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Visual Servoing Feedback based Robust Regulation of Nonholonomic Wheeled Mobile Robots

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Abstract— **This paper investigated the visual servoing regulation of nonholonomic mobile robots with monocular camera. Nonholonomic kinematic systems with visual feedback are uncertain and more involved in comparison with common kinematic systems. Two-phase technique was used to present a robust controller that enabled the mobile robot image pose and the orientation regulation despite the lack of depth information and the lack of precise visual parameters. The most interesting feature of this paper is that the problem was discussed in the image frame and the inertial frame, which made the problem easy and useful. The stabilization of the system by using the proposed method was rigorously proved. The simulation was given to show the effectiveness of the presented controllers.**

Index Terms— **Regulation, nonholonomic mobile robot, visual servoing**

I. Introduction

A mobile robot is one of the well-known systems with nonholonomic constraints[11][12]. By the theorem of R. Brockett(1983)[13], a nonholonomic system cannot be stabilized at a single equilibrium point by a smooth pure state feedback controller. To solve this problem, lots of methods have been considered, such as chained form methods[14][15], tracking control[16] and discontinuous feedback control [17] etc. In the control of nonholonomic mobile robots, it is usually assumed that the robot states are available and exactly reconstructed using proprioceptive and exteroceptive sensor measurements. But in practical mobile robot applications, there are several ideal conditions that can not be satisfied, such as uncertainties in the kinematic model, mechanical limitations, noise and so on. The estimation of the robot state from sensor measurements is much affected by these perturbations.

Visual feedback is an important approach to improve the control performance of manipulators since it mimics the human sense of vision and allows for operating on the basis of noncontact measurement and unconstructure of the environment. Since the late 1980s, tremendous effort has been made to visual servoing and vision-based manipulations[18].

The nonholonomic control problem will become more involved because of the visual feedback. Designing the feedback at the sensor level increases system performances especially when uncertainties and disturbances affect the robot model and the camera calibration, see [19] and therein references.

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Based on the success of image extraction/interpretation technology and advances in control theory, research has focused on the use of monocular camera-based vision systems for navigating a mobile robot [4][8][10]. A significant issue with monocular camera-based vision systems is the lack of depth information. From a review of literature, various approaches have been developed to address the lack of depth information inherent in monocular vision systems. For example, using consecutive image frames and an object database, Kim et al. [5] recently proposed a mobile robot tracking controller based on a monocular visual feedback strategy. To achieve their result, they linearized the system equations using a Taylor series approximation, and then applied extended Kalman filtering (EKF) techniques to compensate for the lack of depth information [5]. Dixon et al. [2] used feedback from an uncalibrated, fixed (ceiling-mounted) camera to develop an adaptive tracking controller for a mobile robot that compensated for the parametric uncertainty in the camera and the mobile robot dynamics. Dixon et al. exploit Lyapunov-based adaptive techniques to compensate for the unknown depth information [2]. However, to employ these techniques, they require the depth from the camera to the mobile robot plane of motion to remain constant (i.e., the camera plane and the mobile robot plane must be parallel). This assumption reduces the nonlinear pinhole camera model to a decoupled linear transformation; however, it also restricts the applicability of the controller. Recently, Chen et al. [1] developed a mobile robot visual servo tracking controller when the camera is onboard. An advantage of the result in [1] is that the mobile robot is not constrained to a planar application and an adaptive estimate is provided to compensate for unknown time-varying depth information. However, the development in [1] and [2] cannot be applied to solve the mobile robot regulation problem due to restrictions on the mobile robot reference velocity (i.e., the reference linear velocity cannot converge to zero). Wang et al. [9] also exploit a Lyapunov-based adaptive technique to compensate for a constant unknown depth parameter for a monocular mobile robot tracking problem. While the approach in [9] may be well suited for tracking applications, the stability analysis requires the same restrictions on the reference trajectory of the mobile robot as in [2], and hence, cannot be applied to solve the regulation problem.

The contribution of this paper is that a two-phase technique was exploited to craft a robust controller that enabled the mobile robot image pose and the orientation regulation despite the lack of depth information and the lack of precise visual parameters provided that the camera plane and the

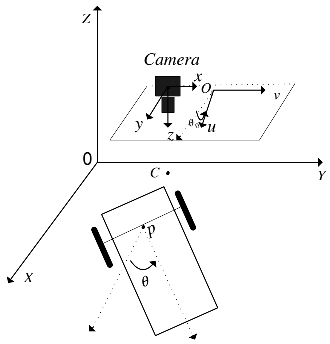

Fig. 1. Wheeled Mobile Robots with Monocular Camera

mobile robot plane were parallel. Due to assumptions on the reference trajectory resulting from the nonholonomic constraint, the aforementioned visual servo tracking control results cannot be applied to solve the regulation problem considered in the current result. See [3][6][7] for a more technically detailed description of the issues and differences associated with developing tracking and regulation controllers for nonholonomic systems. The result in this paper is achieved with a monocular vision system with uncalibrated visual parameters, and the control design approach incorporates the full nonholonomic kinematic equations of motion. Experimental results are provided to illustrate the performance of the developed controller. A practical issue with the presented research is that the feature points may leave the camera's field of view during task execution.

The paper is organized as follows. Section 2 introduces the camera-object visual model in terms of the planar optical flow equations. In Section 3, the controllers are synthesized for several cases. In Section 4, the simulation results carried out to validate the theoretical framework. Finally, in Section 5 the major contribution of the paper is summarized.

II. Problem Statement

A. System Configuration

In the Figure 1, the mobile robot is shown. Assume that a pinhole camera is fixed to the ceiling and the camera plane and the mobile robot plane are parallel. There are three coordinate frames, namely the inertial frame X-Y-Z, the camera frame x-y-z and the image frame u-O_1-v. Assume that the x-y plane of the camera frame is the identical one with the plane of the image coordinate plane. C is the crossing point between the optical axis of the camera and X-Y plane. Its coordinate relative to X-Y plane is (p_x, p_y), the coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c1}, O_{c2}), (x, y) is the coordinate of the mass center of the robot with respective to X-Y plane. Suppose that (x_m, y_m) is the coordinate of (x, y) relative to the image frame. Pinhole camera model yields

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R \left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} p_x \\ p_y \end{bmatrix} \right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \quad (1)$$

where α_1, α_2 are constant which are dependent on the depth information, focus length, scalar factors along x axis and y axis respectively.

$$R = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix} \quad (2)$$

where θ_0 denotes the angle between u axis and X axis with a positive anticlockwise orientation.

B. Problem Description

Assume that the geometric center point and the mass center point of the robot are the same. The nonholonomic constraint is defined by

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \quad (3)$$

By this formula, nonholonomic kinematic equation is written by

$$\begin{cases} \dot{x} = vcos\theta \\ \dot{y} = vsin\theta \\ \dot{\theta} = \omega \end{cases} \quad (4)$$

where v and ω denote the velocity of the heading direction of the robot and the angle velocity of the rotation of the robot, respectively.

In the image frame, the kinematic model can be deduced by (1)

$$\begin{aligned} \begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} &= \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \\ &= \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R\nu \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \\ &= \nu \begin{bmatrix} \alpha_1\cos(\theta-\theta_0) \\ \alpha_2\sin(\theta-\theta_0) \end{bmatrix} \end{aligned} \quad (5)$$

Generally, (x, y) can be obtained from the encoders of motors and other sensors such as ultrasonic sensors, infrared sensors, etc.. However, for complex environment, it is difficult to do it. But vision information can be easily exploited to deal with this problem.

In this paper, the camera is used to measure (x, y) and determine the desired target. A kind of effective method is that the error between the mass center point of the robot and its desired point in the image frame can be used in the closed-loop feedback control of the robot. As for the angle, θ, it can be obtained easily from the angle sensor. Therefore, θ is still included in the error model.

From (5), the regulation problem can be described by

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \nu\alpha_1\cos(\theta-\theta_0) \\ \nu\alpha_2\sin(\theta-\theta_0) \\ \omega \end{bmatrix} \quad (6)$$

In contrary to the general stabilizing model of nonholonomic mobile robots, three new parameters, α_1, α_2 and θ_0, are added. Suppose that the three parameters are available. Then the stabilizing problem can be reduced by the following transformation as

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha_1}\cos(\theta-\theta_0) & \frac{1}{\alpha_2}\sin(\theta-\theta_0) \\ -\frac{1}{\alpha_1}\sin(\theta-\theta_0) & \frac{1}{\alpha_2}\cos(\theta-\theta_0) \end{bmatrix} \begin{bmatrix} x_m \\ y_m \end{bmatrix} \quad (7)$$

Let

$$\left\{\begin{array}{l} u_1 = \omega \\ u_2 = -\omega z_2 + \nu \end{array}\right. \tag{8}$$

(7) can be deduced

$$\begin{aligned} \dot{z}_1 &= -\omega z_2 + \nu = u_2 \\ \dot{z}_2 &= \omega z_1 = u_1 z_1 \\ \dot{\theta} &= u_1 \end{aligned} \tag{9}$$

This is a common nonholonomic chained form system. Lots of methods[14][15][16][17] can be used to investigate it.

But when α_1, α_2 and θ_0 are unknown, the transformation (7) cannot be used for state feedback.

Considered next are controller designs for all cases about α_1, α_2 and θ_0.

III. Controller Design

In the next design of the stabilizing controller, there are two phases. First, ω and ν are designed to make x_m and y_m very small as desired in a limited time. Second, ω and ν are designed again to make θ very much as desired while the small change of x_m and y_m remains.

A. θ_0 known, α_1 and α_2 unknown

1) Suppose $\alpha_1 = \alpha_2 = \alpha$ unknown: Under this case, system (6) can be deduced

$$\left[\begin{array}{c} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{array}\right] = \left[\begin{array}{c} \nu\alpha\cos(\theta-\theta_0) \\ \nu\alpha\sin(\theta-\theta_0) \\ \omega \end{array}\right] \tag{10}$$

Substituting θ by $(\theta-\theta_0)$, it follows that

$$\left[\begin{array}{c} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{array}\right] = \left[\begin{array}{c} \nu\alpha\cos\theta \\ \nu\alpha\sin\theta \\ \omega \end{array}\right] \tag{11}$$

Set

$$\left[\begin{array}{c} z_1 \\ z_2 \end{array}\right] = \left[\begin{array}{cc} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{array}\right] \left[\begin{array}{c} x_m \\ y_m \end{array}\right] \tag{12}$$

Then (11) yields

$$\left\{\begin{array}{rcl} \dot{z}_1 &=& -\omega z_2 + \nu\alpha \\ \dot{z}_2 &=& \omega z_1 \\ \dot{\theta} &=& \omega \end{array}\right. \tag{13}$$

Choosing $\omega = -k\theta$ yields

$$\dot{\theta} = \theta(0)e^{-kt} \tag{14}$$

where $\theta(0)$ is the initial value of θ.

It is clearly seen that $\theta(t) \to 0$ as $t \to \infty$.

Let

$$y_1 = z_1, \quad y_2 = \frac{z_2}{\theta}$$

Then

$$\left\{\begin{array}{rl} \dot{y}_1 = & k\theta^2 y_2 + \nu\alpha \\ \dot{y}_2 = & -ky_1 + ky_2 \end{array}\right. \tag{15}$$

The controller can be taken as

$$\nu = k_1 y_1 + k_2 y_2 \tag{16}$$

where k_1 and k_2 are indeterminate gains. Substituting (16) into (15) yields

$$\left\{\begin{array}{l} \left[\begin{array}{c} \dot{y}_1 \\ \dot{y}_2 \end{array}\right] = \left[\begin{array}{c} k\theta^2 y_2 + \alpha k_1 y_1 + \alpha k_2 y_2 \\ -ky_1 + ky_2 \end{array}\right] \\ \qquad = (A_0 + A_1(t)) \left[\begin{array}{c} y_1 \\ y_2 \end{array}\right] \end{array}\right. \tag{17}$$

where

$$A_0 = \left[\begin{array}{cc} \alpha k_1 & \alpha k_2 \\ -k & k \end{array}\right], A_1(t) = \left[\begin{array}{cc} 0 & k\theta^2 \\ 0 & 0 \end{array}\right] \tag{18}$$

System (17) can be surely stabilized for some chosen k_1 and k_2. First, the following lemma is needed.

Lemma 1[20] Consider time-varying linear system defined by

$$\dot{x} = (B_0 + B_1(t))\,x \tag{19}$$

where $x \in R^n$ is the state vector, $B_0 \in R^{n\times n}$ is a Hurwitz matrix. For every element in $B_1 \in R^{n\times n}$, it satisfies

$$b_{ij} \to (t \to \omega)\, i,j = 1,2.... \tag{20}$$

Then system(19) is asymptotically stable.

The following text is the investigation of asymptotical stable problem of system (17).

The characteristic polynomial of A_0 is

$$|\lambda I - A_0| = \left|\begin{array}{cc} \lambda-\alpha k & -\alpha k_2 \\ k & \lambda - k \end{array}\right| = \lambda^2 - (k+\alpha k_1)\,\lambda + \alpha k k_2 \tag{21}$$

Then matrix A_0 is a Hurwitz matrix if and only if

$$k + \alpha k_1 < 0, \alpha k k_2 > 0$$

.

Assumption 1: $\alpha \ge \alpha_0 > 0$.

Let $k_1 < 0$. Then, $k+\alpha_1 k_1 \le k+\alpha_0 k_1$. The condition $k+\alpha_0 k_1 < 0$ means $k+\alpha k_1 < 0$. Hence, it is only needed that k and k_1 satisfy $k+\alpha_0 k_1 < 0$ or $k_1 < \frac{-k}{\alpha_0}$. But, by $\alpha k k_2 > 0$, we only need $k_2 > 0$. Therefore, A_0 is a $Hurwitz$ matrix if and only if

$$k_2 > 0, k_1 < -\frac{k}{\alpha_0}. \tag{22}$$

From (14), $\theta \to 0$. So $A_1(t) \to 0\,(t\to\infty)$. By using Lemma 1 again, (19) is asymptotically stable. Therefore the controller defined by

$$\omega = -k\theta, \nu = k_1 y_1 + k_2 y_2 \tag{23}$$

can guarantee that the system composed of y_1 and y_2 is asymptotically stable. Therefore, x_m and y_m can be made very small as desired in a limited time.

2) $\alpha_1 \ne \alpha_2$ and unknown case: Let

$$\left[\begin{array}{c} z_1 \\ z_2 \end{array}\right] = \left[\begin{array}{cc} \cos(\theta-\theta_0) & \sin(\theta-\theta_0) \\ \sin(\theta-\theta_0) & -cos\,(\theta-\theta_0) \end{array}\right] \left[\begin{array}{c} x_m \\ y_m \end{array}\right]. \tag{24}$$

Diffferentiating and substituting it into the system equation become to be

$$\left[\begin{array}{c} \dot{z}_1 \\ \dot{z}_2 \end{array}\right] = \omega \left[\begin{array}{c} -z_2 \\ z_1 \end{array}\right] + \nu \left[\begin{array}{c} \alpha_1 q^2 + \alpha_2 r^2 \\ (\alpha_1-\alpha_2)\,rq \end{array}\right], \tag{25}$$

where

$$\left\{\begin{array}{c}\phi=\theta-\theta_0, q=\cos\phi, r=\sin\phi,\\ \alpha_{12}=\alpha_1-\alpha_2, w=qr/\phi.\end{array}\right. \quad (26)$$

Set

$$\omega=-k\left(\theta-\theta_0\right), \quad (27)$$

where k is a positive gain.

Then $\theta-\theta_0=e^{-kt}h$ (where h is the initial value of $\theta\left(t\right)-\theta_0$).

Let

$$y_1=z_1, y_2=\frac{z_2}{\theta-\theta_0}.$$

Then

$$\left\{\begin{array}{ll}\dot{y}_1= & -\omega z_2+\nu\left(\alpha_1 q^2+\alpha_2 r^2\right),\\ \dot{y}_2= & -ky_1+ky_2+\frac{\nu}{\theta-\theta_0}\left(\alpha_1-\alpha_2\right)qr,\end{array}\right. \quad (28)$$

where (26) is used.

The controller is chosen as

$$\nu=k_1 y_1+k_2 y_2. \quad (29)$$

Substituting (27) into (28) has

$$\left[\begin{array}{c}\dot{y}_1\\ \dot{y}_2\end{array}\right] = \left[\begin{array}{c}\nu\alpha_1+k\phi^2 y_2-\nu\alpha_{12}r^2\\ -ky_1+ky_2+\nu\alpha_{12}w\end{array}\right] \quad (30)$$

$$= \left(A_{20}+A_2\left(t\right)\right)\left[\begin{array}{c}y_1\\ y_2\end{array}\right], \quad (31)$$

where

$$A_{20}=\left[\begin{array}{cc}\alpha_1 k_1 & \alpha_1 k_2\\ -k+k_1\alpha_{12} & k+k_2\alpha_{12}\end{array}\right], \quad (32)$$

$$A_2\left(t\right)=\left[\begin{array}{cc}-k_1\alpha_{12}r^2 & k\phi^2-k_2\alpha_{12}r^2\\ k_1\alpha_{12}\left[w-1\right] & k_2\alpha_{12}\left[w-1\right]\end{array}\right]. \quad (33)$$

A_{20} has a characteristic polynomial as follows

$$\left|\lambda I-A_{20}\right| = \left|\begin{array}{cc}\lambda-\alpha_1 k_1 & -\alpha_1 k_2\\ k-k_1\alpha_{12} & \lambda-k-k_2\alpha_{12}\end{array}\right| \quad (34)$$

$$= \lambda^2-\left(k+k_1\alpha_1+k_2\alpha_1-k_2\alpha_2\right)\lambda \quad (35)$$

$$+k\left(k_1+k_2\right)\alpha_1 \quad (36)$$

Then A_0 is a Hurwitz matrix if and only if

$$k+(k_1+k_2)\alpha_1<k_2\alpha_2, \quad k_1+k_2>0. \quad (37)$$

Assumption 2: $\alpha_1\le\alpha^{10}$ and $\alpha_2\ge\alpha_{20}$, where α^{10} and α_{20} are known positive constants.

Then (37) can be rewritten as

$$k+(k_1+k_2)\alpha^{10}<k_2\alpha_{20}, \quad k_1+k_2>0. \quad (38)$$

Remark 1: The inequalities above can be solved. For example, take $\varepsilon=k_2\alpha_{20}/(2\alpha^{10})$, $k_1=-k_2+\varepsilon$, $k=k_2\alpha_{20}/3$. Then it is easily checked that $k_1+k_2=\varepsilon>0$, $k+(k_1+k_2)\alpha^{10}=k+\varepsilon\alpha^{10}=5k_2\alpha_{20}/6<k_2\alpha_{20}$.

The controller composed by (23), or (27) and (29) can make (x_m,y_m) converge to zero as $t\to\infty$. When (x_m,y_m) is very small, let

$$\nu=0,\omega=-k\theta \quad (39)$$

Then $\theta(t)\to 0$ as $t\to\infty$ while the small change of x_m and y_m remains.

B. θ_0 *unknown,* $\alpha_1=\alpha_2=\alpha$ *unknown*

It is noted that (11) cannot be obtained from (10) because θ_0 is unknown. Under this situation, (6) can be reduced to

$$\left\{\begin{array}{lll}\dot{x}_m & = & \nu\alpha\cos\left(\theta-\theta_0\right)\\ \dot{y}_m & = & \nu\alpha\sin\left(\theta-\theta_0\right)\\ \dot{\theta} & = & \omega\end{array}\right. \quad (40)$$

The coordinate transform is written as

$$\left(\begin{array}{c}z_1\\ z_2\end{array}\right)=\left(\begin{array}{cc}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{array}\right)\left(\begin{array}{c}x_m\\ y_m\end{array}\right) \quad (41)$$

It is seen that (z_1,z_2) converges to zero as (x_m,y_m) does so.

The derivative of the two sides of (41) along (40) becomes

$$\begin{aligned}\left(\begin{array}{c}\dot{z}_1\\ \dot{z}_2\end{array}\right)&=\omega\left(\begin{array}{cc}-\sin\theta & \cos\theta\\ \cos\theta & \sin\theta\end{array}\right)\left(\begin{array}{c}x_m\\ y_m\end{array}\right)\\ &+\nu\alpha\left(\begin{array}{cc}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{array}\right)\left(\begin{array}{c}\cos\left(\theta-\theta_0\right)\\ \sin\left(\theta-\theta_0\right)\end{array}\right)\\ &=\omega\left(\begin{array}{c}-z_2\\ z_1\end{array}\right)\\ &+\nu\alpha\left(\begin{array}{c}\cos\theta\cos\left(\theta-\theta_0\right)+\sin\theta\sin\left(\theta-\theta_0\right)\\ \sin\theta\cos\left(\theta-\theta_0\right)-\cos\theta\sin\left(\theta-\theta_0\right)\end{array}\right)\end{aligned}$$

resulting in

$$\left(\begin{array}{c}\dot{z}_1\\ \dot{z}_2\end{array}\right)=\left(\begin{array}{c}-z_2\\ z_1\end{array}\right)\omega+\nu\alpha\left(\begin{array}{c}\cos\theta_0\\ \sin\theta_0\end{array}\right)$$

or

$$\left\{\begin{array}{lll}\dot{z}_1 & = & -z_2\omega+\alpha\nu\cos\theta_0\\ \dot{z}_2 & = & z_1\omega+\alpha\nu\sin\theta_0\\ \dot{\theta} & = & \omega\end{array}\right. \quad (42)$$

Taking the control input

$$\left\{\begin{array}{lll}\nu & = & k_1 z_1+k_2 z_2,\\ \omega & = & \omega(\texttt{constant}),\end{array}\right. \quad (43)$$

we obtain

$$\begin{aligned}\dot{z}_1 &= -z_2\omega+\alpha\cos\theta_0\left(k_1 z_1+k_2 z_2\right)\\ &= k_1\alpha\cos\theta_0 z_1+\left(k_2\alpha\cos\theta_0-\omega\right)z_2\end{aligned}$$

$$\begin{aligned}\dot{z}_2 &= z_1\omega+\alpha\sin\theta_0 k_1 z_1+\alpha\sin\theta_0 k_2 z_2\\ &= \left(\omega+\alpha\sin\theta_0 k_1\right)z_1+\alpha\sin\theta_0 k_2 z_2\end{aligned}$$

Let

$$A=\left[\begin{array}{ll}k_1\alpha\cos\theta_0 & k_2\alpha\cos\theta_0-\omega\\ \omega+\alpha\sin\theta_0 k_1 & k_2\alpha\sin\theta_0\end{array}\right]$$

Then

$$\left(\begin{array}{c}\dot{z}_1\\ \dot{z}_2\end{array}\right)=A\left(\begin{array}{c}z_1\\ z_2\end{array}\right) \quad (44)$$

It is easily seen that the system is asymptotically stable if A is a Hurwitz matrix.

The characteristic polynomial of A becomes

$$|\lambda I - A| = \begin{vmatrix} \lambda - k_1\alpha\cos\theta_0 & \omega - k_2\alpha\cos\theta_0 \\ -\omega - \alpha k_1\sin\theta_0 & \lambda - k_2\alpha\sin\theta_0 \end{vmatrix}$$
$$= \lambda^2 - \alpha\left[k_1\cos\theta_0 + k_2\sin\theta_0\right]\lambda + k_1k_2\alpha^2\sin\theta_0\cos\theta_0$$
$$+ (\omega + \alpha k_1\sin\theta_0)(\omega - k_2\alpha\cos\theta_0)$$
$$= \lambda^2 - \alpha\left[k_1\cos\theta_0 + k_2\sin\theta_0\right]\lambda + \omega^2$$
$$+ \alpha\omega\left[k_1\sin\theta_0 - k_2\cos\theta_0\right]$$

A is a Hurwitz matrix if and only if the following two inequalities

$$\alpha\left(k_1\cos\theta_0 + k_2\sin\theta_0\right) < 0 \tag{45}$$

$$\omega^2 + \alpha\omega\left(k_1\sin\theta_0 - k_2\cos\theta_0\right) > 0 \tag{46}$$

hold.

The following text is to consider what conditions can guarantee that (45) and (46) are valid.

Assumption 3: $0 < \alpha \leq \bar{\alpha}$, $\bar{\alpha}$ is known.

Case 1: $0 \leq \theta_0 \leq \frac{\pi}{2}$,

Take

$$k_1 < 0, k_2 < 0,$$

ω may be chosen as a positive constant such that

$$|\omega| + \bar{\alpha}k_1 > 0$$

Under these conditions, (45) and (46) are valid.

On the one hand, it is clearly seen that (45) is valid because α, $\cos\theta_0$ and $\sin\theta_0$ are positive. On the other hand,

$$\begin{aligned} & \omega^2 + \alpha\omega\left(k_1\sin\theta_0 - k_2\cos\theta_0\right) \\ \geq \quad & \omega^2 + \alpha\omega k_1\sin\theta_0 = \omega^2 + \bar{\alpha}k_1|\omega| \\ \geq \quad & |\omega|\left(|\omega| + \bar{\alpha}k_1\right) > 0 \end{aligned}$$

Hence, (46) are valid.

Case 2: $\frac{\pi}{2} < \theta_0 \leq \pi$.

Take

$$k_1 > 0, k_2 < 0,$$

ω may be chosen as a positive constant such that

$$|\omega| + \bar{\alpha}k_2 > 0 \tag{47}$$

Under these conditions, (45) and (46) are valid.

On the one hand, it is clearly seen that (45) is valid because $\alpha > 0$, $\cos\theta_0 < 0$ and $\sin\theta_0 > 0$. On the other hand, $k_1\sin\theta_0 > 0$ yields

$$\begin{aligned} & \omega^2 + \alpha\omega\left(k_1\sin\theta_0 - k_2\cos\theta_0\right) \\ \geq \quad & \omega^2 - \alpha\omega k_2\cos\theta_0 \\ \geq \quad & \omega^2 + \bar{\alpha}|\omega|k_2 > |\omega|(|\omega| + \bar{\alpha}k_2) > 0 \end{aligned}$$

The last inequality above is valid based on (47). Hence, (41) holds.

For the two cases above, it is necessary to judge aforehand which quadrant(I or II) belongs to for the angle θ_0. Sometimes it is easy, and sometimes it is not easy. Therefore it is better if more relaxed condition for the requirement of θ_0 is founded.

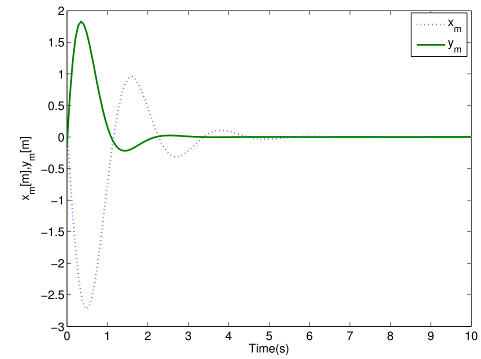

Fig. 2. The trajectories of state x_m and y_m in the image frame with respect to time

Case 3: $\sigma \leq \theta_0 \leq \pi - \sigma$, where σ is a known small positive.

Take

$$k_1 = 0, \quad k_2 < 0.$$

ω may be chosen as a positive constant such that

$$|\omega| + \bar{\alpha}k_2 > 0. \tag{48}$$

It is easily seen that (45) is valid. Otherwise,

$$\omega^2 + \alpha\omega\left(k_1\sin\theta_0 - k_2\cos\theta_0\right) \geq \omega^2 + \bar{\alpha}|\omega|k_2$$
$$\geq |\omega|(|\omega| + \bar{\alpha}k_2) > 0$$

Therefore, (46) holds by using (48).

In section C, the controller is composed of two steps. In the first step, ν and ω are given in three cases, respectively. After the absolute values of $z_1(t)$ and $z_2(t)$ are made small as desired, the controller switches

$$\nu = 0, \qquad \omega = -a\theta, \tag{49}$$

where a is a positive gain.

It is easily clear that $\sqrt{z_1^2(t) + z_2^2(t)}$ keeps constant and are very small in the second step. In the mean time, $\theta(t)$ converges to zero. Summing up, in this section, the system can be stabilized by using the controller proposed here.

IV. Simulation

The simulations are done for the three cases. Here the first case is just shown due to the short pages.

Take the initial value $[1, -1, -\pi/4]$. The controller are chosen as (27), (29) and (39). For system (4), choose k_2, k_1, k. The parameters are chosen as $\theta_0 = \pi/4$, $\alpha_1 = 6, \alpha_2 = 3$, $\alpha^{10} = 8, \alpha_{20} = 1$, $k_1 = -1, k_2 = 1.2, k = 1$. The trajectories of states and movements of robot mass centers are shown in Figure 2-5, respectively.

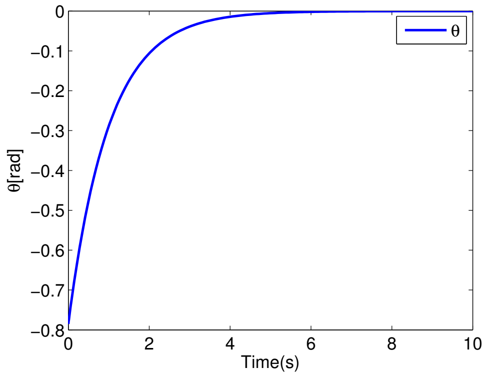

Fig. 3. The trajectories of state θ in the inertial frame with respect to time

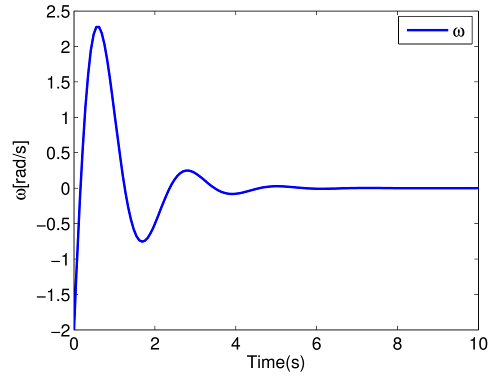

Fig. 5. The trajectories of angular velocity of rotation of the robot around the mass center with respect to time

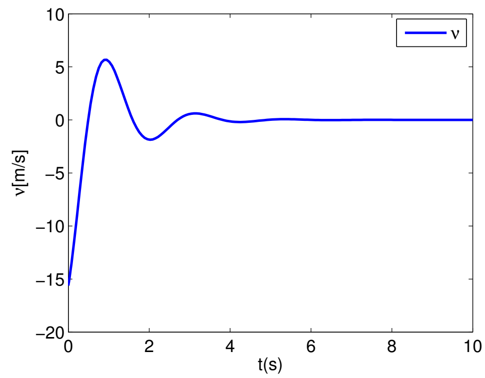

Fig. 4. The trajectories of velocity of heading orientation of the robot with respect to time

V. Conclusion

A new kind of stabilizing problem was proposed for the kinematic model of nonholonomic mobile robots based on visual servoing feedback with uncalibrated vision parameters. The stabilizing controllers were investigated for θ_0 unknown or α_1 and α_2 unknown by using two step techniques, and θ_0, α_1 and α_2 are all unknown but $\alpha_1 = \alpha_2$, As for the other case, the future work will discuss it. In addition, dynamic problems are not neglected for high performance of a practical control systems. It will also be dealt with in the coming period.

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