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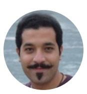

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A Novel Approach for Tracking Control of Differential Drive Robots Subject to Hard Input Constraints

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Abstract— **This paper studies the issue of input saturation in tracking control of differential drive robots. Although the structure of control strategy is a determinant factor for the occurrence of saturation phenomena, having a trajectory for the robot that is compatible with the bounds on the inputs is a necessary condition for avoiding saturation. In this study, analytic conditions for determining the compatibility of a trajectory with the input constraints are derived. Moreover, the problem of refining an incompatible trajectory to make it consistent with the input constraints is studied. This paper also proposes a novel control strategy for trajectory tracking of differential drive robots which can guarantee tracking and preservation of the input constraints provided that the desired trajectory satisfy compatibility conditions. In case of incompatibility of desired trajectory the proposed controller can simultaneously refine the trajectory and use the adjusted version for robot control, guaranteeing convergence of the robot to the geometric profile of trajectory. To demonstrate the power of developed theory, the problem of cooperative load carrying for two input-constrained differential robots has been studied. Simulation results show the validity of the derived control strategy.**

I. Introduction

Control of differential drive robots have been widely studied within the last two decades. Modeled by a unicycle, the vast majority of the published studies are focused on controlling the kinematic behavior of this system under the assumption of rolling without slipping. Basic control tasks for mobile robots in an obstacle free environment can be categorized into posture stabilization (point to point steering) and tracking control problems [1]. Despite the complexities associated with underactuated and nonholonomic systems, various methods have been successfully developed for stabilization and tracking control of differential drive robots. We do not intend not review the rich literature of controlling mobile robots in this manuscript; however, we will cite some of the most common control trends to highlight the diversity of the methods used.

Tracking control of mobile robots has often been studied as a trajectory tracking problem where a geometric path and an associated timing law should be followed by the robot [2]. Initial attempts were focused on using local controllers; however later studies used methods such as nonlinear feedback law [3], dynamic feedback linearization [2,4-6] and back stepping technique to solve the trajectory tracking problem globally [7]. A more recent trend is designing controllers that can solve both stabilization and tracking problems within the same control framework [2, 8, 9]. Despite the diversity and success of the current methods in fulfilling the fundamental requirements of tracking and stabilization, some key aspects of the control of differential drive robots such as control input saturation have rarely been addressed. In fact research studies that take into account the bounds on the magnitude of the inputs are limited to using Model Predictive Control [10, 11]. Studying the effects of hard input constraints on differential drive robots is a realistic step which is necessary from practical as well as theoretical points of view since input saturation can result in loss of stability or performance deterioration of the system.

This paper proposes an analytical framework for studying input saturation issue in tracking control of differential drive robots. The proposed method is an extension to the concept of Reachable Directions [12-14]. This framework associates the achievable velocity data to the reachable direction information and enables analyzing the possibility of input saturation. The developed theory shows that the time element of desired trajectory is the underlying reason for saturation of control inputs and the geometric properties of the desired path only determines the convergence rate of the controllers. Analytical criteria is derived for determining the compatibility of a given trajectory with the input constraints and refining an incompatible trajectory. This paper also presents a novel controller for trajectory tracking problem which can guarantee convergence of the robot to the desired trajectory while satisfying input constraints provided that the desired trajectory is compatible with the bounds on inputs. In case of incompatibility of desired trajectory, the controller can simultaneously refine the trajectory and use it for control guaranteeing convergence of the robot to the geometric profile of trajectory. Based on the developed theory, a new approach for designing steering tasks is introduced. This novel approach integrates the trajectory generation and control into a single unit and can incorporate input constraints into the steering process design. To show the power of developed strategy, the problem of cooperative load carrying for two input constrained differential robots has been solved and simulated.

II. Kinematic Model of Differential Drive Robots

As mentioned earlier, many studies use the first order unicycle model to study differential drive robots. However, we use a comprehensive kinematic model with the angular velocities of the wheels as inputs to incorporate the physics of robot motion into our analysis. Fig.1 shows the schematics of a typical two wheeled differential drive robot:

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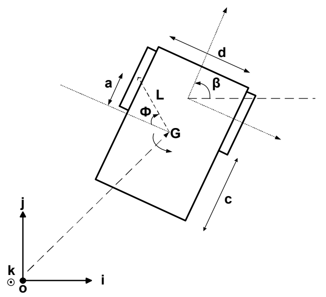

Figure 1. Schematics of the Differential Drive Robot

Using the zero relative velocity constraints imposed by the rolling without slipping phenomenon at the contact points of wheels with the ground, the kinematic model of the robot can be found as [13]:

$$\dot{q} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\beta} \end{bmatrix} = \boldsymbol{B}(\beta) \begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix} \quad (1)$$

$$\boldsymbol{B}(\beta) = \begin{bmatrix} \frac{rLcos(\beta+\phi)}{d} & rcos(\beta) - \frac{rLcos(\beta+\phi)}{d} \\ rsin(\beta) - \frac{rLsin(\beta-\phi)}{d} & \frac{rLsin(\beta-\phi)}{d} \\ -\frac{r}{d} & \frac{r}{d} \end{bmatrix}$$

where q is the vector of generalized coordinates that can uniquely define configuration of robot, (X, Y) denote the position of robot's center of mass, β denote robot's heading and angular velocities of left and right wheels are denoted by $(\dot{\varphi}_L, \dot{\varphi}_R)$ respectively. Other parameters in matrix $\boldsymbol{B}$ are the physical dimensions of the robot specified in Fig.1.

III. Input Saturation and Desired Trajectory Compatibility Conditions

Tracking control of differential drive robots is often formulated as a trajectory tracking problem where a desired path and the corresponding timing law are required to be followed. The timing associated with a desired trajectory determines the desired velocity of robot at each instant of time. Therefore the time element of a desired trajectory is the underlying reason for saturation of input for following a certain path. Compatibility of a trajectory with the constraints on the input can be formally defined as follows.

Definition 1: Let the driftless system $\dot{q}(t) = \boldsymbol{B}u(t)$ represent the differential drive robot with the admissible input set defined as:

$$U_{adm} = \{u \triangleq (u_1, u_2) | u_1, u_2 \in [-U_m, U_m]\} \quad (2)$$

A given trajectory is said to be incompatible with the input constraints in (2), if it is not kinematically possible to achieve the desired velocities associated with the desired trajectory given the constraints on the magnitude of inputs.

From a mathematical point of view, the kinematics of differential drive robot in (1) is a mapping between the space of admissible control input and the configuration space of robot. Theorem I develops a mathematical condition for determining the compatibility of a given trajectory with the constraints on the inputs using the properties of this mapping.

Theorem I: Consider the kinematic model of the differential drive robot in (1) with the admissible input set defined in (2). A given trajectory:

$$\Sigma(t) = (x_d(t), y_d(t))$$

is incompatible with the input constraints if:

$$\max_t \|(\dot{x}_d(t), \dot{y}_d(t))\|_2 > \sqrt{2} U_m \sigma_{max}(B_V(\beta)) \quad (3)$$

where $\sigma_{max}(B_V)$ is the largest singular value of $B_V(\beta)$ defined as:

$$B_V(\beta) = \begin{bmatrix} \frac{rLcos(\beta+\phi)}{d} & rcos(\beta) - \frac{rLcos(\beta+\phi)}{d} \\ rsin(\beta) - \frac{rLsin(\beta-\phi)}{d} & \frac{rLsin(\beta-\phi)}{d} \end{bmatrix} \quad (4)$$

Also, singular values of B_V are independent of β and condition (3) can be expressed explicitly in terms of robot dimensions.

Proof: The space of achievable velocities can be found by mapping the admissible input set (2) with $B_V(\beta)$. To be able to study the mapped set in a precise way, a disc $U_{enclosed}$ is enclosed within the admissible input set.

$$U_{enclosed} = \{u \triangleq (u_1, u_2) | \|u\|_2 = U_m\} \quad (5)$$

Mapping $U_{enclosed}$ with $B_V(\beta)$, supremum of Euclidean norm of members within the mapped set is the induced 2-norm of B_V which is also the largest singular value of the matrix:

$$\sup_{\|u\|_2 = U_m} \|B_V u\|_2 = U_m \sigma_{max}(B_V(\beta)) \quad (6)$$

To find singular values of $B_V(\beta)$, square root of the eigen values of matrix $B_V(\beta) B_V(\beta)^T$ are calculated. Characteristic equation of $B_V(\beta) B_V(\beta)^T$ is found to be:

$$\lambda^2 + p_1(\beta)\lambda + p_2(\beta) = 0 \quad (7)$$

Considering that $d = 2Lcos\phi$ (due to the physical dimensions of the robot) and after cumbersome algebraic calculations $p_1(\beta)$ and $p_2(\beta)$ can be found as:

$$p_1 = \frac{r^2}{2cos\phi^2}, p_2 = \frac{r^4}{4} tan^2 \phi \quad (8)$$

Characteristic equation of $B_V(\beta) B_V(\beta)^T$ can be expressed as:

$$\lambda^2 - \frac{r^2}{2\cos^2\phi}\lambda + \frac{r^4}{4} tan^2 \phi = 0 \quad (9)$$

Thus, singular values of $B_V(\beta)$ are independent of β:

$$\sigma(B_V) = \sqrt{\lambda(B_V(\beta) B_V(\beta)^T)} = \frac{r\sqrt{1 \pm cos(2\phi)}}{2cos(\phi)} \quad (10)$$

Through similar analysis, the input singular vectors corresponding to singular values of $B_V(\beta)$ can be found as:

$$u_{\sigma_{max}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, u_{\sigma_{min}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (11)$$

This means an input in the direction of $u_{\sigma_{max}}$ induces a gain of $\sigma_{max}(B_V)$ in the mapping. Due to the linearity of $B_V(\beta)$ with respect to the inputs, we can infer that:

$$\sup_{U_{adm}} \|B_V u\|_2 = \sqrt{2}\sigma_{max}(B_V) \quad (12)$$

This value is the maximum achievable velocity given the constraints on the magnitude of inputs and any trajectory with the maximum velocity beyond this value is incompatible with the input constraints. ■

Proof of Theorem *I* facilitates formulation of the reachable velocity set. Based on similar analysis it can be shown that an input in $u_{\sigma_{min}}$ direction induces a norm of $\sqrt{2}\sigma_{min}(B_V)$ in the system. To fully identify the reachable velocity set, we need to find the output singular directions corresponding to the singular values of $B_V(\beta)$. *Lemma I* will address this question.

Lemma I: The output singular directions of the $B_V(\beta)$ defined in (5) are:

$$u_{out_{max}} = \begin{bmatrix} cos\beta \\ sin\beta \end{bmatrix}, u_{out_{min}} = \begin{bmatrix} -sin\beta \\ cos\beta \end{bmatrix} \quad (13)$$

Proof: Output singular directions can be found by studying the direction of the mapped input singular vectors:

$$u_{out_{max}} = B_V(\beta)\, u_{in_{max}}$$

$$u_{out_{max}} = \begin{bmatrix} rcos(\beta) \\ rsin(\beta) \end{bmatrix} \xrightarrow{normalize} u_{out_{max}} = \begin{bmatrix} cos(\beta) \\ sin(\beta) \end{bmatrix}$$

Similar steps can be taken for proving the other output singular direction. . ■

Theorem *I* and Lemma *I*, fully describe the reachable velocity set of the differential drive robot in (1). This mapping between the space of the constrained inputs and the robot velocity is graphically shown in Fig. 2:

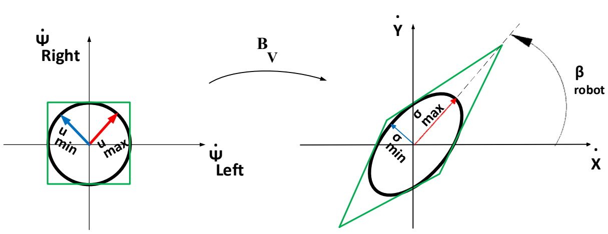

Figure 2. Reachable velocity set

Analyzing the reachable velocity can also be used to derive conditions for making a trajectory kinematically compatible with input constraints. Theorem *II* is the counterpart of Theorem *I* which can be used for this purpose.

Theorem II: Consider the kinematic model of the differential drive robot in (1) with the admissible input set defined in (2). A given trajectory:

$$\Sigma(t) = \big(x_d(t), y_d(t)\big)$$

is compatible with the input constraints if:

$$\max_t \|(\dot{x}_d(t), \dot{y}_d(t))\|_2 < U_m \sigma_{min}(B_V) \quad (14)$$

where $\sigma_{min}(B_V)$ is the smallest singular value of $B_V(\beta)$.

Proof: This theorem is a direct result of Theorem I and *Lemma I.*

Remark: Theorem *II* can be used as a constructive approach for making a trajectory compatible with the input constraints. If a given trajectory $\Sigma(t) = \big(x_d(t), y_d(t)\big)$ be incompatible with the input constraints, according to Theorem *II* a new trajectory can defined as:

$$\Sigma'(t) = \frac{U_m \sigma_{min}(B_V)}{\max_t \|(\dot{x}_d(t), \dot{y}_d(t))\|_2} \big(x_d(t), y_d(t)\big) \quad (15)$$

which is compatible with input constraints. Geometric properties of the new trajectory will be identical to the original; however, temporal aspects are scaled according to bounds.

It should be noted that this is a very conservative result since the velocity of entire trajectory is scaled down to assure its peak value does not exceed the compatibility limit. To improve this, a new method for refining incompatible trajectories is proposed which is in fact a simultaneous trajectory refinement and tracking control algorithm. Separating the geometric and temporal properties of desired trajectory, the proposed control structure guarantees tracking of the path associated with the trajectory. If the desired trajectory be incompatible with the input constraints, the controller adjusts the desired trajectory locally at that certain instant of time according to the input constraints.

IV. A Novel Approach for Simultaneous Trajectory Generation and Control

In this paper, a novel trajectory tracking controller is proposed based on the Constrained Directions method [14]. The proposed controller guarantees error free trajectory tracking while preserving the bounds on the inputs provided that the desired trajectory satisfies compatibility conditions.

Theorem III: Consider the kinematic model of differential drive robots in (1) with the admissible input set U_{adm} defined in (2). Let $\Sigma(t) = (x_d = f(t), y_d(t) = g(t))$ represent the desired trajectory to be followed and let f^{-1} and g^{-1} denote partial inverse functions of the components of desired trajectory. Let function $G_\Sigma(x, y)$ be defined as:

$$G_\Sigma(x, y) = y - gof^{-1}(x) \quad (16)$$

where $(.)o(.)$ operator denotes the composition of functions:

(*a*) If $\Sigma(t)$ be a compatible trajectory, the following control law guarantees exponential convergence of tracking error to zero within finite time while preserving the bounds on the magnitude of inputs:

$$u = v(t) B_v^{-1}(\beta) \begin{bmatrix} cos\big(\theta_{tangent} - \eta sgn(G_\Sigma(X, Y))\big) \\ sin\big(\theta_{tangent} - \eta sgn(G_\Sigma(X, Y))\big) \end{bmatrix} \quad (17)$$

where $v(t) = \left\|\big(\dot{f}(t), \dot{g}(t)\big)\right\|_2$, $\theta_{tangent}$ is the angle of the unit tangent vector in Frenet-Serret apparatus of $G_\Sigma(x, y) = 0$ at $(x = X, y = go\ \ ^{-1}(X))$ with X denoting the *x*-component of the current position of robot, η is an appropriately chosen positive scalar and *sgn(.)* stands for the sign function.

(*b*) If $\Sigma(t)$ be an incompatible trajectory, the control law defined as:

$$u = \gamma u_a \quad (18)$$

$$u_a = voS^{-1}(s)B_v^{-1}(\beta)\begin{bmatrix} cos\left(\theta_{tangent} - \eta sgn\left(G_\Sigma(X,Y)\right)\right) \\ sin\left(\theta_{tangent} - \eta sgn\left(G_\Sigma(X,Y)\right)\right)\end{bmatrix}$$

$$s = S(t) = \int_0^t \sqrt{\left(\dot{f}(\tau)\right)^2 + \left(\dot{g}(\tau)\right)^2}\, d\tau$$

$$\begin{cases} \gamma = 1 & if\ u_a \in U_{adm} \\ 0 < \gamma < 1 \quad s.t.\ \ \gamma u_a \in U_{adm} & if\ u_a \notin U_{adm} \end{cases}$$

guarantees exponential tracking of the geometric path associated with the desired trajectory.

Proof: Substituting the control law in (17) into the kinematics of the robot in (1), it can be shown that the closed-loop equations of differential drive robot are [14]:

$$\begin{cases} \dot{X} = v(t)\, cos\left(\theta_{tangent} - \eta sgn\left(G_\Sigma(X,Y)\right)\right) \\ \dot{Y} = v(t)\, sin\left(\theta_{tangent} - \eta sgn\left(G_\Sigma(X,Y)\right)\right) \end{cases} \quad (19)$$

Let $V(X,Y)$ be a positive definite function defined as:

$$V(X,Y) = \tfrac{1}{2} G_\Sigma^2(X,Y) \quad (20)$$

where (X,Y) is the current position of robot's center of mass. Differentiating $V(X,Y)$ along system trajectories:

$$\dot{V} = \left((\partial_X G_\Sigma)\dot{X} + (\partial_Y G_\Sigma)\dot{Y}\right) G_\Sigma(X,Y)$$

where $\partial_X G_\Sigma = \frac{\partial G_\Sigma}{\partial X}$ and $\partial_Y G_\Sigma = \frac{\partial G_\Sigma}{\partial Y}$.

$$\dot{V} = v(t)\ [\left(\partial_X G_\Sigma\, cos\left(\theta_{tangent} - \eta sgn\left(G_\Sigma(X,Y)\right)\right)\right) + \left(\partial_Y G_\Sigma\, sin\left(\theta_{tangent} - \eta sgn\left(G_\Sigma(X,Y)\right)\right)\right)] G_\Sigma(X,Y) \quad (21)$$

As stated in the theorem, $\theta_{tangent}$ is defined as the angle of the tangent to $G_\Sigma(x,y)$ at $(x = X, y = gof^{-1}(X))$. Therefore:

$$\theta_{tangent} = -tan^{-1}\left(\frac{\partial G_\Sigma/\partial x}{\partial G_\Sigma/\partial y}\right)\Bigg|_{\left(\substack{x=X \\ y=gof^{-1}(X)}\right)} \quad (22)$$

Since $\frac{\partial G_\Sigma/\partial x}{\partial G_\Sigma/\partial y}$ is independent of y:

$$\theta_{tangent} = -tan^{-1}\left(\frac{\partial G_\Sigma/\partial x}{\partial G_\Sigma/\partial y}\right)\Bigg|_{\left(\substack{x=X \\ y=Y}\right)} = -tan^{-1}\left(\frac{\partial_X G_\Sigma}{\partial_Y G_\Sigma}\right)$$

Using trigonometric identities for the inverse tangent function:

$$\dot{V} = -v(t)\ \left(\sqrt{(\partial_X G_\Sigma)^2 + (\partial_Y G_\Sigma)^2}\right) sin(\eta) sgn(G_\Sigma) G_\Sigma$$

$$\dot{V} = -v(t)\left(\sqrt{(\partial_X G_\Sigma)^2 + (\partial_Y G_\Sigma)^2}\right) sin(\eta)\ \left|\sqrt{V(X,Y)}\right| \quad (23)$$

Therefore X and Y converge to $G_\Sigma(X,Y) = 0$ exponentially within finite time. Since the desired trajectory is compatible with the input constraints, the velocity of the robot is always within the reachable velocity set of the robot and input saturation does not happen. This completes the proof of part (*a*). For the case of incompatible trajectory, the desired velocity is formulated as a function of arc length s, and the scaling factor γ is added to the controller to guarantee $u \in U_{adm}$. ∎

It is important to note that the geometric path associated with the desired trajectory can be formulated as $G_\Sigma(x,y) = x - fog^{-1}(x)$. The control defined in Theorem *III* is valid for this case as well and the proof can be easily modified to show it. This intuitive method can be used as a new approach for devising steering control strategies considering the bounds on the system inputs. Inspired by the structure of the tracking control in Theorem *III*, a trajectory generation/tracking problem can be solved as a path following problem where the timing and velocity requirement are adjusted and implemented via control algorithm in accordance with input constraints. To clarify the concept, we study the kinematics of the cooperative load carrying problem. Suppose two identical differential drive robots are supposed to carry a load from an initial position to a final position. Fig. 3 shows a schematics of this problemwhich can be formulated as: "Given $X_1(t_f)$, $Y_1(t_f)$, $X_2(t_f)$ and $Y_2(t_f)$ find $u_1, u_2 \epsilon\ U_{adm}$ such that for $t \in [0, t_f]$:

$$\|(X_1(t) - X_2(t), Y_1(t) - Y_2(t))\|_2 = L \quad (24)$$

where:

$$\|(X_1(0) - X_2(0), Y_1(0) - Y_2(0))\|_2 = L$$
$$\begin{cases} \dot{q}_1 = \boldsymbol{B}(\beta_1)u_1 \\ q_1(0) = q_{10} \end{cases}, \quad \begin{cases} \dot{q}_2 = \boldsymbol{B}(\beta_2)u_2 \\ q_2(0) = q_{20} \end{cases} \quad (25)$$

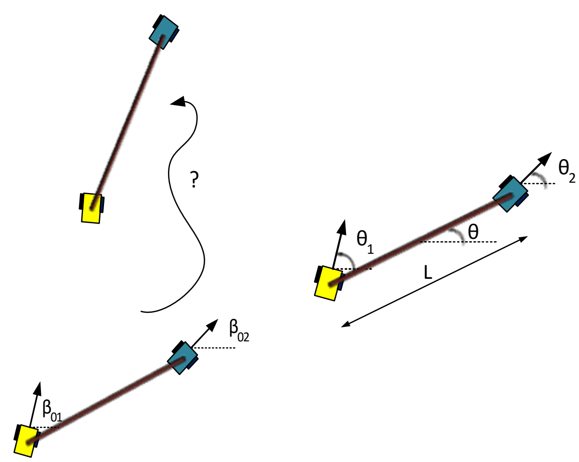

Figure 3. Cooperative Load Carrying

The trajectory tracking control developed in Theorem *III* can be used to tackle this steering challenge. The first step is devising paths that can satisfy the geometric requirement of the problem. This is not a challenging task and various approaches can be used for devising such paths, for example isometric properties of rotation/translation can be useful. For that choice, the geometric path of the robots will be circles centered at pole of the rotation [15]:

$$P_{12} = [I_{2\times2} - R(\phi_{12})]^{-1}\left(\begin{bmatrix} X_1(final) \\ Y_1(final)\end{bmatrix} - R(\phi_{12})\begin{bmatrix} X_1(0) \\ Y_1(0)\end{bmatrix}\right)$$

where $R(.)$ is the rotation matrix and

$$\phi_{12} = tan^{-1}\left(\frac{Y_2(0) - Y_1(0)}{X_2(0) - X_1(0)}\right) - tan^{-1}\left(\frac{Y_2(final) - Y_1(final)}{X_2(final) - X_1(final)}\right) \quad (26)$$

The geometric path is used by the controller for specifying the direction of motion for robots. The time element of the motion of two robots together can be incorporate into the proposed strategy via controlling the velocities of robots. To this end, we study the position of robots within an infinitesimal time interval and derive the velocity constraints for robots. Let $V_1(t)$ and $V_2(t)$ represent the velocities of each robot at each instant of time and let $\theta_1(t)$ and $\theta_2(t)$ represent the desired direction of motion for the robots prescribed by the devised paths. Fig. 3 demonstrates these values. The distance between the robots should always remain L:

$$\left\| \begin{bmatrix} X_2(t+\Delta t) \\ Y_2(t+\Delta t) \end{bmatrix} - \begin{bmatrix} X_1(t+\Delta t) \\ Y_1(t+\Delta t) \end{bmatrix} \right\|_2 = L \qquad (27)$$

Studying (27) for an infinitesimally small Δt, this equation leads to:

$$\frac{V_2(t)}{V_1(t)} = \frac{(X_2 - X_1)cos\theta_1 + (Y_2 - Y_1)sin\theta_1}{(X_2 - X_1)cos\theta_2 + (Y_2 - Y_1)sin\theta_2} \qquad (28)$$

By satisfying the above relationship between the velocities of robots, the timing requirement of the motion is preserved. The last requirement is input saturation considerations which is inherently included in the proposed framework because the tracking control method proposed in Theorem *III* guarantees satisfaction of the input constraints.

V. SIMULATION RESULTS

Two differential drive robots with hard input constraints are supposed to carry a virtual rod with the length of $L = 1^m$ from an initial position to a final position in an environment with obstacles. Let the initial configuration of the robots and the initial (C_0) and desired (C_f) positions of the rod center be:

$$q_1(0) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad q_2(0) = \begin{bmatrix} 3 \\ 1 \\ \pi/2 \end{bmatrix}, C_0 = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}, C_f = \begin{bmatrix} -7.4 \\ 2.9 \end{bmatrix}$$

The input constraint value is set to U_m=20 and Table 1 shows the physical dimensions of the robot used for simulation:

TABLE I. PHYSCIAL DIEMNSIONS OF ROBOT

ϕ(°)	*r(m)*	*a(m)*	*c(m)*	*d(m)*	*L(m)*
0.63	0.0346	0.05	0.06	0.0808	0.05

Rotation formulation in (26) have been used for offline path planning of the couple robots. To avoid obstacle, two consecutive rotations are devised for the robots' paths with the poles at:

$$P_{12} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, P_{23} = \begin{bmatrix} -3.5 \\ 3 \end{bmatrix}$$

The velocity profile associated with the devised path is set to be a constant speed of V_d = 0.35 which satisfy the compatibility condition of Theorem II:

$$\max_t \|(\dot{x}_d(t), \dot{y}_d(t))\|_2 = 0.35 < U_m \sigma_{min}(B_V) = 0.356$$

Fig. 4 shows the trajectory of rod and the carrying robots. As the figure show, the distance between the robots is constant throughout the steering process.

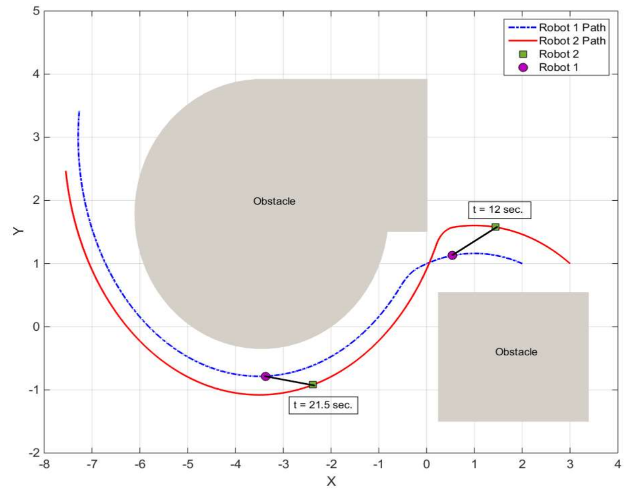

Figure 4. Trajectory of the "Rod" during load carrying scenario

Moreover, Fig. 5 shows the trajectory of robots and Fig. 6 shows the control signals. In accordance with the predictions of Theorem *II*, the control signals in Fig. 8 are below the limit of U_m=20 and the input control is not saturated.

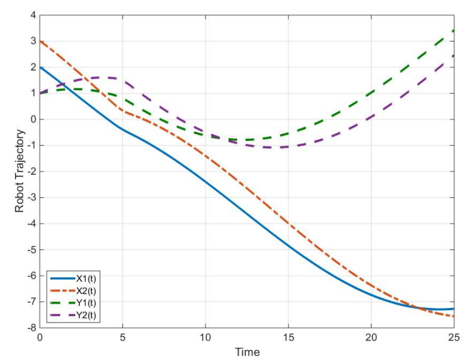

Figure 5. Trajectory of the robots during load carrying scenario

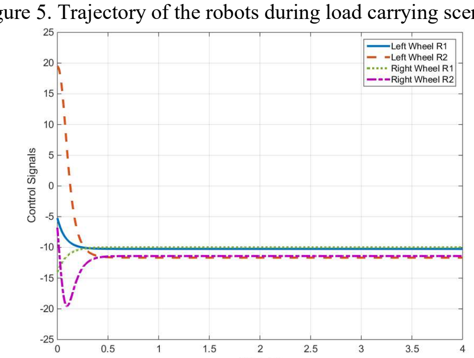

Figure 6. Control Signals

To study the effect of input saturation, the same scenario is simulated with the same settings; however the input bound is set to U_m=10. Fig. 7 shows the saturated control signals. As we expected, the control signal for the robots have reached limit value of U_m=10. This is consistent with the predictions of Theorem *I* since:

$$\max_t \|(\dot{x}_d(t), \dot{y}_d(t))\|_2 = 0.35 > \sqrt{2} U_m \sigma_{max}(B_V) = 0.3465$$

Despite saturation of the input signals, the simulation results show that the steering process has been successfully accomplished as guaranteed by Theorem *III*. To see the effect of saturation on the load carrying, the trajectory of the robots are compared for both cases in figure 8. As the figures show, trajectories for both cases are qualitatively the same. However, for second case where there is lower limit on the input, the controller scales down the desired velocity to guarantee intactness of the tracking performance and preserve the bounds on the inputs. Therefore the geometric path for the load carrying is perfectly tracked but with an adjusted speed and the input constraints are satisfied.

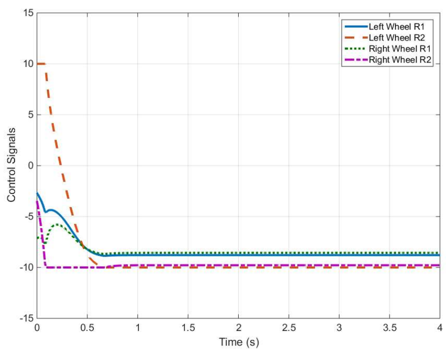

Figure 7. Saturated Control Signals

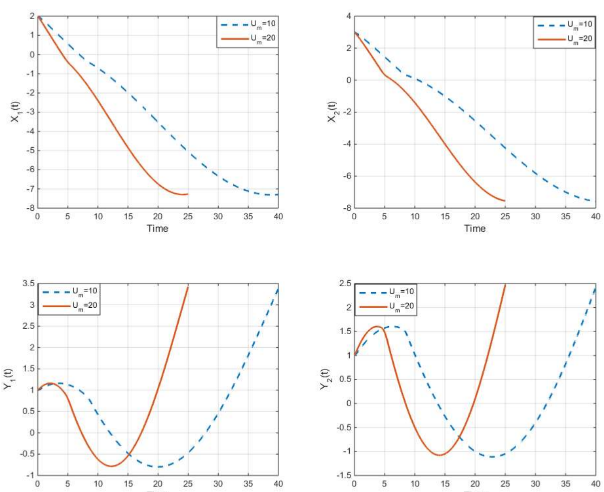

Figure 8. Effect of Saturation on State Trajectories

VI. CONCLUSION

In this paper the issue of input saturation in the control of differential drive robots were studied and analytical conditions were derived for determining the compatibility of a certain trajectory with input constraints. Moreover, a novel control strategy for trajectory tracking of differential drive robots was proposed which can guarantee error free tracking provided that the desired trajectory satisfy compatibility conditions. In case of incompatibility of desired trajectory, the controller can simultaneously refine the trajectory and use it for control. The derived theory introduces a new approach for steering input constrained systems in complicated scenarios such as cooperative load carrying. To demonstrate the power of developed theory, problem of cooperative load carrying for two input-constrained differential robots has been studied. The results of simulations demonstrate the efficiency of derived control strategy.

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