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Adaptive Control of Unicycle-Type Mobile Robots with Longitudinal Slippage*

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Abstract—**This paper studies trajectory tracking control of unicycle-type mobile robots with longitudinal slippage in kinematic level. By introducing a parameter, called "Slipping ratio", into the steering system, an adaptive controller different from [8] is designed. All the tracking errors are guaranteed to converge to zero asymptotically using the proposed controller. At the meantime, with some limitations to the reference trajectory, the slipping ratio estimations are also ensured to converge to their true values. Numerical simulations show the stability and effectiveness of the proposed controller.**

I. INTRODUCTION

Trajectory tracking control of unicycle-type mobile robots has been always an important topic in the fields of automation control and robotics. In the past few years, considerable efforts have been devoted to this control task, for example see [1]-[5] and the references therein. Major difficulties in trajectory tracking lie in the nonholonomic nature of the underactuated robot system and all kinds of requirements, limitations in different applications. To cope with these problems, plenty of methods are investigated, such as adaptive control [6]-[9], sliding mode control [10]-[12], artificial neural network (ANN) [13]-[16], fuzzy logic control [17][18], and dynamic sliding mode control (DSC) [19]-[21].

In current researches, most of the controllers are designed without considering longitudinal slippage or lateral slippage, namely the mobile robots are subject to pure rolling. However, in practice when mobile robots work at a high speed, or the workplace is slippery, slippage is hard to avoid, especially the longitudinal one. In this case, performance of the traditional controllers deteriorates badly, and sometimes part of the controllers cannot even to work. Aiming at this situation, perturbations due to skidding and slipping are categorically classified as input-additive, input multiplicative, or/and matched/unmatched perturbations from control perspective in [22]. And it also points out that longitudinal slippage in the unicycle-type mobile robot is input-additive and matched while lateral slippage is unmatched.

In kinematic level, trajectory tracking of wheeled mobile robots in presence of slipping or skidding has been a key problem attracting many researchers. In [23], based on a modified kinematic model, an adaptive controller is designed to estimate skidding ratio of wheeled mobile robots with unknown skidding. In [6], by dividing skidding into two parts, a variable structure control scheme is proposed to solve the trajectory tracking problem using linear approximation. In [7], an adaptive controller is developed to make mobile robots track reference trajectory via defining slipping ratio as an unknown constant. In [8], an adaptive controller is proposed by similar process, but the convergence of slipping ratio estimations to their true values can be also realized in comparison with [7]. In [24], with the help of GPS and other sensors to measure the mobile robot's posture, velocities, and perturbations for control compensation, a controller is developed to address the tracking control of the car-like wheeled mobile robots in presence of wheel skidding and slipping.

In dynamic level, lots of controllers have been developed with velocity measurements. Under the framework of [10], slippage is taken as perturbations in [9] and a coordinate transformation based adaptive control scheme is proposed to address the trajectory tracking of wheeled mobile robots in presence of unknown slipping and skidding. In [16][17], controllers are designed through a layered architecture strategy used like in [3][4]. In [14], based on a restricted assumption of the slippage, an adaptive neural network controller is proposed to solve trajectory tracking problem. In [15], an adaptive controller is developed to make mobile robots track the desired trajectory by real-time slipping ratio calculating. In [25], adaptive control system is designed for the robot's trajectory tracking via function approximation using neural networks to compensate the observer error.

Overall, controller design based on kinematic model shows its importance in constructing an appropriate controller both in kinematic level and dynamic level. And consider the sensors' cost, volume and weight, controller designed based on kinematic model without complex sensors for the trajectory tracking of mobile robots in presence of slippage is needed.

In this paper, trajectory tracking control in presence of longitudinal slippage is considered in kinematic level for unicycle-type mobile robots. Just as [7][8] does, slipping ratio is introduced into steering system and an adaptive controller is designed. Compared with [8], simpler method of stability proving is addressed as a different controller is conducted. With the proposed controller, all the tracking errors are guaranteed to approach zero asymptotically, and with some limitations to the reference trajectory, slipping ratio can be exact estimated simultaneously. So in comparison with [21], it is more convenient to implement the controller in realistic environment.

II. KINEMATIC MODEL FOR TRAJECTORY TRACKING

Consider the unicycle-type mobile robot shown in Fig. 1. In global coordinates $\{X, Y\}$, robot's gravity center is represented by P, and its posture is represented by $q = [x, y, \theta]^T$, in which (x, y) is robot's position, and θ is

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robot's heading angle. Virtual input $[v, \omega]^T$ denotes robot's linear velocity and angular velocity. The radius of each driving wheel is r, and the distance between two wheels' centers is $2R$.

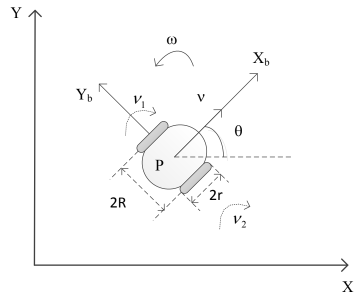

Figure 1. Unicycle-type mobile robot

Thus, the robot's kinematic model is derived as

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \quad (1)$$

In the task of trajectory tracking, desired trajectory is generated by reference robot, whose kinematic model can be described as in (2). Reference robot's posture is represented by $q_r = [x_r, y_r, \theta_r]^T$ and v_r, ω_r denote its linear velocity and rotational velocity respectively.

$$\begin{aligned} \dot{x}_r &= v_r \cos\theta_r \\ \dot{y}_r &= v_r \sin\theta_r \\ \dot{\theta}_r &= \omega_r \end{aligned} \quad (2)$$

The destination of this paper is to find an appropriate input to make the mobile robot track the desired trajectory, which means all the posture errors asymptotically converge to zero, i.e.

$$\lim_{t\to\infty}(q_r - q) = 0. \quad (3)$$

To achieve this goal, coordinate transformation (4), (5) should be introduced first and the tracking error space model (6), called steering system of the mobile robot, is derived.

$$e = [e_1 \quad e_2 \quad e_3]^T = T_e(q_r - q). \quad (4)$$

$$T_e = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

Assumption 1: In the trajectory tracking control of unicycle-type mobile robots, virtual reference inputs v_r and ω_r are bounded and their first-order derivatives, $\dot{v}_r$ and $\dot{\omega}_r$, are also bounded, i.e. $v_r \le B_1$, $\omega_r \le B_2$, $\dot{v}_r \le B_3$, $\dot{\omega}_r \le B_4$. $B_i\ (i = 1,2,3,4)$ are known positive constants. In addition, $v_r > 0$ is also needed.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \omega e_2 + v_r \cos e_3 - v \\ -\omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{bmatrix}. \quad (6)$$

Take longitudinal slipping into consideration, the actual input $[v_1, v_2]^T$ transferred from both wheels to virtual input $[v, \omega]^T$ shall be as following.

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{2}\begin{bmatrix} r & r \\ -\frac{r}{R} & \frac{r}{R} \end{bmatrix}\begin{bmatrix} \frac{1}{a_1} & 0 \\ 0 & \frac{1}{a_2} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \quad (7)$$

In (7), v_1 and v_2 denote rotational velocity of mobile robot's left wheel and right wheel respectively. a_1 and a_2 correspond to each wheel's slipping ratio.

$$a_i = \frac{v_i}{v_{is}}, \quad i = 1,2. \quad (8)$$

In (8), v_{is} is defined as the slippage rotational speed of mobile robot's wheel. And the case $a_i = 1$ means no slippage in the corresponding wheel.

Assumption 2: In this paper, the slipping ratio is bounded as $0 < a_1 \le B_5$ and $0 < a_2 \le B_6$. $B_i\ (i = 5,6)$ are known positive constants.

Remark 1: Assumption 1 guarantees that the desired trajectory is trackable. And $v_r > 0$ means that the mobile robot can only move forward. Assumption 2 satisfies most of the situations in the realistic working environment. The case $v_{is} = 0$, which means mobile robot's corresponding wheel does not move a little. Under this condition, any desired trajectory is no longer trackable, so this situation is not considered in this paper.

III. Adaptive Controller Design

Step 1: In this step, virtual input $[v_d, \omega_d]^T$ is derived assuming that there is no slippage.

Define

$$\bar{e}_3 = e_3 + \arcsin(\frac{ke_2}{\Gamma_1}), \quad (9)$$

where

$$0 < k < 1, \quad \Gamma_1 = \sqrt{1 + e_1^2 + e_2^2}.$$

Define

$$\alpha_1 = \int_0^1 \cos\left[-\arcsin(\frac{ke_2}{\Gamma_1}) + \eta\bar{e}_3\right] d\eta = \begin{cases} \dfrac{\sin e_3 + \frac{ke_2}{\Gamma_1}}{\bar{e}_3}, & \bar{e}_3 \ne 0 \\ \cos e_3, & \bar{e}_3 = 0 \end{cases}. \quad (10)$$

From (9) and (10), the following results can be derived.

$$\sin e_3 = \alpha_1 \bar{e}_3 - \frac{ke_2}{\Gamma_1}. \quad (11)$$

Then the steering system (6)can be rewritten as

$$\begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{\bar{e}}_3\end{bmatrix}=\begin{bmatrix}\omega e_2+v_r\cos e_3-v\\ -\omega e_1+v_r\left(\alpha_1\bar{e}_3-\dfrac{ke_2}{\Gamma_1}\right)\\ \omega_r-\left(1+\dfrac{ke_1}{\Gamma_2}\right)\omega+\alpha_2+\dfrac{ke_1e_2}{\Gamma_1^2\Gamma_2}v\end{bmatrix}, \tag{12}$$

where

$$\Gamma_2=\sqrt{1+e_1^2+(1-k^2)e_2^2},$$
$$\alpha_2=\frac{kv_r\sin e_3-ke_2\left(e_1v_r\cos e_3+e_2v_r\sin e_3\right)}{\Gamma_1^2\Gamma_2}.$$

Define a positive definite function for this system as

$$V_1=\frac{1}{2}e_1^2+\frac{1}{2}e_2^2+\lambda\Gamma_3,\quad \Gamma_3=\sqrt{1+\bar{e}_3^2}. \tag{13}$$

In (13), λ is a positive constant, bounded by $\lambda\le 1$. Thus, the derivative of (13) can be computed by

$$\begin{aligned}\dot{V}_1&=e_1\dot{e}_1+e_2\dot{e}_2+\frac{\lambda\bar{e}_3}{\Gamma_3}\dot{\bar{e}}_3\\ &=e_1\left[-\left(1-\frac{\lambda ke_2\bar{e}_3}{\Gamma_1^2\Gamma_2\Gamma_3}\right)v+v_r\cos e_3\right]-\frac{kv_r}{\Gamma_1}e_2^2\\ &\quad+\frac{\lambda\bar{e}_3}{\Gamma_3}\left[-\left(1+\frac{ke_1}{\Gamma_2}\right)\omega+\omega_r+\alpha_2+\frac{1}{\lambda}e_2\alpha_1v_r\Gamma_3\right].\end{aligned} \tag{14}$$

Then, the virtual input guaranteeing all the tracking errors asymptotically converge to zero can be chosen as

$$\begin{aligned}v_d&=\left(1-\frac{\lambda ke_2\bar{e}_3}{\Gamma_1^2\Gamma_2\Gamma_3}\right)^{-1}\left(v_r\cos e_3+k_1e_1\right)\\ \omega_d&=\left(1+\frac{ke_1}{\Gamma_2}\right)^{-1}\left(\omega_r+\alpha_2+\frac{1}{\lambda}e_2\alpha_1v_r\Gamma_3+k_3\bar{e}_3\right)\end{aligned}. \tag{15}$$

In (15), both k_1 and k_3 are positive constants.

Step 2: In this step, longitudinal slippage is introduced into steering system, and then an adaptive controller is designed to estimate the slipping ratio.

Take $\hat{a}_i\ (i=1,2)$ as the estimation of slipping ratio $a_i\ (i=1,2)$, and then the estimation errors and their derivatives are

$$\tilde{a}_i=a_i-\hat{a}_i,\quad \dot{\tilde{a}}_i=-\dot{\hat{a}}_i,\quad i=1,2. \tag{16}$$

Combined the inverse of (7) with the virtual input (15) acquired from step 1, the actual control input $[v_{1a}\quad v_{2a}]^T$ is given as following.

$$\begin{bmatrix}v_{1a}\\ v_{2a}\end{bmatrix}=\begin{bmatrix}\hat{a}_1 & 0\\ 0 & \hat{a}_2\end{bmatrix}\begin{bmatrix}\dfrac{1}{r} & -\dfrac{R}{r}\\ \dfrac{1}{r} & \dfrac{R}{r}\end{bmatrix}\begin{bmatrix}v_d\\ \omega_d\end{bmatrix}. \tag{17}$$

For the existence of longitudinal slippage, the actual virtual input $[v_a\quad \omega_a]^T$ is no longer the result of (15), $[v_d\quad \omega_d]^T$, and it now turns into (18) according to (7).

$$\begin{bmatrix}v_a\\ \omega_a\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\dfrac{\hat{a}_1}{a_1}+\dfrac{\hat{a}_2}{a_2} & -R\dfrac{\hat{a}_1}{a_1}+R\dfrac{\hat{a}_2}{a_2}\\ -\dfrac{1}{R}\dfrac{\hat{a}_1}{a_1}+\dfrac{1}{R}\dfrac{\hat{a}_2}{a_2} & \dfrac{\hat{a}_1}{a_1}+\dfrac{\hat{a}_2}{a_2}\end{bmatrix}\begin{bmatrix}v_d\\ \omega_d\end{bmatrix}. \tag{18}$$

Further, (18) can be rewritten as

$$\begin{aligned}v_a&=v_d-\frac{1}{2}\left(\frac{\tilde{a}_1}{a_1}+\frac{\tilde{a}_2}{a_2}\right)v_d+\frac{1}{2}R\left(\frac{\tilde{a}_1}{a_1}-\frac{\tilde{a}_2}{a_2}\right)\omega_d\\ \omega_a&=\omega_d+\frac{1}{2}R\left(\frac{\tilde{a}_1}{a_1}-\frac{\tilde{a}_2}{a_2}\right)v_d-\frac{1}{2}\left(\frac{\tilde{a}_1}{a_1}+\frac{\tilde{a}_2}{a_2}\right)\omega_d\end{aligned}. \tag{19}$$

Theorem 1: Consider the kinematic model of unicycle-type mobile robot under assumption 2, which is given in (1). Given a bounded continuous desired trajectory under assumption 1, all the tracking errors can be guaranteed to asymptotically converge to zero with the input (15), (17) and adaptive control law (20). At the same time, the slipping ratio estimations converge to their true values if $v_r\neq\pm\omega_r$.

$$\begin{aligned}\dot{\hat{a}}_1&=\gamma_1\left(\frac{\lambda ke_2\bar{e}_3}{\Gamma_1^2\Gamma_2\Gamma_3}-1\right)\left(-\frac{1}{2}v_d+\frac{1}{2}R\omega_d\right)e_1\\ &\quad-\gamma_1\left(1+\frac{ke_1}{\Gamma_2}\right)\left(\frac{1}{2R}v_d-\frac{1}{2}\omega_d\right)\frac{\lambda\bar{e}_3}{\Gamma_3}\\ \dot{\hat{a}}_2&=\gamma_2\left(\frac{\lambda ke_2\bar{e}_3}{\Gamma_1^2\Gamma_2\Gamma_3}-1\right)\left(-\frac{1}{2}v_d-\frac{1}{2}R\omega_d\right)e_1\\ &\quad+\gamma_2\left(1+\frac{ke_1}{\Gamma_2}\right)\left(\frac{1}{2R}v_d+\frac{1}{2}\omega_d\right)\frac{\lambda\bar{e}_3}{\Gamma_3}\end{aligned}. \tag{20}$$

In (20), both γ_1 and γ_2 are positive constants.

Proof:

Step 1: In this step, the convergence of posture errors e_1, e_2, e_3 to zero will be proved first.

Define a new positive definite function for this system as

$$V_2=\frac{1}{2}e_1^2+\frac{1}{2}e_2^2+\lambda\Gamma_3+\frac{1}{2\gamma_1a_1}\tilde{a}_1^2+\frac{1}{2\gamma_2a_2}\tilde{a}_2^2. \tag{21}$$

Combined with (14), (15), (17), (20), the derivative of (21) along the trajectories of the system is given by

$$\begin{aligned}\dot{V}_2&=e_1\dot{e}_1+e_2\dot{e}_2+\frac{\lambda\bar{e}_3}{\Gamma_3}\dot{\bar{e}}_3-\frac{\tilde{a}_1}{\gamma_1a_1}\dot{\hat{a}}_1-\frac{\tilde{a}_2}{\gamma_2a_2}\dot{\hat{a}}_2\\ &=e_1\left[-\left(1-\frac{\lambda ke_2\bar{e}_3}{\Gamma_1^2\Gamma_2\Gamma_3}\right)v_a+v_r\cos e_3\right]-\frac{kv_r}{\Gamma_1}e_2^2-\frac{\tilde{a}_1}{\gamma_1a_1}\dot{\hat{a}}_1-\frac{\tilde{a}_2}{\gamma_2a_2}\dot{\hat{a}}_2\\ &\quad+\frac{\lambda\bar{e}_3}{\Gamma_3}\left[-\left(1+\frac{ke_1}{\Gamma_2}\right)\omega_a+\omega_r+\alpha_2+\frac{1}{\lambda}e_2\alpha_1v_r\Gamma_3\right]\\ &=-k_1e_1^2-\frac{kv_r}{\Gamma_1}e_2^2-\frac{\lambda k_3}{\Gamma_3}\bar{e}_3^2.\end{aligned} \tag{22}$$

Thus $\dot{V} \le 0$ is derived, which means that V is bounded. According to (21), $e_1, e_2, e_3, \tilde{a}_1, \tilde{a}_2, \Gamma_1, \Gamma_2, \Gamma_3$ are all bounded. Take into consideration the assumption 2, and it can be inferred that slipping ratio estimation $\hat{a}_1, \hat{a}_2$ are all bounded.

Consider in (15),

$$1-\frac{\lambda k e_2 \bar{e}_3}{\Gamma_1^2 \Gamma_2 \Gamma_3} \ge 1-\lambda k > 0, \quad 1+\frac{k e_1}{\Gamma_2} \ge 1-k > 0. \tag{23}$$

Thus, inferred from (15), (19), $v_d, \omega_d, v_a, \omega_a$ are all bounded. Inferred from (6), (12), (20) , $\dot{e}_1$, $\dot{e}_2$, $\dot{e}_3$, $\dot{\bar{e}}_3$, $\dot{\hat{a}}_1$, $\dot{\hat{a}}_2$ are all bounded.

Differentiate Γ_1, Γ_2, Γ_3, then

$$\begin{aligned} \dot{\Gamma}_1 &= \frac{e_1\dot{e}_1 + e_2\dot{e}_2}{\Gamma_1} \le \left|e_1\dot{e}_1 + e_2\dot{e}_2\right| \\ \dot{\Gamma}_2 &= \frac{e_1\dot{e}_1 + \left(1-k^2\right)e_2\dot{e}_2}{\Gamma_2} \le \left|e_1\dot{e}_1 + \left(1-k^2\right)e_2\dot{e}_2\right| . \\ \dot{\Gamma}_3 &= \frac{\bar{e}_3\dot{\bar{e}}_3}{\Gamma_3} \le \left|\bar{e}_3\dot{\bar{e}}_3\right| \end{aligned} \tag{24}$$

So, $\dot{\Gamma}_1$, $\dot{\Gamma}_2$, $\dot{\Gamma}_3$ are bounded. Now consider a continuous function defined in (25), and its derivative defined in (26), in which a, b are all time related functions. By use of Taylor's expansion, it can be easy derived that if $a, b, \dot{a}, \dot{b}$ are all bounded, then $f, \dot{f}$ are also bounded, too.

$$f = \begin{cases} \dfrac{\sin a + \sin b}{a+b}, & a+b \ne 0 \\ \cos a, & a+b = 0 \end{cases}. \tag{25}$$

$$\dot{f} = \begin{cases} \dfrac{\left(\dot{a}\cos a + \dot{b}\cos b\right)\left(a+b\right) - \left(\dot{a}+\dot{b}\right)\left(\sin a + \sin b\right)}{\left(a+b\right)^2}, & a+b \ne 0 \\ \dfrac{1}{2}\left(\dot{b}-\dot{a}\right)\sin a, & a+b = 0 \end{cases}. \tag{26}$$

Similar to $f, \dot{f}$, these terms $\alpha_1, \dot{\alpha}_1, \dot{v}_d, \dot{\omega}_d$, $\dot{v}_a, \dot{\omega}_a$ are all bounded. Then the second-order derivative of (21) is

$$\begin{aligned} \ddot{V}_2 = &-2k_1 e_1 \dot{e}_1 - \frac{k\dot{v}_r \Gamma_1 - \dot{\Gamma}_1 k v_r}{\Gamma_1^2} e_2^2 - 2\frac{k v_r}{\Gamma_1} e_2 \dot{e}_2 \\ &+ \frac{\lambda k_3}{\Gamma_3^2} \dot{\Gamma}_3 \bar{e}_3^2 + 2\frac{\lambda k_3}{\Gamma_3} \bar{e}_3 \dot{\bar{e}}_3. \end{aligned} \tag{27}$$

According to Barbalat's lemma, the limit of $\dot{V}_2$ is zero, which means $e_1, e_2, \bar{e}_3$ asymptotically converge to zero. Consequently, inferred from (9), the limitation e_3 converges to is also zero. Then all the tracking errors in the closed-loop system are guaranteed to converge to zero asymptotically.

Step 2: In this step, the convergence of slipping ratio estimations to their true values will be proved.

Consider the second-order derivative of e_3.

$$\begin{aligned} \ddot{e}_3 = &\dot{\omega}_r - \dot{\omega}_d + \frac{1}{2}R\left(\frac{\dot{\hat{a}}_1}{a_1} - \frac{\dot{\hat{a}}_2}{a_2}\right)v_d - \frac{1}{2}R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)\dot{v}_d \\ &-\frac{1}{2}\left(\frac{\dot{\hat{a}}_1}{a_1} + \frac{\dot{\hat{a}}_2}{a_2}\right)\omega_d + \frac{1}{2}\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right)\dot{\omega}_d. \end{aligned} \tag{28}$$

By use of Barbalat's lemma repeatedly, the limits of $\dot{e}_1$ and $\dot{e}_3$ is derived as

$$\begin{aligned} \lim_{t\to\infty} \dot{e}_1 &= \lim_{t\to\infty}\left[\frac{1}{2}\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right)v_r - \frac{1}{2}R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)\omega_r\right] = 0 \\ \lim_{t\to\infty} \dot{e}_3 &= \lim_{t\to\infty}\left[-\frac{1}{2}R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)v_r + \frac{1}{2}\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right)\omega_r\right] = 0 \end{aligned}. \tag{29}$$

Noted that in the calculation of (28), (29) is used.

$$\lim_{t\to\infty} v_d = v_r, \quad \lim_{t\to\infty} \omega_d = \omega_r. \tag{30}$$

From (29), following results can be derived.

$$\begin{aligned} &\lim_{t\to\infty}\left[\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)^2 - R^2\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right)^2\right]v_r = 0 \\ &\lim_{t\to\infty}\left[\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)^2 - R^2\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right)^2\right]\omega_r = 0 \end{aligned}. \tag{31}$$

Thus

$$\lim_{t\to\infty}\left[\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)^2 - R^2\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right)^2\right]\left(v_r^2 + \omega_r^2\right) = 0. \tag{32}$$

Consider the assumption 1, and following results can be acquired.

$$\lim_{t\to\infty}\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right) = \lim_{t\to\infty} R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right), \tag{33}$$

or

$$\lim_{t\to\infty}\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right) = \lim_{t\to\infty} R\left(-\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right). \tag{34}$$

When (33) satisfied, take (33) into (29), then

$$\lim_{t\to\infty} \dot{e}_1 = \lim_{t\to\infty}\left[R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)v_r - R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right)\omega_r\right] = 0. \tag{35}$$

If $v_r \ne \omega_r$, then

$$\begin{aligned} &\lim_{t\to\infty}\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right) = 0 \\ &\lim_{t\to\infty}\left(\frac{\tilde{a}_1}{a_1} + \frac{\tilde{a}_2}{a_2}\right) = \lim_{t\to\infty} R\left(\frac{\tilde{a}_1}{a_1} - \frac{\tilde{a}_2}{a_2}\right) = 0 \end{aligned}. \tag{36}$$

Thus,

$$\lim_{t\to\infty}\frac{\tilde{a}_1}{a_1}=0,\quad \lim_{t\to\infty}\hat{a}_1=\lim_{t\to\infty}a_1$$
$$\lim_{t\to\infty}\frac{\tilde{a}_2}{a_2}=0,\quad \lim_{t\to\infty}\hat{a}_2=\lim_{t\to\infty}a_2 . \tag{37}$$

When (34) satisfied, take (34) into (29). Then similar conclusion can be drawn: If $v_r \neq -\omega_r$, then (37) can also be acquired, which means the slipping ratio estimations converge to their true values.

Now, the whole proof of theorem 1 is finished.

IV. SIMULATION RESULTS

In this section, performance of the proposed adaptive controller in section III is verified. The trajectory is chosen as $x_r = 0.5\sin 0.6t\cos 0.2t$, $y_r = 0.5\sin 0.6t\sin 0.2t$. The radius of the mobile robot is $r = 0.035[m]$. The distance between two wheels is $2R = 0.235[m]$. Parameters of the proposed controller are chosen as

$$k_1 = 0.9,\ k_3 = 1,\ k = 0.9,\ \lambda = 0.1,\ \gamma_1 = \gamma_2 = 30 . \tag{38}$$

The slipping ratio in corresponding to left wheel and right wheel are chosen as in (49). The black dash-dotted line in Fig. 2 and Fig. 3 show their changes with time. The initial estimations of slipping ratio is $\hat{a}_1 = 1$, $\hat{a}_2 = 1$.

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{cases} [1.5,\quad 1.25]^T, & 0 \le t < 5\pi s \\ [1.25,\quad 1.5]^T, & 5\pi \le t < 10\pi s . \\ [1,\quad 1\ ]^T, & 10\pi \le t \le 15\pi s \end{cases} \tag{39}$$

Reference robot starts from $q_{r0} = [0, 0, 0]^T$, and the actual robot starts from $q_0 = [-0.1, -0.1, \pi/6]^T$.

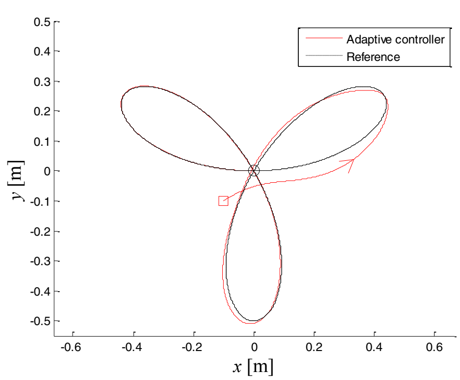

Figure 1. Trcaking trajectory (first loop)

The whole simulation can be divided into 3 segments and Fig. 1, Fig. 2, and Fig. 3 show the tracking error in (X,Y) plane corresponds to each segment respectively. In the first loop, both slippage and initial position errors affect robot's state. In the second loop, only slippage exists while initial positon errors become zero. In the third loop, slippage disappears and robot starts from the same posture with reference robot. Fig. 4-5 show the evolution of slipping ratio estimation and Fig. 6-7 show the tracking errors in details.

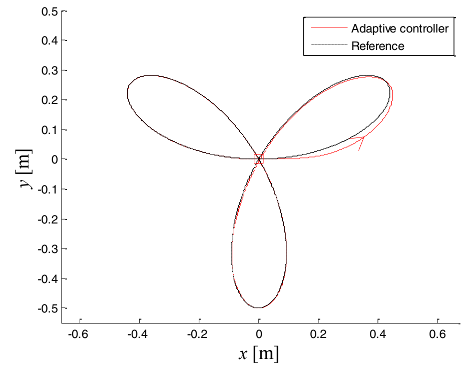

Figure 2. Trcaking trajectory (second loop)

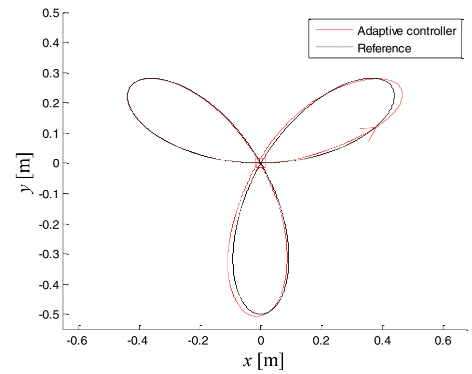

Figure 3. Trcaking trajectory (third loop)

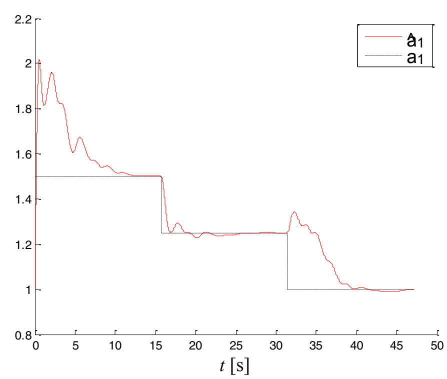

Figure 4. Left wheel slipping ratio estimation

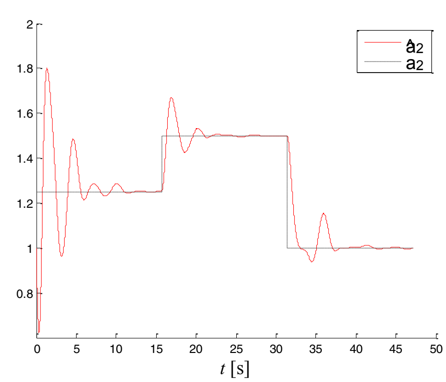

Figure 5. Right wheel slipping ratio estimation

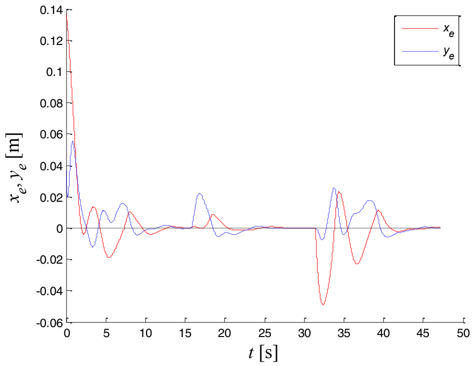

Figure 6. Position tracking errors in local coordinates

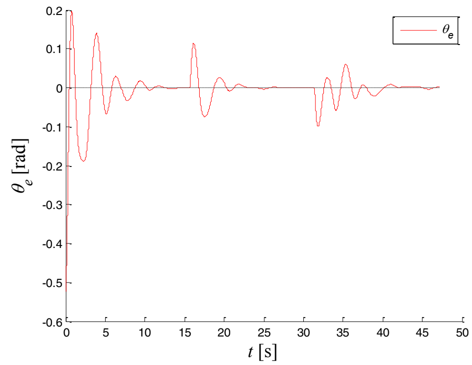

Figure 7. Heading angle tracking error

V. CONCLUSION

Trajectory tracking control of unicycle-type mobile robots in presence of longitudinal slippage is considered in this paper. Firstly, without consideration of slipping, virtual controllers are proposed. Then "Slipping ratio" is introduced into the steering system and adaptive controllers are designed based on the virtual controllers. Under the assumptions 1, 2, and the limitations to reference trajectory, all the tracking errors are guaranteed to converge to zero asymptotically with the slipping ratio estimations converge to their true values by using the controllers proposed in this paper. Finally, simulation results show the stability and effectiveness of the proposed controller. To validate the performance of the proposed controller in practice, experiments based on real wheeled mobile robot will be the future work.

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