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Obstacle Avoidance in Formation

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***Abstract*— In this paper, we present an approach to obstacle avoidance for a group of unmanned vehicles moving in formation. The goal of the group is to move through a partially unknown environment with obstacles and reach a destination while maintaining the formation. We address this problem for a class of dynamic unicycle robots. Using Input-to-State Stability we combine a general class of formation-keeping control schemes with a new dynamic window approach to obstacle avoidance in order to guarantee safety and stability of the formation as well as convergence to the goal position. An important part of the proposed approach can be seen as a formation extension of the configuration space obstacle concept. We illustrate the method with a challenging example.**

I. INTRODUCTION

The problem of controlling formations of unmanned vehicles has received a lot of attention in recent years, [5], [11], [6], [13], [9], [10]. This work has typically focused on formation keeping or coordination along preplanned trajectories. Indeed, very little vehicle formation control work has considered moving the formation through a partially unknown environment with obstacles. Yet, for applications such as search and rescue and terrain data acquisition using ground or low flying vehicles, avoiding obstacles is essential. The papers that do address obstacle avoidance have either taken an approach based on planning and optimal control [13] or a classical reactive approach [8]. The optimal control approaches usually suffer from extensive computational demands, while the purely reactive schemes are often heuristic or dependent on specialized obstacle assumptions.

The obstacle avoidance approach we use is both reactive and deliberate. The reactive part consists of a short-horizon, discretized (and therefore tractable), optimal control scheme that can avoid newly discovered obstacles. The deliberate part relies on a solution to a shortest path problem on a graph approximation of the obstacle-free space. This solution is used to form a navigation function [7], which in turn is used to construct a Lyapunov function guaranteeing convergence.

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The problem we address here is to combine the obstacle avoidance scheme above with a formation-keeping method of a given class. We select one robot in the formation to be the leader. There will typically be some implicit communication, i.e., robots can sense and follow others. Depending on whether or not explicit communication is available we get different problems but they can all be formulated in a disturbance-rejection framework.

Using Input-to-State Stability (ISS), we show how the disturbances affect the geometry of the formation. Given a bound on the disturbances, we compute an *Uncertainty Region*, a region the robots are guaranteed to occupy, around the perfect formation. We further propose how to use uncertainty regions in the obstacle avoidance problem facing the leader. The leader's problem is to determine which actions it can take and still be confident that none of the followers will collide while attempting to maintain the formation. This information is contained and presented in a *formation-leader obstacle map*, a concept similar to configuration space obstacles [7], when viewing the whole Uncertainty Region as one vehicle.

The proposed approach is valid for a general class of asymptotically stabilizing controllers which fix the orientation of the "perfect" formation. Our approach to combine any controller of this class with our obstacle avoidance method ensures that the properties of safety and goal convergence proved for the single-vehicle obstacle avoidance problem carry over to the formation case.

The organization of the paper is as follows. In Section II we provide the construction of the formation-leader obstacle map for a wide class of formation control schemes. In Section III we briefly present the Convergent Dynamic Window Approach to obstacle avoidance [3], [4]. Then, in Section IV we apply our method to a simulation example and draw conclusions in Section V.

II. FORMATION CONTROL

The robot model we consider is the dynamic unicycle [12]. This model is accurate for many indoor robots such as the Nomadic Technologies Super Scout as well as all caterpillar-type outdoor vehicles. The equations of motions are

$$\begin{aligned} \dot{x} &= v\cos\theta, \\ \dot{y} &= v\sin\theta, \\ \dot{\theta} &= \omega, \end{aligned}$$

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$$\dot{v} = F/m,$$
$$\dot{\omega} = \tau/J.$$

where x, y is the position, θ the orientation, v the translational velocity, ω the angular velocity and F/m and τ/J are force per mass and torque per moment of inertia, respectively. A kinematic version of this model (where v and ω are the controls) was used in [8], [13]. It was shown in [11] that the dynamics of the position $r \in \mathbb{R}^2$ of an off-wheel axis point of this model can be feedback linearized to $\ddot{r} = u$, i.e. a two-dimensional double integrator (which is the model used in [2], [5], [10]).

The problem we consider in this paper is to control a set of n vehicles $\ddot{r}_i = u_i,\ i = 1 \dots n$ moving in formation towards some goal point without colliding with obstacles.

Given a formation keeping scheme there are basically two ways for a leader to move the whole formation. One is by just moving and letting the others follow when they try to stay in formation. The other way, when explicit information exchange is possible, is to send the same motion command to all robots and superimpose it on their individual formation controls. Ideally this would just translate the whole formation, but time delays, calibration and other errors will unavoidably cause deviations. Both of these cases can be seen as disturbances to the formation keeping.

We explore in this section how such disturbances influence the formation and how this influence can be quantified in so-called *Uncertainty Regions*. We further show how the influence of the disturbances on the formation can be used in choosing the path of the leader towards the goal with so-called *formation-leader obstacle maps*. We do not specialize to a particular formation-keeping scheme but allow for a general class of such controllers. Before defining that class we need to first define what we mean by a formation.

By a perfect formation we mean that all relative, inter-robot, position vectors are fixed over time

$$r_i(t) - r_j(t) = d_{ij},\ \forall\ t. \tag{1}$$

Note that this means that the orientation of the perfect formation is fixed. Instances where this is not desirable can be imagined and such extensions seem possible but are beyond the scope of this paper.

For clarity it is useful to write the control as the sum of two terms:

$$\ddot{r}_i = u_{iform} + u_{idist},$$

where u_{iform} is the formation-keeping control term and u_{idist} is the remaining input, either a disturbance or a deliberate control term used to translate the formation, as described above. We will be using the concepts of trees and graphs and for clarity we review the definitions below.

Definition 2.1 (Tree): A tree is a directed graph without cycles such that exactly one node called the root has indegree 0 and all others indegree 1. A directed graph G is an ordered triplet (V, E, ϕ), where V is a set of nodes or vertices, E is a set of edges and $\phi : E \to V \times V$, a mapping from the edges to the ordered set of vertex pairs. The *outdegree* of a node $v \in V$ is the number of edges that have v as first element and *indegree* the number of edges that have v as second element. A *path* is a sequence of edges such that the second node of an edge is the first node of the next edge.

Note that the tree definition implies that the graph is connected, i.e. there is a path from the root to all other vertices. A typical tree structure can be found in Figure 1.

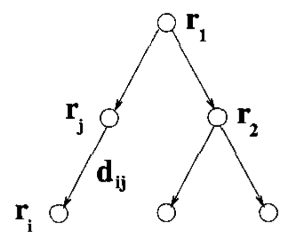

Fig. 1. A tree structure $\mathcal{E}$, we let the robots r_i be the nodes of the tree. The edges can, but do not have to indicate which robot is being tracked by which. In a perfect formation $r_i(t) - r_j(t) = d_{ij}$.

Definition 2.2: (Asymptotically Stable Formation Control) Let the robots $\{r_i\}$ form nodes of a tree $\mathcal{E}$ where the leader r_1 is the root. The edges may correspond to the way the robots sense and follow each other but they may also be randomly assigned as long as they don't violate the tree property. Let new coordinates be given by

$$r_{ij} = r_i - r_j - d_{ij},\ \forall (i,j) \in \mathcal{E}, \tag{2}$$

where d_{ij} are the desired relative position vectors defined in equation (1) and (i,j) are vertex pairs corresponding to edges. Note that adding r_1 to the set $\{r_{ij}\}$ gives a one-to-one correspondence between $\{r_i\}_i$ and $\{r_1, r_{ij}\}_{(i,j)\in\mathcal{E}}$. Let the state of the system be $x = \{r_{ij}, \dot{r}_{ij}\}$. The formation dynamics can now be written as

$$\dot{x} = f(x, u_{form}(x), u_{dist}) \tag{3}$$

where u_{dist} is a disturbance. By an Asymptotically Stable Formation Control u_{form} we mean that $x = 0$ is an asymptotically stable equilibrium of the undisturbed system ($u_{dist} = 0$). We also require the leader to be unaffected by the formation control, $u_{1form} = 0$, and add the technical conditions $f \in C^1$ and $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial u}$ bounded.

Since the tree property makes the graph connected, $x = 0$ implies that the robots are in a perfect formation, i.e. equation (1) holds for all (i,j), not just $(i,j) \in \mathcal{E}$.

Note that for a given formation control scheme the choice of $\mathcal{E}$ is not unique. Choosing a different tree $\mathcal{E}$ gives different coordinates x and thus different ISS bounds and a different Uncertainty Region in Lemma 2.11 below.

The control algorithm presented in [17] satisfies Definition 2.2. There the authors show how to calculate ISS bounds for different interconnections, e.g. cycles and multi-leader following. The control scheme presented there can also be used in the framework of this paper to compute the Uncertainty Regions described below.

Example 2.3: (Asymptotically Stable Formation Control) Consider as an example the tree in Figure 1. We choose to let the implicit communication in the control scheme match the tree structure and apply a simple PD controller at each edge of $\mathcal{E}$. If (i, j) is an edge this corresponds to letting

$$u_{iform} = -k_p(r_i - r_j - d_{ij}) - k_d(\dot{r}_i - \dot{r}_j).$$

In terms of the relative coordinates defined in Equation (2) we get

$$\ddot{r}_{ij} = -k_p r_{ij} - k_d \dot{r}_{ij} - u_j.$$

This can be viewed as a stable system with the disturbance u_j, the control of robot r_j that is being tracked. This part of the system is ISS (see Definition 2.7) and since cascaded ISS systems are again ISS [16], so is the whole formation. With $u_{1dist} = 0$ the system is then asymptotically stable and Definition 2.2 is satisfied. Note that we have argued here from ISS to asymptotical stability while Lemma 2.8 below goes the other way.

Theorem 2.4 (Main): Suppose we are given the system $\ddot{r}_i = u_i = u_{iform} + u_{idist}$, an asymptotically stable formation control u_{iform}, an obstacle set $\mathcal{O} \subset \mathbb{R}^2$ and disturbance bounds $||u_{idist}|| \leq K_i$.

Then, there exists a formation-leader obstacle set $\mathcal{FLO}$, $\mathcal{O} \subset \mathcal{FLO} \subset \mathbb{R}^2$, such that for each t', $r_1(t) \notin \mathcal{FLO}$, $\forall t \leq t'$ implies $r_i(t) \notin \mathcal{O}$, $\forall i, t \leq t'$, i.e. no vehicle will collide with an obstacle if the leader stays out of the $\mathcal{FLO}$.

Furthermore, this set can be computed by the following double integral.

$$\mathcal{FLO}(a, b) = \Theta\left(\iint_{\mathbb{R}^2} \mathcal{O}(a + y_1, b + y_2)\mathcal{UR}(y_1, y_2)dy_1 dy_2\right),$$

where the Uncertainty Region $\mathcal{UR}$ can be calculated using Lemma 2.11, $\mathcal{UR}(\cdot), \mathcal{O}(\cdot), \mathcal{FLO}(\cdot)$, are binary membership functions for the sets $\mathcal{UR}, \mathcal{O}, \mathcal{FLO}$ and Θ is the Heaviside step function ($\Theta(z) = 1$ if $z > 0$, 0 else).

Remark 2.5: The Formation Leader Obstacle $\mathcal{FLO}$ can be seen as an extension of the concept of configuration space obstacles [7], to formation control.

Remark 2.6: In the case of explicit information exchange, as described in the beginning of this section, we have K_1 large and $K_{i\neq 1}$ small. In the other case, (feed forward), time delays and other errors must be accounted for in all K_i bounds.

Before we prove the main theorem we need a few definitions and lemmas. The first two can be found in [16].

Definition 2.7 (Input-to-State Stability (ISS)): The system $\dot{x} = f(t, x, u)$ is locally input-to-state stable if there exists a class KL function β, a class K function γ, constants $k_1, k_2 > 0$ such that whenever $||x(t_0)|| \leq k_1$ and $\max_{\tau \leq t} ||u(\tau)|| \leq k_2$ the solution exists and satisfies

$$||x(t)|| \leq \beta(||x(t_0)||, t - t_0) + \gamma(\sup_{\tau \leq t}||u(\tau)||)$$

for all $t \geq t_0 \geq 0$.

ISS thus means that a bounded input will result in a bounded state.

Lemma 2.8 (Asymptotic Stability to ISS): Consider the system $\dot{x} = f(x, u)$. Suppose that, in some neighborhood of the equilibrium, f is continuously differentiable and that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial u}$ are bounded, uniformly in t. If the unforced system has a uniformly asymptotically stable equilibrium at $x = 0$, then the system is locally ISS.

The ISS concept will be used in Lemma 2.11 to calculate the Uncertainty Region in the general case.

Definition 2.9 (Uncertainty Region ($\mathcal{UR}$)): If there exists a subset $\mathcal{UR} \subset \mathbb{R}^2$ such that $r_j(t) - r_1(t) \in \mathcal{UR}$, $\forall t, r_j$ and $\forall u_{idist} : ||u_{idist}(t)|| \leq K_i$, then this set will be denoted an *Uncertainty Region* of the formation.

When using the uncertainty region in the obstacle avoidance approach, a smaller region is better (less uncertainty). In [17] it is shown how to design a formation control scheme to get low ISS bounds. This can be used in our framework to get a small uncertainty region and thus more effective formation obstacle avoidance.

Example 2.10 (Uncertainty Region): We calculate the Uncertainty Region for a small formation as described in Example 2.3. Let the Lyapunov function candidate be, (with $r = r_{ij}$),

$$V = \frac{1}{2}(r + \dot{r})^T(r + \dot{r}) + r^T r + \dot{r}^T \dot{r}.$$

Let $x = (r, \dot{r})$. Since $||x||^2 \leq V(x) \leq 2||x||^2$, V is clearly positive definite and decrescent. Letting the control be $u = -r - \dot{r} + u_{dist}$ we get

$$\begin{aligned}
\dot{V} &= \frac{-1}{2}||r + \dot{r}||^2 - \frac{1}{2}||r||^2 - \frac{3}{2}||\dot{r}||^2 + (r + 3\dot{r})^T u_{dist} \\
&\leq \frac{-1}{2}||x||^2 + (r + 3\dot{r})^T u_{dist} \\
&\leq \left(\frac{-1}{2}||x|| + \sqrt{10}||u_{dist}||\right)||x|| \\
&\leq \left(\frac{-1}{2}\sqrt{\frac{V}{2}} + \sqrt{10}||u_{dist}||\right)||x||.
\end{aligned}$$

Looking at V such that $\dot{V} = 0$ we see that the region $\{x : V(x) \leq 80||u_{dist}||^2\}$ is invariant. Therefore, $||x|| \leq 9||u_{dist}||$ and $||r|| \leq 9||u_{dist}||$ is an Uncertainty Region. Note that this is with feedback gain $k_d = k_p = 1$ in the PD controller. The higher the gain the smaller the uncertainty region for a given disturbance.

Consider now a triangular formation with $d_{21} = (-1.5, 0.75)$, $d_{31} = (-1.5, -0.75)$ and a disturbance bound of $||u_{dist}|| \leq \frac{1}{24}$. This gives $||r_i - r_1 - d_{1i}|| \leq \frac{3}{8}$ as shown in Figure 2. The leader has a point uncertainty by definition, $r_1 - r_1 = 0$, but to account for the shape of the vehicle we add a small disc around it to the uncertainty region. A similar argument can be made to slightly enlarge the uncertainty region around all the robots.

We now go on to show how to calculate the Uncertainty Region in the general case.

Lemma 2.11 (ISS to Uncertainty Region): Let the unique path from r_1 to r_i in $\mathcal{E}$ be $P_i = \{e_1, e_2, \ldots, e_N\}$,

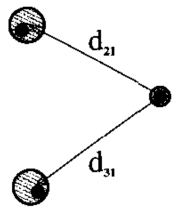

Fig. 2. The Uncertainty Region around a perfect formation, as described in Example 2.10. The small black circles are the robots and the shaded area is the uncertainty region. Note that the leader to the right has low uncertainty (a small disc) in its own position.

where N is the number of edges. Suppose the system (3) is ISS, then the individual error satisfies

$$||r_i - r_{iref}|| \leq \sqrt{N}M(t),$$

where $r_{iref} = r_1 + d_{e_1} + d_{e_2} + \ldots + d_{e_N}$, the desired perfect formation position and $d_{e_k} = d_{ij}$ when $e_k = (i,j)$. $M(t) = \beta(||x(t_0)||, t - t_0) + \gamma(\sup_{\tau \leq t}||u(\tau)||$ from the ISS definition. This bound can furthermore be used to calculate an uncertainty region as

$$\mathcal{UR} = \{r \in \mathbb{R}^2 | \; \exists i, \; ||r - r_{iref}|| \leq \sqrt{N}M(t)\}.$$

Proof: We start by noting that

$$\frac{1}{\sqrt{N}}\Sigma_N |x_i| \leq \sqrt{\Sigma_N |x_i|^2} \leq \Sigma_N |x_i|.$$

Since the system is ISS we have that

$$\begin{aligned}
M(t) &\geq ||x|| = \sqrt{\Sigma_{\mathcal{E}}||r_{ij}||^2 + ||\dot{r}_{ij}||^2} \\
&\geq \sqrt{\Sigma_{\mathcal{E}}||r_{ij}||^2} \\
&= \sqrt{\Sigma_{\mathcal{E}}||r_i - r_j - d_{ij}||^2} \\
&\geq \sqrt{\Sigma_{P_l}||r_i - r_j - d_{ij}||^2} \\
&\geq \frac{1}{\sqrt{N}}\Sigma_{P_l}||r_i - r_j - d_{ij}|| \\
M(t)\sqrt{N} &\geq ||\Sigma_{P_l}(r_i - r_j - d_{ij})|| \\
&= ||(r_2 - r_1 - d_{21}) + (r_3 - r_2 - d_{32}) \\
&\quad + \ldots + (r_{l-1} - r_{l-2} - d_{(l-1)(l-2)}) \\
&\quad + (r_l - r_{l-1} - d_{l(l-1)})|| \\
&= ||r_l - r_1 - (d_{e_1} + d_{e_2} + \ldots + d_{e_N})|| \\
&= ||r_l - r_{lref}||
\end{aligned}$$

which concludes the proof. ■

Proof of Main Theorem: By Lemma 2.8 the system is ISS and by Lemma 2.11 this in turn gives rise to an uncertainty region $\mathcal{UR}$. Writing the sets $\mathcal{O}, \mathcal{FLO}, \mathcal{F}$ as binary memberships functions, $\mathcal{O}(z) = 1$ if $z \in \mathcal{O}$ and $\mathcal{O}(z) = 0$ if $z \notin \mathcal{O}$, we have

$$\mathcal{FLO}(a,b) = \Theta\left(\iint_{\mathbb{R}^2} \mathcal{O}(a + y_1, b + y_2)\mathcal{UR}(y_1, y_2)dy_1dy_2\right).$$

■

If $\mathcal{O}$ or $\mathcal{UR}$ has isolated regions of measure zero this must be handled separately by Dirac delta functions etc.

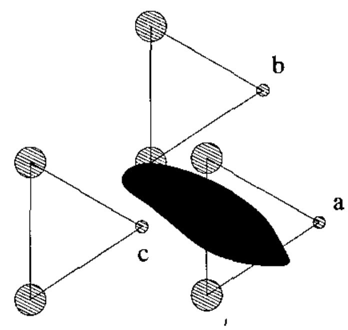

Fig. 3. Calculating the $\mathcal{FLO}$ by checking which positions are guaranteed free.

The double integral equals zero when there is no intersection between the Uncertainty Region $\mathcal{UR}$ and the obstacles $\mathcal{O}$, as in (a) and (c) of Figure 3 . If, however, there is an intersection, as in (b), the integral will not be zero and the Θ function will return 1 and the corresponding leader position will be marked as occupied in the $\mathcal{FLO}$.

The problem is now to drive the leader $\ddot{r}_1 = u_{1dist}$ through the $\mathcal{FLO}$ to the goal point. To do this we will use the convergent dynamic window approach to obstacle avoidance [3], [4].

III. Obstacle Avoidance

The problem of robotic motion planning is a well-studied one, see for instance [7]. Apart from the classical approaches of histograms and vector addition, a few somewhat more recent schemes have emerged. One of them is the Dynamic Window Approach [1], [2]. We will use a revised version of the latter that can be viewed as a synthesis inspired by [15] of the performance-oriented approaches [1], [2] and the convergence-oriented method of exact navigation by artificial potentials presented in[14].

In [4], a Provable Convergent Dynamic Window Approach is proposed. The original approach is reformulated as a combined Receding Horizon Control and Control Lyapunov Function scheme. The problem is stated as

$$\begin{aligned}
&\inf_{u(\cdot)\in C} && V(x(t+T)) && (4)\\
&\text{s.t.} && \ddot{r} = u && (5)\\
& && \dot{V}(x,u) \leq 0 && (6)
\end{aligned}$$

where $x = (r, \dot{r})$ is the state, C is a somewhat technical control sequence set and V is a control Lyapunov function $V(x) = \frac{1}{2}\dot{r}^T\dot{r} + \frac{k}{\sqrt{2}}NF(r)$. $NF(r)$ is the Navigation Function, a continuous approximation of the length of the shortest obstacle-free path to the goal. Below we restate the main results of [4] but refer to that paper for proofs and details.

Theorem 3.1 (Asymptotic Stability): If the control scheme in (4) is used and if there is a traversable path from start to goal in the occupancy grid. Then the robot will reach the goal position.

Remark 3.2: Note that this excludes all so-called local minima problems present in some navigation schemes.

Theorem 3.3 (Safety): If the control scheme in (4) is used and if the robot starts at rest in an unoccupied position. Then, the robot will not run into an obstacle.

IV. Simulation Example

We will now illustrate the approach with a simulation example. In [2] as well as in [4] it is assumed that the sensors can supply us with an occupancy grid map of the surroundings. I.e the state space $\mathbb{R}^2$ is partitioned into small squares, and each square is marked as being occupied or free. The set $\mathcal{O}$ is depicted in Figure 4. The formation uncertainty region $\mathcal{UR}$, from Figure 2, is discretized in the same way as $\mathcal{O}$ and is shown in Figure 5.

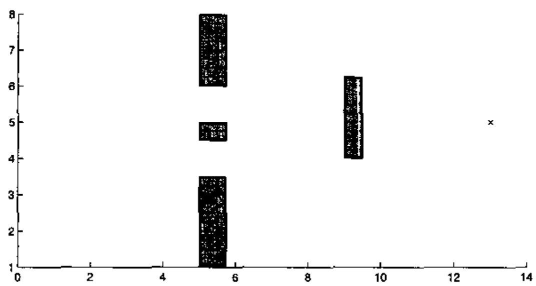

Fig. 4. The obstacle set $\mathcal{O}$.

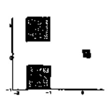

Fig. 5. The uncertainty region $\mathcal{UR}$.

Calculating the two-dimensional integral of Theorem 2.4 using the command conv2 in Matlab we get the formation leader obstacles, $\mathcal{FLO}$, as seen in Figure 6.

Note how the low uncertainty in leader position (only one square) gives rise to an identical copy of $\mathcal{O}$ in $\mathcal{FLO}$. The larger uncertainty regions of the two followers result in larger regions to the right of the $\mathcal{O}$ copy. Note especially how the double passage is turned into a single slot at $(7,5)$.

Running the algorithm we get the trajectories of Figure 7. Since we set the disturbances to zero in the simulation, the robots are in the center of the uncertainty regions and it can be seen how the obstacles are avoided with larger margins for the followers than for the leader, e.g. at $(5,5)$.

In Figure 8 the leader trajectory can be seen not to intersect the set $\mathcal{FLO}$. The level curves of the navigation function are also plotted. Note how the receding horizon control favors going perpendicular to the level curves since it minimizes $V(t+T)$ in Equation (4). At $(8-9,5)$ there is a ridge in the navigation function corresponding to the choice of going above or below the obstacle.

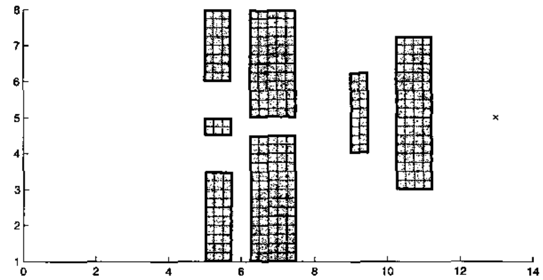

Fig. 6. The $\mathcal{FLO}$ resulting from the $\mathcal{O}$, $\mathcal{UR}$ computation.

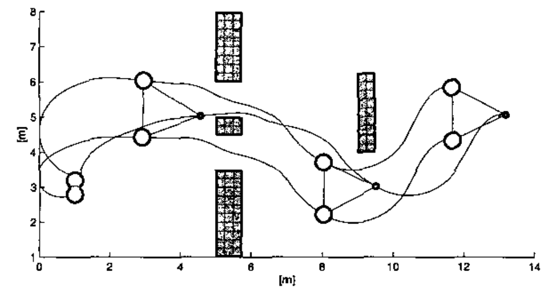

Fig. 7. Trajectories of the three robots. The circles illustrate the size of the Uncertainty Regions. All robots start close to (1,3).

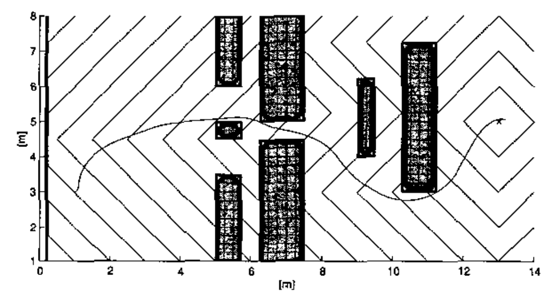

Fig. 8. The $\mathcal{FLO}$ as well as the Navigation Function level curves and resulting leader trajectory.

V. Conclusions and Future Research

By combining a general class of formation schemes with a new convergent dynamic window approach we have shown how to do safe and convergent obstacle avoidance while staying in a formation. The approach is illustrated by a simulation example.

A natural extension of this work is to consider problems of formation rotation and expansion.

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