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Detailed Slip Dynamics for Nonholonomic Mobile Robotic System

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***Abstract* - These It is well known in the literature that wheeled mobile robot undergoes slip while moving through a flat surface, if certain conditions are violated. Slip is a very common phenomena present in all kind of wheeled mobile robotic system and particularly become prominent while taking a sharp bend on a flat ground. The degree of slip mainly depends on the wheel ground interaction properties at the contact point. Slip is absolutely undesirable as it consumes part of the system energy, erodes the wheel surface and also hampers the system performance and stability. While a system undergoes slip eventually it is very difficult to control the system and to get a proper estimate of system's position. It is very much essential to detect the slip when it is taking place and also to estimate the slip parameters with a set of specific sensors. Once the system undergoes slip it needs to go back to no-slip fulfilling certain conditions. The aim of this paper lies in formulating a detailed slip dynamics & its' solution for a differentially driven non-holonomic mobile robot. Certain switching conditions have been proposed for the transition from no-slip to slip & vice versa. The holonomic and non-holonomic constraints of slip and no-slip are taken into account in the model and when there is no slip then the constraints converge to no-slip constraints. A traction model for each wheel has been proposed which is a function of slip velocities. For motion simulation of WMR the traction model is used in conjunction with system dynamic equations. The simulation has been carried out with the data of a real robot and the simulation results illustrate its performance. Simulation results clearly show that the system dynamics is able to switch from slip to no-slip and vice versa.**

Index Terms - Kinetic energy; nonholonomic dynamics; wheeled mobile robot; feedback linearization.

I. Introduction

The study of nonholonomic systems have been initiated long back but the rigorous research on nonlinear coupled dynamics and autonomous motion control of such kind of systems has been initiated hardly two decades ago [1-2]. A general dynamical model using Lie Algebra is derived for mobile robots with nonholonomic constraints [3-4] assuming no-slip. Some of the robust wheel mobile robot controllers based on neural network and sliding mode approaches have been presented in [5-7] which deal with uncertainty without considering the slip phenomena.

Differentially driven Wheeled Mobile Robots (WMRs) represent an important class of nonholonomic mechanical systems [8]. Presently, WMRs are used extensively for strategic applications, visual inspection, service areas, planetary exploration etc. Some of the applications also demand high velocity maneuvering control of the vehicle and appearance of slip phenomena in practice are equivalent to violation of no-slip constraints. If the vehicle performs slow motion with low acceleration maintaining almost zero relative velocity at wheel-terrain contacts, then slip effects are negligibly small and might be neglected. However, in some particular real-life fast motion tasks in an unstructured environment the slip phenomena cannot be neglected because exclusion of them in a system model leads to significant deterioration of control system performance and stability. Apart from that, as most of the autonomous mobile robots rely on Odometry, slip at the wheel-ground contact point leads to localization error and wastage of power.

Several researchers [9-12] has estimated slip states based on kinematic model of wheeled mobile robot of several configurations moving on flat terrain. Wheel slip model on uneven terrain associated with system dynamics has also been addressed in [13-14]. The authors considered small values of slip ratios on which traction force is linearly dependent. In [14] author proposed to use actuated variable length axle to move through uneven terrain without slip. A kinematic control law was developed in [15] for WMR to overcome the disturbance due to slip which is assumed as small, measurable and bounded. In [13] a longitudinal traction force model associated with system dynamics for an Omni-directional WMR has been studied measuring the slip externally.

Slip causes deviation of the robot from the desired path. Active control action along-with switching conditions switch the vehicle from slip to no-slip domain and take back the vehicle to the right track again. There are some works related to the problem of control of wheeled robotic system during slip conditions [16-17].

There are very few work explicitly related to the direct solution of slip dynamics. In [18] wheel slip dynamics was explicitly modeled considering WMR's overall dynamics and a feedback linearization control law has been developed. Both lateral and longitudinal wheel slips has been modeled in a dynamic way. In both the cases traction force models are generated using magic formula [19]. Though a substantial amount of work has been done to understand and control wheel slip phenomena but probably very few researchers has concentrated and solved explicitly the system dynamics associated with slip parameters for the differentially driven wheeled mobile robot (WMR).

In this paper, we have addressed specifically the wheel slip dynamics model associated with equations of motion for a differentially driven WMR. Traction force model for each wheel has been developed which is expressed as a function of slip velocities. The forward slip dynamics problem has been solved uniquely and the model is capable of switching from no-slip to slip domain depending on control input and wheel-ground interaction properties. For motion simulation of WMR the traction model is used with system dynamic equations.

II. Dynamic Modeling Of The Nonholonomic Mobile Robot Considering Wheel Slip

In the present context, a mobile vehicle equipped with two driving wheels and a passive self adjusted castor wheel for balancing the vehicle has been considered. The schematic diagram of the mobile robot is shown in Fig. 1. The two non-deformable driving wheels are independently driven by two actuators (Brushless DC motors) interfaced with a speed reduction gearbox to achieve the desired motion. Robot motion is assumed in a flat terrain only.

The dynamics & control of WMR is studied widely [3-7] by various researchers assuming no slip condition. But, while moving through a very smooth path or variable resistance path with high acceleration or while taking sharp turn mobile robot experience longitudinal & lateral slips depending on wheel ground interaction properties. For the robots, driven by wheels, slip is the relative motion between the tyre and the road surface it is moving on. Practically, slip phenomena is inevitable and it needs to be studied carefully and an appropriate action plan is required to reduce wheel slip so that the system can switch from slip to no-slip domain satisfying certain switching conditions.

A. *Nomenclature of the WMR & associated parameters*

The following notations are used to model the WMR: $\{\mathrm{W: O, X, Y}\}$ & $\{\mathrm{R: O_1, X_1, Y_1}\}$: Denotes the global & body coordinate system of the robot. $\mathrm{C}\,(\mathrm{x_c},\mathrm{y_c})$ & $\mathrm{C_P}\,(\mathrm{x_P},\mathrm{y_P})$ represent Center of Mass (COM) of the overall system & platform respectively. φ - Orientation of the body fixed (local) frame w. r. t. global frame. θ_r - Denotes the rotation of the right wheel. θ_l - Denotes the rotation of the left wheel. L – Length of the robot. 2b - is the distance between the two wheels up-to the contact points. Wheels are located uniformly about the axes of symmetry. r - Radius of the driving wheels (assuming both the wheels are of same radius). d - Distance from point $\mathrm{O_1}$ to the mass centre C of the WMR (assuming the COM lies on the axis of symmetry). m_{P} - Mass of the mobile robot platform without the driving wheels, the motors rotor, and gearbox rotor. m_w - Mass of each driving wheel, and the associated motor rotor, gearbox rotor. m – Mass of the overall system. I_{P} - Moment of Inertia (MOI) of the platform about a vertical axes passing through point $\mathrm{C_P}$. I_w - is equivalent wheel MOI referred to wheel axis. Δ - is the offset between point C & $\mathrm{C_P}$ along $\mathrm{O_1X_1}$. I_m - MOI of each wheel and associated motor & gearbox rotor about a wheel diameter passing through z-axis. ζ - Lateral slip of the right & left wheel tyres (assuming fixed axle length of the robot). δ_r - Longitudinal slip of the right wheel tyre. δ_l - Longitudinal slip of the left wheel tyre. ρ_r - Longitudinal displacement of the centre of right wheel. ρ_l - Longitudinal displacement of the centre of left wheel. $\mathbf{T}_R$ - Input torque to the right wheel. $\mathbf{T}_L$ - Input torque to the left wheel.

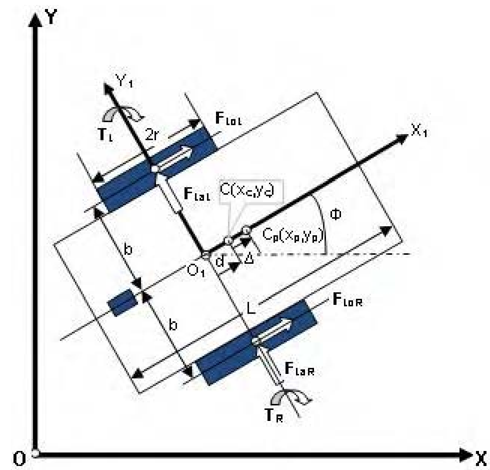

Fig. 1. Free body diagram of the Wheel Mobile Robot

μ_s - Static friction coefficient. $\mathbf{N}_r$ & $\mathbf{N}_l$ - Right & left wheel normal reaction forces. $\mathbf{F}_{LoR}$ & $\mathbf{F}_{LaR}$ - Longitudinal & lateral friction forces respectively acting at the contact point of right wheel. $\mathbf{F}_{LoL}$ & $\mathbf{F}_{LaL}$ - Longitudinal & lateral friction forces respectively acting at the contact point of left wheel. h – Height of the COM from the ground. **g** - Gravitational acceleration.

B. *Kinematics & Constraints*

The pose of the mobile robot in global coordinate frame {W: O, X, Y} is completely specified by the coordinates of the COM of the robot $(\mathrm{x_c},\mathrm{y_c})$ and the heading angle φ . In order to control & calculate the slip parameters for the mobile robotic system $\mathbf{q} = (x_c, y_c, \varphi, \theta_r, \theta_l, \rho_r, \rho_l, \zeta)^T$ have been chosen as the generalized coordinate vector (i.e., *n=8*), which is sufficient to describe the configuration of the robot. The longitudinal displacement of the wheel centre is related with the wheel angular displacement and longitudinal slip of the wheel tyre as follows:

$$\begin{aligned} \rho_r &= r\theta_r - \delta_r \Rightarrow \dot{\rho}_r = r\dot{\theta}_r - \dot{\delta}_r \Rightarrow \dot{\delta}_r = r\dot{\theta}_r - \dot{\rho}_r, \\ \rho_l &= r\theta_l - \delta_l \Rightarrow \dot{\rho}_l = r\dot{\theta}_l - \dot{\delta}_l \Rightarrow \dot{\delta}_l = r\dot{\theta}_l - \dot{\rho}_l. \end{aligned} \tag{1}$$

When no-slip conditions are relaxed in each contact, assumption of pure *rolling motion* is violated and the set of constrained equations are represented as:

$$\dot{x}_c \cos\varphi + \dot{y}_c \sin\varphi + b\dot{\varphi} = r\dot{\theta}_r - \dot{\delta}_r = \dot{\rho}_r, \tag{2}$$

$$\dot{x}_c \cos\varphi + \dot{y}_c \sin\varphi - b\dot{\varphi} = r\dot{\theta}_l - \dot{\delta}_l = \dot{\rho}_l, \tag{3}$$

$$\dot{y}_c\cos\varphi-\dot{x}_c\sin\varphi-\dot{\varphi}d=\dot{\zeta}. \quad (4)$$

C. *Lagrangian & dynamic equations of WMR:*

Lagrangian of any system is defined as:

$L(\mathbf{q},\dot{\mathbf{q}})$ = Kinetic Energy (KE) - Potential Energy (PE).

PE of the WMR is zero, being a planner moving robot. So, Lagrangian is given by: $L(\mathbf{q},\dot{\mathbf{q}})$ = KE of the WMR (T) = Translational KE of the Platform + Rotational KE of the Platform + Translational KE of Right Wheel + Rotational KE of Right Wheel + Translational KE of Left Wheel + Rotational KE of Left Wheel

$$=\left[0.5m_P(\dot{x}_P^2+\dot{y}_P^2)\right]+\left[0.5I_P\dot{\varphi}^2\right]+\left[0.5m_w(\dot{\rho}_r^2+\dot{\zeta}^2)\right]+ \left[0.5I_w\dot{\theta}_r^2+0.5I_m\dot{\varphi}^2\right]+\left[0.5m_w(\dot{\rho}_l^2+\dot{\zeta}^2)\right]+\left[0.5I_w\dot{\theta}_l^2+0.5I_m\dot{\varphi}^2\right] \quad (5)$$

From the "Fig. 1," we can write that,

$$x_P=x_c+\Delta\cos\varphi\Rightarrow\dot{x}_P=\dot{x}_c-\dot{\varphi}\,\Delta\sin\varphi,$$

$$y_P=y_c+\Delta\sin\varphi\Rightarrow\dot{y}_P=\dot{y}_c+\dot{\varphi}\,\Delta\cos\varphi.$$

So, $\dot{x}_P^2+\dot{y}_P^2=\dot{x}_c^2+\dot{y}_c^2+\Delta^2\dot{\varphi}^2-2\Delta\dot{\varphi}(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)$. (6)

Substituting the value $\dot{x}_P^2+\dot{y}_P^2$ refers to "(5)," we get,

$$L=0.5m_P\left\{\dot{x}_c^2+\dot{y}_c^2+\Delta^2\dot{\varphi}^2-2\Delta\,\dot{\varphi}\,(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\right\}+ 0.5\{I_P+2I_m\}\dot{\varphi}^2+0.5m_w(\dot{\rho}_r^2+\dot{\rho}_l^2+2\dot{\zeta}^2)+0.5I_w(\dot{\theta}_r^2+\dot{\theta}_l^2). \quad (7)$$

Now, refer to "(2)," "(3)," & "(4)," we get,

$$\dot{\rho}_r^2+\dot{\rho}_l^2+2\dot{\zeta}^2=2\dot{x}_c^2+2\dot{y}_c^2+2b^2\dot{\varphi}^2+2d^2\dot{\varphi}^2 +4d\dot{\varphi}\,(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi). \quad (8)$$

Referring "(7)," & "(8)," Lagrangian is derived as:

$$L=0.5m_P\left\{\dot{x}_c^2+\dot{y}_c^2+\Delta^2\dot{\varphi}^2-2\Delta\,\dot{\varphi}\,(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\right\} +m_w\left\{\dot{x}_c^2+\dot{y}_c^2+b^2\dot{\varphi}^2+d^2\dot{\varphi}^2+2d\dot{\varphi}\,(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\right\} +0.5\{I_P+2I_m\}\dot{\varphi}^2+0.5I_w(\dot{\theta}_r^2+\dot{\theta}_l^2)\;=0.5m(\dot{x}_c{}^2+\dot{y}_c{}^2)+ 0.5I_w\left(\dot{\theta}_R{}^2+\dot{\theta}_L{}^2\right)+0.5I\dot{\varphi}^2+K\dot{\varphi}(\dot{x}_c s\varphi-\dot{y}_c c\varphi), \quad (9)$$

where, $m=m_P+2m_w$, $I=I_P+m_P\Delta^2+2m_w(b^2+d^2)+2I_m$, $K=\{2m_w d-m_P\Delta\}$, $s\varphi\Rightarrow\sin\varphi$, $c\varphi\Rightarrow\cos\varphi$.

Dynamic equations of the mobile robot are derived from the Euler-Lagrange [20] equation of motion. The *Lagrange-Euler* equations of motion are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\mathbf{q}}}\right)-\frac{\partial L}{\partial\mathbf{q}}=\mathbf{\Gamma},$$

where, $\mathbf{\Gamma}$ = vector of generalized forces = $\mathbf{F}.\frac{\partial\mathbf{V}}{\partial\dot{\mathbf{q}}}+\mathbf{M}.\frac{\partial\boldsymbol{\omega}}{\partial\dot{\mathbf{q}}}$, $\mathbf{F}$ = force vector acting on the robot, $\mathbf{M}$ = moment vector acting on the robot, $\mathbf{V}$ & $\boldsymbol{\omega}$ = Linear and angular velocities.

For the mobile robotic system under consideration the set of dynamic equations of motion are as follows:

$$m\ddot{x}_c+K(\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi)=(\mathbf{F}_{LoR}+\mathbf{F}_{LoL})\cos\varphi -(\mathbf{F}_{LaR}+\mathbf{F}_{LaL})\sin\varphi, \quad (10)$$

$$m\ddot{y}_c-K(\ddot{\varphi}\cos\varphi-\dot{\varphi}^2\sin\varphi)=(\mathbf{F}_{LoR}+\mathbf{F}_{LoL})\sin\varphi +(\mathbf{F}_{LaR}+\mathbf{F}_{LaL})\cos\varphi, \quad (11)$$

$$I\ddot{\varphi}+K(\ddot{x}_c\sin\varphi-\ddot{y}_c\cos\varphi)+K\dot{\varphi}(\dot{x}_c\cos\varphi+\dot{y}_c\sin\varphi)= (\mathbf{F}_{LoR}-\mathbf{F}_{LoL})b-(\mathbf{F}_{LaR}+\mathbf{F}_{LaL})d, \quad (12)$$

$$I_w\ddot{\theta}_r=\mathbf{T}_r-\mathrm{r}\mathbf{F}_{LoR}, \quad (13)$$

$$I_w\ddot{\theta}_l=\mathbf{T}_l-\mathrm{r}\mathbf{F}_{LoL}. \quad (14)$$

Referring again "(2)," "(3)," & "(4)," & taking derivative we get the 2nd order slip kinematics as:

$$\ddot{\rho}_r=\ddot{x}_c\cos\varphi+\ddot{y}_c\sin\varphi-(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\dot{\varphi}+b\ddot{\varphi}, \quad (15)$$

$$\ddot{\rho}_l=\ddot{x}_c\cos\varphi+\ddot{y}_c\sin\varphi-(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\dot{\varphi}-b\ddot{\varphi}, \quad (16)$$

$$\ddot{\zeta}=-\ddot{x}_c\sin\varphi+\ddot{y}_c\cos\varphi-(\dot{x}_c\cos\varphi+\dot{y}_c\sin\varphi)\dot{\varphi}-d\ddot{\varphi}. \quad (17)$$

Referring above eight equations "(10),"to"(17)," the system dynamics associated with slip parameters are represented compactly as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{B}(\mathbf{q})\mathbf{T}+\mathbf{F}(\mathbf{q}), \quad (18)$$

where,

$$\mathbf{M}=\begin{bmatrix} m & 0 & K\sin\varphi & 0 & 0 & 0 & 0 & 0\\ 0 & m & -K\cos\varphi & 0 & 0 & 0 & 0 & 0\\ K\sin\varphi & -K\cos\varphi & I & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & I_w & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & I_w & 0 & 0 & 0\\ \cos\varphi & \sin\varphi & b & 0 & 0 & -1 & 0 & 0\\ \cos\varphi & \sin\varphi & -b & 0 & 0 & 0 & -1 & 0\\ -\sin\varphi & \cos\varphi & -d & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})=\begin{bmatrix} K\dot{\varphi}^2\cos\varphi\\ K\dot{\varphi}^2\sin\varphi\\ K\dot{\varphi}(\dot{x}_c\cos\varphi+\dot{y}_c\sin\varphi)\\ 0\\ 0\\ -(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\dot{\varphi}\\ -(\dot{x}_c\sin\varphi-\dot{y}_c\cos\varphi)\dot{\varphi}\\ -(\dot{x}_c\cos\varphi+\dot{y}_c\sin\varphi)\dot{\varphi} \end{bmatrix},\ \mathbf{B}=\begin{bmatrix} 0&0\\0&0\\0&0\\1&0\\0&1\\0&0\\0&0\\0&0 \end{bmatrix},\ \mathbf{T}=\begin{bmatrix}\mathbf{T}_r\\ \mathbf{T}_l\end{bmatrix},$$

$$\mathbf{F}(\mathbf{q},\dot{\mathbf{q}})=\begin{bmatrix} (\mathbf{F}_{LoR}+\mathbf{F}_{LoL})\cos\varphi-(\mathbf{F}_{LaR}+\mathbf{F}_{LaL})\sin\varphi\\ (\mathbf{F}_{LoR}+\mathbf{F}_{LoL})\sin\varphi+(\mathbf{F}_{LaR}+\mathbf{F}_{LaL})\cos\varphi\\ (\mathbf{F}_{LoR}-\mathbf{F}_{LoL})b+(\mathbf{F}_{LaR}+\mathbf{F}_{LaL})d\\ -\mathbf{F}_{LoR}\ r\\ -\mathbf{F}_{LoL}\ r\\ 0\\ 0\\ 0 \end{bmatrix}.$$

D. *Traction force model while slip is occurring*

Assumptions: 1) Traction force will reach to the maximum limit. 2) The lateral and longitudinal traction forces of each wheels i.e., ($\mathbf{F}_{La}$ & $\mathbf{F}_{Lo}$) are the component of the same force, which is the resultant traction force ($\mathbf{F}$) acting on that wheel. 3) Direction of resultant frictional force ($\mathbf{F}$) is in the direction of velocity of that wheel. 4) If slip occurs on any wheel then other wheel will experience slip definitely. Based on the above assumptions the longitudinal & lateral slip/skid velocities are given by:

$$\dot{\delta}_r = r\dot{\theta}_r - \dot{\rho}_r \tag{19}$$

$$\dot{\delta}_l = r\dot{\theta}_l - \dot{\rho}_l \tag{20}$$

$$\dot{y}_c \cos\varphi - \dot{x}_c \sin\varphi - \dot{\varphi} d = \dot{\zeta} \tag{21}$$

So, for slippage of any wheel in the form of spin/skid to take place while moving through various paths (having different frictional characteristics), following situations may occur:

$$\begin{aligned} &\dot{\rho}_r < r\dot{\theta}_r \text{ or } \dot{\delta}_r > 0 \text{ (case: spinning of right wheel)},\\ &\dot{\rho}_l < r\dot{\theta}_l \text{ or } \dot{\delta}_l > 0 \text{ (case: spinning of left wheel)}, \end{aligned} \tag{22}$$

$$\begin{aligned} &\dot{\rho}_r > r\dot{\theta}_r \text{ or } \dot{\delta}_r < 0 \text{ (case: skidding of right wheel)},\\ &\dot{\rho}_l > r\dot{\theta}_l \text{ or } \dot{\delta}_l < 0 \text{ (case: skidding of left wheel)}, \end{aligned} \tag{23}$$

$$\begin{aligned} &\dot{\zeta} > 0 \text{ (case: side skidding, while left turning)},\\ &\& \ \dot{\zeta} < 0 \text{ (case: side skidding, while right turning)}. \end{aligned} \tag{24}$$

The wheel traction forces are derived from the following set of equations:

$$\begin{aligned} &\mathbf{F}_{LoR} = \mu N_r \left(\frac{\dot{\delta}_r}{\sqrt{\dot{\delta}_r^2 + \dot{\zeta}^2}} \right); \ \mathbf{F}_{LaR} = \mu N_r \left(\frac{\dot{\zeta}}{\sqrt{\dot{\delta}_r^2 + \dot{\zeta}^2}} \right),\\ &\mathbf{F}_{LoL} = \mu N_l \left(\frac{\dot{\delta}_l}{\sqrt{\dot{\delta}_l^2 + \dot{\zeta}^2}} \right); \ \mathbf{F}_{LaL} = \mu N_l \left(\frac{\dot{\zeta}}{\sqrt{\dot{\delta}_l^2 + \dot{\zeta}^2}} \right). \end{aligned} \tag{25}$$

From the free body diagram of WMR the normal reaction forces are calculated from the following relationship:

$$\mathbf{N}_l + \mathbf{N}_r = m\,\mathbf{g}; \quad \mathbf{N}_l - \mathbf{N}_r = (\mathbf{F}_{LaL} + \mathbf{F}_{LaR})\frac{h}{b}. \tag{26}$$

As in the system dynamics lateral traction forces for right & left wheels are appearing in pair so, we have considered them as single parameter and calculated through "(27-29),".

$$\mathbf{F}_{LaR} + \mathbf{F}_{LaL} = \frac{\dfrac{\mu\dot{\zeta} m\mathbf{g}}{2}\left(\dfrac{1}{\sqrt{\dot{\delta}_r^2 + \dot{\zeta}^2}} + \dfrac{1}{\sqrt{\dot{\delta}_l^2 + \dot{\zeta}^2}} \right)}{1 - \dfrac{\mu\dot{\zeta} h}{2b}\left(\dfrac{1}{\sqrt{\dot{\delta}_l^2 + \dot{\zeta}^2}} - \dfrac{1}{\sqrt{\dot{\delta}_r^2 + \dot{\zeta}^2}} \right)}. \tag{27}$$

$$\mathbf{F}_{LoR} = \frac{\mu}{2}\left(\frac{\dot{\delta}_r}{\sqrt{\dot{\delta}_r^2 + \dot{\zeta}^2}} \right)[m\mathbf{g} - \frac{h}{b}(\mathbf{F}_{LaL} + \mathbf{F}_{LaR})]. \tag{28}$$

$$\mathbf{F}_{LoL} = \frac{\mu}{2}\left(\frac{\dot{\delta}_l}{\sqrt{\dot{\delta}_l^2 + \dot{\zeta}^2}} \right)[m\mathbf{g} + \frac{h}{b}(\mathbf{F}_{LaL} + \mathbf{F}_{LaR})]. \tag{29}$$

For slip/skid case, the variables need to be solved are:

$$\mathbf{q} = (x_c, y_c, \varphi, \theta_r, \theta_l, \rho_r, \rho_l, \zeta)^T.$$

Switching conditions: The slip model (spinning or skidding) switch to no-slip model with appropriate control action while fulfilling the following conditions (rolling without slipping):

$$\begin{aligned} &\dot{\delta}_r = 0 \text{ or } \dot{\rho}_r = r\dot{\theta}_r,\\ &\dot{\delta}_l = 0 \text{ or } \dot{\rho}_l = r\dot{\theta}_l,\\ &\dot{\zeta} = 0 \ \& \ \dot{\delta}_r \ddot{\delta}_r = 0. \end{aligned} \tag{30}$$

E. *No-slip model*

The system will come back from slip to no-slip domain depending on the wheel-ground interaction properties upon fulfillment of the switching conditions as stated in "(30)".

Assumptions: 1) Though the value of dynamic friction is slightly less compared to static friction in this case both are assumed as same value.

The constraint equations referring to "(2)," "(3),' & "(4)," converge to *rolling without slipping* constraints i.e.

$$\dot{x}_c \cos\varphi + \dot{y}_c \sin\varphi + b\dot{\varphi} = r\dot{\theta}_r \tag{31}$$

$$\dot{x}_c \cos\varphi + \dot{y}_c \sin\varphi - b\dot{\varphi} = r\dot{\theta}_l \tag{32}$$

$$\dot{y}_c \cos\varphi - \dot{x}_c \sin\varphi - \dot{\varphi} d = 0 \tag{33}$$

In no-slip situation it is always necessary to calculate and monitor the frictional forces according to:

$$\sqrt{(\mathbf{F}_{LoR} + \mathbf{F}_{LoL})^2 + (\mathbf{F}_{LaR} + \mathbf{F}_{LaL})^2} \le \mu_s m\mathbf{g}.$$

For no-slip case the state vector is transformed to:

$$\mathbf{q} = (x_c, y_c, \varphi, \theta_r, \theta_l, \mathbf{F}_{LaR}, \mathbf{F}_{LaL}, \mathbf{F}_{LaR} + \mathbf{F}_{LaL})^T.$$

The system matrices referred to "(18)," take the form:

$$\mathbf{M} = \begin{bmatrix} m & 0 & K\,\mathrm{s}\varphi & 0 & 0 & -\mathrm{c}\varphi & -\mathrm{c}\varphi & \mathrm{s}\varphi \\ 0 & m & -K\,\mathrm{c}\varphi & 0 & 0 & -\mathrm{s}\varphi & -\mathrm{s}\varphi & -\mathrm{c}\varphi \\ K\,\mathrm{s}\varphi & -K\,\mathrm{c}\varphi & I & 0 & 0 & -b & b & d \\ 0 & 0 & 0 & I_w & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & I_w & 0 & r & 0 \\ \mathrm{c}\varphi & \mathrm{s}\varphi & b & -r & 0 & 0 & 0 & 0 \\ \mathrm{c}\varphi & \mathrm{s}\varphi & -b & 0 & -r & 0 & 0 & 0 \\ -\mathrm{s}\varphi & \mathrm{c}\varphi & -d & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where, $\mathrm{c}\varphi = \cos\varphi$; $\mathrm{s}\varphi = \sin\varphi$.

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} K\dot{\varphi}^2 \cos\varphi \\ K\dot{\varphi}^2 \sin\varphi \\ K\dot{\varphi}(\dot{x}_c \cos\varphi + \dot{y}_c \sin\varphi) \\ 0 \\ 0 \\ -(\dot{x}_c \sin\varphi - \dot{y}_c \cos\varphi)\dot{\varphi} \\ -(\dot{x}_c \sin\varphi - \dot{y}_c \cos\varphi)\dot{\varphi} \\ -(\dot{x}_c \cos\varphi + \dot{y}_c \sin\varphi)\dot{\varphi} \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{T} = \begin{bmatrix} \mathbf{T}_r \\ \mathbf{T}_l \end{bmatrix},$$

$$\mathbf{F}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \tag{32}$$

Switching condition:

Transition from no-slip to slip is dependent on the constant monitoring of the frictional forces given by:

$$\sqrt{(\mathbf{F}_{LoR} + \mathbf{F}_{LoL})^2 + (\mathbf{F}_{LaR} + \mathbf{F}_{LaL})^2} \ge \mu_s m\mathbf{g}$$

& fulfilment of the switching condition mentioned in "(30)".

III. Solution Methodology

A. General Algorithm

The slip and no-slip dynamics represent a set of 16 nos. of Ordinary Differential Equations (ODE's) which are solved following a step by step procedure mentioned below:
1) Initial Value Problem (IVP) is solved from [0 tLimit] and broken into patches of many IVPs with time step of 'tStep' i.e. [0 tStep], [tStep 2*tStep], [2*tStep 3*tStep] etc. 2) During the solution of each patch of IVP, condition for switch from slip to no-slip (& vice-versa) is repeatedly checked. If such condition is ever attained, the ongoing IVP is stopped and then a new IVP is formed with initial condition same as the final condition of last IVP patch. 3) This process is iterated until desired time frame to observe dynamics is taken into account.

B. Flowchart:

Flowchart is presented in "Fig. 2," which demonstrates the methodology adopted for solving the slip dynamics. The variables used in the flowchart are described as follows:
tLimit: Time until we need to observe the dynamics. tInitial: Initial time to solve the patch IVP. tFinal: Final time to solve the patch IVP. tStep: Time steps of patch IVP. tBreak: Time at which switch from slip to no-slip (or vice-versa) takes place. tFlag: Time at which switch from one patch of IVP to next patch takes place.

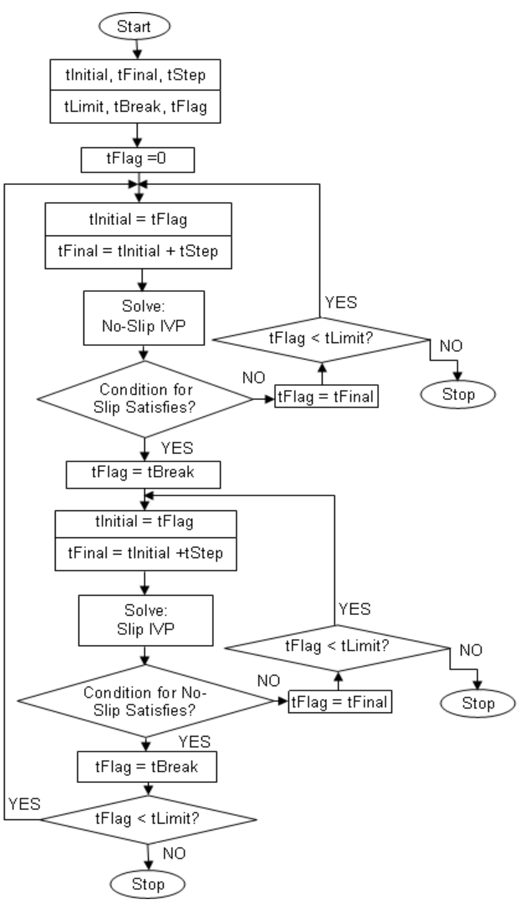

Fig. 2 Flow Chart of the developed algorithm

IV. Simulation Results

The algorithm has been tested with the parameter of a real indoor robot currently fabricated at CMERI, Durgapur, India. The kinematics parameters used for simulation are:
b=0.207m, r = 0.096m, d = 0.116m, Δ = 0.020m, m_p = 34.648 kg, m_w =1.556 kg, I_m =0.0354 kg-m^2, I_w = 0.0708 kg-m^2, $I = I_p + m_p\Delta^2 + 2m_w(b^2 + d^2) + 2I_m$ = 5.716 kg-m^2, g = 9.8 m/s^2. Considering the above parameters the simulation through various flat ground conditions with different wheel torque input has been carried out using suitable initial conditions.

Case I – Equal Wheel Torques: Assuming, μ_s = 0.7, $\mathbf{T}_R$ = 4sin(t) N-m & $\mathbf{T}_L$ =4sin(t) N-m. WMR move in a straight line with no-slip & wheel slip velocities are also zero as demonstrated in "Fig. 3," & "Fig. 4".

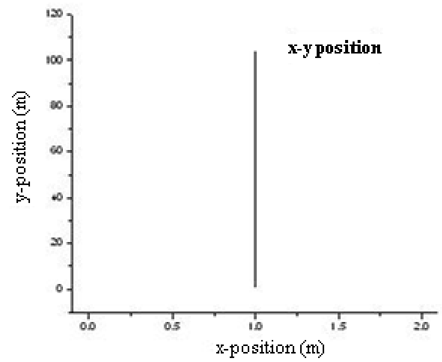

Fig. 3 Plot of x-y position with no-slip

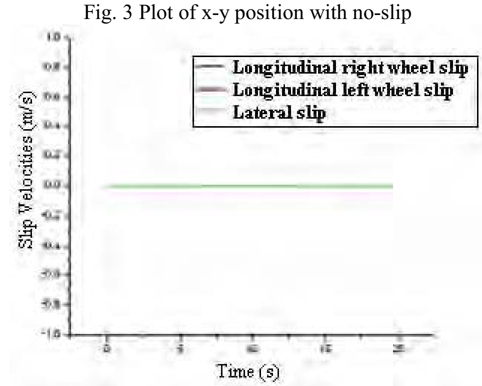

Fig. 4 No-slip occurs while moving straight

Case II- Equal Wheel Torques with reduced friction: Assuming, μ_s = 0.3, $\mathbf{T}_R$ = 4sin(t) N-m & $\mathbf{T}_L$ =4sin(t) N-m. Here, due to low traction WMR slips and related slip velocities are plotted in "Fig. 5," & "Fig. 6". System also switches gradually from no-slip to slip domain.

Case III- Differential Wheel Torques: Assuming, μ_s = 0.7, $\mathbf{T}_R$ = 3 N-m & $\mathbf{T}_L$ = 5 N-m. In this case, as the left wheel torque is more the WMR take right turn. WMR take turn with slip and system dynamics gradually switch from slip to no-slip domain with reduction of wheel torques. The related results are plotted in "Fig. 7," and "Fig. 8".

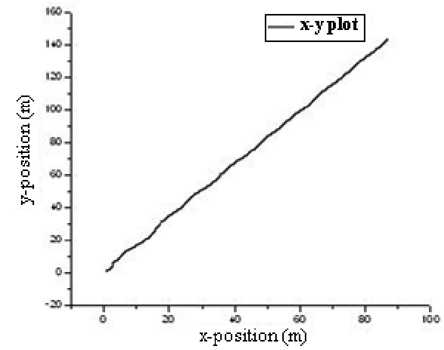

Fig. 5 Plot of x-y position with no-slip

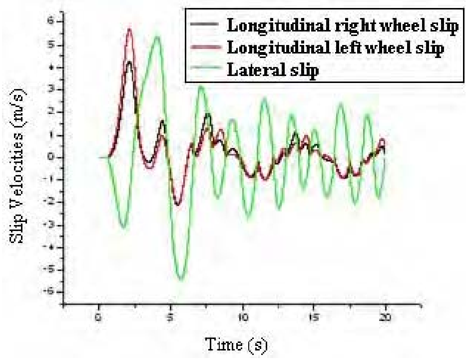

Fig. 6 Plot of slip velocities

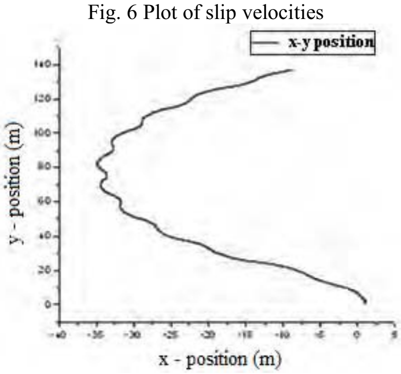

Fig. 7 Plot of x-y position with slip

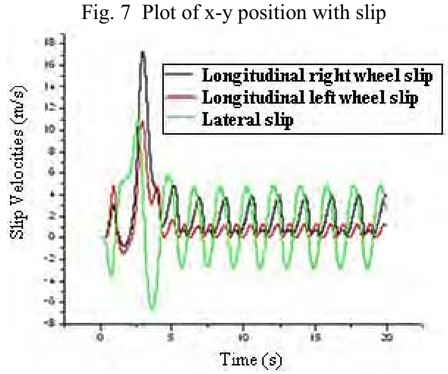

Fig. 8 Plot of slip velocities

IV. CONCLUSION

In this paper, forward no-slip and slip dynamics problem has been solved. Wheel slip phenomenon is captured and the model can switch from no-slip to slip domain and vice-versa depending on appropriate control action & fulfilling some proposed switching conditions. The simulation results demonstrate the effectiveness of the proposed model. This model will be very helpful to device an autonomous controller which can take back the WMR to its desired path while slip occurs. The aim of our future work is concentrated on autonomous control of WMR in presence of slip phenomena using this model.

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