

# Vision-Based Path Control for Differential-Drive Mobile Robots

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**Abstract:** - This paper proposes a vision-based line following controller for a differential-drive mobile robot. First, a kinematic model of the robot suitable for path following applications is derived. A feedback control, which achieves local asymptotic stability of the nonlinear closed-loop system is designed and analysed. A camera model and the perspective point transformations are given. A line detection algorithm based on Hough transform is used in order to extract the position and orientation of the robot with respect to a detected reference line in the task space. Experiments in vision-based path tracking with differential-drive mobile robot Pioneer-3DX have been performed which confirm the validity of the proposed control scheme.

**Key-Words:** - Vision-based control, differential-drive wheeled mobile robot, kinematic model, line detection

## 1 Introduction

In more than three decades, there has been an increasing interest in wheeled mobile robots for industrial [1], public [2, 3], and scientific [4] applications. One of complexity in achieving autonomous navigation of mobile robots is the information for their environments, which must be obtained in real time from sensor systems. The use of visual information is an attractive solution and the visual servoing is a field of extensive research. In indoor applications, computer vision is often used for precise localization and especially, by using landmarks indicating a desired trajectory, which should be tracked [5]. Numerous image processing algorithms for detecting lines in the task space from data provided by cameras are reported in the literature, such as in [6, 7, 8, 12]. In this paper, we consider the problem of controlling the motion of a nonholonomic differential-drive mobile robot Pioneer-3DX (Fig.1), based on the information obtained from a monocular camera mounted on the robot. More specifically, we consider a straight-line following control task. We design a feedback control which achieves local asymptotic stability of the nonlinear closed-loop system. A line detection algorithm based on Hough transform [9] is used in order to extract the position and orientation of the robot with respect to a detected reference line in the task space. The rest of the paper is organized as follows: In Section 2, a kinematic model of the robot in error coordinates is derived. The robot-camera model, the perspective point

transformations, and the line detection algorithm are described in Section 3. In Section 4, the proposed controller and stability analysis is presented. Experimental results in vision-based path tracking are presented in Section 5. Conclusions are given in Section 6.



Fig. 1 The mobile robot Pioneer

## 2 Mobile Robot Kinematics

A plan view of the differential-drive wheeled mobile robot considered in this paper, is shown in Fig. 2. The kinematic scheme of the robot consists

of platform with two independently driven wheels mounted on the same axle and one free wheel (castor). The wheels of the robot are assumed to roll without lateral sliding. The coordinates of point  $P$ , which is located at the centre of the wheel axle, with respect to an inertial frame  $Fxy$ , are denoted by  $(x_P, y_P)$ . The angle  $\theta$  is the orientation angle of the robot with respect to  $Fxy$ . Two coordinate frames  $Px_Py_Pz_P$  and  $Dx_Dy_Dz_D$  are attached firmly to the robot at points  $P$  and  $D$ , respectively, in such way, that the  $x$ -axes are parallel to the longitudinal axis of the robot, and the  $z$ -axes are perpendicular to the ground. The centre of the coordinate system  $Dx_Dy_Dz_D$  is at distance  $h$  from the ground. A monocular camera is placed in front of the mobile in such way that the camera optical axis is perpendicular to the surface of motion and the origin of the camera frame coincides with the centre of the coordinate system  $Dx_Dy_Dz_D$ . Its projection on the ground is at distance  $d$  ahead from the wheel axle. A reference trajectory is defined by a moving reference coordinate frame  $Rx_Ry_R$  such that the  $x_R$  axis is tangent to the desired path and oriented in the direction of motion to follow. leave two blank lines between successive sections as here.

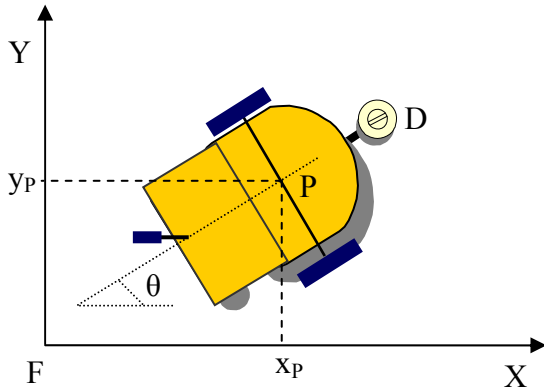


Fig. 2 The mobile robot geometry

Since the mobile robot is moving on a horizontal plane, in that follows, for simplicity of exposition we will use a coordinate frame  $Px_Py_P$  instead of  $Px_Py_Pz_P$ , where it is possible. Using the coordinates  $(x_P, y_P)$  of the reference point  $P$ , the configuration of the system is described by three generalized coordinates

$$q = [x_P \quad y_P \quad \theta]^T \in \mathbb{R}^3 \quad (1)$$

Let us denote by  $p_P = [x_P \quad y_P]^T \in \mathbb{R}^2$  the coordinates of point  $P$  with respect to the frame  $Fxy$ ; the coordinates of point  $D$  with respect to the frame  $Fxy$  by  $p_D = [x_D \quad y_D]^T \in \mathbb{R}^2$ , and the orientation of

$PD$  with respect to  $Fxy$  by  $\theta$ . If the vector  $d_P^D = [d \quad 0]^T \in \mathbb{R}^2$  is the vector from the origin of frame  $Px_Py_P$  to the origin of  $Dx_Dy_D$ , expressed in the coordinate frame  $Px_Py_P$ , then the coordinates of points  $D$  and  $P$  are related by

$$p_D = p_P + R_r d_P^D \quad (2)$$

where  $R_r \in SO(2)$  is orthogonal matrix.

The coordinate frames  $Rx_Ry_R$  and  $Dx_Dy_D$  are defined to describe the error kinematics during the line tracking process. For this purpose, the coordinates and orientation of the frame  $Rx_Ry_R$  in the coordinate frame  $Dx_Dy_D$  can be expressed in the form

$$e = T(p_R - p_D) \quad (3)$$

where  $e = [e_x \quad e_y \quad e_\theta]^T \in \mathbb{R}^3$  is the error posture;

$$p_R = [x_R \quad y_R \quad \theta_R]^T \in \mathbb{R}^3;$$

$$p_D = [x_D \quad y_D \quad \theta]^T \in \mathbb{R}^3,$$

$$\text{and} \quad T = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is a  $3 \times 3$  invertible matrix.

Differentiating (3) with respect to time and taking into account equation (1), after some work the error kinematics for straight-line path following applications can be described as follows

$$\dot{\bar{e}} = f(\bar{e}) + g(\bar{e})u \quad (4)$$

where

$$\bar{e} = [e_y \quad e_\theta]^T \in \mathbb{R}^2;$$

$f(\bar{e}) = [v_x \tan e_\theta \quad 0]^T$  and  $g(\bar{e}) = [-(e_y \tan e_\theta + d) \quad -1]^T$  are  $C^1$  vector functions;  $v_x$  is the longitudinal components of the linear velocities of points  $P$ ;  $u := \omega := \dot{\theta}$  is the angular velocity of the mobile robot and considered as control input, in this paper. We note that the angular velocities of the wheels are easily obtained from the linear and angular velocities of the robot by using algebraic relationships and are omitted in this paper, for brevity of exposition.

### 3 Robot-Camera Model

#### 3.1 Camera Model

A monocular camera is placed in front of the mobile robot, where the origin of the camera frame  $Cx_Cy_Cz_C$  coincides with the centre of the coordinate system  $Dx_Dy_Dz_D$ , at distance  $h$  from the ground. The optical axis of the camera is perpendicular to the surface of motion, as shown in Fig. 3. The focal length of the camera is denoted by  $f$ .

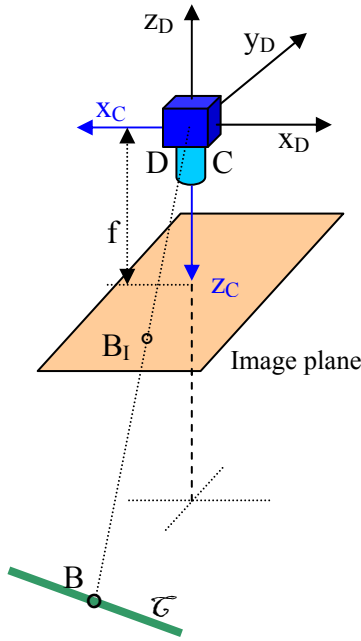


Fig. 3 Robot-camera configuration

The geometric relationships between the onboard camera and a feature point from the reference line, is shown in Fig. 3. Let us denote the position of a feature point  $B$  on the reference line  $\tau$  with respect to the camera frame by

$${}^C p_B = \begin{bmatrix} {}^C x_B \\ {}^C y_B \\ {}^C z_B \end{bmatrix} \in \mathbb{R}^3. \quad (5)$$

The corresponding pixel coordinates in the image plane are obtained as follows

$${}^I p_B = \begin{bmatrix} u_B \\ v_B \\ 1 \end{bmatrix} = \frac{1}{{}^C z_B} T_C {}^C p_B \in \mathbb{R}^3 \quad (6)$$

where the  $3 \times 3$  invertible matrix  $T_C$

$$T_C = \begin{bmatrix} fs_u & 0 & u_0 \\ 0 & fs_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

is the so called intrinsic camera calibration matrix [11] and  $(s_u, s_v)$  are the camera scaling factors. In (7), the angle between the camera axes was set to be  $\pi/2$ . Using (6) and (7), the coordinates of point  $B$  in  $Dx_Dy_Dz_D$  are obtained as follows

$${}^D p_B = \begin{bmatrix} {}^D x_B \\ {}^D y_B \\ {}^D z_B \end{bmatrix} = {}^C z_B R_y T_C^{-1} {}^I p_B \quad (8)$$

where  $R_y \in SO(3)$  is a rotation matrix about the  $y_D$  axis by angle  $\pi$  and  ${}^C z_B = h$ .

Thus, knowing the intrinsic camera parameters given by (7) together with the position and orientation of the camera with respect to the robot and the ground, and the coordinates of feature points from the reference line in the image space, by using (8), it is possible to extract the position of the robot with respect to a detected reference line in the task space.

#### 3.2 Line Detection and Error Posture Estimation

An algorithm based on Hough transform for line detection is developed, in order to extract the posture (position and orientation) of the robot with respect to the detected line in the task space by using data obtained from the monocular onboard camera. In the Hough transform, one involves a transformation of a line in Cartesian coordinate space to a point in polar coordinate space [9]. The method extracts the parameters of the line from its boundary points. In order to apply the Hough transform, an edge detection pre-processing step is performed.

The procedure of isolating a line in an image from the robot camera is illustrated bellow. A sample image containing the reference path is shown in Fig. 4. The Canny edge detector is used to identify edges in the image, as shown in Fig. 5. The detected line is found and automatically indicated in the original image, (Fig. 6).



Fig. 4 The original image from the camera

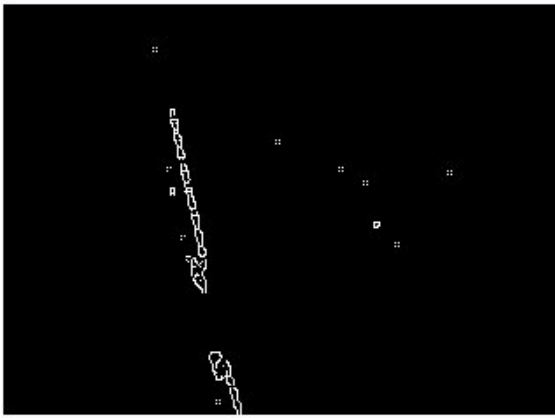


Fig. 5 Canny edge detection

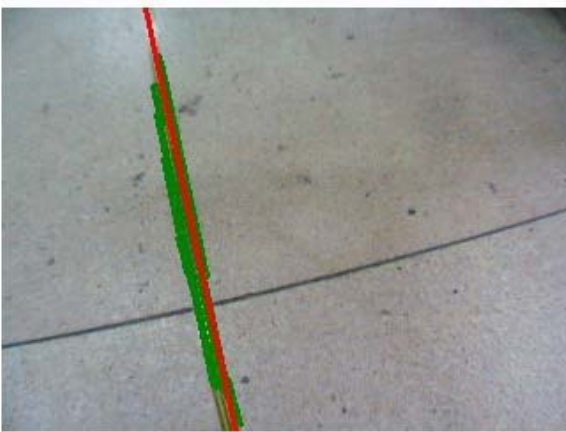


Fig. 6 Line detection

#### 4 Feedback Path Control

The path following geometry used in this paper is represented in Fig. 2. Consider a differential-drive robot moving forward on a flat surface. We assume that the path  $\mathcal{C}$  is a straight line which for simplicity coincides with the  $Fx$ . In this section, we present a path following controller design for the differential-drive mobile robot described by the nonlinear

system (4). We assume that the linear velocity  $v_x$  of the robot is positive and constant. In this case, using the parameterization  $(e_y, e_\theta)$  and given a path  $\mathcal{C}$ , the path following problem consists of finding a feedback control law for the system (4) with control input  $u$ , such that the state vector  $[e_y, e_\theta]^T$  tends to  $[0, 0]^T$ , as  $t \rightarrow \infty$ . We propose the following control law

$$u = k_y v_x e_y \quad (9)$$

where  $k_y$  is a positive constant. The closed-loop system has the form

$$\dot{\bar{e}} = m(\bar{e}) \quad (10)$$

where

$$m = \begin{bmatrix} v_x \tan e_\theta - k_y v_x e_y^2 \tan e_\theta - k_y d v_x e_y & -k_y v_x e_y \end{bmatrix}^T.$$

The point  $\bar{e} = 0$  is an equilibrium point of the closed-loop system. The stability of the equilibrium point will be analyzed based on the stability of the linearization of (10) about the origin. We have

$$\dot{\bar{e}} = M \bar{e} \quad (11)$$

where

$$M = \nabla m|_{\bar{e}=0}.$$

Let  $V: \mathbb{R}^2 \rightarrow \mathbb{R}$  be a  $C^1$  scalar radially unbounded positive definite quadratic function of the state  $\bar{e}$  given by

$$V = \bar{e}^T V_1 \bar{e} \quad (12)$$

where

$$V_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & (1/2)k_y \end{bmatrix}$$

is  $2 \times 2$  diagonal matrix. The derivative of  $V$  results in

$$\dot{V} = -k_y d v_x e_y^2 \leq 0. \quad (13)$$

Using the LaSalle's invariant principle [10], it can be shown that the origin of the linearized system (11) is asymptotically stable. Indeed, from (13), it follows that

$$\dot{V} = 0 \Rightarrow e_y \equiv 0 \Rightarrow \dot{e}_y \equiv 0. \quad (14)$$

Suppose that  $\bar{e}(t)$  is trajectory that belongs to set  $S$ :  $S = \{\bar{e} \in \mathbb{R}^2 \mid \dot{\bar{e}} = 0\}$ . From (11), it follows

$$(e_y = 0) \cap (\dot{e}_y = 0) \Rightarrow (e_\theta = 0) \cap (\dot{e}_\theta = 0). \quad (15)$$

Thus,  $\bar{e} = 0$  is asymptotically (and also exponentially) stable equilibrium point for the linear system (11). Hence, based on the Indirect Method of Lyapunov [10], the origin  $\bar{e} = 0$  is also a locally asymptotically stable equilibrium point for the nonlinear system (10).

The vision-based line tracking algorithm is presented by the flowchart depicted in Fig.7.

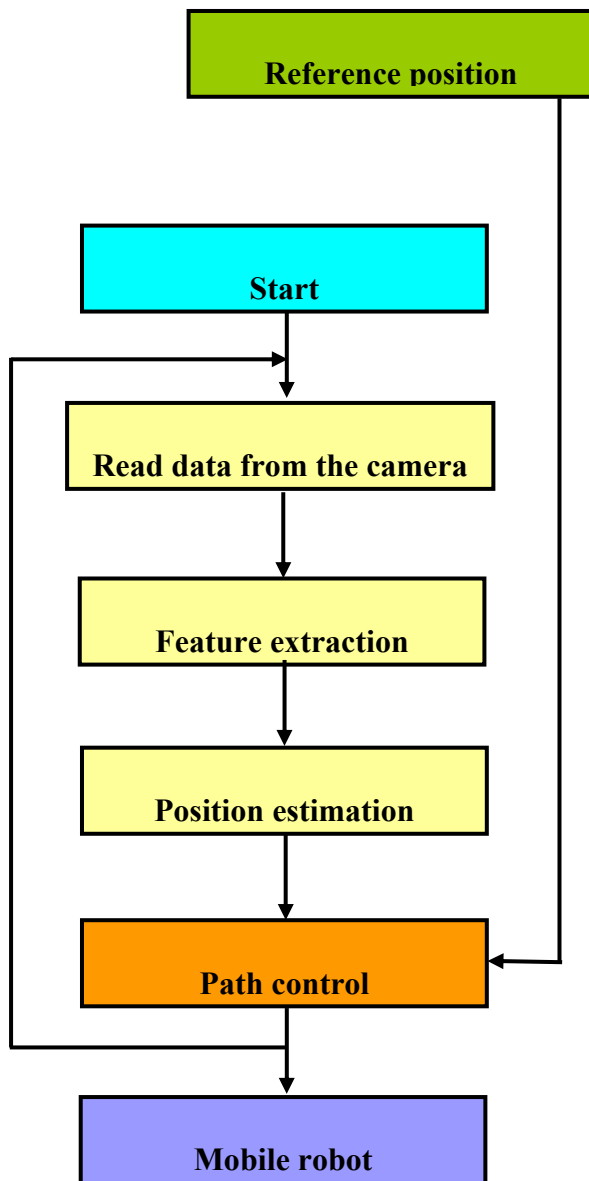


Fig. 7 Flowchart of the vision-based line tracking algorithm

## 5 Experimental Results

Experiments were performed to evaluate the effectiveness of the proposed vision-based line tracking algorithm. The mobile robot was equipped with Creative WebCam Notebook Ultra camera with resolution of  $640 \times 480$  pixels mounted on the robot. The software developed to execute the path tracking control was written in C#. The onboard computer is MSI 710ER and has a AMD Athlon 64 x2 processor. The mobile robot tracks a reference path which consists of series of connected straight-lines with velocity of  $0.3m/s$  with good accuracy (less than  $1cm$  in steady state), and without oscillations. An experimental test is presented in Fig.8. The experimental results confirm the validity of the proposed control scheme.

## 6 Conclusion

In this paper, a vision-based line following controller for a differential-drive mobile robot has been proposed. A kinematic model of the robot suitable for path following applications has been derived. A feedback control, which achieves local asymptotic stability of the nonlinear closed-loop system has been designed and analysed by using Lyapunov stability theory. A camera model and geometrical relationships between the camera and features on the reference line have been presented. A line detection algorithm based on Hough transform has been used in order to extract the position and orientation of the robot with respect to a detected reference line in the task space, in order to implement the proposed feedback control law. Experiments in vision-based path tracking with differential-drive mobile robot Pioneer-3DX have been performed. The experiments confirm the validity of the proposed control scheme.

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Fig. 8 Experiments in vision-based line tracking

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