Joint 48th IEEE Conference on Decision and Control and
28th Chinese Control Conference
Shanghai, P.R. China, December 16-18, 2009

Leader-following Formation Control of Multiple Vision-based Autonomous Robotic Fish

Wei Zhao, Yonghui Hu, and Long Wang

Abstract— **A framework for the deployment of multiple autonomous robotic fish to achieve leader-following formations with Bezier trajectory is presented. Each follower robot estimates the position and orientation angle of its leader with a fast color-tracking vision system, and establishes a Bezier trajectory between its current position and the position of its leader robot. Considering the nonholonomic properties of the robotic fish, the optimization of Bezier curve's curvature to choose appropriate scale factor is conducted by combination of penalty function and one-dimensional search methods to perform a smooth and stable trajectory. The optimal trajectories are accurate enough to estimate the angular velocity of the follower robot, while a fuzzy controller is used to adjust its linear velocity. By the introduction of virtual leaders, formations of different shapes can be generated. Experimental results show the effectiveness of our approach.**

I. INTRODUCTION

Since cooperative multi-robot systems possess many potential advantages over a single robot including higher flexibility, adaptability and robustness, coordinated control of multiple mobile robots has received much attention from the research community due to its wide real-world applications, such as exploring, surveillance, search and rescue, transporting large objects and capturing a prey. By far, most successful related work in coordinated control has been obtained in the terrestrial and aerial fields, while for the underwater case, results are relatively few.

Observations show that fish in nature exhibit group behavior. During schooling behavior of fish, group intelligence emerges in foraging for foods, avoiding predators, cruising, and so on [1]. Based on the previous research about the hydrodynamic mechanism of fish-like swimming, and fish-like vehicles design and motion control [2], [3], coordinated control of multiple robotic fish has recently been an important research topic and attracted considerable attention with applications in the tasks such as underwater cooperative transportation of object [4], coordinated collision and obstacle avoidance [5], and formation control [6]. Among all the topics of study in multi-robot cooperation, we focus on formation control, which means to control a group of robotic fish to move into and keep specified geometrical shapes, as well as to switch between these formation shapes.

Approaches to modeling and solving formation control problems of multiple robots have been categorized as leader-following schemes [6], behavior-based methods [7], and virtual structure techniques [8]. For underwater environment, [6] shows how a group of robotic fish achieve formations based on a global vision positioning and centralized decision-making system. Our platform of interest is an autonomous robotic fish with a single onboard camera. The autonomous robotic fish needs to perceive its environment, make decisions about selection of its behaviors, and finally carry out its behaviors. Our objective is to achieve global-level formation coordination for a group of autonomous robotic fish using only local sensing.

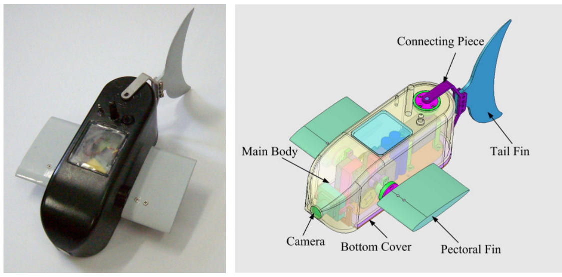
In this paper, we present a framework for the deployment of multiple vision-based autonomous robotic fish to achieve predetermined formations with a leader-following approach. For formation control, a Bezier curve between leader and follower robot, which is tangent to current paths of both leader and follower robots, is introduced based on the pose estimation of the leader with a fast color-tracking vision system of the follower. The contributions of this paper lie in the following. 1) This Bezier curve is optimized by minimizing the curvature's changing. Using the penalty function method, the problem of nonlinear constrained optimization is converted into unconstrained, and one-dimensional search is adopted to solve this unconstrained optimization problem. Based on optimized scale factor, the optimal Bezier curves allow the robotic fish to perform a smooth and stable trajectory, satisfying the nonholonomic constraint of the robotic fish. The desired angular velocity of the follower robot can be obtained with the optimal Bezier trajectory. 2) As most successful related work in formation control has been done with the terrestrial and aerial robots, we apply this approach to the field of underwater robots and it is validated with physical robot experiments. Formation control of multiple underwater robots will help to achieve many complex tasks such as underwater transportation, ocean sampling network, and so on.

The rest of the paper is organized as follows. In Section II, the autonomous robotic fish prototype is introduced by descrbing its mechatronic design and locomotion control. In Section III, as the key point of our algorithm, the Bezier-trajectory-based approach is introduced in leader-following fomation control of multiple robotic fish. Experimental results are given in Section IV. Finally, Section V concludes the paper.

This work was supported by National 863 Program (2006AA04Z258), 11-5 project (A2120061303), NSFC (60674050 and 60528007), and National 973 Program (2002CB312200).

The authors are with Intelligent Control Laboratory, Department of Mechanics and Space Technologies, College of Engineering, Peking University, Beijing 100871, China. email:zhaoweijune@gmail.com.

978-1-4244-3872-3/09/$25.00 ©2009 IEEE



(a) Prototype (b) Mechanical configuration

Fig. 1. Prototype and mechanical configuration of autonomous robotic fish.

II. PROTOTYPE OF AUTONOMOUS ROBOTIC FISH

A. Mechatronic Design

The robotic fish is composed of several elements: a rigid main body, a tail fin, and two pectoral fins. A CMOS camera is installed at the mouth position with a transparent window glued to the hull for waterproof purpose. Fig. 1(a) shows the photograph of the robotic fish prototype and its mechanical configuration is illustrated in Fig. 1(b).

The robotic fish is designed for autonomous operation such that it is equipped with onboard power, embedded processor, image sensor, and a duplex wireless communication module. The embedded microcontroller S3C2440 captures image data in YCbCr 4:2:2 format from onboard camera, does real-time image processing and decision-making, and generates three PWM signals to control the movements of the joints.

B. Locomotion Control

The robotic fish swims by oscillatory movements of the tail and pectoral fins. Since sinusoidal signals can generate smooth oscillations and allow flexible and easy adjustment of joint angles, we model the swimming locomotion as sinusoidal variation of the robot's joint angles. Each joint of the robotic fish oscillates in a harmonic manner according to the following equation:

$$\theta_i(t) = \bar{\theta}_i + A_i \sin(2\pi f_i t + \phi_i) \qquad (i = 1, 2, 3) \tag{1}$$

where $\theta_i(t)$ is the angular position of the i-th joint at time t, $\bar{\theta}_i$ denotes the angular offset, A_i represents the oscillatory amplitude of the joint angle, and f_i indicates the frequency. The swimming speed of the robotic fish can be adjusted by modulating the value of the frequency f and the amplitude A. The angular offset $\bar{\theta}$ can be used as a strategy for maneuvering and three-dimensional swimming of the robotic fish.

III. LEADER-FOLLOWING FORMATION WITH BEZIER TRAJECTORIES

With leader-following method, each robotic fish takes another neighboring fish as a reference point to determine its motion. The referenced robotic fish is called a leader, and the fish following it called a follower. In a group of robots, there are many pairs of leaders and followers and complex formations can be achieved by controlling relative positions of these pairs of robots respectively. This approach has been adopted widely in formation control by the characteristics of simplicity, reliability and no need for global knowledge and computation.

To achieve leader-following formations, a Bezier trajectory between leader and follower robotic fish is introduced for the deployment of multiple autonomous robots. Each follower fish estimates the position and orientation angle of its leader with a real-time color-tracking vision system, and builds a Bezier curve that generates the trajectory from its current position to the position of its leader. The Bezier-trajectory-based approach can be extended to more follower robots for keeping a column formation. By the introduction of virtual leaders, formations of different shapes can also be generated.

As illustrated in Fig. 2, only the position (x_l, y_l) and the orientation angle α of the leader robot need to be estimated with respect to the follower robot by its vision module. The corresponding vision processing for pose estimation is described as follows.

A. Pose Estimation

The leader robotic fish is marked with specified color pattern and its pose estimation is based on the tracking of its attached color regions. Hence the vision processing is based on color information. The underwater images from the camera are digitized in YCbCr color space, and color thresholds that were learned by sampling offline are then applied to the images. Inspired by [9], fast image segmentation, color pattern recognition and localization are processed on the underwater images captured from the onboard camera of the follower robotic fish.

The color pattern attached to the leader robot consists of a red rectangle and a golden rectangle with the same size. The red rectangle provides an estimate to the position based on its lower-right corner. The difference between the perceived heights of the red and golden rectangles provides an estimate of the orientation angle of the pattern with respect to the observing follower robot.

1) Color-based Image Segmentation: Without regard to the luminance component Y that changes dramatically with ambient light, a 2-D threshold table is used to perform the mapping from CbCr pixel values to symbolic color class.

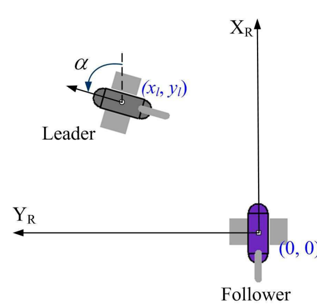


Fig. 2. Position (x_l, y_l) and orientation angle α of the leader robotic fish in the follower's frame of reference (X_R, Y_R).

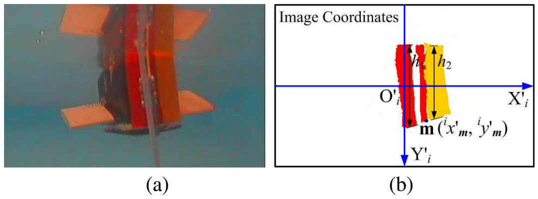


Fig. 3. Color-pattern recognition for follower robotic fish. (a) Sample underwater image. (b) Processed image.

2) Pattern Detection and Feature Extraction: The color pattern, including the red rectangle and the golden rectangle, needs to be recognized. We detect these color regions with several filters according to their characteristics. And the interested information of the regions in the image can be extracted. Fig. 3 shows the result of the color-pattern recognition.

The X_i'-axis and Y_i'-axis of image coordinate system are illustrated in Fig. 3(b). The origin of the image coordinate system is the center of the image. Once the color pattern has been detected, we extract the coordinates of the lower-right corner $\mathbf{m}$ of the red region which is closest to the golden region in the image for its localization. The height h_1 of the red rectangle and h_2 of the golden rectangle in the image are also extracted for an estimate of the orientation angle. As shown in Fig. 3(b), all the interested information, including the perceived heights h_1, h_2, and the coordinates $({}^i x'_{\mathbf{m}}, {}^i y'_{\mathbf{m}})$ of point $\mathbf{m}$, can be obtained by searching the color-pattern region.

3) Localization of Leader Robot: In localization, we use the 2-D regions in the camera image to estimate the position of the leader robotic fish in the 3-D coordinate system of the observing follower robotic fish. With the help of the intrinsic camera parameters, image coordinates can be converted into robot coordinates. The robot coordinate system (X_R, Y_R, Z_R) is fixed to the observing follower robotic fish, and the camera coordinate system (X_C, Y_C, Z_C) is attached to the camera of the follower robotic fish. Fig. 4 shows the relation of the robot coordinate system and the camera coordinate system. Notice that Δl denotes the distance between the origin of the robot coordinate system and the origin of the camera coordinate system in the fish body direction, and the mounted camera has a wide viewing angle of 120°.

In order to estimate the location of the leader, we detect the lower-right corner $\mathbf{m}$ of the red region of its color pattern in the corresponding processed image [see Fig. 3(b)]. Since all the robotic fish are slightly buoyant, the vertical distance h from the observing camera to the lowest point of the color pattern is fixed. This point $\mathbf{m}$ and the optical center of the camera define a viewing ray $\mathbf{rm}$. Its intersection with the lowest horizontal plane of the color pattern attached to the leader yields an estimate of the position of the pattern relative to the observing follower, as illustrated in Fig. 4. The 3-D point $\mathbf{M}$ in camera coordinate system corresponding to the observed image point $\mathbf{m} = [{}^i x'_{\mathbf{m}}, {}^i y'_{\mathbf{m}}]^T$ in Fig. 3(b) is given by

$$\begin{aligned}\mathbf{M_C} &= [{}^C x_{\mathbf{M}}, {}^C y_{\mathbf{M}}, {}^C z_{\mathbf{M}}]^T \\ &= \frac{h}{{}^i y'_{\mathbf{m}}}[{}^i x'_{\mathbf{m}}, {}^i y'_{\mathbf{m}}, f]^T\end{aligned} \tag{2}$$

where f denotes the focal length of the camera. Since the pose of the robot's camera is given in robot coordinate system, the point $\mathbf{M}$ given in camera coordinate system can easily be expressed in the coordinate system of the follower robot as follows

$$\begin{aligned}\mathbf{M_R} &= [{}^R x_{\mathbf{M}}, {}^R y_{\mathbf{M}}, {}^R z_{\mathbf{M}}]^T \\ &= [{}^C z_{\mathbf{M}} + \Delta l, -{}^C x_{\mathbf{M}}, -{}^C y_{\mathbf{M}}]^T\end{aligned} \tag{3}$$

Then the position (x_l, y_l) of the leader robot with respect to the follower robot can be estimated by

$$\begin{aligned}x_l &= {}^R x_{\mathbf{M}} \\ y_l &= {}^R y_{\mathbf{M}}\end{aligned} \tag{4}$$

B. Formation Control with Bezier Trajectory

Defining a Bezier curve between the leader and follower robots is of key importance [10]. As shown in Fig. 5, one of such curves is defined by four points $P_0 - P_3$. The endpoints P_0 and P_3 are determined by the positions of the follower and leader robots respectively, while the control points P_1 and P_2 are chosen along the lines defined by the orientations of the robots. This cubic Bezier curve is given by (5).

$$P(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3 \tag{5}$$

where $t \in [0,1]$ is the curve parameter, and the four points $P_0 - P_3$ are all defined in the 2-D coordinate system (X_R, Y_R) of the follower robot

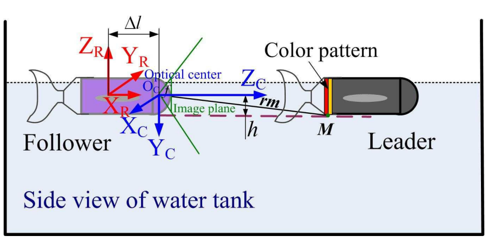


Fig. 4. Estimate of the position of leader robot relative to the follower.

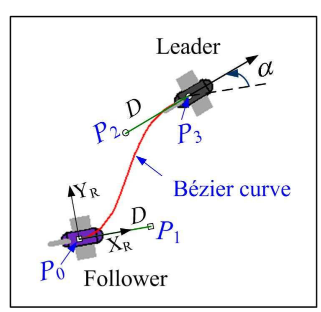


Fig. 5. Definition of Bezier trajectory between leader and follower robots.

$$P_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ P_1 = \begin{bmatrix} D \\ 0 \end{bmatrix},\ P_2 = \begin{bmatrix} x_l - D\cos\alpha \\ y_l - D\sin\alpha \end{bmatrix},\ P_3 = \begin{bmatrix} x_l \\ y_l \end{bmatrix} \tag{6}$$

The values of x_l, y_l, and α are obtained from the vision module described above. D is the distance between P_0 and P_1, also the distance between P_2 and P_3.

Then the expressions of $x(t)$ and $y(t)$ are the following:

$$\begin{cases} x(t) = (3D\cos\alpha - 2x_l + 3D)t^3 + 3(-D\cos\alpha \\ \qquad\qquad + x_l - 2D)t^2 + 3Dt \\ y(t) = (3D\sin\alpha - 2y_l)t^3 + 3(-D\sin\alpha + y_l)t^2 \end{cases} \tag{7}$$

As mentioned above, the well-established Bezier trajectory states two interesting properties: 1) the curve passes through the two endpoints, and 2) the curve is tangent to the vectors $\overrightarrow{P_0P_1}$ and $\overrightarrow{P_2P_3}$ at the endpoints.

1) Orientation Control: To achieve a column formation, the angular velocity of the follower ω_f must correspond to the curvature of the Bezier trajectory at P_0 $(t = 0)$ each time step.

According to the curvature formula of curve parametric equation, on the Bezier curve, the curvature at any point is calculated by

$$\kappa(t) = \frac{|x'(t)y''(t) - y'(t)x''(t)|}{[x'(t)^2 + y'(t)^2]^{3/2}} \tag{8}$$

The first-derivative and second derivative of equation (7) are substituted into (8), let $t = 0$, then the curvature of this curve at P_0 is computed by

$$\kappa(0) = \frac{2(y_l - D\sin\alpha)}{3D^2} \tag{9}$$

So the desired angular velocity of the follower can be calculated by

$$\omega_f = \frac{V_f}{R} = V_f\kappa(0) = \frac{2V_f(y_l - D\sin\alpha)}{3D^2} \tag{10}$$

where the linear velocity of the follower robotic fish V_f can be obtained using fuzzy controller described later.

Equation (10) accords with the nonholonomic constraint. In this method, although the value of D can be set arbitrarily, it does have an effect to define the control points which should be spaced up to a value of D proportional to the distance between the follower and leader robots for invariance to scale. Here for both pairs of endpoint and control point, we set $D = u|P_3 - P_0| = u\sqrt{x_l^2 + y_l^2}$. The scale factor u needs to be regulated, so the optimization problem of Bezier curve's curvature is presented.

Considering the nonholonomic properties and the inherent kinematic constraints of the robotic fish, it needs smooth and stable trajectory, which indicates that the evaluation criterion can be chosen as minimizing the square sum of curvature variation by searching appropriate scale factor u. The evaluation function is set as

$$f(u) = \int \left(\frac{d\kappa(s)}{ds}\right)^2 ds \tag{11}$$

Algorithm 1 Penalty function methods for optimization problem with inequality constraints

Require: $u_{0,0}, \varepsilon > 0, M_0 > 0, c \in [4, 10], k = 0$ {ε is permissible error}

repeat
  $k = k + 1$
  $M_k = cM_{k-1}$
  $F(u, M_k) = f(u) + M_k(\min\{0, u\})^2 + M_k(\min\{0, 1 - u\})^2$
  if $k > 0$ **then**
    $u_{k,0} = u^*_{k-1}$ {$u_{k,0}$ is the starting point for the minimization of $F(u, M_k)$}
  end if
  compute u^*_k {This step involves a complete unconstrained minimization by one-dimensional search}
until $k > 0$ *and* $|u_k - u_{k-1}| < \varepsilon$

where s is the arc length.

Hence, the problem is transformed to be the expression of nonlinear constrained optimization problem as follows

$$\begin{array}{l} \min f(u) \quad u \in R \\ s.t. \quad 0 < u < 1 \end{array} \tag{12}$$

Combining the penalty function and the original objective function [11], a new objective function without any constrained conditions is obtained

$$F(u, M_k) = f(u) + M_k(\min\{0, u\})^2 + M_k(\min\{0, 1 - u\})^2 \tag{13}$$

where M_k is penalty factor, $M_k > 0$, $M_1 < M_2 < \cdots < M_k < M_{k+1} < \cdots$, and $\lim\limits_{k\to\infty} M_k = +\infty$. According to the practical experience, we choose $M_{k+1} = cM_k, c \in [4, 10]$.

Since the constrained optimization problem (12) can be changed into the unconstrained optimization problem (13), one-dimensional search is used to solve the optimization problem without any constraints. Algorithm 1 gives details of the penalty function approach [12]. Finally, the optimal solution $u^*_k \approx 0.4$ when the iteration is stopped. Therefore we choose $D = 0.4\sqrt{x_l^2 + y_l^2}$.

Simulation results are given to confirm the validity of the optimization algorithm. The Bezier curves with different

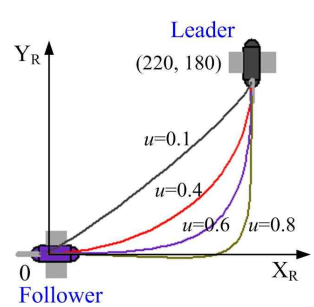


Fig. 6. Bezier curves with different values of the scale factor u.

values of the scale factor u are obtained for a same pair of leader and follower robots, and their curvatures have been optimized and non-optimized, as shown in Fig. 6. This indicates that the optimization process can make the robotic fish perform a smooth and stable trajectory.

In our experiments, the angular velocity of the follower robot is obtained from the optimal Bezier trajectory and updated at each image processing. When the new position and orientation angle of the leader are estimated, the optimized Bezier points are re-defined and the optimal curve is re-drawn, then the angular velocity is recomputed accordingly.

2) Speed Control: To keep formations, the following robotic fish adjusts its speed and tries to maintain a desired distance to its leader. In order to provide a scientific mechanism for reasoning and decision making with uncertain and imprecise information, fuzzy logic controller (FLC) is adopted to adjust the linear velocity of the follower robotic fish V_f. The leader robotic fish is assumed to follow an arbitrary trajectory. The linear velocity of the leader robotic fish V_l is known and can be regarded as an exogenous input of the follower by communications. Let $Dist$ be the distance between the leader and the follower robotic fish, which is obtained by $\sqrt{x_l^2+y_l^2}$. Let D_{set} denote the desired distance between them.

To let the follower robotic fish swim in the expected position relative to its leader, the linear velocity of the follower robotic fish V_f can be computed according to the values including the distance error D_e between the real distance to its leader $Dist$ and the desired distance D_{set}, and the linear velocity of its leader robot V_l, through a set of fuzzy logic rules. Here $V>0$ means forward swimming of the robotic fish while $V<0$ means backward swimming. The inputs of the fuzzy logic rules are D_e and V_l, and the output is V_f. The block diagram of the fuzzy controller is shown in Fig. 7.

The next step in FLC design is to represent the variables by linguistic terms. Let LD_e , LV_l, and LV_f denote fuzzy variable sets associated with linguistic variables D_e, V_l, and V_f respectively. Firstly, LD_e and LV_l are represented by the linguistic fuzzy sets $\{NB,NS,ZE,PS,PB\}$, abbreviated from *negative big*, *negative small*, *zero*, *positive small*, *positive big*, respectively, with the membership functions shown in Fig. 8(a). LV_f is represented by $\{NB,NS,ZE,PS,PB\}$, abbreviated from *negative big*, *negative small*, *zero*, *positive small*, *positive big*, respectively, with the membership function shown in Fig. 8(b). The parameters in the membership functions can be derived and tuned through the experiments.

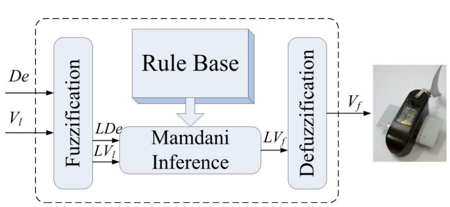


Fig. 7. Fuzzy logic controller.

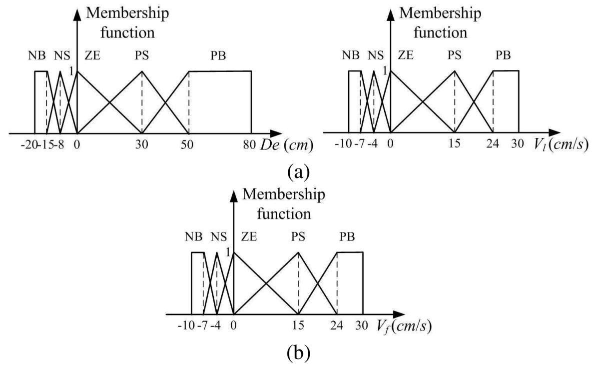


Fig. 8. Membership functions. (a) Membership functions of D_e, V_l. (b) Membership function of V_f.

TABLE I

RULE TABLE FOR SPEED CONTROL OF THE ROBOTIC FISH.

V_f		V_l				
		NB	NS	ZE	PS	PB
D_e	NB	NB	NB	NB	NS	ZE
	NS	NB	NB	NS	ZE	PS
	ZE	NB	NS	ZE	PS	PB
	PS	NS	ZE	PS	PB	PB
	PB	ZE	PS	PB	PB	PB

Based on the experimental experience, a fuzzy rule table which involves the fuzzy rules of the inference engine is designed. The size of rule table is completely dependent on the number of input fuzzy sets of the system. IF-THEN rule is adopted and a 2-D (5×5) rule table shown in Table I is built in our fuzzy control. V_f can be derived using the intuitive rule sets in this table.

Mamdani-type inference is employed here for speed control. V_f is calculated by Mamdani inference with min for intersection and max for union.

At the defuzzification step, the final linear velocity of the follower robotic fish V_f is obtained using the center-of-gravity (Centroid) defuzzification method.

3) Extension to Other Formations: The establishing of Bezier trajectory between leader and follower robots can achieve a column formation directly. To keep specified geometrical formations, the following robotic fish tries to maintain a desired distance and desired angle relative to its leader. Forming arbitrary shapes can be realized by the introduction of virtual leaders. These virtual leaders do not

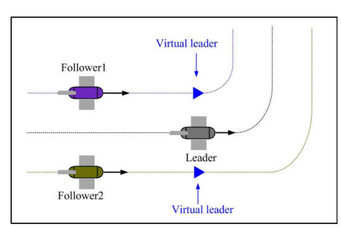
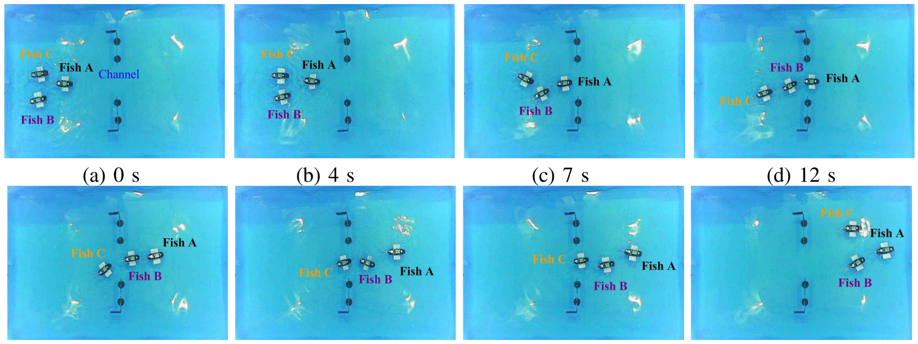


Fig. 9. Triangular formation with virtual leaders.



(a) 0 s (b) 4 s (c) 7 s (d) 12 s

(e) 15 s (f) 17 s (g) 19 s (h) 24 s

Fig. 10. Scenarios: Three robotic fish switch from a triangle to a column and then back to a triangle during passing a channel.

correspond to the exact position of the leader robots, but with an added displacement. For instance, Fig. 9 illustrates a triangular formation where the reference point for the follower robots is not the leader robot but a point with a given offset on its left and right side respectively. Since the position and the orientation angle of the leader robot can be estimated, it is easy to obtain that information of the virtual leaders.

IV. EXPERIMENTAL RESULTS

Experiments with the autonomous robotic fish were carried out in an indoor swimming tank with the size of 3 m$\times$2 m and with still water of 0.25 m in depth. In our experiment, we tested the performance in formation control of three autonomous robotic fish during passing a channel which located at the center of the swimming tank, as shown in Fig. 10(a). Fish A was marked with a color pattern consisting of a red rectangle and a golden rectangle, while fish B was specified with a color pattern of pink and green rectangles.

The three robotic fish, starting from one side of the swimming tank, aimed to converge to a triangular formation, then switched from the triangle to a column passing through the channel, and back again to a triangle after passing. Initially, fish A was designated as the leader of both fish B and C. They first performed a triangular formation. Then, when fish A passed the channel, the three robotic fish were ordered to switch to a column formation so that they could pass through the channel orderly. As a result, fish C broke the relation with fish A and took fish B as the new leader. As the view field of the camera was limited, fish C turned to search for its new leader fish B. Finally, after each fish passed the channel successfully, the robotic fish were ordered again to re-establish the triangular formation. Fig. 10 shows the scenarios of the experiment.

V. CONCLUSIONS

We have presented an effective method for leader-following formation control of multiple vision-based autonomous robotic fish with Bezier trajectory. According to the visual data from the follower's camera, the pose of the leader robot could be estimated with respect to the follower. Then a Bezier curve that describes the trajectory between the current positions of a follower robot and its leader was built and optimized using penalty function method and one-dimensional search. The optimized Bezier curves allowed the follower robotic fish to compute the desired angular velocity for a smooth and stable trajectory, which was compatible with the nonholonomic constraint of the robotic fish. Due to the complexity of the underwater working environment, we employed the fuzzy logic method to control the linear velocity of the follower robotic fish. Experimental results demonstrated the effectiveness of our approach.

REFERENCES

[1] K. Hattori, Y. Narita, Y. Kashimori, and T. Kambara, "Self-organized critical behaviour of fish school and emergence of group intelligence", in *Int. Conf. Neural Information Processing*, 1999, pp. 465-470.

[2] W. Zhao, J. Yu, Y. Fang, and L. Wang, "Development of multi-mode biomimetic robotic fish based on central pattern generator", in *Proc. IEEE/RSJ Int. Conf. Intelligent Rob. and Sys.*, Oct. 2006, pp. 3891-3896.

[3] K.A. Morgansen, B.I. Triplett, and D.J. Klein, "Geometric methods for modeling and control of free-swimming fin-actuated underwater vehicles", *IEEE Trans. Robot.*, vol. 23, no. 6, 2007, pp. 1184-1199.

[4] D. Zhang, L. Wang, J. Yu, M. Tan, "Coordinated transport by multiple biomimetic robotic fish in underwater environment", *IEEE Trans. Control System Technology*, vol. 15, no. 4, 2007, pp. 658-671.

[5] D. Zhang, L. Wang, G. Xie, and W. Zhang, "Coordinated collision avoidance of multiple biomimetic robotic fish", *Lecture Notes in Artificial Intelligence*, Springer-Verlag, Berlin Heidelberg, 2005, pp. 215-224.

[6] J. Shao, J. Yu and L. Wang, "Formation control of multiple biomimetic robotic fish", in *Proc. IEEE/RSJ Int. Conf. Intelligent Rob. and Sys.*, 2006, pp. 2503-2508.

[7] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot team", *IEEE Trans. Robot. Automat.*, vol. 14, no. 6, 1998, pp. 926-939.

[8] K. H. Tan and M. A. Lewis, "Virtual structures for high precision cooperative mobile robot control", *Auton. Robots*, vol. 4, 1997, pp. 387-403.

[9] T. Bandlow, M. Klupsch, R. Hanek, and T. Schmitt, "Fast image segmentation, object recognition and localization in a RoboCup scenario", in *3. RoboCup Workshop*, *IJCAI* 99, 1999, pp. 174-185.

[10] H. Prautzsch, W. Boehm, M. Paluszny, "Bezier and B-Spline techniques", Springer-Verlag, 2002.

[11] H. W. Tang, X. Z. Qin, "Practical optimization method", 3rded., Da Lian University of Technology Press, 2004.

[12] L. E. Scales, "Introduction to non-linear optimization", New York: Springer-Verlag, 1985.