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Wheeled Robot Slip Compensation for Trajectory Tracking Control Problem with Time-Varying Reference Input

J. G. Iossaqui and J. F. Camino

Abstract— **This paper presents the stability analysis of the closed-loop error dynamics obtained using an adaptive kinematic controller for the trajectory tracking control problem of a wheeled mobile robot with longitudinal slip. It is shown that the adaptive kinematic controller is able to compensate in real time for an unknown constant slip whenever the reference trajectory is generated by a slow time-varying reference input. This extends previous results found in the literature where the reference input is assumed to be constant. Moreover, it is also shown that the estimated slip parameters converges to their true values. Numerical results show the performance of the adaptive kinematic controller.**

I. INTRODUCTION

Control design for mobile robots has received considerable attention because of the great variety of applications in unstructured environments, such as forestry, mining, agriculture, military applications, search and rescue, hospital tasks, and space exploration [1]. All of these tasks require an efficient solution to the robot navigation problem. In general, this is a difficult problem, since mobile robot control system must deal with physical limitations of sensors and actuators, nonholonomic constraints, and narrow workspace. Moreover, an important factor that must be taken into account in the robot navigation problem is the slip phenomena.

Most control design techniques for mobile robots are based on the assumption that the wheels roll without slipping. However, the slip has a critical influence that cannot be neglected. Thus, to attain higher performance, it is necessary to incorporate the slip parameters into the model of the robot. Kinematic models for wheeled mobile robots in the presence of wheel skid and slip can be found in [2]. Nonlinear control laws that compensate for the skid-slip effects are presented in [3]–[5]. An LMI-based approach is presented in [6]. Control laws that use an estimation of the slip obtained from the unscented Kalman filter are proposed in [7, 8].

In general, most control techniques in the literature assume that the slip is available in real time. However, it is usually difficult to directly measure the slip and most techniques appeal to an estimator. In this case, if the slip is not precisely estimated, for instance, due to lack of sensor accuracy, the performance of the controllers can be seriously affected [9]. In [10, 11], it is shown that the slip can be estimated from the posture of the robot using an extended Kalman filter. A nonlinear sliding mode observer for the estimation of the slip is presented in [12].

The authors are with the School of Mechanical Engineering, University of Campinas – UNICAMP, 13083-970, Campinas, SP, Brazil (e-mails: jiossaqui@yahoo.com.br; camino@fem.unicamp.br).

Neglecting the slip, a nonlinear controller that drives the posture error to zero is proposed in [13]. To compensate for constant slip parameters, the authors in [14] extended the results from [13] by including an update rule, which ensures that the posture error converges to zero. Both techniques assume that the reference input is constant. In practice, this means that the robot is only able to follow straight and circular reference trajectories, which is a severe limitation.

The main contribution of this paper is to show that the adaptive control law provided in [14] is able to drive the posture error to zero for slow time-varying reference input and constant slip parameters. It is also shown that the estimated slip parameters converge to their true values. Although convergence are only guaranteed for slow time-varying reference input signal, numerical simulations show that the posture error still converges to zero for fast and highly nonlinear time-varying reference input.

The paper is organized as follows. Section II presents the kinematic model that describes the motion of a wheeled mobile robot with longitudinal slip. Section III presents the adaptive kinematic control law that provides convergence to zero of the posture error and of the slip estimation error. Section IV presents the numerical results.

Nomenclature. Let $\mathbb{R}$ denotes the real numbers, $\mathbb{R}^+$ the non-negative real numbers, $\mathbb{R}^+_*$ the strictly positive real numbers, $\mathbb{R}^n$ the n-dimensional space, $\mathbb{R}^{m\times n}$ the set of $m \times n$ real matrices, and $t \in \mathbb{R}$ the continuous time.

II. KINEMATIC MODEL

This section presents the kinematic model of the wheeled mobile robot. For details see [6, 7, 14]. The posture $q = (x, y, \theta)^T$ of the robot is described by its position (x, y) and its orientation θ. Denoting the robot translational velocity by v and its rotational velocity by ω, the kinematic model is given by

$$\dot{q} = S(q)\eta \tag{1}$$

with $\eta = (v, \omega)^T$ and

$$S(q) = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

where the robot posture $q(t) \in \mathbb{R}^3$, the auxiliary control input $\eta(t) \in \mathbb{R}^2$, and the matrix $S(q) \in \mathbb{R}^{3\times 2}$.

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Including the longitudinal slip, the kinematic model becomes

$$\begin{aligned}
\dot{x} &= \frac{r}{2}\left(\omega_l a_l^{-1}(t) + \omega_r a_r^{-1}(t)\right)\cos\theta \\
\dot{y} &= \frac{r}{2}\left(\omega_l a_l^{-1}(t) + \omega_r a_r^{-1}(t)\right)\sin\theta \\
\dot{\theta} &= \frac{r}{b}\left(-\omega_l a_l^{-1}(t) + \omega_r a_r^{-1}(t)\right)
\end{aligned} \tag{3}$$

where $r \in \mathbb{R}_*^+$ is the radius of the wheel, $b \in \mathbb{R}_*^+$ is the spacing between the centerlines of the two wheels, $\omega_l(t) \in \mathbb{R}$ and $\omega_r(t) \in \mathbb{R}$ are, respectively, the angular velocities of the left and right wheels, and $1 \le a_l(t) \in \mathbb{R}$ and $1 \le a_r(t) \in \mathbb{R}$ are, respectively, the longitudinal slip parameters of the left and right wheels.

The kinematic model (3) can be rewritten in the following matrix form

$$\dot{q} = S_a(t,q)\xi \tag{4}$$

where $\xi(t) = (\omega_l, \omega_r)^T \in \mathbb{R}^2$ is the effective control input and the matrix $S_a(t,q) \in \mathbb{R}^{3\times 2}$ is given by

$$S_a(t,q) = \frac{r}{2ba_l(t)a_r(t)}\begin{pmatrix} ba_r(t)\cos\theta & ba_l(t)\cos\theta \\ ba_r(t)\sin\theta & ba_l(t)\sin\theta \\ -2a_r(t) & 2a_l(t) \end{pmatrix}$$

Note that the robot can be controlled by η using (1) or by ξ using (4). Although frequently used in the kinematic design, η can not be directly implementable on a real robot. It is more realistic to control the robot using ξ. The effective control input ξ is related to the auxiliary control input η according to $\eta = T\xi$ with $T \in \mathbb{R}^{2\times 2}$ given by

$$T = \frac{r}{2b}\begin{pmatrix} ba_l^{-1}(t) & ba_r^{-1}(t) \\ -2a_l^{-1}(t) & 2a_r^{-1}(t) \end{pmatrix}$$

and the inverse relation $\xi = T^{-1}\eta$ given by

$$\begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix} = \frac{1}{2r}\begin{pmatrix} 2a_l(t) & -ba_l(t) \\ 2a_r(t) & ba_r(t) \end{pmatrix}\begin{pmatrix} v \\ \omega \end{pmatrix} \tag{5}$$

Notice that if the slip parameters $a_l(t)$ and $a_r(t)$ are precisely known, ξ can always be obtained if η is provided.

III. ADAPTIVE CONTROL LAW

A. Adaptive Control Law for Constant Reference Input

The objective of the control design problem is to provide an effective control input $\xi = (\omega_l, \omega_r)^T$ for the wheeled robot such that

$$\lim_{t\to\infty}(q_{\text{ref}} - q) = 0$$

where the robot posture $q = (x, y, \theta)^T$ is given by (4) and the reference trajectory $q_{\text{ref}} = (x_{\text{ref}}, y_{\text{ref}}, \theta_{\text{ref}})^T$ is generated using the kinematic model

$$\dot{q}_{\text{ref}} = S(q_{\text{ref}})\eta_{\text{ref}} \tag{6}$$

with the reference input given by $\eta_{\text{ref}} = (v_{\text{ref}}, \omega_{\text{ref}})^T$ and the map $S(\cdot)$ given by (2).

To ensure the robot trajectory q will follow the desired reference trajectory q_{ref}, the posture error $e = (e_1, e_2, e_3)^T$ is defined as

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_{\text{ref}} - x \\ y_{\text{ref}} - y \\ \theta_{\text{ref}} - \theta \end{pmatrix} \tag{7}$$

Assuming that the reference input η_{ref} and the unknown slip parameters a_l and a_r are constants, the authors in [14] proposed an update rule to compensate for the slip. For this purpose, Equation (5) was modified to use the estimates

$$\begin{aligned}
\hat{a}_l &= a_l + \tilde{a}_l \\
\hat{a}_r &= a_r + \tilde{a}_r
\end{aligned} \tag{8}$$

with $\tilde{a}_l(t) \in \mathbb{R}$ and $\tilde{a}_r(t) \in \mathbb{R}$ the estimation error of a_l and a_r, respectively. Thus, Equation (5) becomes

$$\begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix} = \frac{1}{2r}\begin{pmatrix} 2\hat{a}_l & -b\hat{a}_l \\ 2\hat{a}_r & b\hat{a}_r \end{pmatrix}\begin{pmatrix} v_d \\ \omega_d \end{pmatrix} \tag{9}$$

where $\eta_d = (v_d, \omega_d)^T$ is the auxiliary control input to be designed.

The dynamics of the posture error e, derived using (4) and (6)-(9), is given by

$$\begin{aligned}
\dot{e}_1 = &\left(1+\frac{\tilde{a}_r}{a_r}\right)\left(\frac{e_2}{b}-\frac{1}{2}\right)\left(v_d+\frac{b}{2}\omega_d\right)+v_{\text{ref}}\cos e_3 \\
&-\left(1+\frac{\tilde{a}_l}{a_l}\right)\left(\frac{e_2}{b}+\frac{1}{2}\right)\left(v_d-\frac{b}{2}\omega_d\right)
\end{aligned} \tag{10}$$

$$\begin{aligned}
\dot{e}_2 = &\left(1+\frac{\tilde{a}_l}{a_l}\right)\left(v_d-\frac{b}{2}\omega_d\right)\frac{e_1}{b}+v_{\text{ref}}\sin e_3 \\
&-\left(1+\frac{\tilde{a}_r}{a_r}\right)\left(v_d+\frac{b}{2}\omega_d\right)\frac{e_1}{b}
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{e}_3 = &\,\omega_{\text{ref}}-\frac{1}{b}\left(1+\frac{\tilde{a}_r}{a_r}\right)\left(v_d+\frac{b}{2}\omega_d\right) \\
&+\frac{1}{b}\left(1+\frac{\tilde{a}_l}{a_l}\right)\left(v_d-\frac{b}{2}\omega_d\right)
\end{aligned} \tag{12}$$

The adaptive kinematic control law is composed by the auxiliary control input $\eta_d = (v_d, \omega_d)^T$, from [13], given by

$$\begin{aligned}
v_d &= v_{\text{ref}}\cos e_3 - k_3 e_3\omega_d + k_1 e_1 \\
\omega_d &= \omega_{\text{ref}} + \frac{v_{\text{ref}}}{2}\left[k_2\left(e_2 + k_3 e_3\right) + \frac{1}{k_3}\sin e_3\right]
\end{aligned} \tag{13}$$

with gains $k_i \in \mathbb{R}_*^+$ and $v_{\text{ref}} > 0$ together with the update rule, from [14], given by

$$\begin{aligned}
\dot{\hat{a}}_l = &\,\gamma_1\left(v_d-\frac{b}{2}\omega_d\right)\left[\left(\frac{e_2}{b}+\frac{1}{2}\right)e_1\right. \\
&\left.-\left(\frac{e_1}{b}+\frac{k_3}{b}\right)\left(e_2+k_3e_3\right)-\frac{1}{bk_2}\sin e_3\right] \\
\dot{\hat{a}}_r = &\,\gamma_2\left(v_d+\frac{b}{2}\omega_d\right)\left[-\left(\frac{e_2}{b}-\frac{1}{2}\right)e_1\right. \\
&\left.+\left(\frac{e_1}{b}+\frac{k_3}{b}\right)\left(e_2+k_3e_3\right)+\frac{1}{bk_2}\sin e_3\right]
\end{aligned} \tag{14}$$

with gains $\gamma_i \in \mathbb{R}_*^+$.

It was shown[1] in [14] that the posture error $e(t)$ converges to zero for constant reference input η_{ref} and constant slip parameters a_l and a_r. However, the convergence of the estimated slip parameters $\hat{a}_l(t)$ and $\hat{a}_r(t)$ to their true values a_l and a_r were not guaranteed. In the next section, using a different approach from that of [14], it is proved that the adaptive kinematic control law (13)-(14) can guarantee that the posture error $e(t)$ and the estimation errors $\tilde{a}_l(t)$ and $\tilde{a}_r(t)$ converge to zero for slow time-varying reference input $\eta_{\mathrm{ref}}(t)$.

B. Stability Analysis for Time-Varying Reference Input

Considering the reference input $\eta_{\mathrm{ref}}(t)$ as been time-varying, the dynamics of the augmented error $e_a = (e_1, e_2, e_3, \tilde{a}_l, \tilde{a}_l)^T$ is given by

$$\dot{e}_a = f(t, e_a) \tag{15}$$

with $f(t, e_a)$ given by (10)-(14). Note that the formulas (10)-(14) do not change if v_{ref} and ω_{ref} are assumed time-varying. Since the slip parameters a_l and a_r in (8) are constants, the time derivatives of $\tilde{a}_l(t)$ and $\tilde{a}_r(t)$ are given, respectively, by $\dot{\tilde{a}}_l(t) = \dot{\hat{a}}_l(t)$ and $\dot{\tilde{a}}_r(t) = \dot{\hat{a}}_r(t)$.

By combining Theorem 1 and Theorem 2, we can prove that the origin $e_a = 0$ of the nonautonomous system (15) is exponentially stable for constant slip parameters a_l and a_r and slow time-varying reference input $\eta_{\mathrm{ref}}(t) = (v_{\mathrm{ref}}, \omega_{\mathrm{ref}})^T$ with $2v_{\mathrm{ref}}(t) \neq \pm b\omega_{\mathrm{ref}}(t)$.

Theorem 1: (See [15, p. 165, Theorem 4.15]) Let $e_a = 0$ be an equilibrium point for the nonlinear system

$$\dot{e}_a = f(t, e_a)$$

where $f : [0, \infty) \times D \to \mathbb{R}^n$ is continuously differentiable, $D = \{e_a \in \mathbb{R}^n \mid \|e_a\|_2 < d\}$, and the Jacobian matrix $[\partial f/\partial e_a]$ is bounded and Lipschitz on D, uniformly in t. Let

$$A(t) = \left.\frac{\partial f}{\partial e_a}(t, e_a)\right|_{e_a=0}$$

Then, the origin $e_a = 0$ is an exponentially stable equilibrium point for the nonlinear system if it is an exponentially stable equilibrium point for the linear time-varying system

$$\dot{e}_a = A(t)e_a \tag{16}$$

To apply the above theorem, we consider the dynamics of the augmented error (15), with the reference input $\eta_{\mathrm{ref}}(t)$ time-varying and the slip parameters a_l and a_r constants. Assuming that $v_{\mathrm{ref}}(t)$, $\omega_{\mathrm{ref}}(t)$, $\dot{v}_{\mathrm{ref}}(t)$, and $\dot{\omega}_{\mathrm{ref}}(t)$ are bounded continuous functions of t, it is readily verified that $f : [0, \infty) \times \mathbb{R}^5 \to \mathbb{R}^5$ is continuously differentiable and the Jacobian matrix $[\partial f/\partial e_a]$ is bounded and Lipschitz on any compact subset of $\mathbb{R}^5$, uniformly in t.

According to Theorem 1, we need to show that the origin $e_a = 0$ is an exponentially stable equilibrium point of (16).

[1]Note that the time derivative of the Lyapunov function in [14] is negative semidefinite and not negative definite. This leads, with further reasoning, to the conclusion that only the posture error $e = (e_1, e_2, e_3)^T$ converges to zero.

The system matrix $A(t)$, obtained by the linearization of (15) around the origin $e_a = 0$, is given by

$$A(t) = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \tag{17}$$

where

$$A_{11} = \begin{pmatrix} -k_1 & \omega_{\mathrm{ref}}(t) & k_3\omega_{\mathrm{ref}}(t) \\ -\omega_{\mathrm{ref}}(t) & 0 & v_{\mathrm{ref}}(t) \\ 0 & -(k_2/2)v_{\mathrm{ref}}(t) & -(k_4/2k_3)v_{\mathrm{ref}}(t) \end{pmatrix}$$

$$A_{12} = \frac{1}{4ba_la_r}\begin{pmatrix} ba_rv_2(t) & -ba_lv_1(t) \\ 0 & 0 \\ -2a_rv_2(t) & -2a_lv_1(t) \end{pmatrix}$$

$$A_{21} = \frac{1}{4bk_2}\begin{pmatrix} -bk_2\gamma_1v_2(t) & 2k_2k_3\gamma_1v_2(t) & 2\gamma_1k_4v_2(t) \\ bk_2\gamma_2v_1(t) & 2k_2k_3\gamma_2v_1(t) & 2\gamma_2k_4v_1(t) \end{pmatrix}$$

$$A_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

with $v_1(t) = 2v_{\mathrm{ref}}(t) + b\omega_{\mathrm{ref}}(t)$, $v_2(t) = -2v_{\mathrm{ref}}(t) + b\omega_{\mathrm{ref}}(t)$, and $k_4 = 1 + k_2k_3^2$.

To show that the origin $e_a = 0$ of the linear time-varying system (16) is exponentially stable, we apply the next theorem.

Theorem 2: (See [16, p. 76, Theorem 1]) Let $\dot{e}_a = A(t)e_a$, where every element $a_{ij}(t)$ of $A(t)$ is differentiable and satisfies $|a_{ij}| \le \sigma$ for some $\sigma > 0$ and every eigenvalue of $A(t)$ satisfies

$$\mathrm{Re}[\lambda(A(t))] \le -\epsilon < 0 \tag{18}$$

Then, there is some $\delta > 0$ (independent of t) such that if every $|\dot{a}_{ij}| \le \delta$, the equilibrium point $e_a = 0$ is uniformly asymptotically stable, which, for linear system, is equivalent to exponential stability [15].

Since $v_{\mathrm{ref}}(t)$, $\omega_{\mathrm{ref}}(t)$, $\dot{v}_{\mathrm{ref}}(t)$, and $\dot{\omega}_{\mathrm{ref}}(t)$ are, by assumption, bounded continuous functions of t, it is readily verified that every element $a_{ij}(t)$ of (17) is differentiable and satisfies $|a_{ij}| \le \sigma$ for some finite constant $\sigma > 0$. Likewise, the entries $\dot{a}_{ij}$ are either zero or depend on the design parameters $\dot{v}_{\mathrm{ref}}(t)$ and $\dot{\omega}_{\mathrm{ref}}(t)$. Thus, it is also verified that $|\dot{a}_{ij}| \le \delta$ for any sufficiently small $\delta > 0$, that is, for a sufficiently slow time-varying reference input $\eta_{\mathrm{ref}}(t)$.

It now remains to show that $\mathrm{Re}[\lambda(A(t))] \le -\epsilon < 0$ for some ϵ. For this purpose, we consider the characteristic polynomial of the matrix $A(t)$, which is given by

$$p(s) = s^5 + \alpha_1s^4 + \alpha_2s^3 + \alpha_3s^2 + \alpha_4s + \alpha_5 \tag{19}$$

whose coefficients α_i are given in Appendix.

The stability criterion of Liénard and Chipart [17] establishes that the necessary and sufficient conditions for all the roots of the real polynomial (19) to have negative real parts can be given by the following conditions:

i) $0 < \alpha_i$, for $i = 1, \ldots, 5$;
ii) $0 < c_2 := \alpha_1\alpha_2 - \alpha_3$;
iii) $0 < c_3 := \alpha_1\alpha_2\alpha_3\alpha_4 - \alpha_3^2\alpha_4 - \alpha_1\alpha_2^2\alpha_5 - \alpha_1^2\alpha_4^2 + \alpha_2\alpha_3\alpha_5 + 2\alpha_1\alpha_4\alpha_5 - \alpha_5^2$.

Assuming that $b > 0$, $k_i > 0$, $\gamma_i > 0$, $a_l \geq 1$, $a_r \geq 1$, and $v_{\text{ref}} > 0$, it is clear, from the formulas in Appendix, that $\alpha_i > 0$, for $i = 1, \ldots, 4$, and that $\alpha_5 > 0$ whenever $v_1 \neq 0$ and $v_2 \neq 0$, that is, whenever $2v_{\text{ref}} \neq \pm b\omega_{\text{ref}}$. The expression c_2 is clearly positive. The expression c_3 is quite large and involve negative terms which makes it difficult to assert its positiveness. Thus, to check whether c_3 is a positive expression, we resort to the following optimization problem

$$\min_{(b,k_i,\gamma_i,a_l,a_r,v_{\text{ref}},\omega_{\text{ref}})} c_3$$

under the constraints

$$b > 0,\ k_i > 0,\ \gamma_i > 0,\ a_l \geq 1,\ a_r \geq 1,\ v_{\text{ref}} > 0$$

This optimization problem is solved using the command Nminimize[] from the computational package Mathematica. The attained optimal cost is a nonnegative number near zero. Hence, we conclude that c_3 is a nonnegative expression. Noticing that the $\det[A(t)] = -(\gamma_1\gamma_2 k_3 v_{\text{ref}} v_1^2 v_2^2)/(16 a_l a_r b^2)$ does not vanishes for $v_1 \neq 0$ and $v_2 \neq 0$, we can further conclude that c_3 is indeed strictly positive.

We have just shown that all conditions of the stability criterion of Liénard and Chipart are satisfied. Consequently, all eigenvalues of $A(t)$ satisfies (18). Hence, the origin $e_a = 0$ of the linear time-varying system (16) is exponentially stable and, from Theorem 1, the origin of the nonlinear system (15) is also exponentially stable. This implies that the robot trajectory $q(t)$ converges to the reference trajectory $q_{\text{ref}}(t)$ and the estimated slip parameters $\hat{a}_l(t)$ and $\hat{a}_r(t)$ converge to their true values a_l and a_r, respectively.

IV. NUMERICAL RESULTS

This section shows the numerical results for the adaptive control strategy given in Section III. The performance of the adaptive kinematic controller (AKC) is compared to the non-adaptive kinematic controller (NAC), without slip compensation, given in [13]. The physical parameters for the model of the wheeled robot, taken from [5], are $b = 0.1624$ m and $r = 0.0825$ m. The parameters of the adaptive controller are heuristically chosen as $k_1 = 1$, $k_2 = 21$ and $k_3 = 1$, and the parameters of the update rule are chosen as $\gamma_1 = \gamma_2 = 7$.

The reference trajectory (6) is generated using the initial condition $q_{\text{ref}}(0) = (0, 0, \pi/6)^T$ and the reference inputs $v_{\text{ref}}(t)$ and $\omega_{\text{ref}}(t)$, taken from [18], as follows:

$$v_{\text{ref}}(t) = \sqrt{\dot{\alpha}^2(t) + \dot{\beta}^2(t)}$$
$$\omega_{\text{ref}}(t) = \frac{\ddot{\beta}(t)\dot{\alpha}(t) - \ddot{\alpha}(t)\dot{\beta}(t)}{\dot{\alpha}^2(t) + \dot{\beta}^2(t)}$$

with $\alpha(t) = \sin(t/2)$ and $\beta(t) = \sin(t/4)$. Fig. 1 shows the reference inputs $v_{\text{ref}}(t)$ and $\omega_{\text{ref}}(t)$.

To show the robot performance under the NAC and AKC schemes, the slip parameters $a_l(t)$ and $a_r(t)$ are chosen as the following nonlinear time-varying signals

$$a_l(t) = 1/\left(1 - 0.3\left(1 - \mathrm{e}^{-0.1t}\cos(1.1t)\right)\right)$$
$$a_r(t) = 1/\left(1 - 0.6\left(1 - \mathrm{e}^{-0.08t}\sin^2(0.02t^2)\right)\right)$$

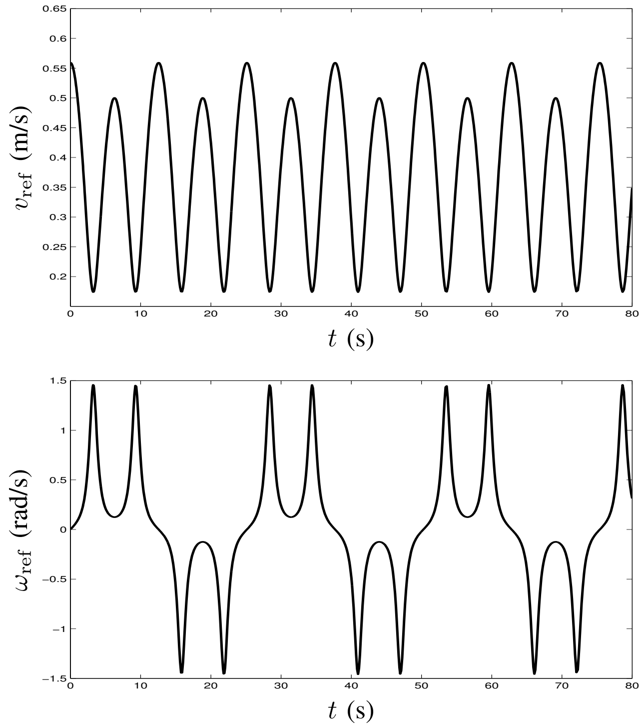

Fig. 1. Time-varying reference inputs $v_{\text{ref}}(t)$ and $\omega_{\text{ref}}(t)$.

The absence of slip on the left and right wheels are represented, respectively, as $a_l = 1$ and $a_r = 1$.

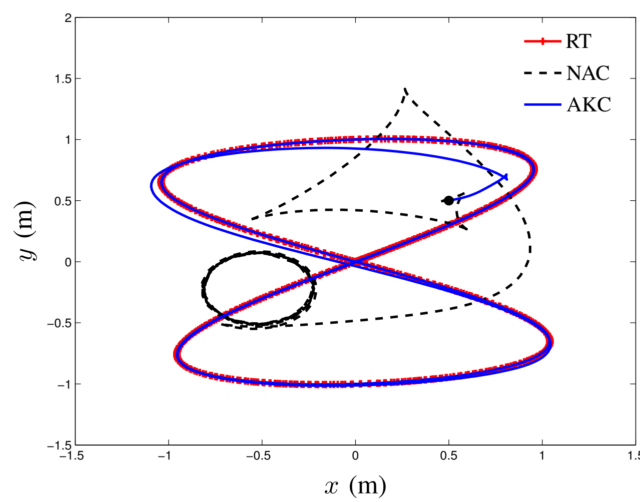

Fig. 2. Reference trajectory (RT) and robot trajectory using the NAC and AKC schemes.

Fig. 2 shows the robot trajectory obtained using the NAC and AKC schemes. The dashdot line, the solid line and the dashed line stand, respectively, for the reference trajectory (RT), the robot trajectory obtained using the NAC scheme, and the robot trajectory obtained using the AKC scheme. The initial conditions of the robot, depicted by a black circle, is $q(0) = (1/2, 1/2, 0)^T$. Note that the robot with the AKC scheme is able to follow the reference trajectory when the slip occurs. On the other hand, the robot trajectory using the NAC scheme is not able to compensate for the slip.

Fig. 3 shows the posture error $e = (e_1, e_2, e_3)^T$. The dashdot line stands for the NAC scheme, while the solid line stands for the AKC scheme. As expected, the AKC scheme achieves significantly better performance compared to the NAC scheme. The posture error of the robot using the NAC scheme attained a large value that cannot be acceptable for navigation in narrow spaces. Note also that the signal $e_3(t)$ goes unbounded around $t = 21$ seconds.

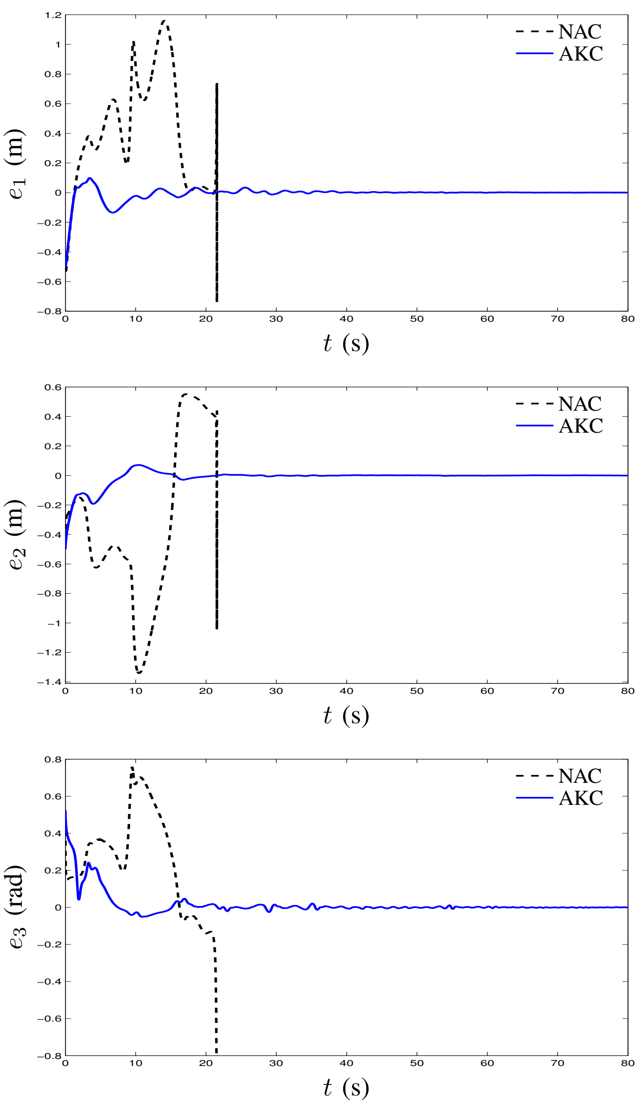

Fig. 3. Posture error $e = (e_1, e_2, e_3)^T$ for the NAC and AKC schemes.

Fig. 4 shows the true values of the time-varying slip parameters a_l and a_r and the respective estimated values $\hat{a}_l$ and $\hat{a}_r$. The dashed line denotes the true values a_l and a_r. The solid line denotes the estimated values $\hat{a}_L$ and $\hat{a}_R$, given by the update rule (14). The initial conditions of the update rule are taken as $\hat{a}_l(0) = 1.6$ and $\hat{a}_r(0) = 1.2$, which differs from the true values $a_l(0) = 1$ e $a_r(0) = 2.5$. Note that an estimation error occurs at the begin. However, this is not surprising, since convergence is only guaranteed for constant slip parameters.

Fig. 5 shows the effective control inputs $\omega_l(t)$ and $\omega_r(t)$ generated by the NAC and AKC schemes. The dashdot line stands for the NAC scheme, while the solid line stands for the AKC scheme. Note that the effective control input $\xi(t)$ generated by the NAC scheme escapes to infinity. Since the

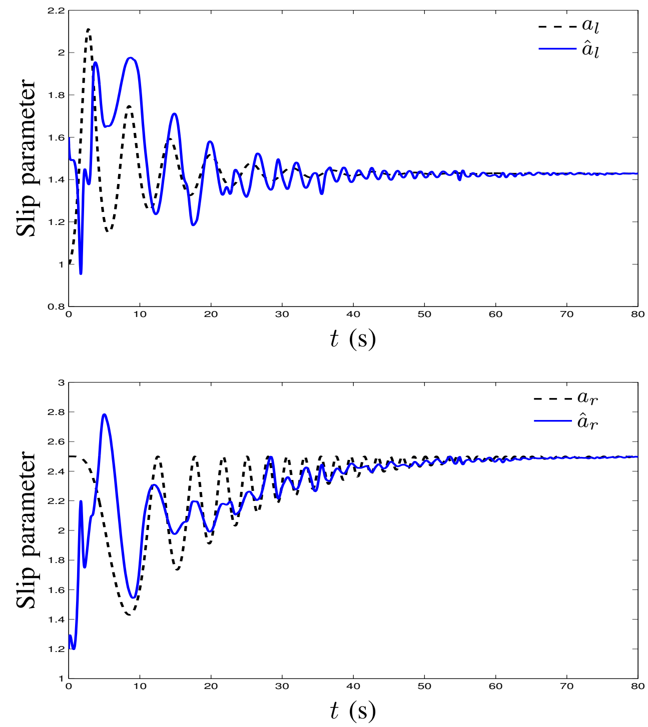

Fig. 4. True values of the time-varying slip parameters a_l and a_r, and the respective estimated values $\hat{a}_l$ and $\hat{a}_r$.

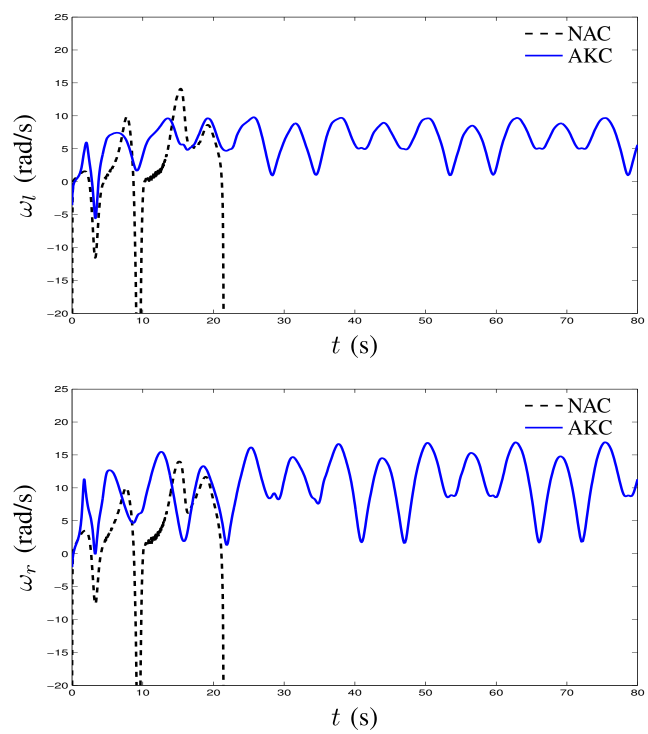

Fig. 5. The angular velocity of the left wheel $\omega_l(t)$ and the angular velocity of the right wheel $\omega_r(t)$ using the NAC and AKC schemes.

auxiliary control input $\eta_d(t)$ converges to the reference input $\eta_{\rm ref}(t)$, the amplitude of the effective control input $\xi(t)$ is ultimately determined by the amplitude of the reference input $\eta_{\rm ref}(t)$.

It was shown in Section III-B that convergence of the augmented error $e_a = (e_1, e_2, e_3, \tilde{a}_l, \tilde{a}_r)^T$ to zero is only guaranteed if $v_1(t) := 2v_{\rm ref}(t) + b\omega_{\rm ref}(t) \neq 0$ and $v_2(t) := 2v_{\rm ref}(t) - b\omega_{\rm ref}(t) \neq 0$. Indeed, if either $v_1(t) = 0$ or $v_2(t) = 0$ an estimation error might occur. Fig. 6 shows the last 20 seconds of the augmented error $e_a(t)$ obtained using the AKC scheme for $v_1(t) = 0$, that is, $2v_{\rm ref}(t) = -b\omega_{\rm ref}(t)$, with $\omega_{\rm ref}(t) = -1.5 + \sin(0.4t)$. The slip parameters are chosen as $a_l = a_r = 1$, which means that the wheels roll without slipping. Clearly the estimation error $\tilde{a}_r$ does not converges to zero. As shown in Fig. 7, similar results are found for $v_2 = 0$, that is, $2v_{\rm ref}(t) = b\omega_{\rm ref}(t)$, with $\omega_{\rm ref}(t) = 1.5 - \sin(0.4t)$. For this case, the estimation error $\tilde{a}_l$ does not converges to zero.

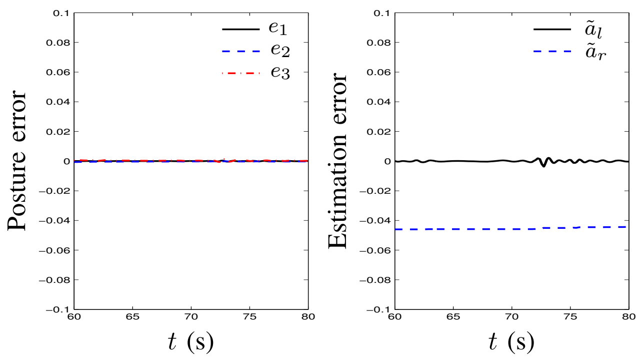

Fig. 6. Augmented error $e_a = (e_1, e_2, e_3, \tilde{a}_l, \tilde{a}_r)^T$ using the AKC scheme for $2v_{\rm ref}(t) = -b\omega_{\rm ref}(t)$ with $\omega_{\rm ref}(t) = -1.5 + \sin(0.4t)$.

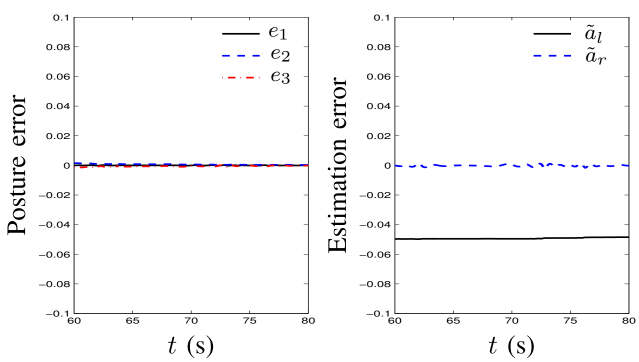

Fig. 7. Augmented error $e_a = (e_1, e_2, e_3, \tilde{a}_l, \tilde{a}_r)^T$ using the AKC scheme for $2v_{\rm ref}(t) = b\omega_{\rm ref}(t)$ with $\omega_{\rm ref}(t) = 1.5 - \sin(0.4t)$.

V. CONCLUSIONS

This paper has shown that the adaptive control strategy, composed by the auxiliary control input from [13] and the update rule from [14], is able to drive the posture error to zero for constant slip parameters and slow time-varying reference input. Under these conditions, it has also been shown that the estimated slip parameters converge to their true values. The slow time-varying assumption on the reference input allows, for instance, the motion planning algorithms to build a greater variety of trajectory, rather than just straight and circular reference trajectories generated by constant reference inputs. Numerical results show that the adaptive kinematic controller can efficiently compensate for a constant slip and can also maintain a satisfactory performance under a time-varying nonlinear slip profile.

APPENDIX

The stability criterion of Liénard and Chipart [17] establishes that the necessary and sufficient conditions for all the roots of the real polynomial

$$p(s) = s^5 + \alpha_1 s^4 + \alpha_2 s^3 + \alpha_3 s^2 + \alpha_4 s + \alpha_5 \tag{20}$$

to have negative real parts can be given by:

i) $0 < \alpha_i$, for $i = 1, \ldots, 5$;
ii) $0 < c_2 := \alpha_1\alpha_2 - \alpha_3$;
iii) $0 < c_3 := \alpha_1\alpha_2\alpha_3\alpha_4 - \alpha_3^2\alpha_4 - \alpha_1\alpha_2^2\alpha_5 - \alpha_1^2\alpha_4^2 + \alpha_2\alpha_3\alpha_5 + 2\alpha_1\alpha_4\alpha_5 - \alpha_5^2$.

The condition (i) consists in verifying the positiveness of the coefficients of the polynomial (20), which are given by

$$\begin{aligned}
\alpha_1 &= k_1 + \frac{k_4}{2k_3} v_{\rm ref} \\
\alpha_2 &= \frac{1}{16 a_l a_r b^2 k_2 k_3} \Big\{ a_l \big[\gamma_2 k_3 (b^2 k_2 + 4k_4) v_1^2 \\
&\quad + 8 a_r b^2 k_2 \big(v_{\rm ref} (k_1 k_4 + k_2 k_3 v_{\rm ref}) + 2 k_3 \omega_{\rm ref}^2 \big) \big] \\
&\quad + a_r \gamma_1 k_3 (b^2 k_2 + 4k_4) v_2^2 \Big\} \\
\alpha_3 &= \frac{1}{32 a_l a_r b^2 k_2 k_3} \Big\{ a_r \Big[16 a_l b^2 k_2 v_{\rm ref} (k_1 k_2 k_3 v_{\rm ref} + \omega_{\rm ref}^2) \\
&\quad + \gamma_1 v_2^2 \Big(k_2 v_{\rm ref} \big(k_3^2 (b^2 k_2 + 8) + b^2 \big) + 8 k_1 k_3 k_4 \Big) \Big] \\
&\quad + a_l \gamma_2 v_1^2 \Big[k_2 v_{\rm ref} \big(k_3^2 (b^2 k_2 + 8) + b^2 \big) + 8 k_1 k_3 k_4 \Big] \Big\} \\
\alpha_4 &= \frac{1}{32 a_l a_r b^2 k_2} \big\{ \gamma_2 v_1^2 \big[a_l \big((b k_2 v_{\rm ref} + 2\omega_{\rm ref})^2 \\
&\quad + 4\omega_{\rm ref}^2 + 8 k_1 k_2 k_3 v_{\rm ref} \big) + 2 \gamma_1 k_4 v_2^2 \big] \\
&\quad + a_r \gamma_1 (b\omega_{\rm ref} - 2v_{\rm ref})^2 \big((b k_2 v_{\rm ref} - 2\omega_{\rm ref})^2 \\
&\quad + 4\omega_{\rm ref}^2 + 8 k_1 k_2 k_3 v_{\rm ref} \big) \big\} \\
\alpha_5 &= \frac{1}{16 a_l a_r b^2} \gamma_1 \gamma_2 k_3 v_{\rm ref} v_1^2 v_2^2
\end{aligned}$$

with $v_1 = 2v_{\rm ref} + b\omega_{\rm ref}$, $v_2 = -2v_{\rm ref} + b\omega_{\rm ref}$, and $k_4 = 1 + k_2 k_3^2$.

The condition (ii) consists in verifying the positiveness of the expression c_2, given by

$$\begin{aligned}
c_2 = \frac{1}{16 a_l a_r b^2 k_2 k_3^2} \big\{ & a_l \big[4 a_r b^2 k_2 \big(k_1 (k_4^2 v_{\rm ref}^2 + 4 k_3^2 \omega_{\rm ref}^2) \\
& + 2 k_1^2 k_3 k_4 v_{\rm ref} + k_2 k_3 v_{\rm ref} (k_4 v_{\rm ref}^2 + 2 k_3^2 \omega_{\rm ref}^2) \big) \\
& + \gamma_2 k_3 v_1^2 \big(b^2 k_1 k_2 k_3 + 2 (k_2^2 k_3^4 v_{\rm ref} + v_{\rm ref}) \big) \big] \\
& + a_r \gamma_1 k_3 v_2^2 \big(b^2 k_1 k_2 k_3 + 2 (k_2^2 k_3^4 v_{\rm ref} + v_{\rm ref}) \big) \big\}
\end{aligned}$$

The condition (iii) consists in verifying the positiveness of the expression c_3, given by

$$
\begin{aligned}
c_3 = & -\frac{p_5^2 p_6^2}{1024 a_l^2 a_r^2 b^4 k_2^2} - \frac{p_6 p_7^2}{32768 a_l^3 a_r^3 b^6 k_2^3 k_3^2} \\
& + \frac{p_5 p_6 p_7 p_9}{16384 a_l^3 a_r^3 b^6 k_2^3 k_3^2} + \frac{\gamma_1 \gamma_2 k_3 p_5 p_6 v_1^2 v_2^2 v_{\mathrm{ref}}}{256 a_l^2 a_r^2 b^4 k_2} \\
& + \frac{\gamma_1 \gamma_2 p_7 p_9 v_1^2 v_2^2 v_{\mathrm{ref}}}{8192 a_l^3 a_r^3 b^6 k_2^2 k_3} - \frac{\gamma_1 \gamma_2 p_5 p_9^2 v_1^2 v_2^2 v_{\mathrm{ref}}}{4096 a_l^3 a_r^3 b^6 k_2^2 k_3} \\
& - \frac{\gamma_1^2 \gamma_2^2 k_3^2 v_1^4 v_2^4 v_{\mathrm{ref}}^2}{256 a_l^2 a_r^2 b^4}
\end{aligned}
$$

with

$$
\begin{aligned}
p_1 &= (b k_2 v_{\mathrm{ref}} + 2\omega_{\mathrm{ref}})^2 + 8 k_1 k_2 k_3 v_{\mathrm{ref}} + 4\omega_{\mathrm{ref}}^2 \\
p_2 &= (b k_2 v_{\mathrm{ref}} - 2\omega_{\mathrm{ref}})^2 + 8 k_1 k_2 k_3 v_{\mathrm{ref}} + 4\omega_{\mathrm{ref}}^2 \\
p_3 &= 8 k_1 (k_3 + k_2 k_3^3) + k_2 \big(b^2 + (8 + b^2 k_2) k_3^2\big) v_{\mathrm{ref}} \\
p_4 &= \gamma_2 k_3 (b^2 k_2 + 4 k_4) v_1^2 \\
&\quad + 8 a_r b^2 k_2 \big(v_{\mathrm{ref}} (k_1 k_4 + k_2 k_3 v_{\mathrm{ref}}) + 2 k_3 \omega_{\mathrm{ref}}^2\big) \\
p_5 &= k_1 + (k_4 v_{\mathrm{ref}})/(2 k_3) \\
p_6 &= a_r \gamma_1 p_2 v_2^2 + \gamma_2 v_1^2 (a_l p_1 + 2\gamma_1 k_4 v_2^2) \\
p_7 &= a_l \gamma_2 p_3 v_1^2 \\
&\quad + a_r \big(\gamma_1 p_3 v_2^2 + 16 a_l b^2 k_2 v_{\mathrm{ref}} (k_1 k_2 k_3 v_{\mathrm{ref}} + \omega_{\mathrm{ref}}^2)\big) \\
p_8 &= \gamma_2 k_3 (b^2 k_2 + 4 k_4) v_1^2 \\
&\quad + 8 a_r b^2 k_2 \big(v_{\mathrm{ref}} (k_1 k_4 + k_2 k_3 v_{\mathrm{ref}}) + 2 k_3 \omega_{\mathrm{ref}}^2\big) \\
p_9 &= a_l p_8 + a_r \gamma_1 k_3 (b^2 k_2 + 4 k_4) v_2^2
\end{aligned}
$$

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