A Relative Measurement based Leader-follower Formation Control of Mobile Robots

Yu. N. Zolotukhin, K. Yu. Kotov, A. S. Maltsev, A. A. Nesterov, M. A. Sobolev and M. N. Filippov
Institute of Automation and Electrometry, Siberian Branch, Russian Academy of Sciences,
pr. Akademika Koptyuga 1, Novosibirsk, 630090, Russia
kotov@idisys.iae.nsk.su

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Abstract: This paper deals with leader-follower formations of nonholonomic mobile robots and introduces a new nonlinear control method for the robot motion in formation. Proposed approach enables to track the target position and is based on using a forced movement along the desired trajectory in the state space. Moreover, this approach requires only relative and local motion sensors data. Simulation results have demonstrated the effectiviness and robustness of the proposed control shemes.

1 INTRODUCTION

During the last years there is an increasing interest in the formation control and coordination of multiple mobile robots. Cooperative using of mobile robots in the group is more efficient than the use of a single robot in various tasks including search, observation, transport, rescue and military operations (Schaub et al., 2000), (Smith et al., 2001), (Lawton et al., 2003), (Burns et al., 2000).

At the present time there are three main approaches to tackle the robot formation control problem: leader-follower approach (Das et al., 2002), behavior based (Lawton et al., 2003) and virtual structures (Lewis and Tan, 1997). In this study, we use the leader-follower approach proposing the division of robot group members on leaders and followers. The followers task is to maintain a desired distance and orientation to the leader. The main drawback of the leader-follower approach is that it depends heavily on the leader for achieving the goal. However this approach is appreciated for its simplicity and scalability.

In contrast to approaches (Lawton et al., 2003), (Consolini et al., 2008), (Zolotukhin et al., 2007), where it is necessary to know the absolute position and/or orientation of the leader for the determination of desired follower position in the group, in this study we use the approach in which follower position is defined in terms of the relationship of robots in local follower frame. This approach is more usable — in the most cases the mobile robot in the group can deal only with sensors data on the relative location of the robots.

In leader-follower formation control, the most widely used control technique is feedback linearization based on the kinematics model of the system (Desai et al., 2001), (Min et al., 2009), (LIU Shi-Cai, 2007). The application of this approach is limited by the complexity of the kinematic model and low robustness to external disturbances.

In this paper, the method of organization of forced motion over a desired trajectory in the plane state space is used to control homogeneous group of differential drive mobile robots. It was developed by us and successfully applied in a number of applications (Zolotukhin et al., 2007), (Belokon et al., 2013).

Simulation results verify the effectiveness of the proposed control system in the presence of measurement noise and external perturbations.

The rest of this paper is organized as follows. In Section 2 we recall the dynamic model of the mobile robot and formulate the formation tracking control problem. The proposed control schemes are presented in Section 3. In Section 4 we show illustrative simulation results and we conclude with some remarks in Section 5.

2 PROBLEM STATEMENT

Let us consider the homogeneous group of mobile robots, which of them have the following equations of motion (Lawton et al., 2003)

$$\left.\begin{aligned} \dot{x}_i &= v_i \sin\phi_i; \\ \dot{y}_i &= v_i \cos\phi_i; \\ \dot{\phi}_i &= w_i; \\ \dot{v}_i &= \frac{F_i}{m_i}; \\ \dot{w}_i &= \frac{M_i}{J_i}. \end{aligned}\right\} \quad (1)$$

Here x_i, y_i are the position coordinates of the robot; v_i, w_i are the linear and angular velocities; ϕ_i is the angle showing the moving direction of the vehicle relative to the axis of ordinates (see Fig. 1); m_i is the mass and J_i is the moment of inertia. The control inputs are the linear propulsive force F_i and the torque moment M_i. In what follows the point above the variable implies its time derivative.

Vehicle control is realized by changing linear and angular accelerations of the vehicle by the parameters $\frac{F_i}{m_i}$ $\frac{M_i}{J_i}$. We define a leader in the group that directs the other members of the group and helps them to determine their positions.

Let us assume that the leader motion along the prescribed trajectory is defined by control algorithm presented in (Zolotukhin et al., 2007). It is necessary to indicate the place of group members against the leader in order to describe their movement. We assume that the navigation system of the follower can determine parameters of the follower position against the leader: d_i — distance to the leader and α_i — direction to the leader relative to the direction of the follower movement, Fig. 1. Such a data is a range measuring data and can be provided from laser and infrared sensors, cameras. In this case, we can uniquely specify the required coordinates for the vehicle and its course in a group.

Let us set a task to the follower to move to the target point T_0, determined by the parameters d_{i_ref}, α_{i_ref} of relative leader-follower arrangement, Fig. 1. It will be shown that after finishing transients in control system, in stationary mode, follower and leader courses are the same and thus this approach to determine follower position is equivalent to determine target position with using leader direction data. Hence, it is unnecessary to apply additional control as in (Zolotukhin et al., 2007) when follower approaches to the target point. We define the error in the follower position against the target point by values, Fig. 1a

$$\left.\begin{aligned} E_{\tau i} &= d_i \cos(\phi_i + \alpha_i) - d_{i\_ref} \cos(\phi_i + \alpha_{i\_ref}); \\ E_{ni} &= d_i \sin(\phi_i + \alpha_i) - d_{i\_ref} \sin(\phi_i + \alpha_{i\_ref}). \end{aligned}\right\} \quad (2)$$

Let us consider the stationary case of the movement, when $\dot{E_{\tau i}} = \dot{E_{ni}} = 0$ and transients of target positioning

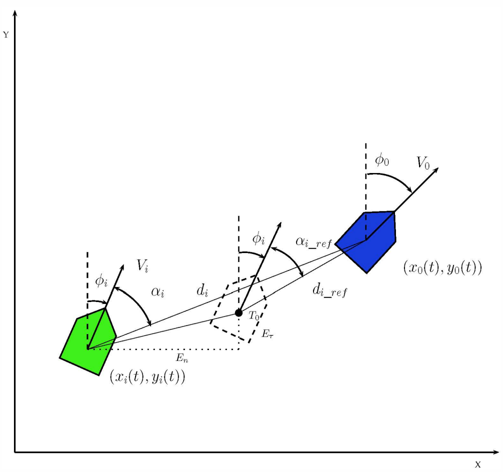

Figure 1: Movement of the follower relative to target position in the group.

are completed. Using the fact that

$$\left.\begin{aligned} d_i \cos(\phi_i + \alpha_i) &= y_0 - y_i; \\ d_i \sin(\phi_i + \alpha_i) &= x_0 - x_i \end{aligned}\right\} \quad (3)$$

and with regard to motion equations (1) compute (2)

$$\left.\begin{aligned} v_i \cos(\phi_i) &= v_0 \cos(\phi_0) + d_{i\_ref} \sin(\phi_i + \alpha_{i\_ref})\dot{\phi}_i; \\ v_i \sin(\phi_i) &= v_0 \sin(\phi_0) - d_{i\_ref} \cos(\phi_i + \alpha_{i\_ref})\dot{\phi}_i. \end{aligned}\right\} \quad (4)$$

Given a constant course angle ϕ_i or $\dot{\phi}_i = 0$, from the ratio of equations in (4) it is not difficult to show that $\phi_i = \phi_0$. It follows that resulting follower position in formation is independent of the definition of the target position parameter α_{i_ref} — relative to the follower or leader course.

3 CONTROL LAW SYNTHESIS

In accordance with the approach described in (Zolotukhin et al., 2007), (Belokon et al., 2013) define the functions

$$\left.\begin{aligned} S_{1i} &= \dot{E}_{\tau i} + k_e E_{\tau i}; \\ S_{2i} &= \dot{E}_{ni} + k_e E_{ni} \end{aligned}\right\} \quad (5)$$

and require the following conditions to be satisfied

$$S_{1i} = 0; S_{2i} = 0. \quad (6)$$

In this case, the errors (2) decrease exponentially, with the time constants $1/k_e$.

Note that the absolute heading angle ϕ_i is used in the equations (2). If considering that x, y coordinate system is a body-fixed system then we can suppose $\phi_i = 0$ in equations (2)

$$\left.\begin{aligned} E_{\tau i} &= d_i \cos(\alpha_i) - d_{i\_ref} \cos(\alpha_{i\_ref}); \\ E_{ni} &= d_i \sin(\alpha_i) - d_{i\_ref} \sin(\alpha_{i\_ref}). \end{aligned}\right\} \quad (7)$$

Thus, control of mobile robot can be relized only on the basis of the relative leader-follower arrangement data, defined by the parameters d_i, α_i.

The fulfillment of the conditions (6) can be provided by

$$\frac{d}{dt}S_{1i}^2 \leq 0; \frac{d}{dt}S_{2i}^2 \leq 0. \quad (8)$$

Let us strengthen conditions (8) by assuming that

$$\dot{S}_{1i} = -\alpha_1 S_{1i}; \dot{S}_{2i} = -\alpha_2 S_{2i}. \quad (9)$$

Here $\alpha_1 > 0$, $\alpha_2 > 0$ determine the time constants $1/\alpha_1$, $1/\alpha_2$ with which S_{1i}, S_{2i} exponentially tend to zero. Differentiating Eqs. (2) and substituting the results to Eqs. (9) with taking into account (1) and $\phi_i = 0$

$$\left.\begin{aligned}
&\frac{F_i}{m_i} = \ddot{y}_0 - d_{i_ref}(\dot{\phi}_i^2 \cos(\alpha_{i_ref}) + \frac{\sin(\alpha_{i_ref})M_i}{J_i}) - \\
&-\alpha_1 S_{1i} - k_e \dot{E}_{\tau i}; \\
&\frac{M_i}{J_i} = \frac{1}{d_{i_ref}\cos(\alpha_{i_ref})}(v_i \dot{\phi}_i - \ddot{x}_0 + \\
&+ d_{i_ref}\sin(\alpha_{i_ref})\dot{\phi}_i^2 + \alpha_2 S_{2i} + k_e \dot{E}_{ni}).
\end{aligned}\right\} \quad (10)$$

Subject to uniform motion, the accelerations values $\ddot{x}_0$, $\ddot{y}_0$ of the leader can be assumed to be zero, therefore when using the equations (10) it is necessary to know the follower speed parameters v_i and $\dot{\phi}_i$. This information can be provided by the inertial measuring unit (accelerometers, gyroscopes) on a robot platform.

We assume that the follower vehicle moves with bounded linear velocity along smooth path and effects due to rotation are expected to be small, hence we can neglect measurement of angular velocity $\dot{\phi}_i = 0$. The equations (10) take the form

$$\left.\begin{aligned}
&\frac{F_i}{m_i} = -d_{i_ref}\sin(\alpha_{i_ref})\frac{M_i}{J_i} - \alpha_1 S_{1i} - k_e \dot{E}_{\tau i}; \\
&\frac{M_i}{J_i} = \frac{1}{d_{i_ref}\cos(\alpha_{i_ref})}(\alpha_2 S_{2i} + k_e \dot{E}_{ni}).
\end{aligned}\right\} \quad (11)$$

In this case the control parameters are computed as rather simple equations, however the control accuracy is reduced.

4 SIMULATION RESULTS

The controlled object is an e-puck mobile robot with a differential drive, which was designed for education in engineering at the EPFL (Switzerland) (Mondada et al., 2009). The discrete equations of the kinematic and dynamic description of the robot have the form

$$\left.\begin{aligned}
&x_i^k = x_i^{k-1} + \Delta t v_i^k \sin\phi_i^k; \\
&y_i^k = y_i^{k-1} + \Delta t v_i^k \cos\phi_i^k; \\
&\phi_i^k = \phi_i^{k-1} + \Delta t w_i^k; \\
&v_i^k = \frac{1}{2a_0}(U_{1i}^k + U_{2i}^k); \\
&w_i^k = \frac{1}{2a_0 l}(U_{1i}^k - U_{2i}^k).
\end{aligned}\right\} \quad (12)$$

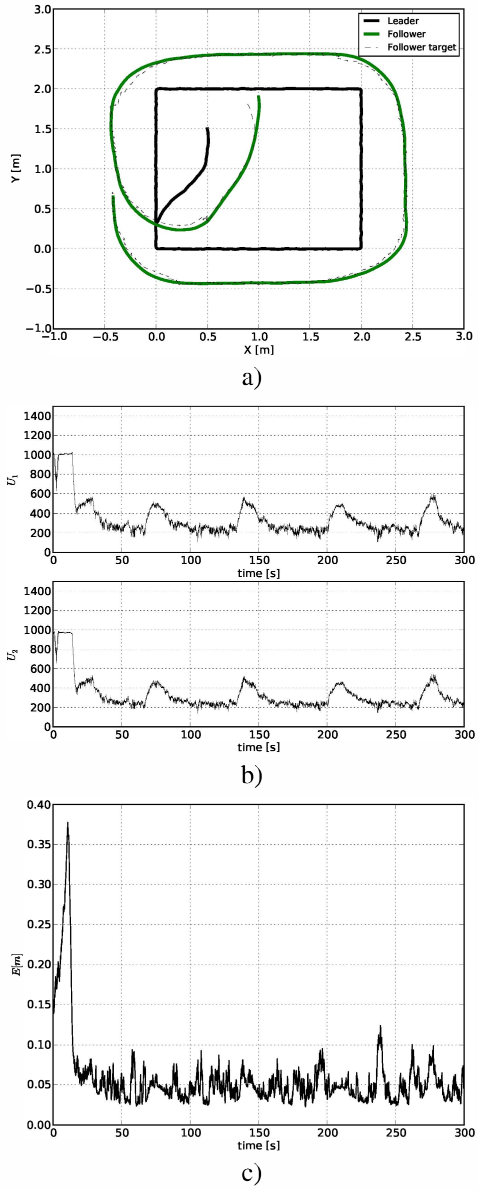

Figure 2: Movement of the leader-follower group by using control algorithm (11) : a) trajectory of the group movement in the plane x, y (solid dashed line, follower target track; solid black line, leader track; solid green line, follower track); b) control variables; c) deviation of the follower position from the target point.

where $1/(2a_0) = 0.0652\text{e-}03$, $2l = 0.05290$ m; Δt — sampling time. Control inputs $U_{1i}^k + U_{2i}^k$, $U_{1i}^k - U_{2i}^k$ correspond to $\frac{F_i}{m_i}$ and $\frac{M_i}{J_i}$. Position coordinates and orientation x_i^k, y_i^k and ϕ_i^k are corrupted by additive gaussian noises $\sigma_x = 0.001$ m, $\sigma_y = 0.001$ m, $\sigma_\phi = 0.05$ rad. The leader motion along the prescribed trajectory is defined by control algorithm described in (Zolotukhin et al., 2007).

The series of simulation results we carried out reflect the smooth trajectory of proposed control for the follower Fig. 2. Follower reference or target trajectory shown on Fig. 2 is calculated as

$$\left.\begin{aligned} x_{i\_ref} &= x_0 - d_{i\_ref}\sin(\phi_i + \alpha_{i\_ref}); \\ y_{i\_ref} &= y_0 - d_{i\_ref}\cos(\phi_i + \alpha_{i\_ref}). \end{aligned}\right\} \tag{13}$$

The control parameters are selected as $k_e = 10$, $\alpha_1 = \alpha_2 = 20$. The parameters of the follower position in the group are $d_{1\_ref} = 0.5$ m, $\alpha_{1\_ref} = 1.046$ rad.

The root-mean-square of the follower position deviations during the uniform motion is about 0.004 m, Fig. 2c. The peak-to-peak amplitude of the control signal oscillations is about 0.1 from the maximum allowable value, Fig. 2b. Defining the follower target position in the follower frame it is possible to provide smaller required follower acceleration, Fig. 2a in contrast with results of (Zolotukhin et al., 2007).

5 CONCLUSIONS

In this paper, we have derived a robust control algorithm for leader-follower formations of mobile robots. The proposed controller does not need global sensor for formation control and use only the relative measurement of the motion states between robots. Simulation results have demonstrated the efficiency of the proposed methods even in the case of significant curvature of the leader trajectory, and presence of measurement noises. In the future, we intend to implement out approach experimentally on mobile robot platform.

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