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Global Uniform Asymptotic Stabilization of an Underactuated Surface Vessel

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Abstract

Explicit formulas of smooth time-varying static feedbacks which make the origin of an underactuated surface vessel globally uniformly asymptotically stable are proposed. The construction of the feedback extensively relies on the backstepping approach. The feedbacks constructed are time periodic functions.

1 Introduction

Dynamic positioning of surface vessels is required in many offshore oil field operations such as drilling, pipe-laying and diving support. Critical to the success of a dynamically positioned surface vessel is its capability for accurate and reliable control, subject to environmental disturbances as well as to configuration related changes, such as a reduced number of available control inputs. This reduction may be the result of an actuator failure or a deliberate decision to limit the number of actuators due to e.g. cost and weight considerations.

In this note we consider the dynamic positioning control problem for a ship that has no side thruster, but two independent main thrusters located at a distance from the center line in order to provide both surge force and yaw moment. The control problem considered in this paper is to find a feedback law that stabilizes both the position variables and the orientation, using only the two available controls. Since we attempt to control three degrees of freedom with only two independent controls, we have an underactuated control problem.

Control of underactuated systems is a continuation of the research on nonholonomic systems. In recent years, nonholonomic systems have been a topic of much interest in the control society. Control of nonholonomic systems has proved to be a challenging problem, inherently nonlinear and not amenable to linear control theory. For the stabilization of nonholonomic systems which do not satisfy the conditions of Brockett [4], several approaches have been proposed. A review of nonholonomic systems control is given in [18]. To mention a few, stabilization of equilibrium manifolds and the use of discontinuous control was proposed in [3] and [2] while [28] was the first to show how continuous time-varying feedback laws could asymptotically stabilize nonholonomic systems, in particular a nonholonomic cart.

Control of underactuated ships is an active topic of research see e.g. [16, 13, 1, 26, 30, 5]. Concerning the stabilization problem, it is seen from results by [4, 10, 33] that the ship is not even locally asymptotically stabilizable by continuous static state feedback. However, the surface vessel is locally strongly accessible and small time locally controllable [23], and by [8] the ship is then locally asymptotically stabilizable in small time by means of an almost smooth periodic time-varying feedback law. However, since the underactuated ship is not a controllable driftless system, the results of [7, 8] do not allow to claim that this system is globally asymptotically stabilizable by time-varying feedbacks.

For the stabilization of the underactuated ship, in [32] a continuous feedback control law is proposed that asymptotically stabilizes an equilibrium manifold. The desired equilibrium point is then stable as all the system variables are bounded by the initial conditions of the system. Furthermore, the position variables with this approach converge exponentially to their desired values. The course angle however converges to some constant value, but not necessarily to zero. In [27] a discontinuous feedback control law is proposed, and this provides exponential convergence to the desired equilibrium point, under certain assumptions on the initial value. In [23] a time-varying feedback control law is proposed that provides exponential (with respect to a given dilation) stability of the desired equilibrium point. However the feedback law only locally stabilizes the desired equilibrium point, and the size of the region of attraction is not known. In [26] a time-varying feedback control law is proposed that provides semi-global practical exponential stability of a simplified model of the ship, where the surge and yaw velocities are considered as controls. In [11] another simplified ship model is considered, a hovercraft, and based on passivity con-

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siderations and Lyapunov theory discontinuous control laws are proposed giving global convergence to the origin. In [6] a geometric framework for controllability analysis and motion control is proposed for mechanical systems on Lie groups, including the underactuated hovercraft.

In this paper, we consider the full ship model with surge force and yaw moment controls. We solve the open problem of determining explicit expressions of smooth time-varying periodic state feedbacks which render the origin globally uniformly asymptotically stable. The result is proven by a Lyapunov function construction which relies extensively on the backstepping technique (see for instance [9, 19, 29, 15]). One of the particular features of our control design is that it exploits recent results of construction of strict Lyapunov functions[1] for time-varying systems (see [20]). Our approach yields several types of control laws.

The work is organized as follows. The ship model is presented in Section 2. In Section 3, the main result is stated. In Section 4, it is proved. Section 5 ends the paper.

Preliminaries.

• A real-valued function $\gamma(s)$ is of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, $\gamma(0) = 0$ and $\lim_{s\to+\infty} \gamma(s) = +\infty$.

• A function $U(X,t)$ is a Lyapunov function of the system $\dot{X} = \varphi(X,t)$ satisfying $\varphi(0,t) = 0$ for all $t \geq 0$, if it is continuously differentiable and there exist two functions $\Gamma_m(\cdot)$, $\Gamma_M(\cdot)$ of class $\mathcal{K}_\infty$ such that, for all X and $t \geq 0$, $\Gamma_m(|X|) \leq U(X,t) \leq \Gamma_M(|X|)$ and $\frac{\partial U}{\partial t}(X,t) + \frac{\partial U}{\partial X}(X,t)\varphi(X,t) \leq 0$. If moreover, there exists a positive definite continuous function $\mathcal{W}(X)$ such that $\frac{\partial U}{\partial t}(X,t) + \frac{\partial U}{\partial X}(X,t)\varphi(X,t) \leq -\mathcal{W}(X)$ we say that $U(X,t)$ is a strict Lyapunov function.

• The arguments of a function will be omitted when no confusion can arise.

2 Ship model

Following [12], the dynamic equations of the ship are

$$\left\{ \begin{array}{rcl} \dot{u} & = & \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_1 \,, \\ \dot{v} & = & -\frac{m_{11}}{m_{22}} ur - \frac{d_{22}}{m_{22}} v \,, \\ \dot{r} & = & \frac{m_{11}-m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_3 \,. \end{array} \right. \tag{1}$$

The variables u, v and r are the velocities in surge, sway and yaw respectively. The parameters $m_{ii} > 0$ are given by the ship inertia and added mass effects. The parameters $d_{ii} > 0$ are given by the hydrodynamic damping. The available controls are the surge control force τ_1, and the yaw control moment τ_3. We do not, however, have available control in the sway direction, and the problem of controlling the ship in three degrees of freedom is therefore an underactuated control problem. variables.When modelling the ship, the dynamics associated with the motion in heave, roll, pitch and terms of second order at the origin in the hydrodynamic terms are assumed to be negligible. It is furthermore assumed that the inertia and damping matrices are diagonal. This is true for ships having port/starboard and fore/aft symmetry. Most ships have port/starboard symmetry. Non-symmetry fore/aft of the ship implies that the off-diagonal terms of the inertia matrix $m_{23} \neq 0$ and $m_{32} \neq 0$, and also for the damping matrix $d_{23} \neq 0$ and $d_{32} \neq 0$. These off-diagonal terms will, however, be small compared to the diagonal elements m_{ii} and d_{ii} $(i = 1 \ldots 3)$ for most ships. Non-symmetry fore/aft will also give some extra cross-terms due to Coriolis and centripetal forces. Control design in the general case where also the off-diagonal terms are taken into account, is trivial to solve for a fully actuated ship while it is still a topic of future research for the underactuated ship.

The kinematics of the ship are described by

$$\left\{ \begin{array}{rcl} \dot{x} & = & \cos(\psi)u - \sin(\psi)v \,, \\ \dot{y} & = & \sin(\psi)u + \cos(\psi)v \,, \\ \dot{\psi} & = & r \,, \end{array} \right. \tag{2}$$

where x, y and ψ give the position and orientation of the ship in the earth-fixed frame. To obtain simpler, polynomial equations we use the same global coordinate transformation as in [23]

$$\begin{array}{rcl} z_1 & = & \cos(\psi)x + \sin(\psi)y \,, \\ z_2 & = & -\sin(\psi)x + \cos(\psi)y \,, \\ z_3 & = & \psi \end{array} \tag{3}$$

which yields

$$\left\{ \begin{array}{rcl} \dot{z}_1 & = & u + z_2 r \,, \\ \dot{z}_2 & = & v - z_1 r \,, \\ \dot{z}_3 & = & r \,. \end{array} \right. \tag{4}$$

3 Stabilization by state feedback: Main result

Before stating the main result of the work, we first transform the dynamics (1), (4) in a form that is easier amenable for stabilization.

3.1 Feedback and coordinate transformations

After the feedback transformations

$$\begin{array}{rcl} \tau_u & = & \frac{m_{22}}{m_{11}} vr - \frac{d_{11}}{m_{11}} u + \frac{1}{m_{11}} \tau_1 \,, \\ \tau_r & = & \frac{m_{11}-m_{22}}{m_{33}} uv - \frac{d_{33}}{m_{33}} r + \frac{1}{m_{33}} \tau_3 \,, \end{array} \tag{5}$$

[1]See the preliminaries for the definition of strict Lyapunov function.

the system (1) simplifies as

$$\left\{\begin{array}{rcl} \dot{u} & = & \tau_u \,, \\ \dot{v} & = & -cur - dv \,, \\ \dot{r} & = & \tau_r \,, \end{array}\right. \tag{6}$$

with $c = \frac{m_{11}}{m_{22}}$, $d = \frac{d_{22}}{m_{22}}$. To remove v from the subsystem (4), we introduce the variable:

$$Z_2 = z_2 + \frac{v}{d} \,. \tag{7}$$

The (z_1, z_2)-subsystem of (4) rewrites

$$\left\{\begin{array}{rcl} \dot{z}_1 & = & u + Z_2 r - \frac{v}{d} r \,, \\ \dot{Z}_2 & = & -c\frac{ur}{d} - z_1 r \,. \end{array}\right. \tag{8}$$

In order to introduce a stabilizing term in the z_1-equation, we use the change of coordinate

$$u = -\frac{d}{c} z_1 - \frac{d}{c}\mu \tag{9}$$

which, with the feedback transformation

$$\tau_\mu = \frac{d}{c} z_1 + \frac{d}{c}\mu - Z_2 r + \frac{v}{d} r - \frac{c}{d}\tau_v \tag{10}$$

eventually leads to the system

$$\left\{\begin{array}{rcl} \dot{z}_1 & = & -\frac{d}{c} z_1 - \frac{d}{c}\mu + Z_2 r - \frac{v}{d} r \,, \\ \dot{Z}_2 & = & \mu r \,, \\ \dot{z}_3 & = & r \,, \\ \dot{v} & = & -dv + d(z_1 + \mu) r \,, \\ \dot{\mu} & = & \tau_\mu \,,\ \dot{r} = \tau_r \,. \end{array}\right. \tag{11}$$

We are ready to state the following result:

Theorem 1 *The system (11) is globally uniformly asymptotically stabilized by any control law which globally uniformly asymptotically stabilizes the system*

$$\left\{\begin{array}{rcl} \dot{Z}_2 & = & \mu r \,, \\ \dot{z}_3 & = & r \,, \\ \dot{\mu} & = & \tau_\mu \,, \\ \dot{r} & = & \tau_r \,. \end{array}\right. \tag{12}$$

Moreover, if an explicit expression of a smooth strict Lyapunov function for the stabilized system (12) is known, then an explicit expression of a smooth strict Lyapunov function for (11) in closed-loop can be constructed.

Proof. By adapting Proposition 4.11 in [29], to the case of time-varying systems, one can easily prove that feedbacks which globally uniformly asymptotically stabilize the system (12) globally uniformly asymptotically stabilize the system (11) as well. The construction of a smooth strict Lyapunov function for (11) can be carried out by borrowing the tools used in [21] to construct Lyapunov functions.

Remark 2 *The benefit of the various feedback transformations and changes of coordinates we have carried out is now manifest: to prove Theorem 3 it remains only to show that (12) is globally uniformly asymptotically stabilizable. the almost*

3.2 The main result

Theorem 3 *Consider the transformed ship dynamics (11). Let k_2, k_3, k_μ, k_r be strictly positive parameters such that $1 \geq k_2 \geq k_3$, the system is globally uniformly asymptotically stabilized by the feedbacks*

$$\begin{array}{rcl} \tau_\mu & = & -k_\mu(\mu - \mu_f) + \dot{\mu}_f - \lambda\left[Z_2 + 2Z_3 k_2 \cos(t)\right] r \,, \\ \tau_r & = & -k_r(r - r_f) + \dot{r}_f \\ & & -\lambda\left[Z_2\mu_f + 2Z_3 + 2Z_3 k_2 \cos(t)\mu_f\right] \,, \end{array} \tag{13}$$

where

$$\lambda = 2 + \frac{k_3}{3} - \frac{k_3 \sin(2t)}{6}\frac{2V_1 + V_1^2}{(1 + V_1)^2} \,, \tag{14}$$

$$Z_3 = z_3 + k_2 \cos(t) Z_2 \,, \tag{15}$$

$$V_1(Z_2, Z_3) = Z_2^2 + 2Z_3^2 \,, \tag{16}$$

$$\mu_f = -\frac{\sin(t) Z_2^2}{2(0.001 + Z_2^2)} \,, \tag{17}$$

$$r_f = \frac{-k_3 Z_3 + k_2 \sin(t) Z_2}{1 + k_2 \cos(t)\mu_f} \,, \tag{18}$$

and $\dot{\mu}_f, \dot{r}_f$ are the derivatives of μ_f, r_f along the solutions of the closed loop system.

4 Proof of Theorem 3

According to Theorem 1, the problem of globally uniformly asymptotically stabilizing the system (11) reduces to the problem of globally uniformly asymptotically stabilizing the system (12). In order to do so, we apply a backstepping approach. In a first step, we determine expressions of stabilizing feedbacks for the (Z_2, z_3)-subsystem of (12) with μ and r as virtual inputs and in a second step we exploit the knowledge of these stabilizing feedbacks to stabilize the system (12). We will construct a strict Lyapunov function for the system (12) because the knowledge of such a function will enable us to exploit robustness backstepping results to determine reasonably simple expressions of stabilizing feedbacks.

4.1 Stabilization of the (Z_2, z_3)-subsystem

Consider the two dimensional system

$$\left\{\begin{array}{rcl} \dot{Z}_2 & = & \mu_f r_f \,, \\ \dot{z}_3 & = & r_f \,, \end{array}\right. \tag{19}$$

with μ_f, r_f as inputs. According to Brockett's theorem [4], we know that there do not exist continuous time invariant feedbacks which locally asymptotically stabilize this system. However, it is well-known that the driftless system (19) is controllable and that it can be globally uniformly asymptotically stabilized by time-varying differentiable feedbacks. To obtain explicit expressions of such feedbacks, we perform the time-varying change of variable (15). A simple calculation yields

$$\dot{Z}_3 = r_f(1 + k_2\cos(t)\mu_f) - k_2\sin(t)Z_2 \; . \qquad (20)$$

We impose a priori $k_2\mu_f$ to be not greater in norm than $\frac{1}{2}$. This property ensures that r_f can be chosen as the feedback defined in (18) where k_3 is a strictly positive tuning parameter. With such a choice, the system (19) becomes

$$\left\{ \begin{array}{rcl} \dot{Z}_2 & = & \frac{-k_3 Z_3 + k_2 \sin(t) Z_2}{1 + k_2 \cos(t)\mu_f}\mu_f \; , \\ \dot{Z}_3 & = & -k_3 Z_3 \; . \end{array} \right. \qquad (21)$$

Clearly, there exist many feedbacks μ_f such that $|k_2\mu_f| \leq \frac{1}{2}$ which globally uniformly asymptotically stabilize the system (21). However, we focus our attention on the particular feedback given in (17).

Using the triangular inequality and the inequalities $1 \geq k_2 \geq k_3$, one can readily check that the derivative of the Lyapunov function defined in (16) along the trajectories of the closed-loop system (21) satisfies

$$\begin{array}{rcl} \dot{V}_1 & = & \frac{k_3\sin(t)Z_3 - k_2\sin^2(t)Z_2}{1 - k_2\cos(t)\sin(t)\frac{Z_2^2}{3(0.001+Z_2^2)}}\frac{Z_2^3}{0.001+Z_2^2} - 4k_3Z_3^2 \\ & \leq & 2k_3|\sin(t)||Z_3|\frac{|Z_2|^3}{0.001+Z_2^2} - \frac{2k_2\sin^2(t)Z_2^4}{3(0.001+Z_2^2)} - 4k_3Z_3^2 \\ & \leq & -\frac{k_2\sin^2(t)Z_2^4}{3(0.001+Z_2^2)} - k_3Z_3^2 \\ & \leq & -\sin^2(t)W_1(Z_2, Z_3) - \frac{k_3}{2}Z_3^2 \end{array} \qquad (22)$$

with

$$\begin{array}{c} W_1(Z_2, Z_3) = \frac{k_2 Z_2^4}{3(0.001+Z_2^2)} + \frac{k_3}{2}Z_3^2 \geq \gamma(V_1(Z_2, Z_3)), \\ \gamma(s) = \frac{k_3 s^2}{8(1+s)}. \end{array} \qquad (23)$$

We use a technique of construction of strict Lyapunov functions for some time-varying systems developed in [20], to determine for the closed-loop system a strict Lyapunov function. Consider the function

$$V_2 = 2V_1 + k(V_1) + \left(\int_0^t (\sin^2(s) - \cos^2(s))ds\right)\gamma(V_1) \qquad (24)$$

where $k(\cdot)$ is a function of class $\mathcal{K}_\infty$ to be chosen later. Since $\displaystyle\int_0^t (\sin^2(s) - \cos^2(s))ds = -\frac{1}{2}\sin(2t)$, we obtain

$$\begin{array}{rcl} \dot{V}_2 & \leq & k'(V_1)\dot{V}_1 - 2\sin^2(t)\gamma(V_1) \\ & & +(\sin^2(t) - \cos^2(t))\gamma(V_1) - \frac{1}{2}\sin(2t)\gamma'(V_1)\dot{V}_1 \\ & \leq & \left[k'(V_1) - \frac{1}{2}\sin(2t)\gamma'(V_1)\right]\dot{V}_1 - \gamma(V_1) \; . \end{array} \qquad (25)$$

Using the inequalities $|\sin(2t)| \leq 1$, $|\gamma'(s)| \leq \frac{k_3}{3}$ one can prove that when $k'(s) = \frac{k_3}{6}$ the inequalities

$$\begin{array}{l} 2V_1 \leq V_2 \leq (2 + k_3)V_1, \\ \dot{V}_2 \leq -\gamma(V_1) < 0, \quad \forall (Z_2, Z_3) \neq (0, 0) \end{array} \qquad (26)$$

are satisfied. According to [17, Theorem 3.8], it follows that the origin of the system (19) in closed-loop with the feedbacks (17)(18) is globally uniformly asymptotically stable.

4.2 Stabilizing feedbacks for the system (12)

In this section, we apply a backstepping approach to obtain explicit expressions of stabilizing control laws for the system (12). According to the calculations of the previous section, we know that the derivative along the trajectories of (12) of the Lyapunov function

$$V_2 = \left(2 + \frac{k_3}{3}\right)V_1 - \sin(2t)\frac{k_3 V_1^2}{6(1 + V_1)} \qquad (27)$$

satisfies

$$\begin{array}{rcl} \dot{V}_2 & = & \frac{\partial V_2}{\partial Z_2}\mu r + \frac{\partial V_2}{\partial Z_3}\left[r - k_2\sin(t)Z_2 + k_2\cos(t)\mu r\right] \\ & + & \frac{\partial V_2}{\partial t} \\ & \leq & -\gamma(V_1) - \frac{k_3}{2}Z_3^2 \\ & + & \left[\frac{\partial V_2}{\partial Z_2} + \frac{\partial V_2}{\partial Z_3}k_2\cos(t)\right](\mu r - \mu_f r_f) \\ & + & \frac{\partial V_2}{\partial Z_3}\left[r - r_f\right] \\ & \leq & -\gamma(V_1) - \frac{k_3}{2}Z_3^2 \\ & + & \left[\frac{\partial V_2}{\partial Z_2} + \frac{\partial V_2}{\partial Z_3}k_2\cos(t)\right]\left[(\mu - \mu_f)r + \mu_f(r - r_f)\right] \\ & + & \frac{\partial V_2}{\partial Z_3}\left[r - r_f\right] \end{array} \qquad (28)$$

and its first partial derivatives are

$$\frac{\partial V_2}{\partial Z_2} = 2\lambda Z_2 \; , \quad \frac{\partial V_2}{\partial Z_3} = 4\lambda Z_3 \qquad (29)$$

where λ is the function defined in (14). We deduce that

$$\dot{V}_2 \leq -\Gamma \qquad (30)$$

with

$$\begin{array}{rcl} \Gamma & = & \gamma(V_1) + \frac{k_3}{2}Z_3^2 \\ & & -2\lambda\left[Z_2 + 2Z_3k_2\cos(t)\right](\mu - \mu_f)r \\ & & -2\lambda\left[Z_2\mu_f + 2Z_3\right](r - r_f) \\ & & -4\lambda Z_3 k_2\cos(t)\mu_f(r - r_f) \; . \end{array} \qquad (31)$$

Consider the function

$$V_3 = V_2 + (\mu - \mu_f)^2 + (r - r_f)^2 \qquad (32)$$

which is a control Lyapunov function satisfying the small control property for the system (12). The derivative of this function along the trajectories of the system (12) satisfies

$$\dot{V}_3 \leq -\Gamma + 2(\mu - \mu_f)(\tau_\mu - \dot{\mu}_f) + 2(r - r_f)(\tau_r - \dot{r}_f) \; . \qquad (33)$$

To deduce from this inequality expressions of stabilizing feedbacks for (12), one has to determine the expressions of $\dot{\mu}_f$ and $\dot{r}_f$. Immediate calculations yield that

$$\begin{aligned} \dot{\mu}_f &= -\frac{\cos(t)}{2}\frac{Z_2^2}{0.001+Z_2^2} - \sin(t)\frac{0.001Z_2}{(0.001+Z_2^2)^2}\mu r, \\ \dot{r}_f &= \frac{-k_3(r-k_2\sin(t)Z_2+k_2\cos(t)\mu r)}{1+k_2\cos(t)\mu_f} \\ &\quad + \frac{k_2(\cos(t)Z_2+\sin(t)\mu r)}{1+k_2\cos(t)\mu_f} \\ &\quad + \frac{k_2(k_3Z_3-k_2\sin(t)Z_2)[-\sin(t)\mu_f+\cos(t)\dot{\mu}_f]}{(1+k_2\cos(t)\mu_f)^2}. \end{aligned} \tag{34}$$

Then choosing the control laws given in (13) we obtain

$$\dot{V}_3 \le -\gamma(V_1) - \frac{k_3}{2}Z_3^2 - 2k_\mu(\mu-\mu_f)^2 - 2k_r(r-r_f)^2 \tag{35}$$

which implies that the origin of the system (12) in closed-loop with the feedback (13) is globally uniformly asymptotically stable.

Remark 4 *It is well-known that the backstepping approach is a very flexible approach: many different types of feedbacks can be deduced from this nonlinear control design strategy. The stabilizing feedbacks (13) are obtained through the cancellation of terms and thereby are given by complicated formulas. Feedbacks much simpler can be obtained via the strategy of domination of terms. This robust backstepping approach can be applied thanks to our knowledge of a strict Lyapunov function for the (Z_2, Z_3)-subsystem. One can find explicit expressions of strictly positive smooth functions $K_\mu(Z_2, Z_3, \mu, r)$, $K_r(Z_2, Z_3, \mu, r)$ such that the control laws*

$$\begin{aligned} \tau_\mu &= -K_\mu(Z_2, Z_3, \mu, r)(\mu-\mu_f), \\ \tau_r &= -K_r(Z_2, Z_3, \mu, r)(r-r_f) \end{aligned} \tag{36}$$

globally uniformly asymptotically stabilize the system (12).

5 Conclusion

A backstepping based method for determining explicit global uniform asymptotically stabilizing feedbacks for an underactuated vessel has been given.

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