i^{-1}\Big(-K_{\varpi_{ei}}\varpi_{ei} + \overline{\mathbf{C}}_i(\dot{\mathbf{p}}_i)\varpi_i + \overline{\mathbf{D}}_i\begin{pmatrix}\alpha_{vi}\\ \alpha_{\phi i}\end{pmatrix} \\ &\quad + \overline{\mathbf{M}}\begin{pmatrix}\dot{\alpha}_{vi}\\ \dot{\alpha}_{\phi i}\end{pmatrix} - \left[\frac{(\mathbf{p}_i-\mathbf{p}_k)^\top}{\sqrt{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2}}\boldsymbol{\rho}(\phi_i), \quad \phi_{ei}\right]^\top\Big) \quad (28)\end{aligned}$$

gives that

$$\dot{V}_{3i} = -k_i\bar{V}_L\frac{\|\mathbf{p}_i-\mathbf{p}_k\|^2}{1+\|\mathbf{p}_i-\mathbf{p}_k\|^2} - k_{\phi_{ei}}\phi_{ei}^2 - \varpi_{ei}^\top(K_{\varpi_{ei}}+D)\varpi_{ei} \quad (29)$$

We now state the main result for the position tracking problem in the following theorem.

Theorem 2: Consider the follower robot with dynamic (1). Under Assumption 1 and the action of the control law (28) the position tracking problem of the follower robot is solved. In particular, the solutions of the closed-loop system are bounded and the tracking errors converge asymptotically to zero.

Proof: The proof of the theorem is a direct application of the Barbalt's lemma to (29). The details are omitted due to space limitation. ■

V. GENERALIZATION TO FORMATION OF n MOBILE ROBOTS

The results so far presented, can be extended to formation of $n > 2$ mobile robots. The structure that can be constructed with a given number of robots represents a rooted tree in the context of graph theory. Let $\mathcal{G}$ be the directed graph modeling the communication links between leaders and follower robots. In $\mathcal{G}$, a vertex $\mathcal{V}_i$ represents a the ith robot and the edge $(\mathcal{V}_i, \mathcal{V}_j)$ symbolizes a direct communication from robot i to robot j. Typical leader follower structure consists of constructing a convoy-like formation. The following is a generalization theorem 2.

Theorem 3: Consider a team of n mobile robots, one of which is considered as the main leader of the formation $\mathcal{V}_0$, the remaining $n-1$ mobile robots are the followers $\mathcal{V}_i, i = 1, \dots, n-1$ each has a dynamic given by (1). A multilayer formation blocks are constructed by a combination of cascades of a chains of leader follower pairs. Using the control input (28) for each of the $n-1$ follower robots, the solutions of the overall closed-loop formation system are bounded and the tracking errors converge asymptotically to zero.

Proof: The proof is quite similar to the proof of Theorem 1 and is omitted. ■

VI. Simulations

In this section we run some simulations to illustrate the effectiveness of our leader-follower controller on three mobile robots forming a platoon formation. The head of the platoon denoted by H_0 is the global leader followed by the two follower robots denoted F_1 and F_2 respectively. The robot F_1 also serves as a leader for the follower robot F_2. The desired distance between all robots are the same and equal to $2m$. The reference paths are generated using polynomial interpolation. Fig. 2 shows the reference and the real trajectories for the robots. Fig. 3 shows the time evolution of the distance error $\tilde{e}_{lk}$ for the leader robot H_0 with respect to its follower F_1. As can be seen in these figures, as leader robot travel along its trajectory the followers robot keep tracking, this leader with inter distance separation and therefore the leader-follower scheme is successful.

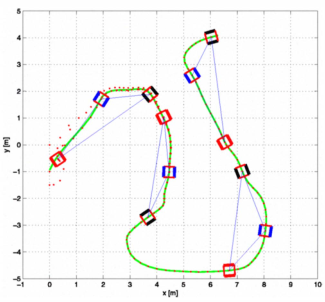

Fig. 2. Formation structure of three mobile robots.

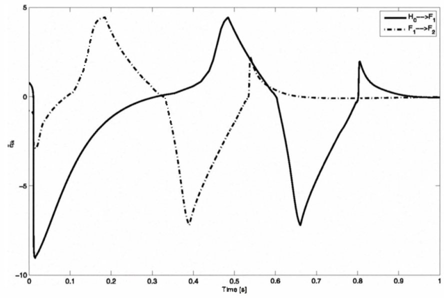

Fig. 3. Distance error between robots.

VII. Conclusion

The paper addressed the leader-follower formation control problem while the velocity of the leader is not available for control design. A virtual vehicle approach is proposed to design the formation control which consists in generating a reference trajectory for a virtual vehicle to track. The control input for the virtual vehicle is designed using only the measurement of the leader vehicle. Once the virtual vehicle converged to its desired position, tracking control for the follower robot is constructed using the backstepping technique to ensure that the follower robot converges to the virtual position. Simulation results have demonstrated the effectiveness of the approach. Further research direction is to consider formation with obstacle and crash avoidance among the vehicles.

References

[1] J. P. Desai, J. Ostrowski, and V. Kumar, "Controlling formations of multiple mobile robots," in Proc. IEEE Int. Conf. Robot. Autom, 1998, pp. 2864-2869

[2] G. Dudek, M. R. M. Jenkin, and D. Wilkes, "A taxonomy for multi-agent robotics," Autonomous Robots, vol. 3, no. 4, pp. 375-397, 1996.

[3] M. Mesbahi and F. Hadaegh, "Formation flying of multiple spacecraft via graphs, matrix inequalities, and switching," Journal of guidance, control, and dynamics, vol. 24, pp. 369-377, Mar. 2001.

[4] A. Howard, L. E. Parker, and G. S. Sukhatme, "Experiments with a Large Heterogeneous Mobile Robot Team: Exploration, Mapping, Deployment and Detection," The International Journal of Robotics Research, vol. 25, no. 5, pp. 431-447, May 2006.

[5] C. R. McInnes, "Autonomous ring formation for a planar constellation of satellites," Journal of Guidance, vol. 18, no. 5, pp. 1215-1217, 1995.

[6] R. Skjetne, S. Moi, and T. I. Fossen, "Nonlinear formation control of marine craft," in Proceedings 41st IEEE Conference on Decision and Control, Las Vegas, Nevada, USA, 2002, pp. 1699-1704.

[7] R. W. Beard, J. Lawton, and F. Y. Hadaegh, "A coordination architecture for spacecraft formation control," IEEE Transactions on Control Systems Technology, vol. 9, no. 6, pp. 777-790, 2001.

[8] T. Balch and R. C. Arkin, "Behavior-based formation control for multi robot teams," IEEE Transactions on Robotics and Automation, vol. 14, no. 6, pp. 926-939, 1998.

[9] J. Huang, S. M. Farritor, A. Qadi, and S. Goddard, "Localization and follow-the-leader control of a heterogeneous group of mobile robots," IEEE/ASME Transactions on Mechatronics, vol. 11, no. 2, pp. 205-215, Apr. 2006.

[10] J. Chen, D. Sun, J. Yang, and H. Chen, "A Leader-follower formation control of multiple nonholonomic mobile robots incorporating receding-horizon scheme," The International Journal of Robotics Research, vol. 29, no. 6, 2009.

[11] M. Defoort, T. Floquet, A. Kokosy, and W. Perruquetti, "Sliding-Mode Formation Control for Cooperative Autonomous Mobile Robots", IEEE Transaction on Industrial Electronics, Vol. 55, No. 11, Nov 2008.

[12] E. Kyrkjebo, K. Pettersen, "A virtual vehicle approach to output synchronization control. In:Proceeding of 45th IEEE Conference on Decision and Control, vol. 1, SanDiego, CA, pp.6016-6021.

[13] K.D. Do, "Output-feedback formation tracking control of unicycle-type mobile robots with limited sensing ranges", Robotics and Autonomous Systems, Vol, 57, no. 1, 2008.